Competitive Online Algorithms for Smoothing Renewable Generation with Energy Storage

Yanfang Mo, *Member, IEEE,* Junyan Su, and Minghua Chen, *Fellow, IEEE*

***Abstract*—We consider a renewable power plant minimizing its peak injection for smoother power output by irrevocable energy storage charging/discharging. This online problem is uniquely challenging due to (i) the non-cumulative peak objective, (ii) coupled charging/discharging decisions across time subject to inventory constraints, and (iii) the nontrivial bilinear constraints impacting the optimal online/offline decision-making. We develop an online algorithm with the best competitive ratio (CR) among deterministic online algorithms. It maintains online-to-offline ratios in each slot to be at most the best CR, which we show, under mild conditions, can be computed beforehand by solving a number of max-min bilinear fractional programs. We also obtain an upper bound of the optimal CR that our algorithm can achieve under all circumstances. We then generalize our approach to adaptively pursue the best competitiveness given observed inputs and online decisions, leading to an anytime-optimal deterministic online algorithm with the optimal worst-case and adaptive average-case performances. Extensive trace-based simulations show that, under typical settings, our adaptive algorithm reduces the peak renewable injection by 17% and the fluctuation range by 18%.**

***Index Terms*—Energy storage management, renewable generation smoothing, competitive online algorithms.**

NOMENCLATURE

T	number of time slots
$\boldsymbol{e}$	generation profile in $\mathbb{R}^T$
$\underline{e}_t, \overline{e}_t$	bounds of renewable generation at slot t
c	energy storage capacity
η_c/η_d	charge/discharge efficiency factor, in $(0, 1]$
$\bar{\delta}_c/\bar{\delta}_d$	charge/discharge rate limit
δ_t^c/δ_t^d	charge/discharge amount at slot t, in $[0, \infty)$
$v(\boldsymbol{e})$	optimal offline peak injection under Profile $\boldsymbol{e}$
$\boldsymbol{\delta}^c(\pi, \boldsymbol{e})$	charge vector of $\mathsf{pCR\text{-}PIM}(\pi)$ under Profile $\boldsymbol{e}$
$\boldsymbol{\delta}^d(\pi, \boldsymbol{e})$	discharge vector of $\mathsf{pCR\text{-}PIM}(\pi)$ under Profile $\boldsymbol{e}$
π^*	optimal CR of all deterministic online algorithms
$[m..n]$	set $\{m, \ldots, n\}$ for $m, n \in \mathbb{N}$, where $[n] := [1..n]$
$[x]^+$	maximum of x and 0 for $x \in \mathbb{R}$

I. INTRODUCTION

The global renewable generation capacity has reached $2,799$ GW, amounting to 125 the Three Gorges Dams by 2020 [1]. Despite the rapid development of renewable generation, it is challenging to efficiently integrate renewable energy into the grid, as wind or solar power is intermittent and volatile. An immediate concern lies in short-term power variations from renewable generators that may cause severe voltage and frequency deviations, leading to poor power quality and even disconnection from the grid [2]. Another concern at longer time scales is curtailment, referring to the involuntary reduction in the output of a generator below what it could otherwise produce given available resources like wind or sunshine [3] and damaging the revenue of renewable plants. From 2010 to 2016, 16% of wind generation in China was curtailed, and the associated opportunity cost exceeds 1.2 billion dollars [4].

We consider a renewable power plant, e.g., an offshore wind farm, using energy storage to smooth its injection to the grid[1]. We emphasize the long-term renewable generation smoothing for more stable supply and reducing the possibility of curtailment [5][2]. Our work is also helpful in filtering power variations at short-term time scales (e.g., milliseconds or seconds) for better grid connection [7]. We focus on minimizing the peak electricity injection into the grid for smoother generation output. The fluctuation level is also commonly measured by the peak-to-mean ratio [8] and range [9]. Our empirical results show that the concise online peak-minimizing practice can also cause significant fluctuation reduction regarding other metrics. Also, in a future 100% renewable electricity system, operators tend to regulate flexible loads to balance time-varying supplies [10]. Lowering peak supply outputs can bring reference curves easier for flexible loads to track [11], [12].

However, achieving our goal is algorithmically nontrivial. Renewable generation, especially the peak one, is hard to predict. The unreliable predictions may cause poor performance for methods such as stochastic optimization [13] and model predictive control (MPC) [14], requiring statistics or exact values of future information. We devote to online optimization requiring little or no future information [15]. Specifically, the renewable generation is revealed sequentially in time, and the operator irrevocably makes charging/discharging decisions to reduce the peak injection to the grid. We adopt a popular online algorithm performance metric – *competitive ratio* (CR). Here, CR refers to the worst-case ratio between the peak renewable injection resulted from an online algorithm with little future information and the optimal (namely, the smallest) one that we can obtain in the omniscient setting with perfect future information [15], [16]. A smaller CR means that the algorithm can achieve performance closer to that of an optimal clairvoyant algorithm for all possible inputs. More importantly, the algorithmic approach in our work can adaptively respond

The work was supported in part by the General Research Fund (11206821) from the Research Grants Council, Hong Kong. All authors are with the School of Data Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China.

[1]The injection refers to DC electricity or active power injection, depending on the generation system design.

[2]When curtailment happens, it is reasonable to give priority to plants with more fluctuating generation and/or higher peak generation [6].

to the real-time information as MPC and concern the average-case performance as stochastic optimization, in addition to emphasizing the robust relative performance quantified by CR.

Mathematically, the online peak-injection minimization with energy storage we propose is a particular online peak minimization problem with inventory constraints. Despite several pioneering results [17]–[20] in the competitive analysis literature, our work differs from previous ones by not requiring to know the value of the peak demand [17], allowing resource replenishment [19], [20], and considering a distinct relation between the peak minimization objective and limited resource [17]–[20]. Notably, there are three main challenges in this work. First, it is hard to identify to what extent the supply at the current time slot will account for the peak injection and whether the corresponding charging/discharging decision can contribute to minimizing the peak supply or not. Second, the online decisions are coupled due to the storage management constraints. Third, there are extra difficulties brought by the bilinear constraints to avoid intentionally using storage inefficiency for peak-injection reduction. This fact distinguishes this work from several peak-cost minimization problems [17], [19], [21] where such bilinear constraints do not affect the optimality of online/offline decisions. Bearing these challenges in mind, we make the following contributions.

▷ We formulate in Section III a peak-injection minimization problem subject to inventory management constraints, and study it under a practical online setting in Section IV. Particularly, we develop an optimal online algorithm with the smallest CR among all deterministic online algorithms. It maintains online-to-offline ratios in each slot to be at most the optimal deterministic CR, which we show can be computed beforehand by solving a number of max-min bilinear fractional programs with bilinear constraints, given a sufficient discharge rate limit. Moreover, we obtain an upper bound of the optimal CR that our algorithm can achieve under all circumstances.

▷ We generalize our online algorithm design to an *anytime-optimal* one in Section V. It addresses the common criticism that online algorithms focus on worst cases which may rarely happen in practice and are not adaptive to the newly observed information at runtime. Our idea is to make online decisions at each round by adaptively pursuing the best deterministic competitiveness regarding the residual uncertainty (referring to the anytime-optimal deterministic CR). In this way, our algorithm achieves adaptive average-case performance while retaining the optimal worst-case performance.

▷ We conduct simulations in Section VI to showcase the empirical performance of our algorithms based on the data from actual wind farms. Typically, our adaptive algorithm can reduce the peak injection by 17%, achieving 72% performance of the optimal clairvoyant algorithm where we know generation curves in advance. It also reduces the fluctuation by 9% and 18%, in terms of peak-to-mean ratio and range, respectively. Also, we compare our algorithms with conceivable alternatives. Particularly, our adaptive algorithm without requiring future information shows comparable performance to the MPC with an accurate 1.25-hour prediction window. These observations verify the effectiveness and robustness of our approach to the decision-making under uncertainty.

Along the way, we develop in Section III a customized bisection method to efficiently solve the offline peak-injection minimization problem with complete future knowledge. We also extend our approach to mitigate the supply-demand mismatch for prosumers [22].

Due to the space limitation, all proofs are in the Appendix.

II. Related Work

A. Using Energy Storage in Renewable Generation

Many results focus on using energy storage for the connection problems of renewable generators. For example, the paper [23] proposes a probabilistic method for the connection of wind generators at weak electricity grids, mainly limited by the voltage rise. The work [24] applies MPC to smoothing the voltage curve of a photovoltaic (PV) system. See [25] for a review on wind power smoothing with short-term applications of energy storage technologies. Researchers also study renewable integration at longer time scales. The work [5] demonstrates the opportunities and challenges of using energy storage to reduce curtailment. An MPC framework is developed in [9] for energy shifting and arbitrage of a PV storage system. The paper [26] studies control algorithms for an energy storage system to improve renewable utilization based on Lyapunov optimization. The authors of [27] exploit the potentials of managing plug-in electric vehicles in facilitating renewable penetration. We aim to minimize the peak injection of a plant with energy storage for reducing the possibility of curtailment.

B. Online Peak Minimization with Inventory Constraints

Online optimization with inventory constraints challenges researchers by the coupling of online decisions across time. The inventory may relate to a warehouse, a buffer [28], or a battery [29]. A special case is called online optimization with budget constraints where replenishment is not allowed. Typical examples include the one-way trading problem [30], the OOIC problem in [31], and the storage (e.g. fuel cells) discharging problem in [20]. Meanwhile, belonging to online minimax optimization, online peak optimization has wide application like load balancing. The peak may refer to the maximum level of resource requirement [32], the maximum job delivery time [33], and the peak power consumption [34]. The nonsmoothness of the peak minimization objective significantly complicates the online algorithm design. Naturally, online peak optimization with inventory constraints is even more difficult due to the twofold concerns. Notwithstanding, we see pioneering results in [17]–[20]. Along this line, we consider a particular peak minimization problem subject to inventory management constraints. The problem differs from the previous ones by not requiring to know the value of the peak demand [17], allowing resource replenishment [19], [20], and considering a distinct relation between the peak minimization objective and limited resource [17]–[20].

C. Competitive Online Optimization in Smart Grids

Competitive online optimization applies to decision-making under uncertainty and the past decade has witnessed its application in smart grids, like electric vehicle charging [18],

[35]–[37], storage management [19], [20], [29], and economic dispatching [38], [39]. Our online algorithm design uses and extends an intuitive algorithmic framework called CR-Pursuit. Basically, the resulting algorithms make decisions by maintaining selected online-to-offline ratios not to exceed given CRs. Similar ideas have been used in [18]–[20], [31], [32], [36]. Nevertheless, there are unique challenges in applying the framework to the particular online problem herein.

Also, our competitive online optimization inherits the merits of other approaches to optimization under uncertainty. Precisely, our approach guarantees the worst-case performance as robust optimization [40], concerns average-case performance as stochastic optimization [41], and adaptively makes decisions in response to newly observed information as MPC [14]. Moreover, this performance metric (CR) distinguishes our approach from the other three methods by stressing the fairness in performance evaluation under different input sequences.

III. System Modeling and Problem Formulation

A. System Modeling

Renewable generation: The renewable generation can be directly injected into the grid or (partially) stored in the energy storage system. We divide the operating cycle into T slots of equal length (e.g. 15 minutes) and let $e_t \geq 0$ denote the amount of electricity generated from the renewable at slot $t \in [T]$, constituting the generation profile $\boldsymbol{e} \in \mathbb{R}^T$.

Energy storage: We consider an energy storage system of capacity c and allowing charging and discharging in operation, like batteries. The charge and discharge efficiency factors are respectively η_c and η_d, taking values in $(0, 1]$. Let $\bar{\delta}_c \geq 0$ and $\bar{\delta}_d \geq 0$ respectively denote the maximum charge and discharge amounts of a slot. See [42] for parameters of typical energy storage systems. We respectively denote the charge and discharge amounts at slot $t \in [T]$ by δ_t^c and δ_t^d. Then, we require $\delta_t^c \leq \bar{\delta}_c$ and $\delta_t^d \leq \bar{\delta}_d$ for all $t \in [T]$. Let s_t denote the state of charge (SoC) of the system at the end of slot $t \in [T]$, that should satisfy $0 \leq s_t \leq c$. As a starting point, let us assume that the energy storage is empty before the operating cycle, namely $s_0 = 0$. It will be clear on how to deal with the case with $s_0 \neq 0$ by similar techniques as we proceed.

B. Problem Formulation

To smooth renewable generation and reduce the possibility of curtailment, we formulate the peak-injection minimization problem with inventory management constraints (PIM) as:

$$\begin{aligned}
\textbf{PIM:} \quad & \min_{\delta_t^c, \delta_t^d, t \in [T]} \quad \max_{t \in [T]} \left(e_t - \delta_t^c + \delta_t^d\right) \\
& \text{subject to} \quad \delta_t^c \leq e_t,\ \delta_t^c \delta_t^d = 0, \\
& \qquad 0 \leq \delta_t^c \leq \bar{\delta}_c,\ 0 \leq \delta_t^d \leq \bar{\delta}_d, \forall t \in [T]; \\
& \qquad 0 \leq \sum_{t \in [\tau]} \left(\eta_c \delta_t^c - \delta_t^d / \eta_d\right) \leq c, \forall \tau \in [T].
\end{aligned}$$

In the above formulation, we aim to reduce the fluctuation by minimizing the peak renewable injection into the grid. At slot t, the operator determines the charge and discharge amounts, namely δ_t^c and δ_t^d of the energy storage system. The constraint $\delta_t^c \leq e_t$ signifies that the charge amount should not exceed the available renewable generation at the slot.

We impose $\delta_t^c \delta_t^d = 0$ so that the injection of each slot t amounts to either the renewable generation deducting the storage recharging ($e_t - \delta_t^c$) or the combination of renewable generation and storage discharging ($e_t + \delta_t^d$). The operator who ignores this constraint will abuse the charging/discharging inefficiency to reduce the peak injection. This practice will significantly increase energy losses and exacerbate the degradation of the energy storage system, and is undesirable. Overall, such a bilinear constraint is deemed nontrivial and differentiates our work from several peak-cost minimization problems [17], [19], [21] where the bilinear constraint can be dispensable in pursuing optimal online/offline decisions.

The remaining constraints are due to the energy storage system. The charge/discharge amounts should respect the rate limits, and the state of charge (SoC) should be within zero and the storage capacity at every slot. Interested readers can find other applications of this formulation. A typical example with $\eta_c = \eta_d = 1$ refers to the scenario where a server equipped with a buffer processes the requests over time with the goal of minimizing the maximum load to serve in a slot.

We aim to study PIM in a practical online setting where the profile $\boldsymbol{e}$ is sequentially revealed and we irrevocably determine the charge/discharge amounts at each slot. This is motivated by the unpredictability of renewable generation and complicates PIM by not knowing future information. Apart from bilinear constraints, the online PIM has two extra challenges. First, like other inventory-related problems, PIM challenges us by the coupling of online decisions across time. Second, it is harder to tackle the non-cumulative (peak-minimizing) objective online than cumulative ones, as later decisions cannot compensate for the previous ones that turn out bad in hindsight. For example, if we use too little generation to charge the energy storage system in an earlier slot, we may obtain a large peak injection which cannot be reduced anymore. In the online setting, we hardly know whether and to what extent the supply at the current slot will account for the ultimate peak supply.

Although PIM is a particular online peak minimization problem with inventory constraints, it differs from previous studies by not requiring to know the value of peak demand [17], allowing resource replenishment [19], [20], and considering a distinct relation between the peak minimization objective and limited resource, thus having unique challenges.

C. Optimal Offline Solution with Perfect Information

Before proceeding, we first examine the offline PIM problem, where we assume to know $\boldsymbol{e}$ in advance. The ideal setting can serve as a benchmark for the online setting. However, the offline PIM problem is also hard to solve because we cannot ignore the non-convex bilinear constraint $\delta_t^c \delta_t^d = 0$ that will be active at the optimum of the offline problem when $\eta_c, \eta_d < 1$. A common method is to reformulate PIM as a mixed integer linear program (MILP) and then resort to standard solvers without guaranteeing optimality, as exemplified in [26].

However, efficiently finding the optimal objective value for the offline PIM is critical to the real-time implementation of

Algorithm 1: A Bisection Method for An Optimal Offline Solution of PIM

Input: Storage parameters $c, \eta_c, \eta_d, \bar{\delta}_c, \bar{\delta}_d$, a generation profile $\boldsymbol{e}$, and the slot number T;

Output: The optimal peak injection $v = v_{ub}$ and an optimal solution: $\delta_t^c, \delta_t^d, t \in [T]$;

$v_{ub} = \max_{t\in[T]} e_t$, $v_{lb} = \max\{0, v_{ub} - \bar{\delta}_c\}$;
if $\sum_{t=1}^{T} e_t \le c/\eta_c$ **then**
  $v = 0$, $\delta_t^c = e_t$, and $\delta_t^d = 0$, for all $t \in [T]$, **return**;
else
  while $v_{ub} - v_{lb} \ge \epsilon$ **do**
    $v = (v_{lb} + v_{ub})/2$, $s = 0$;
    for $t = 1, 2, \ldots, T$ **do**
      $\delta = e_t - v$;
      if $\delta \ge 0$ **then**
        $s = s + \eta_c \min\{\bar{\delta}_c, \delta\}$;
        if $s > c$ **then**
          $v_{lb} = v, v = v_{ub}$, **break**;
        else
          $\delta_t^c = \min\{\bar{\delta}_c, \delta\}$, $\delta_t^d = 0$;
      else
        $\delta_t^c = 0, \delta_t^d = \min\{s\eta_d, \bar{\delta}_d, -\delta\}$,
        $s = s - \delta_t^d/\eta_d$;
    $v_{ub} = v$;

our optimal online algorithm proposed in the next section. We herein exploit the problem structure to develop a bisection-based algorithm (Algorithm 1) for the offline PIM. The key idea is to produce a solution based on an estimate of the optimal peak injection and then check the feasibility of the solution to update the estimated lower or upper bound of the optimal objective value. We continue this process until the termination condition is satisfied. We arrive at the following proposition.

Proposition 1. *Algorithm 1 generates a feasible solution to the PIM problem, attaining the optimal objective value within error ϵ in $\mathcal{O}(\log(\min\{\bar{\delta}_c, \max_{t\in[T]} e_t\}/\epsilon))$ steps.*

IV. First Online Algorithm with the Optimal Deterministic CR

The focus of this section is to propose the first (and optimal) deterministic online algorithm for PIM, with a worst-case performance guarantee measured in CR. A challenge lies in computing the optimal deterministic CR. These results will lay foundation for the next section where we propose an adaptive extension to more tactfully respond to the real-time information for better average-case performance.

A. Online Setting and Worst-Case Performance Metric – CR

In the practical online setting, the renewable generation amounts $\{e_t\}_{t\in[T]}$ are revealed chronologically and we merely know beforehand that $e_t \in [\underline{e}_t, \overline{e}_t]$, for all $t \in [T]$. We can readily obtain the uncertainty intervals $[\underline{e}_t, \overline{e}_t]$ from existing prediction techniques and modeling the uncertainty by intervals can facilitate accommodating possible look-ahead information [19]. Before the operating cycle, the operator identifies the slot number T and gets the prior information including the energy storage parameters $(c, \eta_c, \eta_d, \bar{\delta}_c, \bar{\delta}_d)$. At each slot t, the operator observes e_t and makes irrevocable charging/discharging decisions δ_t^c, δ_t^d.

We use CR to evaluate online algorithms [15]. It concerns robustness by emphasizing the worst-case performance over the uncertainty set, and respects fairness by considering the relative performance of an online algorithm compared to the optimal clairvoyant algorithm for each possible input sequence. For a minimization problem like PIM, the CR of a deterministic online algorithm $\mathfrak{A}$ is defined as the largest ratio between the objective value attained by the algorithm and that under an optimal offline solution with perfect information over all possible input sequences[3], namely $CR_{\mathfrak{A}} = \max_{\boldsymbol{e}\in\mathcal{E}} \frac{v_{\mathfrak{A}}(\boldsymbol{e})}{v(\boldsymbol{e})}$, where $v_{\mathfrak{A}}(\boldsymbol{e})$ refers to the peak injection under the online algorithm $\mathfrak{A}$ with the profile $\boldsymbol{e}$ revealed sequentially in time, and $v(\boldsymbol{e})$ is the optimal objective value of the offline PIM problem knowing the entire profile $\boldsymbol{e}$ beforehand. We prefer online algorithms with smaller CRs, indicating closer performance to the optimal clairvoyant algorithm with perfect information. However, it is fundamentally challenging to design an optimal deterministic online algorithm for PIM because of the peak minimization objective, the inventory constraints coupling decisions across time, and the nontrivial bilinear constraints.

B. CR-Pursuit Framework

Before proceeding, let us introduce the CR-Pursuit algorithmic framework we adopt in this work. As the name indicates, we are first given the CR and then make online decisions in each round by "pursuing" the prescribed CR. The underlying idea is intuitive but the design is problem-specific. Here, we sum up our experience in this work and pioneering results [19], [20], [31], [36], and highlight four critical points.

[P1:] First, to pursue the CR, we need to solve an offline problem in each decision-making round. The offline problems in previous studies [19], [20], [31], [36] are easy to solve. However, the difficulty of solving the offline PIM imposes additional challenges for our algorithm design.

[P2:] Second, the core of a CR-Pursuit algorithm is to maintain an online-to-offline performance ratio to be no more than the CR in each round. Thus, we have to properly select the online-to-offline ratio such that we can maintain the ratio to be no more than a reasonable CR by feasible decisions.

[P3:] Third, given a proper selection of the online-to-offline ratios, the CR-Pursuit framework usually suggests a series of online algorithms parameterized by their respective CRs. We wonder what is the smallest "pursuable" CR under the selection, relating to the best online algorithm among them.

[P4:] Finally, we ask whether the best algorithm under CR-Pursuit attains the best CR among all (deterministic) online

[3]One may question that $v(\boldsymbol{e}) = 0$ holds for some $\boldsymbol{e} \in \mathcal{E}$ and the CR of any online algorithm will approach ∞. To avoid this trouble, in practice, we can select a proper positive constant α (indicating the dead zone for not penalizing small peaks) and replace the denominator with $\max\{v(\boldsymbol{e}), \alpha\}$.

Algorithm 2: pCR-PIM(π) with $s_0 = 0$

Input: Storage parameters $c, \eta_c, \eta_d, \bar{\delta}_c, \bar{\delta}_d$, a generation profile $\boldsymbol{e}$, and the slot number T;

Output: Online decisions $\delta_t^c(\pi, \boldsymbol{e}), \delta_t^e(\pi, \boldsymbol{e}), t \in [T]$;

for $t = 1, 2, \ldots, T$ **do**

  $\delta_t^d(\pi, \boldsymbol{e}) = \min\{[\pi v(\boldsymbol{e}^t) - e_t]^+, \bar{\delta}_d, \eta_d s_{t-1}\}$;

  $\delta_t^c(\pi, \boldsymbol{e}) = [e_t - \pi v(\boldsymbol{e}^t)]^+ \left(\le [e_t - v(\boldsymbol{e}^t)]^+ \le \bar{\delta}_c\right)$;

  $s_t = s_{t-1} + \eta_c \delta_t^c(\pi, \boldsymbol{e}) - \delta_t^d(\pi, \boldsymbol{e})/\eta_d$.

algorithms for the problem. If so, we can significantly reduce the search space of optimal online algorithms to a one-dimensional space by applying the CR-Pursuit framework.

C. Online Algorithm Design

Proposition 1 has addressed **[P1]**. Next, we give a solution to **[P2]**. Note that $v(\boldsymbol{e})$ represents the optimal offline peak injection under the profile $\boldsymbol{e} = [e_1\ e_2\ \cdots\ e_T]'$. At slot $t \in [T]$, given observed inputs and input bounds, we hardly know the complete $\boldsymbol{e}$ but construct a reference input sequence, i.e.,

$$\boldsymbol{e}^t = [e_1\ e_2\ \cdots\ e_t\ \underline{e}_{t+1}\ \cdots\ \underline{e}_T]'.$$

It is clear that $v(\boldsymbol{e}) \ge v(\boldsymbol{e}^t)$ and we choose $v(\boldsymbol{e}^t)$ as the representative of $v(\boldsymbol{e})$ at slot $t \in [T]$. Thus, given a ratio π, we apply the idea of CR-Pursuit and maintain the online-to-offline ratio under the reference input sequence $\boldsymbol{e}^t$ not to exceed π at each slot $t \in [T]$. In this way, we obtain a family of online algorithms for PIM, each of which is parameterized by a given ratio π and called pCR-PIM(π). Let $\boldsymbol{\delta}^c(\pi, \boldsymbol{e})$ and $\boldsymbol{\delta}^d(\pi, \boldsymbol{e})$ respectively denote the charging and discharging decisions of pCR-PIM(π) as the profile $\boldsymbol{e}$ is sequentially revealed in time. Then, at each slot t, the SoC of the system is s_t and pCR-PIM(π) determines $\delta_t^c(\pi, \boldsymbol{e})$ and $\delta_t^d(\pi, \boldsymbol{e})$ such that

$$\frac{\max_{\tau \in [t]} \left(e_\tau - \delta_\tau^c(\pi, \boldsymbol{e}) + \delta_\tau^d(\pi, \boldsymbol{e})\right)}{v(\boldsymbol{e}^t)} \le \pi.$$

See Algorithm 2 for the pseudocode of pCR-PIM(π). The question then arises whether pCR-PIM(π) will always generate a feasible solution for all possible generation profiles. If so, then π is the CR of pCR-PIM(π).

Definition 1. *pCR-PIM(π) is said to be feasible if it generates a feasible solution $\left(\boldsymbol{\delta}^c(\pi, \boldsymbol{e}), \boldsymbol{\delta}^d(\pi, \boldsymbol{e})\right)$ for any $\boldsymbol{e} \in \mathcal{E}$.*

To study the feasibility of pCR-PIM(π), we define an inventory function, which maps a ratio π to the maximum SoC over all possible generation profiles under pCR-PIM(π):

$$\Phi(\pi) = \max_{\boldsymbol{e} \in \mathcal{E}, \tau \in [T]} \sum_{t=1}^{\tau} \left(\eta_c \delta_t^c(\pi, \boldsymbol{e}) - \delta_t^d(\pi, \boldsymbol{e})/\eta_d\right).$$

Lemma 1. *pCR-PIM(π) is feasible if and only if $\Phi(\pi) \le c$, where $\Phi(\pi)$ is non-increasing in π.*

Lemma 1 states that pCR-PIM(π) is feasible if and only if π is no less than a threshold ratio that brings us the best pCR-PIM algorithm. More importantly, we show that this threshold turns out the best possible CR among all deterministic online algorithms for PIM, as stated below.

Theorem 1. *The unique solution π^* to the equation $\Phi(\pi) = c$ is the best possible competitive ratio among all deterministic online algorithms for PIM.*

The above theorem answers **[P4]** and suggests that to design the optimal deterministic online algorithm for PIM, it suffices to search for the smallest ratio such that the resulting pCR-PIM algorithm is feasible.

D. Optimal CR π^* among Deterministic Online Algorithms

To fully address **[P3]**, it remains to find the optimal deterministic CR π^*. Next, we transform finding a lower bound of π^* into a number of max-min bilinear fractional programs with bilinear constraints. The lower bound is tight when the discharge rate limit $\bar{\delta}_d$ is sufficiently large. Moreover, for an arbitrary discharge rate limit, we can obtain an upper bound of the best CR by bilinear fractional programming.

1) A Tight Lower Bound: Recall Algorithm 2 and the feasibility condition ($\Phi(\pi) \le c$). In practice, the discharge rate limit is sufficiently large [42] and thus let us temporarily ignore the discharge rate limit. In this case, it turns out that pCR-PIM(π) is feasible if and only if for any nonempty integer interval $[m..n] \subseteq [T]$ and any profile $\boldsymbol{e} \in \mathcal{E}$, it holds that

$$\sum_{t \in [m..n]} \left(\eta_c [e_t - \pi v(\boldsymbol{e}^t)]^+ - [\pi v(\boldsymbol{e}^t) - e_t]^+/\eta_d\right) \le c,$$

or equivalently,

$$\min_{\eta_t \in \{\eta_c, 1/\eta_d\}} \frac{\sum_{t \in [m..n]} \eta_t e_t - c}{\sum_{t \in [m..n]} \eta_t v(\boldsymbol{e}^t)} \le \pi. \tag{1}$$

One worst-case input sequence for pCR-PIM(π^*) refers to a generation profile $\boldsymbol{e} \in \mathcal{E}$, under which the pCR-PIM(π^*) algorithm will fully charge the storage system at certain moment. That is to say, there exists a worst-case input sequence $\boldsymbol{e}^*$ and an integer interval $[m^*..n^*]$ such that the left-hand side of (1) equals π^*. Hence, we obtain the following proposition.

Proposition 2. *Given $\bar{\delta}_d \ge \max_{t \in [T]} \bar{e}_t - \min_{t \in [T]} \underline{e}_t$, the optimal deterministic CR π^* for the online PIM is given by*

$$\pi^* = \max_{[m..n] \subseteq [T], \boldsymbol{e} \in \mathcal{E}} \min_{\eta_t \in [\eta_c, 1/\eta_d]} \frac{\sum_{t \in [m..n]} \eta_t e_t - c}{\sum_{t \in [m..n]} \eta_t v(\boldsymbol{e}^t)}.$$

Proposition 2 suggests that we can compute π^* by solving a number of max-min bilinear fractional programs. Precisely, we define a class of max-min bilinear fractional programs with bilinear constraints, each of which is parameterized by an integer interval $[m..n] \subseteq [T]$:

$$\max_{\boldsymbol{e} \in \mathcal{E}, u_i, \delta_{ij}^c, \delta_{ij}^d} \ \min_{\eta_t} \ \frac{\sum_{t \in [m..n]} \eta_t e_t - c}{\sum_{t \in [m..n]} \eta_t u_t}$$

$$\text{subject to} \quad 0 \le \sum_{j=1}^{\tau} \left(\eta_c \delta_{ij}^c - \delta_{ij}^d/\eta_d\right) \le c, \forall i, \tau \in [T]; \tag{2}$$

$$\delta_{ij}^c \delta_{ij}^d = 0, \tag{3}$$

$$0 \le \delta_{ij}^d \le \bar{\delta}_d,\ 0 \le \delta_{ij}^c \le \bar{\delta}_c, \forall i, j \in [T]; \tag{4}$$

$$\delta_{ij}^d \le e_j - \delta_{ij}^c + \delta_{ij}^d \le u_i, \forall \le j \le i \le T; \tag{5}$$

$$\delta_{ij}^d \le \underline{e}_j - \delta_{ij}^c + \delta_{ij}^d \le u_i, \forall 1 \le i < j \le T; \tag{6}$$

$$\eta_c \le \eta_t \le 1/\eta_d, \forall t \in [T]. \tag{7}$$

The constraints (2)-(6) are due to the offline PIM solved in each round under pCR-PIM. Precisely, for all $i \in [T]$, the variables $(\delta_{ij}^c, \delta_{ij}^d), j \in [T]$ and u_i are respectively related to the optimal solution and peak injection of PIM with the profile $\boldsymbol{e}^i$. Additionally, the objective is due to Proposition 2[4]. We use CR-C($[m..n]$) to denote the optimal objective value of the problem. Then, the lemma below follows from Proposition 2 and the formulation of the offline PIM problem.

Lemma 2. *For each $[m..n] \subseteq [T]$, it holds that*

$$\textit{CR-C}([m..n]) = \max_{\boldsymbol{e} \in \mathcal{E}} \min_{\eta_t \in [\eta_c, 1/\eta_d]} \frac{\sum_{t \in [m..n]} \eta_t e_t - c}{\sum_{t \in [m..n]} \eta_t v(\boldsymbol{e}^t)}.$$

By Proposition 2 and Lemma 2, we attain the following theorem. It states that we can compute the best deterministic CR by solving $\binom{T}{2}$ max-min bilinear fractional programs with bilinear constraints if $\bar{\delta}_d$ is sufficiently large, as is often the case in practice [42]; otherwise, we obtain a lower bound.

Theorem 2. $\pi^* \geq \max_{[m..n] \subseteq [T]}$ *CR-C*$([m..n])$ *and the equality will hold as $\bar{\delta}_d$ increases.*

2) An Upper Bound: Lemma 1 requires us to find an upper bound π of the optimal CR π^* so that the solutions from the pCR-PIM(π) algorithm are feasible over all possible inputs. However, when the discharge rate limit is small, the previous approach only gives a lower bound of π^*. Thus, for any $\bar{\delta}_d$, we introduce the following disjoint bilinear fractional program that attains its optimum at a vertex of its feasible region [43] and the optimal objective value CRu-C gives an upper bound:

$$\max_{\boldsymbol{e} \in \mathcal{E}, u_i, \delta_{ij}^c, \delta_{ij}^d, \eta_t} \frac{\sum_{t \in [T]} \eta_t e_t - c/\eta_c}{\sum_{t \in [T]} \eta_t u_t}$$
$$\text{subject to} \quad (2); (4); (5); (6);\ 0 \leq \eta_t \leq 1, \forall\, t \in [T].$$

Theorem 3. $\pi^* \leq$ *CRu-C for any $\bar{\delta}_d$.*

The idea behind Theorem 3 is intuitive. When allowing charging only in the online implementation, we can at best maintain an online-to-offline ratio no less than the best CR π^* that is obtained by allowing both charging and discharging. Meanwhile, ignoring the bilinear constraints in a maximization problem leads to a larger optimal objective value.

CR-C and CRu-C programs challenge us by bilinear constraints and bilinear-fractional objectives, which generally cause NP-hardness. Although we can solve such hard problems by tactfully designing reformulation-linearization and branch-and-bound techniques [44], one may argue that these methods are time-consuming for computing the best CR π^* and its bounds. The good news is that the best CR can be computed beforehand; moreover, under typical values of the discharge rate limit $\bar{\delta}_d$, the best CR π^* matches its lower bound in Theorem 2[5]. Thus, the complexity of Algorithm 2 is mainly due to obtaining $v(\boldsymbol{e}^t)$ for each $t \in [T]$. Following Proposition 1, we obtain the following theorem.

Theorem 4. *The time complexity of each decision-making in Algorithm 2 is $\mathcal{O}(\log(\min\{\bar{\delta}_c, \max_{t \in [T]} e_t\}/\epsilon))$ and that of Algorithm 2 is $\mathcal{O}(T \log(\min\{\bar{\delta}_c, \max_{t \in [T]} e_t\}/\epsilon))$.*

V. Adaptive Extension for Improving Average-Case Performance

Most online algorithms are criticized for not being adaptive to the actual inputs gradually revealed in the runtime for the sake of the worst-case performance guarantee. This results in unsatisfactory average-case performance of many online algorithms, despite the good worst-case performance guarantee. We herein extend pCR-PIM algorithm to an adaptive one, in response to the real-time information. The idea is intuitive: As observing parts of the actual input, we prune the uncertainty set and pursue a potentially smaller ratio in the subsequent decision round. This practice will lead to better online-to-offline performance ratios for average-case (non-worst-case) instances, even though we do not require additional statistical information of future uncertainty like stochastic optimization.

Before proceeding, we extend the concept of CR regarding the original uncertainty set $\mathcal{E}$, to a family of anytime CRs regarding the residual uncertainty. Roughly, the anytime CR at slot t of an algorithm refers to the largest ratio between the online and the offline performances, given observed inputs and actions so far. For the online PIM, given observed inputs $e_\tau, \tau \in [t]$, the anytime CR at time slot t of a deterministic online algorithm $\mathfrak{A}$ is defined as $\pi_t^{\mathfrak{A}} = \max_{\boldsymbol{x} \in \mathcal{E}_t} \frac{v_{\mathfrak{A}}(\boldsymbol{x})}{v(\boldsymbol{x})}$, where $\mathcal{E}_t = \{\boldsymbol{x} \in \mathcal{E} \mid x_\tau = e_\tau, \forall \tau \in [t]\}$. Then, the anytime-optimal deterministic CR, defined below, characterizes the best competitiveness a deterministic online algorithm can obtain, given the residual uncertainty and the actions so far.

Definition 2. *Given $e_\tau, \tau \in [t]$ and $\delta_\tau^c, \delta_\tau^d, \tau \in [t-1]$, the anytime-optimal deterministic CR at slot $t \in [T]$ is defined as $\pi_t^* = \min_{\mathfrak{A} \in \mathcal{A}_t} \max_{\boldsymbol{x} \in \mathcal{E}_t} \frac{v_{\mathfrak{A}}(\boldsymbol{x})}{v(\boldsymbol{x})}$, where $\mathcal{A}_t$ is the set of all deterministic online algorithms $\mathfrak{A}$ satisfying $\delta_\tau^c(\mathfrak{A}, \boldsymbol{x}) = \delta_\tau^c, \delta_\tau^d(\mathfrak{A}, \boldsymbol{x}) = \delta_\tau^d$, for all $\tau \in [t-1]$ and $\boldsymbol{x} \in \mathcal{E}_{t-1}$.*

We design the anytime-optimal deterministic pCR-PIM algorithm (see Algorithm 3), where the operator determines the charge/discharge amount at slot t by maintaining an online-to-offline ratio not to exceed the anytime-optimal CR π_t^*, instead of constantly maintaining the ratio π^* as in the pCR-PIM(π^*) algorithm. That is, the formulas in Algorithm 3 ensure that:

$$\frac{\max_{\tau \in [t]} \left(e_\tau - \delta_\tau^c + \delta_\tau^d\right)}{v(\boldsymbol{e}^t)} \leq \pi_t^*.$$

To implement Algorithm 3, it remains to identify the anytime-optimal deterministic CRs. For similar arguments, we can extend the method for computing the best deterministic CR with an empty energy storage ($s_0 = 0$) to obtain the anytime-optimal CRs, which also sheds light on how to compute the best deterministic CR with $s_0 \neq 0$. However, the computation of the anytime-optimal CRs is more difficult because of the non-zero initial SoC s_{t-1} and the fact that it is unnecessary to recharge the battery such that the remaining renewable generation is smaller than the peak injection so far.

The following two observations will give us a range of each anytime-optimal deterministic CR π_t^*. First, it follows from the

[4]As mentioned in Footnote 3, we often set a dead-zone constant $\alpha > 0$ for not penalizing small peaks and thus add a constraint $u_t \geq \alpha$ for each $t \in [T]$.

[5]We assume hereinafter that the discharge rate limit $\bar{\delta}_c$ is large enough, e.g. $\bar{\delta}_d \geq \max_{t \in [T]} \bar{e}_t - \min_{t \in [T]} \underline{e}_t$, unless stated otherwise.

Algorithm 3: Anytime-Optimal pCR-PIM

Input: Parameters $c, \eta_c, \eta_d, \bar{\delta}_c, \bar{\delta}_d, T$, a generation profile $\boldsymbol{e}$, and the initial SoC s_0;
Output: Online decisions $\delta_t^c(\pi, \boldsymbol{e}), \delta_t^e(\pi, \boldsymbol{e}), t \in [T]$;
for $t = 1, 2, \ldots, T$ **do**
  Obtain π_t^* based on observed data and actions;
  $\delta_t^d = \min\{[\pi_t^* v(\boldsymbol{e}^t) - e_t]^+, \bar{\delta}_d, \eta_d s_{t-1}\}$;
  $\delta_t^c = [e_t - \pi_t^* v(\boldsymbol{e}^t)]^+$; $s_t = s_{t-1} + \eta_c \delta_t^c - \delta_t^d / \eta_d$.

definition of anytime-optimal CR that the sequence of anytime-optimal CRs obtained in Algorithm 3 are nonincreasing in t, whatever the generation profile is. Second, assuming that we obtain $\delta_\tau^c, \delta_\tau^d, \tau \in [t-1]$ by the anytime-optimal pCR-PIM algorithm, then a lower bound for the anytime-optimal deterministic CR π_t^* is given by $\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t)$, because the online-to-offline performance ratio for the input $\boldsymbol{e}^t$ is no less than $\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t)$, given the observed inputs and actions so far. Thus, setting $\pi_0^* = \pi^*$, we conclude that $\pi_t^* \in [\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t), \pi_{t-1}^*]$. Based on these, we shall follow a bisection procedure to search for π_t^*. To this end, given the inputs $e_\tau, \tau \in [t]$, actions obtained by the anytime-optimal pCR-PIM algorithm, and SoC s_{t-1}, we define two class of parameterized programs.

The first class corresponds to a possible worst case where the operator uses up the storage space without depleting the storage at an intermediate moment, while the second class relates to the case where the operator uses up the storage space after first depleting the existing energy storage. Each algorithm in the first class is parameterized by $\pi \in [\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t), \pi_{t-1}^*]$ and an integer interval $[t..n]$ with $n \le T$ and we denote its optimal objective value by AOCR-ND$(\pi, [t..n])$:

$$\max_{\boldsymbol{e}\in\mathcal{E}_t, u_i, \delta_{ij}^c, \delta_{ij}^d} \min_{\eta_\tau} \sum_{\tau\in[t..n]} \eta_\tau e_\tau - \pi \sum_{\tau\in[t..n]} \eta_\tau u_\tau$$

$$\text{subject to} \quad (2); (3); (4); (5); (6); (7);$$
$$\max_{\tau\in[t-1]} (e_\tau - \delta_\tau^c + \delta_\tau^d) \le \pi u_i, \forall i \in [n]. \quad (8)$$

Each algorithm in the second class is parameterized by $\pi \in [\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t), \pi_{t-1}^*]$ and $[m..n] \subseteq \{t+1, \ldots, T\}$ and we denote its optimal objective value by AOCR-D$(\pi, [m..n])$:

$$\max_{\boldsymbol{e}\in\mathcal{E}_t, u_i, \delta_{ij}^c, \delta_{ij}^d} \min_{\eta_\tau} \sum_{\tau\in[m..n]} \eta_\tau e_\tau - \pi \sum_{\tau\in[m..n]} \eta_\tau u_\tau$$

$$\text{subject to} \quad (2); (3); (4); (5); (6); (7); (8);$$
$$e_i = \underline{e}_i, \forall i \in [(t+1)...T] \text{ and } i \notin [m..n]. \quad (9)$$

Then, see Algorithm 4 for the bisection procedure to obtain π_t^*. Following are the details. We first check whether we can constantly maintain the online-to-offline performance ratio not to exceed the lower bound $\max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t)$. If so, the lower bound is exactly the anytime-optimal CR at slot t. Otherwise, we use a condition to check whether we overestimate or underestimate π_t^* to update the range. We continue this process until the estimation error is small enough.

Algorithm 4: A Bisection Method for π_t^* in the Anytime-Optimal pCR-PIM Algorithm

Input: (Updated in real time) Observed inputs $e_\tau, \tau \in [t]$, actions $\delta_\tau^c, \delta_\tau^d, \tau \in [t-1]$, π_{t-1}^*, and SoC s_{t-1} under Algorithm 3;
Output: The anytime-optimal CR $\pi_t^* = \pi_{ub}$;
$\pi_{lb} = \max_{\tau\in[t-1]}(e_\tau - \delta_\tau^c + \delta_\tau^d)/v(\boldsymbol{e}^t)$, $\pi_{ub} = \pi_{t-1}^*$;
if $\max_{n\in[t..T]}$ *AOCR-ND*$(\pi_{lb}, [t..n]) \le c - s_{t-1}$ & $\max_{[m..n]\subseteq[t..T]}$ *AOCR-D*$(\pi_{lb}, [m..n]) \le c$ **then**
  $\pi_t^* = \pi_{lb}$, **return**;
else
  while $\pi_{ub} - \pi_{lb} \ge \epsilon$ **do**
    $\pi = (\pi_{lb} + \pi_{ub})/2$;
    if $\max_{n\in[t..T]}$ *AOCR-ND*$(\pi, [t..n]) \le c - s_{t-1}$ & $\max_{[m..n]\subseteq[t..T]}$ *AOCR-D*$(\pi, [m..n]) \le c$ **then**
      $\pi_{ub} = \pi$;
    else
      $\pi_{lb} = \pi$;

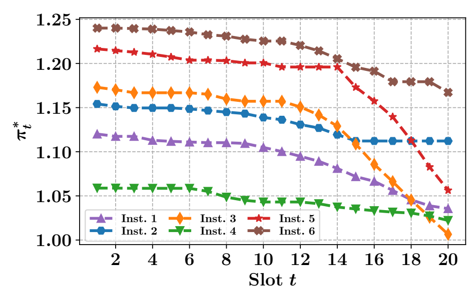

Figure 1: Anytime-optimal CRs over six instances and π_T^* specifies the ultimate online-to-offline performance ratio.

Fig. 1 illustrates the evolution of the anytime-optimal deterministic CRs in Algorithm 3 over six representative instances with $T = 20$. We see that the anytime-optimal CRs for each instance are decreasing over time. This trend signifies that the uncertainty declines after observing new inputs and we can pursue smaller online-to-offline performance ratios. If the instance is close to worst cases, the anytime-optimal CR changes a little over time. However, we observe the great differences between π_1^* and π_T^* for several instances (e.g., Inst. 3 and Inst. 5). This phenomenon indicates that the gradually revealed inputs for these instances can bring great benefits in pruning the uncertainty set, while the adaptive extension can leverage this advantage for better average-case performance. Moreover, we see that our adaptive online algorithm with little future information can even obtain close-to-optimal performance for certain cases as π_T^* approaches one (e.g., Inst. 3).

Overall, by the adaptive extension, we improve the average-case performance, while retaining the optimal worst-case performance. Unlike the pCR-PIM algorithm, the adaptive algorithm requires to update the pursued ratio within limited time, in response to newly observed real-time information. For practical implementations, we propose an efficient modification of the (anytime-)optimal pCR-PIM algorithms (see Appendix E in the supplementary materials), which significantly reduces the computational complexity, provides a theoretical performance guarantee, and achieves strong empirical performance.

VI. Simulation

This section is dedicated to evaluating the empirical performance of our pCR-PIM algorithms in smoothing renewable generation under diverse popular metrics. Notably, we show that the adaptive extension can significantly improve the peak-reduction effect and achieve better empirical performance than conceivable alternatives under typical settings. We also extend our approach to effectively mitigate the mismatch between the supply and uncontrollable demands in Appendix F of the supplementary materials.

A. Data and Setting

We collect one-year wind power generation data from Elia [45]. According to the short-term wind power forecasting, we select the operating cycle of $T = 20$ slots, where each slot corresponds to 15 minutes. For every instance, we identify a uniform lower/upper bound for the renewable generation at a slot, i.e., $\underline{e}_t = \underline{e}, \overline{e}_t = \overline{e}$, for all $t \in [T]$. Note that it is likely for our algorithms to attain better empirical performance with a finer description of the generation bounds. We set $\overline{\delta}_c = \overline{\delta}_d = \overline{e} - \underline{e}$ unless otherwise specified.

We define the empirical online-to-offline performance ratio of an online algorithm as the ratio between the average peak injection under the algorithm and that under the optimal clairvoyant solution. In addition to the ratio, we illustrate the empirical algorithm performance by the reduction on the peak injection and two popular metrics for the fluctuation of electricity injection into the grid with $f_t := e_t - \delta_t^c + \delta_t^d$:

- Peak reduction: $\max_{t\in[T]} e_t - \max_{t\in[T]} f_t$;
- Peak-to-mean ratio: $\frac{\max_{t\in[T]} f_t}{\sum_{t=1}^{T} f_t/T}$;
- Range: $\max_{t\in[T]} f_t - \min_{t\in[T]} f_t$.

B. Empirical Evaluation

We test the performance of our algorithms under different storage capacities, charge/discharge efficiency factors, and rate limits. Note that our adaptive algorithm can utilize look-ahead predictions by simply adjusting the uncertainty intervals. We make comparisons among the optimal clairvoyant algorithm (*ref.* Offline), pCR-PIM(π) (*ref.* Crp), adaptive pCR-PIM (*ref.* AdaCrp $w = 0$), adaptive pCR-PIM with a prediction window (*ref.* AdaCrp $w = 3$), and two classes of conceivable alternatives. The first class refers to two MPC algorithms with different accurate prediction windows ($w = 3$ and $w = 5$, respectively). However, in practice, accurate predictions may not be available. The second is the low pass filter (*ref.* LPF) tracking the moving average of the original generation curves [46]. Note that the offline performance refers to the ideal outcome that we can achieve with perfect information and hope to approach in the online setting.

Table I: The online-to-offline peak-injection ratios.

Capacity (MWh)	30	45	60	75
π^*	1.102	1.115	1.128	1.140
pCR-PIM(π^*)	1.099	1.112	1.120	1.128
Anytime-optimal ($w = 0$)	1.072	1.078	1.083	1.089
Anytime-optimal ($w = 3$)	**1.042**	**1.053**	**1.060**	**1.065**
MPC ($w = 3$)	1.068	1.098	1.122	1.143
MPC ($w = 5$)	1.038	1.059	1.075	1.090
Low pass filter	1.142	1.184	1.224	1.272

1) Performance Ratios with Ideal Charging/Discharging: We first consider the lossless case, where $\eta_c = \eta_d = 1$ and the CR-C, AOCR-ND, and AOCR-D programs become linear or linear-fractional programs that can be readily solved. Based on the collected data, we show in Table I the optimal deterministic CRs and the empirical online-to-offline ratios of involved algorithms. Particularly, we observe that the anytime-optimal pCR-PIM algorithm attains smaller performance ratios than pCR-PIM(π^*). This fact indicates that worst cases may rarely happen and substantiates the importance of our adaptive extension that prunes the uncertainty set in response to real-time information. Moreover, we see that our anytime-optimal algorithm without future information can attain better performance than the MPC algorithm with a smaller window $w = 3$ and has comparable performance the MPC algorithm with a larger window $w = 5$. More importantly, after utilizing the accurate predictions in the smaller window $w = 3$, our anytime-optimal algorithm obtains smaller performance ratios and excels the MPC algorithm with a larger window $w = 5$.

2) Comparison with Inefficient Charging/Discharging: We consider a more practical case, where $\eta_c = \eta_d = 0.85$ [47]. In this situation, we adopt the practical modification of the pCR-PIM algorithm described in Appendix E, using the relaxed versions of involved programs for the offline reference performance and the ratios pursued at each slot. It turns out that our adaptive online algorithm with such modification achieves outstanding performance and can take advantage of look-ahead information, as shown in Fig. 2.

Typically, when the storage capacity is 60 MWh, our adaptive algorithm without using look-ahead information can reduce the peak injection by 17%, achieving almost 72% performance of the optimal clairvoyant algorithm where we know generation curves beforehand. It also reduces the fluctuation level by 9% and 18%, in terms of peak-to-mean ratio and range, respectively. Notably, our adaptive algorithm achieves close-to-optimal performance regarding peak-to-mean ratio over a wide range of capacities. In terms of peak reduction, our adaptive algorithm outperforms the MPC algorithm with a smaller prediction window; moreover, it is of comparable performance to the MPC with a larger accurate prediction window (1.25 hours). In terms of peak-to-mean ratio and range, our adaptive algorithm performs much better than the two MPC algorithms. Compared with the LPF algorithm, our adaptive algorithm and pCR-PIM(π) algorithm both attain better empirical performance in all four metrics.

Furthermore, we see that our adaptive algorithm can benefit from the accurate look-ahead information. Specifically, our adaptive algorithm equipped with a smaller prediction win-

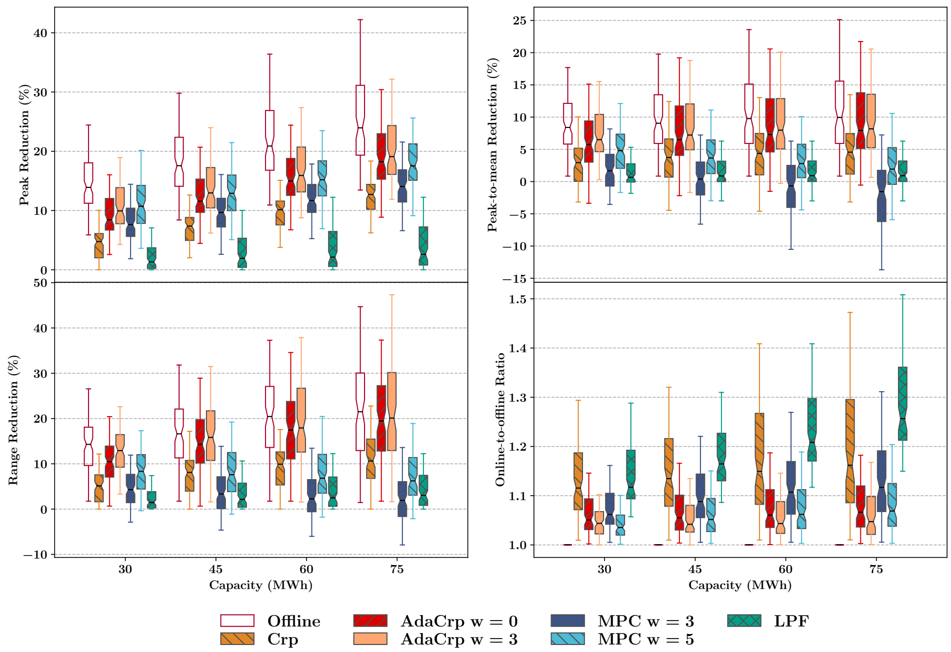

Figure 2: Performance comparison with $\eta_c = \eta_d = 0.85$.

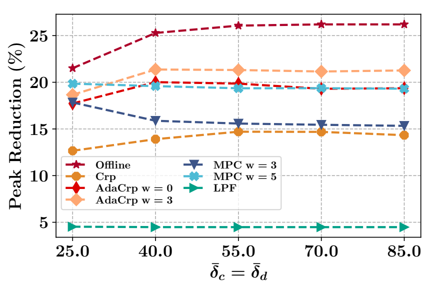

Figure 3: Effects of charge/discharge rate limit $\bar{\delta}_c = \bar{\delta}_d$ with $c = 60$ MWh, $\eta_c = \eta_d = 0.85$, and $T = 20$.

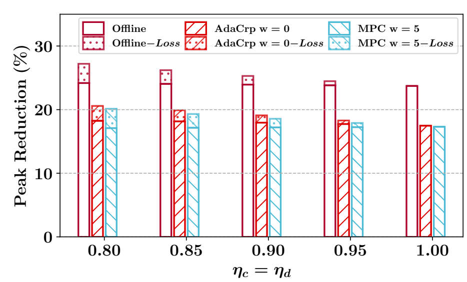

Figure 4: Effects of charge/discharge efficiency $\eta_c = \eta_d$ with $c = 60$ MWh, $\bar{\delta}_c = \bar{\delta}_d = \eta_c(\bar{e} - \underline{e})$, and $T = 20$.

dow ($w = 3$) attains better empirical performance (average values and smaller variances) and excels the MPC algorithm with a longer window ($w = 5$), regarding all four metrics.

We also observe that the effect of increasing the storage capacity is insignificant in improving the fluctuation reduction. This phenomenon indicates that we had better exploit intelligent scheduling rather than purchase more batteries for efficiently smoothing renewable generation.

3) Impact of Charge/Discharge Rate Limit and Efficiency: We respectively illustrate the effects of the charge/discharge rate limit and efficiency in Fig. 3 and Fig. 4. We observe from Fig. 3 that the charge/discharge rate limit has little impact on the performance of our adaptive algorithm. A possible reason should be that the constraints regarding the charge/discharge rate limits are often inactive in computing the offline reference objective values and the charge/discharge decisions under our adaptive algorithm. In practice, renewable power plants can obtain sufficient charge/discharge rate limits by hybridizing different storage technologies. These facts alleviate the difficulty in computing (anytime-)optimal CRs.

Moreover, we see from Fig. 4 that the peak reduction effects of the three algorithms decrease, as the charging/discharging becomes more efficient. A possible reason for this phenomenon lies in that less renewable energy is wasted due to the efficiency improvement, as illustrated by the top bars in Fig. 4. In this work, we aim to smooth the renewable generation by minimizing the peak injection. However, we are not meant to use the charge/discharge inefficiency for reducing the peak injection and thus do not allow charging/discharging simultaneously in a slot. In general, improving the charge/discharge efficiency can help to accommodate more renewable energy, despite the slight increase in the peak renewable injection.

VII. Conclusion

We formulate and study the online peak-injection minimization under inventory constraints (PIM). We develop an optimal online algorithm pCR-PIM(π^*) with the best CR among all deterministic online algorithms. We show that the optimal deterministic CR π^* can be computed beforehand, by solving a number of max-min bilinear-fractional programs with bilinear constraints given a sufficient discharge rate limit. To our best knowledge, these are the first and optimal results for this challenging problem. We further generalize our approach by adaptively pursuing the best deterministic competitiveness regarding the residual uncertainty. This leads to an anytime-optimal online algorithm that has adaptive average-case and optimal worst-case performances. Finally, we corroborate the empirical efficiency of our algorithms in smoothing renewable generation and mitigating supply-demand balances by extensive simulations over real-world traces. In the future, we shall study how randomization can improve the online decision-making for PIM and how to adapt our algorithms with unreliable (e.g., learning-assisted) predictions. It is also interesting to apply our approach to more practical applications.

REFERENCES

[1] IRENA, "World adds record new renewable energy capacity in 2020," https://www.irena.org/newsroom/pressreleases/2021/Apr/World-Adds-Record-New-Renewable-Energy-Capacity-in-2020/.

[2] C. dos Santos, M. J. Rider, and C. Lyra, "Optimized integration of a set of small renewable sources into a bulk power system," *IEEE Trans. Power Syst.*, vol. 36, pp. 248–260, 2021.

[3] L. Bird, J. Cochran, and X. Wang, "Wind and solar energy curtailment: Experience and practices in the United States," National Renewable Energy Lab. (NREL), Golden, CO (United States), Tech. Rep., 2014.

[4] Y. Qi, J. Lu, and M. Zhu, "Wind curtailment in China and lessons from the United States," Brookings, Tech. Rep., 2018.

[5] P. Denholm, "Energy storage to reduce renewable energy curtailment," in *IEEE PES Gen. Meet.*, 2012, pp. 1–4.

[6] M. Andoni, V. Robu, W.-G. Früh, and D. Flynn, "Game-theoretic modeling of curtailment rules and network investments with distributed generation," *Appl. Energy*, vol. 201, pp. 174–187, 2017.

[7] S. Magnússon, G. Qu, and N. Li, "Distributed optimal voltage control with asynchronous and delayed communication," *IEEE Trans. Smart Grid*, vol. 11, pp. 3469–3482, 2020.

[8] Y. Du and F. Li, "Intelligent multi-microgrid energy management based on deep neural network and model-free reinforcement learning," *IEEE Trans. Smart Grid*, vol. 11, pp. 1066–1076, 2019.

[9] T. Wang, H. Kamath, and S. Willard, "Control and optimization of grid-tied photovoltaic storage systems using model predictive control," *IEEE Trans. Smart Grid*, vol. 5, pp. 1010–1017, 2014.

[10] N. Mahdavi, J. H. Braslavsky, M. M. Seron, and S. R. West, "Model predictive control of distributed air-conditioning loads to compensate fluctuations in solar power," *IEEE Trans. Smart Grid*, vol. 8, pp. 3055–3065, 2017.

[11] M. Negrete-Pincetic, A. Nayyar, K. Poolla, F. Salah, and P. Varaiya, "Rate-constrained energy services in electricity," *IEEE Trans. Smart Grid*, vol. 9, pp. 2894–2907, 2016.

[12] Y. Mo, W. Chen, L. Qiu, and P. Varaiya, "Market implementation of multiple-arrival multiple-deadline differentiated energy services," *Automatica*, vol. 116, 2020.

[13] J. C. Spall, *Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control*. John Wiley & Sons, 2005.

[14] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Eng. Pract.*, vol. 11, pp. 733–764, 2003.

[15] A. Borodin and R. El-Yaniv, *Online Computation and Competitive Analysis*. Cambridge University Press, 2005.

[16] N. Buchbinder, S. Chen, J. S. Naor, and O. Shamir, "Unified algorithms for online learning and competitive analysis," *Math. Oper. Res.*, vol. 41, pp. 612–625, 2016.

[17] A. Bar-Noy, Y. Feng, M. P. Johnson, and O. Liu, "When to reap and when to sow – Lowering peak usage with realistic batteries," in *Proc. WEA*. Springer, 2008, pp. 194–207.

[18] S. Zhao, X. Lin, and M. Chen, "Robust online algorithms for peak-minimizing EV charging under multistage uncertainty," *IEEE Trans. Autom. Control*, vol. 62, pp. 5739–5754, 2017.

[19] Y. Mo, Q. Lin, M. Chen, and S. J. Qin, "Optimal peak-minimizing online algorithms for large-load users with energy storage," in *IEEE INFOCOM*, 2021, poster paper.

[20] Y. Mo, Q. Lin, M. Chen, and S. J. Qin, "Optimal online algorithms for peak-demand reduction maximization with energy storage," in *Proc. ACM e-Energy*, 2021, pp. 73–83.

[21] Y. Shi, B. Xu, D. Wang, and B. Zhang, "Using battery storage for peak shaving and frequency regulation: Joint optimization for superlinear gains," *IEEE Trans. Power Syst.*, vol. 33, pp. 2882–2894, 2017.

[22] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nat. Energy*, vol. 1, pp. 1–6, 2016.

[23] J. P. Barton and D. G. Infield, "Energy storage and its use with intermittent renewable energy," *IEEE Trans. Energy Convers.*, vol. 19, pp. 441–448, 2004.

[24] M. Lei, Z. Yang, Y. Wang, H. Xu, L. Meng, J. C. Vasquez, and J. M. Guerrero, "An MPC-based ESS control method for PV power smoothing applications," *IEEE Trans. Power Electron.*, vol. 33, pp. 2136–2144, 2017.

[25] P. H. A. Barra, W. C. de Carvalho, T. S. Menezes, R. A. S. Fernandes, and D. V. Coury, "A review on wind power smoothing using high-power energy storage systems," *Renew. Sustain. Energy Rev.*, 2020, 110455.

[26] X. Chen, K.-C. Leung, and A. Y. S. Lam, "Power output smoothing for renewable energy system: Planning, algorithms, and analysis," *IEEE Syst. J.*, vol. 14, pp. 1034–1045, 2020.

[27] M. Rahmani-Andebili and M. Fotuhi-Firuzabad, "An adaptive approach for PEVs charging management and reconfiguration of electrical distribution system penetrated by renewables," *IEEE Trans. Ind. Informat.*, vol. 14, pp. 2001–2010, 2017.

[28] A. Bar-Noy, M. P. Johnson, and O. Liu, "Peak shaving through resource buffering," in *Proc. WAOA*. Springer, 2008, pp. 147–159.

[29] C.-K. Chau, G. Zhang, and M. Chen, "Cost minimizing online algorithms for energy storage management with worst-case guarantee," *IEEE Trans. Smart Grid*, vol. 7, pp. 2691–2702, 2016.

[30] R. El-Yaniv, A. Fiat, R. M. Karp, and G. Turpin, "Optimal search and one-way trading online algorithms," *Algorithmica*, pp. 101–139, 2001.

[31] Q. Lin, H. Yi, J. Pang, M. Chen, A. Wierman, M. Honig, and Y. Xiao, "Competitive online optimization under inventory constraints," *Proc. SIGMETRICS*, vol. 3, pp. 1–28, 2019.

[32] B. Hunsaker, A. J. Kleywegt, M. W. P. Savelsbergh, and C. A. Tovey, "Optimal online algorithms for minimax resource scheduling," *SIAM J. Discrete Math.*, vol. 16, pp. 555–590, 2003.

[33] J. A. Hoogeveen and A. P. A. Vestjens, "A best possible deterministic on-line algorithm for minimizing maximum delivery time on a single machine," *SIAM J. Discrete Math.*, vol. 13, pp. 56–63, 2000.

[34] Y. Ge, H. Ye, and K. A. Loparo, "Agent-based privacy preserving transactive control for managing peak power consumption," *IEEE Trans. Smart Grid*, vol. 11, pp. 4883–4890, 2020.

[35] W. Tang, S. Bi, and Y. J. A. Zhang, "Online coordinated charging decision algorithm for electric vehicles without future information," *IEEE Trans. Smart Grid*, vol. 5, pp. 2810–2824, 2014.

[36] Q. Lin, H. Yi, and M. Chen, "Minimizing cost-plus-dissatisfaction in online EV charging under real-time pricing," *IEEE Trans. Intell. Transp. Syst.*, 2021, early access.

[37] N. Tucker and M. Alizadeh, "An online admission control mechanism for electric vehicles at public parking infrastructures," *IEEE Trans. Smart Grid*, vol. 11, pp. 161–170, 2019.

[38] M. Badiei, N. Li, and A. Wierman, "Online convex optimization with ramp constraints," in *IEEE Conf. Decis. Control (CDC)*, 2015, pp. 6730–6736.

[39] Y. Zhang, M. H. Hajiesmaili, S. Cai, M. Chen, and Q. Zhu, "Peak-aware online economic dispatching for microgrids," *IEEE Trans. Smart Grid*, vol. 9, pp. 323–335, 2016.

[40] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Rev.*, vol. 53, pp. 464–501, 2011.

[41] H. Alharbi and K. Bhattacharya, "Stochastic optimal planning of battery energy storage systems for isolated microgrids," *IEEE Tran. Sustain. Energy*, vol. 9, pp. 211–227, 2017.

[42] F. Calero, C. A. Cañizares, K. Bhattacharya *et al.*, "A review of modeling and applications of energy storage systems in power grids," *Proc. IEEE*, 2022, early access.

[43] H. Vaish and C. M. Shetty, "The bilinear programming problem," *Nav. Res. Logist. Q.*, vol. 23, pp. 303–309, 1976.

[44] H. D. Sherali and A. Alameddine, "A new reformulation-linearization technique for bilinear programming problems," *J. Glob. Optim.*, vol. 2, pp. 379–410, 1992.

[45] Elia, "Wind power generation," https://www.elia.be/en/grid-data/power-generation/wind-power-generation.

[46] A. Ellis, D. Schoenwald, J. Hawkins, S. Willard, and B. Arellano, "PV output smoothing with energy storage," in *IEEE Photovolt. Spec. Conf.*, 2012, pp. 1523–1528.

[47] J. Qin, Y. Chow, J. Yang, and R. Rajagopal, "Online modified greedy algorithm for storage control under uncertainty," *IEEE Trans. Power Syst.*, vol. 31, pp. 1729–1743, 2015.

APPENDIX

A. Proof of Proposition 1

First, clearly, the optimal objective value v^* satisfies $\max\{0, \max_{t\in[T]} e_t - \bar{\delta}_c\} \le v^* \le \max_{t\in[T]} e_t$. Second, we observe that if there is a feasible solution such that the objective value of the PIM problem is v, then the procedure in Algorithm 1 will also generate a feasible solution attaining the objective value v and we can conclude that $v \ge v^*$. Otherwise, the condition $s > c$ will be triggered. It means that there exists no feasible solution satisfying the constraints of the PIM problem such that the peak injection is v; thus, we have $v \ge v^*$. This completes the proof.

B. Proof of Lemma 1

First, we note that $\delta_t^c(\pi, \boldsymbol{e}) = [e_t - \pi v(\boldsymbol{e}^t)]^+ \le [e_t - v(\boldsymbol{e}^t)]^+ \le \bar{\delta}_c$. Thus, we see that $\delta_t^c(\pi, \boldsymbol{e})$ respects the charge rate limit. Meanwhile, we observe that $\delta_t^d(\pi, \boldsymbol{e}) \le \min\{\bar{\delta}_d, \eta_d s_{t-1}\}$, respecting the charge rate limit and the requirement of nonnegative SoC. Moreover, we see that $\delta_t^c(\pi, \boldsymbol{e}) > 0$ only if $e_t > \pi v(\boldsymbol{e}^t)$, while $\delta_t^d(\pi, \boldsymbol{e}) > 0$ only if $e_t < \pi v(\boldsymbol{e}^t)$. Thus, the operator implementing Algorithm 2 will not charge and discharge simultaneously in the same slot. Note that $\Phi(\pi)$ refers to the maximum SoC over all possible input sequences. It follows that pCR-PIM(π) is feasible if and only if $\Phi(\pi) \le c$. The monotonicity of $\Phi(\pi)$ follows that $\delta_t^c(\pi, \boldsymbol{e})$ is non-increasing in π, while $\delta_t^d(\pi, \boldsymbol{e})$ is non-decreasing in π.

C. Proof of Theorem 1

By Lemma 1, we conclude that π^* is the smallest possible ratio that can be maintained in Algorithm 2. It remains to show that π^* is the best possible CR among all deterministic online algorithms for PIM. To see this, let $\hat{e}$ be a worst-case input sequence such that there exists an index $\hat{T}$ satisfying

$$\sum_{t=1}^{\hat{T}} \left(\eta_c \delta_t^c(\pi, \hat{\boldsymbol{e}}) - \delta_t^d(\pi, \hat{\boldsymbol{e}})/\eta_d\right) = c.$$

Consider any deterministic online algorithm $\mathfrak{A}$, and at slot 1, we present $\hat{e}_1$ to the algorithm. If $\delta_1^c(\mathfrak{A}) < \delta_1^c(\pi^*, \hat{\boldsymbol{e}})$, then we present $\underline{e}_t$ for the remaining time slots. Consequently, we see that the online-to-offline ratio under Algorithm $\mathfrak{A}$ with the input sequence $\hat{\boldsymbol{e}}^1$ exceeds π^*, namely

$$\frac{e_1 - \delta_1^c(\mathfrak{A}) + \delta_1^d(\mathfrak{A})}{v(\hat{\boldsymbol{e}}^1)} \ge \frac{e_1 - \delta_1^c(\mathfrak{A})}{v(\hat{\boldsymbol{e}}^1)} > \frac{e_1 - \delta_1^c(\pi^*, \hat{\boldsymbol{e}})}{v(\hat{\boldsymbol{e}}^1)} = \pi^*.$$

That is, the CR of Algorithm $\mathfrak{A}$ is strictly larger than π^* when $\delta_1^c(\mathfrak{A}) < \delta_1^c(\pi^*, \hat{\boldsymbol{e}})$.

If $\delta_1^c(\mathfrak{A}) \ge \delta_1^c(\pi^*, \hat{\boldsymbol{e}})$, we continue presenting the elements of $\hat{\boldsymbol{e}}$ to Algorithm $\mathfrak{A}$. We observe that if $\delta_1^c(\mathfrak{A}) > \delta_1^c(\pi^*, \hat{\boldsymbol{e}})$, then there exists an index $\hat{t}$ such that $\delta_{\hat{t}}^c(\mathfrak{A}) < \delta_{\hat{t}}^c(\pi^*, \hat{\boldsymbol{e}})$ or $\delta_{\hat{t}}^d(\mathfrak{A}) > \delta_{\hat{t}}^d(\pi^*, \hat{\boldsymbol{e}})$. Thus, for similar reasons, we can show that the CR of Algorithm $\mathfrak{A}$ exceeds π^*, by presenting lower bounds of future renewable generation to the operator. It follows that $\delta_1^c(\mathfrak{A}) = \delta_1^c(\pi^*, \hat{\boldsymbol{e}})$ for any deterministic online algorithm $\mathfrak{A}$ whose CR is no more than π^*. However, when $\delta_1^c(\mathfrak{A}) > 0$, the online-to-offline performance ratio under the algorithm with the input sequence $\hat{\boldsymbol{e}}^1$ is π^*. In this case, the CR of Algorithm $\mathfrak{A}$ is at least π^*.

For similar reasons, we can show that $\delta_t^c(\mathfrak{A}) = \delta_t^c(\pi^*, \hat{\boldsymbol{e}})$, for $t \in [\hat{T}]$, for any deterministic online algorithm $\mathfrak{A}$ whose CR is at most π^*. Because there exists an index $\tilde{t}$ with $\delta_{\tilde{t}}^c(\mathfrak{A}) = \delta_{\tilde{t}}^c(\pi^*, \hat{\boldsymbol{e}}) > 0$, the CR of such an algorithm $\mathfrak{A}$ should be lower bounded by π^*. To summarize, we conclude that π^* is the best possible CR among all deterministic online algorithms.

D. Proof of Proposition 2

The discharge rate limit is so large that $\delta_t^d(\pi^*, \hat{\boldsymbol{e}}) < \bar{\delta}$ is always satisfied. We introduce the interval $[m..n]$ to tackle the case where $\delta_t^d(\pi^*, \hat{\boldsymbol{e}}) = \eta_d s_{t-1}$ for a certain t. It follow from (1) that

$$\pi^* \ge \max_{[m..n]\subseteq[T], e\in\mathcal{E}} \; \min_{\eta_t\in\{\eta_c, 1/\eta_d\}} \frac{\sum_{t\in[m..n]} \eta_t e_t - c}{\sum_{t\in[m..n]} \eta_t v(\boldsymbol{e}^t)}.$$

Moreover, we see that for any interval $[m..n]$ and input sequence $\boldsymbol{e}$, the minimization problem in the proposition will attain its optimal objective value π at a solution $\hat{\eta}_t, t \in [T]$ satisfying

$$\hat{\eta}_t = \begin{cases} \eta_c, & \text{if } e_t \ge \pi v(\boldsymbol{e}^t) \\ 1/\eta_d, & \text{otherwise.} \end{cases}$$

Thus, we complete the proof.

E. Computationally Efficient Modification for Practical Uses

1) Relax the PIM Problem and Obtain An Upper Bound of the Anytime-Optimal Deterministic CR: The complexity of the (anytime-)optimal pCR-PIM algorithm mainly come from the computation of the (anytime-)optimal deterministic CRs, requiring to solve programs with bilinear/bilinear-fractional objectives and constraints. We herein provide a computationally efficient modification of the (anytime-)optimal pCR-PIM algorithms. Such a modification allows us to use linear programming to obtain the ratios to be maintained in the pCR-PIM algorithms and thus facilitates the practical implementation. Note that the obtained ratios also characterize the worst-case performance guarantees of the modified algorithms, which have good empirical performance.

Recall that the PIM problem challenges us by the hard bilinear constraints $\delta_t^c \delta_t^d = 0$. Nevertheless, we can simply use linear programming to solve the relaxed version of PIM that differs from the original PIM only by not considering the hard bilinear constraints. Let $v_{re}(\boldsymbol{e})$ be the optimal objective value of the relaxed PIM with the generation profile $\boldsymbol{e}$ known beforehand. Clearly, we have $v_{re}(\boldsymbol{e}) \le v(\boldsymbol{e})$.

Our modification generalizes the CR-Pursuit algorithmic framework by maintaining the online-to-relaxed-offline performance ratio that is an upper bound of the original online-to-offline performance ratio at every slot. To be specific, instead of maintaining the online-to-offline performance mentioned before, we maintain the online-to-relaxed-offline performance to be no more than a given ratio π_t at slot t, namely

$$\frac{\max_{\tau\in[t]} \left(e_\tau - \delta_\tau^c + \delta_\tau^d\right)}{v_{re}(\boldsymbol{e}^t)} \le \pi_t.$$

To modify the pCR-PIM(π^*) algorithm, we change $v(\boldsymbol{e}^t)$ to $v_{re}(\boldsymbol{e}^t)$ in Algorithm 2 and use the relaxed version of the CR-C program to obtain the smallest possible ratio $\pi^{*,re}$ that can be maintained in the modified pCR-PIM($\pi^{*,re}$) algorithm. Similarly, the relaxed CR-C differs from the original CR-C by dropping the bilinear constraints (3). Since $v_{re}(\boldsymbol{e}) \le v(\boldsymbol{e})$, we conclude that the CR of the modified pCR-PIM($\pi^{*,re}$) algorithm is upper bounded by $\pi^{*,re}$. Thus, the ratio $\pi^{*,re}$ maintained in slots characterizes the worst-case performance guarantee of the modified pCR-PIM algorithm, in the sense that the worst-case online-to-offline ratio is at most $\pi^{*,re}$.

Likewise, we change $v(\boldsymbol{e}^t)$ to $v_{re}(\boldsymbol{e}^t)$ in Algorithm 3 and Algorithm 4 where we use the relaxed versions of AOCR-ND and AOCR-D without considering (3) to obtain the adaptive ratios $\pi_t^{*,re}$ in place of π_t^*. Similarly, we have $\pi_t^* \le \pi_t^{*,re}$ and conclude that the anytime deterministic CR at slot t of the modified anytime-optimal pCR-PIM algorithm is upper bounded by $\pi_t^{*,re}$.

More importantly, we show that the relaxed AOCR-ND and AOCR-D programs, which are max-min problems in original forms, can be exactly solved by linear programming. Precisely, we conduct partial dualization over the variables $\eta_t, t \in [T]$ of the inner minimization problem of the relaxed AOCR-D program and obtain:

$$\begin{aligned}
&\max_{\boldsymbol{e}\in\mathcal{E}_t, u_i, \delta^c_{ij}, \delta^d_{ij}, \alpha_t, \beta_t} \quad \sum_{\tau\in[m..n]} (\eta_c \alpha_\tau - \beta_\tau/\eta_d) \\
&\text{subject to} \quad (2); (4); (5); (6); (7); (8); (9) \\
&\qquad\qquad\quad e_i - \pi u_i - \alpha_i + \beta_i = 0, \forall i \in [m..n].
\end{aligned}$$

Above is a linear program and the variables α_t and β_t respectively refer to the Lagrange multipliers corresponding to the constraints $\eta_t \ge \eta_c$ and $\eta_t \le 1/\eta_d$ in the relaxed AOCR-D program. By the strong duality, the optimal objective values of the above problem and the relaxed AOCR-D program are equivalent. Likewise, we can solve the relaxed AOCR-ND program by linear programming. Thus, we can obtain the parameter ratios $\pi_t^{*,re}$ with much less computational effort than that for π_t^*.

Overall, our modification of the (anytime-)optimal pCR-PIM algorithms in Appendix E can significantly reduce the computational complexity, provide a theoretical performance guarantee, and lead to good empirical performance.

2) Tighten the PIM Problem and Obtain A Lower Bound of the Anytime-Optimal Deterministic CR: Previously, we relax the PIM Problem to avoid the computational complexity brought by the hard bilinear constraints (3), and obtain an upper bound of π_t^*. Now, we tackle the hard bilinear constraints by a complementary approach, namely, tightening the PIM Problem by forbidding discharging in the offline benchmark. Specifically, we obtain the tightened version of the offline PIM problem by replacing the constraint $\delta_t^c \delta_t^d = 0$ in the original one with $\delta_t^d = 0$, for all $t \in [T]$. Correspondingly, we obtain the tightened versions of the CR-C, AOCR-ND and AOCR-D programs by replacing the bilinear constraints (3) with $\delta_{ij}^d = 0$, for all $i, j \in [T]$. Then, we can use the tightened programs to obtain the lower bounds of the optimal deterministic CR π^* and anytime-optimal CRs $\pi_t^*, t \in [T]$.

Similarly, one may argue that we can maintain the online-to-tightened-offline performance ratio at each slot to obtain another computationally efficiently adaptive algorithm. While the resulting algorithm is feasible, we can hardly characterize their CRs or tight upper bounds of the CRs. As a result, we adopt the relaxation-based modification mentioned previously in our numerical study, instead of the tightening-based one.

F. Mismatch between Renewable Generation and Demand

So far, we have shown the theoretical and empirical advantages of our approach to smoothing renewable generation for a power plant. An important relevant problem is to ease the mismatch between the renewable supply and the uncontrollable demand with an energy storage system. This scenario becomes more popular as the surge of prosumers that consume and produce electricity [22]. We show that we can adapt our approach for the relevant scenario, despite the extra challenges.

We use e_t^n (net energy) to denote the difference between the renewable generation and the inflexible demand at slot t. Similarly to before, in the online setting, we are unaware of the exact value of e_t^n before slot t and just know the uncertainty interval $[\underline{e}_t^n, \overline{e}_t^n]$ it belongs to. It is worth noting that the bounds $\underline{e}_t^n$ and $\overline{e}_t^n$ may be negative. Other parameters are the same as those in the main body.

At first sight, one may expect to mitigate the supply-demand imbalances by minimizing the peak over-supply $\max_{t\in[T]} \left(e_t^n - \delta_t^c + \delta_t^d\right)$ with the charging/discharging decision-making strategies proposed before. However, there are two unique challenges for the supply-demand balance in contrast to the renewable generation smoothing. First, the objective value $v(\boldsymbol{e}^n)$ is often negative due to the possible overload or under-supply, but competitive analysis requires positive optimal objective values as indicated by the definition of CR. Second, mitigating the supply-demand mismatch requires the maximum mismatch amount approaches zero, in addition to improving the smoothness of mismatch curves. Nevertheless, our adaptation will address these challenges and we use extensive simulations justify the empirical efficiency of our algorithms.

To address the first challenge, we lift the demands and their bounds by a constant of value $l = \min_{t\in[T]} \underline{e}_t^n + ac/T$, where $a > 1$ is a tunable parameter and we empirically set $a = 6$. That is, we set $e_t = e_t^n + l$, $\underline{e}_t = \underline{e}_t^n + l$, $\overline{e}_t = \overline{e}_t^n + l$. The reasons lie in that $v(\boldsymbol{e}) = v(\boldsymbol{e}^n + l\mathbf{1}) = v(\boldsymbol{e}^n) + l$ and such demand lifting will not affect the optimality of offline solutions and the feasibility of online solutions. We tackle the second challenge by imposing the requirement of nonnegative peak over-supply, namely $\max_{t\in[T]} \left(e_t^n - \delta_t^c + \delta_t^d\right) \ge 0$ in the offline PIM problem, leading to the augmented PIM. Accordingly, we supplement the CR-C, AOCR-ND, and AOCR-D programs by the constraints: $u_i \ge l$, for all $i \in [T]$. Overall, we obtain the online charging/discharging decisions to mitigate the supply-demand mismatch by adapting and implementing the pCR-PIM algorithm with the lifted profile $\boldsymbol{e} = \boldsymbol{e}^n + l\mathbf{1}$ as the input sequence, the augmented PIM for the offline benchmark $v(\boldsymbol{e}^t)$, and the supplemented CR-C, AOCR-ND, and AOCR-D programs for ratios π_t^* maintained in slots.

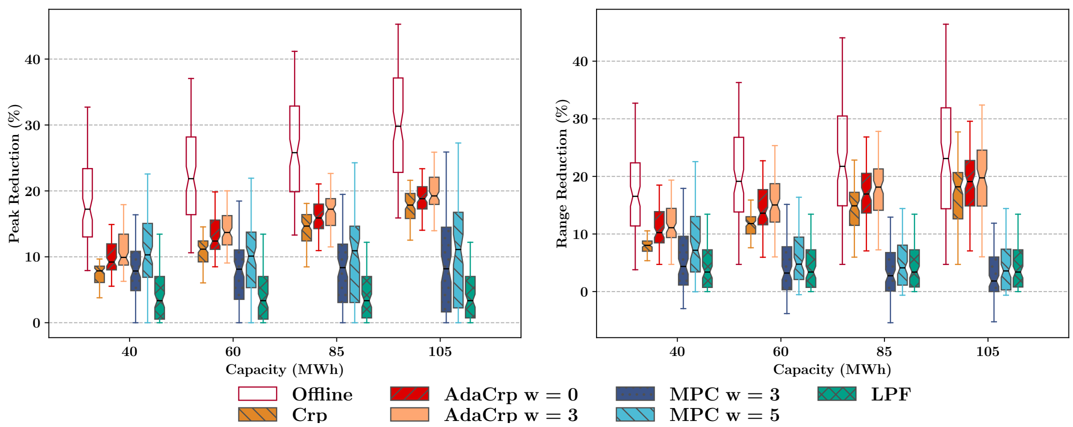

Figure 5: Performance comparison with $\eta_c = \eta_d = 0.85$ with load data.

In Fig. 5, we showcase the good empirical performance of our approach to ease the supply-demand mismatch by the following two metrics with $f_t = \left(e_t^n - \delta_t^c + \delta_t^d\right)$:

- Peak reduction: $\max_{t\in[T]} e_t^n - \max_{t\in[T]} f_t$;
- Mismatch range: $\max_{t\in[T]} f_t - \min_{t\in[T]} f_t$.

Similarly to before, we collect one-year data on renewable generations and demands from Elia [45]. Then, we identify $T = 20$ with 15-minute slots and a uniform lower/upper bound for net energy i.e., $\underline{e}_t^n = \underline{e}^n, \overline{e}_t^n = \overline{e}^n$, for all $t \in [T]$.

We see from Fig. 5 that under the typical storage capacity 60 MWh, our adaptive algorithm without using look-ahead information (*ref.* AdaCrp $w = 0$) can reduce the peak oversupply by 13% and mismatch range by 14%, achieving 74% of the ideal performance (*ref.* Offline). We also see that our adaptive pCR-PIM algorithm outperforms its nonadaptive counterpart (*ref.* Crp), since the former adaptively responds to real-time information for better average-case performance. After equipped with a look-ahead window of size $w = 3$, our look-ahead adaptive algorithm (*ref.* AdaCrp $w = 3$) attains better performance owing to utilizing the additional future information. Again, we see that our approach excels two MPC algorithms and the lower pass filter in Fig. 5. In contrast to the comparison in Fig. 2, we see that the both MPC algorithms are much worse than the algorithms proposed in our work, in terms of peak reduction and range. A possible reason is that the uncertainty herein is dual and due to both load variations and volatile renewable generations; moreover, the short-sighted MPC algorithms are vulnerable to the augmented uncertainty.