46770 Integrated energy grids

Marta Victoria

Lecture 9 – CO2 Constraint

Learning goals for this lecture

– Add learning goals

Cost recovery in optimized markets ("non-profit rule")

Now our optimization variables are the energy $g_{s,t}$ generated by every generator s in every time step t and the installed capacity G_s of every generator

$$\begin{cases} \min_{g_{s},\, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s \, g_{s,t} \right] \\ \text{subject to:} \\ \sum_s g_{s,t} - d_t \; = 0 \leftrightarrow \lambda_t \\ -g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}} \end{cases}$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda^*_{s,t} + \overline{\mu^*_{s,t}} \; = 0 \quad \rightarrow \quad o_s = \lambda^*_{s,t} - \overline{\mu^*_{s,t}}$$

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = \frac{\partial f}{\partial G_s} - \sum_i \lambda_i \frac{\partial h_i}{\partial G_s} - \sum_j \mu_j \frac{\partial g_j}{\partial G_s} = c_s - \sum_t \overline{\mu^*_{s,t}} \cdot (1) = 0 \quad \rightarrow \quad c_s = \sum_t \overline{\mu^*_{s,t}}$$

Cost recovery in optimized markets ("non-profit rule")

Total cost for generator s

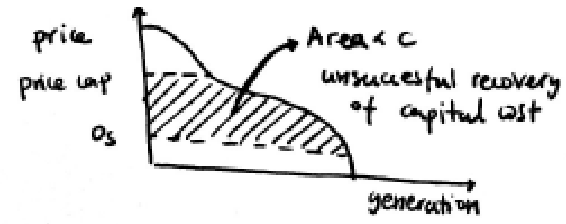
$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = c_s G_s^* + \sum_t (\lambda_{s,t}^* - \overline{\mu_{s,t}^*})\, g_{s,t}^*$$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = c_s G_s^* + \sum_t \lambda_t^* g_{s,t}^* - \sum_t \underbrace{\overline{\mu_{s,t}^*}}_{c_s}\, \underbrace{g_{s,t}^*}_{G_s^*}$$

This is only true if the constraint is binding, but otherwise $\overline{\mu_{s,t}^*} = 0$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = \underbrace{\cancel{c_s G_s^*}}_{0} + \sum_t \lambda_t^* g_{s,t}^* - \underbrace{\cancel{c_s G_s^*}}_{0}$$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = \sum_t \lambda_t^* g_{s,t}^*$$



The generator costs (fixed and variable) are **exactly fully-recovered** form market revenues (generation times market Price). This is known as long-term market equilibrium or non-profit rule.

If a price cap is set in the market, this is not true anymore. Some countries set a price cap and use capacity markets to compensate for the unsuccessful recovery of capital cost.

Global CO_2 constraint

Total cost for generator s

$$\sum_{s,t} \frac{\epsilon_s}{\eta_s} g_{s,t} \leq CAP_{CO2} \quad \leftrightarrow \quad \mu_{CO2}$$

Where ϵ_s are the specific emissions per technology (in kgCO2/$MWh_{thermal}$), and η_s is the efficiency of the generator (in $MWh_{electricity}$ / $MWh_{thermal}$)

μ_{CO2} is the Lagrange or Karush-Kuhn-Tucker (KKT) multiplier associated with the CO2 constraint. It represents the change in the objective function at the optimal solution, with respect to a small change in the constraint.

Small change in constraint : $CO_2\ limit^{*} = CO_2\ limit + 1\ \text{tonne}$

Change in objective function : $System\ cost^{*} = System\ cost + \Delta System\ cost$

Hence μ_{CO2} represents the cost of 1 tone of CO_2

- If $\mu_{CO2} = 0$ the constraint is not binding.
- If $\mu_{CO2} \neq 0$ we would obtain the same solution if we remove the constraint and add a CO_2 price equal to μ_{CO2}

μ_{CO2} is also known as CO_2 price, CO2 tax, shadow price or marginal abatement cost. CO2 price,

Setting a global CO2 constraint is equivalent to adding a CO2 tax

If we repeat the demonstration of cost recovery in optimized markets (“non-profit rule), including the CO2 constraint

$$\min_{g_{s}, G_s} \left[\sum_s c_s G_s + \sum_{s,t} o_s \, g_{s,t} \right]$$

subject to:

$$\sum_s g_{s,t} - d_t \ = 0 \leftrightarrow \lambda_t$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_{s,t}}$$

$$-\sum_{s,t} \frac{\epsilon_s}{\eta_s} g_{s,t} + CAP_{CO2} \geq 0 \quad \leftrightarrow \quad \mu_{CO2}$$

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = \ \frac{\partial f}{\partial g_{s,t}} - \sum_i \lambda_i \frac{\partial h_i}{\partial g_{s,t}} - \sum_j \mu_j \frac{\partial g_j}{\partial g_{s,t}} = o_s - \lambda^*_{s,t} + \overline{\mu^*_{s,t}} + \mu_{CO2} \frac{\epsilon_s}{\eta_s} = 0 \quad \rightarrow \quad o_s = \lambda^*_{s,t} - \overline{\mu^*_{s,t}} - \mu_{CO2} \frac{\epsilon_s}{\eta_s}$$

Setting a global CO2 constraint is equivalent to adding a CO2 tax

If we repeat the demonstration of cost recovery in optimized markets (“non-profit rule)

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = c_s G_s^* + \sum_t \left(\lambda_{s,t}^* - \overline{\mu_{s,t}^*} - \mu_{CO2}\frac{\epsilon_s}{\eta_s}\right) g_{s,t}^*$$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = c_s G_s^* + \sum_t \left(\lambda_t^* - \mu_{CO2}\frac{\epsilon_s}{\eta_s}\right) g_{s,t}^* \;-\; \underbrace{\sum_t}_{c_s} \underbrace{\overline{\mu_{s,t}^*}\, g_{s,t}^*}_{G_s^*}$$

This is only true if the constraint is binding, but otherwise $\overline{\mu_{s,t}^*} = 0$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = \cancelto{0}{c_s G_s^*} + \sum_t \lambda_t^* g_{s,t}^* \;-\; \cancelto{0}{c_s G_s^*}$$

$$c_s G_s^* + \sum_t o_s\, g_{s,t}^* = \underbrace{\sum_t \left(\lambda_t^* - \mu_{CO2}\frac{\epsilon_s}{\eta_s}\right) g_{s,t}^*}_{\text{Market revenues minus cost of CO2 price times generation}}$$

This shows that it is equivalent setting a global CO2 constraint or adding a CO2 price

(i.e. substituting o_s by $o_s + \mu_{CO2}\frac{\epsilon_s}{\eta_s}$

CO_2 price and long-term economic analysis

William D. Nordhaus

Nobel prize in Economy 2018 for integrating climate change into long-run macroeconomic analysis

The Syndrome of Free-Riding : nationalist or non-cooperative policies that seek to maximize the interests of a single country at the expense of others are a poor way to resolve global problems.

Climate clubs: nations can overcome the syndrome of free-riding in international climate agreements if they adopt the club model rather than voluntary arrangements.

Further reading: W. Nordhaus, Climate Change: The Ultimate Challenge for Economics, American Economic Review (2019)

What discount rates should we use when optimizing a transition path from now to 2050?

How do we discount our children's costs?

Further reading: D. Roberts, Discount rates: A boring thing you should know about (with otters!), 2012