

Government College of Engineering, Amravati
(An Autonomous Institute of Government of Maharashtra)

First Year B. Tech. (All Branches)

Winter – 2016

Course Code: SHU101

Course Name: Engineering Mathematics – I

Time: 2 Hrs. 30 Min.

Max. Marks: 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. **Attempt any three :**

12

(a)

Find the rank of
$$\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$$
 by

Echelon form.

- (b) Discuss the consistency of the following system of equations and solve them if possible

(c) $x + y + z = 3, x + 2y + 3z = 4, 2x + 3y + 4z = 7$

Using Cayley Hamilton theorem find A^8

if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

(d) Find non-singular matrices P, Q so that PAQ is a normal form where $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

2. Attempt any three :

12

(a) If $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$. Show that the Jacobian of y_1, y_2, y_3 w.r.to x_1, x_2, x_3 is 4.

(b) If $\theta = t^n e^{\frac{-r^2}{4t}}$, find the value of n which will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$.

(c) If $u = \sin^{-1} \left(\frac{x + 2y + 3z}{\sqrt{x^8 + y^8 + z^8}} \right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} + 3 \tan u = 0.$$

(d) Examine for minimum and maximum values $\sin x + \sin y + \sin(x + y)$

3. Attempt the following :

12

(a) Show that the continued product of all the values

of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ is 1 .

(b) Prove $i^{\theta} = \cos \theta + i \sin \theta$

Where $\theta = \pi \left(2m + \frac{1}{2}\right) e^{-\left(2m + \frac{1}{2}\right)\pi}$

(c) Seprate into real and imaginary parts of $\sin^{-1}(e^{i\theta})$.

4. **Attempt any three :**

12

(a) Arrange $7 + (x+2) + 3(x+2)^3 + (x+2)^4 - (x+2)^5$ in powers of x using Taylors series.

(b) Evaluate $\lim_{x \rightarrow a} \sin^{-1} \sqrt{\frac{a-x}{a+x}} \operatorname{cosec} \sqrt{a^2 - x^2}$.

(c) Find the n^{th} derivative of $\frac{x}{(x-a)(x-b)(x-c)}$.

(d) Find the 15^{th} derivative of $(x^2 + 1)\log(ax + b)$ with respect to x.

5. **Attempt the following :**

12

(a) Show that the eigen vector of a matrix A corresponding to eigen value 4 and eigen vector of matrix $D = 2A^2 - \frac{1}{2}A + 3I$ corresponding to

eigen value 33 are same if $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.

(b) If $V = f(r)$ where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2 V}{dr^2} + \frac{2}{r} \frac{dV}{dr}$$

Government College of Engineering, Amravati
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1 Semester B. Tech.

Summer - 2019

Course Code : FE101

Course Name : Engineering Mathematics-I

Time : 2 hr, 30 min.

Max. Marks : 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumption made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. (a) Solve

$$x^4 - x^3 + x^2 - x + 1 = 0$$

20

(b) If $z = \frac{k}{\sqrt{y}} e^{\frac{-x^2}{4y}}$ then show that

$$\frac{\partial z}{\partial y} = m \frac{\partial^2 z}{\partial x^2}$$

Contd.

(c) Evaluate

$$\lim_{x \rightarrow \infty} \frac{e^x}{\left[\left(1 + \frac{1}{x} \right)^x \right]^x}$$

(d) Find the relation of linear dependence amongst the row vectors of matrix

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 1 \end{pmatrix}$$

10

2. Attempt any TWO

(a) If $\cosh x = \sec \theta$ then show that

$$\theta = \frac{\pi}{2} - 2 \tan^{-1} (e^{-x})$$

(b) Separate the real and imaginary parts of $(1+i)^{2-3i}$

(c) Use Demoivre's theorem to express $\tan 5\theta$ in terms of powers of $\tan \theta$ and hence find value of $5 \tan^4 \frac{\pi}{10} - 10 \tan^2 \frac{\pi}{10}$

10

3. Attempt any TWO

(a) Find n^{th} derivative of $\frac{1}{1+x+x^2+x^3}$

Government College of Engineering, Amravati
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I Semester B. Tech.

Winter - 2009

Course Code : FE101

Course Name : Engineering Mathematics-I

Time : 2 Hrs. 30min.

Max. Marks : 60

Instructions to Candidate

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary and clearly state the assumptions made.
- 3) Diagrams/sketches should be given wherever necessary.
- 4) Use of logarithmic table, drawing instruments and non-programmable calculators is permitted.
- 5) Figures to the right indicate full marks.

1. Attempt any two

12

a) ~~If~~ $\cos\left(\frac{\pi}{4} + ia\right) \cosh\left(b + i\frac{\pi}{4}\right) = 1$ then show
that $2b = \log(2 + \sqrt{3})$

b) Separate real and imaginary parts of $(\sqrt{i})^{\sqrt{i}}$

c) Find all the roots of $x^4 - x^3 + x^2 - x + 1 = 0$

2. Attempt any two

12

- a) Find the rank of the following matrix by reducing to normal form

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

b) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

and use it to the matrix

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

c) Find eigen values & eigen vectors for the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

3. Attempt any two

a) Find n^{th} derivative of $y = \frac{x}{x^2 + a^2}$

b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ then show that

$$y = x + \frac{2x^3}{3} + \frac{8x^5}{15} + \frac{16x^7}{35} + \dots$$

c) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$

4. Attempt any two

a) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$ then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

- b) If $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$ and
 $u = r \sin \theta \cos \phi$, $v = r \sin \theta \sin \phi$, $w = r \cos \theta$
 find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$

- c) Find the extreme values of
 $u = x^3 + y^3 - 63(x + y) + 12xy$,

5. Attempt any two

12

- a) If $u = \frac{x^4 + y^4}{x^2 y^2} + x^6 \tan^{-1}\left(\frac{x^2 + y^2}{x^2 + 2xy}\right)$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \text{ at } x=1, y=2$$

- b) Examine whether the vectors Determine the analytic function $(1, 2, -1, 0)$, $(1, 3, 1, 2)$, $(4, 2, 1, 0)$, $(6, 1, 0, 1)$ are linearly dependent or independent also find the relation between them.

- c) Find z if $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$

(b) Show that

$$e^y = 1 + \sin y + \frac{1}{2} \sin^2 y + \frac{1}{3} \sin^3 y + \dots$$

(c) If $y = e^{2 \sin^{-1} x}$ then show that

$$(1 - x^2) y_{k+2} - (2k + 1) x y_{k+1} - (k^2 + 4) y_k = 0$$

4

Attempt any TWO

10

(a) If

$$u = \frac{x - y}{x + y} \quad \text{and} \quad v = \frac{x + y}{x} \quad \text{verify}$$

whether u and v are functionally dependent? If so find the relation between them.

(b) If

$$z = \left(\frac{x^3 + y^3}{y\sqrt{x}} \right) + x^{-7} \sin^{-1} \left[\frac{x^2 + y^2}{x^2 + 2xy} \right] \quad \checkmark$$

Then find value of

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} \quad \text{at the point } (1, 2)$$

(c) Find the maximum and minimum distances of the point $(3, 4, 12)$ from the surface

$$x^2 + y^2 + z^2 - 1 = 0$$

10

5.

Attempt any TWO

(a) State Cayley Hamilton theorem. And use it to find

Contd..

A^{-2} and A^4 where

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(b) Define rank of matrix.

Find rank of matrix A where

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

(c) Define Eigen vector of the matrix. Find all Eigen vectors of

$$A = \begin{pmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$