FSRM/MSDS 581 Homework 4
Due on Wednesday 11:59pm EST, 10/7

1. Suppose (X_1, X_2) has a joint pdf

$$f_{X_1,X_2}(x_1, x_2) = e^{-(x_1+x_2)}, \text{ for } x_1 > 0, x_2 > 0$$

- (a) Find the marginal distribution of X_1

- (b) Find the conditional distribution of X_2 given $X_1 = x_1$

- (c) Use either (a) or (b) to show that X_1 and X_2 are independent

- (d) Find the joint density of $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$.

2. Suppose the joint pdf of (X, Y) is

$$f_{X,Y}(x, y) = \frac{1}{3}(x + y), \text{ for } 0 \leq x < 1, 0 \leq y < 2$$

Find $Var(2X - 3Y + 8)$

3. Roll a four-sided die twice. The faces of the die are marked 1 to 4. Let X be the smaller number of the two numbers shown and Y be the larger number of the two. ($X = Y$ when the two numbers shown are the same).

- (a) Find the joint distribution of (X, Y).

- (b) Find the conditional distribution of X given $Y = 2$

- (c) Find the covariance of X and Y.

4. [Use computer] Simulate 1,000 independent sets of i.i.d. standard Normal random numbers, with 40 observations in each set. Find the maximum absolute correlation coefficient among all pairs of sets of your simulated data.

   [**Hint:**] *It is easier to generate a long vector of Normal random numbers (e.g. `x=rnorm(1000*40,0,1`*

then put it into a matrix with 1,000 columns (e.g. `xx=matrix(x,ncol,nrow)`). You can subtract a diagonal matrix to remove the diagonal from the correlation matrix (because the diagonal elements are always one), then use the absolute function and the max function on the resulting matrix to find the maximum absolute correlation coefficient in the matrix. Function `cor(x)` calculates the correlation among the **columns***. Function `diag(1,ncol,nrow)` creates a diagonal matrix. Function `abs(y)` gets the absolute value of y – elementwise when y is a matrix.*

- (a) Comment on the results.

- (b) Without computation, what do you expect to see if there are 10,000 such independent sets (of 40 observations) and why?

- (c) Without computation, what do you expect to see if there are 100 such independent sets (of 40 observations) and why?