1.

Una parabola esta dada por la ec.

$$r = \frac{q}{1+\cos\theta}$$

Para hallar los puntos donde se intersecta con la orbita de la tierra se iguala al radio de la orbita de la tierra

$$R_T = \frac{q}{1+\cos\varphi} \rightarrow \cos\varphi = \frac{q}{R_T} - 1$$

$$\varphi = \arccos\left(\frac{q}{R_T} - 1\right)$$

$$\theta_0 = 2\pi - \gamma \qquad y \qquad \theta_\varphi = \varphi$$

Por la segunda Ley de Kepler

$$dA = \frac{1}{2} r^2 d\theta \qquad y \qquad \frac{dA}{dt} = \frac{\ell}{2\mu} \rightarrow dA = \frac{\ell}{2\mu} dt$$

$$\frac{\ell}{2\mu} dt = \frac{1}{2} r^2 d\theta$$

$$dt = \frac{\mu}{\ell} r^2 d\theta = \frac{\mu}{\ell} \frac{q^2}{(1+\cos\theta)^2} d\theta = dt$$

$$t = \frac{\mu}{\ell}\int_{2\pi-\varphi}^{\varphi} \frac{q^2}{(1+\cos\theta)^2} d\theta$$

Con wolfram sale

$$\frac{\mu q^2}{\ell}\left(\frac{\tan^3\left(\frac{\varphi}{2}\right) + 3\tan\left(\frac{\varphi}{2}\right)}{6} - \left(\frac{\tan^3\left(\frac{2\pi-\varphi}{2}\right) + 3\tan\left(\frac{2\pi-\varphi}{2}\right)}{6}\right)\right)$$

4. a. Para la fuerza central usamo la E. D de una fuerza central

$$\frac{l^2}{\mu}\left[\frac{\partial^2 u}{\partial\theta^2}+u\right]=-\frac{\partial V}{\partial u} \quad \text{con} \quad u=\frac{1}{r}=\frac{1}{a(1+\cos\theta)}$$

$$\frac{\partial u}{\partial\theta}=\frac{1}{a}\frac{\sin\theta}{(1+\cos\theta)^2}=a\sin\theta\, u^2 \to \frac{\partial^2 u}{\partial\theta^2}=a\cos\theta\, u^2+a\sin\theta\, 2u\frac{du}{d\theta}$$

$$\frac{\partial^2 u}{\partial\theta^2}=a\cos\theta\, u^2+a^2\sin^2\theta\, 2u^3$$

$$\frac{l^2}{\mu}\left[a\cos\theta\, u^2+a^2\sin^2\theta\, 2u^3+u\right]=-\frac{\partial V}{\partial u}$$

$$\frac{l^2}{\mu}\left[a\cos\theta\frac{1}{r^2}+2a^2\sin\theta\frac{1}{r^3}+\frac{1}{r}\right]=-\frac{\partial V}{\partial r}=\text{Fuerza central}$$

b Con la segunda ley de Kepler

$$\frac{l}{2\mu}dt=\frac{1}{2}r^2 d\theta$$

$$dt=\frac{\mu}{l}r^2 d\theta=\frac{\mu}{l}a^2(1+\cos\theta)^2 d\theta$$

$$t=\int_0^{2\pi}\frac{\mu}{l}a^2(1+\cos\theta)^2$$

$$\frac{\mu}{l}a^2\int_0^{2\pi}\left(1+\cos^2\theta+2\cos\theta\right)d\theta$$

Con wolfram

$$\frac{\mu}{l}a^2\left(\frac{(\cos\theta+4)\sin\theta+3\theta}{2}\right)\Bigg|_0^{2\pi}=3\pi\frac{\mu a^2}{l}$$

Powered by CamScanner

3. $V(r) = -\frac{k}{r}e^{-r/a}$, donde $k > 0$ y $a > 0$

a. Suponiendo que esta en un plano

$$\mathcal{L} = \frac{1}{2}m\dot{\vec{r}}^2 - \frac{k}{r}e^{-r/a} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}e^{-r/a}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = m\dot{r} \quad ; \quad \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

$$E = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{k}{r}e^{-r/a}$$

Como θ es ciclica $\dot{\theta}$ es cte

$$p_\theta = mr^2\dot{\theta} = cte$$

La componente L_z de $\vec{L}$ en coordenadas cartesianas es la misma

$$L_z = m(x\dot{y} - y\dot{x}) = mr^2\dot{\theta} = p_\theta$$

$$\dot{\theta} = \frac{L}{mr^2}$$

Sustituyendo en la energía

$$E = \frac{m\dot{r}}{2} + \frac{L^2}{2mr^2} - \frac{k}{r}e^{-r/a}$$

obtenemos el $V_{ef} = \frac{L^2}{2mr^2} - \frac{k}{r}e^{-r/a}$

Usando la condición de estabilidad $\frac{3f(r_0)}{r_0} + f'(r_0) < 0$

$$-\frac{\partial V}{\partial r} = f(r) = -k\left(\frac{1}{ar} + \frac{1}{r^2}\right)e^{-r/a}$$

$$y \quad \frac{\partial f}{\partial r} = f'(r) = k\left(\frac{2}{r^3} + \frac{2}{ar^2} + \frac{1}{a^2 r}\right)e^{-r/a}$$

La condición de estabilidad se puede reescribir como:

$$3+\frac{r_0 f'(r)}{f(r)}>0$$

$$\frac{r_0 K\left(\frac{1}{a^2 r_0}+\frac{2}{a r_0^2}+\frac{2}{r_0^3}\right)}{-K\left(\frac{1}{a r_0}+\frac{1}{r_0^2}\right)}+3>0$$

$$=\frac{r_0\left(\frac{a r_0^2+2a^2 r_0}{a^3 r_0^3}+\frac{2}{r_0^3}\right)}{-\left(\frac{r_0^2+a r_0}{a r_0^3}\right)}+3>0$$

$$=-\left(\frac{a r_0^5+2a^2 r_0^4+2a^3 r_0^3}{a^2 r_0^3}\right)\frac{1}{r_0+a}+3>0$$

$$=-\frac{r_0^2+2a r_0+2a^3}{a(r_0+a)}+3>0$$

~~Usando wolfram~~

$$=\frac{a^2}{r_0^2}+\frac{a}{r_0}-1>0 \rightarrow \frac{a}{r_0}\gtrsim 0,618$$

b $T_r=2\pi/w_r$

$$w_r^2=\frac{1}{\mu}\left.\frac{\partial^2 V_{ef}}{\partial r^2}\right|_{r_0}=\frac{1}{m}\left[\frac{3l^2}{m r_0^4}-K\left(\frac{2}{r_0^3}+\frac{2}{a r_0^2}+\frac{1}{a^2 r_0}\right)\right]$$

$$T_r=2\pi\left[\frac{1}{m}\left\{\frac{3l^2}{m r_0^4}-K\left(\frac{2}{r_0^3}+\frac{2}{a r_0^2}+\frac{1}{a^2 r_0}\right)\right\}\right]^{-1/2}$$

5. $$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GM_S m}{r}$$

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - GM_S m \frac{1}{r^2}$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = m\ddot{r}$$

$$\ddot{r} = \left(r\dot{\theta}^2 - GM_S \frac{1}{r^2}\right)$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$$

$mr^2\dot{\theta} = \text{ctte} \rightarrow$ Momento asociado a $\theta \equiv l$

$$\dot{\theta} = \frac{l}{mr^2}$$

$$\ddot{r} = \frac{m\,l^2}{m^2 r^4} - \frac{G m_s}{r^2}$$

$$\ddot{r} = \frac{l^2}{m r^4} - \frac{G m_s}{r^2}$$

$$m(\ddot{r} - m r\left(\frac{l}{m r^2}\right)^2 = -\frac{G \cancel{m_1} m_2}{r_2}$$

$$m\ddot{r} = \frac{l^2}{m r^3} - \frac{G m\, m_2}{r^2}$$

$$u(\theta) = A\cos(\theta-\theta_0) + \underbrace{\frac{Gm^2M^2}{l^2}}_{\to U_c}$$

$$u(\theta) = U_c\left(\frac{A}{U_c}\cos(\theta-\theta_0) + 1\right)$$

$$r(\theta) = \frac{1}{U_c\left(\frac{A}{U_c}\cos(\theta-\theta_0) + 1\right)}$$

$$r_{min} = \frac{r_c}{1 + \frac{A}{m}}$$

$$r_{max} = \frac{r_c}{1 - \frac{A}{m}}$$

2. $V(r) = \frac{a}{r} + \frac{b}{r^2}$

a. $V(r) = V\left(\frac{1}{u}\right) = au + bu^2$

Usando la ec. del libro

$$\theta = \theta_0 - \int \frac{du}{\sqrt{\frac{2\mu E}{l^2} - \frac{2\mu}{l^2}(au + bu^2) - u^2}}$$

$$\theta = \theta_0 - \int \frac{du}{\sqrt{\frac{2\mu E}{l^2} - \frac{2\mu a u}{l^2} - u^2\left(\frac{2b\mu}{l^2} + 1\right)}}$$

lo cual tiene la forma $iu^2 + ju + k$ y por la siguiente ec.

$$\int \frac{du}{\sqrt{iu^2 + ju + k}} = \frac{1}{\sqrt{-i}} \cos^{-1}\left[\frac{-(j + 2iu)}{\sqrt{j^2 - 4ik}}\right]$$

$$i = -\frac{2b\mu}{l^2} - 1 \qquad j = -\frac{2\mu a}{l^2} \qquad k = \frac{2\mu E}{l^2}$$

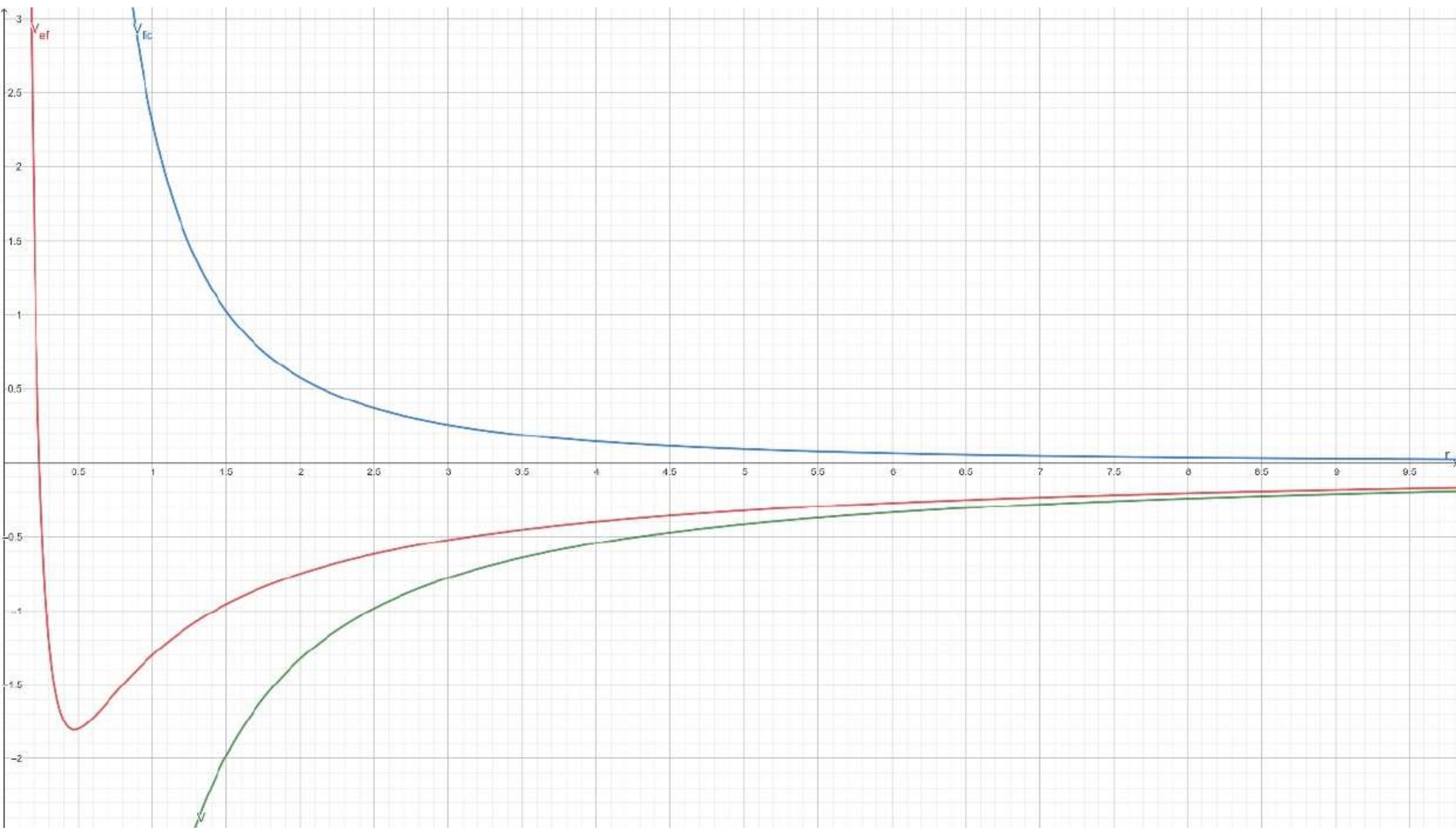
$$j + 2iu = -\frac{2\mu a}{l^2} + u\left(-\frac{4\mu}{l^2}b - 2\right)$$

$$j^2 - 4ik = \left(-\frac{2b\mu}{l^2} - 1\right)^2 + \frac{16\mu^2 bE}{l^4} + \frac{8\mu E}{l^2}$$

$$\theta = \theta_0 - \left(-\frac{2b\mu}{l^2} - 1\right)^{-1/2} \cos^{-1}\left(\frac{-\frac{2\mu a}{l^2} + u\left(-\frac{4\mu b}{l^2} - 2\right)}{\left(-\frac{2b\mu}{l^2} - 1\right) + \frac{16\mu^2 bE}{l^4} + \frac{8\mu E}{l^2}}\right)$$

$$\cos\left([\theta_0 - \theta]\left[-\frac{2b\mu}{l^2} - 1\right]\right) = \frac{-\frac{2\mu a}{l^2} + \frac{1}{r}\left(-\frac{4\mu b}{l^2} - 2\right)}{\left(-\frac{2b\mu}{l^2} - 1\right) + \frac{16\mu bE}{l^2} + \frac{8\mu E}{l^2}}$$

De aquí es posible obtener $r(\theta)$ facilmente haciendo un poco de algebra



V_ef
V_fic
V
r
3
2.5
2
1.5
1
0.5
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5

$a = 1.7$

-5 5

$b = 1.9$

-5 5

$$V(x) = -\frac{a}{x} - \frac{b}{x^2}$$

$$= -\frac{1.7}{x} - \frac{1.9}{x^2}$$

$c = 2.3$

-5 5

$$V_{fic}(x) = \frac{c}{x^2}$$

$$= \frac{2.3}{x^2}$$

$$V_{ef}(x) = V(x) + V_{fic}(x)$$

$$= -\frac{1.7}{x} - \frac{1.9}{x^2} + \frac{2.3}{x^2}$$

Entrada...