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Tarea Parcial 1

Clase 13

Ej 37:

Ej 38:

$$F(x,y) = \left(\frac{1}{2}x + \frac{1}{3}y,\ -\frac{1}{2}x + \frac{1}{2}y + 1\right)$$

$$\frac{\partial F_1}{\partial x} = \frac{1}{2} \qquad \frac{\partial F_1}{\partial y} = \frac{1}{3}$$

$$\frac{\partial F_2}{\partial x} = -\frac{1}{2} \qquad \frac{\partial F_2}{\partial y} = \frac{1}{2}$$

$$J = \begin{pmatrix} 1/2 & 1/3 \\ -1/2 & 1/2 \end{pmatrix} \Rightarrow \|J\|_2 \leq \|J\|_F = \frac{\sqrt{31}}{6} < 1$$

$$\Rightarrow \|J\|_2 < 1.$$

es contracción.

Ej 39:

Dem: $\sqrt{ab} \leq \frac{a+b}{2}$

$\Rightarrow (\sqrt{a} - \sqrt{b})^2 \geq 0$

$\hookrightarrow 0 \leq a + b - 2\sqrt{ab}$

$\sqrt{ab} \leq \frac{a+b}{2}$ $\square$

Ej 40:

$$\begin{cases} x_{n+1} = N/y_{n+1} \\ y_{n+1} = (x_n + y_n)/2 \end{cases} ; \; x_0 = 1, \; y_0 = N = 7$$

$n=0$

$$\begin{cases} x_1 = 7/4 = 1.75 \\ y_1 = 4 \end{cases}$$

$n=1$

$$\begin{cases} x_2 = 56/23 \\ y_2 = 23/8 \end{cases}$$

$n=2$

$$\begin{cases} x_2 = 2576/977 \\ y_2 = 977/368 \end{cases}$$

$n=3$

$$\begin{cases} x_3 = 2.64573578 \\ y_3 = 2.645767 \end{cases}$$

$\sqrt{7} \approx y_3 = x_3 = 2.64$

Escaneado con CamScanner

Ej 41:
$$\begin{cases} x_{n+1} = N/y_{n+1} \\ y_{n+1} = (x_n + y_n)/2 \end{cases}$$

Ya encontramos $y_{n+1} = \dfrac{N + y_n^2}{2y_n}$, $y_0 = N$

$$x_{n+1} = \frac{2N y_n}{N + y_n^2}, \text{ pero } y_n = N/x_n$$

$$x_{n+1} = \frac{2N(N/x_n)}{N + (N/x_n)^2}$$

$$\boxed{x_{n+1} = \frac{2N^2 x_n}{N x_n^2 + N^2}, \quad x_0 = 1}$$

Ej 42:
ya se sabe que $x_{n+1} y_{n+1} = N$

Además $\dfrac{x_n + y_n}{2} \geq \sqrt{x_n y_n}$

$$y_{n+1} \geq \sqrt{N}$$

y como $x_{n+1} y_{n+1} = N$ y $y_{n+1} \geq \sqrt{N}$

$\Rightarrow$ es necesario $x_{n+1} \leq \sqrt{N} \leq y_{n+1}$

Ej 43: Tomando $y_{n+1} = \frac{N + y_n^2}{2y_n}$

Por Ied: $y_* = \frac{N + y_*^2}{2y_*}$

$y_*^2 = N \Rightarrow y_* = \pm\sqrt{N}$

Se descarta $-\sqrt{N}$

$\Rightarrow \lim_{n \to \infty} y_n = \sqrt{N}$

Clase 16

Ej 49:

$f = h^{-1} \circ g \circ h$

para $n=2$

$f^2 = (h^{-1} \circ g \circ h)(h^{-1} \circ g \circ h)$

$= h^{-1} \circ g^2 \circ h$

$n=k$

$f^k = h^{-1} \circ g^k \circ h$

$f^{k+1} = (h^{-1} \circ g^k \circ h)(h^{-1} \circ g \circ h)$

$= h^{-1} \circ g^{k+1} \circ h$

Ej 50:

Ej 51:

Reflexividad

$$h \circ \Phi^t \circ h^{-1} \stackrel{?}{=} \Phi^t \quad \text{si} \quad h = Id.$$

$$\Phi^t = \Phi^t$$

Simetria:

$$h \circ \Psi^t \circ h^{-1} = \Phi^t$$

$$h \circ \Psi^t = \Phi^t \circ h$$

$$\Psi^t = \underset{\substack{\downarrow \\ h'}}{h^{-1}} \circ \Phi^t \circ \underset{\substack{\downarrow \\ h'^{-1}}}{h}$$

$$\Psi^t = h' \circ \Phi^t \circ h'^{-1}$$

Transitividad

$$\Phi^t = h \circ \Psi^t \circ h^{-1} \qquad \Psi^t = H \circ \Lambda^t \circ H^{-1}$$

$$\Phi^t = h \circ H \circ \Lambda^t \circ h^{-1} \circ H^{-1}$$

$$= (h \circ H) \Lambda^t \circ (h \circ H)^{-1}$$

$$\downarrow$$

$$\mathbb{H}$$

$$\Phi^t = \mathbb{H} \circ \Lambda^t \circ \mathbb{H}^{-1}$$

Clase 17

53:

$$f(x) = \begin{cases} -2x+2 & x \leq 1 \\ x-1 & x > 1 \end{cases}$$

$x_0 = 1/5$

$$\mathcal{O}_x = \frac{1}{5}, \frac{8}{5}, \frac{3}{5}, \frac{4}{5}, \frac{2}{5}, \frac{6}{5}, \frac{1}{5}, \ldots$$

$$\omega(1/5) = \left\{ \frac{1}{5}, \frac{8}{5}, \frac{3}{5}, \frac{4}{5}, \frac{2}{5}, \frac{6}{5} \right\}$$