Parcial 1

MC 1

1) $f = \left(\frac{\rho G}{3\pi}\right)^{1/2}$; $\rho = 10^{15}\ kg/m^3$

$\sum F = \cancel{m} a_c = G \frac{M \cancel{m}}{R^2}$ $\quad M = \rho V = \rho \left(\frac{4}{3}\pi R^3\right)$

$a_c = \underbrace{4\pi^2 f^2}_{\omega^2} R$

$4\pi^2 f^2 \cancel{R} = G \frac{\rho \left(\frac{4}{3}\pi R^3\right)}{\cancel{R^2}}$

$\pi f^2 = \frac{G\rho}{3} \longrightarrow f = \sqrt{\frac{G\rho}{3\pi}} = 84.15\ Hz.$

2) $v(0) = 0$

$$\Delta p = \int F\,dt \Rightarrow m v = F t \rightarrow v = \frac{Ft}{m} \qquad (1)$$

Por energía

$$\Delta T = \int F v\,dt = \frac{F^2}{m}\int t\,dt = \frac{1}{2}\frac{F^2 t^2}{m}$$

$$\downarrow$$

$$T_f$$

Otra definición de energía

$$\Delta T = T = \int F\,\underline{dx}$$

$$\frac{dv}{dx}\frac{dx}{dt} = \frac{F}{m} \rightarrow \frac{m}{F}\,dv\,\underbrace{\frac{dx}{dt}}_{v} = dx$$

$$dx = \frac{m}{F} v\,dv$$

$$T = \int \cancel{F}\left(\frac{m}{\cancel{F}} v\right) dv = \frac{1}{2} m v^2 \qquad (2)$$

Sust. (1) en (2)

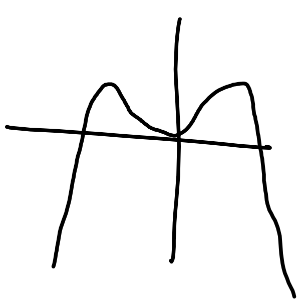
$$T = \frac{1}{2} m \left(\frac{F t}{m}\right)^2 = \frac{1}{2} \frac{F^2 t^2}{m}$$

$\boxed{3}$

$$F(x) = -kx + \frac{kx^3}{a^2}$$

¿$F(x)$ conservativa? $\rightarrow$ Sí.

$$V(x) = -\int -kx + \frac{kx^3}{a^2}\, dx = \frac{1}{2} k x^2 - \frac{1}{4} \frac{k x^4}{a^2}$$



$$t - t_0 = \sqrt{\frac{m}{2}} \int_{x_0}^{x} \left[E - V(x)\right]^{-1/2} dx \; ; \; E = \frac{1}{4} k a^2$$

$$t - t_0 = \sqrt{\frac{m}{2}} \int \left(\frac{1}{4} k a^2 - \frac{1}{2} k x^2 + \frac{1}{4} k \frac{x^4}{a^2} \right)^{-1/2} dx \quad // \text{Fact.} \; \frac{k}{4a^2}$$

$$= \sqrt{\frac{m}{2}} \left(\frac{k}{4a^2} \right)^{-1/2} \int \Big(\underbrace{a^4 - 2x^2 a^2 + x^4}_{(x^2 - a^2)^2} \Big)^{-1/2} dx$$

$$= \sqrt{\frac{m}{2}} \left(\frac{k}{4a^2} \right)^{-1/2} \int \frac{1}{x^2 - a^2} dx \qquad x_0, t_0$$

$$\operatorname{arctan}\left(\frac{x}{a}\right) \qquad \ln\left|\frac{x-a}{x+a}\right|$$