

FSRM/MSDS 581 Homework 2

Due on Wednesday 6:30pm EST, 9/23

1. Toss a fair coin independently until either two Heads are obtained or the maximum number of 6 tosses is reached.

- (a) List all element in the sample space
- (b) Let a random variable X be the number of Tails observed when the experiment ends. Write down the detail mapping of X , from each element of the sample space to \mathbb{R}
- (c) Find the expected value of X . Find the p.m.f of X
- (d) Find the expected value of X .

2. Let X have p.d.f

$$f(x) = \begin{cases} 1/4 & 0 < x < 1 \\ 3/8 & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Verify $f(x)$ is a valid p.d.f
- (b) Find the corresponding c.d.f
- (c) Find the expectation of X

3. *About random number generation*

Show the details of derivations and calculations for the following problems.

- (a) There is a random number generator, which generates 0 with probability p and 1 with probability $1-p$ ($p \neq 0.5$). Using this generator, how to generate 0, 1 with equal probabilities? How to generate 1,2,3 with equal probabilities? How to generate $1, 2, \dots, N$ ($N > 3$) with equal probabilities?

- (b) There is a random number generator, which generates $1, 2, \dots, M$ with equal probability $1/M$. Using this generator, how to generate $1, 2, \dots, N$ ($N \neq M$) with equal probabilities?
- (c) There is a random number generator, which generates integers with certain probabilities. The possible integers and the corresponding probabilities are both unknown. The total number of possible integers can be infinity. Using this generator, how to generate 0, 1 with equal probabilities?
- (d) Toss a fair coin for infinity times. All tosses are independent. For each toss, if it is head, you will get 1 point. If it is tail, you will get 2 points. In the entire process, what is the probability of ever having n points?