TALLER 6

$\boxed{1}$ $$S_n = \sum_{k=1}^{n} \alpha_k e_k \longrightarrow$$

converge
en el espacio

$m > n$

$$\|S_n - S_m\|^2 = \sum_{k=n+1}^{m} \sum_{j=n+1}^{m} \alpha_j \overline{\alpha_k} \cancel{(e_j, e_k)} \to \delta_{jk}$$

$$= \sum_{k=n+1}^{m} \alpha_k \overline{\alpha_k} = \underbrace{\sum_{k=n+1}^{m} |\alpha_k|^2}$$

$\exists\, x \in E$

$$\lim_{n \to \infty} \|S_n - x\| = 0$$

$$\Rightarrow x = \lim_{n \to \infty} S_n$$

$$= \sum_{k=1}^{\infty} \alpha_k e_k \leftarrow$$

Teo. de parseval

$$\|x\|^2 = \sum_{k=1}^{\infty} |\alpha_k|^2 \quad \square$$

$\boxed{2}$

$$f_n(x) = \begin{cases} 0 & 0 \le x \le 1/n \\ x - \frac{1}{n} & 1/n \le x \le 1 \end{cases}$$

$$\lim_{n\to\infty} \| f_n(x) - x \| \quad \text{Si} \quad = 0$$

$$\| \cdot \|_\infty = \sup_{x\in I} \{ | \cdot | \}$$

$$\lim_{n\to\infty} \| \cancel{x} - \frac{1}{n} - \cancel{x} \| = \lim_{n\to\infty} \| -\frac{1}{n} \|$$

0

$$\| 0 \|_\infty = \sup_{x\in I} \{ |0| \} = \boxed{0}$$

$\boxed{4}$ $\boxed{T} = 2L \longrightarrow$ $\qquad\qquad \rightarrow [-L/2, L/2]$ $[0, L]$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi}{L}x + b_n \operatorname{sen}\frac{n\pi}{L}x \right)$$

$a_n, b_n = ?$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(\quad)\,dx \quad \longleftarrow \text{ESTO!}$$

$$\int_{-L}^{L} f(x)\cos\left(\frac{m\pi}{L}x\right)dx = \frac{a_0}{2}\int_{-L}^{L} \cancel{\cos\left(\frac{m\pi}{L}x\right)}\,dx \quad 0$$

$$+\underbrace{\sum_{n=1}^{\infty}\int_{-L}^{L} a_n \cos\left(\frac{n\pi}{L}x\right)\cos\left(\frac{m\pi}{L}x\right)dx}$$

$$+\sum_{n=1}^{\infty}\int_{-L}^{L} \cancel{b_n \cos\left(\frac{m\pi}{L}x\right)\operatorname{sen}\left(\frac{n\pi}{L}x\right)}\,dx \quad 0$$

para $\boxed{m = n}$

$$\int_{-L}^{L} f(x)\cos\left(\frac{m\pi}{L}x\right)dx = a_m \overbrace{\int_{-L}^{L}\cos^2\left(\frac{m\pi}{L}x\right)dx}^{L}$$

$$\frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{m\pi}{L}x\right)dx = a_m$$

$\square$

b_n lo mismo $\sin\left(\frac{m\pi}{L}x\right)$