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CHAPTER 6: INTEGER PROGRAMMING (PART 1)

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Innovating Solutions

Introduction

- Linear programming:
  - Assumes that the decision variables are continuous.
  - In practice, decision variables may need to be integers.
    - Sometimes, binary (i.e., 0 or 1)
- When the decisions variables are integer, we have integer programming problem.
  - Pure integer programming (IP)- All decision variables required to have integer solution values.
  - Binary integer programming (BIP)-All decision variables required to have integer values of zero or one.
  - Mixed integer programming (MIP)-Some of the decision variables (but not all) required to have integer values.

Integer Programming Model

- Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

- A feasible solution is ensured by rounding down non-integer solution values but may result in a less than optimal (sub-optimal) solution.

Example 1

- Machine shop obtaining new presses and lathes.
- Marginal profitability: each press \$100/day; each lathe \$150/day.
- Resource constraints: \$40,000, 200 sq. ft. floor space.
- Machine purchase prices and space requirements:

Machine	Required Floor Space (sq. ft.)	Purchase Price
Press	**15**	**\$8,000**
Lathe	**30**	**4,000**

- How many of each type of machine to purchase to maximize the daily increase in profit?

Example 1 (cont'd)

x_1 = number of presses
x_2 = number of lathes

Integer Programming Model:

Maximize $Z = 100x_1 + 150x_2$

Subject to:

$$8000x_1 + 4000x_2 \leq 40000$$
$$15x_1 + 30x_2 \leq 200$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

Example 1 (cont'd)

- Optimal Solution:

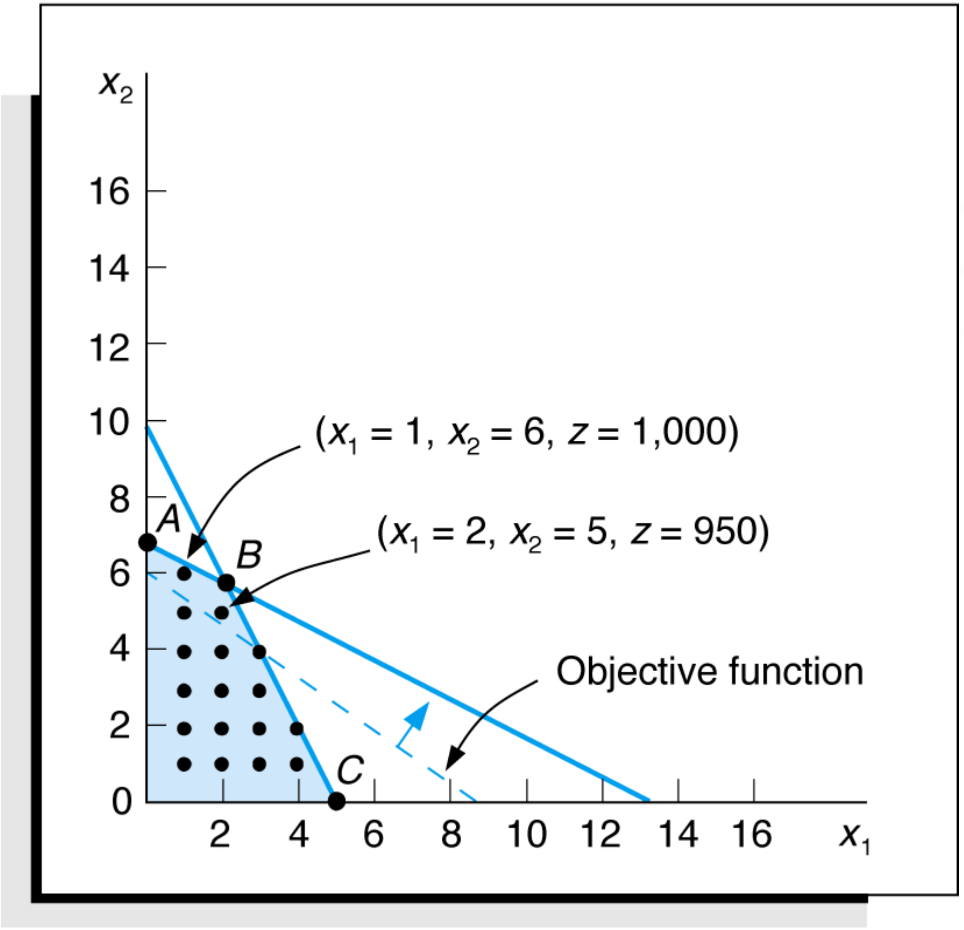
  $Z = 1{,}055.56$

  $x_1 = 2.22$ presses

  $x_2 = 5.55$ lathes

- The dots indicate integer solution points.
- Rounding non-integer solution values up to the nearest integer value ($x_1 = 2$ and $x_2 = 6$) can result in an infeasible solution:

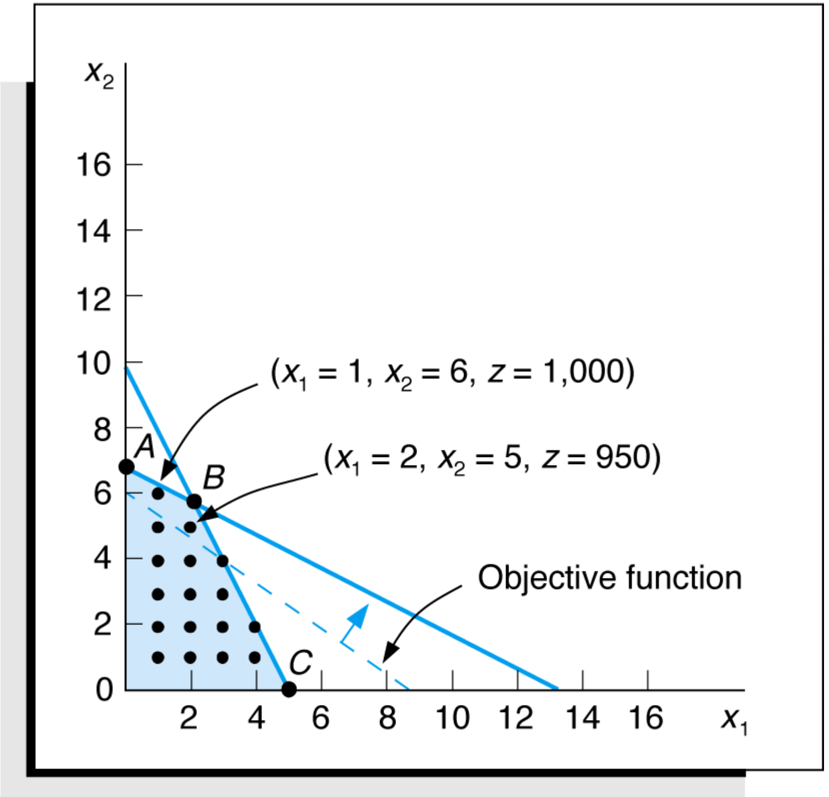
$$15x_1 + 30x_2 \leq 200$$
$$15(2) + 30(6) \leq 200$$
$$210 \nleq 200$$



Feasible Solution Space with Integer Solution Points

Example 1 (cont'd)

- The point $(x_1 = 2 \text{ and } x_2 = 5)$ is the rounded-down solution. Notice that as the objective function edge moves outward through the feasible solution space.
- One of the difficulties of simply rounding down non-integer values is that another integer solution may result in a higher profit $(x_1 = 1 \text{ and } x_2 = 6)$
- Thus, a more direct approach for solving integer problems is required.

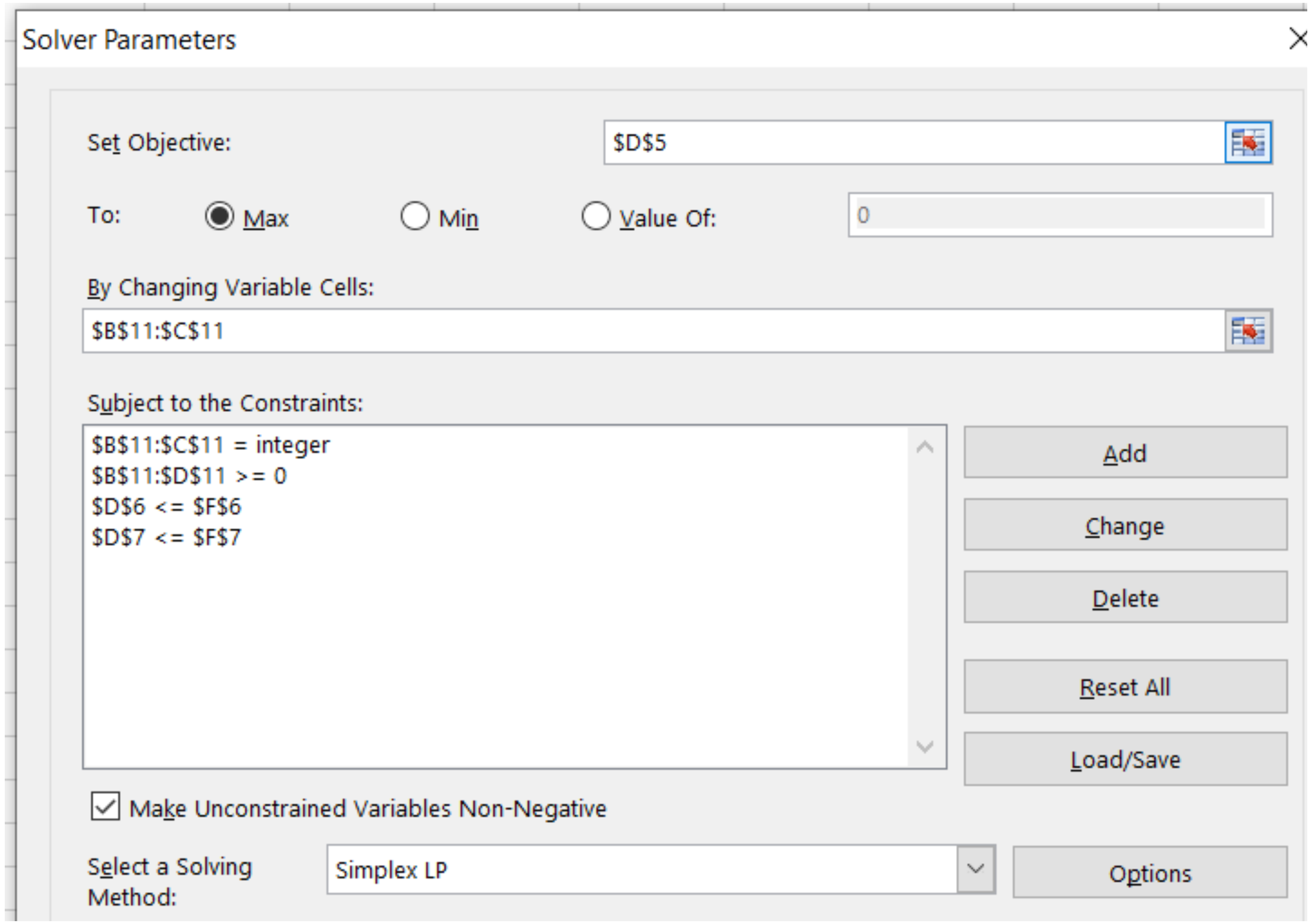


Feasible Solution Space with Integer Solution Points

Integer Programming using Excel Solver

	A	B	C	D	E	F	G
1	**IP Model Example 1**						
2	**Input data:**						
3		**x1**	**x2**				
4		**Press**	**lathe**	**Total**		**Limits**	
5	**Objective**	**100**	**150**	**0**			
6	**Purchase Price**	**8000**	**4000**	**0**	**<=**	**40000**	
7	**Floor Space**	**15**	**30**	**0**	**<=**	**200**	
8							
9	**Output result**						
10		**x1**	**x2**	**Z**			
11	**Solution**	0	0	0			
12							

	A	B	C	D	E	F
1						
2	**Input data:**					
3		**x1**	**x2**			
4		**Press**	**lathe**	**Total**		**Limits**
5	**Objective**	**100**	**150**	**1000**		
6	**Purchase Price**	**8000**	**4000**	**32000**	**<=**	**40000**
7	**Floor Space**	**15**	**30**	**195**	**<=**	**200**
8						
9	**Output result**					
10		**x1**	**x2**	**Z**		
11	**Solution**	1	6	1000		



Integer Programming using Phython

```
# Import PuLP modeler functions
from pulp import *
# Create the 'prob' variable to contain the problem data
prob = LpProblem("Machine",LpMaximize)
# The 2 variables  are created with a lower limit of zero
x1=LpVariable("presses",0,None,'Integer')
x2=LpVariable("lathes",0,None,'Integer')
# The objective function is added to 'prob' first
prob += 100*x1 + 150*x2, "Total daily profit"
# The four constraints are entered
prob += 8000*x1 + 4000*x2 <= 40000, "Purchased Price"
prob += 15*x1 + 30*x2 <= 200, "Floor Space"
```

Integer Programming using Python (cont'd)

```
# The problem data is written to an .lp file
prob.writeLP("machines.lp")
# The problem is solved using PuLP's choice of Solver
prob.solve()
# The status of the solution is printed to the screen
print ("Status:", LpStatus[prob.status])
# Each of the variables is printed with it's resolved optimum value
for v in prob.variables():
    print(v.name, "=", v.varValue)
    # The optimised objective function value is printed to the screen
print ("Total Profit = ", value(prob.objective))
```

Integer Programming using Python (cont'd)

```
Status: Optimal
lathes = 6.0
presses = 1.0
Total Profit =  1000.0
```

Exercise 1

A textbook publishing company has developed two new sales regions and is planning to transfer some of its existing sales force into these two regions. The company has 10 salesperson available for the transfer. Because of the different geographic configurations and the location of schools in each region, the average annual expenses for a salesperson differ in the two regions:

- The average is $10,000 per salesperson in region 1; $7,000 per salesperson in region 2.

The total annual expense budget for the new regions is $72,000. It is estimated that a salesperson in region 1 will generate an average of $85,000 in sales each year, and a salesperson in region 2 will generate $60,000 annually in sales. The company wants to know how many salesperson to transfer into each region to maximize increased sales.

Example 2

- Recreation facilities selection to maximize daily usage by residents.
- Resource constraints: $120,000 budget; 12 acres of land.
- Selection constraint: either swimming pool or tennis center (not both).
- Data:

Recreation Facility	Expected Usage (people/day)	Cost ($)	Land Requirement (acres)
Swimming pool	300	35,000	4
Tennis Center	90	10,000	2
Athletic field	400	25,000	7
Gymnasium	150	90,000	3

Example 2 (cont'd)

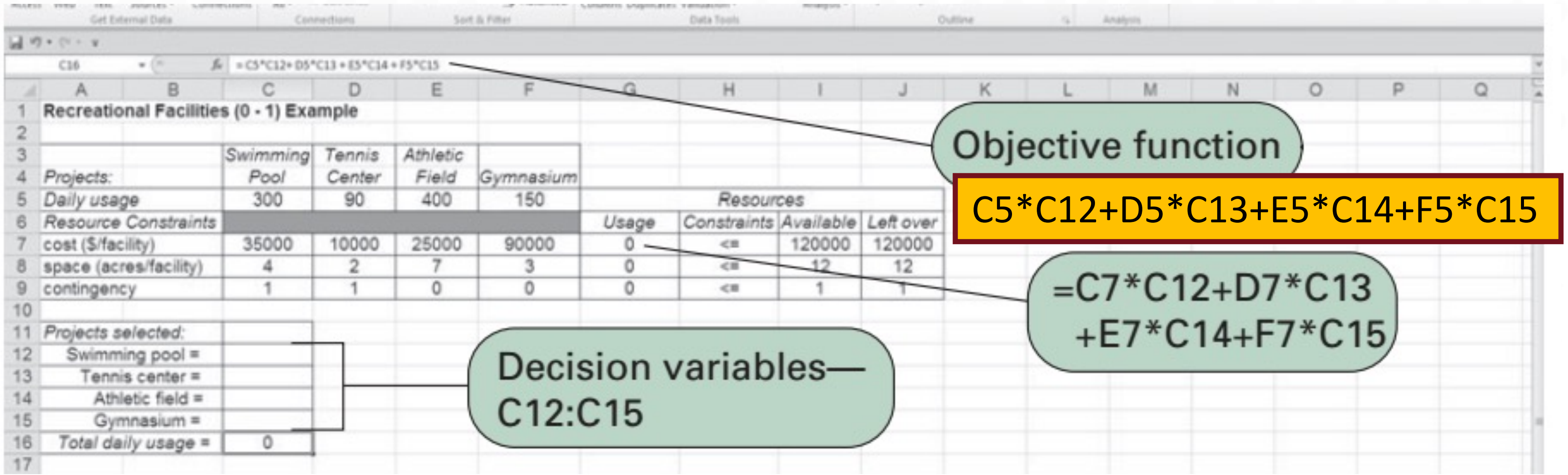
Binary Integer Programming Model:

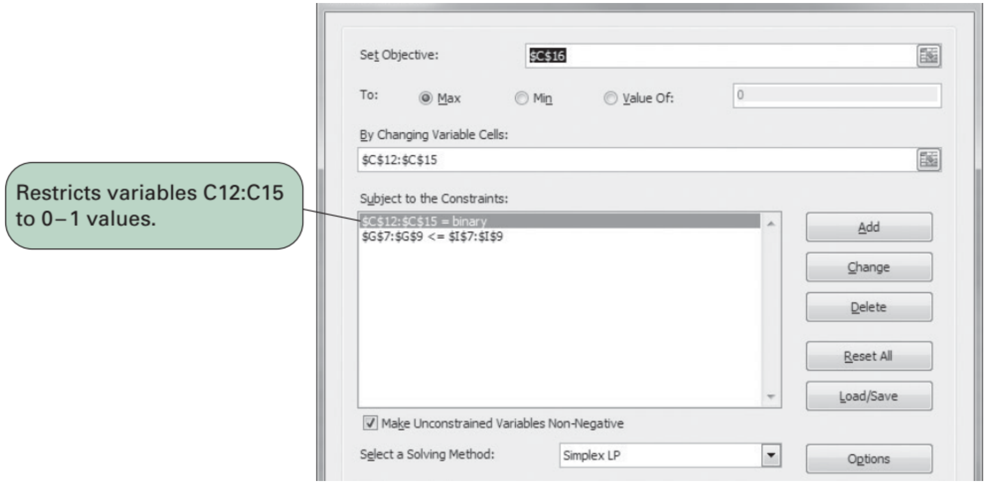
Maximize $Z = 300x_1 + 90x_2 + 400x_3 + 150x_4$

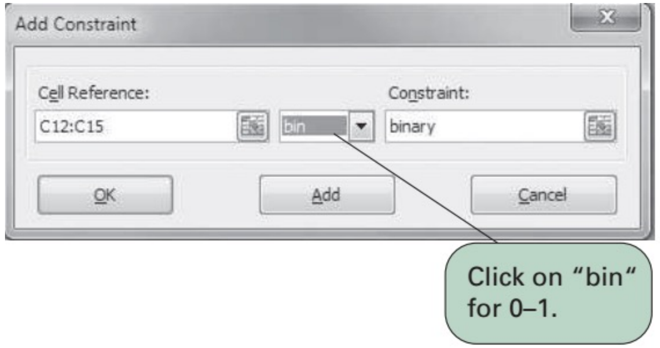
Subject to:

$$35{,}000x_1 + 10{,}000x_2 + 25{,}000x_3 + 90{,}000x_4 \le 120{,}000$$
$$4x_1 + 2x_2 + 7x_3 + 3x_4 \le 12 \qquad (acres)$$
$$x_1 + x_2 \le 1 \qquad (facility)$$
$$x_1, x_2, x_3, x_4 = 0 \; or \; 1$$
$$x_1 = construction\ of\ a\ swimming\ pool$$
$$x_2 = construction\ of\ a\ tennis\ center$$
$$x_3 = construction\ of\ an\ athletic\ field$$
$$x_4 = construction\ of\ a\ gymnasium$$

Example 2 (cont'd) Binary Integer Programming Model – in Excel Solver







Example 2 (cont'd) Binary Integer Programming Model – in Excel Solver

	A	B	C	D	E	F	G	H	I
	Input Data								
	Project:	Swimming Pool	Tennis Centre	Athletic Field	Gymnasium				
	Daily usage	300	90	400	150	Resources			
	Resource Constraints					Usage	Constraints	Available	LeftOver
	Cost($/facility)	35000	10000	25000	90000	60000	<=	120000	120000
	Space (acres/Facility)	4	2	7	3	11	<=	12	12
	Contingency	1	1	0	0	1	<=	1	1
	Output results:								
	Project selected:								
	Swimming Pool =	1							
	Tennis Centre =	0							
	Athletic Field =	1							
	Gymnasium =	0							
	Total daily usage	700							

- Recreation facilities selection to maximize daily usage by residents.

  => Swimming Pool and Athletic Field

- Total daily usage = 700

Example 2 (cont'd) Binary Integer Programming Model – in Python

```
# Import PuLP modeler functions
from pulp import *

# Create the 'prob' variable to contain the problem data
prob = LpProblem("facility",LpMaximize)

# The 4 variables  are created with a lower limit of zero
x1=LpVariable("SP",0,1,'Integer')
x2=LpVariable("TC",0,1,'Integer')
x3=LpVariable("AF",0,1,'Integer')
x4=LpVariable("G",0,1,'Integer')
# The objective function is added to 'prob' first
prob += 300*x1 + 90*x2+400*x3+150*x4, "Total daily usage"

# The constraints are entered
prob += 35000*x1 + 10000*x2+25000*x3+90000*x4 <= 120000, "Costs"
prob += 4*x1 + 2*x2+7*x3+3*x4 <= 12, "acre"
prob += 1*x1 + 1*x2 <= 1, "facility "
```

Example 2 (cont'd) Binary Integer Programming Model – in Python

```
# The problem data is written to an .lp file
prob.writeLP("facilities.lp")

# The problem is solved using PuLP's choice of Solver
prob.solve()

# The status of the solution is printed to the screen
print ("Status:", LpStatus[prob.status])

# Each of the variables is printed with it's resolved optimum value
for v in prob.variables():
    print(v.name, "=", v.varValue)

# The optimised objective function value is printed to the screen
print ("Total Usage = ", value(prob.objective))
```

Example 2 (cont'd) Binary Integer Programming Model – in Python

```
Status: Optimal
AF = 1.0
G = 0.0
SP = 1.0
TC = 0.0
Total Usage =  700.0
```

Mix Integer Programming Model

In a mixed integer model, some solution values for decision variables are integers and others can be non-integer.

Example 3:

Nancy Smith has $250,000 to invest in three alternative investments—condominiums, land, and municipal bonds. She wants to invest in the alternatives that will result in the greatest return on investment after 1 year. Each condominium costs $50,000 and will return a profit of $9,000 if sold at the end of 1 year; each acre of land costs $12,000 and will return a profit of $1,500 at the end of 1 year; and each municipal bond costs $8,000 and will result in a return of $1,000 if sold at the end of 1 year. In addition, there are only 4 condominiums, 15 acres of land, and 20 municipal bonds available for purchase.

Example 3 (cont'd)

x_1 = condominiums purchased
x_2 = acres of land purchased
x_3 = bonds purchased

Maximize $Z = 9000x_1 + 1500x_2 + 1000x_3$

Subject to:

$$50000x_1 + 12{,}000x_2 + 8{,}000x_3 \leq 250{,}000$$

$$x_1 \leq 4\ (condominiums)$$

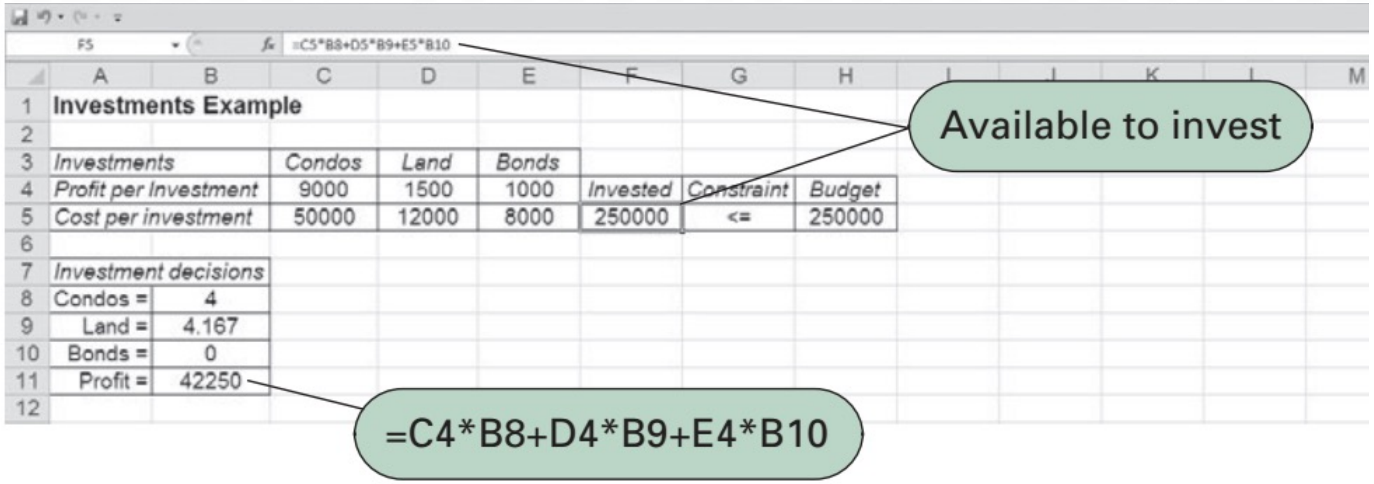
$$x_2 \leq 15\ (acres)$$

$$x_3 \leq 20\ (bonds)$$

$$x_2 \geq 0$$

$$x_1, x_3 \geq 0\ and\ integer$$

Example 3 (cont'd)



	A	B	C	D	E	F	G	H
1	Investments Example							
2								
3	Investments		Condos	Land	Bonds			
4	Profit per Investment		9000	1500	1000	Invested	Constraint	Budget
5	Cost per investment		50000	12000	8000	250000	<=	250000
6								
7	Investment decisions							
8	Condos =	4						
9	Land =	4.167						
10	Bonds =	0						
11	Profit =	42250						
12								

Notice that the constraint values for the availability of each type of investment is enter directly into Solver
(i.e., 4 condos, 15 acres of land, and 20 bonds)

Integer requirement for condos and bonds)