FSRM/MSDS 581 Homework 9
Due on Wednesday 11:59pm EST, 12/2

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d following Uniform(θ_1, θ_2).

(a) Use factorization theorem to show that $T = (X_{(1)}, X_{(n)})$ is sufficient for $\theta = (\theta_1, \theta_2)$, where $X_{(1)} = \min\{X_1, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, \ldots, X_n\}$.

(b) Show $T = (X_{(1)}, X_{(n)})$ is minimal sufficient.

(c) Find MLE of θ_1 and θ_2.

2. Let (x_i, Y_i) $(i = 1, \ldots, n)$ be independent following the **Poisson regression** model (without intercept)

$$Y_i \sim Poission(\lambda x_i)$$

(This is often used to model number of events occurred given a covariate. For example, the number of traffic accidents in a month as Y_i at the i-th intersection vs the number of cars going through the same intersection in the same period, as x_i)

(a) Since $E(Y_i) = \lambda x_i$, we can write $Y_i = \lambda x_i + e_i$ where $E(e_i) = 0$. Find the least square estimator for λ.

(b) Write the likelihood function $L(\lambda)$ (treat $x_1, \ldots, x_n$ as non-random)

(c) Find MLE for λ.

(d) Suppose we use Gamma(α, β) with pdf

$$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \quad \text{for} \quad \lambda > 0$$

as the prior distribution. Find the posterior distribution given observations $(y_1, x_1), \ldots, (y_n, x_n)$. And find the corresponding Bayes estimator for λ. [Hint: The prior is a conjugate prior so the posterior distribution is also a Gamma distribution. The mean of the Gamma(a, b) distribution is a/b]