46770 Integrated energy grids

Marta Victoria

Lecture 8 – Join Capacity and dispatch optimization in one node

Learning goals for this lecture

– Add learning goals

The first part of this lecture provides a review of the electricity market and shows why maximizing profits for independent generators is equivalent to maximizing social welfare.

For a comprehensive discussion and demonstration that the market-clearing problem obtains the Nash equilibrium solution (i.e. no market player desires to deviate from the market-clearing results), check the nice DTU course 46755 Renewables in Electricity Markets

Economic dispatch

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_s . Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d in a certain hour while minimizing the total system cost.

$$\left\{\begin{array}{l} \min\limits_{g_s} \sum\limits_s o_s g_s \\ \\ \text{subject to:} \\ \sum\limits_s g_s - d = 0 \quad \leftrightarrow \quad \lambda \\ \quad g_s \geq 0 \; \leftrightarrow \; \underline{\mu_s} \\ -g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_s} \end{array}\right.$$

For renewable generators, the installed capacity is multiplied by the capacity factor: $g_s - CF_s G_s \leq 0 \; \leftrightarrow \overline{\mu_s}$

	Wind	Solar	Gas
Variable cost o_s (EUR/MWh)	0	0	50
Installed Capacity G_s (MW)	2	1	1
CF_s	0.5	0.5	1
generation g_s (assuming demand d = 1.5 MWh)	1	0.5	0
generation g_s (assuming demand d = 2 MWh)	1	0.5	0.5

We will explore further this topic in Lecture 8.

Economic dispatch

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_s . Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d in a certain hour while minimizing the total system cost.

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda$$

$$g_s \geq 0 \;\leftrightarrow\; \underline{\mu}_s$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu}_s$$

To find a solution, we start by building the Lagrangian function

$$\mathcal{L}(x, \lambda, \mu) = f(x) - \sum_i \lambda_i [h_i(x) - c_i] - \sum_j \mu_j [g_j(x) - d_j] = \sum_s o_s g_s - \lambda \, (d - \sum_s g_s) - \sum_s \mu_s(-g_s - G_s)$$

We derive the Lagrangian and make the derivative equal to zero

$$\frac{\partial \mathcal{L}}{\partial g_s} = \frac{\partial f}{\partial g_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_s} - \sum_j \mu_j \frac{\partial g_j}{\partial g_s} = o_s - \lambda^* + \overline{\mu}_s^* = 0$$

The inequality constraint can be binding ($\mu_s > 0$) when the installed capacity is limiting the generation or not-binding ($\mu_s = 0$).

The most expensive generator s whose capacity is not binding sets the price

$$\overline{\mu}_s^* = 0$$

$$\lambda^* = o_s$$

We will explore further this topic in Lecture 8.

Economic dispatch

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_s . Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d in a certain hour while minimizing the total system cost.

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda$$

$$g_s \geq 0 \leftrightarrow \underline{\mu}_s$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu}_s$$

λ represents the change in the objective function at the optimal solution, with respect to a small change in the constraint.

Small change in constraint : $d^* = d^* + 1\ \mathrm{MWh}$

Change in objective function : $System\ cost^* = System\ cost + \Delta System\ cost$

λ represents the cost of 1 MWh, i.e. spot price in every node and time step

We will explore further this topic in Lecture 8.

Economic dispatch

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_s . Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d in a certain hour while minimizing the total system cost.

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$d - \sum_s g_s = 0 \quad \leftrightarrow \quad \lambda$$

$$g_s \geq 0 \quad \leftrightarrow \quad \underline{\mu_s}$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_s}$$

This equation excludes the consideration of any network constraints (e.g. line limits), and any additional security constraints, and additional generation constraints (CO2 emissions, ramp limits)

Dispatch optimization in one node

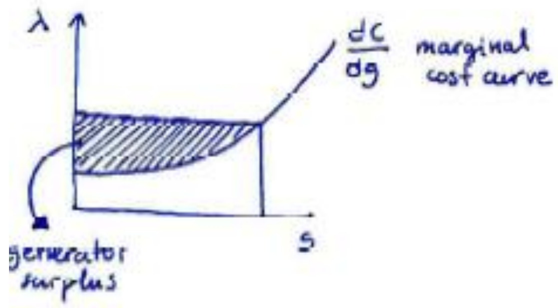
Optimal generator behavior

If we assume that a generator is a price-taker (it cannot influence the market price λ) which wants to maximize its profits (revenues minus expenditures)

$$\max_{g}[\lambda g - C(g)]$$

where g is the energy generated and $C(g)$ the cost curve of the generator

The generation g^* that maximizes the generator's profits fulfills $\lambda - \left.\frac{\partial C}{\partial g}\right|_{g=g^*} = 0 \quad \rightarrow \quad \lambda = \left.\frac{\partial C}{\partial g}\right|_{g=g^*}$



The marginal cost curve $\frac{\partial C}{\partial g}$ is the supply curve for a competitive firm. It shows for each generation g at which price the generator is willing to supply

For a generator with constant marginal costs $C(g) = o_s \cdot g \quad \rightarrow \quad \lambda = \left.\frac{\partial C}{\partial g}\right|_{g=g^*} = o_s$

The maximum generation can be constrained, e.g. by the installed capacity

$g_s \leq G_s \leftrightarrow \overline{\mu_s}$

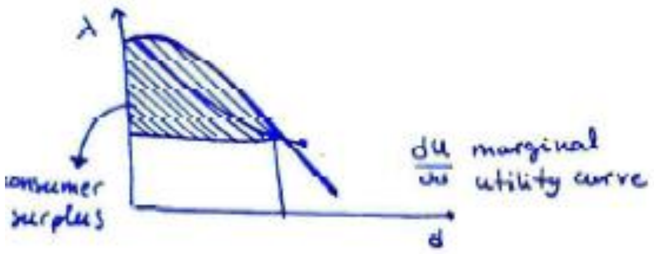
Optimal consumer behavior

If we assume that a consumer is a price-taker (it cannot influence the market price λ) which wants to maximize its net utility or profits (utility minus expenditures)

$$\max_{d}[U(d) - \lambda d]$$

where d is the energy consumed and $\mathrm{U}(d)$ the utility curve of the consumer

The consumption d^* that maximizes the consumer's net utility $\left.\frac{\partial U}{\partial d}\right|_{d=d^*} - \lambda = 0 \quad \rightarrow \quad \lambda = \left.\frac{\partial U}{\partial d}\right|_{d=d^*}$



The marginal utility curve $\frac{\partial U}{\partial d}$ is the demand curve for a rational consumer. It shows for each demand d at which price the consumer is willing to buy.

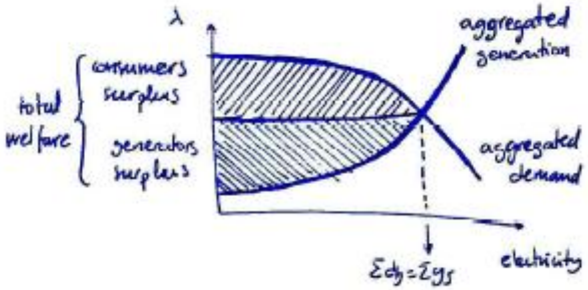
For a consumer with constant marginal utility $\mathrm{U}(d) = q_s \cdot d \quad \rightarrow \quad \lambda = \left.\frac{\partial U}{\partial d}\right|_{d=d^*} = d_s$

The maximum demand can be constrained, e.g. by the capacity of the electrical machinery in the factory $\quad d_b \leq D_b \leftrightarrow \overline{\mu_b}$

Dispatch optimization in one node

The total welfare (consumers and generator surplus) is maximized at the point where aggregated marginal cost (offer) and aggregated marginal utility (demand) meet

$$\begin{cases} \max_{d_b,\, g_s} \left[\sum_b U_b\,(d_b) - \sum_s C_s\,(g_s) \right] \\ \text{subject to:} \\ \sum_b d_b - \sum_s g_s = 0 \leftrightarrow \lambda \end{cases}$$



$$0 = \frac{\partial \mathcal{L}}{\partial d_b} = \frac{\partial f}{\partial d_b} - \sum_i \lambda_i \frac{\partial h_i}{\partial d_b} - \sum_j \mu_j \frac{\partial g_j}{\partial d_b} = \frac{\partial U}{\partial d_b} - \lambda^* = 0 \quad \rightarrow \quad \lambda^* = \left. \frac{\partial U_b}{\partial d_b} \right|_{b=b^*}$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_s} = \frac{\partial f}{\partial g_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_s} - \sum_j \mu_j \frac{\partial g_j}{\partial g_s} = -\frac{\partial C}{\partial g_s} + \lambda^* = 0 \quad \rightarrow \quad \lambda^* = \left. \frac{\partial C_s}{\partial g_s} \right|_{g=g^*}$$

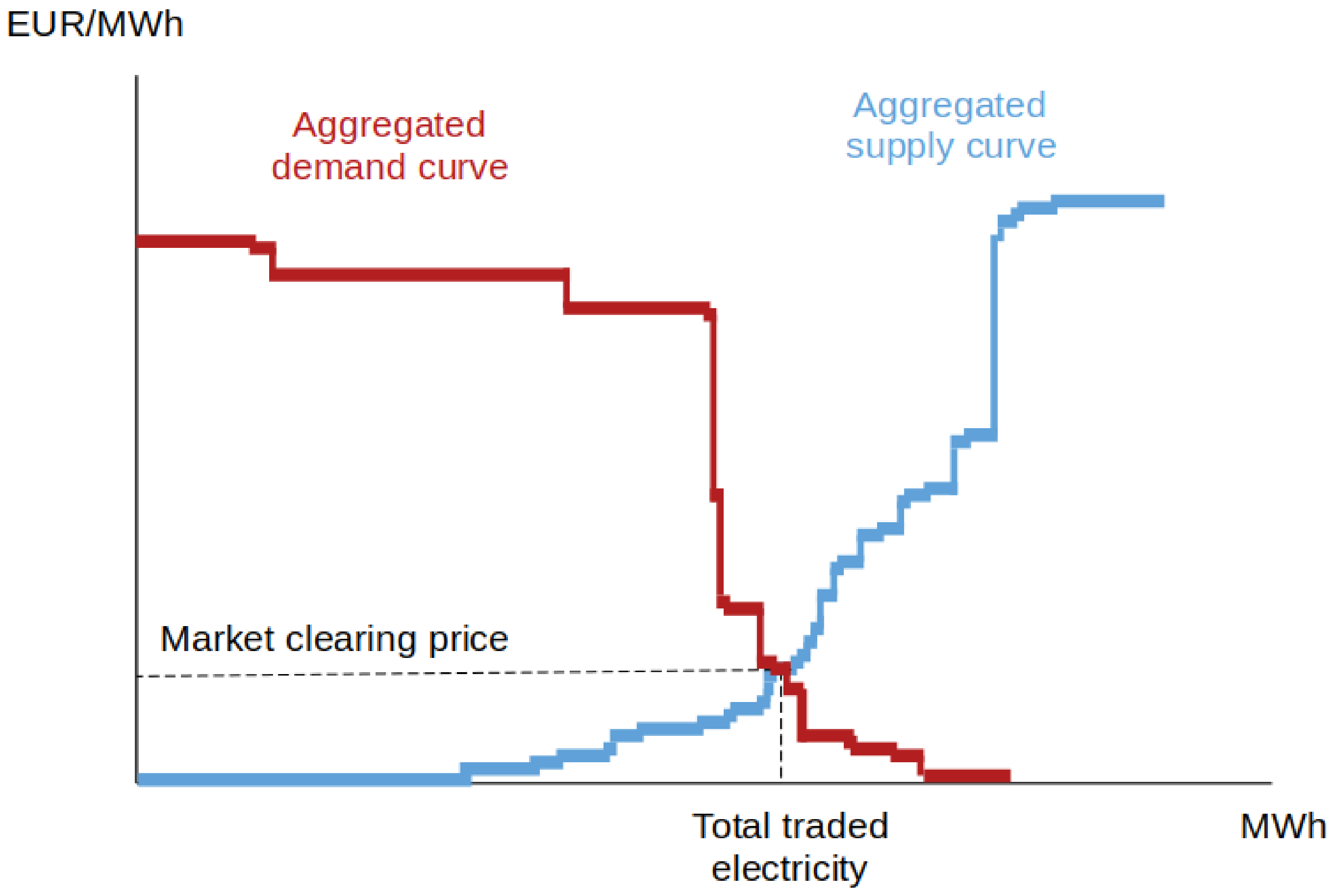
At the optimal point of maximum total economic welfare, we get the same result as if everyone maximizes their own welfare separately → This is a fundamental results in microeconomics that justifies electricity markets

Dispatch optimization in one node – Electricity markets

The electricity market is the same as markets for coffee, wood, or any other goods

There are a few relevant differences:

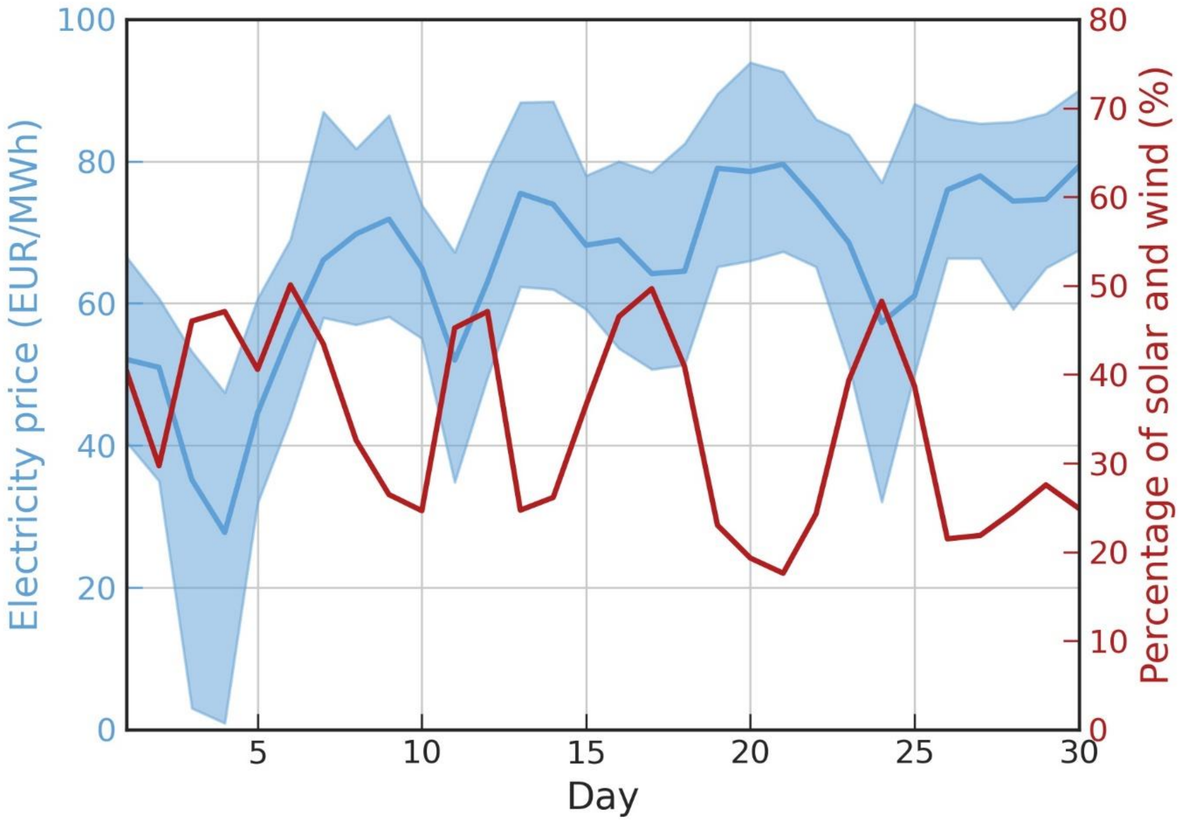
- Supply and demand of electricity must be equal at any time step
- Demand is typically inelastic
- The resulting economic optimal solution must be compatible with power flow equations on the networks



Ref: Victoria and Gallego, 2024, Economics of Photovoltaic Solar Energy (open-license figure)

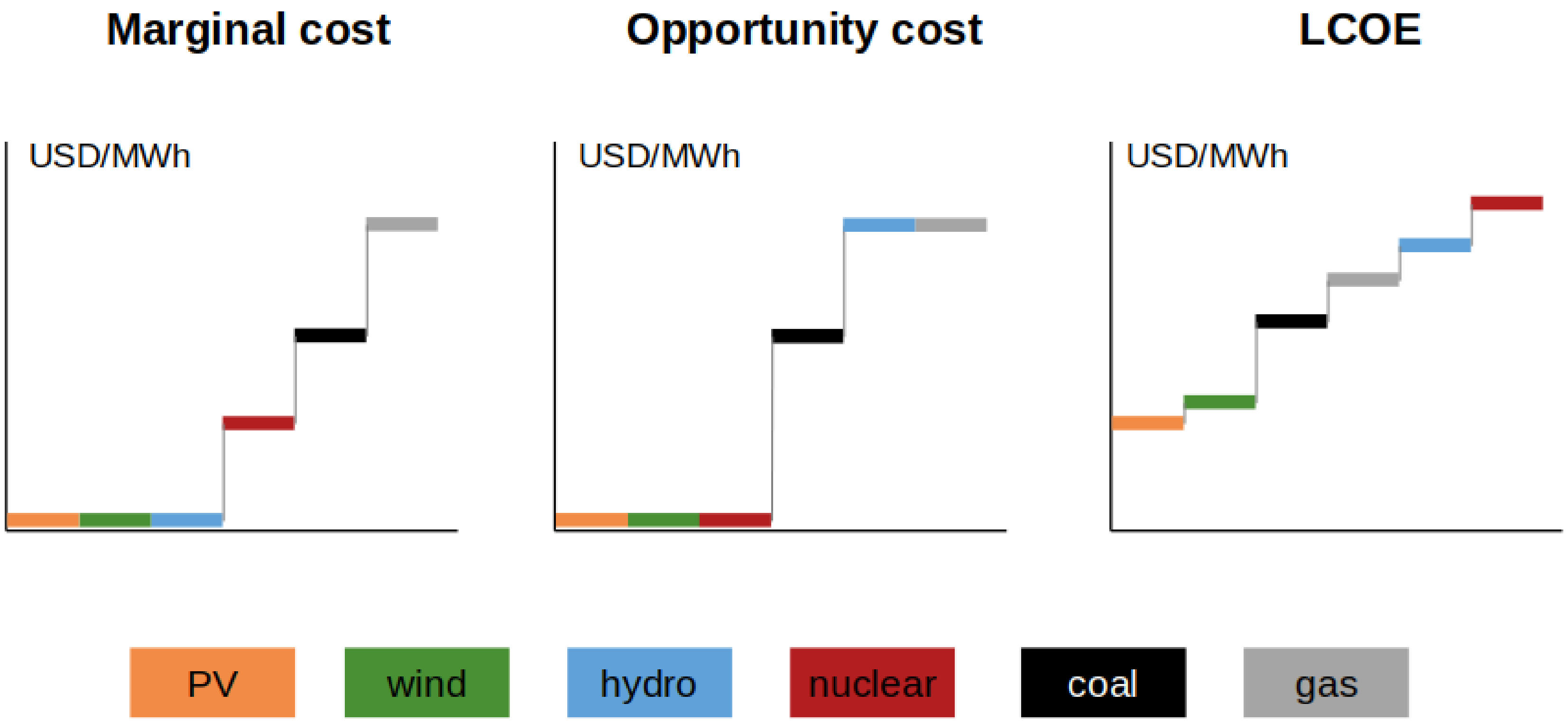
For a comprehensive discussion on electricity markets, check the nice DTU course 46755 Renewables in Electricity Markets

Dispatch optimization in one node – Electricity markets



Ref: Victoria and Gallego, 2024, Economics of Photovoltaic Solar Energy (open-license figure)

Dispatch optimization in one node – Electricity markets



Ref: Victoria and Gallego, 2024, Economics of Photovoltaic Solar Energy (open-license figure)

Dispatch optimization in one node – Electricity markets

Ref: https://www.nordpoolgroup.com/

Dispatch optimization in one node

At the optimal point of maximum total economic welfare, we get the same result as if everyone maximizes their own welfare separately → This is a fundamental results in microeconomics that justifies electricity markets

Welfare can be maximized with decentralized markets:

1º) If the market price is equal to the Lagrange/KKT multiplier of the supply-demand balance constraint

2º) All consumers are price-takers perfect competition)

Maximizing welfare is equivalent to minizing total cost

For the sake of simplicity, we can assume inelastic demand (i.e., it does note respond to price) , consumers with liner utility and generators with linear costs

$$
\left\{
\begin{array}{l}
\max_{g_s} \left[o_d D - \sum_s o_s \, g_s \right] \\
\text{subject to:} \\
D - \sum_s g_s = 0 \leftrightarrow \lambda
\end{array}
\right.
$$

In this simple case, since D is a constant, maximizing welfare is equivalent to minimizing aggregated cost.

Maximizing welfare is equivalent to minizing total cost

we can assume now elastic demand and represent demand elasticity or load shedding by adding a dummy generator so that $d = D - g_d$

$$
\begin{cases}
\max_{g_s} \left[o_d D - o_d g_d - \sum_s o_s \, g_s \right] \\
\text{subject to:} \\
D - \sum_s g_s = 0 \leftrightarrow \lambda
\end{cases}
$$

We can group the dummy generators with the rest of generators and again we can see that maximizing welfare is equivalent to minimizing aggregated cost.

Join capacity and dispatch optimization in one node

Join capacity and dispatch optimization in one node

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$\begin{cases} \min_{g_{s},\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s\, g_{s,t} \right] \\ \text{subject to:} \\ d_t - \sum_s g_{s,t} = 0 \leftrightarrow \lambda_t \\ g_s \geq 0 \leftrightarrow \underline{\mu}_{s,t} \\ -g_s + G_s \geq 0 \leftrightarrow \overline{\mu}_{s,t} \end{cases}$$

Energy balance in every time step

Generation limited by installed capacity in every generator and time step

Join capacity and dispatch optimization in one node

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$\begin{cases} \min_{g_s,\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s\, g_{s,t} \right] \\ \text{subject to:} \\ \sum_s g_{s,t} - d_t \; = 0 \leftrightarrow \lambda_t \end{cases}$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}}$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda^*_{s,t} + \overline{\mu^*_{s,t}} \;=0 \qquad \rightarrow \qquad \lambda^* = \left.\frac{\partial C_s}{\partial g_s}\right|_{g=g^*}$$

The optimal solution for the dispatch is the same as "without capacity optimization" → For every time step t, the generator needed so that the suppy curve intersects the demand sets the price $\lambda^* = \left.\frac{\partial C_s}{\partial g_s}\right|_{g=g^*} = o_s$

Join capacity and dispatch optimization in one node

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$
\begin{cases}
\min_{g_{s},\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s\, g_{s,t} \right] \\
\text{subject to:} \\
\sum_s g_{s,t} - d_t \ = 0 \leftrightarrow \lambda_t \\
-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}}
\end{cases}
$$

From previous slide

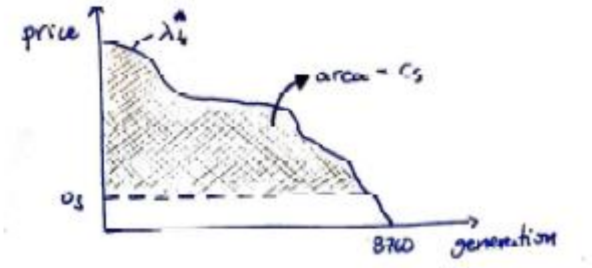
$$
0 = \frac{\partial \mathcal{L}}{\partial G_s} = \ \frac{\partial f}{\partial G_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial G_s} - \sum_j \mu_j \frac{\partial g_j}{\partial G_s} = c_s - \sum_t \overline{\mu_{s,t}^*} \cdot (1) = 0 \qquad \rightarrow \qquad c_s = \sum_t \overline{\mu_{s,t}^*} = \sum_t \lambda_{s,t}^* - o_s
$$

The level of investment in generator capacity is optimal when the gap between the system marginal cost λ and the generator marginal cost o_s is equal to the capital cost of the added generation capacity

Dispatch optimization in one node

$$c_s = \sum_t \overline{\mu^*_{s,t}} = \sum_t \lambda^*_{s,t} - o_s$$

The level of investment in generator capacity is optimal when the gap between the system marginal cost λ and the generator marginal cost o_s is equal to the capital cost of the added generation capacity



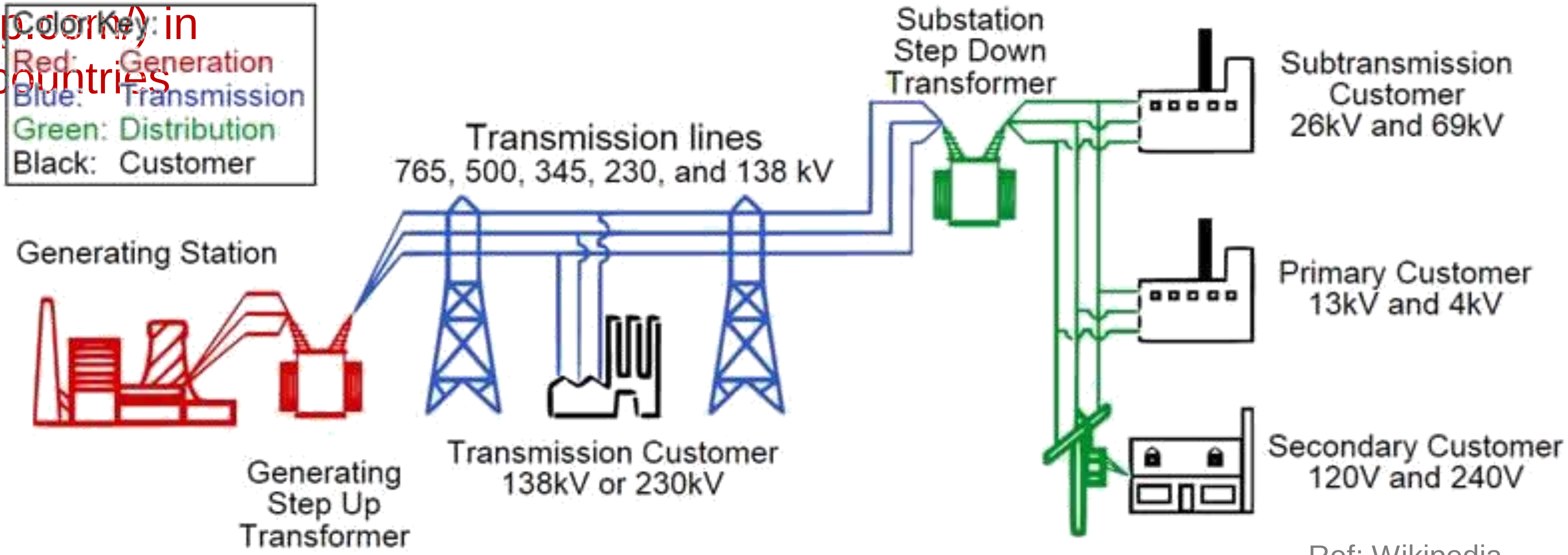
The price of the electricity market must be higher than the variable cost for the most expensive generator for several time steps. In those time steps, the expensive generator can cover its capital cost.

Main actors in the power system

This is covered in "Renewables in Electricity Markets"

Main actors in the power system:

- Generators   Liberalized (sell their energy in the market)
- Consumers   Buy electricity from a retailer
- Transmission & Distribution   Monopoly   (https://en.energinet.dk/),
- Retailers   Buy electricity in the market and supply it to consumers
- Market operator Nord Pool (https://www.nordpoolgroup.com/) in Scandinavian (and Baltic) countries



Ref: Wikipedia

Additional markets

This is covered in “Renewables in Electricity Markets” Lectures 5, 6, 7

Intra-day market

In Nord Pool “Electricity Balancing Adjustment System” (Elbas) (based on financial contracts to be agreed bilaterally, there is no clearing mechanism)

Ancillary Services/Balancing market:

to be cleared by the system (not market) operator, involve re-disaptching

- **FCR** (Frequency Containment Reserve): also known as primary reserve or regulating reserve, you must react in 15 seconds and stay active for 15 minutes, you get paid for being available although you don’t deliver energy

- **AFRR** (Automatic Frequency Restoration Reserve): also known as secondary reserve or operating reserve, it is activated if FCR is active for too long (>15min), you must react in minutes

- **MFRR** (Manual Frequency Restoration Reserve): also known as tertiary reserve or replacement reserve, activated if MFRR is active for too long

Ref: https://www.nordpoolgroup.com/

DTU