

2.

DD

MM

$$X = R \sin \theta \cos \varphi$$

$$Y = R \sin \theta \sin \varphi$$

$$Z = r \cos \theta$$

$$\theta \rightarrow \text{ctte} \rightarrow d\theta = 0$$

$$ds = \sqrt{dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2}$$

$$ds^2 = dr^2 + r^2 \sin^2 \theta d\varphi^2 \frac{d\varphi^2}{d\varphi^2}$$

$$ds^2 = \frac{dr}{d\phi} + r^2 \sin^2 \theta$$

$$ds = \sqrt{\left(\frac{dr}{d\phi}\right)^2 + r^2 \sin^2 \theta} d\phi$$

$$f(r, r', \phi)$$

$$f(r, r', \phi) = \sqrt{r'^2 + r^2 \sin^2 \theta}$$

$$\frac{\partial f}{\partial r} = \frac{r \sin^2 \theta}{\sqrt{r'^2 + r^2 \sin^2 \theta}}$$

$$\frac{\partial f}{\partial r'} = \frac{r'}{\sqrt{r'^2 + r^2 \sin^2 \theta}}$$

$$\frac{d}{d\phi} \left(\frac{\partial f}{\partial r'} \right) = \frac{r'' \sqrt{r'^2 + r^2 \sin^2 \theta} - \frac{r'^2 \cdot r''}{\sqrt{r'^2 + r^2 \sin^2 \theta}}}{r'^2 + r^2 \sin^2 \theta}$$

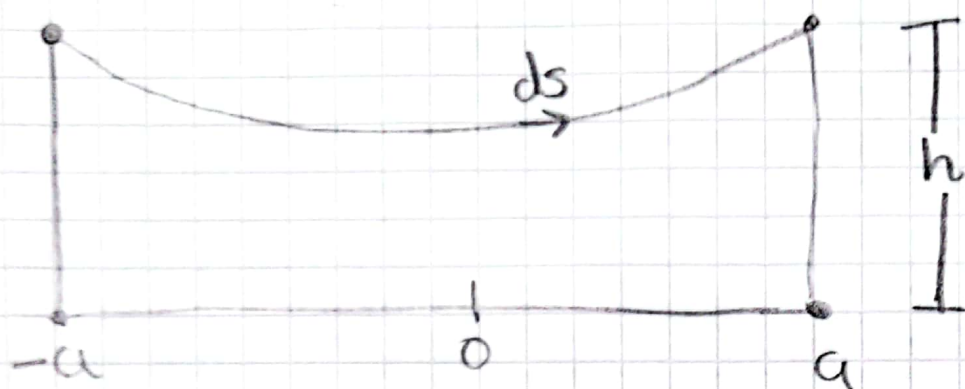
$$= r'' \left(\frac{(r'^2 + r^2 \sin^2 \theta) - r'^2}{\sqrt{r'^2 + r^2 \sin^2 \theta}} \right)$$

$$= \frac{r'' r^2 \sin^2 \theta}{(r'^2 + r^2 \sin^2 \theta)^{3/2}}$$

$$\frac{r \sin^2 \theta}{\sqrt{r'^2 + r^2 \sin^2 \theta}} - \frac{r'' r^2 \sin^2 \theta}{(r'^2 + r^2 \sin^2 \theta)^{3/2}} = 0$$

$$r(\phi) = c_2 e^{c_1 \phi + \frac{\phi^2 \sin^2 \theta}{2}}$$

3.

Cable flexible de longitud L 

$$L = \int_{-a}^a ds$$

$$L = \int_{-a}^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$ds^2 = dx^2 + dy^2 \quad \left) \frac{dx^2}{dx^2} \right.$$

$$ds^2 = \left(\frac{dx^2}{dx^2}\right) dx^2 + \left(\frac{dy^2}{dx^2}\right) dx^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Suponiendo un cable de densidad λ constantePodemos definir la función energía potencial U como

$$U = \int_{-a}^a y g \lambda \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Aplicando principio variacional para la funcion

$$f(y, y', x) = \gamma g \lambda \sqrt{1 + y'^2}$$

Usamos las ecuaciones de euler lagrange

$$\frac{\partial f}{\partial y} = g \lambda \sqrt{1 + y'^2}$$

$$\frac{\partial f}{\partial y'} = \gamma g \lambda \cdot \frac{y'}{\sqrt{1 + y'^2}}$$

$$\frac{1}{2}(1 + y'^2)^{-1/2} \cdot 2y'$$

$$g \lambda \sqrt{1 + y'^2} - \frac{d}{dx} \left(\gamma g \lambda \cdot \frac{y'}{\sqrt{1 + y'^2}} \right) = 0$$

$$y(x) = a \cosh \left(\frac{x - c_1}{a} \right) + c_2$$

Si se quiere minimizar la energia potencial gravitacional, los cables tienden a tomar la forma de una catenaria