# HELICALLY TRAPPED EP IMPLEMENTATION

**Analytical model description:**

The effect of the helically trapped EP are introduced in the model through modification of the average drift velocity operator of the passing EP. Using the expressions 6.72, 6.77 and 6.83 of the (Wakatani, M. et al, Stellarator and Heliotron Devices: Oxford University Press, ISBN:0195078314, 1998), the trapped EP trajectory can be derived in the ψ-θ plane, described by the period of the guiding center motion in the helical ripple (T) and the adiabatic invariant J defined as:

$$J = 2B_\phi \int_{\phi_-}^{\phi_+} \frac{mv_\parallel}{B_0} d\phi = \frac{4\psi'}{a^2\sqrt{g}} \int_{\phi_-}^{\phi_+} \frac{mv_\parallel}{B_0} d\phi$$

The averaged drift equations of the helically trapped EP can be written as:

$$\frac{d\psi}{dt} = \omega_b \frac{4\psi'}{a^2} \frac{\partial}{\partial\theta}\left( \frac{1}{\sqrt{g}} \frac{m}{q} \int_{\phi_-}^{\phi_+} \frac{v_\parallel}{B_0} d\phi \right)$$

$$\frac{d\theta}{dt} = -\omega_b \frac{4\psi'}{a^2} \frac{\partial}{\partial\psi}\left( \frac{1}{\sqrt{g}} \frac{m}{q} \int_{\phi_-}^{\phi_+} \frac{v_\parallel}{B_0} d\phi \right)$$

with $\omega_b = 1/2\pi T$ the averaged bounce frequency of the guiding center. We can also define the square of the averaged bounce length of the guiding center ($d_b$), defined as:

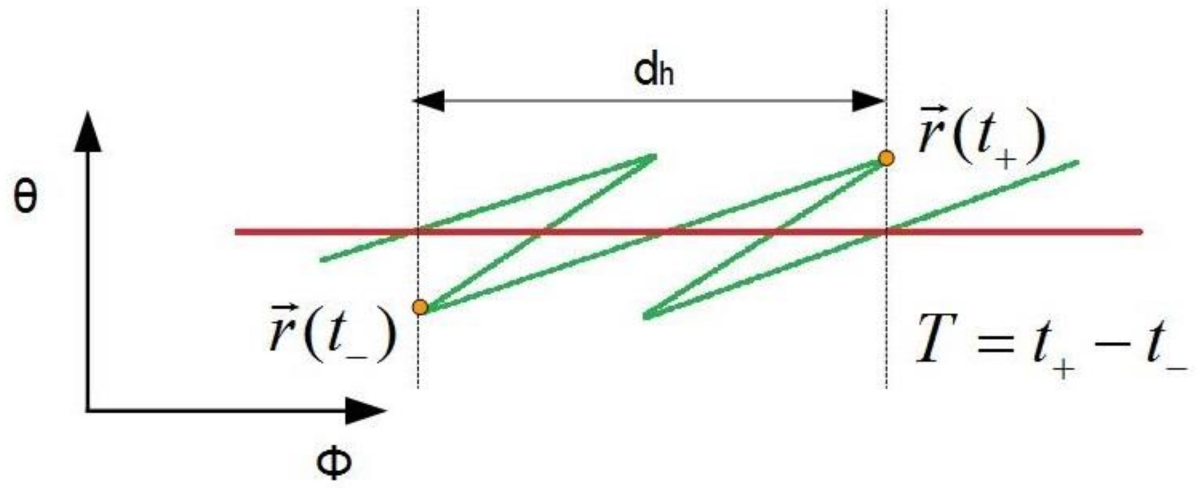
$$d_b^2 = \frac{m}{q} \int_{\phi_-}^{\phi_+} \frac{v_\parallel}{B_0} d\phi = \frac{m}{q} \lim_{\Delta\phi \to \infty} \sum_{\phi_-}^{\phi_+} \frac{v_\parallel}{B_0} \Delta\phi$$

The experimental data indicates that the EP participating in the resonance have defined characteristics, thus we consider for simplicity the next assumption: the EP involved in the resonance have the same averaged guiding center bounce frequency and length. In effect, this implies that we are considering only particles at a single energy and pitch angle $\Delta = \mu B_0 / 2$, with μ the magnetic moment. Consequently, as first order approximation, no radial or angular dependency of the averaged bounce length is included in the model, thus:

$$\frac{d\psi}{dt} = \omega_b d_b^2 \frac{4\psi'}{a^2} \frac{\partial}{\partial\theta}\left( \frac{1}{\sqrt{g}} \right)$$

$$\frac{d\theta}{dt} = -\omega_b d_b^2 \frac{4\psi'}{a^2} \frac{\partial}{\partial\psi}\left( \frac{1}{\sqrt{g}} \right)$$

Following such simplification, next figure shows a representation of the guiding center motion of the helically trapped EPs that participate in the resonance:




*Schematic view of the guiding center motion of the helically trapped EPs that participate in the resonance. The red line indicates the magnetic field line and the green line the motion of the helically trapped EP. The yellow dots indicate the motion of the EP guiding center during one oscillation period.*

If we define the averaged bounce velocity of the guiding center as $v_b = 2\pi\ \omega_b\ d_b$, the averaged drift equations can be reformulated as:

$$v_b = \frac{\rho^2}{4d_b}\left[\int_{t_-}^{t_+} \frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)dt\right]^{-1} = \frac{\theta}{4d_b}\left[\int_{t_-}^{t_+} \frac{\partial}{\partial\rho^2}\left(\frac{1}{\sqrt{g}}\right)dt\right]^{-1}$$

where the Jacobian is an equilibrium variable thus it is not time dependent. Consequently:

$$\int_{t_-}^{t_+} \frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)dt = \frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)\int_{t_-}^{t_+} dt = \frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)T$$

Using the expression of the averaged bounce velocity, we introduce a correction to the averaged drift velocity operator of the passing particles $\Omega_{d,\|}$:

$$\left[\Omega_{d,\|}\right]_{Trapped} = \frac{2\varepsilon\pi^2\rho^2\omega_b}{d_b}\left[\frac{\partial}{\partial\theta}\left(\frac{1}{\sqrt{g}}\right)\right]^{-1}\left[\Omega_{d,\|}\right]_{passing}$$

The new averaged drift velocity operator includes information of the averaged bounce frequency and length of the guiding center of the helically trapped EP.

**Implementation on code:**

**Subroutine VMEC**

Definition of the new averaged drift velocity operator:

```
do l=1,lbmax
        if (mm(l) .eq. 0)  then
                sb1(:,l)=-19.74*eps*r*r*omcyb*sqgeq(:,l)/(rbound)
        else
                sb1(:,l)=- 19.74*eps *r*r*omcyb*sqgeq(:,l)/(rbound*mm(l))
        endif
end do
call multb(omdrprpeq,omdreq,-1,sb1,1,0.0_IDP,1.0_IDP)
```

$$\left[\Omega_{d,\|}\right]_{T,\rho} = \frac{2\varepsilon\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\|}\right]_{P,\rho}$$

```
do l=1,lbmax
        if (mm(l) .eq. 0)  then
                sb1(:,l)=- 19.74*eps *r*r*omcyb*sqgeq(:,l)/(rbound)
        else
                sb1(:,l)=- 19.74*eps *r*r*omcyb*sqgeq(:,l)/(rbound*mm(l))
        endif
end do
call multb(omdtprpeq,omdteq,1,sb1,1,0.0_IDP,1.0_IDP)
```

$$\left[\Omega_{d,\|}\right]_{T,\theta} = \frac{2\varepsilon\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\|}\right]_{P,\theta}$$

```
do l=1,lbmax
    if (mm(l) .eq. 0)  then
        sb1(:,l)=- 19.74*eps *r*r*omcyb*sqgeq(:,l)/(rbound)
    else
        sb1(:,l)=- 19.74*eps *r*r*omcyb*sqgeq(:,l)/(rbound*mm(l))
    endif
end do
call multb(omdzprpeq,omdzeq,1,sb1,1,0.0_IDP,1.0_IDP)
```

$$\left[\Omega_{d,\|}\right]_{T,\zeta} = \frac{2\varepsilon\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\|}\right]_{P,\zeta}$$

**Subroutine linstart/lincheck**

Example of the implementation

```
if(Trapped_on .eq. 0) then
    do l=1,leqmax
        sceq1(:,l)=vfova2*omdr(:,l)/(epsq*omcyd)
        sceq2(:,l)=vfova2*omdt(:,l)/(epsq*omcyd)
        sceq3(:,l)=vfova2*omdz(:,l)/(epsq*omcyd)
    end do
else
    do l=1,leqmax
        sceq1(:,l)=vfova2*omdrprp(:,l)/(epsq*omcyd)
        sceq2(:,l)=vfova2*omdtprp(:,l)/(epsq*omcyd)
        sceq3(:,l)=vfova2*omdzprp(:,l)/(epsq*omcyd)
    end do
end if
```

$$sceq1 = \frac{v_{th,f}^2}{v_{A0}^2}\frac{1}{\varepsilon\omega_{cy}}\frac{2\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\|}\right]_{P,\rho}$$

$$sceq2 = \frac{v_{th,f}^2}{v_{A0}^2}\frac{1}{\varepsilon\omega_{cy}}\frac{2\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\parallel}\right]_{P,\theta}$$

$$sceq3 = \frac{v_{th,f}^2}{v_{A0}^2}\frac{1}{\varepsilon\omega_{cy}}\frac{2\pi^2\rho^2\omega_b\sqrt{g}}{md_b}\left[\Omega_{d,\parallel}\right]_{P,\zeta}$$