KOREA UNIVERSITY

DSBA CS224n 2021 Study

[Lecture 05] Language Models and RNNs
[Lecture 06] Simple and LSTM RNNs

DSBA
Data Science & Business Analytics

고려대학교 산업경영공학과

Data Science & Business Analytics Lab

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00 발표 목차

Contents

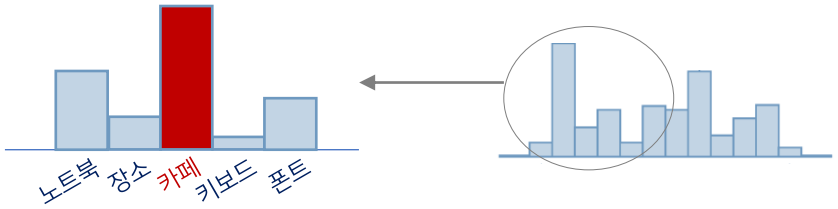
01 Language Model

Description

- 확률 분포를 기반으로 주어진 문맥(sequence) 이후에 위치할 단어 예측
  - 시퀀스의 결합 확률 by multiplication rule

$$P(\boldsymbol{x}^{(1)},\ldots,\boldsymbol{x}^{(T)}) = P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)}|\,\boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)}|\,\boldsymbol{x}^{(T-1)},\ldots,\boldsymbol{x}^{(1)})$$
$$= \prod_{t=1}^{T} P(\boldsymbol{x}^{(t)}|\,\boldsymbol{x}^{(t-1)},\ldots,\boldsymbol{x}^{(1)})$$

  - $P(x^{(3)}|\,\text{코딩하기, 좋은})$



01 Language Model

N-gram LM, Neural LM

N-gram Language Model

$\boldsymbol{P}$(코딩하기, 좋은, 카페, 추천)

uni-gram

P(코딩하기) P(좋은) P(카페) P(추천)

bi-gram

P(코딩하기) P(좋은| 코딩하기) P(카페| 좋은) P(추천| 카페)

$$P(\text{카페}|\ \text{코딩하기, 좋은}) = \frac{count(\text{코딩하기, 좋은, 카페})}{count(\text{코딩하기, 좋은})}$$

- ✓ 이전에 등장한 n개의 단어를 바탕으로 예측
- ✓ count(빈도)를 기반으로 확률 계산
- ✓ (-) sparsity problem: n-gram chunk 문서 내 등장 x

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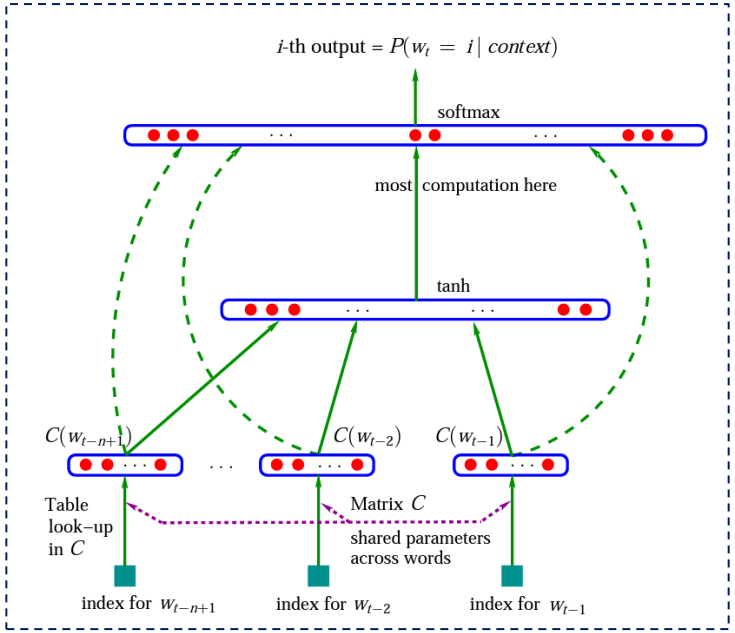
Neural Language Model



A neural probabilistic language model (Bengio et al, 2003)

- ✓ 미리 지정한 window size 이전 단어를 바탕으로 예측
- ✓ word embedding: 단어의 distributed representation 학습
- ✓ (-) window size가 커지면 파라미터 수 증가 -> 연산 부담, 과적합 가능

01 Language Model

N-gram LM, Neural LM

N-gram Language Model

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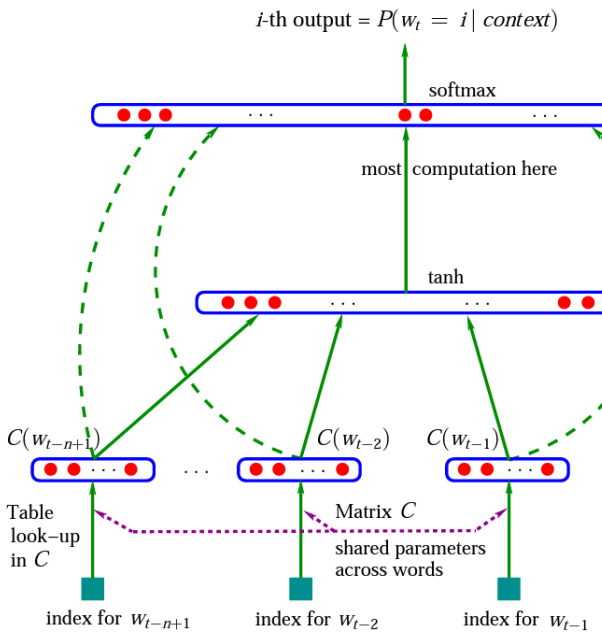
$$P(\text{카페}|\ \text{코딩하기, 좋은}) = \frac{count(\text{코딩하기, 좋은, 카페})}{count(\text{코딩하기, 좋은})}$$

Neural Language Model



A neural probabilistic language model (Bengio et al, 2003)

✓ 입력 길이가 고정되어(N, window size) 이전에 등장하는 모든 단어를 고려할 수 없음

02 RNN Language Model

Architecture

④ Output distribution

$$\hat{y}^{(t)} = \text{softmax}\left(Uh^{(t)} + b_2\right) \in \mathbb{R}^{|V|}$$

③ Hidden States

$$h^{(t)} = \sigma\left(W_h h^{(t-1)} + W_e e^{(t)} + b_1\right)$$

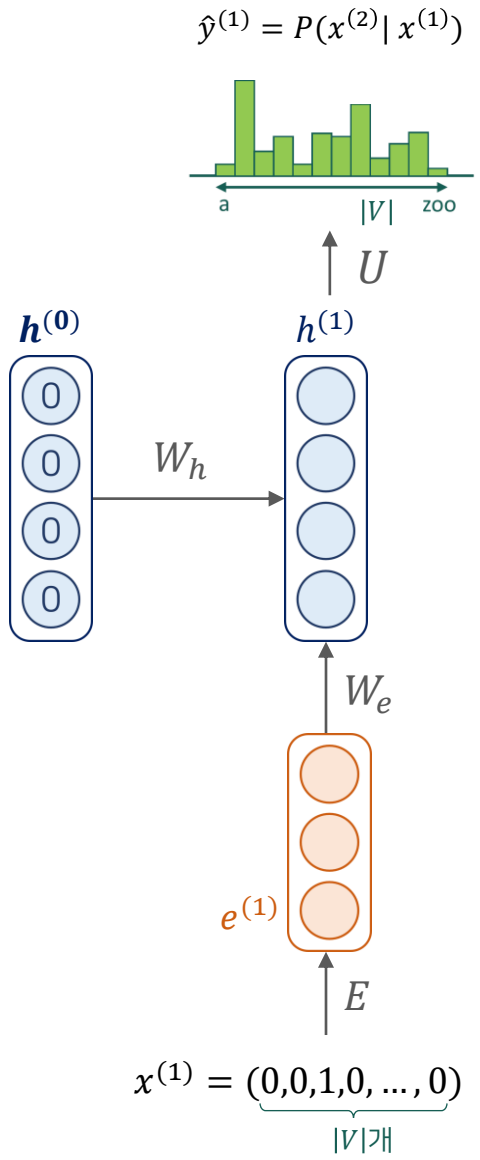
W_h: $(d_h \times d_h)$, W_e: $(d_e \times d_h)$

② Word embedding

$$e^{(t)} = Ex^{(t)} \quad E: (|V| \times d_e)$$

① Input word sequence

- 원핫 벡터 $x^{(t)} \in \mathbb{R}^{|V|}$



02 RNN Language Model

Architecture

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$$\hat{\boldsymbol{y}}^{(t)} = \text{softmax}\left(\boldsymbol{U}\boldsymbol{h}^{(t)} + \boldsymbol{b}_2\right) \in \mathbb{R}^{|V|}$$

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$$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)} \qquad \boldsymbol{E}: (|V| \times d_e)$$

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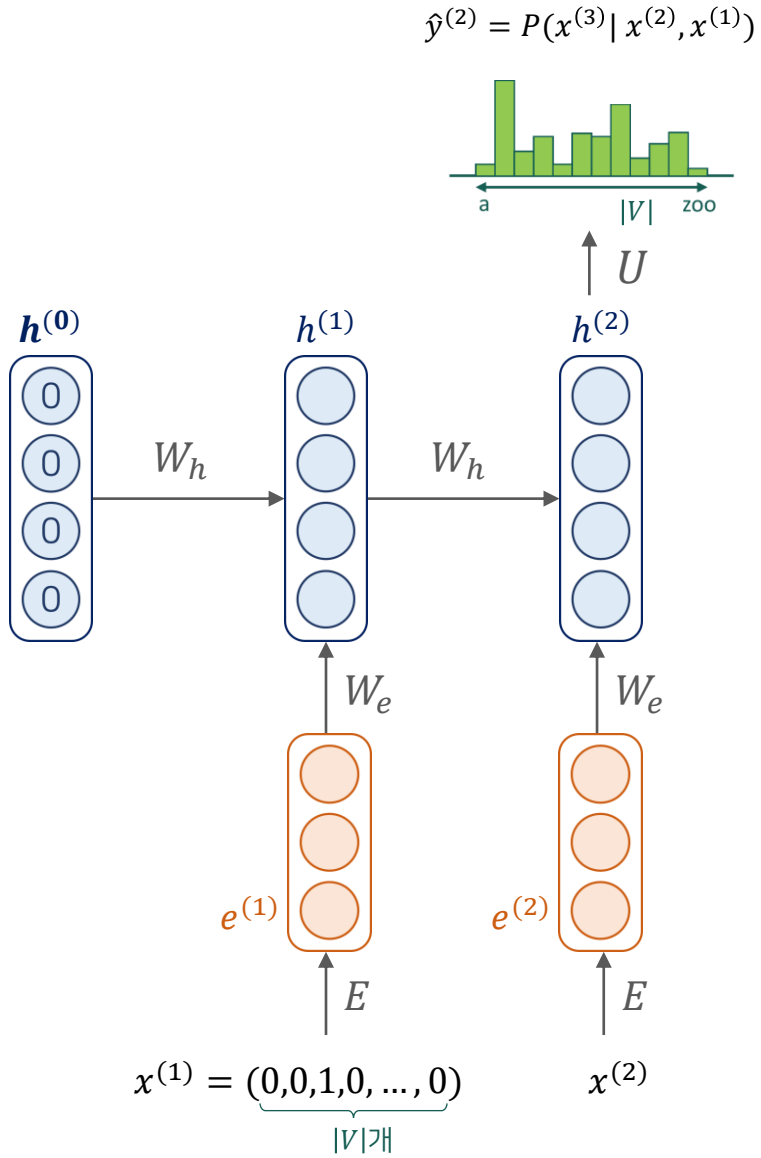

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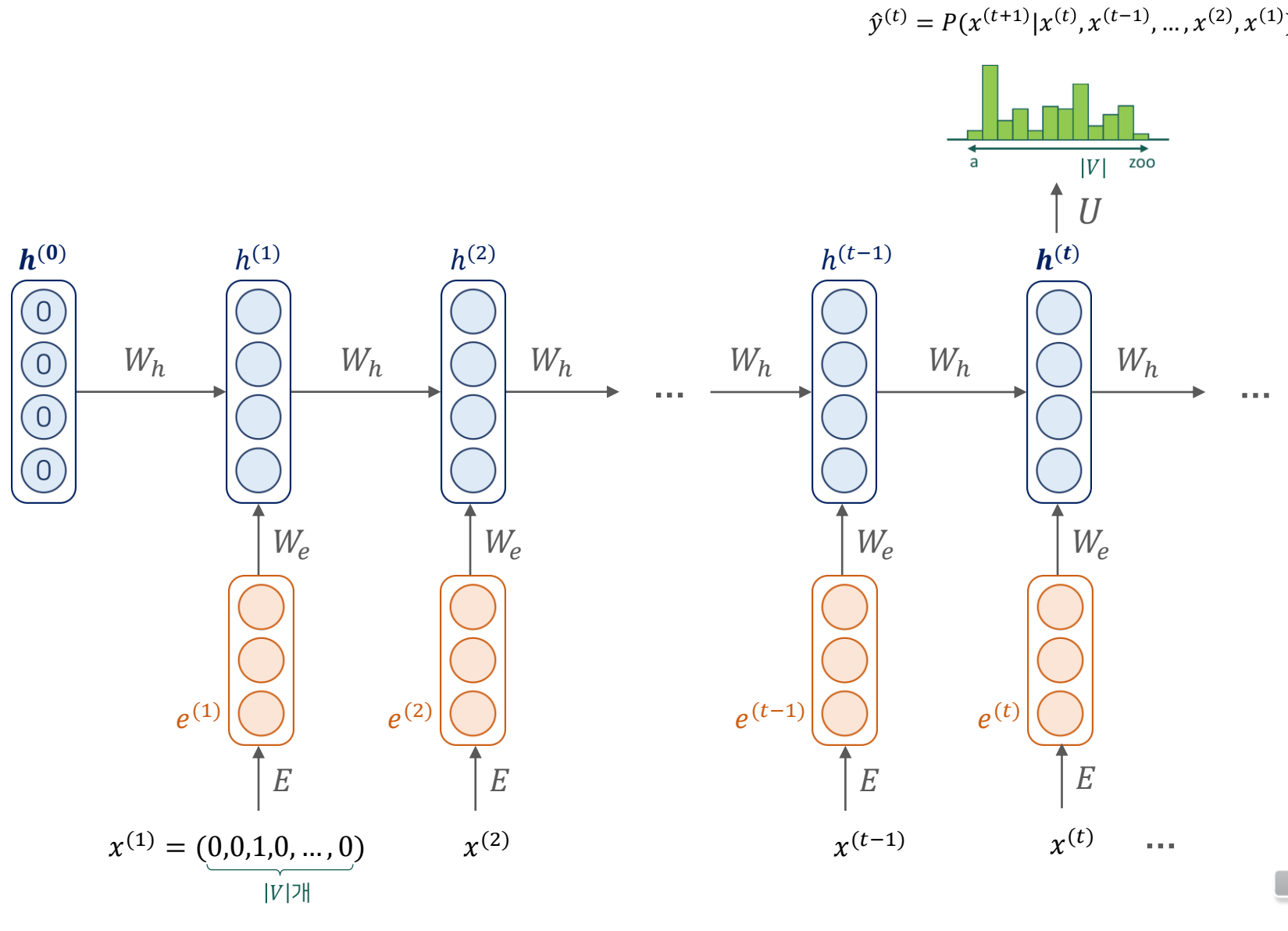
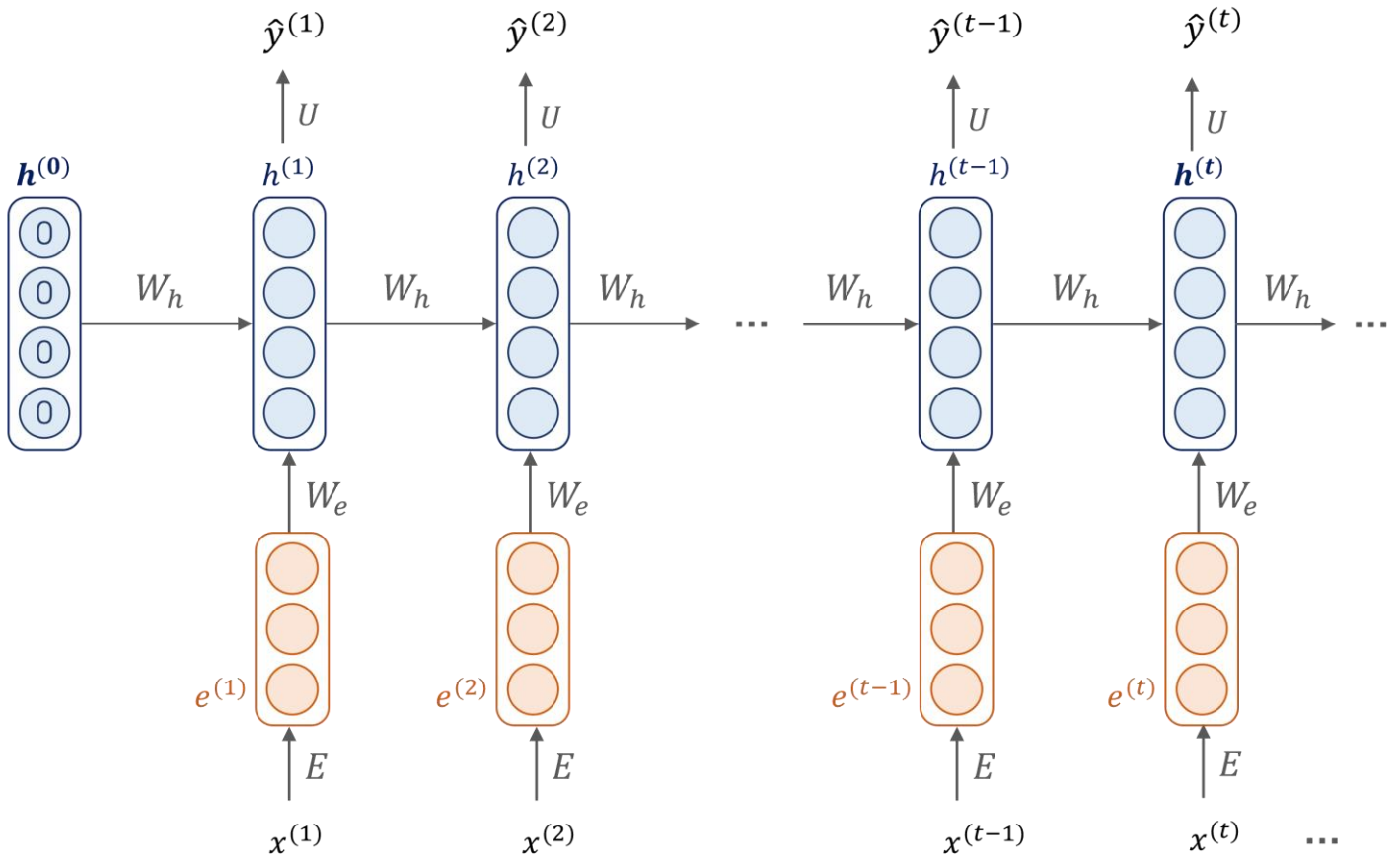


02 RNN Language Model

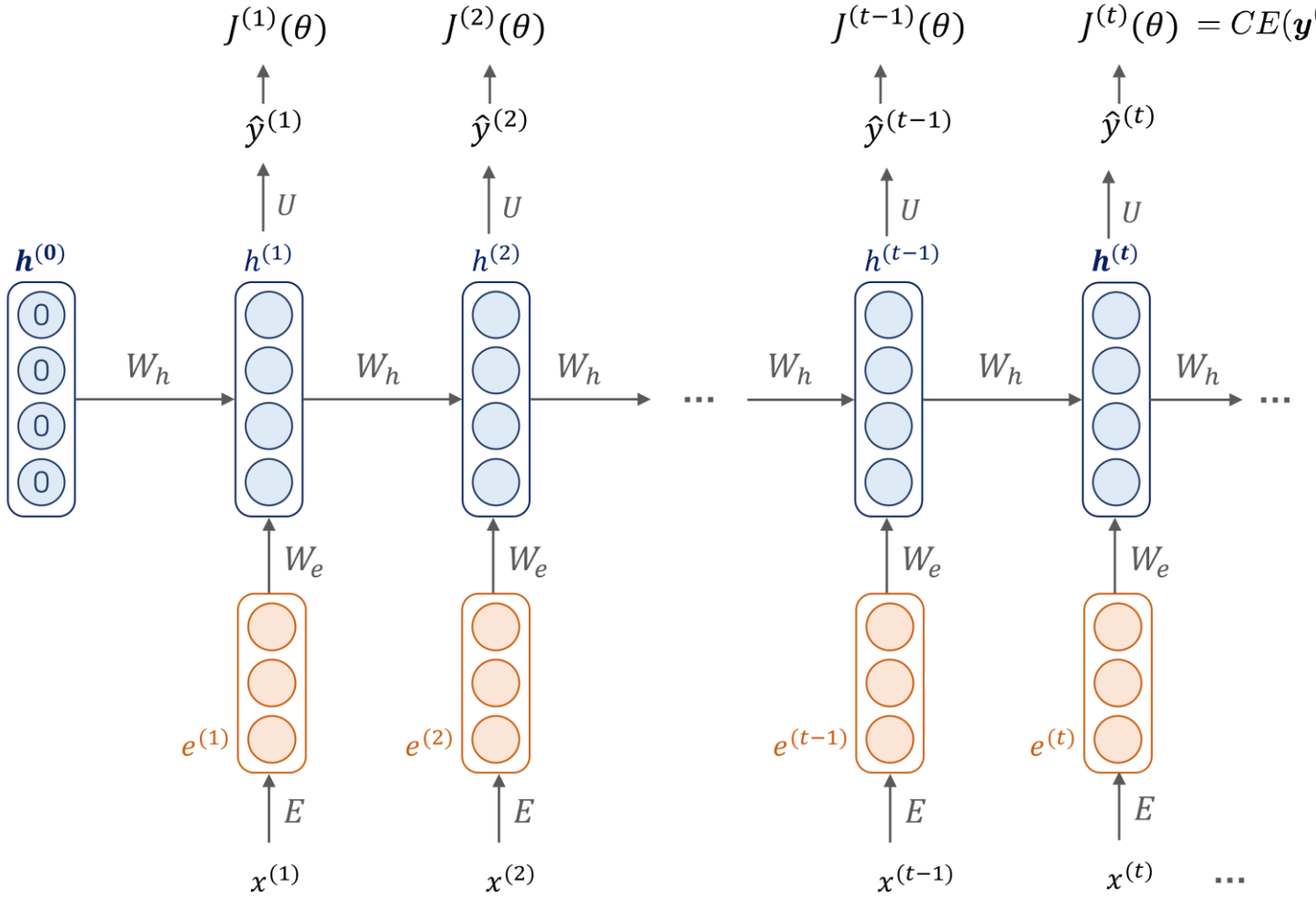
Loss Function



- ✓ many-to-many
- ✓ 매 time step마다 입,출력 존재

02 RNN Language Model

Loss Function



$$J^{(t)}(\theta) = CE(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

Cross Entropy Loss

$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T}\sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

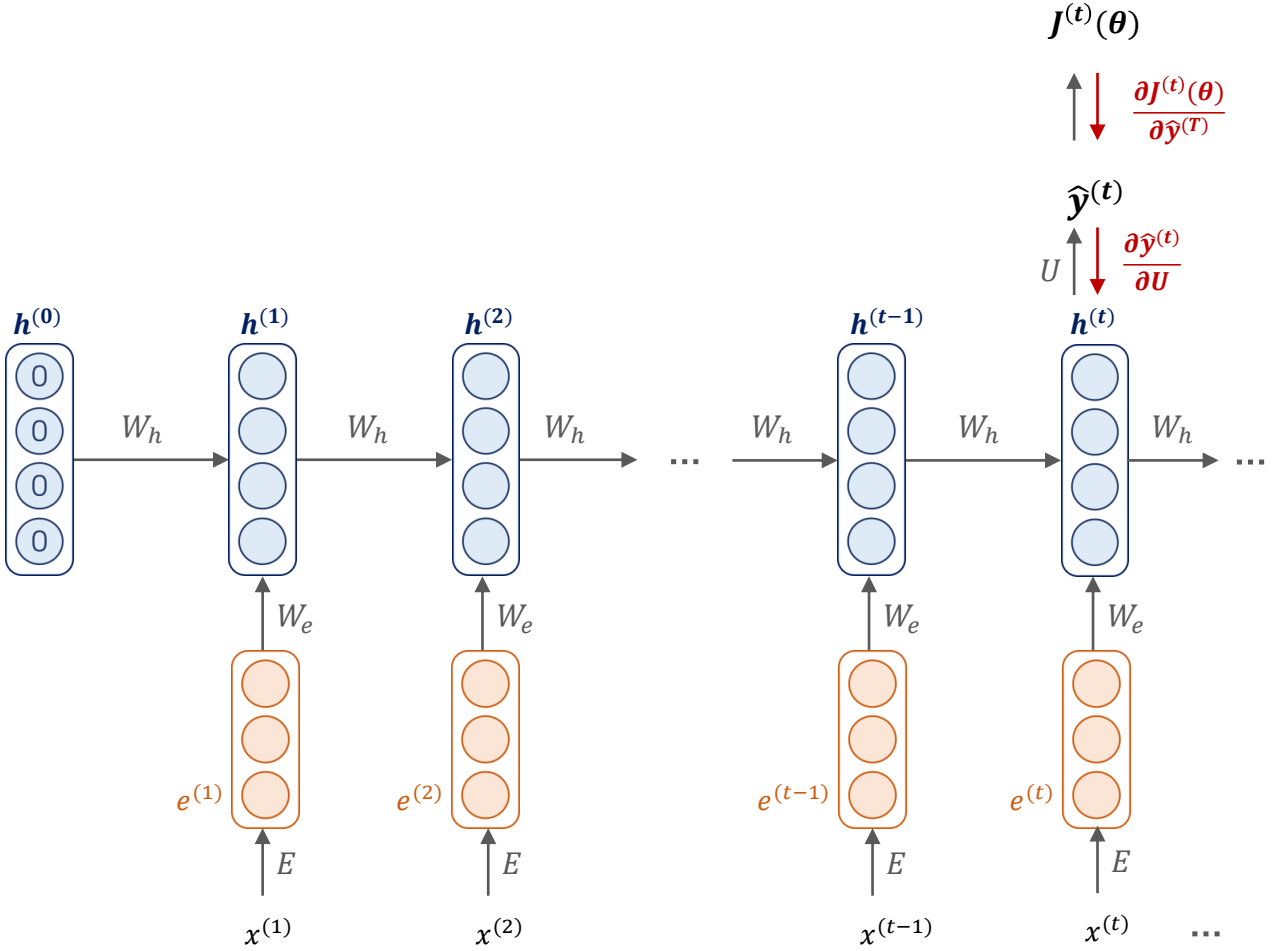
- ✓ many-to-many
- ✓ 매 time step마다 입,출력 존재

02 RNN Language Model

Back Propagation Through Time (BPTT) ①

$$\hat{y}^{(t)} = \text{softmax}\left(Uh^{(t)} + b_2\right) \in \mathbb{R}^{|V|}$$

$$h^{(t)} = \sigma\left(W_h h^{(t-1)} + W_e e^{(t)} + b_1\right)$$



① $\frac{\partial J^{(t)}(\theta)}{\partial U} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial U}$

02 RNN Language Model

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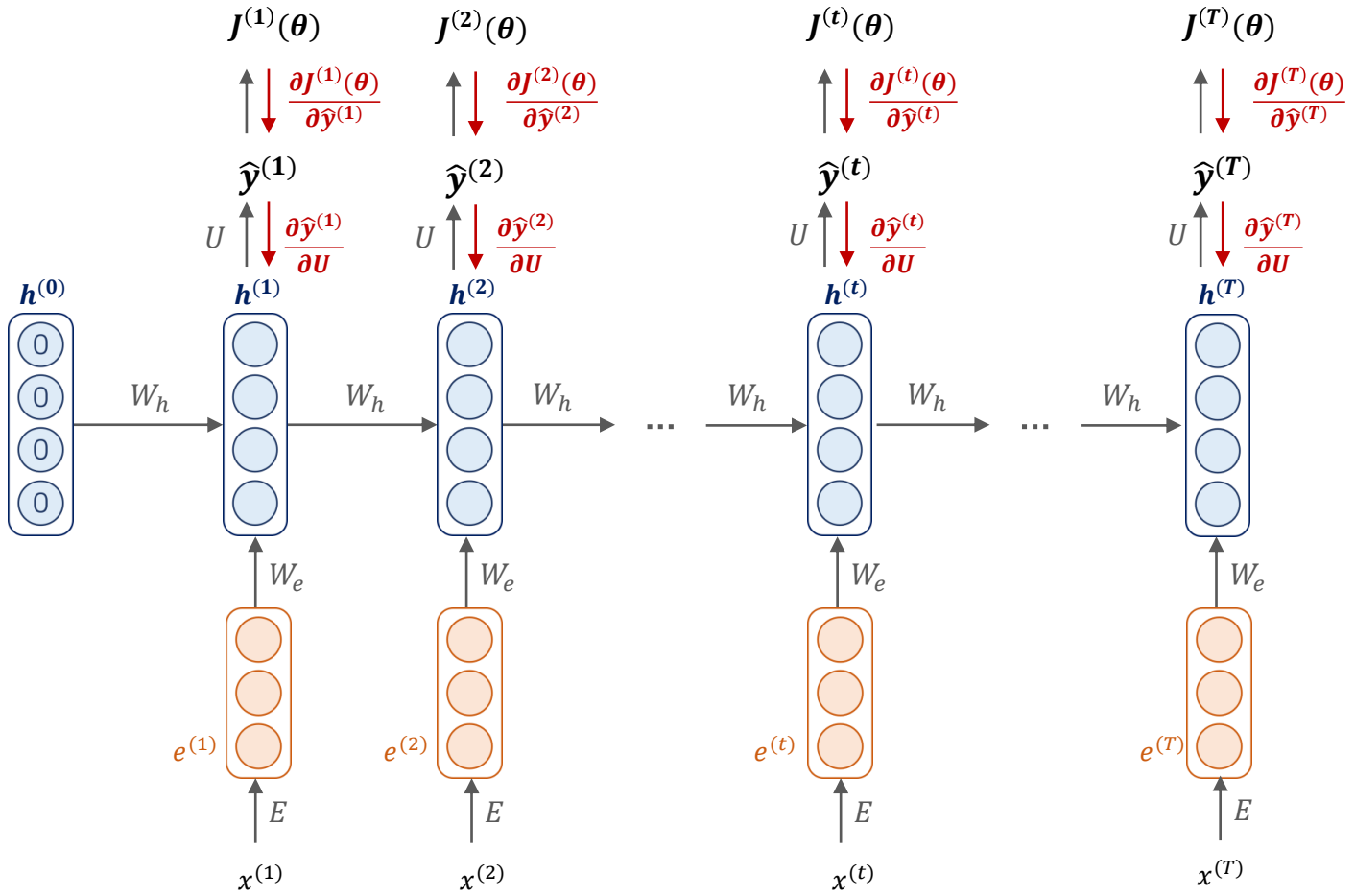

$$① \frac{\partial J^{(t)}(\theta)}{\partial U} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial U}$$

$$\Rightarrow \frac{\partial J(\theta)}{\partial U} = \sum_{t=1}^{T} \frac{\partial J^{(t)}(\theta)}{\partial U}$$

$$U^{new} = U^{old} - \alpha \frac{\partial J(\theta)}{\partial U}$$

02 RNN Language Model

Back Propagation Through Time (BPTT) ②

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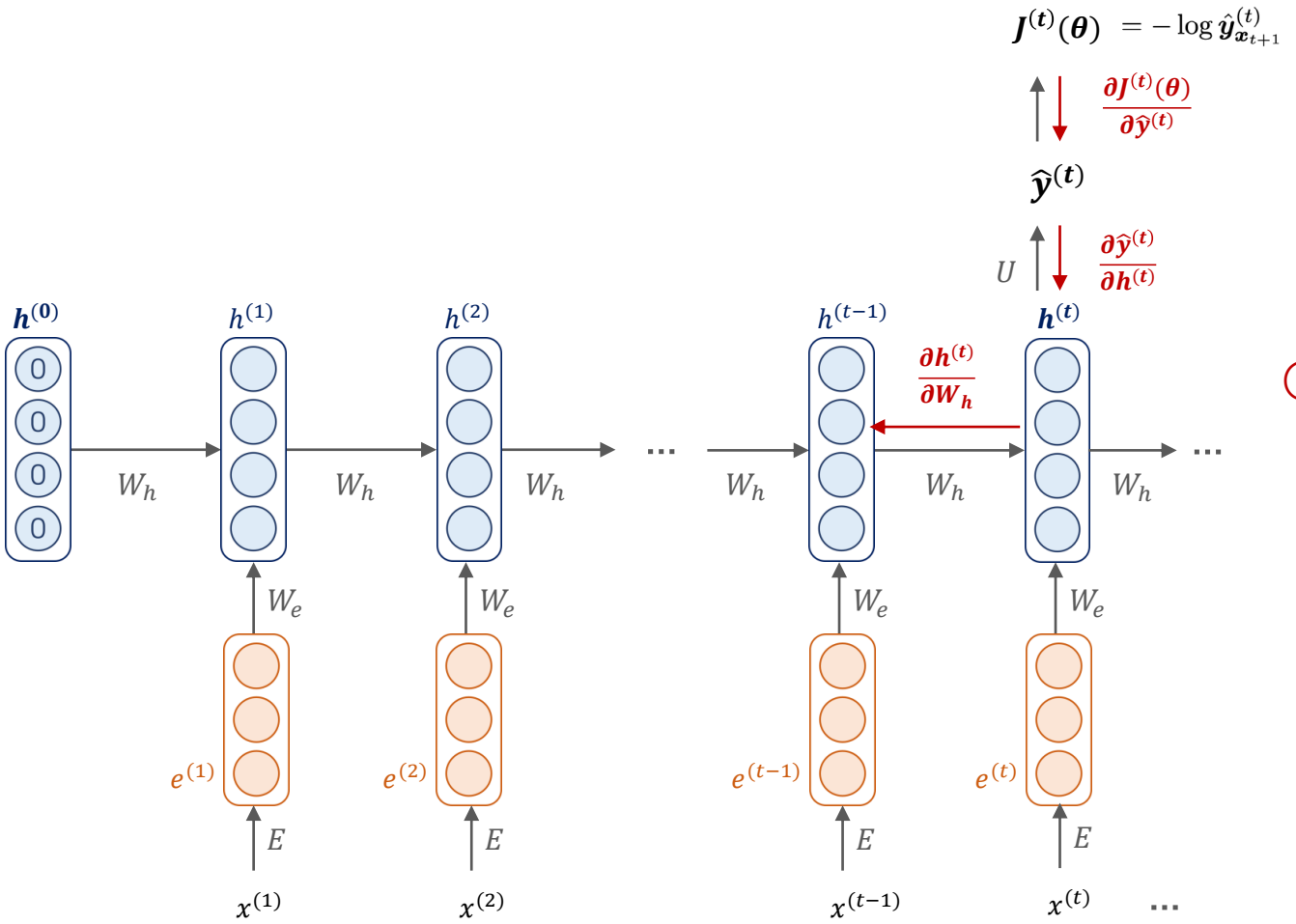

$$② \frac{\partial J^{(t)}(\theta)}{\partial W_h} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial W_h}$$

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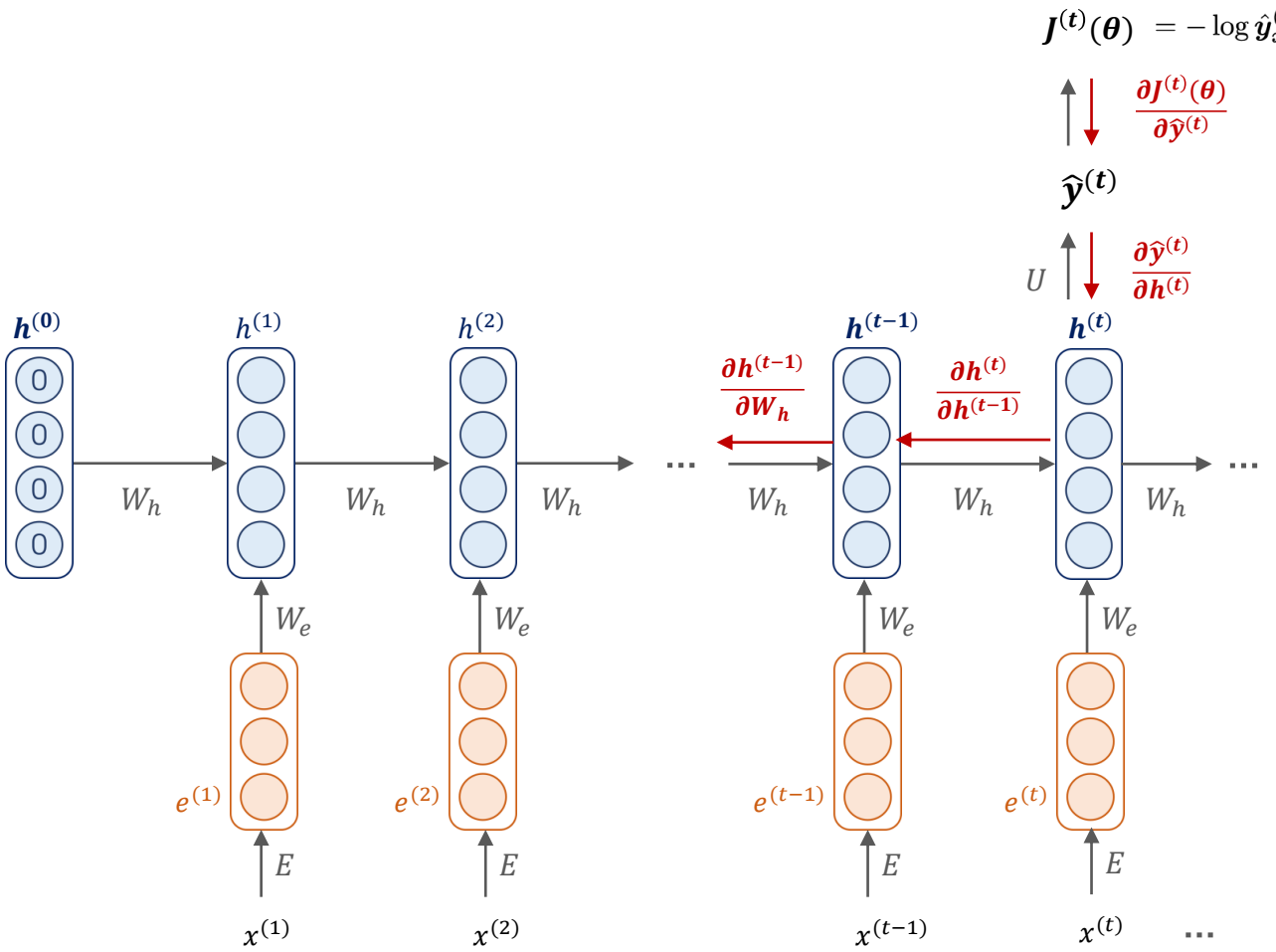

$$② \frac{\partial J^{(t)}(\theta)}{\partial W_h} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial W_h}$$

$$+ \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(t-1)}} \cdot \frac{\partial h^{(t-1)}}{\partial W_h}$$

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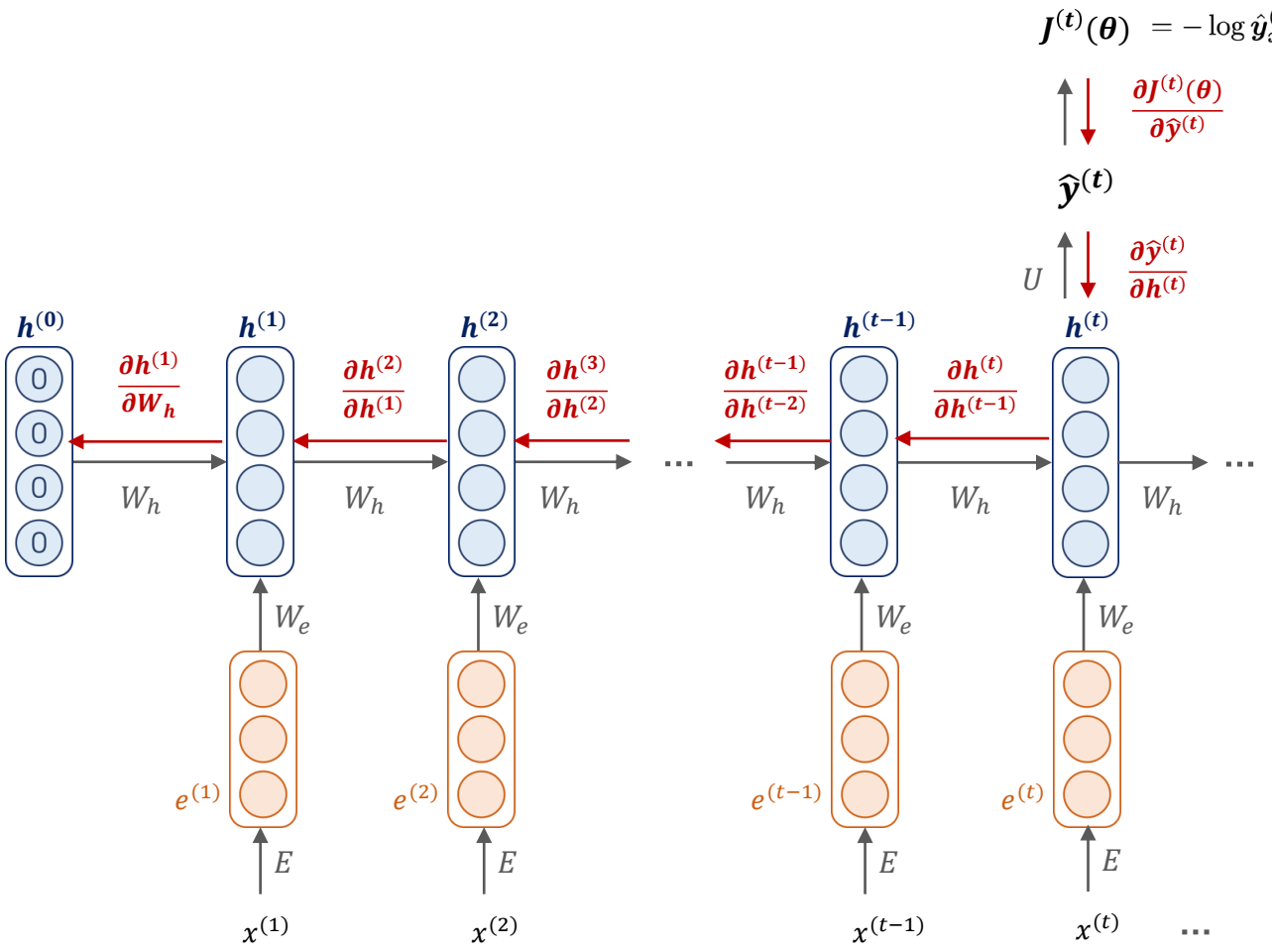

$$② \frac{\partial J^{(t)}(\theta)}{\partial W_h} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial W_h}$$

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$$\vdots$$

$$+ \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(t-1)}} \cdot \cdots \cdot \frac{\partial h^{(2)}}{\partial h^{(1)}} \cdot \frac{\partial h^{(1)}}{\partial W_h}$$

$$= \sum_{i=1}^{t} \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(i)}} \cdot \frac{\partial h^{(i)}}{\partial W_h}$$

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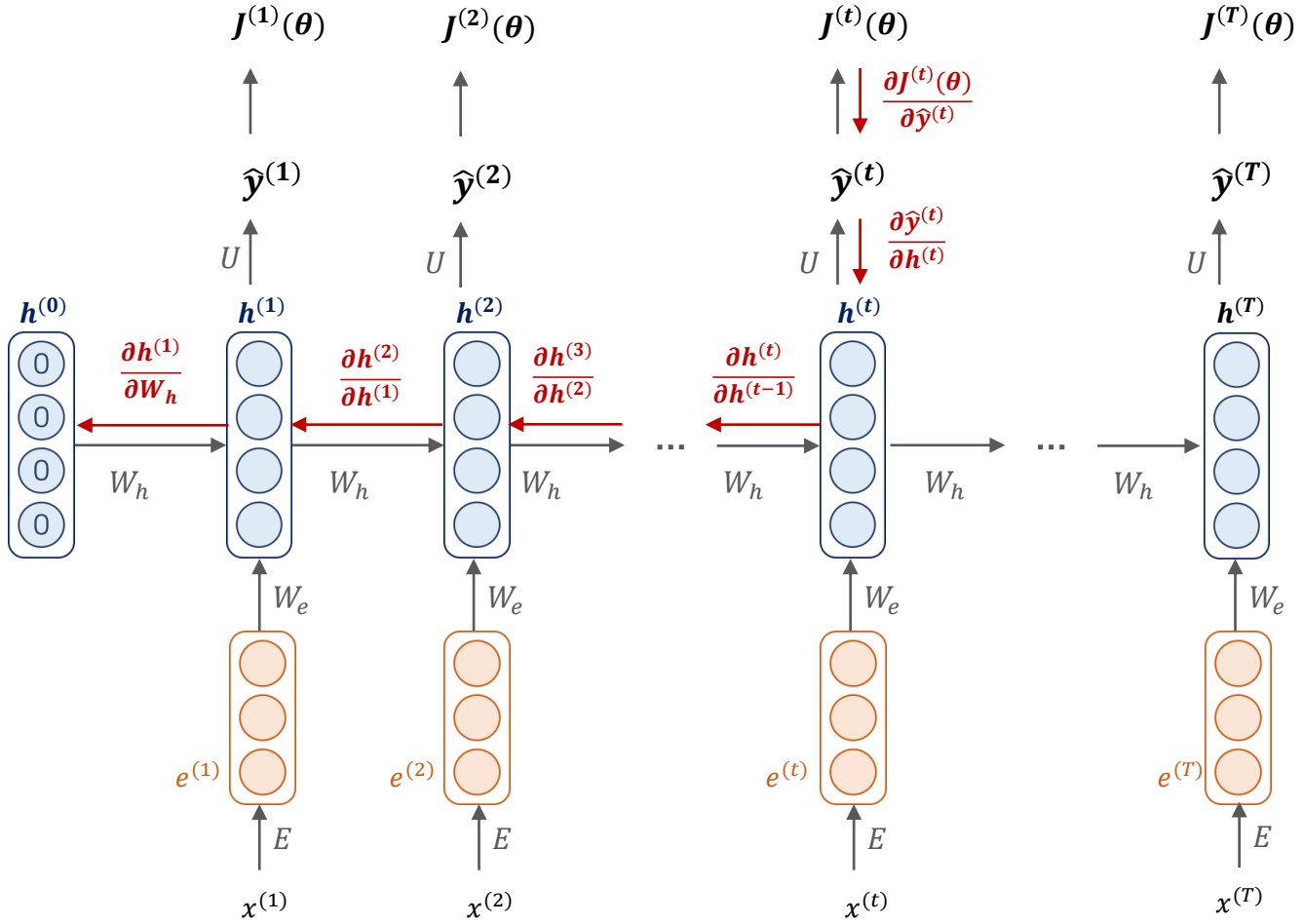

$$② \frac{\partial J^{(t)}(\theta)}{\partial W_h} = \sum_{i=1}^{t} \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(i)}} \cdot \frac{\partial h^{(i)}}{\partial W_h}$$

$$\Rightarrow \frac{\partial J(\theta)}{\partial W_h} =$$

02 RNN Language Model

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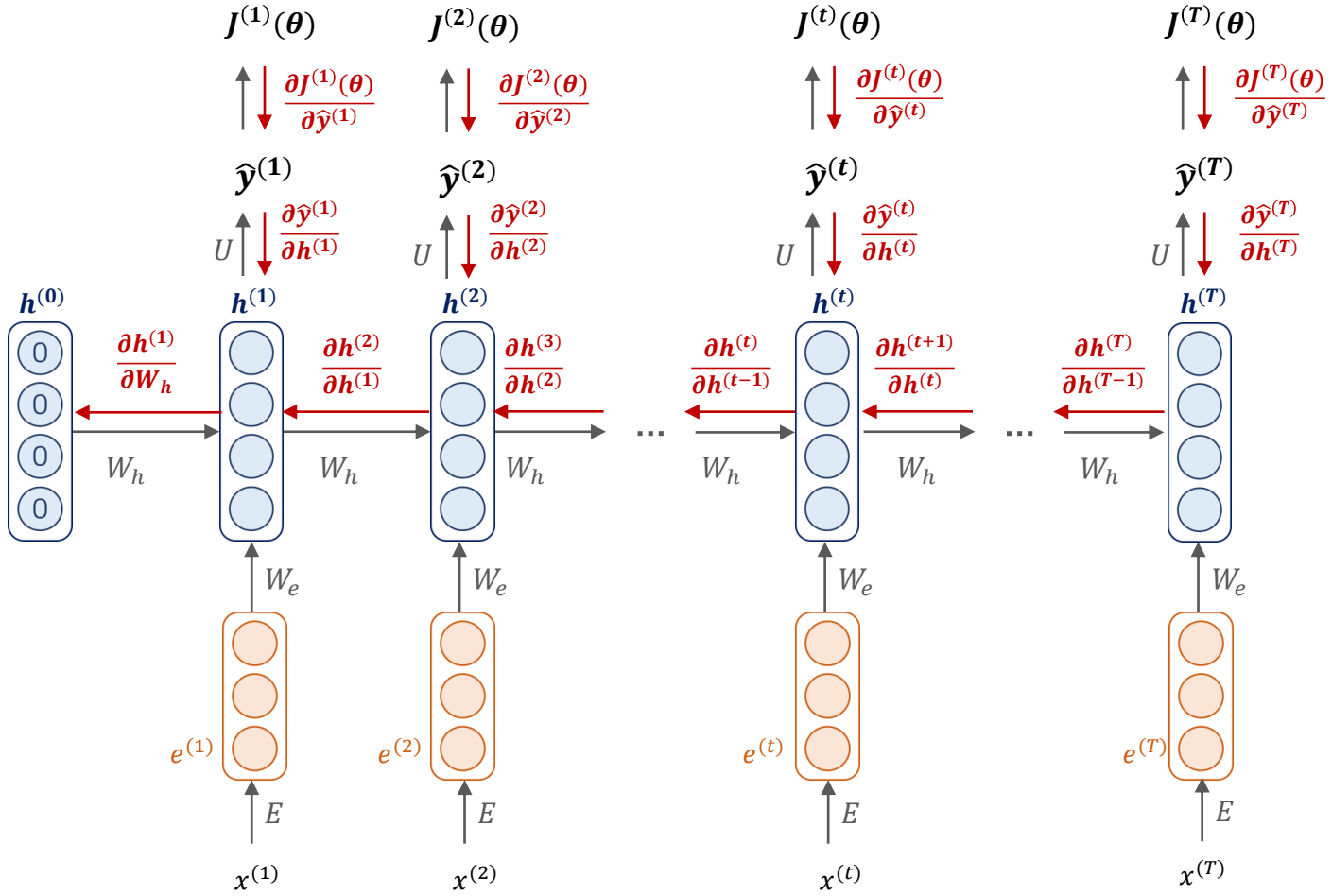

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$$\Rightarrow \frac{\partial J(\theta)}{\partial W_h} = \sum_{t=1}^{T} \frac{\partial J^{(t)}(\theta)}{\partial W_h}$$

$$W_h{}^{new} = W_h{}^{old} - \alpha \frac{\partial J(\theta)}{\partial W_h}$$

02 RNN Language Model

Back Propagation Through Time (BPTT) ③

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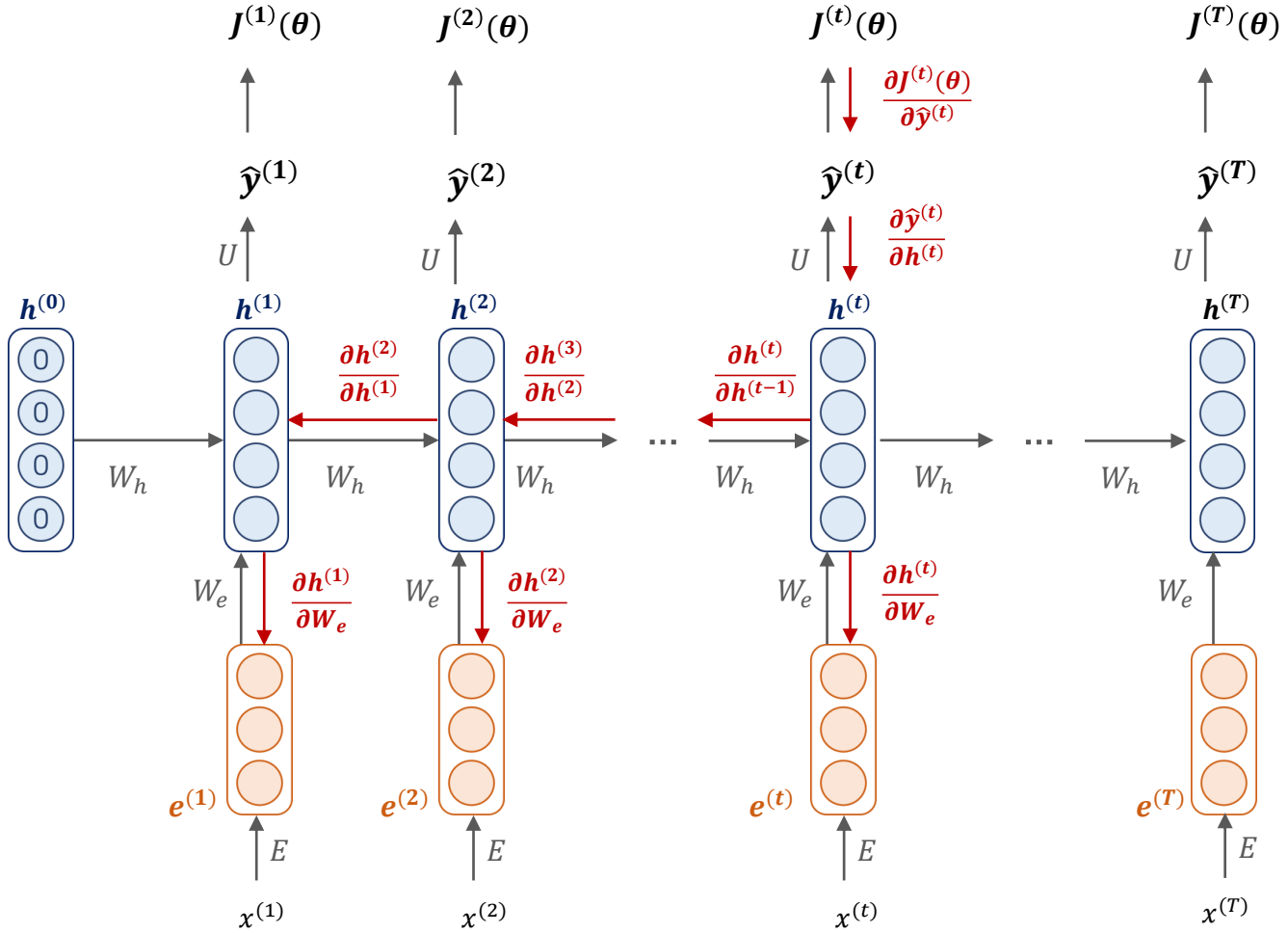

$$③ \frac{\partial J^{(t)}(\theta)}{\partial W_e} = \sum_{i=1}^{t} \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(i)}} \cdot \frac{\partial h^{(i)}}{\partial W_e}$$

02 RNN Language Model

Back Propagation Through Time (BPTT) ③

$$\hat{y}^{(t)} = \text{softmax}\left(Uh^{(t)} + b_2\right) \in \mathbb{R}^{|V|}$$

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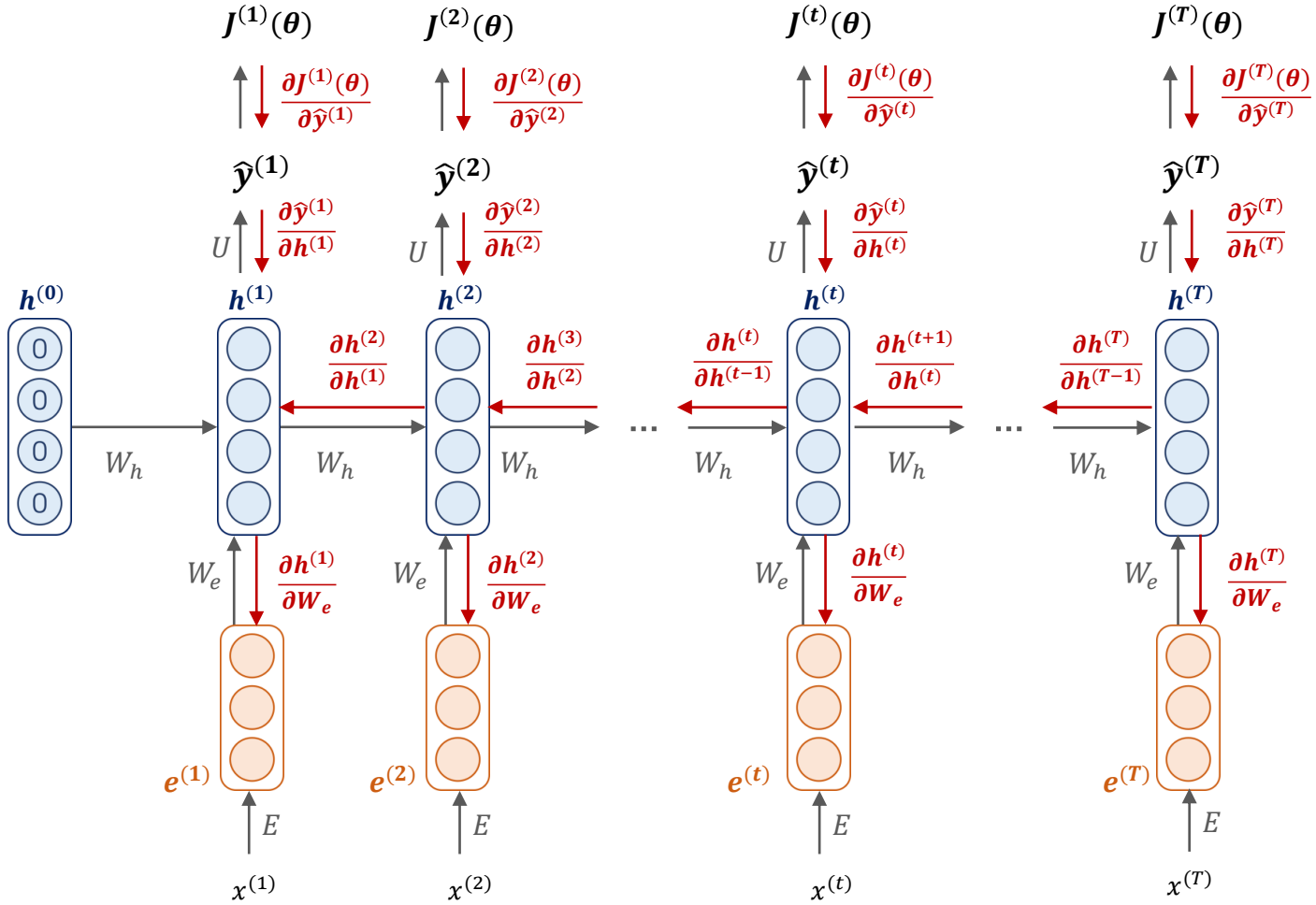


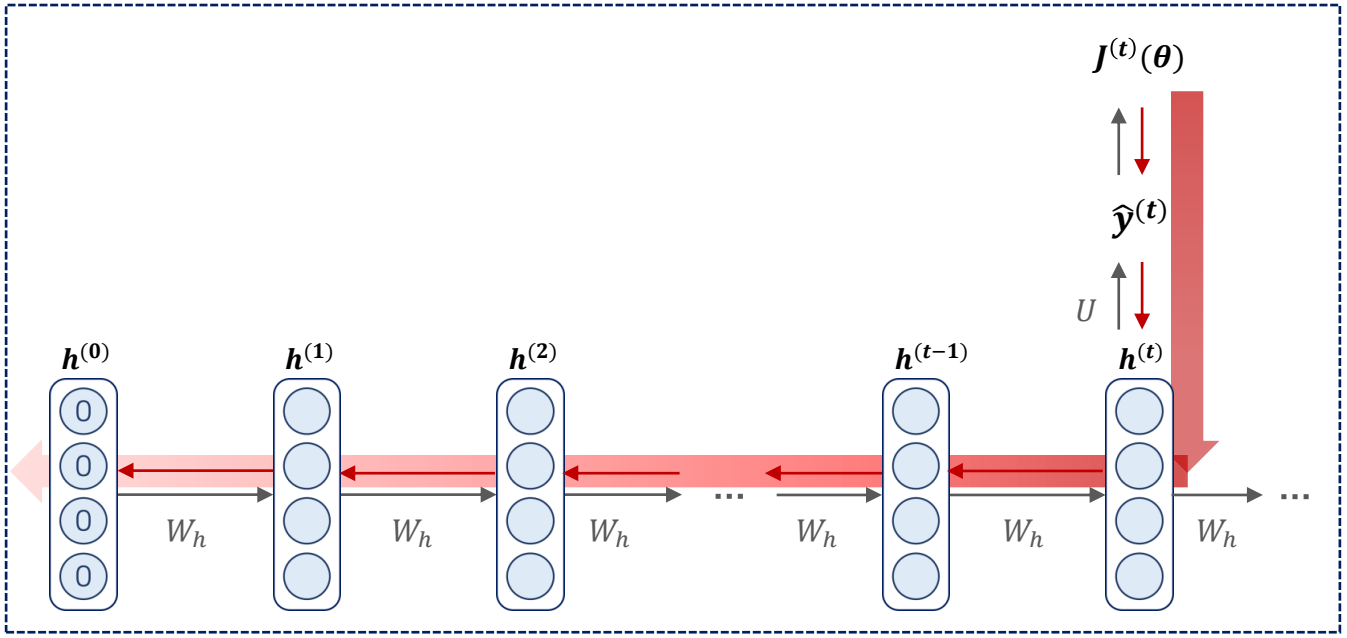
$$③\ \frac{\partial J^{(t)}(\theta)}{\partial W_e} = \sum_{i=1}^{t} \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial h^{(i)}} \cdot \frac{\partial h^{(i)}}{\partial W_e}$$

$$\Rightarrow \quad \frac{\partial J(\theta)}{\partial W_e} = \sum_{t=1}^{T} \frac{\partial J^{(t)}(\theta)}{\partial W_e}$$

$$W_e^{new} = W_e^{old} - \alpha \frac{\partial J(\theta)}{\partial W_e}$$

03 Problem of RNN

① Vanishing/Exploding Gradients



$$\hat{y}^{(t)} = \text{softmax}\left(Uh^{(t)} + b_2\right) \in \mathbb{R}^{|V|}$$

$$h^{(t)} = \sigma\left(\underbrace{W_h h^{(t-1)}}_{=S^{(t)}} + W_e e^{(t)} + b_1\right)$$

$$\frac{\partial h^{(t)}}{\partial h^{(t-1)}} = \frac{\partial h^{(t)}}{\partial s^{(t)}} \cdot \frac{\partial s^{(t)}}{\partial h^{(t-1)}} = \frac{\partial h^{(t)}}{\partial s^{(t)}} \cdot W_h$$

$$\frac{\partial J^{(t)}(\theta)}{\partial W_h} = \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \frac{\partial h^{(t)}}{\partial W_h} + \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \underbrace{\frac{\partial h^{(t)}}{\partial h^{(t-1)}}}_{W_h} \cdot \frac{\partial h^{(t-1)}}{\partial W_h} \cdots + \frac{\partial J^{(t)}(\theta)}{\partial \hat{y}^{(t)}} \cdot \frac{\partial \hat{y}^{(t)}}{\partial h^{(t)}} \cdot \underbrace{\frac{\partial h^{(t)}}{\partial h^{(t-1)}} \cdot \cdots \cdot \frac{\partial h^{(2)}}{\partial h^{(1)}}}_{(W_h)^{t-1}} \cdot \frac{\partial h^{(1)}}{\partial W_h}$$

동일한 가중치(W_h) 공유

- W_h가 작을 수록(< 1) 반복적으로 곱해지는 값이 0에 가까워져 gradient vanishing
- W_h가 클 수록(> 1) 반복적으로 곱해지는 값이 기하급수적으로 커져 gradient exploding

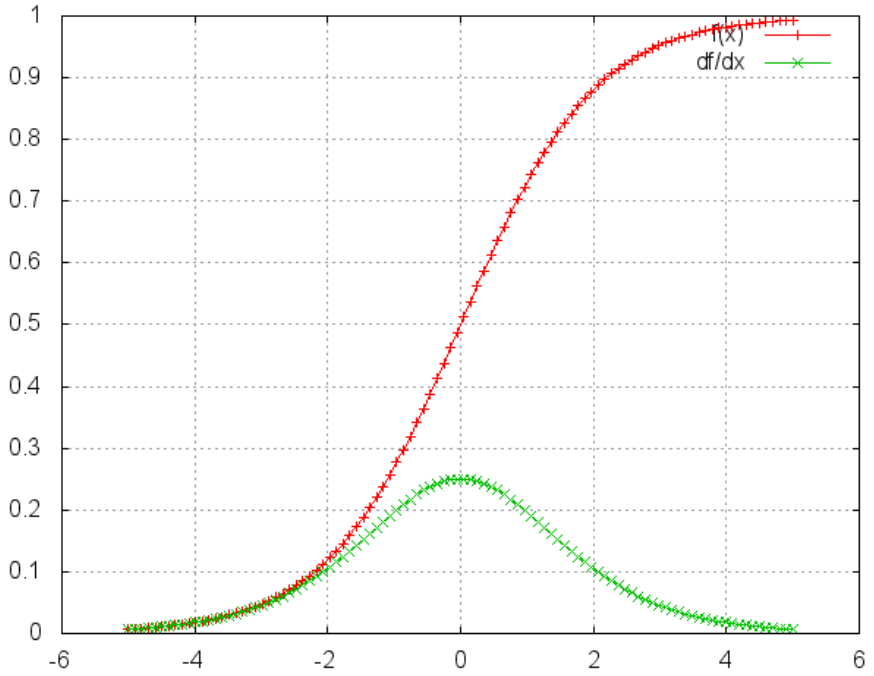
03 Problem of RNN

Sigmoid vs. Tanh

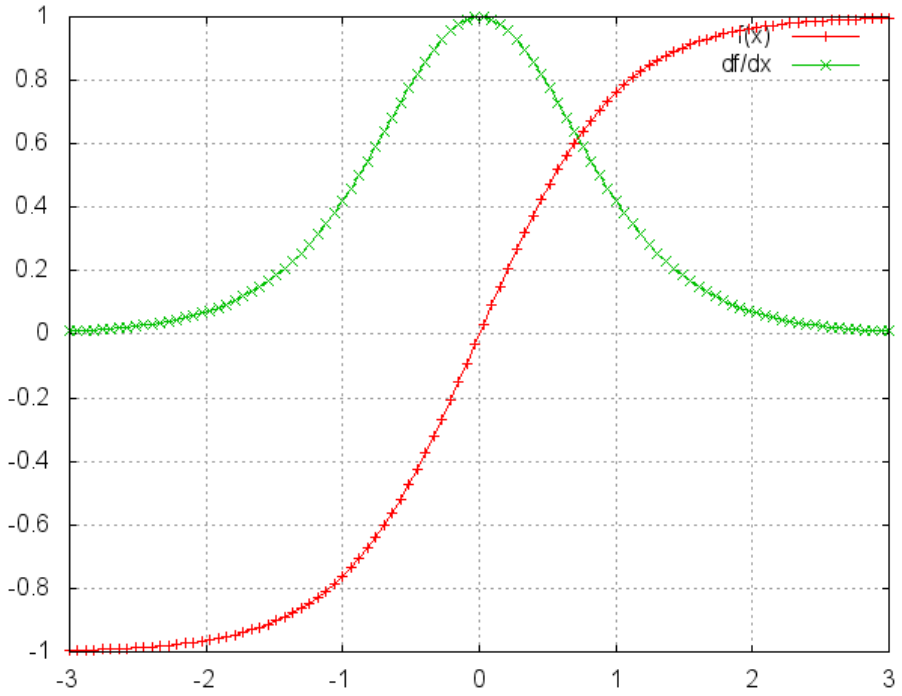
③ Hidden States

$$h^{(t)} = \sigma\left(W_h h^{(t-1)} + W_e e^{(t)} + b_1\right)$$

Sigmoid



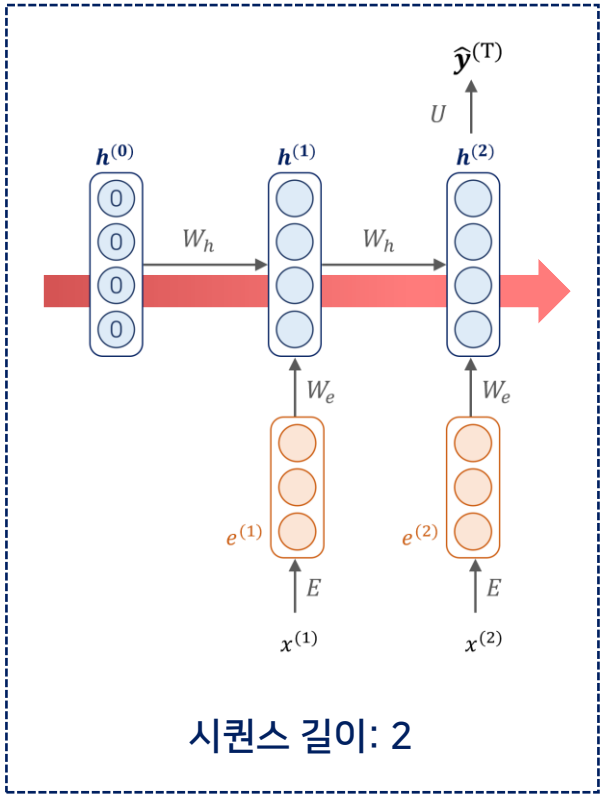
Tanh



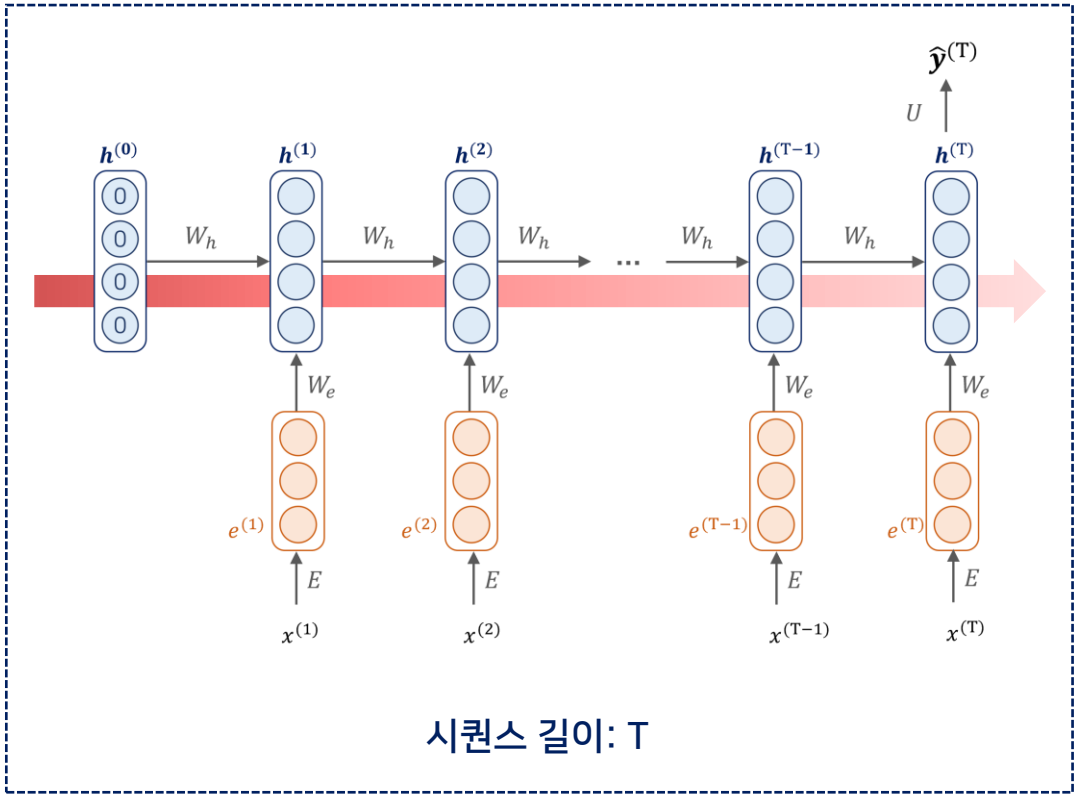
- ✓ **Sigmoid**: 기울기 0에서 약 0.25 사이
- ✓ **Tanh**: 기울기 0에서 1 사이 -> gradient vanishing에 더 강함

03 Problem of RNN

② Long Term Dependency (장기 의존성)



시퀀스 길이: 2

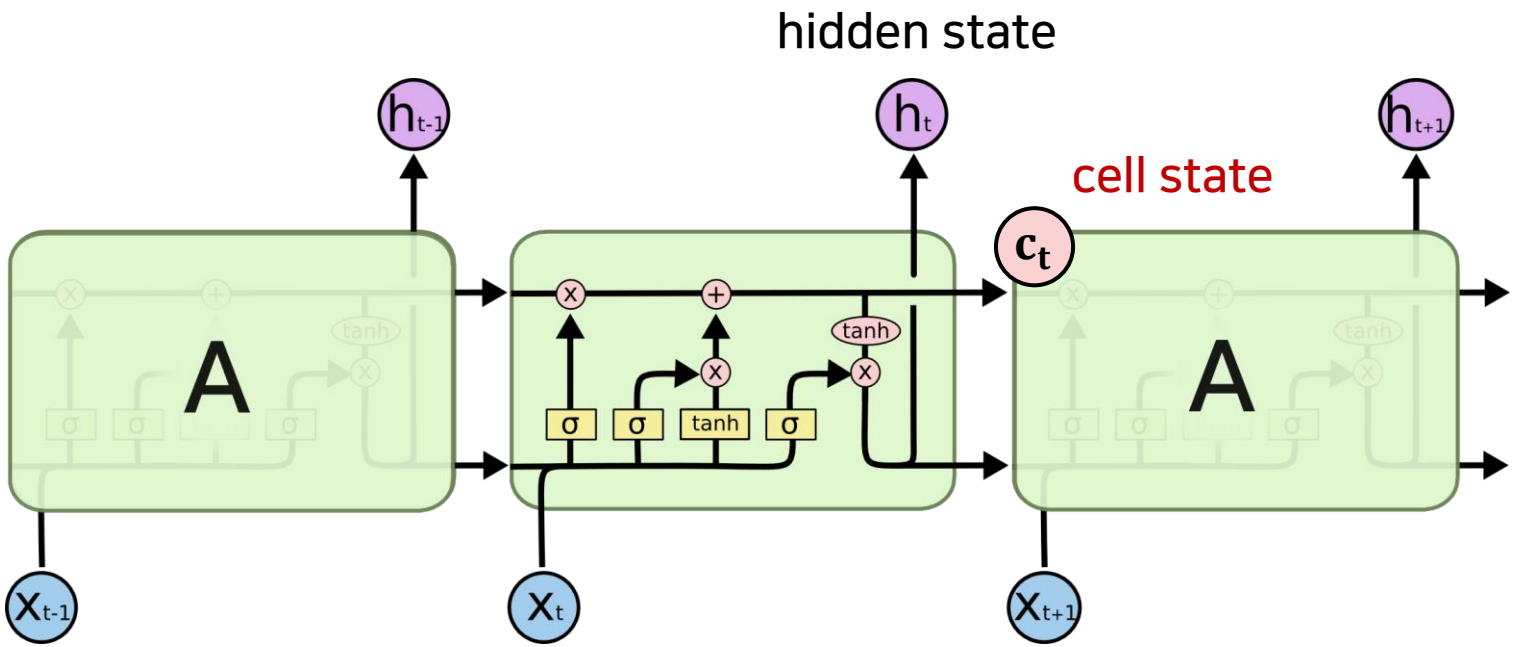


시퀀스 길이: T

- input sequence 길이가 긴 경우, 시퀀스 초반의 정보가 후반 time step의 hidden state를 도출하는데까지 전달되기 어려움 -> **Long term dependency를 잘 반영하지 못함**

04 LSTM: Long Short-Term Memory RNN

RNN with Separate Memory



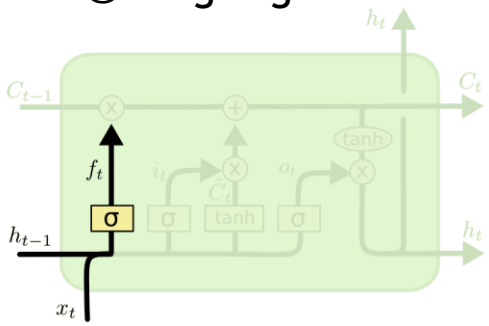
① **hidden state**를 통해 short term memory를 조절하고 **cell state**를 통해 long term memory 보존

② **forget, input, output, 3개의 gate**를 통해 매 time step의 cell state와 hidden state, input에서 취할 정보의 양 결정

04 LSTM: Long Short-Term Memory RNN

Architecture

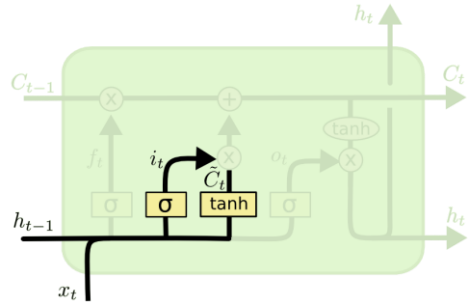
① forget gate



$$f_t = \sigma(W_f h^{(t-1)} + U_f x^{(t)} + b_f)$$

- 입력 정보(새로운 입력 시퀀스와 이전 시점의 hidden state)에 시그모이드를 취함

② input gate, new cell content



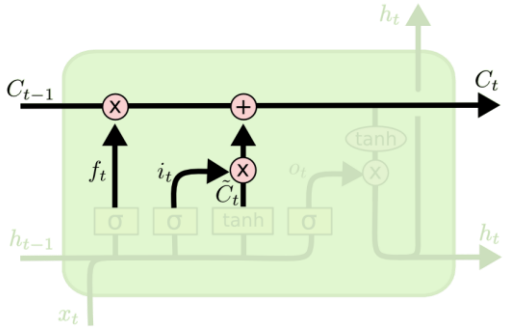
$$i_t = \sigma(W_i h^{(t-1)} + U_i x^{(t)} + b_i)$$

- **input gate**: 입력 정보에 시그모이드를 취함

$$\widetilde{C}_t = \tanh(W_c h^{(t-1)} + U_c x^{(t)} + b_c)$$

- **new cell content**: 입력 정보에 tanh를 취함

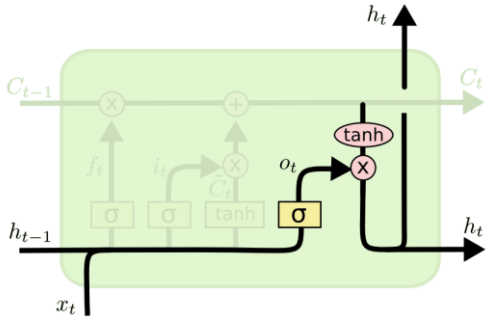
③ Cell state



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

- **forget gate**: 이전 시점의 cell state에서 어느 정도의 정보를 가져갈 것인지 결정
- **input gate**: 입력 정보에서 장기 기억으로 가져갈 정보의 양 결정
- element wise product

④ Output gate, hidden state



$$o_t = \sigma(W_o h^{(t-1)} + U_o x^{(t)} + b_o)$$

- **output gate**: 입력 정보에 시그모이드를 취함

$$h_t = o_t * \tanh(C_t)$$

- **hidden state**: 현재 입력과 대비해서 장기 기억에서 어느 정도의 정보를 단기 기억으로 사용할지 결정

04 LSTM: Long Short-Term Memory RNN

Architecture

① forget gate $f_t = \sigma(W_f h^{(t-1)} + U_f x^{(t)} + b_f)$

② input gate $i_t = \sigma(W_i h^{(t-1)} + U_i x^{(t)} + b_i)$

③ Output gate $o_t = \sigma(W_o h^{(t-1)} + U_o x^{(t)} + b_o)$

new cell content

$\tilde{C}_t = \tanh(W_c h^{(t-1)} + U_c x^{(t)} + b_c)$

$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$ Cell state (장기 기억)

이전 시점의 cell state

$h_t = o_t * \tanh(C_t)$ hidden state (단기 기억)

05 Summary

Conclusion

- ✓ **Language Model** : 확률 분포를 기반으로 주어진 문맥(sequence) 이후에 위치할 단어 예측
  - N-gram LM: 이전에 등장하는 N개의 단어 chunck를 바탕으로 다음에 올 단어 예측 (sparsity problem)
  - Neural LM: 이전에 등장하는 window size만큼의 단어를 바탕으로 다음에 올 단어 예측
  - input length 에 제한 -> 모든 문맥 고려 불가능

- ✓ **RNN**
  - input length 에 제한이 없는 sequential neural network
  - gradient vanishing /exploding
  - long term dependency problem

- ✓ **LSTM**
  - cell state를 통해 long term memory 보존
  - 3개의 gate를 통해 정보의 flow 조절

감사합니다