Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

First Semester B. Tech. First Year (All)

Winter– 2015

Course Code: SHU101

Course Name: Engineering Mathematics-I

Time: 2 Hrs. 30 Min. Max. Marks: 60

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three :** 12

(a) Verify Cayley Hamilton theorem for the matrix

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$ Hence compute A^{-1}.

(b) Show that the only real value of p for which the following equations will have a non-trivial solution is 6:
$x+2y+3z=px,\ 3x+y+2z=py,\ 2x+3y+z=pz$

(c) Find the Eigen values and the corresponding Eigen

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vectors for the matrix $A=\begin{bmatrix}1 & -6 & -4\\ 0 & 4 & 2\\ 0 & -6 & -3\end{bmatrix}$

(d) Show that the vectors $x_1=(1,2,4),\ x_2=(2,-1,3),\ x_3=(0,1,2)\ \&\ x_4=(-3,7,2)$ are linearly dependent and find the relation between them.

2. **Attempt any three :** **12**

(a) If $r=\sqrt{x^2+y^2},\ \theta=\tan^{-1}\left(\frac{y}{x}\right)$, Evaluate $\frac{\partial(r,\theta)}{\partial(x,y)}$.

(b) Find $\frac{dy}{dx}$, when $x^y+y^x=c$

(c) Verify Eulers theorem for the function $u=\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)\left(x^n+y^n\right)$.

(d) Locate the stationary points of $x^4+y^4-2x^2+4xy-2y^2$ and determine their nature.

3. **Attempt the following :** **12**

(a) Show that $Sech^{-1}(\sin\theta)=\log\cot\frac{\theta}{2}$.

(b) Solve the equation $x^7+x^4+x^3+1=0$

(c) Simplify $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4$.

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4. **Attempt any three :** 12

(a) Use Taylors Theorem to find $\sqrt{25.15}$.

(b) Evaluate $\lim\limits_{x \to 0} \dfrac{e^{2x} - (1+x)^2}{x \log(1+x)}$.

(c) Find the n$^{\text{th}}$ derivative of $x^2 e^x \cos x$.

(d) Prove that

$$\log\left[\log(1+x)^{\frac{1}{x}}\right] = \frac{-x}{2} + \frac{5}{24}x^2 - \frac{1}{8}x^3 + \frac{251}{2880}x^4 - - - - -$$

5. **Attempt the following :** 12

(a) Show that the eigen vector of a matrix A corresponding to eigen value 4 and eigen vector of matrix $D = 2A^2 - \frac{1}{2}A + 3I$ corresponding to eigen value 33 are same if $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

0+2

(b) Express $\log(\log i)$ in the form of A+iB.

0+1

$z = \log(\log i)$

$\cancel{\log} z = \cancel{\log} \log \log i$

$= \log \log(e + i\pi/2)$

0·2

$\log \log i\pi/2$

Government College of Engineering, Amravati

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30

First Year B. Tech. (All Branches)

Winter – 2016

Course Code: SHU101

Course Name: Engineering Mathematics – I

Max. Marks: 60

Time: 2 Hrs. 30 Min.

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three :** **12**

(a) Find the rank of $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ by Echelon form.

12

(b) Discuss the consistency of the following system of equations and solve them if possible

(c) $x+y+z=3,\ x+2y+3z=4,\ 2x+3y+4z=7$

Using Cayley Hamilton theorem find A^8

if $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$

(d) Find non-singular matrices P, Q so that PAQ is a

normal form where $A=\begin{bmatrix}2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\\ 1 & 1 & 1 & 2\end{bmatrix}$

2. **Attempt any three :** **12**

(a) If $y_1=\dfrac{x_2 x_3}{x_1}, y_2=\dfrac{x_3 x_1}{x_2}, y_3=\dfrac{x_1 x_2}{x_3}$ .Show that the Jacobian of y_1, y_2, y_3 w.r.to x_1, x_2, x_3 is 4.

4

(b) If $\theta=t^n e^{\frac{-r^2}{4t}}$, find the value of n which will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}.$$

(c) If $u=\sin^{-1}\left(\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$,show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}+3\tan u=0 .$$

(d) Examine for minimum and maximum values

$\sin x+\sin y+\sin(x+y)$

3. **Attempt the following :** **12**

4

(a) Show that the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ is 1 .

(b) Prove $i^{i^{i}} = \cos\theta + i\sin\theta$

Where $\theta = \pi\left(2m+\frac{1}{2}\right)e^{-\left(2n+\frac{1}{2}\right)\pi}$

(c) Seprate into real and imaginary parts of $\sin^{-1}\left(e^{i\theta}\right)$.

4. **Attempt any three :** **12**

(a) Arrange $7+(x+2)+3(x+2)^3+(x+2)^4-(x+2)^5$ in powers of x using Taylors series.

9 (b) Evaluate $\lim_{x\to a}\sin^{-1}\sqrt{\frac{a-x}{a+x}}\,\operatorname{cosec}\sqrt{a^2-x^2}$.

(c) Find the nth derivative of $\frac{x}{(x-a)(x-b)(x-c)}$.

(d) Find the 15th derivative of $(x^2+1)\log(ax+b)$ with respect to x.

5. **Attempt the following :** **12**

(a) Show that the eigen vector of a matrix A corresponding to eigen value 4 and eigen vector of matrix $D = 2A^2 - \frac{1}{2}A + 3I$ corresponding to eigen value 33 are same if $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

(b) If $V = f(r)$ where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{dr^2} + \frac{2}{r}\frac{dV}{dr}$$

JDE10

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

I Semester B. Tech.

Summer - 2010

Course Code :FE101

Course Name : Engineering Mathematics-I

Time : 2 hr.30min. **Max. Marks : 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. (a) Solve 20

$$x^4 - x^3 + x^2 - x + 1 = 0$$

(b) If $z = \frac{k}{\sqrt{y}} e^{\frac{-x^2}{4my}}$ then show that

$$\frac{\partial z}{\partial y} = m \frac{\partial^2 z}{\partial x^2}$$

Contd..

non-program
5) Figures to the

(c) Evaluate

$$\lim_{x\to\infty}\frac{e^x}{\left[\left(1+\frac{1}{x}\right)^x\right]^x}$$

(d) Find the relation of linear dependence amongst the row vectors of matrix

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 1 \end{pmatrix}$$

2. Attempt any TWO **10**

(a) If $\cosh x = \sec\theta$ then show that

$$\theta = \frac{\pi}{2} - 2\tan^{-1}\left(e^{-x}\right)$$

(b) Separate the real and imaginary parts of

$$(1+i)^{2-3i}$$

(c) Use Demoivre's theorem to express $\tan 5\theta$ In terms of powers of $\tan\theta$ and hence find value of $5\tan^4\frac{\pi}{10} - 10\tan^2\frac{\pi}{10}$

3 Attempt any TWO **10**

(a) Find n^{th} derivative of $\dfrac{1}{1+x+x^2+x^3}$

(b) Show that

$$e^y = 1 + \sin y + \frac{1}{2}\sin^2 y + \frac{1}{3}\sin^3 y + \cdots$$

(c) If $y = e^{2\sin^{-1}x}$ then show that

$$\left(1-x^2\right)y_{k+2} - \left(2k+1\right)xy_{k+1} - \left(k^2+4\right)y_k =$$

10

4 Attempt any TWO

(a) If

$$u = \frac{x-y}{x+y} \quad and \quad v = \frac{x+y}{x} \quad \text{verify}$$

whether u and v are functionally dependent? If so find the relation between them.

(b) If

$$z = \left(\frac{x^3+y^3}{y\sqrt{x}}\right) + x^{-7}\sin^{-1}\left[\frac{x^2+y^2}{x^2+2xy}\right]$$

Then find value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} \quad \text{at the point } (1, 2)$$

(c) Find the maximum and minimum distances of the point
(3 , 4 , 12) from the surface
$x^2 + y^2 + z^2 - 1 = 0$

10

5. Attempt any TWO

(a) State Cayley Hamilton theorem. And use it to find

A^{-2} and A^{4} where

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(b) Define rank of matrix.
Find rank of matrix A where

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

(c) Define Eigen vector of the matrix. Find all Eigen vectors
of

$$A = \begin{pmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

Attempt any three :

JVR29

Government College of Engineering, Amravati

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I Semester B. Tech.

Winter - 2009

Course Code :FE101

Course Name :Engineering Mathematics-I

Time : 2 Hrs. 30min. **Max. Marks : 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. Attempt any two **12**

a) If $\cos\left(\frac{\pi}{4}+ia\right)\cosh\left(b+i\frac{\pi}{4}\right)=1$ then show that $2b=\log\left(2+\sqrt{3}\right)$

b) Separate real and imaginary parts of $\left(\sqrt{i}\right)^{\sqrt{i}}$

c) Find all the roots of $x^4-x^3+x^2-x+1=0$

2. Attempt any two **12**

a) Find the rank of the following matrix by reducing to normal form

$$A=\begin{bmatrix}1 & 2 & 3 & -1\\ -1 & -1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$$

b) Verify Cayley Hamilton theorem for $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$

and use it to the matrix

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$$

c) Find eigen values & eigen vectors for the matrix

$$A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$$

3. **Attempt any two** 12

a) Find n[th] derivative of $y=\dfrac{x}{x^2+a^2}$

b) If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$ then show that

$$y=x+\frac{2x^3}{3}+\frac{8x^5}{15}+\frac{16x^7}{35}+\text{..........}$$

c) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$

4. **Attempt any two** 12

a) If $\dfrac{x^2}{a^2+u}+\dfrac{y^2}{b^2+u}+\dfrac{z^2}{c^2+u}=1$ then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$$

b) If $x=\sqrt{vw}$, $y=\sqrt{uw}$, $z=\sqrt{uv}$ and
$u=r\sin\theta\cos\phi$, $v=r\sin\theta\sin\phi$, $w=r\cos\theta$
find $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

c) Find the extreme values of
$u=x^3+y^3-63(x+y)+12xy$,

5. Attempt any two **12**

a) If $u=\dfrac{x^4+y^4}{x^2y^2}+x^6\tan^{-1}\left(\dfrac{x^2+y^2}{x^2+2xy}\right)$, find the value of

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$$ at
$x=1$, $y=2$

b) Examine whether the vectors Determine the analytic function $(1,2,-1,0)$,$(1,3,1,2)$, $(4,2,1,0)$, $(6,1,0,1)$ are linearly dependent or independent also find the relation between them.

c) Find z if $\arg(z+1)=\dfrac{\pi}{6}$ and $\arg(z-1)=\dfrac{2\pi}{3}$