STAT 958:587 Homework 3
Due: 11/18/2021

In class discussion schedule
Problem 1&2 11/04
Problem 3&4 11/11

Problem 1.
Assume that a stock price, say s_t, at time t is a smooth function of time. Denote this smooth function as $g(t)$. Suppose that $g(t)$ satisfies the following stochastic differential equation:

$$\frac{d}{dt}g(t) = \mu g(t) + \sigma g(t)\frac{d}{dt}W(t),$$

where both the drift μ and the volatility σ are constants and the noise $W(t)$ is the standard Brownian motion. Generate and plot a path of s_t on $t \in [0,1]$ using the following inputs

(i) $s_0 = 0.35, \mu = 3, \sigma = 0.1 \text{ and } \Delta t = 0.05,$

(ii) $s_0 = 0.5, \mu = -2.5, \sigma = 0.1 \text{ and } \Delta t = 0.1.$

For each question, use 1) random walk and 2) Brownian bridge to simulate the Brownian motions. (4 plots are required.)

Problem 2.
Suppose that S_1 and S_2 are prices of two stocks and follow a correlated GBM(μ, ∑) process, where $\mu = \binom{r-\sigma^2/2}{r-\sigma^2/2}, \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$ under risk neutral measure. A *spread option* has payoff $(|S_1(T) - S_2(T)| - K)_+$ at time T. Use a Monte-Carlo method to simulate the option price $P = e^{-rT}E(|S_1(T) - S_2(T)| - K)_+$. Use N=5000. Take $\sigma_1 = 0.04, \sigma_2 = 0.06, \rho = 0.6, S_1(0) = 35, S_2(0) = 33$, risk free rate $r = 0.05, K = 4, T = 4, dt = 1$.

Remark: For problem 2, you may resort to page 104 of the textbook "Monte Carlo Methods in Financial Engineering" by Glasserman P. for the construction of multi-dimensional geometric Brownian motions.

Problem 3.
Write a program to implement *Merton's jump-diffusion process*, using two methods introduced in class: 1) fixed jumping dates and 2) jumping times. Suggested parameter values: $T = 300, dt = 1, \lambda = 0.01, S(0) = 5, \mu = 0.01, \sigma = 0.02$. You can try your own parameter values if you find the plot does not look good. Report the plots of generated paths and the time when jumps occur.

Problem 4.
For this problem we assume $T = 4, dt = 1, S(0) = 35, r = 0.05, \sigma = 0.04, K = 40$ under risk neutral measure. Assume that the stock price $S(t)$ follows a GBM(μ, σ^2), where $\mu = r - \sigma^2/2$. The payoff of an *Asian call option* at time *T* is given by $Y = (\bar{S} - K)_+$, where $\bar{S} = \frac{1}{4}\sum_{i=1}^{4} S(t_i)$, so the option price is defined as $P = e^{-rT}E((\bar{S} - K)_+)$. A standard Monte-Carlo approach to estimate *P* is $\hat{P} = e^{-rT}\frac{1}{N}\sum_{j=1}^{N} Y_j$, where each Y_j is a sample from Y. It is known that $\hat{\Delta} = \frac{\partial \hat{P}}{\partial S(0)}$ is an unbiased estimator of *Delta*: $\Delta = \frac{\partial P}{\partial S(0)}$. Try to design a Monte-Carlo method from above to estimate Delta at $S(0) = 35$. Use *N=5000*. (Hint: recall the definition of partial derivatives.)