46770 Integrated energy grids

Marta Victoria

Lecture 3 – Networks

Recap from previous lectures

Balancing renewable generation

We ended last lecture, stating that we need optimization to decide how to combine different flexibility options.

1. Back-up generation and curtailment
2. Storage
3. Regional integration of renewables
4. Demand-side management
5. Sector-coupling

Temporal and spatial balancing (together with demand-side management and sector-coupling) must be simultaneously considered -> **This requires optimization!**

General formulation of optimization problems (I)

$$\begin{cases} \min_x f(x) \\ \text{subject to} \\ h_i(x) = c_i \ \leftrightarrow \ \lambda_i \\ g_j(x) \geq d_j \ \leftrightarrow \ \mu_j \end{cases}$$

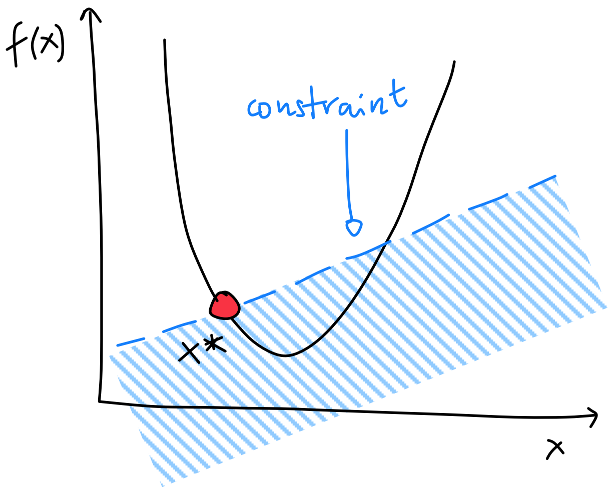
Objective function

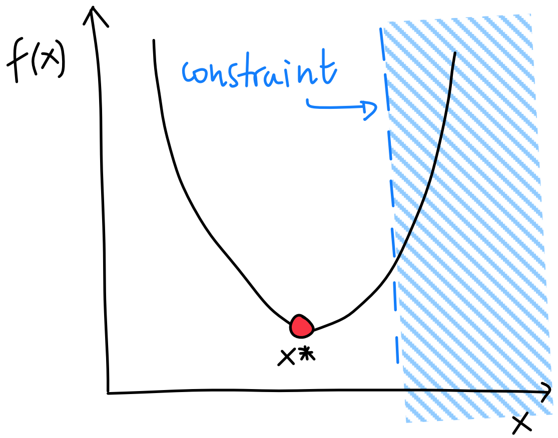
Equality and inequality constraints

λ_i and μ_j are the Lagrange or Karush-Kuhn-Tucker (KKT) multipliers

x, y, z are called primary variables and λ_i , μ_j are called dual variables

A constraint can be binding (affecting the optimal solution x^*) or not-binding





Economic dispatch or one-node dispatch optimization (I)

Assume we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity G_s and a linear variable cost o_S. The economic dispatch consists in calculating the optimal dispatch (how much energy is being produced by each generator g_s) to supply the demand d in a certain hour while minimizing the total system cost.

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0 \quad \leftrightarrow \quad \lambda$$

$$g_s \geq 0 \quad \leftrightarrow \quad \underline{\mu_s}$$

$$-g_s + G_s \geq 0 \quad \leftrightarrow \quad \overline{\mu_s}$$

For renewable generators, the installed capacity is multiplied by the capacity factor: $-g_s + CF_s G_s \leq 0 \ \leftrightarrow \overline{\mu_s}$

Types of optimization problems and course structure

<table>
<tr><th></th><th>One node</th><th colspan="4">Network</th></tr>
<tr><td rowspan="2">One time step</td><td rowspan="2">Economic dispatch or
One-node dispatch optimization
(Lecture 2)</td><td colspan="2">Power</td><td rowspan="2">Gas flow
(Lecture 6)</td><td rowspan="2">Heat flow
(Lecture 7)</td></tr>
<tr><td>Linearized
AC power flow
(Lecture 4)</td><td>AC power flow
(Lecture 5)</td></tr>
<tr><td>Multiple time steps</td><td>Multi-period optimization

Join capacity and dispatch optimization in one node
(Lecture 8)</td><td colspan="4">Join capacity and dispatch optimization in a network
(Lecture 10)</td></tr>
</table>

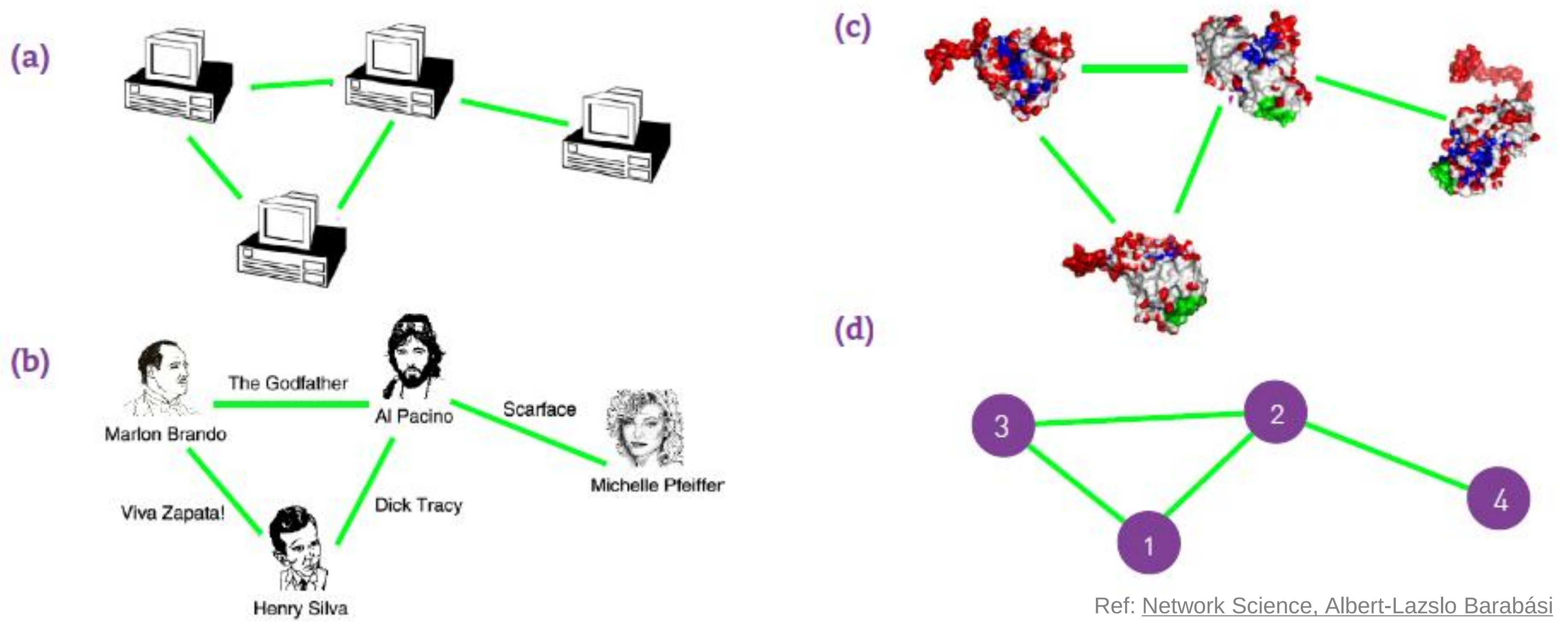
Networks

Learning goals for this lecture

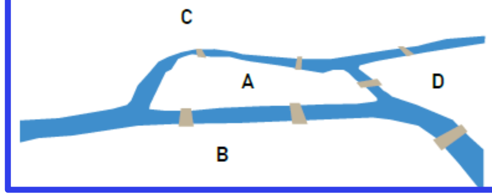
- Describe the Degree, Adjacency, and Laplacian (or bus susceptance) matrix.
- Describe the Incidence and Cycle matrix.
- Obtain the Laplacian (or bus susceptance) matrix describing the topology of a network.
- Calculate the current flows in a DC network by calculating the voltages using the inverse of the weighted Laplacian matrix
- Calculate the current flows in a DC network by using the Power Transfer Distribution Factors (PTDF) matrix

Networks

A network is a catalogue of a system's components often called nodes (**N**) or vertices and the direct interactions between them, called links (**L**) or edges.



Ref: Network Science, Albert-Lazslo Barabási



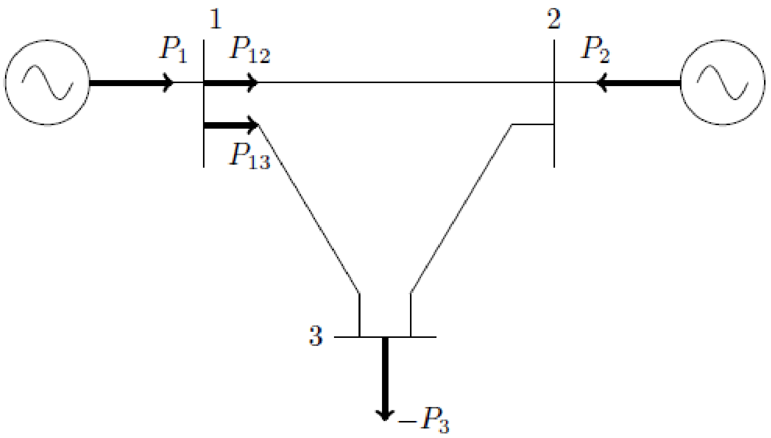
The origin of network science: The Bridges of Königsberg

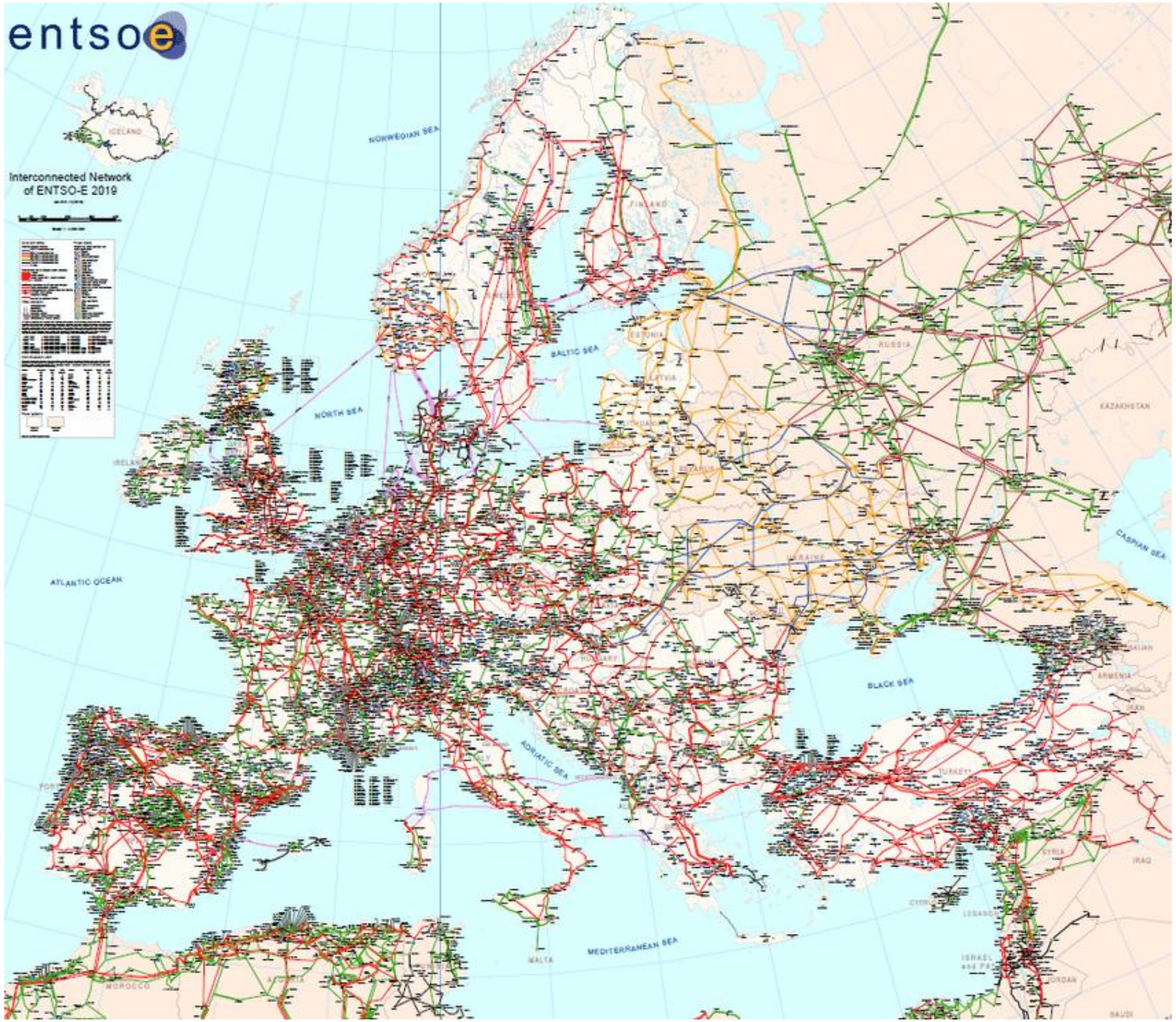
Can one walk across all seven bridges and never cross the same one twice?

https://www.youtube.com/watch?v=nZwSo4vfw6c

Networks

In power systems, nodes (**N**) are called buses and links (**L**) are called lines.

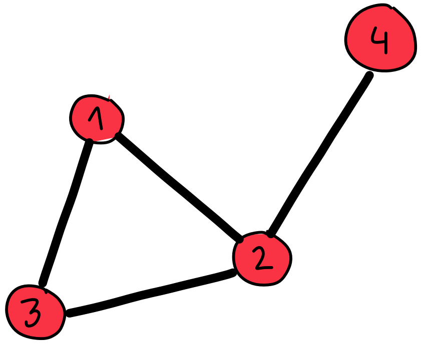




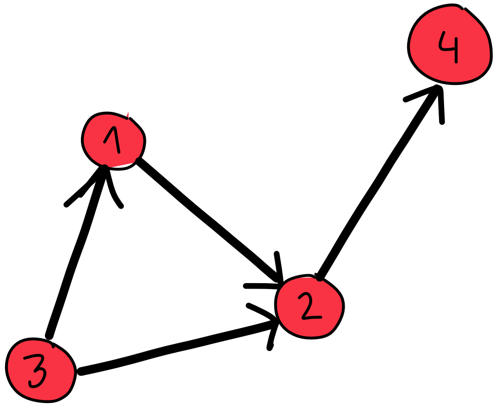
Ref: ENTSOE map

Networks

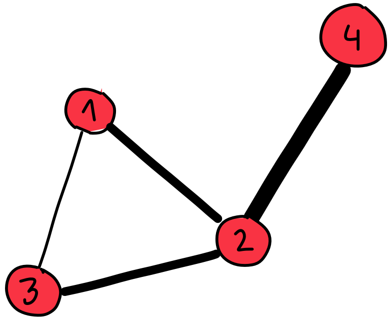
Undirected Network: A network whose links do not have a defined direction. E.g. actor network, power grid …



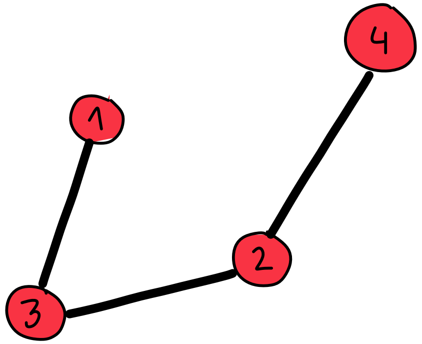
Directed Network: A network whose links have selected directions. E.g. WWW, citation network…



Weighted Network: A network whose links have a defined weight, strength or flow parameter, E.g. power grid, gas networks …



Radial Network: A network that was no cycles



Ref: Network Science, Albert-Lazslo Barabási

Networks

The degree k indicates the number of links that a node has to other nodes.

The average degree $\langle k \rangle$ gives an indication of how meshed is a network.

NETWORK	NODES	LINKS	DIRECTED UNDIRECTED	N	L	⟨k⟩
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

Ref: Network Science, Albert-Lazslo Barabási

Overview of relevant equations to model energy networks

Regardless of the type of network (power, gas, heat) we need:

- Equations that represent the balance of energy (or mass) in every node -> **Nodal balance equations**

- Equations that represent the transport of energy in every link -> **Link equations**

(these will depend on the physical applicable laws when transporting power, gas, heat)

- Information on how the nodes are connected by links -> **Matrices to capture the network topology**

  (we need a “map” of the network)

Notation in this course

- Nodes: i, j, k
- Links, Lines: $l, i \to j$ (line transporting power from node i to node j)
- Matrix X_{ij} with i rows and j columns
- Complex number $\bar{S}$
- To simplify notation, index are not always declared

For example, when we write $-g_{n,s,t} + G_{s,n} \geq 0$

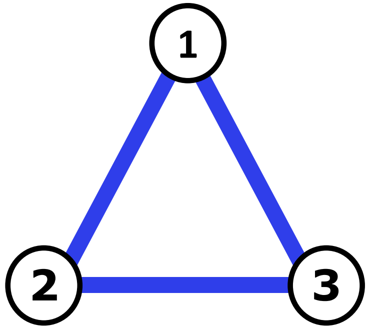
We implicitly mean $-g_{n,s,t} + G_{s,n} \geq 0 \quad \forall n, s, t$ where n, s, t are technologies, nodes and timesteps

Degree matrix

We are going to define five matrices which are very useful in energy networks: **Degree**, Adjacency, Laplacian, Incidence and Cycles matrix.

Degree matrix

$$D_{ij} = \begin{cases} k_i = \sum_{j=1}^{n} A_{ij} & if\ i = j \quad \text{number of links attached to node } i \\ 0 & if\ i \neq j \end{cases}$$

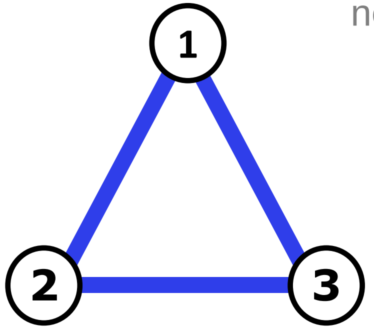


$$\begin{array}{cc} \text{nodes:} \quad 1 \quad 2 \quad 3 & \text{nodes} \\ \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

Adjacency matrix

We are going to define five matrices which are very useful in energy networks: Degree, **Adjacency**, Laplacian, Incidence and Cycles matrix.

Adjacency matrix $A_{ij} = \begin{cases} 1 & \text{if link between node } i \text{ and } j \text{ exists} \\ 0 & \text{if link between node } i \text{ and } j \text{ does not exist} \end{cases}$

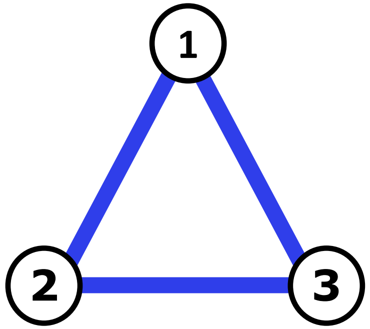


$$\begin{array}{c|c} \text{nodes: } \; 1 \quad 2 \quad 3 & \text{nodes} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

Laplacian matrix

We are going to define 5 matrices which are very useful in energy networks: Degree, Adjacency, **Laplacian**, Incidence and Cycles matrix.

Laplacian matrix $L_{ij} = D_{ij} - A_{ij}$



$$L_{ij} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} = \left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\right]$$

The columns (and rows) of the Laplacian matrix sum to zero $\sum_j L_{ij} = 0$

The Laplacian matrix is a “map” of the network, it contains information on how the nodes are connected.

If the links have different strength, we can add this information in the off-diagonal elements and we get the weighted Laplacian matrix.

In AC power flows, the weighted Laplacian matrix is also called Bus Susceptance matrix.

Laplacian matrix (II)

Why we call it Laplacian matrix?

In mathematics, the Laplacian operator indicates the divergence of the gradient, it is given by the sum of second partial derivatives of the function with respect to each independent variable

$$\Delta f = \nabla^2 f = \varphi \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2}\right) f(x, y, z) = \varphi(x, y, z)$$

If we approach the first and second partial derivatives

$$\frac{\partial}{\partial x} \sim \frac{f_i - f_{i-1}}{\Delta x}$$

$$\frac{\partial^2}{\partial x^2} \sim \frac{\left.\frac{\partial}{\partial x}\right|_{i+1/2} - \left.\frac{\partial}{\partial x}\right|_{i-1/2}}{\Delta x} = \frac{\frac{f_{i+1} - f_i}{\Delta x} - \frac{f_i - f_{i-1}}{\Delta x}}{\Delta x} = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2} \qquad L_{ij} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

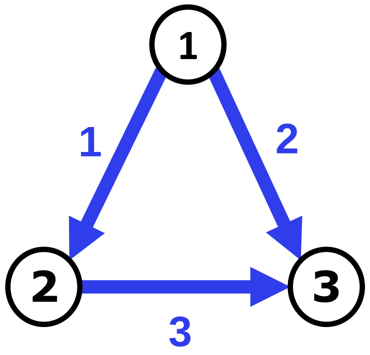
The rows in the Laplacian matrix resembles the Laplacian operator

Incidence matrix

We are going to define 5 matrices which are very useful in energy networks: Degree, Adjacency, Laplacian, **Incidence** and Cycles matrix.

Incidence matrix

$$K_{il} = \begin{cases} 1 & \text{if link } l \text{ starts at node } i \\ -1 & \text{if link } l \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{array}{cc} & \begin{array}{ccc} \text{links: } 1 & 2 & 3 \end{array} \quad \text{nodes} \\ K_{il} = & \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

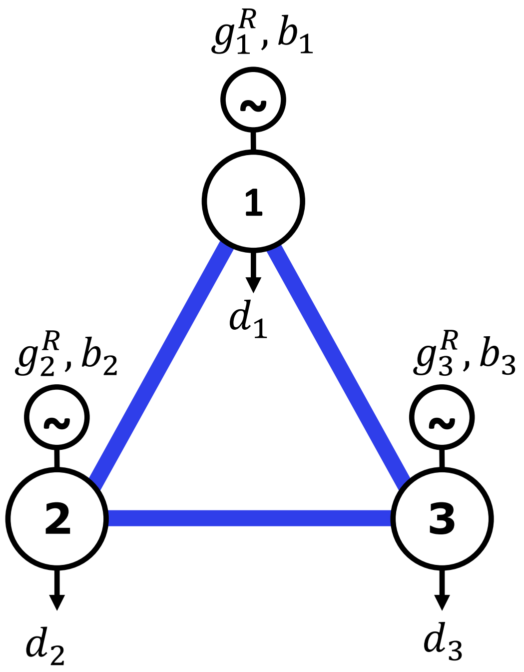
The Laplacian and the Incidence matrix are related by: $L = KK^T$

The Incidence matrix contains information on the links connected to each node and allows us to impose energy conservation or nodal energy balances $I_i = \sum_l K_{il} I_l$

The transpose of the Incidence matrix allows us to express the current flowing through a link as a function of the voltage difference between the nodes $I_l = \sum_j K_{lj} V_j$

Example: Current flows in a 3-node DC network

Energy Networks. Stating the problem (I)



In every node i, mismatch (i.e., renewable generation g_i^R minus demand d_i), is equal to local balance b_i plus injection p_i :

$$\Delta_i = g_i^R - d_i = b_i + p_i$$

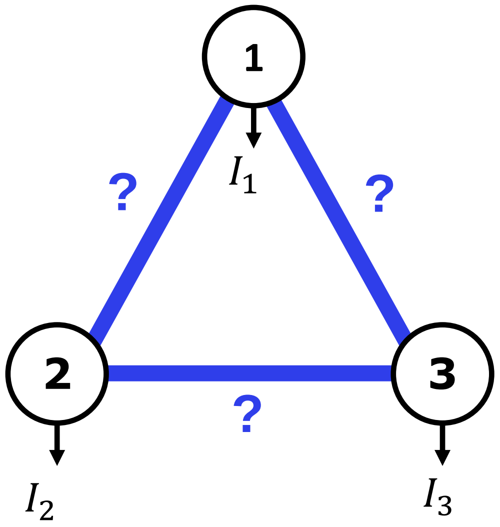
The total sum of energy injection is zero (energy conservation):

$$\sum_i p_i = p_1 + p_2 + p_3 = 0$$

The **goal** of power flow analysis is to **find the power flows through the links of a network given the injection pattern for the nodes.**

More generally, in energy networks, we want to calculate the flows of energy (electricity, gas, heat) given the injection patterns in the nodes

Energy Networks. Stating the problem (I)



In this lecture, we focus on a simplified version using DC circuits.

How can we **find the current flows in the links of a network given the current injection pattern?**

Data: I_1, I_2, I_3 Unknown: $I_{1\to2}, I_{1\to3}, I_{2\to3}$

We follow the convention that positive current is leaving a node.

The total sum of current injection is zero (energy conservation):

$$\sum_i I_i = I_1 + I_2 + I_3 = 0$$

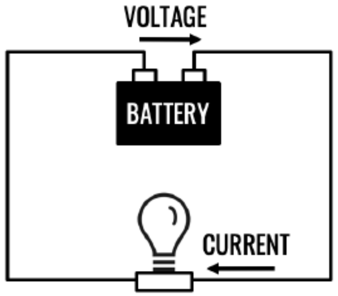
We can use the incidence matrix to impose nodal energy balances $I_i = \sum_l K_{il} I_l$

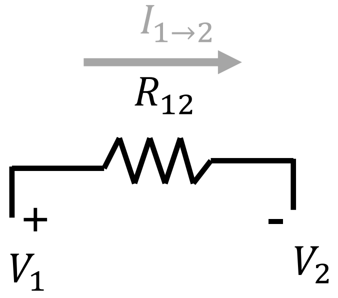
However, to calculate the current flows, we need additional equations otherwise there are many possible solutions (e.g. consider $I_1 = 1,\ I_2 = -1,\ I_3 = 0$)

How do current flow in DC circuits?

DC circuits and Ohm's law

The electric current through a conductor between two points is directly proportional to the voltage difference between the two points





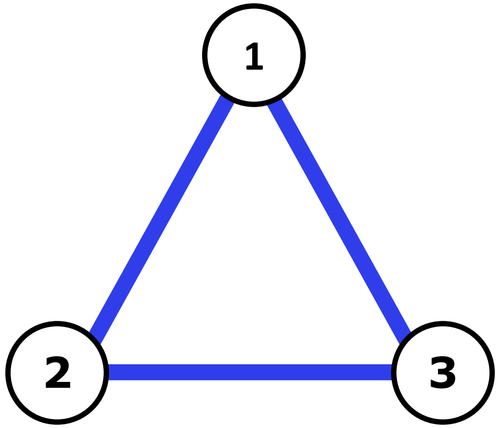
$$I_{1\to 2} = \frac{V_1 - V_2}{R_{12}}$$

Notice here that the current flowing trough a link depends on the voltage difference, so we can always find an equivalent solution adding a constant to every voltage. We will have to choose a slack or reference node for which $V = 0$.

For simplicity, in the following example, we assume $R_{12} = 1$ and hence $I_{1\to 2} = V_1 - V_2$

Example: Current flows in a 3-node network

Problem: Calculate the current flows $I_{i\to j}$ in the links of a network given the current injection pattern for the nodes I_i



$$I_1 = I_{1\to 2} + I_{1\to 3} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

$$I_2 = I_{2\to 1} + I_{2\to 3} = (V_2 - V_1) + (V_2 - V_3) = -V_1 + 2V_2 - V_3$$

$$I_3 = I_{3\to 1} + I_{3\to 2} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}}_{\substack{L_{ij} \\ \text{Laplacian matrix}}} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \left[\underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\substack{D_{ij} \\ \text{degree matrix}}} - \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{\substack{A_{ij} \\ \text{adjacency matrix}}} \right] \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$L_{ij} = D_{ij} - A_{ij}$$

Laplacian matrix = degree matrix − adjacency matrix

The Laplacian matrix relates the current injection I_i and voltages V_j in every node $I_i = \sum_j L_{ij} V_j$

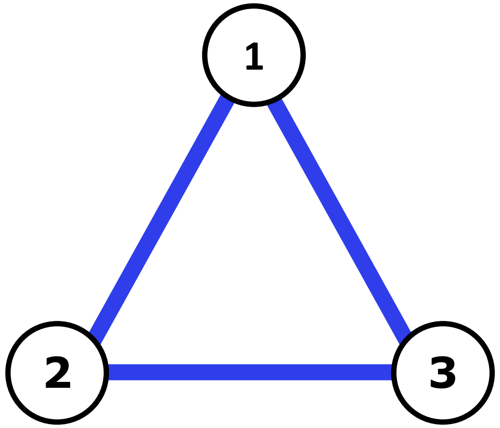
To obtain the currents $I_{i\to j}$ flowing through the links :

1º. Calculate the voltages V_j

2º. Calculate the flows $I_{i\to j}$ using the voltages V_j

Example: Current flows in a 3-node network

Problem: Calculate the current flows $I_{i\to j}$ in the links of a network given the current injection pattern for the nodes I_i



$$I_1 = I_{1\to 2} + I_{1\to 3} = (V_1 - V_2) + (V_1 - V_3) = 2V_1 - V_2 - V_3$$

$$I_2 = I_{2\to 1} + I_{2\to 3} = (V_2 - V_1) + (V_2 - V_3) = -V_1 + 2V_2 - V_3$$

$$I_3 = I_{3\to 1} + I_{3\to 2} = (V_3 - V_1) + (V_3 - V_2) = -V_1 - V_2 + 2V_3$$

There are only two independent variables because $I_1 + I_2 + I_3 = 0$

We select $V_3 = 0$

1º. Calculate the voltages

$$I_1 = 2V_1 - V_2$$

$$I_2 = -V_1 + 2V_2$$

$$\begin{cases} V_1 = \frac{2}{3} I_1 + \frac{1}{3} I_2 \\ V_2 = \frac{1}{3} I_1 + \frac{2}{3} I_2 \\ V_3 = 0 \end{cases}$$

2º. Calculate the current flows using the voltages

$$I_{1\to 2} = (V_1 - V_2)$$

$$I_{1\to 3} = (V_1 - V_3)$$

$$I_{2\to 3} = (V_2 - V_3)$$

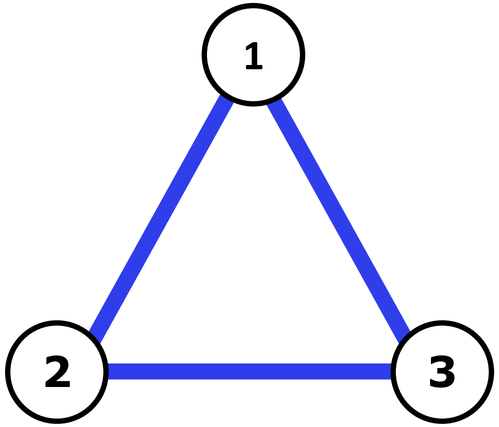
We can also express the result as a matrix that relates the current flows to the current injection

$$\begin{pmatrix} I_{1\to 2} \\ I_{1\to 3} \\ I_{2\to 3} \end{pmatrix} = \begin{pmatrix} 1/3 & -1/3 & 0 \\ 2/3 & 1/3 & 0 \\ 1/3 & 2/3 & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

General formulation: Calculating currents flows knowing current injections

Calculating the current flows in a network

The Laplacian matrix relates the injected currents I_i and voltages V_j in every node $I_i = \sum_j L_{ij} V_j$



To obtain the currents $I_{i\to j}$ flowing through the links, we follow two steps:

1º. Calculate the voltages V_j using the inverse of the Laplacian matrix $V_j = \sum_i (L^{-1})_{ji} I_i$

2º. Calculate the current flows $I_l = I_{i\to j}$ using the voltages and the transpose incidence matrix $I_l = \sum_j K_{lj} V_j$

Inverting the Laplacian matrix

There is a small caveat: the **Laplacian matrix is not invertible**.

because it is singular (its determinant is zero).

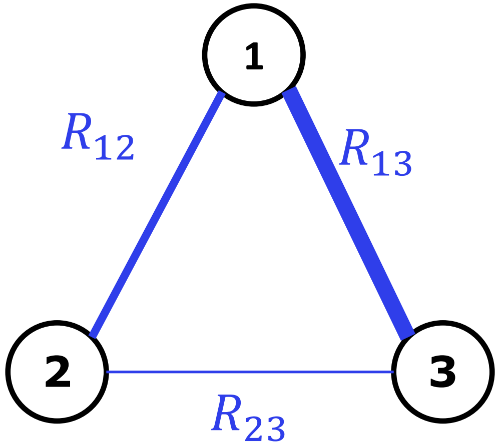
This is related to the fact that the currents flows depend on the voltage differences, so we can always find an equivalent solution adding a constant to every voltage.

To invert it, we delete the row and column corresponding to the slack bus (in the previous example $V_3 = 0$), so that we obtain a matrix with dimension $(i-1)\cdot(j-1)$ that can be inverted

Alternatively, we can use the Moore-Penrose pseudo-inverse

Weighted Laplacian matrix

If we assume now that the resistance is different for every link.



The current flowing through every link $I_{i\to j} = I_l$ depends on the voltage difference across the nodes connected by the link

$$I_{i\to j} = I_l = \frac{V_l}{R_l} = \frac{1}{R_l}\sum_j K_{lj}V_j$$

And the current injected in every node I_i is equal to the sum of the current exported through the links

$$I_i = \sum_l K_{il} I_l = \sum_l \underbrace{K_{il}\frac{1}{R_l}\sum_j K_{lj}}_{L = KBK^T} V_j$$

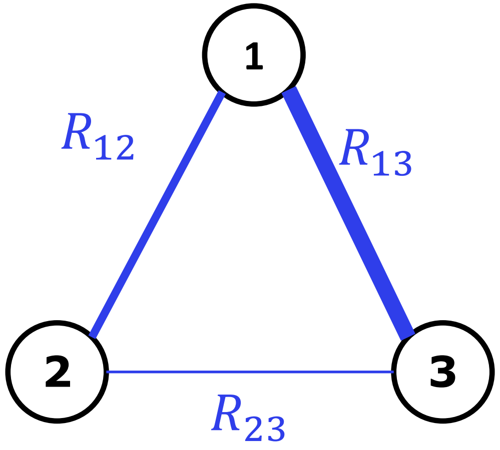
$$B_{ll} = \frac{1}{R_l}$$ is the diagonal matrix of inverse of every link resistance

The **weighted Laplacian matrix** relates the injected currents I_i and voltages V_j in every node.

In AC power flows, the weighted Laplacian matrix is also called Bus Susceptance matrix.

Calculating the current flows in a weighted network

The weighted Laplacian matrix relates the injected currents I_i and voltages V_j in every node $I_i = \sum_j L_{ij} V_j$



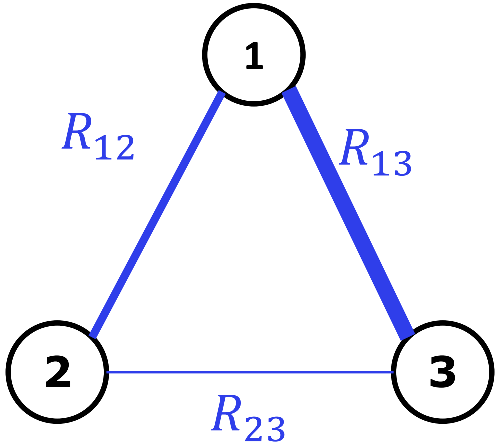
To obtain the currents $I_{i \to j}$ flowing through the links, we follow two steps :

1º. Calculate the voltages V_j using the inverse of the weighted Laplacian matrix $V_j = \sum_i (L^{-1})_{ji} I_i$

2º. Calculate the current flows $I_l = I_{i \to j}$ using the voltages and the transposed incidence matrix $I_l = \frac{1}{R_l} \sum_j K_{lj} V_j$

Power Transfer Distribution Factors (PTDF) matrix

An alternative method consists in using the Power Transfer Distribution Factors (PTDF) matrix



$$I_l = \frac{1}{R_l}\sum_j K_{lj}V_j = \frac{1}{R_l}\sum_{ji} K_{lj}(L^{-1})_{ji}I_i = \underbrace{\sum_i PTDF_{li}I_i}$$

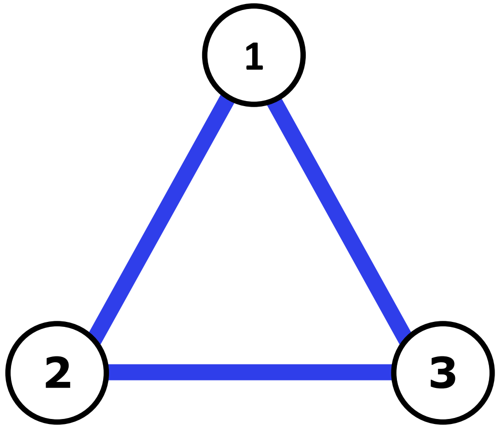
The PTDF is calculated as the transpose incidence matrix times the inverse Laplacian matrix

The PTDF matrix measures the sensitivity of current flows in each link relative to incremental changes in nodal current injections throughout the network.

We calculated the PTDF matrix at the end of the 3-node example (slide 26)

Power Transfer Distribution Factors (PTDF) matrix

The PTDF matrix measures the sensitivity of current flows in each link relative to incremental changes in nodal current injections throughout the network.



$$\begin{pmatrix} I_{1\to 2} \\ I_{1\to 3} \\ I_{2\to 3} \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} 1/3 \\ 2/3 \\ 1/3 \end{matrix}} & \begin{matrix} -1/3 \\ 1/3 \\ 2/3 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

$$I_l = \sum_i PTDF_{li} I_i$$

For a unitary current being transferred from node i to the slack node (3), each element column i indicates the resulting current flows in the links

Alternative formulation using the cycles matrix

Cycle matrix

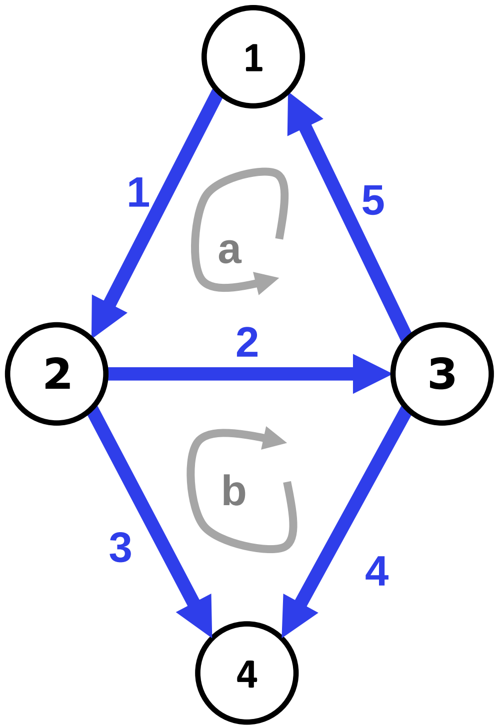
The cycle matrix contains the closed cycles.

Cycle matrix $C_{lc} = \begin{cases} 1 & \text{if link } l \text{ belongs de cycle } c \\ -1 & \text{if reverse link } l \text{ belongs to cycle } c \\ 0 & \text{otherwise} \end{cases}$

The combination of links representing a closed cycle multiplied by incidence matrix is zero: $KC = 0$

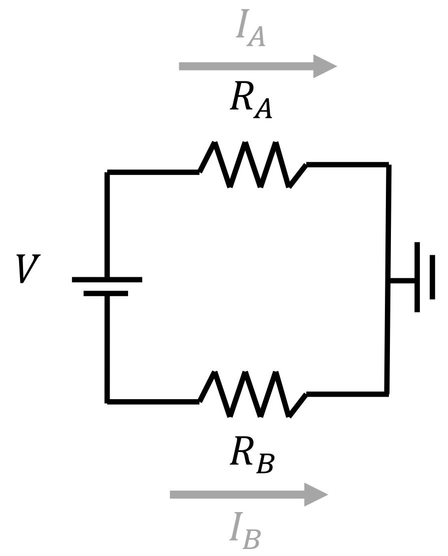
$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} = 0$$



$$\begin{array}{cc} \text{cycles:} & \begin{array}{cc} a & b \end{array} & \text{links} \\ C_{lc} = & \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \end{array}$$

Physical laws of current and voltage in DC circuits

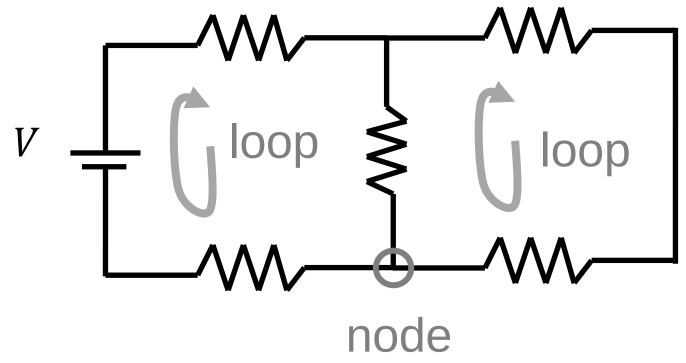


Currents flowing through a circuit depends on the circuit configuration

We use Kirchhoff's laws to determine currents in a circuit:

Kirchhoff's Currents Law — The algebraic sum of currents in a network of conductors meeting at a point is zero.

Kirchhoff's Voltage Law — The directed sum of the voltages around any closed loop is zero.



Kirchhoff's laws

Alternatively, we can solve the problem of calculating the current flows (given the current injection patterns), by imposing both Kirchhoff's laws

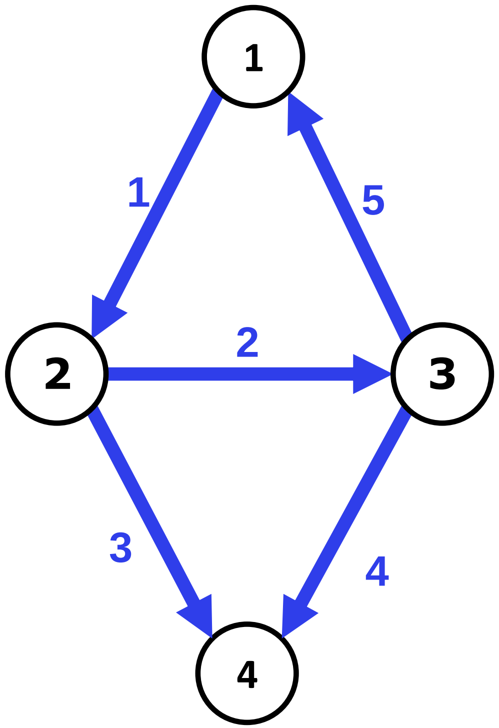
Kirchhoff's Currents Law can be imposed using the incidence matrix

$$I_i = \sum_l K_{il} I_l \qquad \forall i$$

Here, I_i is the current that node i wants to inject, while I_l is the current flowing throughout link l

Kirchhoff's Voltage Law can be imposed using the cycle matrix

$$\sum_l C_{lc} V_l = 0 \qquad \forall c$$



This method is computationally very efficient for large networks*.

Kirchhoff's voltage law is equivalent to the equations imposing Ohm's law that we have used before.

$$I_{1\to 2} = \frac{V_1 - V_2}{R_{12}} \qquad I_l = \frac{1}{R_l} \sum_j K_{lj} V_j$$

*See Hörsch et al, 2018. Linear optimal power flow using cycle flows

Problems for this lecture

Problems 3.1, 3.2 (**Group 6**)

Review tutorial on networkx
https://martavp.github.io/integrated-energy-grids/intro-networkx.html

Problems 3.3, 3.4 (**Group 7**)