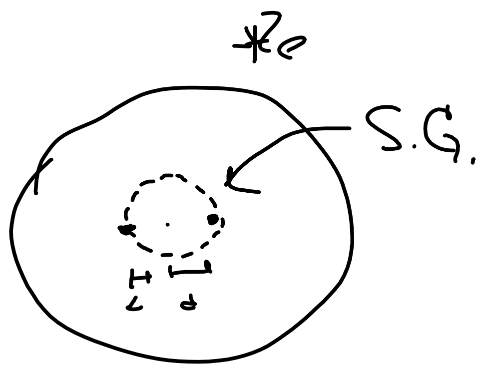
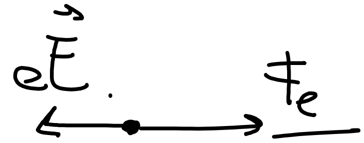
Ej 2)





Ley de Gauss:

$$\rho = \frac{2e}{\frac{4\pi}{\cancel{3}} R^3} = \frac{Q_{enc}}{\frac{4\pi}{\cancel{3}} r^3}$$

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = 2e\left(\frac{r}{R}\right)^3$$

$$E = \frac{e r}{2\pi \epsilon_0 R^3}$$

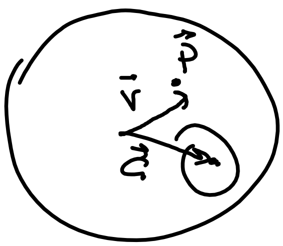
$$\frac{\cancel{e^2} d}{\cancel{2\pi\epsilon_0} R^3} = \frac{\cancel{e^2}}{{}_2\cancel{4\pi\epsilon_0} (2d)^2}$$

$$\frac{4d^3}{R^3} = \frac{1}{2} \longrightarrow \boxed{d = R/2}$$

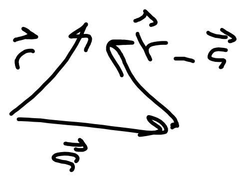
Ej: 4

a) $E(\cancel{4\pi r^2}) = \dfrac{\rho\left(\frac{4}{3}\cancel{\pi} r^{\cancel{3}}\right)}{\epsilon_0} \longrightarrow E(\vec{r}) = \dfrac{\rho \vec{r}}{3\epsilon_0}$

b)



$$\vec{E} = \frac{\rho \cancel{\vec{r}}}{3\epsilon_0} + \frac{(-\rho)(\cancel{\vec{r}} - \vec{a})}{3\epsilon_0}$$



$$= \frac{\rho \vec{a}}{3\epsilon_0}$$

Energía Potencial E.

Def: $$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$ 2 Cargas puntuales.

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{1}{2}\right) \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$$

Potencial Eléctrico

$$V_{(r)} = -\int_{O}^{r} \vec{E} \cdot d\vec{l}. \qquad [V] = \text{Volt}.$$

↑ referencia.

↳ en el infinito.

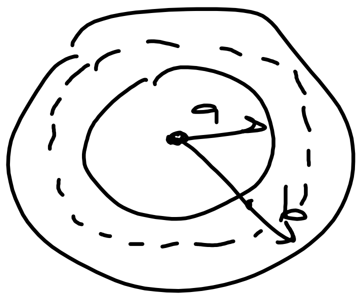
$$V(b) - V(a) = -\int_{O}^{b} \vec{E} \cdot d\vec{l} + \int_{O}^{a} \vec{E} \cdot d\vec{l} = -\int_{a}^{b} \vec{E} \cdot d\vec{l}$$

no son relevantes.

$$\vec{E} = -\vec{\nabla} V$$

gradiente

Ejemplo:



$\rho = \frac{K}{r^2}$, $a \leq r \leq b$.

Potencial en el centro

Para $r<a \rightarrow E=0$.

Para $a<r<b \rightarrow$

$$Q_{enc} = \int_V \rho \, dV = \int_V \frac{K}{\cancel{r^2}} \cancel{r^2} \operatorname{sen}\theta \, dr \, d\theta \, d\phi$$

$$= K \underbrace{\int_a^r dr}_{r-a} \underbrace{\int_0^{\pi} \operatorname{sen}\theta \, d\theta}_{2} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} = K(4\pi)(r-a)$$

$$E(\cancel{4\pi} r^2) = \frac{\cancel{4\pi} k(r-a)}{\epsilon_0} \rightarrow E(r) = \frac{k(r-a)}{\epsilon_0 r^2}$$

$r > b$

$$E(4\pi r^2) = \frac{k}{\epsilon_0} \cancel{4\pi}(b-a) \Rightarrow E = \frac{k(b-a)}{\epsilon_0 r^2}$$

$\boxed{k_e \neq k}$

$$V_{(0)} = -\int_{\infty}^{0} \vec{E} \cdot d\vec{\ell} = -\int_{\infty}^{b} \frac{k(b-a)}{\epsilon_0 r^2} dr - \int_{b}^{a} \frac{k(r-a)}{\epsilon_0 r^2} dr - \cancel{\int_{a}^{0} 0\, dr}$$

$\hookrightarrow \infty$

$$= \frac{k}{\epsilon_0} \frac{(b-a)}{b} - \frac{k}{\epsilon_0} \left[\ln\left(\frac{a}{b}\right) + a \left(\frac{1}{a} - \frac{1}{b}\right) \right]$$

$$= \frac{k}{\epsilon_0} \ln\left(\frac{b}{a}\right)$$

Nota: $\vec{\nabla} \rightarrow$ nabla

Gradiente $\rightarrow$ Razón de cambio $\quad \vec{\nabla} V$

Divergencia $\rightarrow$ Diferencia de flujo $\quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Rotacional $\rightarrow$ Tendencia a rotar $\quad \vec{\nabla} \times \vec{E} = 0$

$dV = dx\,dy\,dz$

$dV = r^2 \operatorname{sen}\theta \, dr \, d\theta \, d\phi$ → azimutal

$0 \leq \phi \leq 2\pi$.

radio

polar

$0 \leq \theta \leq \pi$