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CHAPTER 9:

DECISION ANALYSIS

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Innovating Solutions

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Introduction

- In LP, models are formulated and solved to aid the manager in making a decision.
- The solutions to the models were represented by values for **decision** variables.
- However, these LP models were all formulated under the assumption that certainty existed.
- In actual practice, many decision-making situations occur under conditions of **uncertainty**.
- For example, the demand for a product may be not 100 units next week, but 50 or 200 units, depending on the state of the market (which is uncertain).
- Several decision-making techniques are available to aid the decision maker in dealing with this type of decision situation in which there is uncertainty.
- Decision situations can be categorized into TWO classes:
    1. Situations in which probabilities **cannot** be assigned to future occurrences.
    2. Situations in which probabilities **can** be assigned to future occurrences.

Introduction

Classifying decision-making criteria:

- Decision making under certainty.
  - ➢ The future state-of-nature is assumed known.
- Decision making under risk (with probabilities).
  - ➢ There is some knowledge of the probability of the states of nature occurring.
- Decision making under uncertainty (without probabilities).
  - ➢ There is no knowledge about the probability of the states of nature occurring.

Components of Decision Making

- The field of decision analysis provides a framework for making important decisions.
- Decision analysis allows us to select a decision from a set of possible decision alternatives when uncertainties regarding the future exist.
- The goal is to optimize the resulting payoff in terms of a decision criterion.
- The decision-making situation includes several components:
    1. The **decision** themselves
    2. The actual events that may occur in the future, known as "**state of nature**".
- Example:

    A distribution company is considering purchasing a computer to increase the number of orders it can process and thus increase its business. If economic conditions remain good, the company will realize a large increase in profit; however, if the economic takes a downturn, the company will lose a money.

    - ➢ The possible decision are (1) to purchase the computer or (2) not to purchase the computer
    - ➢ The state of nature: (1) Good economic condition; (2) bad economic condition
    - ➢ The state of nature that occur will determine the outcome and decision maker has no control over which state will occur.

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Payoff Table

- To facilitate the analysis of these types of decision situations so that the best decisions result, they are organized into **payoff tables**.
- Payoff table is a means of organizing and illustrating the payoffs from the different decisions, given the various states of nature in a decision problem.
- A payoff table is constructed as shown in Table 1:

Table 1

Decision	State of Nature	
	a	b
1	Payoff 1a	Payoff 1b
2	Payoff 2a	Payoff 2b

- Each decision, 1 or 2, in Table 1 will result in an outcome, or payoff, for the particular state of nature that will occur in the future.
- Payoff typically expressed in terms of profit revenues or cost (although they can be expressed in terms of a variety of values).

Decision Making Without Probabilities (Uncertainty)

- **Example:** Real Estate Investment

An investor is to purchase one of three types of real estate, as illustrated in Figure 1. The investor must decide among an apartment building, an office building, and a warehouse. The future states of nature that will determine how much profit the investor will make are good economic conditions and poor economic conditions. Table 2 show the profits that will result from each decision in the event of each state of nature.

Figure 1

Table 2

Decision (Purchase)	State of Nature	
	Good economic condition	Poor economic condition
Apartment building	\$50,000	\$30,000
Office building	\$100,000	-\$40,000
Warehouse	\$30,000	\$10,000

Ref: *Introduction to Management Science / Bernard W. Taylor III.—Ed. 11.*

Decision Making Criteria

- Once the decision situation has been organized into a payoff table, several criteria are available for making the actual decision.
- These decision criteria are as follows:
  1. Maximax
  2. Maximin
  3. Minimax regret
  4. Hurwicz
  5. Equal likelihood

Decision Making Criteria - MAXIMAX

- With maximax criterion, the decision maker selects the decision that will result in the maximum of the **maximum payoffs**.
- The maximax criterion is very **optimistic.** The decision maker assumes that the most favourable state of nature for each decision alternative will occur.
- For the example, the decision maker will make a decision to purchase Office building because it has maximum payoff. (Note: If the payoff table consisted of costs, the opposite selection would be indicated.)

Table 2

Decision (Purchase)	State of Nature	
	Good economic condition	Poor economic condition
Apartment building	\$50,000	\$30,000
Office building	\$100,000	-\$40,000
Warehouse	\$30,000	\$10,000

Maximum payoff with optimistically assume that good economic conditions will prevail in the future.

Decision Making Criteria - MAXIMIN

- In contrast with maximax criterion, the maximin criterion is **pessimistic**.
- The decision maker selects the decision that will reflect the maximum of the **minimum payoffs.**
- For each decision alternative, the decision maker assumes that the minimum payoff will occur.

Table 2

Decision (Purchase)	State of Nature	
	Good economic condition	Poor economic condition
Apartment building	$50,000	$30,000
Office building	$100,000	-$40,000
Warehouse	$30,000	$10,000

Maximum payoff among the minimum payoff.

Decision Making Criteria - MINIMAX REGRET

- In our example, suppose the investor decided to purchase the warehouse, only to discover that economic conditions in the future were better than expected.
- Naturally, the investor would be disappointed that she had not purchased the office building because it would have resulted in the largest payoff ($100,000) under good economic conditions.
- In fact, the investor would **regret** the decision to purchase the warehouse, and the **degree of regret** would be $70,000, the difference between the payoff for the investor's choice and the best choice.
- This example demonstrates the principle underlying the decision criterion known as **minimax regret criterion.**
- With this decision criterion, the decision maker attempts to avoid regret by selecting the decision alternative that minimizes the maximum regret.
- To use the minimax regret criterion, a decision maker first selects the maximum payoff under each state of nature, then all other payoffs under the respective states of nature are subtracted from these amounts.

Decision Making Criteria - MINIMAX REGRET

Table 2		
Decision (Purchase)	State of Nature	
	Good economic condition	Poor economic condition
Apartment building	\$100,000 - \$50,000 = \$50,000	\$30,000 - \$30,000 = \$0
Office building	\$100,000 - \$100,000 = \$0	\$30,000 -(-\$40,000) =\$70,000
Warehouse	\$100,000 - \$30,000 = \$70,000	\$30,000 - \$10,000 = \$20,000

- These values represent the regret that the decision maker would experience if a decision were made that resulted in less than the maximum payoff. The values are summarized in a modified version of the payoff table known as a **regret table** (or **opportunity loss** table), shown in Table 3.

Decision Making Criteria - MINIMAX REGRET

Table 3

Decision (Purchase)	State of Nature	
	Good economic condition	Poor economic condition
Apartment building	$50,000	$0
Office building	$0	$70,000
Warehouse	$70,000	$20,000

The minimax regret value.

- To make the decision according to the minimax regret criterion, the maximum regret for **each decision** must be determined.
- The decision corresponding to the minimum of these regret values is then selected.
- For example, if the investor purchased either the office building or the warehouse, $70,000 worth of regret could result; however, the purchase of the apartment building will result in, at most, $50,000 in regret.

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Decision Making Criteria - HURWICZ

- The Hurwicz criterion is a compromise between the maximax and maximin criteria.
- With the Hurwicz criterion, the decision payoffs are weighted by a **coefficient of optimism**, a measure of the decision maker's optimism.
- The coefficient of optimism (α) is between 0 and 1. If α =1.0, then the decision maker is said to be completely optimistic; if α =0, then the decision maker is completely pessimistic.
- The Hurwicz criterion requires that for each decision alternative, the maximum payoff be multiplied by α and the minimum payoff be multiplied by $1-\alpha$.
- For example, if α =0.4 (i.e., the investor is slightly pessimistic), $1-\alpha$ =0.6:

Decision	Values
Apartment building	$50,000(0.4) + 30,000(0.6) = $38,000
Office building	$100,000(0.4) - 40,000(0.6) = $16,000
Warehouse	$30,000(0.4) + 10,000(0.6) = $18,000

The maximum weighted value.

  - The Hurwicz criterion specifies selection of the decision alternative corresponding to the maximum weighted value, which is $38,000 for this example. Thus, the decision would be to purchase the apartment building.

Decision Making Criteria - HURWICZ

- It should be pointed out that when the Hurwicz criterion is actually the maximin criterion; when a = 1.0, it is the maximax criterion.

- A limitation of the Hurwicz criterion is the fact that a must be determined by the decision maker.

- This can be quite difficult for a decision maker to accurately determine his or her degree of optimism.

- Regardless of how the decision maker determines, it is still a completely **subjective** measure of the decision maker’s degree of optimism. Therefore, the Hurwicz criterion is a completely subjective decision-making criterion.

Decision Making Criteria - EQUAL LIKELIHOOD

- When the maximax criterion is applied to a decision situation, the decision maker implicitly assumes that the most favourable state of nature for each decision will occur.
- Alternatively, when the maximin criterion is applied, the least favourable states of nature are assumed.
- The **equal likelihood** or **LaPlace**, criterion weights each state of nature equally, thus assuming that the states of nature are equally likely to occur.
- For the example, because there are two states of nature, we assign a weight of 0.50 to each one.

Decision	Values
Apartment building	\$50,000(0.5) + 30,000(0.5) = \$40,000
Office building	\$100,000(0.5) - 40,000(0.5) = \$30,000
Warehouse	\$30,000(0.5) + 10,000(0.5) = \$20,000

The maximum weighted value.

- As with the Hurwicz criterion, we select the decision that has the maximum of these weighted values. Because \$40,000 is the highest weighted value, the investor's decision would be to purchase the apartment building.

Decision Making Criteria - Summary

- The decisions indicated by the decision criteria examined so far can be summarized as follows:

Criterion	Decision
Maximax	Office building
Maximin	Apartment building
Minimax Regret	Apartment building
Hurwicz	Apartment building
Equal Likelihood	Apartment building

- The decision to purchase the apartment building was designated most often by the various decision criteria.
- Decision to purchase the warehouse was never indicated by any criterion because the payoffs for an apartment building, under either set of future economic conditions, are always better than the payoffs for a warehouse. Thus, the warehouse decision alternative could have been eliminated from consideration under each of our criteria.
- The alternative of purchasing a warehouse is said to be dominated by the alternative of purchasing an apartment building.
- A **dominant decision** is one that has a better payoff than another decision under each state of nature.

Decision Making Criteria - Summary

- A **dominant decision** is one that has a better payoff than another decision under each state of nature.

- The use of several decision criteria often results in a mix of decisions, with no one decision being selected more than the others.

- The criterion or collection of criteria used, and the resulting decision depend on the characteristics and philosophy of the decision maker.

- For example, the extremely optimistic decision maker might eschew most of the foregoing results and make the decision to purchase the office building because the maximax criterion most closely reflects his or her personal decision-making philosophy.

Decision Making Criteria - Excel

Part 1:

	A	B	C	D	E	F	G
1		Real Estate Investment Example					
2							
3		**Decision**	**State of Nature**				
4		**(Purchase)**	Good economic condition	Poor economic condition	**Maximum**	**Minimum**	
5		Apartment building	50,000.00	30,000.00	50,000.00	30,000.00	
6		Office building	100,000.00	-40,000.00	100,000.00	-40,000.00	
7		Warehouse	30,000.00	10,000.00	30,000.00	10,000.00	
8				(Formula)	[=MAX(C5,C7)]	[=MIN(C5,D5)]	
9							
10		Decision Criterion:			(Formula)		
11		**Maximax**	100,000.00	Office building	[=MAX(E5,E7)]		
12		**Maximin**	30,000.00	Apartment building	[=MAX(F5,F7)]		
13							

=IF(C11=C6,B6,(IF(C11=C5,B5),B7))

Decision Making Criteria - Excel

Part 2:

	A	B	C	D	E	F
13						
14		**REGRET TABLE**				
15		**Decision**	**State of Nature**			
16		**(Purchase)**	Good economic condition	Poor economic condition	**Maximum**	(Formula)
17		Apartment building	50,000.00	0.00	50,000.00	[=MAX(C16, D16)]
18		Office building	0.00	70,000.00	70,000.00	[=MAX(C17, D17)]
19		Warehouse	70,000.00	20,000.00	70,000.00	[=MAX(C18, D18)]
20						
21		Decision Criterion:				
22		**Minimax regret**	50,000.00	Apartment building		
23						
24						
25		**HURWICZ**		(Formula)		
26		Aplha	0.4	(insert value)		
27		1- Alpha	0.6	[=1 - alpha]		
28		Apartment building	38000	[=(C5*C25) + (D5*C26)]		
29		Office building	16000	[=(C6*C25) + (D6*C26)]		
30		Warehouse	18000	[=(C7*C25) + (D7*C26)]		
31						
32		Decision Criterion:				
33		**Hurwicz**	38,000.00	Apartment building		

Decision Making Criteria - Excel

Part 3:

	A	B	C	D
34				
35		**EQUAL LIKELIHOOD**		(Formula)
36		Apartment building	40000	[=(C5*0.5) + (D5*0.5)]
37		Office building	30000	[=(C6*0.5) + (D6*0.5)]
38		Warehouse	20000	[=(C7*0.5) + (D7*0.5)]
39				
40		Decision Criterion:		
41		**Equal Likelihood**	40,000.00	Apartment building
42				

=MAX(C36:C38)

Exercise #1

Mewah, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants, located in South Carolina. Three alternative decisions are being considered:

(1) expand the plant and produce lightweight, durable materials for possible sales to the military, a market with little foreign competition;

(2) maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition; or

(3) sell the plant now.

If one of the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends on foreign market conditions, including the status of a trade embargo bill in Congress. The following payoff table describes this decision situation:

	State of Nature	
Decision	*Good Foreign Competitive Conditions*	*Poor Foreign Competitive Conditions*
Expand	$ 800,000	$ 500,000
Maintain status quo	1,300,000	−150,000
Sell now	320,000	320,000

Determine the best decision by using the following decision criteria:

1. Maximax
2. Maximin
3. Minimax regret
4. Hurwicz $(\alpha = 0.3)$
5. Equal likelihood

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Exercise #2

Basyir has inherited RM1000. He has to decide how to invest the money for one year. A broker has suggested five potential investments as follows:

1. Gold
2. Junk Bond
3. Growth Stock
4. Certificate of Deposit
5. Stock Option Hedge

The payoff table as shown in Table 4.

Table 4

Decision Alternatives	States of Nature				
	Large Rise	Small Rise	No Change	Small Fall	Large Fall
Gold	-100	100	200	300	0
Bond	250	200	150	-100	-150
Stock	500	250	100	-200	-600
C/D account	60	60	60	60	60
Stock option	200	150	150	-200	-150

If Basyir knows nothing about probabilities of the state of nature, what is the recommended decision using the Maximax, Maximin, Minimax regret, Hurwicz and Equal likelihood approaches?

Decision Making Criteria With Probabilities
(Under Risk)

- In previous topic, the decision criteria assumed that no information regarding the likelihood of the states of nature was available. Thus, no probabilities of occurrence were assigned to the states of nature, except in the case of the equal likelihood criterion.

- It is often possible for the decision maker to know enough about the future states of nature to assign probabilities to their occurrence.

- Given that probabilities can be assigned, several decision criteria are available to aid the decision maker.

- We will consider two of these criteria:
    1. Expected value (EV)
    2. Expected opportunity loss (EOL)

(Note: Although several others, including the maximum likelihood criterion, are available).

Decision Making Criteria With Probabilities: Expected Value

- To apply the concept of expected value as a decision-making criterion, the decision maker must first estimate the probability of occurrence of each state of nature.

- The expected value is computed by multiplying each outcome (of a decision) by the probability of its occurrence and then summing these products.

$$EV(x) = \sum_{i=1}^{n} x_i P(x_i)$$

where

n = number of values of the random variable x

Decision Making Criteria With Probabilities: Expected Value

- Example - For the real estate investment, suppose based on several economic forecast, the investor is able to estimate 0.60 probability that good economic conditions will prevail and 0.40 probability that poor economic conditions will prevail. This new information is shown in Table 5.

Table 5

Decision (Purchase)	State of Nature	
	Good economic condition (0.60)	Poor economic condition (0.40)
Apartment building	\$50,000	\$30,000
Office building	\$100,000	-\$40,000
Warehouse	\$30,000	\$10,000

- Then, the EV for each decision is computed as follows:

EV (Apartment) = \$50,000 (0.6) + \$30,000 (0.40) = \$42,000
EV (Office) = \$100,000 (0.6) + (-\$40,000 (0.40)) = \$44,000
EV (Warehouse) = \$30,000 (0.6) + \$10,000 (0.40) = \$22,000

Decision Making Criteria With Probabilities: Expected Value

- Then, the EV for each decision is computed as follows:

  EV (Apartment) = \$50,000 (0.6) + \$30,000 (0.4) = \$42,000
  EV (Office) = \$100,000 (0.6) + (-\$40,000 (0.4)) = \$44,000
  EV (Warehouse) = \$30,000 (0.6) + \$10,000 (0.4) = \$22,000

- The best decision is the one with greatest EV. In this case is to purchase the office building.

- This does not mean that \$44,000 will result if the investor purchases the office building; rather, it is assumed that one of the payoff values will result (either \$100,000 or -\$40,000).

- The expected value means that if this decision situation occurred many times, an average payoff of \$44,000 would result.

- Alternatively, if the payoffs were in terms of costs, the best decision would be the one with the lowest expected value.

Decision Making Criteria With Probabilities: Expected Opportunity Loss

- To use this criterion, we multiply the probabilities by the regret (i.e., opportunity loss) for each decision outcome rather than multiplying the decision outcomes by the probabilities of their occurrence, as we did for expected monetary value.

- The concept of regret was introduced in our discussion of the minimax regret criterion. The regret value found for each decision outcome will be repeated with the addition of the probabilities of occurrence for each state of nature, as shown in Table 6.

Table 6

Decision (Purchase)	State of Nature	
	Good economic condition (0.6)	Poor economic condition (0.4)
Apartment building	\$50,000	\$0
Office building	\$0	\$70,000
Warehouse	\$70,000	\$20,000

Decision Making Criteria With Probabilities: Expected Opportunity Loss

- Then, the EOL for each decision is computed as follows:

  EOL (Apartment) = \$50,000 (0.6) + \$0 (0.4) = \$30,000
  EOL (Office) = \$0 (0.6) + \$70,000 (0.4)) = \$28,000
  EOL (Warehouse) = \$70,000 (0.6) + \$20,000 (0.4) = \$50,000

- As with the minimax regret criterion, the best decision results from minimizing the regret, or, in this case, minimizing the expected regret or opportunity loss. Because \$28,000 is the minimum expected regret, the decision is to purchase the office building.

- Notice that the decisions recommended by the expected value and expected opportunity loss criteria were the same—to purchase the office building.

- This is not a coincidence because these two methods always result in the same decision.

- Thus, it is repetitious to apply both methods to a decision situation when one of the two will suffice.

Decision Making Criteria With Probabilities:
Expected Opportunity Loss - Excel

	A	B	C	D	E
1		Real Estate Investment Example			
2					
3		**Expected Value**			
4		**Decision**	**State of Nature**		
5		**(Purchase)**	Good economic condition	Poor economic condition	
6		**Probability**	0.6	0.4	**Expected Value**
7		Apartment building	50,000.00	30,000.00	42,000.00
8		Office building	100,000.00	-40,000.00	44,000.00
9		Warehouse	30,000.00	10,000.00	22,000.00
10					
11		Decision Criterion:	44,000.00	Office building	
12					
13					
14		**Expected Opportunity Loss**			
15		**Decision**	**State of Nature**		
16		**(Purchase)**	Good economic condition	Poor economic condition	
17		**Probability**	0.6	0.4	**Expected Value**
18		Apartment building	50,000.00	0.00	30,000.00
19		Office building	0.00	70,000.00	28,000.00
20		Warehouse	70,000.00	20,000.00	50,000.00
21					
22		Decision Criterion:	28,000.00	Office building	
23					

=(C7*C6) + (D7*D6)

=MAX(E7:E9)

=(C18*C6) + (D18*D6)

=MIN(E18:E20)

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Exercise # 3

Refer to Exercise # 1, assume that it is now possible to estimate a probability of 0.70 that good foreign competitive conditions will exist and a probability of 0.30 that poor conditions will exist. Determine the best decision by using expected value and expected opportunity loss.

	State of Nature	
Decision	*Good Foreign Competitive Conditions*	*Poor Foreign Competitive Conditions*
Expand	$ 800,000	$ 500,000
Maintain status quo	1,300,000	−150,000
Sell now	320,000	320,000

Decision Making Criteria Under Certainty: Expected Value of Perfect Information

- It is often possible to purchase additional information regarding future events and thus make a better decision.

- For example, a real estate investor could hire an economic forecaster to perform an analysis of the economy to more accurately determine which economic condition will occur in the future.

- However, the investor (or any decision maker) would be foolish to pay more for this information than he or she stands to gain in extra profit from having the information.

- That is, the information has some maximum value that represents the limit of what the decision maker would be willing to spend.

- This value of information can be computed as an expected value - hence its name, the expected value of perfect information (also referred to as **EVPI**)

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Decision Making Criteria Under Certainty: Expected Value of Perfect Information

- It is often possible to purchase additional information regarding future events and thus make a better decision.
- For example, a real estate investor could hire an economic forecaster to perform an analysis of the economy to more accurately determine which economic condition will occur in the future.
- However, the investor (or any decision maker) would be foolish to pay more for this information than he or she stands to gain in extra profit from having the information.
- That is, the information has some maximum value that represents the limit of what the decision maker would be willing to spend.
- This value of information can be computed as an expected value - hence its name, the expected value of perfect information (also referred to as **EVPI**).
- The EVPI is the maximum amount a decision maker would pay for additional information.

Decision Making Criteria Under Certainty: Expected Value of Perfect Information

- To compute the expected value of perfect information, we first look at the decisions under each state of nature.
- If we could obtain information that assured us which state of nature was going to occur (i.e., perfect information), we could select the best decision for that state of nature.
- Table 7 show the payoff table with decisions, given perfect information.

Table 7

Decision (Purchase)	State of Nature	
	Good economic condition (0.6)	Poor economic condition (0.4)
Apartment building	$50,000	$30,000
Office building	$100,000	-$40,000
Warehouse	$30,000	$10,000

Decision Making Criteria Under Certainty: Expected Value of Perfect Information

- Even though perfect information enables the investor to make the right decision, each state of nature will occur only a certain portion of the time. Thus, each of the decision outcomes obtained using perfect information must be weighted by its respective probability:

$$\$100{,}000\ (0.6) + \$30{,}000\ (0.40)) = \$72{,}000$$

- The amount $72,000 is the expected value of the decision, given perfect information, not the expected value of perfect information. The expected value of perfect information is the maximum amount that would be paid to gain information that would result in a decision better than the one made without perfect information.

- Recall that the expected value decision without perfect information was to purchase an office building, and the expected value was computed as:

$$EV(\text{Office}) = \$100{,}000\ (0.6) + (-\$40{,}000\ (0.40)) = \$44{,}000$$

Decision Making Criteria Under Certainty: Expected Value of Perfect Information

- The expected value of perfect information is computed by subtracting the expected value without perfect information ($44,000) from the expected value given perfect information ($72,000):

$$\text{EVPI} = \$72{,}000 - \$44{,}000 = \$28{,}000$$

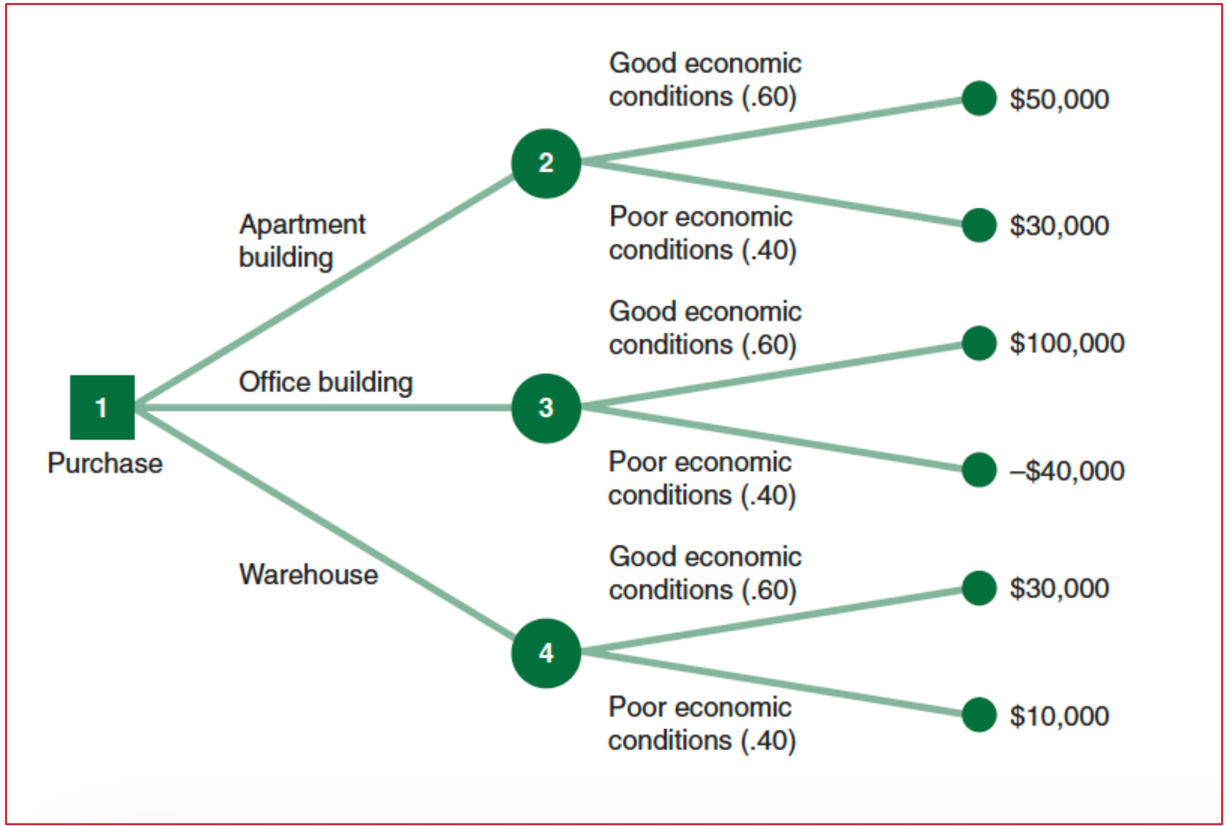
- The expected value of perfect information, $28,000, is the maximum amount that the investor would pay to purchase perfect information from some other source, such as an economic forecaster.

Decision Tree

- Another useful technique for analyzing a decision situation is using a decision tree.
- A decision tree is a diagram consisting of square decision nodes, circle probability nodes, and branches representing decision alternatives.
- In a decision tree, the user computes the expected value of each outcome and makes a decision based on these expected values.
- The primary benefit of a decision tree is that it provides an illustration (or picture) of the decision-making process. This makes it easier to correctly compute the necessary expected values and to understand the process of making the decision.

Decision Tree

Example: The decision tree represents the sequence of events in a decision situation. First, one of the three decision choices is selected at node 1. Depending on the branch selected, the decision maker arrives at probability node 2, 3, or 4, where one of the states of nature will prevail, resulting in one of six possible payoffs.



Decision Tree

Determining the best decision by using a decision tree involves computing the expected value at each probability node. This is accomplished by starting with the final outcomes (payoffs) and working backward through the decision tree toward node 1. Select the branch that comes from the probability node with the highest expected payoff.