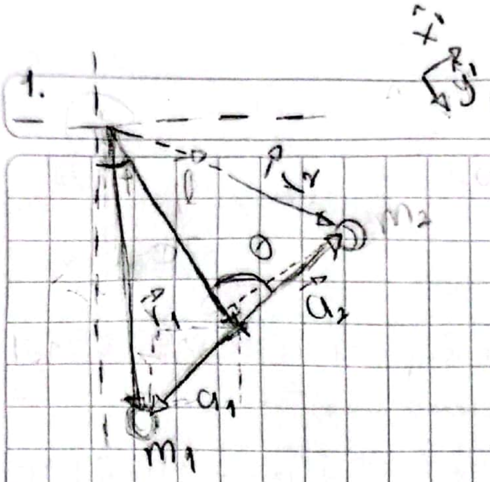
1.



$|\vec{l}| = cte$

$$\vec{r}_1 = \vec{l} + \vec{a}_1 = \vec{l} - \vec{a}_2$$

$$\vec{r}_2 = \vec{l} + \vec{a}_2$$

$$\hat{x}' = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y} \qquad \dot{\hat{x}}' = \dot{\varphi}(-\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}) = -\dot{\varphi}\,\hat{y}'$$

$$\hat{y}' = \sin\varphi\,\hat{x} - \cos\varphi\,\hat{y} \qquad \dot{\hat{y}}' = \dot{\varphi}(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}) = \dot{\varphi}\,\hat{x}'$$

$$\vec{l} = l\,\hat{y}'$$

$$\vec{a}_2 = -\vec{a}_1 \qquad \vec{a}_2 = \frac{a}{2}(\sin\theta\,\hat{x}' - \cos\theta\,\hat{y}')$$

$$\mathcal{L} = \frac{1}{2}m_1\dot{\vec{r}}_1^{\,2} + \frac{1}{2}m_2\dot{\vec{r}}_2^{\,2} - m_1\vec{g}\cdot\vec{r}_1 - m_2\vec{g}\cdot\vec{r}_2$$

$$\dot{\vec{r}}_1 = l\dot{\hat{y}}' - \dot{\vec{a}}_2 = l\dot{\varphi}\hat{x}' - \frac{a}{2}\left[\dot{\theta}(\cos\theta\,\hat{x}' + \sin\theta\,\hat{y}') - \dot{\varphi}(\sin\theta\,\hat{y}' + \cos\theta\,\hat{x}')\right]$$

$$\dot{\vec{r}}_1 = \left[\frac{a}{2}(\dot{\varphi}\cos\theta - \dot{\theta}\cos\theta) + l\dot{\varphi}\right]\hat{x}' + \frac{a}{2}\left[\dot{\varphi}\sin\theta - \dot{\theta}\sin\theta\right]\hat{y}'$$

$$\dot{\vec{r}}_2 = l\dot{\varphi}\hat{x}' + \dot{\vec{a}}_2 = \left[\frac{a}{2}(\dot{\theta}\cos\theta - \dot{\varphi}\cos\theta) + l\dot{\varphi}\right]\hat{x}' + \frac{a}{2}\left[\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta\right]\hat{y}'$$

$$\vec{g} = -g\hat{y} = -g(\sin\varphi\,\hat{x}' - \cos\varphi\,\hat{y}')$$

$$\mathcal{L} = \frac{1}{2}m_1\left[\left(\frac{a}{2}\left[\dot{\varphi}\cos\theta - \dot{\theta}\cos\theta\right] + l\dot{\varphi}\right)^2 + \left(\frac{a}{2}\left[\dot{\varphi}\sin\theta - \dot{\theta}\sin\theta\right]\right)^2\right]$$

$$+ \frac{1}{2}m_2\left[\left(\frac{a}{2}\left[\dot{\theta}\cos\theta - \dot{\varphi}\cos\theta\right] + l\dot{\varphi}\right)^2 + \left(\frac{a}{2}\left[\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta\right]\right)^2\right]$$

$$- m_1 g\left[\frac{a}{2}\sin\theta\sin\varphi + \left(l + \frac{a}{2}\cos\theta\right)\cos\varphi\right] + m_2 g\left[\frac{a}{2}\sin\theta\sin\varphi - \left(l - \frac{a}{2}\cos\theta\right)\cos\varphi\right]$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q}$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m_1\left[\left(\frac{a}{2}\left[\dot{\varphi}\cos\theta - \dot{\theta}\cos\theta\right] + l\dot{\varphi}\right)\left(\frac{a}{2}\cos\theta + l\right) + \left(\frac{a}{2}\left[\dot{\varphi}\sin\theta - \dot{\theta}\sin\theta\right]\right)\frac{a}{2}\sin\theta\right.$$

$$+ m_2\left[\left(\frac{a}{2}\left[\dot{\theta}\cos\theta - \dot{\varphi}\cos\theta\right] + l\dot{\varphi}\right)\left(-\frac{a}{2}\cos\theta + l\right) + \left(\frac{a}{2}\left[\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta\right]\right)\left(-\frac{a}{2}\sin\theta\right)\right]$$

(1) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m_1\left[\left(\frac{a}{2}\left[\ddot{\varphi}\cos\theta - \dot{\varphi}\dot{\theta}\sin\theta - \ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta\right] + l\ddot{\varphi}\right)\left(\frac{a}{2}\cos\theta + l\right)\right.$$

$$+ \left(\frac{a}{2}\left[\dot{\varphi}\cos\theta - \dot{\theta}\cos\theta\right] + l\dot{\varphi}\right)\left(-\frac{a}{2}\dot{\theta}\sin\theta\right) +$$

$$\left(\frac{a}{2}\left[\ddot{\varphi}\sin\theta + \dot{\varphi}\dot{\theta}\cos\theta - \ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta\right]\right)\frac{a}{2}\sin\theta + \left(\frac{a}{2}\left[\dot{\varphi}\sin\theta - \dot{\theta}\sin\theta\right]\right)\frac{a}{2}\dot{\theta}\cos\theta\Big]$$

$$+ m_2\left[\left(\frac{a}{2}\left[\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta - \ddot{\varphi}\cos\theta + \dot{\varphi}\sin\theta\right] + l\ddot{\varphi}\right)\left(-\frac{a}{2}\cos\theta + l\right) +\right.$$

$$\left(\frac{a}{2}\left[\dot{\theta}\cos\theta - \dot{\varphi}\cos\theta\right] + l\dot{\varphi}\right)\left(\frac{a}{2}\dot{\theta}\sin\theta\right) + \left(\frac{a}{2}\left[\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta - \ddot{\varphi}\sin\theta - \dot{\varphi}\dot{\theta}\cos\theta\right]\right)\left(-\frac{a}{2}\sin\theta\right)$$

$$+ \left(\frac{a}{2}\left[\dot{\theta}\sin\theta - \dot{\varphi}\sin\theta\right]\right)\left(-\frac{a}{2}\dot{\theta}\cos\theta\right)\Big]$$

(2) $$\frac{\partial L}{\partial \varphi} = -m_1 g\left[\frac{a}{2}\dot{\varphi}\sin\theta\cos\varphi - \left(l + \frac{a}{2}\cos\theta\right)\dot{\varphi}\sin\varphi\right] + m_2 g\left[\frac{a}{2}\dot{\varphi}\sin\theta\cos\varphi +\right.$$

$$\left.\left(l - \frac{a}{2}\cos\theta\right)\dot{\varphi}\sin\varphi\right]$$

usando la ecuacion (1) y (2) se puede ver $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = \frac{\partial L}{\partial \varphi}$

DD MM AA

$$\frac{\partial L}{\partial \dot\theta} = m_1\left[-\left(\frac{a}{2}\left[\dot\varphi\cos\theta - \dot\theta\cos\theta\right] + l\dot\varphi\right)\frac{a}{2}\cos\theta - \left(\frac{a}{2}\left[\dot\varphi\sin\theta - \dot\theta\sin\theta\right]\frac{a}{2}\sin\theta\right)\right]$$

$$+ m_2\left[\left(\frac{a}{2}\left[\dot\theta\cos\theta - \dot\varphi\cos\theta\right] + l\dot\varphi\right)\frac{a}{2}\cos\theta + \left(\frac{a}{2}\left[\dot\theta\sin\theta - \dot\varphi\sin\theta\right]\right)\frac{a}{2}\sin\theta\right]$$

$$)\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = -m_1\left[\left(\frac{a}{2}\left[\ddot\varphi\cos\theta - \dot\varphi\dot\theta\sin\theta - \ddot\theta\cos\theta + \dot\theta^2\sin\theta\right] + l\ddot\varphi\right)\frac{a}{2}\cos\theta - \right.$$

$$\left(\frac{a}{2}\left[\dot\varphi\cos\theta - \dot\theta\cos\theta\right] + l\dot\varphi\right)\frac{a}{2}\dot\theta\sin\theta + \left(\frac{a^2}{4}\left[\ddot\varphi\sin\theta + \dot\varphi\dot\theta\cos\theta - \ddot\theta\sin\theta - \dot\theta^2\cos\theta\right]\sin\theta\right)$$

$$\left. + \frac{a^2}{4}\left(\dot\varphi\sin\theta - \dot\theta\sin\theta\right)\dot\theta\cos\theta\right] + m_2\left[\left(\frac{a}{2}\left[\ddot\theta\cos\theta - \dot\theta^2\sin\theta - \ddot\varphi\cos\theta + \dot\varphi\dot\theta\sin\theta\right] + l\ddot\varphi\right)\frac{a}{2}\cos\theta\right.$$

$$- \left(\frac{a}{2}\left[\dot\theta\cos\theta - \dot\varphi\cos\theta\right] + l\dot\varphi\right)\frac{a}{2}\dot\theta\sin\theta + \frac{a^2}{4}\left(\ddot\theta\sin\theta + \dot\theta^2\cos\theta - \ddot\varphi\sin\theta - \dot\varphi\dot\theta\cos\theta\right)\sin\theta$$

$$\left. + \frac{a^2}{4}\left(\dot\varphi\sin\theta - \dot\theta\sin\theta\right)\dot\theta\cos\theta\right]$$

$$4)\ \frac{\partial L}{\partial \theta} = m_1\left[\left(\frac{a}{2}\left[\dot\varphi\cos\theta - \dot\theta\cos\theta\right] + l\dot\varphi\right)\left(-\dot\varphi\sin\theta + \dot\theta\sin\theta\right)\frac{a}{2}\right.$$

$$\left. + \left(\frac{a}{2}\left[\dot\varphi\sin\theta - \dot\varphi\sin\theta\right]\right)\left(\dot\varphi\cos\theta - \dot\varphi\cos\theta\right)\frac{a}{2}\right]$$

$$+ m_2\left[\left(\frac{a}{2}\left[\dot\theta\cos\theta - \dot\varphi\cos\theta\right] + l\dot\varphi\right)\frac{a}{2}\left(-\dot\theta\sin\theta + \dot\varphi\sin\theta\right)\right.$$

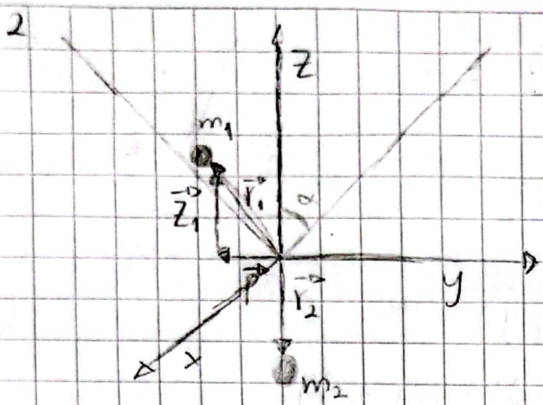
$$\left. + \left(\frac{a}{2}\left[\dot\theta\sin\theta - \dot\varphi\sin\theta\right]\right)\frac{a}{2}\left(\dot\theta\cos\theta - \dot\varphi\cos\theta\right)\right.$$

$$- m_1 g\left[\frac{a}{2}\cos\theta\sin\varphi - \frac{a}{2}\sin\theta\cos\varphi\right] + m_2 g\left[\frac{a}{2}\cos\theta\sin\varphi - \frac{a}{2}\sin\theta\cos\varphi\right]$$

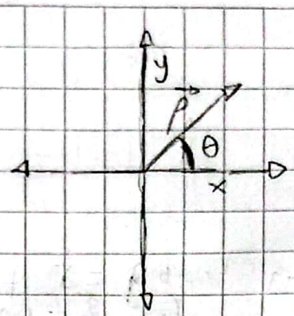
Ahora usando las ec. (3) y (4) se puede ver $\frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right)$

α en radianes cte

2



$\vec{g} = -g\hat{z}$



$\hat{\rho} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$

$\dot{\hat{\rho}} = \dot{\theta}(\sin\theta\,\hat{x} - \cos\theta\,\hat{y})$

$\dot{\hat{\rho}} = \dot{\theta}\hat{\theta}$

$\vec{r}_1 = \vec{\rho} + \vec{z}_1 \quad ; \quad \vec{r}_2 = -(\ell - |\vec{r}_1|)\hat{z}$

$\vec{z}_1 = z_1(\rho)\hat{z} = \rho\tan(\pi/2 - \alpha)\hat{z}$

$\vec{r}_1 = \rho\hat{\rho} + \rho\tan(\pi/2 - \alpha)\hat{z}$

$\vec{r}_2 = \left(\sqrt{\rho^2 + \rho^2\tan^2(\pi/2 - \alpha)} - \ell\right)\hat{z} = \left(\rho\sqrt{1 + \tan^2(\pi/2 - \alpha)} - \ell\right)\hat{z}$

$\vec{r}_2 = [\rho\sec(\pi/2 - \alpha) - \ell]\hat{z}$

$\dot{\vec{r}}_1 = \dot{\rho}\hat{\rho} + \rho\dot{\hat{\rho}} + \dot{\rho}\tan(\pi/2 - \alpha)\hat{z} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{\rho}\tan(\pi/2 - \alpha)\hat{z}$

$\dot{\vec{r}}_2 = \dot{\rho}\sec(\pi/2 - \alpha)\hat{z}$

$\mathcal{L} = \frac{1}{2}m_1[\dot{\rho}^2 + \rho^2 + \dot{\rho}^2\tan^2(\pi/2 - \alpha)] + \frac{1}{2}m_2\dot{\rho}^2\sec^2(\pi/2 - \alpha)$

$+ m_1 g\rho\tan(\pi/2 - \alpha) + m_2 g[\rho\sec(\pi/2 - \alpha) - \ell]$

$\frac{\partial L}{\partial \dot{\rho}} = m_1\dot{\rho}[1 + \tan^2(\pi/2 - \alpha)] + m_2\dot{\rho}\sec^2(\pi/2 - \alpha)$

$\frac{\partial L}{\partial \dot{\rho}} = \dot{\rho}[m_1(1 + \tan^2(\pi/2 - \alpha)) + m_2\sec^2(\pi/2 - \alpha)]$

(1) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\rho}}\right) = \ddot{\rho}[m_1(1 + \tan^2(\pi/2 - \alpha)) + m_2\sec^2(\pi/2 - \alpha)]$

(2) $\frac{\partial L}{\partial \rho} = m_1\rho + m_1 g\tan(\pi/2 - \alpha) + m_2 g\sec(\pi/2 - \alpha)$

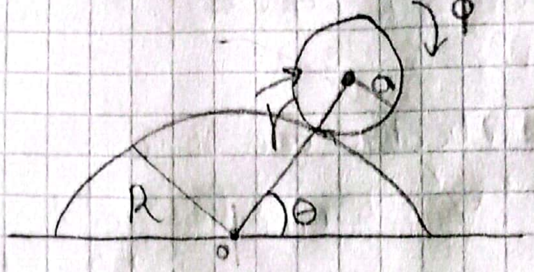
Ahora si usamos la ec. (1) y la ec (2) tendremos $\frac{\partial L}{\partial \rho} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\rho}}\right)$

Para que se encuentre en equilibrio $\ddot{\rho}$ debe ser cero Entonces tengo la ec. (1) igual a cero. Entonces tendremos

$m_1\rho + m_1 g\tan(\pi/2 - \alpha) + m_2 g\sec(\pi/2 - \alpha) = 0$

$\rho = \frac{-m_1 g\tan(\pi/2 - \alpha) - m_2 g\sec(\pi/2 - \alpha)}{m_1}$

3.



$$T = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \qquad U = mgr\,sen\theta$$

$$T = \frac{1}{2} m \left((\dot{r}^2 + r^2\dot{\theta}^2) + a^2\dot{\phi}^2 \right)$$

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + a^2\dot{\phi}^2 \right) - mgr\,sen\theta$$

Mientras el cilindro este rodando sobre el otro tenemos dos ligaduras

$$|\vec{r}| = R + a \qquad r - (R+a) = 0 \rightarrow \begin{array}{l} a_{1r} = 1 \\ a_{1\theta} = 0 \\ a_{1\phi} = 0 \end{array}$$

$$(R+a)\dot{\theta} = -a\dot{\phi} \qquad (R+a)\dot{\theta} + a\dot{\phi} = 0$$

$$\int (R+a)\frac{d\theta}{dt} = \int -a\frac{d\phi}{dt}$$

$$c + (R+a)\theta + a\phi = 0 \rightarrow \begin{array}{l} a_{2r} = 0 \\ a_{2\theta} = (r+a) \\ a_{2\phi} = a \end{array}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q} = \lambda_1 a_{1i} + \lambda_2 a_{2i}$$

Usando multiplicadores de lagrange

- Para r

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - mg\,sen\theta$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = m\ddot{r}$$

$$m\ddot{r} - mr\dot{\theta}^2 + mg\,sen\theta = \lambda_1 \cdot 1 + \lambda_2 \cdot 0$$

- Para θ

$$\frac{\partial L}{\partial \theta} = -mgr\cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$$

$$2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta} + mgr\cos\theta = \lambda_1 \cdot 0 + \lambda_2 \cdot (R+a)$$

- Para ϕ

$$\frac{\partial L}{\partial \phi} = 0$$

$$\frac{\partial L}{\partial \dot{\phi}} = ma^2\dot{\phi} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = ma^2\ddot{\phi}$$

$$ma^2\ddot{\phi} = \lambda_1 \cdot 0 + \lambda_2 a$$

Ec:

$$m\ddot{r} - mr\dot{\theta}^2 + mg\,sen\theta = \lambda_1 \quad (3)$$

$$2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta} + mgr\cos\theta = \lambda_2(R+a) \quad (4)$$

$$ma^2\ddot{\phi} = \lambda_2 a \quad (5)$$

$$r - (R+a) = 0 \quad (1) \qquad r, \theta, \phi, \lambda_1, \lambda_2$$

$$(R+a)\dot{\theta} + a\dot{\phi} = 0 \quad (2)$$

de (1) $r = (r+a)$

$$\dot{r} = 0 \quad (6)$$

$$\ddot{r} = 0 \quad (7)$$

(6) en (3)

$$-m(R+a)\dot{\theta}^2 + mg\,\mathrm{sen}\,\theta = \lambda_1 \quad (8)$$

(6) en (4)

$$m(R+a)^2\ddot{\theta} + mg(R+a)\cos\theta = \lambda_2(R+a) \quad (9)$$

Integrando 2

$$\dot{\phi} = -\frac{(R+a)}{a}\dot{\theta} \rightarrow \ddot{\phi} = -\frac{(R+a)}{a}\ddot{\theta} \quad (10)$$

$$\int \frac{d\phi}{\cancel{dt}} = -\int \frac{(R+a)}{a}\frac{d\theta}{\cancel{dt}}$$

$$\phi = -\frac{R+a}{a}\theta + \phi_0 \quad (11)$$

(10) en (5)

$$ma^{\cancel{2}}\left(-\frac{(R+a)}{\cancel{a}}\ddot{\theta}\right) = \lambda_2 \cancel{a}$$

$$\boxed{-m(R+a)\ddot{\theta} = \lambda_2} \quad (12)$$

Fuerza de rozamiento entre los cilindros

(12) en (9)

$$m(R+a)^2\ddot{\Theta} + mg(R+a)\cos\Theta + m(R+a)\ddot{\Theta}(R+a) = 0$$

$$\ddot{\Theta}\left(m(R+a)^2 + m(R+a)^2\right) = -mg(R+a)\cos\Theta$$

$$\ddot{\Theta} = -\frac{g}{(R+a)}\cos\Theta$$

$\downarrow$

$$\frac{1}{2}\dot{\Theta}^2 + \frac{g}{2(R+a)}\operatorname{sen}\Theta = C$$

Si empieza desde la cima

sabemos que $\dot{\Theta} = 0$ cuando $\Theta = \pi/2$

conociendo C Tenemos

$$\dot{\Theta}^2 = \frac{g}{R+a}(1-\operatorname{sen}\Theta) \quad (13)$$

(13) en (8)

$$-m(R+a)\left(\frac{g}{R+a}(1-\operatorname{sen}\Theta)\right) + mg\operatorname{sen}\Theta = \lambda_1$$

$$-mg(1-\operatorname{sen}\Theta) + mg\operatorname{sen} = \lambda_1$$

$$mg(2\operatorname{sen}\Theta - 1) = \lambda_1$$ $\hookrightarrow$ Fuerza normal entre los cilindros

Para que el cilindro no se caiga $\lambda_1 > 0$

$$mg(2\,\text{sen}\theta - 1) > 0$$

$$2\,\text{sen}\theta - 1 > 0$$

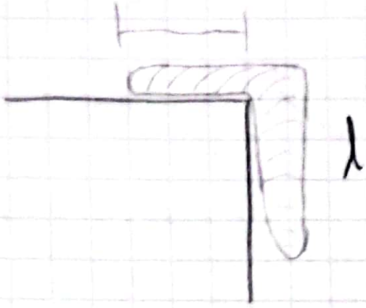
$$\text{sen}\theta > 1/2$$

$$\theta > \text{sen}^{-1}(1/2)$$

$$\theta > 30^\circ$$

Comenzando desde arriba del cilindro (90°) el cilindro pequeño rodara y dejara de estar en contacto con el grande despues de los 30°

4.



λ = densidad lineal de la cuerda $\rightarrow$ cte $= \frac{m}{L}$

m = masa total

$L \rightarrow$ Longitud total

$l(t) \rightarrow$ longitud que cuelga

$W = m_c g$

Masa que cuelga

$m_c = \lambda \cdot l(t)$

$W = \frac{l\, mg}{L}$

$m_c = \frac{ml}{L}$

$F = ma$

$K = 10$

$l_0 = 0{,}05$

$\frac{l\, mg}{L} = m\ddot{l}$

$C_1 = 0{,}025$

$g\, l = L\ddot{l}$

$t = 0$
$l = l_0$

K

$$l(t) = C_1 e^{\left(\sqrt{\frac{g}{L}}\, t\right)} + C_2 e^{\left(-\sqrt{\frac{g}{L}}\, t\right)}$$

$$l_0 = C_1 + C_2$$

$t = 0$
$V = 0$

$$\dot{l}(t) = C_1 K e^{Kt} - C_2 K e^{-Kt}$$

$$0 = C_1 K - C_2 K$$

$$C_1 = C_2 = \frac{l_0}{2}$$

$$C_1 = C_2$$

$$l(t) = \frac{l_0}{2}\left(e^{Kt} + e^{-Kt}\right)$$

$$\dot{l}(t) = \frac{l_0 K}{2}\left(e^{Kt} - e^{-Kt}\right)$$