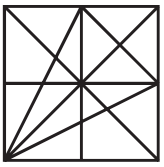
Counting Stretch

221. _____ Hazel wrote the integers 1 through 321 on the board. How many total digits did she write?

222. triangles  How many triangles of any size are in this figure?

223. ways In how many ways can one knife, one fork and one spoon be distributed, in any order, to three people, if each person is given 0, 1, 2 or 3 utensils?

224. ways Using pennies, nickels, dimes and quarters, how many ways can you make 67 cents?

225. scores In the game Fortrix, a player can earn 3, 7 or 11 points on a turn. How many different scores are possible for a single player after six turns?

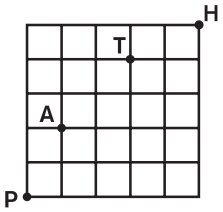
226. integers How many 3-digit integers are divisible by both 5 and 17?

227. integers How many positive integers less than 40 are relatively prime to both 7 and 10?

228. palin-dromes How many palindromes are between 9 and 1009?

229. paths In the 3 × 3 grid shown, a path can begin in any cell and can pass through a cell more than once. How many such paths spell ROTOR?

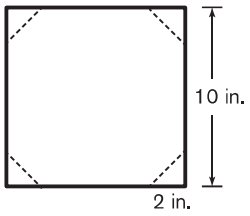
R	O	R
O	T	O
R	O	R

230. paths 

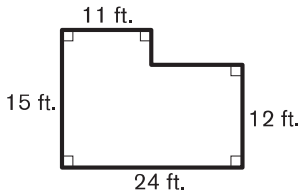
Moving only up and right, how many paths from P to H pass through A and T?

Area Stretch

231. _____ % Norm has a square sheet of paper with 10-inch sides. Along each side, he makes a mark 2 inches from each corner. He then draws a line segment connecting the two marks near each corner. Finally, he cuts along each line segment, removing a triangle from each corner of the square and creating an octagon. What percentage of the area of the square is the area of the octagon?



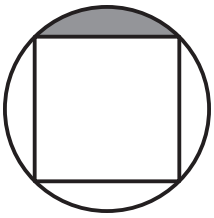
232. _____ ft^2 The figure shows an office floor plan. How many square feet does this office occupy?



233. _____ m^2 A running track consists of two parallel straight segments, each 100 meters long, connected by two semicircular stretches, each with inner diameter 50 meters. What is the total area enclosed by the running track? Express your answer to the nearest hundred.

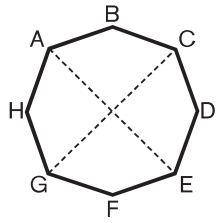
234. _____ $units^2$ What is the greatest possible area of a concave pentagon in the coordinate plane with vertices $(-2, 0)$, $(2, 0)$, $(2, 10)$, $(0, 6)$ and $(-2, 10)$?

235. _____ $units^2$ A square is inscribed in a circle of radius 4 units. The square divides the interior of the circle into five regions, four of which lie outside the square. What is the area of the shaded region? Express your answer in terms of π.

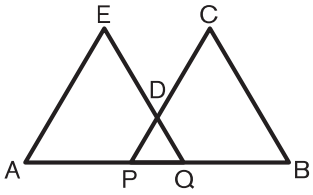


236. _____ in^2 Amy marks two points A and B that are 4 inches apart. She draws one circle that has segment AB as a diameter. She then draws a larger circle, which overlaps the first circle, such that the arc from A to B along its circumference is a quarter-circle. What is the total area covered by the two circles? Express your answer in terms of π.

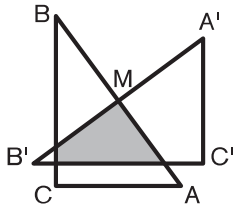
237. _____ $units^2$ In convex octagon ABCDEFGH, shown here, each side has length 6 units, and diagonals AE and CG have length 16 units. If the octagon is symmetric across both diagonals AE and CG, what is its area? Express your answer in simplest radical form.



238. _____ $units^2$ In this figure, AE = EQ = BC = CP = 10 units, and AQ = BP = 12 units. The points A, P, Q and B are collinear. If the perimeter of the concave pentagon ABCDE is 52 units, what is its area? Express your answer as a common fraction.



239. _____ $units^2$ Right triangle ABC with AC = 3 units, BC = 4 units and AB = 5 units is rotated 90° counterclockwise about M, the midpoint of side AB, to create a new right triangle A'B'C'. What is the area of the shaded region where triangles ABC and A'B'C' overlap? Express your answer as a common fraction.



240. _____ units In right triangle ABC, ∠C is a right angle, AC = 10 units and BC = 24 units. If a point X is located inside triangle ABC so that the distance from X to side AB is twice the distance from X to side AC, and the distance from X to side AC is twice the distance from X to side BC, what is the distance from X to side AB? Express your answer as a common fraction.

Modular Arithmetic Stretch

Modular arithmetic is a system of integer arithmetic that enables us obtain information and draw conclusions about large quantities and calculations. It would be extremely helpful, for instance, when asked to find the units digit of 2^{2015} if we didn't really have to calculate the value of the expression to get that information. Modular arithmetic allows us to do just that!

The Basics:

The simplest example of modular arithmetic is commonly referred to as "clock arithmetic." Suppose it is 3 o'clock now and I want to know what time it will be in 145 hours. We could count from 3 o'clock for 145 consecutive hours. We certainly wouldn't be expected to count 145 hours starting with 3 o'clock. Suppose we did counting the hours from 3 o'clock. What happens when we get to 12 o'clock? We continue counting but begin a new 12-hour cycle. Instead of counting 145 hours, we can just see how many of these 12-hour cycles we'd go through counting 145 hours. More importantly, we need to determine how many hours would remain after making it through the last full 12-hour cycle.

In this example, the value 12 is called the ***modulus*** and what is left over is called the remainder. In this case, we can determine fairly quickly that there are 12 full 12-hour cycles in 145 hours, with a remainder of 1 hour (since $12 \times 12 = 144$ and $145 - 144 = 1$).

Standard arithmetic: $145 = 12 \times 12 + 1$

Modular arithmetic we write: $145 \equiv 1 \pmod{12}$ *Read **"145 is congruent to 1 modulo 12"***

The remainder of 1 tells me that it will be the same time 145 hours after 3 o'clock that it will be 1 hour after 3 o'clock. And that time is 4 o'clock.

Here's another example of modular arithmetic. Suppose today is Tuesday. What day of the week will it be 417 days from now? Since the days of the week are on a 7-day repeating cycle, the modulus here is 7. If we divide 417 by 7, we get

Standard arithmetic: $417 = 59 \times 7 + 4$

Modular arithmetic we write: $417 \equiv 4 \pmod{7}$

Thus, 417 days from Tuesday will be the same day of the week as 4 days from Tuesday, Saturday.

TRY THESE

241._____ If the current month is July, what month will it be in 152 months?

242._____ a.m. / p.m. If the time is currently 8 a.m., what time will it be in 255 hours? *Circle a.m. or p.m. in answer blank.*

243._____ m Jennie goes out every morning and jogs on the school track. The track is 400 meters around. If Jennie runs 5310 meters then how far will she be from where she started once she finished her run?

Modular Addition: What is the remainder when 9813 + 7762 + 11252 is divided by 10?

$$9813 + 7762 + 11252 = (981 \times 10 + 3) + (776 \times 10 + 2) + (1125 \times 10 + 2)$$
$$= (981 + 776 + 1125) \times 10 + (3 + 2 + 2)$$

Since we are only interested in the remainder, we need only focus on the last part. We see that the remainder is 3 + 2 + 2 = **7.** Written in modular arithmetic notation it would look like this:

$$9813 + 7761 + 11252 \equiv 3 + 2 + 2 \equiv 7 \pmod{10}$$

Modular Multiplication: What is the remainder whsen 9813×7762 is divided by 10?

$$9813 \times 7762 = (981 \times 10 + 3) \times (776 \times 10 + 2)$$
$$= (981 \times 776 \times 10^2) + (981 \times 2 \times 10) + (776 \times 3 \times 10) + (3 \times 2)$$

The first three terms are multiples of 10, and once again last term is the remainder $3 \times 2 = \mathbf{6}$. Written in modular arithmetic notation would look like this:

$$9813 \times 7762 \equiv 3 \times 2 \equiv 6 \pmod{10}$$

More Mod Shortcuts: There are many useful applications of modular arithmetic. Here are just a few more.

- Consider the powers of 3: $3^0 = 1$; $3^1 = 3$; $3^2 = 9$; $3^3 = 27$; $3^4 = 81$; $3^5 = 243$; $3^6 = 729$
  Notice that the units digits are repeated every four powers of 3, so the modulus is 4. Repeating units digits correspond to remainders 1, 2, 3 and 0.
- Suppose you want the unit digit of 3^{53}. First, we note that $53 \equiv 1 \pmod{4}$ since the remainder 1 corresponds to units digit 3, thus, the expansion of 3^{53} has a units digit of 3.
- The smallest number that has remainder 1 when divided by 2 and 3 is 7. Why?
  $1 \equiv 7 \pmod{2}$ and $1 \equiv 7 \pmod{3}$

MODULAR ARITHMETIC PRACTICE

244._____ What is the last digit of 2^{2015}?

245._____ What is the value of 122×71 modulo 11?

246._____ What is the remainder when $5981 \times 8162 \times 476$ is divided by 5?

247._____ Jon has 29 boxes of donuts with 51 donuts in each box. He wants to divide them into groups of a dozen each. Once he groups them again, how many donuts will be left over?

248._____ What is the least integer greater than 6 that leaves a remainder of 6 when it is divided by 7 and by 11?

249._____ When organizing her pencils, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she always has exactly one pencil left over. If Faith has between 10 and 100 pencils, how many pencils does she have?

250._____ When organizing her pens, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she is always one pen short of being able to make full groups. If Faith has between 10 and 100 pens, how many pens does she have?

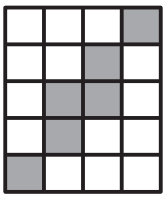
Fractions Stretch

Solve the following problems. Express any non-integer answer as a common fraction.

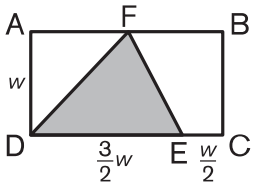
221._____ What fraction of 100 is 25?

222._____ What fraction of $\frac{3}{8}$ is $\frac{9}{16}$?

223._____ What is the value of $\sqrt{\frac{3}{11} \div \frac{11}{12}}$?

224._____  What fractional part of this grid of 20 unit squares is shaded?

225._____ What fraction of the area of rectangle ABCD is the area of inscribed triangle DEF?



226._____ On a number line, what common fraction is $\frac{3}{4}$ of the way from $\frac{1}{2}$ to $\frac{3}{4}$?

227._____ What is the reciprocal of $\frac{1}{2+\frac{1}{3}}$?

228._____ What common fraction is equal to $0.7\overline{5}$?

229._____ If $\frac{1}{\frac{1}{\frac{1}{n}+\frac{1}{3}}+\frac{1}{\frac{1}{3}+\frac{1}{n}}} = \frac{5}{12}$, what is the value of n?

230._____ If $\frac{2x}{x-3} - 2 = \frac{4}{x+2}$, what is the value of x?

Angles and Arcs Stretch

Secant	a line that intersects the circle at two points
Chord	a line segment whose endpoints are two points on the circle
Tangent	a coplanar line that intersects the circle at a single point of tangency
Central Angle	an angle with its vertex at the center of the circle
Inscribed Angle	an angle with its vertex on the circle and whose sides are chords of the circle
Major Arc	an arc of the circle with measure greater than or equal to 180°
Minor Arc	an arc of the circle with measure less than 180°

Angle and Arc Measures

In the figures below, observe how the degree measure of $\angle AXB$ decreases as the distance between the vertex of the angle and the center of the circle increases.

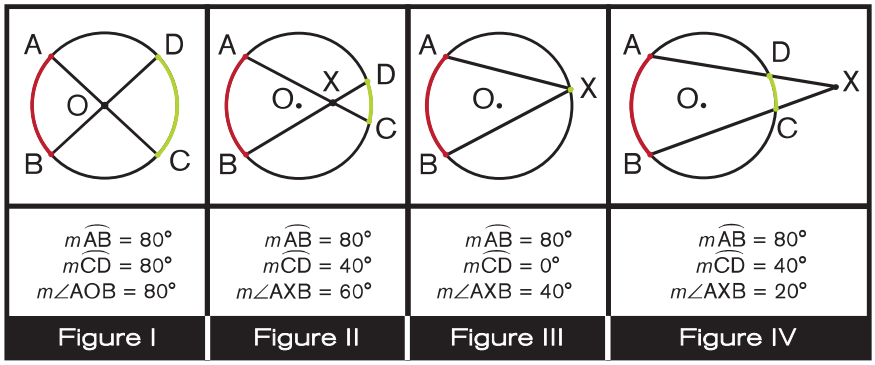


Figure I	Figure II	Figure III	Figure IV
$m\overset{\frown}{AB} = 80°$ $m\overset{\frown}{CD} = 80°$ $m\angle AOB = 80°$	$m\overset{\frown}{AB} = 80°$ $m\overset{\frown}{CD} = 40°$ $m\angle AXB = 60°$	$m\overset{\frown}{AB} = 80°$ $m\overset{\frown}{CD} = 0°$ $m\angle AXB = 40°$	$m\overset{\frown}{AB} = 80°$ $m\overset{\frown}{CD} = 40°$ $m\angle AXB = 20°$

- In Figure I, angles AOB and COD are central angles of circle O that intercept arcs AB and CD, respectively. The degree measure of a central angle and the arc it intercepts are equal.

$$m\angle AOB = m\overset{\frown}{AB} \text{ and } m\angle COD = m\overset{\frown}{CD}$$

- In Figure II, vertical angles AXB and CXD, formed by the intersection of chords AC and BD inside circle O, intercept arcs AB and CD, respectively. The degree measure of vertical angles formed by two chords intersecting inside a circle is half the sum of the measures of their intercepted arcs.

$$m\angle AXB = m\angle CXD = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$$

- In Figure III, $\angle AXB$ is inscribed in circle O. The degree measure of an inscribed angle is half the measure of the intercepted arc.

$$m\angle AXB = \frac{1}{2}m\overset{\frown}{AB}$$

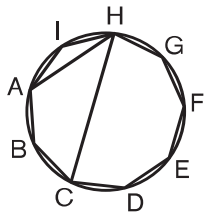
- In Figure IV, $\angle AXB$, formed by the intersection of two secants at point X outside of circle O, intercepts arcs AB and CD. The degree measure of an angle formed by two secants, two tangents or a secant and a tanget is half the difference of the measures of its intercepted arcs.

$$m\angle AXB = \frac{1}{2}\left(m\overset{\frown}{AB} - m\overset{\frown}{CD}\right)$$

It may appear that there are four different formulas for calculating the four types of angles. But in each case, the measure of the angle in question is, essentially, the average of the measures of the intercepted arcs. In Figure IV, note that, with respect to $\angle AXB$, $\overset{\frown}{AB}$ appears concave, while $\overset{\frown}{CD}$ appears convex. So the measure of $\angle AXB$ can be thought of as the average of 80° and −40°.

Solve the following problems by using what you've learned about angles and arcs. Express any non-integer value as a decimal to the nearest tenth.

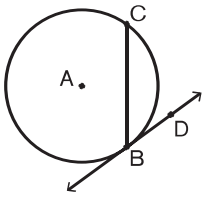
231. _____ °



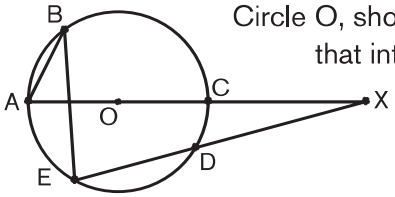
Regular nonagon ABCDEFGHI is inscribed in a circle, as shown. What is $m\angle AHC$?

232. _____ °

In circle A, shown here, $\overleftrightarrow{BD}$ is tangent to the circle at B, and major $\overset{\frown}{BC}$ has measure 230°. What is $m\angle CBD$?



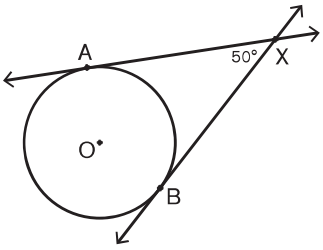
233. _____ °



Circle O, shown here with chords AB and BE, has secants AC and DE that intersect at X. If $m\angle ABE = 35°$ and $m\angle AXE = 15°$, what is the measure of $\overset{\frown}{CD}$?

234. _____ °

In this figure, lines AX and BX are tangent to circle O at A and B, respectively. If $m\angle AXB = 50°$, what is the measure of major $\overset{\frown}{AB}$?



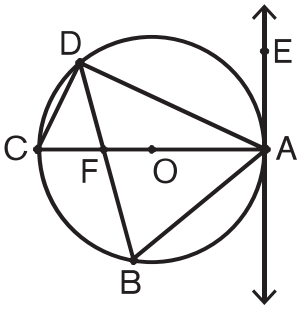
Use the figure at the right for questions 235 through 238.

235. _____ ° What is $m\angle ABD$?

236. _____ ° What is $m\overset{\frown}{AB}$?

237. _____ ° What is $m\angle BAE$?

238. _____ ° What is $m\angle CFD$?



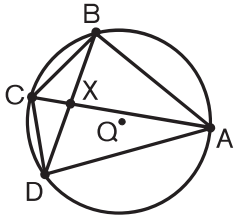
$\overleftrightarrow{AE}$ is tangent to circle O

$\overleftrightarrow{AE} \perp \overline{AC}$

$m\angle BDC = 40°$

$m\overset{\frown}{AD} = 125°$

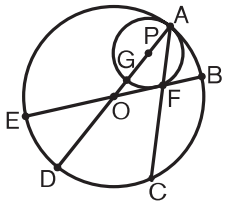
239. _____ °



Quadrilateral ABCD is inscribed in circle Q, as shown, with diagonals intersecting at X. If $m\overset{\frown}{AB} = 110°$, $m\overset{\frown}{BC} = 60°$ and AB = BD, what is $m\angle CXD$?

240. _____ °

Circle P is internally tangent to circle O at A, as shown. $\overline{AC}$ and $\overline{BE}$ intersect at F, which is also the point of tangency between $\overline{BE}$ and circle P. $\overline{AD}$ and $\overline{BE}$ are diameters of circle O, and $\overline{AG}$ is a diameter of circle P. If $m\overset{\frown}{CD} = 50°$, what is the measure of minor $\overset{\frown}{BC}$?



Bases Stretch

The **base 10** number system, the number system we are most familiar with, uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Numerals with these digits in the ones, tens, hundreds and higher places express specific numerical quantities. In base 10, the number 245, for example, is composed of 2 hundreds, 4 tens and 5 ones. That is, $2(10^2) + 4(10^1) + 5(10^0) = 200 + 40 + 5 = 245$.

A **base *b*** number system uses the digits 0, 1, ..., $b - 1$. Numerical quantities are expressed with these digits in the b^0, b^1, b^2 and higher places. In base b, if $b \geq 6$, the numeral 245_b represents the number $2(b^2) + 4(b^1) + 5(b^0)$. In base 8, for example, $245_8 = 2(8^2) + 4(8^1) + 5(8^0) = 2(64) + 4(8) + 5(1) = 128 + 32 + 5 = 165$.

Bases greater than 10 use letters to represent the digits greater than 9. For example, the 12 digits used in base 12 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B. The numeral 10 in base 12 has 1 twelve and 0 ones. That is, $10_{12} = 1(12^1) + 0(12^0) = 1(12) + 0(1) = 12 + 0 = 12$.

Practice Problems

What is the representation of each of the following in base 10?

241._____ 24_9

242._____ 24_8

243._____ 24_7

What is the representation of 24 in each of the following bases?

244._____ base 9

245._____ base 8

246._____ base 7

Now try these.

247._____ What is the representation of 4991 in base 12?

248._____ What is the representation of $3BB_{12}$ in base 6?

249._____ If $523_b = 262$, what is the value of b?

250._____ If $441_b = n^2$ and $351_b = (n - 2)^2$, for some $b < 10$, what is the value of n?

Probability Stretch

1. _____ % Petra randomly selects a card from a standard deck of 52 playing cards. What is the percent probability that the card shows a red number greater than 6? Express your answer to the nearest hundredth.

2. _____ Max has eight identical cups. Each cup contains a different combination of nickels, dimes and quarters, each totaling 45 cents. Max randomly selects a cup. What is the probability that the cup he selects contains at least three dimes? Express your answer as a common fraction.

3. _____ A bag contains five chips numbered 2 through 6. Danya draws chips from the bag one at a time and sets them aside. After each draw, she totals the numbers on all the chips she has already drawn. What is the probability that at any point in this process her total will equal 10? Express your answer as a decimal to the nearest tenth.

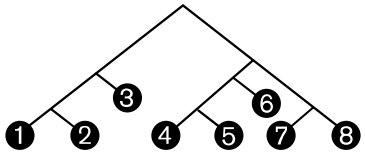
4. _____ A drawer contains five socks: two green and three blue. What is the probability that two socks pulled out of the drawer at random will match? Express your answer as a common fraction.

5. _____ A penny, a nickel and a dime are flipped. What is the probability that at least two coins land heads up and one of them is the nickel? Express your answer as a common fraction.

6. _____ % When the circuit containing blinking lights A and B is turned on, lights A and B blink together. Then A blinks once every 5 seconds and B blinks once every 11 seconds. Lindsey looks at the two lights just in time to see A blink alone. What is the percent probability that the next light to blink will be A blinking alone?

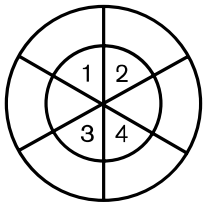
7. _____ % What is the percent probability that a randomly selected multiple of 3 less than or equal to 3000 is also a multiple of 5?

8. _____ Starting at the top and selecting paths randomly as you move downward, what is the probability of ending at an odd number? Express your answer as a common fraction.



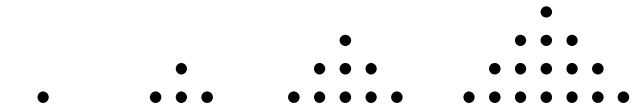
9. _____ A five-digit number is made by randomly ordering the digits 1, 2, 3, 4 and 5. What is the probability that this number is divisible by 4? Express your answer as a common fraction.

10. _____ Pierre throws darts that land randomly in the dartboard shown here. The dartboard is a circle of radius 2 units, with an inner circle of radius 1 unit. Both circles are divided into six congruent sectors. What is the probability that a dart Pierre throws will land in one of the four inner numbered sectors? Express your answer as a decimal to the nearest hundredth.



Patterns Stretch

11. _____ dots The first four stages of a dot pattern are shown. How many more dots are in the figure at Stage 47 than in the figure at Stage 27?



12. _____ The first three terms of a sequence are 1, 2 and 3. Each subsequent term is the sum of the three previous terms. What is the 11th term of this sequence?

13. _____ What is the sum of the terms in the arithmetic series $2 + 5 + 8 + 11 + 14 + \ldots + 89 + 92$?

14. _____ Three consecutive terms in an arithmetic sequence are x, $2x + 11$ and $4x - 3$. What is the constant difference between consecutive terms in this sequence?

15. _____ What is the sum of the terms in the geometric series $1 + 4 + 16 + \ldots + 1024$?

16. _____ What is the sum of the first 51 consecutive odd positive integers?

17. _____ What is the sum of the terms in the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$?

18. _____ What is the sum of the terms in the infinite series $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \ldots$? Express your answer as a common fraction.

19. _____ Let $f(x) = 2x + 3$ and $f^2(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$. If $f^5(x) = ax + b$, what is the value of $a + b$?

20. _____ degrees The degree measures of the interior angles of a quadrilateral form a geometric sequence whose terms have integer values and are all integer multiples of the first term. What is the largest possible degree measure of an angle in this quadrilateral?

Travel Stretch

$\text{speed} = \frac{\text{distance}}{\text{time}}$	$\text{distance} = \text{speed} \times \text{time}$	$\text{time} = \frac{\text{distance}}{\text{speed}}$

21. _____ mi/h Jack and Jill travel up a hill at a speed of 2 mi/h. They travel back down the hill at a speed of 4 mi/h. What is their average speed for the entire trip? Express your answer as a mixed number.

22. _____ : _____ p.m. At 2:20 p.m., Jack is at the top of the hill and starts walking down at the exact same time that Jill, who is at the bottom of the hill, starts walking up. If they maintain the same uphill and downhill speeds from the previous problem, and the distance from the bottom to the top of the hill is 1.5 miles, at what time will Jack and Jill meet?

23. _____ yards When Jack and Jill meet, as described in the previous problem, how many yards will they be from the bottom of the hill?

24. _____ minutes Alysha's average speed when walking from home to the market is 5 mi/h, and it takes her 21 minutes longer than when she drives to the market. If Alysha drives to the market, along the same route, at an average speed that is eight times her average walking speed, how many minutes does it take her to drive from home to the market?

25. _____ miles Based on problem 24, how many miles does Alysha travel to get from home to the market?

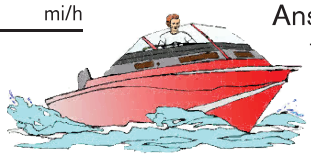
26. _____ minutes

Jana begins jogging along a path and, 5 minutes later, Zhao begins riding his bicycle along the same path, which has a length of 2 miles. Zhao rides his bicycle at a speed of 10 mi/h, and Jana's jogging speed is 6 mi/h. If they both begin at one end of the path and end at the other, how many minutes after Zhao reaches the end of the path will Jana reach the end of the path?

27. _____ minutes Based on problem 26, how many minutes after Zhao begins riding will he catch up with Jana? Express your answer as a mixed number.

28. _____ miles Again, based on problem 26, how many miles will Jana have traveled when Zhao catches up with her? Express your answer as a mixed number.

29. _____ mi/h



Ansel left the dock in his motorboat, traveled 10 miles, and then returned to the dock along the same route. On the return trip, Ansel was traveling against the current of the river, and his average speed relative to the water was 20 mi/h. If the round-trip took Ansel 64 minutes, what is the speed of the river's current?

30. _____ Based on problem 29, what fraction of Ansel's total travel time was spent traveling upstream? Express your answer as a common fraction.

Measurement Stretch

1. _____ units

Merri places weights of 6 units and 28 units on the right side of a balance and weights of 3 units and 19 units on the left side. If she adds an object to the left side that makes the balance level, how many units does the object weigh?

2. _____ tacks

The weight of a small clip is $\frac{2}{3}$ the weight of a large clip. If 2 tacks weigh the same amount as a large clip, how many tacks weigh the same amount as 12 small clips?

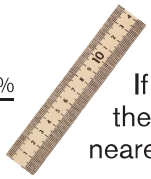
3. _____ Dems

On the planet Klem, 1 Bem plus 7 Dems equals 4 Pems, and 2 Bems plus 1 Dem equals 1 Pem. How many Dems equal 7 Bems?

4. _____ meters

If a race car is traveling at 99 mi/h, how many meters does it travel in a second, given that 0.305 meter = 1 foot? Express your answer as a decimal to the nearest tenth.

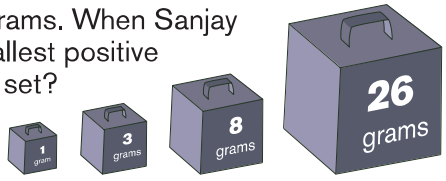
5. _____ %



If the results when reading a measuring stick can be off by at most 1 cm, what is the maximum percent error when 24 cm is measured? Express your answer to the nearest tenth.

6. _____ grams

Vijay gives Sanjay a set of four weights of 1, 3, 8 and 26 grams. When Sanjay places weights on either side of a balance, what is the smallest positive integer number of grams that he **cannot** measure with this set?



7. _____ gallons

If Clem has 2 cups, 7 pints, 8 quarts and 11 half-gallons of lemonade, how many total gallons of lemonade does she have? Express your answer as a mixed number.

8. _____ Klegs

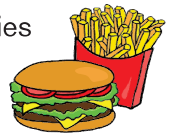
If 2 Blams equal 15 Droms and 5 Droms equal 28 Klegs, how many Klegs are in a Blam?

9. _____

What is the ratio of 1 ounce to 1000 grams, given that 1 pound equals 454 grams? Express your answer as a decimal to the nearest thousandth.

10. _____ times

If one order of fries and five burgers cost twice as much as three orders of fries and two burgers, how many times as much does a burger cost compared to one order of fries?



Expected Value Stretch

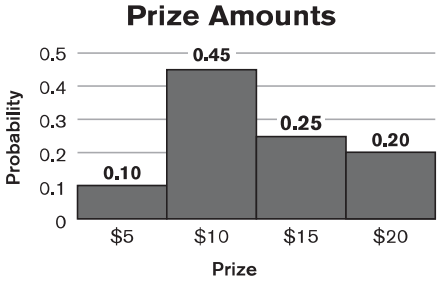
If the outcomes of random variable X have values x_1, x_2, x_3, ..., x_n and the probabilities of these outcomes occurring are p_1, p_2, p_3, ..., p_n, respectively, then the **expected value** of the outcome is the sum of the products of the probability of each outcome and the value of that outcome.

$$E(X) = p_1 x_1 + p_2 x_2 + p_3 x_3 + \cdots + p_n x_n$$

11. _____ An unfair six-sided die with faces labeled 1, 2, 3, 5, 8 and 13 is rolled. The table lists the probability of the die landing with each number showing on the top face. The expected value of the roll is the sum of the products of each face value and its corresponding probability of being rolled. What is the expected value when the die is rolled? Express your answer as a mixed number.

Top Face Value	Probability
1	$\frac{1}{3}$
2	$\frac{1}{15}$
3	$\frac{1}{6}$
5	$\frac{1}{5}$
8	$\frac{2}{15}$
13	$\frac{1}{10}$

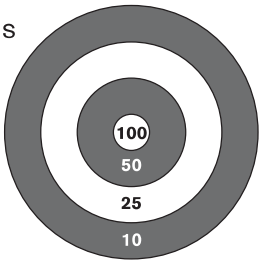
12. $_____ Terry plays a game with prizes of 5, 10, 15 and 20 dollars. The graph shows each possible prize amount and its corresponding probability. The expected value of her prize is the sum of the products of each prize and the probability of winning that prize. What is the expected value of Terry's prize?



13. _____ A fair 10-sided die with one face labeled 1, two faces labeled 2, three faces labeled 3 and four faces labeled 4 is rolled. What is the expected value when this die is rolled?

14. _____ cm^2 Ana has a bowl containing two square tiles, one with side length 2 cm and the other with side length 3 cm. She randomly chooses a tile from the bowl. The expected value of the area of the chosen tile is the sum of the products of each tile's area and its corresponding probability of being chosen. If the probability of choosing a particular tile is proportional to its area, what is the expected value of the area of the tile Ana chooses? Express your answer as a common fraction.

15. _____ points For the dartboard shown, the number of points scored when a dart lands in each region is indicated. The innermost circle of the board has radius 1 inch, and each subsequent circle has a radius 2 inches greater than the previous circle. Kane throws a dart that lands randomly somewhere on the board. What is the expected value of the number of points he scores? Express your answer as a decimal to the nearest tenth.



16. _____ Gwen randomly draws a card from a deck of 40 cards numbered 1 through 40. What is the expected value of the number on the card she draws? Express your answer as a decimal to the nearest tenth.

17. _____ faces Luke paints each face of a 5 × 5 × 5 cube red. He then cuts the cube into 125 unit cubes and randomly chooses a single unit cube. What is the expected value of the number of painted faces on this unit cube? Express your answer as a decimal to the nearest tenth.

A property of E is that it is a linear function of the random variable. So, for random variables X and Y, the expected value of the sum of random variables equals the sum of their expected values.

$$E(X + Y) = E(X) + E(Y)$$

18. _____ points In each round of a particular game, Dinara can win at most one point. If she has a 70% chance of winning a point in each round, what is the expected value of Dinara's total score after three rounds? Express your answer as a decimal to the nearest tenth.

19. _____ Jo and her four friends each secretly pick a random integer from −5 to 5, inclusive. What is the expected value of the sum of the five chosen numbers?

20. _____ jelly beans Allen randomly distributes 1000 jelly beans into 10 jars lined up in a row from left to right. What is the expected value of the number of jelly beans in the leftmost jar?

Transformations Stretch

21. _____ units A point P(−3, 2) is translated right 4 units to its image P′. The point P′ is then translated up 3 units to its image P″. What is the distance from P to P″?

22. _____ units A segment has endpoints A(0, 0) and B(−3, 4). Point C is the image of point B translated down 4 units and left 3 units. What is the perimeter of △ABC?

23. (,) A point Q(−3, 4) is reflected across the x-axis, and then the image Q′ is reflected across the line $x = 2$. What are the coordinates of the image Q″? Express your answer as an ordered pair.

24. _____ A point S(1, 6) is reflected across the line $x - 2y = -6$. What is the sum of the coordinates of the image S′?

25. (,) What are the coordinates of the image of point D(−5, −3) when it is rotated 90 degrees clockwise about the origin? Express your answer as an ordered pair.

26. (,) What are the coordinates of the image of the point E(3, −1) when it is rotated 90 degrees counterclockwise about the point F(5, 4)? Express your answer as an ordered pair.

27. _____ A segment with endpoints G(−2, 3) and H(4, 7) is dilated by a scale factor of $\frac{2}{3}$ with center of dilation (0, 0). What is the sum of all the coordinates of G′ and H′?

28. _____ Point J(4, 8) is dilated by a scale factor of $\frac{3}{2}$ with center of dilation K(2, 2). What is the product of the coordinates of J′?

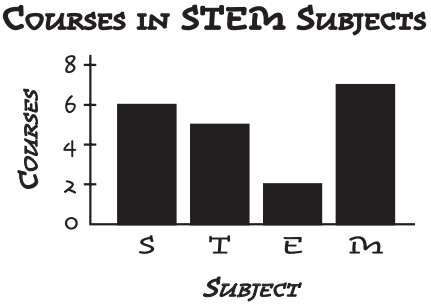
29. _____ units^2 A point L(−2, 4) is rotated 90 degrees clockwise about the point M(3, 2). Point N is the image of L′ dilated by a scale factor of $\frac{3}{2}$ with center of dilation M. What is the area of △LMN? Express your answer as a common fraction.

30. _____ units A point R(−5, 3) is reflected across the line $y = x - 2$, and then the image R′ is rotated 90 degrees clockwise about the origin. What is the distance from R to R″? Express your answer in simplest radical form.

Ratios Stretch

Definition

A ***ratio*** is the comparison of two quantities by division.



This graph shows the number of courses in **S**cience, **T**echnology, **E**ngineering and **M**ath offered at a particular school. According to the graph, the ratio of Engineering to Technology courses is 2 to 5, also written E:T = 2:5. The ratio E:T is a comparison of two of the four "parts" that combine to make up the "whole" group of STEM courses. Some ratios compare a "part" to the "whole." Ratios are often written in the form of a fraction, decimal or percent. According to the graph, this school offers a total of 20 STEM courses. So, the ratio of Math courses to STEM courses is 7 to 20. In other words, Math courses account for $\frac{7}{20} = 0.35 = 35\%$ of the STEM courses at this school.

1. _____ Dave's digital library contains 25 fiction books and 15 nonfiction books. What is the ratio of nonfiction books to fiction books in Dave's digital library? Express your answer as a common fraction.

2. _____ For a 30-60-90 right triangle, what is the ratio of the length of the longer leg to the length of the hypotenuse? Express your answer as a common fraction in simplest radical form.

3. _____ A jar contains seven blue marbles and eight green marbles. Ming adds four yellow marbles and five blue marbles to the jar. What is the ratio of green marbles to non-green marbles in the jar? Express your answer as a common fraction.

Fairy Godmother has granted wishes to Aurora, Belle and Cindi in the ratio 6:8:11. Use this information to solve problems 4 through 7.

4. _____ What fraction of the wishes were granted to Belle? Express your answer as a common fraction.

5. _____ % What percent of the wishes granted by Fairy Godmother were *not* granted to Aurora?

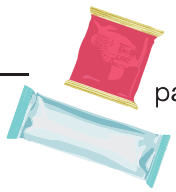
6. _____ % What is the absolute difference between the percents of wishes Fairy Godmother has granted to Aurora and to Cindi?

7. _____ wishes Fairy Godmother has granted at least 20 wishes each to Aurora, Belle and Cindi. What is the least possible number of wishes that she has granted to Cindi?

8. _____ % Jennie put 38 mL of water in a cylinder with a capacity of 60 mL. If she increases the volume of water in the cylinder by 50%, what percent of the cylinder will contain water?

9. _____ % A shop owner increased the price of a jacket by 17%. What percent of the new price is the original price of the jacket? Express your answer to the nearest tenth.

10. _____ New packaging for fruit snacks contains 10% less weight than the original packaging. If the new package costs 15% more than the original package, by what fraction did the unit price increase? Express your answer as a common fraction.



Venn Diagrams Stretch

Set Theory Review

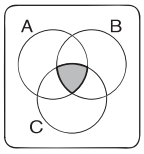
A **set** is a collection of objects or elements, called **members.** Consider two sets A and B.

- The **intersection** of A and B, denoted $A \cap B$, is the set of elements that are in both A and B.
- The **union** of A and B, denoted $A \cup B$, is the set of elements in A or in B or in both.
- The **universal set U** is the set of all possible elements.
- The **relative complement** of A, denoted $B \setminus A$, is the set of all elements in B but not in A.
- The **complement** of A, denoted A', is the set of all elements not in A, in other words, $U \setminus A$.
- The **cardinality** of A, denoted $|A|$, is the number of elements in set A.

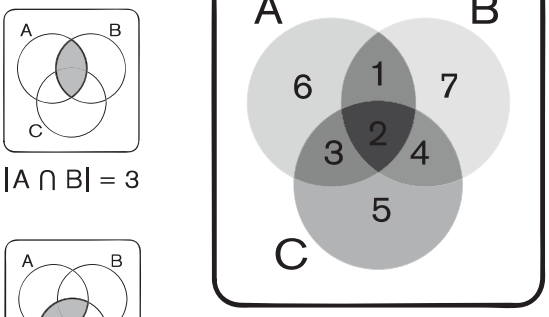
A **Venn diagram** is a useful tool for comparison. It helps us visualize the relationships between two or more sets. The Venn diagrams shown compare sets A, B and C.

$|A \cap (B \cup C)| = |(A \cap B) \cup (A \cap C)| = 6$

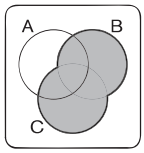
$|A| = 12$



$|A \cap B \cap C| = 2$

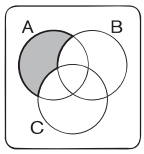


$|A \cap B| = 3$

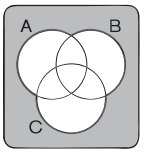


$|B \cup C| = 22$

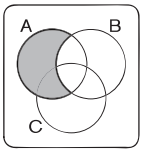
$|A \cap C| = 5$



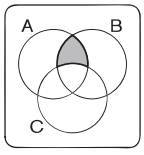
$|A \setminus B \setminus C| = 6$



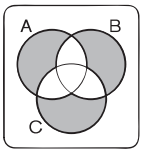
$(A \cup B \cup C)'$



$|A \setminus B| = 9$

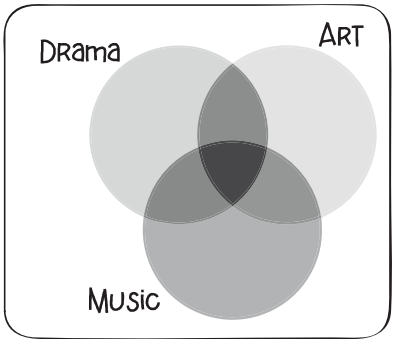


$|(A \cap B) \setminus C| = 1$



$|(A \setminus B \setminus C) \cup (B \setminus A \setminus C) \cup (C \setminus A \setminus B)| = 18$

At Mesa Performing Arts Center, 30 students take courses in one or more of the drama, music and art departments. Five students take courses in exactly one department. Of these students, twice as many take drama courses as take music courses. Five students take courses in exactly two departments. Of these students, twice as many take drama and music courses as take music and art courses. Use the provided Venn diagram to organize this information, and then answer questions 11 through 13.

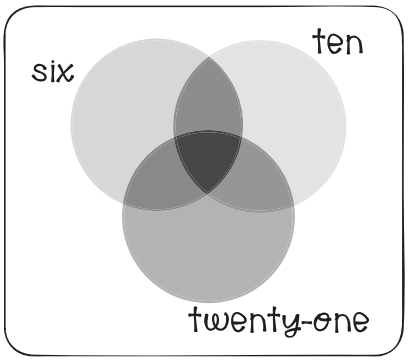


11. _____ students How many students take only art courses?

12. _____ students How many students take courses in all three departments?

13. _____ students How many students take courses in art or music but not both?

The integers from 1 to 630, inclusive, are tested for divisibility by 6, 10 and 21. Use the provided Venn diagram to help determine the cardinality of various sets that contain multiples of 6, 10 and 21, and then answer questions 14 through 16.

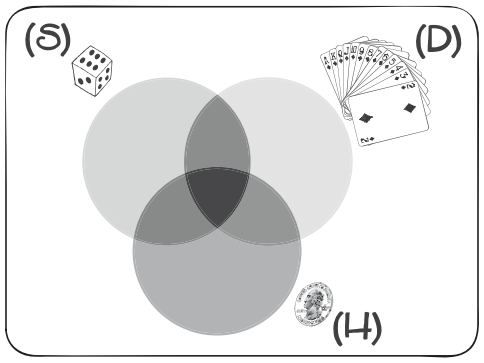


14. _____ integers How many of these integers are divisible by both 6 and 10 but not by 21?

15. _____ integers How many of these integers are divisible by 6 but not by either 10 or 21?

16. _____ integers How many of these integers are not divisible by any of 6, 10 or 21?

A fair coin is flipped, a standard six-sided die is rolled and a card is randomly selected from a standard deck of 52 playing cards. The Venn diagram shown can be used to organize the numbers of outcomes that include flipping heads (H), rolling a 6 (S) and/or selecting a diamond card (D). Use your answers to questions 17 through 19 to fill in this diagram.



17. _____ ways How many ways are there to flip heads, roll a 6 and select a diamond card?

Hint: This is the value of $|H \cap S \cap D|$.

18. _____ ways Since there are $1 \times 1 \times 52 = 52$ ways to roll a 6 and flip heads, how many ways are there to roll a 6 and flip heads but not select a diamond card?

Hint: Use $|S \cap H| = 52$ to find the value of $|(S \cap H) \setminus D|$.

19. _____ ways Since there are $1 \times 6 \times 52 = 312$ ways to flips heads and $13 \times 1 \times 6 = 78$ ways to select a diamond card and flip heads, how many ways are there to flip heads but not select a diamond card and not roll a 6?

Hint: To start, use $|D \cap H| = 78$ to find the value of $|(D \cap H) \setminus S|$. Remember that $|H| = 312$.

20. _____ What is the probability of rolling a 6 and not flipping heads ***or*** flipping heads and not selecting a diamond? Express your answer as a common fraction.

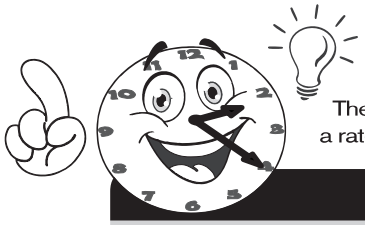
Clocks Stretch

Problems in this stretch involve 12-hour digital and analog clocks, and all time answers should be expressed to the nearest minute, unless otherwise stated.

21. ____ : ____ a.m. What time will it be 47 minutes after 7:37 a.m.?

22. ____ : ____ a.m. What time was it 43 minutes before 9:32 a.m.?

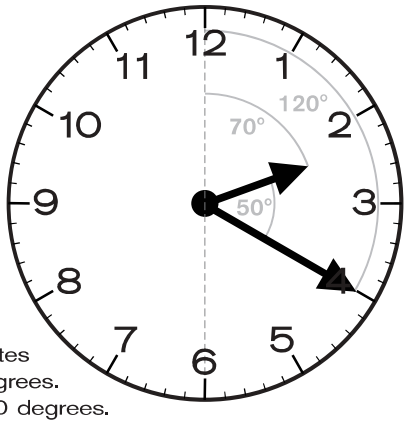
23. ____ seconds A certain clock sounds one chime at 1 o'clock, two chimes at 2 o'clock, three chimes at 3 o'clock, and so on. If this clock behaves in this manner every hour, on the hour so that each chime lasts one second and there is a one-second pause between consecutive chimes, how many seconds long are the chimes that sound at 11 o'clock?



The angle formed by the hour hand and minute hand changes (±) at a rate of $6 - \frac{1}{2} = 11\frac{1}{2} = 5\frac{1}{2}$ degrees each minute.

Rates of Rotation

Minute Hand	Hour Hand
$\frac{360 \text{ degrees}}{1 \text{ hour}} = \frac{6 \text{ degrees}}{1 \text{ minute}} = \frac{1 \text{ degree}}{\frac{1}{6} \text{ minute}}$	$\frac{30 \text{ degrees}}{1 \text{ hour}} = \frac{\frac{1}{2} \text{ degree}}{1 \text{ minute}} = \frac{1 \text{ degree}}{2 \text{ minutes}}$



In the example shown, the time is 2:20. In 2 hours 20 minutes, the hour hand rotates clockwise 70 degrees. In 20 minutes, the minute hand rotates clockwise 120 degrees. The degree measure of the acute angle formed by the clock hands is $120 - 70 = 50$ degrees.

24. ____ degrees What is the degree measure of the acute angle formed by the hour and minute hands at 2:16?

25. ____ degrees What is the absolute difference in the degree measures of the smaller angles formed by the hour and minute hands at 2:08 and 8:02?

26. ____ : ____ After 5:30, when is the next time that the hour and minute hands are aligned so that the angle formed measures 0 degrees?

27. ____ minutes After 4 o'clock, how many minutes elapse between the first and second times that the hour and minute hands form a 38-degree angle? Express your answer as a mixed number.

28. ____ minutes After 3:24, how many minutes have elapsed the first time that the angle formed by the hour and minute hands is twice the measure of the angle formed by the hands at 3:24? Express your answer as a mixed number.

29. ____ What fraction of the times displayed on a digital clock contain the digit 5? Express your answer as a common fraction.

30. ____ times A 24-hour digital clock displays times from 00:00 to 23:59. How many of the times displayed on this clock contain the digit 2?