Searching, Sorting and Greedy Algorithms

Searching, Sorting and Greedy Algorithms

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Searching Algorithms

Linear and Binary Search

Search Algorithm

- **Search algorithm** == an algorithm for finding an item with specified properties among a collection of items
- Different types of searching algorithms:
  - For virtual search spaces
    - Satisfy specific mathematical equations
    - Try to exploit partial knowledge about a structure
  - For sub-structures of a given structure
    - A graph, a string, a finite group
  - Search for the min / max of a function, etc.

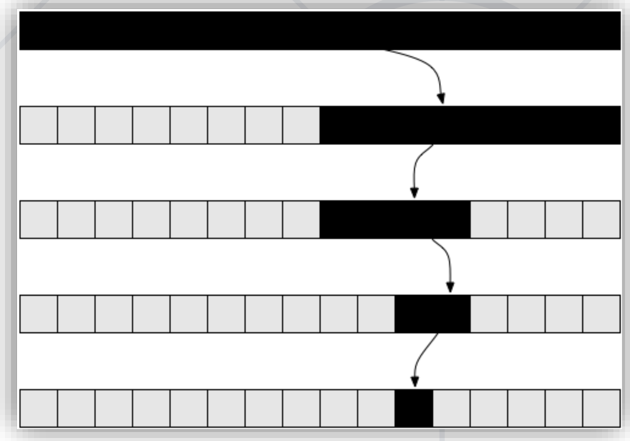
Linear Search

- **Linear search** finds a particular value in a list
  - Checking every one of the elements
  - One at a time, in sequence
  - Until the desired one is found
- Worst & average performance: O(n)

```
for each item in the list:
  if that item has the desired value,
    return the item's location
return nothing
```

Binary Search

- Binary search finds an item within a ordered data structure
- At each step, compare the input with the middle element
  - The algorithm repeats its action to the left or right sub-structure
- Average performance: **O(log(n))**
- See the visualization



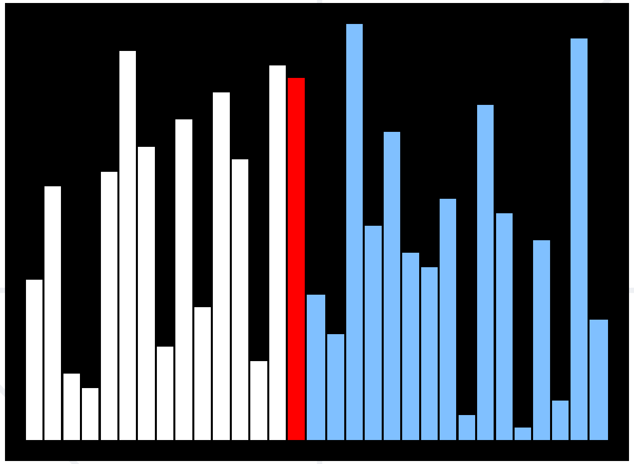
Binary Search (Iterative)

```
static int BinarySearch(int[] numbers, int searchNumber) {
  var left = 0;
  var right = numbers.Length - 1;
  while (left <= right) {
    var mid = (left + right) / 2;
    if (numbers[mid] == searchNumber)
      return mid;
    if (searchNumber > numbers[mid])
      left = mid + 1;
    else
      right = mid - 1;
  }
  return -1;
}
```

Simple Sorting Algorithms

Selection Sort and Bubble Sort

What is a Sorting Algorithm?



- **Sorting algorithm**
  - An algorithm that rearranges elements in a list
    - In non-decreasing order
  - Elements must be **comparable**
- More formally
  - The **input** is a sequence / list of elements
  - The **output** is an rearrangement / **permutation** of elements
    - In non-decreasing order

Sorting – Example

- Efficient sorting algorithms are important for:
  - Producing human-readable output
  - Canonicalizing data – making data uniquely arranged
  - In conjunction with other algorithms, like binary searching
- Example of sorting:

Unsorted list

10	3	7	3	4

→ sorting →

Sorted list

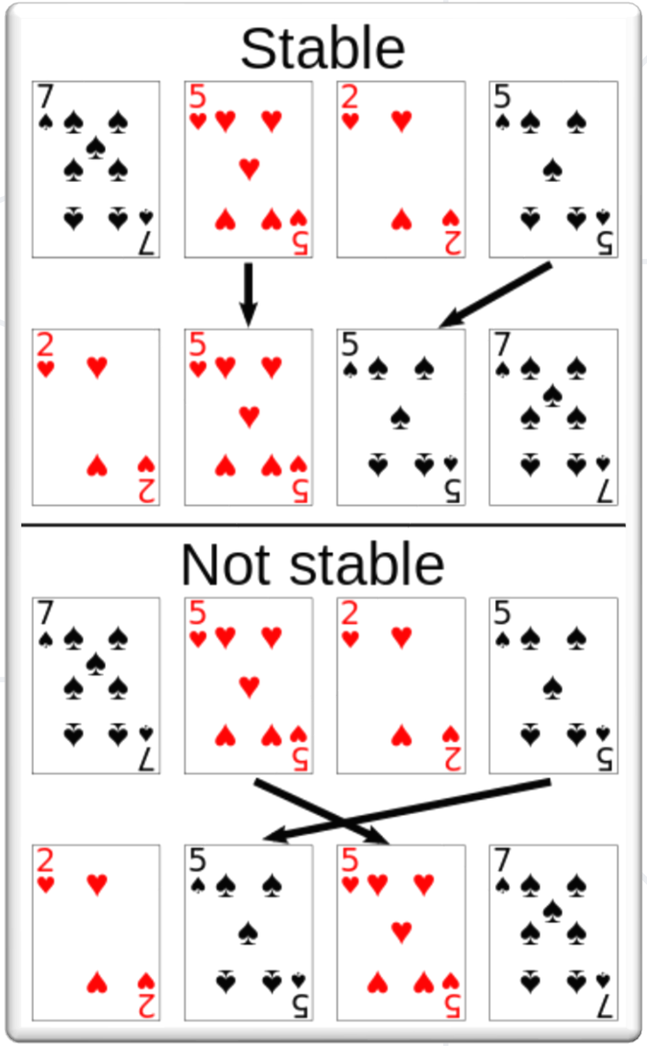
3	3	4	7	10

Sorting Algorithms: Classification

- Sorting algorithms are often classified by:
  - Computational **complexity** and memory usage
    - Worst, average and best-case behavior
  - **Recursive** / non-recursive
  - **Stability** – stable / unstable
  - **Comparison-based** sort / non-comparison based

Stability of Sorting

- **Stable** sorting algorithms
  - Maintain the order of equal elements
  - If two items compare as equal, their relative order is preserved
- **Unstable** sorting algorithms
  - Rearrange the equal elements in unpredictable order
- Often **different elements** have **same key** used for equality comparing

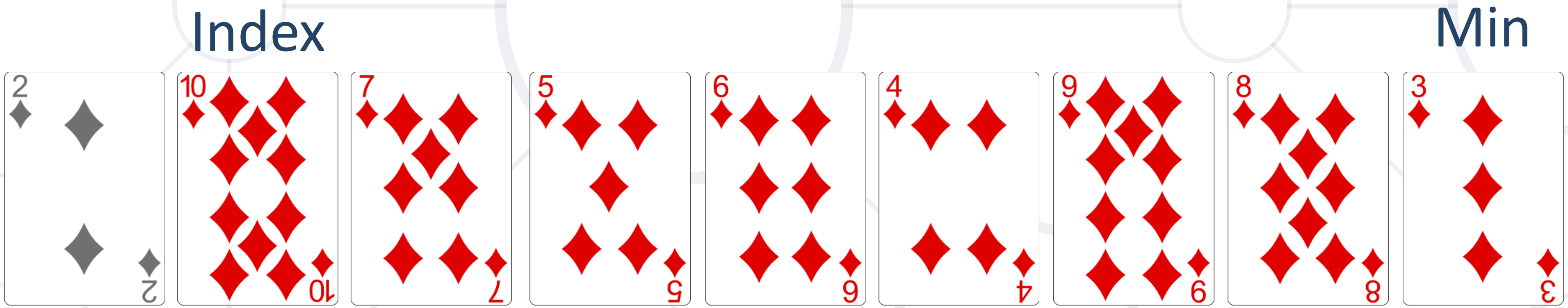


Selection Sort

- **Selection sort** – simple, but inefficient algorithm
  - Swap the first with the min element on the right, then the second, etc.
  - Memory: **O(1)**
  - Time: **$O(n^2)$**
  - Stable: No
  - Method: Selection

Selection Sort Visualization

Selection Sort Visualization



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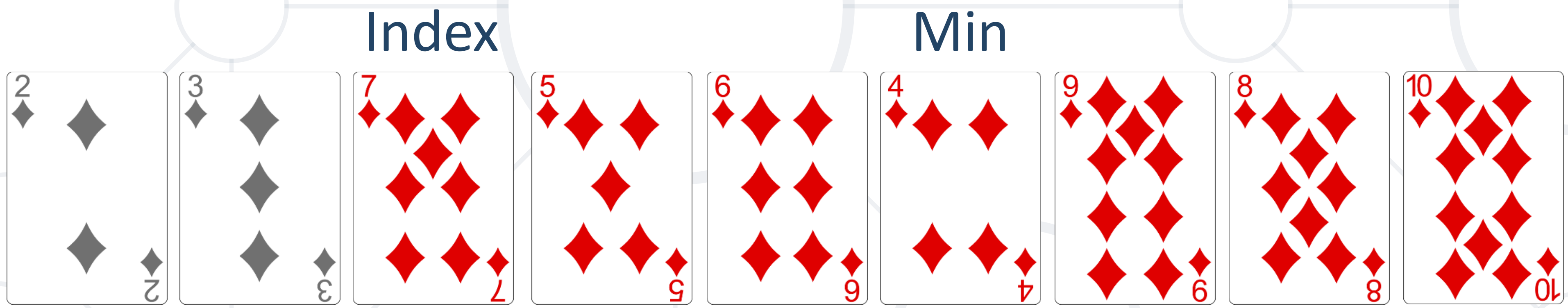
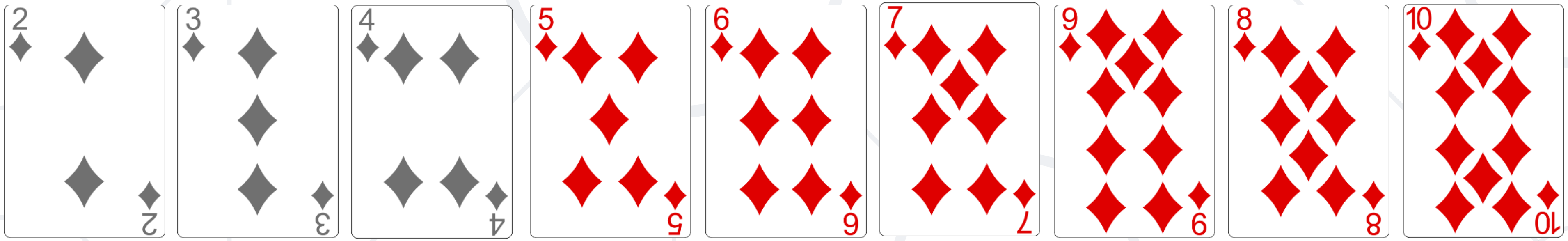


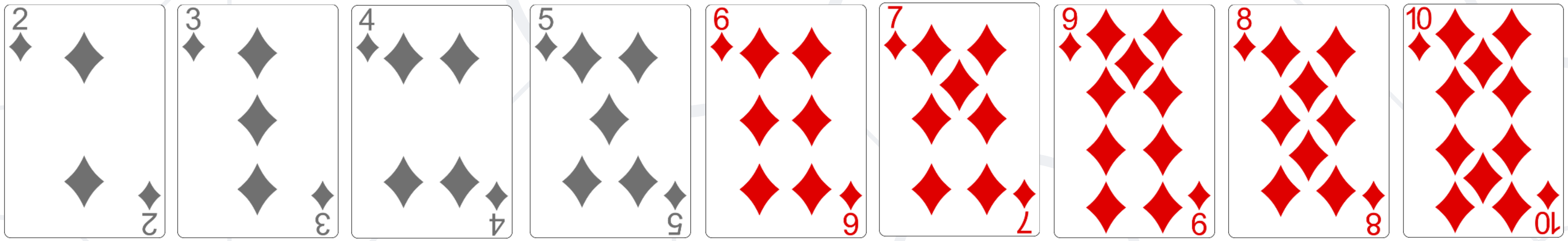
Selection Sort Visualization

Min
Index



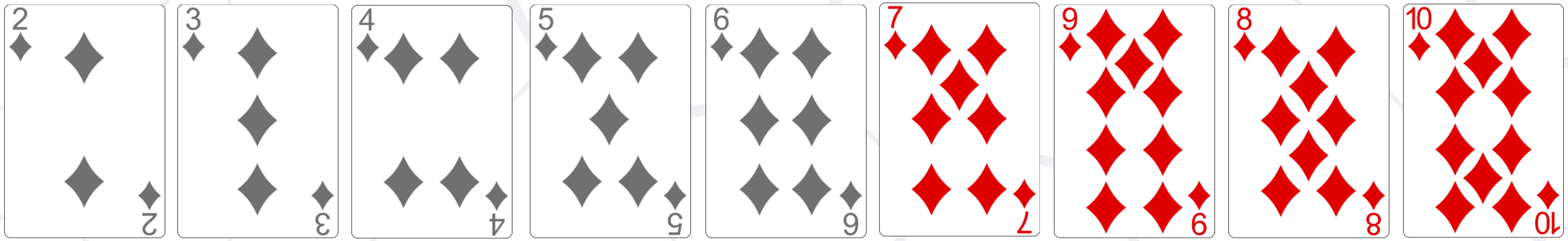
Selection Sort Visualization

Min
Index

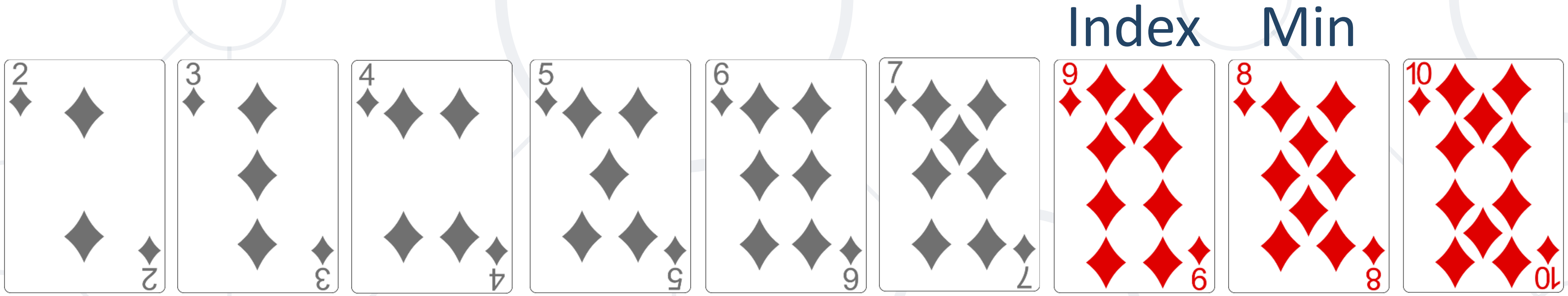


Selection Sort Visualization

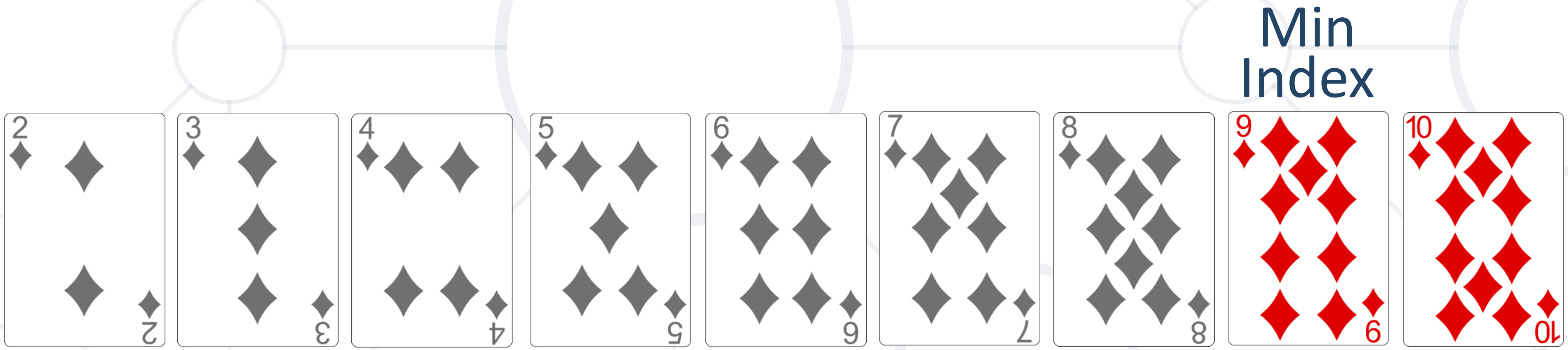
Min Index



Selection Sort Visualization



Selection Sort Visualization



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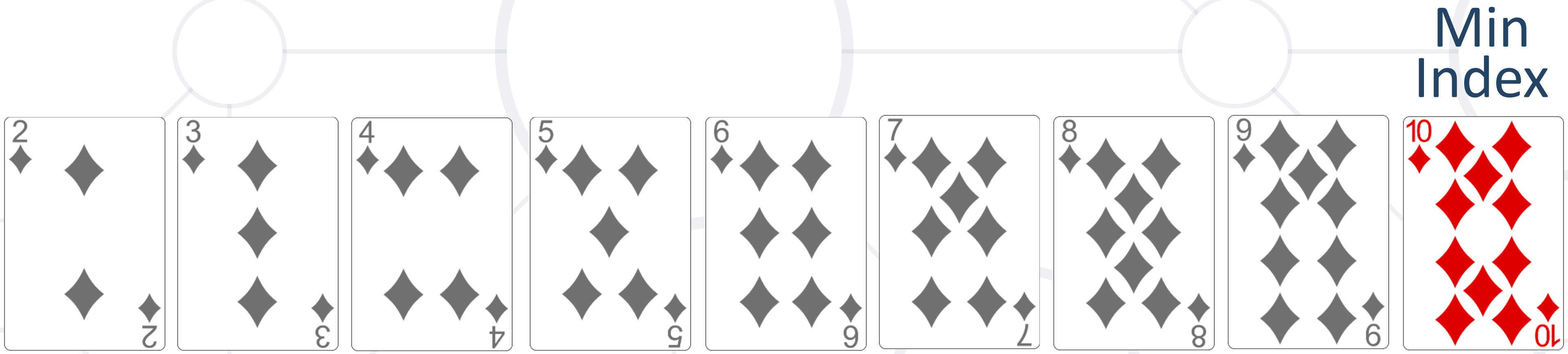

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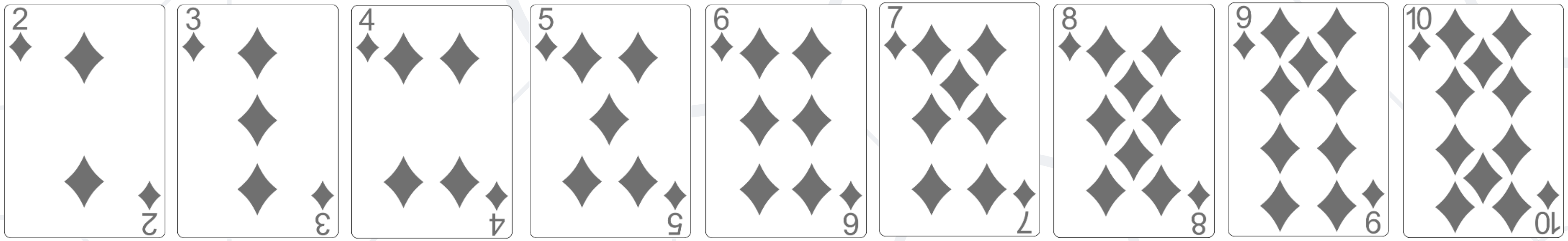

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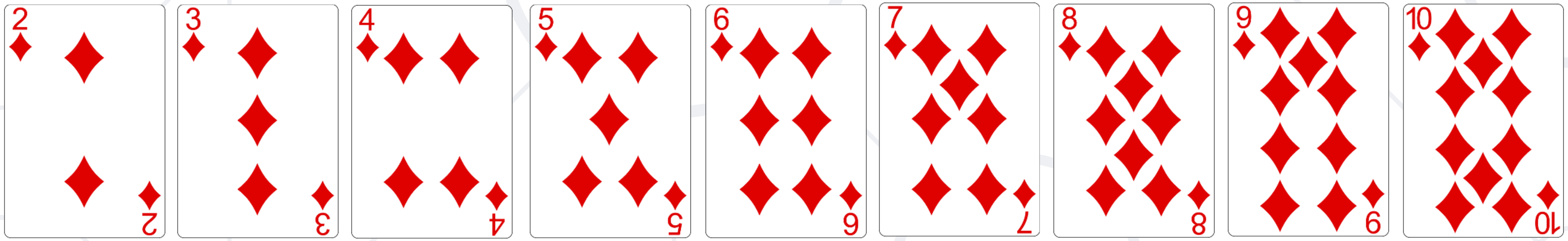


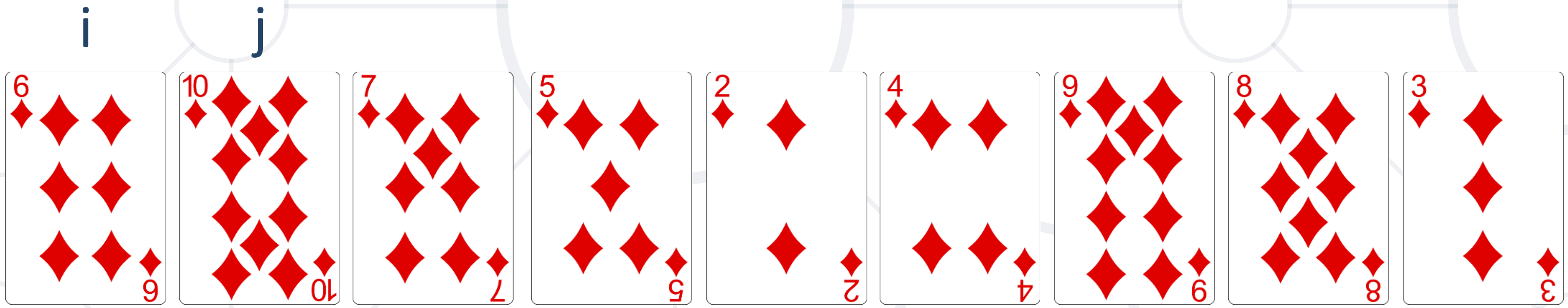
Selection Sort Code

```
for (int index = 0; index < arr.Length; index++) {
    var min = index;
    for (int curr = index + 1; curr < arr.Length; curr++) {
        if (arr[curr] < arr[min]) {
            min = curr;
        }
    }
    Swap(arr, index, min);
}
```

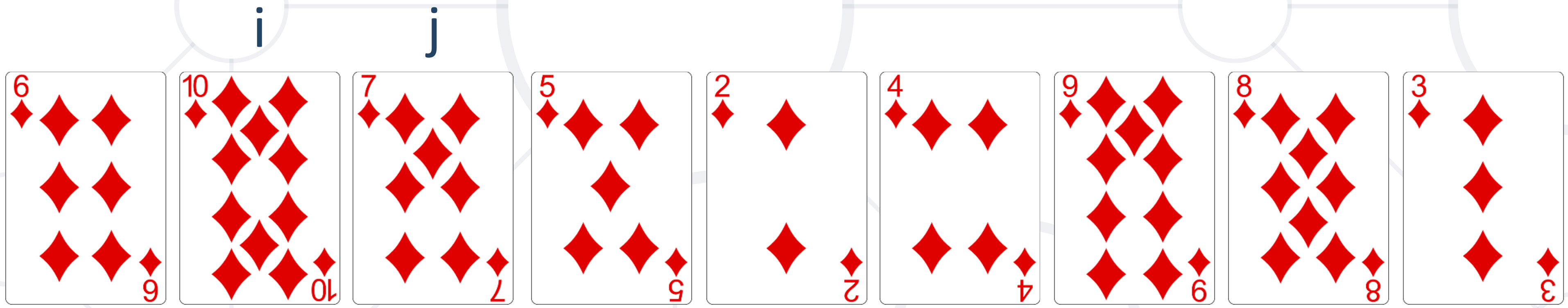
Bubble Sort

- **Bubble sort** – simple, but inefficient algorithm
- Swaps to neighbor elements when not in order until sorted
  - Memory: **O(1)**
  - Time: **$O(n^2)$**
  - Stable: Yes
  - Method: Exchanging

Bubble Sort Visualization



Bubble Sort Visualization



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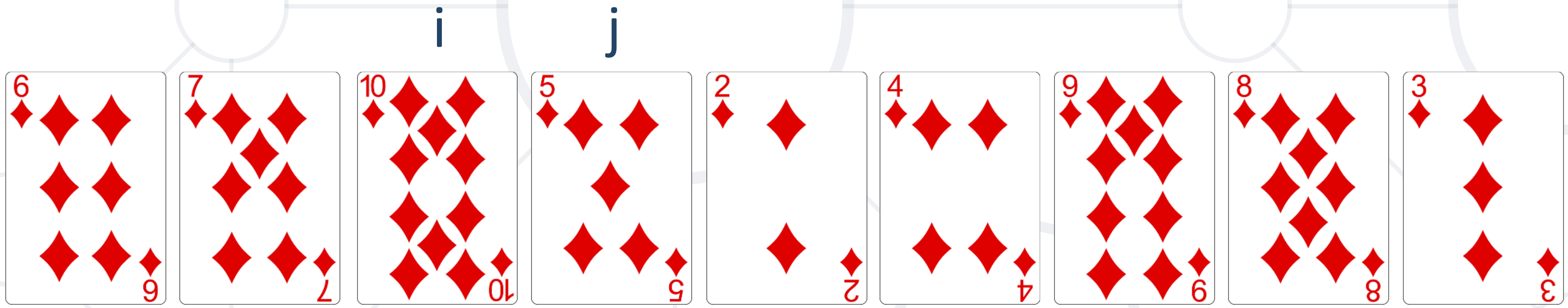

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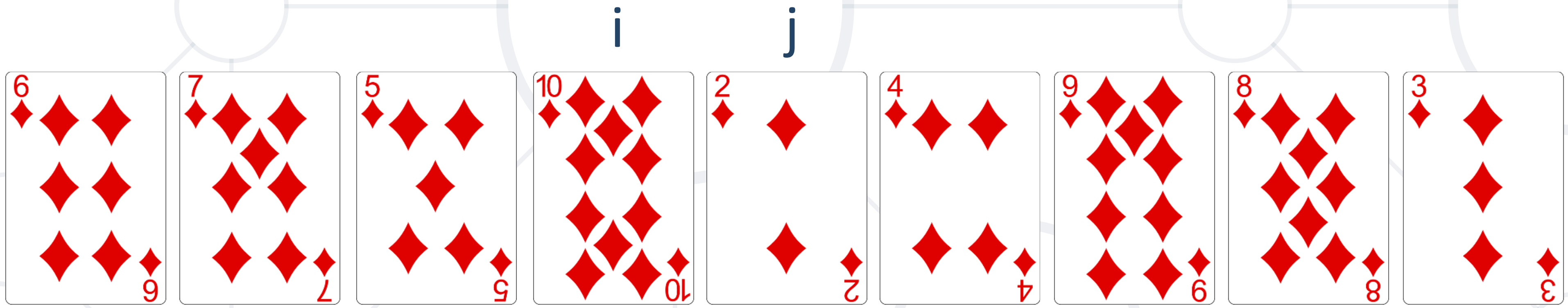

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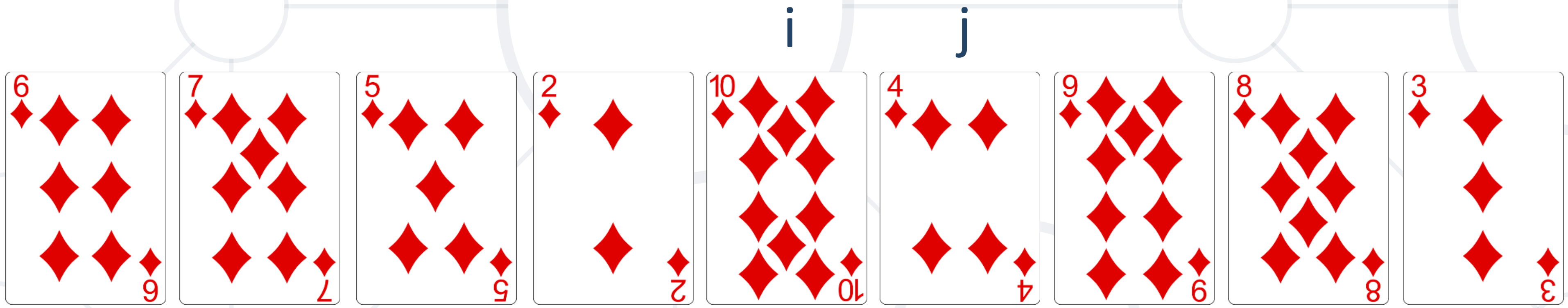

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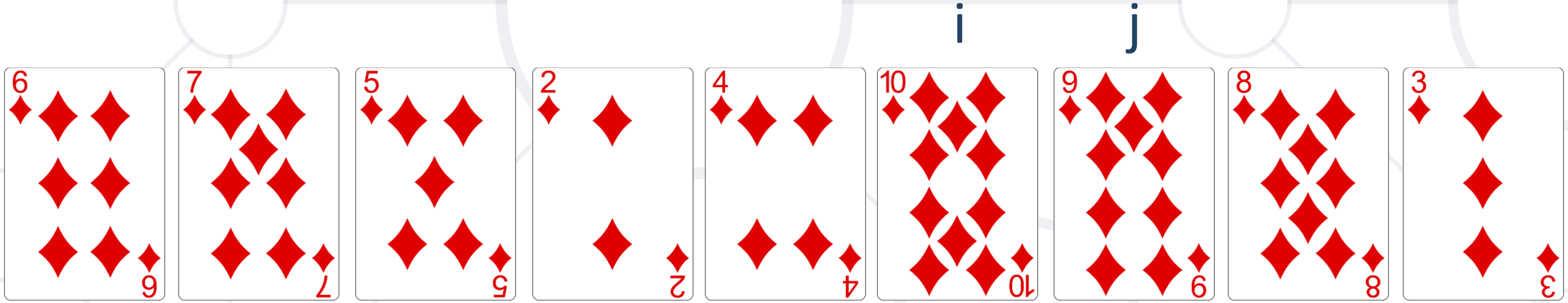

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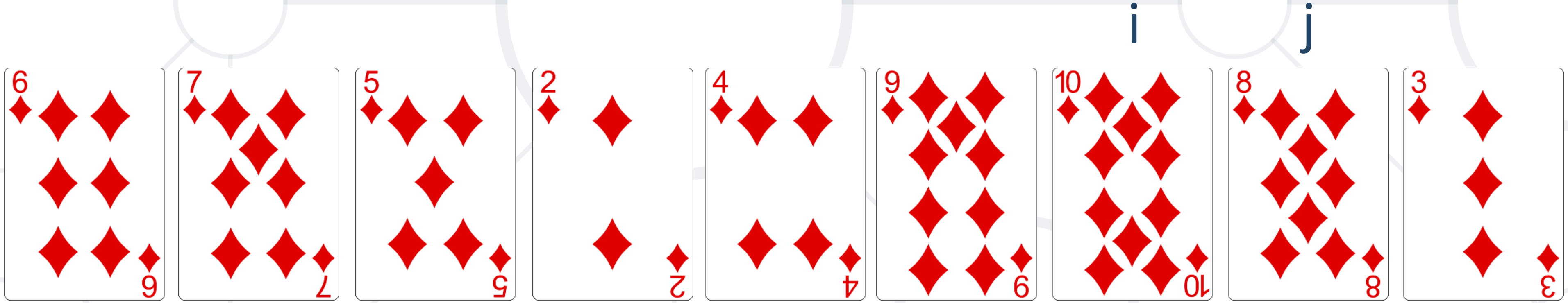

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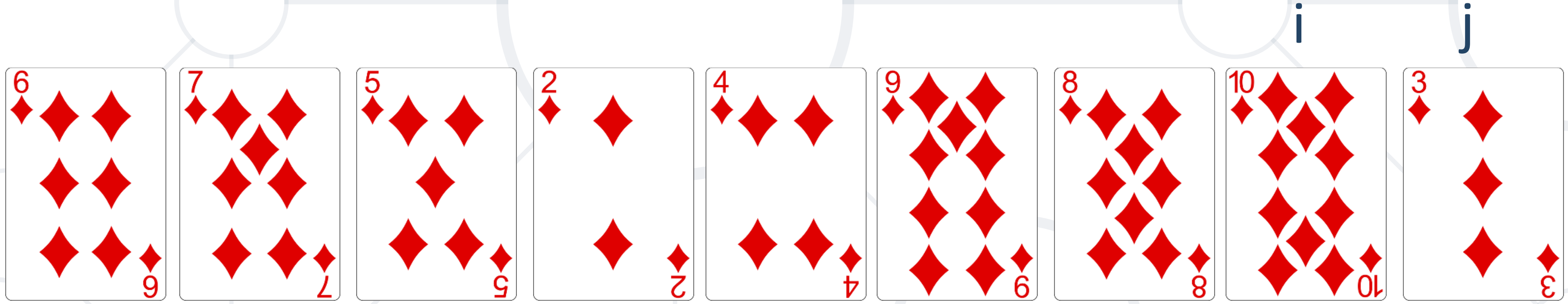

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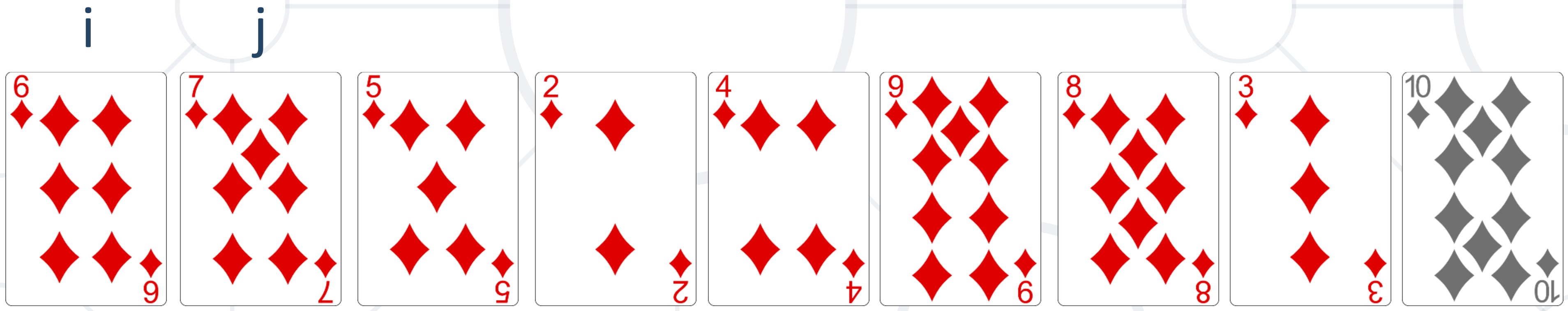

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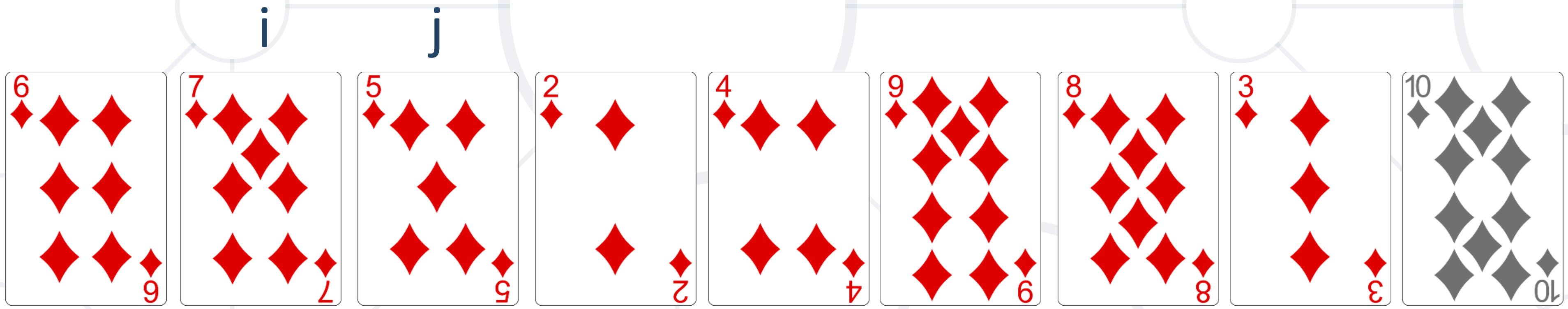

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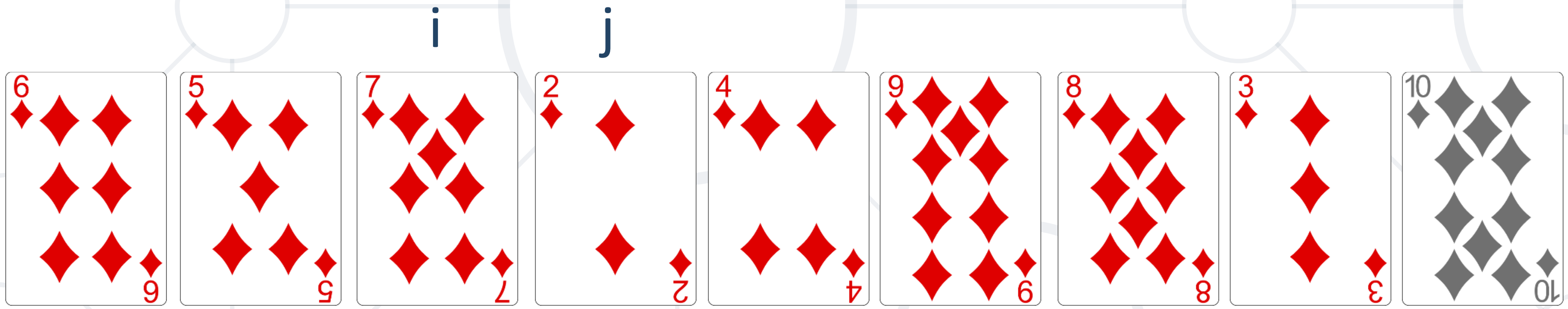

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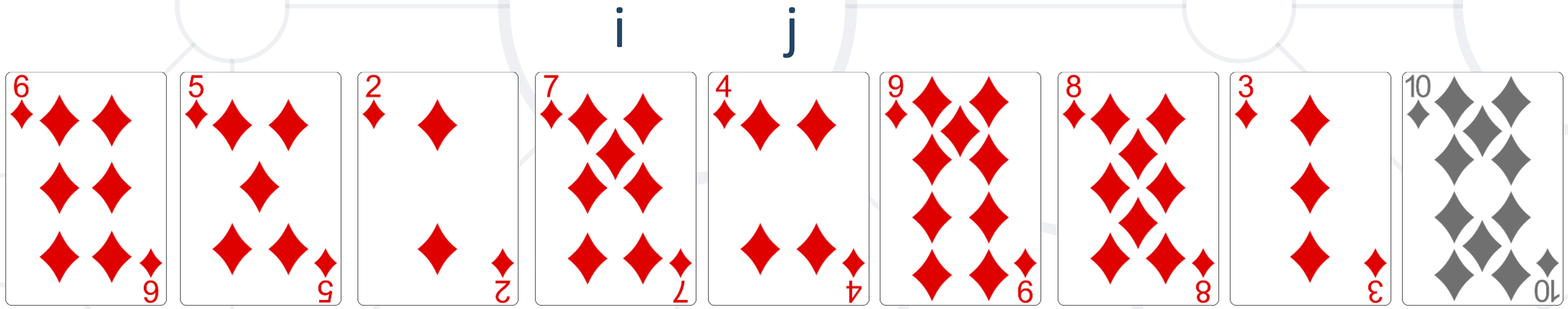

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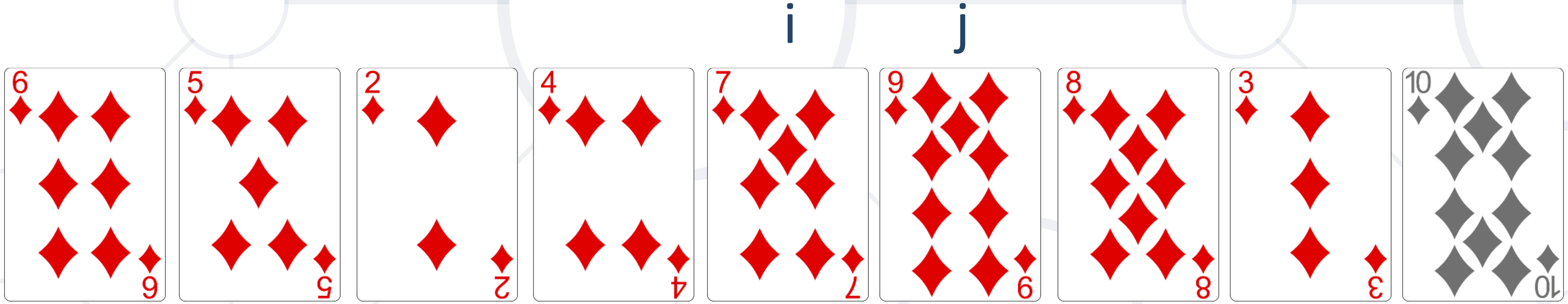

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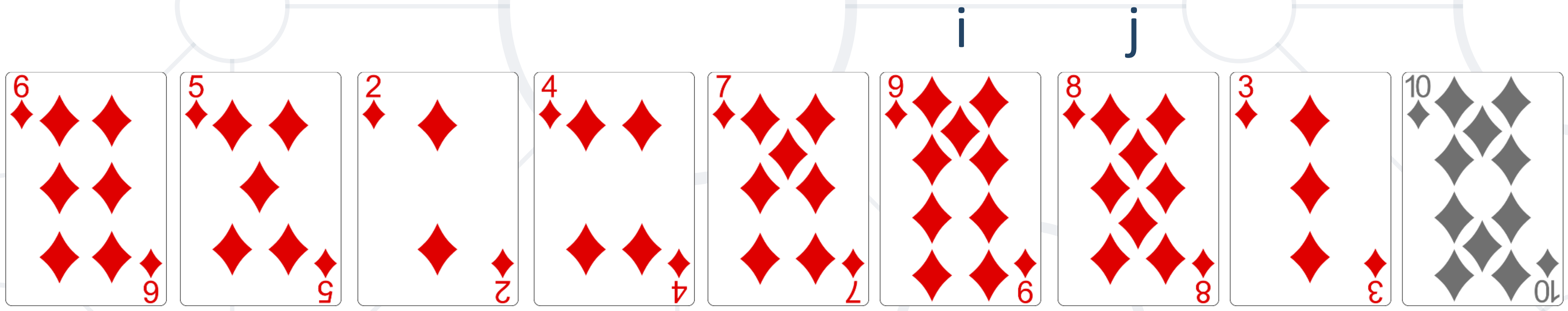

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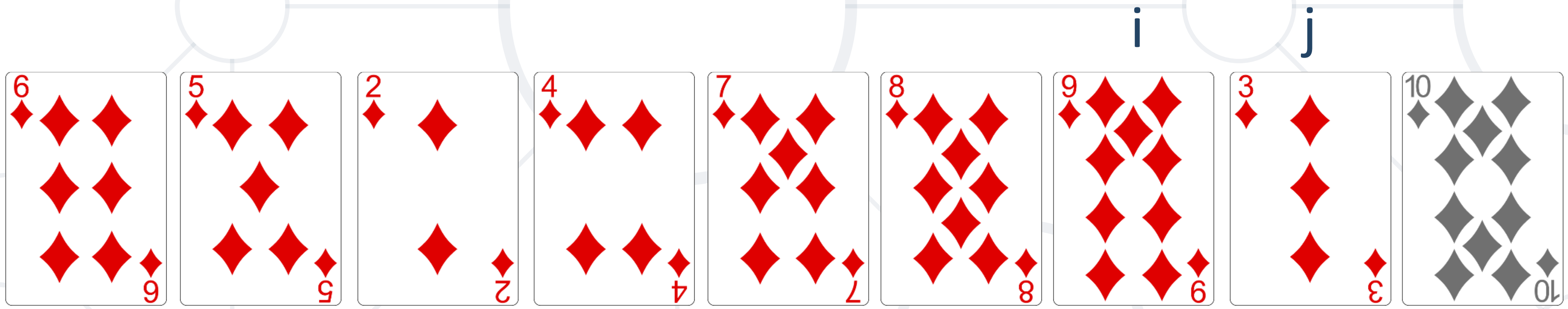

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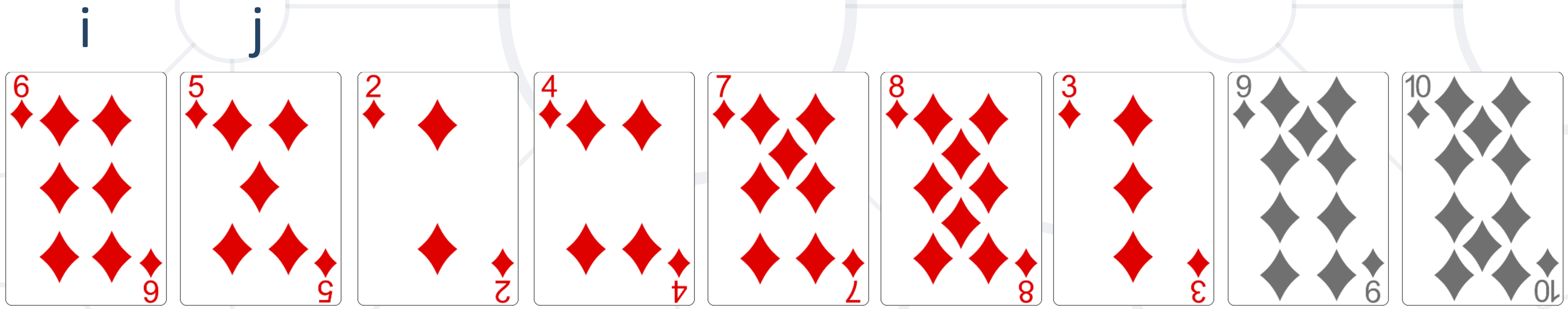

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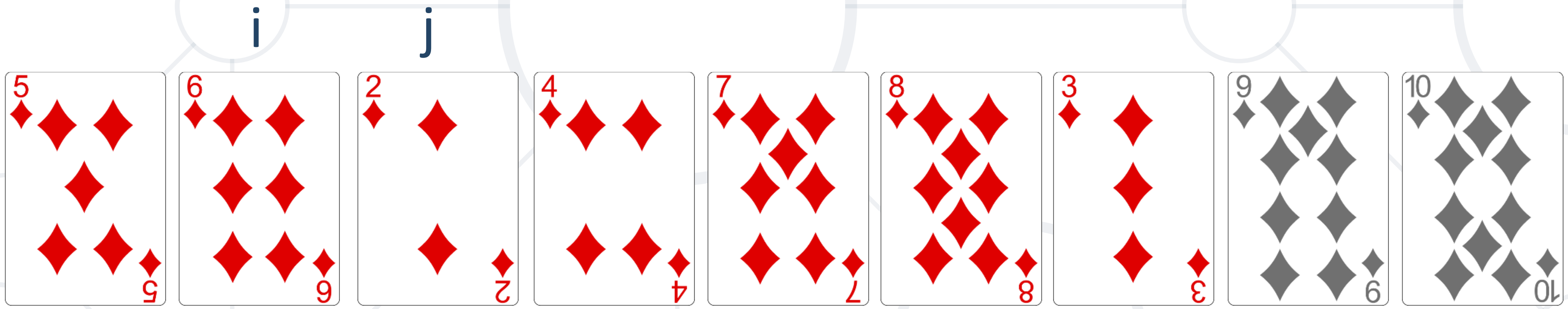

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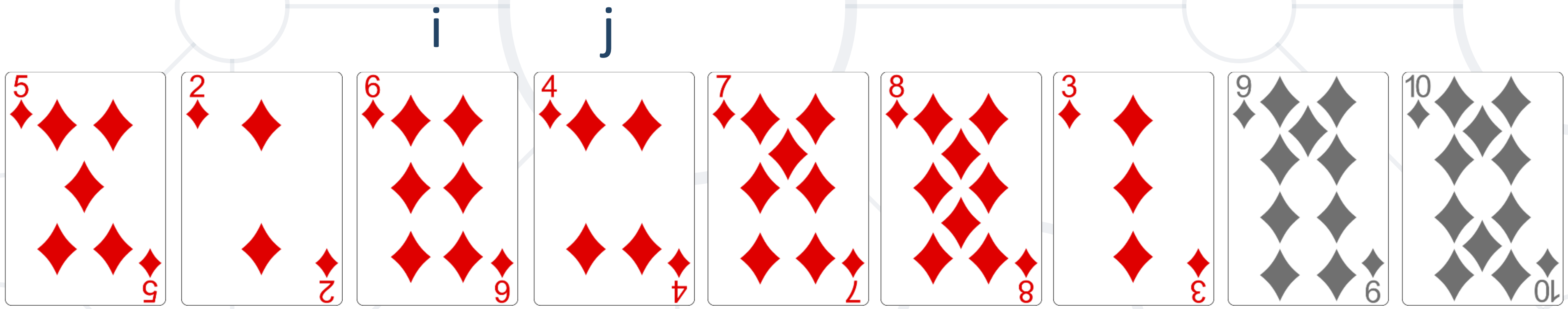

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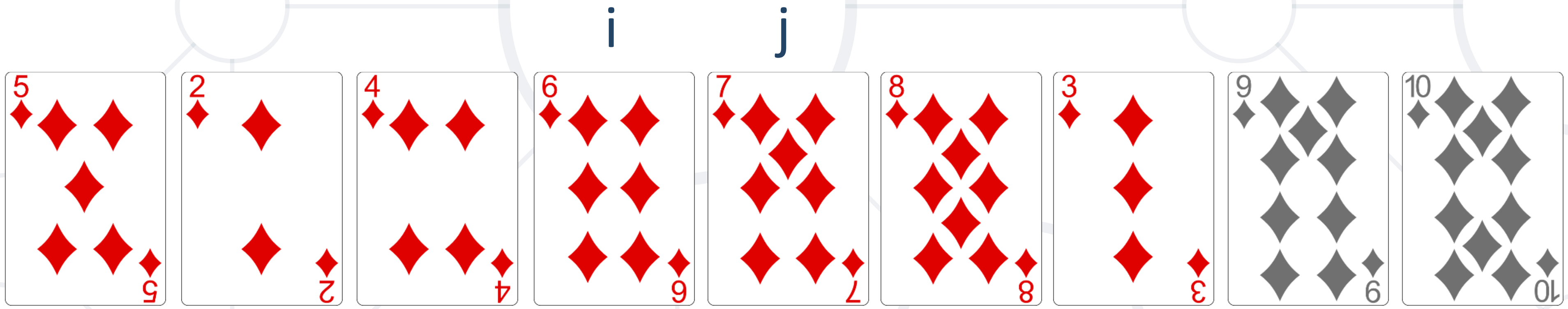

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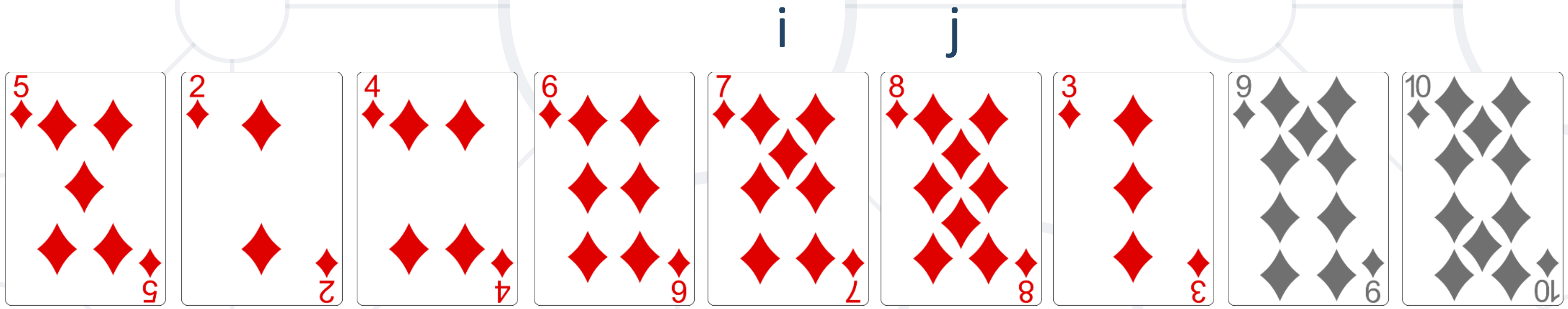

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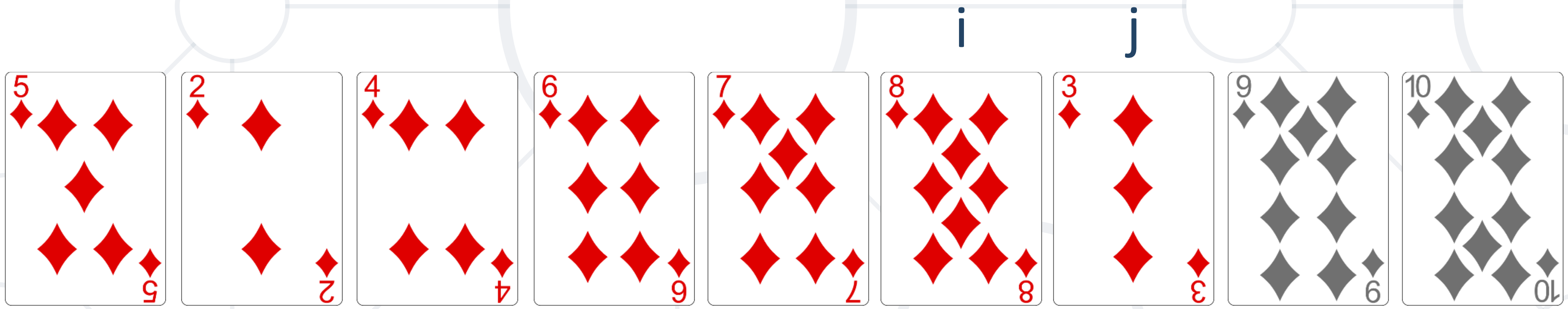

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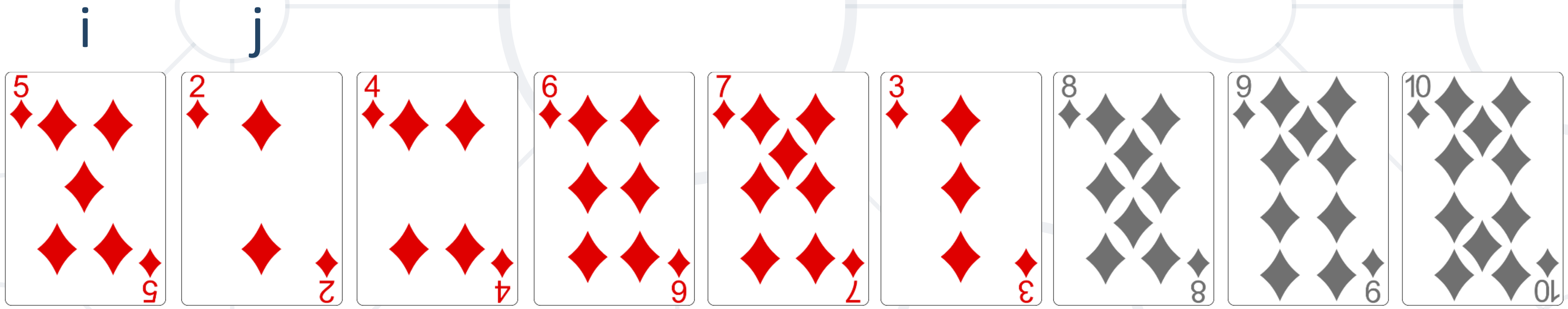

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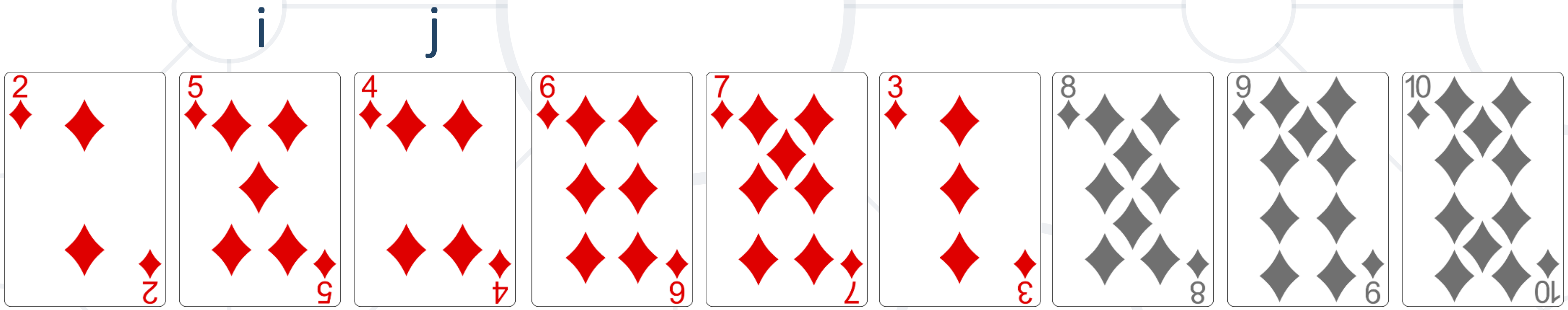

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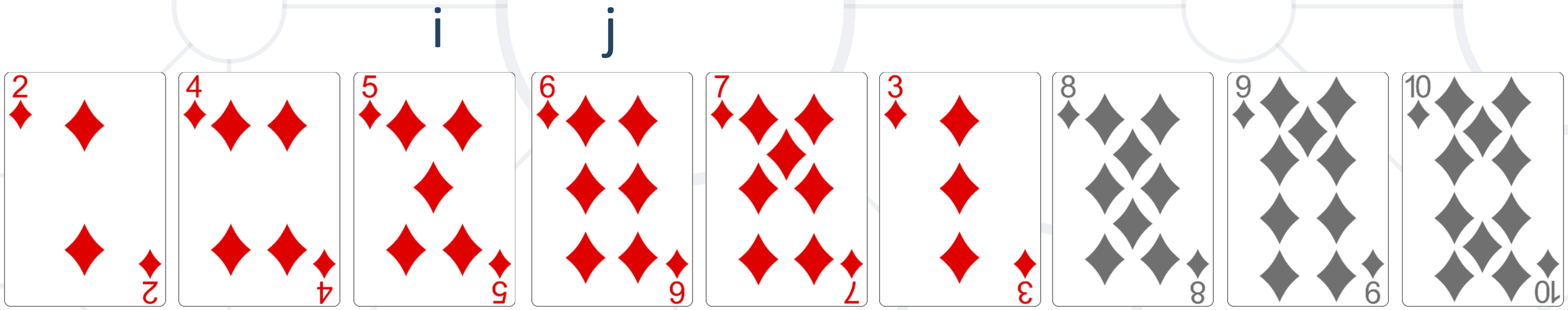

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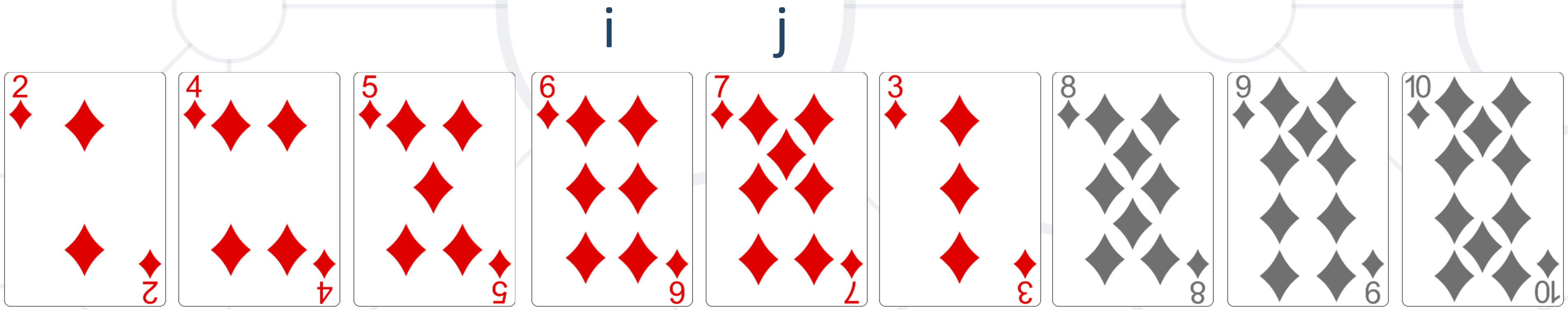

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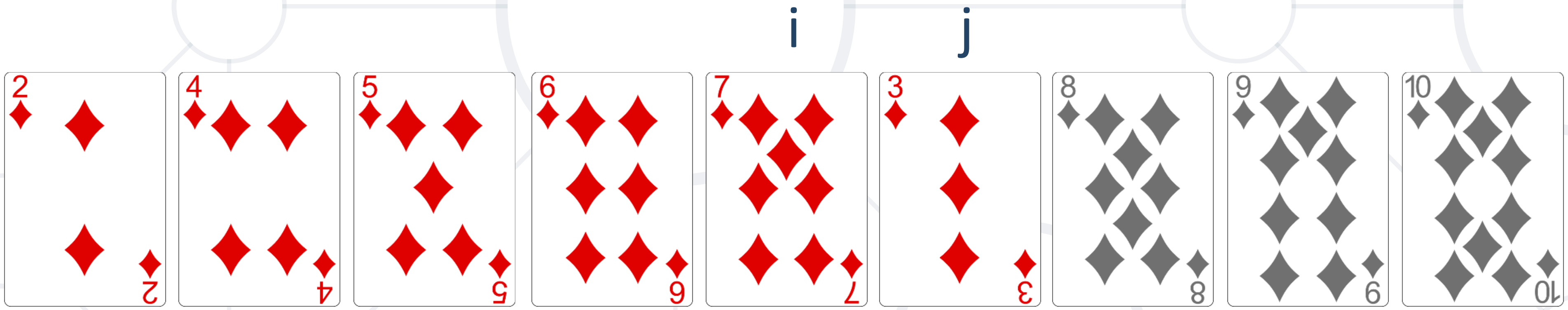

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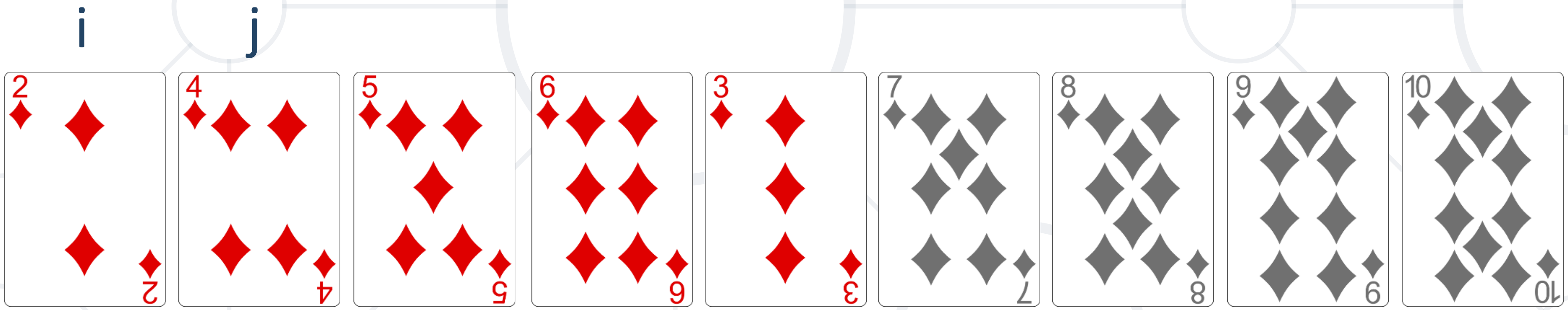

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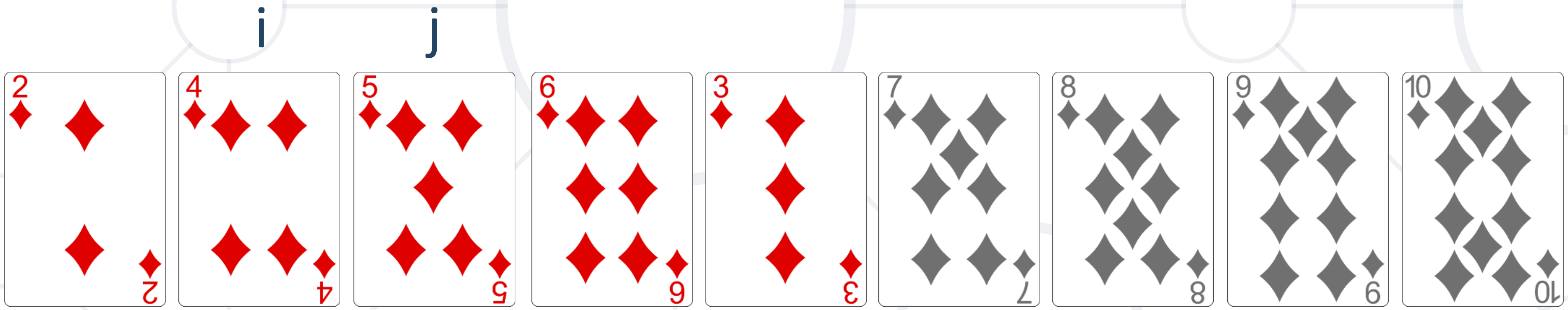

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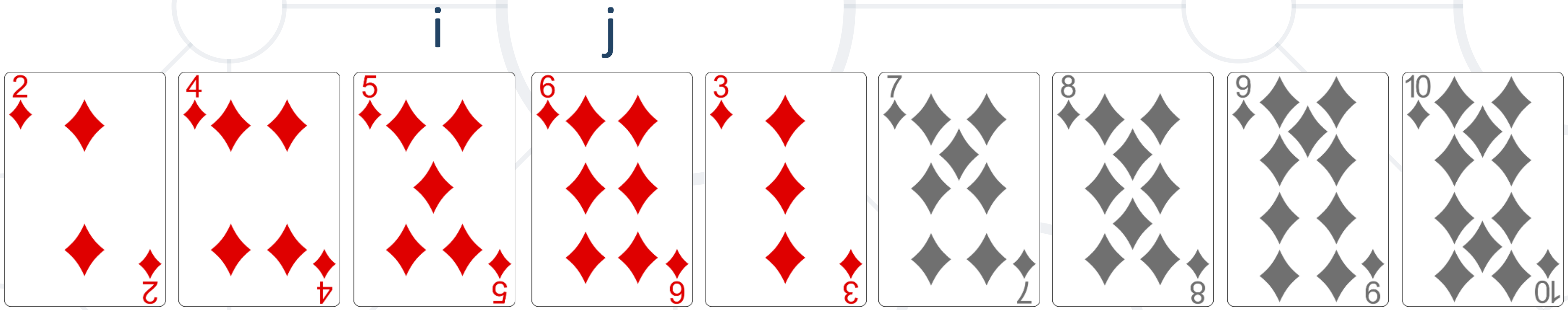

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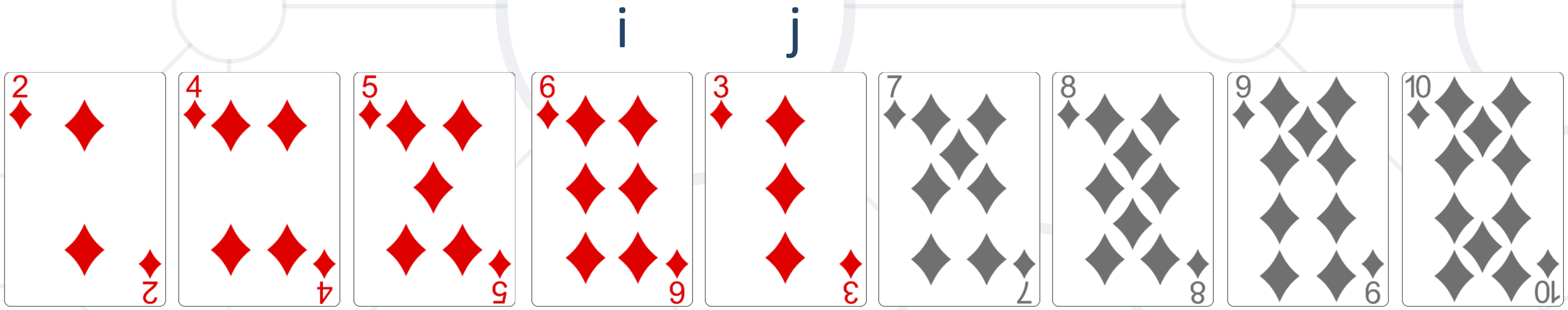

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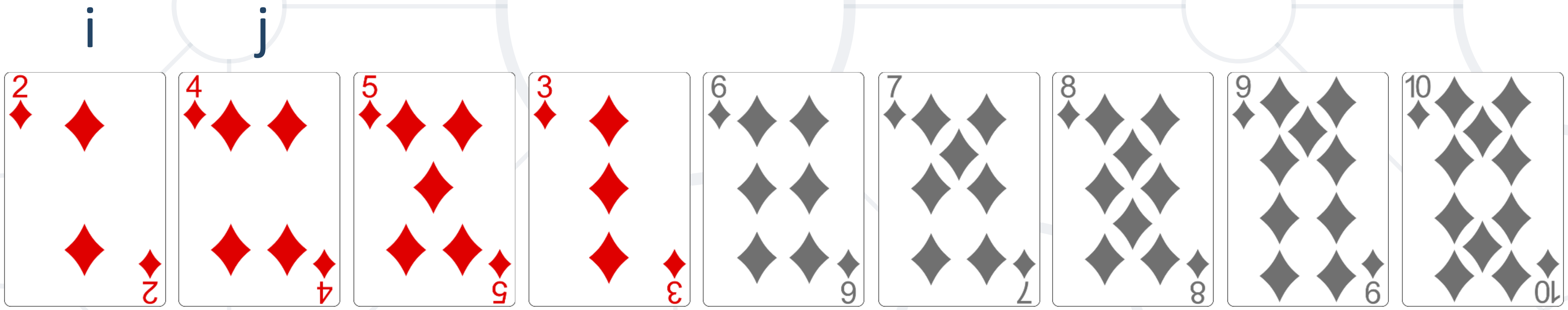
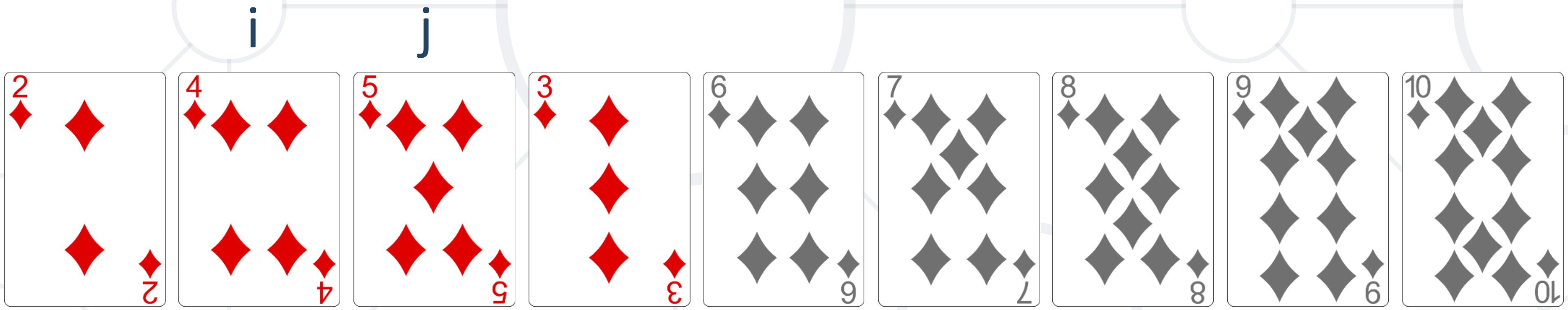


Bubble Sort Visualization



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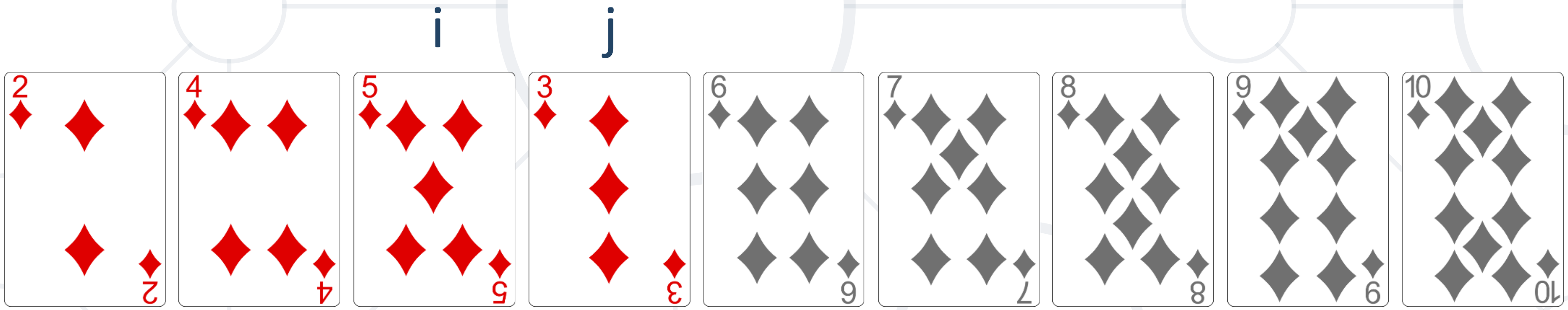

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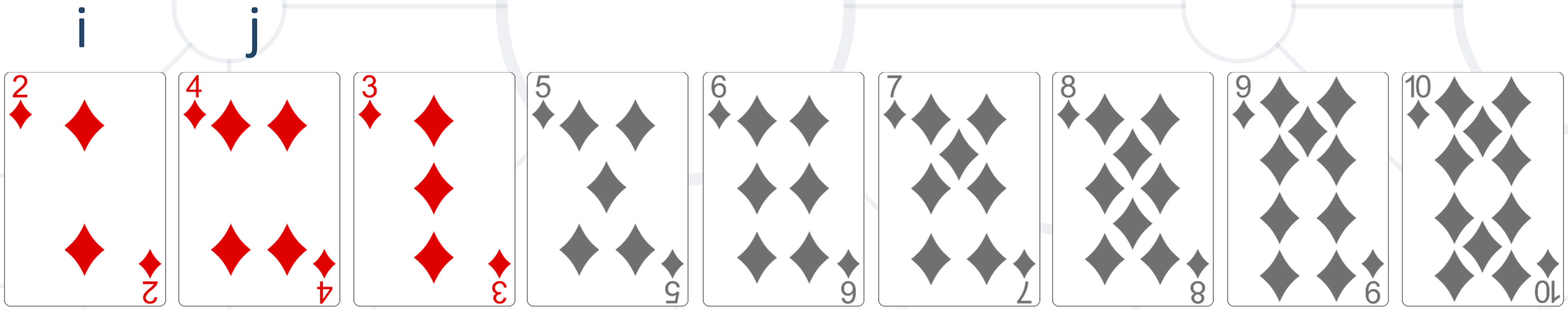

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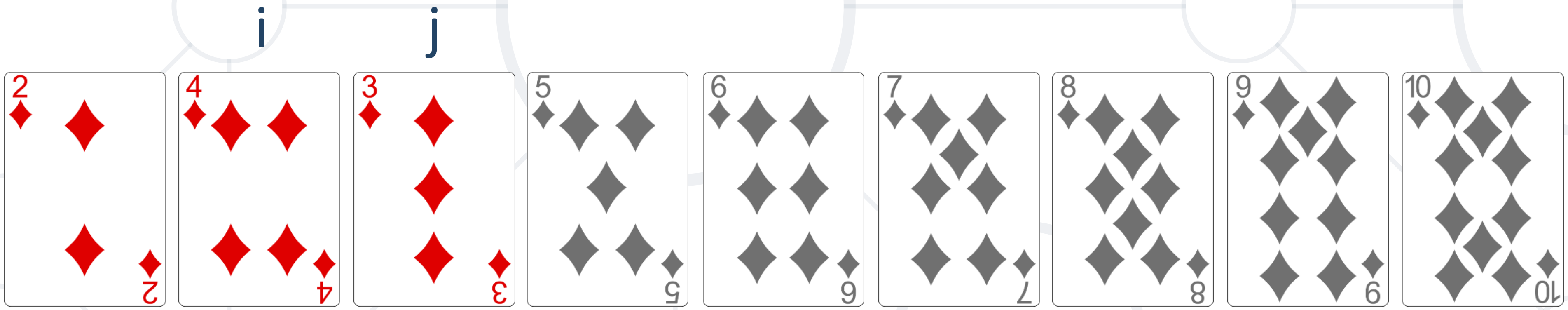

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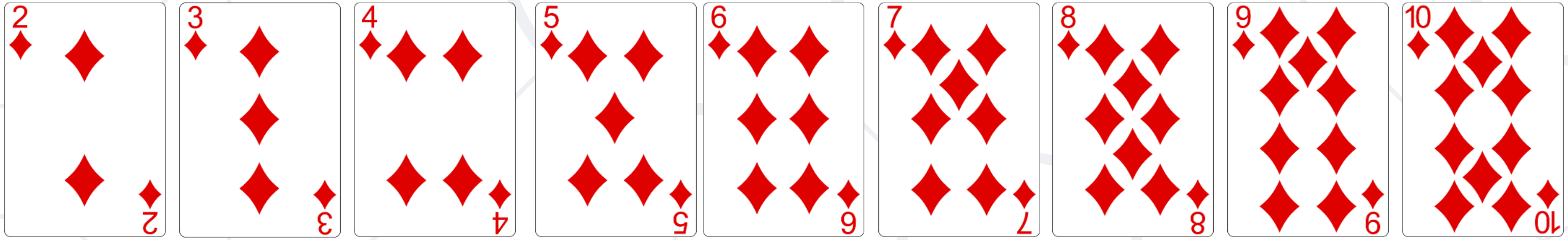


BubbleSort

```
var numbers = new [] {1, 3, 4, 2, 5, 6};
for (int i = 0; i < numbers.Length; i++) {
  for (int j = 1; j < numbers.Length - i; j++) {
    if (numbers[j - 1] > numbers[j]) {
      Swap(numbers, j - 1, j);
    }
  }
}
```

Bubble Sort (2)

```
var numbers = new [] {1, 3, 4, 2, 5, 6};
var isSorted = false;
var i = 0;
while (!isSorted) {
  isSorted = true;
  for (int j = 1; j < numbers.Length - i; j++) {
    if (numbers[j - 1] > numbers[j]) {
      isSorted = false;
      Swap(numbers, j - 1, j);
    }
  }
  i += 1;
}
```

Comparison of Sorting Algorithms

Name	Best	Average	Worst	Memory	Stable	Method
Selection	n^2	n^2	n^2	1	No	Selection
Bubble	n	n^2	n^2	1	Yes	Exchanging

Insertion Sort

- **Insertion Sort** – simple, but inefficient algorithm
  - Move the first unsorted element left to its place
  - Memory: **O(1)**
  - Time: **$O(n^2)$**
  - Stable: **Yes**
  - Method: **Insertion**

Insertion Sort

```
for (int i = 1; i < arr.Length; i++)
{
  var j = i;
  while (j > 0 && arr[j] < arr[j - 1])
  {
    Swap(arr, j, j - 1);
    j--;
  }
```

Comparison of Sorting Algorithms

Name	Best	Average	Worst	Memory	Stable	Method
Selection	n^2	n^2	n^2	1	No	Selection
Bubble	n	n^2	n^2	1	Yes	Exchanging
Insertion	n	n^2	n^2	1	Yes	Insertion

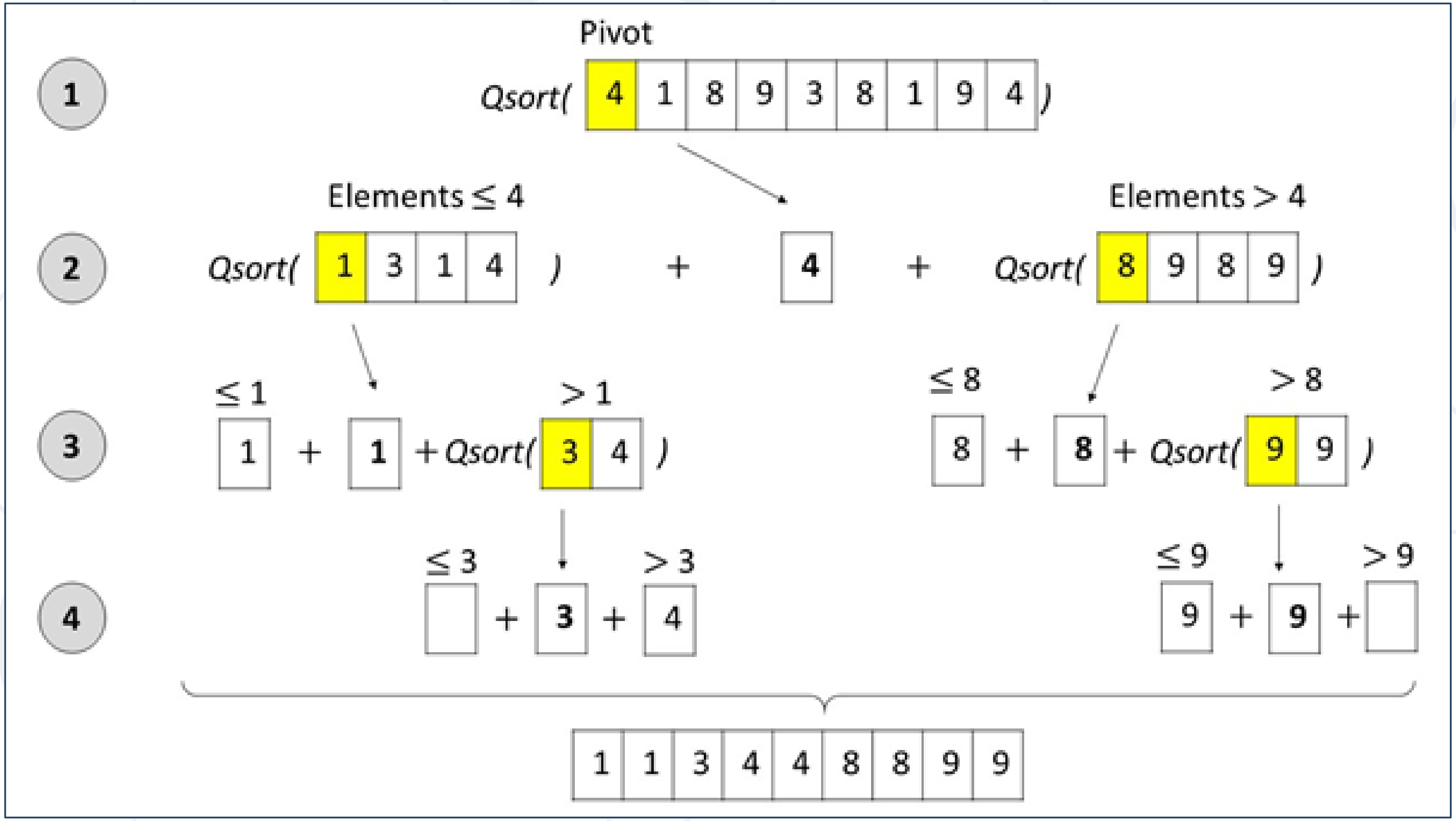
Advanced Sorting Algorithms

QuickSort, MergeSort

Quick Sort

- **QuickSort** – efficient sorting algorithm
  - Choose a pivot; move smaller elements left & larger right; sort left & right
  - Memory: **O(log(n))** stack space (recursion)
  - Time: **$O(n^2)$**
  - Stable: **Depends**
  - Method: **Partitioning**

Quick Sort: Conceptual Overview



Quick Sort (1)

```
public static void QuickSortHelper(
  int[] array, int startIdx, int endIdx)
{
  if (startIdx >= endIdx)
    return;
  var pivotIdx = startIdx;
  var leftIdx = startIdx + 1;
  var rightIdx = endIdx;
  while (leftIdx <= rightIdx) {
    // TODO: Continues on the next slide
  }
  // TODO: Continues on slide Quick Sort (3)
}
```

Quick Sort (2)

```
if (array[leftIdx] > array[pivotIdx] &&
      array[rightIdx] < array[pivotIdx]) {
  Swap(array, leftIdx, rightIdx);
}

if (array[leftIdx] <= array[pivotIdx]) {
  leftIdx += 1;
}

if (array[rightIdx] >= array[pivotIdx]) {
  rightIdx -= 1;
}
```

Quick Sort (3)

```
Swap(array, pivotIdx, rightIdx);

var isLeftSubArraysSmaller =
  rightIdx - 1 - startIdx < endIdx - (rightIdx + 1);
if (isLeftSubArraysSmaller) {
  QuickSortHelper(array, startIdx, rightIdx - 1);
  QuickSortHelper(array, rightIdx + 1, endIdx);
} else {
  QuickSortHelper(array, rightIdx + 1, endIdx);
  QuickSortHelper(array, startIdx, rightIdx - 1);
}
```

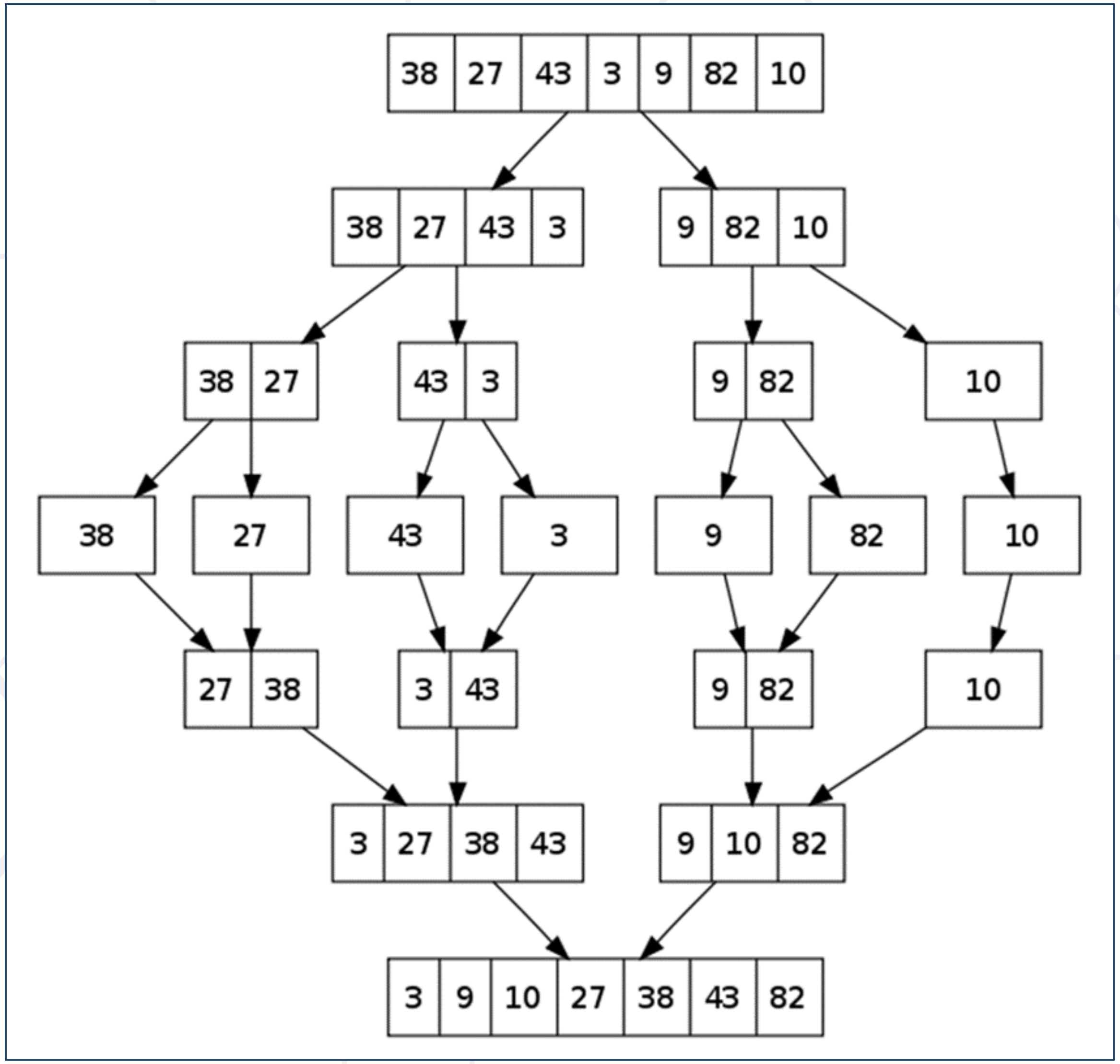
Comparison of Sorting Algorithms

Name	Best	Average	Worst	Memory	Stable	Method
Selection	n^2	n^2	n^2	1	No	Selection
Bubble	n	n^2	n^2	1	Yes	Exchanging
Insertion	n	n^2	n^2	1	Yes	Insertion
Quick	n * log(n)	n * log(n)	n^2	1	Depends	Partitioning

Merge Sort

- **Merge sort** is efficient sorting algorithm
- Divide the list into sub-lists (typically 2 sub-lists)
  1. Sort each sub-list (recursively call merge-sort)
  2. Merge the sorted sub-lists into a single list
- Memory: **O(n)** / **O(n*log(n))**
- Time: **O(n*log(n))**
- Highly parallelizable on multiple cores / machines → up to **O(log(n))**

Merge Sort: Conceptual Overview



Merge Sort (1)

```
// Memory: O(n*log(n))
public static int[] MergeSort(int[] array)
{
  if (array.Length == 1)
    return array;

  var middleIdx = array.Length / 2;
  var leftHalf = array.Take(middleIdx).ToArray();
  var rightHalf = array.Skip(middleIdx).ToArray();

  return MergeArrays(MergeSort(leftHalf), MergeSort(rightHalf));
}
```

Merge Sort (2)

```
public static int[] MergeArrays(int[] left, int[] right) {
  var sorted = new int[left.Length + right.Length];
  var sortedIdx = 0; var leftIdx = 0; var rightIdx = 0;
  while (leftIdx < left.Length && rightIdx < right.Length) {
    if (left[leftIdx] < right[rightIdx]) {
      sorted[sortedIdx++] = left[leftIdx++];
    } else {
      sorted[sortedIdx++] = right[rightIdx++];
    }
  }
  // TODO: Take remaining elements either from the left or right
  return sorted;
}
```

Merge Sort (3)

```
while (leftIdx < left.Length) {
  sorted[sortedIdx] = left[leftIdx];
  sortedIdx += 1;
  leftIdx += 1;
}

while (rightIdx < right.Length) {
  sorted[sortedIdx] = right[rightIdx];
  sortedIdx += 1;
  rightIdx += 1;
}
```

Merge Sort

```
// Memory: O(n)
public static int[] MergeSort(int[] array)
{
  if (array.Length <= 1)
    return array;

  var copy = new int[array.Length];
  Array.Copy(array, copy, array.Length);

  MergeSortHelper(array, copy, 0, array.Length - 1);

  return array;
}
```

Merge Sort (2)

```
public static void MergeSortHelper(
  int[] source, int[] copy, int leftIdx, int rightIdx)
{
  if (leftIdx >= rightIdx)
    return;

  var middleIdx = (leftIdx + rightIdx) / 2;
  MergeSortHelper(copy, source, leftIdx, middleIdx);
  MergeSortHelper(copy, source, middleIdx + 1, rightIdx);

  MergeArrays(source, copy, leftIdx, middleIdx, rightIdx);
}
```

Merge Sort (3)

```
public static void MergeArrays(
  int[] source, int[] copy, int startIdx, int middleIdx, int endIdx)
{
  var sourceIdx = startIdx;
  var leftIdx = startIdx; var rightIdx = middleIdx + 1;
  while (leftIdx <= middleIdx && rightIdx <= endIdx) {
    if (copy[leftIdx] < copy[rightIdx])
      source[sourceIdx++] = copy[leftIdx++];
    else
      source[sourceIdx++] = copy[rightIdx++];
  }
  // TODO: Take remaining elements either from the left or right
}
```

Merge Sort (4)

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```
while (leftIdx <= middleIdx)
{
  source[sourceIdx] = copy[leftIdx];
  leftIdx += 1;
  sourceIdx += 1;
}

while (rightIdx <= endIdx)
{
  source[sourceIdx] = copy[rightIdx];
  rightIdx += 1;
  sourceIdx += 1;
}
```

Comparison of Sorting Algorithms

Name	Best	Average	Worst	Memory	Stable	Method
Selection	n^2	n^2	n^2	1	No	Selection
Bubble	n	n^2	n^2	1	Yes	Exchanging
Insertion	n	n^2	n^2	1	Yes	Insertion
Quick	n * log(n)	n * log(n)	n^2	1	Depends	Partitioning
Merge	n * log(n)	n * log(n)	n * log(n)	1	Yes	Merging

Greedy Algorithms

Picking Locally Optimal Solution

Greedy Algorithms

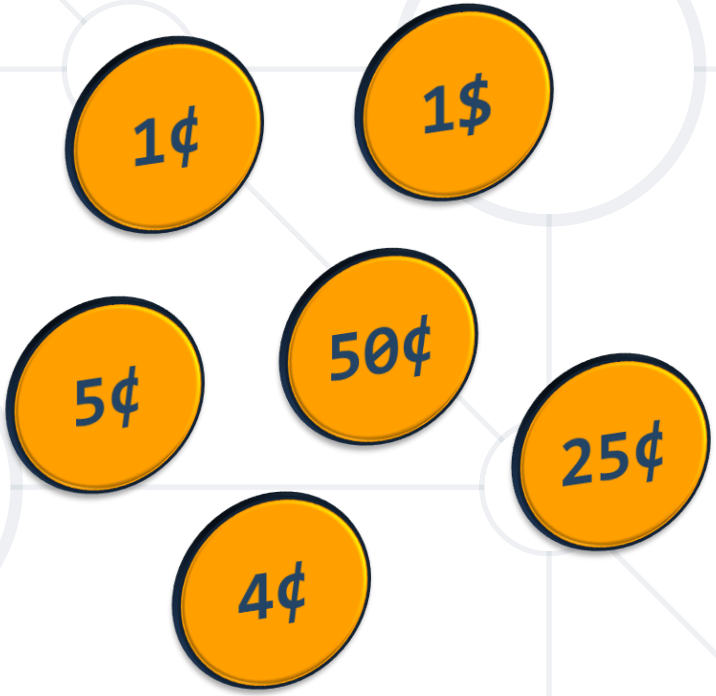
- Used for solving optimization problems
- Usually more efficient than the other algorithms
- Can produce a **non-optimal** (incorrect) result
- Pick the **best local** solution
  - The optimum for a **current** position and point of view
- Greedy algorithms assume that always choosing a **local** optimum leads to the **global** optimum

Optimization Problems

- Finding the best solution from all possible solutions
- Examples:
  - Find the **shortest** path from Sofia to Varna
  - Find the **maximum increasing subsequence**
  - Find the shortest route that visits each city and returns to the origin city

Problem: Sum of Coins

- Write a program, which gathers a sum of money, using the least possible number of coins
- Consider the US **currency coins**
  - **0.01**, **0.02**, **0.05**, **0.10**
- **Greedy algorithm** for "Sum of Coins":
  - Take the largest coin while possible
  - Then take the second largest
  - Etc.



Sum of Coins Visualization

Target: 18

Actual: 0

Sum of Coins Visualization

Target: 18

Actual: 10

Sum of Coins Visualization

Target: 18

10¢ 5¢ 2¢ 2¢ 1¢ 1¢

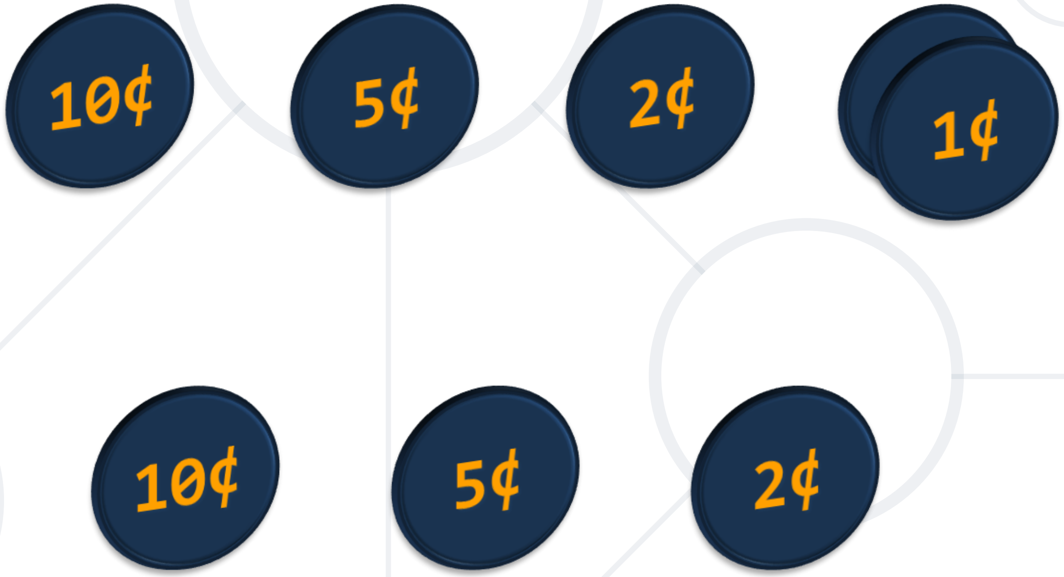
Actual: 15

10¢ 5¢

Sum of Coins Visualization

Target: 18

Actual: 17



Sum of Coins Visualization

Target: 18

Actual: 18

Solution: Sum of Coins (1)

```
var coins = // Read an array of integers and sort it in desc.
var target = int.Parse(Console.ReadLine());
var counter = 0;
var coinsIndex = 0;
var sb = new StringBuilder();
while (target > 0 && coinsIndex < coins.Length) {
  var currentCoin = coins[coinsIndex++];
  var coinsCount = target / currentCoin;
  if (coinsCount > 0) {
    counter += coinsCount;
    target -= currentCoin * coinsCount;
    sb.AppendLine($"{coinsCount} coin(s) with value {currentCoin}");
  }
}
// TODO: Print the output
```

Problem: Set Cover

- Write a program that finds the smallest subset of S, the union of which = **U** (if it exists)
- You will be given a **set** of integers **U** called "**the Universe**"
- And a set **S** of **n** integer sets whose union = **U**

```
1, 2, 3, 4, 5
4
1
2, 4
5
3
```

```
Sets to take (4):
2, 4
1
5
3
```

Solution: Set Cover

```
// Read the input elements – universe and sets
var selectedSets = new List<int[]>();
while (universe.Count > 0) {
  var currentSet = sets
    .OrderByDescending(s => s.Count(e => universe.Contains(e)))
    .FirstOrDefault();
  foreach (var number in currentSet)
  { universe.Remove(number); }

  sets.Remove(currentSet);
  selectedSets.Add(currentSet);
}
// TODO: Pirnt the output
```

Greedy Failure Cases

Greedy Algorithms Often Fail

Sum of Coins Failure

Target: 18

Actual: 0

Sum of Coins Failure

Target: 18

Actual: 10

Sum of Coins Failure

Target: 18

Actual: 15

Sum of Coins Failure

Target: 18

Actual: 16

Sum of Coins Failure

Target: 18

Actual: 17

Sum of Coins Failure

Target: 18

Actual: 18

Sum of Coins Failure

Target: 18

Optimal Greedy Algorithms

Optimal Substructure and Greedy Choice Property

Optimal Greedy Algorithms

- Suitable problems for greedy algorithms have these properties:
  - **Greedy choice property**
  - **Optimal substructure**
- Any problem having the above properties is guaranteed to have an optimal greedy solution

Greedy Choice Property

- **Greedy choice** property
  - **A global optimal solution** can be obtained by greedily selecting a **locally optimal** choice
  - Sub-problems that arise are solved by consequent greedy choices
    - Enforced by optimal substructure

Optimal Substructure Property

- **Optimal substructure** property
  - After each greedy choice the problem remains an optimization problem of the same form as the original problem
  - **An optimal global solution contains the optimal solutions of all its sub-problems**

Greedy Algorithms: Example

- The "**Max Coins**" game
  - You are given a set of coins
  - You play against another player, alternating turns
  - Per each turn, you can take up to three coins
  - Your goal is to have as many coins as possible at the end

Max Coins – Greedy Algorithm

- A simple **greedy strategy** exists for the "Max Coins" game

```
At each turn take the maximum number of coins
```

- Always choose the local maximum (at each step)
  - You don't consider what the other player does
  - You don't consider your actions' consequences
- The **greedy algorithm** works optimally here
  - It takes as many coins as possible

Summary

- **Searching** algorithms
  - Binary Search, Linear Search
- **Slow** sorting algorithms:
  - Selection sort, Bubble sort, Insertion sort
- **Fast** sorting algorithms:
  - Quick sort, Merge sort, etc.
  - How to choose the most appropriate algorithm?
- **Greedy Algorithms**

Questions?

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