Universidad de San Carlos de Guatemala
Escuela de Ciencias Físicas y Matemáticas
Mecánica Estadística
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# Tarea 4

## 1. Problema 1

Dada la varianza $\langle(\Delta E)^2\rangle = \langle(E_i-\varepsilon)^2\rangle = \langle E_i^2\rangle - \varepsilon^2$, entonces, encontramos el segundo momento

$$\frac{\mathrm{d}^2\mathfrak{z}}{\mathrm{d}\beta^2} = \sum E_i^2 e^{-\beta E_i} = \underbrace{\left(\sum p_i E_i^2\right)}_{\langle E_i^2\rangle}\mathfrak{z}. \tag{1}$$

Ahora, tomando la segunda derivada de $\ln\mathfrak{z}$, se tiene

$$\frac{\mathrm{d}^2\ln\mathfrak{z}}{\mathrm{d}\beta^2} = \frac{\mathrm{d}}{\mathrm{d}\beta}\left(\frac{1}{\mathfrak{z}}\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right) = -\frac{1}{\mathfrak{z}^2}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^2 + \frac{1}{\mathfrak{z}}\frac{\mathrm{d}^2\mathfrak{z}}{\mathrm{d}\beta^2},$$

despejando y reemplazando en la fórmula de la varianza

$$\langle(\Delta E)^2\rangle = \frac{\mathrm{d}^2\ln\mathfrak{z}}{\mathrm{d}\beta^2} + \frac{1}{\mathfrak{z}^2}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^2 - \left(-\frac{1}{\mathfrak{z}}\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^2$$

$$\boxed{\langle(\Delta E)^2\rangle = \frac{\mathrm{d}^2\ln\mathfrak{z}}{\mathrm{d}\beta^2}.}$$

## 2. Problema 2

Dada la definición de tercer momento, se realiza la expación

$$\langle(\Delta E)^3\rangle = \sum \underbrace{(E_i-\varepsilon)^3}_{E_i^3-3E_i^2\varepsilon+3E_i\varepsilon^2-\varepsilon^3} p_i = \langle E_i^3\rangle - 3\varepsilon\langle E_i^2\rangle + 3\varepsilon^3 - \varepsilon^3 = \langle E_i^3\rangle - 3\varepsilon\langle E_i^2\rangle + 2\varepsilon^3. \tag{2}$$

Ahora, siguiendo la idea del problema anterior

$$-\frac{1}{\mathfrak{z}}\frac{\mathrm{d}^3\mathfrak{z}}{\mathrm{d}\beta^3} = \langle E_i^3\rangle,$$

entontrando la tercera derivada de $\ln\mathfrak{z}$,

$$\frac{\mathrm{d}^3\ln\mathfrak{z}}{\mathrm{d}\beta^3} = \frac{2}{\mathfrak{z}^3}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^3 - \frac{3}{\mathfrak{z}^2}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)\frac{\mathrm{d}^2\mathfrak{z}}{\mathrm{d}\beta^2} + \frac{1}{\mathfrak{z}}\frac{\mathrm{d}^3\mathfrak{z}}{\mathrm{d}\beta^3}. \tag{3}$$

Sustituyendo (1), (3) y $\varepsilon$ en (2)

$$\langle(\Delta E)^3\rangle = -\frac{\mathrm{d}^3\ln\mathfrak{z}}{\mathrm{d}\beta^3} + \frac{2}{\mathfrak{z}^3}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^3 - \frac{3}{\mathfrak{z}^2}\left(\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)\frac{\mathrm{d}^2\mathfrak{z}}{\mathrm{d}\beta^2} - 3\left(-\frac{1}{\mathfrak{z}}\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)\left(\frac{1}{\mathfrak{z}}\frac{\mathrm{d}^2\mathfrak{z}}{\mathrm{d}\beta^2}\right) + 2\left(-\frac{1}{\mathfrak{z}}\frac{\mathrm{d}\mathfrak{z}}{\mathrm{d}\beta}\right)^3,$$

$$\boxed{\langle(\Delta E)^3\rangle = -\frac{\mathrm{d}^3\ln\mathfrak{z}}{\mathrm{d}\beta^3}.}$$