RMSMC

Redmond Middle School Math Club

1920B

2019-2020 Benchmark Exam

Friday, October 25, 2019

INSTRUCTIONS

1. DO NOT BEGIN THIS EXAM UNTIL YOUR PROCTOR TELLS YOU.
2. This is a thirty question SHORT ANSWER test. All answers must be recorded in the correct location on the separate answer sheet.
3. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer. Ties will be broken for top placement positions based on the highest numbered question answered correctly. If students are still tied, the process is repeated for the remainder of questions in reverse order. Exact ties will be broken at the sole discretion of the Math Club chair.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Units are not necessary unless the question asks for time, where AM or PM should be specified.
7. Give all answers in simplest form, rationalizing the denominator if necessary. If you get a fractional answer, express it as a common fraction unless otherwise indicated. If the answer is dealing with money, then round to the nearest hundredth.
8. Please make sure to write your name where indicated.
9. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to finish your exam.
10. When you finish the exam, please go over your answers again to check your work.

Questions for this exam were authored by Evan Kim, Joseph Kaim, and Conor Kennedy.

ANSWER SHEET

Name	
Grade	

Score 1	Score 2	Final
Initial 1	Initial 2	

Do not write in shaded regions.

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
1-15 Total			

	Answer	1 or 0	1 or 0
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
16-30 Total			

1. The operation $a \, \Omega \, b$ is defined as $a \, \Omega \, b = (a - b)!$. What is $2 \, \Omega \, 2$?

2. How many feet are in 57 miles?

3. If a rhombus has diagonals of length 10 and 24, then what is the area of the rhombus?

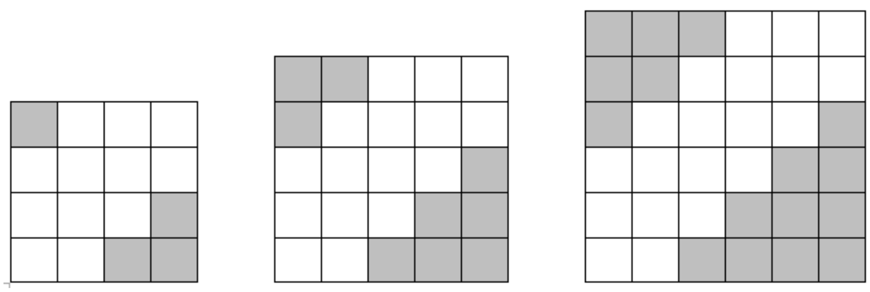
4. If $n = 10201200_3$ is a number in base 3, then what is $\frac{n}{9}$ expressed in base 3?

5. On Mars, Martians use strange objects as coins. Johnny the Martian flips three **3-sided** coins. What is the probability that he gets the same side on all three coins?

6. Jerry has 6 white marbles and 5 black marbles, which he needs to put in two boxes labeled A and B. If Jerry needs at least 3 white marbles in box A and at least 2 black marbles in box B, then how many different ways are there for him to put the marbles in the boxes? Assume that same color marbles are indistinguishable from one another.

7. If $35A62$ is a five-digit number that is divisible by 3, then what is the sum of all possible values of A?

8. How many gray squares are in stage 5 of the following pattern?



Stage 1 Stage 2 Stage 3

9. Let ΔABC be a triangle and let P be a point on side $\overline{BC}$. If $\angle BAP = 50°$ and $\angle ABC = 70°$, what is the measure of $\angle APC$?

10. John has a broken clock that moves 69 minutes for every 60 minutes on a correct clock. His clock first broke at exactly 2:35 PM. What time is it in real life if his clock says that it is 7:57 PM on the same day that it broke?

11. There are five tall towers in Redmond. Tower A, B, C, D, and E. Tower A is 2.7 times as tall as Tower B, Tower B is 2.7 times as tall as Tower C, and so on all the way to Tower E. How tall is Tower E, if Tower A is 1500 feet tall? Each tower is the whole number height closest to the true quotient of the taller tower.

12. On the planet Minetune, 25 gold ingots can be traded for 29 emeralds, and 16 emeralds can be traded for 27 iron ingots. How many iron ingots can John trade for if he has 18 64-item stacks of gold ingots and 48 extra gold ingots?

13. Find the sum of the infinite series $\frac{1}{3}+\frac{4}{9}+\frac{9}{27}+\frac{16}{81}+\cdots$.

14. At Redmond Middle School there are sixth graders, seventh graders, and eighth graders. There are fifteen sixth graders in Redmond Middle School, and there are equal amounts of sixth and seventh graders in math club, with one-fifth of the sixth graders being in math club. The amount of seventh graders that are not in math club is equal to the amount of eighth graders that are in math club. The ratio of seventh graders not in math club to eighth graders that aren't in math club is 2:7. How many students are at Redmond Middle School, if the ratio of sixth and seventh graders is 5:7?

15. Evaluate x if

$$x = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}$$

and $x > 0$.

16. Let O be the center of a circle with radius 6. Let A, B, and G be points on the circumference of O. If the area of the minor sector AOB is 8π, and G does not lie on $\widehat{AB}$, find the measure of $\angle AGB$.

17. Suppose you are given that $232131_4 = 5635_b$ for some $b \in \mathbb{Z}^+$. What is b? (Express your answer in base 10).

18. Mr. Lester commutes to work on the weekdays. On Monday, he got up on time and left his home at 8:00 PM. He got on his normal bus, which travels at 30 mph, and arrived to work 5 minutes early. On Tuesday however, he got up late and left his home at 8:30 PM. Because he was late, he rode a turbo penguin, which travels at 40 mph, but takes a 5-minute break at some point during the commute. He arrived 10 minutes late on Tuesday. How far is his house from work?

19. What is the shorter measure between the hands of a 12-hour clock at 3:19 PM?

20. Let ΔABC be a triangle with integer side lengths, with $AB = 7$ and $BC = 10$. Circle w is constructed such that its center is A, and point C lies on the circumference of w. What is the positive difference between the area of the largest possible right triangle and the area of the smallest possible right triangle that can be inscribed in w?

21. John went to the carnival, and being the gambler he is, decided to bet \$20 on a game where he would win \$48 (profit of \$28) if a six was rolled, win \$20 (break even) if a three, four, or five was rolled, and win nothing (lose \$20 overall) if a one or two was rolled. What's John's expected profit on this gamble?

22. An equilateral triangle has vertices with coordinates $(1,0)$, $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$. Every second, the equilateral triangle is translated vertically by up or down 1 unit with equal probability and it is translated horizontally right or left 1 unit with equal probability. After

2019 seconds, the expected location of the vertices are (a, b), (c, d), and (e, f). Find $a + b + c + d + e + f$.

23. John is gambling again. There are four horses that John can bet on.

    Horse A has a 40% chance of winning, where if he bets $50 he would receive $90 (profit $40).

    Horse B has a 30% chance of winning, where if he bets $30 he would receive $88 (profit $58).

    Horse C has a 20% chance of winning, where if he bets $20 he would receive $85 (profit $65).

    Horse D has a 10% chance of winning, where if he bets $10 he would receive $95 (profit $85).

    Give John's greatest possible return of investment (in dollars) if he may only bet on one horse.

24. What is the smallest 4-digit palindrome that is divisible by both 5 and 19?

25. If one meter is approximately 3.28 feet, how many centimeters are in an inch? No one cares if you memorized it, just use the measurement that was given to find the answer, or you run the risk of being wrong. Express to the nearest hundredth.

26. The point $A = (a, b)$ where $a, b \neq 0$ is reflected across $y = \frac{1}{\sqrt{3}}x$ and then it is reflected across $y = \sqrt{3}x$. The new coordinates of A are given by (c, d). The sum $c + d$ can be expressed as $ma + nb$ where $m, n \in \mathbb{R}$ and are constants. What is $m + n$?

27. Jerry and his marbles are back. Jerry has 8 red marbles and 14 blue marbles, which he needs to put into two boxes labeled A and B. If Jerry cannot have more red than blue marbles in either box and Jerry cannot have more than 10 marbles in box A, then how many ways are there for him to put the marbles in the boxes?

28. Andy the ant is at $(5, 5)$ on the Cartesian plane. Andy can only move left or down along the grid lines (i.e. from (x, y) he can only go to $(x, y - 1)$ or $(x - 1, y)$). What is the probability that when he first reaches the x or y axis, it is at $(1,0)$ or $(0,1)$?

29. There are 2 hippos, Jack and Jill. Jack is 200 meters behind Jill. Jimmy the flying man starts on top of Jack. The 3 begin moving forward simultaneously. Jack moves at 20 m/s and Jill moves at 15 m/s. Jimmy moves forward at 30 m/s and backward at 25 m/s due to a wind. Jimmy will move forward until he reaches Jill and he will turn and go backward to Jack and he will continue going back and forth in this fashion until Jack and Jill meet. What is the distance covered by Jimmy?

30. On the planet Meanus, there are 2 intelligent monkeys, Mr. Mitch and Mr. Fitch. They play a game called Doom. In Doom, the players sit in chairs A and B. Both players randomly choose a number between 0 and 1 at the start of each round. Then the player in chair A flips a coin. The player in chair A wins if the coin flip was heads and the numbers chosen differ by less than 0.25. If this did not happen, but the coin flip was heads, then the players switch seats and start the next round. If the coin flip was tails, the players just start the next round (without switching seats). Mr. Mitch starts in chair A and Mr. Fitch starts in chair B. What is the probability that Mr. Mitch wins?