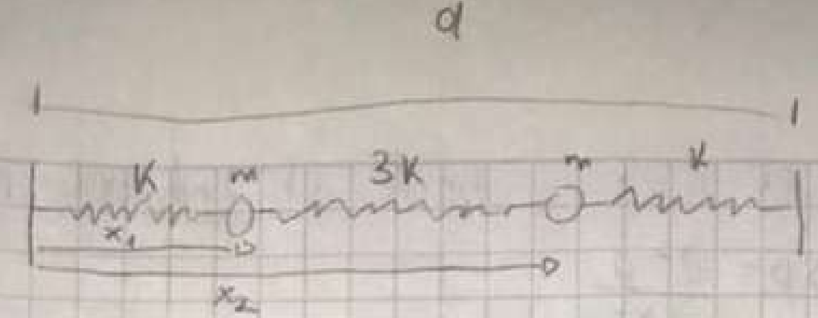
1. 

$c_1 + c_2 + c_3 = d$

$$T = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 \quad ; \quad V = \frac{1}{2} k (x_1 - c_1)^2 + \frac{3}{2} k (x_2 - x_1 - c_2)^2 + \frac{1}{2} k (d - x_2 - c_3)^2$$

$$V = \frac{1}{2} k (x_1 - c_1)^2 + \frac{3}{2} k (x_2 - x_1 - c_2)^2 + \frac{1}{2} k (c_1 + c_2 - x_2)^2$$

$$V = \frac{1}{2} \sum_{i,j} V_{ij} \eta_i \eta_j, \quad \text{donde} \quad V_{ij} = \left. \frac{\partial^2 V}{\partial y_i \partial y_j} \right|_{(c_1, c_2)}$$

$$\left. \frac{\partial^2 V}{\partial x_1^2} \right|_{(c_1, c_2)} = \frac{\partial}{\partial x_1} \Big(k (x_1 - c_1) - 3k (x_2 - x_1 - c_2) \Big) \Big|_{(c_1, c_2)} = k + 3k = 4k$$

$$\left. \frac{\partial^2 V}{\partial x_2^2} \right|_{(c_1, c_2)} = \frac{\partial}{\partial x_2} \Big(3k (x_2 - x_1 - c_2) - k (c_1 + c_2 - x_2) \Big) = 4k$$

$$\left. \frac{\partial^2 V}{\partial x_1 \partial x_2} \right|_{(c_1, c_2)} = -3k = \frac{\partial^2 V}{\partial x_2 \partial x_1}$$

$V = 2K\eta_1^2 + 2K\eta_2^2 - 3K\eta_1\eta_2 \qquad T = \frac{1}{2}m\dot{\eta}_1^2 + \frac{1}{2}m\dot{\eta}_2^2$

$$\begin{vmatrix} V_{11} - w^2 T_{11} & V_{12} - w^2 T_{12} \\ V_{21} - w^2 T_{21} & V_{22} - w^2 T_{22} \end{vmatrix}$$

$$= \begin{vmatrix} 4K - w^2 m & -3K \\ -3K & 4K - w^2 m \end{vmatrix} = 0$$

$(4K - w^2 m)^2 - 9K^2 = 0$

$4K - w^2 m = \pm 3K \qquad w_1^2 = \frac{K}{m} \rightarrow f_k = \frac{2\pi}{w_k} \qquad f_1 = 2\pi\sqrt{\frac{m}{K}}$

$\frac{4K \mp 3K}{m} = w^2 \qquad w_2^2 = \frac{7K}{m} \qquad f_2 = 2\pi\sqrt{\frac{m}{7K}}$

$(V_{11} - w_K^2 T_{11})a_1 + (V_{12} - w_K^2 T_{12})a_2 = 0$

$(V_{21} - w_K^2 T_{21})a_1 + (V_{22} - w_K^2 T_{22})a_2 = 0$

$(4K - w_K^2 m)a_1 - 3Ka_2 = 0$

$-3Ka_1 + (4K - w_K^2 m)a_2 = 0$

para $K=1$

$3Ka_1 - 3Ka_2 = 0$

$-3Ka_1 + 3Ka_2 = 0$

$a_1 = a_2$

$\frac{a_1}{a_2} = 1$

Para $K=2$

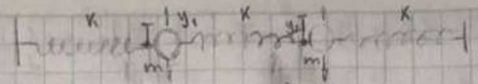
$-3Ka_1 - 3Ka_2 = 0$

$-3Ka_1 - 3Ka_2 = 0$

$a_1 = -a_2$

$\frac{a_1}{a_2} = -1$

2.



$$T = \frac{1}{2} m\dot{y}_1 + \frac{1}{2} m\dot{y}_2^2$$

$$V = \frac{1}{2} k y_1^2 + \frac{1}{2} k y_2^2 + \frac{1}{2} k (y_1 - y_2)^2$$

$$V = k y_1^2 + k y_2^2 - k y_1 y_2 = \tfrac{1}{2}\left(2k y_1^2 + 2k y_2^2 - 2k y_1 y_2 \right)$$

$$T_{ij} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad V_{ij} = \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \qquad \det\left[V - T w^2 \right] = 0$$

$$\begin{vmatrix} 2k - w^2 m & -k \\ -k & 2k - w^2 m \end{vmatrix} = 0$$

$$(2k - w^2 m)^2 - k^2 = 0 \rightarrow 2k - w^2 m = \pm k$$

$$w^2 = \frac{2k \mp k}{m} \begin{array}{l} \rightarrow w_1^2 = k/m \rightarrow f_1 = 2\pi\sqrt{m/k} \\ \rightarrow w_2^2 = 3k/m \rightarrow f_2 = 2\pi\sqrt{m/3k} \end{array}$$

$$(V_{11} - w_k^2 T_{11})\, a_1 + (V_{12} - w_k^2 T_{12})\, a_2 = 0$$

$$(V_{21} - w_k^2 T_{21})\, a_1 + (V_{22} - w_k^2 T_{22})\, a_2 = 0$$

$$(2k - w_k^2 m)\, a_1 - k a_2 = 0$$

$$-k a_1 + (2k - w_k^2 m)\, a_2 = 0$$

$k = 1$

$k a_1 - k a_2 = 0$ Claramente $a_1 = x$ y $a_2 = x$

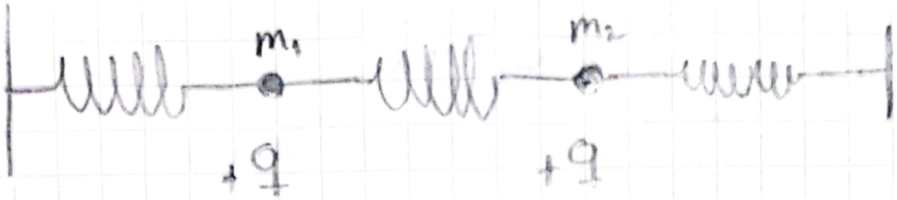
$-k a_1 + k a_2 = 0$ con $k = 1$ $\frac{a_1}{a_2} = 1$

$k = 2$

$-k a_1 - k a_2 = 0$ Claramente $a_1 = x$ y $a_2 = -x$

$-k a_1 - k a_2 = 0$ con $k = 2$ $\frac{a_1}{a_2} = -1$

3.



$$T = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$V(X_2, X_1) = \frac{K e q^2}{|X_2 - X_1|}$$

$$V_R = \begin{bmatrix} 2K & -K \\ -K & 2K \end{bmatrix}$$

$$V_q = \left.\frac{2K q^2}{\ell^3}\right] = \mu$$

$$V_q = \mu \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 2K + \mu & -(K + \mu) \\ -(K + \mu) & 2K + \mu \end{bmatrix}$$

$$\det\left(V - \omega^2 T\right) = 0$$

$$\begin{vmatrix} 2K + \mu - \omega^2 m & -(K + \mu) \\ -(K + \mu) & 2K + \mu - \omega^2 m \end{vmatrix} = 0$$

$$(2K + \mu - \omega^2 m)^2 = (K + \mu)^2$$

$$\omega^2 = \frac{2K + \mu \pm (K + \mu)}{m}$$

$$\omega_1 = \sqrt{\frac{K}{m}} \qquad \omega_2 = \sqrt{\frac{3K + 2\mu}{m}}$$

$$f_1 = \frac{2\pi}{\sqrt{\frac{K}{m}}} \qquad f_2 = \frac{2\pi}{\sqrt{\frac{3K + 2\mu}{m}}}$$

$$\begin{pmatrix} 2K+\mu & -K-\mu \\ -K-\mu & 2K+\mu \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \omega_1^2 m \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$a_1(2K+\mu) + a_2(-K-\mu) = \omega_1^2 m a_1$$

$3K/m$

$$a_1(-K-\mu) + a_2(2K+\mu) = \omega_1^2 m a_2$$

despejando a_2

$$a_2 = \frac{\omega_1^2 m a_1 - a_1(2K+\mu)}{-K-\mu}$$

sabiendo que $\omega_1^2 = \frac{K}{m}$

$$a_2 = \frac{K a_1 - a_1 2K - a_1 \mu}{-K-\mu}$$

$$a_2 = \frac{a_1(-K-\mu)}{-K-\mu}$$

$a_2 = a_1 \rightarrow$ Fase

sabiendo que $\omega_2^2 = \frac{3K+2\mu}{M}$

$$a_2 = \frac{3K a_1 + 2\mu a_1 - 2K a_1 - \mu a_1}{-K-\mu}$$

$$a_2 = a_1 \frac{(K+\mu)}{-K-\mu}$$

$a_2 = -a_1 \rightarrow$ desfase