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1 Multiples of 3 and 5

[Problem]

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

2 Even Fibonacci numbers

[Problem]

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

3 Largest prime factor

[Problem]

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

4 Largest palindrome product

[Problem]

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \times 99$.

Find the largest palindrome made from the product of two 3-digit numbers.

5 Smallest multiple

[Problem]

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

6 Sum square difference

[Problem]

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \cdots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \cdots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.
Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

7 10001st prime

[Problem]

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10001st prime number?

8 Largest product in a series

[Problem]

The four adjacent digits in the 1000-digit number that have the greatest product are $9 \times 9 \times 8 \times 9 = 5832$.

```
73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450
```

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

9 Special Pythagorean triplet

[Problem]

A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$. Find the product abc.

10 Summation of primes

[Problem]

The sum of the primes below 10 is $2 + 3 + 5 + 7 = 17$.

Find the sum of all the primes below two million.

11 Largest product in a grid

[Problem]

In the 20×20 grid below, four numbers along a diagonal line have been marked in red. The product of these numbers is $26 \times 63 \times 78 \times 14 = 1788696$.

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

12 Highly divisible triangular number

[Problem]

The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The first ten terms would be:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \cdots$$

Let us list the factors of the first seven triangle numbers:

$$\begin{aligned}
1 &: \texttt{1} \\
3 &: \texttt{1, 3} \\
6 &: \texttt{1, 2, 3, 6} \\
10 &: \texttt{1, 2, 5, 10} \\
15 &: \texttt{1, 3, 5, 15} \\
21 &: \texttt{1, 3, 7, 21} \\
28 &: \texttt{1, 2, 4, 7, 14, 28}
\end{aligned}$$

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

13 Large sum

[Problem]

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

```
37107287533902102798797998220837590246510135740250
74324986199524741059474233309513058123726617309629
23067588207539346171171980310421047513778063246676
28112879812849979408065481931592621691275889832738
47451445736001306439091167216856844588711603153276
62176457141856560629502157223196586755079324193331
92575867718337217661963751590579239728245598838407
80181199384826282014278194139940567587151170094390
86515506006295864861532075273719591914205175255829
54370070576826684624621495650076471787294438377604
36123272525000296071075082563815656710885258350721
17423706905851860660448207621209813287860733969412
51934325451728388641918047049293215058642563049483
15732444386908125794514089057706229429197107928209
18336384825330154686196124348767681297534375946515
78182833757993103614740356856449095527097864797581
48403098129077791799088218795327364475675590848030
59959406895756536782107074926966537676326235447210
41052684708299085211399427365734116182760315001271
35829035317434717326932123578154982629742552737307
88902802571733229619176668713819931811048770190271
36270218540497705585629946580636237993140746255962
91430288197103288597806669760892938638285025333403
23053081172816430487623791969842487255036638784583
63783299490636259666498587618221225225512486764533
95548255300263520781532296796249481641953868218774
37774242535411291684276865538926205024910326572967
29798860272258331913126375147341994889534765745501
38298203783031473527721580348144513491373226651381
40957953066405232632538044100059654939159879593635
41698116222072977186158236678424689157993532961922
23189706772547915061505504953922979530901129967519
11306739708304724483816533873502340845647058077308
97623331044818386269515456334926366572897563400500
55121603546981200581762165212827652751691296897789
75506164965184775180738168837861091527357929701337
32924185707147349566916674687634660915035914677504
73267460800591547471830798392868535206946944540724
97142617910342598647204516893989422179826088076852
10848802521674670883215120185883543223812876952786
62184073572399794223406235393808339651327408011116
60661826293682836764744779239180335110989069790714
66024396409905389607120198219976047599490197230297
16730393931987275027546890690370753941304265235011
78639167021187492431995700641917969777599028300699
40789923115535562561142322423255033685442488917353
41503128880339536053299340368006977710650566631954
82616570773948327592232845941706525094512325230608
77158542502016545090413245809786882778948721859617
20849603980134001723930671666823555245252804609722
46376937677490009712648124896970078050417018260538
91942213363574161572522430563301811072406154908250
89261670696623633820136378418383684178734361726757
44274228917432520321923589422876796487670272189318
70386486105843025439939619828917593665686757934951
64906352462741904929101432445813822663347944758178
58203565325359399008402633568948830189458628227828
35398664372827112653829987240784473053190104293586
71693888707715466499115593487603532921714970056938
53282654108756828443191190634694037855217779295145
45876576172410976447339110607218265236877223636045
81142660418086830619328460811191061556940512689692
62467221648435076201727918039944693004732956340691
55037687525678773091862540744969844508330393682126
80386287592878490201521685554828717201219257766954
16726320100436897842553539920931837441497806860984
87086987551392711854517078544161852424320693150332
69793950679652694742597709739166693763042633987085
65378607361501080857009149939512557028198746004375
94953759765105305946966076831565743771674018752 75
25267680276078003013678680992525463401061632866526
24074486908231174977792365466257246923322810917141
34413065578016127815921815005561868836468420090470
11487696932154902810424020138335124462181441773470
67720186971698544312419572409913959008952310058822
76085327132285723110424803456124867697064507995236
23701913275725675285653248258265463092207058596522
18495701454879288984856827726077713721403798879715
34829543829199918180278916522431027392251122869539
29746152185502371307642255121183693803580388584903
62467957194401269043877107275048102390895523597457
86188088225875314529584099251203829009407770775672
82959174767140363198008187129011875491310547126581
42846280183517070527831839425882145521227251250327
32238195734329339946437501907836945765883352399886
62177842752192623401942399639168044983993173312731
99518671430235219628894890102423325116913619626622
76841822524674417161514036427982273348055556214818
87783646182799346313767754307809363333018982642090
71329612474782464538636993009049310363619763878039
66627891981488087797941876876144230030984490851411
85786944089552990653640447425576083659976645795096
64913982680032973156037120041377903785566085089252
94809377245048795150954100921645863754710598436791
15368713711936614952811305876380278410754449733078
44889911501440648020369068063960723221932041495535
81234880673210146739058568557934581403627822703280
22918802058777319719839450180888072429661980811197
72107838435069186155435662884062257473692284509516
53503534226472524250874054075591789781264330331690
```

14 Longest Collatz sequence

[Problem]

The following iterative sequence is defined for the set of positive integers:

$$n \to n/2 \qquad (n \text{ is even})$$
$$n \to 3n + 1 \quad (n \text{ is odd})$$

Using the rule above and starting with 13, we generate the following sequence:

$$13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

15 Lattice paths

[Problem]

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.

How many such routes are there through a 20×20 grid?

16 Power digit sum

[Problem]

$2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$.

What is the sum of the digits of the number 2^{1000}?

17 Number letter counts

[Problem]

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3+3+5+4+4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of ”and” when writing out numbers is in compliance with British usage.

18 Maximum path sum I

[Problem]

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

```
   3
  7 4
 2 4 6
8 5 9 3
```

That is, $3 + 7 + 4 + 9 = 23$.

Find the maximum total from top to bottom of the triangle below:

```
                75
               95 64
              17 47 82
             18 35 87 10
            20 04 82 47 65
           19 01 23 75 03 34
          88 02 77 73 07 63 67
         99 65 04 28 06 16 70 92
        41 41 26 56 83 40 80 70 33
       41 48 72 33 47 32 37 16 94 29
      53 71 44 65 25 43 91 52 97 51 14
     70 11 33 28 77 73 17 78 39 68 17 57
    91 71 52 38 17 14 91 43 58 50 27 29 48
   63 66 04 68 89 53 67 30 73 16 69 87 40 31
  04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
```

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

19 Counting Sundays

[Problem]

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September, April, June and November.
- All the rest have thirty-one, Saving February alone, Which has twenty-eight, rain or shine. And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

20 Factorial digit sum

[Problem]

$n!$ means $n \times (n-1) \times \cdots \times 3 \times 2 \times 1$

For example, $10! = 10 \times 9 \times \cdots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number 10! is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$.

Find the sum of the digits in the number 100!