Mov. Amortiguado

MAS. ⟶ Oscilaciones Libres.

Recordando: $F(v) = bv^n(-\vec{v})$, $n=1$

$F(x) = -kx$

EDO.

$$m\ddot{x} = -kx - b\dot{x}$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

Frec. Ang. Característica / Natural

$$\omega_0^2 = \frac{k}{m}$$

$$\beta = \frac{b}{2m}$$

Parámetro de Amortiguamiento.

$\beta = 0 \rightarrow$ MAS.

$\beta^2 < \omega_0^2 \rightarrow$ Subamortiguado

$\beta^2 = \omega_0^2 \rightarrow$ Críticamente Amortiguado

$\beta^2 > \omega_0^2 \rightarrow$ Sobreamortiguado

$$r_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \rightarrow x(t) = e^{-\beta t}\left[A_1 e^{\sqrt{\beta^2-\omega_0^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega_0^2}\,t}\right]$$

Mov. Subamortiguado

$\beta^2 < \omega_0^2 \rightarrow r_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$

$r_{1,2} = -\beta \pm \underbrace{\sqrt{\omega_0^2 - \beta^2}\, i}_{\omega_1} = -\beta \pm \omega_1 i$

$\omega_1 \rightarrow$ Frec. Ang.

$$x(t) = e^{-\beta t}\underbrace{\left[A_1 e^{i\omega_1 t} + A_2 e^{-i\omega_1 t}\right]}_{\text{M.A.S.}}$$

$$x(t) = \underbrace{A_1 e^{-\beta t}}_{x_{amp}(t)} \underbrace{\cos(\omega_1 t - \delta)}.$$

Diagrama de Fase

$$x(t) = A_1 e^{-\beta t}\cos(\omega_1 t - \delta)$$

$$\dot{x}(t) = -A_1 e^{-\beta t}\left[\beta\cos(\omega_1 t - \delta) + \omega_1 \operatorname{sen}(\omega_1 t - \delta)\right]$$

T.C.

$u(t) = \omega_1 x(t)$

$v(t) = \beta x + \dot{x}$

$$u(t) = \omega_1 A_1 e^{-\beta t} \cos(\omega_1 t - \delta)$$

$$v(t) = -\omega_1 A_1 e^{\beta t} \operatorname{sen}(\omega_1 t - \delta)$$

→ Ver Script Mathematica.

Amortiguamiento Crítico

$$\beta^2 = \omega_0^2 \rightarrow r_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$$\sqrt{\beta^2 - \omega_0^2} \rightarrow 0$$

$$r_{1,2} = -\beta$$

$$X(t) = A_1 e^{-\beta t} + A_2 t e^{-\beta t}$$

$$X(t) = (A_1 + A_2 t) e^{-\beta t}$$

Sobreamortiguado.

$$\beta^2 > \omega_0^2 \rightarrow r_{1,2} = -\beta \pm \underbrace{\sqrt{\beta^2 - \omega_0^2}}_{\omega_2}$$

$$X(t) = e^{-\beta t}\left(A_1 e^{\omega_2 t} + A_2 e^{\omega_2 t}\right)$$