



INTRODUCTION TO STATISTICS

MODULE 2

MODULE 2 OVERVIEW



DESCRIPTIVE STATISTICS

PAST DATA
MAKE DECISIONS
THIS IS NO ERROR OR
UNCERTAINTY ASSOCIATED IN
THIS PROCESS

ADVANCED TOPICS

MONTECARLO SIMULATIONS
TIME SERIES
CURVE FITTING
STOCHASTIC CALCULUS



INFERRENTIAL STATISTICS

CONCLUSIONS BEYOND
IMMEDIATE AVAILABLE DATA
BASED ON SAMPLE CONCLUDE
FOR POPULATION

PROBABILISTIC APPROACH
PARAMETRIC ESTIMATION
CENTRAL LIMIT THEOREM
GOODNESS OF FIT TESTING

EXAMPLE OF INTUITION

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuclear demonstrations. Which of the two alternatives is more probable?

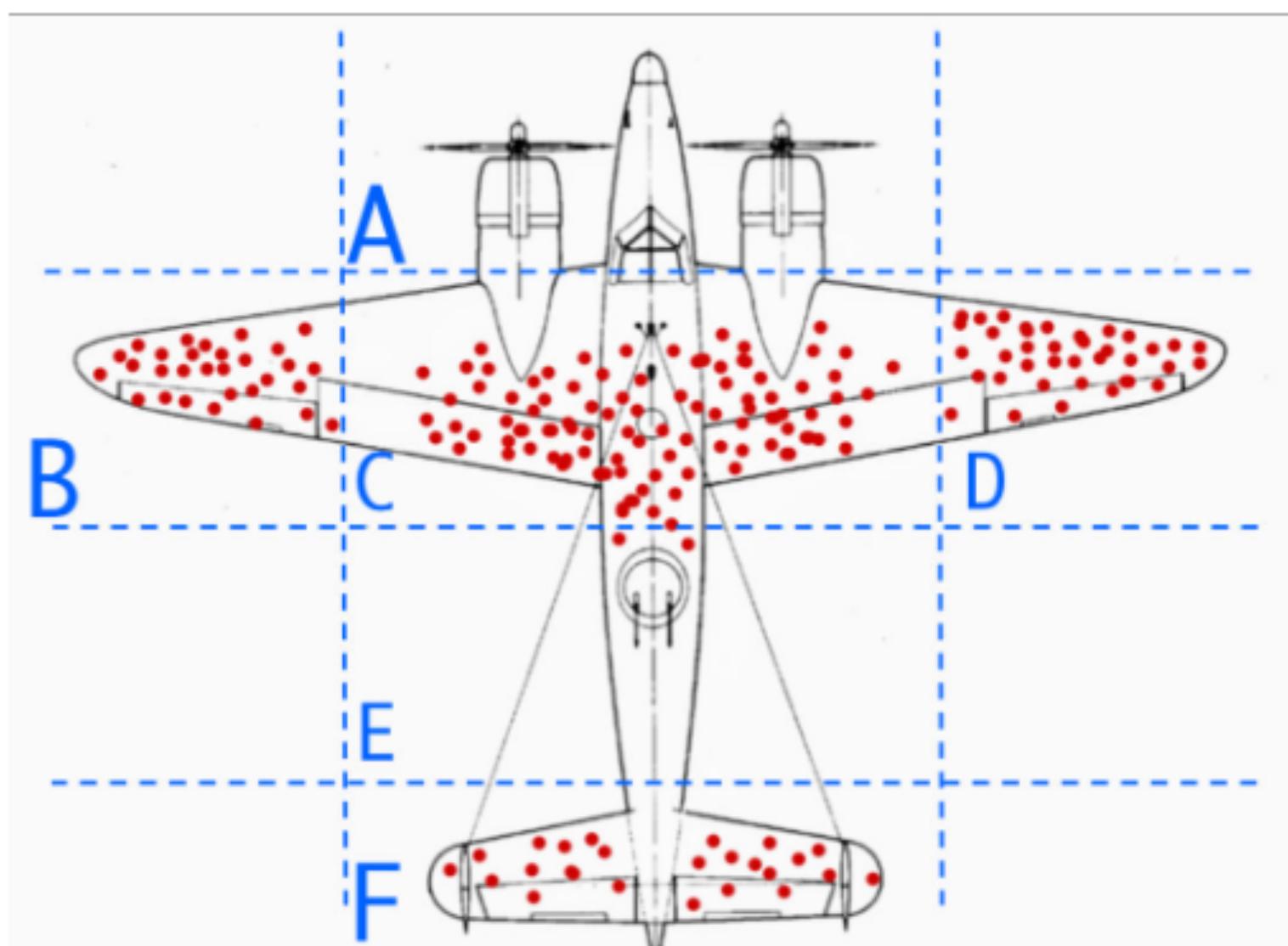
(a) Linda is a banker

(b) Linda is a banker and active in the feminist movement

EXAMPLE OF INTUITION

During WWII, the Navy tried to determine where they needed to armor their aircraft to ensure they came back home. They ran an analysis of where planes had been shot up, and came up with the figure on the right.

What places of the plane (areas A to F) do you think would need the **most** armor?

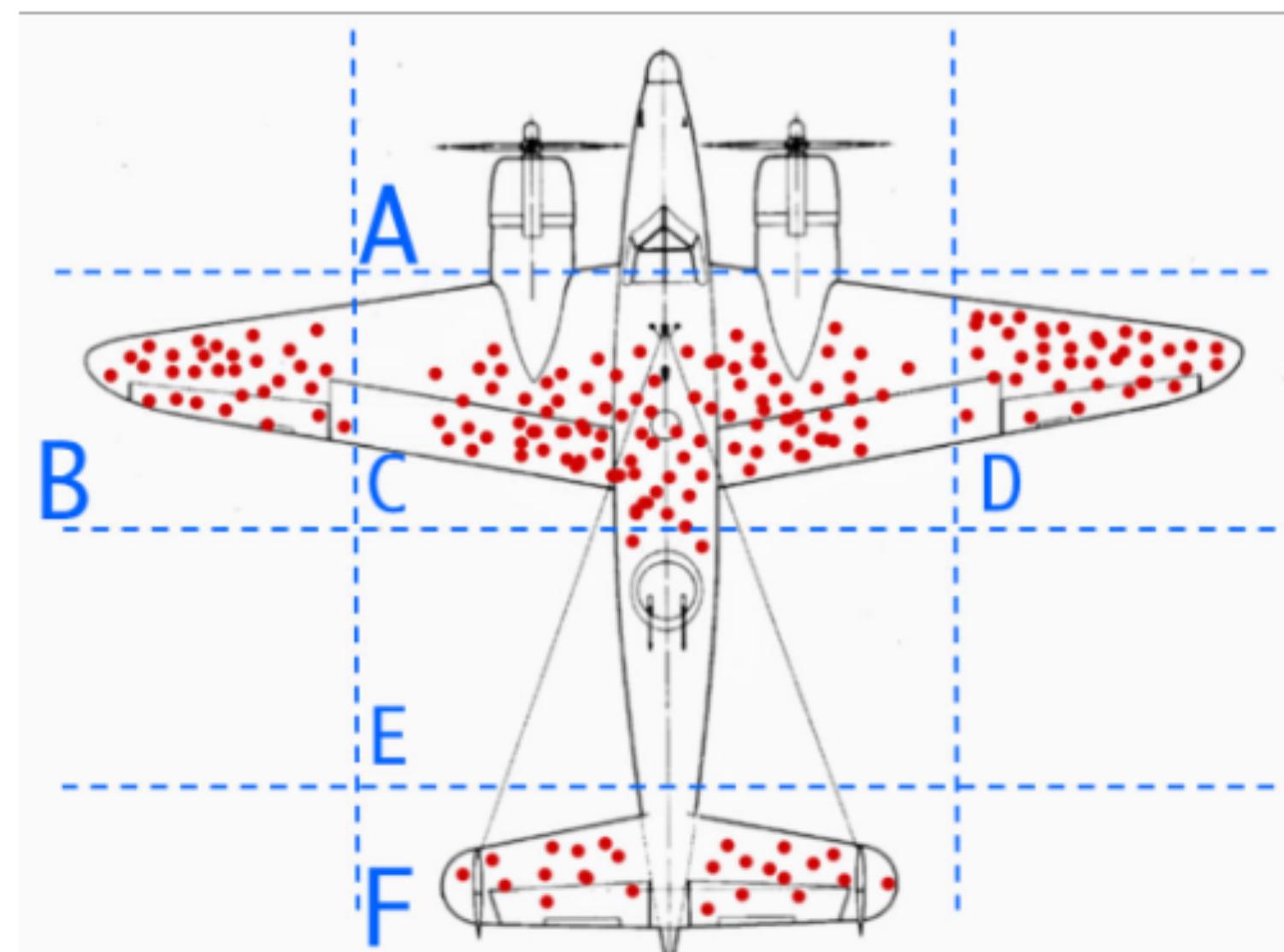


This problem is related to the famous mathematician Albert Wald

STATISTICS INTUITION - SURVIVORSHIP BIAS

During WWII, the Navy tried to determine where they needed to armor their aircraft to ensure they came back home. They ran an analysis of where planes had been shot up, and came up with the figure on the right.

What places of the plane (areas A to F) do you think would need the **most** armor?



BUT THE REAL REASON AGAINST ADDITIONAL ARMOR IS THAT THE HOLES PRECISELY SHOWED THE STRONGEST PARTS OF THE PLANE SINCE THE PLANE SURVIVED DESPITE THE DAMAGE IT SUFFERED.
TO PUT IT SIMPLY, THE PLANE SURVIVED DESPITE BEING HIT ON THOSE SPECIFIC PARTS, AND THAT MEANS THAT THOSE BADLY-HIT PARTS CAN WITHSTAND ENEMY FIRE.

WHAT IS STATISTICS?

**STATISTICS IS THE GRAMMAR OF NATURE'S
LANGUAGE, RANDOMNESS.**

MEASURES OF DESCRIPTIVE STATISTICS

WHY DO WE NEED STATISTICAL INDICATORS OF A DATASET?

TWO TYPES: LOCATION & DISPERSION

LOCATION MEASURES CAN BE CENTRAL OR NON CENTRAL

MEASURES OF CENTRAL TENDENCY AND LOCATION

ARITHMETIC MEAN

Definition 3.3 ((Arithmetic) Mean) The arithmetic mean (or simply mean) of a data set is given by the sum of its observations divided by the number of observations. For a sample of size n , we write

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

MEDIAN

Definition 3.4 (Median) The median is the observation located in the middle of a data set after this latter has been arranged in increasing or decreasing order.

If the sample size n is an odd number, then the median is middle observation. If n is even, the median is the average of the two middle observations.

The median will be the number located in the

$$0.5(n + 1)\text{th ordered position}$$

[60, 63, 65, 67, 70, 72, 75, 75, 80, 82, 84, 85]

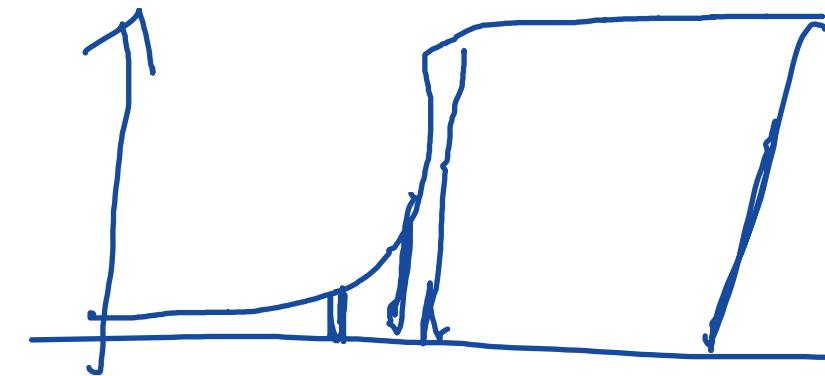


$$\frac{72 + 75}{2} = 73.5$$

MODE

Definition 3.5 (Mode) The mode, if it exists, is the most frequently occurring value. If many exist, then the variable is said bimodal (two) or multimodal (several). This measure fits best categorical data.

PERCENTILES AND QUARTILES



UPPER/LOWER QUARTILE

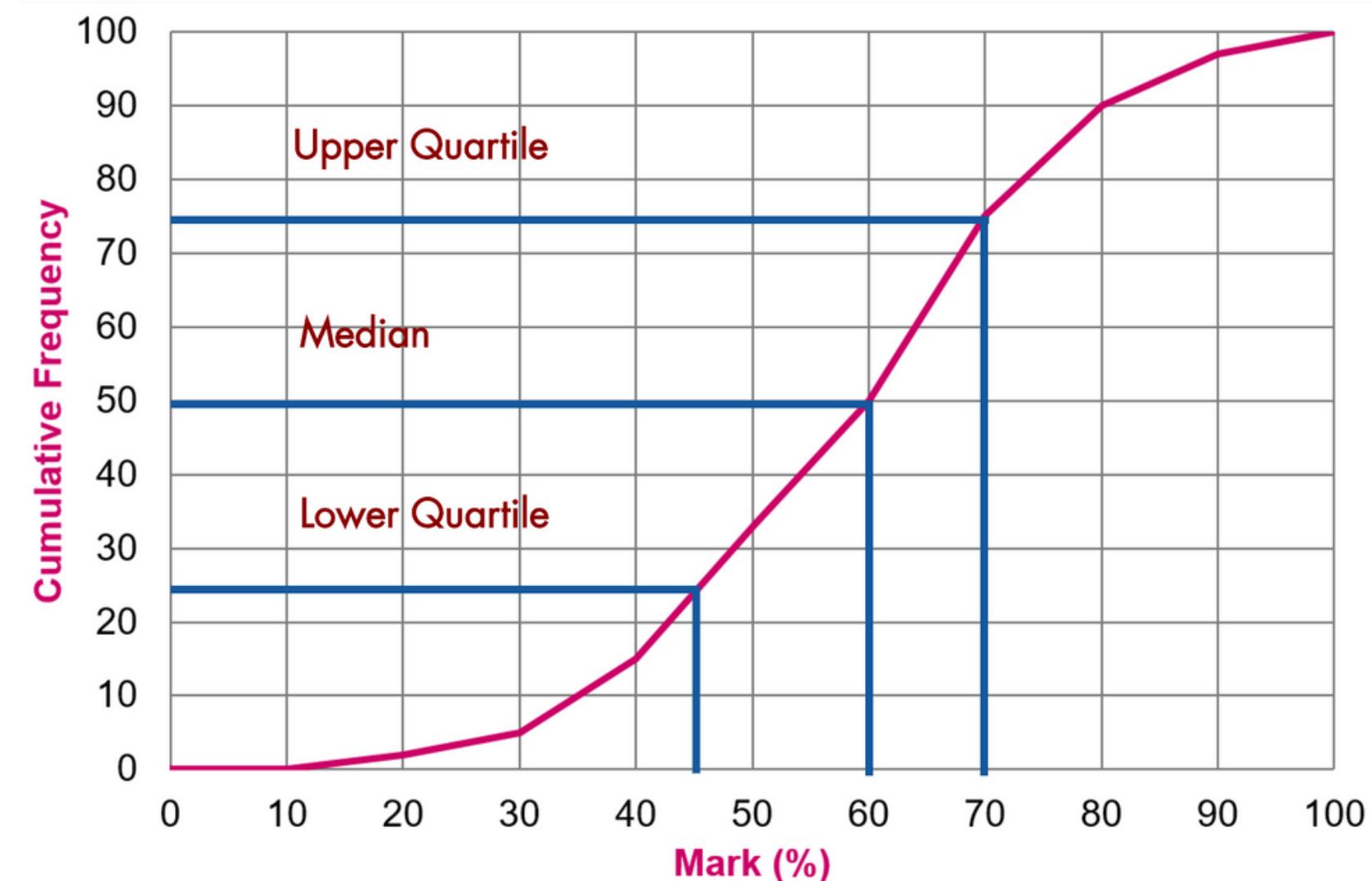
Q_1 = value in the $0.25(n + 1)$ th ordered position

Q_2 = value in the $0.50(n + 1)$ th ordered position

Q_3 = value in the $0.75(n + 1)$ th ordered position

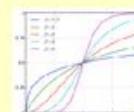
PERCENTILE

P th percentile = value located in the $\frac{P}{100}(n + 1)$ th ordered position

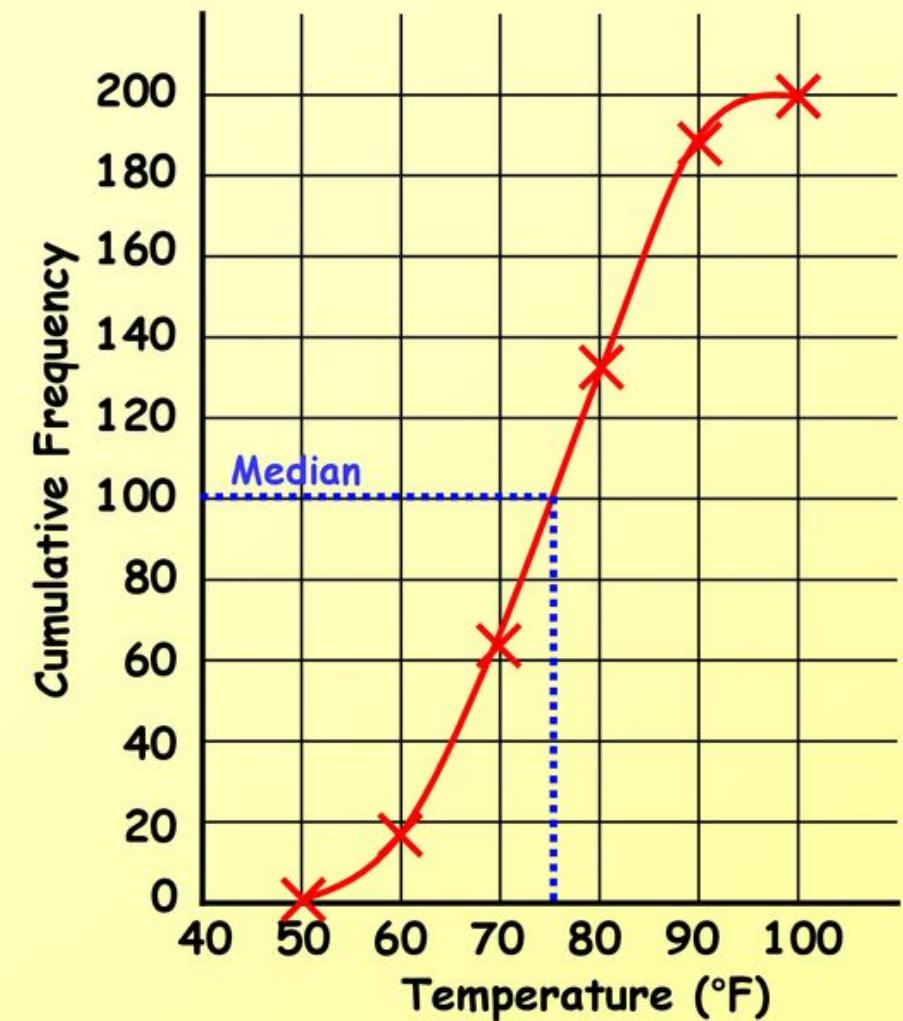


$\text{minimum} < Q_1 < \text{median} < Q_3 < \text{maximum}$

PERCENTILES AND QUARTILES



Cumulative Frequency and Quartiles



- The **median** value is the middle number of a set of data
- This can be estimated from the **Cumulative Frequency** curve.
- The median here will be the 100th value (out of 200)
- This will be roughly 75-76°F

LET'S TEST

Exercise 3.5 Consider a sample of $n = 9$ observations that are not all equal. For the following propositions, if they are true, briefly explain why they are true. If they are false, provide a counter-example to prove they are not generally true.

- a. A 10th observation is collected. If it is equal to the mean in the 9-observation sample, then the mean does not change. ✓
- b. A 10th observation is collected. If it is equal to the median in the 9-observation sample, then the median does not change. ✓
- c. A 10th observation is collected. If it is equal to the mode in the 9-observation sample, then the mode does not change. ✓
- d. A 10th and 11th observations are collected. If they are equal to the minimum and the maximum of the 9-observation sample, then the mean does not change. ↗
- e. A 10th and 11th observations are collected. If they are equal to the minimum and the maximum of the 9-observation sample, then the mode does not change. ↘

LET'S TEST

Exercise 3.5 Consider a sample of $n = 9$ observations that are not all equal. For the following propositions, if they are true, briefly explain why they are true. If they are false, provide a counter-example to prove they are not generally true.

- a. A 10th observation is collected. If it is equal to the mean in the 9-observation sample, then the mean does not change. True
- b. A 10th observation is collected. If it is equal to the median in the 9-observation sample, then the median does not change. ~~True~~
Generally False
- c. A 10th observation is collected. If it is equal to the mode in the 9-observation sample, then the mode does not change. True
- d. A 10th and 11th observations are collected. If they are equal to the minimum and the maximum of the 9-observation sample, then the mean does not change. False
- e. A 10th and 11th observations are collected. If they are equal to the minimum and the maximum of the 9-observation sample, then the mode does not change. True

DISPERSION/VARIABILITY MEASURES

RANGE

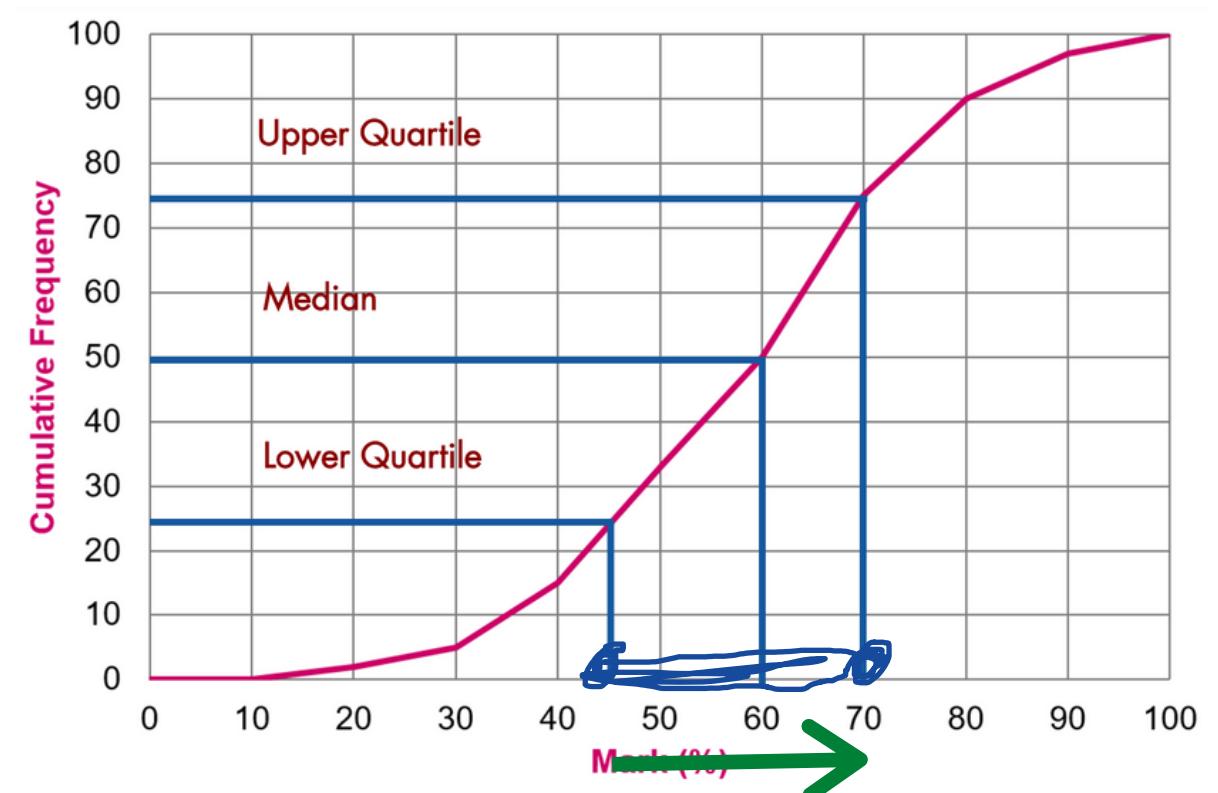
Definition 4.1 (Range) The range of a variable is the difference between the largest and the smallest observation.

INTERQUARTILE RANGE

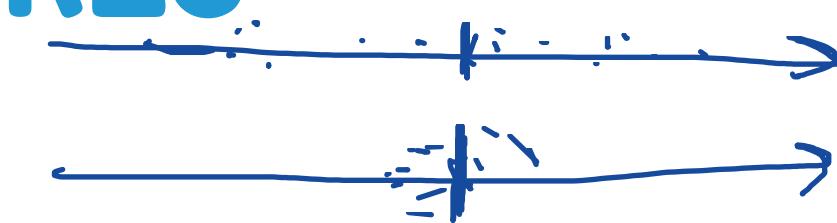
Definition 4.2 (Interquartile range) The interquartile range (IQR) measures the spread in the middle 50% of the data.

It is the difference between the value at the third quartile, Q_3 , and the observation at the first quartile, Q_1 .

$$\text{IQR} = Q_3 - Q_1$$



DISPERSION/VARIABILITY MEASURES



VARIANCE (SAMPLE AND POPULATION)

Definition 4.3 (Variance of a population) The variance of a population, σ^2 , is the sum of the squared differences of each observation with respect to the mean, divided by the population size, N .

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2$$

The variance of a sample of size n is based on the same differences, but the division is by $n - 1$.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

WHY DO WE MANY TIMES
SEE THE VARIANCE
ASSOCIATED TO AN
EXPECTED VALUE?

DISPERSION/VARIABILITY MEASURES

STANDARD DEVIATION (SAMPLE AND POPULATION)

Definition 4.4 (Standard deviation) For a population, the standard deviation is the square root of the variance

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

The sample's standard deviation, s , is the square root of the sample's variance.

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

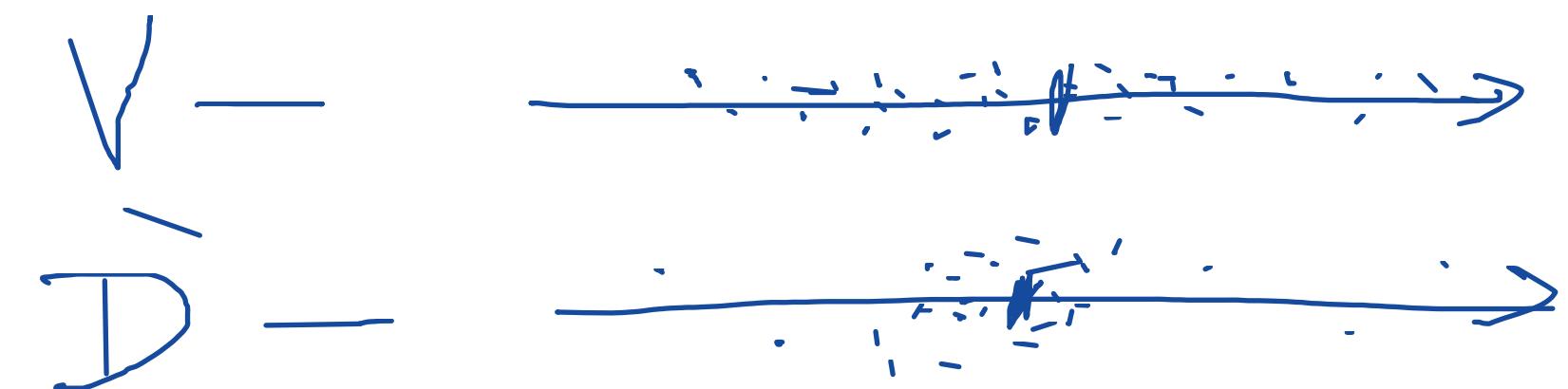
COEFFICIENT OF VARIATION

Definition 4.5 (Coefficient of variation) The coefficient of variation, CV , is a measure of relative dispersion that measures the standard deviation as a percentage of the mean (provided that the mean is positive). For a population, if $\mu > 0$,

$$CV = \frac{\sigma}{\mu}$$

For a sample, if $\bar{x} > 0$,

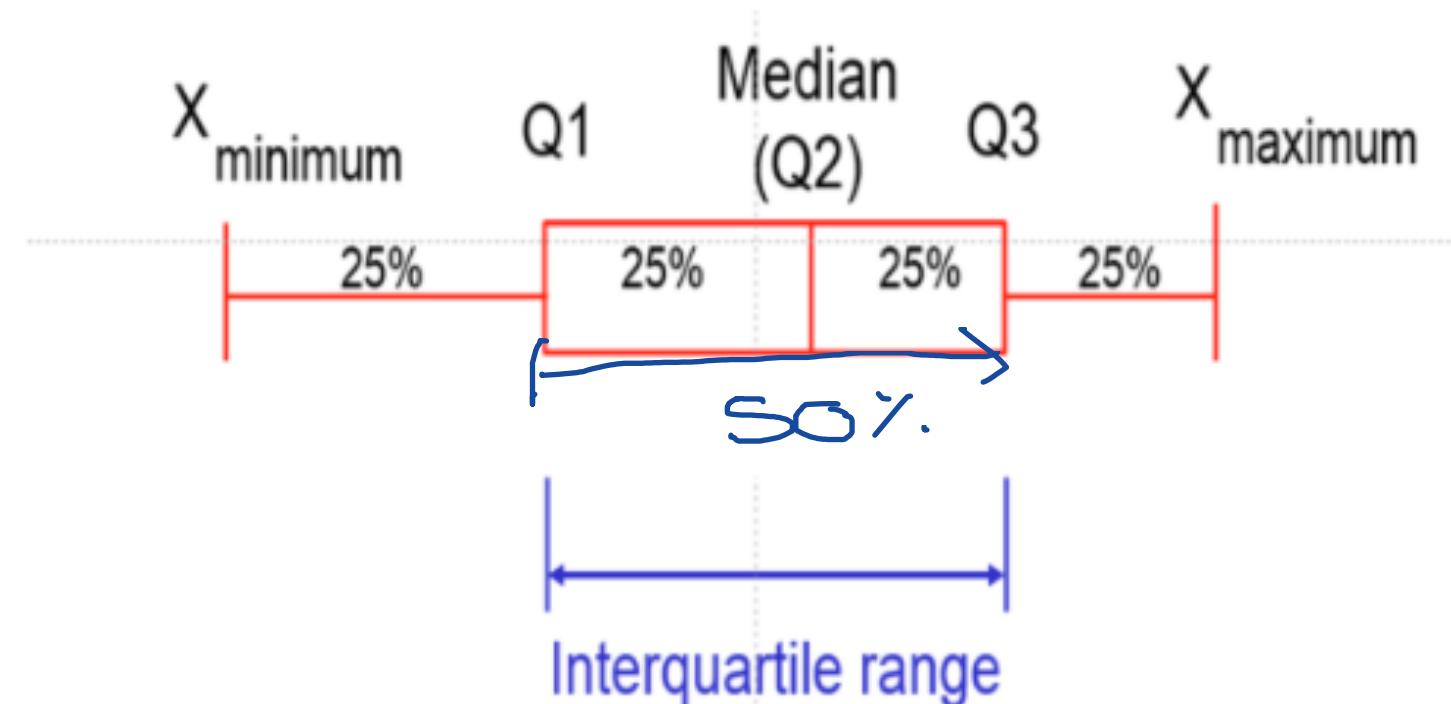
$$CV = \frac{s}{\bar{x}}$$



VISUAL REPRESENTATION

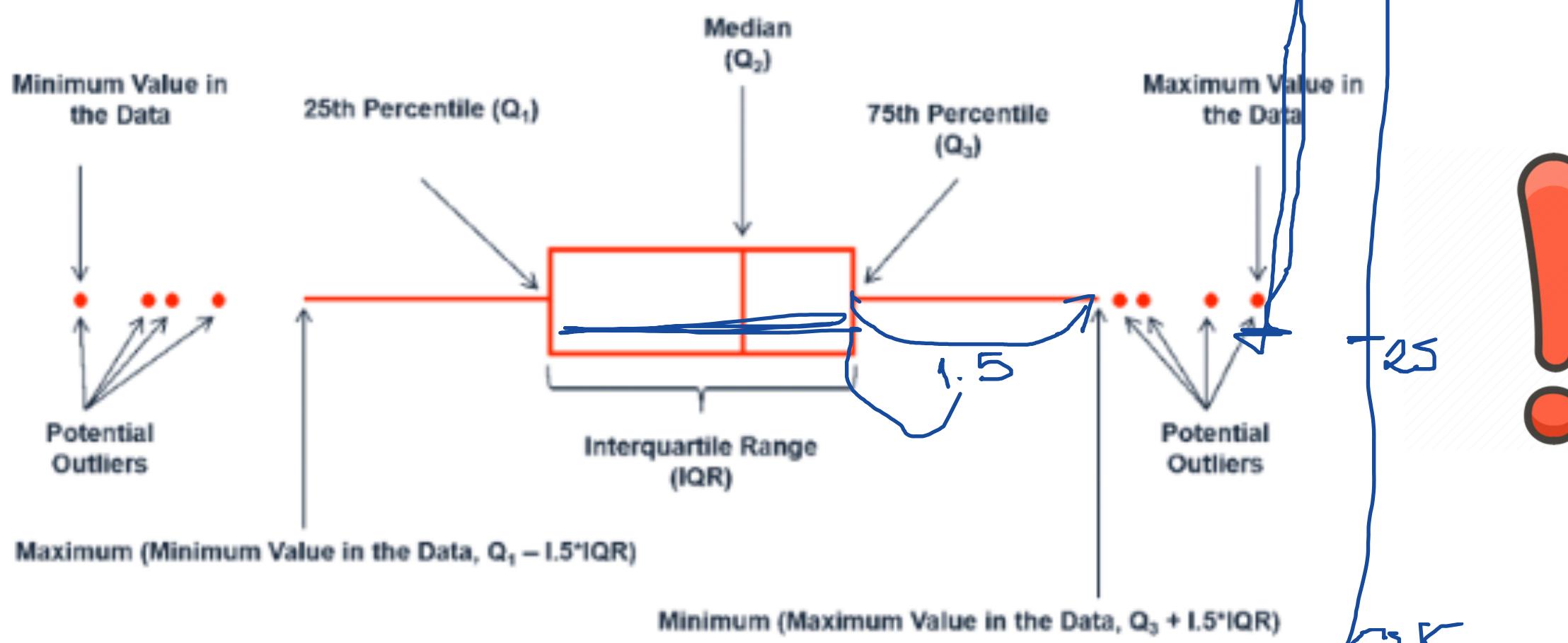
THE BOX-PLOT

This type of plot is also known as the box-and-whisker plot. There are two main alternatives for building it.



VISUAL REPRESENTATION

THE BOX-PLOT



DEFINITION OF OUTLIER

OUTLIER:
 $\text{POINT} > Q_3 + 1.5 \times \text{IQR}$

$\text{POINT} < Q_1 - 1.5 \times \text{IQR}$

COVARIANCE

COVARIANCE IS A WAY TO DESCRIBE THE **LINEAR RELATIONSHIP** BETWEEN TWO VARIABLES

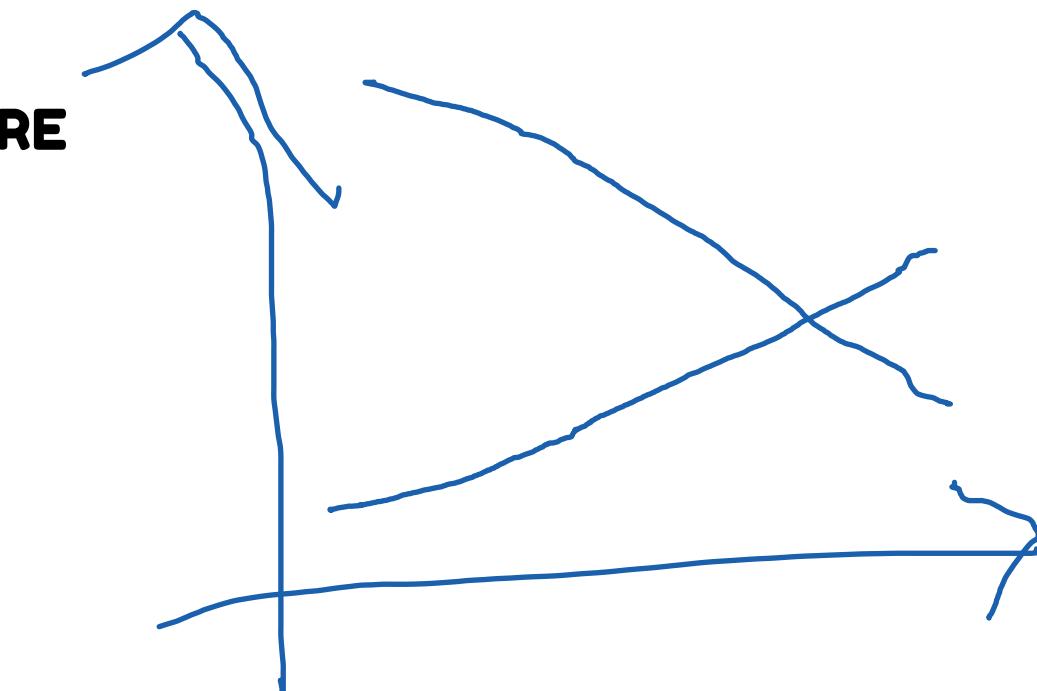
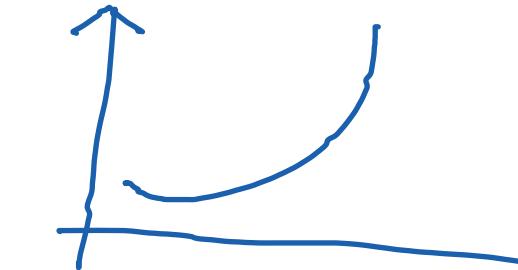
$$Cov(x, y) = \sigma_{xy} = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{N}$$

THE COVARIANCE DESCRIBES ONLY THE DIRECTION OF THE LINEAR CORRELATION BETWEEN BOTH VARIABLES AND NOT THE STRENGTH

+VE COVARIANCE -> POSITIVE RELATIONSHIP BETWEEN VARIABLES

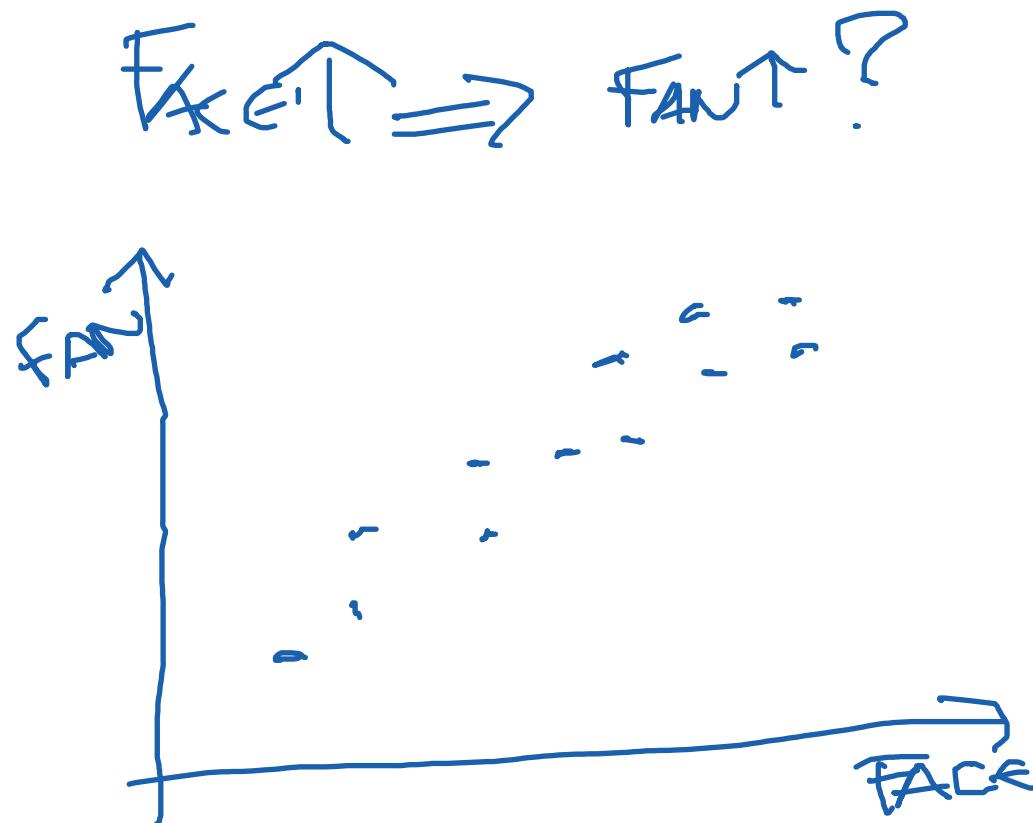
-VE COVARIANCE -> ANTI-CORRELATION BETWEEN VARIABLES. ONE GOES UP, THE OTHER GOES DOWN

THINK OF THE UNITS! ARE THEY RELATIVE OR ABSOLUTE?



COVARIANCE

EXAMPLE FROM TEXTBOOK



Example 2.19 Facebook Posts and Interactions (Covariance and Correlation Coefficient)

RELEVANT Magazine (a culture magazine) keeps in touch and informs their readers by posting updates through various social networks. These updates take up a large part of both the marketing and editorial teams' time. Because these updates take so much time, marketing is interested in knowing whether reducing posts (updates) on Facebook (a specific site) will also lessen their fan interaction; if not, both departments may pursue using their time in more productive ways. The weekly number of posts (updates) and fan interactions for Facebook during a 9-week period are recorded in Table 2.10. Compute the covariance and correlation between Facebook posts (site updates) and fan interactions. The data are stored in the data file RELEVANT Magazine.

COVARIANCE

EXAMPLE FROM TEXTBOOK

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$$Cov(x, y) = \sigma_{xy} = \frac{\sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y)}{N}$$

Table 2.10 Facebook Posts (site updates) and Fan Interactions

Facebook posts (updates), x	16	31	27	23	15	17	17	18	14
Fan interactions, y	165	314	280	195	137	286	199	128	462

Solution The computation of covariance and correlation between Facebook posts (site updates) and fan interactions are illustrated in Table 2.11. The mean and the variance in the number of Facebook posts are found to be approximately

$$\bar{x} = 19.8 \quad \text{and} \quad s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1} = 34.694$$

and the mean and the variance in the number of fan interactions are found to be approximately

$$\bar{y} = 240.7 \quad \text{and} \quad s_y^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1} = 11,369.5$$

Table 2.11 Facebook Posts and Fan Interactions (Covariance and Correlation)

x	y	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
16	165	-3.8	14.44	-75.7	5,730.49	287.66
31	314	11.2	125.44	73.3	5,372.89	820.96
27	280	7.2	51.84	39.3	1,544.49	282.96
23	195	3.2	10.24	-45.7	2,088.49	-146.24
15	137	-4.8	23.04	-103.7	10,753.69	497.76
17	286	-2.8	7.84	45.3	2,052.09	-126.84
17	199	-2.8	7.84	-41.7	1,738.89	116.76
18	128	-1.8	3.24	-112.7	12,701.29	202.86
14	462	-5.8	33.64	221.3	48,973.69	-1,283.54
$\bar{x} = 19.8 \quad \bar{y} = 240.7$						$\Sigma = 652.34$

COVARIANCE

APPLYING THE EQUATION ABOVE (FOR SAMPLE)

$$Cov(x, y) = s_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{652.34}{8} = 81.542$$

BECAUSE
IT IS A SAMPLE

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μ_{FP}

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and the mean and the variance in the number of fan interactions are found to be approximately

μ_{FI}

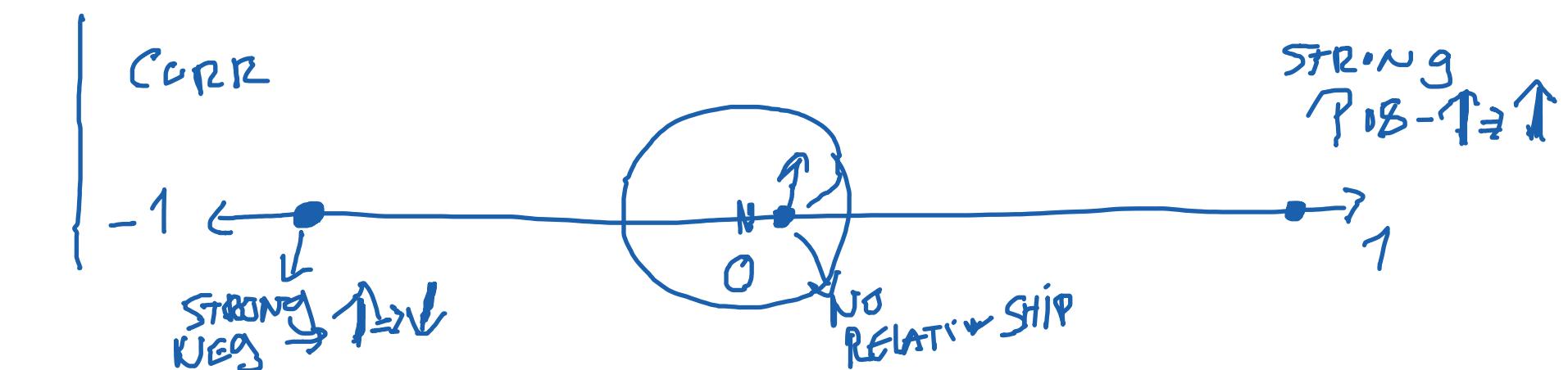
$$\bar{y} = 240.7 \text{ and } s_y^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1} = 11,369.5$$

$(16 - \mu_x)(65 - \mu_y)$
 19.8

Table 2.11 Facebook Posts and Fan Interactions (Covariance and Correlation)

x	y	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
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PEARSON CORRELATION

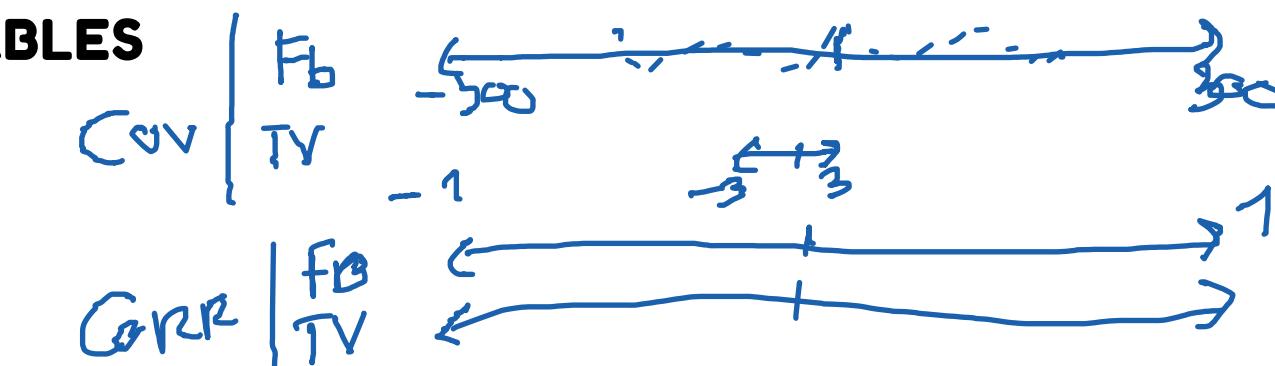


THE CORRELATION PROVIDES THE DIRECTION AND STRENGTH OF THE LINEAR RELATIONSHIP BETWEEN TWO VARIABLES

$$\rho = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y} = \frac{\text{COVARIANCE}}{\text{STANDARDIZATION FACTOR}}$$

UNLIKE THE COVARIANCE, THE CORRELATION IS A RELATIVE MEASURE AND THEREFORE CAN BE USED AS COMPARISON BETWEEN DIFFERENT MEASURES/DATASETS

CORRELATION GIVES US A STANDARDIZED MEASURE OF THE LINEAR RELATIONSHIP BETWEEN THE TWO VARIABLES

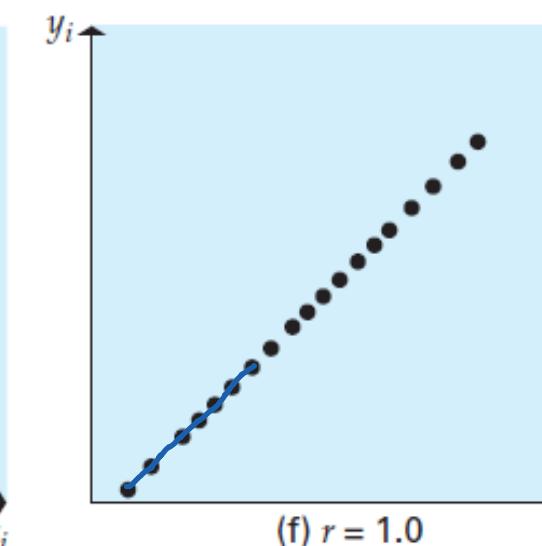
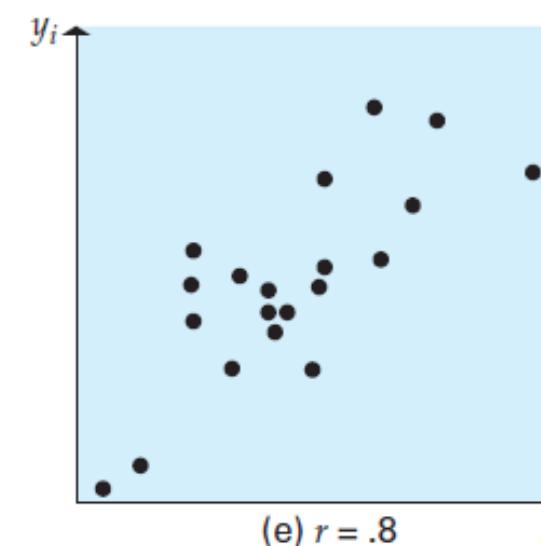
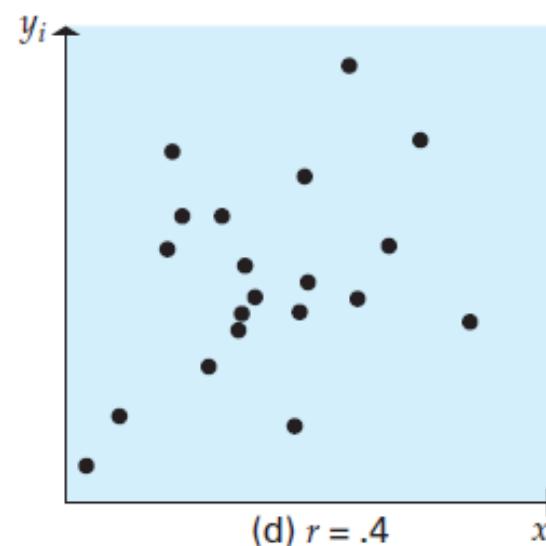
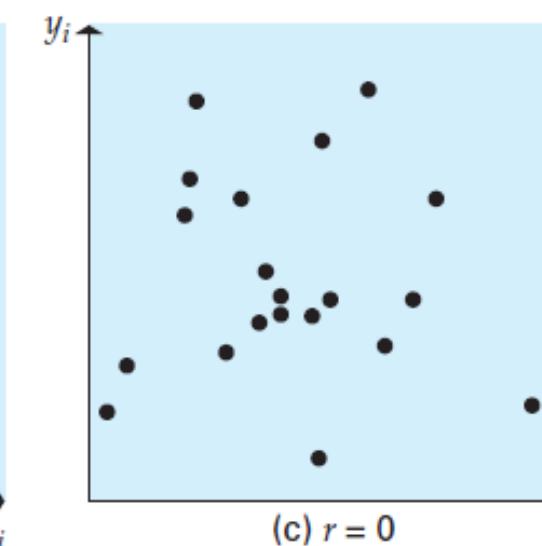
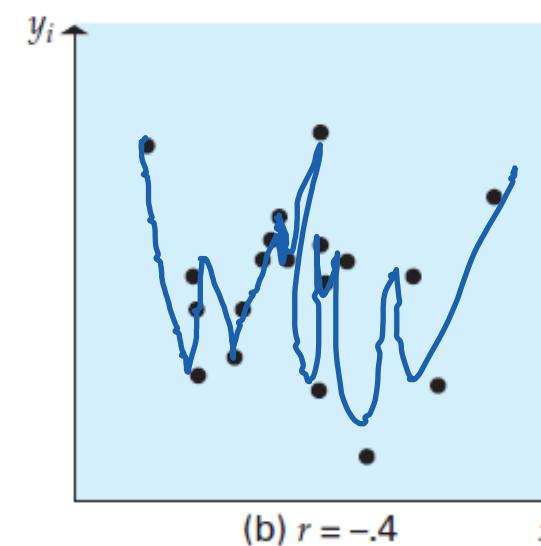
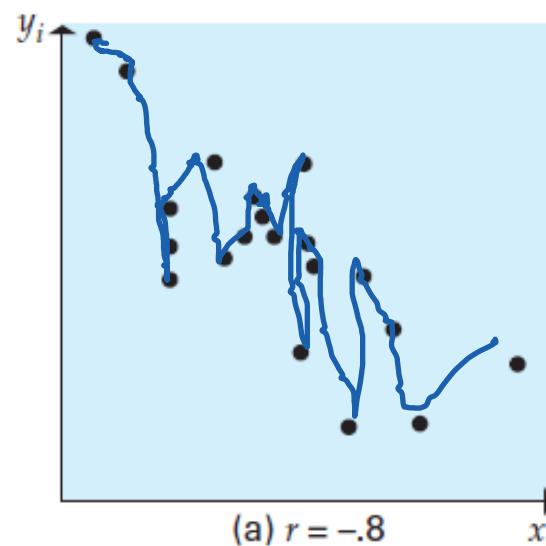


+1 INDICATES A PERFECT LINEAR CORRELATION

-1 INDICATES A PERFECT LINEAR ANTI-CORRELATION

PEARSON CORRELATION

THE CORRELATION PROVIDES THE DIRECTION AND STRENGTH OF THE LINEAR RELATIONSHIP BETWEEN TWO VARIABLES



WRAPPING IT UP

Corr $\not\Rightarrow$ Causality

$$Cov(x, y) = s_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{652.34}{8} = 81.542$$

$$r = \frac{Cov(x, y)}{s_x s_y} = \frac{81.542}{\sqrt{34.694} \sqrt{11,369.5}} = 0.1298$$

**DATA DOES NOT SUPPORT STRONG LINEAR
RELATIONSHIP BETWEEN POSTS AND FAN
INTERACTION**

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and the mean and the variance in the number of fan interactions are found to be approximately

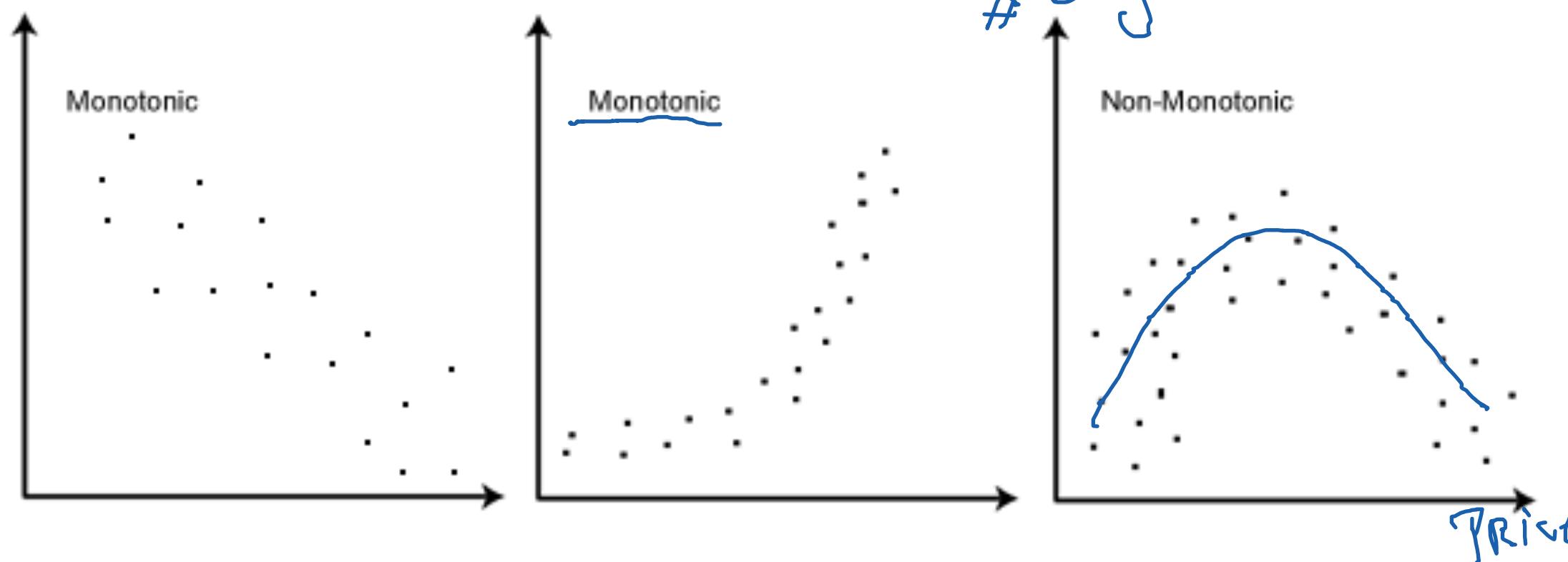
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Table 2.11 Facebook Posts and Fan Interactions (Covariance and Correlation)

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SPEARMAN CORRELATION

THE SPEARMAN CORRELATION DOESN'T LOOK FOR A LINEAR RELATIONSHIP BUT RATHER A MONOTONIC RELATIONSHIP (IN THE SAME DIRECTION). IT DOES SO BY COMPARING THE RANK OF THE POINTS

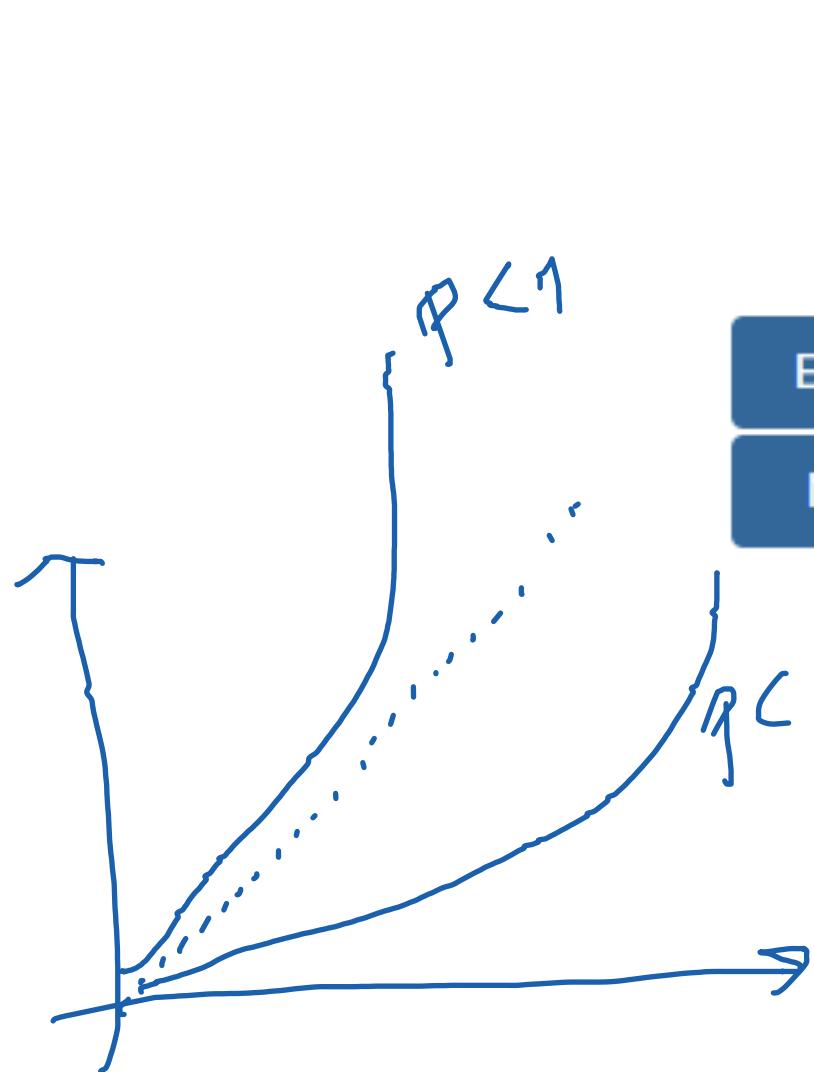


$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

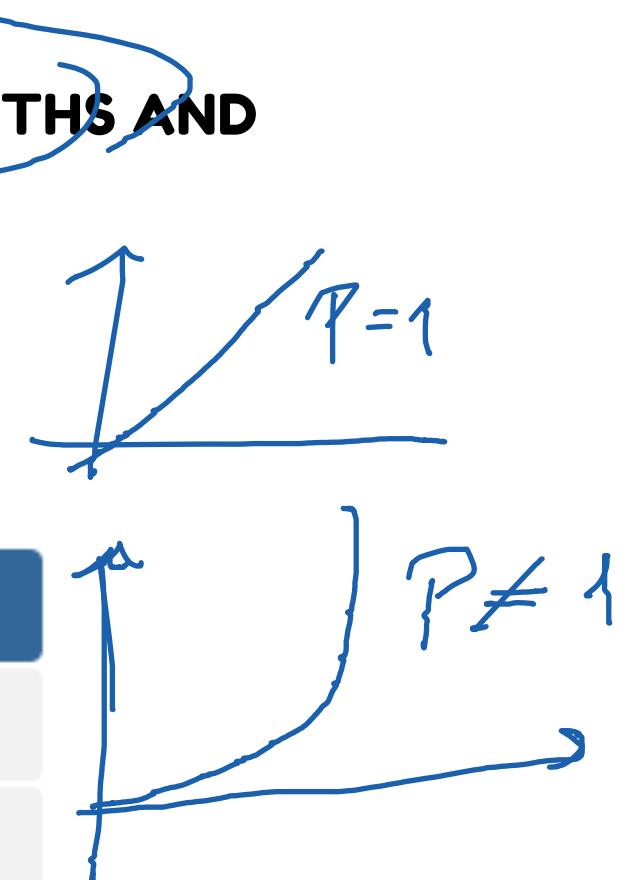
IN SPEARMAN THE RANK OF EACH POINT IS WHAT MATTERS RATHER THAN THE ABSOLUTE VALUE

SPEARMAN CORRELATION

LET'S CALCULATE THE SPEARMAN CORRELATION BETWEEN THE GRADES OF A CLASS IN MATHS AND ENGLISH



Marks									
English	56	75	45	71	61	64	58	80	76
Maths	66	70	40	60	65	56	59	77	67



$$\rho = \frac{\text{COV}(\text{ENG}, \text{MATH})}{\sigma_{\text{ENG}} \times \sigma_{\text{MATH}}} =$$

$$\frac{\sum (e_i - \bar{e}_i) \cdot (m_i - \bar{m}_i)}{(n-1) \sigma_e \cdot \sigma_m}$$

SPEARMAN CORRELATION

START BY RANKING THEM WITHIN THE RESPECTIVE COLUMN

English (mark)	Maths (mark)	Rank (English)	Rank (maths)
56	66	9	4
75	70	3	2
45	40	10	10
71	60	4	7
61	65	6.5	5
64	56	5	9
58	59	8	8
80	77	1	1
76	67	2	3
61	63	6.5	6

SPEARMAN CORRELATION

CALCULATE THE DIFFERENCE OF THE RANK FOR EACH STUDENT

English (mark)	Maths (mark)	Rank (English)	Rank (maths)	d	d^2
56	66	9	4	5	25
75	70	3	2	1	1
45	40	10	10	0	0
71	60	4	7	3	9
62	65	6	5	1	1
64	56	5	9	4	16
58	59	8	8	0	0
80	77	1	1	0	0
76	67	2	3	1	1
61	63	7	6	1	1

$$\sum d_i^2 = 25 + 1 + 9 + 1 + 16 + 1 + 1 = 54$$

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

$$\rho = 1 - \frac{6 \times 54}{10(10^2 - 1)}$$

$$\rho = 1 - \frac{324}{990}$$

$$\rho = 1 - 0.33$$

- 4 → $\rho = 0.67$ → 1

EXTRA - KENDALL TAU CORRELATION

SIMILARLY TO SPEARMAN'S CORRELATION KENDAL TAU'S DOESN'T LOOK FOR A LINEAR RELATIONSHIP BUT RATHER A MONOTONIC RELATIONSHIP (IN THE SAME DIRECTION). IT DOES SO BY COMPARING THE RANK OF THE POINTS

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{\binom{n}{2}}.$$

Where $\binom{n}{2} = \frac{n(n-1)}{2}$ is the binomial coefficient for the number of ways to choose two items from n items.

Let $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be a set of observations of the joint random variables X and Y respectively, such that all the values of (x_i) and (y_i) are unique. Any pair of observations (x_i, y_i) and (x_j, y_j) , where $i < j$, are said to be *concordant* if the ranks for both elements (more precisely, the sort order by x and by y) agree: that is, if both $x_i > x_j$ and $y_i > y_j$; or if both $x_i < x_j$ and $y_i < y_j$. They are said to be *discordant*, if $x_i > x_j$ and $y_i < y_j$; or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

EXTRA - KENDALL TAU CORRELATION

SIMILARLY TO SPEARMAN'S CORRELATION KENDAL TAU'S DOESN'T LOOK FOR A LINEAR RELATIONSHIP BUT RATHER A MONOTONIC RELATIONSHIP (IN THE SAME DIRECTION). IT DOES SO BY COMPARING **THE RANK OF THE POINTS**

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{\binom{n}{2}}. [3]$$

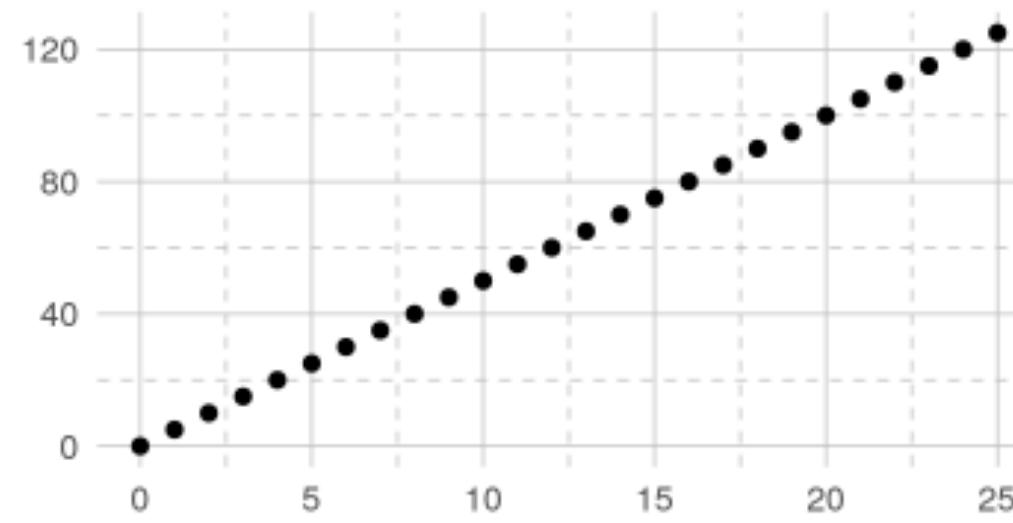
Where $\binom{n}{2} = \frac{n(n - 1)}{2}$ is the **binomial coefficient** for the number of ways to choose two items from n items.



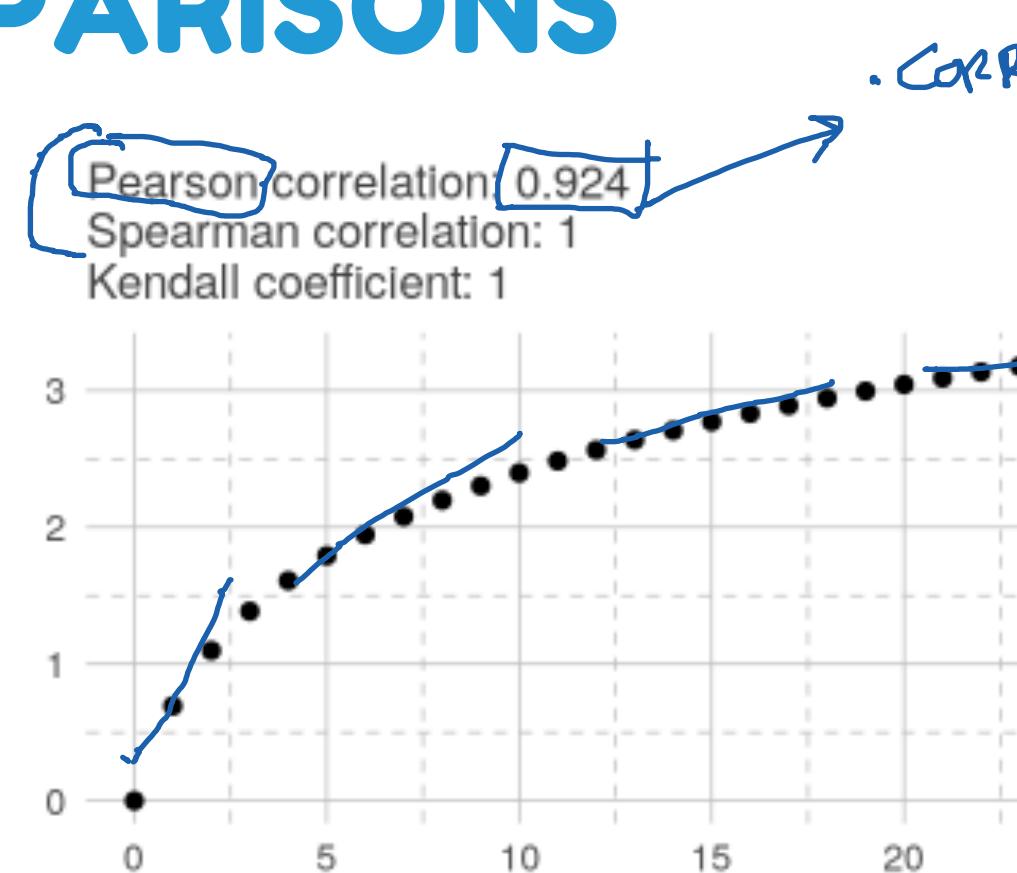
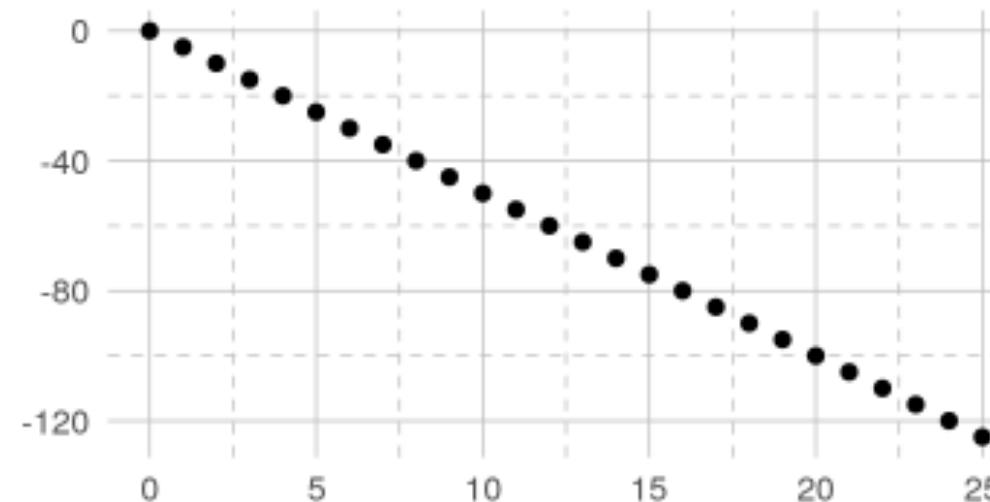
Let $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be a set of observations of the joint random variables X and Y respectively, such that all the values of (x_i) and (y_i) are unique. Any pair of observations (x_i, y_i) and (x_j, y_j) , where $i < j$, are said to be *concordant* if the ranks for both elements (more precisely, the sort order by x and by y) agree: that is, if both $x_i > x_j$ and $y_i > y_j$; or if both $x_i < x_j$ and $y_i < y_j$. They are said to be *discordant*, if $x_i > x_j$ and $y_i < y_j$; or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

CORRELATION COMPARISONS

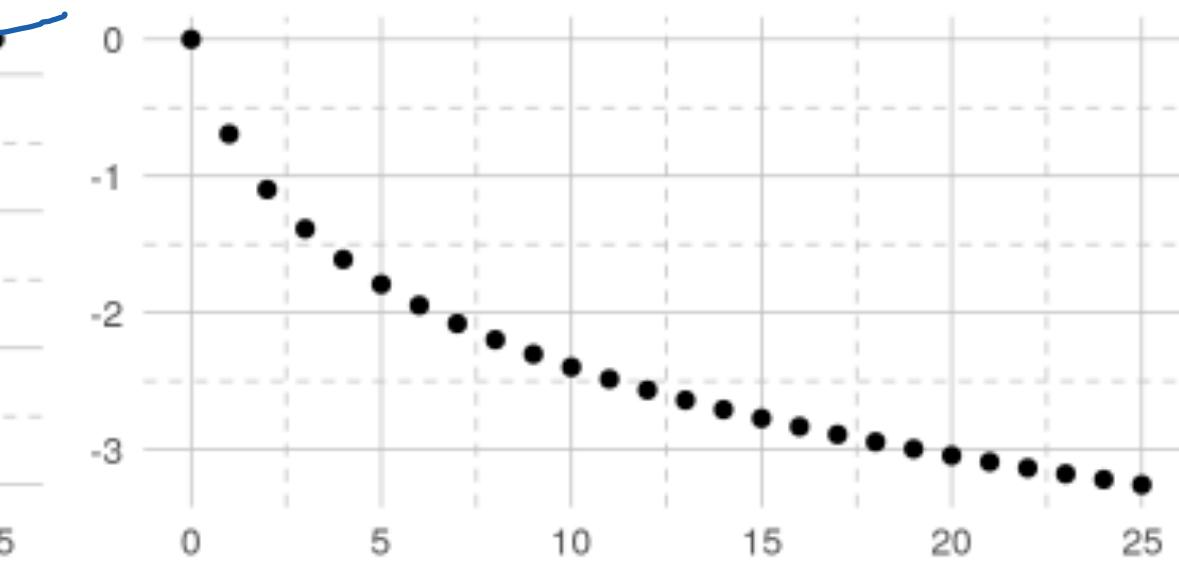
Pearson correlation: 1
Spearman correlation: 1
Kendall coefficient: 1



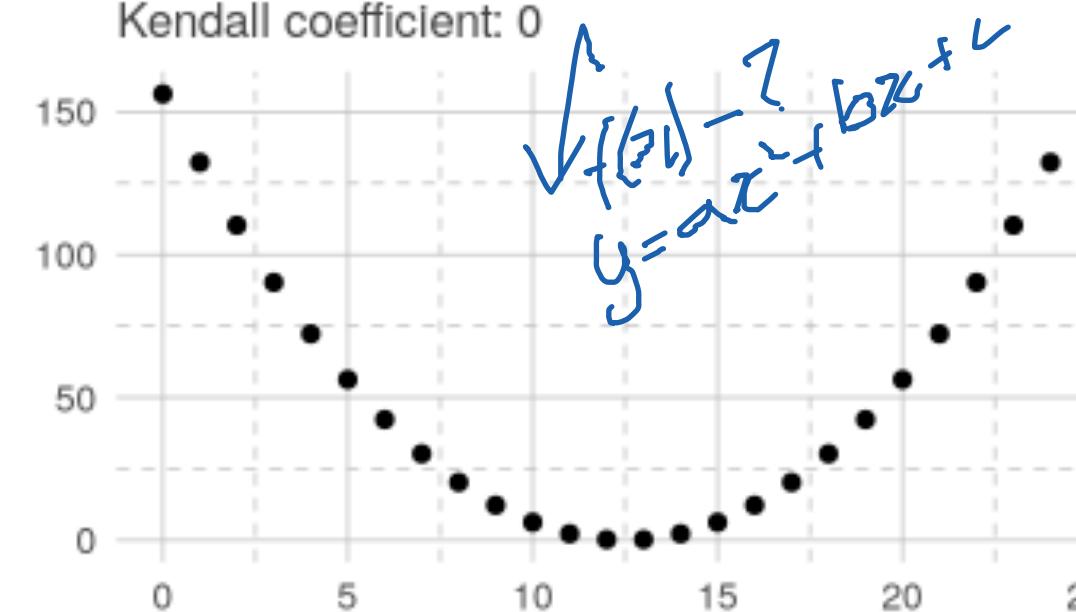
Pearson correlation: -1
Spearman correlation: -1
Kendall coefficient: -1



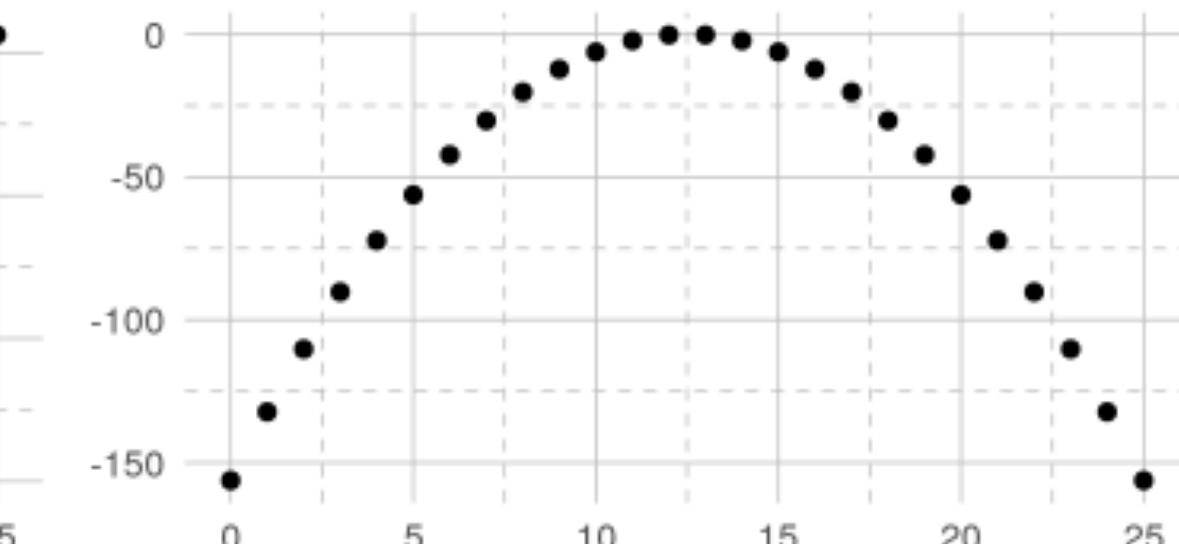
Pearson correlation: -0.924
Spearman correlation: -1
Kendall coefficient: -1



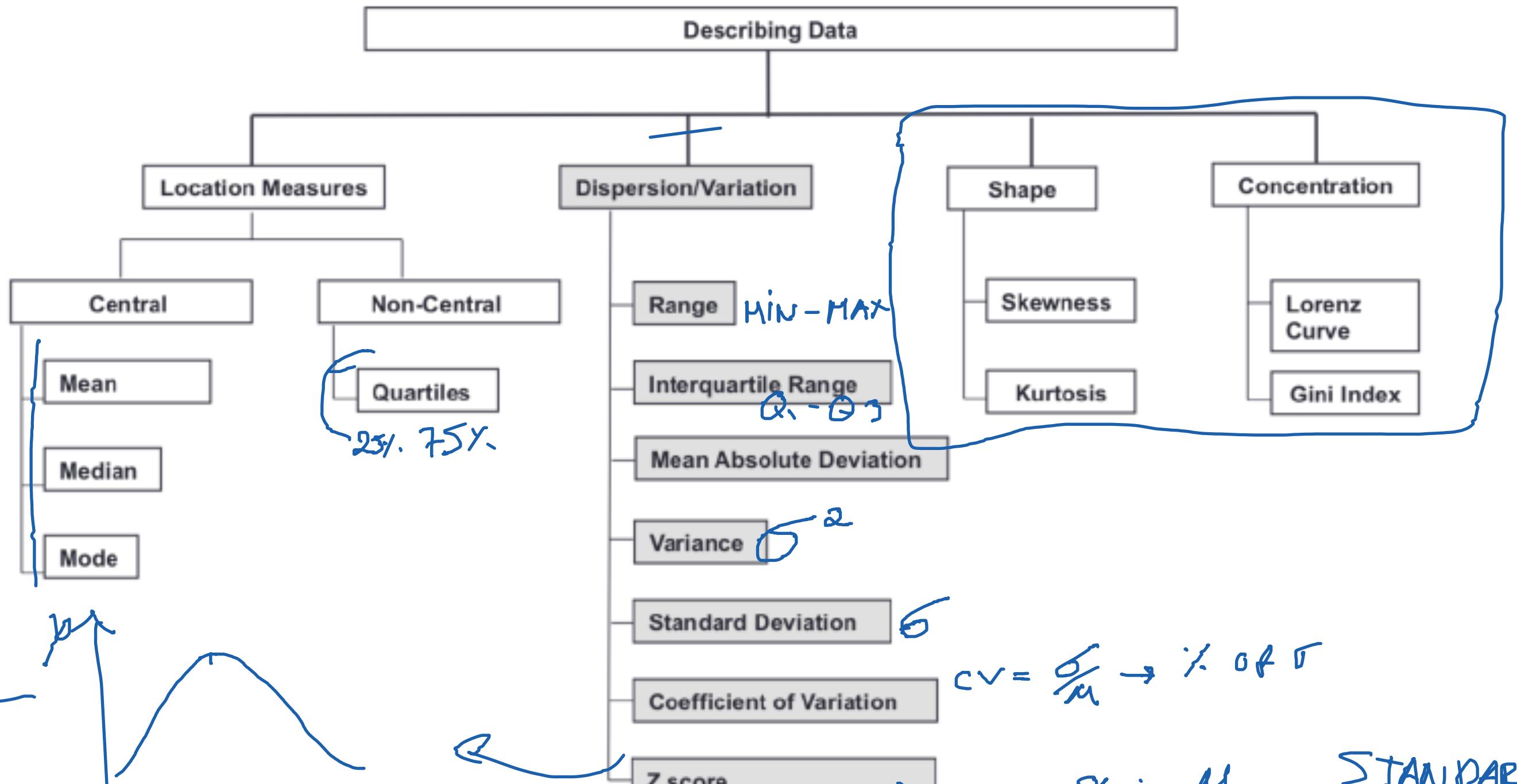
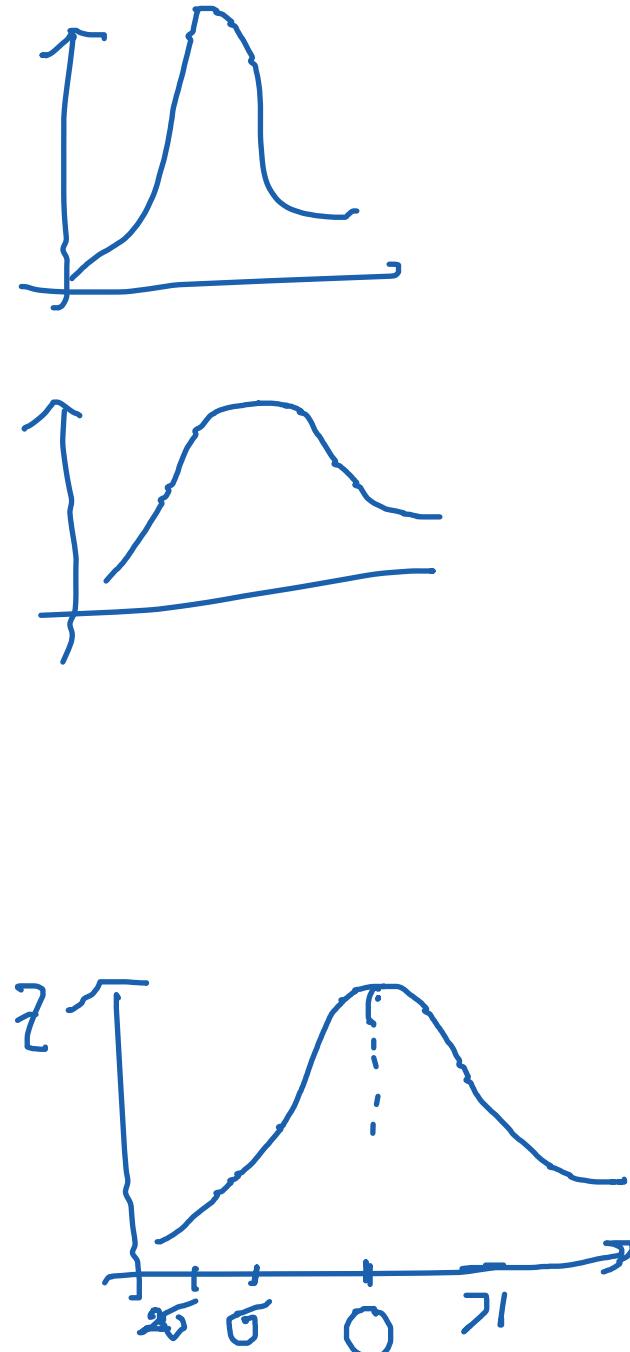
Pearson correlation: 0
Spearman correlation: 0
Kendall coefficient: 0



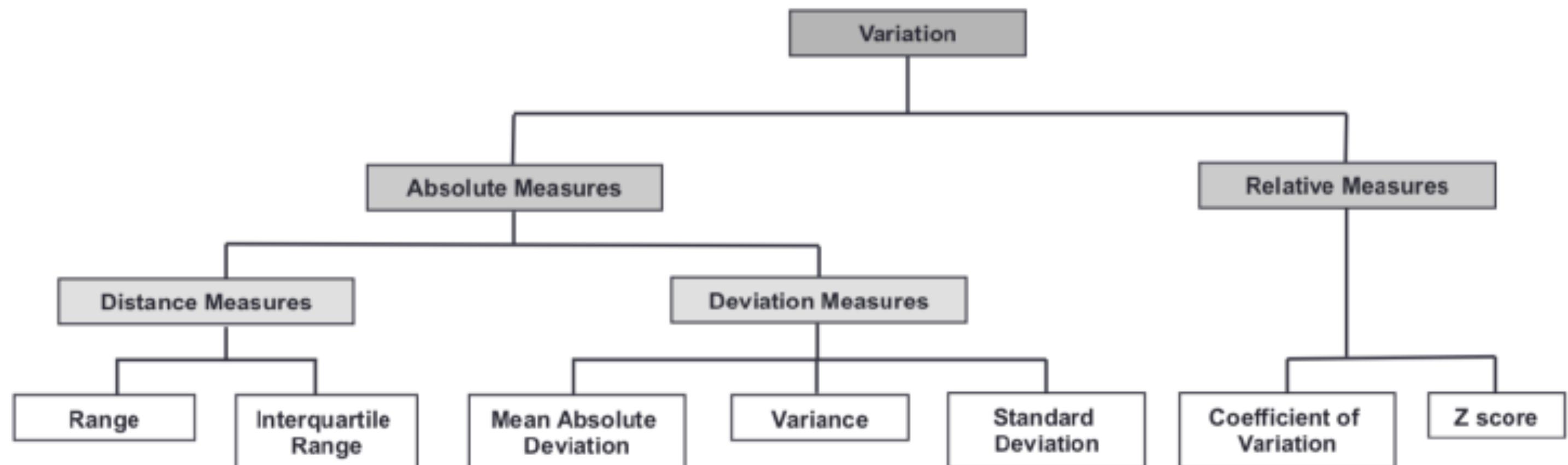
Pearson correlation: 0
Spearman correlation: 0
Kendall coefficient: 0



A SUMMARY



A SUMMARY - VARIABILITY



REFERENCES

CHAPTER 2, NEWBOLD, CALSON, THORNE, STATISTICS FOR BUSINESS & ECONOMICS

<https://statistics.laerd.com/statistical-guides/spearmans-rank-order-correlation-statistical-guide.php>

<https://towardsdatascience.com/do-you-have-a-trustworthy-gut-the-most-counterintuitive-probability-problems-7b76aff941cb>

ANY
QUESTIONS ?