Electromagnetismo 2

Escuela de Ciencias Físicas y Matemáticas

1. Identidades y teoremas vectoriales

1.1 $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

1.2 $\int_a^b \boldsymbol{\nabla}\varphi\, d\mathbf{l} = \varphi(b) - \varphi(a)$

1.3 $\int_V \boldsymbol{\nabla}\cdot\mathbf{F}\, dv = \oint_S \mathbf{F}\cdot\mathbf{n}\, da$

1.4 $\int_S \boldsymbol{\nabla}\times\mathbf{F}\cdot\mathbf{n}\, da = \oint_C \mathbf{F}\cdot d\mathbf{l}$

1.5 $\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\varphi = \nabla^2\varphi$

1.6 $\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\mathbf{F} = 0$

1.7 $\boldsymbol{\nabla}\times\boldsymbol{\nabla}\varphi = 0$

1.8 $\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{F} = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

1.9 $\boldsymbol{\nabla}(\varphi\psi) = (\boldsymbol{\nabla}\varphi)\psi + \varphi\boldsymbol{\nabla}\psi$

1.10 $\boldsymbol{\nabla}(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\boldsymbol{\nabla})\mathbf{G} + \mathbf{F}\times(\boldsymbol{\nabla}\times\mathbf{G}) + (\mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F} + \mathbf{G}\times(\boldsymbol{\nabla}\times\mathbf{F})$

1.11 $\boldsymbol{\nabla}\cdot(\varphi\mathbf{F}) = (\boldsymbol{\nabla}\varphi)\cdot\mathbf{F} + \varphi\boldsymbol{\nabla}\cdot\mathbf{F}$

1.12 $\boldsymbol{\nabla}\cdot(\mathbf{F}\times\mathbf{G}) = (\boldsymbol{\nabla}\times\mathbf{F})\cdot\mathbf{G} - (\boldsymbol{\nabla}\times\mathbf{G})\cdot\mathbf{F}$

1.13 $\boldsymbol{\nabla}\times(\varphi\mathbf{F}) = (\boldsymbol{\nabla}\varphi)\times\mathbf{F} + \varphi\boldsymbol{\nabla}\times\mathbf{F}$

1.14 $\boldsymbol{\nabla}\times(\mathbf{F}\times\mathbf{G}) = (\boldsymbol{\nabla}\cdot\mathbf{G})\mathbf{F} - (\boldsymbol{\nabla}\cdot\mathbf{F})\mathbf{G} + (\mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F} - (\mathbf{F}\cdot\boldsymbol{\nabla})\mathbf{G}$

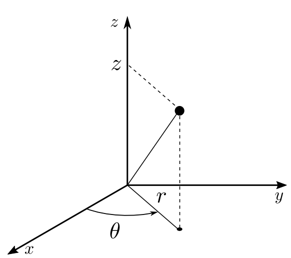
1.15 $\int_S \mathbf{n}\times\boldsymbol{\nabla}\varphi\, da = \oint_C \varphi\, d\mathbf{l}$

1.16 $\int_V \boldsymbol{\nabla}\varphi\, dv = \oint_S \varphi\mathbf{n}\, da$

1.17 $\int_V \boldsymbol{\nabla}\times\mathbf{F}\, dv = \oint_S \mathbf{n}\times\mathbf{F}\, da$

1.18 $\int_V (\boldsymbol{\nabla}\cdot\mathbf{G} + \mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F}\, dv = \oint_S \mathbf{F}(\mathbf{G}\cdot\mathbf{n})\, da$

2. Coordenadas cilíndricas



2.1
$$\begin{aligned}
\hat{\mathbf{r}} &= \cos\theta\,\hat{\mathbf{\imath}} + \operatorname{sen}\theta\,\hat{\mathbf{\jmath}} & \hat{\mathbf{\imath}} &= \cos\theta\,\hat{\mathbf{r}} - \operatorname{sen}\theta\,\hat{\boldsymbol{\theta}}\\
\hat{\boldsymbol{\theta}} &= -\operatorname{sen}\theta\,\hat{\mathbf{\imath}} + \cos\theta\,\hat{\mathbf{\jmath}} & \hat{\mathbf{\jmath}} &= \operatorname{sen}\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\boldsymbol{\theta}}\\
\hat{\mathbf{k}} &= \hat{\mathbf{k}}
\end{aligned}$$

2.2 $\boldsymbol{\nabla}\varphi = \hat{\mathbf{r}}\dfrac{\partial\varphi}{\partial r} + \hat{\boldsymbol{\theta}}\dfrac{1}{r}\dfrac{\partial\varphi}{\partial\theta} + \hat{\mathbf{k}}\dfrac{\partial\varphi}{\partial z}$

2.3 $\boldsymbol{\nabla}\cdot\mathbf{F} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rF_r) + \dfrac{1}{r}\dfrac{\partial F_\theta}{\partial\theta} + \dfrac{\partial F_z}{\partial z}$

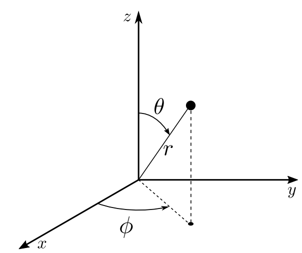
2.4 $\boldsymbol{\nabla}\times\mathbf{F} = \hat{\mathbf{r}}\left(\dfrac{1}{r}\dfrac{\partial F_z}{\partial\theta} - \dfrac{\partial F_\theta}{\partial z}\right) + \hat{\boldsymbol{\theta}}\left(\dfrac{\partial F_r}{\partial z} - \dfrac{\partial F_z}{\partial r}\right) + \hat{\mathbf{k}}\dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(rF_\theta) - \dfrac{\partial F_r}{\partial\theta}\right)$

2.5 $\nabla^2\varphi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\varphi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2\varphi}{\partial\theta^2} + \dfrac{\partial^2\varphi}{\partial z^2}$

armónicos cilíndricos

1	$\ln r$
$r^n\cos n\theta$	$r^{-n}\cos n\theta$
$r^n\operatorname{sen} n\theta$	$r^{-n}\operatorname{sen} n\theta$

3. Coordenadas esféricas



3.1
$$\begin{aligned}
\hat{\mathbf{r}} &= \operatorname{sen}\theta\cos\phi\,\hat{\mathbf{\imath}} + \operatorname{sen}\theta\operatorname{sen}\phi\,\hat{\mathbf{\jmath}} + \cos\theta\,\hat{\mathbf{k}} & \hat{\mathbf{\imath}} &= \operatorname{sen}\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \operatorname{sen}\phi\,\hat{\boldsymbol{\phi}}\\
\hat{\boldsymbol{\theta}} &= \cos\theta\cos\phi\,\hat{\mathbf{\imath}} + \cos\theta\operatorname{sen}\phi\,\hat{\mathbf{\jmath}} - \operatorname{sen}\theta\,\hat{\mathbf{k}} & \hat{\mathbf{\jmath}} &= \operatorname{sen}\theta\operatorname{sen}\phi\,\hat{\mathbf{r}} + \cos\theta\operatorname{sen}\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}\\
\hat{\boldsymbol{\phi}} &= -\operatorname{sen}\phi\,\hat{\mathbf{\imath}} + \cos\phi\,\hat{\mathbf{\jmath}} & \hat{\mathbf{k}} &= \cos\theta\,\hat{\mathbf{r}} - \operatorname{sen}\theta\,\hat{\boldsymbol{\theta}}
\end{aligned}$$

3.2 $\boldsymbol{\nabla}\varphi = \hat{\mathbf{r}}\dfrac{\partial\varphi}{\partial r} + \hat{\boldsymbol{\theta}}\dfrac{1}{r}\dfrac{\partial\varphi}{\partial\theta} + \hat{\boldsymbol{\phi}}\dfrac{1}{r\operatorname{sen}\theta}\dfrac{\partial\varphi}{\partial\phi}$

3.3 $\boldsymbol{\nabla}\cdot\mathbf{F} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2F_r) + \dfrac{1}{r\operatorname{sen}\theta}\dfrac{\partial}{\partial\theta}(F_\theta\operatorname{sen}\theta) + \dfrac{1}{r\operatorname{sen}\theta}\dfrac{\partial F_\phi}{\partial\phi}$

3.4
$$\begin{aligned}
\boldsymbol{\nabla}\times\mathbf{F} = {}& \hat{\mathbf{r}}\frac{1}{r\operatorname{sen}\theta}\left[\frac{\partial}{\partial\theta}(F_\phi\operatorname{sen}\theta) - \frac{\partial F_\theta}{\partial\phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{r}\left[\frac{1}{\operatorname{sen}\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\\
& + \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]
\end{aligned}$$

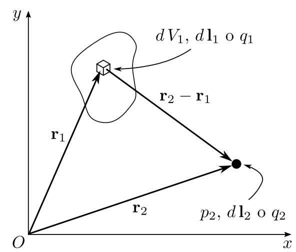
3.5 $\nabla^2\varphi = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\varphi}{\partial r}\right) + \dfrac{1}{r^2\operatorname{sen}\theta}\dfrac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\dfrac{\partial\varphi}{\partial\theta}\right) + \dfrac{1}{r^2\operatorname{sen}^2\theta}\dfrac{\partial^2\varphi}{\partial\phi^2}$

armónicos esféricos $\varphi_n = r^n P_n(\theta)$ o $\varphi_n = r^{-(n+1)} P_n(\theta)$

n	$P_n(\theta)$
0	1
1	$\cos\theta$
2	$\frac{1}{2}(3\cos^2\theta - 1)$
3	$\frac{1}{2}(5\cos^3\theta - 3\cos\theta)$

4. Sistema de coordenadas y vectores

La figura muestra el sistema de coordenadas y los vectores utilizados en las fórmulas de las siguentes secciones.



5. Magnetostática

5.1 $\mathbf{F}_m = \dfrac{\mu_0}{4\pi}\dfrac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}\mathbf{v}_2 \times \left(\mathbf{v}_1 \times \dfrac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}\right)$

5.2 $\mathbf{B}(\mathbf{r}_2, \ldots) = \dfrac{\mu_0}{4\pi}\dfrac{q_1}{|\mathbf{r}_2 - \mathbf{r}_1|^2}\mathbf{v}_1 \times \dfrac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$

5.3 $\mathbf{F_L} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

5.4 $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$

5.5 $\mathbf{m} = I\mathbf{A} = \dfrac{1}{2} I \oint_C \mathbf{r} \times d\mathbf{l}$ (Aquí $\mathbf{A}$ es el vector de área)

5.6 $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

5.7 $\displaystyle\int_{V_1} (\cdots)\,\mathbf{J}(\mathbf{r}_1)\,dV_1 \to \oint_{C_1} (\cdots)\,I_1\,d\mathbf{l}_1$

5.8 $\mathbf{B}(\mathbf{r}_2) = \dfrac{\mu_0}{4\pi}\displaystyle\int_{V_1} \dfrac{\mathbf{J}(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3}\,dV_1$

5.9 $\nabla \cdot \mathbf{B} = 0 \qquad \oint_s \mathbf{B} \cdot \hat{\boldsymbol{n}}\,da = 0$

5.10 $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$

5.11 $\mathbf{B} = \nabla \times \mathbf{A}$

5.12 $\mathbf{A}(\mathbf{r}_2) = \dfrac{\mu_0}{4\pi}\displaystyle\int_{V_1} \dfrac{\mathbf{J}(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|}\,dV_1$

5.13 $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\mathbf{m} \times \mathbf{r}}{|\mathbf{r}|^3}$ (Para $\mathbf{r}$ mucho mayor que las dimensiones del circuito)

5.14 $\mathbf{B} = -\mu_0 \nabla\varphi^*$ con $\nabla^2\varphi^* = 0$ (Para regiones donde $\mathbf{J} = 0$)

5.15 $\mathbf{M} = \lim\limits_{\Delta V \to 0} \sum\limits_i \dfrac{\mathbf{m}_i}{\Delta V}$

5.16 $\mathbf{J}_M = \nabla \times \mathbf{M}, \quad \mathbf{j}_M = \mathbf{M} \times \hat{\mathbf{n}}$

5.17 $\mathbf{A}(\mathbf{r}_2) = \dfrac{\mu_0}{4\pi}\displaystyle\int_{V_1} \dfrac{\mathbf{J}_M(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|}\,dV_1 + \dfrac{\mu_0}{4\pi}\oint_{S_1} \dfrac{\mathbf{j}_M(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|}\,da_1$

5.18 $\varphi^*(\mathbf{r}_2) = -\dfrac{1}{4\pi}\displaystyle\int_{V_1} \dfrac{\nabla_1 \cdot \mathbf{M}(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|}\,dV_1 + \dfrac{1}{4\pi}\oint_{S_1} \dfrac{\mathbf{M}(\mathbf{r}_1) \cdot \hat{\mathbf{n}}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}\,da_1, \quad \nabla^2\varphi^* = \nabla \cdot \mathbf{M}$

5.19 $\mathbf{B}(\mathbf{r}_2) = -\mu_0 \nabla\varphi^*(\mathbf{r}_2) + \mu_0 \mathbf{M}(\mathbf{r}_2)$

5.20 $\mathbf{H} = \dfrac{1}{\mu_0}\mathbf{B} - \mathbf{M}$

5.21 $\mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{B} = \mu\mathbf{H}$

5.22 $\mathbf{H} = -\nabla\varphi^*$

5.23 $\nabla \times \mathbf{H} = \mathbf{J} \qquad \oint_C \mathbf{H} \cdot d\mathbf{l} = I$

5.24 $B_{1n} = B_{2n}, \quad (\mathbf{H}_1 - \mathbf{H}_2)_t = \mathbf{j} \times \hat{\mathbf{n}}$

6. Inducción electromagnética

6.1 $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \qquad \oint_C \mathbf{E} \cdot d\mathbf{l} = -\dfrac{d\Phi}{dt} = \mathcal{E}$

6.2 $L = \dfrac{d\Phi}{dI}$

6.3 $\mathcal{E} = -L\dfrac{dI}{dt}$

6.4 $M_{ij} = \dfrac{d\Phi_{ij}}{dI_j} = M_{ji} \qquad M_{ii} = L_i \qquad M_{ij} = k\sqrt{L_i L_j}, \quad -1 < k < 1$

7. Energía magnética

7.1 $U = \dfrac{1}{2}\sum\limits_i I_i \Phi_i$ con $\Phi_i = \sum\limits_j M_{ij} I_j$

7.2 $U = \dfrac{1}{2}\displaystyle\int_V \mathbf{H} \cdot \mathbf{B}\,dV$

7.3 $\mathbf{F} = \nabla U$ para I constante, $\quad \mathbf{F} = -\nabla U$ para Φ constante

8. Circuitos C.A.

8.1 $V = V_0 e^{i\omega t}, \qquad I = I_0 e^{i\omega t}$

8.2 $V = ZI, \qquad |I_0| = V_0/|Z|$

8.3 $Z_R = R, \qquad Z_L = i\omega L, \qquad Z_C = \dfrac{-i}{\omega C}$

8.4 $\bar{P} = \dfrac{1}{2}|I_0||V_0|\cos\phi$

9. Ecuaciones de Maxwell

9.1 $\boldsymbol{\nabla} \cdot \mathbf{D} = \rho$

9.2 $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$

9.3 $\boldsymbol{\nabla} \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

9.4 $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$

9.5 $u = \dfrac{1}{2}\left(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}\right)$

9.6 $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

9.7 $\boldsymbol{\nabla} \cdot \mathbf{S} + \dfrac{\partial \rho}{\partial t} = -\mathbf{J} \cdot \mathbf{E}$