**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: Solved using Jupyter Note Book (Python)- File attached

0.2676(B)

*μ* = 45

*σ* = 8

X= 60-10 =50

Z = (X - µ) / σ

Z=(50-45)/8= 0.625

probability P(X>50) = 1-stats.norm.cdf(abs(z\_score))

1-stats.norm.cdf(abs(0.625))

=0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: Solved using Jupyter Note Book (Python)- File attached

Mean (μ) = 38

Standard Deviation (*σ* )=6

1. More employees at the processing center are older than 44 than between 38 and 44. (False)

The probability of employee age more than 44 is 15.87 %

The probability of employee age between 38 and 44 is 34.13 %

Code:

1-stats.norm.cdf(44,loc=38,scale=6)

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. (True)

Code:

stats.norm.cdf(30,38,6)

400\*stats.norm.cdf(30,38,6)

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer:**

The difference between https://tex.z-dn.net/?f=2%20X_1 and https://tex.z-dn.net/?f=X_1%20%2B%20X_2 is https://tex.z-dn.net/?f=N(%200%2C6%20%5Csigma%5E2).

**Step-by-step explanation:**

According to the **Central Limit Theorem**, any **large sum** of **independent**, **identically distributed(iid)** random variables is approximately **Normal**.

The **Normal distribution** is defined by two parameters, the **mean**, https://tex.z-dn.net/?f=%5Cmu, and the **variance**, https://tex.z-dn.net/?f=%5Csigma%5E%7B2%7D and written as https://tex.z-dn.net/?f=X%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2).

Given https://tex.z-dn.net/?f=X_1%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)~~%5Cmbox%7Band%7D~~%20X_2%20%5Csim%20N(%5Cmu%2C%20%5Csigma%5E2)  are two independent identically distributed random variables.

From the properties of **normal random variables**,

if https://tex.z-dn.net/?f=X%20%5Csim%20N(%5Cmu_1%2C%20%5Csigma_1%5E2) and https://tex.z-dn.net/?f=Y%20%5Csim%20N(%5Cmu_2%2C%20%5Csigma_2%5E2) are two independent identically distributed random variables then

* the **sum** of normal random variables is given by

https://tex.z-dn.net/?f=X%20%2B%20Y%20%5Csim%20N(%5Cmu_1%20%2B%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20),

* and the **difference** of normal random variables is given by

https://tex.z-dn.net/?f=X%20-%20Y%20%5Csim%20N(%5Cmu_1%20-%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20)

* When  https://tex.z-dn.net/?f=Z%20%3D%20aX, the **product** of X is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2C%20a%5E2%5Csigma_1%5E2%20)

* When  https://tex.z-dn.net/?f=Z%20%3D%20aX%20%2B%20bY, the **linear combination** of X and Y is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2B%20b%5Cmu_2%2C%20a%5E2%5Csigma_1%5E2%20%2B%20b%5E2%5Csigma_2%5E2%20)

Given to find, https://tex.z-dn.net/?f=2X_1

Thus, following the property of multiplication, we get

https://tex.z-dn.net/?f=2X_1%20%5Csim%20N(2%5Cmu%2C%202%5E2%5Csigma%5E2)%5Cimplies2X_1%20%5Csim%20N(2%5Cmu%2C%204%5Csigma%5E2)

and following the property of addition,

https://tex.z-dn.net/?f=X_1%2BX_2%5Csim%20N(%5Cmu%20%2B%20%5Cmu%2C%20%5Csigma%5E2%20%2B%20%5Csigma%5E2%20)%20%5Csim%20N(2%5Cmu%2C%202%5Csigma%5E2%20)

And the difference between the two is given by

https://tex.z-dn.net/?f=2X_1-(X_1%2BX_2)%20%5Csim%20N(2%5Cmu%20-%202%5Cmu%2C%202%5Csigma_1%5E2%20%2B%204%5Csigma_2%5E2%20)%5Csim%20N(%200%2C6%20%5Csigma%5E2)

The mean of https://tex.z-dn.net/?f=2X_1 and https://tex.z-dn.net/?f=X_1%2BX_2 is same but the var(https://tex.z-dn.net/?f=%5Csigma%5E2) of  https://tex.z-dn.net/?f=2X_1 is 2 times more than the variance of https://tex.z-dn.net/?f=X_1%2BX_2.

The difference between the two says that the two given variables are **identically** and **independently** distributed.

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Answer:** Solved using Jupyter Note Book (Python)- File attached

stats**.**norm**.**interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

( D. 48.5, 151.5 )

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer:** Solved using Jupyter Note Book (Python)- File attached

Mean Profit is Rs 540 Million

Standard Deviation is Rs 225.0 Million

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Answer:**

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

**Answer:**

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer:** Division 1

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.0477903522728147

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.040059156863817086