Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Winter – 2014

Course Code: SHU304

Course Name: Engineering Mathematics – III

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. Attempt any three: **12**

(a) Prove that Particular integral of $(D-m)y = f(x)$ is $e^{mx}\int e^{-mx} f(x)\,dx$ and hence find the same of the differential equation $(D^2+2D+1)y = e^{-x}$

(b) Solve : $(D^4-2D^3-3D^2+4D+4)y = x^2e^x$

(c) Solve: $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}-4y = x^2+2\log x$

(d) Solve by method of variation of parameters the equation $y''-2y'+2y = e^x\tan x$.

2. Attempt any three: **12**

(a) i) Solve $p+q=\sin x+\sin y$

Cont…

ii) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$

(b) Solve: $(x - y)(px - qy) = (p - q)^2$

(c) Using method of separation of variables, solve
$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x,0) = 4e^{-x}$$

(d) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

3. **Attempt any three:** 12

(a) Using first shifting theorem prove that
$$L\{t\cos at\} = \frac{s^2 - a^2}{(s^2 + a^2)^2}$$

(b) Evaluate

i) $\int_0^{\infty} te^{-2t}\sin t\,dt$ ii) $L\left\{\int_0^t \frac{e^t \sin t}{t}\,dt\right\}$

(c) Find the inverse Laplace transform of

i) $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$ ii) $\tan^{-1}(2/s^2)$

(d) Define Unit impulse function, Heaviside unit step function and find the relation between them

(e) State Convolution theorem and verify it for the pair of functions $f(t) = t,\ g(t) = e^{at}$.

4. **Attempt any three:** 12

(a) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at (1,1,1) has maximum magnitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Find a,b,c.

(b) Show that $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$.

(c) Show that $\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$ is solenoidal field, where $r = |\bar{r}|$, $\bar{r} = xi + yj + zk$ and $\bar{a}$ be the constant vector.

(d) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the field of force given by
$\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$
Is the field conservative?

5. **Attempt the following:**

(a) If $\bar{r} \times \frac{d\bar{r}}{dt} = 0$, show that $\bar{r}$ has a constant direction. **04**

(b) The small oscillations of a certain system with two degrees of freedom are given by the equations

$$D^2 x + 3x - 2y = 0$$
$$D^2 y + D^2 x - 3x + 5y = 0$$

Where $D = \frac{d}{dt}$. If $x = 0, y = 0, Dx = 3, Dy = 2$ when $t = 0$. Find x and y when $t = \pi / 2$ **06**

(c) Given $L\{J_0(t)\} = \frac{1}{\sqrt{s^2 + 1}}$, then

i) $\int_0^\infty J_0(t)dt = ------$

ii) $\int_0^\infty e^{-t} J_0(t)dt = ------$ **02**

Government College of Engineering, Amravati

(An Autonomous Institute of Government of Maharashtra)

Third Semester B. Tech. (CS / IT)

Winter – 2016

Course Code: SHU304

Course Name: Engineering Mathematics-III

Time: 2 Hrs. 30 Min. **Max. Marks: 60**

Instructions to Candidate

1) All questions are compulsory.
2) Assume suitable data wherever necessary and clearly state the assumptions made.
3) Diagrams/sketches should be given wherever necessary.
4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
5) Figures to the right indicate full marks.

1. **Attempt any three:** **12**

(a) State the formula for particular integral of the differential equation $\phi(D)y = e^{ax} V$ and hence use it to find the same for $(D^2+2D-3)y = e^x \cos x$

(b) Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \sin^2 x$

(c) Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$

(d) Solve by method of variation of parameters the equation $y'' - y = e^{-2x}\cos(e^{-x})$.

2. **Attempt any three:** **12**

(a) i) Solve $p^2 + q = x + \sin y$

Cont.

ii) Solve $\sqrt{1+p^2-q^2}=0$

(b) Solve: $pqz = p^2(xq+p^2)+q^2(yp+q^2)$

(c) Using method of separation of variables, solve
$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\ u(x,0)=4e^{-x}$$

(d) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$

3. **Attempt any three:** 12

(a) State first shifting theorem. Find the Laplace transform of $\sinh 3t\cos^2 t$

(b) Find the Laplace transform of $\frac{1-\cos t}{t}$ and hence find $L\left(\frac{1-\cos t}{t^2}\right)$.

(c) Find the Laplace transform of the triangular wave given by
$$f(t)=\begin{cases} t,\ 0<t<a \\ 2a-t,\ a<t<2a \end{cases}\quad \text{where } f(t+2a)=f(t).$$

(d) Find the inverse Laplace transform of

i) $\dfrac{s^2-10s+13}{(s+1)(s^2-5s+6)}$ ii) $\log\left(1-\dfrac{a^2}{s^2}\right)$

4. **Attempt any three:** 12

(a) Show that
$\overline{F}=(y^2\cos x+z^3)i+(2y\sin x-4)j+(3xz^2+2)k$
is a conservative vector field and find a function ϕ such that $\overline{F}=\nabla\phi$.

(b) Find the directional derivative of
$\phi=x^2y+2y^2z+3z^2x$ at (1,1,1) in the direction

parallel to $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$.

(c) Show that $\nabla^2\left[\nabla\cdot\left(\frac{\bar{r}}{r^4}\right)\right]=\frac{-12}{r^6}$

(d) Find the work done in moving a particle once around the circle C in the x-y plane if the circle has centre at the origin and radius 3 and the force field $\bar{F}=(2x-y+z)i+(x+y-z^2)j+(3x-2y+4z)k$

5. **Attempt the following:** **12**

(a) Prove that $\bar{r}.\frac{d\bar{r}}{dt}=0$ iff $\bar{r}$ has a constant magnitude.

(b) Solve the following differential equation using Laplace transform

$$\frac{d^2y}{dt^2}-6\frac{dy}{dt}+9y=t^2e^{3t} \quad with\ y(0)=2,\ y'(0)=6$$

(c) If r and $\bar{r}$ have their usual meanings then prove that

i) $\nabla\cdot(r^n\bar{r})=(n+3)r^n$

ii) $\nabla^2 r^n=n(n+1)r^{n-2}$