Hoja de Trabajo 6

Diego Sarceño, 201900109

Calculos realizados para la hoja.

Problema 1

In[ ]:= `n =.`

In[ ]:= `$Assumptions := Element[a, Reals] && Element[p0, Reals] && Element[ℏ, Reals];`

$$\psi 1 = n * \frac{\text{Exp}\left[\frac{p0*x*I}{\hbar}\right]}{\sqrt{x^2 + a^2}}$$

Out[ ]= $$\frac{e^{\frac{i\, p0\, x}{\hbar}}\, n}{\sqrt{a^2 + x^2}}$$

Encontrando la constante de normalización

In[ ]:= $$n = \sqrt{\frac{1}{\text{Integrate}\left[\left(\frac{1}{\sqrt{x^2+a^2}}\right)^2, \{x, -\infty, \infty\}\right]}}$$

Out[ ]= $$\frac{1}{\left(\frac{1}{a^2}\right)^{1/4}\sqrt{\pi}} \quad \text{if } \text{Re}[a^2] \geq 0 \,||\, a^2 \notin \mathbb{R}$$

Ahora, hallamos la probabilidad de encontrar a la partícula en el intervalo $\left(\frac{-a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$.

In[ ]:= $$\text{Integrate}\left[n^2 * \frac{1}{x^2 + a^2}, \left\{x, \frac{-a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right\}\right]$$

Out[ ]= $$\frac{1}{3\sqrt{\frac{1}{a^2}}\; a} \quad \text{if } \left(\text{Re}[a^2] \geq 0 \,||\, a^2 \notin \mathbb{R}\right) \,\&\&\, \text{Re}[a] \geq 0 \,\&\&\, \text{Im}[a] == 0$$

Encontramos el valor esperado del momentum lineal

In[ ]:= $D\left[\sqrt{\frac{a}{\pi}} * \frac{\text{Exp}\left[\frac{p0*x*I}{\hbar}\right]}{\sqrt{x^2+a^2}}, x\right]$

Out[ ]= $-\frac{\sqrt{a}\, e^{\frac{i\, p0\, x}{\hbar}}\, x}{\sqrt{\pi}\, (a^2+x^2)^{3/2}} + \frac{i \sqrt{a}\, e^{\frac{i\, p0\, x}{\hbar}}\, p0}{\sqrt{\pi}\, \sqrt{a^2+x^2}\, \hbar}$

In[ ]:= $\text{Integrate}\left[\sqrt{\frac{a}{\pi}} * \frac{\text{Exp}\left[-\frac{p0*x*I}{\hbar}\right]}{\sqrt{x^2+a^2}} * D\left[\sqrt{\frac{a}{\pi}} * \frac{\text{Exp}\left[\frac{p0*x*I}{\hbar}\right]}{\sqrt{x^2+a^2}}, x\right], \{x, -\infty, \infty\}\right]$

Out[ ]= $\frac{i\, p0}{\hbar}$ if $\text{Re}[a] > 0$

.

Problema 3

Problema 5

Inciso (b).

In[ ]:= $\text{DSolve}\left[\left\{x'[t] == \frac{1}{m} * p[t],\ p'[t] - m * \omega_1 * x[t] == -\frac{1}{2} * m * \omega_2,\ x[0] == a_0,\ p[0] == b_0\right\}, \{x[t], p[t]\}, t\right]$ // ExpToTrig

Out[ ]= $\Big\{\Big\{p[t] \to \frac{1}{4\sqrt{\omega_1}} \left(\text{Cosh}\left[t\sqrt{\omega_1}\right] - \text{Sinh}\left[t\sqrt{\omega_1}\right]\right) \Big(2\, b_0 \sqrt{\omega_1} + 2\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] b_0 \sqrt{\omega_1} + 2\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] b_0 \sqrt{\omega_1} - 2\, m\, a_0\, \omega_1 + 2\, m\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] a_0\, \omega_1 + 2\, m\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] a_0\, \omega_1 + m\, \omega_2 - m\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] \omega_2 - m\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] \omega_2\Big),\ x[t] \to \frac{1}{4\, m\, \omega_1} \left(\text{Cosh}\left[t\sqrt{\omega_1}\right] - \text{Sinh}\left[t\sqrt{\omega_1}\right]\right) \Big(-2\, b_0 \sqrt{\omega_1} + 2\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] b_0 \sqrt{\omega_1} + 2\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] b_0 \sqrt{\omega_1} + 2\, m\, a_0\, \omega_1 + 2\, m\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] a_0\, \omega_1 + 2\, m\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] a_0\, \omega_1 - m\, \omega_2 + 2\, m\, \text{Cosh}\left[t\sqrt{\omega_1}\right] \omega_2 - m\, \text{Cosh}\left[2\, t\sqrt{\omega_1}\right] \omega_2 + 2\, m\, \text{Sinh}\left[t\sqrt{\omega_1}\right] \omega_2 - m\, \text{Sinh}\left[2\, t\sqrt{\omega_1}\right] \omega_2\Big)\Big\}\Big\}$

Problema 6

Constante de normalización

In[ ]:= $n = 1 \Big/ \left(\sqrt{\left(\text{Integrate}\left[\text{Exp}\left[-\left(\frac{\text{Abs}[x]}{a}\right)\right], \{x, -\infty, \infty\}\right] * \text{Integrate}\left[\text{Exp}\left[-\left(\frac{\text{Abs}[y]}{b}\right)\right], \{y, -\infty, \infty\}\right] * \text{Integrate}\left[\text{Exp}\left[-\left(\frac{\text{Abs}[z]}{c}\right)\right], \{z, -\infty, \infty\}\right]\right)}\right)$

Out[ ]= $$1 \Big/ \left(\sqrt{\left(\left(\begin{cases} 2\,a & \text{Re}[a] > 0 \\ \text{Integrate}\left[e^{-\frac{\text{Abs}[x]}{a}}, \{x, -\infty, \infty\}, \text{Assumptions} \to \text{Re}[a] \le 0\right] & \text{True} \end{cases}\right) \left(\begin{cases} 2\,b & \text{Re}[b] > 0 \\ \text{Integrate}\left[e^{-\frac{\text{Abs}[y]}{b}}, \{y, -\infty, \infty\}, \text{Assumptions} \to \text{Re}[b] \le 0\right] & \text{True} \end{cases}\right) \left(\begin{cases} 2\,c & \text{Re}[c] > 0 \\ \text{Integrate}\left[e^{-\frac{\text{Abs}[z]}{c}}, \{z, -\infty, \infty\}, \text{Assumptions} \to \text{Re}[c] \le 0\right] & \text{True} \end{cases}\right)\right)}\right)$$

In[ ]:= $\psi 6 = \frac{1}{\sqrt{8\,a * b * c}}\ \text{Exp}\left[-\left(\frac{\text{Abs}[x]}{2 * a} + \frac{\text{Abs}[y]}{2 * b} + \frac{\text{Abs}[z]}{2 * c}\right)\right]$

Out[ ]= $$\frac{e^{-\frac{\text{Abs}[x]}{2\,a} - \frac{\text{Abs}[y]}{2\,b} - \frac{\text{Abs}[z]}{2\,c}}}{2\,\sqrt{2}\,\sqrt{a\,b\,c}}$$

In[ ]:= $\text{Integrate}\left[\frac{e^{-\frac{\text{Abs}[x]}{a}}}{8\,a\,b\,c}, \{x, 0, a\}\right]$

Out[ ]= $$\frac{\left(1 - e^{-\frac{1}{\text{Sign}[a]}}\right) \text{Sign}[a]}{8\,b\,c}$$

$\text{Integrate}\left[\frac{e^{-\frac{\text{Abs}[y]}{b}}}{2\,a\,b\,c}, \{y, -b, b\}\right]$

Out[ ]= $$\frac{\left(1 - e^{-\frac{\text{Conjugate}[b]\,\text{Sign}[b]}{b}}\right) \text{Sign}[b]}{a\,c}$$

$$\frac{\left(1 - e^{-\frac{\text{Conjugate}[b]\,\text{Sign}[b]}{b}}\right) \text{Sign}[b]}{4\,a\,c}$$

In[ ]:= $\text{Integrate}\left[e^{-\frac{\text{Abs}[z]}{c}}, \{z, -c, c\}\right]$

Out[ ]= $2\,c \left(1 - e^{-\frac{\text{Conjugate}[c]\,\text{Sign}[c]}{c}}\right) \text{Sign}[c]$