TALLER 12

Recordar:

$$r_{cm} = R = \frac{\sum_{k=1}^{N} m_k \underbrace{r_k}_{\xi}}{M} \; ; \quad M = \sum_{k=1}^{N} m_k$$

$r_k \rightarrow (x_k, y_k, z_k)$

$$R = (X, Y, Z)$$

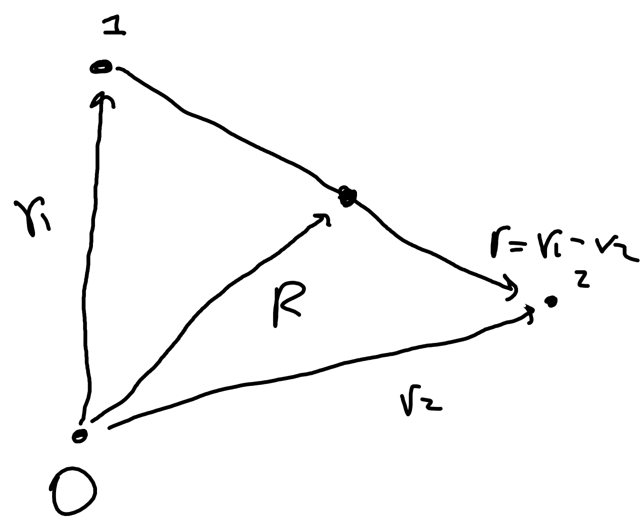
Ec. de movimiento

$$F = M\ddot{R}$$

Problema de los 2 cuerpos.

$\Rightarrow$ Fuerzas Internas $\quad F_1^i = -F_2^i$

$$\begin{cases} m_1 \ddot{r}_1 = F_1^i + F_1^e \\ m_2 \ddot{r}_2 = F_2^i + F_2^e \end{cases}.$$



$r = r_1 - r_2$

$$R = \frac{\sum m_k r_k}{M} \Rightarrow \begin{cases} r_1 = R + \dfrac{m_2}{m_1+m_2}\, r \\[2ex] r_2 = R - \dfrac{m_1}{m_1+m_2}\, r. \end{cases}$$

$$\begin{cases} (m_1+m_2)\ddot{R} = F_1^e + F_2^e \\[2ex] m_1 m_2 \ddot{r} = (m_1+m_2) F_1^i + m_1 m_2 \underbrace{\left(\dfrac{F_1^e}{m_1} - \dfrac{F_2^e}{m_2} \right)}_{0.} \end{cases}$$

$$\frac{F_1^e}{m_1} = \frac{F_2^e}{m_2} \rightarrow \text{Cond.}$$

Reescribir:

$$\begin{cases} \underbrace{(m_1+m_2)}\ddot{R} = \underbrace{F_1^e + F_2^e} \rightarrow \text{Solo el movimiento del centro de masa} \\[2ex] \underbrace{\dfrac{m_1 m_2}{(m_1+m_2)}}\ddot{r} = F_1^i \rightarrow \text{Esto reduce el sistema a uno de 1 sola partícula de masa } \dfrac{m_1 m_2}{m_1+m_2}. \end{cases}$$

$$\mu = \frac{m_1 m_2}{m_1+m_2} \rightarrow \text{Masa Reducida.}$$

Tomar la energía cinética.

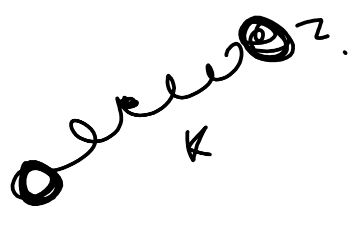
$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M \dot{R}^2 + \frac{1}{2} \mu \dot{r}^2$$

$$\dot{r}_1 = \dot{R} + \frac{\mu}{m_1} \dot{r}$$

$$\dot{r}_2 = \dot{R} - \frac{\mu}{m_2} \dot{r}$$

<u>Ejemplo 1:</u>

Una molécula



a) $$r_{cm_1} = \frac{\cancel{m_1 r_1}^{\,0} + m_2 r_2}{m_1 + m_2} = \frac{m_2 r_2}{m_1 + m_2}.$$

b) Dado que rota alrededor de cm.

$$T = \frac{1}{2} I \omega^2 \Rightarrow T = \frac{1}{2} \mu r_0^2 \omega^2.$$

masa puntual → $m r_0^2$

$\Downarrow$

μ

c) $\mu \ddot{r} = -kr \rightarrow$ MAS.

$\ddot{r} + \frac{k}{\mu} r = 0$

$\hookrightarrow \omega_0^2 \longrightarrow \omega_0 = \sqrt{\frac{(m_1+m_2)k}{m_1 m_2}}$