# TALLER 3

## HT1

(3) $\langle u, 0 \rangle$ , $u \in E$

$\langle u, v - v \rangle$, $u, v \in E$

$\langle u, v \rangle - \langle u, v \rangle = 0$ $\quad \therefore \langle u, 0 \rangle = 0$ , $u \in E$

$\in \mathbb{F}$ $\quad \in \mathbb{F}$

1a) $\langle a, b \rangle = a_1 b_1 - a_1 b_2 - a_2 b_1 + K a_2 b_2$

Linealidad $\rightarrow$ $\langle a, b + c \rangle = a_1 (b_1 + c_1) - a_1 (b_2 + c_2)$

$- a_2 (b_1 + c_1) + K a_2 (b_2 + c_2)$

$= (a_1 b_1 - a_1 b_2 - a_2 b_1 + K a_2 b_2)$

$+ (a_1 c_1 - a_1 c_2 - a_2 c_1 + K a_2 c_2)$

$= \langle a, b \rangle + \langle a, c \rangle$ $\square$

Simetría $\rightarrow$ trivial

Semi. Pos.

$$\langle a,a\rangle \geq 0$$

$$\langle a,a\rangle = a_1^2 - 2a_1a_2 + ka_2^2$$

$$\uparrow$$

$k=1$

$k<1$

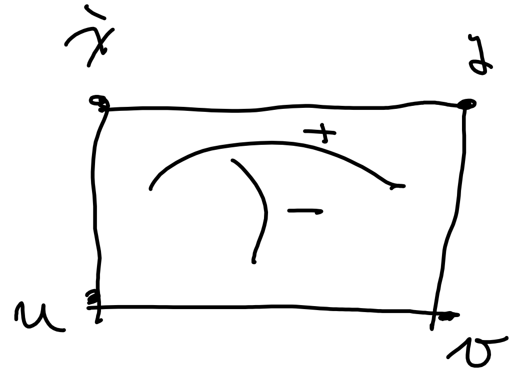
$\forall k \geq 1$

7) $x, y, u, v \in E$

$$|\rho(x,y) - \rho(x,u)| \leq \rho(u,y)$$

$\Leftrightarrow$

$$-\rho(u,y) \leq \underbrace{\rho(x,y) - \rho(x,u) \leq \rho(u,y)}$$




$$\rho(x,y) \leq \rho(u,y) + \rho(x,u)$$

$$\rho(x,y) - \rho(x,u) \leq \rho(u,y)$$

$$\langle \cdot, \cdot \rangle \longrightarrow \|\cdot\| \longrightarrow \rho(\cdot,\cdot)$$

10) $\|\cdot\| : C(a,b) \longrightarrow \mathbb{R}$

$$f(x) \longmapsto \max_{x\in[a,b]} |f(x)|$$

Semi. Pos. $\rightarrow$ $\|f(x)\| > 0$ y $\|f(x)\| = 0$

$\Leftrightarrow f(x) = 0(x)$

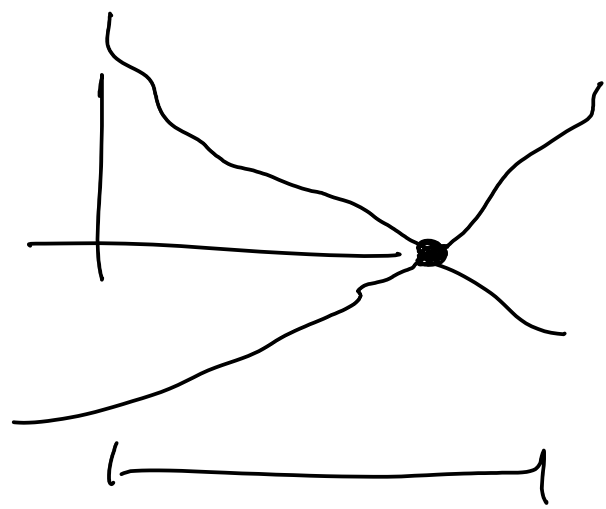
$\downarrow$

trivial

Tomamos $\|f(x)\| = 0$

$0 = \max |f(x)|$

$\Rightarrow f(x) = 0(x)$



Homogeneidad $\rightarrow$ $|\lambda f(x)| = |\lambda| \, |f(x)|$

✓

Des. triangular $\rightarrow$ $\|f(x) + g(x)\| = \max |f(x) + g(x)|$

$\leq \max |f(x)| + \max |g(x)|$

✓

8)

$$\rho(n,m) = \begin{cases} 0, & m = n \\ 1 + \dfrac{1}{n+m}, & m \neq n \end{cases} \quad , n,m \in \mathbb{N}$$

Simetría $\rightarrow$ $\rho(m,n) = \rho(n,m)$ ✓

I.I. $\rightarrow$ $\rho(n,n) = 0$ ✓

Semi. Pos. $\rightarrow$ $\rho(n,m) \geq 0$

Triangular $\rightarrow$ $\rho(n,m) \leq \underbrace{\rho(n,l) + \rho(l,m)}$

$$\underline{1 + \frac{1}{n+m}} \qquad\qquad 1 + \frac{1}{n+l} + 1 + \frac{1}{l+m}$$

$$1 + \frac{1}{2} \rightarrow \frac{3}{2} \leq$$

maximo

$$\delta_{xy} = \rho(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases} \qquad \text{Discreta}$$