ACOUSTIC MODES IMPLEMENTATION

New model equations:

New parallel thermal velocity equation:

$$n_0 m_p \frac{\partial v_{\parallel,th}}{\partial t} = -\hat{b}\cdot\vec{\nabla}p - \frac{\partial\vec{B}}{|B|}\cdot\vec{\nabla}p_0$$

$$= -\frac{2\psi'}{\sqrt{g}}\frac{1}{a^2 R_0 B_0}\left(\frac{\partial}{\partial\zeta}+\tau\frac{\partial}{\partial\theta}\right)p - \frac{\vec{\nabla}\psi\wedge\hat{e}_\zeta}{R_0 B_0}\cdot\vec{\nabla}p_0$$

$$\vec{\nabla}\psi\wedge\hat{e}_\zeta = \vec{\nabla}\wedge\left(\psi\hat{e}_\zeta\right) - \cancel{\psi\left(\vec{\nabla}\wedge\hat{e}_\zeta\right)}$$

$$\Rightarrow \vec{\nabla}\wedge\left(\psi\hat{e}_\zeta\right) = \frac{1}{\sqrt{g}}\begin{vmatrix}\hat{e}_\rho & \hat{e}_\theta & \hat{e}_\zeta \\ \frac{\partial}{\partial\rho} & \frac{1}{\rho}\frac{\partial}{\partial\theta} & \frac{1}{R_0}\frac{\partial}{\partial\zeta} \\ 0 & 0 & \psi\end{vmatrix} = \frac{1}{\sqrt{g}}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\theta}\hat{e}_\rho - \frac{\partial\psi}{\partial\rho}\hat{e}_\theta\right)$$

$$\Rightarrow n_0 m_p \frac{\partial v_{\parallel,th}}{\partial t} = -\frac{2\psi'}{\sqrt{g}}\frac{1}{a^2 R_0 B_0}\left(\frac{\partial}{\partial\zeta}+\tau\frac{\partial}{\partial\theta}\right)p - \frac{1}{\sqrt{g}}\left(\frac{1}{\rho}\frac{\partial\psi}{\partial\theta}\hat{e}_\rho - \frac{\partial\psi}{\partial\rho}\hat{e}_\theta\right)\cdot\left(\frac{dp_0}{d\rho}\hat{e}^\rho\right)$$

Normalization:

$$\frac{a n_{00}\tilde{n}_0 m_p}{\tau_A^2}\frac{\partial\tilde{v}_{\parallel,th}}{\partial\tilde{t}} = -\frac{a^2 B_{00}}{\sqrt{g}}\frac{p_{00}}{a^2 R_0 B_{00}\tilde{B}_0}\left(\frac{\partial}{\partial\zeta}+\tau\frac{\partial}{\partial\theta}\right)\tilde{p} - \frac{a^2 B_{00}}{R_0 a^2 B_{00}\tilde{B}_0}\frac{p_{00}}{\sqrt{g}}\frac{1}{\tilde{\rho}}\frac{\partial\tilde{\psi}}{\partial\theta}\frac{dp_0}{d\rho}$$

$$\Rightarrow \frac{\partial\tilde{v}_{\parallel,th}}{\partial\tilde{t}} = -\frac{\beta_0}{2}\frac{\tilde{B}_0}{\tilde{n}_0\left(\tilde{J}-\tau\tilde{I}\right)}\left[\left(\frac{\partial}{\partial\zeta}+\tau\frac{\partial}{\partial\theta}\right)\tilde{p} + \frac{1}{\tilde{\rho}}\frac{\partial\tilde{\psi}}{\partial\theta}\frac{dp_0}{d\rho}\right]$$

New terms in the pressure equation, normalized:

$$\frac{\partial\tilde{p}}{\partial\tilde{t}} = -\frac{\Gamma\tilde{p}_0\tilde{B}_0}{\tilde{J}-\tau\tilde{I}}\left(\frac{\partial}{\partial\theta}+\frac{\partial}{\partial\zeta}\right)v_{\parallel th} - \frac{\Gamma\tilde{p}_0\tilde{B}_0\sqrt{g}}{\tilde{J}-\tau\tilde{I}}\left[\tau\frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)+\frac{\partial}{\partial\zeta}\left(\frac{1}{\sqrt{g}}\right)\right]v_{\parallel th}$$

Implementation on code:

New parallel thermal velocity equation terms

! Perturbed pressure gradient term

```
do l=1,leqmax
        sceq1(:,l)=bet0*r*bmod(:,l)/(2.*denseq*(feq-qqinv*cureq))
end do
```

$$sceq1 = \frac{\beta_0}{2}\frac{\rho \tilde{B}_0}{\tilde{n}_0\left(\tilde{J} - \tau\tilde{I}\right)}$$

```
call block(sceq1,1,7,3,1,0,0,1.0_IDP)
```

$$Svp_1 = \frac{\beta_0}{2}\frac{\tau\tilde{B}_0}{\tilde{n}_0\left(\tilde{J} - \tau\tilde{I}\right)}\frac{\partial p}{\partial \theta}$$

```
do l=1,leqmax
        sceq1(:,l)=-bet0 *bmod(:,l)/(2.*denseq*(feq-qqinv*cureq))
end do
```

$$sceq1 = -\frac{\beta_0}{2}\frac{\tilde{B}_0}{\tilde{n}_0\left(\tilde{J} - \tau\tilde{I}\right)}$$

```
call block(sceq1,1,7,3,0,0,1,1.0_IDP)
```

$$Svp_1 = \frac{\beta_0}{2}\frac{\tilde{B}_0}{\tilde{n}_0\left(\tilde{J} - \tau\tilde{I}\right)}\frac{\partial p}{\partial \zeta}$$

! Perturbed magnetic field term

```
call dbydr0(sd1,preq,0.0_IDP,1.0_IDP,0)
```

$$sd1 = \frac{dp_0}{d\rho}$$

```
do l=1,leqmax
        sceq2(:,l)=-sceq1(:,l)*sd1(:)
end do
```

$$sceq2 = -\frac{dp_0}{d\rho}\frac{\beta_0}{2}\frac{\tilde{B}_0}{\varepsilon\tilde{n}_0\left(\tilde{J} - \tau\tilde{I}\right)}$$

call blockj(sceq2,1,7,1,1,0,0,1.0_IDP)

$$Svp_2 = -\frac{dp_0}{d\rho}\frac{\beta_0}{2}\frac{\tilde{B}_0}{\tilde{n}_0\left(\tilde{J}-\tau\tilde{I}\right)}\frac{1}{\rho}\frac{\partial\psi}{\partial\theta}$$

New terms in the pressure equation:

do l=1,leqmax

sceq1(:,l)=preq*r*qqinv*bmod(:,l)/(feq(:)-qqinv(:)*cureq(:))

end do

$$sceq1 = \frac{p_0\rho\tau B_0}{J-\tau I}$$

call blockj(sceq1,1,3,7,1,0,0,-gamma)

$$Sp_1 = -\frac{\Gamma\tau p_0 B_0}{J-\tau I}\frac{\partial v_{\|th}}{\partial\theta}$$

do l=1,leqmax

sceq1(:,l)=preq*bmod(:,l)/(feq(:)-qqinv(:)*cureq(:))

end do

$$sceq1 = \frac{p_0 B_0}{J-\tau I}$$

call blockj(sceq1,1,3,7,0,0,1,-gamma)

$$Sp_2 = -\frac{\Gamma p_0 B_0}{J-\tau I}\frac{\partial v_{\|th}}{\partial\zeta}$$

do l=1,leqmax

sceq1(:,l)=preq*r*qqinv*sqgibmodith(:,l)/(feq-qqinv*cureq)

end do

$$sceq1 = \frac{p_0\rho\tau B_0\sqrt{g}}{J-\tau I}\frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)$$

call blockj(sceq1,1,3,7,0,0,0,-gamma)

$$Sp_3 = -\frac{\Gamma p_0 \rho \tau B_0 \sqrt{g}}{J - \tau I} \frac{\partial}{\partial \theta}\left(\frac{1}{\sqrt{g}}\right) v_{\parallel, th}$$

```
do l=1,leqmax
        sceq1(:,l)=preq*sqgibmodizt(:,l)/(feq-qqinv*cureq)
end do
```

$$sceq1 = \frac{p_0 B_0 \sqrt{g}}{J - \tau I} \frac{\partial}{\partial \zeta}\left(\frac{1}{\sqrt{g}}\right)$$

call blockj(sceq1,1,3,7,0,0,0,-gamma)

$$Sp_4 = -\frac{\Gamma p_0 B_0 \sqrt{g}}{J - \tau I} \frac{\partial}{\partial \zeta}\left(\frac{1}{\sqrt{g}}\right) v_{\parallel, th}$$