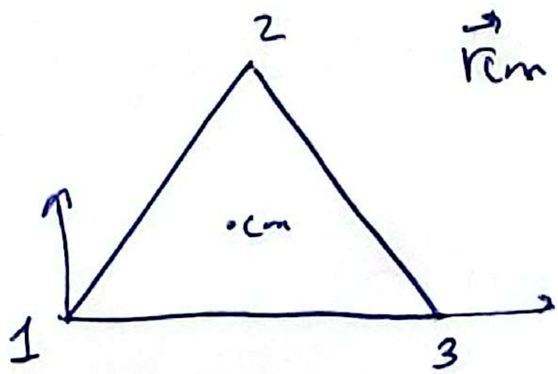
HT1. Sol

1



$$\vec{r}_{cm} = \frac{m\left(\frac{L}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) + m(L\hat{y})}{3m}$$

$$\vec{r}_{cm} = \frac{L}{2}\hat{x} + \frac{\sqrt{3}}{6}\hat{y}$$

$$|r_{cm}| = \frac{L}{\sqrt{3}}$$

$$F_{2,3\to 1} = G\frac{m^2}{L^2}\left(\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\right) + G\frac{m^2}{L^2}\hat{x}$$

$$= G\frac{m^2}{L^2}\left(\frac{3}{2}\right)\hat{x} + G\frac{m^2}{L^2}\left(\frac{\sqrt{3}}{2}\right)\hat{y}$$

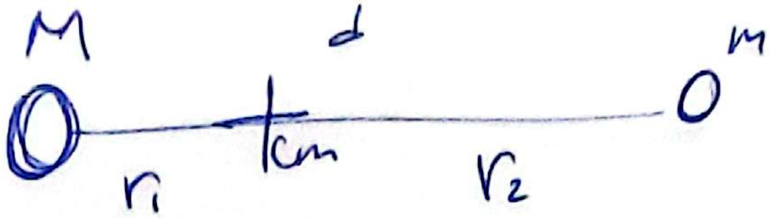
$$|F| = G\frac{m^2}{L^2}\sqrt{3}$$

Fcentripeta $\rightarrow$ $F_c = m\omega^2\frac{L}{\sqrt{3}}$

$$|F| = F_c \Rightarrow G\frac{m^{\cancel{2}}\sqrt{3}}{L^2} = \frac{\cancel{m}\omega^2 L}{\sqrt{3}}$$

$$\boxed{\omega = \sqrt{\frac{3Gm}{L^3}}}$$

2



$$0 = \frac{M r_2 - m r_1}{M+m} \rightarrow M r_2 = m r_1$$

Para cada cuerpo

$$\cancel{M} r_1 \omega^2 = \frac{G\cancel{M}m}{d^2} \quad (*) \; ; \quad \cancel{m} r_2 \omega^2 = \frac{GM\cancel{m}}{d^2} \quad (**)$$

Sumando $(*) + (**)$

$$\underbrace{(r_1+r_2)}_{d}\,\omega^2 = \frac{G(M+m)}{d^2} \quad ; \quad \omega = \frac{2\pi}{T}$$

$$\frac{4\pi^2}{T^2} = \frac{G(M+m)}{d^3}$$

$$\boxed{\therefore\; T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$$

3

Dado que tiene densidad uniforme, una pequeña esfera dentro de la tierra posee la misma densidad que la tierra.

$$\rho = \frac{M_T}{\frac{4}{3}\pi R_T^3} \quad ; \quad \rho = \frac{M^*}{\frac{4}{3}\pi r^3}$$

$$M^* = \left(\frac{r}{R_T}\right)^3 M$$

$$g(r) = \frac{GM^*}{r^2} \Rightarrow \boxed{g(r) = \frac{GMr}{R_T^3}}$$

4

Para la esfera original

$$F = G\frac{Mm}{d^2}$$

Tomamos la contribución de la esfera hueca, tomando la idea del ejercicio anterior

$$M^* = M/8$$

$$\Rightarrow F^* = \frac{G(M/8)m}{(d-R/2)^2} = \frac{GMm}{8d^2(1-R/2d)^2}$$

Fuerza de la esfera hueca sobre 'm'.

$$\vec{F}_h = F - F^* = \frac{GMm}{d^2}\left[1 - \frac{1}{8(1-R/2d)^2}\right]$$