

TALLER 2

Ej. 5:

$$\|(x, y)\| = \max\{|x|, |y|\}$$

Refuter. $v = (1, 0), u = (0, 1)$

$$\text{Dada } \|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$$

$$\|(1, 1)\|^2 + \|(-1, 1)\|^2 = 2(\|(0, 1)\|^2 + \|(1, 0)\|^2)$$

$$1^2 + 1^2 = 2(1^2 + 1^2)$$

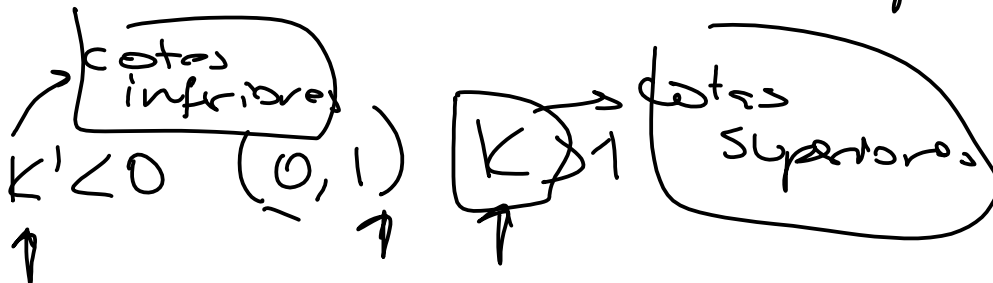
$$\boxed{2 \neq 4}$$

→ no tiene prod. interno asociado.

Espacios Normados:

$C[a, b] \rightarrow$

$$\|f\|_{\infty} = \sup_{x \in [a, b]} f(x)$$



$$\sup(0,1) = 1$$

$$\inf(0,1) = 0$$

Ej 8:

$$\rightarrow (n+1)P_{n+1} = (Z_{n+1}) \times P_n - nP_{n-1} \quad (1) \quad (Z_{n-1})P_{n-1}$$

$$n = n-1$$

$$\rightarrow nP_n = (Z_n - 1) \times \underline{P_{n-1}} - (n-1)P_{n-2} \quad (2) \quad (Z_{n+1})P_n$$

$$(Z_{n+1})nP_n^2 - (Z_{n+1})(n+1)P_{n+1}P_{n-1} =$$

$$(Z_{n-1})nP_{n-1}^2 - (Z_{n+1})(n-1)P_nP_{n-2}$$

integrando

$$(Z_{n+1}) \gamma \int_{-1}^1 P_n^2 dx = (Z_{n-1}) \gamma \int_{-1}^1 P_{n-1}^2 dx$$

$$\int_{-1}^1 P_n^2 dx = \frac{Z_{n-1}}{Z_{n+1}} \int_{-1}^1 P_{n-1}^2 dx$$

$$\int_{-1}^1 P_1^2 dx = \frac{1}{3} (2)$$

$$\int_{-1}^1 P_2^2 dx = \frac{2(2)-1}{2(2)+1} \left(\frac{2}{3} \right) = \frac{2}{5} \left(\frac{2}{3} \right)$$

$$\int_{-1}^1 P_n^2 dx = \frac{2}{2n+1}$$

$$(P_n, P_n) = \frac{2}{2n+1} \delta_{nn}$$

$$E_j: \quad 16 \leq (a+b+c+d) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)$$

\forall numbers positive a, b, c, d .

Cauchy-Schwarz

$$u = (\sqrt{a}, \sqrt{b}, \sqrt{c}, \sqrt{d}) \rightarrow \|u\|^2 = (a+b+c+d)$$

$$v = \left(\frac{1}{\sqrt{a}}, \frac{1}{\sqrt{b}}, \frac{1}{\sqrt{c}}, \frac{1}{\sqrt{d}} \right)$$

$$|\langle u, v \rangle|^2 \leq \|u\|^2 \|v\|^2$$

$$\underbrace{(1+1+1+1)}_{16} \leq (a+b+c+d) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)$$