$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$, $\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$, $\vec{g} = -g\hat{y}$

1. 

$$\vec{r} = r\hat{r} \to \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\dot{\vec{r}}^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$L = \frac{1}{2}m\dot{\vec{r}}^2 - m\vec{g}\cdot\vec{r}$$

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + mg\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = p_\theta = mr^2\dot{\theta} \qquad \frac{\partial L}{\partial \dot{r}} = p_r = m\dot{r}$$

$$E = m[r^2\dot{\theta}^2 + \dot{r}^2] - \frac{1}{2}m[\dot{r}^2 + r^2\dot{\theta}^2] - mg\sin\theta$$

$$= \frac{1}{2}m[r^2\dot{\theta}^2 + \dot{r}^2] - mg\sin\theta$$

2. $$L = \frac{1}{2}(m\dot{q}^2 - kq^2)e^{\frac{\alpha}{m}t} \qquad (1)$$

a. $$m\ddot{q} = -kq$$

Claramente describe un m.a.s. donde k podría ser la constante elastica de un resorte y m la masa.

b. $$Q = e^{\frac{\alpha}{2m}t}q \to \dot{Q} = e^{\frac{\alpha}{2m}t}\frac{\partial q}{\partial q}\dot{q} + q\frac{\partial e^{\frac{\alpha}{2m}t}}{\partial t}$$

$$\dot{Q} = e^{\frac{\alpha}{2m}t}\dot{q} + \frac{q\alpha}{2m}e^{\frac{\alpha}{2m}t} = e^{\frac{\alpha}{2m}t}\left(\dot{q} + \frac{q\alpha}{2m}\right)$$

$$\dot{Q}^2 = e^{\frac{\alpha}{m}t}\left(\dot{q}^2 + \frac{q^2\alpha^2}{4m^2} + \frac{\dot{q}q\alpha}{m}\right)$$

$$\dot{Q}Q = e^{\frac{\alpha}{m}t}\left(\dot{q}q + \frac{q^2\alpha}{2m}\right)$$

$$\dot{Q}^2 - \frac{\alpha\dot{Q}Q}{m} = e^{\frac{\alpha}{m}t}\left(\dot{q}^2 + \frac{q^2\alpha^2}{4m^2} + \frac{\dot{q}q\alpha}{m} - \frac{\alpha\dot{q}q}{m} - \frac{q^2\alpha^2}{2m^2}\right)$$

$$= e^{\frac{\alpha}{m}t}\left(\dot{q}^2 - \frac{q^2\alpha^2}{4m^2}\right) \to \frac{m\dot{Q}^2}{2} - \frac{\alpha\dot{Q}Q}{2} = \frac{1}{2}\left(m\dot{q}^2 - \frac{\alpha^2q^2}{4m}\right)e^{\frac{\alpha}{m}t}$$

$$L = \frac{m\dot{Q}^2}{2} - \frac{\alpha\dot{Q}Q}{2} + \frac{Q^2}{2}\left(\frac{\alpha^2}{4m} - k\right) \qquad (2)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{Q}}\right) = m\ddot{Q} - \frac{\alpha \dot{Q}}{2} \quad (3) \qquad \frac{\partial \mathcal{L}}{\partial Q} = \frac{-\alpha \dot{Q}}{2} + Q\left(\frac{\alpha^2}{4m} - k\right) \quad (4).$$

Ahora se iguala (3) con (4).

$$-\frac{\alpha \dot{Q}}{2} = \frac{-\alpha \dot{Q}}{2}$$

$$m\ddot{Q} = Q\left(\frac{\alpha^2}{4m} - k\right)$$

$$\ddot{Q} = \frac{Q}{2}\left(\frac{\alpha^2}{2m^2} - \frac{2k}{m}\right) \rightarrow Q = C_1 e^{zt} + C_2 e^{-zt}$$

$$z = \sqrt{\frac{\alpha^2}{4m^2} - \frac{2k}{m}}$$

Tonces $q = \left(C_1 e^{zt} + C_2 e^{-zt}\right) e^{-\frac{\alpha}{2m}t}$

Para el segundo Lagrangeano de la Ec. 2. claramente se conserva la energía, debido a que no depende de t explícitamente.

$$E = \frac{m}{2}\dot{Q}^2 - \frac{Q^2}{2}\left(\frac{\alpha^2}{4m} - k\right) = cte$$

3. $\mathcal{L} = T - V = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + F_x x + F_y y + F_z z$

Haciendo una transformación $t' = t + \delta t$, con δt siendo una función biyectiva creciente tenemos

$$\delta \mathcal{L} = \sum_i \frac{\partial \mathcal{L}}{\partial q_i} \delta q_i + \sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \delta \dot{q}_i + \frac{\partial \mathcal{L}}{\partial t} \delta t$$

Como $\mathcal{L}$ no depende explicitamente de t, entonces $\partial \mathcal{L}/\partial t = 0$

$$\delta \mathcal{L} = \sum_i \frac{\partial \mathcal{L}}{\partial q_i} \delta q_i + \sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \delta \dot{q}_i$$

Con $\frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right]$ y $\delta \dot{q}_i = \frac{d}{dt} \delta q_i$

$$\delta \mathcal{L} = \sum_i \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right] \delta q_i + \sum_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \frac{d}{dt}\left[\delta q_i\right]$$

$$\delta \mathcal{L} = \sum_i \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \delta q_i\right]$$

4

a) $V(\vec{r},\vec{v}) = U(r) + \vec{n}\cdot\vec{l}$

$\vec{F} = -\nabla V = -\nabla U(r) - m\nabla[n_x(y\dot{z} - z\dot{y}) + n_y(z\dot{x} - x\dot{z}) + n_z(x\dot{y} - y\dot{x})]$

$\vec{F} = -\nabla U(r) - m[(n_z\dot{y} - n_y\dot{z})\hat{i} + (n_x\dot{z} - n_z\dot{x})\hat{j} + (n_y\dot{x} - n_x\dot{y})\hat{k}]$

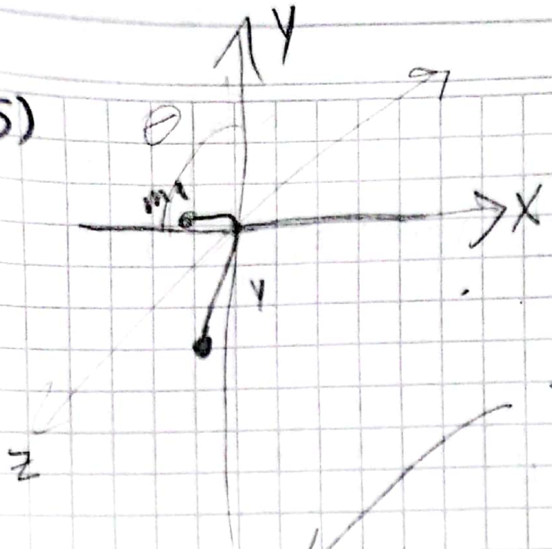
b. $\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U(r) - \vec{n}\cdot\vec{l}$

Con las ecuaciones de Euler-Lagrange, llegamos a

$m\ddot{x} = F_x$; $m\ddot{y} = F_y$; $m\ddot{z} = F_z$

c. La energía se conserva en este sistema

5)



$r_1 = L - r_2 \qquad \dot{r}_1 = -\dot{r}_2$

$\theta_1 = \pi/2 \Leftrightarrow \dot{\theta}_1 = 0$

$T_1 = \frac{m_1}{2}\left(\dot{r}_1^2 + r_1^2 \dot{\phi}_1^2\right)$

$T_2 = \frac{m_2}{2}\left(\dot{r}_2^2 + r_2^2 \dot{\theta}_2^2 + r_2^2 \operatorname{sen}^2\theta\, \dot{\phi}^2\right)$

$T_1 = \frac{m_1}{2}\left(\dot{r}_2^2 + (L - r_2)^2 \dot{\phi}_1^2\right)$

$U_1 = 0$

$U_2 = m_2 g \cos\theta_2$

$$L = \frac{m_1}{2}\left(\dot{r}_2^2 + (L-r_2)^2\dot{\phi}_1^2\right) + \frac{m_2}{2}\left(\dot{r}_2^2 + r_2^2\dot{\theta}_2^2 + r_2^2 \operatorname{sen}^2\theta_2\, \dot{\phi}_2^2\right)$$
$$- m_2 g \cos\theta_2$$

4 coord generalizadas $\phi_1, r_2, \theta_2, \phi_2$

Para ϕ_1

$\frac{\partial L}{\partial \phi_1} = 0 \qquad \frac{\partial L}{\partial \dot{\phi}_1} = m_1 (L - r_2)^2 \dot{\phi}_1$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}_1}\right) = -2m_1(L - r_2)\dot{r}_2\dot{\phi}_1 + m_1(L-r_2)^2\ddot{\phi}_1$$

$$-m_1(L - r_2)\left(2\dot{r}_2\dot{\phi}_1 - (L-r_2)\ddot{\phi}_1\right) = 0$$

Para r_2

$$\frac{\partial L}{\partial r_2} = -m_1(l-r_2)\dot{\phi}_1^2 + m_2 r_2 \dot{\theta}_2^2 + m_2 r_2 \operatorname{Sen}^2\theta_2 \dot{\phi}_2^2$$

$$\frac{\partial L}{\partial \dot{r}_2} = m_1 \dot{r}_2 + m_2 \dot{r}_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_2}\right) = \ddot{r}_2 (m_1 + m_2)$$

$$-m_1(l-r_2)\dot{\phi}_1^2 + m_2 r_2 (\dot{\theta}_2^2 + \operatorname{Sen}^2\theta_2 \dot{\phi}_2^2) = \ddot{r}_2(m_1+m_2)$$

Para θ_2

$$\frac{\partial L}{\partial \theta_2} = m_2 r_2^2 \operatorname{Sen}\theta_2 \operatorname{Cos}\theta_2 + m_2 g \operatorname{Sen}\theta_2$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 r_2^2 \dot{\theta}_2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = 2m_2 r_2 \dot{r}_2 \dot{\theta}_2 + m_2 r_2^2 \ddot{\theta}_2$$

$$m_2 \operatorname{Sen}\theta_2 (r_2^2 \operatorname{Cos}\theta_2 + g) = m_2 r_2 (2\dot{r}_2\dot{\theta}_2 + r_2\ddot{\theta}_2)$$

Para ϕ_2

$$\frac{\partial L}{\partial \phi_2} = 0$$

$$\frac{\partial L}{\partial \dot{\phi}_2} = m_2 r_2^2 \operatorname{Sen}^2\theta_2 \dot{\phi}_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}_2}\right) = m_2 \dot{\phi}_2 (2r_2\dot{r}_2 \operatorname{Sen}^2\theta_2 + 2r_2^2 \operatorname{Sen}\theta_2 \operatorname{Cos}\theta_2 \dot{\theta}_2)$$
$$+ m_2 r_2^2 \operatorname{Sen}^2\theta_2 \ddot{\phi}_2$$

$$m_2 r_2 \operatorname{Sen}\theta_2 \left(2 \operatorname{Sen}\theta_2 \dot{r}_2 \dot{\phi}_2 + 2 r_2 \cos\theta \dot{\theta}_2 \dot{\phi}_2 + r_2 \operatorname{Sen}\theta_2 \ddot{\phi}_2 \right) = 0$$

b) Se conserva

1) el momento asociado a $\phi_1 \rightarrow \frac{\partial L}{\partial \dot{\phi}_1}$

2) el momento asociado a $\phi_2 \rightarrow \frac{\partial L}{\partial \dot{\phi}_2}$

3) La energía $\rightarrow \frac{\partial L}{\partial \dot{q}_i} - L$

3 constantes 4 grados de libertad, no es integrable.

d) $v_f^2 = v_0^2 + 2ay \rightarrow a$

$v_f = \sqrt{19{,}6\, a}$

6. En "x" En y

$F_x = -K_1 x$ $F_y = -K_2 y$

$-K_1 x = m\ddot{x}$ $-K_2 y = m\ddot{y}$

$\ddot{x} + \omega_1^2 x = 0$ $\ddot{y} + \omega_2^2 y = 0$

Ecuaciones de movimiento

$x(t) = A_x \cos(\omega_1 t + \phi_x)$

$y(t) = A_y \cos(\omega_2 t + \phi_y)$

Para pequeñas oscilaciones consideremos

$x' = x$ y $y' = y$

Donde x' y y' son las pequeñas desviaciones de la masa con respecto al punto de equilibrio

$\ddot{x}' + \omega_1^2 x' = 0$ $\ddot{y}' + \omega_2^2 y' = 0$

analisis para

$K_1 = K_2 \longrightarrow \omega_1^2 = \omega_2^2$

Esto muestra una relación entre las oscilaciones mostrando un movimiento sincronizado

Para

$K_1 \neq K_2 \longrightarrow \omega_1^2 \neq \omega_2^2$

Genera trayectorias mas desordenados