Problema 65

Dado que $\mu(x)$ aumenta uniformemente

$$\mu(x) = mx + b.$$

por condiciones de frontera.

$$\mu(0) = \mu_0 \quad y \quad \mu(L) = \mu_L$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mu_0 = b \qquad\qquad m = \frac{\mu_L - \mu_0}{L}$$

$$\therefore \mu(x) = \left(\frac{\mu_L - \mu_0}{L}\right) x + \mu_0$$

$$\forall x \in [0, L].$$

El tiempo para que un pulso recorra dx es $\frac{dx}{v}$

$$\rightarrow \int dt = \int_0^L \frac{dx}{v} = \frac{1}{\sqrt{T}} \int_0^L \sqrt{\mu(x)}\, dx$$

$$\Delta t = \frac{1}{\sqrt{T}} \int_0^L \left[\frac{\mu_L - \mu_0}{L} x + \mu_0 \right]^{1/2} dx$$

por sustitución en u.

$$u = \frac{\mu_L - \mu_0}{L} x + \mu_0$$

$$du = \frac{\mu_L - \mu_0}{L} dx$$

$$\Delta t = \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \int_0^L u^{1/2} dx$$

$$\frac{2}{3} \left(\frac{\mu_L - \mu_0}{L} x + \mu_0 \right)^{3/2} \Big|_0^L$$

$$\Delta t = \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \left(\frac{2}{3} \right) \left(\mu_L^{3/2} - \mu_0^{3/2} \right)$$

$$\Delta t = \frac{2L}{3\sqrt{T}} \quad \frac{\mu_L^{3/2} - \mu_0^{3/2}}{\mu_L - \mu_0} \begin{array}{l} \rightarrow \text{Dif. de cubos} \\ \\ \rightarrow \text{Dif. de cuadrados.} \end{array}$$

$$\Delta t = \frac{2L\cancel{(\sqrt{\mu_L} - \sqrt{\mu_0})}(\mu_L + \mu_0 + \sqrt{\mu_L \mu_0})}{3\sqrt{T}\cancel{(\sqrt{\mu_L} - \sqrt{\mu_0})}(\sqrt{\mu_L} + \sqrt{\mu_0})}$$

$$\Delta t = \frac{2L(\mu_L + \mu_0 + \sqrt{\mu_L \mu_0})}{3\sqrt{T}(\sqrt{\mu_L} + \sqrt{\mu_0})}$$