TALLER 10

1) $u_t = k u_{xx}$ $\quad x \in (0,3)$

$u(0,t) = u_x(3,t) = 0$; $u(x,0) = \operatorname{sen} \frac{\pi}{2} x - \operatorname{sen} \frac{5}{6} \pi x.$

$u(x,t) = v(x) w(t)$

$v(x) \dot{w}(t) = k w(t) v''(x)$

$$\frac{\dot{w}(t)}{k w(t)} = \frac{v''(x)}{v(x)} = -\lambda^2$$

a) $\dot{w}(t) + \lambda^2 k w(t) = 0$

b) $v''(x) + \lambda^2 v(x) = 0$

a) $w(t) = C e^{-\lambda^2 k t}$

b) $v(x) = a \cos(\lambda x) + b \operatorname{sen}(\lambda x)$

$$\lambda_n = \frac{(2n+1)\pi}{6}$$

$a = 0$; $v'(3) = b \cos(3\lambda) = 0$

$$u_n(x,t) = \underbrace{c\,b}_{B_n} \operatorname{sen}(\lambda_n x)\, e^{-(\lambda_n)^2 k t}$$

$$u(x,t) = \sum_{n=1}^{\infty} B_n \operatorname{sen}(\lambda_n x)\, e^{-(\lambda_n)^2 k t}.$$

$$u(x,0) = \underbrace{\operatorname{sen}\tfrac{\pi}{2}x - \operatorname{sen}\tfrac{5\pi}{6}x}_{f(x)} = \sum_{n=1}^{\infty} B_n \operatorname{sen}(\lambda_n x)$$

$$B_n = \frac{\int_0^3 f(x) \operatorname{sen}(\lambda_n x)\, dx}{\int_0^3 \operatorname{sen}^2(\lambda_n x)\, dx}$$

$$\lambda_n = \frac{(2n+1)}{6}\pi$$

$$B_1 = 1, \quad B_2 = -1, \quad B_n = 0 \ \forall n \geq 3.$$

$$u(x,t) = \operatorname{sen}\left(\frac{\pi}{2}x\right) e^{-\frac{\pi^2 k t}{4}} - \operatorname{sen}\left(\frac{5}{6}\pi x\right) e^{-\frac{25}{36}\pi^2 k t}$$

2 A autoadjunto y acotado

Usando lema 1.7 Folari-

$$A^2 u = \|A\|^2 u$$

$$\underbrace{A^2 u - \|A\|^2 u}=0$$

$$(A - \|A\|I)(A + \|A\|I)u = 0$$

o bien $(A - \|A\|I)u = 0 \Rightarrow Au = \|A\|u \therefore \lambda = \|A\|$

o $(A + \|A\|I)u = 0 \Rightarrow Au = -\|A\|u \therefore \lambda = -\|A\|$ $\square$

3 $A = (a_{ij}) \in M^{m\times n}$

$$\|A\|_1 = \max_{1\le j\le n} \sum_{i=1}^{m} |a_{ij}|$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\|A\|_\infty = \max_{1\le i\le m} \sum_{j=1}^{n} |a_{ij}|$$

a)

$\|A\|_1 = \max\{|a|+|c|, |b|+|d|\}$ ✓

$\|A\|_\infty = \max\{|a|+|b|, |c|+|d|\}$ ✓

$\|A\|_2 = \sqrt{\rho(A^{\dagger}A)}$ ✓

$\hookrightarrow \max_i\{|\lambda_i|\}$

b) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\|A\|_1 = \max\{|0|+|1|, |1|+|0|\} \boxed{= 1}$.

$\|A\|_\infty = \max\{|0|+|1|, |1|+|0|\} \boxed{= 1}$

$\|A\|_2 \Rightarrow A^{\dagger}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\rho(A^{\dagger}A) = \max\{|1|, |1|\} \boxed{= 1}$