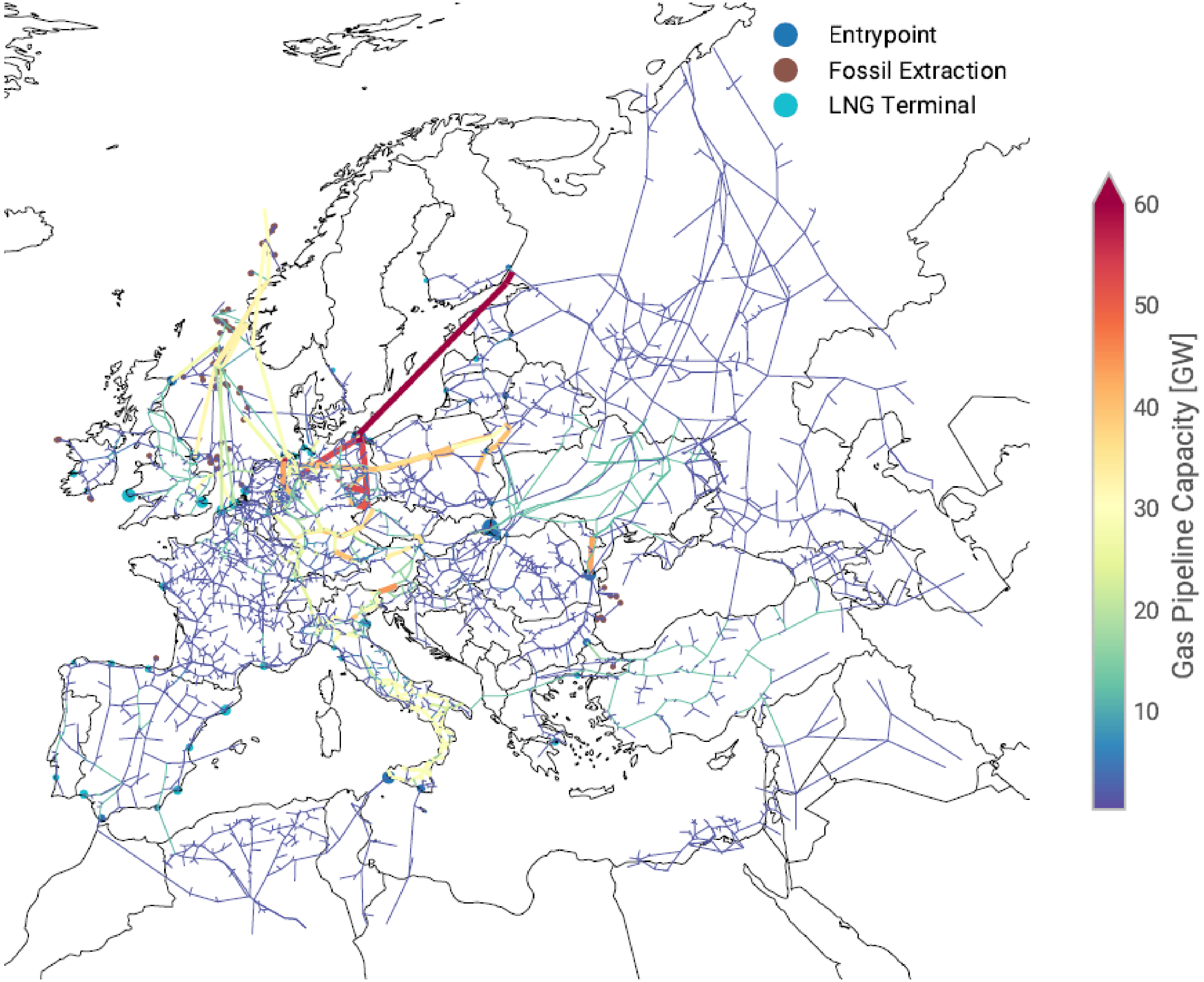
46770 Integrated energy grids

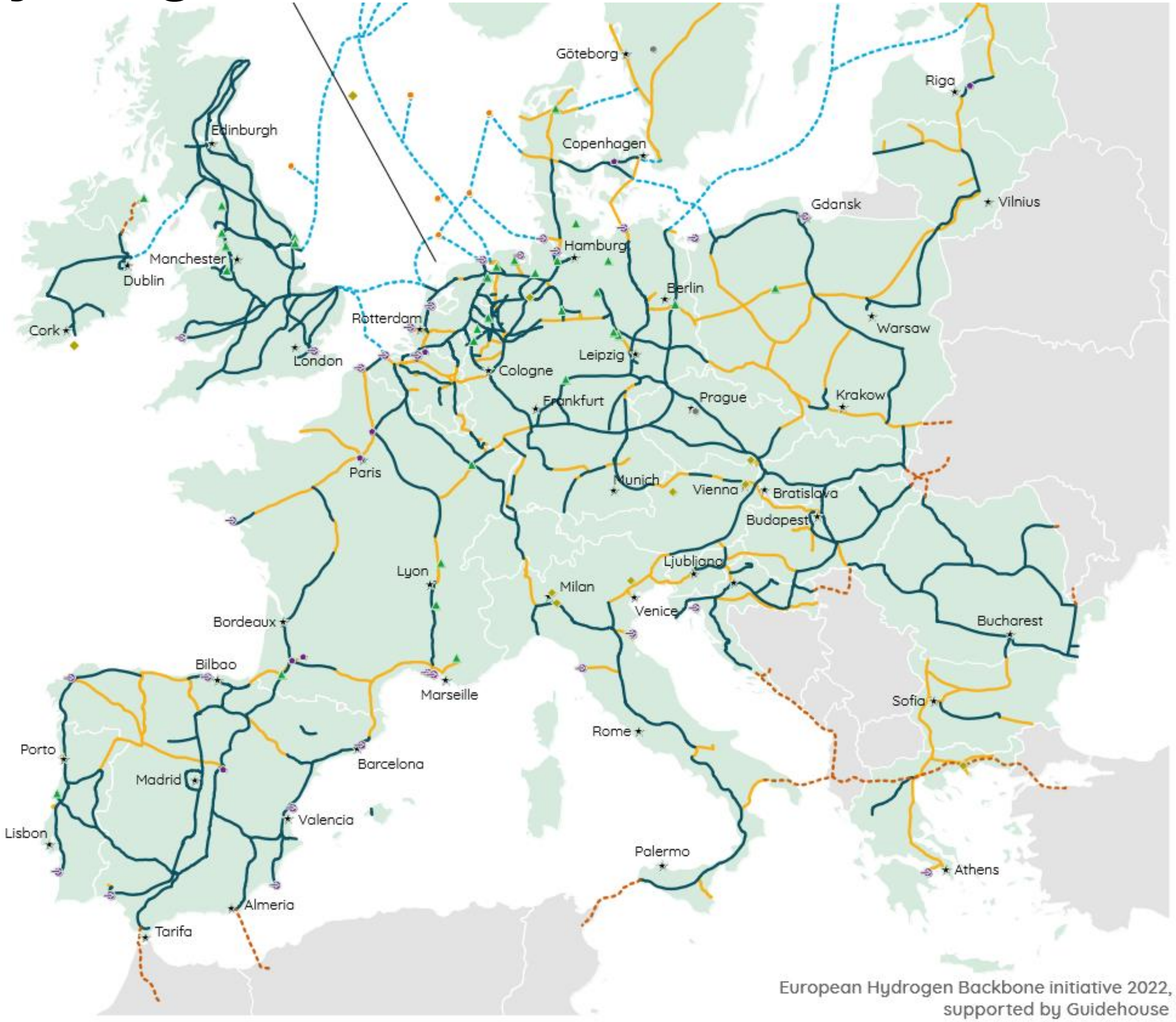
Lecture 6 – Gas Networks

European gas network



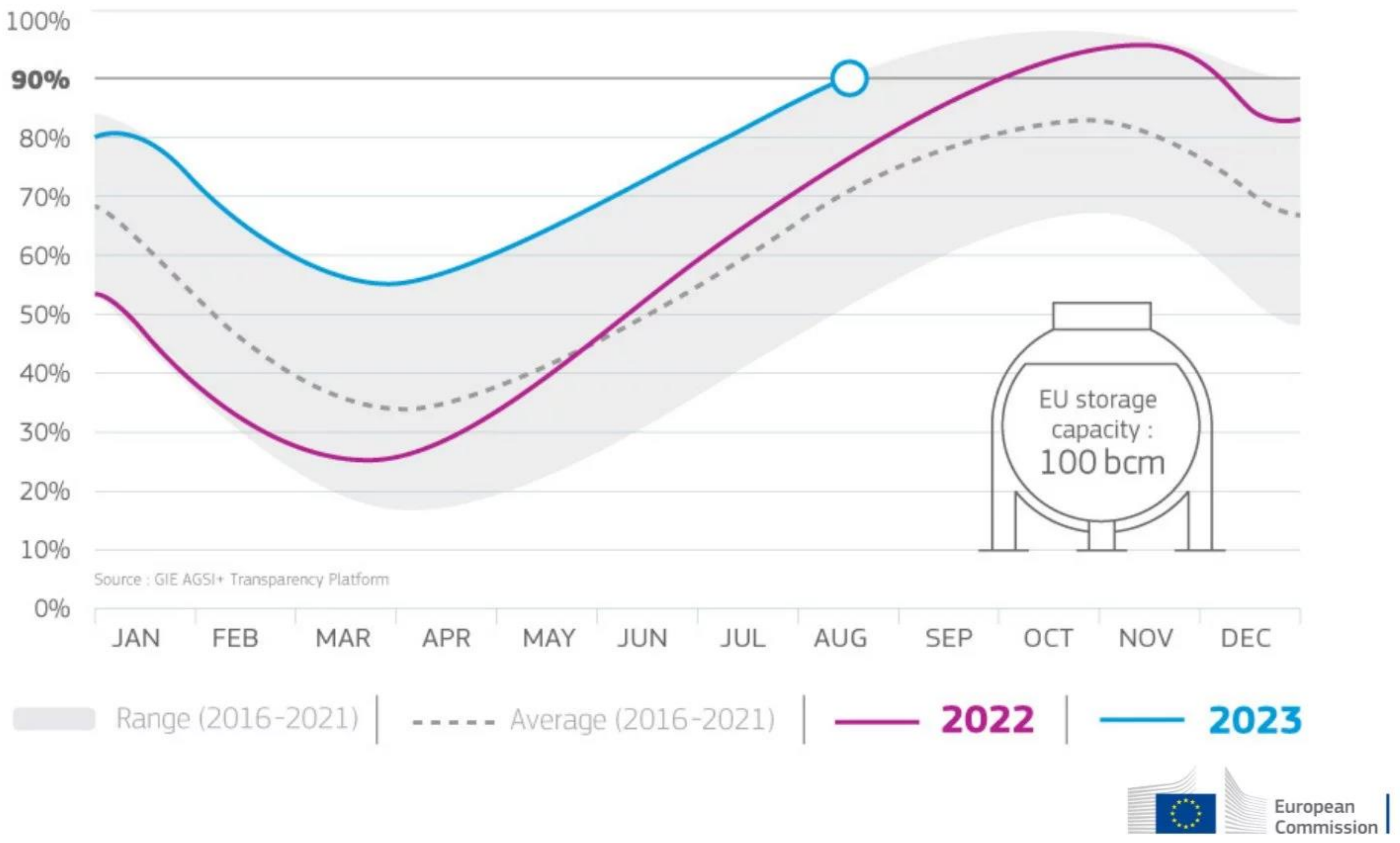
Neuman, Zeyen, Victoria, Brown, Joule, 2023

A potential European hydrogen network



European Hydrogen Backbone initiative 2022, supported by Guidehouse

Gas storage



100%
90%
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Source : GIE AGSI+ Transparency Platform
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
EU storage capacity : 100 bcm
Range (2016-2021)
Average (2016-2021)
2022
2023
European Commission

Learning goals

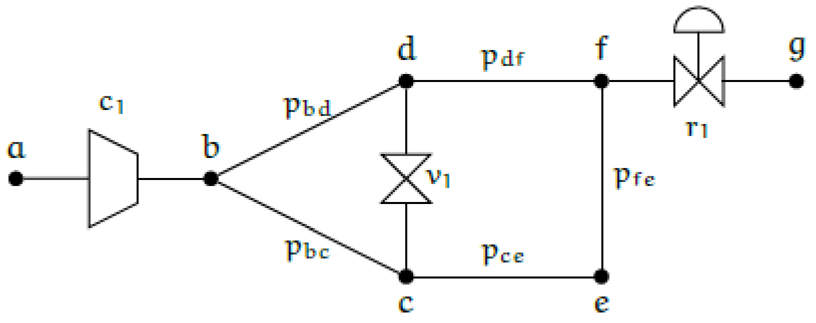
- Calculate the mass flow capacity and energy capacity of a pipeline
- Estimate the energy stored as linepack in a pipeline
- Derive the Weymouth equation that relates pressure and flow rates in gas pipelines
- Write the system cost minimization problem including gas optimal power flow
- Describe approaches the discretize and linearized the gas flow equations

Outline

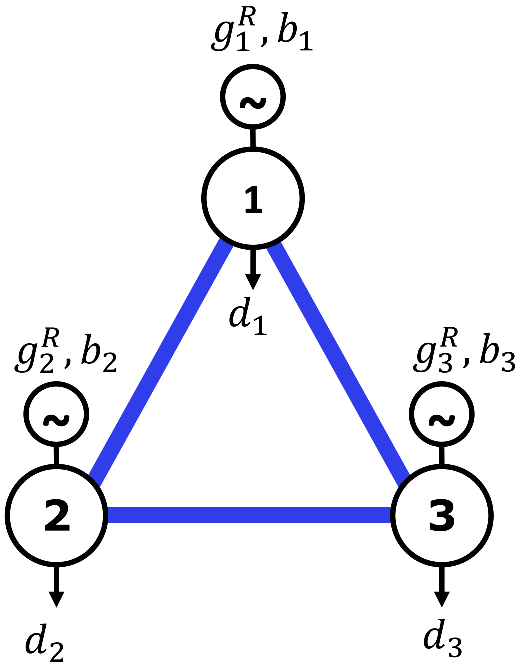
1. Gas networks and components
2. Gas flow in pipelines. Weymouth equation that relates pressure and flow rates in gas pipelines
3. Dynamic, quasi-dynamic, steady-state
4. Linepack
5. Discretization
6. Solve by Newton-Raphson or linearization
7. Optimal gas flow formulation
8. Technologies: CCGT, OCGT, electrolyzers, fuel cells, gas storage

Gas networks and components

Gas network and components



A gas network is directed (because gas flow has a direction) and weighted (because pipelines have different capacities). The network contains links (pipelines) and nodes. In the nodes, we can have gas gas demand (for example, gas power plants), gas supply, compressors (increase pressure), regulators (decrease pressure), valves (reconfigure the network topology by opening or closing).



The gas flows can be **balanced (steady-state conditions)** or **dynamic (non-stationary)**

Linepack is the volume of gas that can be "stored" in a gas pipeline by increasing the pressure. Compare to the fast dynamic of power systems the slow dynamics of gas pipelines function as short-term storage.

Nodal balance and pipeline equations

In every node, there should be a balance between the nodal supply, demand, and the mass flowing in and out.

$$g_{i,t} - d_{i,t} = \sum_j m_{i\rightarrow j} - \sum_j m_{j\rightarrow i}$$

mass nodal balance

Every pipeline has a certain capacity

$$|m_{i\rightarrow j}| \leq M_{i\rightarrow j}$$

Pipeline capacity

The mass flow in a pipeline is related to the pressure difference

FIGURE 2.5: Subpipe Diagram

$$a_{ij} m_{i\rightarrow j}^2 = \pi_i^2 - \pi_j^2$$

In the next slides we will derive the equation that relates mass flows and pressure in a gas network (momentum conservation)

Note: We use π to represents pressure (because we use p for power flows).

Compressors

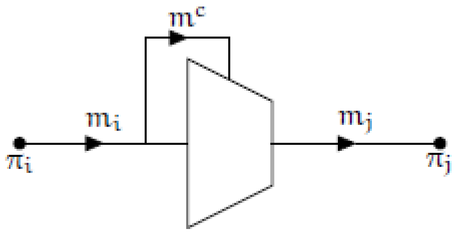
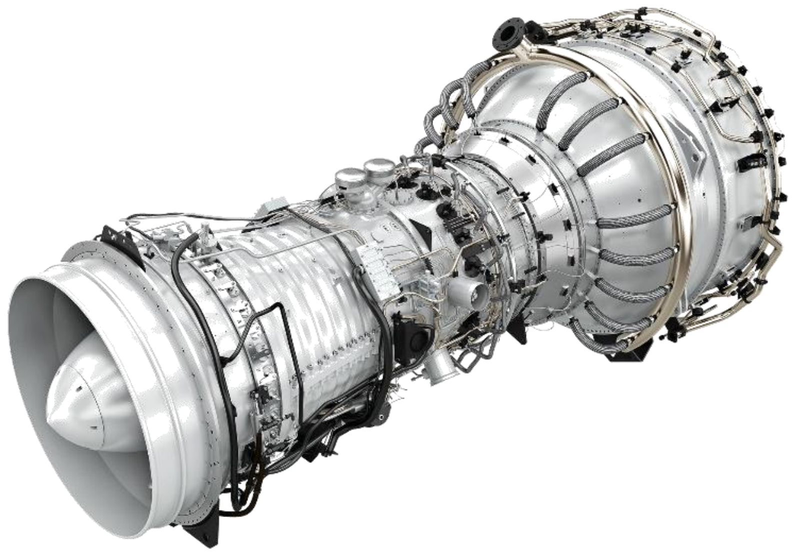


FIGURE 2.7: Compressor Diagram



A compressors increases pressure by $c_{i\rightarrow j,t}$

$$\pi_{j,t} = c_{i\rightarrow j,t}\pi_{i,t}$$

To represent the compressor energy demand, we can follow different approaches

a) Assume that the compressors consumes a certain gas mass flow

$$m_{i\rightarrow j,t}^{compressor} = |m_{j,t}| \quad A_{i\rightarrow j}\left(\left(c_{i\rightarrow j,t}\right)^{\gamma} - 1\right)$$

$$m_{i,t} = m_{i\rightarrow j,t}^{compressor} + m_{j,t}$$

b) Assume that the compressors consumes electricity

c) Assume that a percentage of the energy transported by the pipeline needs to be consumed to maintain the high pressure, typically 2%/1000 km

Gas flow in pipelines

Gas flow in pipelines

Voltage and current in AC transmission lines are equivalent to pressure and mass flow in gas networks

Gas flow in pipelines

Mass flow transport capacity of a gas pipeline

$$m = \rho A u$$

where ρ represents the density, u the velocity of the gas and A the cross-sectional area of the pipeline.

Using the ideal gas equation, density can be expressed as function of the pressure in the pipeline and the speed of sound in gas c

$$\frac{\pi}{\rho} = \frac{ZRT}{M} = c^2 \qquad\qquad m = \rho A u = \frac{\pi}{c^2} A u$$

Energy transport capacity of a gas pipeline

$$q = me = \frac{\pi}{c^2} A u e$$

where e is the energy content in GJ/tonnes or in MWh/kg

Note: m represents a mass flow and q represents an energy flow

How to model gas flow?

Navier-Stokes equations are partial differential equations which describe the motion of fluids.

Navier-Stokes equations in one dimension

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\text{change}} + \underbrace{\frac{\partial (\rho u)}{\partial x}}_{\text{net mass outflow}} = 0$$

mass conservation

$$\underbrace{\frac{\partial (\rho u)}{\partial t}}_{\text{inertia}} + \underbrace{\frac{\partial (\rho u^2)}{\partial x}}_{\text{momentum net inflow}} + \underbrace{\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} - \underbrace{\frac{\partial \tau}{\partial x}}_{\text{Friction force}} - \underbrace{f(x,t)}_{\text{External force}} = 0$$

momentum conservation

where $u(x,t)$ is the velocity of the fluid particle in location x and time step t, $\rho(x,t)$ is the density of the fluid, $\pi(x,t)$ is the pressure, and $f(x,t)$ is some external force per unit area (e.g. gravity)

How to model gas flow?

Navier-Stokes equations are partial differential equations which describe the motion of fluids.

We use **Euler's equations** (a particularization of Navier-Stokes with zero viscosity and zero thermal conductivity) to impose conservation of mass and momentum

Navier-Stokes equations in one dimension

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\text{change}} + \underbrace{\frac{\partial (\rho u)}{\partial x}}_{\text{net mass outflow}} = 0$$

mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$$

$$\underbrace{\frac{\partial (\rho u)}{\partial t}}_{\text{inertia}} + \underbrace{\frac{\partial (\rho u^2)}{\partial x}}_{\text{momentum net inflow}} + \underbrace{\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} - \underbrace{\frac{\partial \tau}{\partial x}}_{\text{Friction force}} - \underbrace{\cancel{f(x,t)}}_{\text{External force}} = 0$$

0

momentum conservation

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + \pi)}{\partial x} + \frac{f^D \rho u |u|}{2D} = 0$$

Main hypotheses

1. We model a single pipe not tilted (gravity neglected) $f(x,t) = 0$

2. Darcy-Weisbach empirical formula is used to represent the pressure loss due to friction $\frac{\partial \tau}{\partial x} = \frac{f^D \rho u |u|}{2D}$

where D is the diameter of the pipe and f^D is the Darcy friction coefficient (also named λ) that depends on the characteristics of the pipe (diameter, roughness) and the fluid (Reynolds number)

where $u(x,t)$ is the velocity of the fluid particle in location x and time step t, $\rho(x,t)$ is the density of the fluid, $\pi(x,t)$ is the pressure, and $f(x,t)$ is some external force per unit area (e.g. gravity)

How to model gas flow?

We use **Euler's equations** to impose conservation of mass and momentum

mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$

momentum conservation

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + \pi)}{\partial x} + \frac{f^D \rho u|u|}{2D} = 0$$

Ideal gas equation

$$\frac{\pi}{\rho} = \frac{ZRT}{M} = c^2$$

where c is the speed of sound in gas

Main hypotheses

3. We consider an ideal gas, and the gas flow is **isothermal**. Hence, pressure and density in linear relation

The velocity written $u|u|$ as to note that the model allows bidirectional flows.

How to model gas flow?

We use **Euler's equations** to impose conservation of mass and momentum

mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0 \qquad \frac{1}{c^2}\frac{\partial \pi}{\partial t} + \frac{1}{A}\frac{\partial m}{\partial x} = 0 \qquad \frac{\partial \pi}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$

momentum conservation

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + \pi)}{\partial x} + \frac{f^D \rho u |u|}{2D} = 0 \qquad \frac{1}{A}\frac{\partial m}{\partial t} + \frac{\partial (\cancelto{0}{\frac{\pi}{c^2}u^2} + \pi)}{\partial x} + \frac{f^D m|m|}{2DA^2 \frac{\pi}{c^2}} = 0 \qquad \frac{\partial m}{\partial t} + A\frac{\partial \pi}{\partial x} + \frac{f^D c^2}{2DA}\frac{m|m|}{\pi} = 0$$

Ideal gas equation

$$\frac{\pi}{\rho} = \frac{ZRT}{M} = c^2$$

Main hypotheses

4. Negligible expansion or contraction of the pipe wall and constant cross-sectional area A, then mass flow $m = \rho u A \qquad \rho u = \frac{m}{A}$

5. We ignore fast transients: the gas flow velocity is much lower than the speed of sound in gas c, $(u \ll c)$

How to model gas flow? Dynamic model

In the **dynamic model** we keep all the elements in the mass and momentum conservation equations

$$\underbrace{\frac{\partial \pi}{\partial t}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\frac{\partial m}{\partial t}}_{\text{inertia}} + \underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f^D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

How to model gas flow? Quasi-dynamic model

In the **quasi-dynamic model,** we consider the inertia term negligible.

$$\underbrace{\frac{\partial p}{\partial t}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\cancel{\frac{\partial m}{\partial t}}}_{\text{inertia}} + \underbrace{A\frac{\partial p}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f^D c^2}{2DA}\frac{m|m|}{p}}_{\text{Friction force}} = 0$$

0

Generally, the inertia term is much smaller than the friction force.

The inertia term is only relevant when fast dynamics occur, for example, when there is a sudden shut-down of a gas-fired power plant.

How to model gas flow? Steady-state model

In the **steady-state model**, we consider steady-state conditions (the derivative with respect to time zero)

$$\underbrace{\cancel{\frac{\partial p}{\partial t}}}_{\text{Linepack change}}^{0} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0 \qquad \frac{\partial m}{\partial x} = 0 \qquad m_{in} = m_{out}$$

In this model, mass conservation equation states that outflow is equal to inflow.

This means that **linepack** is neglected in this model

$$\underbrace{\cancel{\frac{\partial m}{\partial t}}}_{\text{inertia}}^{0} + \underbrace{A\frac{\partial p}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f^D c^2}{2DA}\frac{m|m|}{p}}_{\text{Friction force}} = 0$$

How to model gas flow? Weymouth equation

We apply the **steady-state** momentum conservation to the flow in a pipeline to obtain the equation that relates gas flows and pressure in the optimal gas flow problem. This is known as **Weymouth equation**.

$$\underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f^D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

We approach mass flow and pressure in the pipeline by average values and the pressure derivative as the in-out difference

$$m_{ij} = \frac{m_{ij}^{in} + m_{ij}^{out}}{2} \qquad \pi_{ij} = \frac{\pi_i + \pi_j}{2} \qquad \frac{\partial \pi}{\partial x} = \frac{\pi_i - \pi_j}{2}$$

$$\frac{f^D c^2}{2DA^2} m|m| = \pi \frac{\partial \pi}{\partial x}$$

$$\frac{f^D c^2}{2DA^2} m_{ij}|m_{ij}| = \frac{\pi_i + \pi_j}{2}\frac{\pi_i - \pi_j}{\Delta x}$$

$$\boxed{\frac{f^D c^2}{2DA^2} m_{ij}|m_{ij}| = \frac{(\pi_i^2 - \pi_j^2)}{L}}$$

Weymouth equation

$$a_{ij} m_{ij}^2 = \pi_i^2 - \pi_j^2 \qquad a_{ij} = \frac{L f^D c^2}{DA^2}$$

a_{ij} depends on the physical properties of the pipeline (length L, diameter D, cross-sectional area A), friction coefficient f^D and the speed of sound in gas c

Linepack

Linepack is the volume of gas that can be stored/discharged in a gas pipeline by increasing/decreasing the pressure. This is why changes in average pressure also are referred to as changes in linepack

Linepack is the amount of gas (in kg) pipeline at step t

where π is the average pressure

$$Linepack(kg) = \rho AL = \frac{\pi}{c^2} AL$$

We can also express linepack in MWh by multiplying by the energy content e (in M2Wh/kg or GJ/tonnes) pipeline at step t

$$Linepack(MWh) = \rho ALe = \frac{\pi}{c^2} ALe$$

Mass-conservation equation can be modified to include an explicit representation of Linepack.

We get an equation similar to a storage constraint

$$\frac{\partial \pi}{\partial t} + \frac{c^2}{A} \frac{\partial m}{\partial x} = 0$$

$$\underbrace{\frac{A}{c^2} \frac{\partial \pi}{\partial t}}_{\text{Change in linepack}} = \frac{\partial m}{\partial x}$$

$$Linepack_t = Linepack_{t-1} + \Delta t (m_t^{in} - m_t^{out})$$

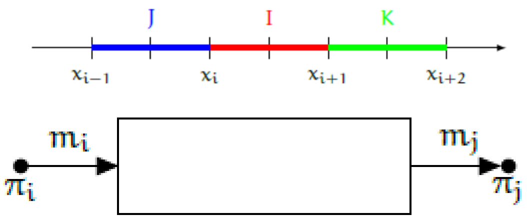
Modelling approaches for power flow in AC networks

Non-discretized pipelines	Discretized pipelines		
	Steady-state	Quasi-dynamic	Dynamic
	$$\frac{\partial m}{\partial x} = 0$$ $$A\frac{\partial p}{\partial x} + \frac{f^D c^2}{2DA}\frac{m\lvert m\rvert}{p} = 0$$	$$\frac{\partial p}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$ $$A\frac{\partial p}{\partial x} + \frac{f^D c^2}{2DA}\frac{m\lvert m\rvert}{p} = 0$$	$$\frac{\partial p}{\partial t} + \frac{c^2}{A}\frac{\partial m}{\partial x} = 0$$ $$\frac{\partial m}{\partial t} + A\frac{\partial p}{\partial x} + \frac{f^D c^2}{2DA}\frac{m\lvert m\rvert}{p} = 0$$
Linear Compressors demand modelled as link efficiency Linepack modelled as energy storage	Can be linearized or not No representation of Linepack	Not good enough to model fast dynamics, e.g shut-down of a gas-fired power plant	

Discretization

Discretization

This system of partial differential equations has no analytical solution so we must use numerical methods to solve them. We discretize the spatial domain, the temporal domain or both. We only need to discretize the pipes by dividing them into segments of equal length Δx and use the finite different methods (cell-centered method). Temporal derivatives are discretivzed using a uniform length Δt



Average pressure and mass flow for subpipe $i \rightarrow j$ at time step t

$$\pi_{ij,t} = \frac{\pi_{i,t}+\pi_{j,t}}{2} \quad m_{ij,t} = \frac{m_{i,t}+m_{j,t}}{2}$$

Partial Differential Equations

$$\underbrace{\frac{\partial \pi}{\partial t}}_{\text{Linepack change}} + \underbrace{\frac{c^2}{A}\frac{\partial m}{\partial x}}_{\text{net mass outflow}} = 0$$

$$\underbrace{\frac{\partial m}{\partial t}}_{\text{inertia}} + \underbrace{A\frac{\partial \pi}{\partial x}}_{\text{Pressure gradient}} + \underbrace{\frac{f^D c^2}{2DA}\frac{m|m|}{\pi}}_{\text{Friction force}} = 0$$

Discrete versions

$$\frac{\pi_{ij,t} - \pi_{ij,t-1}}{\Delta t} + \frac{c^2}{A}\frac{m_{j,t} - m_{i,t}}{\Delta x} = 0$$

$$\frac{m_{ij,t} - m_{i,jt-1}}{\Delta t} + A\frac{\pi_{j,t} - \pi_{i,t}}{\Delta x} + \frac{f^D c^2}{2DA}\frac{m_{ij,t}|m_{ij,t}|}{\pi_{ij,t}} = 0$$

Discretization

In the **steady-state model**, we consider steady-state conditions (the derivative with respect to time zero)

$$\frac{\cancel{\pi_{ij,t}-\pi_{ij,t-1}}^{\,0}}{\Delta t}+\frac{c^2}{A}\frac{m_{j,t}-m_{i,t}}{\Delta x}=0$$

$$\frac{\cancel{m_{ij,t}-m_{ij,t-1}}^{\,0}}{\Delta t}+A\frac{\pi_{j,t}-\pi_{i,t}}{\Delta x}+\boxed{\frac{f^D c^2}{2DA}\frac{m_{ij,t}|m_{ij,t}|}{\pi_{ij,t}}}=0$$

The discrete equations are still not linear.

1. We can use Newton-Raphson algorithm
2. We can use linear approximations

1.a We can approximate the non-linear term with values from the previous time step

$$\frac{m_{ij,t}|m_{ij,t}|}{\pi_{ij,t}}\approx\frac{m_{ij,t-1}|m_{ij,t-1}|}{\pi_{ij,t-1}}$$

2. We can approximate the non-linear term using firs order Taylor series expansion

$$\frac{m_{ij,t}|m_{ij,t}|}{\pi_{ij,t}}\approx\frac{|m_{ij,t-1}|}{\pi_{ij,t-1}}\left(2m_{ij,t}-\frac{m_{ij,t-1}}{\pi_{ij,t-1}}\pi_{ij,t}\right)$$

Optimal gas flow

Nodal balance and pipeline equations

We recall here the nodal balance and pipeline equations.

In every node, there should be a balance between the nodal supply, demand, and the mass flowing in and out.

$$g_{i,t} - d_{i,t} = \sum_j m_{i \to j} - \sum_j m_{j \to i}$$

mass nodal balance

We can use the incidence matrix to compute all the pipelines connected to a node.

$$g_{i,t} - d_{i,t} = \sum_l K_{il} m_l$$

Every pipeline has a certain capacity

$$|m_{i \to j}| \leq M_{i \to j}$$

Pipeline capacity

The mass flow in a pipeline is related to the pressure difference

$$a_{ij} m_{i \to j}^2 = \pi_i^2 - \pi_j^2 \qquad a_{ij} = \frac{L f^D c^2}{D A^2}$$

Momentum conservation in pipelines

where m_i represents the injection in node i and a_{ij} depends on the physical properties of the pipeline (length L, diameter D, cross-sectional area A), friction coefficient f^D and the speed of sound in gas c

Economic dispatch with gas optimal power flow

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_S. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_n in a certain hour and the optimal gas flows while minimizing the total system cost.

Economic dispatch in one node

$$\left\{\begin{aligned} &\min_{g_s} \sum_s o_s g_s \\ &\text{subject to:} \\ &\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda \\ &0 \leq g_s \leq G_s \end{aligned}\right.$$

Economic dispatch with AC power flow

$$\left\{\begin{aligned} &\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ &\text{subject to:} \\ &\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} m_l \quad \leftrightarrow \quad \lambda_i \\ &0 \leq g_s \leq G_s \\ &|m_l| \leq M_l \\ &|q_l| \leq Q_l \\ &a_{ij} m_{ij}^2 = \pi_i^2 - \pi_j^2 \quad a_{ij} = \frac{L f^D c^2}{D A^2} \end{aligned}\right.$$

Nodal power balance

Pipelines capacities

m represents a mass flow and q represents an energy flow (i.e. $q = m \cdot u$ where u is the energy content in MWh/kg or GJ/tonne

Technologies connecting power and gas networks

Technologies connecting power and gas networks

CCGT

OCGT

CHP

Electrolyzers

Fuel cells

Gas storage: salt caverns + steel tanks

Add descriptions

DTU