Analytical Formulation and Python Implementation for an Extended Carrión/Arroyo SCUC/SCED Optimization Formulation

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1 Introduction

Centrally-managed wholesale power markets in the United States rely on *Security-Constrained Unit Commitment (SCUC)* and *Security Constrained Economic Dispatch (SCED)* optimizations to determine commitments and scheduled dispatch levels for generating units for the provision of power in future real-time operating conditions. In an earlier report [1], a complete analytically-formulated combinec SCUC/SCED optimization was presented that extends the well-known SCUC/SCED optimization model developed by Carrión and Arroyo [2] in five key ways:

- Inclusion of non-dispatchable generation
- Inclusion of energy storage units
- Inclusion of nodal power balance constraints with possible transmission congestion
- Inclusion of zonal as well as system-wide spinning reserve requirements
- Inclusion of penalty terms in the objective function for slack in power balance and reserve constraints

In addition, the earlier report [1] discusses a software implementation of this extended SCUC/SCED optimization formulation by means of the *Python Optimization Modeling Objects (Pyomo)* package [3, 4, 5]. The Pyomo package is an open-source tool for optimization applications. This Pyomo software implementation has been incorporated into the AMES Wholesale Power Market Test Bed [6], starting with AMES V4.0 [7]. It has also been used to implement the agent-based 8-Zone ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion [8, 9], and an extension of this test system by Li and Tesfatsion [10] that incorporates wind power in the form of physically-modeled wind turbine agents.

The current report provides a substantially revised version of the earlier report [1] in order to improve the readability and understanding of the extended SCUC/SCED optimization formulation. First, the ordering of presented materials has been changed to facilitate the logical progression of ideas. Second, the presentation of technical model terms (nomenclature) has been augmented with explanatory notes to facilitate understanding of these terms. Third, the presentation of technical model components (objective function, choice variables, constraints) has been augmented with detailed notes to explain the meaning and/or derivation of these components. Fourth, sections discussing possible stochastic (scenario-based) extensions of the extended SCUC/SCED optimization formulation have been omitted.

Hereafter, the extended SCUC/SCED optimization formulation presented in this report will be referred to as the *Extended Carrión-Arroyo (ECA)* Model. Also, the Pyomo software implementation of the ECA Model will be referred to as the *Pyomo Model*.

Section 3 provides complete nomenclature for the ECA Model, grouped by similar elements with accompanying explanatory notes. Section 4 provides analytical formulations for the ECA Model objective function and system constraints, again with accompanying explanatory notes. Pyomo Model piece-wise linear approximations for the quadratic ECA Model objective function are explained in Section 5. Pyomo Model flags permitting the user to implement the SCUC/SCED optimization either with or without energy storage units, and either with or without non-dispatchable generation, are discussed in Section 6. Finally, Section 7 provides a mapping among corresponding input names for the ECA Model, AMES V5.0, and the Pyomo Model, together with Pyomo Model input default values.

2 The ECA Model: Overview

The ECA Model provides a complete analytical formulation for a SCUC/SCED optimization undertaken by an *Independent System Operator (ISO)* tasked with management of wholesale power system operations. The participants in the SCUC/SCED optimization include dispatchable and non-dispatchable generator units, energy storage units, and load-serving entities functioning as intermediaries for retail power customers.

Given initial system conditions, together with forecasts for loads and non-dispatchable generation, the SCUC/SCED optimization determines cost-minimizing solution values for dispatchable generator unit commitments, energy storage unit commitments, dispatchable generator power outputs, energy storage power outputs (discharge levels), energy storage power absorptions (charge levels), and locational marginal prices (LMPs) over a finite planning horizon subject to system constraints. These system constraints include:

- transmission line power constraints;
- power balance constraints;
- generator capacity constraints;
- dispatchable generator ramp constraints for start-up, normal, and shut-down conditions;
- dispatchable generator minimum up-time/down-time constraints;
- dispatchable generator hot-start constraints;
- dispatchable generator start-up/shut-down cost constraints;
- storage unit limit constraints;
- storage unit charge/discharge constraints;
- storage unit ramping constraints;
- storage unit energy conservation constraints;
- storage unit end-point constraints;
- system-wide spinning reserve requirement constraints;

- zonal spinning reserve requirement constraints.

The ECA Model formulation for the power balance constraints relies on a standard DC Optimal Power Flow (DC-OPF) approximation. Consequently, it relies on the following three assumptions. First, the resistance for each transmission line is negligible compared to the reactance, hence the resistance for each transmission line is set to 0. Second, the voltage magnitude at each bus is equal to a common base voltage magnitude. Third, the voltage angle difference $\Delta\theta(\ell)$ across any line ℓ is sufficiently small that the following approximations can be used: $\cos(\Delta\theta(\ell)) \approx 1$ in size and $\sin(\Delta\theta(\ell)) \approx \Delta\theta(\ell)$ in size.

3 Nomenclature for the ECA Model

3.1 Sets

$\mathcal{B}$	Set of buses
$\mathcal{G}$	Set of dispatchable generators j
$\mathcal{G}(b) \subset \mathcal{G}$	Subset of dispatchable generators located at bus b
$\mathcal{G}(z) \subset \mathcal{G}$	Subset of dispatchable generators located in reserve zone z
K	Set of indices for time periods $k = 1, \ldots, \lvert K\rvert$
$\mathcal{L} \subset \mathcal{B} \times \mathcal{B}$	Set of transmission lines ℓ
$\mathcal{L}_O(b) \subset \mathcal{L}$	Subset of lines outgoing at bus b
$\mathcal{L}_I(b) \subset \mathcal{L}$	Subset of lines incoming at bus b
$\mathcal{NG}$	Set of non-dispatchable generators i
$\mathcal{NG}(b) \subset \mathcal{NG}$	Subset of non-dispatchable generators located at bus b
$\mathcal{S}$	Set of energy storage units s
$\mathcal{S}(b) \subset \mathcal{S}$	Subset of energy storage units located at bus b
$\mathcal{Z}$	Set of reserve zones z

3.2 User-Specified Parameters

User-Specified Parameters for Non-Dispatchable Generation Physical Attributes:

$\overline{NP}_i$	Maximum possible output (MW) for non-dispatchable generator $i \in \mathcal{NG}$
$\underline{NP}_i$	Minimum possible power output (MW) for non-dispatchable generator $i \in \mathcal{NG}$

User-Specified Parameters for Dispatchable Generation Physical Attributes:

DT_j	Minimum down-time (h) for generator $j \in \mathcal{G}$
UT_j	Minimum up-time (h) for generator $j \in \mathcal{G}$
NRD_j	Nominal ramp-down rate (MW/τ) for generator $j \in \mathcal{G}$
NRU_j	Nominal ramp-up rate (MW/τ) for generator $j \in \mathcal{G}$
NSD_j	Shut-down ramp rate (MW/τ) for generator $j \in \mathcal{G}$
NSU_j	Start-up ramp rate (MW/τ) for generator $j \in \mathcal{G}$
$\overline{P}_j$	Maximum power output (MW) for generator $j \in \mathcal{G}$ in a synchronized state
$\underline{P}_j$	Minimum power output (MW) for generator $j \in \mathcal{G}$ in a synchronized state

User-Specified Parameters for Dispatchable Generator Start-Up/Shut-Down Costs:

CSC_j	Cold-start cost (\$) for generator $j \in \mathcal{G}$
CSH_j	Cold-start hours (h) for generator $j \in \mathcal{G}$
HSC_j	Hot-start parameter (\$) for generator $j \in \mathcal{G}$ (required to satisfy $HSC_j \leq CSC_j$)
SDC_j	Shut-down cost (\$) for generator $j \in \mathcal{G}$

Remarks on the Cold-Start Hours Parameter: The cold-start hours parameter CSH_j has the following meaning. If a dispatchable generator j at the beginning of a time period k has been off-line for at least CSH_j consecutive hours *immediately prior* to k, then j in time period k is in a *cold-start state* and any start-up of j in k incurs the cold-start cost CSC_j. Otherwise, j in time period k is in a *hot-start state* and j incurs no start-up cost in k.[1]

User-Specified Parameters for Dispatchable Generator Total Production Cost Functions:

a_j	Production cost function coefficient (\$) for generator $j \in \mathcal{G}$
b_j	Production cost function coefficient (\$/MW) for generator $j \in \mathcal{G}$
c_j	Production cost function coefficient (\$/(MW)^2) for generator $j \in \mathcal{G}$
NL	Number of blocks for piecewise-linear approximations for generator production cost functions
T_{ij}	Maximum power output (MW) for generator $j \in \mathcal{G}$ in block $i = 1, \ldots, NL-1$
G_{ij}	Total production cost (\$) for generator $j \in \mathcal{G}$ at power level T_{ij}, $i = 1, \ldots, NL-1$
$G_{NL,j}$	Total production cost (\$) for generator $j \in \mathcal{G}$ at power level $\overline{P}_j$

Remarks on the Formulation of the Total Production Cost Function: As explained more fully in Section 5, the total production cost function for a dispatchable generator $j \in \mathcal{G}$ can be input into the Pyomo Model using *either* an automated piecewise-linear approximation option or a user-directed piecewise-linear approximation option. The first option only requires user-specified settings for the quadratic coefficients (a_j, b_j, c_j) and the block number NL. The second option additionally requires a user to specify settings for (T_{ij}, G_{ij}), $i = 1, \ldots, NL$.

User-Specified Parameters for Energy Storage Units $s \in \mathcal{S}$:

$EPSOC_s$	Target charge state (decimal percent) for storage unit s at end of planning horizon
$\overline{ES}_s$	Maximum energy storage capacity (MWh) of storage unit s during each time period
$NRDIS_s$	Nominal charge ramp-down rate (MW/τ) for storage unit s
$NRUIS_s$	Nominal charge ramp-up rate (MW/τ) for storage unit s
$NRDOS_s$	Nominal discharge ramp-down rate (MW/τ) for storage unit s
$NRUOS_s$	Nominal discharge ramp-up rate (MW/τ) for storage unit s
$\overline{PIS}_s$	Maximum charge power (MW) for storage unit s
$\underline{PIS}_s$	Minimum charge power (MW) for storage unit s
$\overline{POS}_s$	Maximum discharge power (MW) for storage unit s
$\underline{POS}_s$	Minimum discharge power (MW) for storage unit s
$\underline{SOC}_s$	Minimum state of charge (decimal percent) for storage unit s
η_s	Round-trip efficiency (decimal percent) for storage unit s

Other User-Specified Parameters:

[1] Carrión and Arroyo [2, Sec. II] propose a "stairwise startup function" to model the manner in which start-up costs increase for a dispatchable generator j as a function of the number of consecutive hours immediately prior to k during which j was offline.

$BF(\ell)$	Start-bus for transmission line ℓ
$BT(\ell)$	Terminal bus for transmission line ℓ
$RE(\ell)$	Reactance (ohms) on transmission line ℓ, restricted to be non-zero
$TL(\ell)$	Capacity limit (MW) for transmission line ℓ
$R(k)$	System-wide spinning reserve requirement (MW) in time period k
$ZR(z,k)$	Zonal spinning reserve requirement (MW) at reserve zone z in time period k
S_o	Base power (in three-phase MVA)
V_o	Base voltage magnitude (kV)
Λ	Penalty weight ($/MW) for non-zero slack variables in the total cost objective function
τ	Length (h) of each time period k

3.3 Derived Parameters (Calculated from User-Specified Parameters)

A_j	Min total production cost ($) for generator $j \in \mathcal{G}$ in any time period for which it is committed
$B(\ell)$	Inverse of reactance (pu) on transmission line ℓ
$re(\ell)$	Reactance (pu)
SDT_j	Scaled minimum down-time (number of time periods) for generator $j \in \mathcal{G}$
SUT_j	Scaled minimum up-time (number of time periods) for generator $j \in \mathcal{G}$
$SNRDIS_s$	Scaled nominal charge ramp-down limit (MW) for storage unit s (ramp-down per time period)
$SNRUIS_s$	Scaled nominal charge ramp-up limit (MW) for storage unit s (ramp-up per time period)
$SNRDOS_s$	Scaled nominal discharge ramp-down limit (MW) for storage unit s (ramp-down per time period)
$SNRUOS_s$	Scaled nominal discharge ramp-up limit (MW) for storage unit s (ramp-up per time period)
SRD_j	Scaled nominal ramp-down limit (MW) for generator $j \in \mathcal{G}$
SRU_j	Scaled nominal ramp-up limit (MW) for generator $j \in \mathcal{G}$
SSD_j	Scaled shut-down ramp limit (MW) for generator $j \in \mathcal{G}$
SSU_j	Scaled start-up ramp limit (MW) for generator $j \in \mathcal{G}$
Z_o	Base impedance (ohms)

Calculations for Derived Parameters:

- $A_j = a_j + b_j \underline{P}_j + c_j \underline{P}_j^2$
- $B(\ell) = 1/re(\ell)$
- $re(\ell) = RE(\ell)/Z_o$
- $SDT_j = \texttt{round}(DT_j/\tau)$
- $SUT_j = \texttt{round}(UT_j/\tau)$
- $SNRDIS_s = \tau \times NRDIS_s$
- $SNRUIS_s = \tau \times NRUIS_s$
- $SNRDOS_s = \tau \times NRDOS_s$
- $SNRUOS_s = \tau \times NRUOS_s$
- $SRD_j = \min\{\overline{P}_j, \tau \times NRD_j\}$

- $SRU_j = \min\{\overline{P}_j, \tau \times NRU_j\}$
- $SSD_j = \min\{\overline{P}_j, \tau \times NSD_j\}$
- $SSU_j = \min\{\overline{P}_j, \tau \times NSU_j\}$
- $Z_o = (V_o)^2/S_o$

  Remarks on the "Round" Function: In the above calculations for SUT_j and SDT_j, "`round`" denotes Python's function round(), used to round a number to a certain decimal point. Round() takes in two numbers as inputs. The first number is interpreted as the number to be rounded, and the second number is interpreted as the number of decimal places to be included in this rounding. The number 5 is the cut-off for rounding up. Thus, for example, round(17.750,1) = 17.8 whereas round(17.749,1) = 17.7. If nothing is received for the second number input, round() rounds off the first number input to the nearest integer. For example, round(15.59159) = 16 whereas round(15.49321) = 15.

3.4 User-Specified Initial State Conditions

$p_j(0)$	Initial power output (MW) for dispatchable generator $j \in \mathcal{G}$
$\hat{v}_j(0)$	Initial up-time/down-time status (number of hours) for dispatchable generator $j \in \mathcal{G}$
$SOC_s(0)$	Initial state of charge (decimal percent) for storage unit s
$\bar{x}_s(0)$	Initial power output (MW) for storage unit s
$\underline{x}_s(0)$	Initial power absorption (MW) for storage unit s

Remarks on the Meaning of $\hat{v}_j(0)$: If the value of $\hat{v}_j(0)$ is positive (negative) for some dispatchable generator $j \in \mathcal{G}$, it indicates the number of consecutive hours prior to *and including* time period 0 that j has been turned on (off). Note that $\hat{v}_j(0)$ cannot be zero, by definition.

3.5 Derived Initial State Conditions

ITF_j	Number of time periods dispatchable generator $j \in \mathcal{G}$ must be offline *initially*
ITO_j	Number of time periods dispatchable generator $j \in \mathcal{G}$ must be online *initially*
$v_j(0)$	Initial ON/OFF (1/0) status for dispatchable generator $j \in \mathcal{G}$

Calculations for Derived Initial State Conditions:

- If $\hat{v}_j(0) < 0$, $ITF_j = \min\left(|K|, \max\left(0, \texttt{round}((DT_j + \hat{v}_j(0))/\tau)\right)\right)$; otherwise, $ITF_j = 0$.
- If $\hat{v}_j(0) > 0$, $ITO_j = \min\left(|K|, \max\left(0, \texttt{round}((UT_j - \hat{v}_j(0))/\tau)\right)\right)$; otherwise, $ITO_j = 0$.
- If $\hat{v}_j(0) > 0$, $v_j(0) = 1$; otherwise, $v_j(0) = 0$.

3.6 User-Specified External Forcing Terms for Pyomo Model Implementation

OPTION 1: *Separately Specify Forecasts for Loads and for Non-Dispatchable Generation (NDG)*

Step 1: Set the Pyomo Model flag for NDG to 1 (i.e., set NDGFlag = 1), indicating that forecasts for loads and NDG are *separately* represented within the SCUC/SCED optimization constraints.

Step 2: Specify values for forecasted loads:

$D(b,k)$ Forecast for load (MW) at bus b in time period k, $\forall b \in \mathcal{B},\ k \in K$

$D(b,k) \geq 0 \quad \forall b \in \mathcal{B},\ k \in K$

Step 3: Specify values for NDG parameters:

$\overline{NP}_i$ Maximum possible power output for NDG i, $\forall i \in \mathcal{NG}$

$\underline{NP}_i$ Minimum possible power output for NDG i, $\forall i \in \mathcal{NG}$

Step 4: Specify values for forecasted power outputs for each NDG unit:

$np_i(k)$ Forecasted power output (MW) for NDG i in time period k, $\forall i \in \mathcal{NG},\ k \in K$

$\underline{NP}_i \leq np_i(k) \leq \overline{NP}_i \quad \forall i \in \mathcal{NG},\ k \in K$

Remarks on External Forcing Terms under Option 1:

- A SCUC/SCED optimization is a forward-market planning tool for ensuring suitable resource availability for subsequent real-time operations. If a SCUC/SCED optimization is conducted several hours in advance of real-time operations, it would generally not be credible to assume real-time loads and non-dispatchable generation are known with certainty at the time of this optimization.
- In US ISO/RTO-managed day-ahead markets, the ISO/RTO is required to use load-serving entity demand bids as forecasted next-day loads. Forecasts for non-dispatchable generation are typically formulated by the ISO/RTO itself. Spinning reserve requirements are then included in the SCU/SCEDC optimization constraints to protect against the possibility of forecast errors.
- The SCUC/SCED optimization formulated by Carrión and Arroyo [2] does not include non-dispatchable generation and does not consider transmission congestion. Consequently, the only external forcing term in each time period k is forecasted power demand $D(k)$ aggregated across all transmission grid buses b.

OPTION 2: *Specify Forecasts for Net Loads, with NDG Treated as Negative Load*

Step 1: Set the Pyomo Model flag for NDG to 0 (i.e., set NDGFlag = 0), indicating that NDG (if any) is treated as negative load and only forecasts for *net* loads appear within the SCUC/SCED optimization constraints.

Step 2: Specify values for net load forecasts, where *net load = load - NDG*:

$D(b,k)$ Forecast for net load (MW) at bus b in time period k, $\forall b \in \mathcal{B},\ k \in K$

3.7 ISO Choice Variables and Derived Solution Variables

Binary-Valued ISO Choice Variables:

$v_j(k)$ 1 if dispatchable generator $j \in \mathcal{G}$ is committed for time period k; 0 otherwise

$hs_j(k)$ 1 if dispatchable generator $j \in \mathcal{G}$ is in a hot-start state in period k; 0 otherwise

$\bar{u}_s(k)$ 1 if storage unit s is committed for power output (discharge) in time period k; 0 otherwise;

$\underline{u}_s(k)$ 1 if storage unit s is committed for power absorption (charging) in time period k; 0 otherwise.

Continuously-Valued ISO Choice Variables:

$p_j(k)$	Power output (MW) for dispatchable generator $j \in \mathcal{G}$ in time period k
$\theta_b(k)$	Voltage angle (radians) for bus b during time period k
$\bar{x}_s(k)$	Power output (MW) for storage unit s in time period k
$\underline{x}_s(k)$	Power absorption (MW) for storage unit s in time period k
$\alpha_b(k)$	Slack variable (MW) for power balance at bus b in time period k
$\beta(k)$	Slack variable (MW) for system-wide spinning reserve requirement in time period k

Solution Variables Derived from Choice Variables and System Constraints:

$c_j^P(k)$	Total production cost (\$) for dispatchable generator $j \in \mathcal{G}$ in time period k
$c_j^u(k)$	Start-up cost (\$) for dispatchable generator $j \in \mathcal{G}$ in time period k
$c_j^d(k)$	Shut-down cost (\$) for dispatchable generator $j \in \mathcal{G}$ in time period k
$\overline{p}_j(k)$	Max available power output (MW) of dispatchable generator $j \in \mathcal{G}$ in time period k
$w_\ell(k)$	Power flow (MW) on transmission line ℓ in time period k
$z_s(k)$	State of charge (decimal percent) for storage unit s in time period $k = 0, \ldots, \lvert K \rvert$
$\alpha_b^+(k), \alpha_b^-(k)$	Power balance slack variable terms (MW) for bus b in time period k
$\beta^+(k), \beta^-(k)$	System-wide spinning reserve slack variable terms (MW) for time period k

Remarks on the Slack Variable Terms: For any real variable x, there exist unique non-negative values x^+ and x^- satisfying $x^+ - x^- = x$ and $x^+ + x^- = |x|$. As will be seen below in Section 4, the total cost objective function (1) for the ECA Model decomposes the slack variables $\alpha_b(k)$ and $\beta(k)$ into $(\alpha_b^+(k), \alpha_b^-(k))$ and $(\beta^+(k), \beta^-(k))$ in order to impose equal penalties on (i) positive and negative deviations from power balance, and (ii) positive and negative deviations from system-wide spinning reserve requirements.

4 Objective Function and System Constraints for the ECA Model

4.1 Overview

The ECA Model is an analytical formulation for a SCUC/SCED optimization in which an ISO selects admissible choice variables to minimize total cost subject to system constraints. This section provides the analytical formulations for the total cost objective function and the system constraints, making use of the nomenclature presented in Section 3. The start-up and shut-down cost functions $c_j^u(k)$ and $c_j^d(k)$ included in the total cost objective function are carefully formulated in the constraint relationships (29) and (30); the production cost function $c_j^P(k)$ included in the total cost objective function is carefully explained in Section 5.

For concreteness, the system constraints for this analytical formulation include explicit forecasts $np_i(k)$ for the power output of non-dispatchable generation (NDG) units i in each time period k as well as separately represented forecasted loads (demands) $D(b,k)$ for each bus b in each time period k. That is, Option 1 in Section 3.6 is used for the representation of these external forcing terms.

4.2 Total Cost Objective Function

$$\sum_{k \in K} \sum_{j \in \mathcal{G}} c_j^P(k) + c_j^u(k) + c_j^d(k) + \Lambda \left(\sum_{b \in \mathcal{B}} \sum_{k \in K} (\alpha_b^+(k) + \alpha_b^-(k)) + \sum_{k \in K} (\beta^+(k) + \beta^-(k)) \right) \tag{1}$$

The total cost objective function (1) is constructed as the summation of production cost, start-up cost, shut-down cost, and penalty terms imposing costs for any non-zero slack variables appearing in the power balance constraints and the system-wide spinning reserve requirement constraints.[2]

4.3 ISO Admissible Choice Variables

$$v_j(k) \in \{0,1\} \qquad \forall j \in \mathcal{G}, k \in K \tag{2}$$
$$hs_j(k) \in \{0,1\} \qquad \forall j \in \mathcal{G}, k \in K \tag{3}$$
$$\bar{u}_s(k) \in \{0,1\} \qquad \forall s \in \mathcal{S}, k \in K \tag{4}$$
$$\underline{u}_s(k) \in \{0,1\} \qquad \forall s \in \mathcal{S}, k \in K \tag{5}$$
$$p_j(k) \geq 0 \qquad \forall j \in \mathcal{G}, k \in K \tag{6}$$
$$\theta_b(k) \in [-\pi, \pi] \qquad \forall b \in \mathcal{B}, k \in K \tag{7}$$
$$\bar{x}_s(k) \geq 0 \qquad \forall s \in \mathcal{S}, k \in K \tag{8}$$
$$\underline{x}_s(k) \geq 0 \qquad \forall s \in \mathcal{S}, k \in K \tag{9}$$
$$\alpha_b(k) \in \mathbb{R} \qquad \forall b \in \mathcal{B}, k \in K \tag{10}$$
$$\beta(k) \in \mathbb{R} \qquad \forall k \in K \tag{11}$$

4.4 System Constraints

Line power constraints:

$$w_\ell(k) = S_0 B(\ell) \left[\theta_{BF(\ell)}(k) - \theta_{BT(\ell)}(k)\right], \qquad \forall \ell \in \mathcal{L},\ k \in K \tag{12}$$
$$-TL(\ell) \leq w_\ell(k) \leq TL(\ell), \qquad \forall \ell \in \mathcal{L},\ k \in K \tag{13}$$

Power balance constraints (with slack variables):

$$\begin{aligned} &\sum_{j \in \mathcal{G}(b)} p_j(k) \\ &+ \sum_{s \in \mathcal{S}(b)} \bar{x}_s(k) - \sum_{s \in \mathcal{S}(b)} \underline{x}_s(k) \\ &+ \sum_{i \in \mathcal{NG}(b)} np_i(k) \\ &+ \sum_{\ell \in \mathcal{L}_I(b)} w_\ell(k) - \sum_{\ell \in \mathcal{L}_O(b)} w_\ell(k) + \alpha_b(k) \\ &= D(b,k), \qquad \forall b \in \mathcal{B}, k \in K \end{aligned} \tag{14}$$

Dispatchable generator capacity constraints:

$$\underline{P}_j v_j(k) \leq p_j(k) \leq \bar{p}_j(k),\ \forall j \in \mathcal{G},\ \forall k \in K \tag{15}$$
$$0 \leq \bar{p}_j(k) \leq \bar{P}_j v_j(k),\ \forall j \in \mathcal{G},\ \forall k \in K \tag{16}$$

[2]The SCUC/SCED optimization formulated by Carrión and Arroyo [2] does not include slack variables and penalty terms.

Dispatchable generator ramping constraints for start-up, normal, and shut-down conditions:

$$\bar{p}_j(k) \leq \; p_j(k-1) \; + \; SRU_j v_j(k-1) \; + \; SSU_j[v_j(k) - v_j(k-1)] \; + \; \bar{P}_j[1 - v_j(k)]\,, \quad \forall j \in \mathcal{G},\; \forall k \in K\,; \tag{17}$$

$$\bar{p_j}(k) \leq \; \bar{P}_j v_j(k+1) \; + \; SSD_j[v_j(k) - v_j(k+1)]\,, \quad \forall j \in \mathcal{G},\; \forall k = 1 \cdots |K| - 1\,. \tag{18}$$

$$p_j(k-1) - p_j(k) \leq \; SRD_j v_j(k) \; + \; SSD_j[v_j(k-1) - v_j(k)] \; + \; \bar{P}_j[1 - v_j(k-1)]\,, \quad \forall j \in \mathcal{G},\; \forall k \in K \tag{19}$$

Dispatchable generator minimum up-time constraints:

$$\sum_{k=1}^{ITO_j} [1 - v_j(k)] = 0 \;\text{ for all } j \in \mathcal{G} \text{ with } ITO_j \geq 1 \tag{20}$$

$$\sum_{n=k}^{k+SUT_j-1} v_j(n) \geq SUT_j[v_j(k) - v_j(k-1)], \quad \forall j \in \mathcal{G},\; \forall k = ITO_j + 1, \cdots, |K| - SUT_j + 1 \tag{21}$$

$$\sum_{n=k}^{|K|} \{v_j(n) - [v_j(k) - v_j(k-1)]\} \geq 0, \quad \forall j \in \mathcal{G},\; \forall k = |K| - SUT_j + 2, \cdots, |K| \tag{22}$$

Remarks on the derivation of the minimum up-time constraints: To derive these constraints, consider the following. If $ITO_j \geq 1$ for generator j, then by definition of ITO_j it must hold that $v_j(k) = 1$ for all time periods k satisfying $1 \leq k \leq ITO_j$. For $ITO_j + 1 \leq k$, suppose a start-up event occurs for generator j in period k; i.e., suppose $v_j(k-1) = 0$ and $v_j(k) = 1$, implying generator j is turned off in period $k-1$ and on in period k. Then, by definition of SUT_j, generator j must remain on for $SUT_j - 1$ additional periods, or until the end of the final modeled period $|K|$ if $|K| \leq k + SUT_j - 1$. The above minimum up-time constraints express these requirements in concise form.

Dispatchable generator minimum down-time constraints:

$$\sum_{k=1}^{ITF_j} v_j(k) = 0 \;\text{ for all } j \in \mathcal{G} \text{ with } ITF_j \geq 1 \tag{23}$$

$$\sum_{n=k}^{k+SDT_j-1} [1 - v_j(n)] \geq SDT_j[v_j(k-1) - v_j(k)], \quad \forall j \in \mathcal{G},\; \forall k = ITF_j + 1, \cdots, |K| - SDT_j + 1 \tag{24}$$

$$\sum_{n=k}^{|K|} \big[1 - v_j(n) - [v_j(k-1) - v_j(k)]\big] \geq 0, \quad \forall j \in \mathcal{G},\; \forall k = |K| - SDT_j + 2, \cdots, |K| \tag{25}$$

Remarks on the derivation of the minimum down-time constraints: The derivation of the above minimum down-time constraints is similar to the derivation of the minimum up-time constraints, except that one considers shut-down events with $v_j(k-1) = 1$ and $v_j(k) = 0$ rather than start-up events.

Dispatchable generator hot-start constraints:

$$hs_j(k) = 1, \qquad \forall j \in \mathcal{G}, 1 \le k \le CSH_j : k - CSH_j \le \hat{v}_j(0) \tag{26}$$

$$hs_j(k) \le \sum_{t=1}^{k-1} v_j(t), \qquad \forall j \in \mathcal{G}, 1 \le k \le CSH_j : k - CSH_j > \hat{v}_j(0) \tag{27}$$

$$hs_j(k) \le \sum_{t=k-CSH_j}^{k-1} v_j(t), \qquad \forall j \in \mathcal{G}, k = CSH_j + 1, \ldots, |K| \tag{28}$$

Remarks on Generator Hot-Start Constraints: Constraint (26) ensures that, if k does not exceed CSH_j, then generator j is in a hot-start state ($hs_j(k) = 1$) as long as j was "on" either during time period 0 or during a time period prior to time period 0 that is sufficiently close to time period 0. Constraint (27) ensures that, if k does not exceed CSH_j, and j was *not* "on" during time period 0 or during a time period prior to time period 0 that is sufficiently close to time period 0, then j is in a cold-start state ($hs_j(k) = 0$) unless j was on during some time period between 1 and $k-1$. Finally, constraint (28) ensures that, for k larger than CSH_j, j will be in a cold-start state ($hs_j(k) = 0$) if j was not committed during any of the CSH_j time periods immediately preceding time period k. For reasons explained in the remarks following the next set of constraints (i.e., the generator start-up cost constraints), if generator j has a positive cold-start cost CSC_j, the cost-minimizing ISO will set $hs_j(k) = 1$ unless the generator hot-start constraints (26) through (28) force the ISO to set $hs_j(k) = 0$.

Dispatchable generator start-up cost constraints:

$$\begin{aligned} c_j^u(k) &= \max\{0, U(k)\}\,; \\ U(k) &= CSC_j - [CSC_j - HSC_j]hs_j(k) - CSC_j\big[1 - [v_j(k) - v_j(k-1)]\big], \ \forall j \in \mathcal{G}, k \in K \end{aligned} \tag{29}$$

Remarks on Dispatchable Generator Start-Up Cost: Definitions of a cold-start state versus a hot-start state for any dispatchable generator j are provided in Section 3.2. Also recall from this previous section that the user-specified parameters CSC_j and HSC_j are required to satisfy $\mathrm{CSC}_j \ge \mathrm{HSC}_j$. Consequently, (29) implies for any time period k that $c_j^u(k) = \mathrm{CSC}_j$ if j is in a cold-start state in k and $c_j^u(k) = 0$ if j is in a hot-start state in k. Thus, assuming $\mathrm{CSC}_j > 0$, in attempting to minimize total costs the ISO will strive to avoid starting up generator j in a cold-start state, all else equal. In particular, unless ruled out by the hot-start constraints, the cost-minimizing ISO will set $hs_j(k) = 1$ if it commits generator j for time period k.

Dispatchable generator shut-down cost constraints:

$$\begin{aligned} c_j^d(k) &= \max\{0, D(k)\}\,; \\ D(k) &= SDC_j[v_j(k-1) - v_j(k)], \quad \forall j \in \mathcal{G}, k \in K \end{aligned} \tag{30}$$

Storage unit limit constraints:

$$\underline{u}_s(k)\underline{PIS}_s \le \underline{x}_s(k) \le \underline{u}_s(k)\overline{PIS}_s, \ \forall s \in \mathcal{S}, k \in K \tag{31}$$

$$\bar{u}_s(k)\underline{POS}_s \le \bar{x}_s(k) \le \bar{u}_s(k)\overline{POS}_s, \ \forall s \in \mathcal{S}, k \in K \tag{32}$$

Storage unit charge/discharge constraint (cannot charge and discharge at same time):

$$\underline{u}_s(k) + \bar{u}_s(k) \leq 1,\ \forall s \in \mathcal{S}, k \in K \tag{33}$$

Storage unit ramping constraints:

$$\bar{x}_s(k) \leq \bar{x}_s(k-1) + SNRUOS_s,\ \forall s \in \mathcal{S}, k \in K \tag{34}$$

$$\bar{x}_s(k) \geq \bar{x}_s(k-1) - SNRDOS_s,\ \forall s \in \mathcal{S}, k \in K \tag{35}$$

$$\underline{x}_s(k) \leq \underline{x}_s(k-1) + SNRUIS_s,\ \forall s \in \mathcal{S}, k \in K \tag{36}$$

$$\underline{x}_s(k) \geq \underline{x}_s(k-1) - SNRDIS_s,\ \forall s \in \mathcal{S}, k \in K \tag{37}$$

Storage unit energy conservation constraints:

$$z_s(k) = z_s(k-1) + \frac{[-\bar{x}_s(k) + \eta_s \underline{x}_s(k)] \cdot \tau}{\overline{ES}_s},\ \forall s \in \mathcal{S}, k \in K \tag{38}$$

$$z_s(0) = SOC_s(0),\ \forall s \in \mathcal{S} \tag{39}$$

Storage unit end-point constraints:

$$z_s(|K|) = EPSOC_s,\ \forall s \in \mathcal{S} \tag{40}$$

System-wide spinning reserve requirement constraints (with slack variables):

$$\sum_{j \in \mathcal{G}} \overline{p}_j(k) + \sum_{i \in \mathcal{NG}} np_i(k) + \sum_{s \in \mathcal{S}} \bar{x}_s(k) = \sum_{b \in \mathcal{B}} D(b,k) + R(k) + \beta(k) \qquad \forall k \in K \tag{41}$$

Zonal spinning reserve requirement constraints:

$$\sum_{j \in \mathcal{G}(z)} \overline{p}_j(k) - \sum_{j \in \mathcal{G}(z)} p_j(k) \geq ZR(z,k) \qquad \forall z \in \mathcal{Z}, k \in K \tag{42}$$

Slack variable constraints:

$$\alpha_b^+(k) - \alpha_b^-(k) = \alpha_b(k) \qquad \forall b \in \mathcal{B}, k \in K \tag{43}$$

$$\alpha_b^+(k) + \alpha_b^-(k) = |\alpha_b(k)| \qquad \forall b \in \mathcal{B}, k \in K \tag{44}$$

$$\beta^+(k) - \beta^-(k) = \beta(k) \qquad \forall k \in K \tag{45}$$

$$\beta^+(k) + \beta^-(k) = |\beta(k)| \qquad \forall k \in K \tag{46}$$

Voltage angle specification at angle reference bus 1:

$$\theta_1(k) = 0,\quad \forall k \in K \tag{47}$$

5 Pyomo Formulations for ECA Model Total Production Costs

The Pyomo Model assumes that the total production cost function (\$) for a dispatchable generator $j \in \mathcal{G}$ in each time period $k \in K$ has the following general quadratic form:

$$c_j^P(k) = a_j v_j(k) + b_j p_j(k) + c_j p_j^2(k) \quad . \tag{48}$$

As seen in Section 4.4, the system constraints for the ECA Model imply that the total production cost (48) is zero in any time period k for which j is not committed. That is, if $v_j(k) = 0$, then $p_j(k) = 0$, hence $c_j^P(k) = 0$. On the other hand, if j is committed for some time period k, i.e., if $v_j(k) = 1$, then j must run at a power level that is greater or equal to $\underline{P}_j$ during k, where $\underline{P}_j$ is the user-specified minimum power output of j in a synchronized state.[3] The ECA Model presumption is that generator j must then be paid at least its minimum total production cost during k, given by $A_j = a_j + b_j\underline{P}_j + c_j\underline{P}_j^2$.[4]

As will next be explained, the Pyomo Model offers users two different options for producing piecewise-linear approximations for the total production cost functions (48). Each approximation yields the same minimum total production cost A_j for a dispatchable generator j during each period k for which j is committed.

Option 1: Automated Approximation

The user selects the automated approximation option by specifying quadratic coefficients (a_j, b_j, c_j) for each dispatchable generator $j \in \mathcal{G}$ together with a block number NL. The Pyomo Model then uses these inputs to generate a piecewise-linear approximation for the total production cost function (48) for each $j \in \mathcal{G}$ and $k \in K$.

More precisely, given (a_j, b_j, c_j) and NL, Pyomo computes a piecewise approximation for the total production cost function (48) as follows: (a) The width of each segment is $w_j = (\overline{P}_j - \underline{P}_j)/NL$; (b) the ith power value is $T_{ij} = \underline{P}_j + iw_j$, $i = 1, \ldots, NL-1$, with $T_{NL,j} \equiv \overline{P}_j$; and (c) the ordinate corresponding to T_{ij} is $G_{ij} = a_j + b_jT_{ij} + c_jT_{ij}^2$, $i = 1, ..., NL$. This piecewise-linear approximation is accomplished via a Pyomo `Piecewise` construct.

Option 2: User-Directed Approximation

Alternatively, a user selects the user-directed approximation option by specifying a block number NL, quadratic coefficients (a_j, b_j, c_j) for each dispatchable generator $j \in \mathcal{G}$, and piecewise-linear parameters T_{ij} and G_{ij} as follows:

- T_{ij} Max power output (MW) for j in block i, $\forall j \in \mathcal{G}$ and $i = 1, \ldots, NL-1$;
- G_{ij} Total production cost (\$) for j at power level T_{ij}, $\forall j \in \mathcal{G}$ and $i = 1, \ldots, NL-1$;
- $G_{NL,j}$ Total production cost (\$) for j at power level $\overline{P}_j$, $\forall j \in \mathcal{G}$.

Then, letting $\delta_i(j,k)$ denote the power (MW) produced in block i by dispatchable generator $j \in \mathcal{G}$ in time period k, the Pyomo Model employs these user specifications to construct a piecewise-linear approximation for

[3] A dispatchable generator j operating over a transmission grid is said to be in a "synchronized state" relative to this grid if it is connected to the grid and capable of injecting power into the grid without delay, i.e., capable of producing "power output." The commitment of j for a particular time period k obligates j to ensure it is synchronized to the grid during k.

[4] *No-load costs* are costs that a committed dispatchable generator must incur in order to maintain itself in a synchronized state during its commitment interval, even in the absence of any actual power injection into the transmission grid. *Lost opportunity costs* are the costs incurred by a committed dispatachable generator due to its inability to deploy its generation capacity in a next-best alternative use during its commitment interval. In the ECA model, time-invariant no-load and lost-opportunity costs incurred by a dispatchable generator j during each period k within its commitment interval $1 \leq k \leq K$ can be included within a_j.

the total production cost function (48) for each $j \in \mathcal{G}$ and $k \in K$ as follows:

$$c_j^P(k) = A_j v_j(k) + \sum_{i=1}^{NL} F_i(j)\delta_i(j,k) \tag{49}$$

$$p_j(k) = \sum_{i=1}^{NL} \delta_i(j,k) + \underline{P}_j v_j(k) \tag{50}$$

$$\delta_1(j,k) \leq T_{1j} - \underline{P}_j \tag{51}$$

$$\delta_i(j,k) \leq T_{ij} - T_{i-1,j}, \ \forall i = 2 \cdots NL - 1 \tag{52}$$

$$\delta_{NL}(j,k) \leq \bar{P}_j - T_{NL-1,j} \tag{53}$$

$$\delta_i(j,k) \geq 0, \ \forall i = 1 \cdots NL \tag{54}$$

where

$$A_j = a_j + b_j \underline{P}_j + c_j \underline{P}_j^2 \, ; \tag{55}$$

$$F_{1j} = \frac{G_{1j} - A_j}{T_{1j} - \underline{P}_j} \, ; \tag{56}$$

$$F_{ij} = \frac{G_{ij} - G_{i-1,j}}{T_{ij} - T_{i-1,j}}, \ i = 2, \ldots, NL - 1 \, ; \tag{57}$$

$$F_{NL,j} = \frac{G_{NL,j} - G_{NL-1,j}}{\overline{P}_j - T_{NL-1,j}} \, , \tag{58}$$

See [2, Sec. II.A] for a more detailed discussion of this piecewise-linear approximation for total production cost.

6 Pyomo Model Flags for Storage and NDG

The Pyomo Model permits the user to implement the SCUC/SCED optimization either with or without energy storage units. This is done by setting a flag named StorageFlag either to 1 (*with* storage) or to 0 (*without* storage). This flag setting is used by the Pyomo Model either to include or exclude the appearance of storage variables and storage constraints in the implemented SCUC/SCED optimization.

In addition, the Pyomo Model permits the user to implement the SCUC/SCED optimization either with or without the explicit presence of forecasts $np_i(k)$ for the power outputs of non-dispatchable generation (NDG) units i in the SCUC/SCED optimization constraints for each time period k. This is done by setting a Pyomo Model flag named NDGFlag either to 1 (*with* explicit NDG forecasts) or to 0 (*without* explicit NDG forecasts).

However, as detailed in Section 3.6, even if the user sets NDGFlag=0, the user can still *implicitly* include NDG forecasts in the SCUC/SCED optimization constraints by treating NDG as negative load and by inputting values for the "Demand" variables $D(b,k)$ as forecasted *net* load at bus b in time period k, where *net* load is load minus NDG.

7 Input Name Mapping

Each of the user-specified inputs (parameters, initial state conditions, and external forcing terms) named in Section 3 for the analytical ECA Model has a corresponding name in the Pyomo Model implementation of this ECA Model. Table 1, reproduced from ref. [1, Sec. 3], provides a partial mapping between these names, and also between these names and named variables in AMES V5.0. In addition, Table 1 lists any default values assigned by the Pyomo Model for these inputs. If no default value is assigned, the field is left blank.

In the Pyomo Model, `model` is the name of the variable used to define the model. All variable names are defined as instance variables of the model and may be programmatically accessed via `model.VariableName`. The "`model.`" syntax is elided for both brevity and clarity.

The order in Table 1 groups the parameters for dispatchable generator units before the parameters for energy storage units, which differs from their ordering in the Pyomo Model. This change in ordering, permitting related elements to be displayed together, is made to facilitate understanding.

ECA Model	**AMES V5.0**	**Pyomo Model**	**Pyomo Default**
$D(b, k)$	Demand[5]	Demand[5]	0.0
$\overline{NP}_i$		MaxNondispatchablePower	0.0
$\underline{NP}_i$		MinNondispatchablePower	0.0
$RE(\ell)$	Reactance	Reactance	
$R(k)$	ReserveRequirement[6]	ReserveRequirement	
$ZR(z, k)$		ZonalReserveRequirement	
$TL(\ell)$	MaxCap	ThermalLimit	
$\lvert K\rvert$	NumTimePeriods[6]	NumTimePeriods	
τ		TimePeriodLength	1
Λ		LoadMismatchPenalty	1.0×10^6
Dispatchable Generation Parameters			
DT_j	MinDownTime	MinimumDownTime	0
UT_j	MinUpTime	MinimumUpTime	0
NRD_j	NominalRampDown	NominalRampDownLimit	
NRU_j	NominalRampUp	NominalRampUpLimit	
NSD_j	ShutdownRampLim	ShutdownRampLimit	
NSU_j	StartupRampLim	StartupRampLimit	
$\overline{P}_j$	capU	MaximumPowerOutput	0.0
$\underline{P}_j$	capL	MinimumPowerOutput	0.0
$p_j(0)$	PowerT0	PowerGeneratedT0	
$\hat{v}_j(0)$		UnitOnT0State	
$v_j(0)$	UnitOnT0[7]	UnitOnT0[7]	
CSC_j		ColdStartCost	0.0
CSH_j		ColdStartHours	0

[5]The user can specify this external forcing term to represent either forecasted load or forecasted *net* load; see Section 3.6.
[6]Currently this variable is written by AMES V5.0 into ReferenceModel.dat for the Pyomo Model. If it's value needs to be changed, the user should change this value inside the AMES V5.0 software
[7]This value is computed from $\hat{v}_j(0)$, hence it does not need to be specified.

ECA Model	AMES V5.0	Pyomo Model	Pyomo Default
HSC_j		HotStartCost	0.0
SDC_j		ShutdownCostCoefficient	0.0
a_j	FCost	ProductionCostA0	10.0
b_j	a	ProductionCostA1	0.0
c_j	b	ProductionCostA2	0.0
NL		NumGeneratorCostCurvePieces	2
T_{ij}		PowerGenerationPiecewisePoints[8]	
G_{ij}		PowerGenerationPiecewiseValues[8]	
Energy Storage Parameters			
$EPSOC_s$		EndPointSocStorage	0.5
$\overline{ES}_s$		MaximumEnergyStorage	0.0
$NRDIS_s$		NominalRampDownLimitStorageInput	
$NRUIS_s$		NominalRampUpLimitStorageInput	
$NRDOS_s$		NominalRampDownLimitStorageOutput	
$NRUOS_s$		NominalRampUpLimitStorageOutput	
$\overline{PIS}_s$		MaximumPowerInputStorage	0.0
$\underline{PIS}_s$		MinimumPowerInputStorage	0.0
$\overline{POS}_s$		MaximumPowerOutputStorage	0.0
$\underline{POS}_s$		MinimumPowerOutputStorage	0.0
$\underline{SOC}_s$		MinimumSocStorage	0.0
$SOC_s(0)$		StorageSocOnT0	0.5
η_s		EfficiencyEnergyStorage	1.0
$\overline{x}_s(0)$		StoragePowerOutputOnT0	0.0
$\underline{x}_s(0)$		StoragePowerInputOnT0	0.0
Flags for Storage and Forecasted Non-Dispatchable Generation			
		StorageFlag	0
		NDGFlag	0

Table 1: Input name-mapping for the ECA Model, AMES V5.0, and the Pyomo Model, plus default values for the Pyomo Model

[8] As explained in Section 5, these values do not need to be specified if (a_j, b_j, c_j) and NL are specified.

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