FSRM/MSDS 581 Homework 1
Due on Wednesday 6:30pm EST, 9/16

1. *Some (tricky) probability problems*

   Show the details of derivations and calculations for the following problems.

   (a) A hat contains 3 index cards: one with both sides blue, one with both sides red, and one with one side blue and the other side red. Assume that the cards are well shuffled. Picking one at random, you see that the front side is blue. What is the chance that the back side of the card is red?

   (b) There is a circle with radius R. Randomly pick three points on the circle (each point on the circle has an equal chance to be picked) and connect them by lines to form a triangle. What is the probability that such a triangle is an acute triangle?

   (c) There is a rope. Randomly pick two different points on the rope (each point on the rope has an equal chance to be picked). Cut the rope into three segments according to these two points. What is the probability that these three segments can form a triangle?

   (d) Toss a fair coin for infinity times. All tosses are independent. For each toss, if it is head, you will get 1 point. If it is tail, you will get 2 points. In the entire process, what is the probability of ever having n points?

   (e) Toss a fair coin 1000 times. All tosses are independent. Using computer, estimate the probability that there is at least one segments of ten consecutive Heads in the sequence.

2. *Monty Hall Problem*

   The Monty Hall problem is a brain teaser, in the form of a probability puzzle, loosely based on the American television game show Let's Make a Deal and named after

its original host, Monty Hall. There are many versions of solutions to this problem, but many of them are not complete. The answer actually depends on some crucial assumptions. In this problem, we will go over details of this famous problem.

The question: Suppose you are on a game show, and you are given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Let "You" be the number of door you chooses, say You= 1 means that you chooses door No. 1. Let "Car" be the number of door which the car is behind, say Car= 2 means the car is behind door No. 2. Let "Host" be the number of door which the host opens, say Host= 3 means the host opens door No. 3.

Show detailed derivations and calculations for the following questions.

(a) Show that

$$P(Car = 2|Host = 3, You = 1) = \frac{P(Host = 3|Car = 2, You = 1)P(Car = 2|You = 1)}{\sum_{i=1}^{3} P(Host = 3|Car = i, You = 1)P(Car = i|You = 1)}$$

(b) What are $P(Car = i|You = 1), i = 1, 2, 3$?

(c) If the host does not know the location of the car and opens the door randomly, what are $P(Host = 3|Car = i, You = 1),\ i = 1, 2, 3$? What is $P(Car = 2|Host = 3, You = 1)$?

(d) If the host knows the location of the car and always opens door No. 3 when the car is not behind door No. 3, what are $P(Host = 3|Car = i, You = 1),\ i = 1, 2, 3$? What is $P(Car = 2|Host = 3, You = 1)$?

(e) If the host knows the location of the car and opens door No. 2 and No. 3 with equal chance if the car is behind door No. 1, , what are $P(Host = 3|Car =$

$i, You = 1),\ i = 1, 2, 3?$ What is $P(Car = 2|Host = 3, You = 1)?$