Midterm Practice Problems

1) Interpolating between data points in a common problem in data science. In the attached csv file there are 21 (x, y) pairs. Your job is to find a 20-degree polynomial that goes through all 21 points. The format of the polynomial is $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{20}x^{20}$. This means you need to find $a_0, \ldots, a_{20}$ such that $a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 + \cdots + a_{20}x_1^{20} = y_1$ and so on. Set this up as a linear system of equations. You have 21 unknowns: $a_0, \ldots, a_{20}$. You also have 21 equations that set the polynomial evaluated at $x_i$ equal to the corresponding value of $y_i$. Set this up as a matrix vector problem and solve for $a_0, \ldots, a_{21}$. Evaluate this polynomial at x = -1.9 (-1.9 is not a grid point, but you know all the a's so you can plug -1.9 into the polynomial). Is this close to the known y-values at x=-2 or x=-1.8? Do you think this is a good method for interpolation?

2) There are 5 tasks you have been assigned to work on each week at your job. You work 40 hours per week. Each task must receive at least 4 hours of your time. No one task can occupy 2 times the amount of time of any other task. The total combined time spent on tasks 1 and 3 must be at least 15 hours. The total combined time spent on task 4 and 5 cannot exceed 15 hours. The revenue to your company for each hour spent on each of the 5 tasks are (100, 200, 150, 250, 225) dollars per hour. You need not work on tasks an integer number of hours. How should you spend your time? You get paid $150 per hour, are you profitable to the company? If you work more than 40 hours per week you get paid time and a half. Should the company ask you to work more hours?