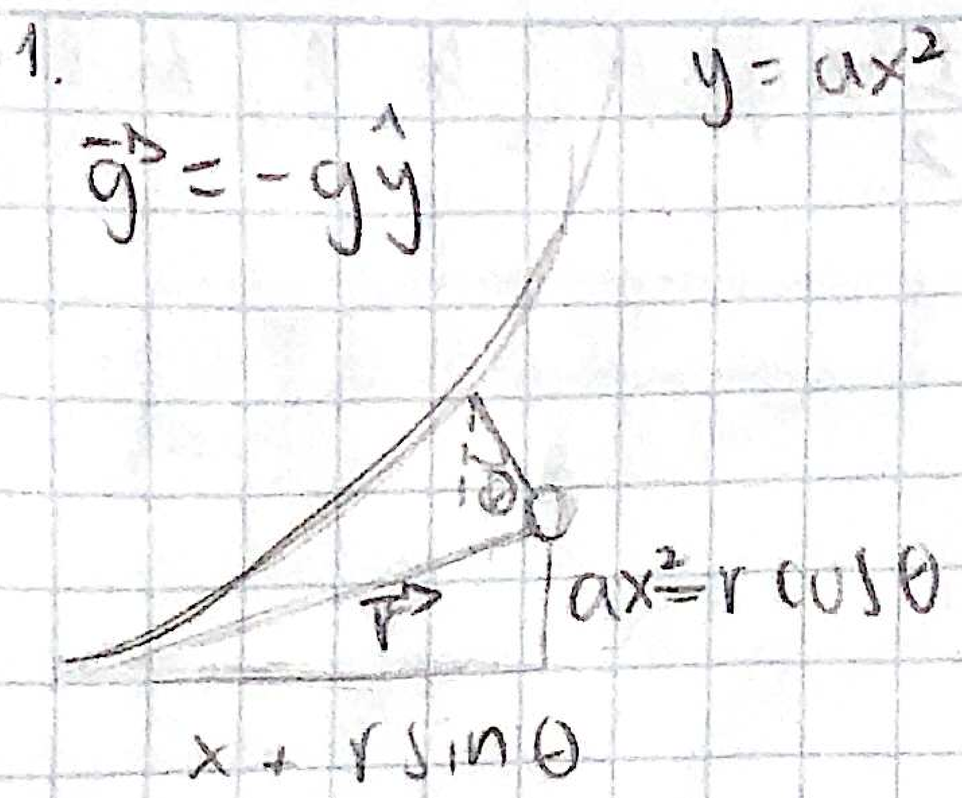
1.

$\vec{g} = -g\hat{y}$ $\qquad y = ax^2$ $\qquad \mathcal{L} = \frac{1}{2} m \dot{\vec{r}}^2 - m\vec{g}\cdot\vec{r}$



$$\dot{\vec{r}} = (\dot{x} + r\dot{\theta}\cos\theta)\hat{x} + (2ax\dot{x} + r\dot{\theta}\sin\theta)\hat{y}$$

$$m\vec{g}\cdot\vec{r} = -mg(ax^2 - r\cos\theta)$$

$$\mathcal{L} = \frac{1}{2} m\left[(\dot{x} + r\dot{\theta}\cos\theta)^2 + (2ax\dot{x} + r\dot{\theta}\sin\theta)^2\right] + mg(ax^2 - r\cos\theta)$$

$$\mathcal{L} = \frac{1}{2} m\left[\dot{x}^2 + r^2\dot{\theta}^2\cos^2\theta + 2\dot{x}r\dot{\theta}\cos\theta + 4a^2x^2\dot{x}^2 + r^2\dot{\theta}^2\sin^2\theta\right.$$
$$\left. + 4ax\dot{x}r\dot{\theta}\sin\theta\right] + mg(ax^2 - r\cos\theta)$$

$$\mathcal{L} = \frac{1}{2} m\left[\dot{x}^2 + r^2\dot{\theta}^2 + 2r\dot{x}\dot{\theta}(\cos\theta + 2ax\sin\theta) + 4a^2\dot{x}^2x^2\right] + mg(ax^2 - r\cos\theta)$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} + mr\dot{\theta}(\cos\theta + 2ax\sin\theta) + 4ma^2\dot{x}x^2)$$

$$p_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mr^2\dot{\theta} + mr\dot{x}(\cos\theta + 2ax\sin\theta)$$

$$\frac{p_x}{m} = \dot{x}(1 + 4a^2x^2) + r\dot{\theta}(\cos\theta + 2ax\sin\theta)$$

$$\left[\frac{p_x}{m} - r\dot{\theta}(\cos\theta + 2ax\sin\theta)\right]\frac{1}{1 + 4a^2x^2} = \dot{x}$$

$$p_\theta = mr^2\dot{\theta} + mr\left[\frac{p_x}{m} - r\dot{\theta}(\cos\theta + 2ax\sin\theta)\right]\frac{\cos\theta + 2ax\sin\theta}{1 + 4a^2x^2}$$

$$p_\theta - r p_x\left[\frac{\cos\theta + 2ax\sin\theta}{1 + 4a^2x^2}\right] = m\dot{\theta}r^2\left[1 - \frac{(\cos\theta + 2ax\sin\theta)^2}{1 + 4a^2x^2}\right]$$

$$\left(p_\theta - r p_x\left[\frac{\cos\theta + 2ax\sin\theta}{1 + 4a^2x^2}\right]\right)\frac{1}{mr^2}\left[1 - \frac{(\cos\theta + 2ax\sin\theta)^2}{1 + 4a^2x^2}\right]^{-1} = \dot{\theta}$$

$$H(x, \theta, p_x, p_\theta) = \dot{\theta}(x, \theta, p_x, p_\theta)p_\theta + \dot{x}(x, \theta, p_x, p_\theta)p_x - \dots$$
$$\dots - \mathcal{L}(x, \theta, \dot{x}(x, \theta, p_x, p_\theta), \dot{\theta}(x, \theta, p_x, p_\theta))$$

2. $H = \frac{p^2}{2m} - A\left(\frac{p}{m}\cos\gamma t + \gamma q \sin\gamma t\right) + \frac{1}{2}Kq^2$ $\quad A, \gamma, K$ cte

$L = p\dot{q} - H(q, \dot{q}, t)$

$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} - \frac{A\cos\gamma t}{m}$

$m\dot{q} + A\cos\gamma t = p$

$L = m\dot{q}^2 + A\dot{q}\cos\gamma t - \left(\frac{m\dot{q}^2 + 2m\dot{q}A\cos\gamma t + A^2\cos^2\gamma t}{2m}\right)$

$+ \frac{A}{m}\left(m\dot{q}\cos\gamma t + A\cos^2\gamma t\right) + A\gamma q\sin\gamma t - \frac{1}{2}Kq^2$

(1) $L = \frac{m\dot{q}^2}{2} + \frac{A^2\cos^2\gamma t}{2m} + A\dot{q}\cos\gamma t + A\gamma q\sin\gamma t - \frac{1}{2}Kq^2$

$L = \frac{m}{2}\dot{q}^2 - \frac{1}{2}Kq^2 + \frac{A^2\cos^2\gamma t}{2m} + A\dot{q}\cos\gamma t + A\gamma q\sin\gamma t$

$L' = \frac{m}{2}\dot{q}^2 - \frac{1}{2}Kq^2 + A\dot{q}\cos\gamma t + A\gamma q\sin(\gamma t)$

$f(q,\dot{q},t) = A\dot{q}\cos\gamma t + A\gamma q\sin\gamma t = A\left(\frac{dq}{dt}\cos\gamma t - q\frac{d\cos\gamma t}{dt}\right)$

Si $\int f(q,\dot{q},t)\,dt = F(q,t) \rightarrow L - L' = f(q,\dot{q},t)$

Pero $f(q,\dot{q},t)$ no es facilmente integrable, entonces no podemos decir que existe un L'' de la siguiente manera, que sea equivalente sin dependencia temporal.

$L'' = \frac{m\dot{q}^2}{2} - \frac{1}{2}Kq^2$

3. $\mathcal{H} = q + te^{p}$

$Q = q + e^{p}, \quad P = p$

$\{Q, P\} = \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} = 1 \cdot 1 - 0 = 1$

$\{Q, Q\} = 0 \qquad \{P, P\} = 0$

Por lo que es transformación canónica

Supongamos que la función generatriz $F = F_2(q, P)$

$p = \frac{\partial F_2}{\partial q} \qquad Q = \frac{\partial F_2}{\partial P} \rightarrow q + e^{P} = \frac{\partial F_2}{\partial P} \rightarrow F_2 = qP + \underbrace{e^{P}}_{F}$

$P = \frac{\partial F_2}{\partial q} \rightarrow F_2 = Pq + f$

$F_2 = qP + e^{P}$

$\mathcal{H} = Q - e^{P} + te^{P} = Q + (t-1)e^{P}$

$-\frac{\partial \mathcal{H}}{\partial Q} = \dot{P} = -1 \qquad \frac{\partial \mathcal{H}}{\partial P} = \dot{Q} = (t-1)e^{P}$

$P = -t + c_1 \qquad \dot{Q} = c_1(t-1)e^{-t}$

$\int \frac{dQ}{dt}dt = \int c_1(t-1)e^{-t}\,dt$

$Q = c_1 \int (te^{-t} - e^{-t})\,dt =$

$Q = c_1\left(-te^{-t} - \cancel{e^{-t}} + \cancel{e^{-t}}\right) + c_2$

$Q = c_1 te^{-t} + c_2$

4. $H = \frac{1}{2}\left(qp^3 + \frac{q}{p}\right) \rightarrow H = \frac{P^2}{2m} + \frac{1}{2}kq^2$

$kQ^2 = \frac{q}{p}$ $\quad \frac{P^2}{m} = qp^3$

$Q = \frac{q}{\sqrt{pk}}$ $\quad P = \sqrt{mqp^3}$

$\{Q, P\} = \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\frac{\partial P}{\partial q}$

$\frac{\partial Q}{\partial q} = \frac{1}{2\sqrt{pkq}}$ $\quad \frac{\partial Q}{\partial p} = -\frac{1}{2}\sqrt{\frac{q}{p^3 k}}$

$\frac{\partial P}{\partial p} = \frac{3}{2}\sqrt{mqp}$ $\quad \frac{\partial P}{\partial q} = \frac{1}{2}\sqrt{\frac{mp^3}{q}}$

$\frac{3}{4}\sqrt{\frac{m}{k}} + \frac{1}{4}\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{k}}$ si $m = k \rightarrow \sqrt{\frac{m}{k}} = 1$

$\{Q, Q\} = 0 \quad \{P, P\} = 0$

$H = \frac{P^2}{2m} + \frac{1}{2}mQ^2$

E.M

$\frac{\partial H}{\partial Q} = mQ = -\dot{P}$ $\quad \frac{\partial H}{\partial P} = \frac{P}{m} = \dot{Q}$

$\cancel{m}Q = -\cancel{m}\ddot{Q}$ $\quad P = m\dot{Q} \rightarrow \dot{P} = m\ddot{Q}$

$Q = -\ddot{Q}$

$Q = c_1 \sin(t) + c_2 \cos(t)$