FSRM/MSDS 581 Homework 7
Due on Wednesday 11:59pm EST, 11/4

1. Let X_1, $X_2, \ldots, X_n$ be a random sample, following the Bernoulli $Ber(p)$ distribution. Let $\bar{X}$ be an estimator of the parameter p.

   (a) Is $\bar{X}$ a consistent estimator of p? Give a reason (you may just cite a theorem)

   (b) What is the (approximate) sampling distribution of $\bar{X}$ when n is sufficiently large?

   (c) Is $\bar{X}$ unbiased?

   (d) What is the standard deviation of $\bar{X}$ (in the expression of n and p)?

   (e) If $n = 1000$ and $\bar{x} = 0.45$, what is the approximate standard error of $\bar{X}$ estimating p?

   (f) Find a 95% confidence interval of p, when $n = 1000$ and $\bar{x} = 0.45$.

2. Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$ and let $\hat{\lambda}_n = \sum_{i=1}^n X_i/n$. Find the bias, se and MSE of this estimator.

3. Let $X_1, \ldots, X_n \sim \text{Uniform}(0, \theta)$.

   (a) Let $\hat{\theta}_n = X_{(n)} = \max\{X_1, \ldots, X_n\}$. Find the bias, se and MSE of this estimator.

   (b) Let $\hat{\theta}_n = X_{(n)} = 2\sum_{i=1}^n X_i/n$. Find the bias, se and MSE of this estimator.

4. Is a consistent estimator always unbiased? If yes, prove it. If no, give a counter example. Is an unbiased estimator always consistent? If yes, prove it. If no, give a counter example.