TALLER 2

Ej. 5:

$$\|(x,y)\| = \max\{|x|, |y|\}$$

Refutar. $v = (1,0)$, $u = (0,1)$

Dada $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$

$$\|(1,1)\|^2 + \|(-1,1)\|^2 = 2(\|(0,1)\|^2 + \|(1,0)\|^2)$$

$$1^2 + 1^2 = 2(1^2 + 1^2)$$

$$\boxed{2 \neq 4}$$ $\rightarrow$ no tiene prod. interno asociado.

Espacio Normado:

$C([a,b]) \longrightarrow \|f\|_\infty = \sup(f),\ x \in [a,b]$

cotas inferiores

$K' < 0 \quad (0,1) \quad \boxed{K} > 1$ $\rightarrow$ cotas superiores

$\sup(0,1) = 1$

$\inf(0,1) = 0$

Ej 8:

$$\rightarrow (n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1} \quad (1) \quad (2n-1)P_{n-1}$$

$n = n-1$

$$\rightarrow nP_n = (2n-1)x\underline{P_{n-1}} - (n-1)P_{n-2} \quad (2) \quad (2n+1)P_n$$

$$(2n+1)nP_n^2 - (2n+1)(n+1)P_{n+1}P_{n-1} =$$

$$(2n-1)nP_{n-1}^2 - (2n+1)(n-1)P_nP_{n-2}$$

integrando

$$(2n+1)\cancel{n}\int_{-1}^{1} P_n^2\,dx = (2n-1)\cancel{n}\int_{-1}^{1} P_{n-1}^2\,dx$$

$$\int_{-1}^{1} P_n^2\,dx = \frac{2n-1}{2n+1}\int_{-1}^{1} P_{n-1}^2\,dx$$

$$\int_{-1}^{1} P_1^2 \, dx = \frac{1}{3}(2)$$

$$\int_{-1}^{1} P_2^2 \, dx = \frac{2(2)-1}{2(2)+1}\left(\frac{2}{3}\right) = \frac{\cancel{3}}{5}\left(\frac{2}{\cancel{3}}\right)$$

$$\int_{-1}^{1} P_n^2 \, dx = \frac{2}{2n+1}$$

$$(P_n, P_m) = \frac{2}{2n+1}\delta_{nm}$$

Ej 2: $$16 \leq (a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

$\forall$ número positivo a, b, c, d.

Cauchy-Schwarz

$$u = (\sqrt{a}, \sqrt{b}, \sqrt{c}, \sqrt{d}) \Rightarrow \|u\|^2 = (a+b+c+d)$$

$$v = \left(\frac{1}{\sqrt{a}}, \frac{1}{\sqrt{b}}, \frac{1}{\sqrt{c}}, \frac{1}{\sqrt{d}}\right)$$

$$|\langle u, v\rangle|^2 \leq \|u\|^2 \|v\|^2$$

$$\underbrace{(1+1+1+1)^2}_{16} \leq (a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

$$16 \leq$$