46770 Integrated energy grids

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Lecture 3 – Networks

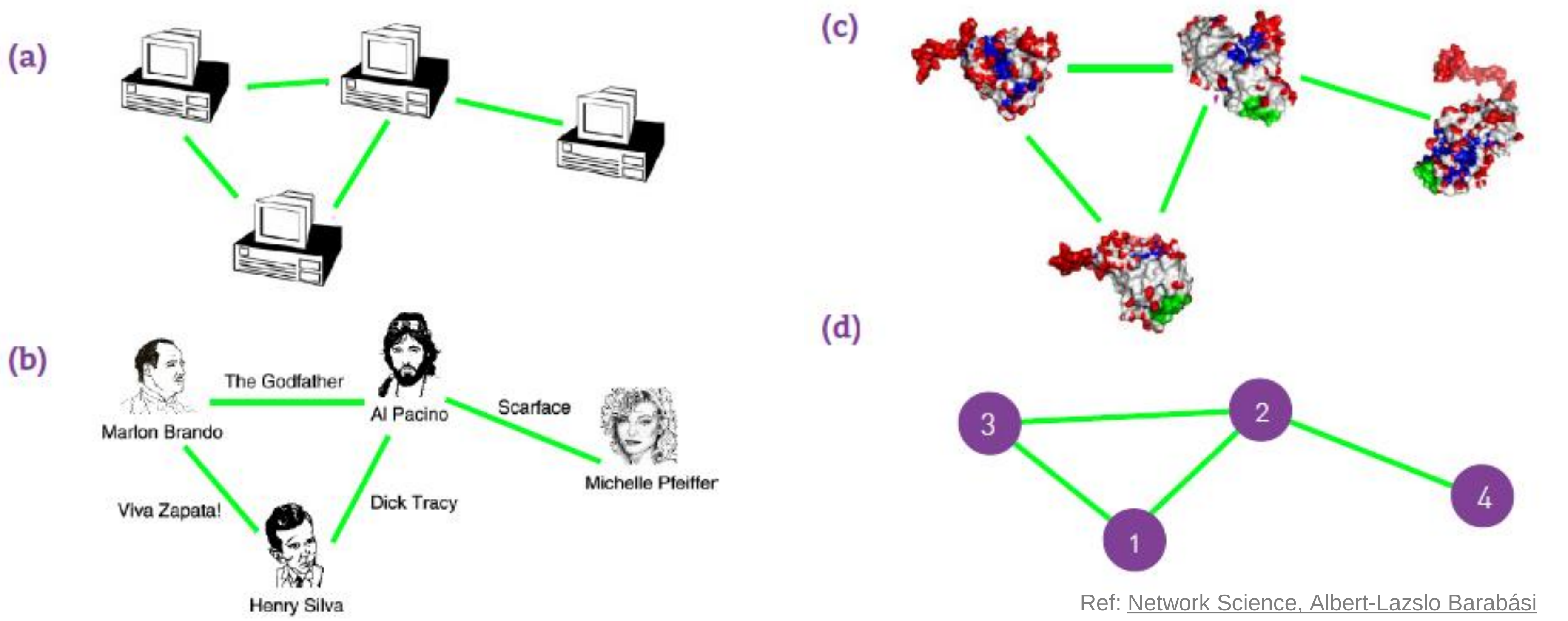
Learning goals for this lecture

- Describe the Degree, Adjacency, and Laplacian (or bus susceptance) matrix.
- Describe the Incidence and Cycle matrix.
- Obtain the Laplacian (or susceptance) matrix describing the topology of a network.
- Describe the objective of power flow analysis.
- Calculate the current flows in a DC network by calculating the voltages using the inverse of the weighted Laplacian matrix
- Calculate the current flows in a DC network by using the Power Transfer Distribution Factors (PTDF) matrix

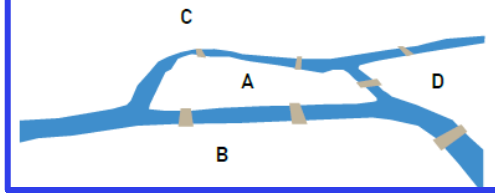
Networks

Networks

A network is a catalogue of a system's components often called nodes (**N**) or vertices and the direct interactions between them, called links (**L**) or edges.



Ref: [Network Science, Albert-Lazslo Barabási]



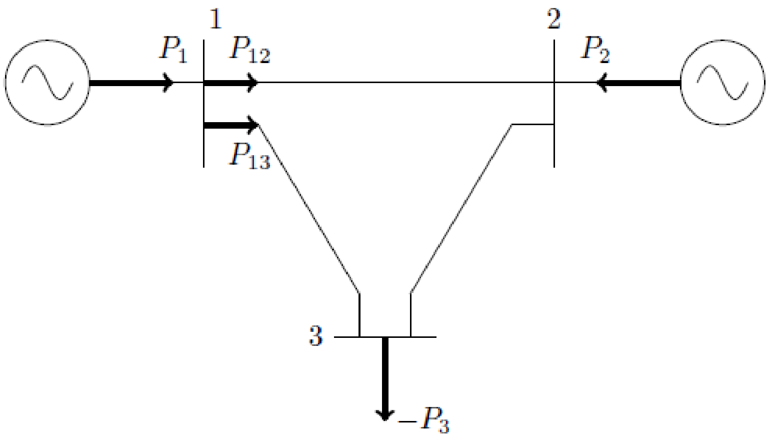
The origin of network science: The Bridges of Königsberg

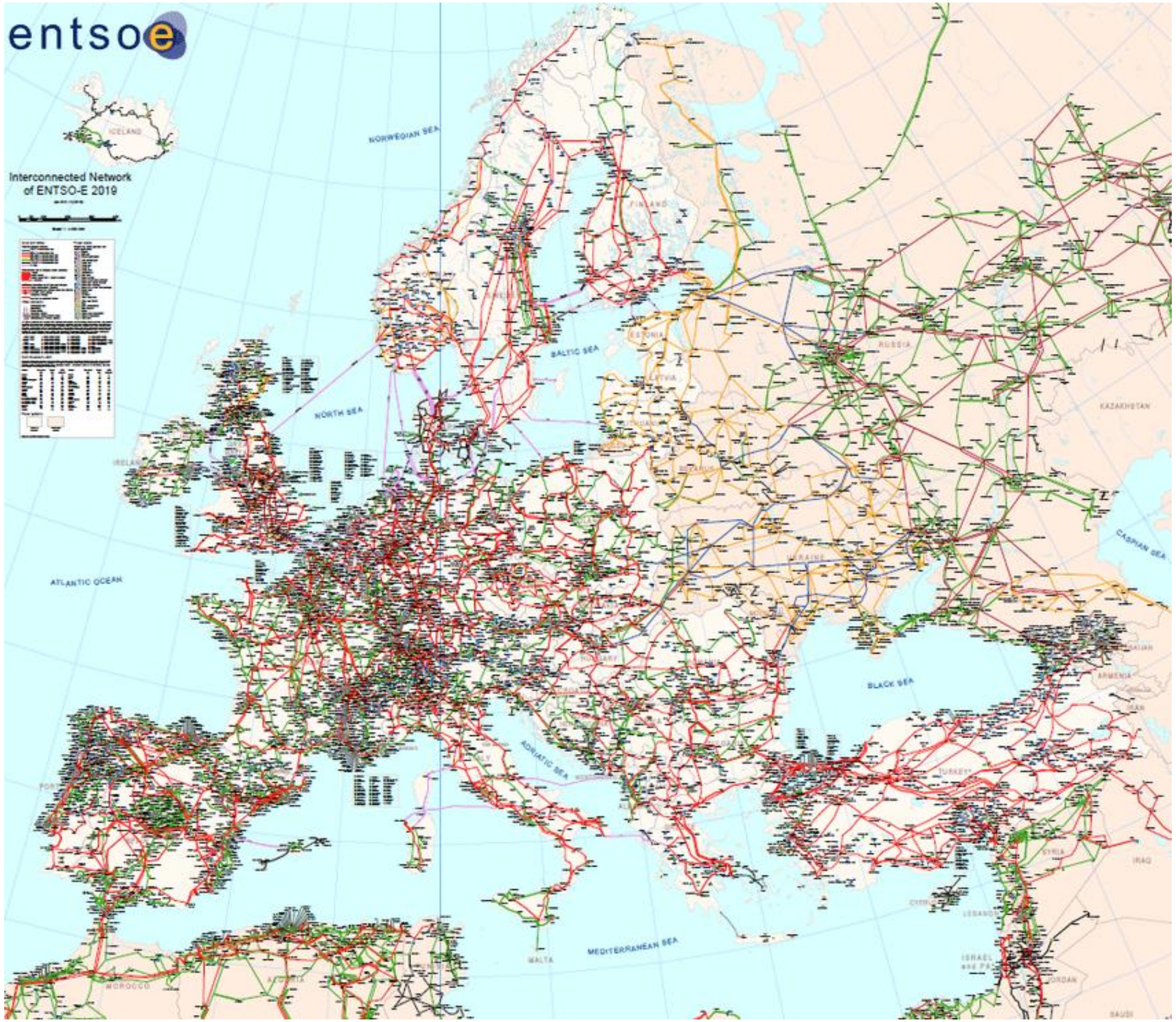
Can one walk across all seven bridges and never cross the same one twice?

https://www.youtube.com/watch?v=nZwSo4vfw6c

Networks

In power systems, nodes (**N**) are called buses and links (**L**) are called lines.

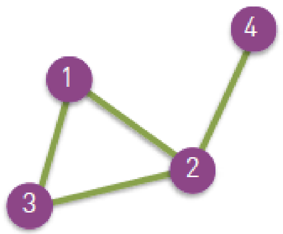




Ref: ENTSOE map

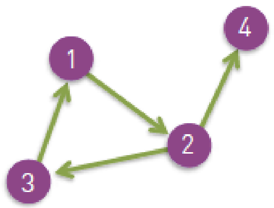
Networks

Undirected



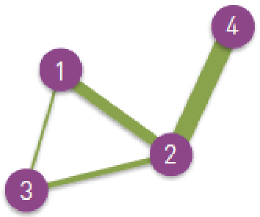
Undirected Network: A network whose links do not have a defined direction. E.g. internet, power grid …

Directed



Directed Network: A network whose links have selected directions. E.g. WWW, citation network…

Weighted
(undirected)



Weighted Network: A network whose links have a defined weight, strength or flow parameter, E.g. power grid, gas networks …

Ref: Network Science, Albert-Lazslo Barabási

Networks

The degree k indicates the number of links that a node has to other nodes.

The average degree $\langle k \rangle$ gives an indication of how meshed is a network.

NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L	⟨k⟩
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

Ref: Network Science, Albert-Lazslo Barabási

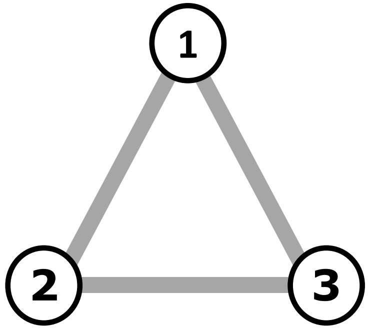
Degree matrix

We are going to define five matrices which are very useful in power networks: **Degree**, Adjacency, Laplacian, Incidence and Cycles matrix.

Degree matrix

$$D_{ij} = \begin{cases} k_i = \sum_{j=1}^{n} A_{ij} & if\ i = j \\ 0 & if\ i \neq j \end{cases}$$

number of links attached to node *i*

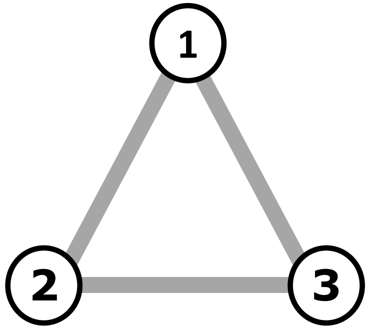


$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Adjacency matrix

We are going to define five matrices which are very useful in power networks: Degree, **Adjacency**, Laplacian, Incidence and Cycles matrix.

Adjacency matrix $$A_{ij} = \begin{cases} 1 & \text{if link between node } i \text{ and } j \text{ exists} \\ 0 & \text{if link between node } i \text{ and } j \text{ does not exist} \end{cases}$$

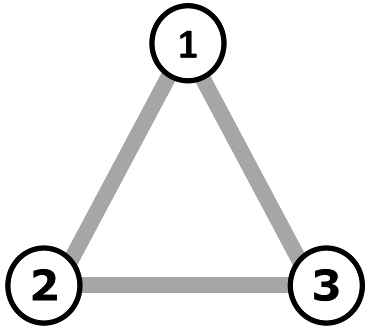


$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Laplacian matrix

We are going to define 5 matrices which are very useful in power networks: Degree, Adjacency, **Laplacian**, Incidence and Cycles matrix.

Laplacian matrix $L_{ij} = D_{ij} - A_{ij}$



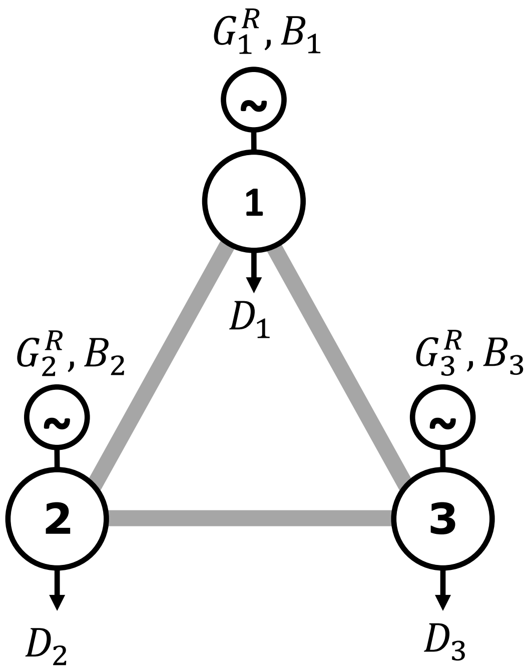
$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} = \left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\right]$$

The columns (and rows) of the Laplacian matrix sum to zero.

The Laplacian matrix is a “map” of the network, it contains information on how the nodes are connected.

Power networks

Power Networks



In every node i, mismatch (i.e., renewable generation G_i^R minus demand D_i), is equal to local balance B_i plus injection P_i :

$$\Delta_i = G_i^R - D_i = B_i + P_i$$

The total sum of power injection is zero (energy conservation):

$$\sum_i P_i = P_1 + P_2 + P_3 = 0$$

Power flow analysis:

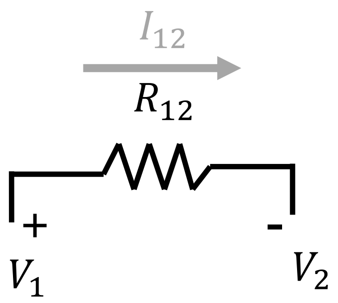
Find the flows in the links of a network given the injection pattern for the nodes.

DC circuits

$$I_{12} = \frac{V_1 - V_2}{R_{12}}$$ Ohm's law, assuming R_{12} =1

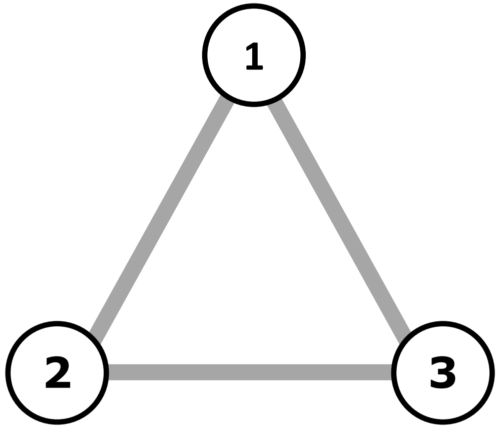
Ohm's law

The electric current through a conductor between two points is directly proportional to the voltage across the two points



$$I_{12} = \frac{V_1 - V_2}{R_{12}}$$

DC power flow in a 3-node network



$$I_{12} = \frac{V_1 - V_2}{R_{12}}$$ Ohm's law, assuming R_{12} =1

$$I_1 = I_{12} + I_{13} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

$$I_2 = I_{21} + I_{23} = (V_2 - V_1) + (V_2 - V_3) = -V_1 + 2V_2 - V_3$$

$$I_3 = I_{31} + I_{32} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \left[\underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{D_{ij}} - \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{A_{ij}} \right] \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$L_{ij} = D_{ij} - A_{ij}$$

Laplacian matrix | degree matrix | adjacency matrix

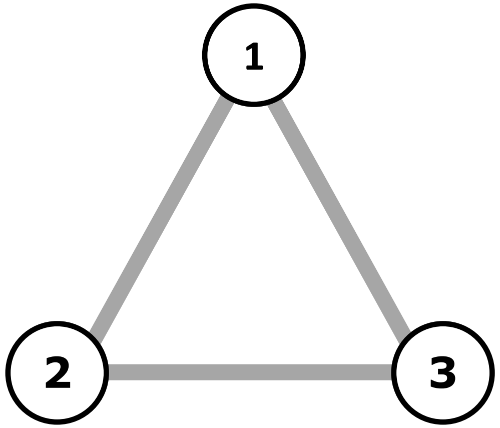
The Laplacian matrix relates the injection I_i and voltages V_j in every node $I_i = \sum_j L_{ij} V_j$

To obtain the currents I_{ij} flowing through the links :

1º. Calculate the voltages V_j using the inverse of the Laplacian matrix.

2º. Calculate the flows I_{ij} using the voltages V_j

DC power flow in a 3-node network



$$I_1 = I_{12} + I_{13} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

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$$I_3 = I_{31} + I_{32} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

There are only two independent variables because $I_1 + I_2 + I_3 = 0$

We select $V_3 = 0$

1º. Calculate the voltages using the inverse of the Laplacian matrix.

$$I_1 = 2V_1 - V_2$$

$$I_2 = -V_1 + 2V_2$$

$$\begin{cases} V_1 = \frac{2}{3} I_1 + \frac{1}{3} I_2 \\ V_2 = \frac{1}{3} I_1 + \frac{2}{3} I_2 \\ V_3 = 0 \end{cases}$$

2º. Calculate the current flows using the voltages

$$I_{12} = (V_1 - V_2)$$

$$I_{13} = (V_1 - V_3)$$

$$I_{23} = (V_2 - V_3)$$

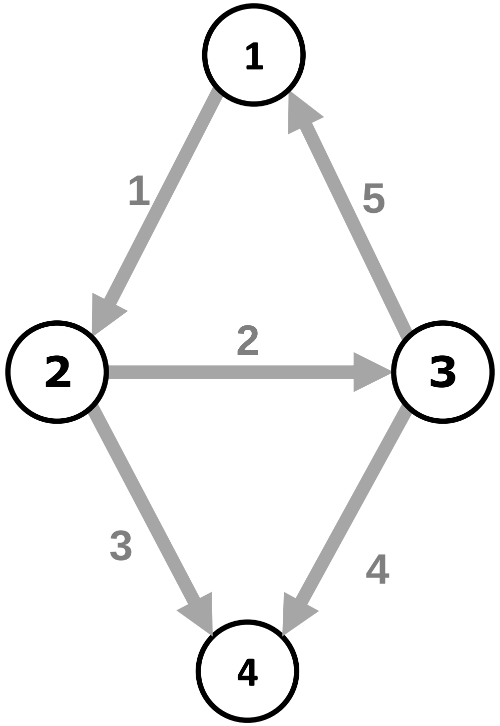
We can also express the result as a matrix that relates the current flows to the current injection

$$\begin{pmatrix} I_{12} \\ I_{13} \\ I_{23} \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

This is equivalent to calculating the PTDF matrix by inverting the Laplacian and multiplying by the incidence matrix

Incidence matrix

Incidence matrix $K_{il} = \begin{cases} 1 & \text{if link } l \text{ starts at node } i \\ -1 & \text{if link } l \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases}$



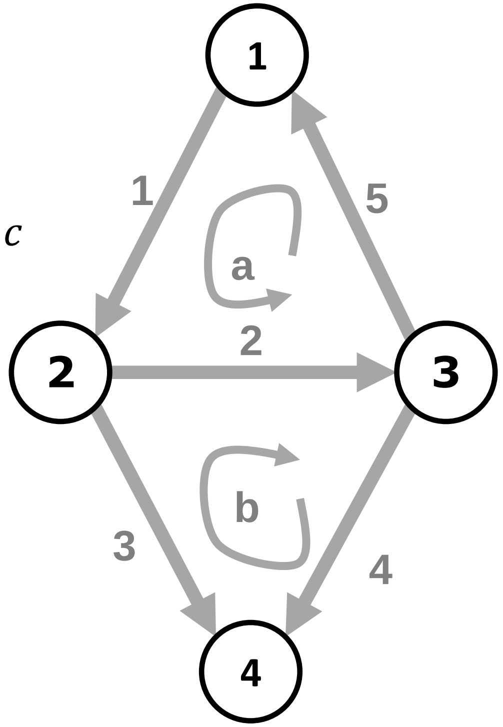
$$K = \begin{array}{c} \text{links:} \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \end{array} \begin{array}{c} \text{nodes} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

The Laplacian and the incidence matrix are related by : $L = KK^T$

Cycle matrix

The cycle matrix contains the closed cycles.

Cycle matrix $C_{lc} = \begin{cases} 1 & \text{if link } l \text{ belongs de cycle } c \\ -1 & \text{if reverse link } l \text{ belongs to cycle } c \\ 0 & \text{otherwise} \end{cases}$



$$\begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ C_{lc} = & \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array} \begin{array}{c} \text{links} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

cycles: a b

The combination of links representing a closed cycle multiplied by incident matrix is zero: $KC = 0$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

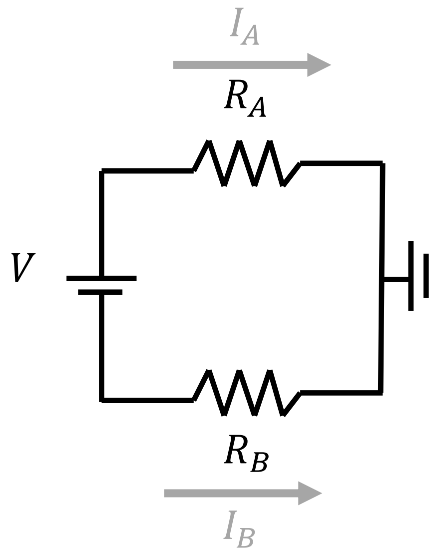
$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} = 0$$

The dimension of the cycle matrix is equal to:
L– N + 1

In the example, 5-4+1=2

Kirchhoff's laws

Currents flowing through a circuit depends on the circuit configuration



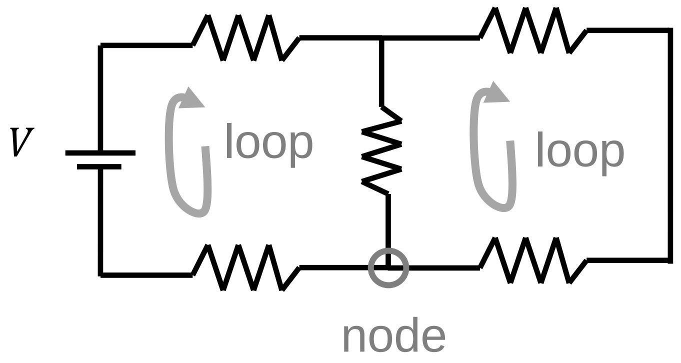
We use Kirchhoff's laws to determine currents in a circuit:

Kirchhoff's Currents Law

The algebraic sum of currents in a network of conductors meeting at a point is zero.

Kirchhoff's Voltage Law

The directed sum of the voltages around any closed loop is zero.



Kirchhoff's laws

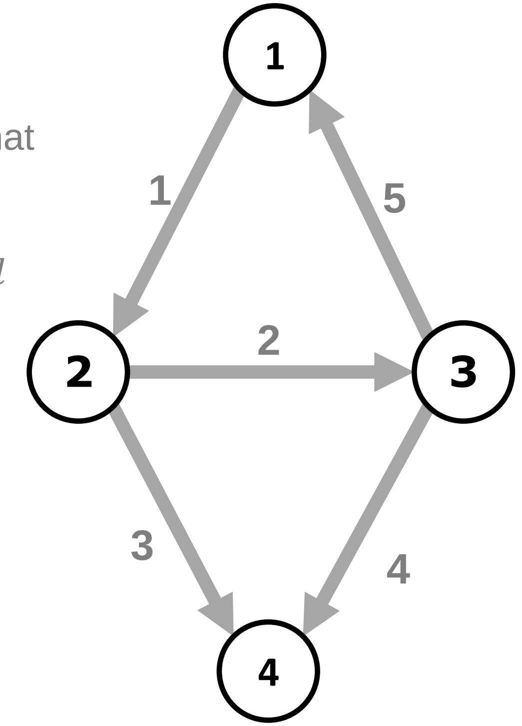
Kirchhoff's Currents Law can be imposed using the incidence matrix

$$I_i = \sum_l K_{il} I_l \qquad \forall i$$

Here, I_i is the current that node i wants to inject, while I_l is the current flowing throughout link l

Kirchhoff's Voltage Law can be imposed using the cycle matrix

$$\sum_l C_{lc} V_l = 0 \qquad \forall c$$



Alternative way of expressing Kirchhoff's Current Law

$$I_l = \frac{1}{R_l} V_l = \frac{1}{R_l} \sum_j K_{lj} V_j$$

$$I_i = \sum_l K_{il} I_l = \sum_l \underbrace{K_{il} \frac{1}{R_l} \sum_j K_{lj}}_{L = KBK^T} V_j$$

B_{kl} is the diagonal matrix of inverse lines series resistance $\qquad B_{ll} = \frac{1}{R_l}$

The weighted Laplacian matrix relates the injected currents I_i and voltages V_j in every node

DC power flow in a 3-node network

Power flow analysis:

Find the flows in the links of a network given the injection pattern for the nodes.

$$I_i = \sum_l K_{il} I_l = \sum_l \underbrace{K_{il} \frac{1}{R_l} \sum_j K_{lj}}_{L = KBK^T} V_j$$

The weighted Laplacian matrix relates the injected currents I_i and voltages V_j in every node $I_i = \sum_j L_{ij} V_j$

To obtain the current flows I_l flowing through the links:

1º. Calculate the voltages using the inverse of the weighted Laplacian matrix. $V_j = \sum_i (L^{-1})_{ji} I_i$

2º. Calculate the current flows I_l using the voltages $I_l = \frac{1}{R_l} \sum_j K_{lj} V_j$

DC power flow in a 3-node network

Power flow analysis:

Find the flows in the links of a network given the injection pattern for the nodes.

An alternative method consists in using the Power Transfer Distribution Factors (PTDF) matrix

$$I_l = \frac{1}{R_l}\sum_j K_{lj}V_j$$

$$I_l = \frac{1}{R_l}\sum_j K_{lj}V_j = \frac{1}{R_l}\sum_{ji} \underbrace{K_{lj}(L^{-1})_{ji}}_{\text{PTDF= transpose incidence matrix times inverse Laplacian matrix}} I_i = \sum_i PTDF_{li}I_i$$

The PTDF matrix measures the sensitivity of power flows in each transmission line relative to incremental changes in nodal power injections throughout the electricity network.

Problems for this lecture

Problems 3.1, 3.2

Review tutorial on networkx

https://martavp.github.io/integrated-energy-grids/intro-python.html

Problems 3.3, 3.4