46770 Integrated energy grids

Lecture 4 – DC Optimal power flow

DC power flow

A clarification on naming:

The term DC optimal power flow does not require that we are using DC circuits, but it refers to a linearization of the AC optimal power flow

Ref: Wikipedia

Learning goals

- Review operating principles of AC power
- Write the AC power flow equations and linearlize them to obtain the DC Optimal power flow equations
- Explain the analogy between DC systems and DC optimal power flow.
- Calculate the linearlized power flows in an AC network by calculating the voltages using the inverse of the weighted Laplacian matrix (or susceptance matrix)
- Calculate the linearlized power flows in an AC network by using the Power Transfer Distribution Factors (PTDF) matrix
- Write the system cost minimization problem including AC power flows
- Formulate the optimal power flow problem on a computer.

Power flow vs Optimal Power Flow

	Power flow analysis	Optimal power flow
Objective	• Find the flows in the links of a network given the injection pattern for the nodes. • Determines voltage magnitudes and angles at each bus • Computes real and reactive power flows for lines connecting buses, as well as lines losses	• Find the optimal flows in the links of a network that minimizes the total system costs (while supplying demand in every node)
Applications	• Feasibility analysis: for dispatch (obtained in market clearing, identify grid overlading) • Security analysis: simulate outages to identify possible grid overloading • Transmission adequacy analysis: identify possible bottlenecks in long-term planning	• Find optimal dispatch that minimizes system cost and ensures power flow feasibility • Find optimal capacities to avoid bottlenecks (long-term planning) • Find optimal redispatch after an outage to restore feasibility

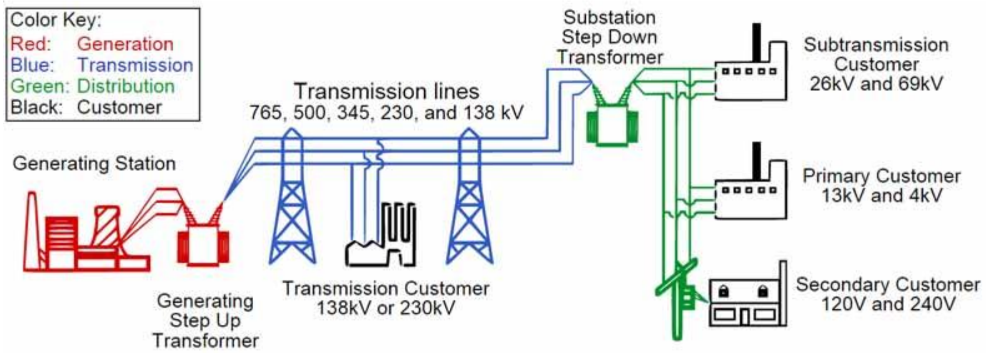
Modelling approaches for power flow in AC networks

Net Transfer Capacities	Linearized AC power flow (DC Power flow)	AC Power flow
$\sum_{s,i} g_{s,i} - d_i = \sum_{l} K_{il} p_l$ $\lvert p_l \rvert \leq P_l$	$\sum_{s,i} g_{s,i} - d_i = \sum_{l} K_{il} p_l$ $\lvert p_l \rvert \leq P_l$ $\sum_{l} C_{lc}\, x_l\, p_l = 0$	$\sum_{s,i} g_{s,i} - d_i = \bar{V}_i \left(\sum_{J} \overline{Y_{bus,ij}}\, \bar{V}_j \right)^*$ $\lvert \bar{V}_i \bar{Y}^*_{line,i\to j} \bar{V}_j^* \rvert \leq S_{i\to j,max}$ $\lvert \bar{V}_i \bar{Y}^*_{line,j\to i} \bar{V}_j^* \rvert \leq S_{j\to i,max}$
	✓ Linear ✓ Optimality guaranteed ✓ Computational tractable ✓ Good enough for long-term planning	✓ Feasible AC power flows
	× Not guarantee feasible power flows × No representation of power losses × Not good enough for heavily-loaded systems, fast changes, restart from blackout, network splitting, distribution networks, etc.	× Non-linear × No optimality guaranteed × High computational complexity

AC Power
(a short review)

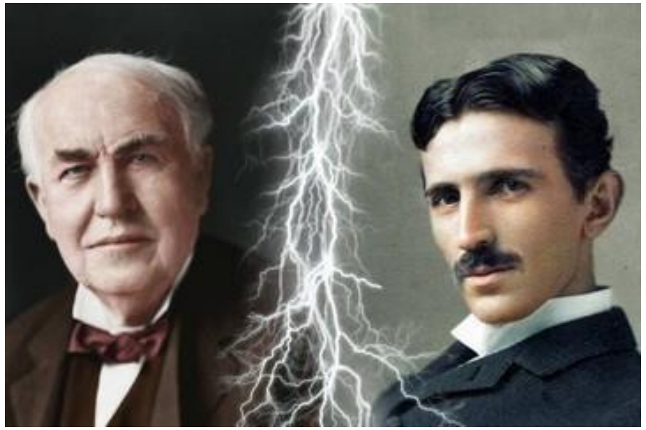
Why do we use alternating current (AC) ?

We need high voltage to transport power long-distances while keeping current (and power losses) low.

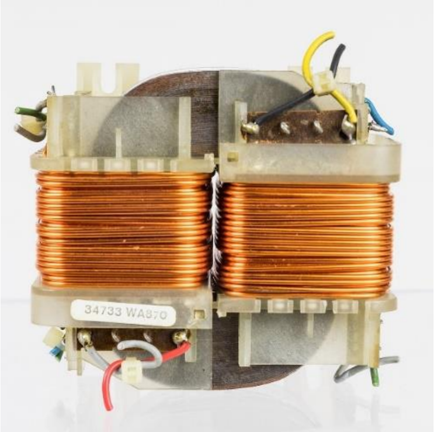


Ref: Wikipedia

The war of currents

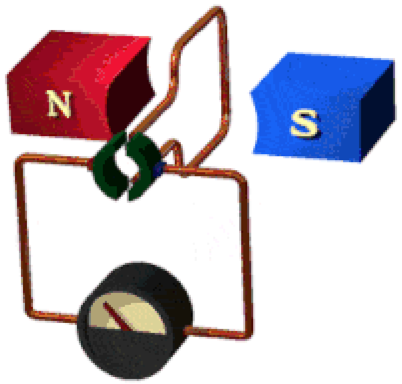


Transformers allow high-efficiency voltage conversion



Ref: Wikipedia

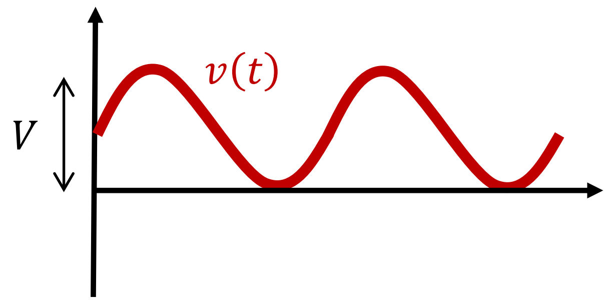
Generators convert mechanical rotational energy into AC electricity



AC voltage and current

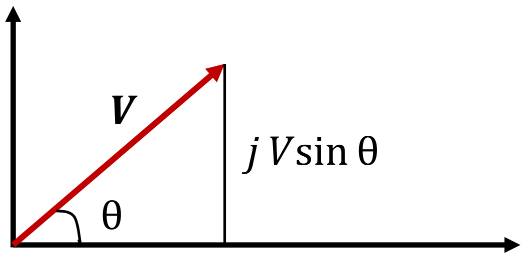
$v\,(t) = V\cos(\omega t + \theta)$ Lower letter indicates instantaneous values,

Upper letter indicates peak values



Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$ where $j = \sqrt{-1}$

$\bar{V} = Ve^{j(\omega t+\theta)}$ Upper bar indicates imaginary number



$I(t) = I\cos(\omega t + \varphi)$ Current and Voltage are not necessarily in phase $\varphi \neq \theta$

Resistive, capacity and inductive loads

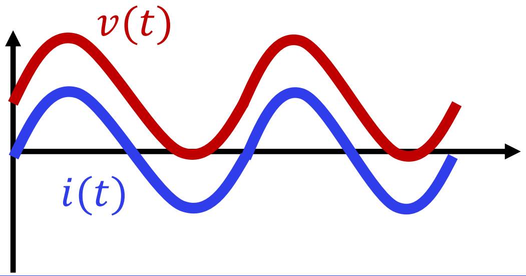
Resistive loads

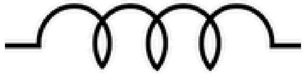
$$v(t) = Ri(t)$$

$$v(t) = Ve^{j\omega t}$$

$$i(t) = \frac{1}{R} Ve^{j\omega t}$$

$$I = \frac{V}{R}$$





Inductive loads

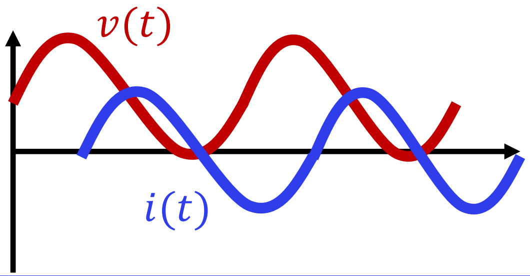
$$v(t) = L\frac{di(t)}{dt}$$

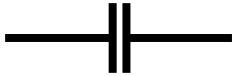
$$v(t) = Ve^{j\omega t}$$

$$i(t) = V\frac{1}{j\omega L}e^{j\omega t} = \frac{1}{j\omega L} \cdot v(t)$$

$$I = \frac{1}{j\omega L}V$$

inductive reactance $X_L = \omega L$





Capacity loads

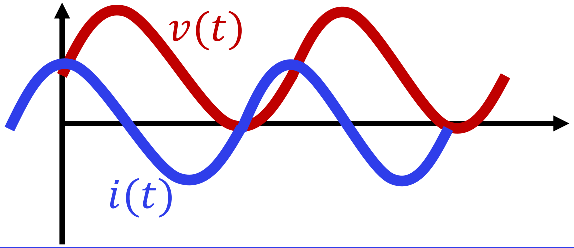
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = Ve^{j\omega t}$$

$$i(t) = Vj\omega Ce^{j\omega t} = j\omega C \cdot v(t)$$

$$I = j\omega C \cdot V$$

capacitance reactance $X_C = \frac{1}{\omega C}$



Generic loads

- **Impedance** (how much opposition to current in a line). It includes the resistance R associated to resistive loads and the reactance X associated the capacitive and inductive loads

$$\mathrm{Z} = R + jX = \left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$I_{j\to i} = \frac{1}{Z_{ji}}\left(V_j - V_i\right)$$

- **Admittance** (it is the inverse impedance, i.e.g, how easily a line allows current to flow). It includes the conductance G and the susceptance B

$$\mathrm{Y} = G + jB$$

$$Y = \frac{1}{Z}$$

Complex Power

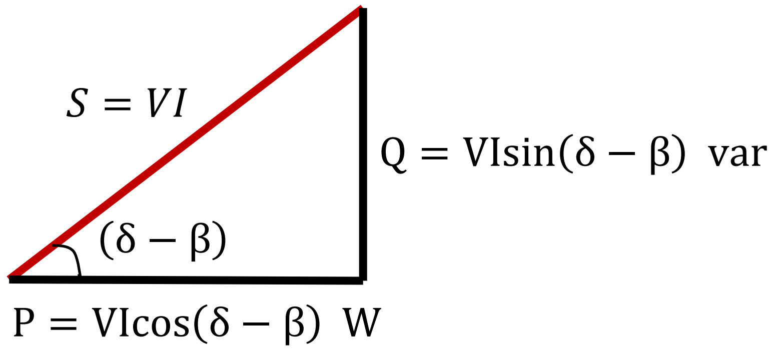
Active and reactive power can be calculated from complex power:

$$V = V\angle\delta \qquad I = I\angle\beta$$

$$\bar{S} = \bar{V}\bar{I}^* = [V\angle\delta][I\angle\beta]^* = VI\angle\delta - \beta = \mathrm{VIcos}(\delta - \beta) + \mathrm{jVIsin}(\delta - \beta)$$

$$\bar{S} = P + \mathrm{jQ} = \underbrace{\mathrm{VIcos}(\delta - \beta)}_{\text{active power}} + \mathrm{j}\,\underbrace{\mathrm{VIsin}(\delta - \beta)}_{\text{reactive power}}$$

S is named apparent power



Power factor

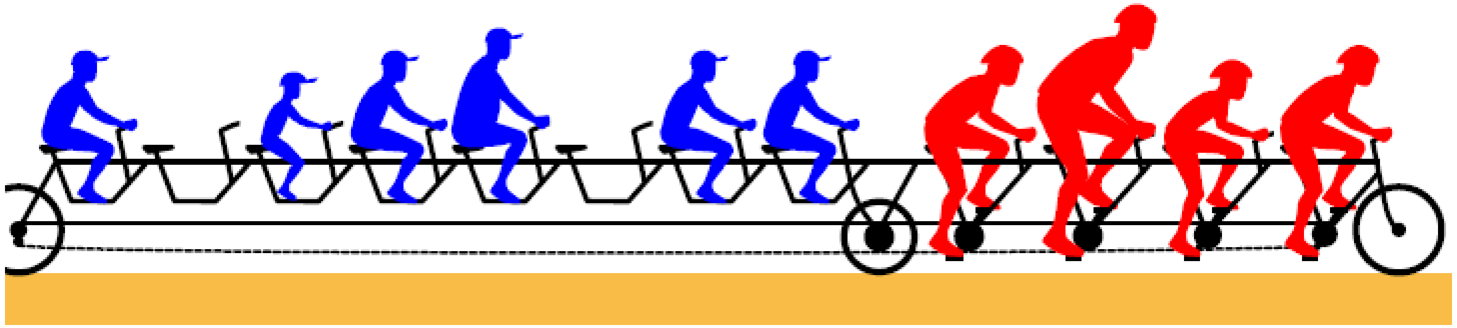
$$\underbrace{\cos(\delta - \beta)}_{\cos\varphi} = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

For resistive loads, we get the active power.

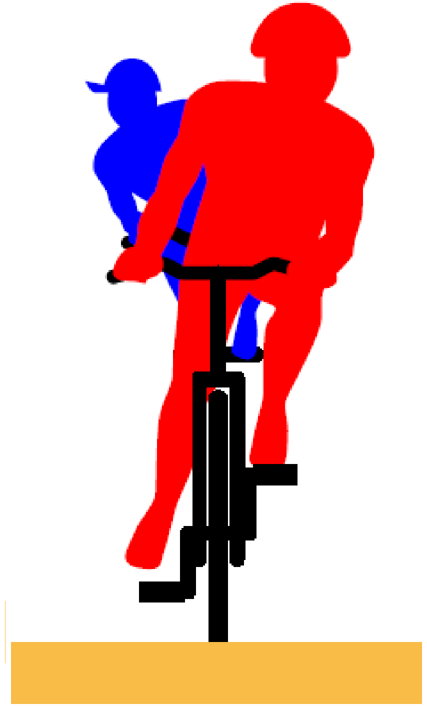
For loads where the current is not in phase with the voltage, we also get a reactive power.

Alternating current

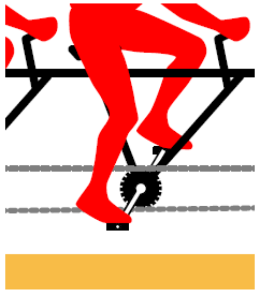
The tandem bicycle analogy



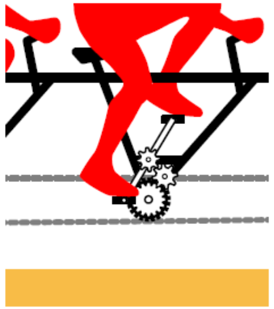
Reactive power



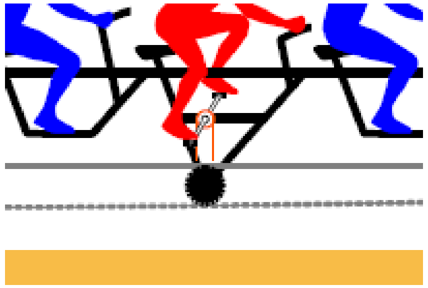
Synchronous generators using the grid frequency



Synchronous generators using a different frequency



Other generators, e.g., solar PV



Further readings: Fassbinder 2005, [The electrical system as a tandem bicycle]

AC transmission lines

Overhead: naked metal and suspended on insulators, lower cost and easy maintenance, aluminum conductors (low cost, light weight)

Underground: need insulating cover

Transmission lines are a combination of resistive, capacitive, and inductive loads

Resistance

$$R_{DC} = \frac{\rho l}{A}$$

AC resistance is higher than DC resistance due to skin effect which forces more current flow near the outer surface of the conductor.

Inductance

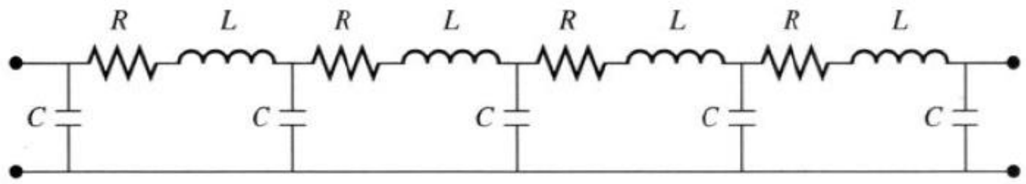
A conductor carrying a current that varies in time produces a variable magnetic flux.

The greater the spacing between the phases of a transmission line, the greater the inductance of the line.

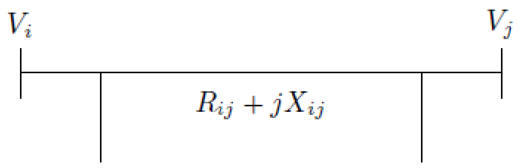
Capacitance

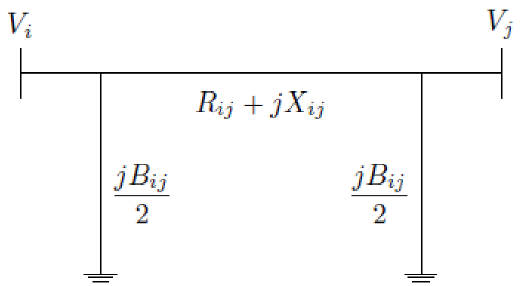
Because we have a pair of conductors separated by a dielectric (air)

The greater the spacing between the phases of a transmission line, the lower the capacitance of the line.

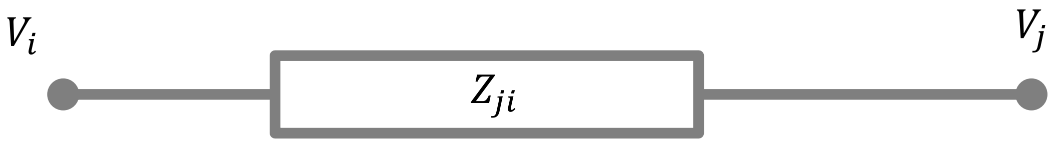


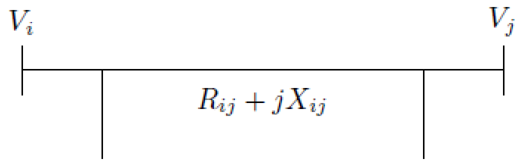
AC transmission lines





AC transmission lines





$$\bar{I}_{j\to i} = \frac{1}{\bar{Z}_{ji}}\left(\bar{V}_j - \bar{V}_i\right) = \frac{1}{R + jX}\left(V_j e^{j(\omega t + \theta_j)} - V_i e^{j(\omega t + \theta_i)}\right) = \frac{1}{R + jX} V_i e^{j(\omega t + \theta_i)} \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} - 1\right)$$

In AC systems, the net complex power flowing through a line is equal to the product of the voltage and the conjugate of the current (both complex numbers)

$$\bar{S}_{ji} = \bar{V}_i \bar{I}_{ji}^* = P_{ji} + jQ_{ji} = \frac{1}{R + jX} V_i^2 \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} - 1\right)$$

$$S_{ji} = V_i I_{ji}^* \neq S_{ij} = V_i I_{ij}^*$$

power losses

This equation is nonlinear and we can simplify it under certain assumptions.

DC Power flow (Linearized AC power flow)

Linear power flow

1º. Power flows primarily according to angle differences

$V_i \approx V_j$

2º. No significant voltage shift occurs between the nodes

$\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$ $\cos(\theta_i - \theta_j) \approx 1$

3º. Resistance of the links is neglected. $R \ll X$

Hence, power losses are not included

Step 3 also implies that reactive power flow is neglected $Q_i \sim 0$ (reactive power flow delivered by line is proportional to voltage drop, it is controlled so that it does not “travel” far)

We define a normalized reactance. $x_L = \frac{X}{V_i^2}$

$$\bar{S}_{ij} = P_{ij} + \mathrm{j}Q_{ij} = \frac{1}{R + jX} V_i^2 \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} - 1 \right)$$

$$S_{ij} = \frac{1}{R + jX} V_i^2 \left(\cos(\theta_i - \theta_j) + j\sin(\theta_i - \theta_j) - 1 \right)$$

$$S_{ij} = \frac{1}{R + jX} V_i^2 j(\theta_i - \theta_j)$$

$$p_{ij} = p_l = \frac{V_i^2}{X}(\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_L}$$

θ_i is in radians and p_{ij} is in per unit!

This ensure consistent tolerance used in the optimization problem for optimality (objective function) and feasibility (constraints)

The same result is discussed in Optimization in Modern Power Systems (Lecture 3)

Analogy between DC and linearized AC power flow

$$I_l = \frac{V_i - V_j}{R_l}$$

$$p_l = \frac{\theta_i - \theta_j}{x_l}$$

DC POWER FLOW	LINEARIZED AC POWER FLOW
Current flow I_l	Active power flow p_l
Voltage V_i	Voltage angle θ_i
Resistance R_l	Reactance x_l

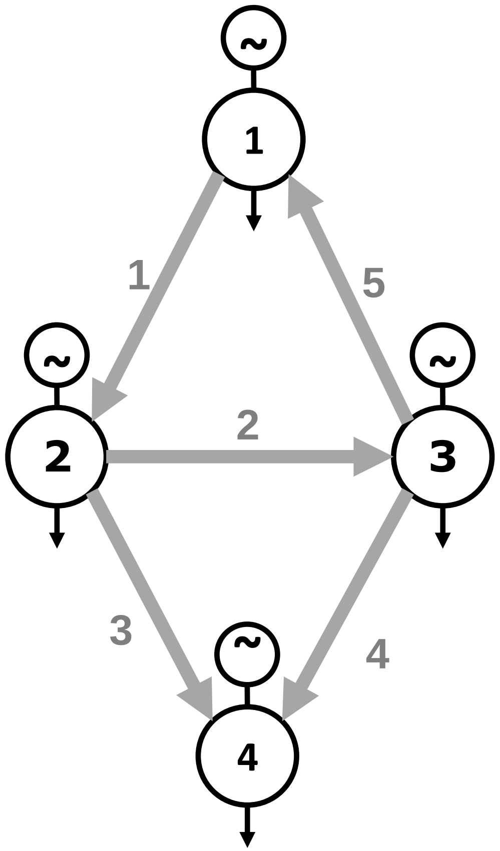
This is approximation is called linearized AC power flow or DC approximation.

Linearized AC power flow

Kirchhoff's Currents Law $$p_i = \sum_l K_{il} p_l \qquad \forall i$$

Kirchhoff's Voltage Law $$\sum_l C_{lc}\, \theta_l = 0 \qquad \forall c$$

Here, P_i is the power that node i wants to inject, while P_l is the power flowing throughout link l



Linearized AC power flow

Alternative way of expressing Kirchhoff's Current Law

$$p_l = \frac{1}{x_l}\theta_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$$

$$p_i = \sum_l K_{il} P_l = \sum_l \underbrace{K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

B_{kl} is the diagonal matrix of inverse lines series reactance $B_{ll} = \frac{1}{x_l}$

The weighted Laplacian matrix relates the injected power p_i and voltage angles θ_j in every node

The weighted Laplacian matrix is also called the Bus Susceptance Matrix

Linearized AC power flow

Power flow analysis:

Find the flows in the links of a network given the injection pattern for the nodes.

$$p_i = \sum_l K_{il} p_l = \underbrace{\sum_l K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

The weighted Laplacian matrix relates the injected power p_i and voltage angles θ_j in every node $p_i = \sum_j L_{ij}\, \theta_j$

To obtain the power flows p_l :

1º. Calculate the voltage angles using the inverse of the weighted Laplacian matrix. $\theta_j = \sum_i (L^{-1})_{ji}\, p_i$

2º. Calculate the flows p_l using the voltage angles $p_l = \frac{1}{x_l} \sum_j K_{lj} \theta_j$

Power Transfer Distribution Factors (PTDF) matrix

Power flow analysis:

Find the flows in the links of a network

given the injection pattern for the nodes.

An alternative method consists in using the Power Transfer Distribution Factors (PTDF) matrix

$$p_l = \frac{1}{x_l}\theta_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$$

$$p_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j = \frac{1}{x_l}\sum_{ji} K_{lj}(L^{-1})_{ji}p_i = \sum_i PTDF_{li}p_i$$

The PTDF matrix can be understood as a linear sensitivity that represents the marginal change of the active power flow on a line if we apply a marginal increase of the power injection at a node.

For every tuple <line, node> we have a different power transfer distribution factor

$$\Delta p_l = PTDF_{li}p_i$$

The linearized AC power flow based on PTDF formulation is used for flow-based market coupling of the European markets, each node corresponds to a biding zone and PTDFs are derived for the interconnections between countries

Linearized AC power flow

$$p_i = \sum_l K_{il} p_l = \sum_l \underbrace{K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

We need the inverse of the weighted Laplacian matrix. $\theta_j = \sum_i (L^{-1})_{ji}\, p_i$

The weighted Laplacian is a singular matrix and thus not invertible. To invert it, we follow the procedure:

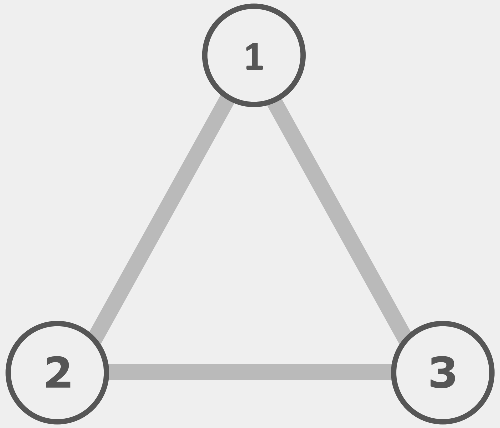
a. Delete the row and column corresponding to the slack bus, so that we obtain a matrix with dimension $(i-1) \cdot (j-1)$

b. Invert the matrix $(i-1) \cdot (j-1)$

c. Add a row and column of zeros at the row and column corresponding to the slack bus to obtain the matrix with dimension $i \cdot j$

This is what we did on previous lecture (slide 17)

DC power flow in a 3-node network



$$I_1 = I_{1\to 2} + I_{1\to 3} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

$$I_2 = I_{2\to 1} + I_{2\to 3} = (V_2 - V_1) + (V_2 - V_3) = -V_1 + 2V_2 - V_3$$

$$I_3 = I_{3\to 1} + I_{3\to 2} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

There are only two independent variables because $I_1 + I_2 + I_3 = 0$

We select $V_3 = 0$

1º. Calculate the voltages using the inverse of the Laplacian matrix.

$$I_1 = 2V_1 - V_2$$

$$I_2 = -V_1 + 2V_2$$

$$\begin{cases} V_1 = \frac{2}{3} I_1 + \frac{1}{3} I_2 \\ V_2 = \frac{1}{3} I_1 + \frac{2}{3} I_2 \\ V_3 = 0 \end{cases}$$

2º. Calculate the current flows using the voltages

$$I_{1\to 2} = (V_1 - V_2)$$

$$I_{1\to 3} = (V_1 - V_3)$$

$$I_{2\to 3} = (V_2 - V_3)$$

We can also express the result as a matrix that relates the current flows to the current injection

$$\begin{pmatrix} I_{1\to 2} \\ I_{1\to 3} \\ I_{2\to 3} \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

This is equivalent to calculating the PTDF matrix by inverting the Laplacian and multiplying by the incidence matrix

Limitations of DC Optimal Power flow

- The linearized AC approximation is only valid for stable situations, where voltage angle differences are small. It can not be used to analyze heavily-loaded systems, fast changes in the systems, restart from blackout, network splitting, distribution networks, etc.

- With the linear power flow approximation, we cannot estimate losses in the network because we have assumed resistance to be zero.

- We can use the linear power flow approach for long-term planning.

Economic dispatch with DC optimal power flow

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_s. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_n in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch in one node

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda$$

$$0 \leq g_s \leq G_s$$

Economic dispatch with AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l \quad \leftrightarrow \quad \lambda_i$$

$$0 \leq g_s \leq G_s$$

$$|p_l| \leq P_l$$

$$\sum_l C_{lc}\, x_l\, p_l = 0$$

Nodal power balance

(Kirchoff's Current Law)

Power flow Capacities

Kirchoff's Voltage Law

Economic dispatch with lines net transfer capacities

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_s. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_n in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch in one node

$$\begin{cases} \min_{g_s} \sum_s o_s g_s \\ \text{subject to:} \\ \sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda \\ 0 \leq g_s \leq G_s \end{cases}$$

Economic dispatch with AC power flow

$$\begin{cases} \min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ \text{subject to:} \\ \sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l \quad \leftrightarrow \quad \lambda_i \\ 0 \leq g_s \leq G_s \\ |p_l| \leq P_l \end{cases}$$

Nodal power balance

(Kirchoff’s Current Law)

Power flow Capacities

Complex Power

		Active power	Reactive power	
resistor	$\bar{S} = \bar{V}\bar{I}^* = [V\angle\delta]\left[\frac{V}{R}\angle\delta\right]^* = \frac{V^2}{R}$	$P = \frac{V^2}{R}$	$Q = 0$	
inductor	$\bar{S} = \bar{V}\bar{I}^* = [V\angle\delta]\left[\frac{V}{jX_C}\angle-\delta\right]^* = j\frac{V^2}{X_L}$	$P = 0$	$Q = \frac{V^2}{X_L}$	Inductor absorbs positive reactive power
capacitor	$\bar{S} = \bar{V}\bar{I}^* = [V\angle\delta]\left[\frac{-V}{jX_C}\angle-\delta\right]^* = -j\frac{V^2}{X_C}$	$P = 0$	$Q = \frac{-V^2}{X_C}$	Capacitor absorbs negative reactive power or deliver positive reactive power

DTU