

# Measurement of higher moments of transverse momentum of particles produced in proton-proton collisions

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A review of “Skewness of mean transverse momentum fluctuations in heavy-ion collisions” [1] for p-p collisions at centre-of-mass energy 13 TeV is presented. Corresponding transverse momentum and mean transverse momentum distributions of particles produced have been plotted for 6 multiplicity classes. Mean transverse momentum distributions are compared against a Gaussian distribution and corresponding intrinsic and standardised skewness plotted against multiplicities using the STAR method. Positive skew is observed for mean transverse momentum distributions as concluded in the paper.

## I. INTRODUCTION

Event generator : Pythia 8 Monte Carlo event generator.  
Number of events : 2 million  
Collisions System : p + p at centre-of-mass energy 13 TeV.

The following variables will be used throughout the text.

$p_i$  : Transverse momentum of  $i$ th particle  
 $N_{ch}$  : The number of particles in an even  
 $\langle p_T \rangle$  : The mean transverse momentum of emitted particles for a given collision centrality  
 $\langle \langle p_T \rangle \rangle$  : Average of  $\langle p_T \rangle$  over events in a centrality class  
 $\langle \Delta p_i \Delta p_j \rangle$  : Variance of dynamical  $p_T$  fluctuations  
 $\langle \Delta p_i \Delta p_j \Delta p_k \rangle$  : The skewness or the third centered moment

p-p collisions produce various particles depending on the energy of colliding particles. Following is a geometrical analysis of a typical particle collision:

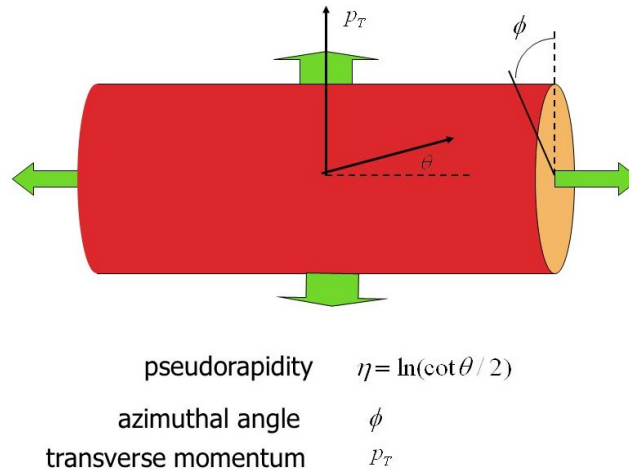


FIG. 1: Collision geometry

The paper proposes presence of positive skew for the  $\langle p_T \rangle$  distributions for heavy ion collisions. We provide an analysis for the same for p-p collisions. We study the  $p_T$  and  $\langle p_T \rangle$  distributions where  $\langle p_T \rangle$  is computed as,

$$\langle p_T \rangle = \sum_{i=1}^{N_{ch}} \frac{p_i}{N_{ch}} \quad (1)$$

and the STAR definition of  $\langle \langle p_T \rangle \rangle$  is used as:

$$\langle \langle p_T \rangle \rangle = \left\langle \sum_{i=1}^{N_{ch}} \frac{p_i}{N_{ch}} \right\rangle \quad (2)$$

Skewness depends on standardised variance. Hence, to derive an expression for the same, the following expression is used:

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} = \left\langle \frac{\sum_{i,j,i \neq j} (p_i - \langle \langle p_T \rangle \rangle)(p_j - \langle \langle p_T \rangle \rangle)}{N_{ch}(N_{ch} - 1)} \right\rangle \quad (3)$$

standardised variance then becomes

$$\frac{\sqrt{\langle \Delta p_i \Delta p_j \rangle}}{\langle p_T \rangle} \quad (4)$$

Taking:

$$Q_n = \sum_{i=1}^{N_{ch}} (p_i)^n \quad (5)$$

To compute the intensive ( $\gamma_{p_t}$ ) and standardised ( $\Gamma_{p_t}$ ) skewness the expressions for the same are:

$$\gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \quad (6)$$

$$\Gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle \langle p_T \rangle \rangle}{\langle \Delta p_i \Delta p_j \rangle^2} \quad (7)$$

where

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} = \left\langle \frac{Q_1^2 - Q_2}{N_{ch}(N_{ch} - 1)} \right\rangle - \left\langle \frac{Q_1}{N_{ch}} \right\rangle^2 \quad (8)$$

and

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle_{STAR} = \left\langle \frac{Q_1^3 - 3Q_2Q_1 + 2Q_3}{N_{ch}(N_{ch} - 1)(N_{ch} - 2)} \right\rangle - 3 \left\langle \frac{Q_1^2 - Q_2}{N_{ch}(N_{ch} - 1)} \right\rangle \left\langle \frac{Q_1}{N_{ch}} \right\rangle + 2 \left\langle \frac{Q_1}{N_{ch}} \right\rangle^3 \quad (9)$$

## II. EXPERIMENTAL OBSERVATIONS

### A. $p_T$ distributions

The following are plots of distribution of transverse momentum in logarithmic scale and have been fitted against an Exponential plot (in red). The transverse momentum values used were in  $GeV/c$ .

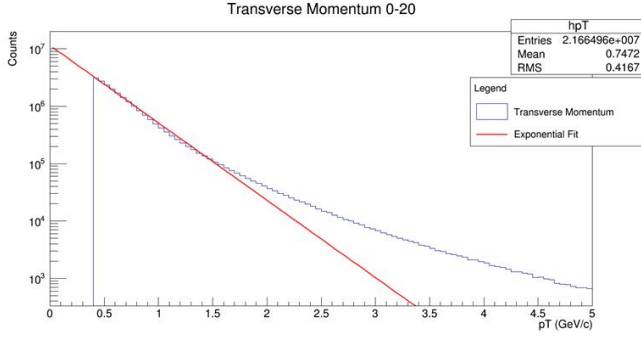


FIG. 2: Distribution of  $p_T$  for Multiplicity Class 0-20

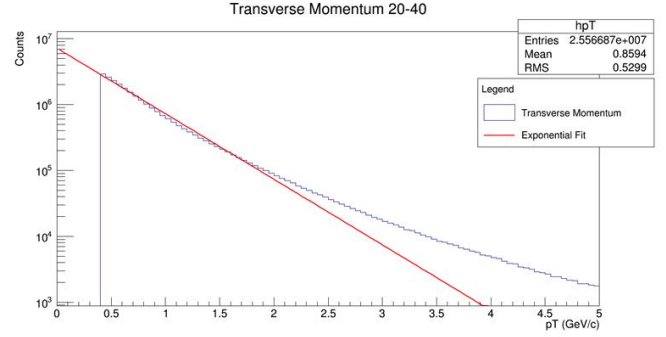


FIG. 3: Distribution of  $p_T$  for Multiplicity Class 20-40

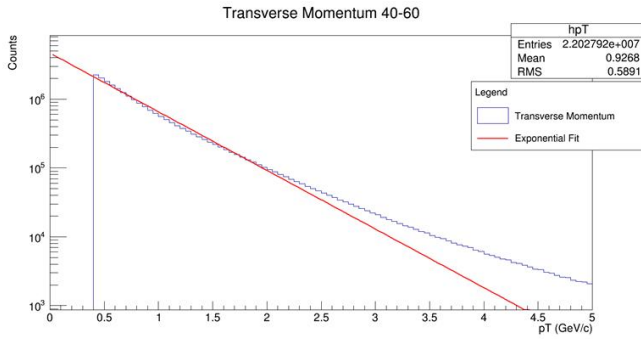


FIG. 4: Distribution of  $p_T$  for Multiplicity Class 40-60

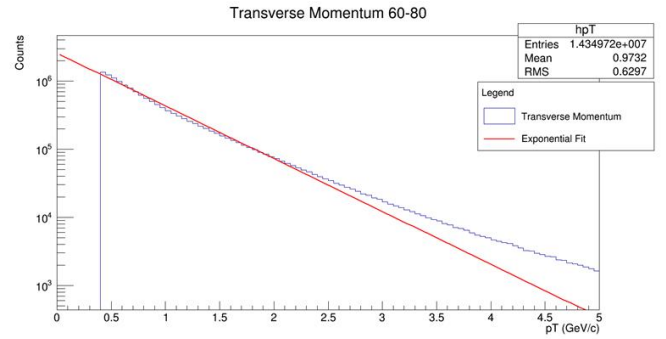


FIG. 5: Distribution of  $p_T$  for Multiplicity Class 60-80

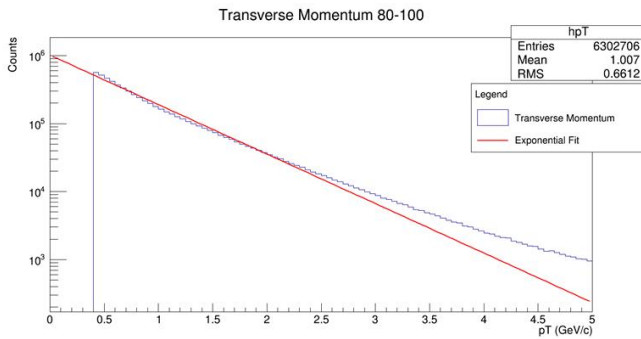


FIG. 6: Distribution of  $p_T$  for Multiplicity Class 80-100

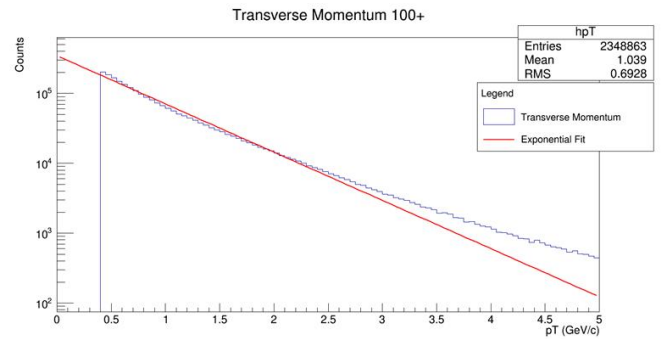


FIG. 7: Distribution of  $p_T$  for Multiplicity Class 100+

## B. $\langle p_T \rangle$ distributions

The following are plots of averaged transverse momentum in logarithmic scale and have been fitted against a Gaussian curve (in red). The mean transverse momentum values used were in  $GeV/c$ .

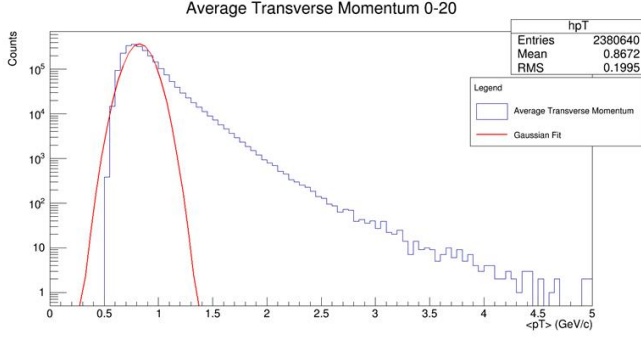


FIG. 8: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 0-20

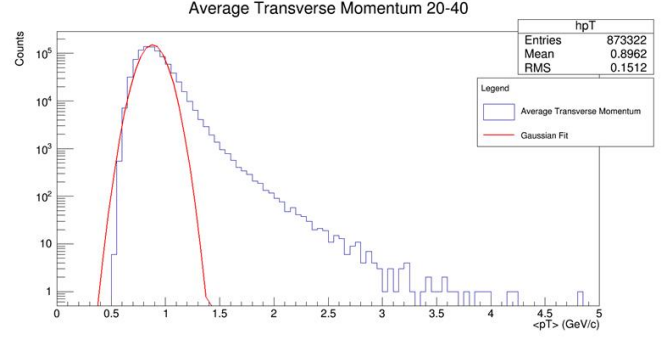


FIG. 9: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 20-40

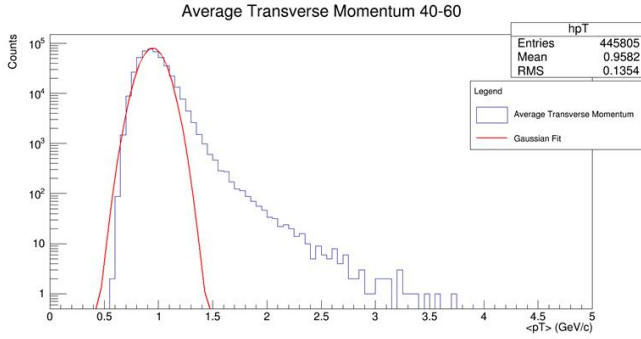


FIG. 10: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 40-60

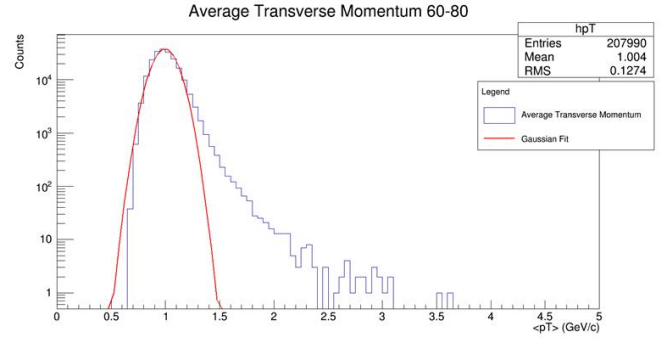


FIG. 11: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 60-80

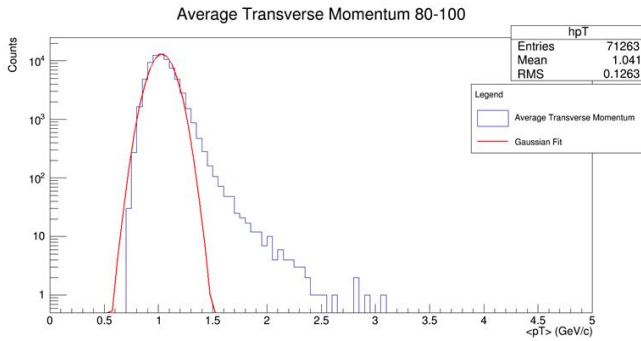


FIG. 12: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 80-100

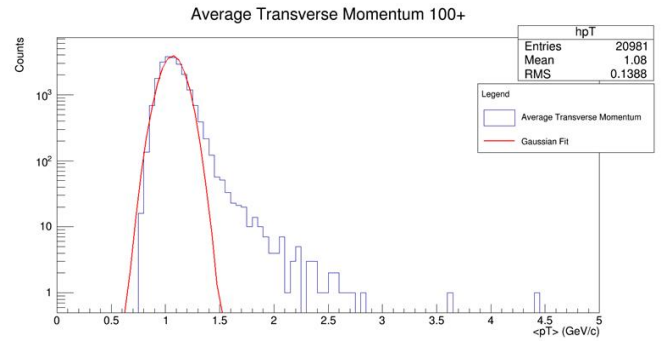


FIG. 13: Distribution of  $\langle p_T \rangle$  for Multiplicity Class 100+

### C. Plot for $\langle\langle p_T \rangle\rangle$

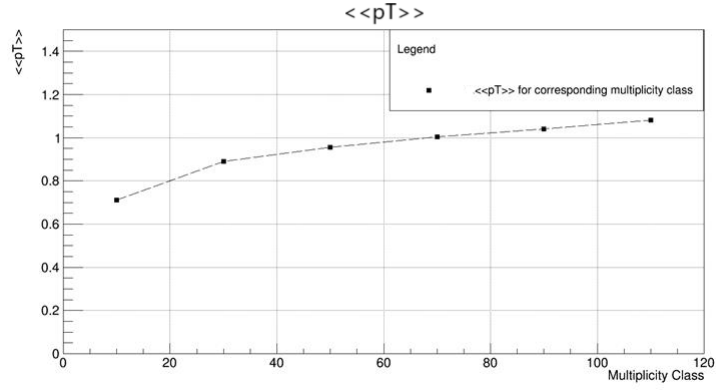


FIG. 14: Variation in  $\langle\langle p_T \rangle\rangle$  across different Multiplicity Classes

### D. Central Moments of $\langle p_T \rangle$

Using Eq (4), (6), (7) the graphs of Standardised Skewness, Intensive Skewness and Standardised Variance are obtained with  $|\eta \leq 2.5|$ .

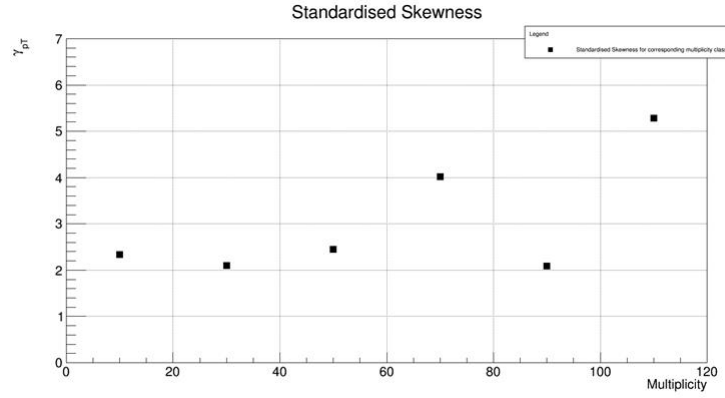


FIG. 15: Variation in Standardised Skewness across different Multiplicity Classes

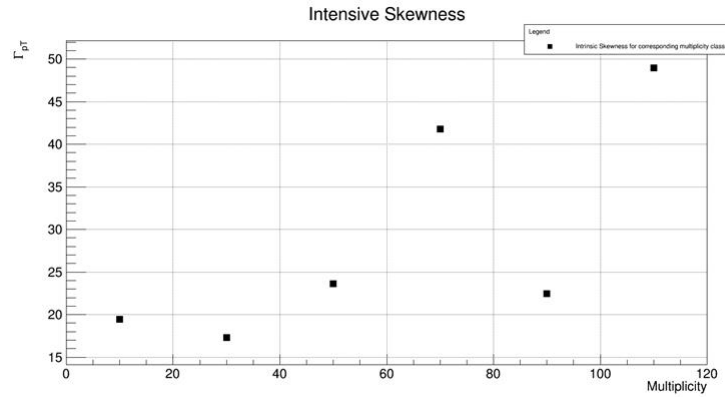


FIG. 16: Variation in Intensive Skewness across different Multiplicity Classes

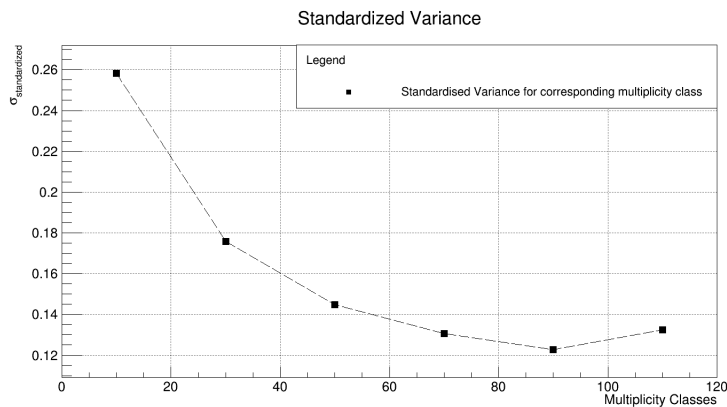


FIG. 17: Variation in Standardised Variance across different Multiplicity Classes

### III. SUMMARY

ROOT Data Analysis Framework by CERN and partner institutions is a great tool for handling and visualising large data files. We started with a study and development of plots ranging from 3-D histograms to line plots; transforming huge lines of data to beautiful results. ROOT Forum and ROOT manual were our constant sources of reference.

1. We understood the p-p collision model and corresponding data storage in TTree files and applied ROOT to extract the data to plots.
2. We produced  $pT$  and  $\langle pT \rangle$  distributions for multiplicity classes of bin size 20 and range 0 to 100 and fit them against Exponential and Gaussian distributions.
3. As the paper proposed observation of positive skewness in  $\langle pT \rangle$  distributions for heavy-ion collisions; we observed similar results for p-p collision data generated with Pythia 8 Monte Carlo event generator in ROOT.
4. We further plot standardised variance v/s multiplicity classes and observed a decrease with an increase in the range of  $N_{ch}$  depicting an inverse relation to the same. The decrease of variance is a result of Central Limit Theorem.
5. Plot for  $\langle \langle pT \rangle \rangle$  v/s multiplicity classes had a steep rise.
6. Standardized and intrinsic skewness plots present a more random picture for the data.

### IV. LINK TO REPOSITORY

[PH-219 ROOT Project Github Repository](#)

### REFERENCES

- [1] Giuliano Giacalone **and others**. *Skewness of mean transverse momentum fluctuations in heavy-ion collisions*. 2020. arXiv: [2004.09799 \[nucl-th\]](#).