MCSD1133 Operations Research & Optimization

UTM
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CHAPTER 1: INTRODUCTION TO OPERATIONS RESEARCH (OR)

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Innovating Solutions

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INTRODUCTION TO OPERATIONS RESEARCH

OR is a multidisciplinary field that uses mathematical and analytical methods to solve complex decision-making problems in various sectors and industries.

OR aims to optimize the use of resources, improve processes, and make better decisions to achieve specific objectives.

Key Aspect of OR:

OBJECTIVE

- The primary goal of OR is to find the **best possible solution** or decision among various alternatives.
- It often involves balancing conflicting objectives, such as minimizing costs while maximizing efficiency or profit.

Key Aspect of OR:

HISTORY

- OR emerged during World War II when scientists and mathematicians were tasked with solving military logistics and planning problems.
- Since then, it has evolved and expanded to address a broad spectrum of decision-making challenges in both the public and private sectors.

Key Aspect of OR:

PROBLEM FORMULATION

- A critical initial step in OR is defining the problem.
- This involves identifying the decision variables, constraints, and objectives.
- Proper problem formulation is essential for finding effective solutions.

Key Aspect of OR:

DECISION SUPPORT

- OR provides decision support tools and techniques for organizations and individuals to make informed decisions.

- It helps in choosing the most optimal course of action under uncertainty.

- Solution requires answering three questions:

  1. What are the decision alternatives?
  2. What is an appropriate objective criterion for evaluating the alternatives?
  3. Under what restrictions is the decision made?

Key Aspect of OR:

SCIENTIFIC APPROACH

- OR applies scientific methods, techniques and tools for the purpose of analysis and solution of the complex problems.
- In this approach there is no place for guesswork and the person bias of the decision maker.

Key Aspect of OR:

SYSTEM APPROACH

- The main aim of the system approach is to trace out all significant and indirect effects for each proposal on all sub-system on a system and to evaluate each action in terms of effects for the system as a whole.

- The inter-relationship and interaction of each sub-system can be handled with the help of mathematical or analytical models of OR to obtain acceptable solution.

Key Aspect of OR:

INTERDISCIPLINARY APPROACH

- OR draws from various disciplines, including mathematics, statistics, computer science, economics, engineering, and management science.
- This interdisciplinary approach allows it to address a wide range of real-world problems.

Key Aspect of OR:

MATHEMATICAL MODELLING

- OR often uses mathematical models to represent real-world systems and problems.

- These models can be linear or nonlinear, deterministic or stochastic, and can take various forms, including linear programming, dynamic programming, and simulation.

Key Aspect of OR:

OPTIMIZATION

- Optimization is a central theme in OR.
- It involves finding the best solution within the defined constraints.
- This can include minimizing costs, maximizing profits, optimizing resource allocation, or other objectives.

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Key Aspect of OR:

USE OF COMPUTERS

- The models of OR need lot of computation and therefore, the use of computers becomes necessary.
- With the use of computer, it is possible to handle complex problems requiring large amounts of calculations.
- The development of increasingly powerful personal computers accompanied by good software packages for doing OR.

Key Aspect of OR:

APPLICATION

- OR is applied in diverse fields, including logistics, supply chain management, transportation, healthcare, finance, manufacturing, energy, and military operations.

- It can be used to optimize production schedules, manage inventory, design efficient transportation networks, and improve healthcare resource allocation, among other applications.

Key Aspect of OR:

CHALLENGE

- OR can be challenging due to the complexity of real-world problems, the need for accurate data, and the requirement to balance multiple objectives. It also involves a fair amount of modelling and computation.
- In deriving a Quantitative Solutions, we do not consider human factors, which doubtlessly play a great role in the problems.
- However, study of OR is incomplete without study of human factors.

Key Aspect of OR:

BENEFIT

The benefits of OR include cost reduction, improved efficiency, better decision-making, increased competitiveness, and enhanced resource allocation.

METHODOLOGY OF OPERATIONS RESEARCH

1. PROBLEM DEFINITION
2. MODEL FORMULATION
3. MODEL SOLUTION
4. MODEL VALIDATION
5. IMPLEMENTATION

PROBLEM DEFINITION

Involves delineating the scope of the problem under investigation. This function should be carried out by the entire OR team.

The aim is to identify **three principal** elements of the decision problem:

1. Description of the decision alternatives.
2. Determination of the objective of the study.
3. Specification of the limitations under which the modelled system operates.

MODEL FORMULATION

Construct a **mathematical model** that represents the essence of the problem. Mathematical model of a business problem is the system of equations and related mathematical expressions.

If there are *n* related quantifiable decisions to be made, they are represented as **decision variables.**

The appropriate measure of performance (e.g., profit) is then expressed as a mathematical function of these decision variables called as **objective function.**

Any restrictions on the values that can be assigned to the decision variables called **constraints.**

If the mathematical relationships are too complex to allow the determination of an analytic solution, the OR team may opt to simplify the model and use a **heuristic approach**, or the team may consider the use of **simulation**, if appropriate.

In some cases, mathematical, simulation, and heuristic models may be **combined** to solve the decision problem, as some of the end-of-chapter case analyses demonstrate.

Basic Component of OR Model

- **Decision variables** that we seek to determine
- **Objective Function** that we need to optimize (maximize or minimize).
- **Constraints** that the solution must satisfy.

Example 1

A company manufactures two products A and B, with 4 and 3 units of price. A and B take 3 and 2 minutes respectively to be machined. The total time available at machining department is 800 hours (100 days or 20 weeks). A market research showed that at least 10000 units of A and not more than 6000 units of B are needed. It is required to determine the number of units of A and B to be produced to maximize profit.

Example 1 - Solution

- **Decision variables**

x_1= number of units produced of A.

x_2 = number of units produced of B.

- **Objective Function**

Maximize, $Z = 4x_1 + 3x_2$

- **Constraints**

$3x_1 + 2x_2 \leq (800 \times 60\ mins)$
$x_1 \geq 10000$
$x_2 \leq 6000$
$x_1, x_2 \geq 0$

Example 2

A farmer is interested in feeding his cattle at minimum cost. Two feeds are used A and B. Each cow must get at least 400 grams/day of protein, at least 800 grams/day of carbohydrates, and not more than 100 grams/day of fat. Given that A contains 10% protein, 80% carbohydrates and 10% fat while B contains 40% protein, 60% carbohydrates and no fat. A costs RM 20/kg, and B costs RM 50 /kg. Formulate the problem to determine the optimum amount of each feed to minimize cost.

Example 2 - Solution

- **Decision variables**

x_1= weight of feed A kg/day/animal

x_2= weight of feed B kg/day/animal

Objective Function

Maximize $Z = 20x_1 + 50x_2$

- **Constraints**

$0.1x_1 + 0.4x_2 \geq 0.4$

$0.8x_1 + 0.6x_2 \geq 0.8$

$0.1x_1 \leq 0.1$

$x_1 , x_2 \geq 0$

MODEL SOLUTION

Develop a procedure (usually a computer-based procedure) for deriving solutions to the problem from the mathematical model.

OR is applied on a computer by using one of several readily available software packages.

Search for an optimal, or best solution with respect to the model being used.

Conducting sensitivity analysis to determine behavior of the optimum solution when the model undergoes some parameter changes. .

MODEL VALIDATION

Process of testing and improving a model to increase its validity.

Checks whether the proposed model does what it purports to do—that is, does it adequately predict the behaviour of the system under study?

Retrospective test-using historical data to reconstruct the past and then determining how well the model and the resulting solution would have performed if they had been used. The model is valid if, under similar input conditions, it reasonably duplicates past performance.

MODEL IMPLEMENTATION

The translation of the results into **understandable operating instructions** to be issued to the people who will administer the recommended system.

Depends a great deal upon the **support** of both top management and operating management.

Good communications help to ensure that the study accomplishes what management wanted and give management a greater sense of ownership of the study, which encourages their support for implementation.

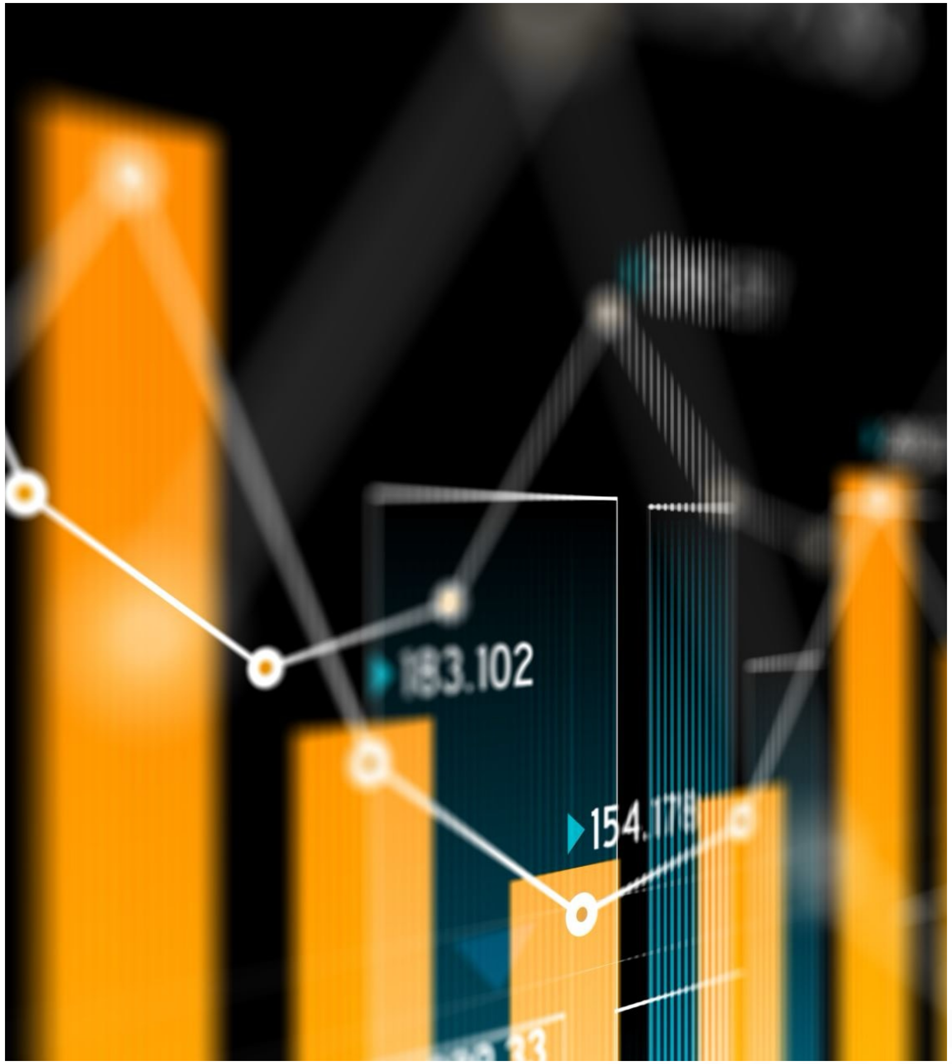
It is important to continue to obtain **feedback** on how well the new system is working and whether the assumptions of the model continue to be satisfied.

TECHNIQUES @ TOOLS OF OPERATIONS RESEARCH

- Linear Programming
- Integer Programming- Transportation Problems, Assignment Problems
- Nonlinear Programming
- Queuing Theory
- Sequencing
- Decision Theory and Games Theory

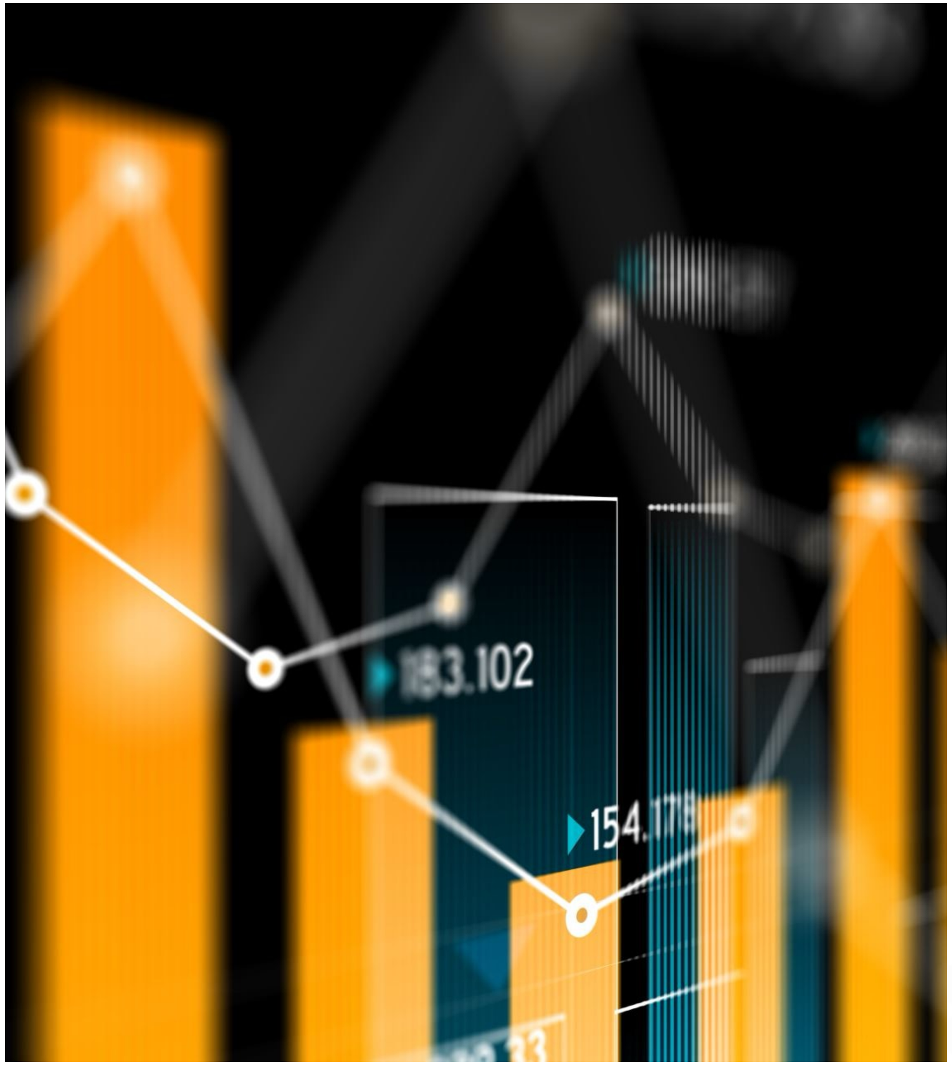
LINEAR PROGRAMMING

- This technique is used to find a solution for optimizing given objective.
- Objective may be maximizing profits or minimizing costs.
- Objective function and Boundary conditions are linear in nature.
- Solve Product-Mix and Distribution problems of enterprise. Its also used to allocate Scarce Resources in optimum manner in problems of scheduling, product mix, etc.



INTEGER PROGRAMMING

- Integer means complete or whole number. The variable assume integer values.
- By using the Integer Programming Algorithm, a series of continuous linear programming problem are solved in such a way that the solution containing unacceptable noninteger value are ruled out and the best higher programming solution is obtained.



TRANSPORTATION PROBLEMS

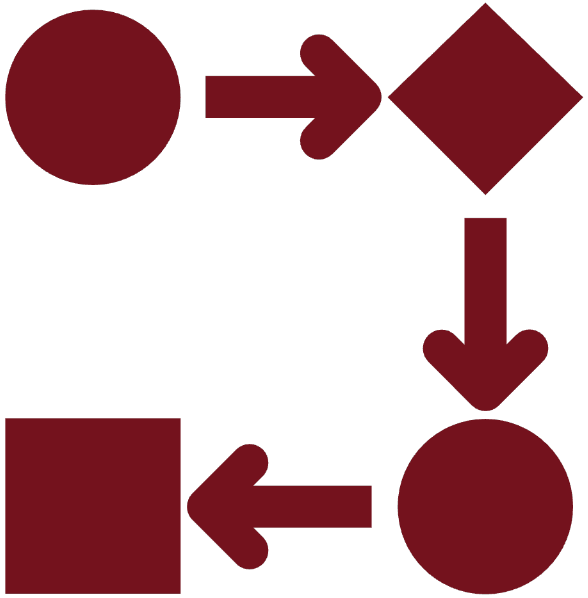
- Transportation problems deals with transportation of a product:
  - From several sources
  - With limited supplies
  - To number of destinations
  - With specified demands
  - At the total transportation cost.
- The main objective of transportation is to schedule shipment from sources to destinations in such a way to minimize the total transportation cost.

ASSIGNMENT PROBLEMS

- It deals in allocating the various resources or items to various activities in a one-to-one basis in such a way that the time or cost involved in minimized and the sale or profit is maximized.
- Example: Manager may like to know which job should be assigned to which person so that all jobs can be completed in the shortest possible time.

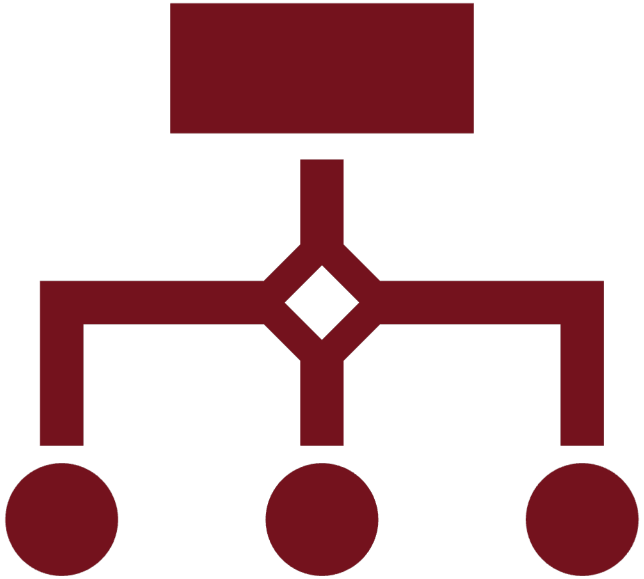
QUEUING THEORY

- This theory deals with the situations in which queue is formed, e.g., customers waiting for services, machines waiting for repairmen, and aircrafts waiting for landing strips, etc.
- If the Queue will be long the cost will be high due to long waiting hour.
- This technique is used to analyze the feasibility of adding facilities and to access the amount and cost of waiting time.
- This calculations can then be used to determine the desirable number of service facilities.



SEQUENCING

- Models have been developed to find a sequence for Processing Jobs so that the total elapsed time for all the jobs will be minimum.
- The models also help to resolve the conflict between the objectives of maximizing machines utilization and complying with predetermined delivering rates.



DECISION THEORY AND GAME THEORY

- Decision Theory is primarily considered with decision making under the conditions of:
  - Risk
  - Uncertainty

- Game Theory is concerned with:
  - Decision Making under conflict

- Hence, both Decision Theory and Game Theory assist the Decision-Maker in analyzing problems with numerous alternative course of action and consequences.