MCSD1133 Operations Research & Optimization

UNIVERSITI TEKNOLOGI MALAYSIA

CHAPTER 7: ILP – TRANSSHIPMENT MODEL

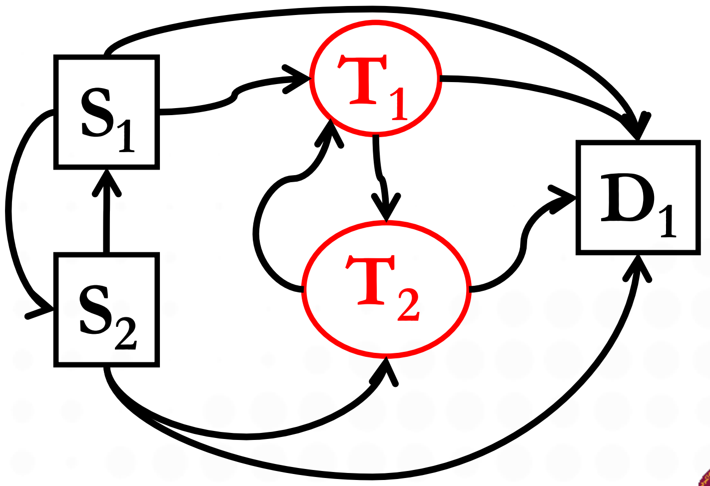
UTM JOHOR BAHRU

nzah@utm.my-20232024(S2)

Innovating Solutions

The Transshipment Model

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations. An example of a transshipment point is a distribution center or warehouse located between plants and stores.
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these



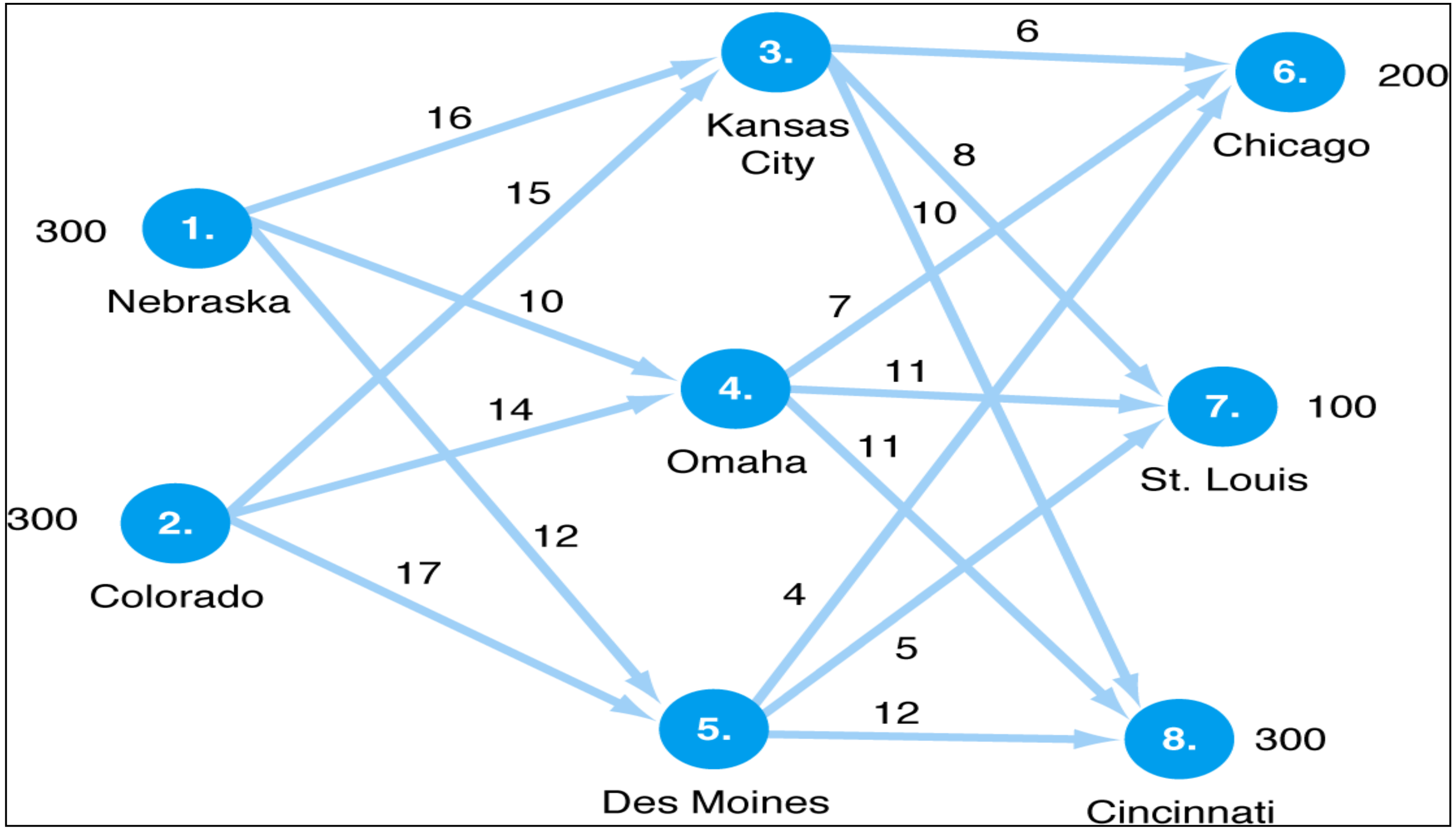
Example 1

Wheat is harvested at farms in Nebraska and Colorado before being shipped to the three grain elevators in Kansas City, Omaha, and Des Moines, which are now transshipment points. The amount of wheat harvested at each farm is 300 tons. The wheat is then shipped to the mills in Chicago, St. Louis, and Cincinnati. The shipping costs from the grain elevators to the mills remain the same, and the shipping costs from the farms to the grain elevators are as follows:

	Grain Elevator		
Farm	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	$16	10	12
2. Colorado	15	14	17

Transport Cost from Grain Elevator to Mill ($/ton)			
Grain Elevator	6. Chicago	7. St. Louis	8. Cincinnati
3. Kansas City	$ 6	$ 8	$ 10
4. Omaha	7	11	11
5. Des Moines	4	5	12

Example 1: Transshipment Network Routes

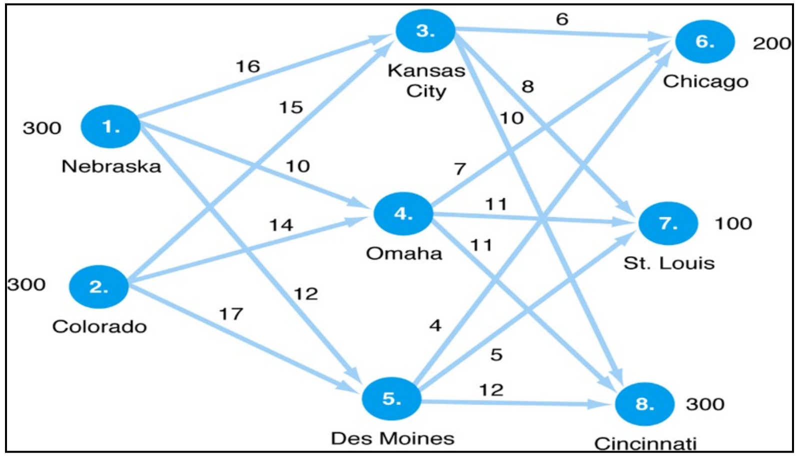


Example 1: Model Formulation

Minimize, Z $= 16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} + 17x_{25} +$
$6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} + 11x_{48} +$
$4x_{56} + 5x_{57} + 12x_{58}$

subject to:

$x_{13} + x_{14} + x_{15} = 300$(Availability supply in Nebraska)
$x_{23} + x_{24} + x_{25} = 300$(Availability supply in Colorado)
$x_{36} + x_{46} + x_{56} = 200$ (Demand at Chicago)
$x_{37} + x_{47} + x_{57} = 100$ (Demand at St. Louis)
$x_{38} + x_{48} + x_{58} = 300$(Demand at Cincinnati)
$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$ (shipped into =out Kansas City)
$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$ (shipped into =out Omaha)
$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$ (shipped into =out Des Moines)
$x_{ij} \geq 0$



Example 1: Solution with Excel Solver (1 of 3)

Objective Function

C24 =SUMPRODUCT(B6:D7,I6:K7)+SUMPRODUCT(C13:E15,J13:L15)

The Wheet Shipping Transshipment Example

=SUM(B6:D6)

	Grain Elevatorts				Grain
Farms	3.Kansas City	4. Omaha	5.Des Moined	Supply	Shipped
1.Nebraska	0	0	0	300	0
2. Colorado	0	0	0	300	0
Shipped	0	0	0		

Shipping Cost

	Grain Elevatorts		
Farms	3.Kansas Cit	4. Omaha	5.Des Moined
1.Nebraska	16	10	12
2. Colorado	15	14	17

	Mills			Grain
Grain Elevatorts	6.Chicago	7.St Louis	8.Cincinnati	Shipped
3.Kansas City	0	0	0	0
4. Omaha	0	0	0	0
5. Des Moined	0	0	0	0
Demand	200	100	300	
Shipped	0	0	0	

		Mills		
Grain Elevatorts		Chicago	St Louis	Cincinnati
	Kansas City	6	8	10
	Omaha	7	11	11
	Des Moined	4	5	12

=SUM(C13:E13)

=SUM(C13:C15

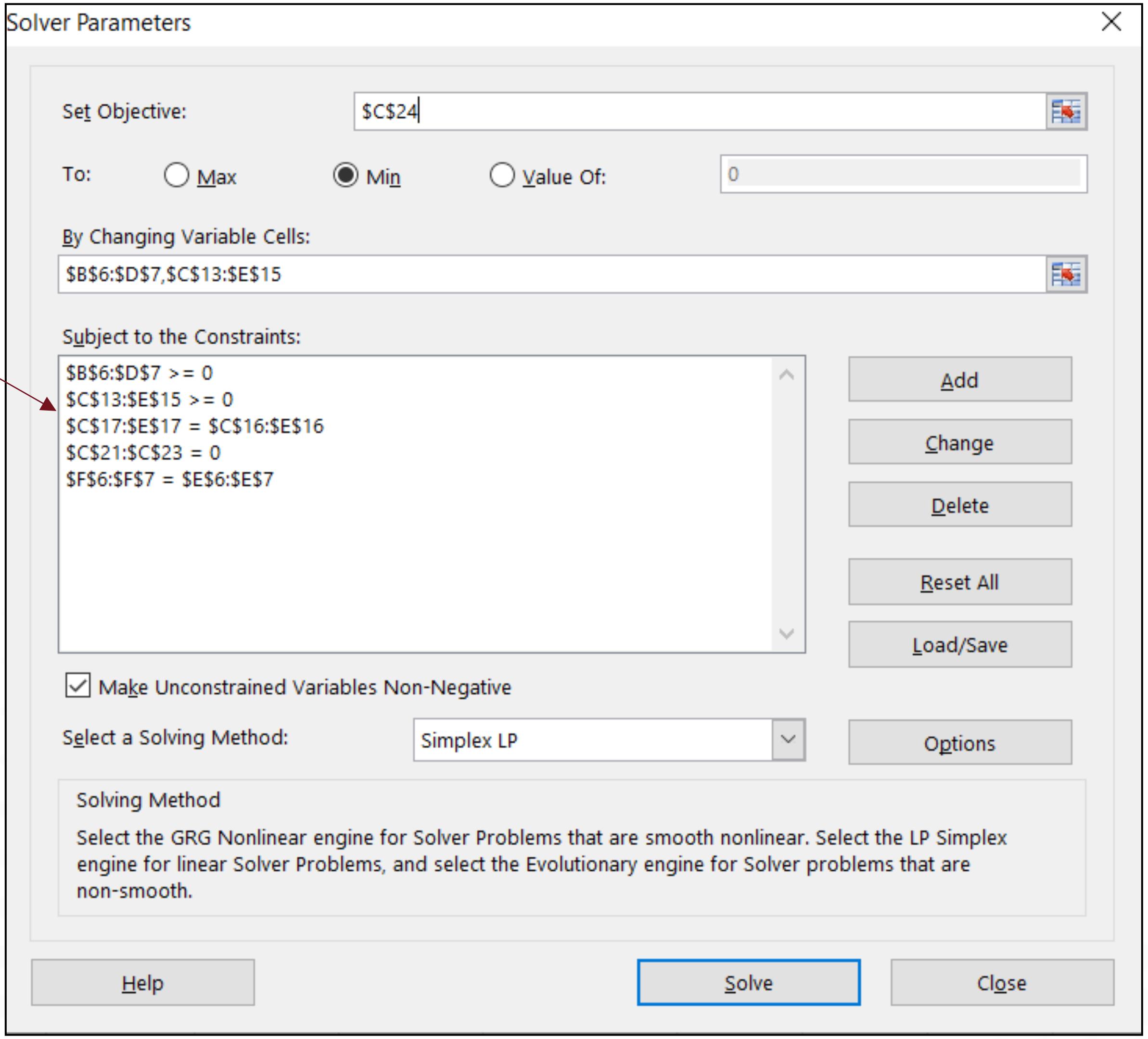
Transshipment flows

3.Kansas City	0
4. Omaha	0
5. Des Moined	0
Cost	0

=B8-F13

Example 1: Solution with Excel Solver (2 of 3)

Transshipment constraints in cells C21:C23



Example 1: Solution with Excel Solver (3 of 3)

	A	B	C	D	E	F
1	**The Wheet Shipping Transshipment Example**					
2						
3						
4		Grain Elevatorts				Grain
5	**Farms**	3.Kansas City	4. Omaha	5.Des Moined	Supply	Shipped
6	1.Nebraska	**0**	**0**	**300**	**300**	300
7	2. Colorado	**0**	**300**	**0**	**300**	300
8	Shipped	0	300	300		
9						
10						
11				Mills		Grain
12	Grain Elevatorts		6.Chicago	7.St Louis	8.Cincinnati	Shipped
13	3.Kansas City		**0**	**0**	**0**	0
14	4. Omaha		**0**	**0**	**300**	300
15	5. Des Moined		**200**	**100**	**0**	300
16	Demand		200	100	300	
17	Shipped		200	100	300	
18						
19	Transhipment flows					
20						
21	3.Kansas City		0			
22	4. Omaha		0			
23	5. Des Moined		0			
24		**Cost**	**12400**			
25						
26						

Variable Cells

Cell	Name	Original Value	Final Value
B6	1.Nebraska 3.Kansas City	0	0
C6	1.Nebraska 4. Omaha	0	0
D6	1.Nebraska 5.Des Moined	0	300
B7	2. Colorado 3.Kansas City	0	0
C7	2. Colorado 4. Omaha	0	300
D7	2. Colorado 5.Des Moined	0	0
C13	3.Kansas City 6.Chicago	0	0
D13	3.Kansas City 7.St Louis	0	0
E13	3.Kansas City 8.Cincinnati	0	0
C14	4. Omaha 6.Chicago	0	0
D14	4. Omaha 7.St Louis	0	0
E14	4. Omaha 8.Cincinnati	0	300
C15	5. Des Moined 6.Chicago	0	200
D15	5. Des Moined 7.St Louis	0	100
E15	5. Des Moined 8.Cincinnati	0	0

Exercise 1

The manager of Harley's Sand and Gravel Pit has decided to utilize two intermediate nodes as transshipment points for temporary storage of topsoil.

Table 1 Cost of Shipping One Unit from the Farms to Warehouses

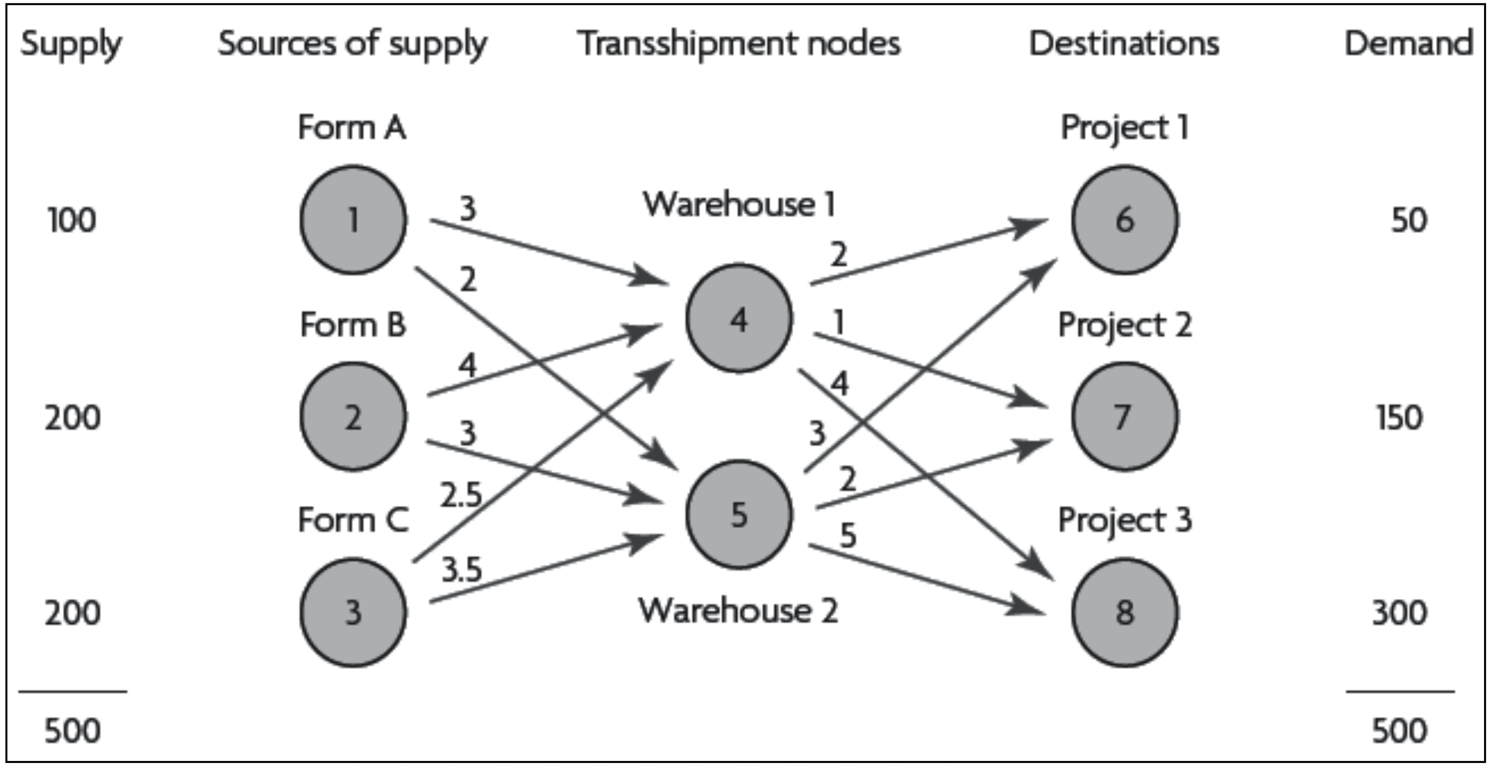
From/To	Warehouse 1 (4)	Warehouse 2 (5)
Farm A (1)	3	2
Farm B (2)	4	3
Farm C (3)	2.5	3.5

Table 2 Cost of Shipping One Unit from the Warehouses to Projects

From/To	Project 1 (6)	Project 2 (7)	Project 3 (8)
Warehouse 1 (4)	2	1	4
Warehouse 2 (5)	3	2	5

Exercise 1 (cont'd)

A Network Diagram of Harley's Sand and Gravel Pit Transshipment



(a) Formulate the problem as a transshipment model.
(b) Determine the optimal solution using Excel Solver.

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CHAPTER 7: ILP – ASSIGNMENT MODEL

UTM JOHOR BAHRU

nzah@utm.my-20232024(S2)

Innovating Solutions

The Assignment Model

- Special form of linear programming model similar to the transportation model.
- ***Supply*** at each source and ***demand*** at each destination ***limited to one unit***. Workers represent sources and jobs represent destinations.
- Involve the matching or pairing of two sets of items such as jobs and machines, secretaries and reports, lawyers and cases, and so forth. Have different cost, distance or time requirements for different pairings.
- Use in a **balanced** model (supply equals demand).
- The goal is to determine the minimum cost assignment of workers to jobs.

The Assignment Model

- The assignment problem in which *i*-workers are assigned to *j*-jobs can be represented as an LP model .
- Let c_{ij} be the cost of assigning worker *i* to job *j*

$$x_{ij} = \begin{cases} 1, & \text{if worker } i \text{ is assigned to job } j \\ 0, & \text{otherwise} \end{cases}$$

Then the LP model is given as

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n$$

$$x_{ij} = 0 \text{ or } 1$$

Example 2

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials; demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Example 2: Model Formulation

$$\text{Minimize } Z = 210x_{A1} + 90x_{A2} + 180x_{A3} + 160x_{A4} + 100x_{B1} + 70x_{B2} + 130x_{B3} + 200x_{B4} + 175x_{C1} + 105x_{C2} + 140x_{C3} + 170x_{C4} + 80x_{D1} + 65x_{D2} + 105x_{D3} + 120x_{D4}$$

subject to:

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1$$
$$x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1$$
$$x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1$$
$$x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1$$
$$x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1$$
$$x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1$$
$$x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1$$
$$x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1$$

$$x_{ij} \geq 0$$

	Game Sites			
	1	2	3	4
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Example 2: Assignment Model Example with Excel Solver (1 of 3)

Objective Function

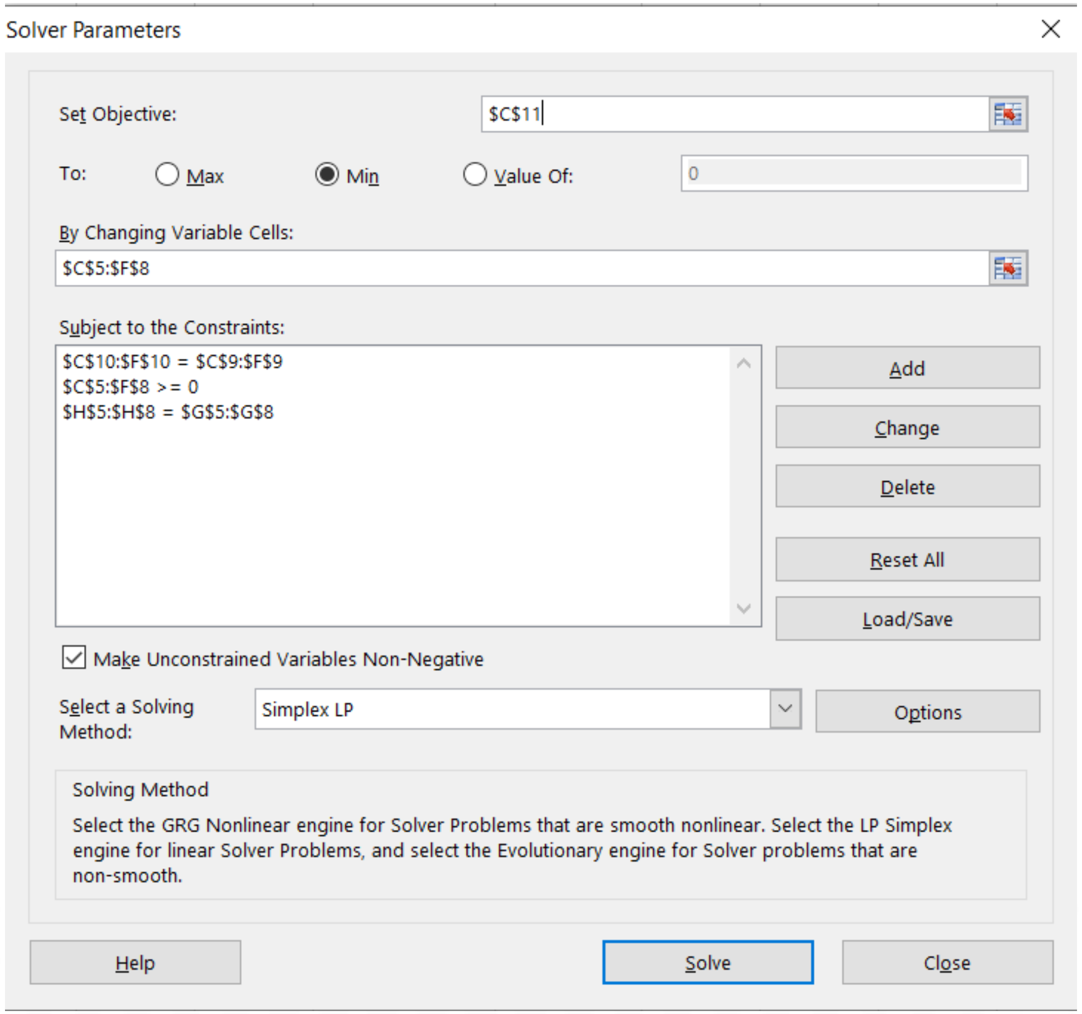
C11 | =SUMPRODUCT(C5:F8,C17:F20)

=SUM(C5:F5)

=SUM(C5:C8)

	A	B	C	D	E	F	G	H
1	**The ACC Basketball Example**							
2								
3			Game Sites				Team	Team
4	Official Team		1.Raleigh	2.Atlanta	3.Durham	4.Clelmson	Available	Assigned
5		A	**0**	**0**	**0**	**0**	**1**	0
6		B	**0**	**0**	**0**	**0**	**1**	0
7		C	**0**	**0**	**0**	**0**	**1**	0
8		D	**0**	**0**	**0**	**0**	**1**	0
9	Teams Demanded		**1**	**1**	**1**	**1**	**4**	4
10	Teams Assigned		0	0	0	0		
11	**Total Mileage**		0					
12								
13								
14								
15			Game Sites					
16	Official Team		1.Raleigh	2.Atlanta	3.Durham	4.Clelmson		
17		A	210	90	180	160		
18		B	100	70	130	200		
19		C	175	105	140	170		
20		D	80	65	105	120		

Example 2: Assignment Model Example with Excel Solver (2 of 3)



Example 2: Assignment Model Example with Excel Solver (3 of 3)

	A	B	C	D	E	F	G	H
1	**The ACC Basketball Example**							
2								
3			Game Sites				Team	Team
4	Official Team		1.Raleigh	2.Atlanta	3.Durham	4.Clelmson	Available	Assigned
5		A	**0**	**1**	**0**	**0**	**1**	1
6		B	**1**	**0**	**0**	**0**	**1**	1
7		C	**0**	**0**	**1**	**0**	**1**	1
8		D	**0**	**0**	**0**	**1**	**1**	1
9	Teams Demanded		**1**	**1**	**1**	**1**	**4**	4
10	Teams Assigned		1	1	1	1		
11	**Total Mileage**		450					
12								
13								
14								
15			Game Sites					
16	Official Team		1.Raleigh	2.Atlanta	3.Durham	4.Clelmson		
17		A	210	90	180	160		
18		B	100	70	130	200		
19		C	175	105	140	170		
20		D	80	65	105	120		

Variable Cells

Cell	Name	Original Value	Final Value
C5	A 1.Raleigh	0	0
D5	A 2.Atlanta	0	1
E5	A 3.Durham	0	0
F5	A 4.Clelmson	0	0
C6	B 1.Raleigh	0	1
D6	B 2.Atlanta	0	0
E6	B 3.Durham	0	0
F6	B 4.Clelmson	0	0
C7	C 1.Raleigh	0	0
D7	C 2.Atlanta	0	0
E7	C 3.Durham	0	1
F7	C 4.Clelmson	0	0
C8	D 1.Raleigh	0	0
D8	D 2.Atlanta	0	0
E8	D 3.Durham	0	0
F8	D 4.Clelmson	0	1

Assignment Problem - Hungarian Method.

Step 1: Determine *pi*, the minimum cost element of row-*i* in the original cost matrix and subtract it from all the elements of row *i*, *i* = 1, 2, 3.

Step 2. For the matrix created in step 1, determine *qj*, the minimum cost element of column *j*, and subtract it from all the elements of column *j*, *j* = 1, 2, 3.

Step 3. From the matrix in step 2, attempt to find a *feasible* assignment among all the resulting zero entries.

3a. If such an assignment can be found, it is optimal.

3b. Else, additional calculations are needed (Extension Hungarian Method)

$$
\begin{array}{cc}
 & \text{Jobs} \\
 & \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\text{Worker} \begin{array}{c} 1 \\ 2 \\ \vdots \\ N \end{array} & \begin{array}{|cccc|c} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{array} \begin{array}{c} \mathbf{1} \\ \mathbf{1} \\ \vdots \\ \mathbf{1} \end{array} \\
 & \begin{array}{cccc} \mathbf{1} & \mathbf{1} & \cdots & \mathbf{1} \end{array}
\end{array}
$$

Assignment Problem - Hungarian Method.

3b. If no feasible zero-element assignments can be found,

i. Draw the *minimum* number of horizontal and vertical lines in the last reduced matrix to cover *all* the zero entries.

ii. Select the *smallest uncovered* entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

iii. If no feasible assignment can be found among the resulting zero entries, repeat step **3b.**

Example 3

Joe Ken's three children, John, Karen, and Terri, want to earn some money for personal expenses. Mr. Ken has chosen three chores for his children: mowing the lawn, painting the garage door and washing the family cars.

To avoid anticipated sibling competition, he asks them to submit individual (secret) bids for what they feel is fair pay for each of the three chores. Table 1 summarizes the bids received. The children will abide by their father's decision regarding the assignment of chores.

Table 1

	Mow	Paint	Wash
John	\$15	\$10	\$9
Karen	\$9	\$15	\$10
Terri	\$10	\$12	\$8

Example 3: Solution using Hungarian Method

Step 1:

	Mow	Paint	Wash	Row min
John	15	10	9	$p_1 = 9$
Karen	9	15	10	$p_2 = 9$
Terri	10	12	8	$p_3 = 8$

Step 2:

$\Rightarrow$

	Mow	Paint	Wash
John	6	1	0
Karen	0	6	1
Terri	2	4	0
Column min	$q_1 = 0$	$q_2 = 1$	$q_3 = 0$

Step 3:

	Mow	Paint	Wash
John	6	$\underline{\mathbf{0}}$	0
Karen	$\underline{\mathbf{0}}$	5	1
Terri	2	3	$\underline{\mathbf{0}}$

The cells with underscored zero provide the (feasible) optimum solution:

John gets the paint job,
Karen gets to mow the lawn,
Terri gets to wash the family cars.

The total cost to Mr. Ken is 9 + 10 + 8 = $27.

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Example 4

Suppose that the situation discussed in Example 3 is extended to four children and four chores. Table 2 summarizes the cost elements of the problem.

Table 2

		Chore 1	Chore 2	Chore 3	Chore 4
Child	1	\$1	\$4	\$6	\$3
	2	\$9	\$7	\$10	\$9
	3	\$4	\$5	\$11	\$7
	4	\$8	\$7	\$8	\$5

Example 4 - Solution

Step 1

		Chores				
		1	2	3	4	Row min
	1	**1**	4	6	3	p1=1
Child	2	9	**7**	10	9	p2=7
	3	**4**	5	11	7	p3=4
	4	8	7	8	**5**	p4=5

Step 2

		1	2	3	4
	1	**0**	3	5	2
Child	2	2	**0**	**3**	2
	3	**0**	1	7	3
	4	3	2	**3**	**0**
Col min		q1=0	q2=0	q2=3	q4=0

Step 3

		Chores			
		1	2	3	4
	1	**0**	3	2	2
Child	2	2	**0**	**0**	2
	3	**0**	1	4	3
	4	3	2	**0**	**0**

The locations of the zero entries do not allow assigning unique chores to all the children.

Example 4 - Solution

Step 3b(i)

		Chores			
		1	2	3	4
Child	1	**0**	3	2	2
	2	2	**0**	**0**	2
	3	**0**	1	4	3
	4	3	2	**0**	**0**

Draw the *minimum* number of horizontal and vertical lines in the last reduced matrix to cover *all* the zero entries.

Step 3b(ii)

		Chores			
		1	2	3	4
Child	1	**0**	2	1	1
	2	3	**0**	**0**	2
	3	**0**	0	3	2
	4	4	2	**0**	**0**

Select the *smallest uncovered* entry, subtract it from every uncovered entry, and then add it to every entry at the intersection of two lines.

Step 3b(iii)

		Chores			
		1	2	3	4
Child	1	**0**	2	1	1
	2	3	0	**0**	2
	3	0	**0**	3	2
	4	4	2	0	**0**

The cells with underscore zero provide the (feasible) optimum solution:
Child 1 to chore 1, Child 2 to chore 3
Child 3 to chore 2, Child 4 to chore 4

The total cost to Mr. Ken is 1 + 10 + 5+ 5 = $21.

Exercise 2

A manager has prepared a table that shows the cost of performing each of five jobs by each of five employees .

	Employee				
Job	Al	Bill	Cindy	David	Earl
1	15	20	18	24	19
2	12	17	16	15	14
3	14	15	19	17	18
4	11	14	12	13	15
5	13	16	17	18	16

The manager has stated that his goal is to develop a set of job assignments that will minimize the total cost of getting all four jobs done. It is further required that the jobs be performed simultaneously, thus requiring one job being assigned to each employee.

(a) Formulate the problem as an assignment model.
(b) Determine the optimal solution using Excel solver.
(c) Obtain the optimal solution using Hungarian method.