

Vulnerability Modelling with Functional Programming and Dependent Types

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Outline

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1 Context

2 Vulnerability

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Richard Bird (1943-2022) (image from Wikipedia)

Ideas:

- *Category theory* is an extensible language for precisely formulating mathematical theories (concepts, models, etc.).
- One such extension is particularly well-suited for formulating *functional programs*. Further extensions allow us to formulate *specifications*.
- There are other frameworks with similar properties: higher-order logics (dependent types), Hoare logics, etc. The categorical framework is good for *equational reasoning*.

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Parallel Computing
Volume 33, Issue 12, December 2007, Pages 795-821




Relation-based computations in a monadic BSP model

[N. Botta](#)  [C. Ionescu](#)


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Abstract


We propose a Haskell monadic model of bulk synchronous parallel programs and apply it to the analysis of relation-based computations.

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...The complexity of the climate, ecological, social and economic systems that researchers are modelling means that the validity of scenario results will inevitably be subject to ongoing criticism.

...What this criticism does, however, is emphasize the need for a strong foundation upon which scenarios (i.e., modelling) can be applied, a foundation that provides a basis for managing risk despite uncertainties associated with future climate changes.

This foundation lies in the concept of vulnerability.

(from “Climate Change Impacts and Adaptation: A Canadian Perspective”, 2004)

The paper was drafted for DTP10 *Dependently-Typed Programming*, Edinburgh.

DTP10

The first version was submitted in December 2010, the final version was published online in ...December 2014, and the paper journal version came out January 2016.

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Definitions (or “Definitions”)

- UNDP Annual Report, 2004:
...a human condition or process resulting from physical, social and environmental factors which determine the likelihood and damage from the impact of a given hazard
- Cannon et al. 2004
Vulnerability [...] is a way of conceptualizing what may happen to an identifiable population under conditions of particular risk and hazards.
- The Intergovernmental Panel on Climate Change, 1995
vulnerability: the extent to which climate change may damage or harm a system.”
- Institute for Environment and Security (United Nations University), 2004
Vulnerability is the intrinsic and dynamic feature of an element at risk (community, region, state, infrastructure, environment, etc.) that determines the expected damage or harm resulting from a given hazardous event, and is often affected by the harmful event itself. Vulnerability changes continuously over time and is driven by physical, social, economic and environmental factors.

...many, many others (Thywissen 2006 lists 32 more)

vulnerable (adj.):

- ① exposed to the possibility of being attacked or harmed, either physically or emotionally: *we were in a vulnerable position* | *small fish are **vulnerable** to predators*
- ② Bridge (of a partnership) liable to higher penalties, either by convention or through having won one game towards a rubber.

(Oxford Dictionary of English, 2010)

The Structure of Vulnerability

```
possible      :: Functor f => State -> f Evolution

harm          :: Preorder v => Evolution -> v

measure       :: Functor f, Preorder v, Preorder w => f v -> w

vulnerability :: Preorder w => State -> w

vulnerability = measure . fmap harm . possible
```

Calvo and Dercon (2005):

- Vulnerability measures a set of outcomes across possible states of the world. These states of the world are assumed to be in finite number and come with associated probabilities.
- Poverty is defined in terms of a threshold, which has the same value in all states of the world.
- The states of the world in which the outcomes are above the threshold do not enter in the vulnerability measurement (this is called the “axiom of focus”).
- Monotonicity requirements: the likelier the outcomes below the threshold, and the greater their distance to the threshold, the greater the vulnerability.

Example (from paper)

```
> V = sum [p i * v (x i) | i <- [1 .. n]]
```

where

```
> n :: Nat           -- the number of possible states of the world
> p :: Nat -> [0, 1] -- p i is the probability of state i
> v :: Real -> Real  -- a monotonically decreasing and convex function
> x i = (y i) / z     -- relative distance to threshold of outcome i
> y :: Nat -> Real   -- y i is the outcome in state i if below the
>                      -- threshold, 0 otherwise
> z :: Real          -- the threshold
```

Exercise: Does this fit the structure of vulnerability? How / why not?

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“Small fish are vulnerable to predators.”

```
possible :: State -> [Evolution]
```

```
predators :: Evolution -> Bool
```

```
wounded :: Evolution -> Bool
```

```
harm :: Evolution -> Bool
```

```
harm = wounded  $\wedge$  predators
```

- Vulnerability is here a predicate on *states* (transferred epithet)
- Factors of interest (*predators*) and impacts to assess (*wounded*) play a symmetric role

Monotonicity Condition for Vulnerability Measures

Vulnerability measures should be “monotonous”. If every harm value in the structure increases, the structure remaining the same, then the aggregated value should also increase (“increasing” is **non-strict**).

For all $\text{inc} : V \rightarrow V$ increasing ($\text{inc } v \sqsupseteq v$), we have for all $v \in F \ V$

$$\text{measure } (\text{fmap } \text{inc } v) \sqsupseteq \text{measure } v$$

“Most common harm value” does not fulfil this condition!

E.g., $[1, 2, 3, 10, 10]$ worse than $[4, 4, 4, 11, 11]$!

- natural transformation between representations of possibility

$$\tau :: F1\ a \rightarrow F2\ a$$

- vulnerability measures of the “same kind”

$$m1 :: F1\ V \rightarrow W$$
$$m2 :: F2\ V \rightarrow W$$

- compatibility w.r.t. τ

$$m1\ v1 \sqsupseteq m1\ v2 \Rightarrow m2\ (\tau\ v1) \sqsupseteq m2\ (\tau\ v2)$$

- reuse: $m2$. τ is compatible with $m2$

Exercise: What are the “minimal” categories involved?

Enter dependent types (under the influence of Patrik Jansson).

We started moving the categorical machinery to the new framework, leading to many developments (see the rest of the FPClimate papers).

There is still a certain “friction” between categorical frameworks and the dependently typed ones which have largely superseded them, as illustrated by Richard Bird’s last book (together with Jeremy Gibbons).

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Main idea: formulate problems as programming correctness problems!

This can be useful in contexts where simulation is used to replace experiments (climate science, social science, political theory, etc.).

Type theory can formulate both correctness conditions and the programs themselves. Moreover, the correctness proofs are then mechanically checked.

Meta-exercise: find and correct all the errors in the article.