## DIAMAGNETIC TERMS

From Ohm's law and Faraday's law, in the direction parallel to the magnetic field,

$$E_{\parallel} = -\mathbf{b} \cdot \frac{\partial \mathbf{A}_{\parallel}}{\partial t} - \mathbf{b} \cdot \nabla \Phi = \eta J_{\parallel} - \frac{\mathbf{b} \cdot \nabla p_e}{en}, \tag{1}$$

where

$$\mathbf{A}_{\parallel} \equiv -\frac{\Psi}{R_0} \mathbf{e}^{\zeta}. \tag{2}$$

The last term in (1) is the diamagnetic contribution to the evolution equation for the poloidal flux  $\Psi$ . In dimensionless units, the equation is

$$\frac{\partial \tilde{\Psi}}{\partial t} = \sqrt{g}B\nabla_{\parallel}\Phi - \frac{\beta_{0e}}{2\varepsilon^{2}\omega_{cy}n}\sqrt{g}B\nabla_{\parallel}p + \frac{\eta}{S}\tilde{J}^{\zeta},\tag{3}$$

Here,  $\omega_{cy} = \omega_{c0}\tau_A$ , where  $\omega_{c0} = eB_0/m_i$  is the cyclotron frequency of the thermal ions.

Contribution to the momentum balance:

$$\frac{\partial U}{\partial t} = \sqrt{g} \frac{\partial}{\partial t} \left[ \nabla \times (\rho_m \sqrt{g} \mathbf{v}) \right]^{\zeta} = \dots - \sqrt{g} \left[ \nabla \times (\rho_m \sqrt{g} (\mathbf{v}_{*i} \cdot \nabla) \mathbf{v}_{\perp}) \right]^{\zeta}, \tag{4}$$

where

$$\mathbf{v}_{*i} = \frac{\mathbf{B} \times \nabla p_i}{enB^2} \tag{5}$$

For the calculation of the components of  $(\mathbf{B} \times \nabla p \cdot \nabla)\mathbf{v}_{\perp}$ , we need either the Christoffel symbols  $\Gamma_{ij}^k$  or the use of vector identities. Instead of calculating the Christoffel symbols, we get, from vector identities,

$$(\mathbf{B} \times \nabla p \cdot \nabla)\mathbf{v}_{\perp} = \frac{1}{2} \left\{ \nabla \left[ (\mathbf{B} \times \nabla p) \cdot \mathbf{v}_{\perp} \right] + \nabla \times \left[ \mathbf{v}_{\perp} \times (\mathbf{B} \times \nabla p) \right] - (\mathbf{B} \times \nabla p) \times (\nabla \times \mathbf{v}_{\perp}) \right.$$
$$\left. - \mathbf{v}_{\perp} \times \left[ \nabla \times (\mathbf{B} \times \nabla p) \right] + \mathbf{B} \times \nabla p \left( \nabla \cdot \mathbf{v}_{\perp} \right) - \mathbf{v}_{\perp} \nabla \cdot (\mathbf{B} \times \nabla p) \right\}$$
(6)

In dimensionless units,

$$\begin{aligned} (\mathbf{B} \times \nabla p)^i &= \frac{B_0 p(0)}{a} \frac{1}{\sqrt{g}} \frac{dp_{eq}}{d\rho} \left( 0, J, -\frac{I}{\varepsilon \rho} \right) \\ (\mathbf{B} \times \nabla p) \cdot \mathbf{v}_{\perp} &= \frac{B_0 p(0)}{\tau_A} \frac{dp_{eq}}{d\rho} \left[ J \left( -\frac{g_{\rho\theta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\theta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) - \frac{I}{\varepsilon \rho} \left( -\frac{g_{\rho\zeta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\zeta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ & \left[ \mathbf{v}_{\perp} \times (\mathbf{B} \times \nabla p) \right]_i = \frac{B_0 p(0)}{\tau_A} \frac{dp_{eq}}{d\rho} \left( -\frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho}, -\frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta}, -J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \end{aligned}$$

$$\{\nabla \times [\mathbf{v}_{\perp} \times (\mathbf{B} \times \nabla p)]\}^{i} = -\frac{B_{0}p(0)}{a\tau_{A}} \frac{1}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right), \\
\varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right), \\
\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) \right]$$

where  $p' = dp_{eq}/d\rho$ .

$$\begin{split} (\nabla \times \mathbf{v}_{\perp})^{i} &= \frac{1}{\tau_{A}} \frac{1}{\sqrt{g}} \left( \frac{1}{\rho} \frac{\partial v_{\zeta}}{\partial \theta} - \varepsilon \frac{\partial v_{\theta}}{\partial \zeta}, \varepsilon \frac{\partial v_{\rho}}{\partial \zeta} - \frac{\partial v_{\zeta}}{\partial \rho}, \frac{1}{\rho} \frac{\partial (\rho v_{\theta})}{\partial \rho} - \frac{1}{\rho} \frac{\partial v_{\rho}}{\partial \theta} \right) \\ &= \frac{1}{\tau_{A}} \frac{1}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Phi}{\partial \rho} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right), \\ &\varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\rho\theta} \frac{\partial \Phi}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Phi}{\partial \rho} \right), \\ &\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( -\rho g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \rho g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\rho\theta} \frac{\partial \Phi}{\partial \rho} \right) \right] \end{split}$$

$$\begin{split} \left[\nabla\times\left(\mathbf{B}\times\nabla p\right)\right]^{i} &= \frac{B_{0}p(0)}{a^{2}}\frac{1}{\sqrt{g}}\left[\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p'J\frac{g_{\theta\zeta}}{\sqrt{g}} - p'\frac{I}{\varepsilon\rho}\frac{g_{\zeta\zeta}}{\sqrt{g}}\right) - \varepsilon\frac{\partial}{\partial\zeta}\left(p'J\frac{g_{\theta\theta}}{\sqrt{g}} - p'\frac{I}{\varepsilon\rho}\frac{g_{\theta\zeta}}{\sqrt{g}}\right),\\ &\varepsilon\frac{\partial}{\partial\zeta}\left(p'J\frac{g_{\rho\theta}}{\sqrt{g}} - p'\frac{I}{\varepsilon\rho}\frac{g_{\rho\zeta}}{\sqrt{g}}\right) - \frac{\partial}{\partial\rho}\left(p'J\frac{g_{\theta\zeta}}{\sqrt{g}} - p'\frac{I}{\varepsilon\rho}\frac{g_{\zeta\zeta}}{\sqrt{g}}\right),\\ &\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho p'J\frac{g_{\theta\theta}}{\sqrt{g}} - p'\frac{I}{\varepsilon}\frac{g_{\theta\zeta}}{\sqrt{g}}\right) - \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p'J\frac{g_{\rho\theta}}{\sqrt{g}} - p'\frac{I}{\varepsilon\rho}\frac{g_{\rho\zeta}}{\sqrt{g}}\right)\right]\\ &\nabla\cdot\mathbf{v}_{\perp} = \frac{1}{\tau_{A}}\frac{1}{\sqrt{g}}\left(-\frac{\partial\sqrt{g}}{\partial\rho}\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta} + \frac{1}{\rho}\frac{\partial\sqrt{g}}{\partial\theta}\frac{\partial\Phi}{\partial\rho}\right)\\ &\nabla\cdot\left(\mathbf{B}\times\nabla p\right) = 0 \end{split}$$

So, except for a factor  $B_0p(0)/(a\tau_A)$ , the covariant components of  $\mathbf{G} = 2(\mathbf{B} \times \nabla p \cdot \nabla)\mathbf{v}_{\perp}$  can be written as

$$\begin{split} G_{\rho} &= \frac{\partial}{\partial \rho} \left[ p' J \left( -\frac{g_{\rho\theta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\theta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) - p' \frac{I}{\varepsilon \rho} \left( -\frac{g_{\rho\zeta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\zeta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \frac{g_{\rho\rho}}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &- \frac{g_{\rho\theta}}{\sqrt{g}} \left[ \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &- \frac{g_{\rho\zeta}}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \frac{p' J}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( -\rho g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \rho g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\rho\theta} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \frac{p' I}{\varepsilon \rho \sqrt{g}} \left[ \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\rho\theta} \frac{\partial \Phi}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho p' J \frac{g_{\theta\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon \rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \right] \frac{\partial \Phi}{\partial \rho} \\ &+ p' \left( J \frac{g_{\rho\theta}}{\sqrt{g}} - \frac{I}{\varepsilon \rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \frac{1}{\sqrt{g}} \left( -\frac{\partial\sqrt{g}}{\partial \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{1}{\rho} \frac{\partial\sqrt{g}}{\partial \theta} \frac{\partial \Phi}{\partial \rho} \right) \end{split}$$

$$\begin{split} G_{\theta} = & \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ p' J \left( -\frac{g_{\rho\theta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\theta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) - p' \frac{I}{\varepsilon \rho} \left( -\frac{g_{\rho\zeta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\zeta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ & - \frac{g_{\rho\theta}}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ & - \frac{g_{\theta\theta}}{\sqrt{g}} \left[ \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ & - \frac{g_{\theta\zeta}}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ & + \frac{p' I}{\varepsilon \rho \sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Phi}{\partial \rho} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ & - \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho p' J \frac{g_{\theta\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon \rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \right] \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ & + p' \left( J \frac{g_{\theta\theta}}{\sqrt{g}} - \frac{I}{\varepsilon \rho} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) \frac{1}{\sqrt{g}} \left( -\frac{\partial\sqrt{g}}{\partial \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{1}{\rho} \frac{\partial\sqrt{g}}{\partial \theta} \frac{\partial \Phi}{\partial \rho} \right) \end{split}$$

Then,

$$F^{\zeta} = 2\sqrt{g} \left[ \nabla \times \left( \frac{\sqrt{g}}{B^2} \left( \mathbf{B} \times \nabla p \cdot \nabla \right) \mathbf{v}_{\perp} \right) \right]^{\zeta} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\sqrt{g}}{B^2} G_{\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} G_{\rho} \right)$$

so, except for a factor  $p(0)/(a^2B_0\tau_A)$ ,

$$\begin{split} F^{\zeta} &= \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ p' J \left( -\frac{g_{\rho\theta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\theta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) - p' \frac{I}{\varepsilon \rho} \left( -\frac{g_{\rho\zeta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\zeta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \frac{\partial}{\partial \rho} \left[ p' J \left( -\frac{g_{\rho\theta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\theta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) - p' \frac{I}{\varepsilon \rho} \left( -\frac{g_{\rho\zeta}}{\sqrt{g}} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \frac{g_{\theta\zeta}}{\sqrt{g}} \frac{\partial \Phi}{\partial \rho} \right) \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{g_{\rho\theta}}{B^2} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \right. \\ &+ \rho \frac{g_{\theta\theta}}{B^2} \left[ \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) - \frac{\partial}{\partial \rho} \left( p' J \frac{1}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &+ \frac{g_{\rho\theta}}{B^2} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( p' J \frac{1}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &+ \frac{g_{\rho\theta}}{B^2} \left[ \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &+ \frac{g_{\rho\theta}}{B^2} \left[ \varepsilon \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \\ &+ \frac{g_{\rho\theta}}{B^2} \left[ \varepsilon \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) - \varepsilon \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon \rho} \frac{\partial \Phi}{\partial \theta} \right) \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{g_{\rho\rho}}{B^2} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( -\rho g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \rho g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{B^2} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\phi} \frac{\partial \Phi}{\partial \rho} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Phi}{\partial \rho} \right) \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{\varepsilon \rho} \left[ \frac{1}{\varepsilon \rho} \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\phi} \frac{\partial \Phi}{\partial \rho} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\phi} \frac{\partial \Phi}{\partial \rho} \right) \right] \right\} \\ &- \frac{\partial}{\partial \rho} \left\{ \frac{g'J}{g^2} \left[ \frac{1}{\varepsilon \rho} \frac{\partial}{\partial \zeta} \left( -g_{\rho\rho} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + g_{\theta\phi} \frac{\partial \Phi}{\partial \rho} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\theta}}{g^2} - p' \frac{I}{\varepsilon \rho} \frac{g_{\rho\phi}}{\sqrt{g}} \right) \right] \right\} \frac{\partial \Phi}{\partial \rho} \\ &+ \frac{\partial}{\partial \theta} \left\{ \frac{g'J}{g^2} \left[ \frac{1}{\varepsilon \rho} \frac{\partial}{\partial \rho} \left$$

$$\begin{split} F^{\zeta} &= \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ - \left( p'J \frac{g_{\rho\theta}}{\sqrt{g}} - \frac{p'I}{\varepsilon\rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \frac{1}{\rho} \frac{\partial \theta}{\partial \theta} + \left( p'J \frac{g_{\theta\theta}}{\sqrt{g}} - \frac{p'I}{\varepsilon\rho} \frac{g_{\phi\zeta}}{\sqrt{g}} \right) \frac{\partial \Phi}{\partial \rho} \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \frac{\partial}{\partial \rho} \left[ - \left( p'J \frac{g_{\rho\theta}}{\sqrt{g}} - \frac{p'I}{\varepsilon\rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} + \left( p'J \frac{g_{\theta\theta}}{\sqrt{g}} - \frac{p'I}{\varepsilon\rho} \frac{g_{\phi\zeta}}{\sqrt{g}} \right) \frac{\partial \Phi}{\partial \rho} \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[ \rho \frac{g_{\rho\theta}}{B^2} \left( p'J \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta} - \frac{p'I}{\rho} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} \right) + \rho \frac{g_{\theta\theta}}{B^2} \left( \frac{p'I}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} - p'J \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \theta} \right) \\ &- \rho \frac{g_{\theta\theta}}{B^2} \left( p'J \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta} - \frac{p'I}{\rho} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} \right) + \frac{g_{\theta\theta}}{B^2} \left( \frac{p'I}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} - p'J \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \right) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{g_{\theta\theta}}{B^2} \left( p'J \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} - \frac{p'I}{\rho} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} \right) + \frac{g_{\theta\theta}}{B^2} \left( \frac{p'I}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} - p'J \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \right) \\ &- \frac{g_{\rho\theta}}{B^2} \left( \frac{p'J}{\rho} \right) \frac{1}{\partial \Phi} + \frac{g_{\rho\zeta}}{\varepsilon B^2} \left( \frac{p'I}{\rho} \right) \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \right] \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'J}{B^2} \left[ \frac{g_{\rho\zeta}}{\rho} - \frac{1}{\rho} \frac{\partial g_{\rho\phi}}{\partial \theta^2} - \frac{1}{\rho} \frac{\partial g_{\rho\phi}}{\rho \partial \theta} + \frac{g_{\theta\zeta}}{\rho} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} - \frac{1}{\rho} \frac{\partial g_{\rho\phi}}{\partial \theta} \right) \frac{\partial \Phi}{\partial \rho} \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{B^2} \left[ \frac{g_{\rho\zeta}}{\rho \partial \theta} - \frac{\partial g_{\rho\phi}}{\partial \theta} + \frac{1}{\rho} \frac{\partial^2 \Phi}{\rho \partial \theta} + \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} - \frac{g_{\theta\theta}}{\rho \partial \theta} \right) \frac{\partial \Phi}{\partial \rho} \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{\rho \partial \theta} - \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} + \frac{g_{\rho\zeta}}{\rho \partial \theta} \frac{\partial^2 \Phi}{\rho \partial \phi} + \frac{g_{\rho\zeta}}{\rho} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} - \frac{g_{\theta\phi}}{\rho \partial \theta} \right) \frac{\partial \Phi}{\partial \rho} \right\} \\ &- \left( \frac{1}{2} \frac{1}{\rho} \frac{\partial g_{\rho\zeta}}{\partial \theta} - \frac{\partial g_{\rho\zeta}}{\partial \theta} \right) \frac{\partial \Phi}{\rho \partial \phi} + \left( \frac{1}{\rho} \frac{1}{\rho} \frac{\partial g_{\rho\zeta}}{\partial \theta} - \frac{\partial g_{\rho\zeta}}{\partial \phi} \right) \frac{\partial \Phi}{\partial \rho} \right] \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{\rho \partial \theta} \left( \rho p'J \frac{g_{\theta\phi}}{\rho \partial \phi} - \frac{p'I}{\rho} \frac{g_{\theta\zeta}}{\rho \partial \theta} \right) \frac{\partial \Phi}{\rho \partial \zeta} + \frac{1}{\rho} \frac{\partial g_{\rho\zeta}}{\partial \rho} \right) \frac{\partial \Phi}{\partial \rho} \right\} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left\{ \frac{p'I}{\beta \partial \theta} \left( \rho p'J \frac{g_{\theta\phi}}{\rho \partial \phi} - \frac{p'I}{\rho} \frac{g_{\theta\zeta}$$

$$\begin{split} F^{\zeta} &= \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \left[ - \left( p' J \frac{g_{\rho\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial^2 \Phi}{\partial \theta^2} + \left( p' J \frac{g_{\phi\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \left[ - \left( p' J \frac{g_{\rho\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} + \left( p' J \frac{g_{\phi\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial^2 \Phi}{\partial \rho^2} \right] \\ &+ \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \left[ - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial}{\rho} \frac{\partial}{\partial \theta} + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\phi\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\phi\phi}}{\sqrt{g}} \right) \frac{\partial^2 \Phi}{\partial \rho} \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \left[ - \frac{\partial}{\partial \rho} \left( p' J \frac{g_{\rho\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\kappa}}{\sqrt{g}} \right) \frac{\partial}{\partial \theta} + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\phi\phi}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\phi\phi}}{\sqrt{g}} \right) \frac{\partial^2 \Phi}{\partial \rho} \right] \\ &- \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{D^2} \right) \frac{\partial^2 \Phi}{\rho^2 \partial \theta^2} - \frac{p' I}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta \partial \zeta} \right) - \frac{g_{\phi\phi}}{B^2} \left( p' J \frac{\partial^2 \Phi}{\partial \rho^2 \partial \zeta} - p' J \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \theta} \right) \\ &- \frac{g_{\phi\phi}}{\partial \rho} \left( p' J \frac{g_{\phi\phi}}{B^2} \right) \frac{1}{\rho^2 \partial \phi^2} + \frac{\partial}{\partial \rho} \left( p' \frac{I}{D^2} \frac{g_{\phi\phi}}{\rho^2 \partial \theta^2} \right) - \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} - \frac{p' I}{\rho} \frac{\partial^2 \Phi}{\rho^2 \partial \zeta} - p' J \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \phi} \right) \\ &+ \frac{\partial}{\partial \rho} \left( p' J \frac{J}{B^{\phi\phi}} \right) \frac{\partial^2 \Phi}{\partial \rho^2 \partial \theta} + \left[ \rho \left( \frac{p' J}{\rho} \right)' \frac{g_{\phi\phi}}{B^2} \right) \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta} \right) \\ &+ \frac{2}{\rho^2 \rho} \left( p' J \frac{1}{\rho^3} \frac{\partial^3 \Phi}{\partial \theta^3} - \frac{p' I}{\rho^2} \frac{1}{\rho^2 \partial^2 \partial \zeta} \right) + \frac{g_{\phi\phi}}{B^2} \left( \frac{p' I}{\rho} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \partial \zeta} - p' J \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \rho \partial \theta} \right) \\ &+ \frac{2}{\rho^2 \rho} \left( p' J \frac{1}{\rho^3} \frac{\partial^3 \Phi}{\partial \theta^3} - \frac{p' I}{\rho^2} \frac{1}{\rho^2 \partial^2 \partial \zeta} \right) + \frac{g_{\phi\phi}}{B^2} \left( \frac{p' I}{\rho} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \partial \zeta} - p' J \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \rho \partial \theta} \right) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\phi\phi}}{B^2} \right) \frac{1}{\rho^2 \partial \Phi} - \frac{1}{\rho} \frac{\partial}{\partial \theta} \frac{\partial^2 \Phi}{\partial \theta^2 \partial \zeta} \right) + \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta} + \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \right) \\ &+ \frac{1}{\rho^2 \rho} \frac{\partial}{\partial \theta} \left[ p' J \frac{g_{\phi\phi}}{\rho^2 \partial \theta} - \frac{p' I}{\rho^2 \partial \theta} \right) \frac{\partial^2 \Phi}{\rho^2 \partial \theta} - \frac{p' I}{\rho^2 \partial \theta} \frac{\partial^2 \Phi}{\partial \rho} \right) - \frac{1}{\rho^2 \partial \theta} \frac{\partial^2$$

$$\begin{split} &+\frac{p'I}{\rho B^2}\left[-g_{\rho\rho}\frac{1}{\rho^2}\frac{\partial^3\Phi}{\partial\theta^2\partial\zeta}+g_{\rho\theta}\frac{1}{\rho}\frac{\partial^3\Phi}{\partial\rho\rho\partial\theta\zeta}+\frac{g_{\rho\zeta}}{\varepsilon}\frac{1}{\rho^2}\frac{\partial^3\Phi}{\partial\rho\partial\theta^2}-\frac{g_{\theta\zeta}}{\varepsilon}\frac{1}{\rho}\frac{\partial^3\Phi}{\partial\rho^2\partial\theta}\right]\\ &-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p'I}{\rho B^2}g_{\rho\rho}\right)\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\theta\zeta}+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p'I}{\rho B^2}g_{\rho\theta}\right)\frac{\partial^2\Phi}{\partial\rho\partial\zeta}+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p'I}{\rho B^2}\frac{g_{\rho\zeta}}{\varepsilon}\right)\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\rho\partial\theta}\\ &-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p'I}{\rho B^2}\frac{g_{\theta\zeta}}{\varepsilon}\right)\frac{\partial^2\Phi}{\partial\rho^2}-\frac{p'I}{\rho B^2}\left[\left(\frac{\partial g_{\rho\rho}}{\partial\zeta}-\frac{\rho}{\varepsilon}\frac{\partial}{\partial\rho}\left(\frac{g_{\rho\zeta}}{\rho}\right)\right)\frac{1}{\rho^2}\frac{\partial\Phi}{\partial\theta^2}-\left(\frac{\partial g_{\rho\theta}}{\partial\zeta}-\frac{1}{\varepsilon}\frac{\partial g_{\theta\zeta}}{\partial\rho}\right)\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\rho\theta}\right]\\ &-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'I}{\rho B^2}\left(\frac{\partial g_{\rho\rho}}{\partial\zeta}-\frac{\rho}{\varepsilon}\frac{\partial}{\partial\rho}\left(\frac{g_{\rho\zeta}}{\rho}\right)\right)\right]\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'I}{\rho B^2}\left(\frac{\partial g_{\rho\rho}}{\partial\zeta}-\frac{1}{\varepsilon}\frac{\partial g_{\theta\zeta}}{\partial\rho}\right)\right]\frac{\partial\Phi}{\partial\rho}\\ &-\frac{\partial}{\partial\rho}\left\{\frac{\sqrt{g}}{B^2}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho p'J\frac{g_{\theta\theta}}{\sqrt{g}}-p'\frac{I}{\varepsilon}\frac{g_{\theta\zeta}}{\sqrt{g}}\right)-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p'J\frac{g_{\rho\theta}}{\sqrt{g}}-p'\frac{I}{\varepsilon}\frac{g_{\rho\zeta}}{\sqrt{g}}\right)\right]\right\}\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left\{\frac{\sqrt{g}}{B^2}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho p'J\frac{g_{\theta\theta}}{\sqrt{g}}-p'\frac{I}{\varepsilon}\frac{g_{\theta\zeta}}{\sqrt{g}}\right)-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p'J\frac{g_{\rho\theta}}{\sqrt{g}}-p'\frac{I}{\varepsilon}\frac{g_{\rho\zeta}}{\sqrt{g}}\right)\right]\right\}\frac{\partial\Phi}{\partial\rho}\\ &+\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\theta\zeta}\right)\left(-\frac{\partial\sqrt{g}}{\partial\rho}\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\rho}\right)+\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\theta\zeta}\right)\left(-\frac{\partial\sqrt{g}}{\partial\rho}\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\theta}\right)+\frac{1}{\rho}\frac{\partial\sqrt{g}}{\partial\theta}\right)\right]\frac{\partial\Phi}{\partial\rho}\\ &-\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\left(-\frac{\partial\sqrt{g}}{\partial\rho}\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\theta}\right)+\frac{1}{\rho}\frac{\partial\sqrt{g}}{\partial\theta}\right)\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\left(-\frac{\partial\sqrt{g}}{\partial\rho}\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\theta}\right)+\frac{1}{\rho}\frac{\partial\sqrt{g}}{\partial\theta}\right)\right]\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\sqrt{g}}{\partial\rho}\right]\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\sqrt{g}}{\partial\rho}\right]\frac{\partial\Phi}{\partial\rho}\\ &-\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\sqrt{g}}{\partial\rho}\right]\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\sqrt{g}}{\partial\rho}\right]\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\sqrt{g}}{\partial\rho}\right]\frac{\partial\Phi}{\partial\rho}\\ &+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{p'}{\sqrt{g}B^2}\left(Jg_{\theta\theta}-\frac{I}{\varepsilon\rho}g_{\rho\zeta}\right)\frac{\partial\Phi}{\partial\rho}\right]\frac{\partial\Phi}{\partial$$

$$\begin{split} & + \left\{ -\frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\theta}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) + \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \frac{\partial}{\partial \rho} \left( \frac{p' J}{\rho} \frac{g_{\rho\theta}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho^2} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \\ & - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \rho \left( \frac{p' J}{\rho} \right)' \frac{g_{\rho\theta}}{B^2} - \left( \frac{p' I}{\rho} \right)' \frac{g_{\rho\zeta}}{\varepsilon B^2} \right] + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p'}{\sqrt{g} B^2} \left( J g_{\rho\theta} - \frac{I}{\varepsilon \rho} g_{\rho\zeta} \right) \frac{\partial \sqrt{g}}{\partial \rho} \right] \\ & + \frac{\partial}{\partial \rho} \left[ \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\rho\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon \rho} \frac{g_{\rho\zeta}}{\sqrt{g}} \right) \right] - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' J}{B^2} \left( \frac{\partial g_{\rho\theta}}{\partial \rho} - \frac{1}{\rho} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) \right] \\ & - \frac{\partial}{\partial \rho} \left[ \frac{p' I}{\rho B^2} \left( \frac{1}{\varepsilon} \frac{1}{\rho} \frac{\partial g_{\rho\zeta}}{\partial \theta} - \frac{\partial g_{\rho\theta}}{\partial \zeta} \right) \right] - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' I}{\rho B^2} \left( \frac{\partial g_{\rho\rho}}{\partial \zeta} - \frac{\rho}{\varepsilon} \frac{\partial}{\partial \rho} \left( \frac{g_{\rho\zeta}}{\rho} \right) \right) \right] \\ & + \frac{\partial}{\partial \rho} \left[ \rho \left( \frac{p' J}{\rho} \right)' \frac{g_{\theta\theta}}{B^2} - \left( \frac{p' I}{\rho} \right)' \frac{g_{\theta\zeta}}{\varepsilon B^2} \right] - \frac{\partial}{\partial \rho} \left[ \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho p' J \frac{g_{\theta\theta}}{g^2} - p' \frac{I}{\varepsilon} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) \right] \\ & - \frac{\partial}{\partial \rho} \left[ \frac{p'}{\sqrt{g} B^2} \left( J g_{\theta\theta} - \frac{I}{\varepsilon \rho} g_{\theta\zeta} \right) \frac{\partial \sqrt{g}}{\partial \rho} \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ & + \left\{ \frac{\partial}{\partial \rho} \left( \sqrt{g} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \sqrt{g} \right) \frac{\partial}{\partial \rho} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} - \frac{p' I}{\varepsilon \rho} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho p' J \frac{g_{\theta\theta}}{\sqrt{g}} - p' \frac{I}{\varepsilon} \frac{g_{\theta\zeta}}{\sqrt{g}} \right) \right] + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[ \frac{p' I}{B^2} \left( J g_{\theta\theta} - \frac{I}{\varepsilon \rho} g_{\theta\zeta} \right) \frac{\partial \sqrt{g}}{\partial \theta} \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' J}{B^2} \left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial g_{\rho\theta}}{\partial \theta} \right) \right] - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' I}{B^2} \left( \frac{1}{\varepsilon} \frac{1}{\rho} \frac{\partial g_{\theta\zeta}}{\partial \theta} - \frac{\partial g_{\theta\zeta}}{\partial \zeta} \right) \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' J}{B^2} \left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho g_{\theta\theta} \right) - \frac{1}{\varepsilon} \frac{\partial g_{\rho\zeta}}{\partial \rho} \right) \right] - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' J}{B^2} \left( \frac{1}{\rho} \frac{\partial g_{\theta\zeta}}{\partial \theta} - \frac{\partial g_{\theta\zeta}}{\partial \zeta} \right) \right] \\ & - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p' J}{B^2} \left( \frac{1}{\rho} \frac{\partial g_{\theta\zeta}}{\partial \zeta} - \frac{1}{\varepsilon} \frac{\partial g_{\theta\zeta}}{\partial \zeta} \right) \right] - \frac$$

$$\begin{split} F^{\zeta} &= 2\frac{p^{\prime}J}{B^{2}}\left(g_{\rho\rho}\frac{1}{\rho^{2}}\frac{\partial^{3}\Phi}{\partial\theta^{3}} - 2g_{\rho\theta}\frac{1}{\rho^{2}}\frac{\partial^{3}\Phi}{\partial\rho\partial\theta^{2}} + g_{\theta\theta}\frac{1}{\rho^{2}}\frac{\partial^{3}\Phi}{\partial\rho^{2}\partial\phi}\right) \\ &- 2\frac{p^{\prime}I}{\rho B^{2}}\left(g_{\rho\rho}\frac{1}{\rho^{2}}\frac{\partial^{3}\Phi}{\partial\theta^{2}\partial\zeta} - 2g_{\rho\theta}\frac{1}{\rho}\frac{\partial^{3}\Phi}{\partial\rho\partial\theta\partial\zeta} + g_{\theta\theta}\frac{\partial^{3}\Phi}{\partial\rho^{2}\partial\zeta}\right) \\ &+ \left[-\frac{\partial}{\partial\rho}\left(\frac{\sqrt{g}}{B^{2}}\right)\left(p^{\prime}J\frac{g_{\rho\theta}}{\sqrt{g}} - \frac{p^{\prime}J}{\varepsilon\rho}\frac{g_{\rho\phi}}{\sqrt{g}}\right) - \rho\frac{\partial}{\partial\rho}\left(\frac{p^{\prime}J}{\rho^{2}B^{2}}\right) + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p^{\prime}J\frac{g_{\rho\rho}}{B^{2}}\right) - \rho\left(\frac{p^{\prime}J}{\rho}\right)\frac{g_{\rho\theta}}{B^{2}} \\ &+ \left(\frac{p^{\prime}I}{\rho}\right)^{\prime}\frac{g_{\rho\xi}}{\varepsilon^{2}} + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p^{\prime}J}{B^{2}}\right)g_{\rho\rho} - \frac{p^{\prime}J}{B^{2}}\left(\partial g_{\rho\rho} - \frac{1}{\rho}\frac{\partial g_{\rho\rho}}{\partial\rho}\right) - \rho\frac{\partial}{\partial\rho}\left(\frac{p^{\prime}J}{\rho^{2}B^{2}}\frac{g_{\kappa\zeta}}{\partial\rho}\right) \\ &- \frac{p^{\prime}I}{\rho B^{2}}\left(\frac{\partial g_{\rho\rho}}{\partial\zeta} - \frac{\rho}{\varepsilon}\frac{\partial}{\partial\rho}\left(\frac{p^{\prime}J}{\rho\rho}\right)\right) + \frac{p^{\prime}}{\sqrt{g}B^{2}}\left(Jg_{\rho\theta} - \frac{1}{\rho}\frac{\partial g_{\rho\rho}}{\partial\rho}\right) - \rho\frac{\partial}{\partial\rho}\left(\frac{p^{\prime}J}{\rho^{2}B^{2}}\frac{g_{\kappa\zeta}}{\partial\rho}\right) \\ &+ \left[\frac{\partial}{\partial\rho}\left(\frac{\sqrt{g}}{B^{2}}\right)\left(p^{\prime}J\frac{g_{\theta\phi}}{\sqrt{g}} - \frac{p^{\prime}I}{\rho^{2}}\frac{g_{\kappa\zeta}}{\partial\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p^{\prime}J\frac{g_{\rho\theta}}{\partial\rho}\right) - \rho\frac{\partial}{\partial\rho}\left(\frac{p^{\prime}J}{\rho^{2}}\frac{g_{\kappa\zeta}}{\partial\rho}\right) \\ &+ \left[\frac{\partial}{\partial\rho}\left(\frac{\sqrt{g}}{B^{2}}\right)\left(p^{\prime}J\frac{g_{\theta\phi}}{\sqrt{g}} - \frac{p^{\prime}I}{\rho^{2}}\frac{g_{\kappa\zeta}}{\partial\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(p^{\prime}J\frac{g_{\rho\theta}}{\partial\rho}\right) - \frac{1}{\rho^{2}}\frac{\partial}{\partial\theta}\left(\frac{p^{\prime}J}{B^{2}}\right) + \frac{\partial}{\partial\rho}\left(p^{\prime}J\frac{g_{\rho\theta}}{\partial\rho}\right) \\ &+ \frac{p^{\prime}J}{\rho^{2}}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho g_{\theta\theta}\right) - \frac{1}{\rho^{2}}\frac{\partial g_{\rho\zeta}}{\partial\rho}\right) + \frac{1}{\rho^{2}}\frac{\partial}{\partial\rho}\left(p^{\prime}J\frac{g_{\rho\phi}}{B^{2}}\right) - \frac{p^{\prime}I}{\rho^{2}}\frac{1}{\partial\theta}\left(\frac{p^{\prime}J}{B^{2}}\right)g_{\rho\theta} \\ &+ \frac{p^{\prime}J}{\rho^{2}}\left(\frac{1}{\rho^{2}}\frac{\partial}{\partial\rho}\left(p^{\prime}g_{\theta\theta}\right) - \frac{1}{\rho^{2}}\frac{\partial g_{\rho\zeta}}{\partial\zeta}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{1}{\rho^{2}}\frac{1}{\rho^{2}}\frac{1}{\rho^{2}}\frac{\partial}{\partial\zeta}\right) \\ &+ \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right) + \frac{p^{\prime}J}{\rho^{2}}\frac{1}{\rho^{2}}\frac{\partial}{\partial\rho}\left(p^{\prime}J\frac{g_{\rho\phi}}{\partial\rho}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{1}{\rho^{2}}\frac{\partial}{\partial\rho}\left(p^{\prime}J\frac{g_{\rho\phi}}{\partial\rho}\right) \\ &+ \left[\frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right]\left(p^{\prime}J\frac{g_{\rho\phi}}{\partial\rho}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho} \\ &+ \left[\frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\left(p^{\prime}J\frac{g_{\rho\phi}}{\partial\rho}\right) - \frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right] - \frac{p^{\prime}J}{\rho^{2}}\frac{g_{\rho\phi}}{\partial\rho}\right)$$

$$\begin{split} & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{1}{\rho} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) + \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\varepsilon \rho} \frac{g_{\rho\varsigma}}{\sqrt{g}} \right) - \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \frac{\partial}{\partial \rho} \left( \frac{p'I}{E^{\rho}} \frac{g_{\rho\varsigma}}{\sqrt{g}} \right) \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \left( \frac{p'I}{\rho} \right)' \frac{g_{\rho\varsigma}}{\varepsilon B^2} + \frac{\sqrt{g}p'}{B^2} \frac{I}{\varepsilon_{\rho}} g_{\rho\varsigma} \frac{\partial}{\partial \rho} \left( \frac{1}{\sqrt{g}} \right) + \frac{p'I}{\rho B^2} \frac{\rho}{\varepsilon} \frac{\partial}{\partial \rho} \left( \frac{g_{\rho\varsigma}}{\rho} \right) \right] \\ & - \frac{\partial}{\partial \rho} \left[ \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon \rho} \frac{g_{\rho\varsigma}}{\sqrt{g}} \right) + \frac{p'I}{\rho B^2} \frac{1}{\varepsilon} \frac{1}{\rho} \frac{\partial g_{\rho\varsigma}}{\partial \theta} \right] \\ & - \frac{\partial}{\partial \rho} \left[ \left( \frac{p'I}{\rho} \right)' \frac{g_{\theta\varsigma}}{\varepsilon B^2} + \frac{\sqrt{g}p''I}{B^2\varepsilon\rho} g_{\theta\varsigma} \frac{\partial}{\partial \rho} \left( \frac{1}{\sqrt{g}} \right) - \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( p' \frac{I}{\varepsilon} \frac{g_{\theta\varsigma}}{\sqrt{g}} \right) \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ & + \left\{ \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\sqrt{g}}{B^2} \right) \frac{\partial}{\partial \rho} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} \right) \right\} + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} \right) \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p'J}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho g_{\theta\theta} \right) + \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' J \frac{g_{\theta\theta}}{\sqrt{g}} \right) + \frac{p'}{\sqrt{g}B^2} J g_{\rho\theta} \frac{1}{\rho} \frac{\partial\sqrt{g}}{\partial \theta} \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'I}{B^2} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\theta\varsigma}}{\partial \zeta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'I}{B^2} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{B^2} \frac{\partial g_{\theta\varsigma}}{\partial \theta} - \frac{p'}{\sqrt{g}B^2} \frac{I}{\varepsilon \rho} \frac{\partial}{\partial \theta} \frac{\partial\sqrt{g}}{\partial \theta} \right) \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{B^2} \frac{1}{\rho} \frac{\partial g_{\theta\varsigma}}{\partial \theta} - \frac{p'}{\sqrt{g}B^2} \frac{I}{\varepsilon \rho} \frac{\partial g_{\theta\varsigma}}{\partial \theta} \right) \\ & - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{g}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon} \frac{g_{\theta\varsigma}}{\sqrt{g}} \right) + \frac{p'I}{\rho B^2} \frac{1}{\varepsilon} \frac{\partial g_{\theta\varsigma}}{\partial \rho} \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{g}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon} \frac{g_{\theta\varsigma}}{\sqrt{g}} \right) + \frac{p'I}{\rho B^2} \frac{1}{\varepsilon} \frac{\partial g_{\theta\varsigma}}{\partial \rho} \right] \\ & + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{g}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( p' \frac{I}{\varepsilon} \frac{g_{\theta\varsigma}}{\sqrt{g}} \right) + \frac{p'I}{\rho B^2} \frac{1}{\varepsilon} \frac{\partial g_{\theta\varsigma}}{\partial \rho} \right] \right] \right\} \frac{\partial\Phi}{\partial \rho} \end{aligned}$$

$$\begin{split} F^{\zeta} &= 2 \frac{p'J}{B^2} \left( g_{\rho\rho} \frac{1}{\rho^3} \frac{\partial^3 \Phi}{\partial \theta^3} - 2 g_{\rho\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \theta} + g_{\theta\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \Phi} \right) \\ &- 2 \frac{p'J}{\rho B^2} \left( g_{\rho\rho} \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \theta^2 \partial \zeta} - 2 g_{\rho\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \theta \partial \zeta} + g_{\theta\theta} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \zeta} \right) \\ &+ 2 \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\theta} \right) - \rho \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} \\ &+ 2 \left[ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \rho^2} \\ &- \frac{p'J}{\rho B^2} \left( \frac{\partial g_{\rho\rho}}{\partial \rho} \frac{1}{\sigma^2} \frac{\partial^2 \Phi}{\partial \theta^2} - 2 \frac{\partial g_{\rho\theta}}{\partial \rho^2} \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial g_{\theta\theta}}{\partial \rho^2} \frac{\partial^2 \Phi}{\partial \rho^2} \right) \\ &+ 2 \left[ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho B^2} g_{\theta\theta} \right) \right] \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \rho^2} \\ &- 2 \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho B^2} g_{\theta\theta} \right) \right] \frac{\partial^2 \Phi}{\partial \rho^2} \\ &+ \left\{ -\frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho B^2} g_{\theta\theta} \right) \right\} \frac{\partial^2 \Phi}{\partial \rho^2} \\ &+ \left\{ -\frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho B^2} g_{\theta\theta} \right) \right\} \frac{\partial^2 \Phi}{\partial \rho^2} \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{\sqrt{g}}{B^2} \frac{1}{\partial \rho} \left( p'J \frac{g_{\rho\theta}}{\sqrt{g}} \right) \right] - \frac{\sqrt{g}}{B^2} \frac{1}{\rho^2} \frac{\partial}{\partial \theta} \left( p'J \frac{g_{\rho\theta}}{\sqrt{g}} \right) + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) \right] \\ &+ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\theta}}{\partial \rho} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\theta}}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \rho} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \rho} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) - \frac{\partial}{\rho} \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \left( \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) \\ &+ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \rho} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} \frac{g_{\rho\theta}}{\partial \rho} \right) - \frac{\partial}{\partial \rho} \left( \frac{$$

$$\begin{split} F^{\zeta} &= 2 \frac{p'J}{B^2} \left( g_{\rho\rho} \frac{1}{\rho^3} \frac{\partial^3 \Phi}{\partial \theta^3} - 2 g_{\rho\theta} \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \rho \partial \theta^2} + g_{\theta\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \theta} \right) \\ &- 2 \frac{p'I}{\rho B^2} \left( g_{\rho\rho} \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \theta^2 \partial \zeta} - 2 g_{\rho\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \theta \zeta} + g_{\theta\theta} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \zeta} \right) \\ &+ 2 \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\rho} \right) - \rho \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} \\ &+ 2 \left[ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \\ &- \frac{p'I}{\rho B^2} \left( \frac{\partial g_{\rho\rho}}{\partial \zeta} \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} - 2 \frac{\partial g_{\rho\theta}}{\partial \zeta} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} + \frac{\partial g_{\theta\theta}}{\partial \zeta} \frac{\partial^2 \Phi}{\partial \rho^2} \right) \\ &+ 2 \left[ \frac{\partial}{\partial \rho} \left( \frac{p'I}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} g_{\rho\rho} \right) \right] \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \theta \partial \zeta} \\ &- 2 \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'I}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} g_{\rho\theta} \right) \right] \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} \\ &+ \left\{ - \frac{\partial}{\partial \rho} \left[ \frac{p'J}{\rho^2 B^2} \frac{\partial}{\partial \rho} \left( \rho^2 g_{\theta\theta} \right) \right] + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{1}{\rho} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) + \frac{\partial}{\partial \rho} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\rho\theta}}{\partial \zeta} \right) \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ &+ \left\{ p'J \frac{\partial}{\partial \rho} \left( \frac{\sqrt{g}}{B^2} \right) \frac{1}{\rho} \frac{\partial g_{\theta\theta}}{\partial \theta} - \frac{\sqrt{g}}{B^2} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[ p'J g_{\theta\theta} \frac{\partial}{\partial \theta} \left( \frac{1}{\sqrt{g}} \right) \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ &+ \left\{ \frac{\rho}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) - \frac{\partial}{\partial \rho} \left( \frac{p'I}{B^2} \frac{\partial g_{\rho\phi}}{\partial \theta} \right) + \frac{\partial}{\partial \rho} \left[ \frac{p'I}{\rho B^2} \frac{\partial}{\partial \theta} \left( \rho g_{\theta\zeta} \right) \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ &+ \left\{ \frac{\rho}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\theta\theta}}{\partial \rho} + 2 \frac{p'J}{\rho B^2} g_{\theta\theta} \right) + \frac{\sqrt{g}}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho \partial \theta} \left( \frac{p'J}{\rho \partial \theta} \frac{\partial}{\partial \zeta} \right) \right) \right\} \\ &- \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\theta\theta}}{\partial \theta} \right) + \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) - \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho \partial \theta} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) \\ &- \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\theta\theta}}{\partial \theta} \right) + \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho \partial \theta} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) - \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho \partial \theta} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) \\ &- \frac{\partial}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\rho \partial \theta} \frac{\partial g_{\theta\theta}}{\partial \zeta} \right) + \frac{\partial}{\rho}$$

Finally, we get that, in dimensionless units,

$$\frac{\partial U}{\partial t} = \dots - \frac{\beta_{0i}}{2\varepsilon^2 \omega_{cu}} \sqrt{g} \left[ \nabla \times \left( \frac{\sqrt{g}}{B^2} \left( \mathbf{B} \times \nabla p \cdot \nabla \right) \mathbf{v}_{\perp} \right) \right]^{\zeta}$$
 (7)

where

$$\begin{split} & \sqrt{g} \left[ \nabla \times \left( \frac{\sqrt{g}}{B^2} (\mathbf{B} \times \nabla p \cdot \nabla) \mathbf{v}_{\perp} \right) \right]^{\zeta} = \\ & \frac{p'J}{B^2} \left( g_{\rho\rho} \frac{1}{\rho^3} \frac{\partial^3 \Phi}{\partial \theta^3} - 2g_{\rho\theta} \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \rho \partial \theta^2} + g_{\theta\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \theta} \right) \\ & - \frac{p'I}{\rho B^2} \left( g_{\rho\rho} \frac{1}{\rho^2} \frac{\partial^3 \Phi}{\partial \theta^2 \partial \zeta} - 2g_{\rho\theta} \frac{1}{\rho} \frac{\partial^3 \Phi}{\partial \rho \partial \theta \partial \zeta} + g_{\theta\theta} \frac{\partial^3 \Phi}{\partial \rho^2 \partial \zeta} \right) \\ & + \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\rho} \right) - \rho \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} \\ & + \left[ \frac{\partial}{\partial \rho} \left( \frac{p'J}{B^2} g_{\theta\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} g_{\rho\theta} \right) \right] \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} \\ & - \frac{1}{2} \frac{p'I}{\rho B^2} \left( \frac{\partial g_{\rho\rho}}{\partial \zeta} \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \theta^2} - 2 \frac{\partial g_{\rho\theta}}{\partial \zeta} \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \theta} + \frac{\partial g_{\theta\theta}}{\partial \zeta} \frac{\partial^2 \Phi}{\partial \rho^2} \right) \\ & + \left[ \frac{\partial}{\partial \rho} \left( \frac{p'I}{\rho B^2} g_{\rho\theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} g_{\rho\rho} \right) \right] \frac{1}{\rho} \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} \\ & - \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} g_{\rho\theta} \right) \right] \frac{\partial^2 \Phi}{\partial \rho \partial \zeta} \\ & + \frac{1}{2} \left\{ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{B^2} \frac{\partial g_{\rho\rho}}{\partial \theta} \right) - \frac{\partial}{\partial \rho} \left[ \frac{p'J}{\rho^2 B^2} \frac{\partial}{\partial \rho} \left( \rho^2 g_{\theta\theta} \right) \right] + \frac{\partial}{\partial \rho} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\rho\theta}}{\partial \zeta} \right) \\ & - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} \frac{\partial g_{\rho\theta}}{\partial \zeta} \right) - \frac{\partial}{\partial \rho} \left( \frac{p'I}{\varepsilon \rho B^2} \frac{1}{\rho} \frac{\partial g_{\rho\zeta}}{\partial \theta} \right) + \frac{\partial}{\partial \rho} \left[ \frac{p'I}{\varepsilon \rho B^2} \frac{1}{\rho} \frac{\partial g_{\rho\theta}}{\partial \rho} \left( \rho g_{\theta\zeta} \right) \right] \right\} \frac{1}{\rho} \frac{\partial \Phi}{\partial \theta} \\ & + \frac{1}{2} \left\{ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'J}{\rho B^2} \frac{\partial g_{\theta\theta}}{\partial \theta} \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p'J}{\rho B^2} \frac{1}{\partial \rho} \left( \rho^2 g_{\theta\theta} \right) \right] \right] \\ & - \frac{2}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'J}{\beta B^2} \frac{1}{\rho} \frac{\partial g_{\rho\theta}}{\partial \theta} \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\rho B^2} \frac{1}{\partial \zeta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{p'I}{\beta B^2} \frac{1}{\partial \theta} \right) \right\} \frac{\partial \Phi}{\partial \rho} \\ & - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left[ \frac{p'I}{\beta B^2} \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \rho g_{\theta\zeta} \right) \right] + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\varepsilon \rho B^2} \frac{1}{\rho} \frac{\partial}{\partial \zeta} \right) - \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{p'I}{\beta B^2} \frac{1}{\partial \theta} \right) \right\} \frac{\partial}{\partial \rho} \end{aligned}$$

Contribution to the evolution equation for the pressure:

$$\frac{\partial p}{\partial t} = \dots - \Gamma \frac{p_i}{en} \nabla p \cdot \nabla \times \frac{\mathbf{B}}{B^2}$$
(9)

$$\nabla \tilde{p} \cdot \nabla \times \frac{\mathbf{B}}{B^2} = \frac{\mu_0 \mathbf{J} \cdot \nabla \tilde{p}}{B^2} - \mathbf{B} \times \nabla \left(\frac{1}{B^2}\right) \cdot \nabla \tilde{p} = \frac{\mu_0 \mathbf{J} \cdot \nabla \tilde{p}}{B^2} + \frac{2}{B^3} \mathbf{B} \times \nabla B \cdot \nabla \tilde{p}$$

In dimensionless units.

$$\nabla \tilde{p} \cdot \nabla \times \frac{\mathbf{B}}{B^2} = \frac{p(0)}{B_0 a^2} \left\{ \frac{1}{B^2 \sqrt{g}} \left[ -\left(\frac{dJ}{d\rho} - \rho \frac{\partial \beta_*}{\partial \zeta}\right) \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \theta} + \left(\frac{1}{\rho} \frac{dI}{d\rho} - \frac{\partial \beta_*}{\partial \theta}\right) \frac{\partial \tilde{p}}{\partial \zeta} \right] + 2\Omega_d(\tilde{p}) \right\}$$

$$\begin{split} & \nabla p_{eq} \cdot \nabla \times \frac{\tilde{\mathbf{B}}}{B^2} = \frac{1}{\sqrt{g}} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( \frac{\tilde{B}_{\zeta}}{B^2} \right) - \frac{1}{R_0} \frac{\partial}{\partial \zeta} \left( \frac{\tilde{B}_{\theta}}{B^2} \right) \right] \frac{dp_{eq}}{d\rho} \\ & = \frac{1}{B^2 g} \frac{1}{R_0} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Psi}{\partial \rho} \right) - \frac{1}{R_0} \frac{\partial}{\partial \zeta} \left( -g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Psi}{\partial \rho} \right) \right] \frac{dp_{eq}}{d\rho} \end{split}$$

In dimensionless units,

$$\begin{split} \nabla p_{eq} \cdot \nabla \times \frac{\tilde{\mathbf{B}}}{B^2} = & \frac{p(0)}{B_0 R_0 a} \frac{1}{B^2 g} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho \zeta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta \zeta} \frac{\partial \Psi}{\partial \rho} \right) \right. \\ & \left. - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho \theta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta \theta} \frac{\partial \Psi}{\partial \rho} \right) \right] \frac{dp_{eq}}{d\rho} \end{split}$$

Then

$$\frac{\partial p}{\partial t} = \dots - \frac{\Gamma \beta_{0i}}{2\varepsilon^2 \omega_{cy}} \frac{p_{i\,eq}}{n} \nabla p \cdot \nabla \times \frac{\mathbf{B}}{B^2}$$
(10)

where

$$\nabla p \cdot \nabla \times \frac{\mathbf{B}}{B^{2}} = \frac{1}{B^{2}\sqrt{g}} \left[ -\left(\frac{dJ}{d\rho} - \rho \frac{\partial \beta_{*}}{\partial \zeta}\right) \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \theta} + \left(\frac{1}{\rho} \frac{dI}{d\rho} - \frac{\partial \beta_{*}}{\partial \theta}\right) \frac{\partial \tilde{p}}{\partial \zeta} \right] + 2\Omega_{d}(\tilde{p})$$

$$+ \frac{\varepsilon}{B^{2}g} \left[ \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( -g_{\rho\zeta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta\zeta} \frac{\partial \Psi}{\partial \rho} \right) - \varepsilon \frac{\partial}{\partial \zeta} \left( -g_{\rho\theta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + g_{\theta\theta} \frac{\partial \Psi}{\partial \rho} \right) \right] \frac{dp_{eq}}{d\rho}$$

$$\tag{11}$$

There is a possible third contribution:

$$\tilde{p} \nabla p \cdot \nabla \times \frac{\mathbf{B}}{B^2} = 2\tilde{p} \Omega_d(p_{eq}) = \frac{\tilde{p}}{B^4 \sqrt{g}} \left( \frac{I}{\rho} \frac{\partial B^2}{\partial \zeta} - J \frac{1}{\rho} \frac{\partial B^2}{\partial \theta} \right) \frac{dp_{eq}}{d\rho}$$