Analytical Formulation and Python Implementation for an Extended Carrión/Arroyo SCUC/SCED Optimization Formulation

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Last Revised: 14 August 2019 Rev D

1 Introduction

Centrally-managed wholesale power markets in the United States rely on ISO/RTO-managed *Security-Constrained Unit Commitment (SCUC)* and *Security Constrained Economic Dispatch (SCED)* optimizations to determine unit commitments, reserve, and scheduled dispatch levels for generating units during future operating periods. In an earlier report [4], an analytically-formulated combined SCUC/SCED optimization was presented that extends the well-known SCUC/SCED optimization model developed by Carrión and Arroyo [2] in five key ways:

- Inclusion of non-dispatchable generation
- Inclusion of energy storage units
- Inclusion of nodal power balance constraints with possible transmission congestion
- Inclusion of zonal as well as system-wide reserve requirements
- Inclusion of imbalance penalty terms in the objective function for slack in power balance constraints

This extended SCUC/SCED optimization formulation will hereafter be referred to as the Basic Extended Carrión-Arroyo Model, or the *Basic ECA Model* for short.

In addition, the earlier report [4] discusses a software implementation of the Basic ECA Model by means of the *Python Optimization Modeling Objects (Pyomo)* package [5, 12, 13]. The Pyomo package is an open-source tool for optimization applications. This Pyomo software implementation has been incorporated into the AMES Wholesale Power Market Test Bed [14], starting with AMES V4.0 [7]. It has also been used to implement the agent-based 8-Zone ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion [8, 9], and an extension of this test system by Li and Tesfatsion [10] that incorporates wind power in the form of physically-modeled wind turbine agents. Hereafter this Pyomo software implementation of the Basic ECA Model will be referred to as the *Pyomo Model*.

The current report provides a substantially revised version of the earlier report [4] in order to improve its readability and clarity. First, the ordering of presented materials has been changed to facilitate the logical progression of ideas. Second, the presentation of nomenclature for the Basic ECA Model has been augmented with explanatory notes to facilitate understanding. Third, the presentation of the Basic ECA Model components (objective function, decision variables, constraints) has been augmented with detailed notes to explain the meaning and/or derivation of these components. Fourth, the Basic ECA Model is further generalized to include price-sensitive demand bids. Fifth, the sections of the original report [4] proposing a stochastic (scenario-based) extension of the Basic ECA Model have been omitted.

This report is organized as follows. Section 2 provides a broad overview of the Basic ECA Model. Section 3 provides complete nomenclature for the Basic ECA Model, grouped by similar elements with accompanying explanatory notes. The manner in which the Pyomo Model implements piece-wise linear approximations for the total production cost functions of dispatchable generators is explained in Section 4.

Section 5 is the heart of this report; it provides a complete analytical formulation for the three SCUC/SCED components of the Basic ECA Model: namely, the ISO's objective function, the ISO's decision variables, and the system constraints. Each component is accompanied by explanatory notes. Section 6 then shows how the Basic ECA Model can be extended to permit load-serving entities to submit demand bids with accompanying price (valuation) information permitting the measurement of customer benefits. Three cases are considered: demand bids with Time-of-Use (TOU) pricing; demand bids in the form of price-quantity demand schedules; and demand bids directly expressed as benefit functions.

The final sections of this report focus on Pyomo Model implementation issues. Section 8 describes how the Pyomo Model determines locational marginal prices. Section 9 provides a mapping among corresponding input names for the Basic ECA Model, AMES V5.0 [14], and the Pyomo Model, together with Pyomo Model input default values.

Finally, an important terminological clarification needs to be stressed. Throughout these notes, *power output* refers to the amount of power (MW) that a generator is injecting into a transmission grid at a particular point in time. In contrast, *total power generation* refers to the total amount of power (MW) that a generator is producing at a particular point in time. This total power generation can include local (behind-the-meter) power that the generator needs to produce and use locally in order to bring itself into a "synchronized state". A *synchronized state* is an operating state in which a generator is ready to inject power into the grid, even if no actual injection is currently taking place.

2 The Basic ECA Model: Overview

The Basic ECA Model provides a complete analytical formulation for a SCUC/SCED optimization undertaken by an *Independent System Operator (ISO)* tasked with ensuring the efficiency and reliability of wholesale power system operations. The participants in the SCUC/SCED optimization include dispatchable and non-dispatchable generator units, energy storage units, and load-serving entities functioning as intermediaries for retail power customers with fixed (non-price-sensitive) loads.

The objective of the ISO is to minimize the expected total cost of securing sufficient resources to ensure the balancing of net fixed load during a future operating period T, where *net fixed load* is fixed load minus non-dispatchable generation. The future operating period T is partitioned into consecutive time-steps $k = 1, \ldots NK$. Total cost is the summation of production cost, start-up cost, shut-down cost, and imbalance cost incurred over all NK time-steps. These costs are "expected" costs in the sense that they are conditioned on forecasts for next-day net fixed loads.[1]

Given initial system conditions, together with forecasts for period-T net fixed loads, the SCUC/SCED optimization determines cost-minimizing solution values for dispatchable generator unit commitments, energy storage unit commitments, dispatchable generator power outputs, energy storage power outputs (discharge

[1] That is, certainty equivalence is used to approximate expectations.

levels), energy storage power absorptions (charge levels), and locational marginal prices (LMPs) for each time-step k, subject to system constraints. These system constraints include:

- transmission line power constraints;
- power balance constraints;
- generator capacity constraints;
- dispatchable generator ramp constraints for start-up, normal, and shut-down conditions;
- dispatchable generator minimum up-time/down-time constraints;
- dispatchable generator hot-start constraints;
- dispatchable generator start-up/shut-down cost constraints;
- storage unit limit constraints;
- storage unit charge/discharge constraints;
- storage unit ramping constraints;
- storage unit energy conservation constraints;
- storage unit end-point constraints;
- system-wide reserve requirement constraints;
- zonal reserve requirement constraints.

The Basic ECA Model formulation for the power balance constraints relies on a standard DC Optimal Power Flow (DC-OPF) approximation. Consequently, it relies on the following three assumptions. First, the resistance for each transmission line is negligible compared to the reactance, hence the resistance for each transmission line is set to 0. Second, the voltage magnitude at each bus is equal to a common base voltage magnitude. Third, the voltage angle difference $\Delta\theta(\ell)$ across any line ℓ is sufficiently small that the following approximations can be used: $\cos(\Delta\theta(\ell)) \approx 1$ in size and $\sin(\Delta\theta(\ell)) \approx \Delta\theta(\ell)$ in size.

3 Nomenclature for the Basic ECA Model

3.1 Sets and Subsets

$\mathbb{B} = \{1, \ldots, NB\}$	Index set for the buses b of a transmission grid
$\mathbb{B}(z) \subseteq \mathbb{B}$	Subset of buses constituting reserve zone z
$\mathbb{G}$	Index set for participant dispatchable generators g
$\mathbb{G}(b) \subseteq \mathbb{G}$	Subset of dispatchable generators located at bus b
$\mathbb{G}(z) \subseteq \mathbb{G}$	Subset of dispatchable generators located in reserve zone z
$\mathbb{K} = \{1, \ldots, NK\}$	Index set for time-steps k forming a partition of the operating period T

$\mathbb{L} \subseteq \mathbb{B} \times \mathbb{B}$	Index set for the lines ℓ of a transmission grid
$\mathbb{L}_{O(b)} \subseteq \mathbb{L}$	Subset of transmission lines originating at bus b
$\mathbb{L}_{E(b)} \subseteq \mathbb{L}$	Subset of transmission lines ending at bus b
$\mathbb{LS}$	Index set for participant load-serving entities j
$\mathbb{LS}(b) \subseteq \mathbb{LS}$	Subset of load-serving entities serving customers at bus b
$\mathbb{LS}(z) \subseteq \mathbb{LS}$	Subset of load-serving entities serving customers in zone z
$\mathbb{M} = \mathbb{G} \cup \mathbb{S}$	Set of dispatchable market participants m
$\mathbb{NG}$	Index set for participant non-dispatchable generators n
$\mathbb{NG}(b) \subseteq \mathbb{NG}$	Subset of non-dispatchable generators located at bus b
$\mathbb{NG}(z) \subseteq \mathbb{NG}$	Subset of non-dispatchable generators located in zone z
$\mathbb{NS}_g(k) = \{1, \ldots, NS_g(k)\}$	Index set for the segments i used to form a piecewise-linear approximation for the total production cost function of g at time-step k
$\mathbb{S}$	Index set for participant energy storage units s
$\mathbb{S}(b) \subseteq \mathbb{S}$	Subset of energy storage units located at bus b
$\mathbb{S}(z) \subseteq \mathbb{S}$	Subset of energy storage units located in zone z
$\mathbb{Z}$	Set of indices $z = 1, \ldots, NZ$ for reserve zones $\mathbb{B}(z)$, which form a partition of $\mathbb{B}$, i.e., $\cup_{z \in \mathbb{Z}} \mathbb{B}(z) = \mathbb{B}$, and $\mathbb{B}(z_i) \cap \mathbb{B}(z_j) = \emptyset$ for any z_i and z_j in $\mathbb{Z}$ with $i \neq j$

3.2 User-Specified Parameters

User-Specified Parameters for the Physical Attributes of Dispatchable Generators $g \in \mathbb{G}$:

$DT_g \geq 0$	Minimum down-time (h) for g
$UT_g \geq 0$	Minimum up-time (h) for g
NRD_g	Nominal ramp-down rate (MW/Δt) for g
NRU_g	Nominal ramp-up rate (MW/Δt) for g
NSD_g	Shut-down ramp rate (MW/Δt) for g
NSU_g	Start-up ramp rate (MW/Δt) for g
$P_g^{\max}(k) \geq 0$	Max power output (MW) for g at time-step k
$P_g^{\min}(k) \geq 0$	Min power output (MW) for g at time-step k (must satisfy $P_g^{\min}(k) \leq P_g^{\max}(k)$)

User-Specified Parameters for the Start-Up/Shut-Down Costs of Dispatchable Generators $g \in \mathbb{G}$:

$CSC_g \geq 0$	Cold-start cost (\$) for g
$CSH_g \geq 0$	Cold-start hours (h) for g
$HSC_g \geq 0$	Hot-start cost (\$) for g (must satisfy $HSC_g \leq CSC_g$)
$SDC_g \geq 0$	Shut-down cost (\$) for g

Remarks on the Cold-Start Hours Parameter: The cold-start hours parameter CSH_g has the following meaning. If a dispatchable generator g at the start of a time-step k has been off-line for at least CSH_g consecutive hours *immediately prior* to k, then g is in a *cold-start state* and any start-up of g at the start of k incurs the cold-start cost CSC_g. Otherwise, g is in a *hot-start state* at the start of k and any start-up of g at the start of k incurs the hot-start cost HSC_g.[2]

[2] Carrión and Arroyo [2, Sec. II] propose a "stairwise startup function" to model the manner in which start-up costs increase for a dispatchable generator g as a function of the number of consecutive hours immediately prior to k during which g was offline.

User-specified parameters for the approximation of the total production cost function for a dispatchable generator $g \in \mathbb{G}$ at a time-step $k \in \mathbb{K}$:

$a_g(k) \geq 0$	Production cost function coefficient (\$/h) for g
$b_g(k) > 0$	Production cost function coefficient (\$/MWh) for g
$c_g(k) > 0$	Production cost function coefficient (\$/[MW]^{2}h) for g
$NS_g(k) \geq 1$	Number of segments i used for the piecewise-linear approximation of g's production cost function for time-step k

Remark: The construction of an approximate measure for the total production cost of each dispatchable generator g incurred during each time-step k is carefully explained in Section 4, below.

User-Specified Parameters for Energy Storage Units $s \in \mathbb{S}$:

$EPSOC_s \geq 0$	Target charge state (decimal percent) for s at end of the operating period T
$ES_s^{\mathsf{max}} \geq 0$	Maximum energy storage capacity (MWΔt) of s during each time-step k
$NRDIS_s$	Nominal charge ramp-down rate (MW/Δt) for s
$NRUIS_s$	Nominal charge ramp-up rate (MW/Δt) for s
$NRDOS_s$	Nominal discharge ramp-down rate (MW/Δt) for s
$NRUOS_s$	Nominal discharge ramp-up rate (MW/Δt) for s
PIS_s^{max}	Maximum charge power (MW) for s
$PIS_s^{\mathsf{min}} \geq 0$	Minimum charge power (MW) for s
POS_s^{max}	Maximum discharge power (MW) for s
$POS_s^{\mathsf{min}} \geq 0$	Minimum discharge power (MW) for s
$SOC_s^{\mathsf{min}} \geq 0$	Minimum state of charge (decimal percent) for s
$\eta_s \geq 0$	Round-trip efficiency (decimal percent) for s

User-Specified Parameters for Down/Up Reserve Requirements:

$RD(k) \geq 0$	System-wide down-reserve requirement (decimal percent) at time-step k
$RU(k) \geq 0$	System-wide up-reserve requirement (decimal percent) at time-step k
$RD(z,k) \geq 0$	Zonal down-reserve requirement (decimal percent) for reserve zone z at time-step k
$RU(z,k) \geq 0$	Zonal up-reserve requirement (decimal percent) for reserve zone z at time-step k

Remarks on Reserve Requirements:

The system-wide down/up reserve requirements $RD(k)$ and $RU(k)$ appear on the right-hand side of the system-wide down/up reserve requirement constraints (76) and (77) for time-step k as decimal percentages of the forecasted system-wide net fixed load $\widehat{NL}^{\mathsf{f}}(k)$ at time-step k. The zonal down/up reserve requirements $RD(z,k)$ and $RU(z,k)$ appear on the right-hand side of the zonal down/up reserve requirement constraints (79) and (80) for zone z at time-step k as decimal percentages of the forecasted net fixed load $\widehat{NL}_z^{\mathsf{f}}(k)$ in zone z at time-step k.

Other User-Specified Parameters:

$F^{\max}(\ell) \geq 0$	Capacity limit (MW) for transmission line ℓ
$E(\ell)$	End bus for transmission line ℓ
$O(\ell)$	Originating bus for transmission line ℓ
$RE(\ell) \geq 0$	Reactance (ohms) on transmission line ℓ, restricted to be non-zero
$V_o > 0$	Base voltage magnitude (kV)
$\Delta k > 0$	Length of each time-step k, measured in minutes
$\Lambda^-, \Lambda^+ \geq 0$	Imbalance penalty weights ($/MWh) for power-balance slack terms

3.3 Derived Parameters (Calculated from User-Specified Parameters)

$Y_g(k)$	Minimum possible production cost ($/h) for dispatchable generator g at time-step k for which g is committed
$P_{i,g}(k) \geq 0$	Maximum possible power output (MW) for g during segment i for time-step k
$C_{i,g}(k) \geq 0$	Production cost ($/h) associated with power output $P_{i,g}(k)$
$B(\ell)$	Inverse of reactance (pu) on transmission line ℓ
$re(\ell) > 0$	Reactance (pu) on transmission line ℓ
SDT_g	Scaled minimum down-time (number of time-steps) for dispatchable generator g
SUT_g	Scaled minimum up-time (number of time-steps) for dispatchable generator g
$SNRDIS_s$	Scaled nominal charge ramp-down limit (MW) for storage unit s (ramp-down per time-step)
$SNRUIS_s$	Scaled nominal charge ramp-up limit (MW) for storage unit s (ramp-up per time-step)
$SNRDOS_s$	Scaled nominal discharge ramp-down limit (MW) for storage unit s (ramp-down per time-step)
$SNRUOS_s$	Scaled nominal discharge ramp-up limit (MW) for storage unit s (ramp-up per time-step)
$SRD_g(k)$	Scaled nominal ramp-down limit (MW) for dispatchable generator g at time-step k
$SRU_g(k)$	Scaled nominal ramp-up limit (MW) for dispatchable generator g at time-step k
$SSD_g(k)$	Scaled shut-down ramp limit (MW) for dispatchable generator g at time-step k
$SSU_g(k)$	Scaled start-up ramp limit (MW) for dispatchable generator g at time-step k
$S_o > 0$	Positive base power (in three-phase MVA)
Z_o	Base impedance (ohms)
Δt	Length of each time-step k, measured in hours (e.g., 2h, 1h/12, etc.)

Calculations for Derived Parameters:

- $Y_g(k) = a_g(k) + b_g(k)P_g^{\min}(k) + c_g(k)[P_g^{\min}(k)]^2$
- $B(\ell) = 1/re(\ell)$
- $re(\ell) = RE(\ell)/Z_o$
- $SDT_g = \texttt{round}(DT_g/\Delta t)$
- $SUT_g = \texttt{round}(UT_g/\Delta t)$
- $SNRDIS_s = \Delta t \times NRDIS_s$
- $SNRUIS_s = \Delta t \times NRUIS_s$
- $SNRDOS_s = \Delta t \times NRDOS_s$
- $SNRUOS_s = \Delta t \times NRUOS_s$

- $SRD_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta t \times NRD_g\}$
- $SRU_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta t \times NRU_g\}$
- $SSD_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta t \times NSD_g\}$
- $SSU_g(k) = \min\{P_g^{\mathsf{max}}(k), \Delta t \times NSU_g\}$
- $Z_o = (V_o)^2/S_o$
- $\Delta t = \Delta k \times$ [1h/60min]

Remarks on the "Round" Function: In the above calculations for SUT_g and SDT_g, "`round`" denotes Python's function round(), used to round a number to a certain decimal point. Round() takes in two numbers as inputs. The first number is interpreted as the number to be rounded, and the second number is interpreted as the number of decimal places to be included in this rounding. The number 5 is the cut-off for rounding up. Thus, for example, round(17.750,1) = 17.8 whereas round(17.749,1) = 17.7. If nothing is received for the second number input, round() rounds off the first number input to the nearest integer. For example, round(15.59159) = 16 whereas round(15.49321) = 15.

3.4 User-Specified Initial State Conditions

$p_g(0)$	Initial power output (MW) for dispatchable generator g
$\hat{v}_g(0)$	Initial up-time/down-time status (number of hours) for dispatchable generator g
$SOC_s(0)$	Initial state of charge (decimal percent) for storage unit s
$\bar{x}_s(0)$	Initial power output (MW) for storage unit s
$\underline{x}_s(0)$	Initial power absorption (MW) for storage unit s

Remarks on the Meaning of $\hat{v}_g(0)$: If the value of $\hat{v}_g(0)$ is positive (negative) for some dispatchable generator $g \in \mathbb{G}$, it indicates the number of consecutive hours prior to *and including* time-step 0 that g has been turned on (off). Note that $\hat{v}_g(0)$ cannot be zero, by definition.

3.5 Derived Initial State Conditions

ITF_g	Number of time-steps dispatchable generator g must be offline *initially*
ITO_g	Number of time-steps dispatchable generator g must be online *initially*
$v_g(0)$	Initial ON/OFF (1/0) status for dispatchable generator g

Calculations for Derived Initial State Conditions:

- If $\hat{v}_g(0) < 0$, $ITF_g = \min(NK, \max(0, \texttt{round}((DT_g + \hat{v}_g(0))/\Delta t)))$; otherwise, $ITF_g = 0$.
- If $\hat{v}_g(0) > 0$, $ITO_g = \min(NK, \max(0, \texttt{round}((UT_g - \hat{v}_g(0))/\Delta t)))$; otherwise, $ITO_g = 0$.
- If $\hat{v}_g(0) > 0$, $v_g(0) = 1$; otherwise, $v_g(0) = 0$.

3.6 External Forcing Terms: Load and Non-Dispatchable Generation Forecasts

Step 1: *Specify Fixed Load Forecasts* $\forall i \in \mathbb{LS}, b \in \mathbb{B}, z \in \mathbb{Z}, k \in \mathbb{K}$:

$\widehat{p}^{\mathsf{f}}_i(k) \geq 0$	Forecast (MW) by load-serving entity i for the fixed power usage of its customers at time-step k
$\widehat{L}^{\mathsf{f}}_b(k) = \sum_{i \in \mathbb{LS}(b)} \widehat{p}^{\mathsf{f}}_i(k)$	Forecast (MW) for fixed load at bus b at time-step k
$\widehat{L}^{\mathsf{f}}_z(k) = \sum_{i \in \mathbb{LS}(z)} \widehat{p}^{\mathsf{f}}_i(k)$	Forecast (MW) for fixed load in zone z at time-step k
$\widehat{L}^{\mathsf{f}}(k) = \sum_{i \in \mathbb{LS}} \widehat{p}^{\mathsf{f}}_i(k)$	Forecast (MW) for system-wide fixed load at time-step k

Step 2: *Specify Forecasts for Non-Dispatchable Generation* $\forall n \in \mathbb{NG}, b \in \mathbb{B}, z \in \mathbb{Z}, k \in \mathbb{K}$:

$\widehat{p}_n(k) \geq 0$	Forecast (MW) for the power output of non-dispatchable generator n at time-step k
$\widehat{NG}_b(k) = \sum_{n \in \mathbb{NG}(b)} \widehat{p}_n(k)$	Forecast (MW) for non-dispatchable generation at bus b at time-step k
$\widehat{NG}_z(k) = \sum_{n \in \mathbb{NG}(z)} \widehat{p}_n(k)$	Forecast (MW) for non-dispatchable generation in zone z at time-step k
$\widehat{NG}(k) = \sum_{n \in \mathbb{NG}} \widehat{p}_n(k)$	Forecast (MW) for system-wide non-dispatchable generation at time-step k

Step 3: *Specify Net Fixed Load Forecasts* $\forall i \in \mathbb{LS}, n \in \mathbb{NG}, b \in \mathbb{B}, z \in \mathbb{Z}, k \in \mathbb{K}$:

$\widehat{NL}^{\mathsf{f}}_b(k) = [\widehat{L}^{\mathsf{f}}_{\mathsf{b}}(k) - \widehat{NG}_b(k)]$	Forecast (MW) for net fixed load at bus b at time-step k
$\widehat{NL}^{\mathsf{f}}_z(k) = [\widehat{L}^{\mathsf{f}}_z(k) - \widehat{NG}_z(k)]$	Forecast (MW) for net fixed load in zone z at time-step k
$\widehat{NL}^{\mathsf{f}}(k) = [\widehat{L}^{\mathsf{f}}(k) - \widehat{NG}(k)]$	Forecast (MW) for system-wide net fixed load at time-step k

Remarks on Net Fixed Load Forecasts:

- Load is said to be *fixed* if it is not sensitive to price.
- *Net (fixed) load* for a designated region (e.g., bus, zone, entire system) is defined to be (fixed) load for this region minus non-dispatchable generation for this region. Thus, non-dispatchable generation is treated as negative load.
- A SCUC/SCED optimization is a forward-market planning tool for ensuring suitable resource availability for the balancing of net load in subsequent real-time operations. If a SCUC/SCED optimization is conducted several hours in advance of real-time operations, it would generally not be credible to assume real-time fixed loads and non-dispatchable generation are known with certainty at the time of this optimization.
- In US ISO/RTO-managed day-ahead markets, each LSE's demand bid for next-day operations is permitted to include both a 24-hour fixed load profile and 24 hourly price sensitive demand schedules. The 24-hour fixed load profile is generally interpreted to be the LSE's forecast for the next-day power usage of its customers.
- Forecasts for non-dispatchable generation are typically formulated by the ISO/RTO itself.
- The ISO/RTO is required to use LSE demand bids to determine forecasted next-day loads.
- The ISO/RTO includes reserve requirements in its SCUC/SCED optimization constraints to protect against the possibility of net load forecast errors.

- The SCUC/SCED optimization formulated by Carrión and Arroyo [2] for a day-ahead wholesale power market does not include non-dispatchable generation and does not consider transmission congestion. Consequently, the only external forcing term for each hour k for next-day operations is "demand" $D(k)$, where $D(k)$ denotes forecasted system-wide net fixed load for hour k.

3.7 ISO Decision Variables and Derived Solution Variables

Integer-Valued ISO Decision Variables:

$v_g(k)$	1 if dispatchable generator g is committed for time-step k; 0 otherwise
$hs_g(k)$	1 if dispatchable generator g is in a hot-start state in time-step k; 0 otherwise
$\bar{u}_s(k)$	1 if storage unit s is committed for power output (discharge) in time-step k; 0 otherwise;
$\underline{u}_s(k)$	1 if storage unit s is committed for power absorption (charging) in time-step k; 0 otherwise.

Continuously-Valued ISO Decision Variables:

$\bar{x}_s(k)$	Power output (MW) for storage unit s at time-step k
$\underline{x}_s(k)$	Power absorption (MW) for storage unit s at time-step k
$\delta_{i,g}(k)$	Variable used to determine the power output $p_g(k)$ of generator g at time-step k in the total production cost approximation method for g
$\delta_{i,j}(k)$	Variable used to determine the price-sensitive demand $p^{\mathsf{s}}_j(k)$ of LSE j at time-step k in the total benefit approximation method for j
$\theta_b(k)$	Voltage angle (radians) for bus $b \in \mathbb{B}/\{1\}$ at time-step k

Solution Variables Derived from ISO Decision Variables and System Constraints:

$c^{\mathsf{p}}_g(k)$	Total production cost (\$) for dispatchable generator g for time-step k
$c^{\mathsf{u}}_g(k)$	Start-up cost (\$) for dispatchable generator g for time-step k
$c^{\mathsf{d}}_g(k)$	Shut-down cost (\$) for dispatchable generator g for time-step k
$p_g(k)$	Power output (MW) for dispatchable generator g at time-step k
$p^{\mathsf{s}}_j(k)$	Price-sensitive demand (MW) for the customers of LSE j at time-step k
$w_\ell(k)$	Power flow (MW) on transmission line ℓ at time-step k
$z_s(k)$	State of charge (decimal percent) for storage unit s at time-step k
$\beta^-_b(k), \beta^+_b(k)$	Power-imbalance slack terms (MW) for bus b at time-step k
$\beta_b(k)$	Power-imbalance slack variable (MW) for bus b at time-step k
$\theta_1(k)$	Voltage angle (radians) for angle reference bus 1 at time-step k

Remarks on the Slack Variable Terms: For any real variable x, there exist unique non-negative values x^+ and x^- satisfying $x^+ - x^- = x$ and $x^+ + x^- = |x|$. As will be seen below in (9), the objective function for the Basic ECA Model decomposes the slack variable $\beta_b(k)$ into $(\beta^-_b(k), \beta^+_b(k))$ in order to permit the imposition of different penalties on positive and negative deviations from power balance at bus b in time-step k.

4 Approximation of Generator Total Production Costs

As will be seen in Section 5, the total production cost (\$) incurred by each dispatchable generator g during each time-step k appears in the objective function (9) for the Basic ECA Model SCUC/SCED optimization in approximate form as $c^{\mathsf{p}}_g(k)$ (\$). This approximation relies on the following four assumptions:

- Each time-step $k \in \mathbb{K}$ has an equal user-set length Δk, measured in minutes. The length of k measured in hours is then given by $\Delta t = \Delta k \times$ [1h/60min].
- The set of feasible power levels that g can maintain during time-step k is given by an interval $\mathbb{P}_g(k) = [P_g^{\min}(k), P_g^{\max}(k)]$ with $0 \leq P_g^{\min}(k)$.
- The ISO dispatches generation by means of dispatch set points. Specifically, the ISO's dispatch instruction conveyed to a committed generator g for a time-step k consists of a single dispatch set point $p_g(k) \in \mathbb{P}_g(k)$ signaled to g at the start-time k^{s} for time-step k, which indicates the power level that g is to maintain during time-step k.
- The production cost (\$/h) of each dispatchable generator g for each time-step k can be expressed as a non-decreasing convex function of p taking the following quadratic form:

$$\mathrm{C}_{g,k}(p) \;=\; a_g(k) + b_g(k)p + c_g(k)p^2 \;, \quad \forall p \in \mathbb{P}_g(k) \;, \tag{1}$$

  where the cost coefficient $a_g(k)$ has units \$/h, the cost coefficient $b_g(k) > 0$ has units \$/MWh, and the cost coefficient $c_g(k) > 0$ has units \$/[MW]^{2}h.

Given these four assumptions, the Pyomo Model constructs a piecewise-linear approximation for g's production cost function (1) for any time-step k by connecting finitely many power-cost points $\{(P_i, C_i) \mid i = 0, \ldots, NS_g(k)\}$ in the power-cost plane satisfying

$$P_g^{\min}(k) = P_0 \;<\; P_1 \;<\; P_2 \;\ldots\; < P_{NS_g(k)} \;=\; P_g^{\max}(k) \tag{2}$$

and $C_i = C_{g,k}(P_i)$ for each i. See, for example, Fig. 1. As will be clarified below, the user can permit the Pyomo Model to automatically set these power-cost points by internal calculations as a function of the user-designated number $NS_g(k)$ of line segments to be used in the approximation.

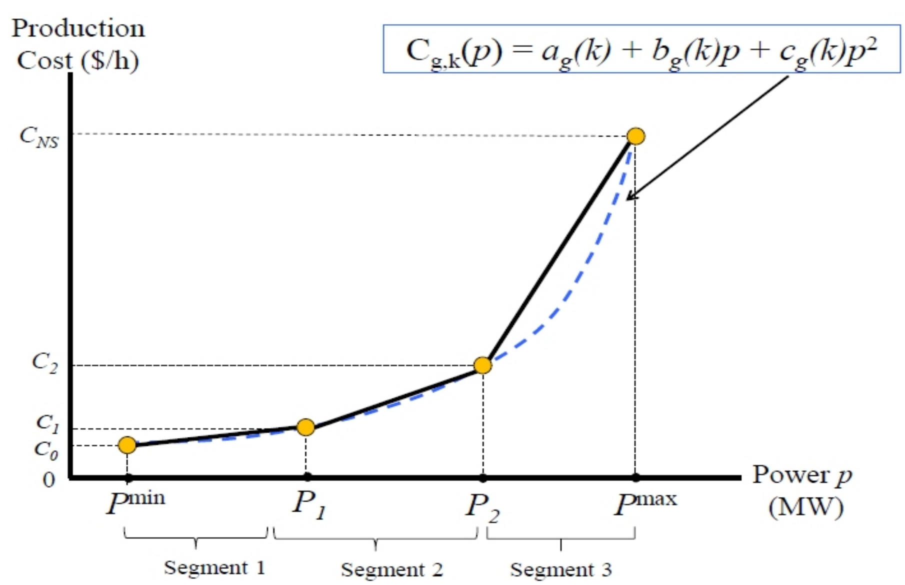

Figure 1: Illustration of the piecewise-linear approximation for the per-hour production cost function (1) of a dispatchable generator g for any time-step k. The depicted approximation uses $NS = 3$ line segments.

Suppose for the moment that these power-cost points have been set. The approximate total production cost $c_g^{\mathsf{p}}(k)$ ($) incurred by g during time-step k is then determined by the system of equations (3)-(7), below, as a function of the ISO's optimal selection of the continuously-valued decision variables $\{\delta_{i,g}(k) \mid i = 1, \dots NS_g(k)\}$:[3]

Total Production Cost ($) Approximation for a Dispatchable Generator g at a Time-Step k:

$$c_g^{\mathsf{p}}(k) = C_0 \Delta t v_g(k) + \sum_{i=1}^{NS_g(k)} \big(MC_i \cdot \delta_{i,g}(k)\big)\Delta t\,; \tag{3}$$

$$p_g(k) = P_0 v_g(k) + \sum_{i=1}^{NS_g(k)} \delta_{i,g}(k)\,; \tag{4}$$

$$\delta_{i,g}(k) \leq P_i - P_{i-1}\,,\ \forall i = 1, \cdots, NS_g(k)\,; \tag{5}$$

$$\delta_{i,g}(k) \geq 0,\ \forall i = 1, \cdots, NS_g(k)\,, \tag{6}$$

where:

$$MC_i \ = \ \frac{C_i - C_{i-1}}{P_i - P_{i-1}},\ \forall i = 1, \dots, NS_g(k)\,. \tag{7}$$

The unit commitment indicator $v_g(k)$ in (3) indicates whether (1) or not (0) the ISO commits generator g for time-step k, and Δt denotes the length of time-step k measured in hours. If the ISO commits g for time-step k, the power level $p_g(k)$ in (4) denotes the ISO's choice of a dispatch set point for g for time-step k.[4]

For each segment $i \in \{1, ..., NS_g(k)\}$ the marginal cost ($/MWh) of generator g is approximated by MC_i in (7). The variables $\delta_{i,g}(k)$ (MW) appearing in constraints (3)-(6) are incorporated into the SCUC/SCED optimization as continuously-valued ISO decision variables. For example, suppose $v_g(k) = 1$ and there exists a segment $n \in \{1, \dots, NS_g(k)\}$ such that $\delta_{i,g}(k)$ takes on its maximum possible value for $i = 1, \dots, n$ and $\delta_{i,g}(k) = 0$ for $i = n+1, \dots, NS_g(k)$. Then $p_g(k) = P_n$ and $c_g^P(k) = C_n \Delta t = C_{g,k}(P_n)$. On the other hand, suppose $v_g(k) = 1$ but $\delta_{i,g}(k) = 0$ for all $i = 1, \dots, NS_g(k)$. Then $p_g(k) = P_0$ and $c_g^P(k) = C_0 \Delta t \equiv \mathrm{C}_{g,k}(P_0)\Delta t \equiv \mathrm{C}_{g,k}(P_g^{\mathsf{min}}(k))\Delta t$.

Finally, the Pyomo Model automatic construction of the power-benefit points $\{(P_0, C_0), \dots, (P_{NS_g(k)}, C_{NS_g(k)})\}$ satisfying restrictions (2) with $C_i = \mathrm{C}_{g,k}(P_i)$ for each i is given below. This method partitions generator g's power generation domain $[P_g^{\mathsf{min}}(k), P_g^{\mathsf{max}}(k)]$ into power segments having equal lengths. The resulting power-cost points are treated as exogenous inputs to the ISO's SCUC/SCED optimation.

Automated Power-Cost Point Setting Method for a Dispatchable Generator g at a Time-Step k:

This automated method requires the following inputs: (i) admissible values for the production cost coefficients $(a_g(k), b_g(k), c_g(k))$ in (1); (ii) the minimum and maximum sustainable power levels $P_g^{\mathsf{min}}(k)$ and $P_g^{\mathsf{max}}(k)$; and (iii) a positive integer value $NS_g(k)$ for the total number of line segments i to be used in the approximation. The Pyomo Model then uses these inputs to compute[5] the power-cost points $\{(P_i, C_i) \mid i = 0, \dots NS_g k\}$ for use in the Total Production Cost Approximation Method, as follows:

(a) The initial power-cost points are set to $P_0 = P_g^{\mathsf{min}}(k)$ and $C_0 = \mathrm{C}_{g,k}(P_0)$;

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[3] The following Total Production Cost Approximation Method is adapted from [2, Sec. II.A].

[4] The SCUC/SCED constraints presented in Section 5 imply that $c_g^{\mathsf{p}}(k)$ is zero for any time-step k for which g is not committed. That is, if $v_g(k) = 0$, then $p_g(k) = 0$; hence, constraints (3), (4), and (6) together with the assumption $P_0 = P_g^{\mathsf{min}}(k) \geq 0$ imply that $c_g^{\mathsf{p}}(k) = 0$ as well.

[5] This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

(b) The power-width of each segment $i = 1, \ldots, NS_g(k)$ is set equal to

$$w_g(k) \;\equiv\; \frac{[P_g^{\max}(k) - P_g^{\min}(k)]}{NS_g(k)} \; ; \tag{8}$$

(c) For each segment $i = 1, \ldots NS_g(k)$, the power point P_i is set equal to $P_0 + iw_g(k)$;

(d) For each segment $i = 1, \ldots, NS_g(k)$, the cost point C_i is set equal to $\mathrm{C}_{g,k}(P_i)$.

Figure 2 illustrates the energy requirements for a dispatchable generator g that maintains a constant power output $p_g(k) = P_2$ during a time-step k. In this figure, g^{sync} denotes the non-negative power generation level at which the generator attains a synchronized state, ready to inject power into the grid but not yet injecting any power into the grid. The generator's total power generation level sustained during time-step k is thus given by $g(t) = P_2 + g^{\mathsf{sync}}$ for all $t \in k$.

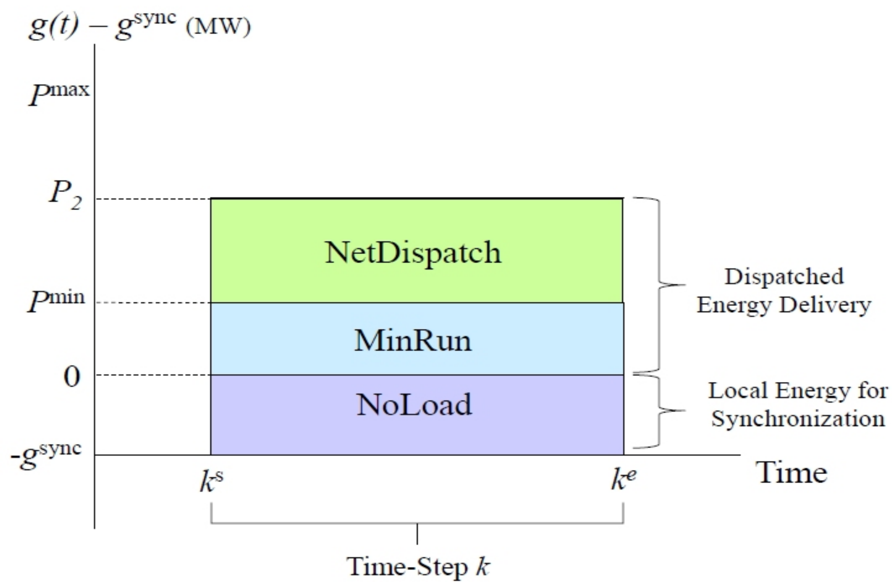

Figure 2: Illustrative depiction of the energy requirements for a dispatchable generator g that maintains a constant power output P_2 during a time-step k.

The amount of energy that generator g expends locally (behind-the-meter) during time-step k in order to maintain itself in a synchronized state is denoted by NoLoad in Fig. 2. The amount of energy that g injects into the grid during time-step k in order to support its minimum sustainable power output P^{min} during time-step k is denoted by MinRun. The remaining amount of energy that g injects into the grid during time-step k is denoted by NetDispatch.

Generator g's *no-load cost* (\$) for time-step k is the cost g incurs in order to maintain itself in a synchronized state during k. Generator g's *lost opportunity cost* (\$) for time-step k is the net earnings that g could have obtained from the deployment of its generation capacity in a next-best alternative use during k. Finally, g's *dispatch cost* (\$) for time-step k is the cost that g incurs for the dispatched delivery of power during k.

For the case depicted in Fig. 2, generator g's no-load cost for time-step k includes the energy cost incurred for NoLoad. The production cost function (1) can account for this no-load cost for time-step k, along with any lost opportunity cost for time-step k, by appropriate specification of $\mathrm{C}_{g,k}(0)\Delta t = a_g(k)\Delta t$. Generator g's dispatch cost for time-step k includes the energy cost incurred for MinRun and NetDispatch. These costs can be accounted for by appropriate specification of $[\mathrm{C}_{g,k}(P_2)$ - $\mathrm{C}_{g,k}(0)]\Delta t$.

5 SCUC/SCED Optimization Formulation for the Basic ECA Model

5.1 Overview

The Basic ECA Model provides a complete analytical MILP modeling of a SCUC/SCED optimization for a future operating period T. The objective of the ISO is to select admissible decision variables to minimize the expected total cost ($) of achieving a balancing of net fixed load during period T, subject to system constraints.

The future operating period T is partitioned into NK consecutive time-steps k of equal length Δt. Total cost is the summation of production cost $c^{\mathsf{p}}_g(k)$, start-up cost $c^{\mathsf{u}}_g(k)$, shut-down cost $c^{\mathsf{d}}_g(k)$, and imbalance cost summed over all dispatchable generators $g \in \mathbb{G}$ and all time-steps $k \in \mathbb{K} = \{1, \ldots, NK\}$. These costs are "expected" costs in the sense that they are conditioned on forecasts for next-day net fixed loads.[6]

All notation appearing in this optimization formulation is carefully explained in preceding sections of this report.

5.2 Complete Analytical SCUC/SCED Optimization Formulation

ISO Objective:

Select decision variables to minimize forecasted total cost, subject to system constraints, where forecasted total cost is given by:

$$\widehat{\mathrm{C}}(\mathrm{T}) \;=\; \sum_{k\in\mathbb{K}}\sum_{g\in\mathbb{G}}\Big[c^{\mathsf{p}}_g(k) \;+\; c^{\mathsf{u}}_g(k) \;+\; c^{\mathsf{d}}_g(k)\Big] \;+\; \sum_{b\in\mathbb{B}}\sum_{k\in\mathbb{K}}\big[\Lambda^-\beta^-_b(k) \;+\; \Lambda^+\beta^+_b(k)\big]\Delta t \tag{9}$$

ISO Decision Variables:

ISO Integer Decision Variables: $\forall g \in \mathbb{G}, s \in \mathbb{S},$ and $k \in \mathbb{K}$,

$$v_g(k) \in \{0,1\} \tag{10}$$
$$hs_g(k) \in \{0,1\} \tag{11}$$
$$\bar{u}_s(k) \in \{0,1\} \tag{12}$$
$$\underline{u}_s(k) \in \{0,1\} \tag{13}$$

ISO Continuously-Valued Decision Variables: $\forall i \in \mathbb{NS}_g(k), k \in \mathbb{K}_g, g \in \mathbb{G},$ and $s \in \mathbb{S}$,

$$\bar{x}_s(k) \tag{14}$$
$$\underline{x}_s(k) \tag{15}$$
$$\delta_{i,g}(k) \tag{16}$$
$$\theta_b(k) \tag{17}$$
$$\tag{18}$$

[6]That is, certainty equivalence is used to approximate expectations.

Other Solution Variables:

Variables Determined by ISO Decisions and System Constraints: $\forall b \in \mathbb{B}, \ell \in \mathbb{L}, g \in \mathbb{G}, s \in \mathbb{S},$ and $k \in \mathbb{K}$,

$$c_g^{\mathsf{p}}(k),\ c_g^{\mathsf{u}}(k),\ c_g^{\mathsf{d}}(k) \tag{19}$$
$$p_g(k),\ \overline{p}_g(k),\ \underline{p}_g(k) \tag{20}$$
$$w_\ell(k) \tag{21}$$
$$z_s(k) \tag{22}$$
$$\beta_b(k),\ \beta_b^-(k),\ \beta_b^+(k) \tag{23}$$
$$\theta_1(k) \tag{24}$$
$$\tag{25}$$

System Constraints:

Transmission line power flow constraints: For all $\ell \in \mathbb{L}$ and $k \in \mathbb{K}$,

$$w_\ell(k) = S_0 B(\ell)\left[\theta_{O(\ell)}(k) - \theta_{E(\ell)}(k)\right]\ ; \tag{26}$$
$$-F^{\mathsf{max}}(\ell) \le w_\ell(k) \le F^{\mathsf{max}}(\ell)\,. \tag{27}$$

Power balance constraints (with slack variables): For all $b \in \mathbb{B}$ and $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}(b)} p_g(k) + \Big[\sum_{\ell \in \mathbb{L}_{\mathsf{E}}(b)} w_\ell(k) \ - \sum_{\ell \in \mathbb{L}_{\mathsf{O}}(b)} w_\ell(k)\Big] + \Big[\sum_{s \in \mathbb{S}(b)} \big(\overline{x}_s(k) - \underline{x}_s(k)\big)\Big] \ = \ \widehat{NL}_b^{\mathsf{f}}(k) + \beta_b(k)\,. \tag{28}$$

Slack variable constraints: For all $b \in \mathbb{B}$ and $k \in \mathbb{K}$,

$$\beta_b^-(k) = \max\{0, -\beta_b(k)\}\,; \tag{29}$$
$$\beta_b^+(k) = \max\{0,\ \beta_b(k)\}\,. \tag{30}$$

Dispatchable generator capacity constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\underline{p}_g(k) \ \le\ p_g(k) \ \le\ \overline{p}_g(k)\,; \tag{31}$$
$$\overline{p}_g(k) \ \le\ P_g^{\mathsf{max}}(k) v_g(k)\,; \tag{32}$$
$$\underline{p}_g(k) \ \ge\ P_g^{\mathsf{min}}(k) v_g(k)\,. \tag{33}$$

Dispatchable generator ramping constraints for start-up, normal, and shut-down conditions:

$$\bar{p}_g(k) \le\ p_g(k-1) \ + \ SRU_g(k) v_g(k-1) \ + \ SSU_g(k)[v_g(k) - v_g(k-1)] \ + \ P_g^{\mathsf{max}}(k)[1 - v_g(k)]\,, \quad \forall g \in \mathbb{G},\ \forall k \in \mathbb{K}\,; \tag{34}$$

$$\bar{p}_g(k) \le\ P_g^{\mathsf{max}}(k) v_g(k+1) \ + \ SSD_g(k)[v_g(k) - v_g(k+1)]\,, \quad \forall g \in \mathbb{G},\ \forall k = 1 \cdots NK-1\,; \tag{35}$$

$$p_g(k-1) - \underline{p}_g(k) \leq \; SRD_g(k)v_g(k) \; + \; SSD_g(k)[v_g(k-1) - v_g(k)] \; + \; P_g^{\mathsf{max}}(k)[1 - v_g(k-1)]\,, \qquad \forall g \in \mathbb{G}, \; \forall k \in \mathbb{K}\,. \tag{36}$$

Dispatchable generator minimum up-time constraints:

$$\sum_{k=1}^{ITO_g} [1 - v_g(k)] = 0 \;\text{ for all } j \in \mathbb{G} \text{ with } ITO_g \geq 1\,; \tag{37}$$

$$\sum_{n=k}^{k+SUT_g-1} v_g(n) \geq SUT_g[v_g(k) - v_g(k-1)], \quad \forall g \in \mathbb{G}, \; \forall k = ITO_g + 1, \cdots, NK - SUT_g + 1\,; \tag{38}$$

$$\sum_{n=k}^{NK} \{v_g(n) - [v_g(k) - v_g(k-1)]\} \geq 0, \quad \forall g \in \mathbb{G}, \; \forall k = NK - SUT_g + 2, \cdots, NK\,. \tag{39}$$

Remarks on the minimum up-time constraints: To derive these constraints, consider the following. If $ITO_g \geq 1$ for generator g, then by definition of ITO_g it must hold that $v_g(k) = 1$ for all time-steps k satisfying $1 \leq k \leq ITO_g$. For $ITO_g + 1 \leq k$, suppose a start-up event occurs for generator g in time-step k; i.e., suppose $v_g(k-1) = 0$ and $v_g(k) = 1$, implying generator g is turned off in period $k-1$ and on in time-step k. Then, by definition of SUT_g, generator g must remain on for $SUT_g - 1$ additional periods, or until the end of the final modeled period NK if $NK \leq k + SUT_g - 1$. The above minimum up-time constraints express these requirements in concise form.

Dispatchable generator minimum down-time constraints:

$$\sum_{k=1}^{ITF_g} v_g(k) = 0 \;\; \forall g \in \mathbb{G} \text{ with } ITF_g \geq 1\,; \tag{40}$$

$$\sum_{n=k}^{k+SDT_g-1} [1 - v_g(n)] \geq SDT_g[v_g(k-1) - v_g(k)], \quad \forall g \in \mathbb{G}, \; \forall k = ITF_g + 1, \cdots, NK - SDT_g + 1\,; \tag{41}$$

$$\sum_{n=k}^{NK} \big[1 - v_g(n) - [v_g(k-1) - v_g(k)]\big] \geq 0, \quad \forall g \in \mathbb{G}, \; \forall k = NK - SDT_g + 2, \cdots, NK\,. \tag{42}$$

Remarks on the minimum down-time constraints: The derivation of the above minimum down-time constraints is similar to the derivation of the minimum up-time constraints, except that one considers shut-down events with $v_g(k-1) = 1$ and $v_g(k) = 0$ rather than start-up events.

Dispatchable generator hot-start constraints:

$$hs_g(k) = 1, \qquad \forall g \in \mathbb{G}, 1 \leq k \leq CSH_g : k - CSH_g \leq \hat{v}_g(0)\,; \tag{43}$$

$$hs_g(k) \leq \sum_{n=1}^{k-1} v_g(n), \qquad \forall g \in \mathbb{G}, 1 \leq k \leq CSH_g : k - CSH_g > \hat{v}_g(0)\,; \tag{44}$$

$$hs_g(k) \leq \sum_{n=k-CSH_g}^{k-1} v_g(n), \qquad \forall g \in \mathbb{G}, k = CSH_g + 1, \ldots, NK\,. \tag{45}$$

Remarks on Generator Hot-Start Constraints:

As previously explained, a positive (negative) value for $\hat{v}_g(0)$ for some dispatchable generator

$g \in \mathbb{G}$ indicates the number of consecutive hours prior to *and including* time-step 0 that g has been turned ON (OFF). Note that $\hat{v}_g(0)$ cannot be zero, by definition. Also, $v_g(0)$ denotes the initial ON/OFF (1/0) status for generator g. If $\hat{v}_g(0) > 0$, then $v_g(0) = 1$; otherwise, $v_g(0) = 0$.

Constraint (43) ensures that, if k does not exceed CSH_g, then generator g is in a hot-start state ($hs_g(k) = 1$) as long as g was ON either during time-step 0 or during a time-step prior to time-step 0 that is sufficiently close to time-step 0. Constraint (44) ensures that, if k does not exceed CSH_g, and g was *not* ON during time-step 0 or during a time-step prior to time-step 0 that is sufficiently close to time-step 0, then g is in a cold-start state ($hs_g(k) = 0$) unless g was ON during some time-step between 1 and $k-1$. Finally, constraint (45) ensures that, for k larger than CSH_g, g will be in a cold-start state ($hs_g(k) = 0$) if g was not committed during any of the CSH_g time-steps immediately preceding time-step k. For reasons explained in the remarks following the next set of constraints (i.e., the generator start-up cost constraints), if generator g has a positive cold-start cost CSC_g, the cost-minimizing ISO will set $hs_g(k) = 1$ unless the generator hot-start constraints (43) through (45) force the ISO to set $hs_g(k) = 0$.

Dispatchable generator start-up cost constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\begin{aligned} c_g^{\mathsf{u}}(k) &= \max\{0, U(k)\}\,; \\ U(k) &= CSC_g - [CSC_g - HSC_g]hs_g(k) - CSC_g\big[1 - [v_g(k) - v_g(k-1)]\big]\,. \end{aligned} \tag{46}$$

Remarks on Dispatchable Generator Start-Up Cost: Definitions of a cold-start state versus a hot-start state for any dispatchable generator g are provided in Section 3.2. Also recall from this previous section that the user-specified costs CSC_g and HSC_g are required to satisfy $\mathrm{CSC}_g \geq \mathrm{HSC}_g$. Consequently, (46) implies: (i) $c_g^{\mathsf{u}}(k) = \mathrm{CSC}_g$ if g starts up at k (i.e., $v_g(k-1) = 0$ and $v_g(k) = 1$) while in a cold-start state ($hs_g(k) = 0$); and (ii) $c_g^{\mathsf{u}}(k) = \mathrm{HSC}_g$ if g starts up at k while in a hot-start state ($hs_g(k) = 1$). Otherwise, $c_g^{\mathsf{u}}(k) = 0$.

Thus, assuming $\mathrm{CSC}_g > 0$, in attempting to minimize total costs the ISO will strive to avoid starting up generator g in a cold-start state, all else equal. In particular, unless ruled out by the hot-start constraints (43) - (45), the cost-minimizing ISO will set $hs_g(k) = 1$ if it commits generator g for time-step k.

Dispatchable generator shut-down cost constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$\begin{aligned} c_g^{\mathsf{d}}(k) &= \max\{0, D(k)\}\,; \\ D(k) &= SDC_g[v_g(k-1) - v_g(k)]\,. \end{aligned} \tag{47}$$

Storage unit limit constraints: For all $s \in \mathbb{S}$ and $k \in \mathbb{K}$,

$$\underline{u}_s(k) PIS_s^{\mathsf{min}} \leq \underline{x}_s(k) \leq \underline{u}_s(k) PIS_s^{\mathsf{max}}\,; \tag{48}$$

$$\bar{u}_s(k) POS_s^{\mathsf{min}} \leq \bar{x}_s(k) \leq \bar{u}_s(k) POS_s^{\mathsf{max}}\,. \tag{49}$$

Storage unit charge/discharge constraint (cannot charge and discharge at same time):

$$\underline{u}_s(k) + \bar{u}_s(k) \leq 1,\ \forall s \in \mathbb{S}, k \in \mathbb{K}\,. \tag{50}$$

Storage unit ramping constraints: For all $s \in \mathbb{S}$ and $k \in \mathbb{K}$,

$$\bar{x}_s(k) \leq \bar{x}_s(k-1) + SNRUOS_s\,; \tag{51}$$

$$\bar{x}_s(k) \geq \bar{x}_s(k-1) - SNRDOS_s\,; \tag{52}$$

$$\underline{x}_s(k) \leq \underline{x}_s(k-1) + SNRUIS_s\,; \tag{53}$$

$$\underline{x}_s(k) \geq \underline{x}_s(k-1) - SNRDIS_s. \tag{54}$$

Storage unit energy conservation constraints: For all $s \in \mathbb{S}$ and $k \in \mathbb{K}$,

$$z_s(k) = z_s(k-1) + \frac{[-\bar{x}_s(k) + \eta_s \underline{x}_s(k)] \cdot \Delta t}{ES_s^{\mathsf{max}}}\,; \tag{55}$$

$$z_s(0) = SOC_s(0)\,. \tag{56}$$

Storage unit end-point constraints: For all $s \in \mathbb{S}$,

$$z_s(NK) = EPSOC_s\,. \tag{57}$$

System-wide down/up reserve requirement constraints: For all $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}} \underline{p}_g(k) \;\leq\; [1 - RD(k)] \cdot \widehat{NL}^{\mathsf{f}}(k)\,; \tag{58}$$

$$\sum_{g \in \mathbb{G}} \overline{p}_g(k) \;\geq\; [1 + RU(k)] \cdot \widehat{NL}^{\mathsf{f}}(k)\,. \tag{59}$$

Zonal down/up reserve requirement constraints: For all $z \in \mathbb{Z}$ and $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}(z)} \underline{p}_g(k) \;\leq\; [1 - RD(z,k)] \cdot \widehat{NL}_z^{\mathsf{f}}(k)\,; \tag{60}$$

$$\sum_{g \in \mathbb{G}(z)} \overline{p}_g(k) \;\geq\; [1 + RU(z,k)] \cdot \widehat{NL}_z^{\mathsf{f}}(k)\,. \tag{61}$$

Voltage angle specifications: For all $b \in \mathbb{B}/\{1\}$ and $k \in \mathbb{K}$,

$$\theta_b(k) \in [-\pi, \pi] \tag{62}$$

$$\theta_1(k) = 0\,. \tag{63}$$

Total Production Cost Approximation Constraints: For all $g \in \mathbb{G}$ and $k \in \mathbb{K}$,

$$c_g^{\mathsf{p}}(k) = C_0 \Delta t v_g(k) + \sum_{i=1}^{NS_g(k)} \big(MC_i \cdot \delta_{i,g}(k)\big)\Delta t\,; \tag{64}$$

$$p_g(k) = P_0(k) v_g(k) + \sum_{i=1}^{NS_g(k)} \delta_{i,g}(k)\,; \tag{65}$$

$$\delta_{i,g}(k) \leq P_{i,g}(k) - P_{i-1,g}(k)\,,\ \forall i = 1, \cdots, NS_g(k) - 1\,; \tag{66}$$

$$\delta_{NS_g(k),g}(k) \leq P_g^{\mathsf{max}}(k) - P_{NS_g(k)-1,g}(k)\,; \tag{67}$$

$$\delta_{i,g}(k) \geq 0,\ \forall i = 1 \cdots NS_g(k)\,, \tag{68}$$

where:

$$MC_i \ = \ \frac{C_i - C_{i-1}}{P_i - P_{i-1}},\ i = 1, \ldots, NS_g(k)\,. \tag{69}$$

6 Incorporation of Price-Sensitive Demand Bids

6.1 Overview

Demand bids submitted by LSEs into current U.S. ISO/RTO-managed Day-Ahead Markets (DAMs) are demands for the delivery of power for end-use customers, with or without accompanying price information indicating willingness to pay. If a demand bid submitted by an LSE into a SCUC/SCED optimization for a day-ahead market is cleared, the LSE must compensate the ISO for the resulting delivery of power to its customers. These LSE payments are determined in part through locational marginal price assessments (which take into account any LSE submitted price information) and in part through subsequent ISO/RTO allocations of its net costs across LSEs on the basis of their load shares.[7]

Currently in these DAMs, most LSE demand bids take a fixed form. A *fixed demand bid* submitted into a day-ahead market held on day D is a load profile designating a forecasted demand $\widehat{p}^{\mathsf{f}}(k)$ (MW) for power usage during each hour k of day D+1, with no accompanying price information indicating willingness to pay.

In economic terms, a forecasted demand $\widehat{p}^{\mathsf{f}}(k)$ for power usage effectively represents a vertical demand curve in a power-price plane, as if customers had an infinite willingness to pay for these amounts. Although the effective[8] maximum willingness to pay for this power usage is necessarily finite, it cannot be determined from this fixed-bid form. Consequently, the presence of fixed demand bids hinders an ISO/RTO's ability to ensure that SCUC/SCED optimization solutions result in an efficient allocation of resources.

These concerns arise for the SCUC/SCED optimization formulation presented for the Basic ECA Model in Section 5. All LSE demand bids are assumed to take a fixed form. Consequently, all LSE demand bids are entered into power balance constraints as must-meet load obligations. Since benefits cannot be measured, the usual stated SCUC/SCED optimization objective, maximization of expected net benefit, is replaced by the goal of minimizing expected cost.

This section discusses how the SCUC/SCED optimization formulation for the Basic ECA Model can be generalized to permit LSEs to submit demand bids with accompanying price (valuation) information on behalf

[7] To preserve its independent status, an ISO/RTO cannot have a financial stake in the market operations it manages. Thus, an ISO/RTO must pass through to market participants any profits or losses resulting from these market operations.

[8] *Effective* willingness to pay is willingness to pay back-stopped by actual purchasing power.

of a collection of end-use customers. Three cases are considered: demand bids with Time-of-Use (TOU) pricing; demand bids in the form of price-quantity demand schedules; and demand bids in the form of general benefit functions.[9]

6.2 Incorporation of Benefits into the Basic ECA Model

The Basic ECA Model assumes that an ISO-managed SCUC/SCED optimization is conducted in order to secure net-load balancing resources for a future operating period T. The operating period T is partitioned into time-steps $k = 1, \ldots, NK$, where each time-step k has equal length Δt measured in hours (e.g., 2h, 1h/12, ...).

During each time-step k, each LSE j services the power usage of end-use customers at its bus location b_j. This servicing consists of the submission into the SCUC/SCED optimization of a fixed demand bid expressed as a forecasted power-usage amount $\widehat{p}^{\mathsf{f}}_j(k)$ (MW) for its end-use customers at bus b_j, with no accompanying price information.

Now suppose that each LSE j also submits into this SCUC/SCED optimization a price-sensitive demand bid for each time-step k. This price-sensitive reserve bid expresses a possible power-usage amount $p^{\mathsf{s}}_j(k)$ (MW) at bus b_j together with some type of price (valuation) metric for measuring the benefit of this power usage to LSE j's end-use customers.

Let the benefit assigned by LSE_j to each possible power-usage sequence

$$\mathbf{p}^{\mathsf{s}}_j \;=\; \{p^{\mathsf{s}}_j(k) \mid k \in \mathbb{K}\} \tag{70}$$

be denoted by $\mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j)$ (\$). The total benefit (\$) from these price-sensitive demand bids is then given by

$$\mathrm{B(T)} \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j)\,. \tag{71}$$

In order to incorporate these price-sensitive demand bids and benefit calculations into the ISO-managed SCUC/SCED optimization presented in Section 5 for the Basic ECA Model, the following modifications must be made.

Required ISO Modifications:

First, the ISO objective function (9) for the Basic ECA Model, expressed solely in terms of forecasted cost $\widehat{\mathrm{C}}(\mathrm{T})$ for operating period T, needs to be modified to represent forecasted total net benefit for T. This forecasted total net benefit is expressed as

$$\widehat{\mathrm{NB}}(\mathrm{T}) \;=\; \mathrm{B(T)} - \widehat{\mathrm{C}}(\mathrm{T}) \tag{72}$$

Second, the objective of the ISO needs to be changed from the minimization of period-T forecasted cost to the maximization of period-T forecasted total net benefit. Third, the price-sensitive power-usage amounts $\{p^{\mathsf{s}}_j(k) \mid j \in \mathbb{LS}, k \in \mathbb{K}\}$ need to be included among the ISO's continuously-valued decision variables, subject to domain constraints of the form

$$p^{\mathsf{s}}_j(k) \in \mathbb{P}_j(k)\,,\ \forall j \in \mathbb{LS}, k \in \mathbb{K}\,, \tag{73}$$

where each domain $\mathbb{P}_j(k)$ includes 0, i.e., the zero-power option.[10]

[9] For the latter two cases, participant end-use customers would need to have installed real-time telemetry permitting them to respond to real-time price or power-usage set points communicated to them by the LSE or some other designated scheduling entity.

[10] If the domains $\mathbb{P}_j(k)$ are not required to include a zero-power option, the ISO would have no way of refusing to service price-sensitive demand bids at LSE-set prices whose servicing results in a lowering of total net benefit. To avoid this forced servicing, it would be necessary to modify further the form of the SCUC/SCED optimization to include an ISO binary 0/1 unit commitment decision with regard to each submitted price-sensitive demand bid, thus enlarging the number of ISO integer decision variables.

Required System Constraint Modifications:

Fourth, the price-sensitive power-usage amounts $p^{\mathsf{s}}_j(k)$ need to be appropriately entered into the power balance constraints (28). The resulting generalized constraints take the following form:

Generalized power balance constraints (with slack variables): For all $b \in \mathbb{B}$ and $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}(b)} p_g(k) + \Big[\sum_{\ell \in \mathbb{L}_{\mathsf{E}}(b)} w_\ell(k) - \sum_{\ell \in \mathbb{L}_{\mathsf{O}}(b)} w_\ell(k) \Big] + \Big[\sum_{s \in \mathbb{S}(b)} \big(\overline{x}_s(k) - \underline{x}_s(k) \big) \Big] \;=\; \widehat{NL}_b(k) + \beta_b(k)\,, \tag{74}$$

where

$$\widehat{NL}_b(k) \;=\; \widehat{NL}^{\mathsf{f}}_b(k) + \sum_{j \in \mathbb{LS}(b)} p^{\mathsf{s}}_j(k)\,. \tag{75}$$

Fifth, the price-sensitive power-usage amounts $p^{\mathsf{s}}_j(k)$ need to be appropriately entered into the reserve requirement constraints (76) through (80). The resulting generalized constraints take the following form:

Generalized system-wide down/up reserve requirement constraints: For all $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}} \underline{p}_g(k) \;\leq\; [1 - RD(k)] \cdot \widehat{NL}(k)\,; \tag{76}$$

$$\sum_{g \in \mathbb{G}} \overline{p}_g(k) \;\geq\; [1 + RU(k)] \cdot \widehat{NL}(k)\,, \tag{77}$$

where

$$\widehat{NL}(k) \;=\; \widehat{NL}^{\mathsf{f}}(k) + \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{LS}} p^{\mathsf{s}}_j(k)\,. \tag{78}$$

Generalized zonal down/up reserve requirement constraints: For all $z \in \mathbb{Z}$ and $k \in \mathbb{K}$,

$$\sum_{g \in \mathbb{G}(z)} \underline{p}_g(k) \;\leq\; [1 - RD(z,k)] \cdot \widehat{NL}_z(k)\,; \tag{79}$$

$$\sum_{g \in \mathbb{G}(z)} \overline{p}_g(k) \;\geq\; [1 + RU(z,k)] \cdot \widehat{NL}_z(k)\,, \tag{80}$$

where

$$\widehat{NL}_z(k) \;=\; \widehat{NL}^{\mathsf{f}}_z(k) + \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{LS}(z)} p^{\mathsf{s}}_j(k)\,. \tag{81}$$

6.3 Modeling of Price-Sensitive Demand Bids

In general economic terms, a *price-sensitive demand bid* submitted by an LSE j at the start of a time-step k on behalf of a collection $\mathbb{C}$ of end-use customers serviced at a bus b_j can be represented as a non-increasing demand function

$$D_{j,k} : \mathbb{P}_j(k) \to R_+\,, \tag{82}$$

where $0 \in \mathbb{P}_j(k) \subset R_+$. The power-usage domain $\mathbb{P}_j(k)$ denotes the possible aggregate power-usage levels $p^{\mathsf{s}}_j(k)$ that the customers in $\mathbb{C}$ could maintain at bus b_j during time-step k.

Given (82), the price-sensitive demand schedule for LSE j at the start of time-step k consists of all power-price combinations (p, π) satisfying

$$\pi \ = \ D_{j,k}(p) \, , \quad p \in \mathbb{P}_j(k) \, . \tag{83}$$

Each power-price combination (p, π) has the following interpretation: π ($/MWh) is the maximum price that LSE j is willing to pay at the start of time-step k for an incremental increase in the aggregate power usage of the customers in $\mathbb{C}_j$, given that the current aggregate power usage of these customers is at level p (MW). For example, the demand schedule (83) could take the simple linear form

$$\pi \ = \ e_j(k) - 2f_j(k) \cdot p \ , \qquad 0 \ \leq \ p \ \leq \ e_j(k)/2f_j(k) \ , \tag{84}$$

where the coefficients $e_j(k)$ ($/MWh) and $f_j(k)$ ($/[MW]^{2}h) are positively valued.

6.3.1 Price-sensitive Demand Bids with Time-of-Use Pricing

Consider, first, the case in which the demand schedule (83) for each LSE j designates a single price $\pi_j(k)$ ($/MW) for each time-step k, regardless of the power-usage level p (MW). In this case the benefit ($) that an LSE j assigns to a price-sensitive power-usage sequence $\mathbf{p}_j^{\mathsf{s}}$ in (70) takes an extremely simple form, as follows:

$$\mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \ = \ \sum_{k \in \mathbb{K}} \pi_j(k) p_j^{\mathsf{s}}(k) \Delta t \, , \tag{85}$$

where Δt denotes the length of each time-step k measured in hours. The total benefit ($) to be included in the ISO's objective function (72) for the entire future operating period T then takes the form:

$$\mathrm{B(T)} \ = \ \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \tag{86}$$

6.3.2 Price-Sensitive Demand Bids with Price Swing

Consider, instead, the case in which each LSE j submits a price-sensitive demand bid consisting of a demand schedule (83) at each time-step k for which the price π is sensitive to changes in the power-usage level p. How will this affect the expression for total benefit B(T) in the ISO's net-benefit objective function (72)?

For example, Figure 3 illustrates the physical aspects of such a price-sensitive demand bid for an operating period T partitioned into twelve time-steps $k1, \ldots, k12$. Each time-step k has a common length Δt, measured in hours; and the demand-function domain $\mathbb{P}_j(k)$ for each time-step k is a finite set consisting of seven possible power levels $\{0, p_1, \ldots, p_6\}$. Figure 4 depicts a power-price demand schedule (83) that LSE j could designate for a particular time-step k.[11]

Given this form of price-sensitive demand bid, the benefit ($) that LSE j assigns to a power-usage sequence $\mathbf{p}_j^{\mathsf{s}}$ in (70) takes the following form:

$$\mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \ = \ \sum_{k \in \mathbb{K}} \pi_{j,n(k)}(k) p_{j,n(k)}^{\mathsf{s}}(k) \cdot \Delta t \, , \tag{87}$$

[11]This type of price-sensitive demand schedule has the form commonly required in U.S. ISO/RTO-managed DAMs. See, for example, the form required by the ERCOT DAM [3, Module 4].

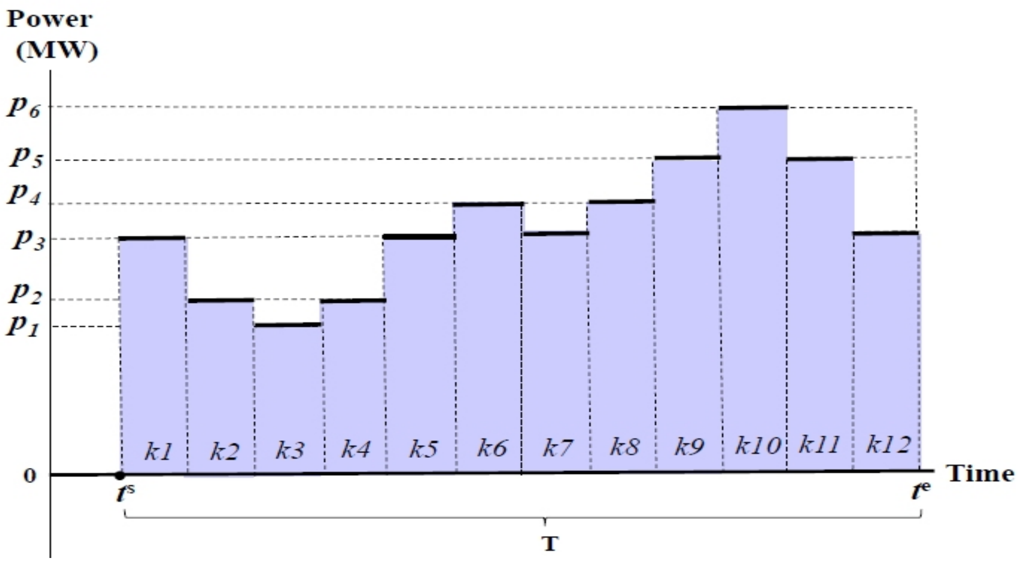

Figure 3: Illustration of the physical aspects of a price-sensitive demand bid submitted by an LSE into an ISO-managed SCUC/SCED optimization for a future operating period T partitioned into 12 time-steps $k1, \ldots, k12$. The same seven possible power demands $0, p_1, \ldots, p_6$ are specified for each time-step k. The shaded region denotes one possible load profile the ISO could clear for period T.

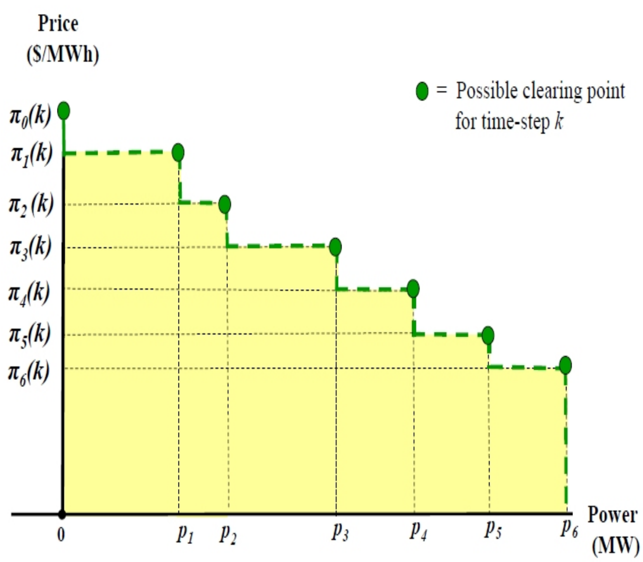

Figure 4: A possible demand schedule for some time-step k that could be designated by the price-sensitive demand bid whose physical aspects are depicted in Fig. 3.

where $n(k)$ is an index that denotes the particular power-usage level $p^{\mathsf{s}}_{j,n(k)}(k)$ (MW) selected from $\mathbb{P}_j(k)$ for time-step k, and $\pi_{j,n(k)}$ ($/MWh) denotes the corresponding price for power usage during time-step k. The total benefit B(T) ($) to be included in the ISO's objective function (72) for the entire operating period T then takes the form

$$\mathrm{B(T)} \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}^{\mathsf{s}}_j) \tag{88}$$

6.4 Demand Bids Directly Expressed as Benefit Functions

More generally, suppose each LSE $j \in \mathbb{LS}$ assigns a benefit (\$/h) to each possible power-usage level $p_j^{\mathsf{s}}(k) \in \mathbb{P}_j(k)$ for each time-step $k \in \mathbb{K}$ by means of a non-decreasing concave benefit function

$$\mathrm{B}_{j,k} : \mathbb{P}_j(k) \rightarrow R \,, \tag{89}$$

where $\mathbb{P}_j(k) = [0, P_j^{\mathsf{max}}(k)]$. For example, (89) might take the quadratic form

$$\mathrm{B}_{j,k}(p) \;=\; d_j(k) \;+\; e_j(k) \cdot p \;-\; f_j(k) \cdot p^2 \tag{90}$$

with ordinate $d_j(k)$ (\$/h), positive coefficients $e_j(k)$ (\$/MWh) and $f_j(k)$ (\$/[MW]^{2}h), and a function domain given by

$$\mathbb{P}_j(k) \;=\; [0,\, e_j(k)/2f_j(k)]\,. \tag{91}$$

If the benefit function (89) is differentiable, the maximum willingness of LSE j to pay for an incremental increase in the aggregate power usage of its customers at the start of time-step k, given that the current aggregate power usage of these customers is at level p, can be expressed by the marginal benefit function[12]

$$\pi_{j,k}(p) \;\equiv\; \frac{\partial B_{j,k}(p)}{\partial p} \;\geq\; 0\,. \tag{92}$$

Note that the price $\pi_{j,k}(p)$ (\$/MWh) depends on the power usage level p.

The benefit (\$) assigned by LSE j to each possible power-usage sequence $\mathbf{p}_j^{\mathsf{s}} = \{p_j^{\mathsf{s}}(k) \mid k \in \mathbb{K}\}$ then takes the form

$$\mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \;=\; \sum_{k \in \mathbb{K}} \mathrm{B}_{j,k}(p_j^{\mathsf{s}}(k)) \cdot \Delta t\,. \tag{93}$$

Consequently, the total benefit B(T) (\$) to be included in the ISO's objective function (72) for the entire future operating period T takes the form:

$$\mathrm{B(T)} \;=\; \sum_{j \in \mathbb{LS}} \mathrm{ben}_j(\mathbf{p}_j^{\mathsf{s}}) \tag{94}$$

7 MILP Tractable Approximation of Benefit Functions

The technique described in Section 4 for obtaining piecewise-linear approximations for non-decreasing *convex* production cost functions $\mathrm{C}_{g,k}(p)$ (\$/h) can similarly be applied to obtain piecewise-linear approximations for non-decreasing *concave* benefit functions $\mathrm{B}_{j,k}(p)$ (\$/h) taking form (89). For completeness, this section presents the latter method in full analytical form.

Suppose the benefit function used by each LSE $j \in \mathbb{LS}$ at each time-step $k \in \mathbb{K}$ to measure the benefit (\$/h) to its customers of different possible price-sensitive power usage levels $p_j^{\mathsf{s}}(k)$ is given by a non-decreasing concave function $\mathrm{B}_{j,k}$:$\mathbb{P}_j(k) \rightarrow R$, where $\mathbb{P}_j(k) = [0, P_j^{\mathsf{max}}(k)]$. A piecewise linear approximation for this benefit function can then be obtained in three simple steps.

[12] In economics, a benefit function $\mathrm{U}(q)$ measuring the benefit of consuming a good q in terms of utliity (utils) is referred to as a *utility function*. Standard budget-constrained utility-maximization problems include as first-order necessary conditions the requirement that $\lambda\pi = \partial\mathrm{U}(q)/\partial q$, where λ (utils/\$) denotes the marginal utility of money and π denotes the price of q measured in dollars per unit of q. In short, at optimal solution points, prices converted into utils per unit of good are expressed as rates of change for benefit functions.

First, select points $\{P_0, P_1, \ldots, P_{NS_j(k)}\}$ from the domain $[0, P_j^{\mathsf{max}}(k)]$, subject to the following restriction:

$$0 = P_0 < P_1 < P_2 < \ldots < P_{NS_j(k)} = P_j^{\mathsf{max}}(k)\ . \tag{95}$$

Second, plot the power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots, NS_j(k)\}$ in the power-benefit plane, where $B_i = \mathrm{B}_{j,k}(P_i)$. Third, connect these power-benefit points by line segments whose slopes, by construction, are non-increasing in i. For example, Fig. 5 illustrates a five-segment piecewise-linear approximation for a non-decreasing concave benefit function taking the quadratic form (90).

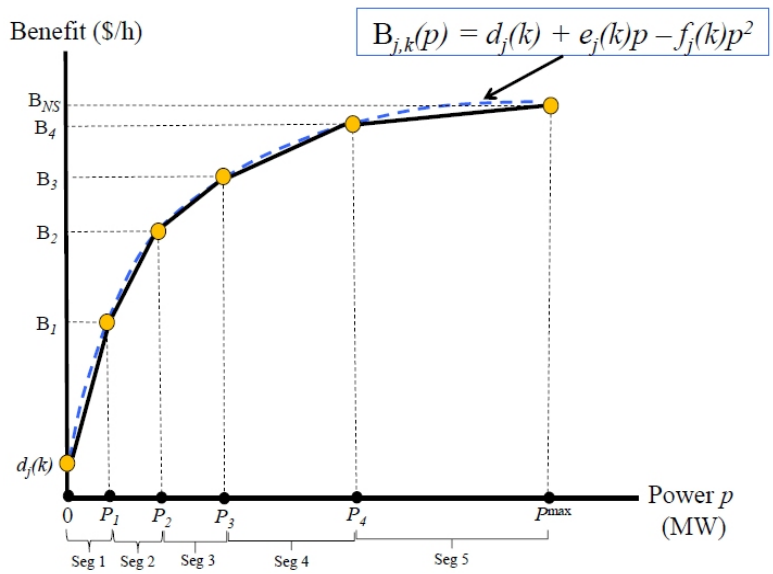

Figure 5: Piecewise-linear approximation of a benefit function (90) used by an LSE j to measure the benefit of its customers at some time-step $k \in \mathbb{K}$. The number of line segments specified for this approximation is $NS = 5$.

Given these power-benefit points (P_i, B_i), the total benefit ($) attained by the customers of each LSE j during each time-step k can be incorporated into the Basic ECA Model SCUC/SCED formulation in an approximate piecewise-linear form, $\mathrm{ben}_j(k)$ ($), that preserves the MILP form of this optimization. This incorporation is similar to the incorporation of total production costs explained in Section 4. It proceeds as follows.

- The ISO's optimal selection of a price-sensitive power usage level $p_j^{\mathsf{s}}(k)$ for the customers of LSE j at time-step k, with corresponding benefit approximation $\mathrm{ben}_j(k)$ ($), is determined for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}$ by means of the linear constraints (97) through (101), below. These constraints must be incorporated into the system constraints for the ISO's optimization problem.

- The variables $\{\delta_{i,j}(k) \mid i = 1, \ldots, NS_j(k)\}$ that appear in the constraints (97) through (101) must be incorporated into the ISO's optimization problem as continuously-valued ISO decision variables, for each $j \in \mathbb{LS}$ and each $k \in \mathbb{K}$.

- The price-sensitive power usage levels $\{p_j^{\mathsf{s}}(k) \mid j \in \mathbb{LS}, k \in \mathbb{K}\}$ must be included among the derived variables determined by the ISO's decision variables and the system constraints.

- As explained with care in Section 6.2, the power usage level $p_j^{\mathsf{s}}(k)$ must be incorporated into the power balance constraint at LSE j's bus location b_j and into the forecasted net loads appearing in the system-wide and zonal reserve constraints, for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}$.

- The ISO's objective function (9) must be extended to include total benefit B(T) ($) for operating period T, calculated as the summation of approximate benefits across all LSEs $j \in \mathbb{LS}$ and all time-steps $k \in \mathbb{K}$ as follows:

$$\mathrm{B(T)} \;=\; \sum_{j \in \mathbb{LS}} \sum_{k \in \mathbb{K}} \mathrm{ben}_j(k) \tag{96}$$

As will be shown, below, given a user-set value for the number $NS_j(k)$ of line segments to be used for the approximation of the benefit function $\mathrm{B}_{j,k}(p)$ in (89) for each $j \in \mathbb{LS}$ and $k \in \mathbb{K}$, plus analytical forms for these benefit functions, the Pyomo Model can automatically construct power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j k\}$ satisfying restriction (95) with $B_i = \mathrm{B}_{j,k}(P_i)$ for each i. Suppose, for the moment, that these power-domain points have already been constructed.

The approximate total benefit $\mathrm{ben}_j(k)$ ($/h) attained by the customers of LSE j during time-step k is then determined by the system of equations (97) - (101), below, as a function of the ISO's optimal selection of the continuously-valued decision variables $\{\delta_{i,j}(k) \mid i = 1, \ldots, NS_j(k)\}$:

Benefit Approximation Method for an LSE j at a Time-Step k:

$$\mathrm{ben}_j(k) = \mathrm{B}_{j,k}(0)\Delta t \;+\; \sum_{i=1}^{NS_j(k)} \big(MB_i \cdot \delta_{i,j}(k)\big) \cdot \Delta t\,; \tag{97}$$

$$p^{\mathsf{s}}_j(k) \;=\; \sum_{i=1}^{NS_j(k)} \delta_{i,j}(k)\,; \tag{98}$$

$$\delta_{i,j}(k) \;\leq\; P_i - P_{i-1}\,,\ \forall i = 1 \cdots NS_j(k)\,; \tag{99}$$

$$\delta_{i,j}(k) \;\geq\; 0,\ \forall i = 1 \cdots NS_j(k)\,, \tag{100}$$

where:

$$MB_i \;=\; \frac{B_i - B_{i-1}}{P_i - P_{i-1}},\ \forall i = 1, \ldots, NS_j(k)\,. \tag{101}$$

The marginal benefit ($/MWh) of LSE j's customers, evaluated at any power-usage level in the interval from P_{i-1} to P_i, is approximated by MB_i in (101). The benefit ($) attained by LSE j's customers at the ISO-cleared price-sensitive power usage level $p^{\mathsf{s}}_j(k)$ in (98) is approximated by $\mathrm{ben}_j(k)$ in (97). For example, suppose there exists a segment $n \in \{1, \ldots, NS_j(k)\}$ such that each $\delta_{i,j}(k)$ takes on its maximum possible value for $i = 1, \ldots, n$ and $\delta_{i,j}(k) = 0$ for $i = n+1, \ldots, NS_j(k)$. Then $p^{\mathsf{s}}_j(k) = P_n$ and $\mathrm{ben}_j(k) = B_n\Delta t = \mathrm{B}_{j,k}(P_n)\Delta t$. On the other hand, suppose $\delta_{i,j}(k) = 0$ for all $i = 1, \ldots, NS_j(k)$. Then $p^{\mathsf{s}}_j(k) = 0$ and $\mathrm{ben}_j(k) = B_0\Delta t \equiv \mathrm{B}_{j,k}(0)\Delta t$.

Finally, the automatic Pyomo Model construction of power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j k\}$ satisfying restriction (95) with $B_i = \mathrm{B}_{j,k}(P_i)$ for each i is given below. This method partitions LSE j's benefit function domain $[0, P^{\mathsf{max}}_j(k)]$ into power segments having equal lengths.

Automated Power-Benefit Point Setting Method for an LSE j at a Time-Step k:

This automated method requires the following inputs: (i) the analytical form of the benefit function (89); (ii) maximum power-usage level $P^{\mathsf{max}}_j(k)$; and (iii) a positive integer value $NS_j(k)$ for the total number of line segments i to be used in the approximation. The Pyomo Model then uses these inputs to compute[13] the power-benefit points $\{(P_i, B_i) \mid i = 0, \ldots NS_j k\}$ for use in the Benefit Approximation Method, as follows:

[13]This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

(a) The initial power-benefit points are set to $P_0 = 0$ and $B_0 = \mathrm{B}_{j,k}(0)$;

(b) The power-width of each segment $i = 1, \ldots, NS_j(k)$ is set equal to

$$w_j(k) \;\equiv\; \frac{P_j^{\mathsf{max}}(k)}{NS_j(k)}\,; \tag{102}$$

(c) For each segment $i = 1, \ldots NS_j(k)$, the power point P_i is set equal to $P_0 + iw_j(k)$;

(d) For each segment $i = 1, \ldots, NS_j(k)$, the benefit point B_i is set equal to $\mathrm{B}_{j,k}(P_i)$.

8 Pyomo Model Calculation of Locational Marginal Prices

The Pyomo Model implementation of the Basic ECA Model SCUC/SCED optimization problem set out in Section 5 determines unit commitments and scheduled dispatch levels for successive future time-steps $k = 1, \ldots NK$. Consistent with actual practice, settlements for these scheduled dispatch levels can be determined in accordance with locational marginal pricing, i.e., the pricing of power in accordance with the location and timing of its injection into, or withdrawal from, a physical grid.

Specifically, *Locational Marginal Prices (LMPs)* can be derived as follows from a Pyomo Model SCUC/SCED optimal solution. First, fix all unit commitment variables at their optimal binary (0/1) solution values. Second, re-run the optimization as a pure SCED optimization, conditional on these optimal unit commitment solution values. Third, calculate the LMP for each bus b at each time-step k as the dual variable for the power balance constraint (28) corresponding to this b and k.

The dual variable for a power balance constraint measures the change in the optimized value of the SCED objective function with respect to a change in the constraint constant for this power balance constraint. This constraint constant is typically taken to be the forecasted amount of fixed (non-price-sensitive) load appearing in the power balance constraint. A unique dual variable solution exists for a power balance constraint with constraint constant cc if the optimized SCED objective function is a differentiable function of cc at the optimal SCED solution point. A range of dual variable solutions exists if the optimized SCED objective function is right and left differentiable with respect to cc at the optimal SCED solution point but not differentiable with respect to cc.[14]

9 Flags and Input Name Mapping

The Pyomo Model permits the user to implement the SCUC/SCED optimization either with or without energy storage units. This is done by setting a flag named StorageFlag either to 1 (*with* storage) or to 0 (*without* storage). This flag setting is used by the Pyomo Model either to include or exclude the appearance of storage variables and storage constraints in the implemented SCUC/SCED optimization.

Each of the user-specified inputs (parameters, initial state conditions, and external forcing terms) named in Section 3 for the analytical Basic ECA Model has a corresponding name in the Pyomo Model implementation of this Basic ECA Model. Table 1, reproduced from ref. [4, Sec. 3], provides a partial mapping between these names, and also between these names and named variables in AMES V5.0. In addition, Table 1 lists any default values assigned by the Pyomo Model for these inputs. If no default value is assigned, the field is left blank.

[14] For detailed discussions of LMP determination in U.S. ISO/RTO-managed wholesale power markets, see [1, 11].

In the Pyomo Model, `model` is the name of the variable used to define the model. All variable names are defined as instance variables of the model and may be programmatically accessed via `model.VariableName`. The "`model.`" syntax is elided for both brevity and clarity.

The order in Table 1 groups the parameters for dispatchable generator units before the parameters for energy storage units, which differs from their ordering in the Pyomo Model. This change in ordering, permitting related elements to be displayed together, is made to facilitate understanding.

Basic ECA Model	**AMES V5.0**	**Pyomo Model**	**Pyomo Default**
$NL^{\mathsf{f}}_b(k)$	NetFixedLoadForecast[15]	NetLoadForecast[6]	0.0
$RE(\ell)$	Reactance	Reactance	
$RD(k)$	DownReservePercent[16]	DownReservePercent	0.05
$RU(k)$	UpReservePercent[7]	UpReservePercent	0.05
$RD(z,k)$	ZonalDownReservePercent	ZonalDownReservePercent	0.05
$RU(z,k)$	ZonalUpReservePercent	ZonalUpReservePercent	0.05
$F^{\mathsf{max}}(\ell)$	MaxCap	ThermalLimit	
NK	NumTimeSteps[7]	NumTimePeriods	
Δt	Time-Step Length	TimePeriodLength	1
Λ^-, Λ^+	Imbalance Penalty Weights	LoadMismatchPenalty	1.0×10^6
	Dispatchable Generation Parameters		
DT_g	MinDownTime	MinimumDownTime	0
UT_g	MinUpTime	MinimumUpTime	0
NRD_g	NominalRampDown	NominalRampDownLimit	
NRU_g	NominalRampUp	NominalRampUpLimit	
NSD_g	ShutdownRampLim	ShutdownRampLimit	
NSU_g	StartupRampLim	StartupRampLimit	
$P^{\mathsf{max}}_g(k)$	capU	MaximumPowerOutput	0.0
$P^{\mathsf{min}}_g(k)$	capL	MinimumPowerOutput	0.0
$p_g(0)$	PowerT0	PowerGeneratedT0	
$\hat{v}_g(0)$		UnitOnT0State	
$v_g(0)$	UnitOnT0[17]	UnitOnT0	
CSC_g		ColdStartCost	0.0
CSH_g		ColdStartHours	0
HSC_g		HotStartCost	0.0

[15] See Section 3.6.

[16] Currently this variable is written by AMES V5.0 into ReferenceModel.dat for the Pyomo Model. If its value needs to be changed, the user should change this value inside the AMES V5.0 software

[17] This value is computed from $\hat{v}_g(0)$, hence it does not need to be specified.

SDC_g		ShutdownCostCoefficient	0.0
$a_g(k)$	FCost	ProductionCostA0	10.0
$b_g(k)$	a	ProductionCostA1	0.0
$c_g(k)$	b	ProductionCostA2	0.0
$NS_g(k)$		NumGeneratorCostCurvePieces	2
$T_{ig}(k)$		PowerGenerationPiecewisePoints[18]	
$G_{ig}(k)$		PowerGenerationPiecewiseValues[9]	
Energy Storage Parameters			
$EPSOC_s$		EndPointSocStorage	0.5
ES_s^{max}		MaximumEnergyStorage	0.0
$NRDIS_s$		NominalRampDownLimitStorageInput	
$NRUIS_s$		NominalRampUpLimitStorageInput	
$NRDOS_s$		NominalRampDownLimitStorageOutput	
$NRUOS_s$		NominalRampUpLimitStorageOutput	
PIS_s^{max}		MaximumPowerInputStorage	0.0
PIS_s^{min}		MinimumPowerInputStorage	0.0
POS_s^{max}		MaximumPowerOutputStorage	0.0
POS_s^{min}		MinimumPowerOutputStorage	0.0
$\underline{SOC}_s$		MinimumSocStorage	0.0
$SOC_s(0)$		StorageSocOnT0	0.5
η_s		EfficiencyEnergyStorage	1.0
$\overline{x}_s(0)$		StoragePowerOutputOnT0	0.0
$\underline{x}_s(0)$		StoragePowerInputOnT0	0.0
Flag for Storage			
		StorageFlag	0

Table 1: Input name-mapping for the Basic ECA Model, AMES V5.0, and Pyomo Model, plus default Pyomo Model values

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[18] As explained in Section 4, these values do not need to be specified if ($a_g(k)$, $b_g(k)$, $c_g(k)$) and $NS_g(k)$ are specified for each dispatchable generator g and time-step k.

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