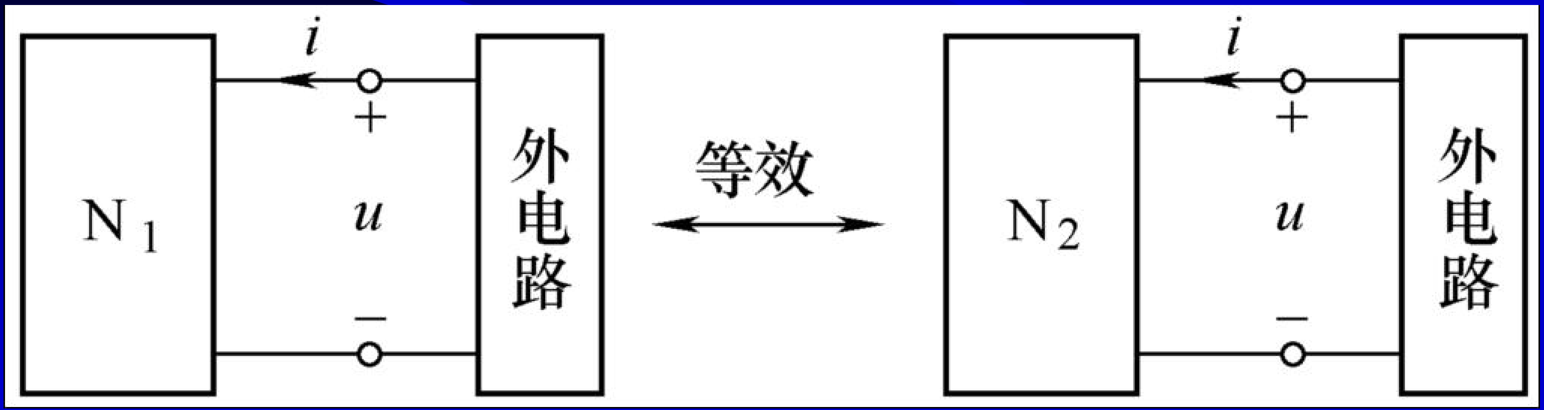
第2章 电路的等效变换

电路的等效是电路分析中一个很重要的概念，应用等效这个概念，可以将结构复杂的电路化简为极其简单的电路。

2.1 等效电路的概念

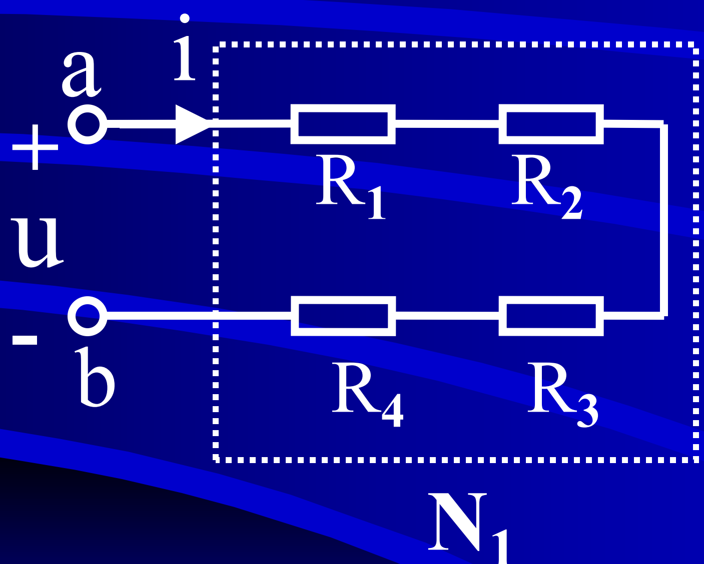
如果一个二端网络N_1**端口**的伏安关系和另一个二端网络N_2**端口**的伏安关系完全相同，则这两个二端网络便是等效的。

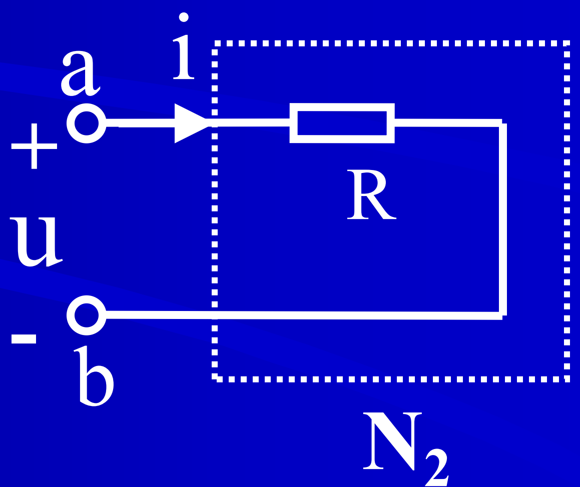


2.2 电阻的串联和并联

2.2.1 电阻的串联

1. 等效电阻





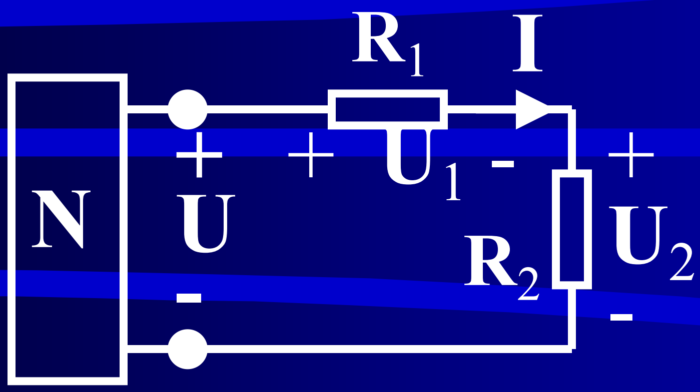
$$u = R i$$

$$u = R_1 i + R_2 i + R_3 i + R_4 i$$
$$= (R_1 + R_2 + R_3 + R_4)\ i$$

若： $R_1 + R_2 + R_3 + R_4 = R$

则： N_1和N_2完全等效

2. 分压公式



$$由KVL: U = U_1 + U_2$$

$$= R_1 \cdot I + R_2 \cdot I$$

$$\therefore I = \frac{U}{R_1 + R_2}$$

$$\because U_1 = R_1 \cdot I \qquad \because U_2 = R_2 \cdot I$$

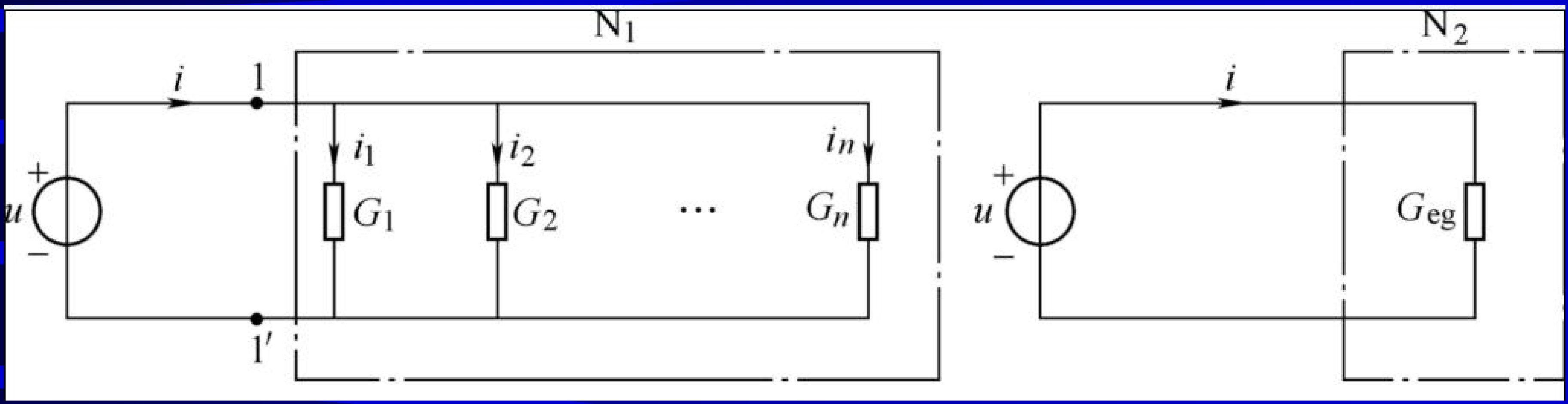
$$\therefore U_1 = \frac{R_1}{R_1 + R_2} U \qquad \therefore U_2 = \frac{R_2}{R_1 + R_2} U$$

由此得出：

若有n个电阻串联，第K个电阻的电压为：

$$\boxed{U_K = \frac{R_K}{\sum_{K=1}^{n} R_K} U}$$

2.2.4 电阻的并联



1.等效电导

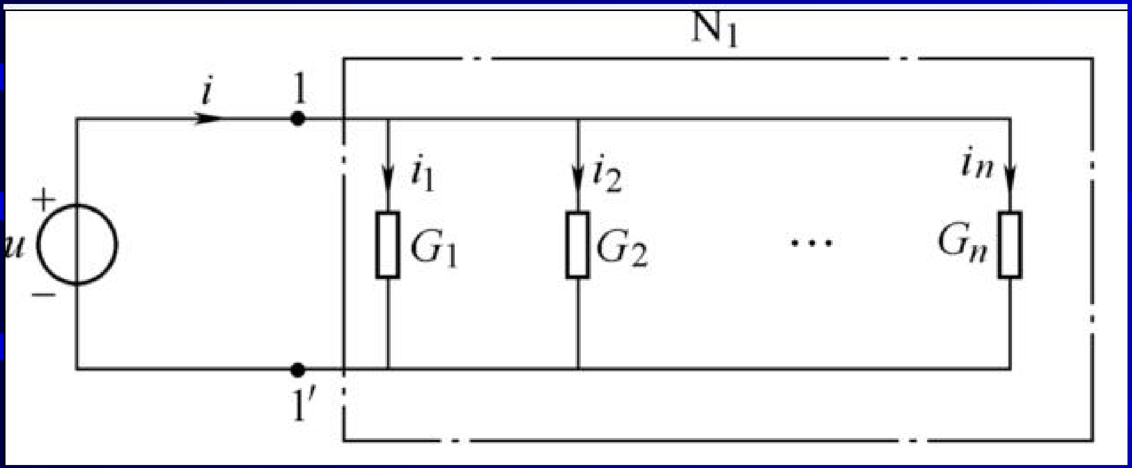
由KCL $i = i_1 + i_2 + \cdots + i_k + \cdots i_n$

$$i = G_1 u + G_2 u + \cdots + G_k u + \cdots G_n u = (G_1 + G_2 + \cdots + G_k + \cdots G_n)\, u = G_{eq}\, u$$

$$G_{eq} = G_1 + G_2 + \cdots + G_k + \cdots G_n = \sum_{k=1}^{n} G_k$$

两个电阻并联时 $R_{eq} = R_1 // R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$

2. 分流公式



$$u=\frac{i}{G_1+G_2+\cdots+G_n}=\frac{i}{\sum_{k=1}^{n}G_k}=\frac{i}{G_{eq}}$$

第k个电阻上的分流公式为

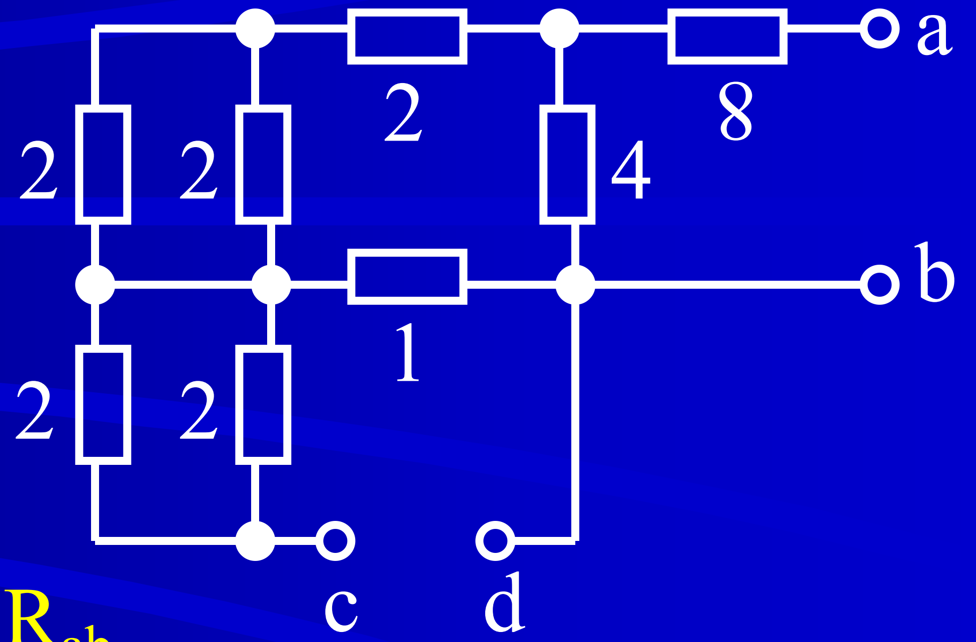
$$i_k=G_k u=G_k\frac{i}{\sum_{k=1}^{n}G_k}=\frac{G_k}{G_{eq}}i$$

两个电阻并联，分流公式为

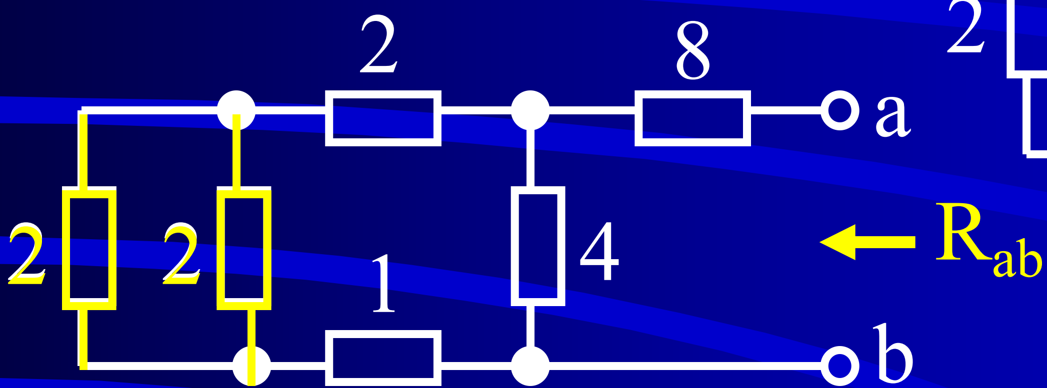
$$i_1=\frac{R_2}{R_1+R_2}i \qquad i_2=\frac{R_1}{R_1+R_2}i$$

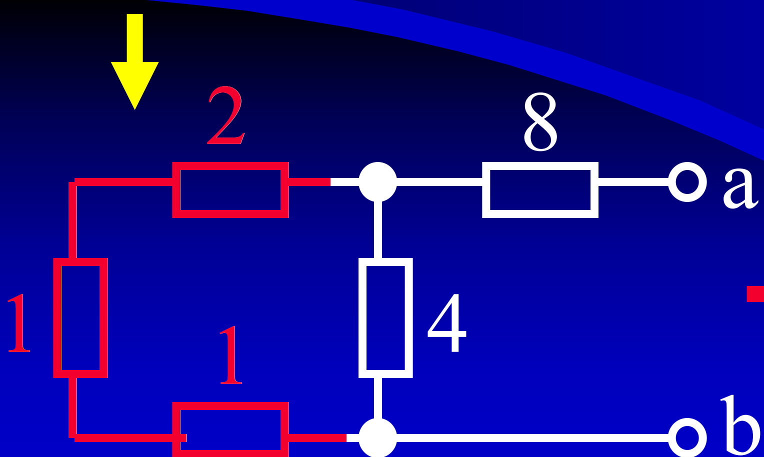
2.2.5 电阻串并联混合电路

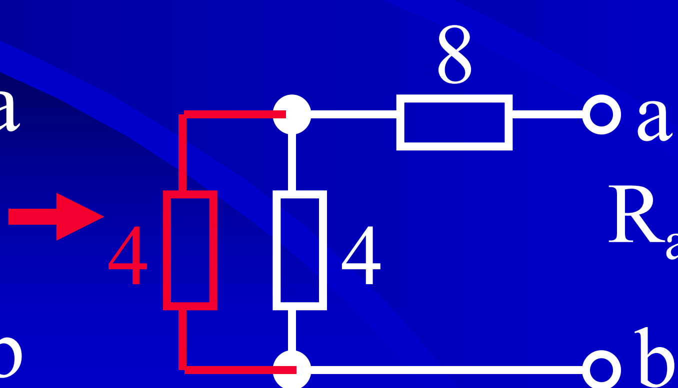
例：求等效电阻R_{ab}、R_{cd} 。



解：求等效电阻R_{ab}

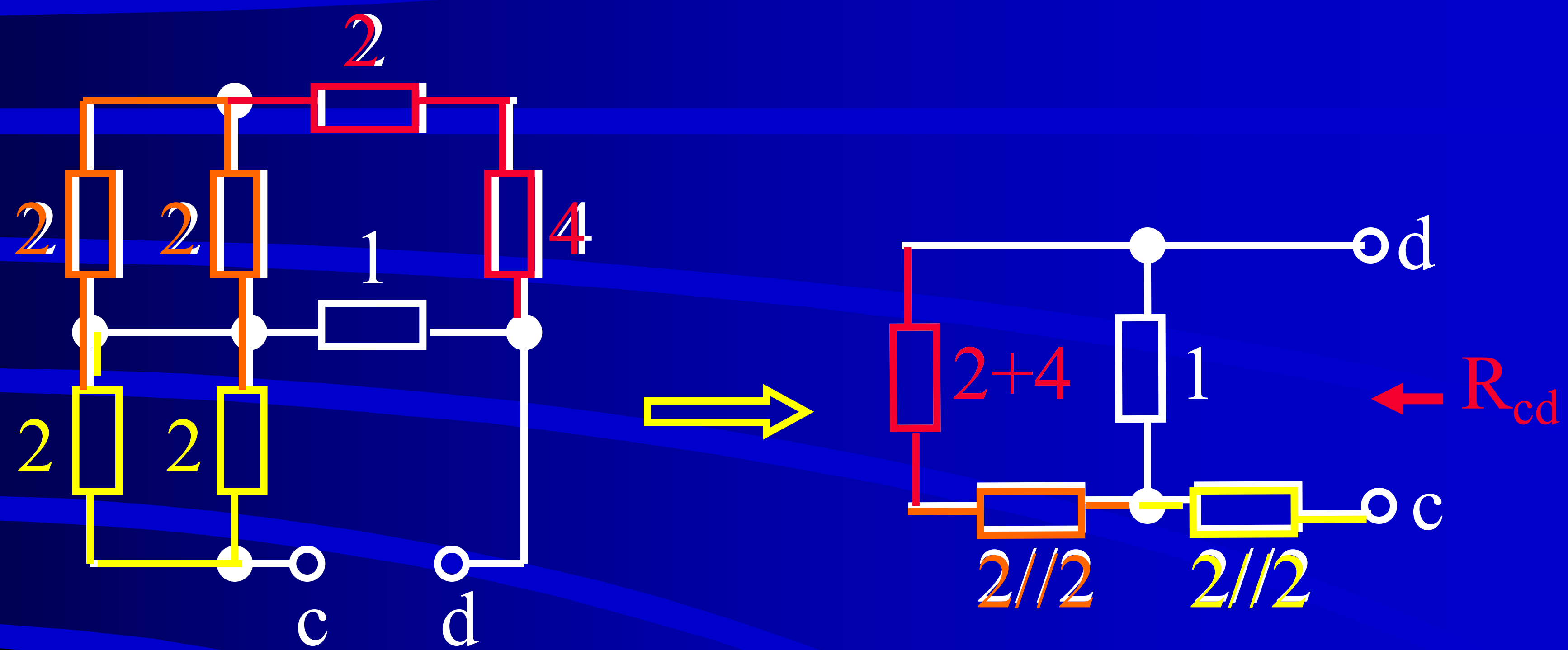






$R_{ab}= 8+4//4=10$

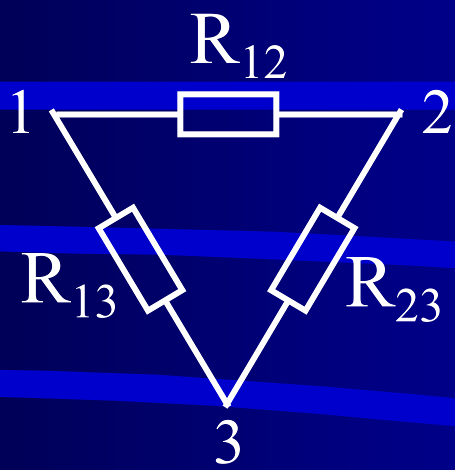
求等效电阻R_{cd}

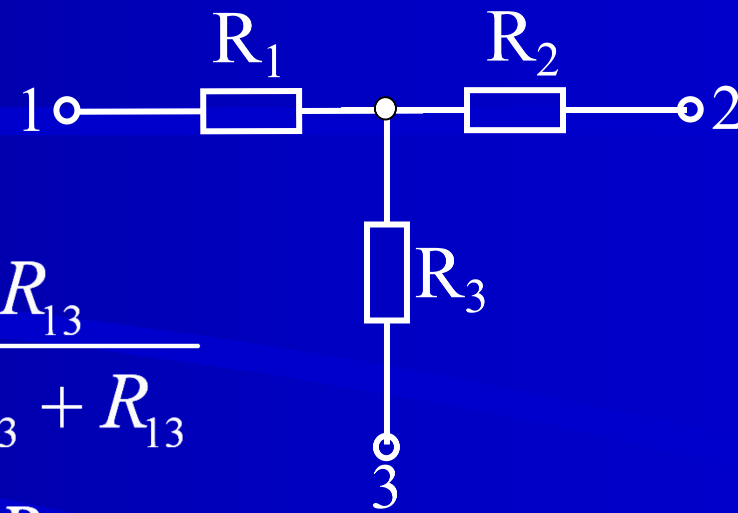


$$R_{cd}=1+1//7=1.875$$

2.3 电阻Y-Δ等效变换

1. Δ- Y变换





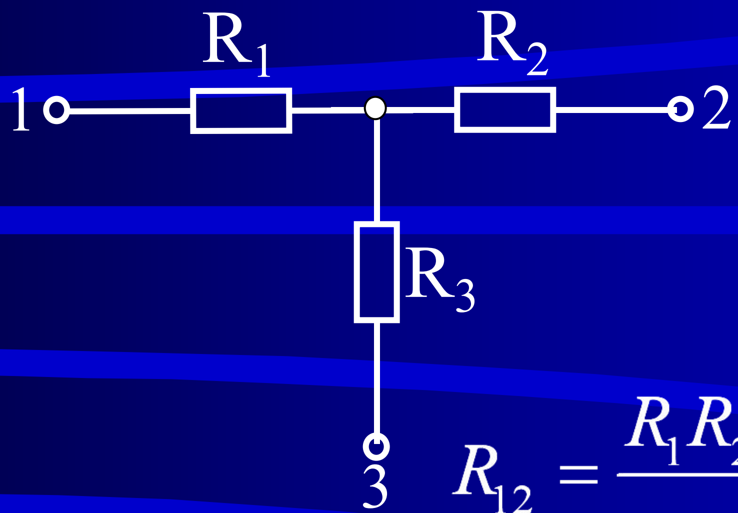
$$R_1 = \frac{R_{12} \cdot R_{13}}{R_{12} + R_{23} + R_{13}}$$

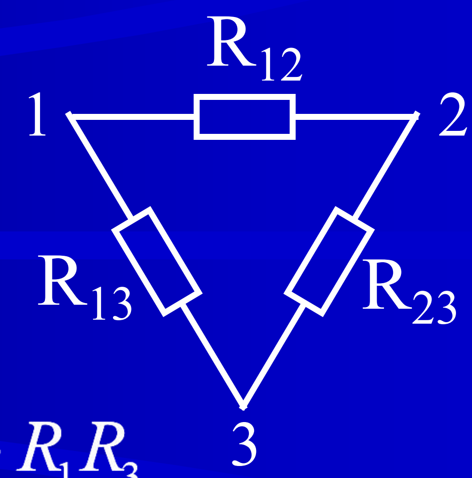
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{13}}$$

$$R_3 = \frac{R_{23} \cdot R_{13}}{R_{12} + R_{23} + R_{13}}$$

$$R_n = \frac{\text{接在}n\text{节点的两电阻乘积}}{\text{三个电阻之和}}$$

2. Y - Δ 变换





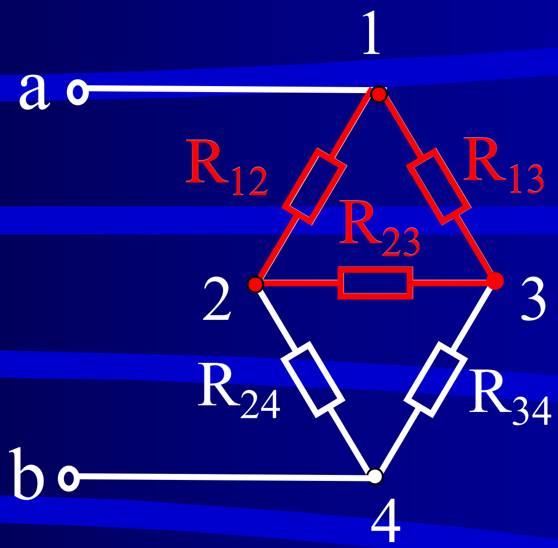
$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

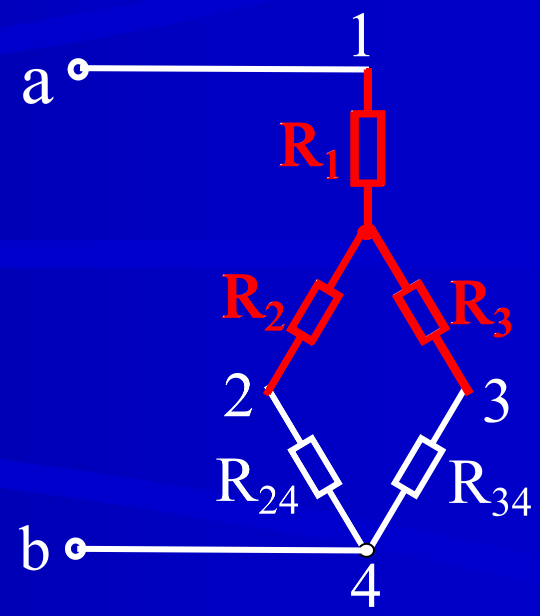
$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_{13} = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_{mn} = \frac{\text{电阻两两乘积之和}}{\text{下标不为}m\text{，}n\text{的电阻}}$$

例：求等效电阻 R_{ab}





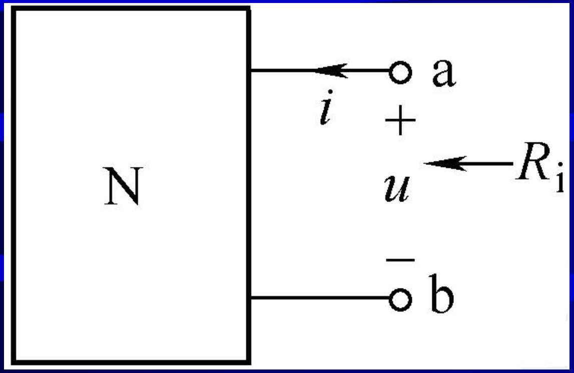
$$R_1 = \frac{R_{12} \cdot R_{13}}{R_{12} + R_{23} + R_{13}}$$

$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{13}}$$

$$R_3 = \frac{R_{23} \cdot R_{13}}{R_{12} + R_{23} + R_{13}}$$

$$\therefore\ R_{ab} = R_1 + \left((R_2 + R_{24}) // (R_3 + R_{34})\right)$$

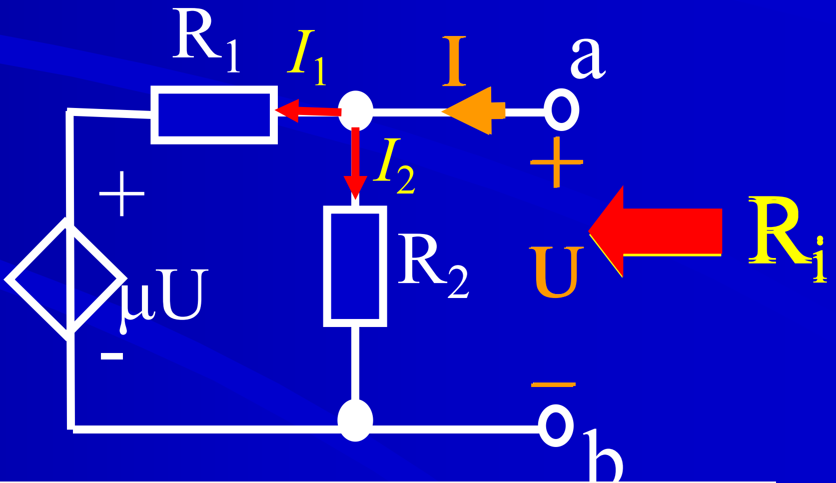
2.5 输入电阻



从(输入) 端口两端看进去的等效电阻。

$$R_i = R_{ab} = \frac{u}{i}$$

例： 求输入电阻R_i。



解： $I = I_2 + I_1$

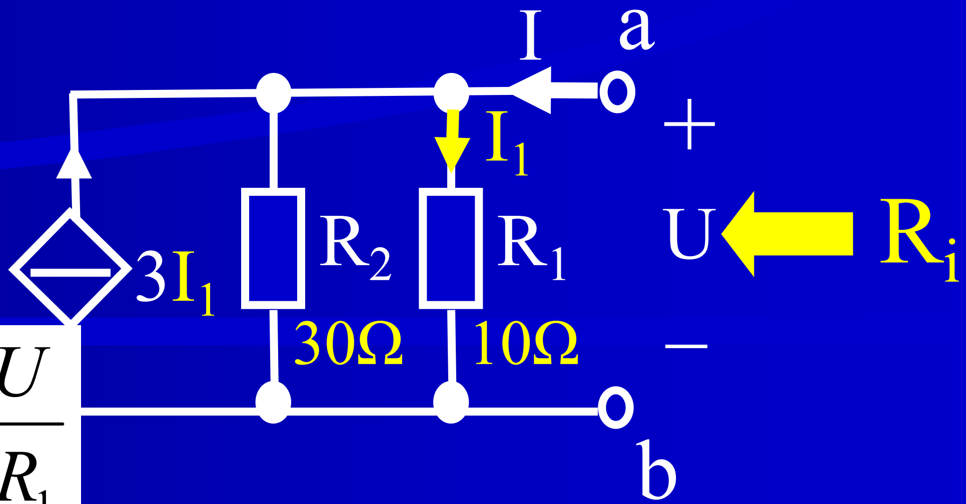
$$= \frac{U}{R_2} + \frac{U - \mu U}{R_1}$$

$$= U(\frac{1}{R_2} + \frac{1-\mu}{R_1})$$

$$= U\left[G_2 + G_1(1-\mu)\right]$$

$$\therefore R_i = \frac{U}{I} = \frac{1}{G_2 + G_1(1-\mu)}$$

例：求输入电阻R_i。



解：

$$I = I_1 + \frac{U}{R_2} - 3I_1$$

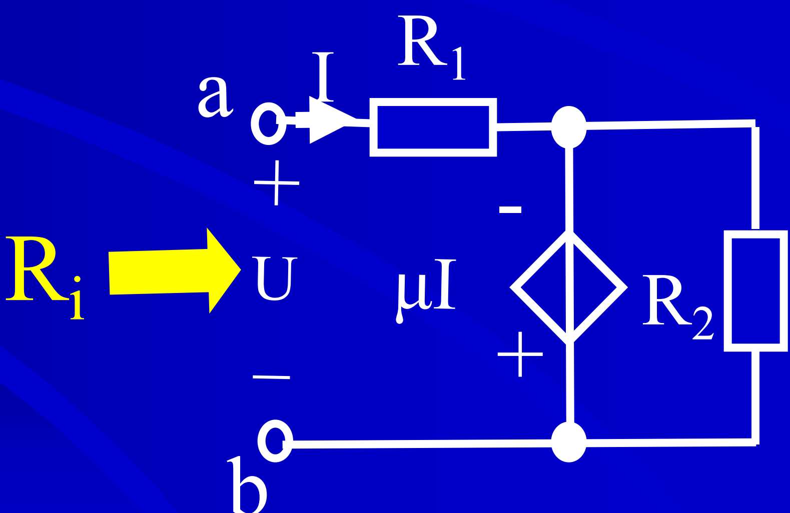
$$= U\left(\frac{1}{R_2} - \frac{2}{R_1}\right)$$

$$I_1 = \frac{U}{R_1}$$

$$\therefore R_i = \frac{U}{I} = \frac{1}{\dfrac{1}{R_2} - \dfrac{2}{R_1}} = \frac{R_1 R_2}{R_1 - 2R_2}$$

$$R_i = \frac{10 \times 30}{10 - 2 \times 30} = \frac{300}{-50} = -6\Omega$$

例：求输入电阻R_i。



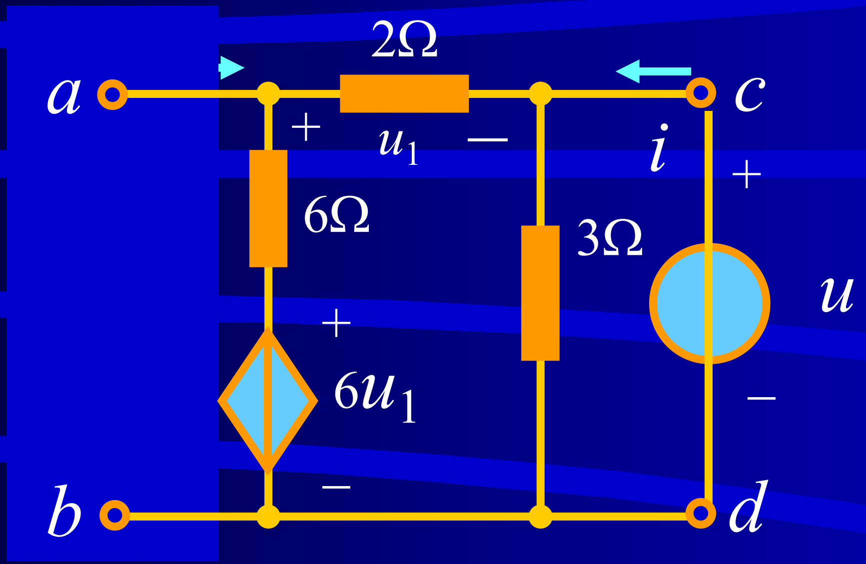
解：

$$U = R_1 \cdot I - \mu I$$

$$= I(R_1 - \mu)$$

$$\therefore R_i = \frac{U}{I} = R_1 - \mu$$

例 求R_{ab}和R_{cd}



$$u = u_1 + 3u_1/2 = 2.5u_1$$

$$u_1 = u/2.5 = 0.4u$$

$$i = \frac{u_1}{2} + \frac{u-6u_1}{6} = \frac{-u}{30}$$

$$R_{ab} = \frac{u}{i} = -30\Omega$$

$$u = -u_1 + 6u_1 + 6\times(-\frac{u_1}{2}) = 2u_1$$

$$i = -u_1/2 + u/3 = u_1/6$$

$$R_{cd} = \frac{u}{i} = 12\Omega$$

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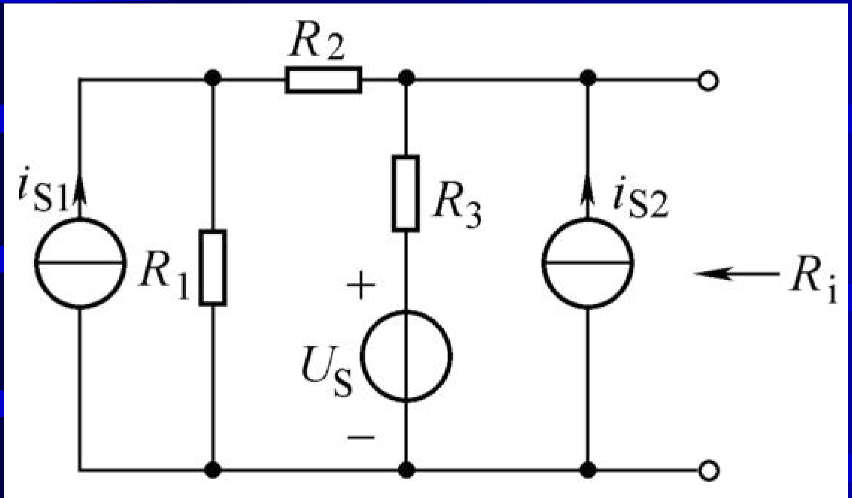
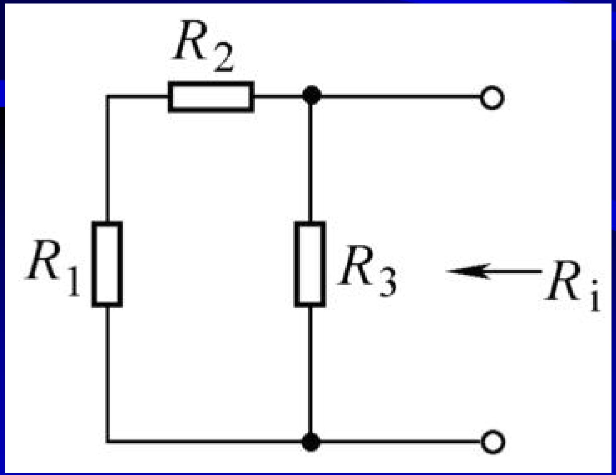
例2-12 计算电路的输入电阻 R_i 。



解： 电路中含有独立电源时，先将独立电源置零，然后再用电阻的串、并联关系进行计算。



电流源置零用开路代替，

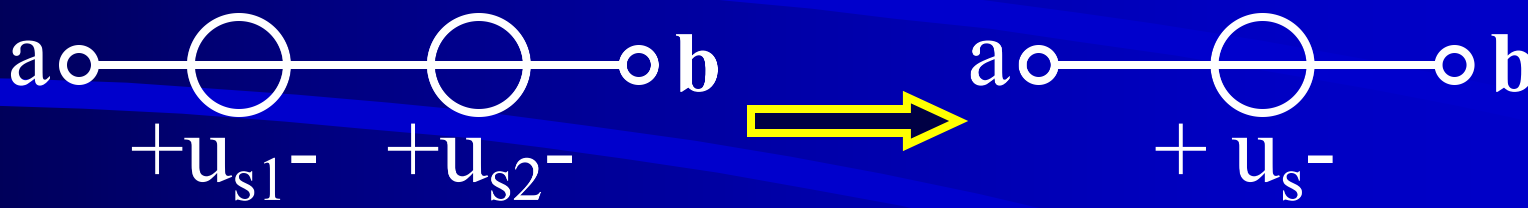
电压源置零用短路代替。

$$R_i = R_3 // (R_1 + R_2)$$

2.6 电源的等效变换

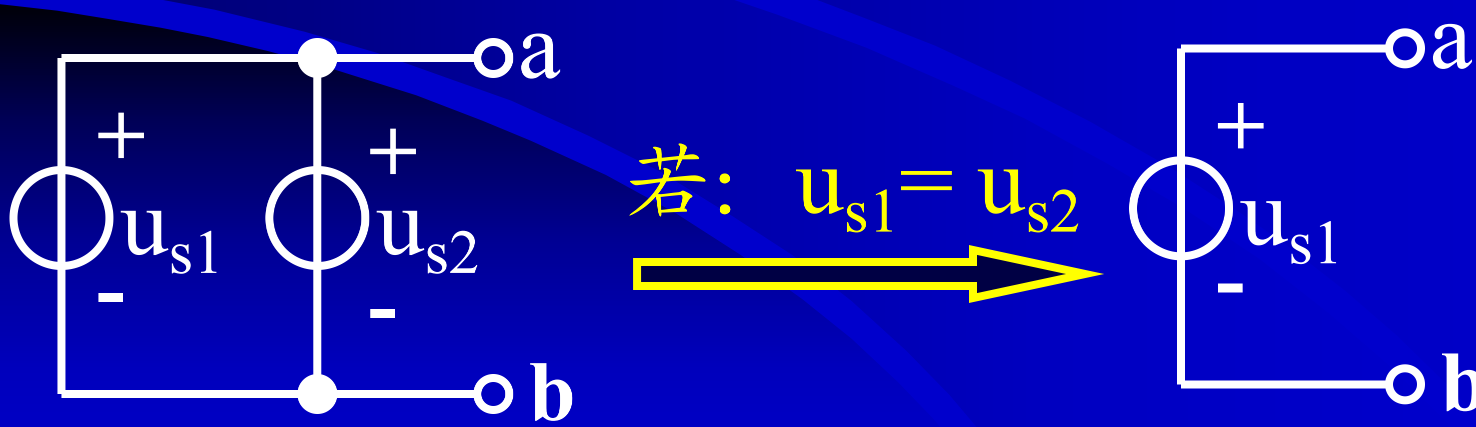
2.6.1 电压源的串联与并联

1.电压源串联



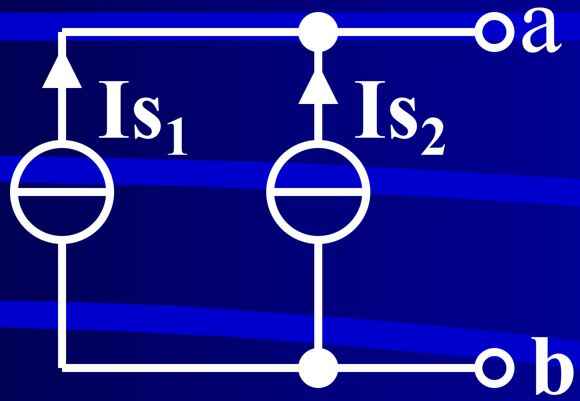
$u_s = u_{s1} + u_{s2}$

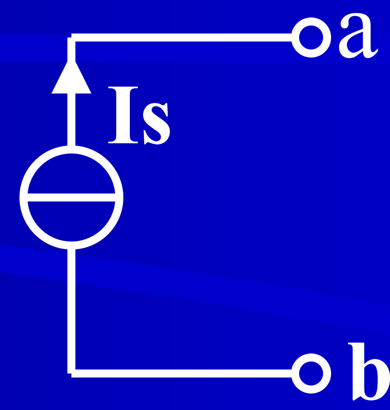
2. 电压源并联



2.6.2 电流源的串联与并联

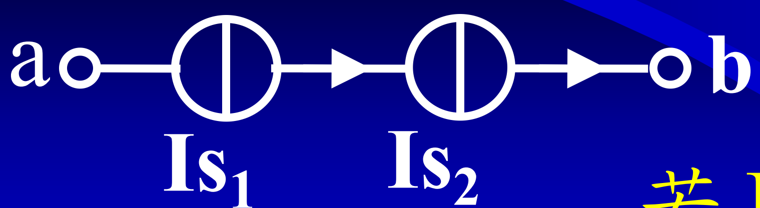
1. 电流源并联



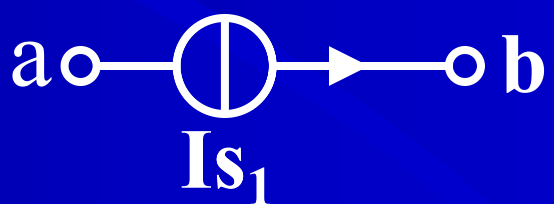


$$Is = Is_1 + Is_2$$

2.电流源串联

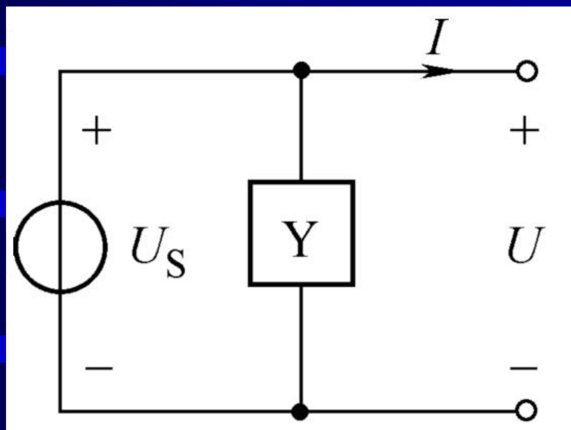


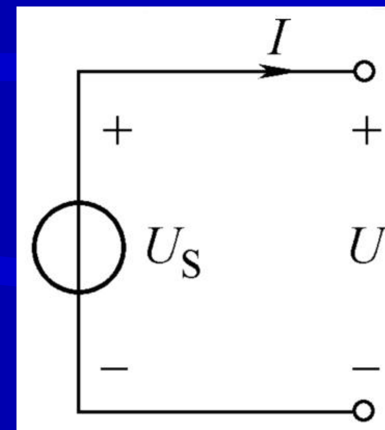
若 $I_{s1} = I_{s2}$



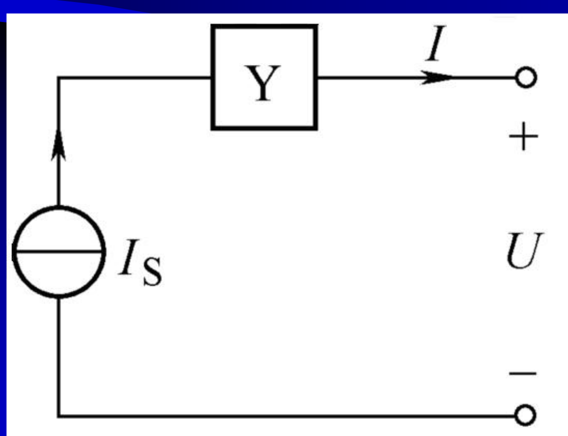
2.6.3 电源与元件的串并联

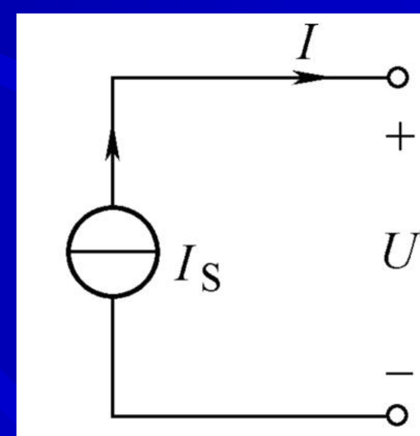
1. 电压源与元件并联





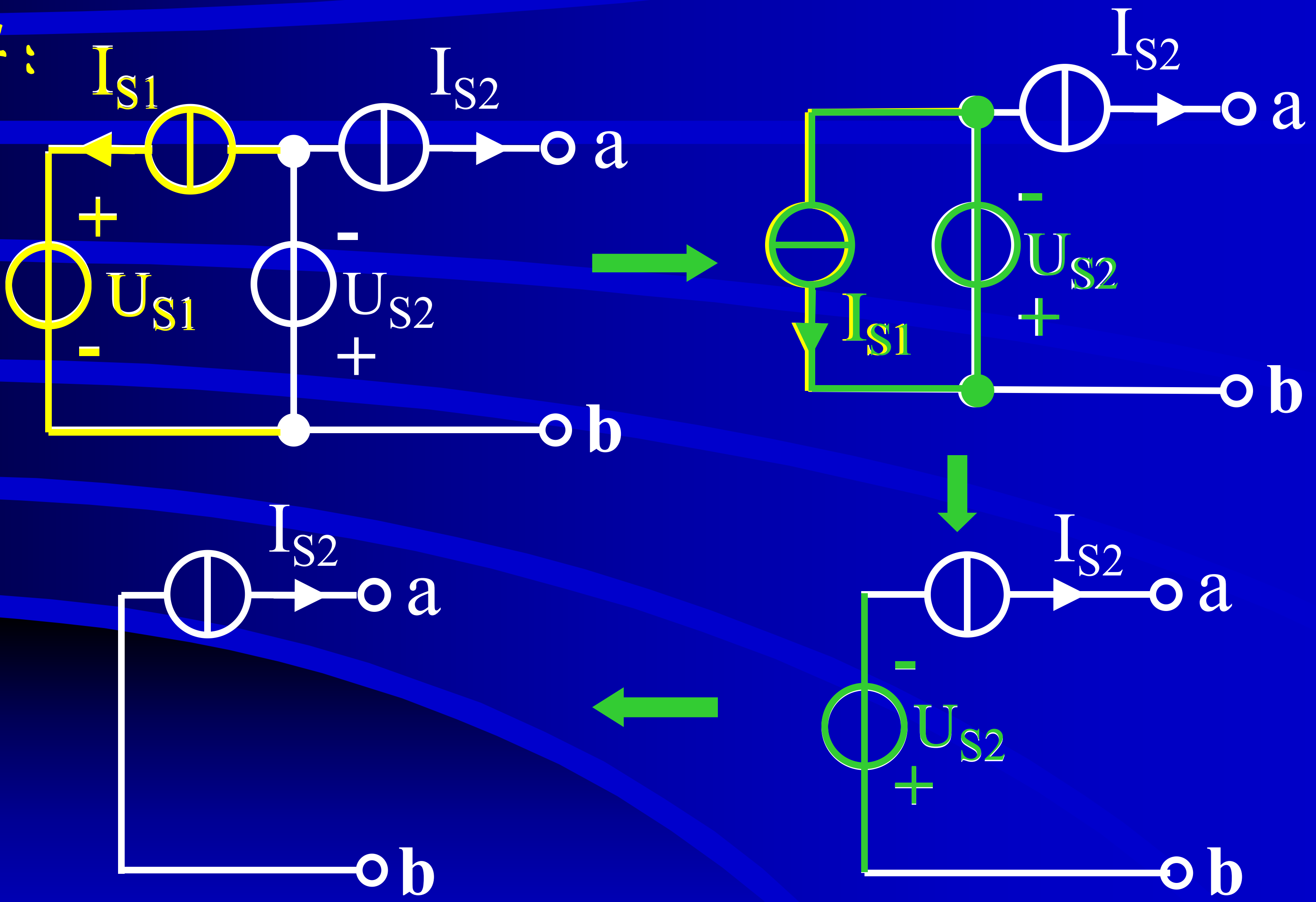
2. 电流源与元件串联



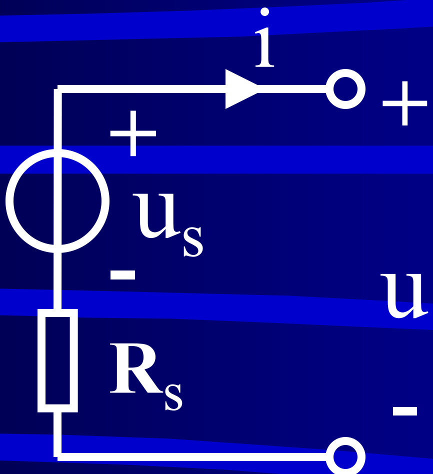


例6： 化简电路。

解：

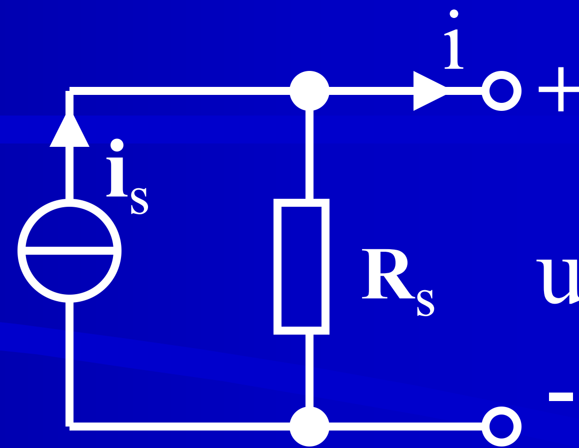


2.6.4 实际电源的等效



$i_s = u_s / R_s$

$u_s = i_s \cdot R_s$



$$u = u_S - R_S\, i$$

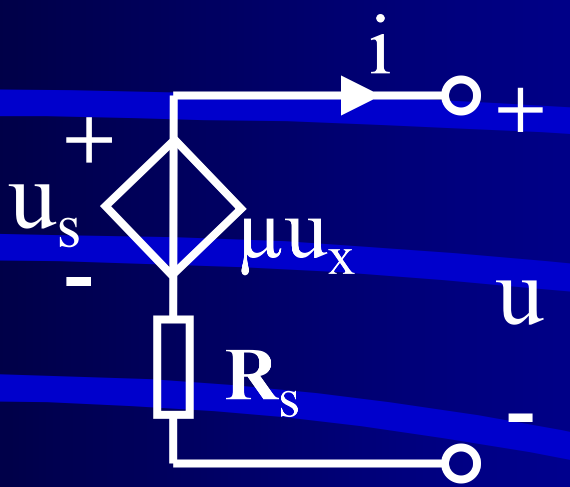
$$i = i_S - \frac{u}{R_S}$$

$$i = \frac{u_S}{R_S} - \frac{u}{R_S} = i_S - \frac{u}{R_S}$$

$$u = R_S\, i_S - R_S\, i = u_S - R_S\, i$$

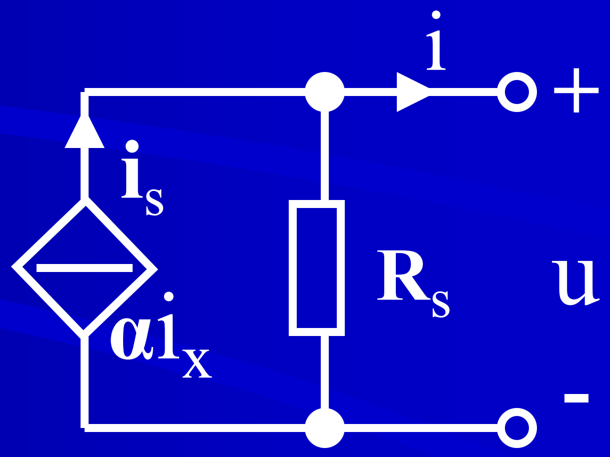
等效时要注意电源方向

受控源的等效公式



$i_s = u_s / R_s$

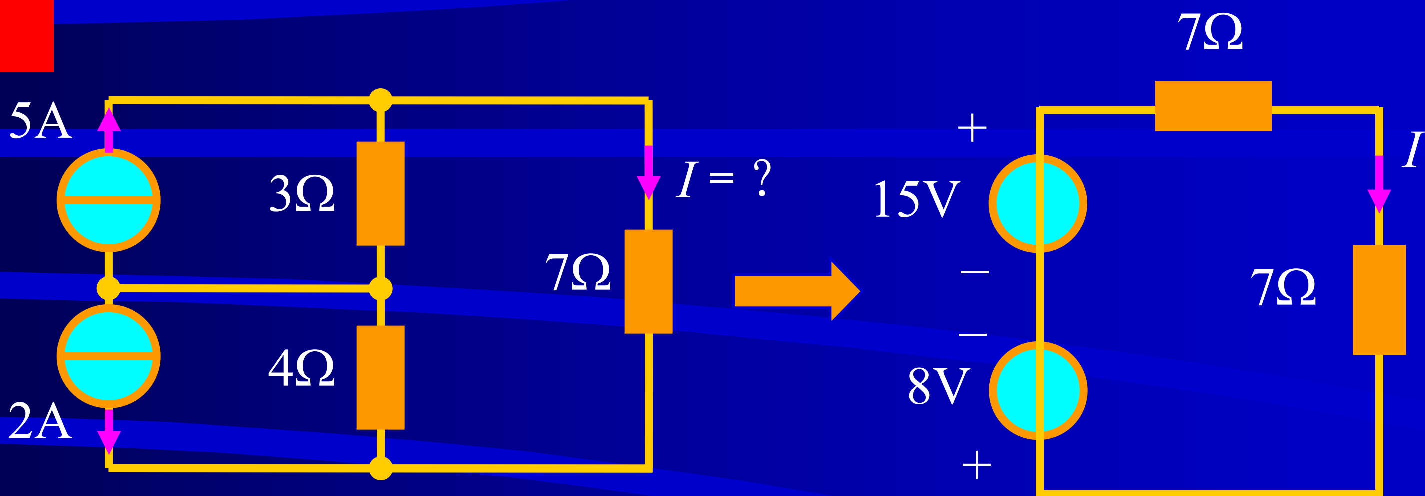
$u_s = i_s \cdot R_s$



等效时要注意受控源方向

利用电源转换简化电路计算。

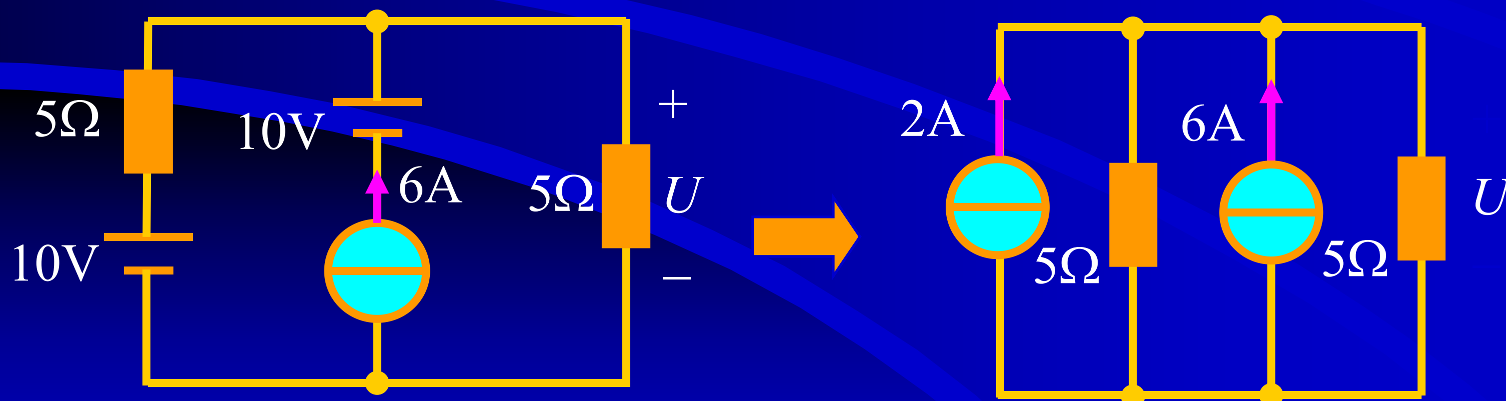
例



I=0.5A

例

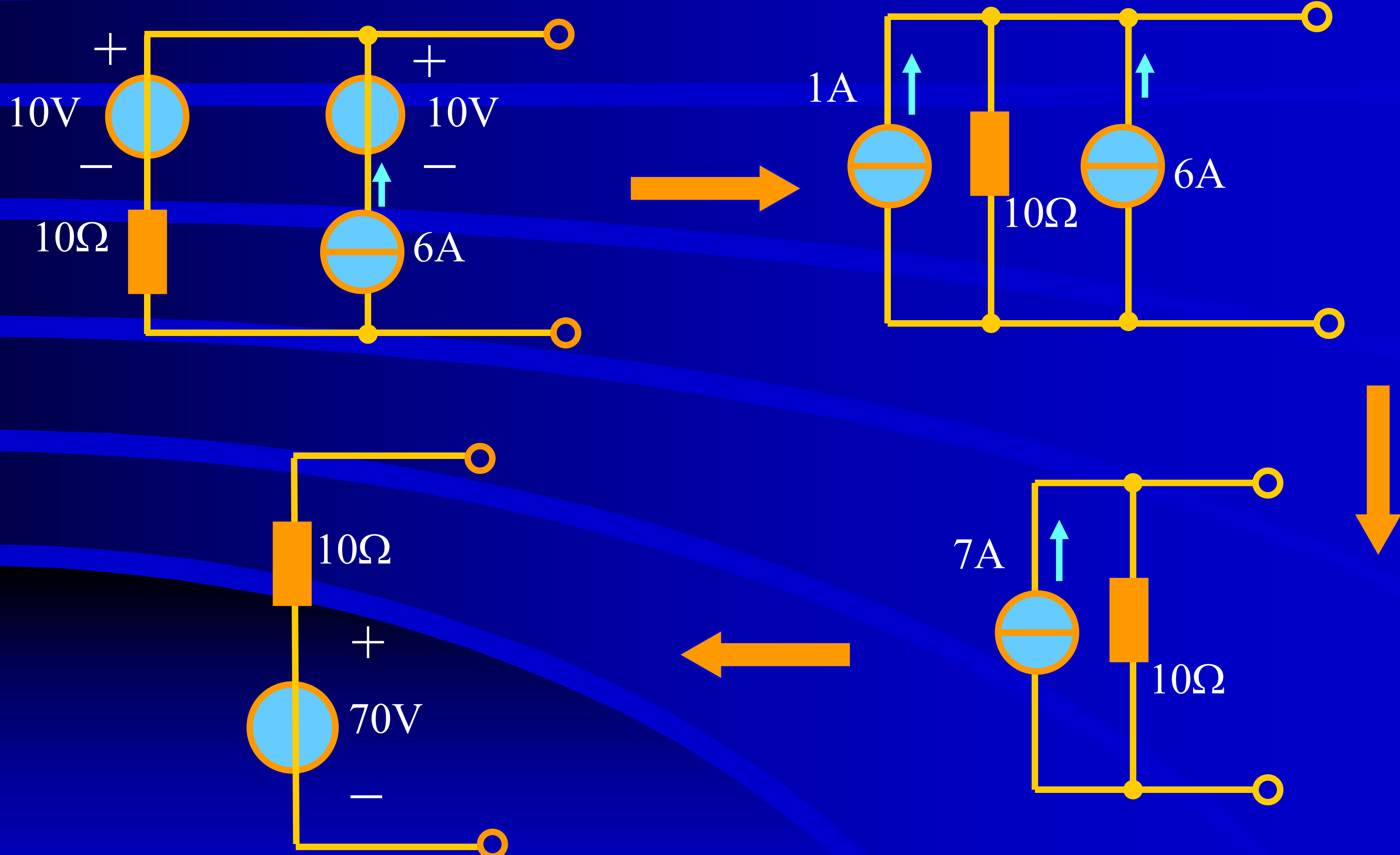
U=?



U=20V

例

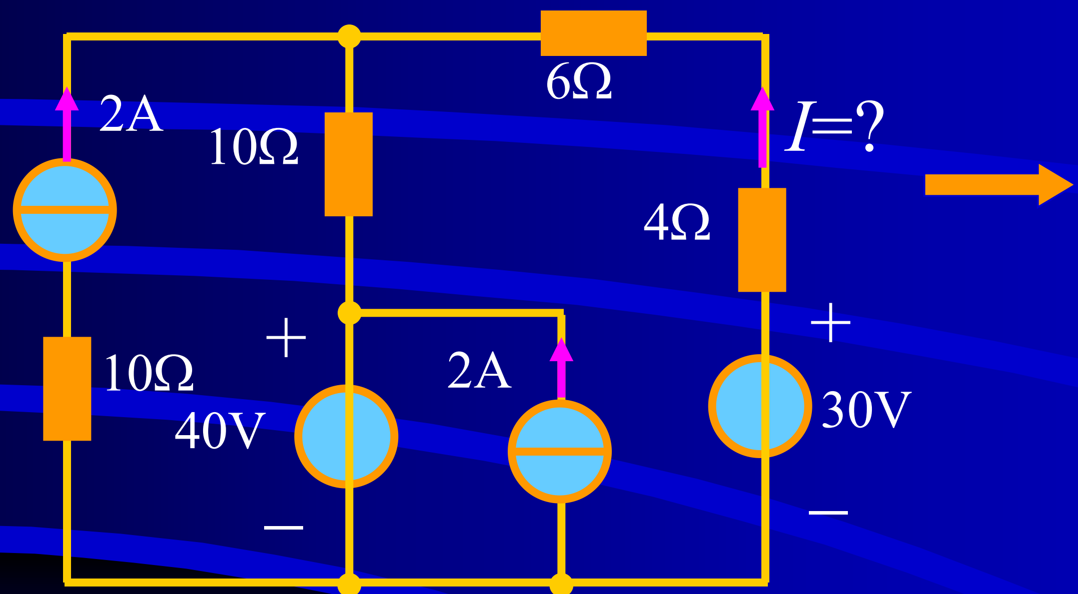
把电路化简成一个电压源和一个电阻的串联。

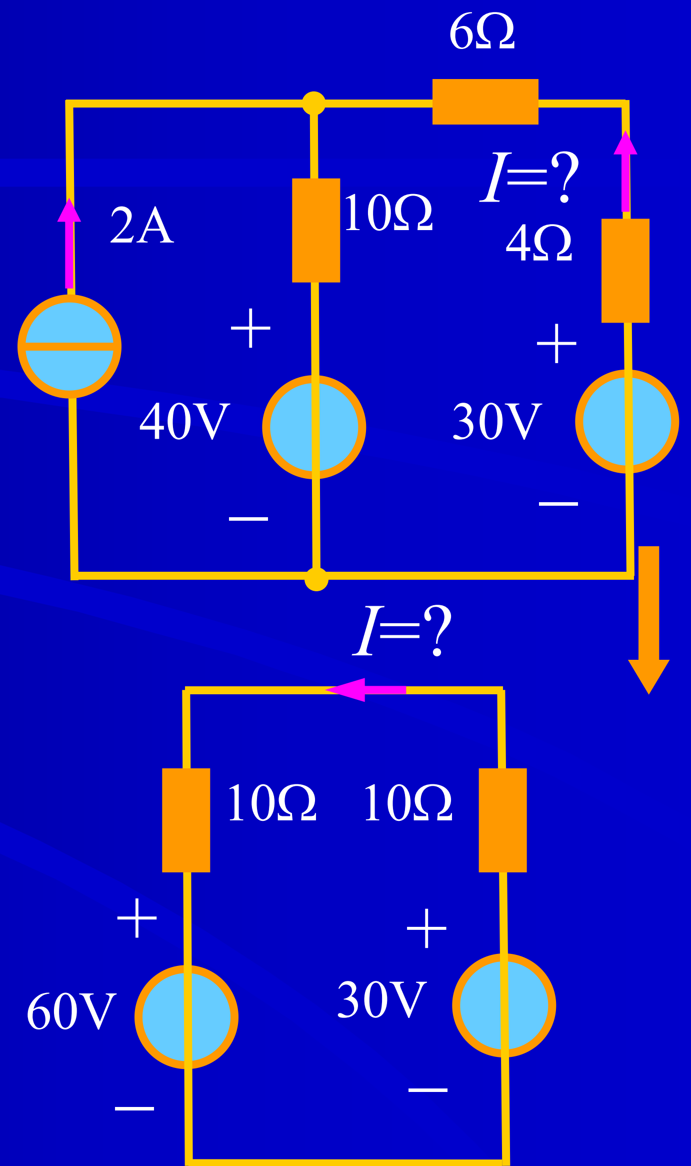


根据KVL，理想电压源与任何负载（含电流源）并联，其对外端口电压不变，故所并联的电流源在计算中无法起作用，故可看成“无效”。但这只是对外而已，对电压源所提供的功率还是有变化的。

根据KCL，理想电流源串联任何电气元件对无法改变电流源的输出电流，与之串联的电压源在对其它元件计算中无法起作用，故也可视为“无效”，但同样会对电流源功率产生影响。

例

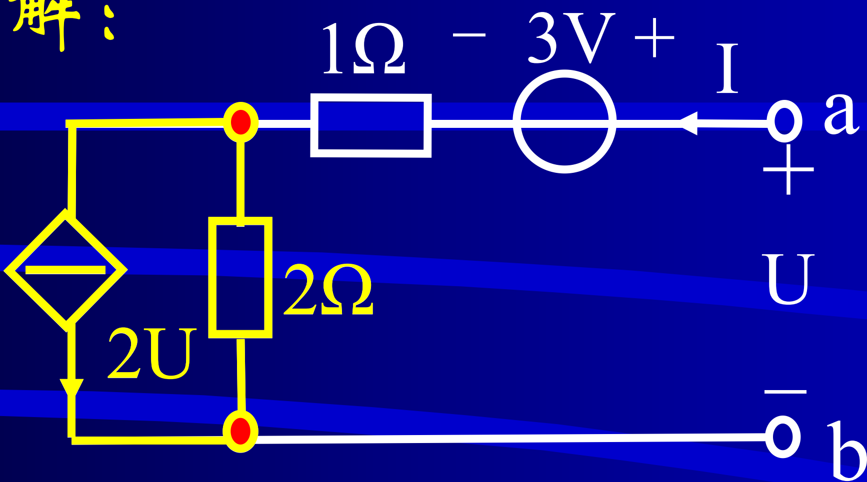


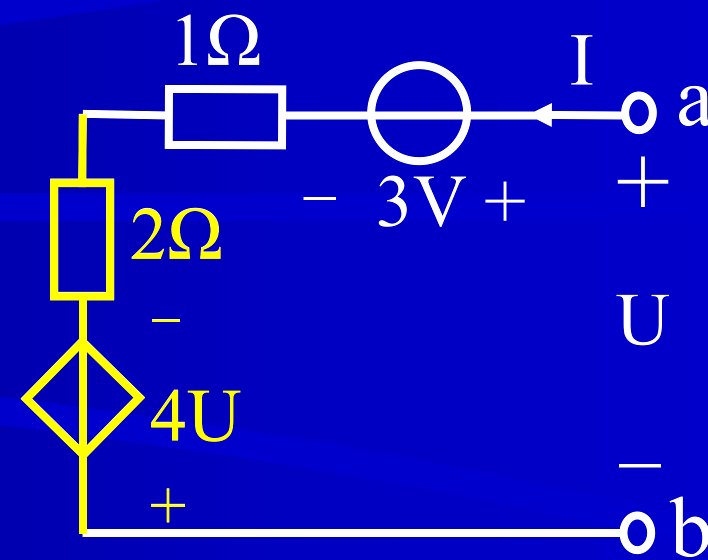


$$I=\frac{30-60}{20}=-1.5A$$

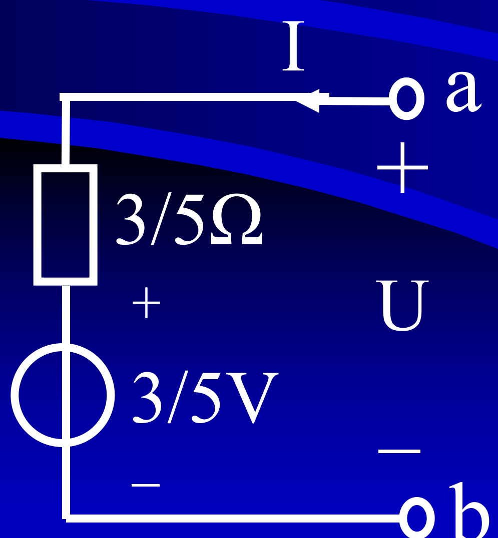
例7：　求电路的VAR及等效电路。

解：





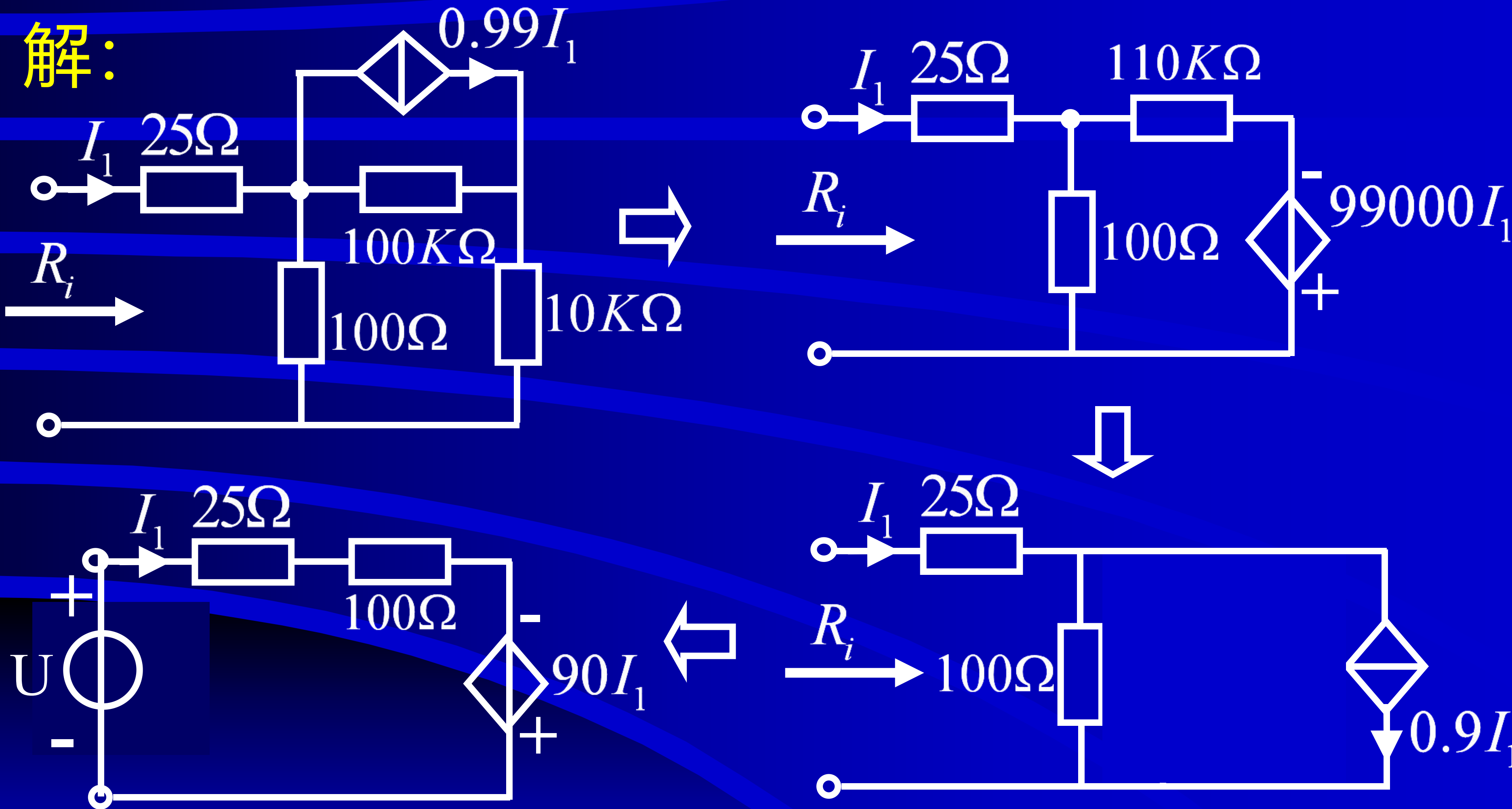
$$U = 3 + (1+2)I - 4U$$



$$U = \frac{3}{5}I + \frac{3}{5}$$

例8:　　求输入电阻Ri。

解:



$$U=(25+100)I_1-90I_1=35I_1 \quad \therefore R_i=\frac{U}{I_1}=35\Omega$$


谢谢大家！
再见
再见！