Oscilaciones Forzadas

$$\vec{f} = -b\dot{x} - kx + F(t)$$

$$\hookrightarrow f_0 \cos(\omega t)$$

$$\rightarrow \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos(\omega t) \quad ; \quad A = f_0/m$$

Sol. Complementaria $\quad$ Particular.

Homogenea

$$\rightarrow \quad x_h(t) = e^{-\beta t}\left[A_1 \exp\left(\sqrt{\beta^2-\omega_0^2}\, t\right) + A_2 \exp\left(-\sqrt{\beta^2-\omega_0^2}\, t\right)\right]$$

$$\rightarrow \quad x_p(t) = D\cos(\omega t - \delta)$$

Reemplazando en la ED.

$$\left(A - D\left[(\omega_0^2 - \omega^2)\cos\delta + 2\omega\beta\sin\delta\right]\right)\cos(\omega t)$$

$$+ \; D\left[(\omega_0^2-\omega^2)\sin\delta - 2\omega\beta\cos\delta\right]\sin(\omega t) = 0$$

Por Independencia Lineal:

$$\operatorname{sen}\delta = \frac{2\omega\beta}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}}$$

$$\cos\delta = \frac{\omega_0^2-\omega^2}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}}$$

$$\tan\delta = \frac{2\omega\beta}{\omega_0^2-\omega^2} \longrightarrow$$

$$D = \frac{A}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}} \longrightarrow$$

Reescribiendo

$$X_p(t) = \frac{A}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}}\cos(\omega t-\delta)$$

con $\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2-\omega^2}\right)$.

Sol. General:

$$X(t) = X_h(t) + X_p(t)$$

Estado Transitorio. ← Estado Estacionario.

$$x(t >> 1/\beta) \simeq x_p(t)$$

Resonancia

Maximizamos D.

$$\left(\frac{dD}{d\omega}\right)_{\omega=\omega_R} = 0$$

$$\Rightarrow \omega_R = \sqrt{\omega_0^2 - 2\beta^2}$$

Amplitude Resonance Frequency.

$\omega_0/2 < \beta$

↳ Rgamos detener Resonancia.

Factor de Calidad.

$$Q = \frac{\omega_R}{2\beta}.$$