Stat 958:587 HW No. 1

Due 9/30/2021

Note: You are only allowed to use random number generator for a uniform distribution to complete this assignment

Programming: R, C++, Python or JAVA

In class discussion schedule –

- Problems 1 and 2: 9/9
- Problems 3 and 4: 9/16
- Problem 5 and amendments: 9/23

Problem 1: (a) Use Monte-Carlo method to directly approximate π ("random hit" method). To get a single estimation, use N=10000 samples. Generate 1000 estimated $\hat{\pi}$.
(b) Use Monte-Carlo method to calculate an integral which leads to an approximation of π (Hint: $\pi = 4 \int_0^1 \sqrt{1-x^2}\, dx$). To do the integration, use N=10000 samples. Generate 1000 estimated $\hat{\pi}$.
(c) Compare the two methods in (a) and (b) in terms of variance and running time. Which one do you prefer?

Problem 2: Write your own code to simulated 100 samples from

(a) Exponential distribution: Exp(λ) with λ = 2.8.
(b) Normal distribution using BOTH Box-Muller transformation AND central limit theorem: N(μ, σ^2) with (a) (μ, σ^2) = (0, 1) and (b) (μ, σ^2) = (3.5, 2).
(c) Log-normal distribution LnN(μ, σ^2) with (a) (μ, σ^2) = (0, 1) and (b) (μ, σ^2) = (-4, 2).
(d) Binomial Distribution: Binomial(n, p) with n =10, p = 0.24.

In each case, plot the density of sample sets to illustrate (validate) your simulated samples.

Problem 3: Write a program to calculate the integration of $\int_{-5}^{5}(x^3 - x^2)e^{-x^2/2}dx$ using following methods:

(a) Monte Carlo simulation with N samples from a uniform distribution, for $N = 10,\ 100,\ 1000$. For each choice of N, repeat the experiment for 500 times, compute the average and variance; visualize the relationship between the variance and N.
(b) Numerical integration with N partitions, for $N = 10,\ 100,\ 1000$.
(c) Comment on the two methods

Problem 4: Re-write the integral:

$$I = \int_{1}^{+\infty} \int_{-\infty}^{+\infty} (1 + x^2 + \sin(x))^{-|y|^3 - 2} dy dx$$

as some expected value, and estimate the integral using Monte-Carlo simulation.

Problem 5. Write a program to simulate a set of 500 samples from the following distribution with density

$$f(x) = \begin{cases} \frac{1}{c} f_{t_5}(x)\{1 - \sin(20x)/4\} & \text{if} \quad |x| < 3 \\ 0 \qquad\qquad \text{if} \quad |x| > 3. \end{cases}$$

where $f_{t_5}(x)$ is the density function of the t-distribution with 5 degrees of freedom and

$$c = \int_{-3}^{+3} f_{t_5}(x)\{1 - \sin(20x)/4\} dx.$$

Plot the density function of your simulated samples