Pandit Deendayal Petroleum University

School of Technology

Artificial Intelligence

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Lab3: Unconstrained Optimisation

Preamble: In this lab exercise we consider the famous Rosenbrock function for study.

Document Source: https://www.scilab.org/introduction-unconstrained-optimization#gallery

Unconstrained Optimisation Problem

We consider an optimisation problem to minimise a cost function

$$\min_{\mathbf{x}\in\mathbb{R}^n} f(x)$$

The variable $\mathbf{x} \in \mathbb{R}^n$ is known as unknowns, or the decision variables.

The function $f: \mathbb{R}^n \to \mathbb{R}^n$ is known as cost function or objective function.

Additionally the gradient of the objective function is defined as

$$\mathbf{g}(\mathbf{x}) = \nabla f(\mathbf{x}) = \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n}\right)^T$$

The Hessian matrix is denoted by $H(\mathbf{x})$ and it is defined as follows

$$H_{ij}(\mathbf{x}) = (\nabla^2 f)_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

Rosenbrock function

A Rosenbrock function is defined as

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

Exercise I:

Show that gradient is $\mathbf{g}(\mathbf{x}) = \begin{pmatrix} 400x_1^3 - 400x_1x_2 + 2x_1 - 2 \\ -200x_1^2 + 200x_2 \end{pmatrix}$

And Hessian $H(\mathbf{x}) = \begin{pmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{pmatrix}$

Exercise II:

Write a Rosenbrock function in Scilab that computes the function value f, the gradient g, and the Hessian H of the Rosenbrock's function.

Find $f(\mathbf{x}_0)$ at $\mathbf{x}_0 = [-1.2, 1]^T$ and $f(\mathbf{x}^*)$ at $\mathbf{x}^* = [1, 1]^T$. Comment on the results.

Exercise III:

Find the derivative and Hessian computed by Rosenbrock function at $\mathbf{x}_0 = [-1.2, 1]^T$. Compute numerical derivatives by using Scilab function **derivative**.

Exercise IV

Cross check and compare the results. Comment on the relative error between the derivative and Hessian computed by Rosenbrock function and Scilab function.

References

https://en.wikipedia.org/wiki/Rosenbrock_function

https://en.wikipedia.org/wiki/Hessian

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