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1 Multiples of 3 and 5

[Problem]

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

⇒ Multiples of 3 and 5

2 Even Fibonacci numbers

[Problem]

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

⇒ Even Fibonacci numbers

3 Largest prime factor

[Problem]

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

4 Largest palindrome product

[Problem]

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \times 99$.

Find the largest palindrome made from the product of two 3-digit numbers.

5 Smallest multiple

[Problem]

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

6 Sum square difference

[Problem]

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \cdots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \cdots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

7 10001st prime

[Problem]

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.
What is the 10001st prime number?

8 Largest product in a series

[Problem]

The four adjacent digits in the 1000-digit number that have the greatest product are $9 \times 9 \times 8 \times 9 = 5832$.

```
73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450
```

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

9 Special Pythagorean triplet

[Problem]

A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$. Find the product abc .

10 Summation of primes

[Problem]

The sum of the primes below 10 is $2 + 3 + 5 + 7 = 17$.

Find the sum of all the primes below two million.

11 Largest product in a grid

[Problem]

In the 20×20 grid below, four numbers along a diagonal line have been marked in red. The product of these numbers is $26 \times 63 \times 78 \times 14 = 1788696$.

| | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 08 | 02 | 22 | 97 | 38 | 15 | 00 | 40 | 00 | 75 | 04 | 05 | 07 | 78 | 52 | 12 | 50 | 77 | 91 | 08 |
| 49 | 49 | 99 | 40 | 17 | 81 | 18 | 57 | 60 | 87 | 17 | 40 | 98 | 43 | 69 | 48 | 04 | 56 | 62 | 00 |
| 81 | 49 | 31 | 73 | 55 | 79 | 14 | 29 | 93 | 71 | 40 | 67 | 53 | 88 | 30 | 03 | 49 | 13 | 36 | 65 |
| 52 | 70 | 95 | 23 | 04 | 60 | 11 | 42 | 69 | 24 | 68 | 56 | 01 | 32 | 56 | 71 | 37 | 02 | 36 | 91 |
| 22 | 31 | 16 | 71 | 51 | 67 | 63 | 89 | 41 | 92 | 36 | 54 | 22 | 40 | 40 | 28 | 66 | 33 | 13 | 80 |
| 24 | 47 | 32 | 60 | 99 | 03 | 45 | 02 | 44 | 75 | 33 | 53 | 78 | 36 | 84 | 20 | 35 | 17 | 12 | 50 |
| 32 | 98 | 81 | 28 | 64 | 23 | 67 | 10 | 26 | 38 | 40 | 67 | 59 | 54 | 70 | 66 | 18 | 38 | 64 | 70 |
| 67 | 26 | 20 | 68 | 02 | 62 | 12 | 20 | 95 | 63 | 94 | 39 | 63 | 08 | 40 | 91 | 66 | 49 | 94 | 21 |
| 24 | 55 | 58 | 05 | 66 | 73 | 99 | 26 | 97 | 17 | 78 | 78 | 96 | 83 | 14 | 88 | 34 | 89 | 63 | 72 |
| 21 | 36 | 23 | 09 | 75 | 00 | 76 | 44 | 20 | 45 | 35 | 14 | 00 | 61 | 33 | 97 | 34 | 31 | 33 | 95 |
| 78 | 17 | 53 | 28 | 22 | 75 | 31 | 67 | 15 | 94 | 03 | 80 | 04 | 62 | 16 | 14 | 09 | 53 | 56 | 92 |
| 16 | 39 | 05 | 42 | 96 | 35 | 31 | 47 | 55 | 58 | 88 | 24 | 00 | 17 | 54 | 24 | 36 | 29 | 85 | 57 |
| 86 | 56 | 00 | 48 | 35 | 71 | 89 | 07 | 05 | 44 | 44 | 37 | 44 | 60 | 21 | 58 | 51 | 54 | 17 | 58 |
| 19 | 80 | 81 | 68 | 05 | 94 | 47 | 69 | 28 | 73 | 92 | 13 | 86 | 52 | 17 | 77 | 04 | 89 | 55 | 40 |
| 04 | 52 | 08 | 83 | 97 | 35 | 99 | 16 | 07 | 97 | 57 | 32 | 16 | 26 | 26 | 79 | 33 | 27 | 98 | 66 |
| 88 | 36 | 68 | 87 | 57 | 62 | 20 | 72 | 03 | 46 | 33 | 67 | 46 | 55 | 12 | 32 | 63 | 93 | 53 | 69 |
| 04 | 42 | 16 | 73 | 38 | 25 | 39 | 11 | 24 | 94 | 72 | 18 | 08 | 46 | 29 | 32 | 40 | 62 | 76 | 36 |
| 20 | 69 | 36 | 41 | 72 | 30 | 23 | 88 | 34 | 62 | 99 | 69 | 82 | 67 | 59 | 85 | 74 | 04 | 36 | 16 |
| 20 | 73 | 35 | 29 | 78 | 31 | 90 | 01 | 74 | 31 | 49 | 71 | 48 | 86 | 81 | 16 | 23 | 57 | 05 | 54 |
| 01 | 70 | 54 | 71 | 83 | 51 | 54 | 69 | 16 | 92 | 33 | 48 | 61 | 43 | 52 | 01 | 89 | 19 | 67 | 48 |

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

12 Highly divisible triangular number

[Problem]

The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

1 : 1

3 : 1, 3

6 : 1, 2, 3, 6

10 : 1, 2, 5, 10

15 : 1, 3, 5, 15

21 : 1, 3, 7, 21

28 : 1, 2, 4, 7, 14, 28

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

13 Large sum

[Problem]

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

```
37107287533902102798797998220837590246510135740250 46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629 91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676 89261670696623633820136378418383684178734361726757
28112879812849979408065481931592621691275889832738 44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276 70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331 64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407 58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390 35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829 71693888707715466499115593487603532921714970056938
5437007057682684624621495650076471787294438377604 53282654108756828443191190634694037855217779295145
36123272525000296071075082563815656710885258350721 45876576172410976447339110607218265236877223636045
17423706905851860660448207621209813287860733969412 81142660418086830619328460811191061556940512689692
151934325451728388641918047049293215058642563049483 62467221648435076201727918039944693004732956340691
15732444386908125794514089057706229429197107928209 55037687525678773091862540744969844508330393682126
18336384825330154686196124348767681297534375946515 80386287592878490201521685554828717201219257766954
78182833757993103614740356856449095527097864797581 16726320100436897842553539920931837441497806860984
48403098129077791799088218795327364475675590848030 87086987551392711854517078544161852424320693150332
59959406895756536782107074926966537676326235447210 69793950679652694742597709739166693763042633987085
41052684708299085211399427365734116182760315001271 65378607361501080857009149939512557028198746004375
35829035317434717326932123578154982629742552737307 9495375976510530594696607683156574377167401875275
88902802571733229619176668713819931811048770190271 25267680276078003013678680992525463401061632866526
36270218540497705585629946580636237993140746255962 24074486908231174977792365466257246923322810917141
9143028819710328859780669760892938638285025333403 34413065578016127815921815005561868836468420090470
23053081172816430487623791969842487255036638784583 11487696932154902810424020138335124462181441773470
6378329949063625966498587618221225225512486764533 67720186971698544312419572409913959008952310058822
95548255300263520781532296796249481641953868218774 76085327132285723110424803456124867697064507995236
37774242535411291684276865538926205024910326572967 23701913275725675285653248258265463092207058596522
29798860272258331913126375147341994889534765745501 18495701454879288984856827726077713721403798879715
38298203783031473527721580348144513491373226651381 34829543829199918180278916522431027392251122869539
40957953066405232632538044100059654939159879593635 29746152185502371307642255121183693803580388584903
41698116222072977186158236678424689157993532961922 62467957194401269043877107275048102390895523597457
23189706772547915061505504953922979530901129967519 8618808822587531452958409925120382900940770775672
75506164965184775180738168837861091527357929701337 82959174767140363198008187129011875491310547126581
73267460800591547471830798392868535206946944540724 42846280183517070527831839425882145521227251250327
97142617910342598647204516893989422179826088076852 32238195734329339946437501907836945765883352399886
10848802521674670883215120185883543223812876952786 62177842752192623401942399639168044983993173312731
62184073572399794223406235393808339651327408011116 99518671430235219628894890102423325116913619626622
60661826293682836764744779239180335110989069790714 7684182252467441716151403642798227334805556214818
66024396409905389607120198219976047599490197230297 87783646182799346313767754307809363333018982642090
16730939319872750275468906903707539413042652315011 71329612474782464538636993009049310363619763878039
78639167021187492431995700641917969777599028300699 66627891981488087797941876876144230030984490851411
40789923115535562561142322423255033685442488917353 85786944089552990653640447425576083659976645795096
4150312888033953605329934036800697710650566631954 64913982680032973156037120041377903785566085089252
82616570773948327592232845941706525094512325230608 94809377245048795150954100921645863754710598436791
77158542502016545090413245809786882778948721859617 15368713711936614952811305876380278410754449733078
20849603980134001723930671666823555245252804609722 44889911501440648020369068063960672322193204149535
81234880673210146739058568557934581403627822703280 82918802058777319719839450180888072429661980811197
72107838435069186155435662884062257473692284509516 72107838435069186155435662884062257473692284509516
53503534226472524250874054075591789781264303331690
```

14 Longest Collatz sequence

[Problem]

The following iterative sequence is defined for the set of positive integers:

$$n \rightarrow n / 2 \quad (n \text{ is even})$$

$$n \rightarrow 3n + 1 \quad (n \text{ is odd})$$

Using the rule above and starting with 13, we generate the following sequence:

$$13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

15 Lattice paths

[Problem]

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.

How many such routes are there through a 20×20 grid?

16 Power digit sum

[Problem]

$2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$.

What is the sum of the digits of the number 2^{1000} ?

17 Number letter counts

[Problem]

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are $3 + 3 + 5 + 4 + 4 = 19$ letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

18 Maximum path sum I

[Problem]

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

```
      3
     7 4
    2 4 6
   8 5 9 3
```

That is, $3 + 7 + 4 + 9 = 23$.

Find the maximum total from top to bottom of the triangle below:

```
      75
     95 64
    17 47 82
   18 35 87 10
  20 04 82 47 65
 19 01 23 75 03 34
 88 02 77 73 07 63 67
 99 65 04 28 06 16 70 92
 41 41 26 56 83 40 80 70 33
 41 48 72 33 47 32 37 16 94 29
 53 71 44 65 25 43 91 52 97 51 14
 70 11 33 28 77 73 17 78 39 68 17 57
 91 71 52 38 17 14 91 43 58 50 27 29 48
 63 66 04 68 89 53 67 30 73 16 69 87 40 31
 04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
```

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

19 Counting Sundays

[Problem]

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September, April, June and November.
- All the rest have thirty-one, Saving February alone, Which has twenty-eight, rain or shine.
And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

20 Factorial digit sum

[Problem]

$n!$ means $n \times (n - 1) \times \cdots \times 3 \times 2 \times 1$

For example, $10! = 10 \times 9 \times \cdots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$.

Find the sum of the digits in the number $100!$

21 Amicable numbers

[Problem]

Let $d(n)$ be defined as the sum of proper divisors of n (numbers less than n which divide evenly into n).

If $d(a) = b$ and $d(b) = a$, where $a \neq b$, then a and b are an amicable pair and each of a and b are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore $d(220) = 284$. The proper divisors of 284 are 1, 2, 4, 71 and 142; so $d(284) = 220$.

Evaluate the sum of all the amicable numbers under 10000.

22 Names scores

[Problem]

Using names.txt (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth $3 + 15 + 12 + 9 + 14 = 53$, is the 938th name in the list. So, COLIN would obtain a score of $938 \times 53 = 49714$.

What is the total of all the name scores in the file?

23 Non-abundant sums

[Problem]

A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be $1 + 2 + 4 + 7 + 14 = 28$, which means that 28 is a perfect number.

A number n is called deficient if the sum of its proper divisors is less than n and it is called abundant if this sum exceeds n .

As 12 is the smallest abundant number, $1 + 2 + 3 + 4 + 6 = 16$, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

24 Lexicographic permutations

[Problem]

A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

012 021 102 120 201 210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

25 1000-digit Fibonacci number

[Problem]

The Fibonacci sequence is defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2}, \text{ where } F_1 = 1 \text{ and } F_2 = 1.$$

Hence the first 12 terms will be:

$$F_1 = 1$$

$$F_2 = 1$$

$$F_3 = 2$$

$$F_4 = 3$$

$$F_5 = 5$$

$$F_6 = 8$$

$$F_7 = 13$$

$$F_8 = 21$$

$$F_9 = 34$$

$$F_{10} = 55$$

$$F_{11} = 89$$

$$F_{12} = 144$$

The 12th term, F_{12} , is the first term to contain three digits.

What is the index of the first term in the Fibonacci sequence to contain 1000 digits?

26 Reciprocal cycles

[Problem]

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$$1/2 = 0.5$$

$$1/3 = 0.(3)$$

$$1/4 = 0.25$$

$$1/5 = 0.2$$

$$1/6 = 0.1(6)$$

$$1/7 = 0.(142857)$$

$$1/8 = 0.125$$

$$1/9 = 0.(1)$$

$$1/10 = 0.1$$

Where $0.1(6)$ means $0.166666\dots$, and has a 1-digit recurring cycle. It can be seen that $1/7$ has a 6-digit recurring cycle.

Find the value of $d < 1000$ for which $1/d$ contains the longest recurring cycle in its decimal fraction part.

27 Quadratic primes

[Problem]

Euler discovered the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive integer values $0 \leq n \leq 39$. However, when $n = 40$, $40^2 + 40 + 41 = 40(40 + 1) + 41$ is divisible by 41, and certainly when $n = 41$, $41^2 + 41 + 41$ is clearly divisible by 41.

The incredible formula $n^2 - 79n + 1601$ was discovered, which produces 80 primes for the consecutive values $0 \leq n \leq 79$. The product of the coefficients, -79 and 1601 , is -126479 .

Considering quadratics of the form:

$$n^2 + an + b, \text{ where } |a| < 1000 \text{ and } |b| < 1000$$

where $|n|$ is the modulus/absolute value of n

e.g. $|11| = 11$ and $|-4| = 4$

Find the product of the coefficients, a and b , for the quadratic expression that produces the maximum number of primes for consecutive values of n , starting with $n = 0$.

28 Number spiral diagonals

[Problem]

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

| | | | | |
|----|----|----|----|----|
| 21 | 22 | 23 | 24 | 25 |
| 20 | 7 | 8 | 9 | 10 |
| 19 | 6 | 1 | 2 | 11 |
| 18 | 5 | 4 | 3 | 12 |
| 17 | 16 | 15 | 14 | 13 |

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

29 Distinct powers

[Problem]

Consider all integer combinations of ab for $2 \leq a \leq 5$ and $2 \leq b \leq 5$:

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

$$4^5 = 1024$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$$5^5 = 3125$$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by ab for $2 \leq a \leq 100$ and $2 \leq b \leq 100$?

30 Digit fifth powers

[Problem]

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

$$8208 = 8^4 + 2^4 + 0^4 + 8^4$$

$$9474 = 9^4 + 4^4 + 7^4 + 4^4$$

As $1 = 1^4$ is not a sum it is not included.

The sum of these numbers is $1634 + 8208 + 9474 = 19316$.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

31 Coin sums

[Problem]

In England the currency is made up of pound, £, and pence, p, and there are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1(100p) and £2(200p).

It is possible to make £2 in the following way:

$$1 \times £1 + 1 \times 50p + 2 \times 20p + 1 \times 5p + 1 \times 2p + 3 \times 1p$$

How many different ways can £2 be made using any number of coins?

32 Pandigital products

[Problem]

We shall say that an n -digit number is pandigital if it makes use of all the digits 1 to n exactly once; for example, the 5-digit number, 15234, is 1 through 5 pandigital.

The product 7254 is unusual, as the identity, $39 \times 186 = 7254$, containing multiplicand, multiplier, and product is 1 through 9 pandigital.

Find the sum of all products whose multiplicand/multiplier/product identity can be written as a 1 through 9 pandigital.

HINT: Some products can be obtained in more than one way so be sure to only include it once in your sum.

33 Digit cancelling fractions

[Problem]

The fraction $49 / 98$ is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that $49 / 98 = 4 / 8$, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, $30 / 50 = 3 / 5$, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

34 Digit factorials

[Problem]

145 is a curious number, as $1! + 4! + 5! = 1 + 24 + 120 = 145$.

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: as $1! = 1$ and $2! = 2$ are not sums they are not included.

35 Circular primes

[Problem]

The number, 197, is called a circular prime because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular primes are there below one million?

36 Double-base palindromes

[Problem]

The decimal number, $585 = 1001001001_2$ (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

(Please note that the palindromic number, in either base, may not include leading zeros.)

37 Truncatable primes

[Problem]

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7. Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

38 Pandigital multiples

[Problem]

Take the number 192 and multiply it by each of 1, 2, and 3:

$$192 \times 1 = 192$$

$$192 \times 2 = 384$$

$$192 \times 3 = 576$$

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the concatenated product of 192 and (1,2,3)

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and (1,2,3,4,5).

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with (1,2, ... , n) where $n > 1$?

39 Integer right triangles

[Problem]

If p is the perimeter of a right angle triangle with integral length sides, $\{a, b, c\}$, there are exactly three solutions for $p = 120$.

$$\{20, 48, 52\}, \{24, 45, 51\}, \{30, 40, 50\}$$

For which value of $p \leq 1000$, is the number of solutions maximised?

40 Champernowne's constant

[Problem]

An irrational decimal fraction is created by concatenating the positive integers:

$$0.12345678910\textcolor{brown}{1}112131415161718192021\cdots$$

It can be seen that the 12th digit of the fractional part is 1.

If d_n represents the n th digit of the fractional part, find the value of the following expression.

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

41 Pandigital prime

[Problem]

We shall say that an n -digit number is pandigital if it makes use of all the digits 1 to n exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest n -digit pandigital prime that exists?

42 Coded triangle numbers

[Problem]

The n th term of the sequence of triangle numbers is given by, $t_n = (1/2)n(n + 1)$; so the first ten triangle numbers are:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is $19 + 11 + 25 = 55 = t_{10}$. If the word value is a triangle number then we shall call the word a triangle word.

Using words.txt (right click and 'Save Link/Target As...'), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

43 Sub-string divisibility

[Problem]

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let d_1 be the 1st digit, d_2 be the 2nd digit, and so on. In this way, we note the following:

- $d_2d_3d_4 = 406$ is divisible by 2
- $d_3d_4d_5 = 063$ is divisible by 3
- $d_4d_5d_6 = 635$ is divisible by 5
- $d_5d_6d_7 = 357$ is divisible by 7
- $d_6d_7d_8 = 572$ is divisible by 11
- $d_7d_8d_9 = 728$ is divisible by 13
- $d_8d_9d_{10} = 289$ is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

44 Pentagon numbers

[Problem]

Pentagonal numbers are generated by the formula, $P_n = n(3n - 1)/2$. The first ten pentagonal numbers are:

$$1, 5, 12, 22, 35, 51, 70, 92, 117, 145, \dots$$

It can be seen that $P_4 + P_7 = 22 + 70 = 92 = P_8$. However, their difference, $70 - 22 = 48$, is not pentagonal.

Find the pair of pentagonal numbers, P_j and P_k , for which their sum and difference are pentagonal and $D = |P_k - P_j|$ is minimised; what is the value of D ?

45 Triangular, pentagonal, and hexagonal

[Problem]

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

$$\text{Triangle} \quad T_n = n(n+1)/2 \quad 1, 3, 6, 10, 15, \dots$$

$$\text{Pentagonal} \quad P_n = n(3n-1)/2 \quad 1, 5, 12, 22, 35, \dots$$

$$\text{Hexagonal} \quad H_n = n(2n-1) \quad 1, 6, 15, 28, 45, \dots$$

It can be verified that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangle number that is also pentagonal and hexagonal.

46 Goldbach's other conjecture

[Problem]

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$$9 = 7 + 2 \times 1^2$$

$$15 = 7 + 2 \times 2^2$$

$$21 = 3 + 2 \times 3^2$$

$$25 = 7 + 2 \times 3^2$$

$$27 = 19 + 2 \times 2^2$$

$$33 = 31 + 2 \times 1^2$$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

47 Distinct primes factors

[Problem]

The first two consecutive numbers to have two distinct prime factors are:

$$14 = 2 \times 7$$

$$15 = 3 \times 5$$

The first three consecutive numbers to have three distinct prime factors are:

$$644 = 2^2 \times 7 \times 23$$

$$645 = 3 \times 5 \times 43$$

$$646 = 2 \times 17 \times 19$$

Find the first four consecutive integers to have four distinct prime factors each. What is the first of these numbers?

48 Self powers

[Problem]

The series, $1^1 + 2^2 + 3^3 + \dots + 10^{10} = 10405071317$.

Find the last ten digits of the series, $1^1 + 2^2 + 3^3 + \dots + 1000^{1000}$.

49 Prime permutations

[Problem]

The arithmetic sequence, 1487, 4817, 8147, in which each of the terms increases by 3330, is unusual in two ways: (i) each of the three terms are prime, and, (ii) each of the 4-digit numbers are permutations of one another. There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence. What 12-digit number do you form by concatenating the three terms in this sequence?

50 Consecutive prime sum

[Problem]

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?