Laplace

Linear transformation

$$L : V \to W$$

$$L\left(\alpha \triangleleft f + \beta \triangleleft g\right)$$

Laplace

sin & cos from exp

Laplace

$$F(s) = \mathcal{L}\{f(t)\}$$

Math notation

$f(t)$	$F(s)$
e^{at}	$\frac{1}{s-a}$
$\cos t$	$\frac{s}{s^2+1}$
$f'(t)$	$sF(s)-f(0)$
$\vdots$	

$(\cdot) = \text{scale}$

$$f'' + 4\cdot f' + f = 6\cdot\cos\ ,\quad f\,0 = 0\ ,\quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s\cdot L\,f\,s \qquad L\,f\,s = 6\cdot\frac{s}{(s^2+1)\cdot(s^2+4s+1)}$$

$$L\,(f'' + 4\cdot f' + f)\,s = L\,(6\cdot\cos)\,s$$

$$L\,f''\,s + 4\cdot L\,f'\,s + L\,f\,s = 6\cdot L\cos s$$

$$s^2\cdot L\,f\,s + 4\cdot s\cdot L\,f\,s + L\,f\,s = 6\cdot\frac{s}{s^2+1}$$

$$(s^2+4s+1)\cdot L\,f\,s = -\!\!\;\;''\!-$$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \, , \quad f\,0 = 0 \, , \quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s$$

$$L\,f\,s = 6 \cdot \frac{s}{(s^2+1)\cdot(s^2+4s+1)} = \frac{3}{2} \cdot \Bigg(\overbrace{\frac{1}{s^2+1} - \frac{1}{s^2+4s+1}}^{4s} \Bigg)$$

$$(s - s_1) \cdot (s - s_2)$$

$$s_1 = -2 + \sqrt{3} \, , \quad s_2 = -2 - \sqrt{3}$$

$$\frac{1}{p \cdot q} = \frac{A}{p} + \frac{B}{q}$$

$$L \sin s$$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos, \quad f\,0 = 0, \quad f'0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s$$

$$\frac{3}{2} \cdot \frac{1}{s^2+4s+1} = \frac{A}{s-s_1} + \frac{B}{s-s_2} \qquad s_1 = -2+\sqrt{3}, \; s_2 = -2-\sqrt{3}$$

$$3 = 2 \cdot A \cdot (s - s_2) + 2 \cdot B \cdot (s - s_1)$$

$$s = s_1: \quad 3 = 2 \cdot A \cdot (s_1 - s_2) = 2 \cdot A \cdot 2 \cdot \sqrt{3} \Rightarrow A = \frac{\sqrt{3}}{4}$$

$$s = s_2: \quad 3 = 2 \cdot B \cdot (s_2 - s_1) = 2 \cdot B \cdot (-2\sqrt{3}) \Rightarrow B = -A$$

$$L\,f\,s = \frac{3}{2} \cdot L\sin s - \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{s-s_1} - \frac{1}{s-s_2} \right)$$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \,, \quad f\,0 = 0 \,, \quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s$$

$s_1 = -2 + \sqrt{3} \approx -0{,}3$, $s_2 = -2 - \sqrt{3} \approx -3{,}7$

$$L\,f\,s = \frac{3}{2} \cdot L\sin\, s - \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{s - s_1} - \frac{1}{s - s_2} \right)$$

$$f\,t = \frac{3}{2} \cdot \sin t - \frac{\sqrt{3}}{4} \cdot \left(\exp(s_1 \cdot t) - \exp(s_2 \cdot t) \right)$$

$$f'\,t = \frac{3}{2} \cdot \cos t - \frac{\sqrt{3}}{4} \cdot \left(s_1 \cdot \qquad - s_2 \cdot \qquad \right)$$

$$f''\,t = -\frac{3}{2} \sin t - \frac{\sqrt{3}}{4} \left(s_1^2 \cdot \qquad - s_2^2 \cdot \qquad \right)$$

$$f'' + 4\cdot f' + f = 6\cdot\cos, \quad f\,0=0, \quad f'\,0=0$$

$s_1 = -2+\sqrt{3}, \quad s_2 = -2-\sqrt{3}$

$$f\,t = \frac{3}{2}\cdot\sin t - \frac{\sqrt{3}}{4}\cdot\left(\exp(s_1\cdot t) - \exp(s_2\cdot t)\right)$$

$$f'\,t = \frac{3}{2}\cdot\cos t - \frac{\sqrt{3}}{4}\cdot\left(s_1\cdot\exp(s_1\cdot t) - s_2\cdot\exp(s_2\cdot t)\right)$$

$$f''\,t = -\frac{3}{2}\cdot\sin t - \frac{\sqrt{3}}{4}\cdot\left(s_1^2\cdot\exp(s_1\cdot t) - s_2^2\cdot\exp(s_2\cdot t)\right)$$

$$\mathrm{dot}(\;)(1,4,1) = 4\cdot\frac{3}{2}\cdot\cos t = 6\cdot\cos t = RHS$$

$$\mathrm{dot}\;(1,4,1) = (1+4\cdot s_1+s_1^2)\cdot\exp(s_1\cdot t) = 0$$

$$\mathrm{dot}\;(1,4,1) = (1+4s_2+s_2^2)\cdot\exp(s_2\cdot t) = 0$$

OK!

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \,,\quad f\,0 = 0 \,,\quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s \qquad s_1 = -2 + \sqrt{3} \,,\; s_2 = -2 - \sqrt{3}$$

$$L\,f\,s = \frac{3}{2} \cdot L\sin\, s - \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{s - s_1} - \frac{1}{s - s_2} \right)$$

$$f\,t = \frac{3}{2} \cdot \sin t - \frac{\sqrt{3}}{4} \cdot \left(\exp(s_1 \cdot t) - \exp(s_2 \cdot t) \right)$$

$$f'\,t = \frac{3}{2} \cdot \cos t - \frac{\sqrt{3}}{4} \cdot \left(s_1 \cdot \exp(s_1 \cdot t) - s_2 \cdot \exp(s_2 \cdot t) \right)$$

$$f''\,t = -\frac{3}{2} \cdot \sin t - \frac{\sqrt{3}}{4} \cdot \left(s_1^2 \cdot \exp(s_1 \cdot t) - s_2^2 \cdot \exp(s_2 \cdot t) \right)$$

$$\left.\begin{aligned} f\,0 &= 0 - \frac{\sqrt{3}}{4} \cdot (1 - 1) = 0 \\ f'\,0 &= \frac{3}{2} \cdot 1 - \frac{\sqrt{3}}{4} \cdot (s_1 \cdot 1 - s_2 \cdot 1) = \frac{3}{2} - \frac{\sqrt{3}}{4} \cdot 2 \cdot \sqrt{3} = \frac{3}{2} - \frac{3}{2} = 0 \end{aligned}\right\} \text{OK}$$