NEW CONGESTION CONTROL SCHEMES OVER WIRELESS NETWORKS: DELAY SENSITIVITY ANALYSIS AND SIMULATIONS [1]

Alessandro Abate * **Minghua Chen** *
Shankar Sastry *

* *Department of Electrical Engineering and Computer Science*
University of California at Berkeley
Berkeley, CA 94720
{*aabate, minghua, sastry*}*@eecs.berkeley.edu*

Abstract: This paper proposes two new congestion control schemes for wireless networks. Starting from the seminal work of Kelly, we consider the decentralized flow control model for the TCP-like algorithm and apply it to a wireless scenario. The presence of channel errors motivates the idea of introducing some updates in a specific part of the model; this assumption has some important physical interpretation. We proposes two updates: the first is through a static law, while the second evolves according to a dynamic relation. The global stability of both the schemes has been already proved; also, a stochastic stability study and the computation of the rate of convergence of the two algorithms have formerly been concluded. This paper focuses on the delay sensitivity for both schemes. A stability condition on the parameters of the system is introduced and proved. Moreover, some deeper insight on the structure of the oscillations of the system is attained. As support, simulations are provided.

Keywords: Communication networks and protocols, flow control, large-scale systems, stability robustness, delay analysis.

1. INTRODUCTION

In these past years the control community has tried to systematically model Communication Networks in order to analyze their structure, properties and behavior in deep. Among the algorithms that have been employed for controlling the flow of information in a network, TCP has been the most successful one (Jacobson, 1998). The seminal work of Kelly has proposed a model (Kelly, 2003)-(Kelly *et al.*, Dec 1999) based on an optimization problem; other authors have studied and interpreted this or similar models (Alpcan and Basar, Dec 2003)-(Kunniyur and Srikant, Mar 2001)-(Kunniyur and Srikant, Oct 2003). The important notion of stability has been considered in (Paganini *et al.*, n.d.), while robustness, in particular with respect to delays, is the focus of (Johari and Tan, Dec 2001)-(Vinnicombe, 2001)-(Vinnicombe, 2002). All these efforts have been devoted on the wired case. The wireless scenario presents more subtleties then the wired one: here the packet loss is due both to congestion at the link and to channel error. (Chen *et al.*, 2005) [2] proposed two schemes, a static and a dynamic one, to fix the sub optimality of the equilibrium

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[2] The paper is currently under review; the reviewers can access it via the following link: http://www-video.eecs.berkeley.edu/~minghua/papers/infocom.2005.pdf

point of the network. In that work, which looks at the problem through the underlying optimization procedure, global stability of the two schemes was proved, and delay sensitivity analysis studied. In another paper by the same authors, (Abate *et al.*, 2005) [3], a stochastic stability analysis is derived, and the rate of convergence of these two schemes computed.

This paper focuses instead on the presence of delays. As commonly known, delays are one of the main causes of instability, as well as of oscillatory dynamics. To tackle the first problem, a condition is introduced to ensure stability for the trajectories of the system. Furthermore, this paper aims at shedding some light on the structure of the oscillations in the system induced by delays. Simulations will close out the paper.

2. THE NETWORK MODEL

A network is described via its J *resources*, its links, and its R *users* (sender-receiver pairs), which can also be conceived as subsets of J (the *routes*). Each link j has a finite capacity C_j. The connections of the network are described via a matrix $A = (a_{jr}, j \in J, r \in R)$, where $a_{jr} = 1$ if $j \in r$. Every user is endowed with a sending rate $x_r \geq 0$ and a utility function $U_r(x_r)$, assumed to be increasingly, strictly concave and $\mathbb{C}^1$.

Kelly (Kelly *et al.*, Dec 1999) introduced the following primal scheme, which is a more general, continuous-time version of the TCP-like additive increase, multiplicative decrease scheme:

$$\frac{d}{dt}x_r(t) = k_r \left(w_r^o - x_r(t) \sum_{j \in r} \mu_j(t) \right), \quad r \in R \tag{1}$$

with k_r being a positive scale factor affecting the adaptation rate; the congestion signal is generated at a link j as

$$\mu_j(t) = p_j \left(\sum_{s: j \in s} x_s(t) \right). \tag{2}$$

Here $p_j(y)$'s are the prices at the links and are assumed to be non-negative, continuous and increasing functions; moreover, they are expected to depend on the aggregate rate passing through the link. Throughout this paper we shall stick to the following form for the price, the "packet loss rate",

$$p_j(y) = \frac{(y - C_j)^+}{y}. \tag{3}$$

The end-to-end packet loss rate for user r is $1 - \prod_{j \in r} p_j(\sum_{s:j \in s} x_s)$, which is approximately $\sum_{j \in r} p_j(\sum_{s:j\in s} x_s)$ when $p_j(\sum_{s:j\in s} x_s)$ is small (we shall assume this in the following). With this primal scheme (1)-(2), the unique, globally asymptotically stable points of the entire network, denoted by $x^o = (x_r^o, r \in R)$ [4], are given by

$$x_r^o = \frac{w_r^o}{\sum_{j \in r} p_j \left(\sum_{s:j\in s} x_s^o \right)}, \quad r \in R. \tag{4}$$

As already stated, one of the main differences between the wired case and the wireless one is the presence, in this latter case, of physical channel errors; these affect the packet loss rate, which in the wired case depended just on the congestion measure. Say every link j is affected by the error ϵ_j; then, the new price function ν_j is:

$$\begin{aligned} \nu_j(t) &= p_j \left(\sum_{s:j\in s} x_s(t) \right) + \left(1 - p_j \left(\sum_{s:j \in s} x_s(t) \right) \right) \epsilon_j \\ &\doteq q_j \left(\sum_{s:j\in s} x_s(t) \right) \geq p_j \left(\sum_{s:j\in s} x_s(t) \right). \end{aligned} \tag{5}$$

The primal scheme (1) then will adapt according to this new price functions q_j, which have the same structural properties as the old p_j; a close look will show that the equilibrium point, compared to the wired case, is sub optimal. We introduce the following two schemes to fix this problem.

3. TWO NEW CONTROL SCHEMES

3.1 Static Update

Assume the term ω_r is time dependent, $w_r(t)$, and is adjusted according to the following law:

$$w_r(t) = w_r^o \frac{\sum_{j \in r} \nu_j(t)}{\sum_{j \in r} \mu_j(t)}. \tag{6}$$

Then, the source rate for user r then is given by:

$$\frac{d}{dt}x_r(t) = k_r \left(w_r(t) - x_r(t) \sum_{j \in r} \nu_j(t) \right). \tag{7}$$

A rapid calculation shows how, under this change, the equilibrium of the system is again x^o. Intuitively, as can be seen in (6), if the noise is large, i.e. $v_j(t) > u_j(t)$, an increase in $w_r(t)$ counteracts it.

[3] The paper has been submitted to the IFAC05 conference and is therefore currently under review. It is complementary to this one; the reviewers are invited to access it via the following link: http://www-video.eecs.berkeley.edu/∼minghua/papers/ifac.2005a.pdf

[4] To lighten the notation, throughout the whole paper users or links variables with no subscript will directly denote vectorial quantities. We charge the reader with the easy task to identify them.

3.2 Dynamic Update

Rather than a simultaneous adaptation rule, we advance a dynamic update for w_r:

$$\frac{d}{dt}w_r(t) = c_r\left(w_r^o - w_r(t)\frac{\sum_{j\in r} p_j(\sum_{s:j\in s} x_s(t))}{\sum_{j\in r} q_j(\sum_{s:j\in s} x_s(t))}\right). \tag{8}$$

The equilibrium points of the new, extended system are composed by a first part given by the vector x^o and a second, for the new dynamics, given by $w_r^o \frac{\sum_{j\in r}\nu_j(t)}{\sum_{j\in r}\mu_j(t)}$. The system of coupled equations (1)-(2)-(8) is strongly nonlinear and asymmetric.

4. DELAY SENSITIVITY ANALYSIS

Delays in the system are one of the first causes of oscillations. As a consequence, being oscillations one of the main concerns with TCP schemes, an analysis of delay sensitivity is necessary. The setting we will adhere to is that developed by Johari and Tan, (Johari and Tan, Dec 2001). The relations (1)-(2) for the primal scheme in the presence of delays can be expressed as:

$$\frac{d}{dt}x_r(t) = k_r w_r^o - k_r x_r(t-T_r)\sum_{j\in r}\mu_j(t - d_2(j,r)) \tag{9}$$

and

$$\mu_j(t) = p_j\left(\sum_{s:j\in s} x_s(t - d_1(j,s))\right), \tag{10}$$

where

$$d_1(j,r) + d_2(j,r) = T_r \qquad \forall r \in R; \tag{11}$$

here $d_1(j,r)$ is the forward delay from sender of route r to link j, and $d_2(j,r)$ is the return delay from the link j to the sender of route r. Hence T_r is the round trip time on route r, which is assumed to be fixed. In the following, two conditions for enforcing stability under delays are introduced (the proof for the first one can be found on (Chen *et al.*, 2005)).

4.1 Static Update

Theorem 1. The system (6)-(7) is locally stable if $\forall r \in R$,

$$k_r\frac{\sum_{j\in r} q_j}{\sum_{j\in r} p_j}\left(\sum_{j\in r} p_j + \sum_{j\in r} p_j' \sum_{s:j\in s} x_s^o\right) < \frac{\pi}{2T_r}, \tag{12}$$

where p_j, q_j are the values of $p_j(\cdot)$ and $q_j(\cdot)$ evaluated at the equilibrium point; p_j' is the derivative of $p_j(\cdot)$ evaluated at the equilibrium point.

4.2 Dynamic Update

Lemma 1. Let $P = P^* \succ 0$, $Q = Q^* \succ 0$, $L = diag\{l_i\}$, $l_i \in \mathbb{C}$, $\forall i$. Then

$$\sigma(Q^{-1}LP) \subset \rho(Q^{-1}P)Co(0 \cup \{\pm l_i\}).$$

Proof: Let v be a right eigenvector of P corresponding to the eigenvalue λ and such that the vector Pv is normalized. Then $Q^{-1}LPv = \lambda v$. Then $LPv = \lambda Qv \Rightarrow (Pv)^*L(Pv) = \lambda(Pv)^*Qv = \lambda v^*P^*(QP^{-1})Pv$. Therefore, naming $k = \rho(Q^{-1}P)((Pv)^*QP^{-1}(Pv))$ and observing that $|k| \geq 1$,

$$\lambda = \frac{(Pv)^*L(Pv)}{v^*P^*(QP^{-1})Pv} = \rho(Q^{-1}P)\Big(\sum_i \frac{|Pv_i|^2}{k}(\pm l_i) + 0\Big). \qquad \square$$

Theorem 2. The system (1)-(2)-(5)-(8) is locally stable if the following two conditions hold $\forall r \in R$:

$$k_r\left(\sum_{j\in r} q_j + \sum_{j\in r} q_j' \sum_{s:j\in s} x_s^o\right) < \frac{\pi}{2T_r}; \tag{13}$$

$$c_r\frac{\left(\sum_{j\in r} p_j + \sum_{j\in r} p_j' \sum_{s:j\in s} x_s^o\right)}{min_{r\in R}\sum_{j\in r} q_j} < \frac{\pi}{2T_r}. \tag{14}$$

Proof: As in the proof of global stability for the dynamic update case (Chen *et al.*, 2005), we shall exploit the idea of the two time scales. The first condition indeed refers to the boundary layer. The linearization of the relation for the reduced system around its equilibrium and the manifold equation, comprehensive of the delays, are:

$$\dot{\omega_r}(t) = c_r\omega_r^o - c_r x_r(t-T_r)\sum_{j\in r} p_j(\sum_{s:j\in s} x_s(t - d_1(j,s) - d_2(j,r)));$$

$$\omega_r(t-T_r) = x_r(t-T_r)\sum_{j\in r} q_j(\sum_{s:j\in s} x_s(t - d_1(j,s) - d_2(j,r))).$$

Taking the derivative of the second term, after a proper shift in time, gives:

$$\dot{\omega_r}(t) = \dot{x}_r(t)\sum_{j\in r} q_j\Big(\sum_{s:j\in s}\tilde{x}_s(j,r)\Big) + x_r(t)\sum_{j\in r} q_j'\Big(\sum_{s:j\in s}\tilde{x}_s(j,r)\Big)\sum_{s:j\in s}\dot{\tilde{x}}_s(j,r),$$

where $\tilde{x}_s(j,r) = x_s(t - d_1(j,s) - d_2(j,r) + T_r)$. Substituting this last relation within the former, letting $x_r(t) = x_r^o + y_r(t), \dot{x}_r(t) = 0 + \dot{y}_r(t)$, and linearizing around these equilibrium points gives:

$$\sum_{j\in r} q_j(\cdot)\dot{y}_r(t) = -c_r x_r^o \sum_{j\in r} p_j'(\cdot)\sum_{s:j\in s} y_s(t - d_1(j,s) - d_2(j,r)) - c_r y_r(t-T_r)\sum_{j\in r} p_j(\cdot) - x_r^o\sum_{j\in r} q_j'(\cdot)\cdot \sum_{s:j\in s}\dot{y}_s(t - d_1(j,s) - d_2(j,r) + T_r).$$

Taking he Laplacian transform and simplifying out common matrix terms, we obtain:

$$\Big(diag\{x_r^o\}^{-1}diag\Big\{\sum_{j\in r} q_j\Big\} + N(s)\Big)sY(s) = -diag\{c_r\}\cdot diag\{e^{-sT_r}\}\Big(diag\{x_r^o\}^{-1}diag\Big\{\sum_{j\in r} p_j\Big\} + M(s)\Big)Y(s),$$

where $N(s)$ and $M(s)$ are composed of

$$N_{rq}(s) = \sum_{j\in r\cap q} q'_j exp(-s(d_1(j,q) - d_1(j,r)));$$
$$M_{rq}(s) = \sum_{j\in r\cap q} p'_j exp(-s(d_1(j,q) - d_1(j,r))).$$

Then, naming the quantities inside the two big parentheses $\mathbf{Q}(s)$ and $\mathbf{P}(s)$, we have

$$sY(s) = -\mathbf{Q}(s)^{-1} diag\{c_r\} diag\{e^{-sT_r}\} \mathbf{P}(s) Y(s). \quad (15)$$

We are interested in checking the stability of this interconnection, and pose conditions on the c_r terms to enforce it. Name $L = diag\Big\{c_r \frac{\pi}{2} \frac{exp(-j\omega T_r)}{j\omega T_r}\Big\}, P = diag\{\sqrt{x_r}\}\mathbf{P}(j\omega) diag\{\sqrt{x_r}\}, Q = diag\{\sqrt{x_r}\}\mathbf{Q}(j\omega) diag\{\sqrt{x_r}\}$. Then, employing the observation that the matrices $\mathbf{Q}(s)^{-1} diag\{c_r\} diag\{e^{-sT_r}\}\mathbf{P}(s)$ and $Q^{-1}LP$ are similar, we derive, according to the lemma,

$$\sigma(Q^{-1}LP) \subset \rho(Q^{-1}P) Co\Big(0 \cup \Big\{ \pm c_r \frac{\pi}{2} \frac{exp(-j\omega T_r)}{j\omega T_r} \Big\}\Big).$$

The introduction of the additional negative terms in the convex hull does not change its structural property of excluding the -1 point in the complex plane. The problem then boils down to posing conditions on the term $\rho(Q^{-1}P)$; we know that $\rho(Q^{-1}P) \le \rho(Q^{-1})\rho(P) \le \frac{\rho(P)}{min_\lambda \sigma(Q)}$. Notice that $N(s) = A^T(-s) diag \sum_{j\in s} q'_j A(s)$ and that $N(s) = N(-s)$, $N(j\omega) \succ 0, \forall\omega$. From a linear algebra fact, given two matrices $A = A^* \succeq 0$ and $B = B^*$, then the eigenvalues of their sum, ranked increasingly, are correspondingly lower bounded by those of B. Therefore, focusing on the structure of matrix $Q = diag\{x^o_r\}^{-1} diag\{\sum_{j\in r} q_j\} + N(s)$, we claim that $min_r(\sum_{j\in r} q_j) \le min_\lambda \sigma(Q)$. This finally translates into the condition, $\forall r \in R$:

$$\rho(Q^{-1}P) < c_r \frac{\Big(\sum_{j\in r} p_j + \sum_{j\in r} p'_j \sum_{s:j\in s} x^o_s\Big)}{min_{r\in R} \sum_{j\in r} q_j} < \frac{\pi}{2T_r}.$$

We include a simulation, in figure (1), to show that the condition ensures stability. The network topology we analyzed was that of two users with one shared link (see figure (2) with $n = 2$). The link error rate was set to 2%. We assumed the following parameters: $w^o_1 = 60$, $w^o_2 = 30$; $T_1 = 0.1s$, $T_2 = 0.16s$; $k_1 = 0.35$, $k_2 = 0.2$. The condition forced us to choose $c_1 = 0.004$, $c_2 = 0.004$.

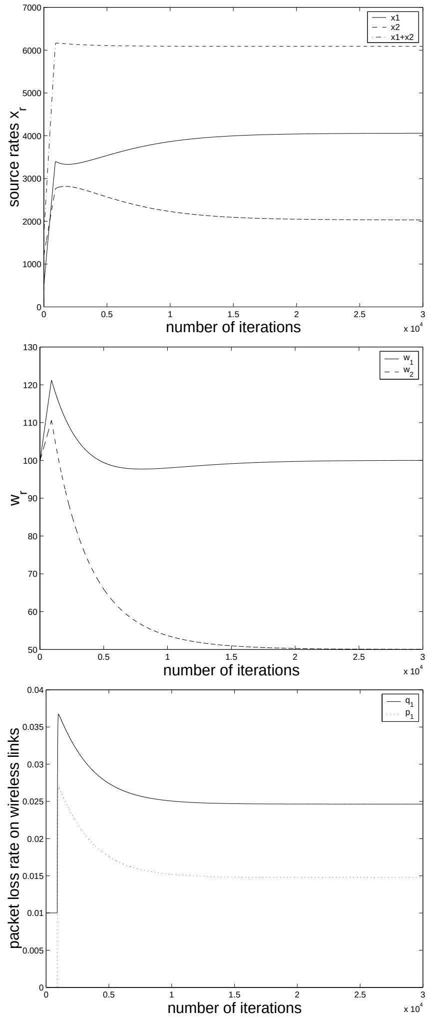

Fig. 1. Dynamic update system with delay: convergence of (a) rates $x_r(t), r = 1, 2$, (b) $w_r(t), r = 1, 2$, (c) packet loss rate $p_j(\cdot)$ and $q_j(\cdot), j = 1, 2$, with initial rate set to 0.

5. THE STRUCTURE OF THE DELAYS

It is quite important to understand the properties of the oscillations present in a system. In our rather complex setting, it is only locally possible to get quantitative results, with a linearized version of the model. Consider a system of delay differential equations of the following kind

$$\dot{x}(t) = \sum_{i=1}^{n} A_i x(t - \tau_i). \quad (16)$$

The above *delay differential equation* (DDE), which is defined for say $t \ge t_0$, needs to be specified also in the interval $[t_0 - T, t_0]$ through a *pre-shape function* $x(t) = \phi(t)$. The frequency of the oscillations of the different modes can be derived from the expression of the poles of the system, that are the solutions of its characteristic equation:

$$det(\lambda I - \sum_{i=1}^{n} A_i e^{-\lambda\tau_i}) = 0. \quad (17)$$

If we make the simplifying assumption, as in (Johari and Tan, Dec 2001), that the delay is the same for every user, then $n = R$, otherwise in general $n = J\binom{R}{2} + R$.

Unfortunately the roots of the characteristic equation (17), which is non linear and transcendental, cannot be expressed in a closed form. As a matter of fact, we need to resort to a class of functions $W(s)$ known as *Lambert* functions. A detailed description of them can be found in (Asl and Ulsoy, 2003). The general solution of the DDE can be expressed, similarly to the case of the solution of an ODE in terms of the state transition matrix, as:

$$x(t) = \sum_{k=-\infty}^{+\infty} e^{1/T W_k(-AT)t} C_k. \tag{18}$$

W_k denotes the k^{th} branch of the function. The coefficients C_k are $x \times 1$ vectors determined by the preshape functions. In our particular case, given that the initial condition for the state equations (rates) is less than the optimal rate [5], then the dynamics will experience no delay as long as all the rates do not congest the network. In fact, in this situation both the price function and its derivative will be zero for all the links. As soon as one link gets congested, the users that rely on it will start experiencing a delay. This implies that the preshape functions in our setting are actually those trajectories that describe the transient of the sending rates. In some special cases (depending on the form of the preshape functions), it could happen that only few of the coefficients C_k are different from zero, which may simplify the analysis of the oscillation frequencies. Unfortunately, it should be clear from the reasoning how this does not happen in our quite involved case.

In the following, we shall focus on two very simple topologies; exploiting some approximations in the first case, we will try to describe how oscillations are characterized in the second, more general case. For the sake of simplicity, we shall focus on the dynamic equations with no update, which structure actually encompasses the case of the static update. The first network topology we study

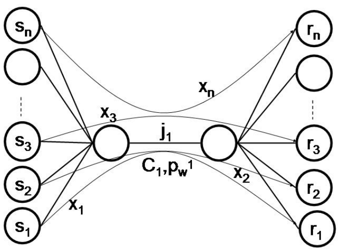

Fig. 2. Multi-user, one-link topology.

is that of one user, with one single link (see figure(2), with $n = 1$). Considering the linearized expression

$$\dot{y}_1(t) = -k_1(p_1 + x_1^o p_1')y_1(t - T),$$

we can infer that the two terms into the parenthesis will have different weight depending on the operating point of the system. Assuming the classical shape for the price, $p_1(x_1) = \frac{(x_1 - C_1)^+}{x_1}$, then the equilibrium of the system will be at the point $x_1^o = C_1 + \omega_1^o$. In the case of light congestion, $\omega_1^o \ll C_1$, then $p_1 \ll x_1^o p_1'$; in the opposite case of heavy congestion, $\omega_1^o \gg C_1$, then $p_1 \gg x_1^o p_1'$. This fact can be extended and used for the more general structure of the one bottleneck link, with n users. It is represented in figure (2). We assume that the forward and backward delays for each user are the same: $d_i(1, j) = d_j(1, i), \forall i, j = 1, \ldots, n$. Therefore, the round trip times are also the same, $T_i = T_j = T, \forall i, j = 1, \ldots, n$. The linearized dynamic equations in this case are:

$$\underline{\dot{y}}(t) = \begin{pmatrix} p_1 + \hat{y}_1 p_1' & \hat{y}_1 p_1' & \cdots & \hat{y}_1 p_1' \\ \hat{y}_2 p_1' & p_1 + \hat{y}_2 p_1' & \vdots & \hat{y}_2 p_1' \\ \vdots & \cdots & \ddots & \vdots \\ \hat{y}_n p_1' & \hat{y}_n p_1' & \cdots & p_1 + \hat{y}_n p_1' \end{pmatrix} \underline{y}(t - T) \tag{19}$$

where we named $\underline{y} = [y_1, \ldots, y_n]^T$. It is possible to extend the reasoning we advanced in the single user case to the multiuser case: if the network works in a highly congested region, then the linearization can be approximated as:

$$\underline{\dot{y}}(t) = p_1 I_{n \times n} \underline{y}(t - T), \tag{20}$$

The matrix has its n eigenvalues equal to p_1 and, as eigenvectors, the orthonormal basis. Therefore, denoting with Λ the diagonal matrix of the eigenvalues, we have that

$$W(A) = W(\Lambda).$$

For the principal branch, $k = 0$, we obtain

$$W_0(\Lambda) = diag\{W_0(\lambda_i)\} = diag\Big\{\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} p_1^n\Big\}.$$

The solution of the DDE equation then can be expressed as

$$\underline{\dot{y}}(t) = e^{diag\Big\{\sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} p_1^n\Big\} Tt} \begin{bmatrix} C_0^1 \\ \vdots \\ C_0^n \end{bmatrix} + \text{conj. branches.}$$

If instead the network has the optimum in a lightly congested point, then the linearization can be approximated as:

$$\underline{\dot{y}}(t) = p_1' diag\{\hat{y}_i\} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \vdots & 1 \\ \vdots & \cdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix} \underline{y}(t - T) \tag{21}$$

The matrix has a special structure, and it has $n - 1$ eigenvalues equal to 0, plus one equal to $p_1' \sum_{i=1}^{n} \hat{y}_i$. In this case, if we name V the matrix with the corresponding eigenvectors on its columns, we have

$$W_0(-AT) = V^{-1} W(-AT) V =$$
$$V^{-1} \begin{bmatrix} 0 & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & \sum_{m=1}^{\infty} \frac{(-m)^{m-1}}{m!} \Big(p_1' \sum_{i=1}^{n} \hat{y}_i\Big)^m \end{bmatrix} VT.$$

[5] This is typically what happens when a user starts sending packets through the network.

At this point, it would be interesting to check for sufficient conditions to avoid oscillations in a system of DDE's. Unfortunately the literature provides only sufficient conditions for the existence of components with oscillating dynamics, or necessary and sufficient conditions for the oscillations of all the components of the solutions of the DDE. All of them hinge upon the following fact (Gopalsamy, 1992)-(Gyory and Ladas, 1991):
Every solution of equation (16) oscillates componentwise if and only if its characteristic equation (17) has no real roots.

From this, we derive that both in the case of highly congested network, as well as in that of lightly congested, there will be some solutions with components that will not oscillate, but which will rather converge to the equilibrium exponentially.

6. DISCUSSION AND CONCLUSION

In this paper we have completed the analysis of the structural properties of two new schemes for flow control over wireless networks. Both algorithms modify the number of connection that a single user establishes with the network; the first scheme employs a static algorithm, while the second applies a dynamic scheme. In a first paper (Chen *et al.*, 2005) we motivated the structure of both schemes and analyzed their global stability; moreover, for the static scheme, we suggested a condition on some of the coefficients to obtain stability in the presence of heterogeneous delays at the users. This paper extends the same idea to the more complex dynamic scheme; simulations for this case are provided. Furthermore, we attempted to get some insight on the structure of the oscillations in the presence of delays. This is motivated by the fact that, in real world TCP schemes, oscillations represent one issue. We should however mention that in real TCP schemes oscillations are as well due to other important causes, for instance the discretization of the schemes. Resorting to the theory of Delay Differential Equations, and posing some simplifying assumptions, we managed to get some early results on the problem, which has never been systematically considered in the literature before. Nevertheless, it is been increasingly clear how hard the topic is to be studied analytically. Future research will stand on these results to proceed further.

Another paper (Abate *et al.*, 2005) focused on the stochastic stability analysis, and the calculation of the rate of convergence of both schemes. The authors are already working on the application of this scheme to a real TCP setting, extending the theory and getting more understanding from the simulations.

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