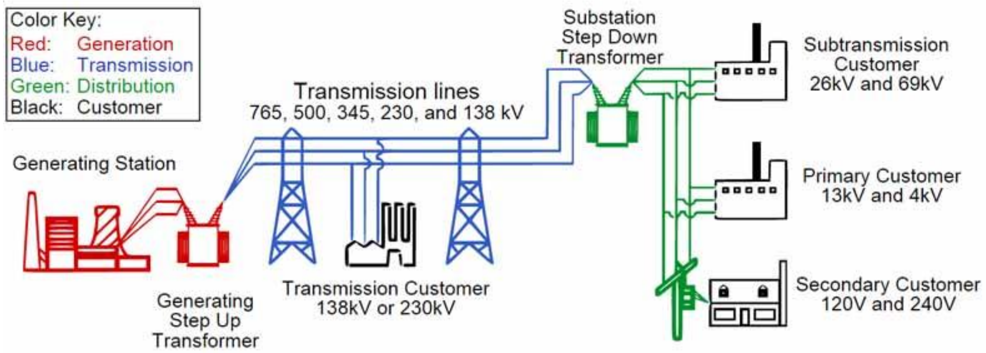
46770 Integrated energy grids

Lecture 4 – AC Power networks

Alternating current (AC)

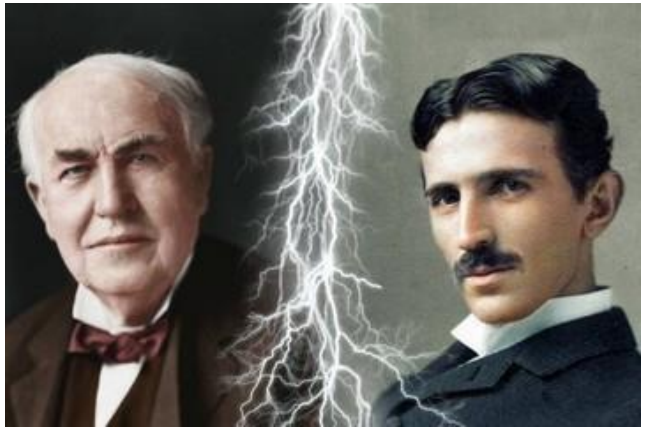
Why do we use alternating current (AC) ?

We need high voltage to transport power long-distances while keeping current (and power losses) low.

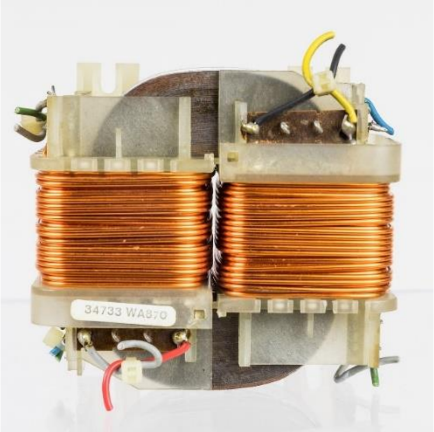


Ref: Wikipedia

The war of currents

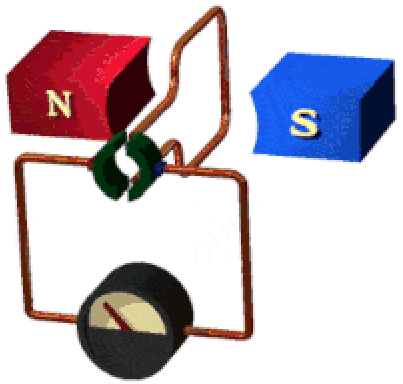


Transformers allow high-efficiency voltage conversion



Ref: Wikipedia

Generators convert mechanical rotational energy into AC electricity



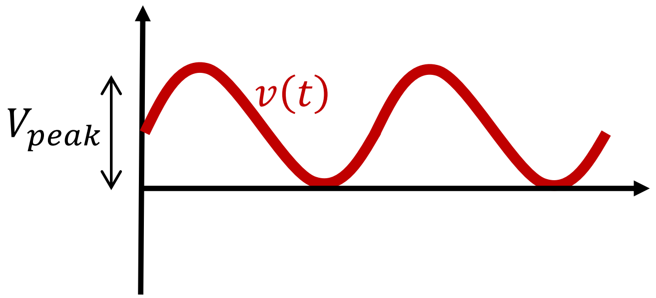
Phasors

$v\ (t) = V_{peak}\cos(\omega t + \delta)$ Lower letter indicate instantaneous values, e.g. $v\ (t)$

$$V = V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Root-Mean-Squared (RMS) voltage/current produces the same thermal effect that its equivalent in DC

Upper letter indicate rms values, e.g. V



$$v\ (t) = \sqrt{2}V\cos(\omega t + \delta)$$

Euler's identity: $e^{j\varphi} = \cos\varphi + j\sin\varphi$ where: $j = \sqrt{-1}$

$$v\ (t) = V_{peak}\cos(\omega t + \delta) = Re\left[\sqrt{2}Ve^{j(\omega t+\delta)}\right]$$

Resistive, capacity and inductive loads

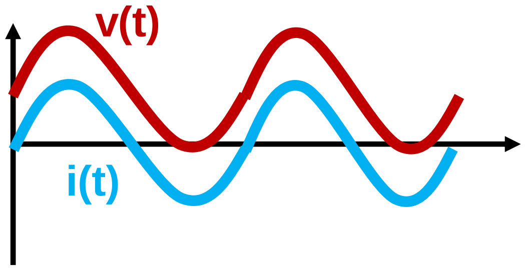
Resistive loads

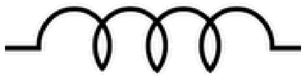
$$v(t) = Ri(t)$$

$$v(t) = \sqrt{2}Ve^{j\omega t}$$

$$i(t) = \frac{1}{R}\sqrt{2}Ve^{j\omega t}$$

$$I = \frac{V}{R}$$





Inductive loads

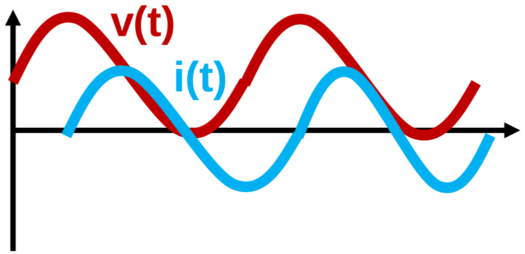
$$v(t) = L\frac{di(t)}{dt}$$

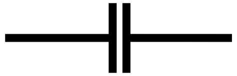
$$v(t) = \sqrt{2}Ve^{j\omega t}$$

$$i(t) = \sqrt{2}V\frac{1}{j\omega L}e^{j\omega t} = \frac{1}{j\omega L}\cdot v(t)$$

$$I = \frac{1}{j\omega L}V$$

inductive reactance $X_L = \omega L$





Capacity loads

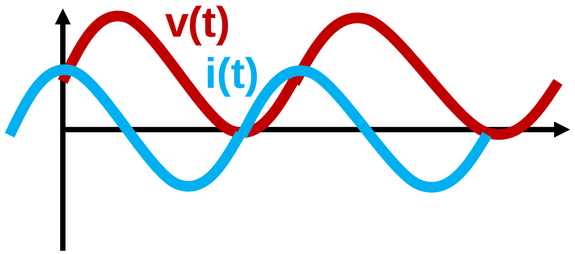
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \sqrt{2}Ve^{j\omega t}$$

$$i(t) = \sqrt{2}Vj\omega Ce^{j\omega t} = j\omega C\cdot v(t)$$

$$I = j\omega C\cdot V$$

capacitance reactance $X_C = \frac{1}{\omega C}$



Active power

$$p(t) = VI\cos(\delta - \beta)\left[1 + \cos\left(2(\omega t + \delta)\right)\right] + VI\sin(\delta - \beta)\sin(2(\omega t + \delta))$$

$$P = VI\cos(\delta - \beta)$$

P is named real power, average power or active power.

The unit for P is W

Power factor

$$\cos(\delta - \beta)$$

Reactive power

$$p(t) = VI\cos(\delta - \beta)\big[1 + \cos\big(2(\omega t + \delta)\big)\big] + VI\sin(\delta - \beta)\sin(2(\omega t + \delta))$$

The instantaneous power absorbed by the reactive component of the load

$$Q = VI\sin(\delta - \beta)$$

The unit for Q is var

Reactive power is alternatively positive and negative. It expresses the reversible flow of energy to and from the reactive component of the load.

Complex Power

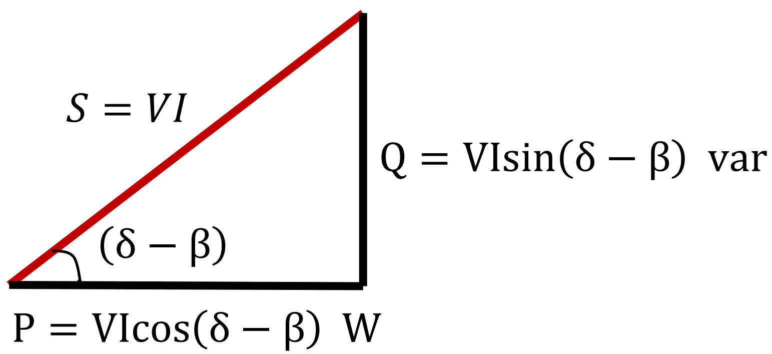
Active and reactive power can be calculated from complex power:

$V = V\angle\delta \qquad\qquad I = I\angle\beta$

$$S = VI^* = [V\angle\delta][I\angle\beta]^* = VI\angle\delta - \beta = \mathrm{VIcos}(\delta - \beta) + \mathrm{jVIsin}(\delta - \beta)$$

$$S = P + \mathrm{jQ} = \underbrace{\mathrm{VIcos}(\delta - \beta)}_{\text{active power}} + \mathrm{j}\underbrace{\mathrm{VIsin}(\delta - \beta)}_{\text{reactive power}}$$

S is named apparent power



Power factor $\underbrace{\cos(\delta - \beta)}_{\cos\varphi} = \dfrac{P}{S} = \dfrac{P}{\sqrt{P^2 + Q^2}}$

For resistive loads, we get the active power.

For loads where the current is not in phase with the voltage, we also get a reactive power.

Complex Power

		Active power	Reactive power	
resistor	$S = VI^* = [V\angle\delta]\left[\frac{V}{R}\angle\delta\right]^* = \frac{V^2}{R}$	$P = \frac{V^2}{R}$	$Q = 0$	
inductor	$S = VI^* = [V\angle\delta]\left[\frac{V}{jX_C}\angle-\delta\right]^* = j\frac{V^2}{X_L}$	$P = 0$	$Q = \frac{V^2}{X_L}$	Inductor absorbs positive reactive power
capacitor	$S = VI^* = [V\angle\delta]\left[\frac{-V}{jX_C}\angle-\delta\right]^* = -j\frac{V^2}{X_C}$	$P = 0$	$Q = \frac{-V^2}{X_C}$	Capacitor absorbs negative reactive power or deliver positive reactive power

AC transmission lines

Overhead: naked metal and suspended on insulators, lower cost and easy maintenance, aluminum conductors (low cost, light weight)

Underground: need insulating cover

Transmission lines are a combination of resistive, capacitive, and inductive loads

Resistance

$$R_{DC} = \frac{\rho l}{A}$$

AC resistance is higher than DC resistance due to skin effect which forces more current flow near the outer surface of the conductor.

Inductance

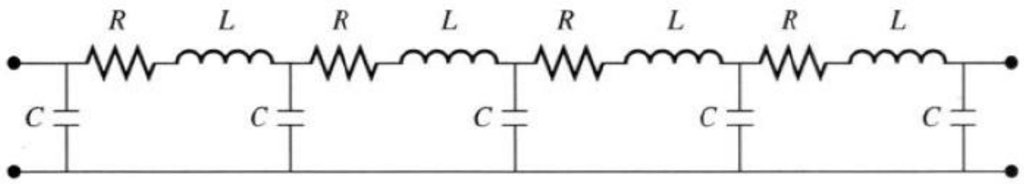
A conductor carrying a current that varies in time produces a variable magnetic flux.

The greater the spacing between the phases of a transmission line, the greater the inductance of the line.

Capacitance

Because we have a pair of conductors separated by a dielectric (air)

The greater the spacing between the phases of a transmission line, the lower the capacitance of the line.



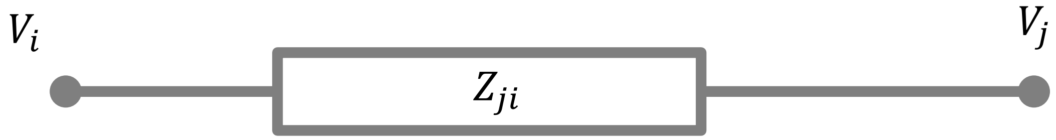
AC transmission lines

Transmission lines are a combination of resistive, capacitive, and inductive loads

$$Z = \left(R + j\omega L + \frac{1}{j\omega C}\right) = R + jX$$

Z is called impedance and includes the resistance associated to resistive loads and the reactance associated the capacitive and inductive loads

$$v(t) = \left(R + j\omega L + \frac{1}{j\omega C}\right) i(t)$$



$$I_{ji} = \frac{1}{Z_{ji}}(V_j - V_i) = \frac{1}{R + jX}(V_j - V_i) = \frac{1}{R + jX}\left(V_j e^{j(\omega t + \theta_j)} - V_i e^{j(\omega t + \theta_i)}\right) = \frac{1}{R + jX} V_i e^{j(\omega t + \theta_i)} \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} - 1\right)$$

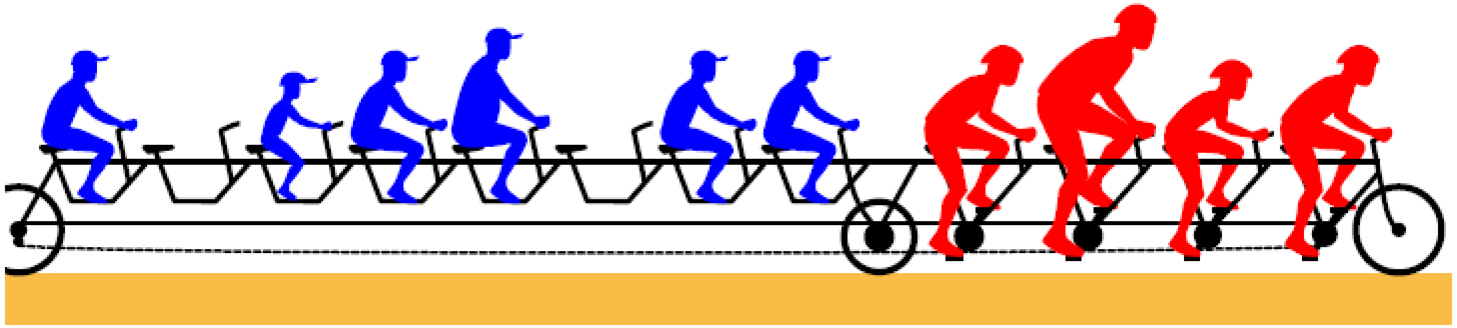
Now the power injected at node i is:

$$S_i = V_i I_{ji} = P_i + jQ_i = \frac{1}{R + jX} V_i^2 \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} - 1\right)$$

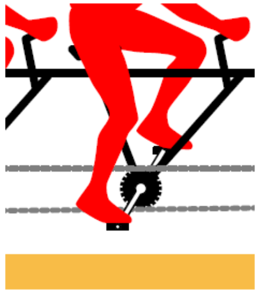
This equation is nonlinear and we can simplify it under certain assumptions.

Alternating current

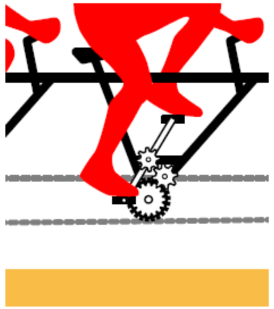
The tandem bicycle analogy



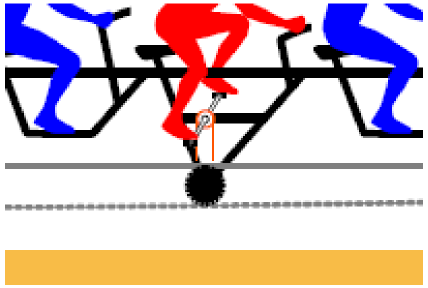
Synchronous generators using the grid frequency



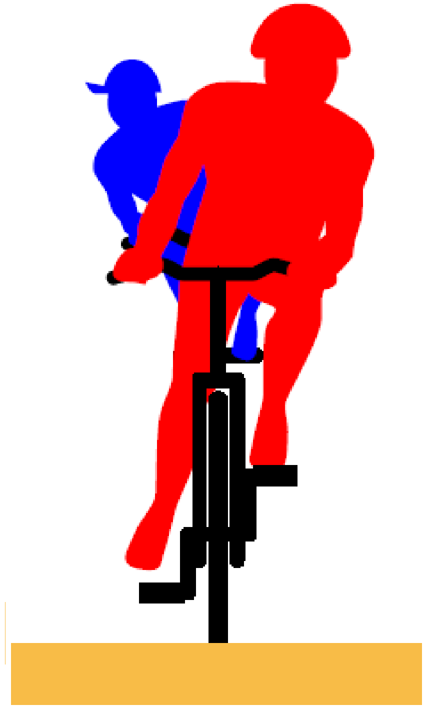
Synchronous generators using a different frequency



Other generators, e.g., solar PV



Reactive power



Further readings: Fassbinder 2005, [The electrical system as a tandem bicycle]

Linearized power flow equations for AC networks

Linear power flow

$$S_{ij} = \frac{1}{R + jX} V_i^2 \left(\frac{V_j}{V_i} e^{j(\theta_j - \theta_i)} \quad - 1 \right)$$

1º. Power flows primarily according to angle differences

$V_i \approx V_j$

$$S_{ij} = \frac{1}{R + jX} V_i^2 \left(\cos(\theta_i - \theta_j) + j sin(\theta_i - \theta_j) - 1 \right)$$

2º. No significant voltage shift occurs between the nodes

$\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$ $\qquad cos(\theta_i - \theta_j) \approx 1$

$$S_{ij} = \frac{1}{R + jX} V_i^2 j(\theta_i - \theta_j)$$

3º. Resistance of the links is neglected.
Hence, power losses are not included $R \ll X$

$$P_{ij} = P_l = \frac{V_i^2}{X} (\theta_i - \theta_j) = \frac{\theta_i - \theta_j}{x_L}$$

Step 3 also implies that reactive power flow is neglected $Q_i \sim 0$
(reactive power flow delivered by line is proportional to voltage drop, it is controlled so that it does not "travel" far)

We define a normalized reactance. $x_L = \frac{X}{V_i^2}$

θ_i is in radians and P_{ij} is in per unit!

This ensure consistent tolerance used in the optimization problem for optimality (objective function) and feasibility (constraints)

The same result is discussed in Optimization in Modern Power Systems (Lecture 3)

Analogy between DC and linearized AC power flow

$$I_l = \frac{V_i - V_j}{R_l}$$

$$P_l = \frac{\theta_i - \theta_j}{x_l}$$

DC POWER FLOW	LINEARIZED AC POWER FLOW
Current flow I_l	Active power flow P_l
Voltage V_i	Voltage angle θ_i
Resistance R_l	Reactance x_l

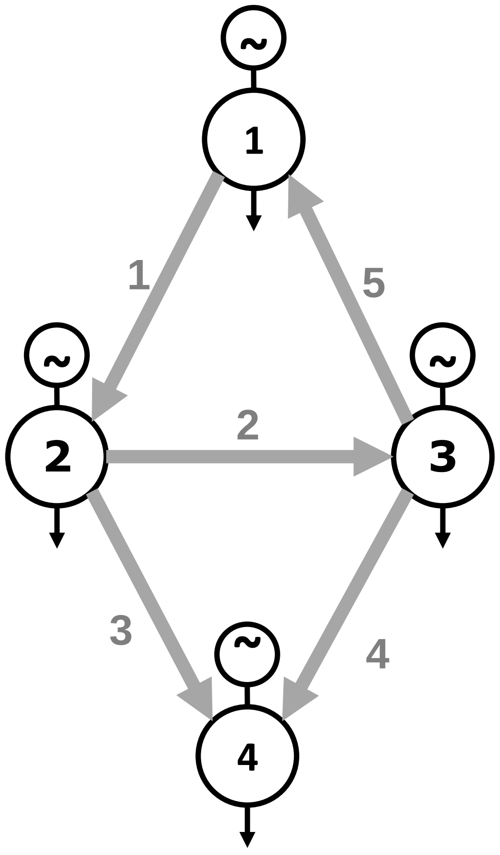
This is approximation is called linearized AC power flow or DC approximation.

Linearized AC power flow

Kirchhoff's Currents Law $$P_i = \sum_l K_{il} P_l \qquad \forall i$$

Here, P_i is the power that node i wants to inject, while P_l is the power flowing throughout link l

Kirchhoff's Voltage Law $$\sum_l C_{lc}\, \theta_l = 0 \qquad \forall c$$



Alternative way of expressing Kirchhoff's Current Law

$$P_l = \frac{1}{x_l}\theta_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$$

$$P_i = \sum_l K_{il} P_l = \underbrace{\sum_l K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

B_{kl} is the diagonal matrix of inverse lines series reactance $B_{ll} = \frac{1}{x_l}$

The weighted Laplacian matrix relates the injected power P_i and voltage angles θ_j in every node

The weighted Laplacian matrix is also called the Bus Susceptance Matrix

Linearized AC power flow

Power flow analysis:

Find the flows in the links of a network given the injection pattern for the nodes.

$$P_i = \sum_l K_{il} P_l = \sum_l \underbrace{K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

The weighted Laplacian matrix relates the injected power P_i and voltage angles θ_j in every node

$$P_i = \sum_j L_{ij}\, \theta_j$$

To obtain the power flows P_l :

1º. Calculate the voltage angles using the inverse of the weighted Laplacian matrix.

$$\theta_j = \sum_i (L^{-1})_{ji}\, P_i$$

2º. Calculate the flows P_l using the voltage angles

$$P_l = \frac{1}{x_l} \sum_j K_{lj} \theta_j$$

Power Transfer Distribution Factors (PTDF) matrix

Power flow analysis:

Find the flows in the links of a network given the injection pattern for the nodes.

An alternative method consists in using the Power Transfer Distribution Factors (PTDF) matrix

$$P_l = \frac{1}{x_l}\theta_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$$

$$P_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j = \frac{1}{x_l}\sum_{ji} K_{lj}(L^{-1})_{ji}P_i = \sum_i PTDF_{li}P_i$$

The PTDF matrix can be understood as a linear sensitivity that represents the marginal change of the active power flow on a line if we apply a marginal increase of the power injection at a node.

For every tuple <line, node> we have a different power transfer distribution factor

$$\Delta P_l = PTDF_{li}P_i$$

The linearized AC power flow based on PTDF formulation is used for flow-based market coupling of the European markets, each node corresponds to a biding zone and PTDFs are derived for the interconnections between countries

Linearized AC power flow

$$P_i = \sum_l K_{il} P_l = \underbrace{\sum_l K_{il} \frac{1}{x_l} \sum_j K_{lj}}_{L = KBK^T} \theta_j$$

We need the inverse of the weighted Laplacian matrix. $$\theta_j = \sum_i (L^{-1})_{ji} P_i$$

The weighted Laplacian is a singular matrix and thus not invertible. To invert it, we follow the procedure:

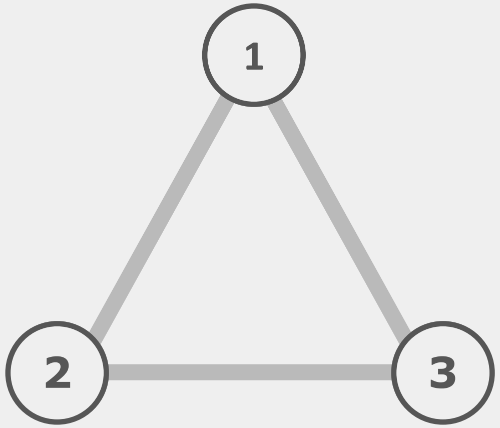
a. Delete the row and column corresponding to the slack bus, so that we obtain a matrix with dimension $(i-1) \cdot (j-1)$

b. Invert the matrix $(i-1) \cdot (j-1)$

c. Add a row and column of zeros at the row and column corresponding to the slack bus to obtain the matrix with dimension $i \cdot j$

This is what we did on previous lecture (slide 17)

DC power flow in a 3-node network



$$I_1 = I_{12} + I_{13} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

$$I_2 = I_{21} + I_{23} = (V_2 - V_1) + (V_2 - V_3) = -V_1 + 2V_2 - V_3$$

$$I_3 = I_{31} + I_{32} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

There are only two independent variables because $I_1 + I_2 + I_3 = 0$

We select $V_3 = 0$

1º. Calculate the voltages using the inverse of the Laplacian matrix.

$$I_1 = 2V_1 - V_2$$

$$I_2 = -V_1 + 2V_2$$

$$\begin{cases} V_1 = \frac{2}{3} I_1 + \frac{1}{3} I_2 \\ V_2 = \frac{1}{3} I_1 + \frac{2}{3} I_2 \\ V_3 = 0 \end{cases}$$

2º. Calculate the current flows using the voltages

$$I_{12} = (V_1 - V_2)$$

$$I_{13} = (V_1 - V_3)$$

$$I_{23} = (V_2 - V_3)$$

We can also express the result as a matrix that relates the current flows to the current injection

$$\begin{pmatrix} I_{12} \\ I_{13} \\ I_{23} \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

This is equivalent to calculating the PTDF matrix by inverting the Laplacian and multiplying by the incidence matrix

Additional constraints for the flow

- The linearized AC approximation is only valid for stable situations, where voltage angle differences are small. It can not be used to analyze heavily-loaded systems, fast changes in the systems, restart from blackout, network splitting, distribution networks, etc.

- With the linear power flow approximation, we cannot estimate losses in the network because we have assumed resistance to be zero.

- We can use the linear power flow approach for long-term planning.

Modelling approaches for power flow in AC networks

- Net Transfer Capacities (NTC)
- Linearized AC Power Flow
- AC Power Flow

$$power\ flow_l \leq Capacity_l$$

$$power\ injection_n = \sum_l K_{nl}\ power\ flow_l$$

$$\sum_l C_{lc}\ x_l\ power\ flow_l = 0$$

Add description/pros and cons

AC power flows (without linearization in next lecture)