# Math Review – Exam for Multivariable Calculus

## Fall 2020

1. The exam lasts **3.5 hours: 8:30am-12pm 11/14/2020 EST**.
2. The exam is closed-book and closed-notes.
3. Please place your camera facing toward you and your desk and unmute your mic.
4. Write your answers on white blank papers (no grid or lines).
5. Scan your answers into **ONE pdf file** and submit it by **12:20pm**.
6. Show all your work; answers without procedure will NOT be credited.
7. Write CLEARLY!!! Leave some blank spaces when necessary.
8. 55 points in total.

1. (10 points) If the limit exists, find the limit. If not, explain why.

   (a) (5 points) $\lim_{x\to 0} \dfrac{\cos(x) - e^{-x^2/2}}{x^4}$

   (b) (5 points) $\lim_{x\to 0} \dfrac{2^{1/x}+1}{2^{1/x}-1}$

2. (10 points) Gamma function is defined by the integral

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\mathrm{d}x, \text{ where } \alpha > 0$$

   and Beta function by

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\mathrm{d}t, \text{ where } \alpha,\beta > 0.$$

   (a) (4 points) Show $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

   (b) (6 points) Show $B(\alpha,\beta)\Gamma(\alpha+\beta) = \Gamma(\alpha)\Gamma(\beta)$. (Hint: use double integral with variable transformation)

3. (8 points) Calculate the integral

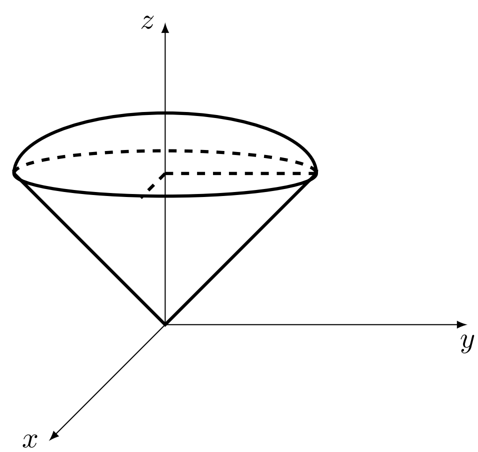
$$\int\cdots\int_A (x_1+x_2)\,\mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_n$$

   where $A = \left\{(x_1,\ldots,x_n)\middle|\boldsymbol{-1 \le x_1 \le x_2 \le 0},\ 0\le x_3\le 1,\ 0\le x_4\le 1,\ldots,\ 0\le x_n\le 1\right\}$

4. (9 points) Find critical point(s) and use second derivative test to determine if the corresponding function value is minimum or maximum for

$$f(x,y) = xy + \frac{a^3}{x} + \frac{b^3}{y} \qquad \text{where } a, b > 0$$

5. (9 points) Calculate $\iiint_\Omega z e^{-(x^2+y^2+z^2)}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where $\Omega$ is the region bounded by cone surface $z = \sqrt{x^2+y^2}$ and sphere $x^2+y^2+z^2 = 1$, that is, $\Omega = \big\{(x,y,z)\big| z \geq \sqrt{x^2+y^2} \text{ and } x^2+y^2+z^2 \leq 1\big\}$.




6. (9 points) Find the Taylor series expansion for $f(x) = \dfrac{1}{(1-x)^2}$ around $x = 0$ up to the third order. Specify the remainder term.