ANUPAM SHUKLA



# ELECTRICAL-NETWORK-THEORY

DAET-25/07/14

Network

Sagar Sen
8871453536

**ANUPAM SHUKLA**

# POLYPHASE AC CKT ANALYSIS

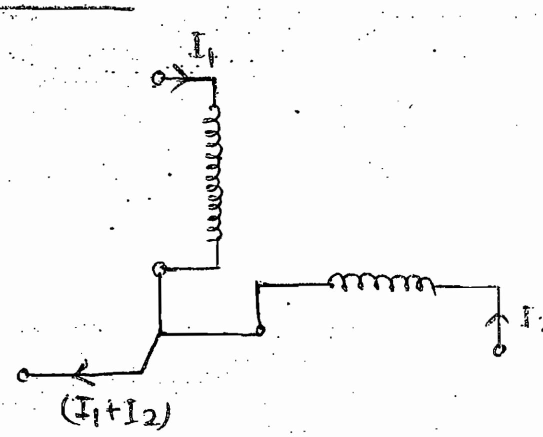
Phase → "A current carrying condr." (or) Live wire (or) Hot wire.

Poly → more than one.

Introduction →

* Y connected genr.
* Δ Connected genr. /mesh
* Power Calculation.
* Concept of P, Q, S & PF.
* Measurement of 3-$\phi$ power.
* Analysis of 3-$\phi$ balanced load.
* Analysis of 3-$\phi$ Unbalanced load.
* Floating point Neutral. (Millman's theorem).
* Application of 3-$\phi$

2 phase wire →




* The 3rd condr gets more amount of current. so the cross section area will be more for 3rd condr
* so 2phase wire sys. is not is used because of uneconomical

3 phase system →

(1.) 3 wire are used of same thickness.

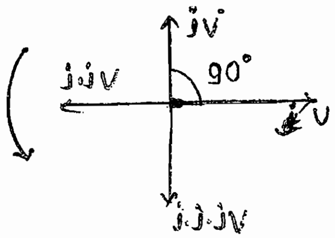
(2.) Power handling capacity increases with little amount.

$$P = \sqrt{3}\, VI\cos\phi$$

$$= 1.732\, VI\cos\phi$$

## Mathmatical Operators →

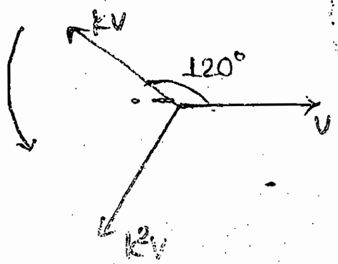
(1.) "j" operator (2.) "k" operators.




AP ∠+90°

→ +ve

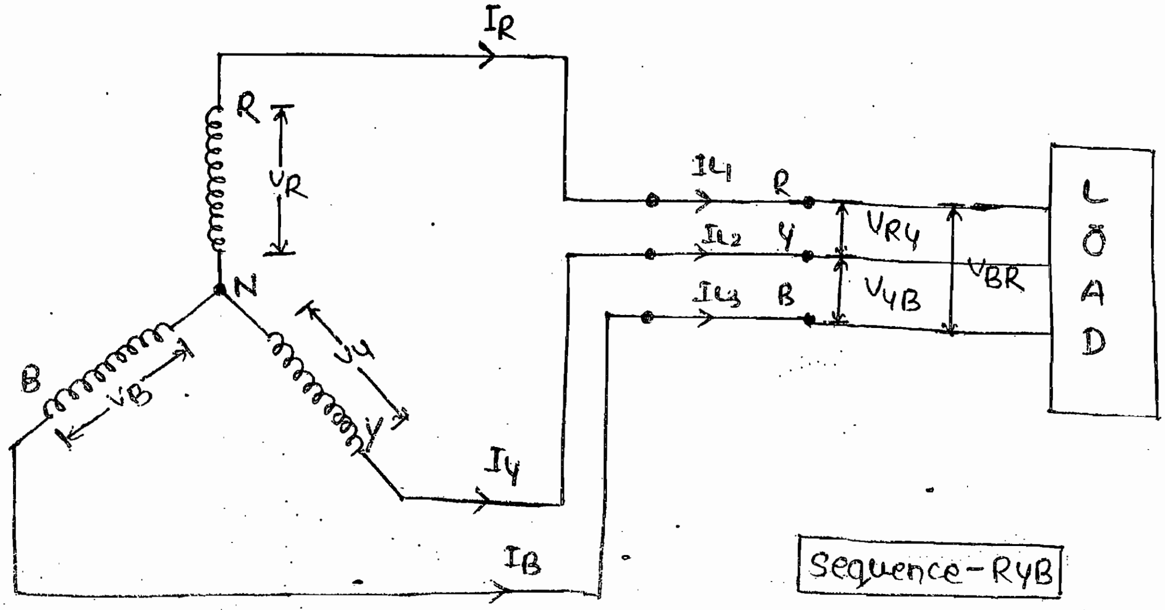
→ Anticlockwise




$k = a = \alpha$

* $2\phi$ electric power was an earlier polyphase AC power distribution. Two cond^r were used in this type.
* The advantageous of 2 ph wire over $1\phi$ wire is its self starting electric motors.
* Two-$\phi$ ckt typically uses 2 pairs of cond^r, sometimes they are required to be interconnected but the common cond^r carries the vector sum of $\phi$ currents which requires a larger cond^r.
* But in $3\phi$ 3 cond^r's are of same size, the most obeous ad. of $3\phi$ power x^n using 3 wires as compare to $1\phi$–2 wires is the power xmitted in $3\phi$ is 73% more but uses only 50% of additional wire.
* Earlier $2\phi$ is also implemented by 3 wire but it introduces a symmetry & the voltage drop in a neutral does not make the phases 90° apart.
* For a given frame size of a m/c a $3\phi$ m/c will have larger capacity than a $1\phi$ m/c. The torque produce in a $3\phi$ motor will be more uniform whereas in $1\phi$ motor it is pulsating.
* The amt of cond^r material needed to xmit same amt of power is lesser for $3\phi$ sys. Therefore it is economical.

## * Star Connected Gen$^n$ →




Conclusions →

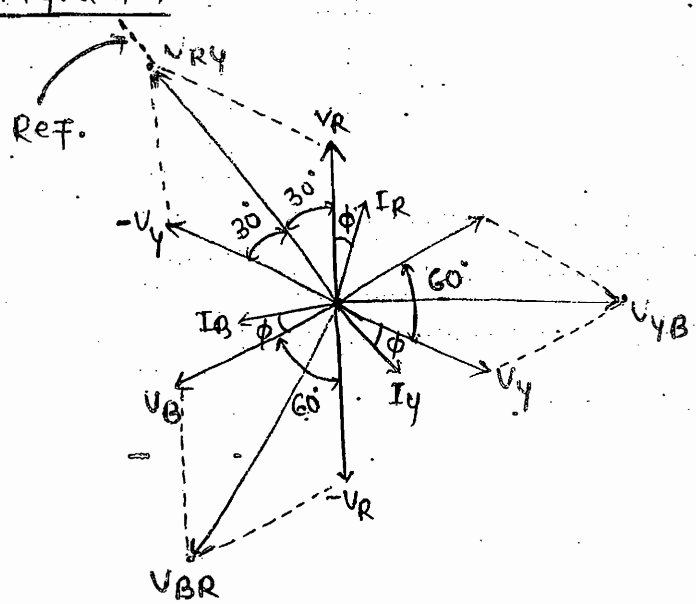
(1.) $V_R = V_y = V_B = V_{PH}$. $V_{RY} = V_{YB} = V_{BR} = V_L$

$I_R = I_y = I_B = I_{PH}$. $I_{L1} = I_{L2} = I_{L3} = I_L$

(2.) Line current = Phase current

$I_L = I_{ph}$.

(3.) Phasor diagram →




$V_{RY} = V_R + V_y$
$= V_R + (-V_y)$.

$V_{YB} = V_y - V_B$
$= V_y + (-V_B)$

$V_{BR} = V_B - V_R$
$= V_B + (-V_R)$

$$V_{RY} = \sqrt{|V_R|^2 + |-V_y|^2 + 2|V_R||-V_y|\cos 60^\circ}$$

$$V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2 V_{ph}^2 \times \frac{1}{2}}$$

$\boxed{V_L = \sqrt{3}\, V_{ph}}$

4.) Angle between $V_L$ & $I_L$ →

For load R-L load $= 30° + \phi$

R load $= 30°$

RC load $= 30 - \phi$

{ From phasor diagram $V_{RY} = V_L$ }

5.) In λ Connection b/w any 2 adjecent phase vol. the angles always 60°.

(6.) w.r.t Ref $V_{RY}, V_{YB}, V_{BR}$

$V_{RY} = V_L \angle 0°$

$V_{YB} = V_L \angle -120°$

$V_{BR} = V_L \angle -240°$ (or) $\angle 120°$

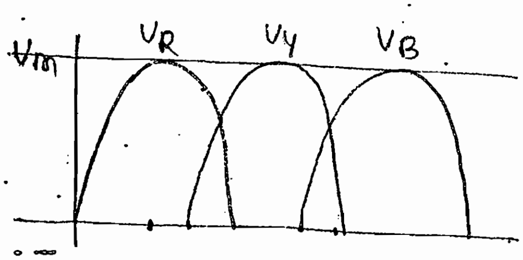
(7.) w.r.t. Ref $V_R, V_Y, V_B$

$V_R = V_{ph} \angle -30°$

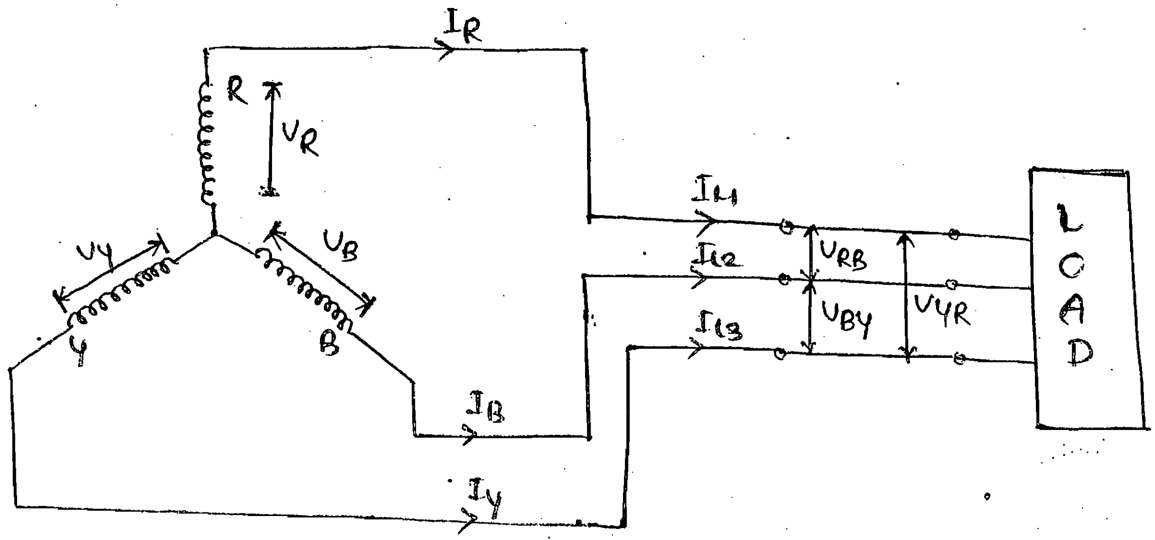
$V_Y = V_{ph} \angle -150°$

$V_B = V_{ph} \angle -270°$ (or) $90°$

Phase Sequence → * It is the order by which the individual φ Voltages attains there peak value.




Analysis for -ve φ seq →

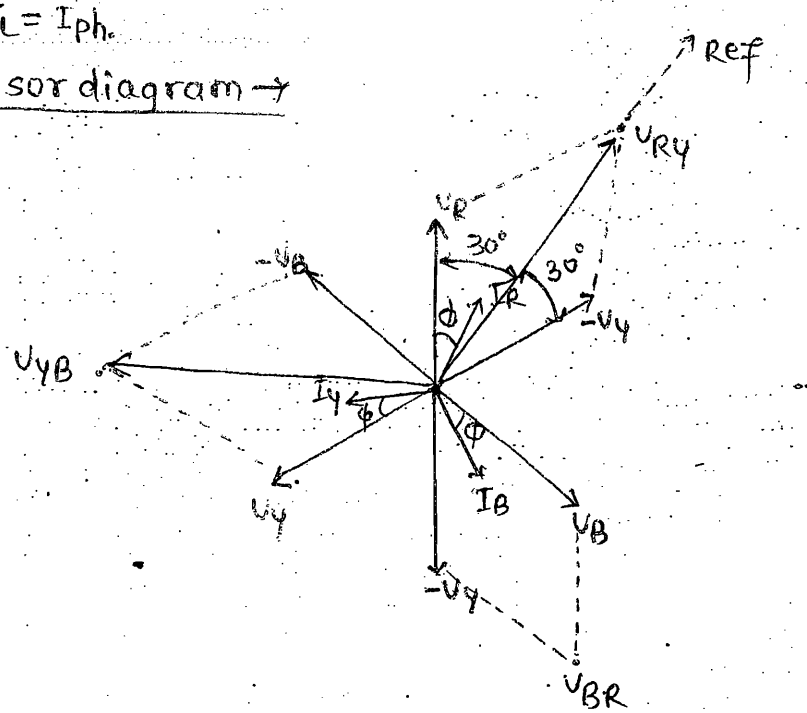



<u>Conclusions →</u>

(1.) No change

(2.) $I_L = I_{ph.}$

(3.) <u>Phasor diagram →</u>




RYB

$V_{RY} = V_R - V_Y$

$\quad = V_R + (-V_Y)$

$V_{YB} = V_Y - V_B$

$\quad = V_Y + (-V_B)$

$V_{BR} = V_B - V_R$

$\quad = V_B + (-V_R)$

$$V_{RY} = \sqrt{|V_R|^2 + |-V_Y|^2 + 2|V_R||-V_Y| \cos 60^\circ}$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

(4.) <u>Angle b/n $V_L$ & $I_L$ →</u>

$RL = 30^\circ - \phi$

$R = 30^\circ$

$RC = 30 + \phi$

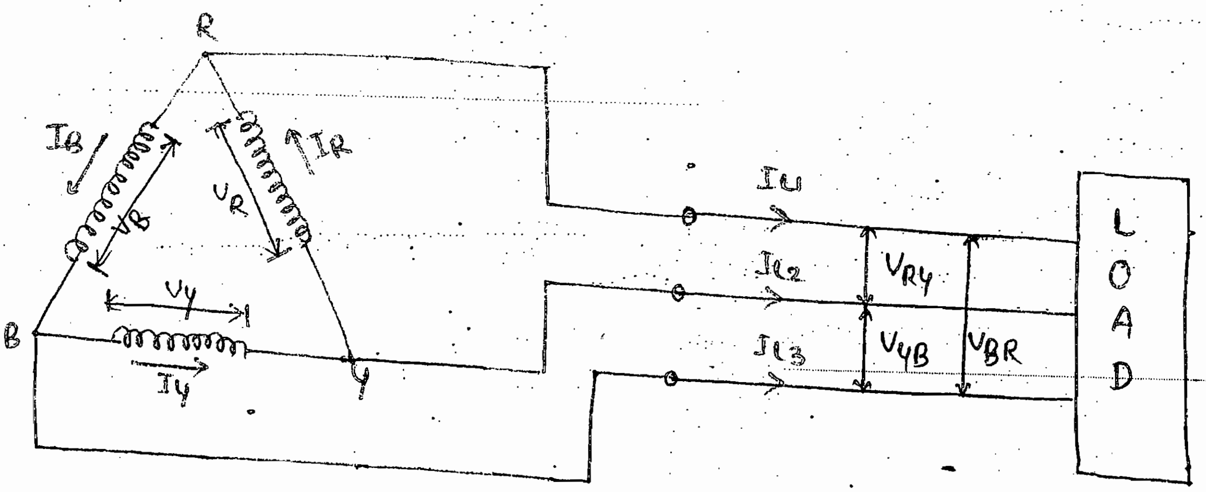
(5.) WRT Ref $\angle V_{RY}, V_{YB}, V_{BR}$

$$V_{RY} = V_L \angle 0^\circ$$
$$V_{YB} = V_L \angle +120^\circ$$
$$V_{BR} = V_L \angle +240 \text{ (or) } -120^\circ$$

(6.) WRT Ref $\angle V_R, V_Y, V_B$

$$V_R = V_{ph} \angle +30^\circ$$
$$V_Y = V_{ph} \angle +150^\circ$$
$$V_B = V_{ph} \angle -90^\circ$$

## Δ Connected genʳ (or) mesh connected genʳ →




Conclusions →

1.) $V_R = V_Y = V_B = V_{ph}$ $\qquad V_{RY} = V_{YB} = V_{BR} = V_L$

$I_R = I_Y = I_B = I_{ph}$ $\qquad I_{L1} = I_{L2} = I_{L3} = I_L$

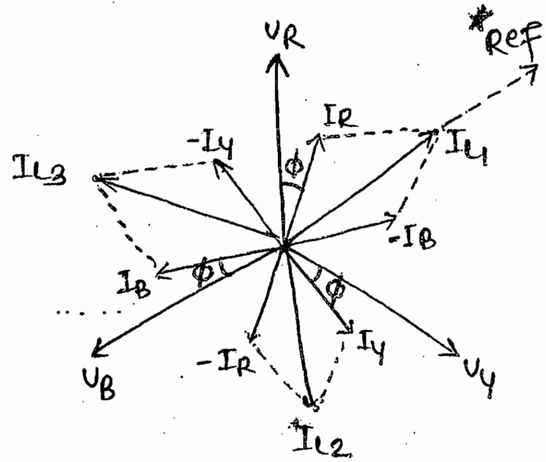
2.) $V_L = V_{ph}$

line vol. = ph. voltage

3.) Applying KCL at R, Y & B

$$I_R = I_{L1} + I_B$$
$$I_B = I_Y + I_{L3}$$
$$I_Y = I_R + I_{L2}$$

Phasor diagram →




$$I_{L1} = I_R - I_B$$
$$I_{L2} = I_Y - I_R$$
$$I_{L3} = I_B - I_Y$$

$$I_{L1} = \sqrt{|I_R|^2 + |-I_B|^2 + 2|I_R||-I_B|\cos 60^\circ}$$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

*** In Δ connection between any 2 adjecent phase currents the angle is always 60°.

(4.) Angle between $V_L$ & $I_L$ →

RL :- $30+\phi$

R :- $30^\circ$

RC :- $30-\phi$

(5.) WRT Ref* $\angle I_{L1}, I_{L2}, I_{L3}$

$$I_{L1} = I_L \angle 0^\circ$$
$$I_{L2} = I_L \angle -120^\circ$$
$$I_{L3} = I_L \angle -240^\circ \text{ (or) } +120^\circ$$

(6.) WRT Ref $\angle I_R, I_Y, I_B$

$$I_R = I_{ph} \angle +30^\circ$$
$$I_Y = I_{ph} \angle +270^\circ \text{ (or) } -90^\circ$$
$$I_B = I_{ph} \angle +150^\circ$$

Analysis for -ve phase seq:→

(1.)
(2.) No change
(3.)

(4.) Angle b/n $V_L$ & $I_L$ →

$$R_L = 30 - \phi$$
$$R = 30^\circ$$
$$R_C = 30 + \phi$$

(5.) WRT $I_{L1}$ $\angle I_{L1}, I_{L2}, I_{L3}$

$$I_{L1} = I_L \angle 0^\circ$$
$$I_{L2} = I_L \angle +120^\circ$$
$$I_{L3} = I_L \angle -120^\circ$$

(6.) WRT Ref $\angle I_R, I_Y, I_B$

$$I_R = I_{ph} \angle +30^\circ$$
$$I_Y = I_{ph} \angle +270^\circ \text{ (OR) } +90^\circ$$
$$I_B = I_{ph} \angle -150^\circ$$

* Power Calculation→

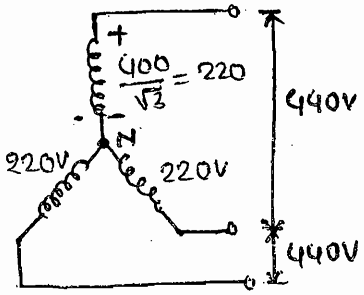
1-$\phi$ :- $P = VI\cos\phi$

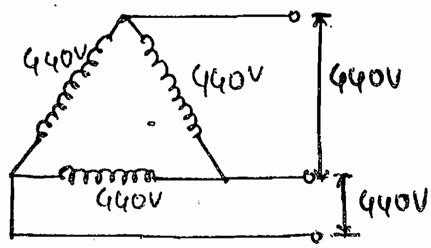
3-$\phi$ →

$$P_{3\text{-}\phi} = 3 V_{ph} I_{ph} \cos\phi$$

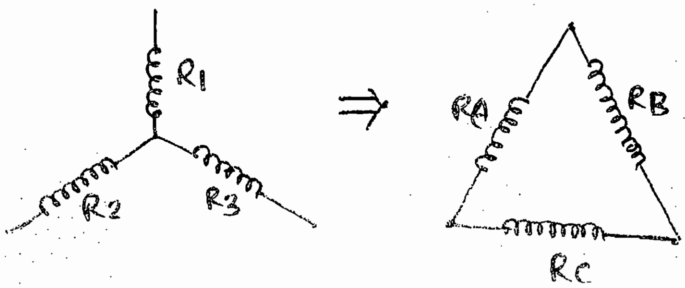
$$P_\lambda = 3 V_{ph} I_{ph} \cos\phi$$
$$= 3 \cdot \frac{V_L}{\sqrt{3}} I_L \cos\phi$$
$$\boxed{P_\lambda = \sqrt{3} V_L I_L \cos\phi}$$

$$P_\Delta = 3 V_{ph} \cdot I_{ph} \cdot \cos\phi$$
$$= 3 V_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi$$
$$\boxed{P_\Delta = \sqrt{3} V_L I_L \cos\phi}$$




Formula →

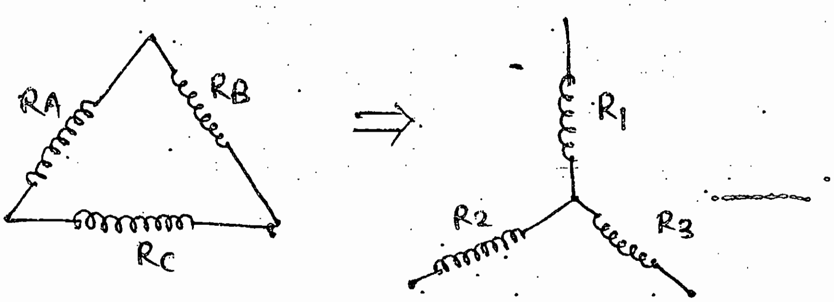



$$S \xrightarrow[\text{Remain}]{\text{OR}} \Delta$$

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$




$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$

$$\Delta \xrightarrow[\text{Remain}]{\text{PUS}} S$$

$$\frac{\text{Product}}{\text{Sum}}$$

$$R_2 = \frac{R_AR_C}{R_A + R_B + R_C}$$

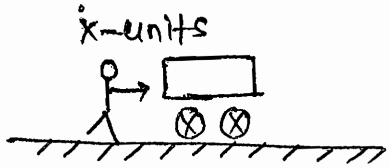
$$R_3 = \frac{R_BR_C}{R_A + R_B + R_C}$$

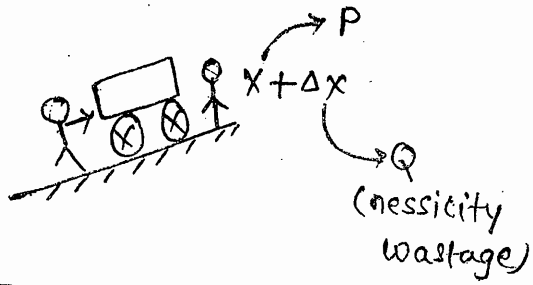
Value or resistance

(3)

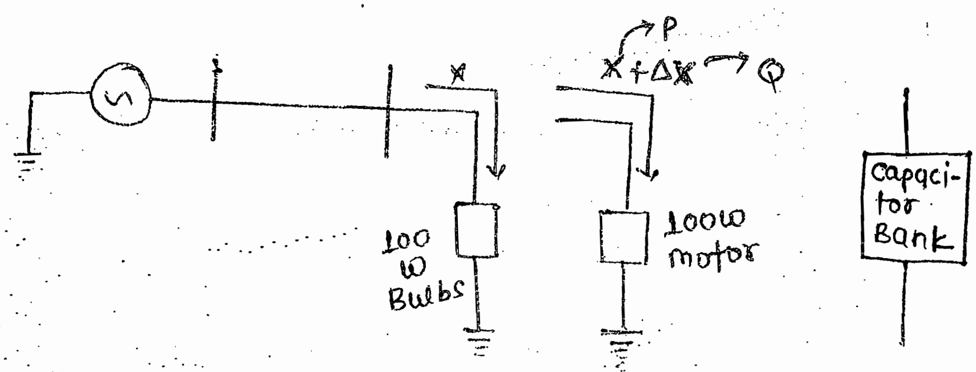
STAR → DELTA

(4) (5.)




| | |
|---|---|
| P → | Active Power |
| Q → | Reactive Power |




(1.) Over load

(2.) $I^2R \uparrow$ $\eta \downarrow$

(3.) Poor Vol. Regn

* A capacitor bank is required for the removing of extra wastage.
* The value of load increases use of capacitor Bank.

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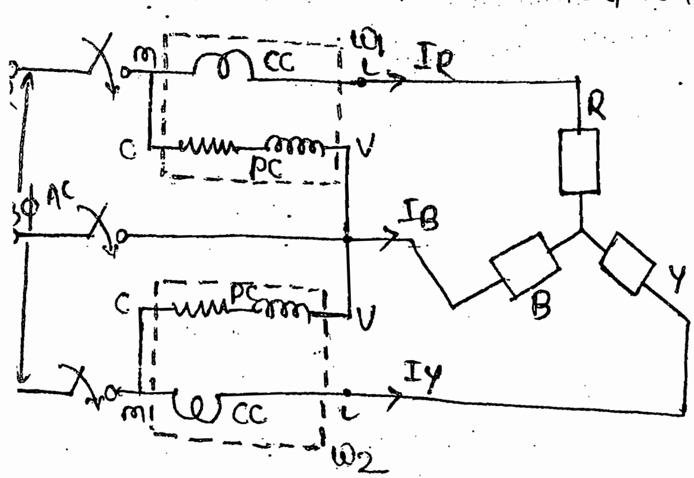
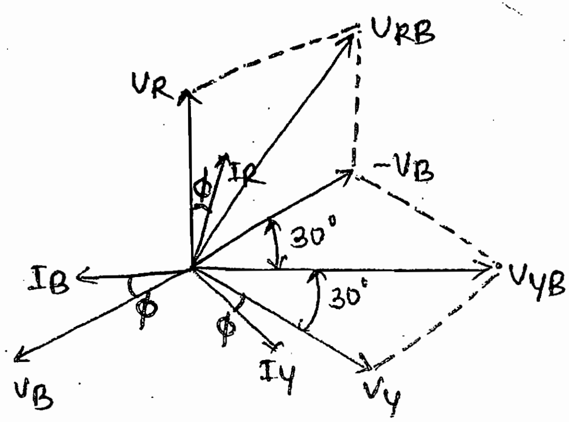
★ Measurement of 3φ power using 2-wattmeter method →
(can be used for bal. & unbalanced load)






$$\omega_1 = V_{RB} I_R \cos(30^\circ - \phi)\ ,\quad \omega_2 = V_{YB} I_Y \cos(30+\phi)$$

For active power consumption

$$\omega_1 + \omega_2 = V_L I_L \left[\cos(30-\phi) + \cos(30+\phi)\right]$$

$$= V_L I_L \cdot (\sqrt{3}\cos\phi)$$

$$\therefore \omega_1 + \omega_2 = \sqrt{3} V_L I_L \cos\phi = P$$

$$\boxed{P = \omega_1 + \omega_2}$$

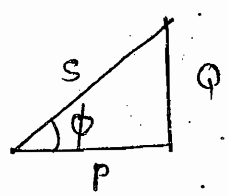
For Reactive power consumption

$$\omega_1 - \omega_2 = V_L I_L \sin\phi$$

$$\boxed{Q = \sqrt{3}(\omega_1 - \omega_2)}$$

From power triangle :-

$$\tan\phi = \frac{Q}{P}$$




$$\phi = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\boxed{\phi = \tan^{-1}\left[\frac{\sqrt{3}(\omega_1 - \omega_2)}{\omega_1 + \omega_2}\right]}$$

* Effect of PE on wattmeter readings →

(i) $\phi = 0°$

$$W_1 = V_L I_L \cos 30°, \quad W_2 = V_L I_L \cos 30°$$

$$\boxed{W_1 = W_2}$$

(ii) $\phi = 30°$

$$W_1 = V_L I_L [\cos(30-30)] = V_L I_L$$

$$W_2 = V_L I_L \cos(30+30) = \frac{V_L I_L}{2}$$

$$\boxed{\frac{W_1}{2} = W_2}$$

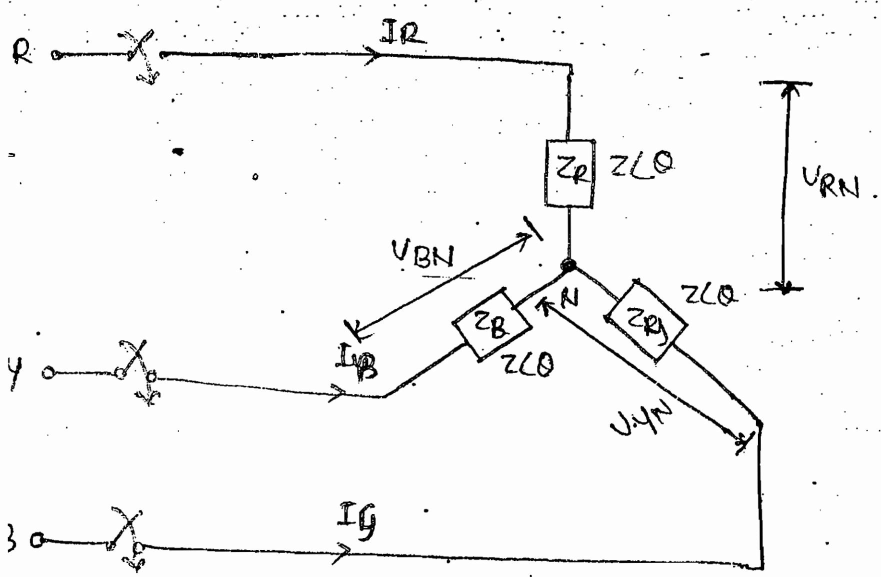
(iii) $\phi = 60°$

$$W_1 = V_L I_L \cos(30-60) = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30+60) = 0$$

Analysis of 3φ balanced ckt →

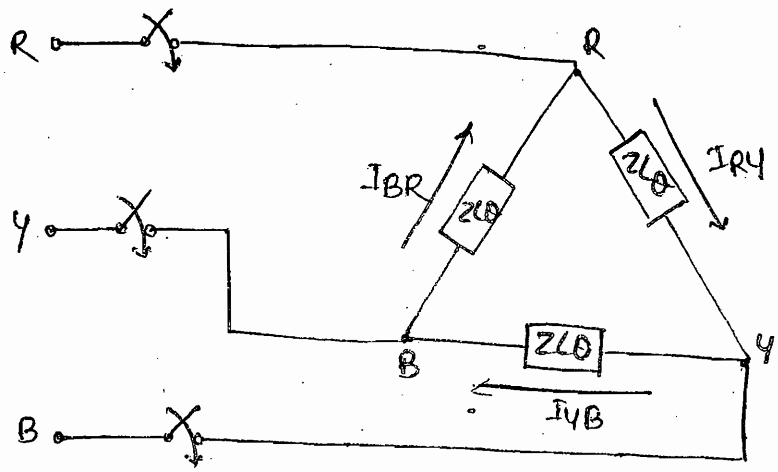
Case (i) → $Y_S$ & $Y_L$ 3φ 3 wire balanced load:-




$$I_R = \frac{V_{RN}}{Z\angle\theta} \quad I_Y = \frac{V_{YN}}{Z\angle\theta} \quad I_B = \frac{V_{BN}}{Z\angle\theta}$$

$$V_{RN} = V_{YN} = V_{BN} = V_{ph} = \frac{V_L}{\sqrt{3}}$$

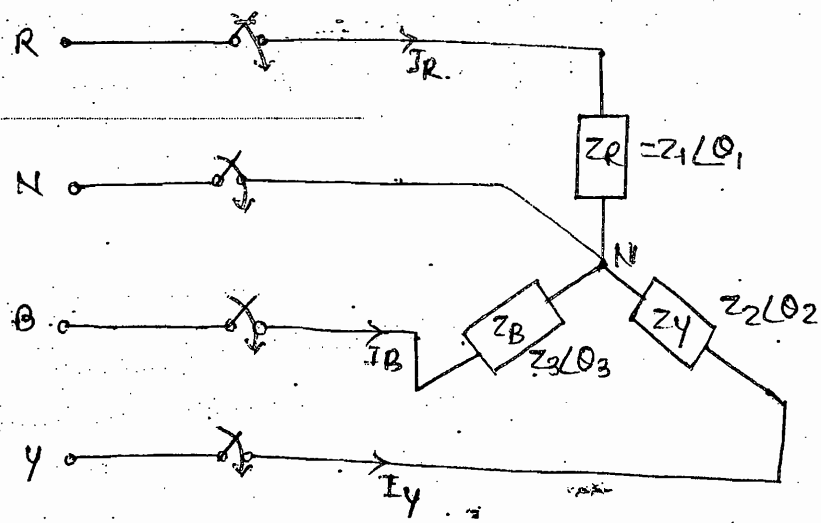
(ii) <u>case(ii)</u> $Y_S - \Delta_L$ (3$\phi$, 3 wire balanced load)




$$\boxed{\begin{aligned} I_{RY} &= \frac{V_{RY}\angle 0^\circ}{Z\angle\theta} \\ I_{YB} &= \frac{V_{YB}\angle -120^\circ}{Z\angle\theta} \\ I_{BR} &= \frac{V_{BR}\angle +120^\circ}{Z\angle\theta} \end{aligned}}$$

* Analysis of 3$\phi$ Unbalanced ckt →

(i) 3$\phi$; 4 wire; Unbalanced Y-load. ($Y_S - Y_L$)



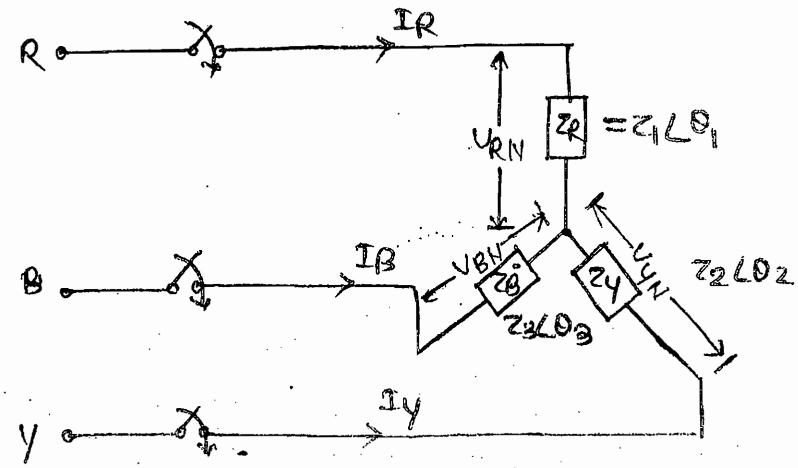

$$\boxed{\begin{aligned} I_R &= \frac{V_{RN}\angle 0^\circ}{Z_R} \\ I_Y &= \frac{V_{YN}\angle -120^\circ}{Z_Y} \\ I_B &= \frac{V_{BN}\angle +120^\circ}{Z_B} \end{aligned}}$$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{V_{RN}}{Z_1\angle\theta_1}\angle 0$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{V_{YN}}{Z_2\angle\theta_2}\angle -120^\circ$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{V_{BN}}{Z_3\angle\theta_3}\angle +120^\circ$$

## (ii) 3ϕ, 3 wire Unbalanced Y-load



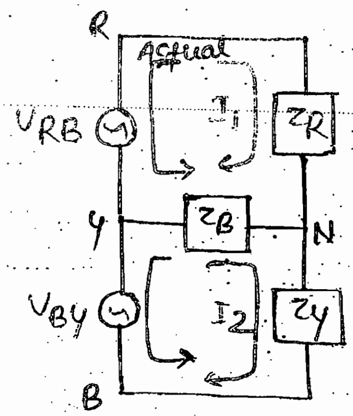

* This is most problematic case because the vol. distribution at every impedance is different which is not acceptable case.

$$V_{RB} = I_1 Z_R + (I_1 - I_2) Z_B$$

$$= I_1 (Z_R + Z_B) - I_2 Z_B \quad \text{---- (i)}$$

$$V_{BY} = (I_2 - I_1) Z_B + I_2 Z_Y$$

$$= -I_1 Z_B + I_2 (Z_Y + Z_B) \quad \text{---- (2)}$$




$$\begin{bmatrix} V_{RB} \\ V_{BY} \end{bmatrix} = \begin{bmatrix} Z_R + Z_B & -Z_B \\ -Z_B & Z_Y + Z_B \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta Z = \left[ (Z_R + Z_B)(Z_Y + Z_B) - Z_B^2 \right]$$

$$\Delta Z = \left[ Z_R Z_Y + Z_R Z_B + Z_B Z_Y + \cancel{Z_B^2} - \cancel{Z_B^2} \right]$$

$$\Delta Z = \left[ Z_R Z_Y + Z_R Z_B + Z_B Z_Y \right]$$

$$I_2 = \frac{1}{\Delta Z} \begin{bmatrix} Z_R + Z_B & V_{RB} \\ -Z_B & V_{BY} \end{bmatrix}$$

$$\boxed{I_2 = \frac{V_{BY}(Z_R + Z_B) + V_{RB} Z_B}{Z_R Z_Y + Z_R Z_B + Z_B Z_Y}}$$

$$I_1 = -\frac{1}{\Delta Z}\begin{bmatrix} V_{RB} & -Z_B \\ V_{BY} & Z_Y+Z_B \end{bmatrix}$$

$$\boxed{I_1 = \frac{V_{RB}(Z_Y+Z_B) + V_{BY}Z_B}{Z_R Z_Y + Z_R Z_B + Z_B Z_Y}}$$

From the eq$^n$ of $I_1$ :-

Our given Ref. vol. are $V_{RY}, V_{YB}, V_{BR}$

$$I_R = I_1 = \frac{-V_{BR}(Z_Y+Z_B) - V_{YB}Z_B}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}$$

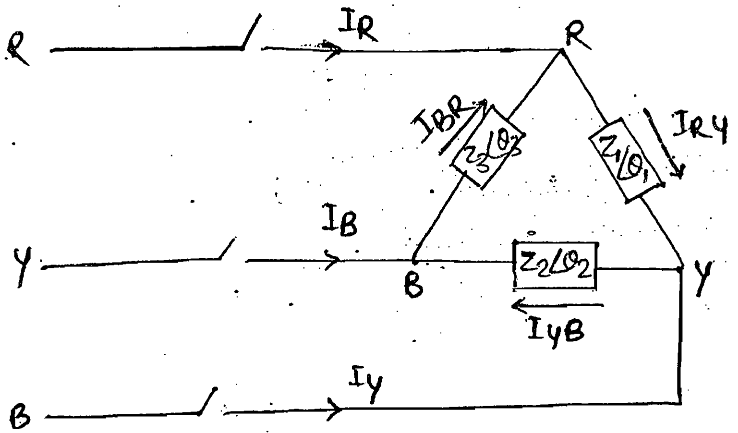
$$I_R = I_1 = -\left[\frac{V_{BR}(Z_Y+Z_B) + V_{YB}Z_B}{Z_Y Z_R + Z_Y Z_B + Z_B Z_R}\right]$$

From the eq$^n$ of $I_2$ :-

$$I_Y = I_2 = \frac{-V_{YB}(Z_R+Z_B) - V_{BR}Z_B}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}$$

$$I_Y = I_2 = -\left[\frac{V_{BY}(Z_R+Z_B) + V_{BR}Z_B}{Z_Y Z_R + Z_Y Z_B + Z_B Z_R}\right]$$

(iii) $3\phi$, 3wire Unbalanced load ( $Y_S - \Delta_L$ ) →

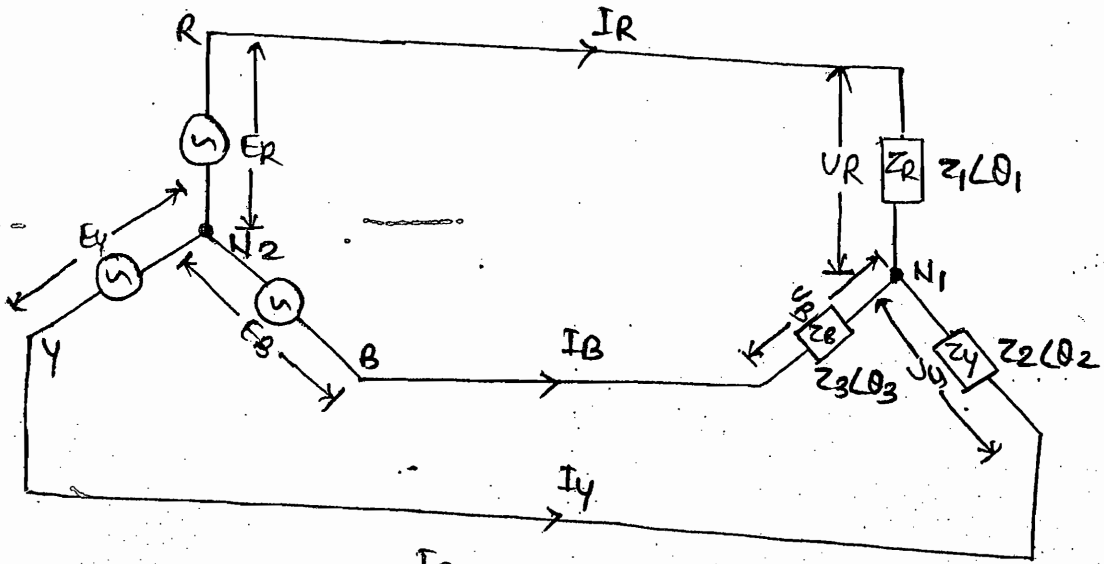


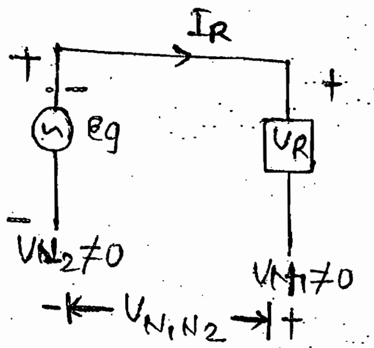

$$I_{RY} = \frac{V_{RY}}{Z_1\angle\theta_1}\angle 0^\circ$$

$$I_{YB} = \frac{V_{YB}}{Z_2\angle\theta_2}\angle -120^\circ$$

$$I_{BR} = \frac{V_{BR}}{Z_3\angle\theta_3}\angle +120^\circ$$

* **Floating Point Neutral →** when the load neutral is at $N_1$ & the source neutral is at $N_2$

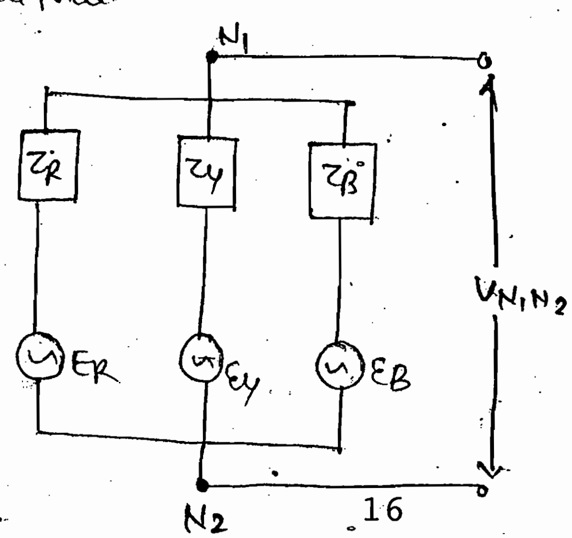



$$-V_{N_1N_2} + E_R - V_R = 0$$

$$V_R = E_R - V_{N_1N_2}$$

→ In 3 wire Y connected load the neutral is not connected to source neutral. Hence when the load is unbalanced, the load will not be at zero volt potential. Hence the vol. of load neutral wrt source neutral is known neutral shift voltage or neutral displacement voltage & the respective load neutral is called as floating point neutral.



By millman's Theorem

$$V_{N_1N_2} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B}$$

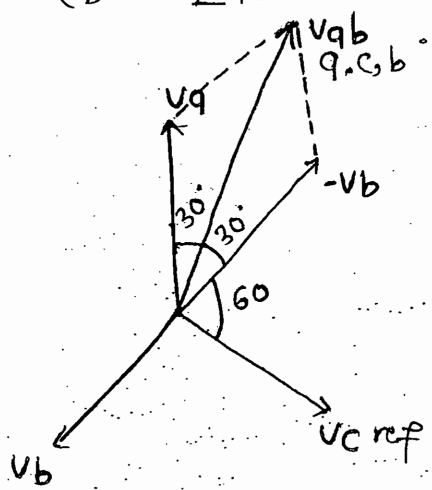
where $Y_R = \frac{1}{Z_R}$, $Y_Y = \frac{1}{Z_Y}$, $Y_B = \frac{1}{Z_B}$

Q.→ The c-$\phi$ voltage of a balanced $\lambda$ connected Supply system is $230\angle{-25^\circ}$. What is the value of $V_{AB}$. If the phase seq. is ACB.
(a) $398\angle{-25^\circ}$ (b) $398\angle{65^\circ}$ (c) $398\angle{-5^\circ}$ (d) $398\angle{-120^\circ}$

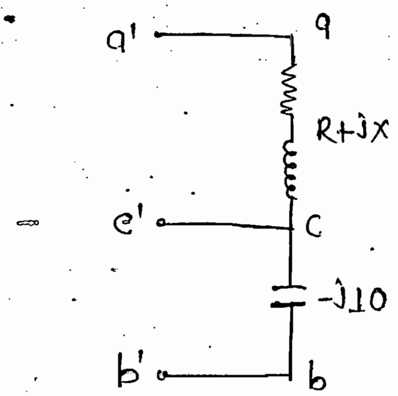
soln→ $V_c = 230\angle{-25^\circ}$

$V_{ab} = \sqrt{3}\,230\angle{-25+90}$

$\boxed{V_{ab} = 398\angle{65}}$




Q.→ The ckt shown in the fig is supplied by a symmetrical 3$\phi$ sys. of voltages. Find the value of R & X; if the current flowing c$\phi$ is 0. Assume abc phase seq.




soln→ phase seq. is abc, so

$V_{ab}\ V_{bc}\ V_{ca}$.

Current flowing through c is 0

$$I = \frac{V_{ca}}{R+jX}\angle{+120^\circ}$$

$$I = \frac{V_{bc}\angle -120}{-j10} = \frac{V_{bc}\angle -120}{10\angle 90°}$$

$$\frac{V_{bc}\angle -120}{10\angle 90} = \frac{V_{ca}\angle 120}{R+jX}$$

$$\frac{\angle -120+90}{10} = \frac{1\angle 120}{R+jX}$$

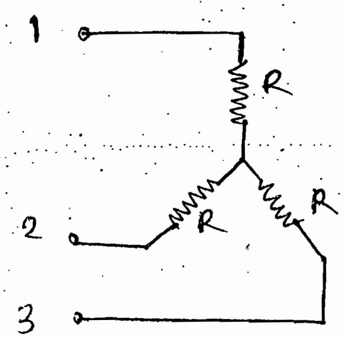
$$\frac{1}{10\angle 30} = \frac{1\angle 120}{R+jX}$$

$$R+jX = (1\angle 120)(10\angle 30)$$

$$R+jX = -8.66+5j$$

$$\boxed{|R| = 8.66\,\Omega,\ |X| = 5\,\Omega}$$

P→ In the ckt shown below find the vol. across each branch as % of $V_L$. Assume 1 2 3 phase seq.



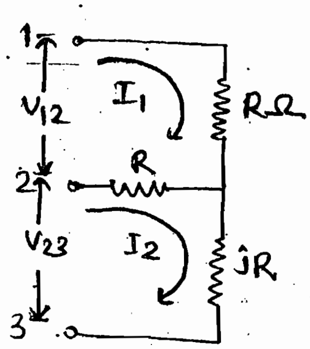

Soln→ Voltage seq. is $V_{12}\ V_{23}\ V_{31}$

$$V_{12} = I_1R + (I_1 - I_2)R$$

$$V_{12}\angle 0° = I_1(2R) - I_2R \text{ ------(i)}$$

$$V_{23}\angle -120 = (I_2 - I_1)R + I_2 jR$$

$$V_{23}\angle -120 = -I_1R + I_2(R+jR) \text{ ----(ii)}$$




$$\begin{bmatrix} V_{12}\angle 0° \\ V_{23}\angle -120 \end{bmatrix} = \begin{bmatrix} 2R & -R \\ -R & R+jR \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta R = 2R(R+\hat{j}R) - R^2$$

$$= 2R^2 + 2\hat{j}R^2 - R^2$$

$$\Delta R = R^2 + 2\hat{j}R^2$$

$$\boxed{\Delta R = R^2(1+2\hat{j}R)}$$

$$I_1 = \frac{1}{\Delta R}\begin{bmatrix} V_{12}\angle 0 & -R \\ V_{23}\angle -120^\circ & R+\hat{j}R \end{bmatrix}$$

$$= \frac{V_{12}\angle 0(R+\hat{j}R) + V_{23}\angle -120^\circ(R)}{R^2(1+2\hat{j})}$$

$$= \frac{V_{12}\angle 0^\circ(1+\hat{j}) + V_{23}\angle -120^\circ}{R(1+2\hat{j})}$$

$$I_1 = \frac{V_L}{R}\left[\frac{1\angle 0^\circ(1+\hat{j}) + 1\angle -120^\circ}{1+2\hat{j}}\right]$$

$$I_1 = (0.231\angle -48^\circ)\frac{V_L}{R}$$

$$I_2 = \frac{1}{\Delta R}\begin{bmatrix} 2R & V_L\angle 0^\circ \\ -R & V_L\angle -120^\circ \end{bmatrix}$$

$$= \frac{1}{\Delta R}\left[V_L\angle -120^\circ(2R) + V_L\angle 0^\circ R\right]$$

$$I_2 = \frac{V_L\angle -120^\circ \times 2 + V_L\angle 0^\circ}{R(1+2\hat{j})}$$

$$= \frac{V_L}{R}\left[\frac{2\angle -120^\circ + 1\angle 0^\circ}{1+2\hat{j}}\right]$$

$$I_2 = (0.774\angle -153^\circ)\frac{V_L}{R}$$

$$(I_1 - I_2) = \frac{V_L}{R}(0.866\angle 12.01^\circ)$$

$V_{1st\,ph.} = 0.23V_L = 23\%$ $\quad$ $V_{3rd\,ph.} = 86\%$

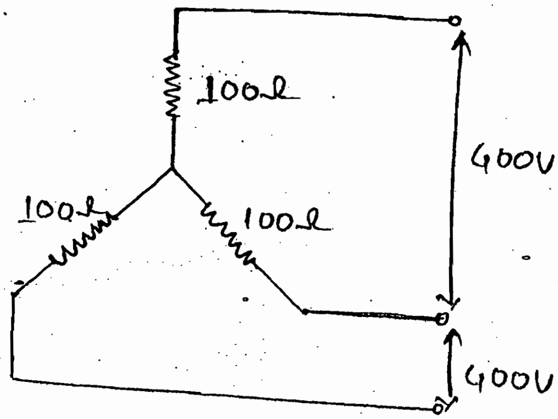
$V_{2nd\,ph.} = 77.4\%$

Q.→ 3, 100Ω resistors are connected to 3ϕ, 400V AC supply. Find Power consume when resistors are connected in

(i) Y Configuration (ii) Δ Configuration

(iii) Open Y (iv) open Δ.

Soln → (i) Y Configuration




$$P = V_L I_L = 400 \times \frac{400}{100} = 1600W$$

(OR)

$$P = 3 V_{ph} I_{ph} \cos\theta$$

$$= 3 \times \frac{400}{\sqrt{3}} \times \frac{4}{\sqrt{3}} \times 1$$

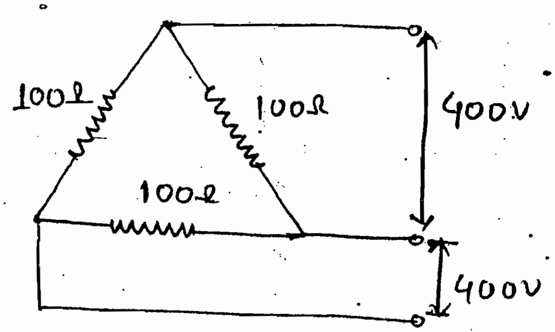
$$\boxed{P = 1600W}$$

(ii)

$$P = 3 V_{ph} I_{ph} \cos\phi$$

$$= 3 \times 400 \times 4$$

$$\boxed{P = 4800W}$$

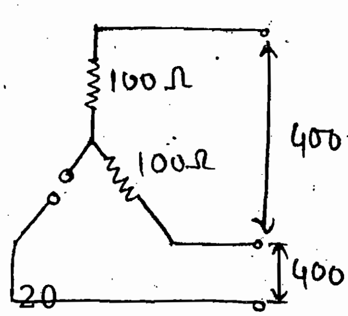



(iii)

$$P_{\curlywedge} = \frac{400^2}{200}$$

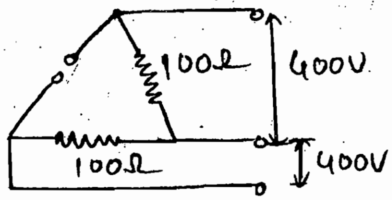
$$= \frac{1600}{200}$$

$$\boxed{P = 800W}$$

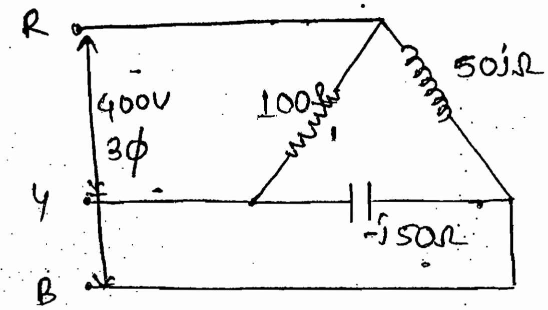


(iv) $P_\Delta = \frac{800^2}{200}$

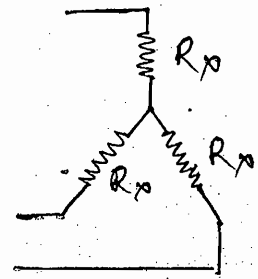
$\boxed{P_\Delta = 3200W}$




Q→ A set of 3 equal resistor of value $R_x$ are connected in $\lambda$ across RYB consumes the same power as the unbalanced Δ connected load as shown in the fig. Find the value of $R_x$.




⇒




Sol^n →

$$P_\Delta = \frac{400^2}{100} = 1600W$$

$$P_\lambda = \frac{400^2}{3R_x}$$

$$1600 = \frac{400^2}{3R_x}$$

$$\boxed{R_x = 100\Omega}$$

Q.→ 3 identical $\lambda$ connected resistors at 1 pu are connected to an unbalanced 3φ supply. The load neutral is isolated. The symmetrical componant of the line vol. in pu are

$$V_{ab_1} = X\angle\theta_1 \;;\; V_{ab_2} = Y\angle\theta_2$$

If pu cal. are with the respectives base values, the phase to neutral seq; voltages are:-

(a.) $V_{an_1} = X\angle\theta_1 + 30^\circ$
$V_{an_2} = Y\angle\theta_2 - 30^\circ$

(b) $V_{an_1} = X\angle\theta_1 - 30$
$V_{an_2} = Y\angle\theta_2 + 30$

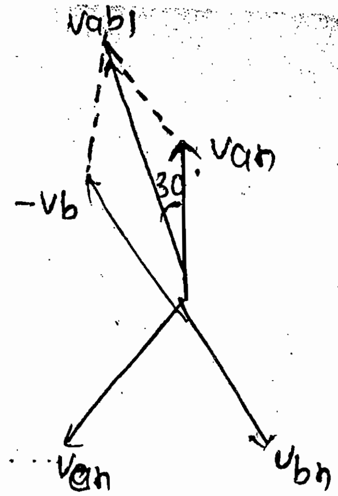
(c) $V_{an_1} = \frac{X}{\sqrt{3}}\angle\theta_1 - 30^\circ$
$V_{an_2} = \frac{Y}{\sqrt{3}}\angle\theta_2 + 30^\circ$

(d) $V_{an_1} = \frac{X}{\sqrt{3}}\angle\theta_1 - 60$
$V_{an_2} = \frac{Y}{\sqrt{3}}\angle\theta_2 - 60^\circ$

sol$^n \rightarrow$ +ve phase seq.

$$V_{ab1} = \sqrt{3} \cdot V_{an1}$$

$$V_{an1} = \frac{X \angle \theta_1 - 30}{\sqrt{3}}$$




-ve phase seq

$$V_{an2} = \frac{Y}{\sqrt{3}} \angle \theta_2 + 30$$

Q.→ A 3$\phi$ balanced $\curlywedge$ connected vol. source with freq. $\omega$ rad/sec. is connected to a $\curlywedge$ connected balanced load which is purely inductive. The instantaneous line current & phase to neutral voltages are denoted by $I_a, I_b ; I_c$ & $V_{an}, V_{bn}, V_{cn}$ respectively & their Rms values are denoted by $V$ & $I$.

If $R = \begin{bmatrix} V_{an} & V_{bn} & V_{cn} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$

sol$^n \rightarrow$

$$V_{an} = V_{ph} \angle 0$$

$$V_{bn} = V_{ph} \angle -120$$

$$V_{cn} = V_{ph} \angle +120$$

$$I_a = I \angle -90$$

$$I_b = I \angle -210$$

$$I_c = I \angle +30$$

$$R = \begin{bmatrix} \frac{-V_{bn}}{\sqrt{3}} + \frac{V_{cn}}{\sqrt{3}}, & \frac{V_{an}}{\sqrt{3}} - \frac{V_{cn}}{\sqrt{3}}, & \frac{-V_{an}}{\sqrt{3}} + \frac{V_{bn}}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$R = \left[ \frac{-V_{bn}}{\sqrt{3}} i_a + \frac{V_{cn}}{\sqrt{3}} i_a + \frac{V_{an}}{\sqrt{3}} i_b - \frac{V_{cn}}{\sqrt{3}} i_b - \frac{V_{an}}{\sqrt{3}} i_c + \frac{V_{bn}}{\sqrt{3}} i_c \right]$$

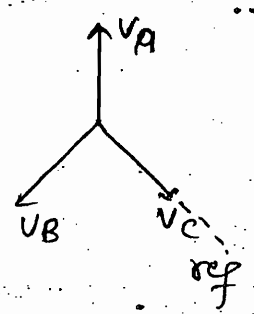
$$R = \frac{1}{\sqrt{3}}\left[V_{bn}(i_c - i_a) + V_{an}(i_a - i_b) + V_{an}(i_b - i_c)\right]$$

$$R = \frac{IV}{\sqrt{3}}\left\{\left[(1\angle -120)(1\angle 30 - 1\angle -90)\right] + \left[(1\angle 120)(1\angle -90 - 1\angle -210)\right] + \left[(1\angle 0)(1\angle -210 - 1\angle 30)\right]\right\}$$

$$R = \frac{VI}{\sqrt{3}}(0)$$

$$\boxed{R = 0}$$

Q.→ A 3$\phi$, 4 wire CBA system with an effective line voltage of 169.7V has 3 impedances of $20\angle -30°$ in a $\lambda$ connection. Determine the line current.




Soln →

$V_L = 169.7V$

$Z = 20\angle -30°$

$$I_A = I_{L_1} = \frac{V_L/\sqrt{3}}{Z} = \frac{169.7/\sqrt{3}\angle +120}{20\angle -30°} = 4.9\angle 150°$$

$$I_B = I_{L_2} = \frac{98\angle -120°}{20\angle -30°} = 4.9\angle -90$$

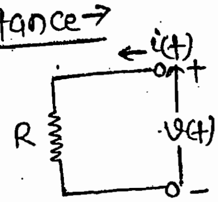
$$I_C = I_{L_3} = \frac{98\angle 0}{20\angle -30°} = 4.9\angle 30°$$

Sagar Sen
8871453536

DATE-28/11/14

***Basic componants of any ele. N/w→**

**(1.) Resistance→**




* Linear
* Bilateral.

$$\boxed{\begin{array}{l} v(t) = R\,i(t) \\ i(t) = \frac{1}{R}\cdot v(t) \end{array}}$$ ...In time domain

$$\boxed{\begin{array}{l} V = R.I \\ I = \frac{1}{R}.V \end{array}}$$ .....using phasors

.....For sinusoidal excitation.

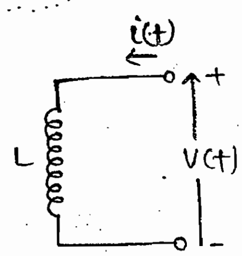
$$\boxed{\begin{array}{l} V(s) = R\,I(s) \\ I(s) = \frac{V(s)}{R} \end{array}}$$ ...in s-domain

$s = \sigma + j\omega$
complex Freq.

$I = |I|\angle\theta$
$V = |V|\angle\theta$

* If the supply is given sinusoidal; then analysis is done only in the phasors.

**(2) Inductor→**




$\boxed{\phi_m \propto i}$

$\boxed{\phi_m = Li}$ .... linear inductor.

$$\boxed{\begin{array}{l} v(t) = L\frac{di(t)}{dt} \\ i(t) = \frac{1}{L}\int_0^t v(t) \end{array}}$$

$$\boxed{\begin{array}{l} V(s) = SL\cdot I(s) \\ I(s) = \frac{1}{LS}V(s) \end{array}}$$ ......in s-domain

assuming zero initial condition.

$$\boxed{\begin{array}{l} Z_L = SL \\ Y_L = \frac{1}{SL} \end{array}}$$

$$\boxed{\begin{array}{l} V = j\omega L\cdot I \\ I = \frac{1}{j\omega L}V \end{array}}$$ .... for sinusoidal excitation

....using phasors $(s = j\omega)$

$$\boxed{\begin{array}{l} Z_L = j\omega L \\ Y_L = \frac{1}{j\omega L} \end{array}}$$

* Inductor; the voltage lead current by 90° $(V(s) = j\omega I(s))$.

## *some points→

* In a linear element the variation b/n the terminal vol. & the terminal current is linear either in the time domain (or) in s-domain (or) in both domains.

* In a bilateral element the terminal current i(t) flows in either dirn irrespective of the type of polarity of the vol. source applied b/n the 2 terminals of the element.

* The excitation to the ele. n/w may be non-sinusoidal (or) sinusoidal in nature.

When the excitation is not sinusoidal any given n/w can be analysed either in the time domain (or) in the Laplace domain.

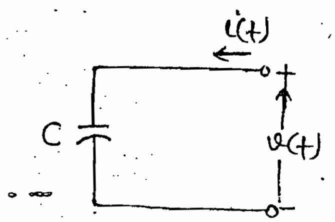
When the excitation is sinusoidal then any given n/w can be analysed only by using phasors by substituting $s=j\omega$.

When the ele. n/w is analysed in the s-domain the advantages are:-

i) Any differential (or) integral eqn. is xformed to a linear eqn & the mathematical manipulation becomes simpler

ii) The initial condn if any present in the ckt elements are automatically taken care of.

### i Capacitor →




$\boxed{q \propto v}$

$\boxed{q = Cv}$ ..... linear element.

$$\boxed{\begin{aligned} i(t) &= C\frac{dv(t)}{dt} \\ v(t) &= \frac{1}{C}\int_0^t i(t)\,dt \end{aligned}}$$ ..... in time domain

$$\boxed{\begin{aligned} I(s) &= Cs\,V(s) \\ V(s) &= \frac{1}{sC}\cdot I(s) \end{aligned}}$$ ..... in s-domain ..... assumin zero initial condn

$$\boxed{\begin{aligned} I &= j\omega C\cdot V \\ V &= \frac{1}{j\omega C}\cdot I \end{aligned}}$$ ..... using phasors ..... for sinusoidal excitation ($s=j\omega$)

$$\boxed{\begin{aligned} Z_C &= \frac{1}{SC} \\ Y_C &= SC \end{aligned}}$$

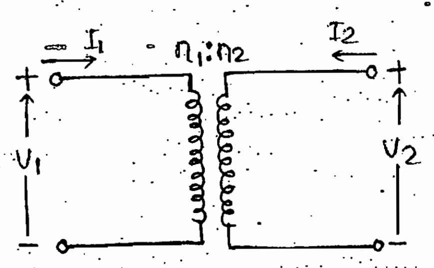
$$\boxed{\begin{array}{l} Z_C = \frac{1}{j\omega C} \\ Y_C = j\omega C \end{array}}$$

* Voltage ~~lead~~ ~~or~~ lags by current by 90°.

| $\omega$ | L | C |
|---|---|---|
| $\omega=\omega$ | $j\omega L$ | $\frac{1}{j\omega C}$ |
| $\omega=0$ | SC | OC |
| $\omega=\infty$ | OC | SC |

* Electrolytic capacitor is non linear capacitor.
  In the n/w we will talk about other then electrolytic capacitor.
* The advantage of the this capacitor is that small size.

## (4.) Transformer→




$$\boxed{\frac{V_1}{V_2} = \frac{n_1}{n_2} \; ; \; \frac{I_1}{I_2} = \frac{n_2}{n_1}}$$

$n_2 > n_1$ ..... step up

$n_2 < n_1$ ..... step down

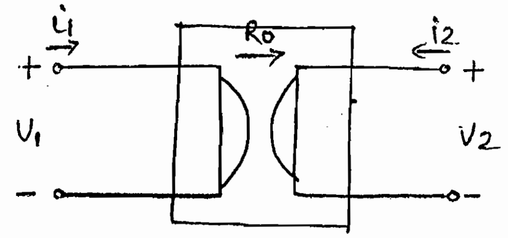
If $n_1 = 1$, $n_2 = 10$

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} = \frac{10}{1} \; ; (>1)$$

$$\frac{I_2}{I_1} = \frac{n_1}{n_2} = \frac{1}{10} \qquad \frac{V_2}{V_1} \cdot \frac{I_2}{I_1} = 1 \qquad \boxed{V_2 I_2 = V_1 I_1}$$

* Any step up Xmer does not work as an ampr since the power at the o/p terminal is same as the power at the i/p terminal.
* For any device to work as a ampr the power at the o/p terminal must always be more than the power at the i/p terminal.

## (5.) Generator→




$R_0$ = Cofficient of gyrator

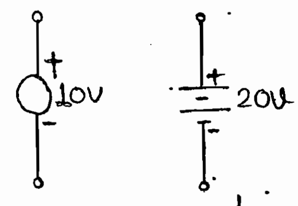
$$\boxed{\begin{aligned} v_1 &= +R_0 i_2 \\ v_2 &= -R_0 i_1 \end{aligned}}$$

* Impedance inversion.
* A gyrator is a 4 terminal (or) a 2 port device.
* It can be made by using an OPAMP along with some externaly connected R & C componants.
* The cofficient of the gyrator depends upon the parameters of the opamp & numerical values of externaly connected R & C componants.

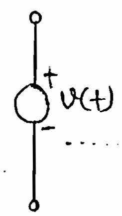
  This cofficient has a fixed value for a given gyrator.
* It can be used to simulate eq. value of inductance L.
* The gyrator shows an impedance inversion.

  Therefore for a capacitive load the i/p impedance is inductive & vice-versa.

## (6.) Voltage Sources →




dc voltage source

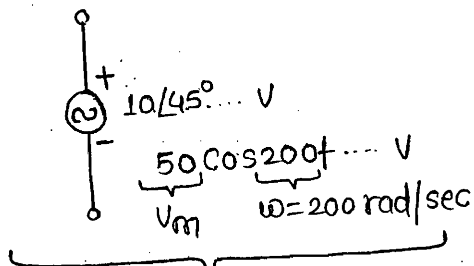



$2u(t)$

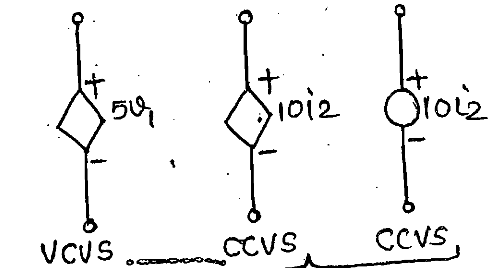
$5e^{-t}u(t)$

$20te^{-5t}\cos 200t$

time-varying non-sinusoidal voltage sources.




sinusoidal voltage sources




... Controlled (or) dependent voltage source

* while analysing any ele. n/w using KVL & KCL eqn the independent & dependent sources are handled exactly in the same manner except in the following 2 cases:-

(i) Application of superposition theorem.

(ii) Application of Thevenin's & Nortons therom.

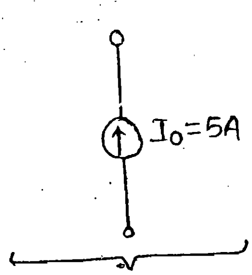
In such cases:-

(1) All independent vol. sources are SC (or) replaced by there internal impedances.

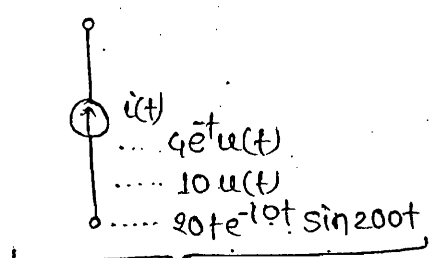
(2) All independent current sources are OC (or) replaced by there internal impedances.

(3) All dependent vol. & current sources remain as they are & therefore this sources are neither SC nor OC.

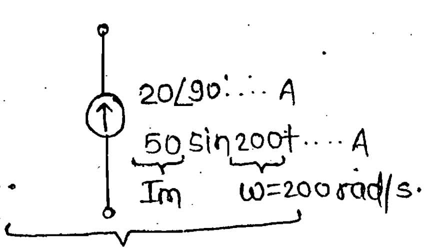
(7) Current sources →



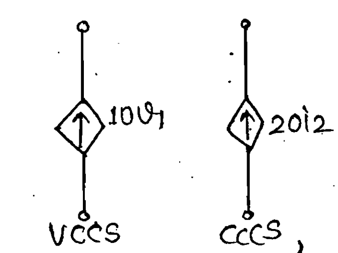

dc current source




timevarying non-sinusoidal current sources



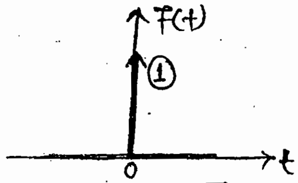

sinusoidal current sources




controlled (or) dependant current sources.

## * Basic fns →

### (1.) Unit impulse $f^n$ →




$$f(t) = \boxed{\begin{array}{l} \delta(t) = \infty \\ \quad\;\; = 0 \end{array}} \begin{array}{l} ; t=0 \\ ; t\neq 0 \end{array}$$

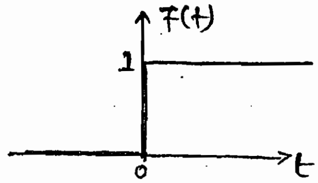
* $\displaystyle\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

* $\displaystyle\int_{-\infty}^{\infty} g(t) \cdot \underbrace{\delta(t)}_{\text{exists at } t=0} dt = g(t)\Big|_{t=0} = g(0)$

* $\displaystyle\int_{-\infty}^{\infty} g(t) \cdot \underbrace{\delta(t-T)}_{\text{exists at } t=T} \cdot dt = g(t)\Big|_{t=T} = g(T)$

* $\mathcal{L}[\delta(t)] = 1$

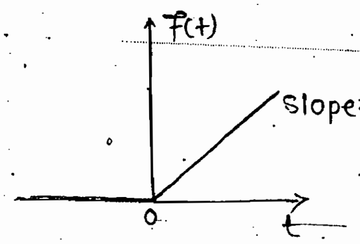
### (2) Unit step $f^n$ →




$$f(t) = \boxed{\begin{array}{l} u(t) = 0 \\ \quad\;\; = 1 \end{array}} \begin{array}{l} ; t<0 \\ ; t>0 \end{array}$$

* $\mathcal{L}[u(t)] = \frac{1}{s}$

### ) Unit Ramp $f^n$ →



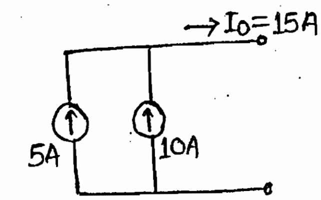

$$\boxed{f(t) = \begin{array}{l} 0 \\ t \end{array}} \begin{array}{l} ; t<0 \\ ; t>0 \end{array}$$

* $\boxed{f(t) = t \cdot u(t) \text{ (OR) } r(t)}$

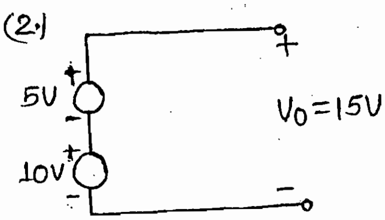
* $\boxed{\mathcal{L}[r(t)] = \frac{1}{s^2}}$

## * Basic Concepts →

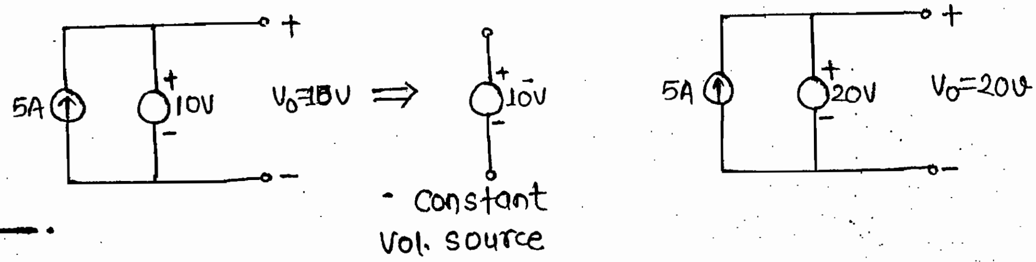
(1.)

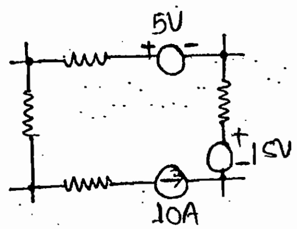



(2.)




(3.)



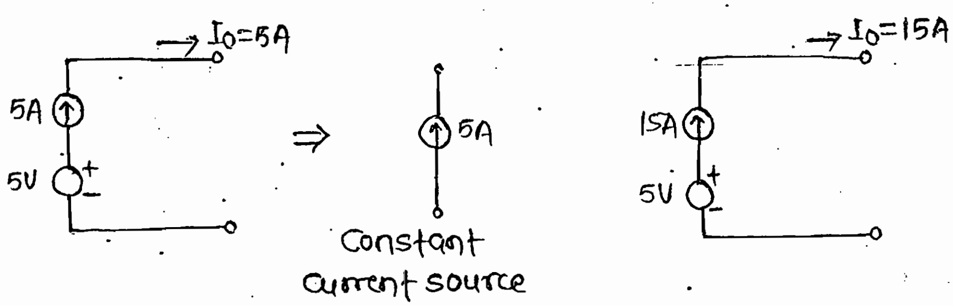



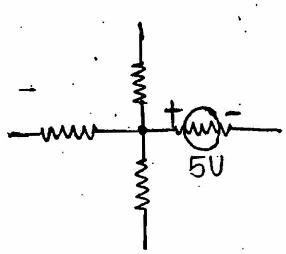

* A parallel combination of a vol. source & a current source behave as a const vol. source.

* The vol. across any current source is purely arbitary & depends only upon externaly connected vol. source (or) externaly connected elements.

* We can't write a KVL eqn in a closed loop which includes a const. current source since the vol. across it is purely arbitary & is unknown.

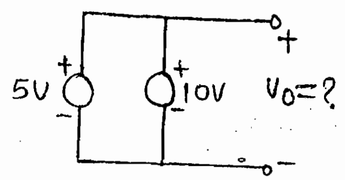
(4.)

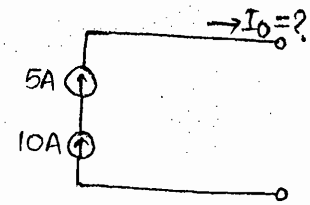


* A series combination of a current source & a vol. source behave as const. current source.
* The current through any vol. source is purely arbitary & depands only upon externaly connected current source (or) externaly connected elements.
* We can't write KCL eqⁿ at a node with which a const. vol. source is connected.
Since current through it is arbitory & is unknown.

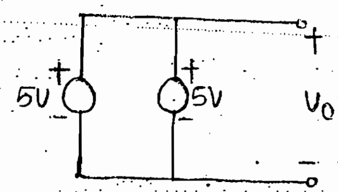
5.)

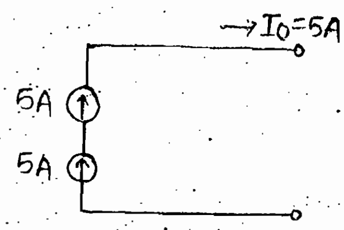



NOT allowed.

)

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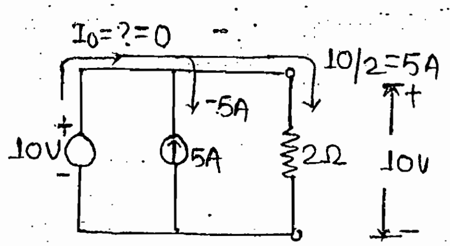


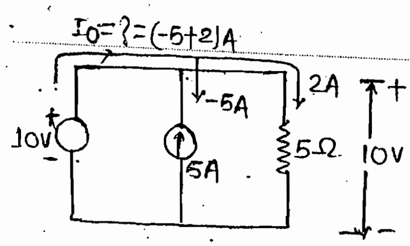


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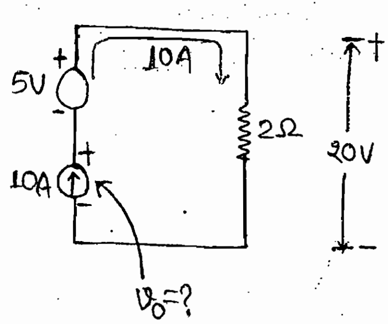
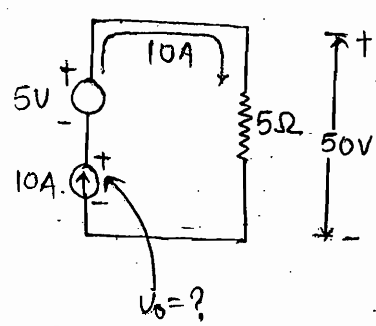



By KVL; $-V_0-5V+20=0$

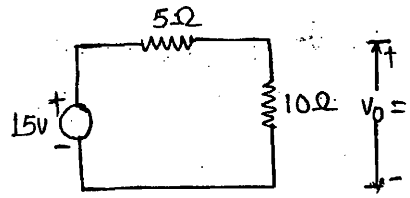
$\boxed{V_0=+15V}$




By KVL; $-V_0-5+50=0$

$\boxed{V_0=45V}$

(9.)

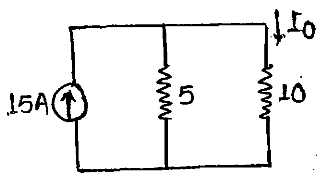



$V_0 = \left(\frac{10}{10+5}\right) \times 15 \ldots V$ * This is applicable only when Resistances are in series.

$V = IR$ Potential deviding Theorem

$\boxed{V \propto R}$

(10.)

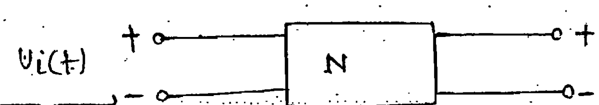



$I_0 = \left(\frac{5}{5+10}\right) \times 15 \ldots A$

$V = IR$ Current deviding theorem.

$\boxed{I \propto \frac{1}{R}}$

## * Impulse responce of any elect. N/ω →




$\underbrace{V_i(t)}_{\text{Excitation}}$ $\underbrace{V_0(t)}_{\text{Responce}}$

Excitation
- $V_i(s)$ Non-Sinusoidal
- $V_i(\omega)$ Sinusoidal

Responce
- $V_0(s)$
- $V_0(\omega)$

## * Transfer $F^n$ →

$$\boxed{H(s) = \frac{V_0(s)}{V_i(s)}}$$

$$V_0(s) = H(s) \cdot V_i(s)$$

If; $V_i(t) = \delta(t)$

$V_i(s) = 1$

$V_0(s) = H(s)$

$V_0(t) = \mathcal{L}^{-1}[H(s)]$

$= h(t) \ldots\ldots$ Impulse Responce of N/ω

* The TF of any n/w represents the ratio of the o/p vol. to the i/p vol. each in the s-domain assuming all the initial condn in various element of the n/w to be 0.

* The TF helps to cal. the corrosponding responce of N/w for specified excitation. For a given n/w the TF has a fixed value.

* The impulse responce fn represents the o/p vol. of the n/w when the i/p vol. represents a unit impulse fn.

* The impulse responce also represents inverse lapalace Xform of the TF of the n/w.

Eg:- If $v_i(t) = e^{-t}u(t)$ ; $v_o(t) = e^{-2t}u(t)$

To find $V_o(t)$ if $V_i(t) = e^{-3t}u(t)$

soln→

$v_i(t) = e^{-t}u(t)$ ; $V_o(t) = e^{-2t}u(t)$

$v_i(s) = \frac{1}{s+1}$ ; $V_o(s) = \frac{1}{s+2}$

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{s+1}{s+2}$$

$$\boxed{H(s) = \frac{s+1}{s+2}} = 1 - \frac{1}{s+2}$$

$$h(t) = \mathcal{L}^{-1}H(s)$$

$$h(t) = \delta(t) - e^{-2t}u(t)$$

Now; $H(s) = \frac{s+1}{s+2}$

$$V_i(t) = e^{-3t}u(t)$$

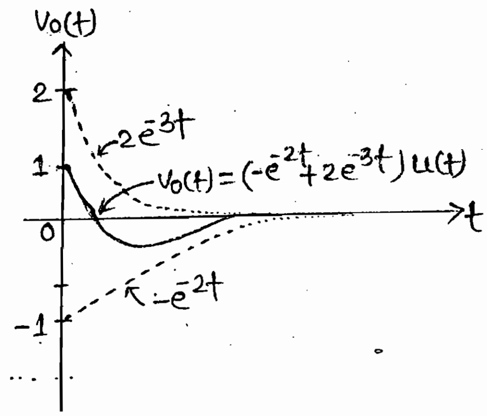
$$V_i(s) = \frac{1}{(s+3)}$$

$$V_o(s) = H(s) \cdot V_i(s)$$

$$= \frac{(s+1)}{(s+2)(s+3)}$$

$$V_o(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

$$v_o(t) = -e^{-2t}u(t) + 2e^{-3t}u(t) = (-e^{-2t} + 2e^{-3t})\,u(t)$$




$V_o(0^+) = 1$ ...... Initial value

...... Initial condⁿ

$V_o(\infty) = 0$ ..... Final value

...... Steady state value

..... Final condⁿ

## * Initial & Final value theorem →

$$f(t) \rightarrow v(t),\ i(t)$$

**Initial value →**

$$f(0^+) = f(t)\Big|_{t=0^+}$$

$$\boxed{f(0^+) = \lim_{s\to\infty}[sF(s)]}$$ ...... Initial value theorem.

**Final value →**

$$f(\infty) = f(t)\Big|_{t=\infty}$$

$$\boxed{f(\infty) = \lim_{s\to 0}[sF(s)]}$$ ...... Final value theorem.

* This is valid for other than oscillatory fⁿ.

DATE- 29/11/14

## * Laplace Xform of some basic $f^n$ →

(1.) $\delta(t) = 1$

(2.) $u(t) = \frac{1}{s}$

(3.) $r(t) = \frac{1}{s^2}$

(4.) $t^n u(t) = \frac{\lfloor n}{s^{n+1}}$

(5.) $e^{-at} = \frac{1}{s+a}$

(6.) $f(t-T) = F(s)e^{-sT}$

(7.) $\sin bt = \frac{b}{s^2+b^2}$

(8.) $\cos bt = \frac{s}{s^2+b^2}$

(9.) $\frac{df(t)}{ds} = sF(s) - f(0^+)$

$f(0^+) = f(t)\big|_{t=0^+}$

(10.) $\int_0^t f(t)dt = \frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}$

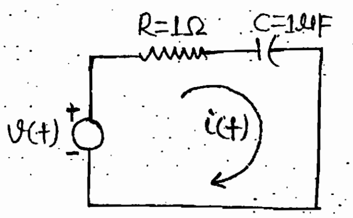
$f^{-1}(0^+) = \left[\int_0^t f(t)dt\right]_{t=0^+}$

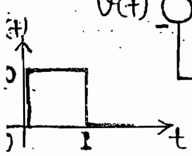
(11.) $e^{-at}\sin bt = \frac{b}{(s+a)^2+b^2}$

(12.) $e^{-at}\cos bt = \frac{s+a}{(s+a)^2+b^2}$

(13.) $te^{-at} = \frac{1}{(s+a)^2}$

Example → Series RC ckt



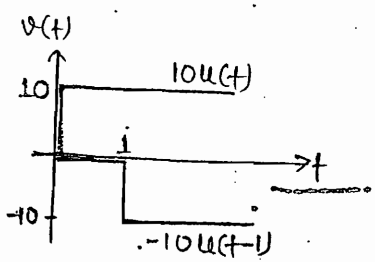




Excitation

$\underbrace{i(t)}$
Responce

To find $i(t)$ for $t>0$

assume: zero IC.




$v(t) = 10u(t) - 10u(t-1)$

$v(s) = \frac{10}{s} - \frac{10}{s}e^{-s}$

$= \frac{10}{s}(1-e^{-s})$

applying KVL on the given ckt

$$R\,i(t) + \frac{1}{C}\int_0^t i(t)dt = v(t)$$

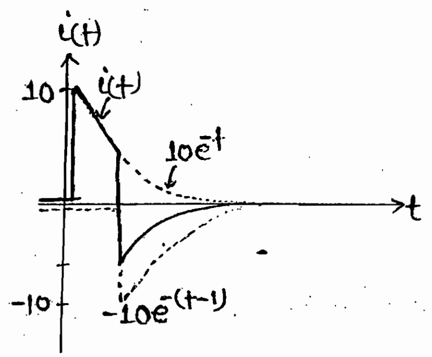
$$\Rightarrow I(s) + \frac{I(s)}{s} + \cancel{\frac{i^{-1}(0^+)}{s}} = \frac{10}{s}(1-e^{-s})$$

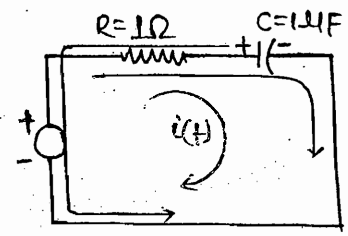
$$\rightarrow \left(\int_0^t i(t)dt\right)\Big|_{t=0^+} = Q(t)\Big|_{t=0^+} = Q(0^+) = 0$$

$$I(s) \times \frac{s+1}{s} = \frac{10}{s}(1-e^{-s})$$

$$I(s) = \frac{10}{(s+1)} - \frac{10}{(s+1)} e^{-s}$$

$$\boxed{i(t) = 10e^{-t}u(t) - 10e^{-(t-1)} \cdot u(t-1)}$$



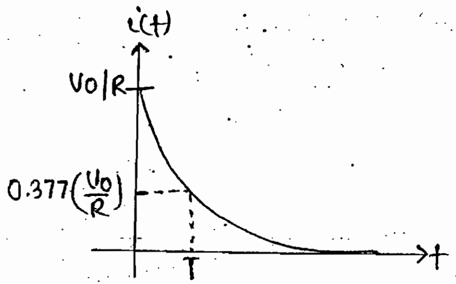

∗ If $v(t) = V_0 u(t)$ ; then $i(t) = \frac{V_0}{R} e^{-t/T} \cdot u(t)$

$T = RC$ ....... Time constant

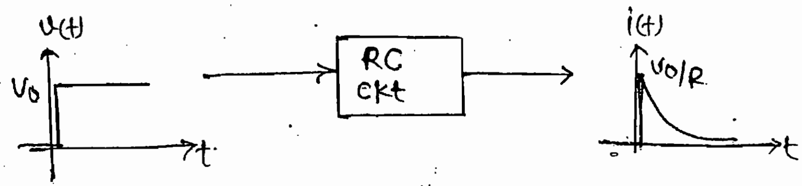
Time constant →

$$i(t) = \frac{V_0}{R} e^{-t/T} u(t)$$

at $t=T$; $i(t) = \frac{V_0}{R} e^{-1} \triangleq 0.377 \frac{V_0}{R}$




Summary → Series RC ckt




$$i(t) = \frac{V_0}{R} e^{-t/T} \cdot u(t)$$

$$T = RC$$

* Initialy at $t=0$ any capacitor acts as a sc, so that the current through the RC n/w is max$^m$.

At $t=\infty$ (or) in the steady state cond$^n$ any capacitor acts as an oc; so that the current through the n/w is 0.

* For pulse excitation if reversal of current takes place in series RC n/w.

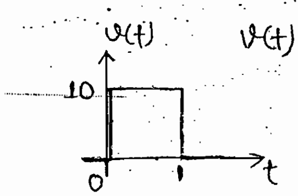
* For step excitation the current through RC n/w decreases exponantialy whose rate is controlled by the time const. of n/w.

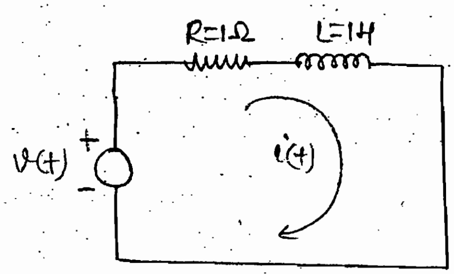
* The time cons. of RC n/w represents the time at which the current through the n/w is decreased to 37.7% of its initial value.

This time const depands only upon the numerical values of R & C componants only.

$$T = R_{eq} \cdot C_{eq}$$ | Voltage source ⇒ sc
Current source ⇒ oc

Example → Series RL ckt




To find
i(t) for t>0
assume: zero initial cond$^n$

applying KVL;

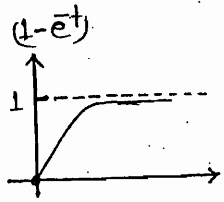
$$\underset{=1}{R}\, i(t) + \underset{=1}{L}\frac{di(t)}{dt} = v(t)$$

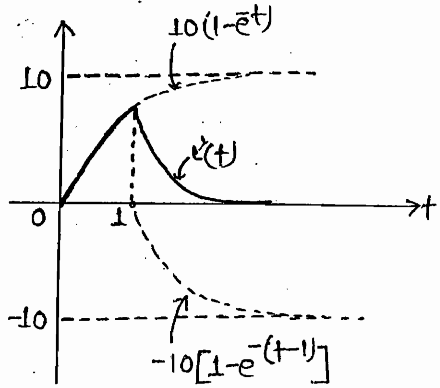
$$\Rightarrow I(s) + \underset{=1}{L}\, s I(s) - \overset{=0}{i(0^+)} = \frac{10}{s}(1-e^{-s})$$

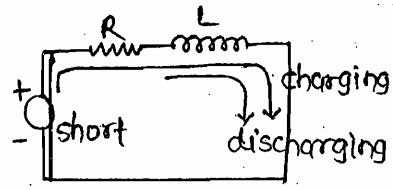
$$\Rightarrow I(s) = \frac{10}{s(s+1)}(1-e^{-s}) = 10\left(\frac{1}{s}-\frac{1}{s+1}\right)(1-e^{-s})$$

$$\Rightarrow I(s) = 10\left[\frac{1}{s}-\frac{1}{s+1}\right] - 10\left[\frac{1}{s}-\frac{1}{s+1}\right]e^{-s}$$

$$i(t) = 10(1-e^{-t})u(t) - 10(1-e^{-(t-1)})u(t-1)$$



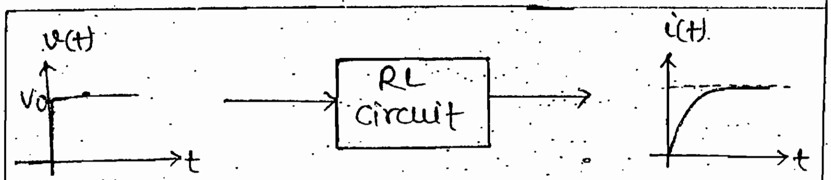




* If $v(t) = V_0 u(t)$ ; R=R & L=L

$$\boxed{\begin{aligned} i(t) &= \frac{V_0}{R}(1-e^{-t/T})u(t) \\ T &= \frac{L}{R} \text{ ...... Time constant} \end{aligned}}$$

* Summary →




$$i(t) = \frac{V_0}{R}(1-e^{-t/T})\,u(t)$$

$$T = \frac{L}{R} \text{ ....... Time constant}$$

* Initialy at t=0 an ind$^r$ acts as OC. so that the current through series RL n/w is 0.
* For pulse excitation reversal of current takes place in series RC n/w where as no such reversal takes place in a series RL n/w.
* For step excitation the current through the ind$^r$ increases exponantialy with time constant T.

This time constant T controls the rate of increment of the current through the RL n/w & depands only upon numerical values of R-L componants.

## * Analysis of an electrical n/w using KVL & KCL eqⁿ →

**KVL →**

| |
|---|
| Voltage-drop ⇒ +ve |
| Voltage Rise ⇒ −ve |

........ convention

no. of KVL eqⁿ = $b-(n-1)$

where; $b$ = no. of branches

$n$ = no. of nodes.

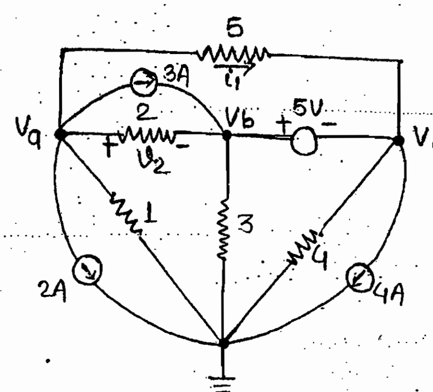
**KCL →**

| |
|---|
| •——→—— ⇒ +ve |
| •——←—— ⇒ −ve |

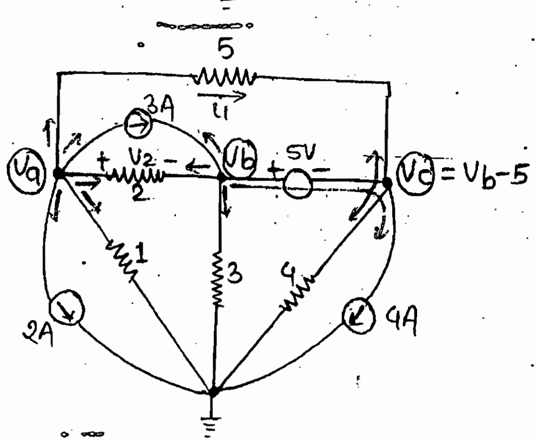
...... convention

no. of KCL eqⁿ = $n-1$

**Example:-**




To find $V_a, V_b, V_c$




dependant OR Redundant OR Super-node

$$\boxed{\begin{array}{l} V_b - V_c = 5 \\ V_b - 5 = V_c \end{array}}$$

**KCL ($V_a$)**

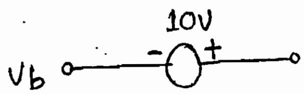
$$+2+\frac{V_a-0}{1}+\frac{V_a-V_b}{2}+3+\frac{V_a-\overset{\to(V_b-5)}{V_c}}{5}=0 \text{ -------(i)}$$

**KCL ($V_b$)**

$$-3+\frac{V_b-V_a}{2}+\frac{V_b-0}{3}+\left[\frac{\overset{\to(V_b-5)}{V_c}-0}{4}+4+\frac{\overset{\to(V_b-5)}{V_c}-V_a}{5}\right]=0 \text{ ------(ii)}$$

$\frac{V_a}{1}$

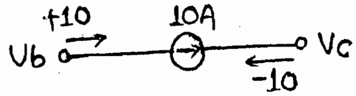
Case(i) →




$V_c = V_b + 10$

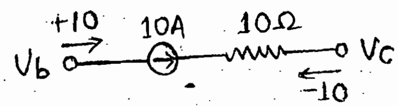
$\frac{2KCL}{V_a, V_b}$

Case(ii) →




$\frac{3KCL}{V_a, V_b, V_c}$

Case(iii) →

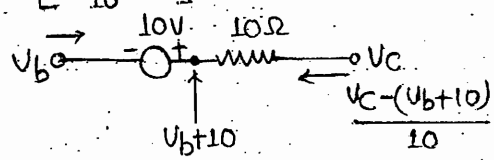



$\frac{3KCL}{V_a, V_b, V_c}$

------ same as case(ii)

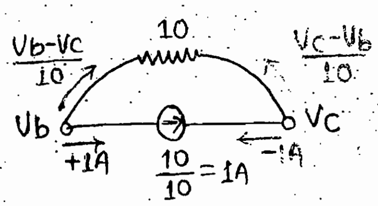
Case(iv) →

$-\left[\frac{V_c-(V_b+10)}{10}\right]$



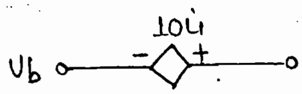

$\frac{3KCL}{V_a, V_b, V_c}$

Case(iv)(a) →




$\frac{3KCL}{V_a, V_b, V_c}$

Case(v) →




$V_c = V_b + 10 i_1$

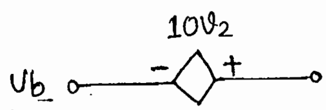
$\frac{2KCL}{V_a, V_b}$

$= V_b + 10\left(\frac{V_a - V_c}{5}\right)$

$3V_c = 2V_a + V_b$

$V_c = \frac{1}{3}(2V_a + V_b)$

Case(vi) →




$V_c = V_b + 10V_2$

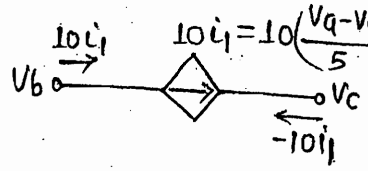
$= V_b + 10(V_a - V_b)$

$= 10V_a - 9V_b$

$\frac{2KCL}{V_a, V_b}$

Case(7) →

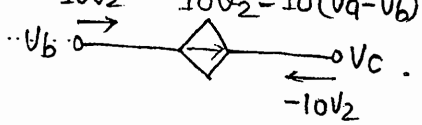
$10i_1 = 10\left(\frac{V_a - V_c}{5}\right) = 2(V_a - V_c)$




$\frac{3\,KCL}{V_a, V_b, V_c}$

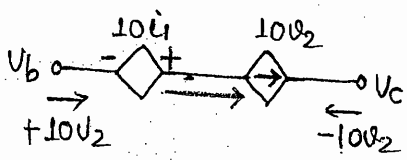
Case(8) →

$10v_2 = 10(V_a - V_b)$




$\frac{3\,KCL}{V_a, V_b, V_c}$

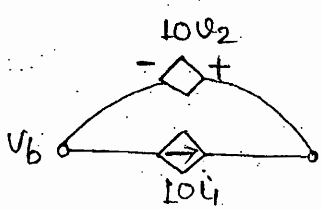
Case(9) →




$\frac{3\,KCL}{V_a, V_b, V_c}$

...... same as case(8)

Case(10) →

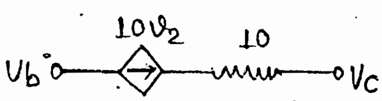



$V_c = V_b + 10v_2$

$\frac{2\,KCL}{V_a, V_b}$

...... same as case(6)

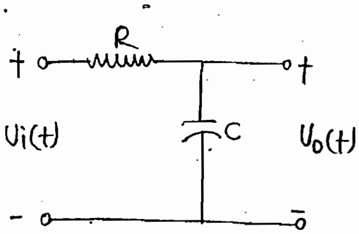
Case(11) →




$\frac{3\,KCL}{V_a, V_b, V_c}$

...... same as case(8)

Que(34)
chap.(1)




$RC = 1\,ms.$

$v_i(t) = \sqrt{2}\sin 10^3 t$

$V_0(t) = ?$

Soln →

$v_i(t) = \underbrace{\sqrt{2}}_{v_i}\sin \underbrace{10^3}_{\omega = 10^3\,rad/s.} t$

$V_0(t) = \underbrace{V_0}_{= ?}\sin 10^3 t$

$$\frac{V_o}{V_i} \equiv H(\omega) = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{1 + j10^3 \times 10^{-3}}$$

$$= \frac{1}{1 + j1}$$

$$= \frac{1}{\sqrt{2}\angle 45^\circ}$$

$$V_o = \frac{1}{\sqrt{2}\angle 45^\circ} \times V_i$$

$$= \frac{1}{\cancel{\sqrt{2}}\angle 45^\circ} \times \cancel{\sqrt{2}}$$

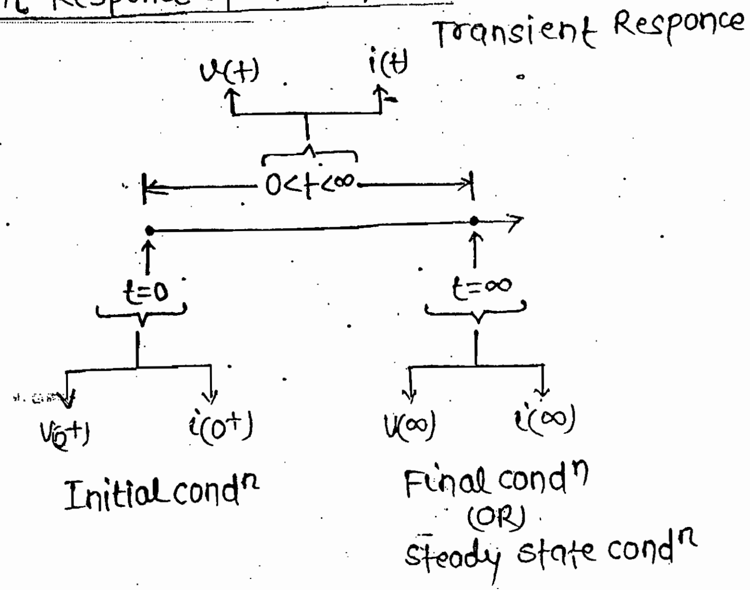
$$V_o = \frac{1}{\angle 45^\circ}$$

$$V_o(t) = V_o \sin 10^3 t$$

$$= \frac{1}{\angle 45^\circ} \sin 10^3 t$$

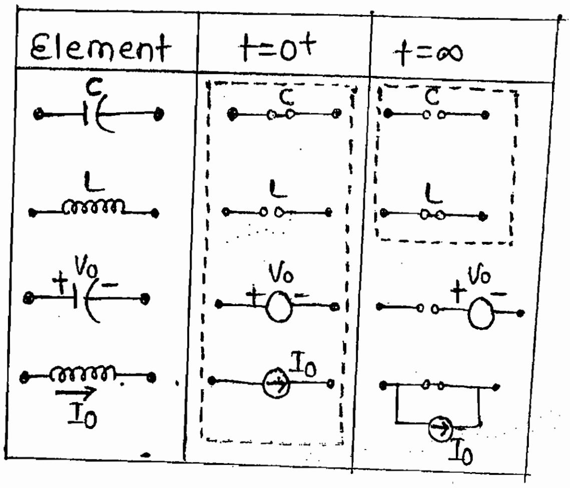
$$\boxed{V_o(t) = \sin(10^3 t - 45^\circ)}$$

## * Transient Responce of elect N/w →



## * Behaviour of L & C in any ckt →

Case(1) → In time domain;



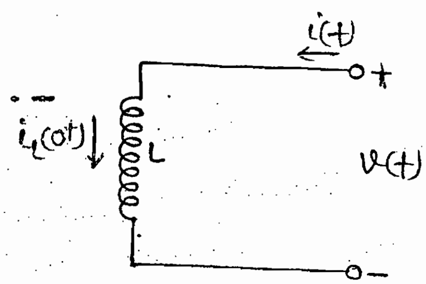

* Initialy at t=0 any capacitor acts as a sc where as any ind$^r$ acts as open ckt.

* At t=∞ (OR) in the steady state condn any capacitor acts as an oc & an ind$^r$ acts as a sc.

* Initialy at t=0 any capacitor with initial vol. acts as a constant vol. source where as any ind$^r$ with initial current acts as a constant current source.

Case(2) → In s-domain;

(a) Inductor →

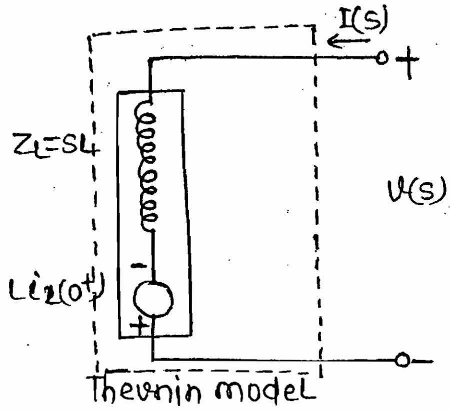



* current i(t) & $i_L(0^+)$ are different with each other.

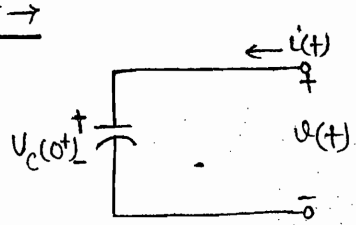
$$v(t) = L\frac{d\,i(t)}{dt}$$

$$V(s) = L[s\,I(s) - i_L(0^+)]$$

$$\boxed{V(s) = sLI(s) - Li_L(0^+)}$$ ..... KVL ..... Series ckt




(b.) Capacitor →

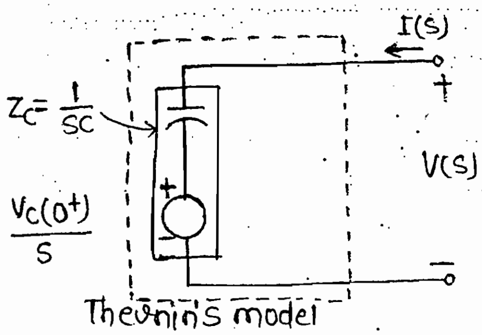



$$i(t) = C \frac{dv(t)}{dt}$$

$$I(s) = C[sV(s) - v_C(0^+)]$$

$$\boxed{V(s) = \frac{1}{sC} \cdot I(s) + \frac{V_C(0^+)}{s}}$$ ...... KVL

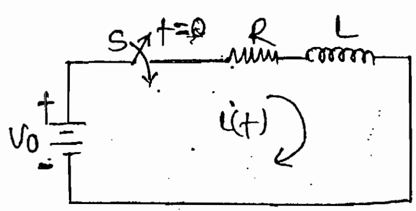
...... series ckt




Que → In the ckt shown the switch is initialy open. The switch is closed at $t=0$. Find the current & the rate of change of current through the n/w just after the switch has been closed?

To find → $i(0^+)$

$\frac{di}{dt}(0^+)$



Sol$^n$→ at $t=0^-$

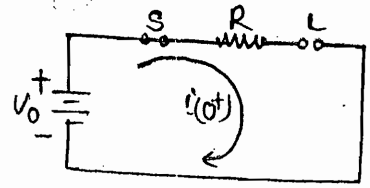
S..... open

$\boxed{i(0^-)=0}$

at $t=0^+$

S..... closed.

L..... OC




$\boxed{i(0^+)=0}$ .....A

***
$$\boxed{\begin{array}{l} i_L(0^+) = i_L(0^-) \\ V_C(0^+) = V_C(0^-) \end{array}}$$

**** The current through ind$^r$ & vol. across a capacitor can't change instantaneouly.

Therefore these value remain same just before & just after the switch has been closed.

(b) at $t>0$ (because it is not at 0 & not at ∞ then not OC & not SC)

S..... closed

general eq$^n$:-

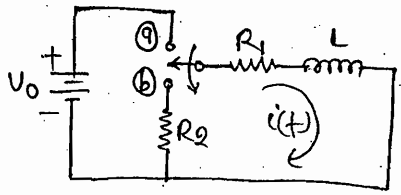
$$i(t)R + L\frac{di(t)}{dt} = V_0$$

$t=0^+$

$$\underbrace{i(0^+)}_{=0}.R + L\frac{di}{dt}(0^+) = V_0$$

$$\boxed{\frac{di}{dt}(0^+) = +\frac{V_0}{L}} \text{.....A/s}$$

ue.→ In the ckt shown the switch is initialy at position a for a long time till the steady state is reached. At $t=0$ the switch is moved to pos$^n$ b. Find a general exp$^n$ of the current $i(t)$ through the n/w for $t>0$.




To find $i(t)$ for $t>0$

DATE-30/11/14

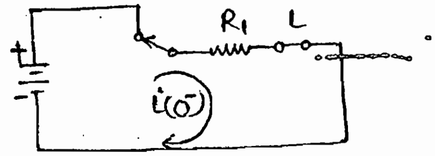
**Soln →** $t=0^-$

S.... at ⓐ

ckt is in steady-state (FC)

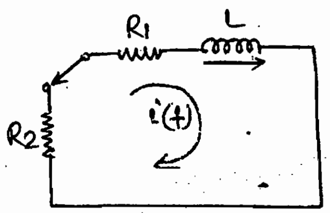
$L \to SC$.

$$i(0^-) = \boxed{\frac{V_0}{R_1} = i(0^+)}$$

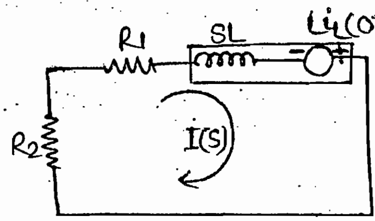



**t>0 →**

S.....at ⓑ




$$i(0^+) = i_L(0^+) = \frac{V_0}{R_1}$$




$$L i_L(0^+) = \frac{LV_0}{R_1}$$

**By KVL →**

$$I(s)(R_1+R_2+sL) - \frac{V_0 L}{R_1} = 0$$

$$I(s) = \frac{V_0 L}{R_1} \cdot \frac{1}{L(s+a)} \qquad \because a = \frac{R_1+R_2}{L} = \frac{Req}{L}$$

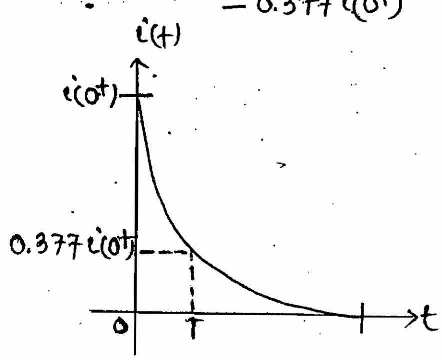
$$i(t) = \frac{V_0}{R_1} e^{-at} u(t).$$

$$\boxed{i(t) = i(0^+) e^{-t/T} . u(t)} \qquad i(0^+) = \frac{V_0}{R_1}$$

$T = \frac{1}{a} = \frac{1}{Req}$ ..... time constant.

at $t=T$; $i(t) = i(0^+) e^{-1}$

$\qquad \qquad \triangleq 0.377\, i(0^+)$



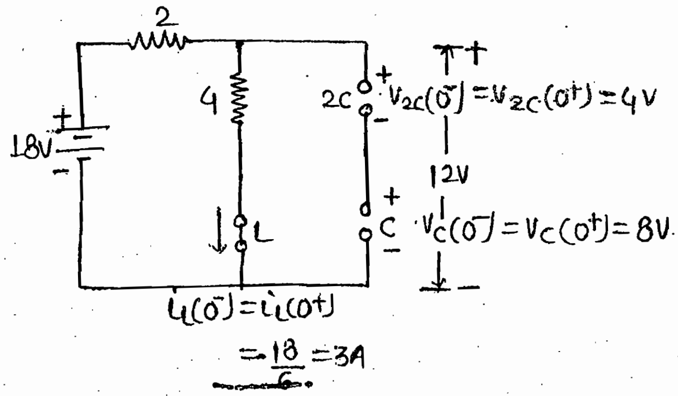
(7/13) at $t=0^-$

S.... open

ckt is in steady state FC.

$2C$ & $C \rightarrow$ OC.

$L \rightarrow$ SC.

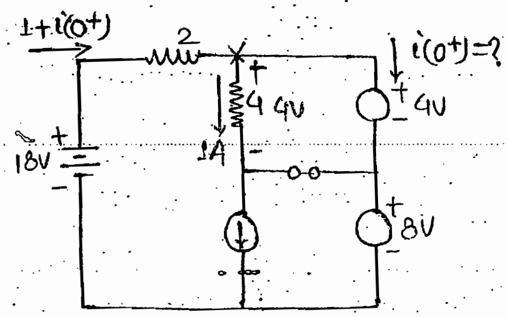



$Q = CV$

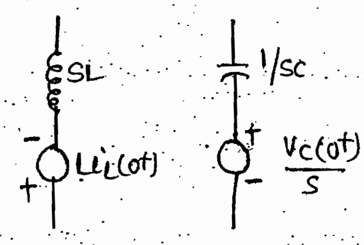
$V \propto \frac{1}{C}$

at $t=0^+$

S.... closed




$i(t)$ for $t>0$




$-18 + 2[1 + i(0^+)] + 4 + 8 = 0$

$\boxed{i(0^+) = 2A}$

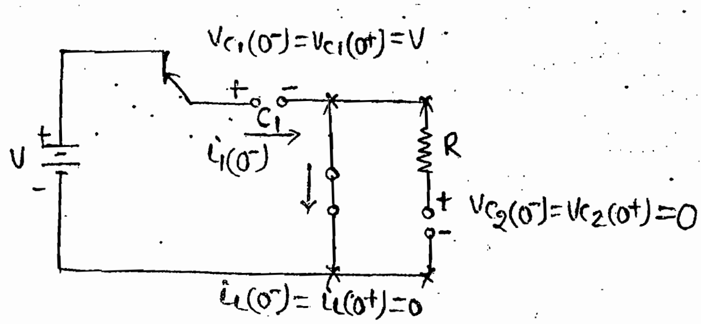
(22/15) at $t=0^- \rightarrow$

S...... at (1)

steady state/ FC

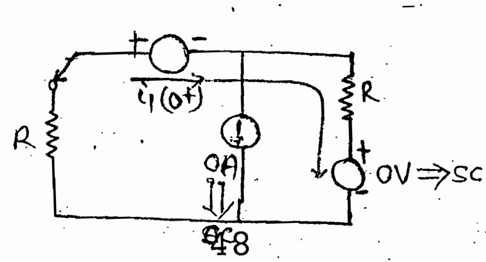
C.... OC

L.... SC




at $t=0^+ \rightarrow$

S... at (2)



$i_1(0^+) \neq i_1(0^-)$ Because of capacitor current..

KVL→

$(R+R)\, i_1(0^+) + V = 0$

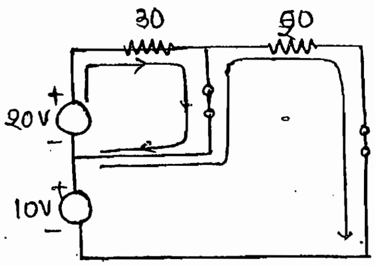
$$\boxed{i_1(0^+) = \frac{-V}{2R}}$$

(25/16) at $t=0^-$ →

S.... closed

SS/FC

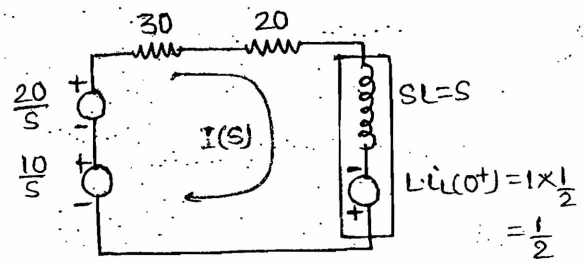
L ⇒ SC



$i_L(0^-) = i_L(0^+) = \frac{10}{20} = 0.5A$

at $t>0$ →

S.... open



$$\frac{-10}{S} - \frac{20}{S} - \frac{1}{2} + I(s)[30+20+s] = 0$$

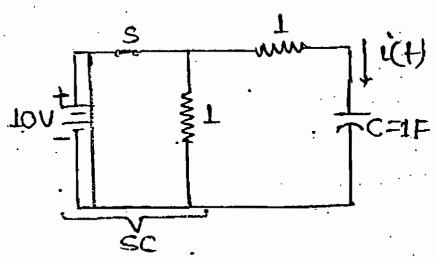
$$I(s)(s+50) = \frac{s+60}{2s}$$

$$I(s) = \frac{1}{2} \cdot \frac{(s+60)}{s(s+50)}$$

$$\boxed{i(t) = 0.6 - 0.1e^{-50t}}$$

(19/15) at $t>0$ →

S.... closed



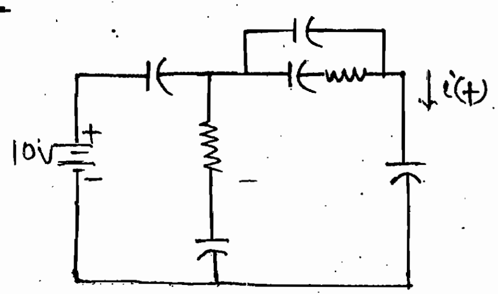
$T = R_{eq} \cdot C_{eq} \Big|_{10V \Rightarrow SC}$

$= 1 \times 1$

$\boxed{T = 1s}$

* Because of TC does not depand on the vol. then so it will be shorted.

Example→



$T = R_{eq} C_{eq} \Big|_{10V \Rightarrow SC}$

Because of many C & R the above method is not applicable for caln of TC.

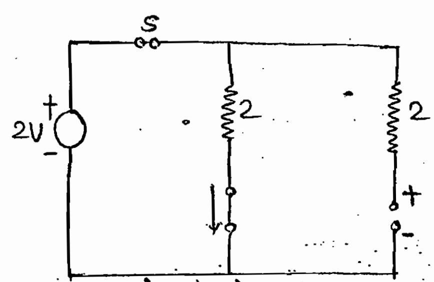
$$\boxed{\begin{aligned} T &= RC \;\cdots\cdots\; \text{for RC ckt} \\ &= \frac{L}{R} \qquad \text{for RL ckt} \\ &= ? \qquad \text{for RLC ckt} \end{aligned}}$$

$\frac{4}{12}$ at $t=0^- \rightarrow$

S..... closed

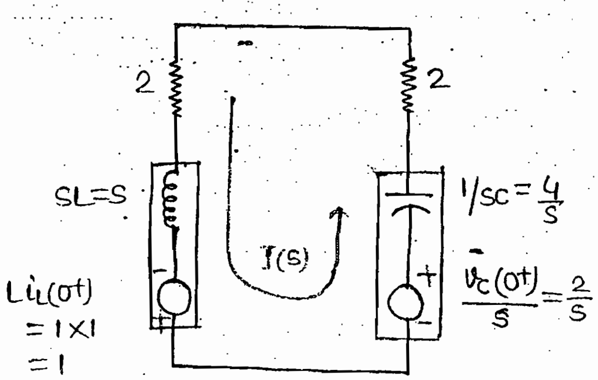
SS/FC

$L \Rightarrow SC$, $C \Rightarrow OC$




at $t>0 \rightarrow$

S.... open




By KVL $\rightarrow$

$$I(s)\,(2+s+\frac{4}{s}+2) - 1 - \frac{2}{s} = 0$$

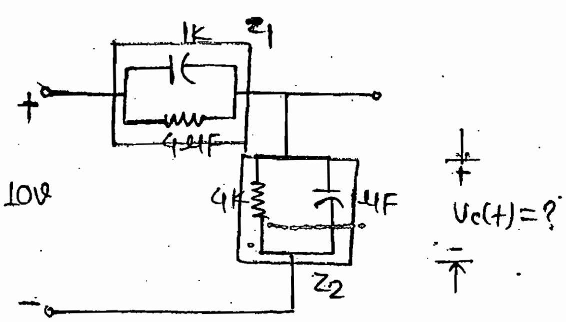
$$I(s) \cdot \frac{\overbrace{s^2+4s+4}^{(s+2)^2}}{s} = \frac{s+2}{s}$$

$$I(s) = \frac{1}{s+2}$$

$$i(t) = e^{-2t} = e^{-t/T}$$

$$\boxed{T = 1/2 \text{ sec}}$$

$\frac{32}{17}$




$V_c(0^+) = 0$ (Given)

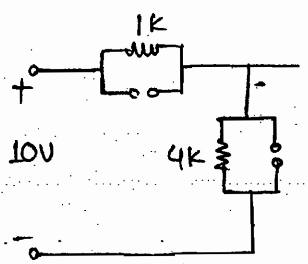
$$\frac{V_c(s)}{10/s} = H(s) = \frac{Z_2}{Z_1+Z_2} \quad \text{........ NOT applicable}$$

* Because of value of c will be erased & conclusion is that we can't use above method (And also value of $R_1C_1 = R_2C_2$)

* If RC-type of n/w is given then;

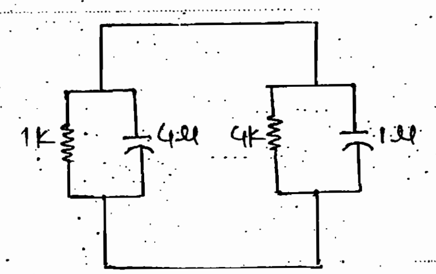
$$V_c(t) = V_c(\infty) - [V_c(\infty) - \underbrace{V_c(0^+)}_{\equiv 0}]\, e^{-t/T} \quad \text{..... only for RC ckt}$$

$V_c(\infty)$ → at steady state capacitor act as oc.




$$V_c(\infty) = \frac{4}{4+1} \times 10 = 8V$$

$$T = R_{eq} \cdot C_{eq}; \Big|_{10 \Rightarrow SC}$$




$R_{eq} = (1 \| 4)K = 0.8K$

$C_{eq} = 4+1 = 5\mu F$

$T = 0.004S$

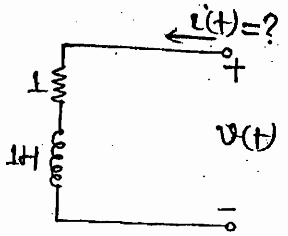
$$V_c(t) = 8 - (8-0)\, e^{-t/0.004}$$

$$\boxed{V_c(t) = 8[1 - e^{-t/0.004}]}$$

* If RL ckt is given;

$$\boxed{i_L(t) = i_L(\infty) - [i_L(\infty) - i_L(0^+)]\, e^{-t/T}}$$

23/15




$v(t) = 10\sqrt{2}\cos(t+10^\circ) + 10\sqrt{5}\cos(2t+10^\circ)$

$\omega = 1\ \text{rad/s}. \qquad \omega = 2\ \text{rad/s}.$

$Z = R + j\omega L = 1 + j\omega = 1 + j1 = \sqrt{2}\angle 45^\circ \ldots \omega = 1$

$= 1 + j2 = \sqrt{5}\angle\theta \ldots \omega = 2$

$\theta = \tan^{-1}\left(\frac{2}{1}\right)$

$i(t) = \frac{10\sqrt{2}}{\sqrt{2}\angle 45^\circ}\cos(t+10^\circ) + \frac{10\sqrt{5}}{\sqrt{5}\angle\theta}\cos(2t+10^\circ)$

$i(t) = 10\cos(t+10^\circ-45^\circ) + 10\cos(2t+10^\circ-\theta)$

$i(t) = 10\cos(t+10^\circ-45^\circ) + 10\cos(2t+10^\circ-\tan^{-1}2)$

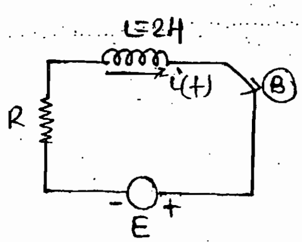
20/15

$i(0) = -8A$

$\frac{di(0)}{dt} = 3A/s$

$i(\infty) = 4A$

B:-




$RL \rightarrow$

$i(t) = \underbrace{i(\infty)}_{4} - [i(\infty) - \underbrace{i(0^+)}_{=-8}]e^{-t/T}$

$i(t) = 4 - 12e^{-t/T}$

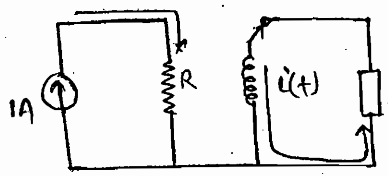
$\frac{di(t)}{dt} = 0 + 12\left(\frac{1}{T}\right)e^{-t/T}$

$t=0; \quad \frac{di(0)}{dt} = 3 = \frac{12}{T}$

$T = 4 = \frac{L}{R} = 2$

$R = 1/2$

(13/14) at position A →

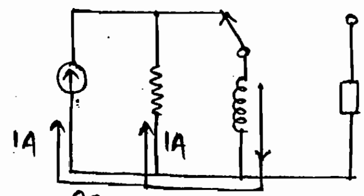



$P = I^2 R$

$i(t) = I\sin(\omega t + \underbrace{30^\circ}_{\text{leading}})$

$X \rightarrow$ capacitor

at position B → (t=0)

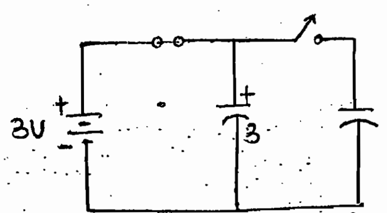



Because of same power $I\sin(0+30^\circ) = \frac{1}{2}I$

$\frac{1}{2}I = 1+1$

$\boxed{I = 4A}$

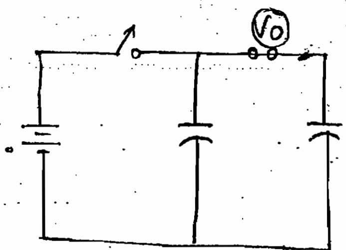
(39/18) case(1.)




$\underbrace{C_1 = 1F,}$ $C_2 = 2F$

$Q_{total} = C_1 V = 1 \times 3 = 3C$

case(2) →




$C_1 = 1F$ $C_2 = 2F$

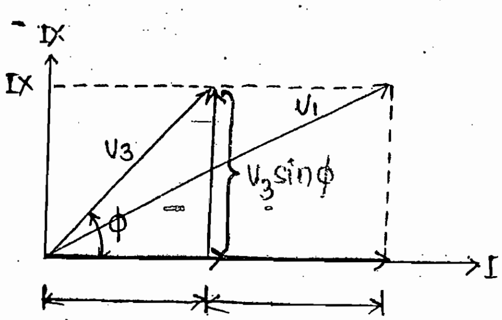
$Q_{total} = Q_1 + Q_2$

$3 = C_1 V_0 + C_2 V_0$

$= 1 \times V_0 + 2 \times V_0$

$\boxed{V_0 = 1V}$

(52/11)



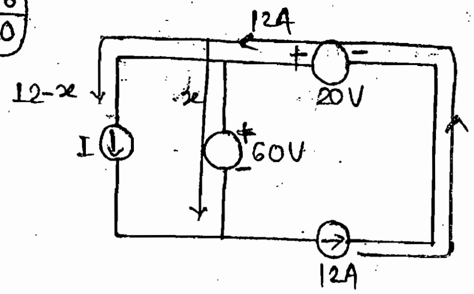

$\underbrace{V_{RL} = I \cdot R_L}_{V_3\cos\phi}$ $V_R = V_2 = 122V$

$V_1^2 = (V_3\cos\phi + V_2)^2 + (V_3\sin\phi)^2$

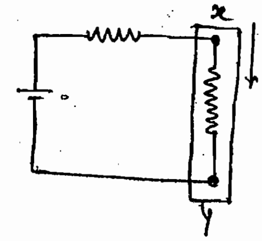
$\cos\phi = PF = 0.45$

(53/11) $P = \frac{V_{RL}^2}{R_L} = \frac{(V_3\cos\phi)^2}{R_L} = 750\omega$

46/10

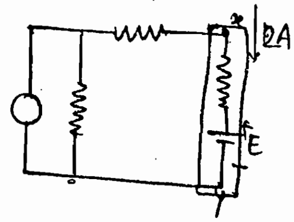



41/9



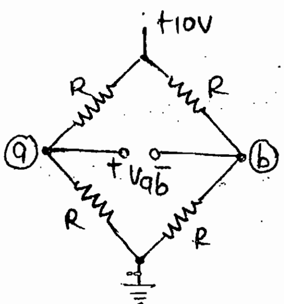

$I = \frac{18}{9} = 2A$

$V_{xy} = 6I = 12V$




$12V = 2R + E$

40/9




$V_{ab} = V_a - V_b$

$= \left(\frac{R}{R+R}\right) \times 10 - \frac{1.1R}{1.1R+R} \times 10$

$= 5 - \frac{1.1R}{1.1R+R} \times 10$

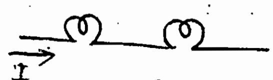
23/6

40W 60W

$P = \frac{V^2}{R} \leftarrow 220V$

$P \propto \frac{1}{R}$

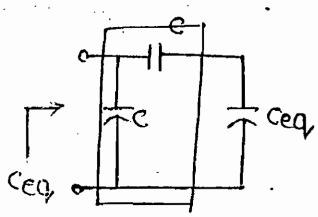
$R \uparrow$ Power Rating $\downarrow$




$P = I^2 R$

40W glow brighter because $R \uparrow$

21/5




$C_{eq} = C + \frac{C \cdot C_{eq}}{C + C_{eq}}$

$(C + C_{eq})(C_{eq}) = C^2 + C \cdot C_{eq} + C \cdot C_{eq}$

$\cancel{C \cdot C_{eq}} + C_{eq}^2 = C^2 + \cancel{C \cdot C_{eq}} + C \cdot C_{eq}$

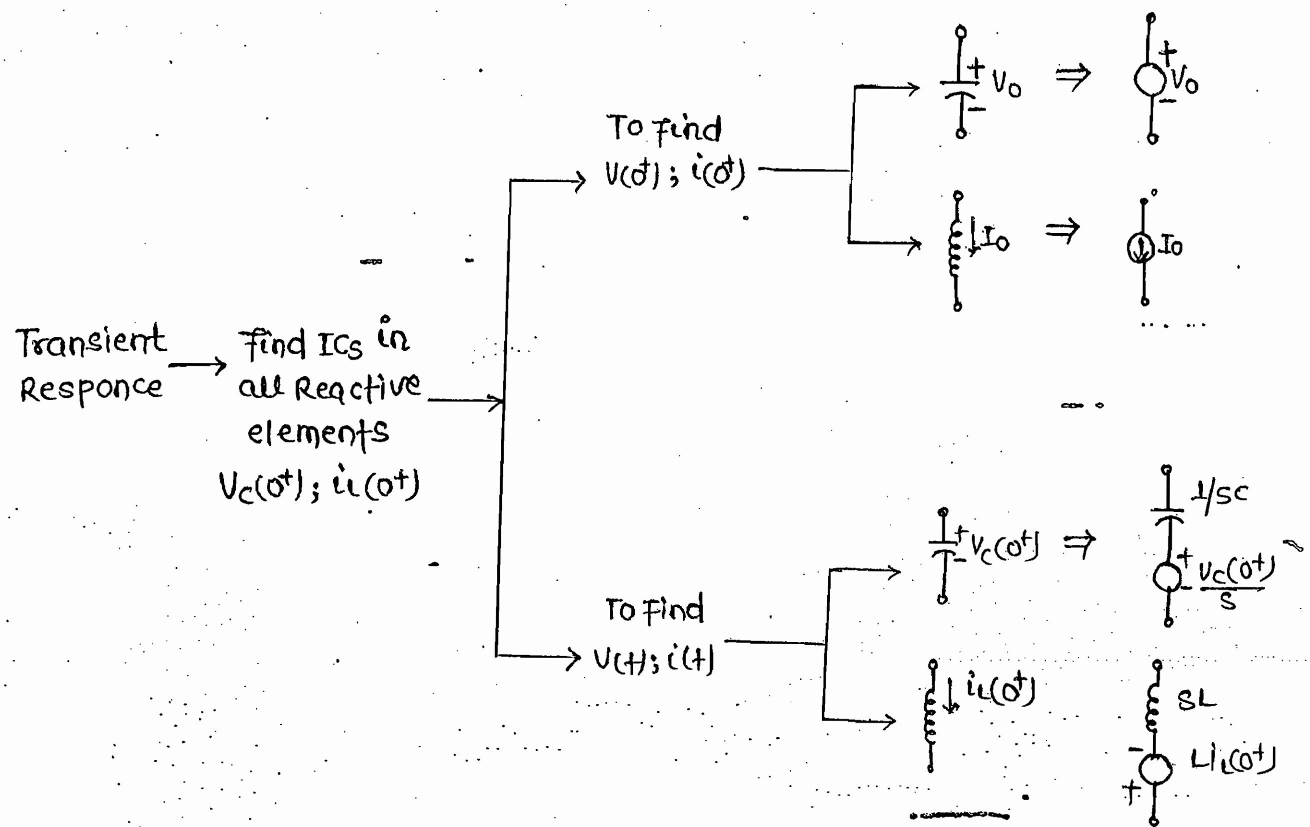
$C_{eq}^2 - C \cdot C_{eq} - C^2 = 0$

$C_{eq} = \frac{+(C) \pm \sqrt{C^2 - 4 \times 1 \times (-C^2)}}{2}$

$= \frac{C \pm \sqrt{5C^2}}{2}$

$= \frac{C(1 + \sqrt{5})}{2}$

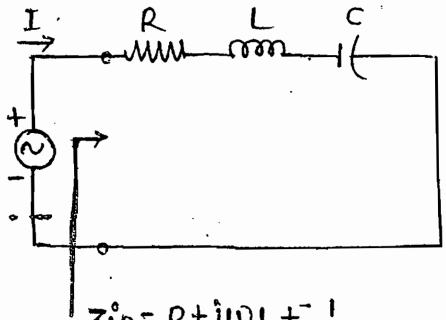
## * Summary → Transient Responce



# RESONANCE

## * Series RLC ckt →

...... Series Resonance




Sinusoidal vol. source (Fixed amplitude & variable freq.)

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

| time | C | L |
|---|---|---|
| t=t | C | L |
| t=0 | SC | OC |
| t=∞ | OC | SC |
| ω=ω | $1/j\omega C$ | $j\omega L$ |
| ω=0 | OC | SC |
| ω=∞ | SC | OC |

.... Transient Responce
.... Effect of variation of $v(t), i(t)$
.... fixed freq.
.... Resonance
.... Effect of variation of parameters w.r.t. freq.
.... at Fixed time instant

## * at $\omega = \omega_0$

j term = 0

$$\Rightarrow \left(\omega_0 L - \frac{1}{\omega_0 C}\right) j = 0$$

$$\Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{.... rad/sec} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{.... Hz}}$$

..... Freq. of resonance

$$Z_{in}\Big|_{\omega=\omega_0} = R + j0 \quad \text{..... min.}$$

$$I_0 = \frac{V}{Z_0} \quad \text{..... max}$$

* at $\omega=\omega_0$

$$|V_L|=|V_C|$$

& are out of phase by 180°

$$Z_{in}=R+j\left(\omega L-\frac{1}{\omega C}\right)$$

→ $\omega=\omega_0$ ; Resistance

→ $\omega>\omega_0$ ; Inductive

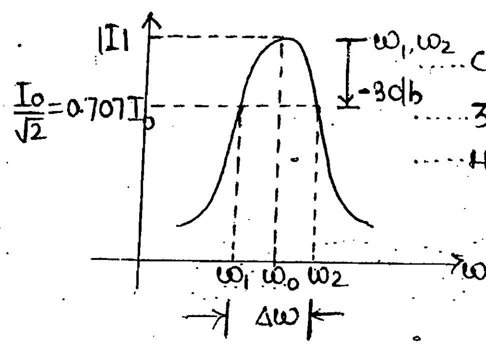
→ $\omega<\omega_0$ ; capacitive

*** Freq. Responce →**

$|I|$ vs $\omega$

$$|I|=\frac{V}{|Z_{in}|}=\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

$$I_0=\frac{V}{R} \cdots \text{max} ; \omega=\omega_0$$




$\omega_1, \omega_2$ ..... Cut off Freq

..... 3 db freq.

..... Half power Freq.

*** Features →**

* At Resonance the Img. part of the i/p impedance is 0.

* At resonance the I/p impedance is purely resistive has a $\text{min}^m$ value, i/p vol. & current are in phase so that the PF is unity.

* At resonance the current drawn by the n/w from the i/p vol. source has a $\text{max}^m$ value & therefore any series RLC n/w behaves as band pass ~~filter.~~

* At resonance the vol. across the $\text{ind}^r$ & across the capacitor are equal in magnitude & are phase shifted by 180° so that net vol. across LC combination is 0.

* The ckt behaves differently at different freq. of operation:-

(a.) At $\omega_0$ n/w is resistive & has a UPF.

(b.) Above $\omega_0$ n/w is inductive & PF is lagging.

(c.) Below $\omega_0$ n/w is capacitive & PF is leading

* The Freq. of resonance represents a rate at which ele. energy stored in the capacitor is Xformed to the magnetic energy stored in the ind$^r$ & vice-versa.

The phenomina of resonance then represents the transformation b/n ele. & magnetic energy at the Freq. of resonance.

* <u>Quality Factor ($Q_0$) →</u>

$$Q_0 = \frac{|V_L|}{V} = \frac{|V_C|}{V} \quad \text{.......voltage amplification factor.}$$

$$|V_L| = |I_0 . j\omega_0 L| = |I_0 . \omega_0 L| = I_0 . \omega_0 L$$

$$|V_C| = \left| I_0 . \frac{1}{j\omega_0 C} \right| = I_0 \frac{1}{\omega_0 C}$$

$$V = I_0 R$$

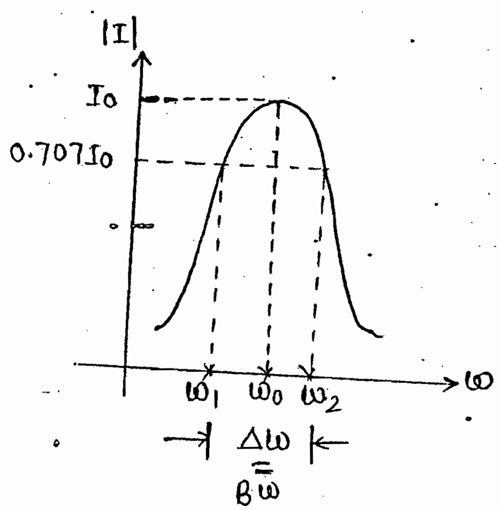
$$Q_0 = \frac{|V_L|}{V} = \frac{I_0 \omega_0 L}{I_0 R} = \frac{\omega_0 L}{R}$$

$$= \frac{\omega_0 L}{R}$$

$$= \frac{1}{\omega_0 RC}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad Q_0 >> 1$$




$$\Delta\omega = \omega_2 - \omega_1$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} \; ; (>>1)$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 \cong \omega_0 + \frac{1}{2}\Delta\omega$$

$$\omega_1 \cong \omega_0 - \frac{1}{2}\Delta\omega$$

$$\boxed{\begin{array}{l} \Delta\omega = \omega_2 - \omega_1 \\ \omega_0 = \sqrt{\omega_1 \omega_2} \end{array}}$$

*** $\boxed{Q_0 \uparrow \longrightarrow \Delta\omega \downarrow}$

## * Bandwidth →

$$Q_0 = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \frac{\omega_0}{Q_0} = \frac{1/\sqrt{LC}}{\frac{1}{R}\frac{\sqrt{L}}{\sqrt{C}}}$$

$$\boxed{\begin{array}{l} \Delta\omega = \frac{R}{L} \\ \Delta f = 2\pi\frac{R}{L} \end{array}}$$

.... rad/sec

..... Hz

$Q_0\uparrow \rightarrow \Delta\omega\downarrow$ — → $R\downarrow$

→ $L\uparrow$ ..... Not advisable (Because of large size of ind$^r$)

* For any tuned n/w the QF must be high & therefore the n/w should have small BW.

High QF of any tuned n/w has a the ability to select a perticular signal thereby rejecting all other unwanted signal.

## * Series RLC ckt →

* $\omega > \omega_0$ ; Inductive

Vol. across the L is max

* $\omega < \omega_0$ ; capacitive

Vol. across the c is max

$$I = \frac{V}{|Z_{in}|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$|V_L| = |I|\omega L$$

$$|V_L|^2 = \frac{(\omega L V)^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

for max $V_L$;

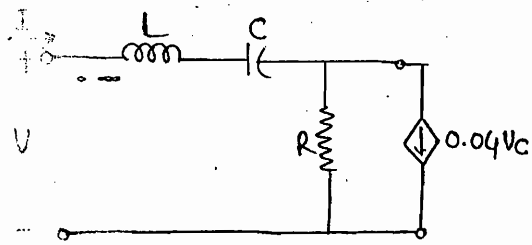
$$\frac{\partial}{\partial\omega}\left[|V_L|^2\right] = 0$$

Find $\omega$

$\Downarrow$

$\omega > \omega_0$

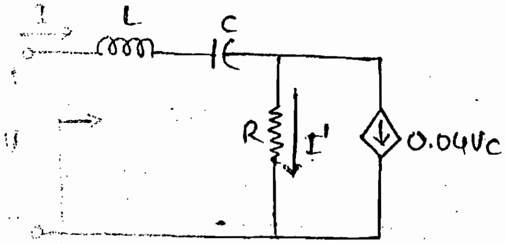
Example →




$R = 20\,\Omega$, $L = 0.1H$, $C = 0.1\mu F$

To find; $Z_{in}$, $\omega_0$, $Q_0$, $\Delta\omega$, $\omega_1$, $\omega_2$

Sol$^n$ →




$$I' = I - 0.04V_c$$

$$= I - 0.04 \times I \times \frac{1}{j\omega C} = I\left(1 - \frac{0.04}{j\omega C}\right)$$

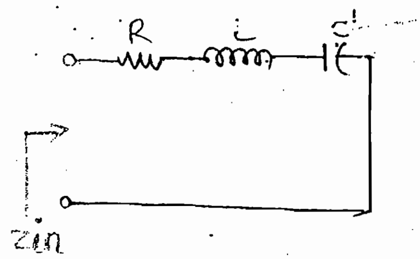
By KVL →

$$\Rightarrow V = j\omega L \cdot I + \frac{1}{j\omega C} \cdot I + I\left(1 - \frac{0.04}{j\omega C}\right)R$$

$$\Rightarrow \frac{V}{I} = Z_{in} = R + j\omega L + \underbrace{\frac{1}{j\omega C} - \frac{0.04R}{j\omega C}}_{\frac{1}{j\omega C}\left[1 - 0.04R\right]}$$

$$\frac{1}{j\omega \times \left(\frac{C}{1-0.04R}\right)} = \frac{1}{j\omega C'} \qquad \left(C' = \frac{C}{1-0.04R}\right)$$

$$\boxed{Z_{in} = R + j\omega L + \frac{1}{j\omega C'}}$$ ...... series RLC ckt



(1.) $\omega_0 = \frac{1}{\sqrt{LC'}} = 4470$ rad/s

(2.) $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C'} = \frac{1}{R}\sqrt{\frac{L}{C'}} = 22.4$

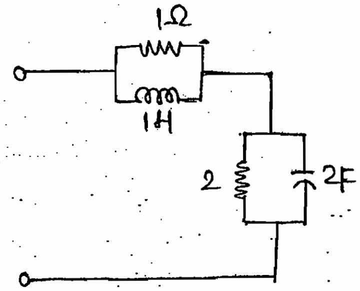
(3.) $BW = \frac{\omega_0}{Q_0} = \frac{R}{L}$ ..... rad/s.

$= 200$ rad/s.

(4.) $\omega_2 \cong \omega_0 + \frac{1}{2}\Delta\omega = 4570$ rad/s

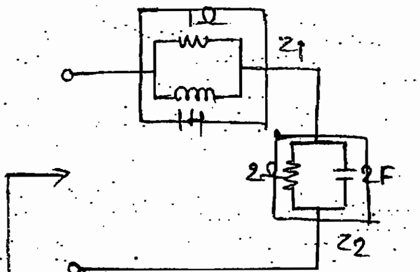
$\omega_1 \cong \omega_0 - \frac{1}{2}\Delta\omega = 4370$ rad/s.

Example →




To find → $\omega_0$

Soln →




$Z_{in} = Z_1 + Z_2$

$$Z_1 = \frac{R \times j\omega L}{R + j\omega L} = \frac{j\omega}{1+j\omega} \times \frac{1-j\omega}{1-j\omega} = \frac{j\omega}{1+\omega^2} + \frac{\omega^2}{1+\omega^2} \quad \text{----(i)}$$

$$Z_2 = \frac{R \times 1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{R}{1+j\omega C} = \frac{2}{1+j\omega 4} \times \frac{1-j\omega 4}{1-j\omega 4}$$

$$= \frac{2}{1+16\omega^2} - \frac{j\omega 8}{1+16\omega^2} \quad \text{----(ii)}$$

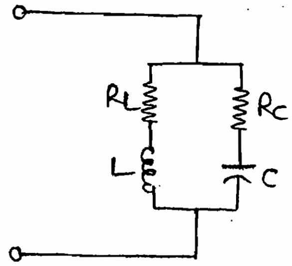
at $\omega = \omega_0$ ; $j$ term $= 0$

then from eqn (i) & (ii)

$$\frac{\omega_0}{1+\omega_0^2} - \frac{\omega_0 8}{1+16\omega_0^2} = 0$$

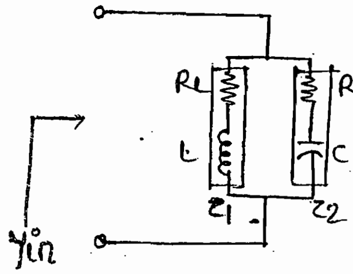
$$\boxed{\omega_0 = \sqrt{\frac{7}{8}}} \text{ rad/s.}$$

Example→



To find :- $\omega_0$

Sol$^n$→



$$Y_{in} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$X_L = \omega L$
$X_C = 1/\omega C$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2} \quad \text{------(i)}$$

at $\omega = \omega_0$ ; j term = 0

$$\Rightarrow \frac{-X_L}{R_L^2 + X_L^2} + \frac{X_C}{R_C^2 + X_C^2} = 0$$

$$\Rightarrow \frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2}$$

$$\Rightarrow X_L(R_C^2 + X_C^2) = X_C(R_L^2 + X_L^2)$$

***

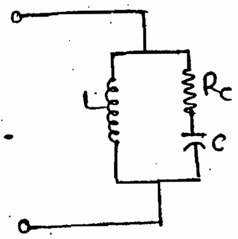
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}}$$

$Z_{in} = (\ ) + j(\ )$
$\equiv 0$ at $\omega = \omega_0$

$Y_{in} = (\ ) + j(\ )$
$\equiv 0$ at $\omega = \omega_0$

Because i/p vol. & i/p current both are in phase so we put imag. of upper part as 0. not $\infty$.

**Case(1) → $R_L = 0$**

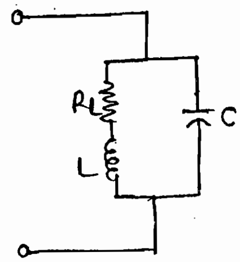



$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{4C}{4C - R_C^2}}}$$

$$\frac{L}{C} - R_C^2 > 0$$

$$\boxed{R_C < \sqrt{\frac{L}{C}}}$$ ...... cond$^n$ of Resonance

**Case(2) → $R_C = 0$**

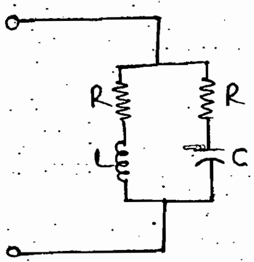



$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{R_L^2 C}{L}}}$$

$$1 - \frac{R_L^2 C}{L} > 0$$

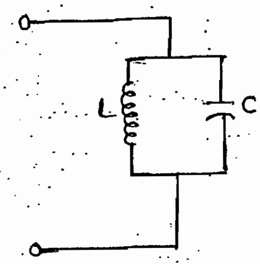
$$\boxed{R_L < \sqrt{\frac{L}{C}}}$$ .... cond$^n$ of Resonance

**Case(3) → $R_L = R_C = R$**




$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

**Case(4) → $R_L = R_C = 0$**

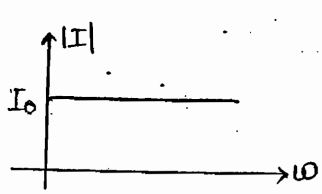



$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

* The freq. of resonance does not depend upon type of connection b/n L & C. Therefore the freq. of resonance will remain same for purely series LC & parallel LC n/w.

**Case(5) → $R_L = R_C = \sqrt{\frac{L}{C}}$**

$Y_{in} = (\text{Real}) + j(0)$

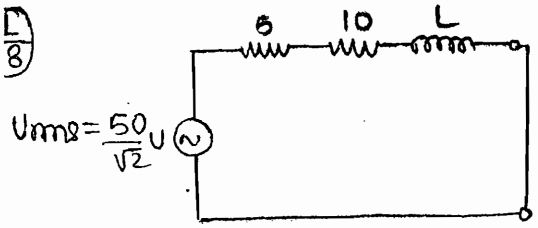



* For the specified values of $R_L$ & $R_C$ the i/p admittance has only the real part & the imag. part is always 0.
Therefore irrespective of the freq. of operation the i/p vol. & the i/p current are always in phase. Then the current drawn by the n/w is independent of

the freq. of operation.

Therefore no specific freq. of resonance will exist & n/w is resonating at all freq.

8)




$P_5 = 10W$

$PF = \cos\phi = \frac{R_t}{|Z|} = \frac{I_{rms} \cdot R_t}{I_{rms} \cdot |Z|} = \frac{I_{rms} \cdot R_t}{V_{rms}}$

$P_5 = I_{rms}^2 \times 5$

$10 = I_{rms}^2 \times 5$

$I_{rms} = \sqrt{2}$

$\boxed{PF = \cos\phi = 0.6}$

8)

$v = 200 \sin(2000t + 50°)$

$= 200 \sin(2000t + 90 \underbrace{- 90° + 50}_{-40})$

$= 200 \cos(2000t \underbrace{- 40}_{})$

$i = 4\cos(2000t \underbrace{+ 13.2}_{})$

$\underbrace{i \text{ leads } v}_{}$

capacitor

R, C

$\underbrace{R, L, C,}_{X_C > X_L}$

3/9)

Parallel RLC ckt:-

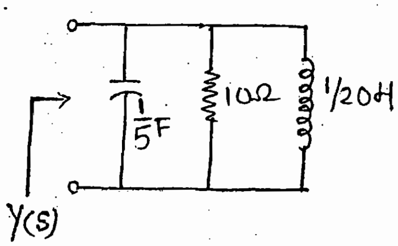
$\frac{1}{Z_S} = Y(s) = \frac{s^2 + 0.5s + 100}{5s}$

$= \frac{s}{5} + 0.1 + \frac{20}{s}$

$= Y_1 + Y_2 + Y_2$

$= sC_1 + \frac{1}{R_2} + \frac{1}{sL_3}$

$\boxed{\begin{array}{l} C_1 = 1/5F \\ R_2 = 10\Omega \\ L_3 = \frac{1}{20}H \end{array}}$



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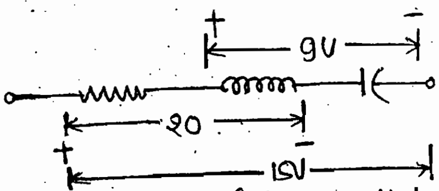
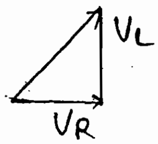



$V_R + V_L = 20V$ (This can't be written because both are not in same phase)




$\sqrt{V_R^2 + V_L^2} = 20$ ------ (i)

* $V_L - V_C = 9$ ------ (ii) (same reason as above) (~~180° phase diff~~)

$\sqrt{V_R^2 + (V_L - V_C)^2} = 15$ ------ (iii)

$\boxed{V_C = 7V}$

*Reason for written of $(V_L - V_C)$ not $(V_C - V_L)$ →*

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

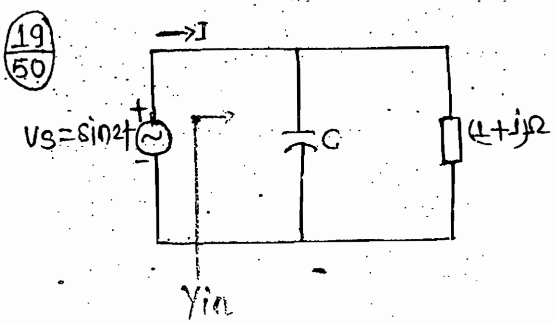
$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

$$V = IZ = IR + j(IX_L - IX_C)$$

$$\boxed{V = V_R + (V_L - V_C)j}$$

19/50




$V_S$ & $RI$ are in phase

* ckt is in resonance at $\omega = 2$ rad/s.

$$Y_{in} = Y_1 + Y_2$$

$$= j\omega C + \frac{1}{1+j} \times \frac{1-j}{1-j}$$

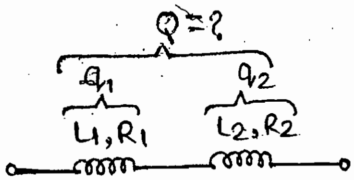
$$= j\omega C + \frac{1}{2} - j\frac{1}{2}$$

$$= j2C + \frac{1}{2} - j\frac{1}{2}$$

at reso. → $2C - \frac{1}{2} = 0$

$2C = \frac{1}{2}$ $\quad \boxed{C = \frac{1}{4}F}$

1/1




$q_1 = \frac{\omega L_1}{R_1}$ , $q_2 = \frac{\omega L_2}{R_2}$ where; $\omega$ = Freq. of operation (operating Freq (or) self resonating Freq.

* If 2 ind$^r$ is present then there will be no resonance.

$$Q = \frac{\omega L_{eq}}{R_{eq}} = \frac{1}{R_1+R_2} \cdot \omega(L_1+L_2) \quad \text{(Because of series combination)}$$

$$Q = \frac{1}{R_1+R_2}\left[\frac{\omega L_1}{R_1}R_1 + \frac{\omega L_2}{R_2}\cdot R_2\right]$$

$$Q = \left(\frac{1}{R_1+R_2}\right)\left[q_1R_1 + q_2R_2\right]$$

3/0 Series RLC ckt

$R = 20\Omega$ , $\phi = -45^\circ$ $V_L = 2V_C$

$IX_L = 2IX_C$

$X_L = 2X_C$

$$I = \frac{V}{Z} = \frac{V}{R+j(X_L-X_C)}$$

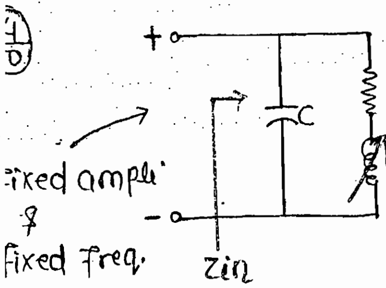
$$\phi = -\tan^{-1}\left(\frac{X_L-X_C}{R}\right) = -45^\circ$$

$$\frac{X_L-X_C}{R} = 1$$

$$X_L - \frac{1}{2}X_L = 20$$

$$\boxed{X_L = 40\Omega}$$

1/0




$$Z_{in} = \frac{(-jX_C)(R+jX_L)}{-jX_C+(R+jX_L)}$$

$$= \frac{(-jX_C)(R+jX_L)}{R+j(X_L-X_C)} \times \frac{R-j(X_L-X_C)}{R-j(X_L-X_C)}$$

$$Z_{in} = (\ \ ) + j\underbrace{(\ \ )}_{\equiv 0}$$

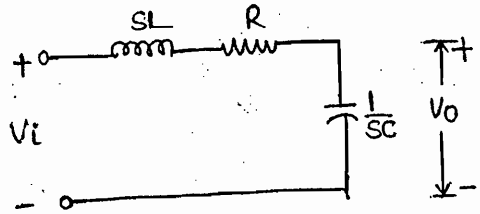
$$X_L = \frac{1}{2}\left[X_C \pm \sqrt{X_C^2 - 4R^2}\right]$$

$\longrightarrow X_C^2 = 4R^2$; one value of $X_L$ ✗ One resonance occurs.

$\longrightarrow X_C^2 < 4R^2$; $X_L = a+jb$ ✗ Inductive Reactance ($Z = jX_L$) is always imag.

$\longleftarrow$ ✓ $X_C^2 > 4R^2$ 66 $X_C > 2R$

15/50 + 12/49




∗ If series RLC is given then o/p can be taken from any elements.

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{1/sC}{sL+R+1/sC}$$

$$= \frac{1}{s^2LC+RCs+1}$$

$$= \frac{1/LC}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$D(s) = s^2+\frac{R}{L}s+\frac{1}{LC}$$

$$= s^2+2\xi\omega_n s+\omega_n^2$$

15

$$2\xi\omega_n = \frac{R}{L} \quad \& \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$\& \ \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

for no oscillations;

$$\xi \geq 1$$

$$\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$$

$$\boxed{R \geq 2\sqrt{\frac{L}{C}}}$$

12

$$H(s) = \frac{10^6}{s^2+20s+10^6} = \frac{\omega_n^2}{s^2+\underbrace{2\xi\omega_n}_{\Delta\omega}s+\underbrace{\omega_n^2}_{\omega_0^2}}$$

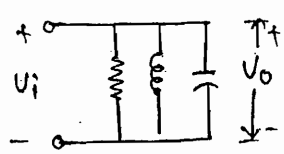
∗∗∗ $\frac{R}{L} = 2\xi\omega_n = \Delta\omega\ (BW)$

$$\frac{1}{\sqrt{LC}} = \omega_n = \omega_0$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} \qquad \Delta\omega = 20 \quad \omega_0 = 10^3$$

$$Q_0 = \frac{\omega_0}{\Delta\omega} = 50$$

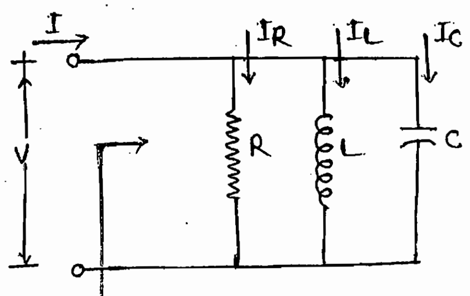
$$\boxed{Q_0 = 50}$$




$$\boxed{\frac{V_o(s)}{V_i(s)} = 1 = H(s)}$$

## * Parallel RLC ckt →

...... Parallel Resonance.




$$Y_{in} = Y_1 + Y_2 + Y_2$$

$$= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

<u>at $\omega = \omega_0$;</u> j term = 0

$$\left(\omega C - \frac{1}{\omega L}\right) = 0$$

$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$

<u>at $\omega = \omega_0$;</u>

$Y_{in} = Y_0 = \frac{1}{R} + j(0)$ ...... min

$Z_0 = R$ ...... max

$I_0 = V Y_0$ ...... min

$|I_L| = |I_C|$

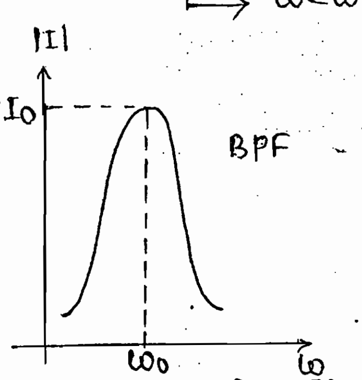
& are out of phase by $180^\circ$

$$Y_{in} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

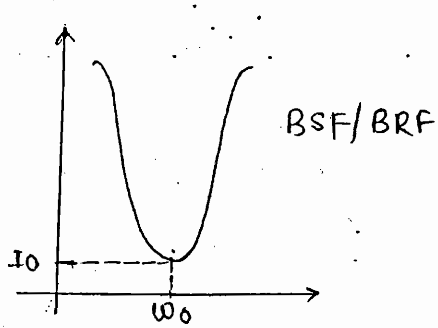
→ $\omega = \omega_0$; Resistive.

→ $\omega > \omega_0$; capacitive.

→ $\omega < \omega_0$; Inductive.




series RLC




parallel RLC

DATE-01/12/14

* Features →

- At resonance the imag. part of the i/p admittance is 0.
- At resonance i/p admittance is purely real; has min^m value, i/p impedance is max^m & therefore the i/p vol. & current are in phase.

At resonance the current drawn by the n/w is min^m & therefore a parallel LLC n/w behaves as a band stop (or) Band reject filter.

At resonance the current through ind^r L & current through cap. (C) are equal in magnitude & are phase shifted by 180° so that net current drawn by the C combination is 0.

The n/w behaves differently at different freq. of operation:—

1) at $\omega = \omega_0$, the n/w is resistive & has a UPF.
2) At Above $\omega_0$ n/w is capacitive & has a leading PF.
3) Below $\omega_0$ n/w is inductive & has a lagging PF.

- The QF represents the current amplification factor & for any tuned n/w it must have a high value.

Quality Factor ($Q_0$) →

$$Q_0 = \frac{|I_L|}{I_0} = \frac{|I_C|}{I_0} \; ; \; (>>1)$$

$$\boxed{Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC = R\sqrt{\frac{C}{L}}}$$

$$|I_L| = \left|\frac{V}{j\omega_0 L}\right| = \frac{V}{\omega_0 L}$$

$$|I_C| = \left|\frac{V}{1/j\omega_0 C}\right| = \omega_0 CV$$

$$I_0 = \frac{V}{R}$$

General Results →

$$Q_0 = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \omega_2 - \omega_1 \qquad \omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_2 \cong \omega_0 + \frac{1}{2}\Delta\omega$$

$$\omega_1 \cong \omega_0 - \frac{1}{2}\Delta\omega$$

Bandwidth →

$$\cancel{\Delta\omega} = \frac{\cancel{\omega_0}}{\cancel{\omega_0}}$$

$$\Delta\omega = \frac{\omega_0}{Q_0}$$

$$= \frac{1/\sqrt{LC}}{R\sqrt{\frac{C}{L}}}$$

$$\boxed{\begin{array}{l} \Delta\omega = \frac{1}{RC} \\ \Delta f = \frac{1}{2\pi RC} \end{array}}$$

$\Delta\omega = \frac{1}{RC}$ …rad/s

$\Delta f = \frac{1}{2\pi RC}$ …Hz

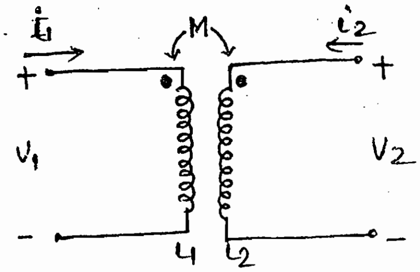
$Q_0\uparrow \rightarrow \Delta\omega\downarrow$ — → $R\uparrow$ …. Not desirable.
→ $C\uparrow$

| Series RLC | Parallel RLC |
|---|---|
| $\Delta\omega = \frac{R}{L}$ | $\Delta\omega = \frac{1}{RC}$ |
| $\Delta f = \frac{R}{2\pi L}$ | $\Delta f = \frac{1}{2\pi RC}$ |

# $\boxed{\text{Coupled circuit}}$

...... Effect of mutual inductance (M)

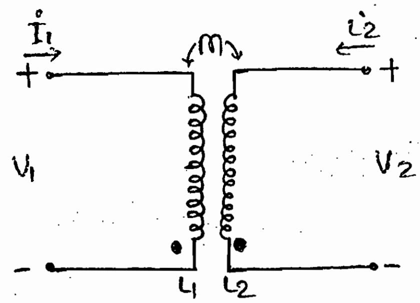
Case(1) →



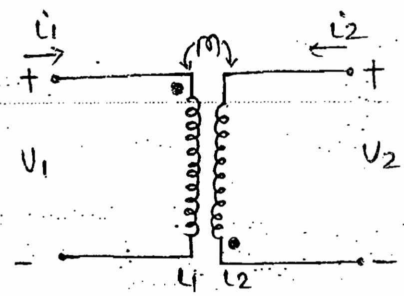

| |
|---|
| Effect of M is +ve |
| Einduced is additive |

..... Case(1), Case(2)

Case(2) →



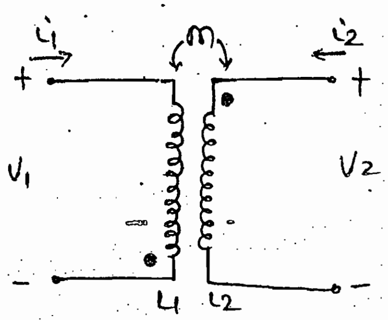

Case(3) →




| |
|---|
| Effect of M is -ve |
| Einduced is subtractive |

...... Case (3), (4)

Case(4) →




$$\left.\begin{aligned} V_1(t) &= L_1\frac{di_1(t)}{dt} \pm M\frac{di_2(t)}{dt} \\ V_2(t) &= \pm M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt} \end{aligned}\right\}$$

..... In time domain

..... For non-sinusoidal excitation

$$V_1(s) = sL_1 I_1(s) \pm sM I_2(s).$$

$$V_2(s) = \pm sM I_1(s) + sL_2 I_2(s).$$

...... In s-domain assuming zero Ic's

...... For non-sinusoidal excitation.

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & \pm sM \\ \pm sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} j\omega L_1 & \pm j\omega M \\ \pm j\omega M & j\omega L_2 \end{bmatrix}}_{\text{Impedance matrix}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

..... for sinusoidal excitation

.... Using phasors.

* Effect of Mutual Impedances →

(1.) In terms of M.

(2.) In terms of K

∵ K is coff. of coupling. ($K \leq 1$)

(3.) $\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$ ; $\boxed{M = K\sqrt{L_1 L_2}}$

Case(i) → $K < 1$, $M < \sqrt{L_1 L_2}$

...... Loose coupling

...... Under coupled ckts.

Case(ii) → $K = 1$, $M = \sqrt{L_1 L_2}$

...... Critical coupling

....... Criticaly coupled.

3.) In terms of turns Ratio

$$\boxed{\begin{gathered} \frac{V_1}{V_2} = \frac{n_1}{n_2} \\ \frac{I_1}{I_2} = \frac{n_2}{n_1} \end{gathered}}$$

* The effect of mutual inductance is considered whenever 2 ind$^r$ are placed physicaly close to each other.

* when the current flows in the 2nd ind$^r$ some time varying magnetic flux is produced. Part of this magnetic flux will link with the 1st ind$^r$.

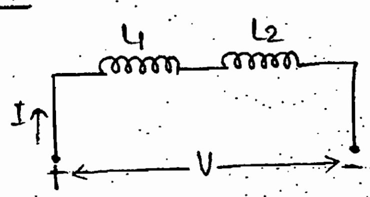
Due to Faraday's law of electromagnetic ind$^n$ (or) due to Rate of change of magnetic flux some emf is induced b/n 2 ~~terminal~~ of 1st ind$^r$.

The polarity of this induced vol. depends upon the dot convention & therefore depends upon relative sense of wdg of 2 ind$^r$.

Then all the KVL eq$^n$ are modified depanding upon the excitation.

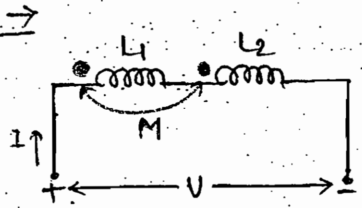
Example →

Case(1) →




$$\boxed{L_{eq} = L_1 + L_2}$$

Case(2) →

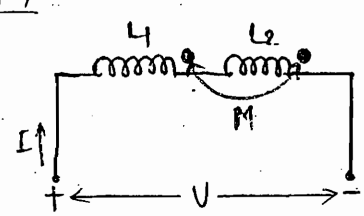



$$V = j\omega L_1 I + j\omega L_2 I + j\omega M I + j\omega M I$$

$$\frac{V}{I} = z_{eq} = j\omega(L_1 + L_2 + 2M) = j\omega L_{eq}$$

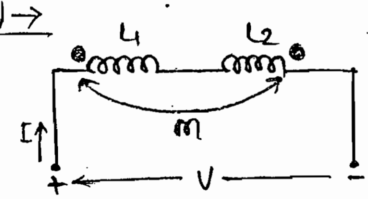
$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

Case(3) →



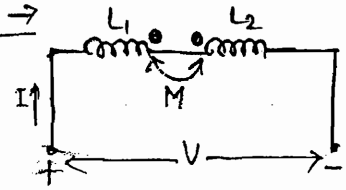

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

Case(4) →



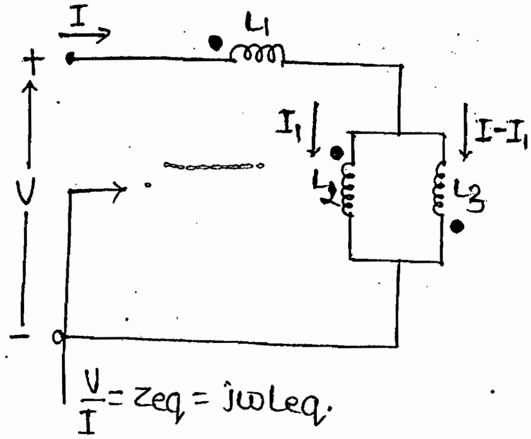

$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$

Case(5) →




$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$

Example →




$L_1 = L_2 = L_3 = 10H$

$M_{12} = M_{21} = 1H$

$M_{13} = M_{31} = 2H$

$M_{23} = M_{32} = 3H$

Find : Leq.

$\frac{V}{I} = Z_{eq} = j\omega L_{eq}$.

KVL-(1) →

$$\left.\begin{aligned} V = j\omega L_1 I + j\omega L_2 \cdot I_1 + \\ + j\omega I_1 M_{21} + j\omega(I-I_1)M_{31} \\ + j\omega M_{12} I - j\omega M_{32}(I-I_1) \end{aligned}\right\} V = j\omega(\ \ )I + j\omega(\ \ )I_1 \cdots (i)$$

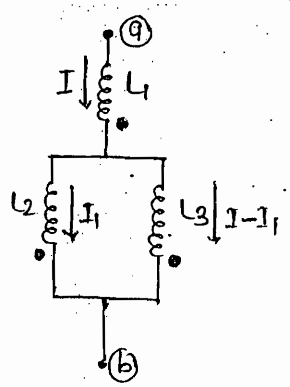
KVL-(2) →

$$\left.\begin{aligned} V = j\omega L_1 \cdot I + j\omega L_3(I-I_1) \\ + j\omega M_{21} I_1 - j\omega M_{31}(I-I_1) \\ + j\omega M_{13} I_1 - j\omega M_{23} I_1 \end{aligned}\right\} V = j\omega(\ \ )I + j\omega(\ \ )I_1 \cdots (ii)$$

Eliminate $I_1$, find $\frac{V}{I} = Z_{eq} = j\omega L_{eq}$

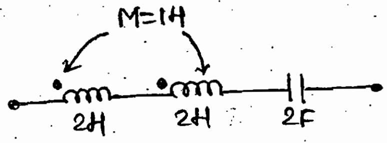
find Leq.

Example →




To find Leq. b/n a & b.

6/49

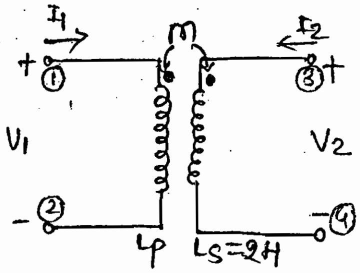



$$F_0 = \frac{1}{2\pi\sqrt{L_{eq}C}}$$

$$L_{eq} = L_1 + L_2 + 2M$$
$$= 2+2+2(1)$$
$$= 6H$$

$$F_0 = \frac{1}{2\pi\sqrt{6\times2}} = \frac{1}{4\pi\sqrt{3}}\ Hz.$$

15/4




$$\because k = \frac{M}{\sqrt{L_p L_s}}$$

3-4 ⇒ OC $I_2=0$

$$\frac{V_1}{I_1} = z_{eq} = j\omega L_{eq} = j\omega 4 \cdots (i)$$

$$V_1 = j\omega L_p I_1 + 0 \cdots (ii)$$

$$\frac{V_1}{I_1} = j\omega L_p = j\omega 4$$

$$\boxed{L_p = 4}$$

3-4 ⇒ SC $V_2=0$

$$\frac{V_1}{I_1} = z_{eq} = j\omega L_{eq} = j\omega 3 \cdots (a)$$

$$V_1 = j\omega L_p \cdot I_1 + j\omega M I_2 \cdots (b)$$

$$V_2 = 0 = j\omega L_s I_2 + j\omega M I_1 \cdots (c)$$

$$\boxed{M=\sqrt{2}}$$

$$k = \frac{M}{\sqrt{L_s L_p}} = 4/8 = 1/2\ (<1)$$

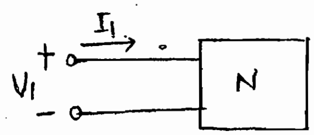
.....loose coupling..

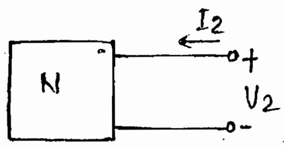
..... Under coupled ckt.

# Network Functions

..... Used to represent any elet n/w mathematically.

(1.) Case(1) → One port networks




$\frac{V_1}{I_1} = z_{11}$ ..... driving pt. i/p impedance fn

$\frac{I_1}{V_1} = Y_{11}$ ..... driving pt i/p admittance fn

$\frac{V_2}{I_2} = z_{22}$ ....... driving pt. o/p impedance fn

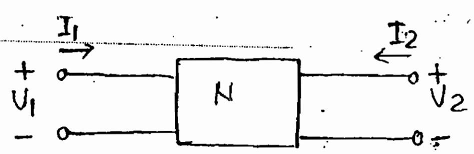
$\frac{I_2}{V_2} = Y_{22}$ ..... driving pt. o/p admittance fn.

driving point immitance fns.

Impedance

admittance

(2.) Case(ii) → Two port networks.




i) Driving pt. immittance fns →

$$Z_{11} = \frac{V_1}{I_1} \quad ; \quad Z_{22} = \frac{V_2}{I_2}$$

$$Y_{11} = \frac{I_1}{V_1} \quad ; \quad Y_{22} = \frac{I_2}{V_2}$$

ii) Transfer impedance Ratio →

$$Z_{12} = \frac{V_1}{I_2} \quad ; \quad Z_{21} = \frac{V_2}{I_1}$$

(iii) Transfer admittance Ratio →

$$Y_{12} = \frac{I_1}{V_2} \;;\; Y_{21} = \frac{I_2}{V_1}$$

(iv) Transfer vol. Ratio →

$$G_{12} = \frac{V_1}{V_2} \;;\; G_{21} = \frac{V_2}{V_1}$$

(v) Transfer Current Ratio →

$$\alpha_{12} = \frac{I_1}{I_2} \;;\; \alpha_{21} = \frac{I_2}{I_1}$$

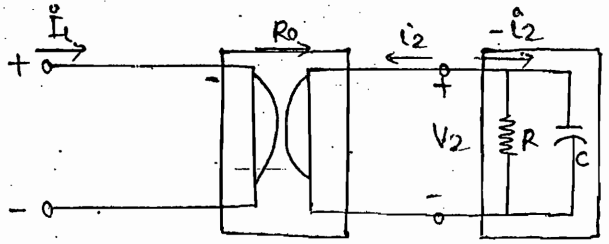
Network fns.
F(s)

$Z_{11}, Z_{22}$, $Y_{11}, Y_{22}$ → driving point immittance fn

$Z_{12}, Z_{21}, Y_{12}, Y_{21}$, $G_{12}, G_{21}, \alpha_{12}, \alpha_{21}$ → Transfer fns.

* For immitance fn all the poles & zeros lie in LHP. For TFn all the poles still lie in LHP where as some of the zeros may also lie in RHP.

Example →




$$Z_{in} = Z_{11} = \frac{V_1}{I_1} = ?$$

$$Z = \frac{R \times \frac{1}{sC}}{R + 1/sC} = \frac{R}{1 + sRC}$$

Soln →

$$v_1 = R_0 i_2 \ldots\ldots (i)$$

$$v_2 = -R_0 i_1 \ldots\ldots (ii)$$

$$v_2 = -i_2 Z = -i_2 \cdot \frac{R}{1 + sRC} \ldots\ldots (iii)$$

From eqn (ii) & (iii)

$$i_2 \cdot \frac{R}{1+SRC} = R_0 i_1$$

$$i_2 = \frac{R_0(1+SRC)}{R} \cdot i_1$$

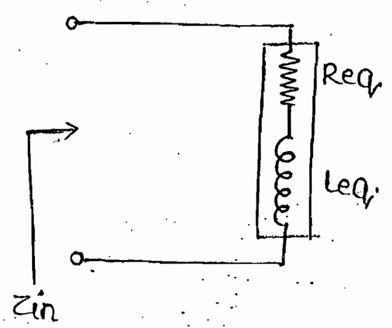
$$V_1 = \frac{R_0^2(1+SRC) \cdot i_1}{R}$$

$$\frac{V_1}{i_1} = Z_{in} = \boxed{Z_{11} = \frac{R_0^2 + SR_0^2 C}{R}}$$

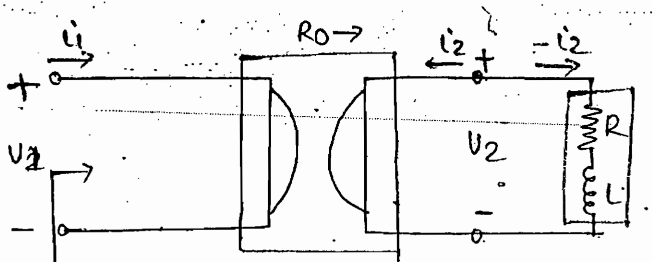
$$= Z_1 + Z_2$$

$$= R_{eq} + S L_{eq}$$

$$\boxed{R_{eq} = \frac{R_0^2}{R} \; ; \; L_{eq} = R_0^2 C}$$




Example →




$$Z_{in} = Z_{11} = \frac{V_1}{i_1}$$

$$Z = R + SL$$

Sol$^n$ →

$$V_1 = +R_0 i_2 \cdots (i)$$

$$V_2 = -R_0 i_1 \cdots (ii)$$

$$V_2 = -i_2 \cdot Z = -i_2(R+SL) \cdots (iii)$$

from eq$^n$ (ii) & (iii)

$$-R_0 \cdot i_1 = -i_2(R+SL)$$

$$i_2 = \frac{R_0}{R+SL} i_1$$

$$V_1 = \frac{R_0^2}{R+sL} \cdot i_1$$

$$\frac{v_i}{i_1} = \boxed{Z_{in} = \frac{R_0^2}{R+sL}} = \frac{1}{Y_{in}}$$

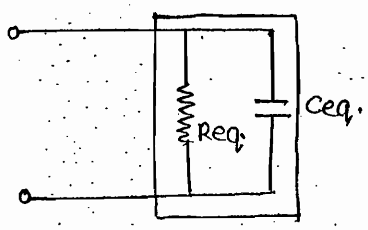
$$Y_{in} = \frac{R+sL}{R_0^2}$$

$$= \frac{R}{R_0^2} + \frac{sL}{R_0^2}$$

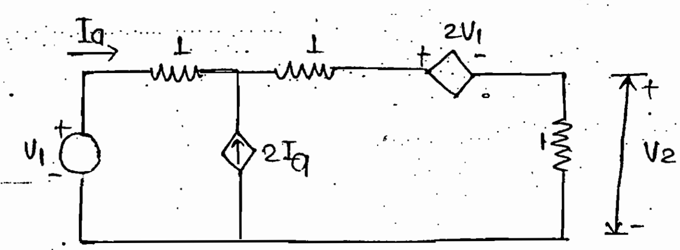
$$= Y_1 + Y_2$$

$$= \frac{1}{R_{eq}} + sC_{eq}$$

$$\boxed{R_{eq} = \frac{R_0^2}{R} \,;\; C_{eq} = \frac{L}{R_0^2}}$$

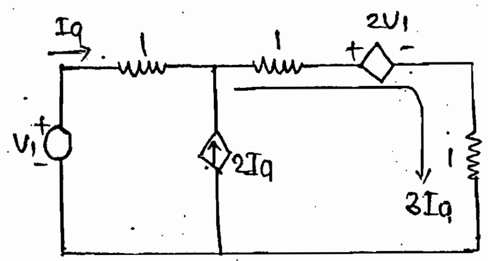



Example→




To find $G_{21} = \frac{V_2}{V_1}$

Soln→




$$-V_1 + I_a + 1\times 3I_a + 2V_1 + 1\times 3I_a = 0$$

$$V_1 = -7I_a \;\ldots\ldots(i)$$

$$V_2 = 1\times 3I_a \;\ldots\ldots(ii)$$

$$\boxed{\frac{V_2}{V_1} = G_{21} = \frac{-3}{7}}$$

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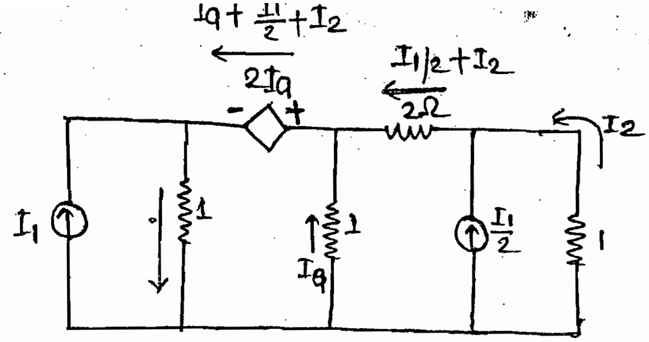



To find →
$\alpha_{21} = \frac{I_2}{I_1}$

$I_a + I_1 + \frac{I_1}{2} + I_2 = \frac{3}{2} I_1 + I_a + I_2$

KVL-(1)

$$1 \times I_2 + 2 \times \left(\frac{I_1}{2} + I_2\right) - 1 \times I_a = 0$$

$$I_a = 3I_2 + I_1 \quad \ldots\ldots (i)$$

KVL-(2)

$$I_a \times 1 + 2I_a + 1 \times \left(\frac{3}{2} I_1 + I_a + I_2\right) = 0$$

$$4I_a + \frac{3}{2} I_1 + I_2 = 0 \quad \ldots\ldots (ii)$$

From eqn (i) & (ii)

$$4(3I_2 + I_1) + \frac{3}{2} I_1 + I_2 = 0$$

$$\boxed{\alpha_{21} = \frac{I_2}{I_1} = \frac{-11}{26}}$$ Ans.

Que → A n/w fn is given by the following expn. Draw the pole-zero pattern & explain the significance of poles & zeros of the n/w.

$$F(s) = \frac{\cancel{(s+1)}(s+3)}{s(s+2)(s+4)}$$

Soln →

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{\text{Num. poly}}{\text{den. poly}}$$

$Z_{11}, Z_{22}$
$Y_{11}, Y_{22}$

$Z_{12}, Z_{21}, Y_{21}, Y_{12}$
$G_{12}, G_{21}, \alpha_{12}, \alpha_{21}$

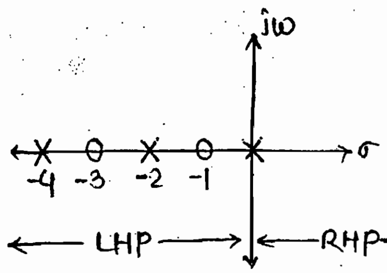
Zeros

Num. poly = 0, F(s) = 0 } $s = -1, -3$

Poles

deno. poly = 0, $F(s) = \infty$ } $s = 0, -2, -4$

…… poles & zeros
…… Complex freq.
…… Singularities.
…… Natural freq.




$$F(S) = \frac{A}{S} + \frac{B}{(S+2)} + \frac{C}{(S+4)}$$

$$f(t) = (A + Be^{-2t} + Ce^{-4t})u(t)$$

A, B, C.

...... coeff. of practical fraction expansion

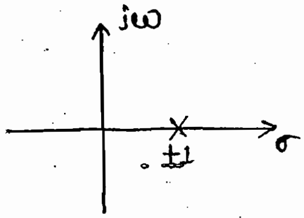
....... Residues.

* The zeros represent complex freq. when the numerator poly. of given n/w f^n becomes zero.
  Therefore at this freq. the n/w f^n becomes zero. The zeros control the magnitude of the responce of any given ele. n/w
* Poles represent the complex freq. at which the denominator poly. of n/w f^n becomes zero.
  Therefore at this freq. the n/w f^n becomes ∞. The poles control the nature (or) shape of responce of the ele. n/w.
  Therefore this poles than control the overall stability of the given n/w.
* In any n/w no. of zeros & no. of poles are always equal.
* For stable n/w :-
  (1) No pole should exist in RHP.
  (2) No multiple poles should exist along the jω axis including the origin.
* The loc^n of poles & zeros control the type of elements contain in the given n/w.
* For driving pt. immitance f^n all the poles & zeros lie in LHP. For TFs all the poles lie in LHP where as some of zeros may also lie in RHP.

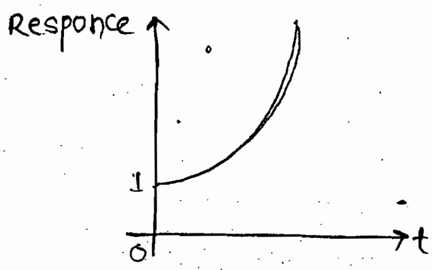
ITE-02/12/14

* Effect of $loc^n$ of poles on the nature of responce of ele n/w →

Case(1) →

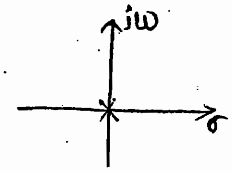


$$\frac{1}{S-1} \longrightarrow e^{t}u(t)$$

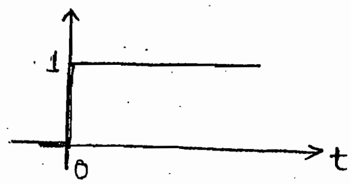


Unstable n/w

case(2) →

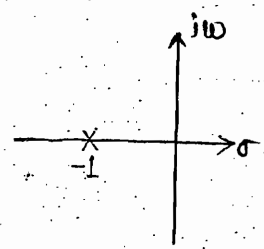


$$\frac{1}{S} \longrightarrow u(t)$$



ideal (OR) lossless n/w

ase(3) →

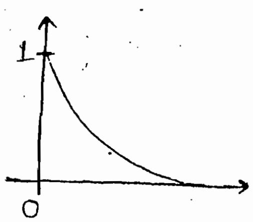


$$\frac{1}{S+1} \longrightarrow e^{-t}u(t)$$

(OR)

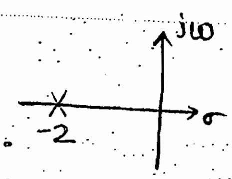
$e^{-t/T}u(t)$

$(T = 1S)$



RC (OR) RL n/w

Case(4) →

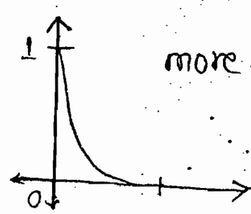


$$\frac{1}{S+2} \longrightarrow e^{-2t}u(t)$$

(OR)

$e^{-t/T}u(t)$

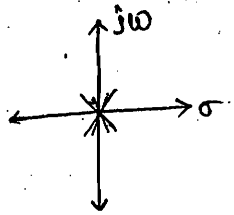
$(T = 1/2\ sec)$



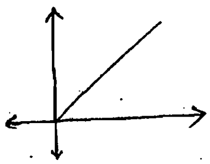
more stable n/w

Stable↑ TC↓

Case(5) →




$\frac{1}{S^2} \longrightarrow t \cdot u(t)$



$\boxed{\text{unstable n/w}}$

* some points →

* For a stable n/w :-

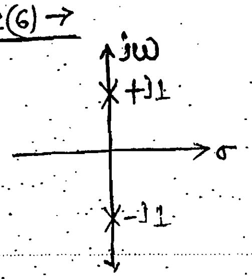
(1.) No pole should exist in RHP.

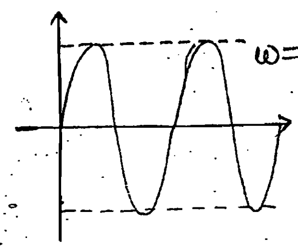
(2.) No multiple pole should exist at the origin.

* When any pole lies along the -ve real axis the responce is exponantialy decaying & the rate of decay depands upon the exact locn of the pole along the -ve real axis.

* When the pole is shifted away from origin along the -ve real axis the rate of decay of the exponantial responce increases, n/w will become more stable & the TC of n/w decreases.

Case(6) →



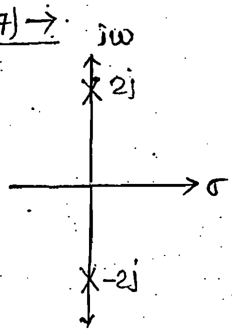

$\frac{1}{(S-j1)(S+j1)} \longrightarrow \frac{1}{S^2+1} \longrightarrow \sin t$ $(\omega=1\ \text{rad/s})$

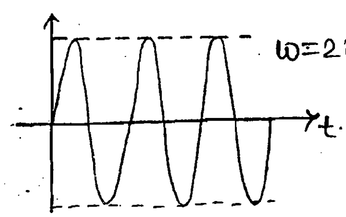
$\boxed{\text{LC network}}$

$S = 0 \pm j1$

$\sigma = 0$

Case(7) →



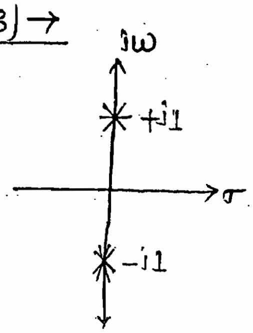




$\frac{1}{(S-2j)(S+2j)} \longrightarrow \frac{1}{S^2+(4)} \longrightarrow \frac{1}{(2)} \cdot \frac{2}{(S^2+4)}$

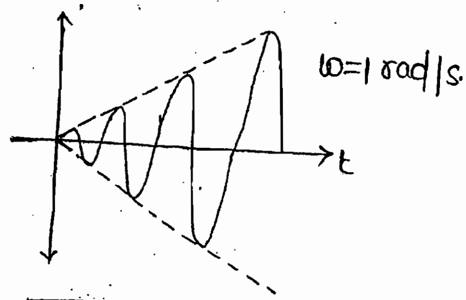
$\Rightarrow \frac{1}{2} \sin 2t$

$(\omega = 2)$

Case(8) →

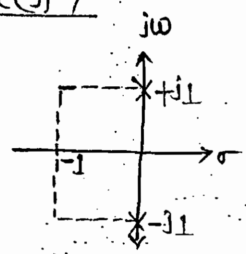


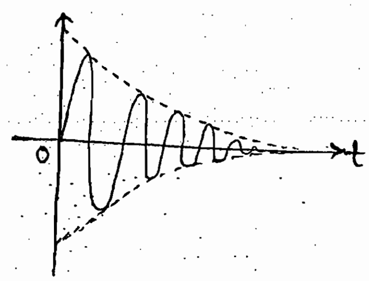

$$\frac{1}{(s^2+1)^2} \longrightarrow t \cdot \sin t$$




Unstable n/w

Case(9) →




RLC n/w

* Some points →

* When the poles are located along the jω axis the responce is oscillatory in nature whose freq. is controlled by the exact locn of the poles along the jω axis.
* When the poles are shifted away from origin along the jω axis the freq. of oscallatory responce increases.
* For a stable n/w multiple poles should not exist along the jω axis.
* When the poles are complex conjugate in nature & lie in LHP. then the responce is oscillatory in nature whose amplitude decreases in an exponantial manner.

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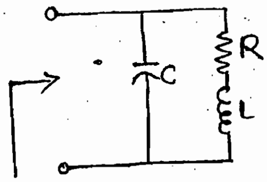



$Z(s)$

$Z(0) = 1, \ Z(s)|_{s=0}$

Zero: $s = -1$

Poles: $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$$Z(s) = \frac{\left(\frac{1}{sC}\right)(R+sL)}{\left(\frac{1}{sC}\right) + (R+sL)}$$

$$= \frac{(R+sL)}{1 + sRC + s^2LC}$$

$$= \frac{L(s+R/L)}{LC\left(s^2 + s\cdot\frac{R}{L} + \frac{1}{LC}\right)}$$

$$Z(0) = \frac{R+0}{1+0+0} = 1$$

$$\boxed{R = 1\Omega}$$

Zeros → form $s + \frac{R}{L} = 0$

$$s = -\frac{R}{L}$$

$$-1 = -\frac{R}{L} \qquad \boxed{L = 1H}$$

Poles → $s^2 + s\cdot\frac{R}{L} + \frac{1}{LC} = 0$

$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\times\frac{1}{LC}}}{2}$$

$$s_1, s_2 = \frac{-1 \pm \sqrt{\left(1 - \frac{4}{C}\right)}}{2}$$

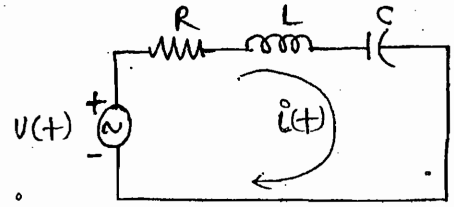
$$= \frac{-1 \pm 2j\sqrt{\cancel{\tfrac{4}{4}}\left(\frac{1}{C} - \frac{1}{4}\right)}}{2}$$

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = \frac{-1 \pm 2j\sqrt{\left(\frac{1}{C} - \frac{1}{4}\right)}}{2}$$

$$\frac{\sqrt{3}}{2} = \sqrt{\frac{1}{C} - \frac{1}{4}}$$

$$\frac{3}{4} = \frac{1}{C} - \frac{1}{4} \qquad \boxed{C = 1F}$$

## ← Effect of variation of R on the Responce on series RLC n/w →




KVL →

$$R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = v(t)$$

$$R\frac{d}{dt}(i(t)) + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}\,i(t) = \frac{dv(t)}{dt}$$

c/s eqⁿ →

$$R\cdot s + Ls^2 + \frac{1}{C} = 0$$

$$s^2 + \frac{R}{L}\cdot s + \frac{1}{LC} = 0 \quad \Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow \quad 2\xi\omega_n = \frac{R}{L}$$

$$\Rightarrow \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

finding the roots of c/s eqⁿ →

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4\omega_n^2}}{2}$$

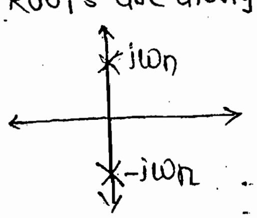
$$= -\xi\omega_n \pm \underbrace{j\omega_n\sqrt{1-\xi^2}}_{\omega_d}$$

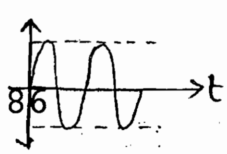
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad s_1, s_2 = -\xi\omega_n \pm j\sqrt{1-\xi^2}\,\omega_n$$

Case(1) → $\xi = 0$, $R = 0$ (Undamped Responce)

$s_1, s_2 = \pm j\omega_n$ $\qquad \frac{1}{s^2+\omega_n^2} \rightarrow \frac{1}{\omega_n}\sin\omega_n t$

..... Roots are along jω axis $\qquad$ ....... sinusoidal Responce with constant amplitude.



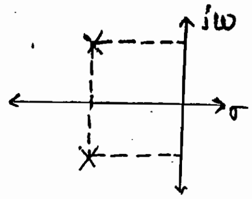



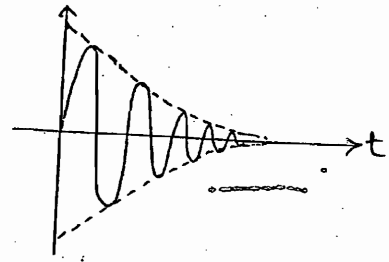
**Case(2) →** $\xi < 1 \; (0 < \xi < 1)$

$R < 2\sqrt{\frac{L}{C}}$

$s_1, s_2 = a \pm jb$

...... Complex conjugate Root




...... sinsusoidal Responce with decreasing amplitude

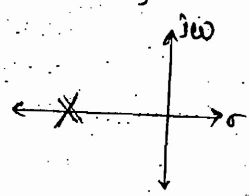
$\boxed{\text{Underdamped Responce}}$

**Case(3) →** $\xi = 1$

$R = 2\sqrt{\frac{L}{C}}$

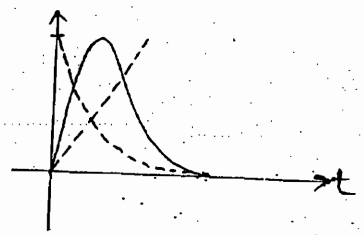
$s_1, s_2 = -\omega_n$

...... Double (equal) poles along -ve real axis




$\frac{1}{(s+\omega_n)^2}$

$\Rightarrow te^{-\omega_n t}$



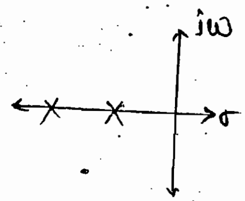

$\boxed{\text{Criticaly damped}}$

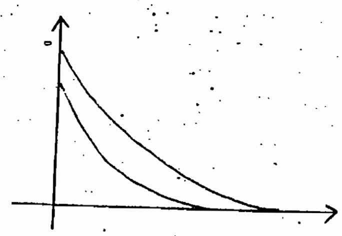
**Case(4) →** $\xi > 1$

$R > 2\sqrt{\frac{L}{C}}$

$s_1, s_2 = -a' \pm b'$

...... Unequal poles along -ve real axis



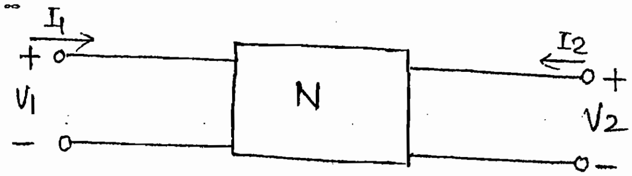



$\boxed{\text{Over-damped}}$

$\boxed{\text{To avoid Oscillations } \xi \geq 1}$

# Two Port n/w Parameters

....... Used to represent any 2 port n/w mathematicaly.




Case(1) → Impedance (OR) Z (OR) OC parameter.

$$\boxed{\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}}$$ ..... Series ..... Series

$\underbrace{}_{x,y}$ $y = mx + c$ — depandant ($y$), independant ($x$)

| $I_2=0$ | $I_1=0$ |
|---|---|
| $Z_{11} = \frac{V_1}{I_1}\,\Omega$ | $Z_{12} = \frac{V_1}{I_2}\,\Omega$ |
| $Z_{21} = \frac{V_2}{I_1}\,\Omega$ | $Z_{22} = \frac{V_2}{I_2}\,\Omega$ |

The n/w Fns are calculated w/o any pre condn whereas the n/w parameters are calc. under some precondn either by oc (OR) sc the i/p OR o/p port. A single n/w fn is req. to specify any given ele. n/w where as to represent he same n/w all the 4 parameters are req. simultaneously.

ise(2) → Admittance (OR) Y (OR) SC parameters.

$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}}$$ ...... parallel .... parallel.

| $V_2=0$ | $V_1=0$ |
|---|---|
| $Y_{11} = \frac{I_1}{V_1}\,S$ | $Y_{12} = \frac{I_1}{V_2}\,S$ |
| $Y_{21} = \frac{I_2}{V_1}\,S$ | $Y_{22} = \frac{I_2}{V_2}\,S$ |

Case(3) → Hybrid (OR) h-parameters

$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$

$V_1 = h_{11} I_1 + h_{12} V_2$ ...... series

$I_2 = h_{21} I_1 + h_{22} V_2$ ...... parallel

| $V_2 = 0$ | $I_1 = 0$ |
|---|---|
| $h_{11} = \frac{V_1}{I_1}\,\Omega$ | $h_{12} = \frac{V_1}{V_2}$ |
| $h_{21} = \frac{I_2}{I_1}$ | $h_{22} = \frac{I_2}{V_2}\,S$ |

* Because of mixed value of units it is known as hybrid parameter.

* This is only parameters used in the BJT because of this gives all the values of vol. Gain, current gain, i/p & o/p imp. req. for BJT as ampʳ.

Case(4) → g-parameters.

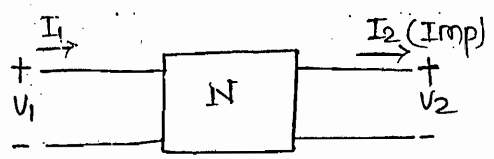
$I_1 = g_{11} V_1 + g_{12} I_2$ ...... parallel

$V_2 = g_{21} V_1 + g_{22} I_2$ ...... series

| $I_2 = 0$ | $V_1 = 0$ |
|---|---|
| $g_{11} = \frac{I_1}{V_1}$, S | $g_{12} = \frac{I_1}{I_2}$ |
| $g_{21} = \frac{V_2}{V_1}$ | $g_{22} = \frac{V_2}{I_2}\,\Omega$ |

* Here $V_1 = 0$, i.e. i/p is shorted. So it is not used in BJT for ampʳ.

Case(5) → Transmission (OR) T (OR) ABCD parameters.




$V_1 = A V_2 + B I_2$

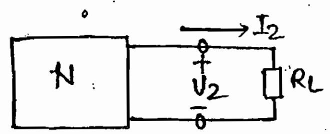
$I_1 = C V_2 + D I_2$

* If the value of vol. is doubled then B will be twice & C will halved.

| $I_2 = 0$ | $V_2 = 0$ |
|---|---|
| $A = \frac{V_1}{V_2}$ | $B = \frac{V_1}{I_2}\,\Omega$ |
| $C = \frac{I_1}{V_2}\,S$ | $D = \frac{I_1}{I_2}$ |

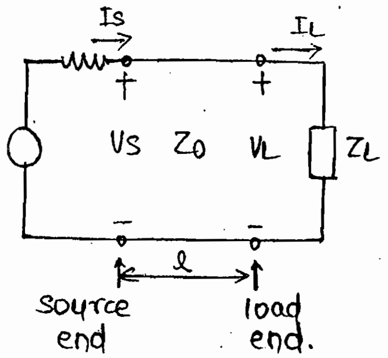
## * Reason behind taking dirn of current opposite →

(1.)

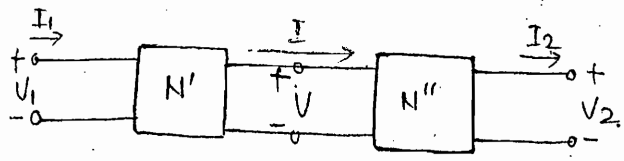



$V_2 = I_2 R_L$

(2.)

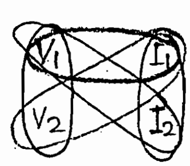



(3.) Cascaded N/ω →




## * Condition for any n/ω to be reciprocal & symmetrical →

| | Reciprocal | Symmetrical |
|---|---|---|
| * | $Z_{12} = Z_{21}$ | $Z_{11} = Z_{22}$ |
| * | $Y_{12} = Y_{21}$ | $Y_{11} = Y_{22}$ |
| * | $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$, $\Delta = 1$<br>$AD - BC = 1$ | $A = D$ |
| * | $h_{12} = -h_{21}$ | $\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$, $\Delta h = 1$<br>$h_{11} \cdot h_{22} - h_{12} \cdot h_{21} = 1$ |
| * | $g_{12} = -g_{21}$ | $\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$, $\Delta g = 1$<br>$g_{11} \cdot g_{22} - g_{21} \cdot g_{12} = 1$ |



## * Conversion b/n parameters →

*(1) Given: $[Z]$ ; To Find :- $[Y]$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta Z}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \; ; \; Y_{21} = \frac{-Z_{21}}{\Delta Z}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} \; ; \; Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$\Delta_Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11}Z_{22} - Z_{12} \cdot Z_{21}$$

*(2) Given :- $h_{11} = 2$, $h_{12} = -2$, $h_{21} = 3$, $h_{22} = 2$

To find :- $[Z]$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\boxed{\begin{aligned} V_1 &= 2I_1 - 2V_2 \\ I_2 &= 3I_1 + 2V_2 \end{aligned}}$$

$\Rightarrow 2V_2 = I_2 - 3I_1$

$\Downarrow$

$$V_2 = \underbrace{\left(\frac{-3}{2}\right)}_{Z_{21}} I_1 + \underbrace{\left(\frac{1}{2}\right)}_{Z_{22}} I_2$$

$\Rightarrow V_1 = 2I_1 - (I_2 - 3I_1)$

$\Downarrow$

$$V_1 = \underbrace{5}_{Z_{11}}I_1 + \underbrace{(-1)}_{Z_{12}}I_2$$

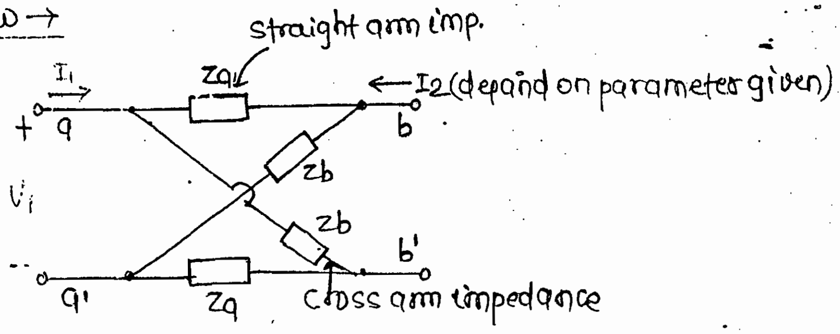
$$\boxed{\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}}$$

$$[Z] = \begin{bmatrix} 5 & -1 \\ -\frac{3}{2} & +1/2 \end{bmatrix} ; \Omega$$

$Z_{11} \neq Z_{22} \Rightarrow$ Not symmetrical

$Z_{12} \neq Z_{21} \Rightarrow$ Not reciprocal.

## * Lattice n/w →




To find →

(1.) $[Z]$

(2.) $Z_a, Z_b$ in terms of $[Z]$

* A Lattice n/w said to be symmetrical if the 2 straight arm imp. are equal i.e.

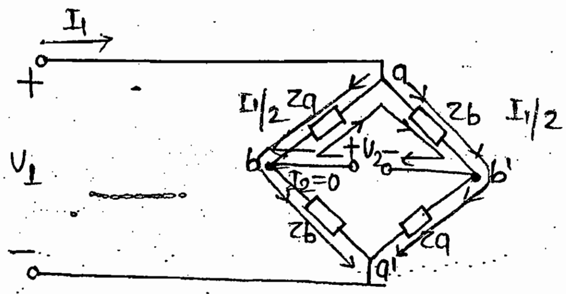
$Z_{11}=Z_{22}$ ...... Symmetrical N

* A Lattice n/w is said to be reciprocal if the 2 cross arm imp. are equal i.e.

$Z_{12}=Z_{21}$ ........ Reciprocal N

<u>$I_2=0$</u>...

$Z_{11}=\frac{V_1}{I_1}$ $\quad$ $Z_{21}=\frac{V_2}{I_1}$




<u>By KVL →</u>

$$V_1=\frac{I_1}{2}(Z_a+Z_b) \text{ ------ (i)}$$

$$0=-V_2-\frac{I_1}{2}Z_a+\frac{I_1}{2}(Z_b)$$

$$V_2=\frac{I_1}{2}(Z_b-Z_a) \text{ ------ (ii)}$$

$$Z_{11}=\frac{V_1}{I_1}=\frac{Z_b+Z_a}{2}=Z_{22}$$

$$Z_{21}=\frac{V_2}{I_1}=\frac{Z_b-Z_a}{2}=Z_{12}$$

$$\boxed{\begin{aligned} Z_{11}=Z_{22}&=\frac{Z_b+Z_a}{2} \\ Z_{21}=Z_{12}&=\frac{Z_b-Z_a}{2}\end{aligned}}$$

$$Z_b+Z_a=2Z_{11}$$

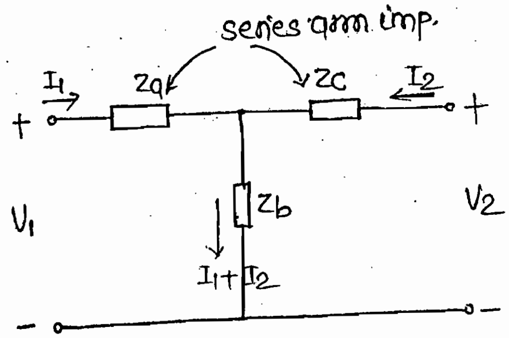
$$Z_b-Z_a=2Z_{12}$$

$$\boxed{\begin{aligned} &\text{add: } Z_b=Z_{11}+Z_{12} \\ &\text{subtract: } Z_a=Z_{11}-Z_{12}\end{aligned}}$$

<u>Lattice N/W</u> $\quad$ <u>T-N/W</u> $\quad$ <u>π-N/W</u>

* $[Z]=\frac{1}{2}\begin{bmatrix} Z_b+Z_a & Z_b-Z_a \\ Z_b-Z_a & Z_b+Z_a\end{bmatrix}$ $\quad$ * $[Z]=\begin{bmatrix} Z_a+Z_b & Z_b \\ Z_b & Z_c+Z_b\end{bmatrix}$ $\quad$ * $[Y]=\begin{bmatrix} Y_a+Y_b & -Y_b \\ -Y_b & Y_c-Y_b\end{bmatrix}$

Example →




To find → $[Z]$

....... T (OR) Y are star n/w

By KVL →

$$V_1 = \underbrace{(Z_a+Z_b)}_{Z_{11}} I_1 + \underbrace{Z_b}_{Z_{12}} I_2$$

$$V_2 = \underbrace{Z_b}_{Z_{21}} I_1 + \underbrace{(Z_b+Z_c)}_{Z_{22}} I_2$$

* Any general T-n/w is always a reciprocal n/w but may not be a symmetrical N/w. Such n/w is symm. in nature is the 2 series arm impedances $Z_a$ & $Z_c$ are equal.

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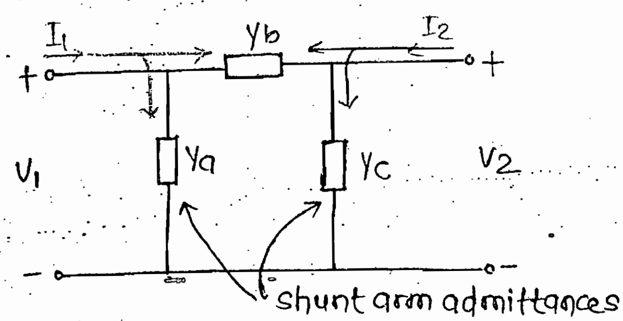
Example → $\pi$ (OR) $\Delta$ n/w



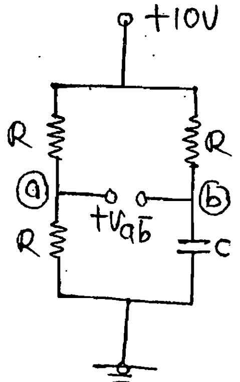

To find $[Y]$

$$I_1 = V_1 Y_a + (V_1 - V_2) Y_b = V_1 \underbrace{(Y_a+Y_b)}_{Y_{11}} + \underbrace{(-Y_b)}_{Y_{12}} V_2$$

$$I_2 = V_2 Y_c + (V_2 - V_1) Y_b = \underbrace{(-Y_b)}_{Y_{21}} V_1 + \underbrace{(Y_c - Y_b)}_{Y_{22}} V_2$$

* Any general $\pi$-n/w is always a reciprocal n/w but may not be symmetrical in nature. Such n/w is a symmetrical n/w if the 2 shunt arm admittances $Y_a$ & $Y_c$ are equal.

(4/34) By rearranging the given n/w




...... All pass filter.

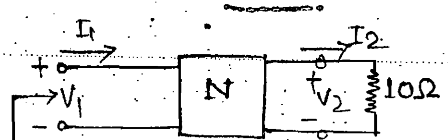
$$|V_{ab}| = |V_a - V_b|$$

$$V_{ab} = 5 - \frac{1/j\omega C}{R + 1/j\omega C} \times 10$$

$$= 5\left[\frac{-1 + j\omega RC}{1 + j\omega RC}\right]$$

$$\boxed{|V_0| = 5\,V}$$

(5/35)




~~$I_1 = AV_2 + BI_2$~~

~~$V_1 = AV_2 + DI_2$~~

$Z_{in} = Z_{11} = \frac{V_1}{I_1}$

$$N;\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$V_1 = AV_2 + BI_2 = V_2 + 2I_2 = 10I_2 + 2I_2 = 12I_2$

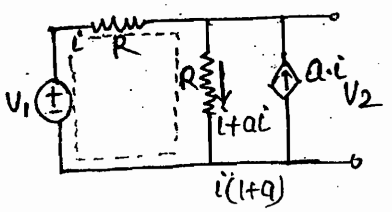
$I_1 = CV_2 + DI_2 = V_2 + 3I_2 = 10I_2 + 3I_2 = 13I_2$

$V_2 = 10I_2$

$V_1 = 12I_2$ ; $I_1 = 13I_2$

$$\boxed{\frac{V_1}{I_1} = \frac{12}{13} = Z_{11}}$$

10/35




$V_1 = iR + i(1+a)R$ ----- (i)

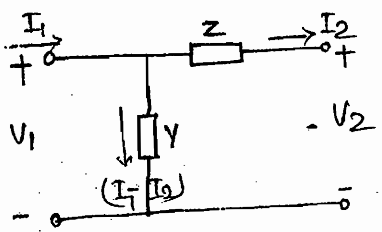
$V_2 = R(1+a)i$ ------ (ii)

$$\frac{V_1}{V_2} = \frac{\cancel{i}[\cancel{R} + (1+a)\cancel{R}]}{\cancel{i}(1+a)\cancel{R}} = \frac{2+a}{1+a}$$

$$\frac{V_1}{V_2} = \frac{2+a}{1+a}$$

$$\boxed{\frac{V_2}{V_1} = \frac{1+a}{2+a}}$$ Ans.

21/37




For ABCD parameters

$V_1 = AV_2 + BI_2$

$I_1 = CV_2 + DI_2$

$(I_1 - I_2)\cancel{Y} = V_1 Y$ ----- (i)

$V_1 = I_2 Z + V_2$ ------ (ii)

$$V_1 = V_2 + ZI_2 = \underbrace{1}_{A} \cdot V_2 + \underbrace{Z}_{B} I_2$$

From eq$^n$(i)

$I_1 = V_1 Y + I_2 = Y(I_2 Z + V_2) + I_2 = YV_2 + I_2(1+YZ)$

$$I_1 = \underbrace{Y}_{C} V_2 + \underbrace{(1+YZ)}_{D} I_2$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+YZ \end{bmatrix}$$ Ans.

$A \neq D$ (Not symmetrical)

$AD - BC = 1(1+YZ) - YZ = 1 + YZ - YZ = 1$ (Reciprocal)

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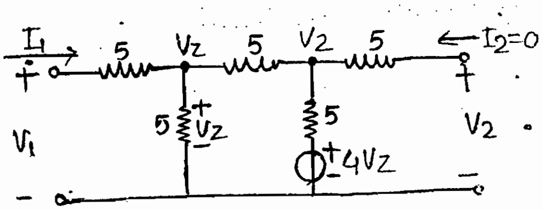


$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

$$Z_{11} = \frac{V_1}{I_1}$$

In this eq$^n$ if we calculate either n/w f$^n$ or n/w parameter both will be same.

$$\frac{V_Z-V_1}{5}+\frac{V_Z-0}{5}+\frac{V_Z-V_2}{5}=0$$

$$3V_Z=V_1+V_2 \text{ ---(i)}$$

$$\frac{V_2-V_Z}{5}+\frac{V_2-4V_Z}{5}=0$$

$$2V_2 \pm 5V_Z \text{ -----(ii)}$$

$$\frac{V_1-V_Z}{5}=I_1 \text{ .....(iii)}$$

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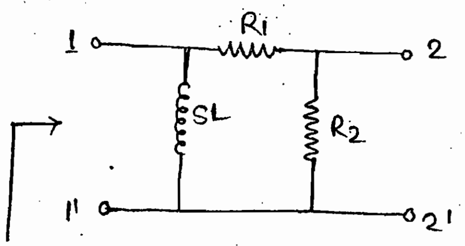
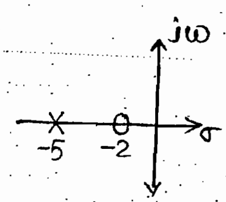



$$Z_{11}=\frac{(SL)(R_1+R_2)}{(SL+R_1+R_2)}=\frac{L(R_1+R_2)}{L}\cdot\frac{S}{S+q}$$

where; $q=\frac{R_1+R_2}{L}$

$$Z_{22}=\frac{(R_2)\times(R_1+SL)}{(R_2+R_1+SL)}$$

$$=\frac{R_2\cdot L}{L}\cdot\frac{S+R_1/L}{S+\left(\frac{R_1+R_2}{L}\right)}=k_2\frac{S+b}{S+q}$$

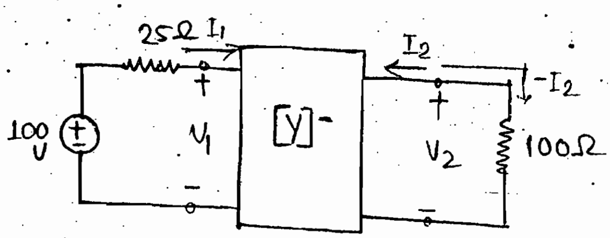



$Z=\frac{V}{I}$

$$[Z_{11},Z_{22}]=K_1\frac{S}{S+q},\ k_2\frac{S+b}{S+q}$$

RL

4/a




$$[Y]=\begin{bmatrix}0.1S & -0.01S\\ 0.01S & 0.1S\end{bmatrix}$$

To Find:- $\frac{V_2}{V_1}=g_{21}$

$$I_1=0.1V_1-0.01V_2 \text{ ------(i)}$$

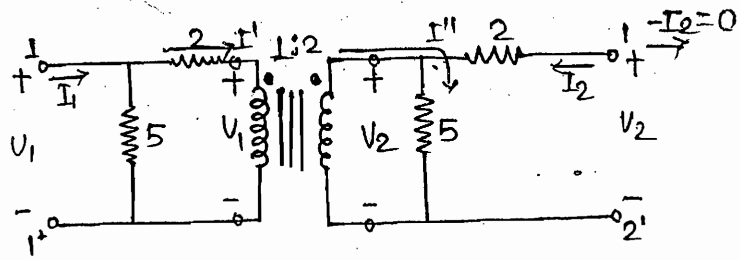
$$I_2=0.01V_1+0.01V_2 \text{ -----(ii)}$$

$$100=25I_1+V_1 \text{ ------(iii)}$$

$$V_2=-I_2 100 \text{ ------(4.)}$$

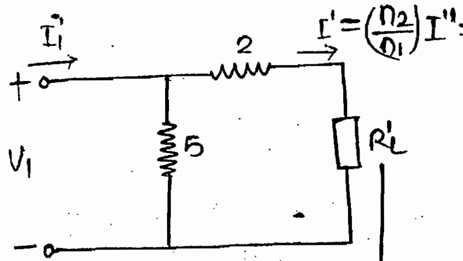
Eliminate $I_2$ from this 2 eq$^n$

36/40




$$V_1 = AV_2 + BI_2$$

$$A = \frac{V_1}{V_2}\Bigg|_{I_2=0}$$




$$I' = \left(\frac{n_2}{n_1}\right)I'' = \left(\frac{2}{1}\right)\left(\frac{V_2}{5}\right) = \left(\frac{2}{5}\right)V_2$$

$$V' = \left(\frac{n_1}{n_2}\right)V_2 = \frac{V_2}{2}$$

$$\left(\frac{n_1}{n_2}\right)^2 \cdot R_L = \left(\frac{1}{2}\right)^2 \times 5 = \frac{5}{4}$$

By KUL →

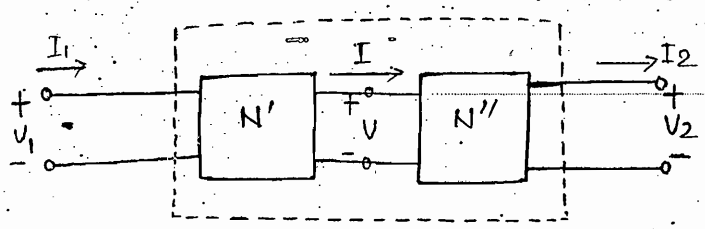
$$V_1 = 2I' + V'$$

$$= 2\times\frac{2}{5}V_2 + \frac{1}{2}V_2$$

$$\boxed{\frac{V_1}{V_2} = A = \frac{13}{10}}$$

## * Inter connection b/n various 2-port n/w →

### case(1) → Cascaded N/w →




$$N': \begin{cases} V_1 = A'V + B'I \\ I_1 = C'V + D'I \end{cases}$$

$$N'': \begin{cases} V = A''V_2 + B''I_2 \\ I = C''V_2 + D''I_2 \end{cases}$$

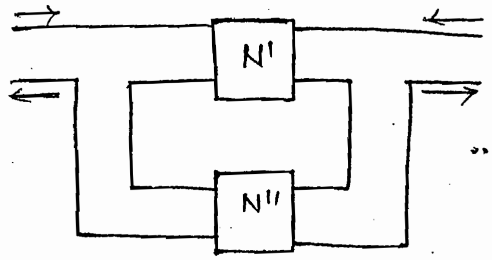
Overall N/w →

$$\begin{cases} V_1 = AV_2 + BI_2 \\ I_1 = CV_2 + DI_2 \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

* when 2 n/w are connected in cascade then the ABCD parameters of the individual are multiplied to obtain overall ABCD parameters of the resulting n/w.

Case(2) → Series N/w →



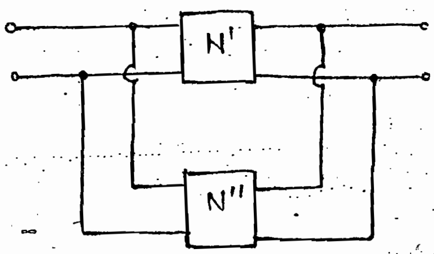

......Current series F/b

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z'_{11}+Z''_{11} & Z'_{12}+Z''_{12} \\ Z'_{21}+Z''_{21} & Z'_{22}+Z''_{22} \end{bmatrix}$$

$$[Z] = [Z'] + [Z'']$$

∴ When 2 n/w are connected in series at the i/p & at series at o/p then the respective Z-parameters of the 2 n/w are added to obtain overall Z-parameters of the resulting n/w.

Case(3) → Parallel n/w →




......Voltage shunt f/b

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y'_{11}+Y''_{11} & Y'_{12}+Y''_{12} \\ Y'_{21}+Y''_{21} & Y'_{22}+Y''_{22} \end{bmatrix}$$

$$[Y] = [Y'] + [Y'']$$

When 2 n/w are connected in parallel (shunt) in the i/p & in shunt at o/p then the ~~resulting~~ respective Y parameters of the 2 n/w are added to obtain the overall Y parameters of resulting n/w.

Case(4) → Series parallel N/w →

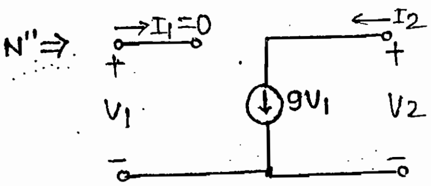
$$[h] = [h'] + [h'']$$

Case(5) → Parallel-series N/w →

$$[g] = [g'] + [g'']$$

19/36 $N' \Rightarrow$ : $y_{11} = y_{22} = -y_{12} = -y_{21} = y$

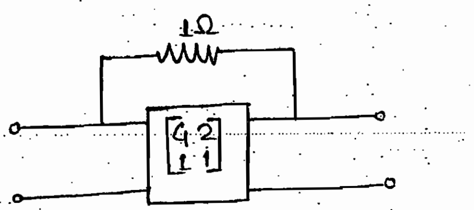
$N'' \Rightarrow$

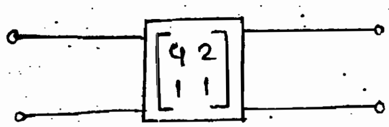


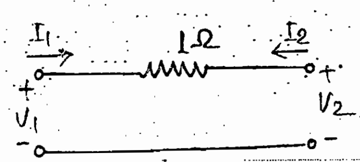

| $V_2 = 0$ | $V_1 = 0$ |
|---|---|
| $y_{11} = \frac{I_1}{V_1} = 0$ | $y_{12} = \frac{I_1}{V_2} = 0$ |
| $y_{21} = \frac{I_2}{V_1} = \frac{gV_1}{V_1}$ | $y_{22} = \frac{I_2}{V_2} = \frac{g(V_1)}{V_2} = 0$ |

$$y_{21}(\text{overall}) = y_{21(N')} + y_{21(N'')}$$
$$= -y + g$$
$$\boxed{y_{21} = -y + g}$$

1/34




$I_1 = -I_2$ ----- (i)

$V_1 = I_1 \times 1 + V_2$

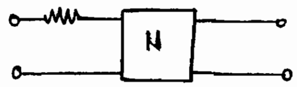
$I_1 = \underbrace{1}_{y_{11}} \times V_1 + \underbrace{(-1)}_{y_{12}} V_2$

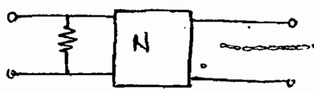
$I_2 = \underbrace{(-1)}_{y_{21}} \times V_1 + \underbrace{(+1)}_{y_{22}} V_2$

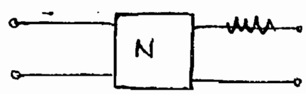
$$[y] = [y'] + [y'']$$

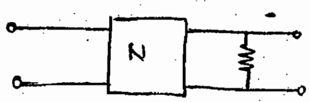
$$[y] = \begin{bmatrix} 4+1 & 2+(-1) \\ 1+(-1) & 1+(1) \end{bmatrix} ; S$$

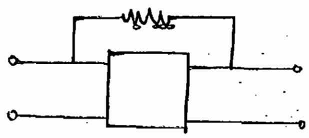
Circuit — Parameters affected

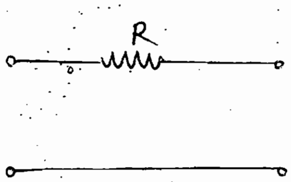
*  $\longrightarrow Z_{11}$

*  $\longrightarrow Y_{11}$

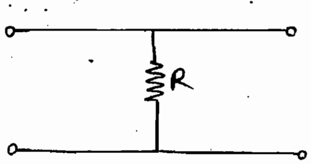
*  $\longrightarrow Z_{22}$

*  $\longrightarrow Y_{22}$

*  $\longrightarrow Y_{11}, Y_{12}, Y_{21}, Y_{22}$ & Hence also all Z parameters

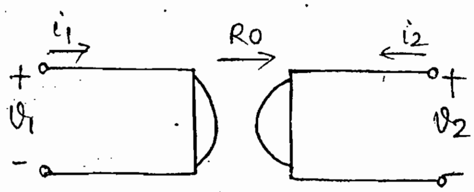
* 

$\longrightarrow$ $[Y]$ ✓ $[Z]$ ✗

* 

$\longrightarrow$ $[Y]$ ✗ $[Z]$ ✓

Example →



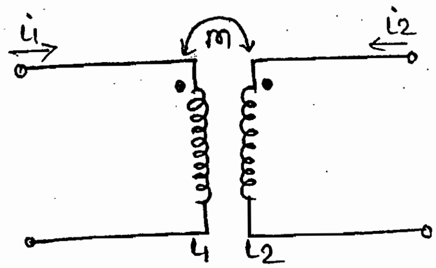

$v_1 = +R_0 i_2 \Rightarrow v_1 = (0)\, i_1 + (+R_0)\, i_2$

$v_2 = -R_0 i_1 \Rightarrow v_2 = (-R_0)\, i_1 + (0)\, i_2$

$$[Z] = \begin{bmatrix} 0 & +R_0 \\ -R_0 & 0 \end{bmatrix} ; \Omega$$

....... symmetrical & not reciprocal.

Example →




$$[Z] = \begin{bmatrix} j\omega L_1 & +j\omega M \\ +j\omega M & j\omega L_2 \end{bmatrix}$$

..... Reciprocal & not symmetrical

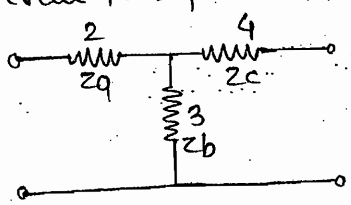
Example → A General Resistive T-n/w is shown below. 2 such identical n/w are connected :-

(1) cascade

(2) series

Calculate overall ABCD parameters & the z parameters.

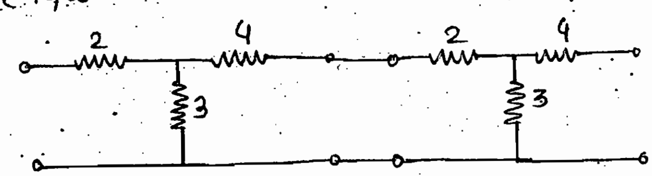
Soln →

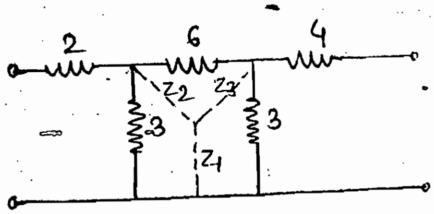


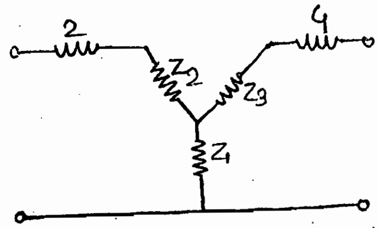

Soln → $z_{11} = Z_a + Z_b = 5\Omega$ $\quad$ $Z_{12} = Z_{21} = Z_b = 3\Omega$

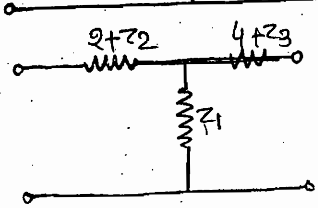
$Z_{22} = Z_b + Z_c = 7\Omega$

Case(1) Cascade N/w



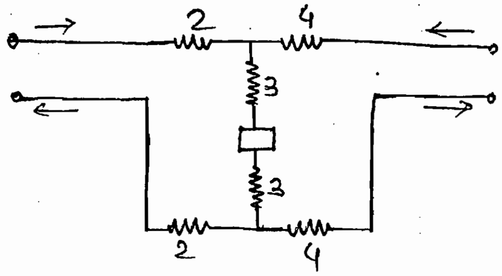



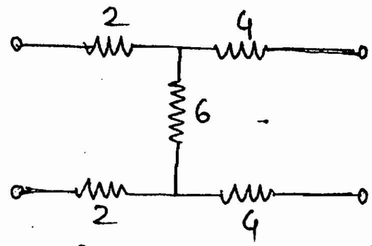

$Z_{11} = (2 + Z_2) + Z_1$

$Z_{22} = (4 + Z_3) + Z_1$.

$Z_{12} = Z_{21} = Z_1$

Case(2) Series N/W →



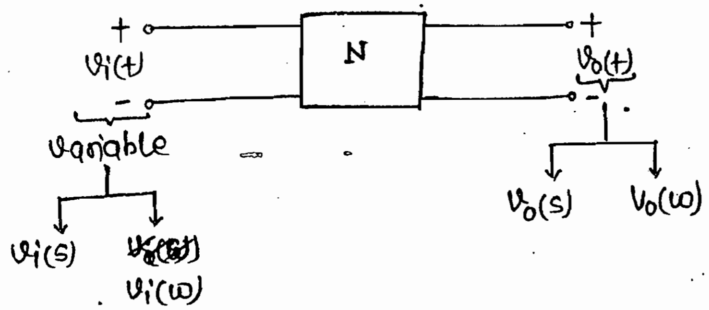

$Z_{11} = 2+6+2 = 10\Omega$ , $Z_{22} = 4+4+6 = 14\Omega$

$Z_{12} = Z_{21} = 6\Omega$

# Filters

............As a freq. selective N/w

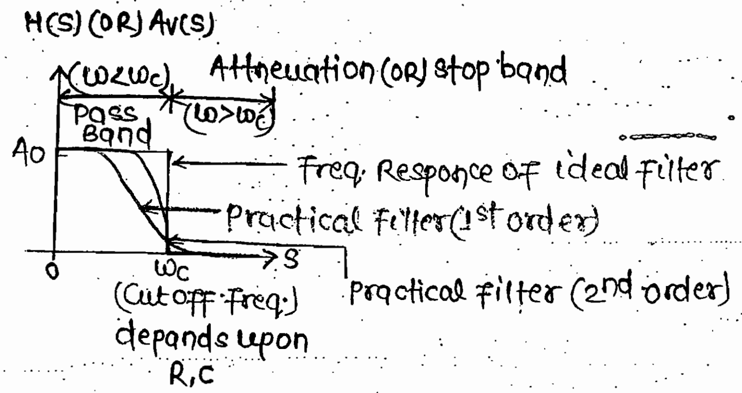
*** Principle →**




$$H(s) = \frac{V_o(s)}{V_i(s)} = A_V(s)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = A_V(\omega)$$

**Case(1) →** Low pass Filter (LPF)




C/s of a practical Filter

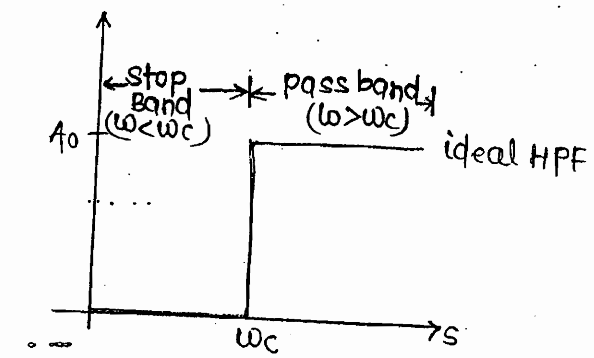
* Cut off freq. of the filter can be adjusted by suitabily selecting the numerical values of R & C componants.

* The gain during the pass band can be adjusted as per requirement by including an active device such as a variable gain OPAMP.
The filter which include OPAMP as active device are called active filter.

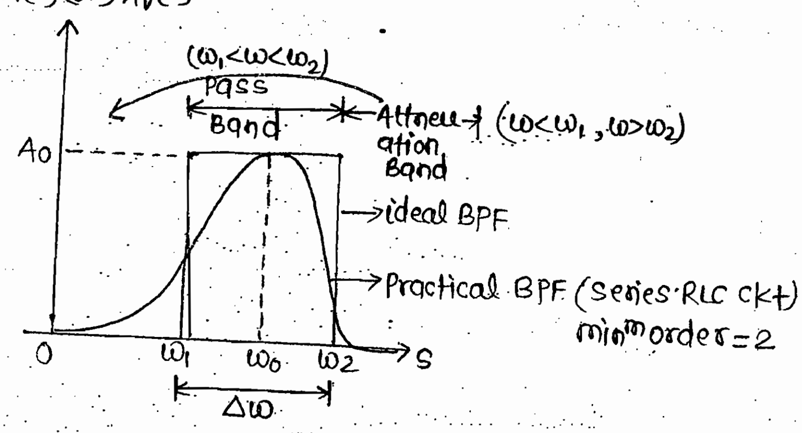
* An ideal filter has sharp cutoff c/s which can't be obtain practicaly. Since the responce of any R-C n/w is limited by its finite time const.
Therefore any practical filter has gradual cut off c/s. This c/s can be approached to those of ideal filter by increasing the order of filter.
This is done the no. of stages which are used in cascade in any type of filter.

DATE-04/12/14

Case(2)→ High pass Filter (HPF)



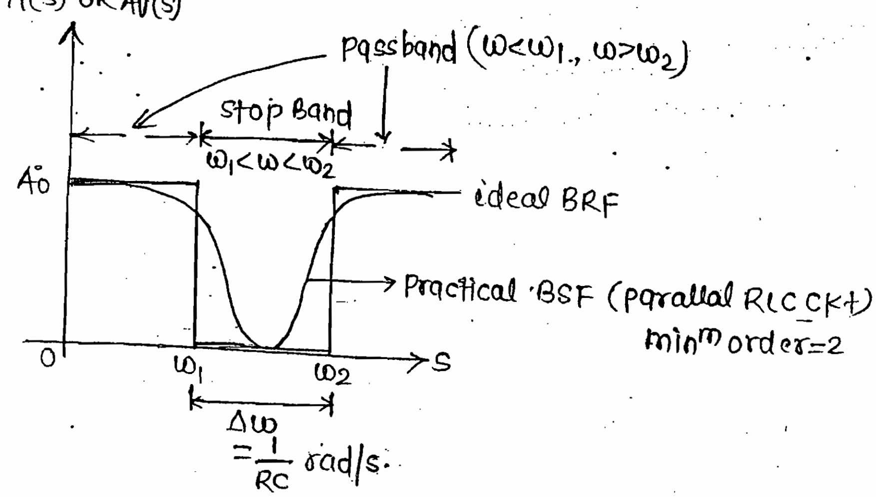

Case(3)→ Band pass Filter (BPF)




$\Delta\omega=\omega_2-\omega_1=BW=\frac{R}{L}$ rad/sec

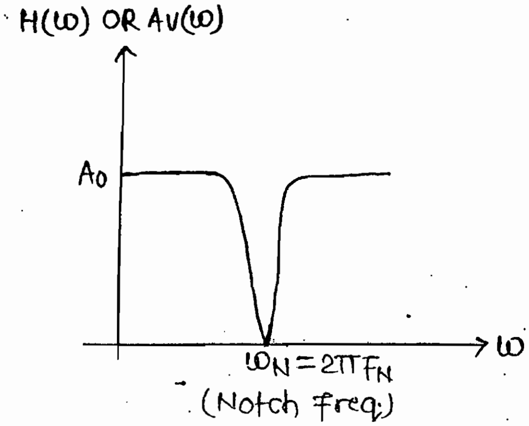
& also $Q_0=\frac{\omega_0}{\Delta\omega}$

Case(4)→ Band stop Filter (OR) BRF (Reject)



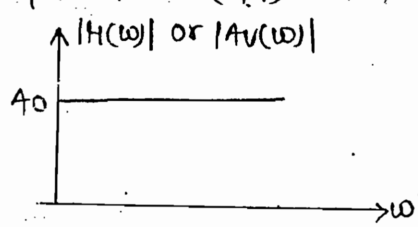
Case(5) → Notch Filter

Because of appⁿ of sinusoidal ($s \to \omega$) in x-axis

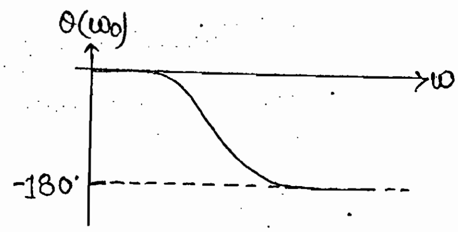



* This filter is used for noise (OR) harmonics minimization. (like 3rd harmonic distortion).
* A Notch filter is special case of a BSF where a single freq; called the Notch freq; is not allowed to pass through sys.
* This filter is always used as the i/p state of any audio sys. where 50Hz main freq; is not allow to pass through the sys to avoid the distortion & the noise in the audio sys.
* This filter is also used to minimise the 3rd harmonic distortion in any sys. & therefore such filter is used as a i/p stage of that sys.

Case(6) → All pass filter (APF)




...... magnitude Responce

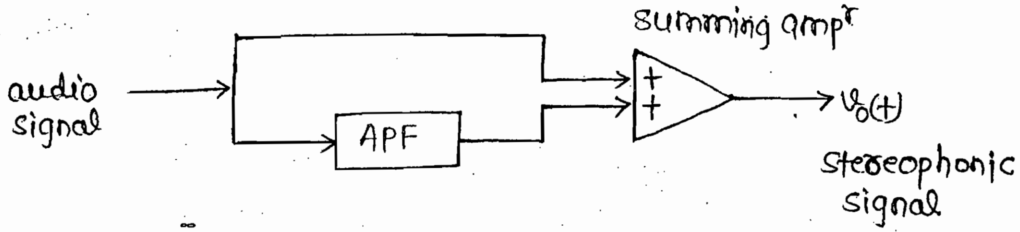



...... Phase responce

* An APF passes all the freq; componant with const. gain but various freq; componant are subjected to diff. amt of phase shift having a minᵐ value of 0° & a maxᵐ phase shift of -180°.

* Such filter has all the poles in LHP & all the zeros lie in RHP.
* Since in any n/w the no. of poles & no. of zeros are always equal then the no. of poles in LHP or the no. of zeros in RHP will control the order of the filter.

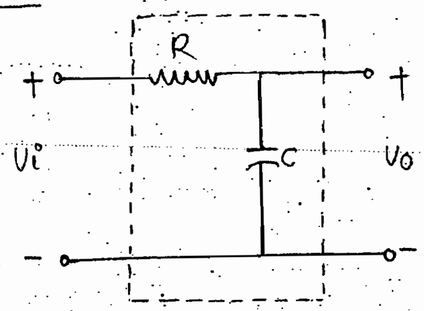
## Application →




* An APF is used to produce stereophonic signal in any audio sys.

## Example →

### (1.) 1st order LPF →




To Find:-

(1.) TF $H(S) = \frac{V_o(S)}{V_i(S)}$

(2.) Cut off freq. ($\omega_c$)

(3.) Pole-zero pattern

(4.) Freq. responce $H(S)$ vs $S$

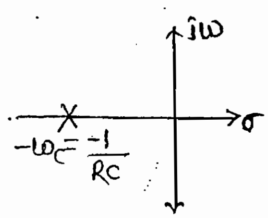
(5.) Magnitude res. $|H(\omega)|$ vs $\omega$

(6.) Justification ***

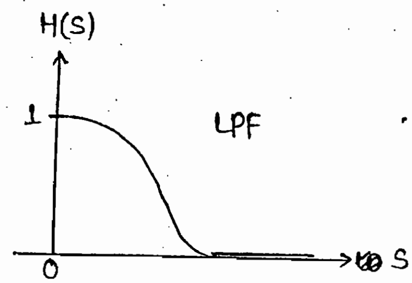
* Based on the time const (RC) this ckt is also known as integrator ckt.

$$\frac{V_o(S)}{V_i(S)} = \frac{1/SC}{R + 1/SC} = \frac{1}{1+SRC} = \boxed{\frac{1}{1+\frac{S}{\omega_c}} = H(S)}$$

$$\left.\begin{aligned} \text{where } \omega_c &= \frac{1}{RC} \cdots \text{rad/s} \\ f_c &= \frac{1}{2\pi RC} \cdots \text{Hz} \end{aligned}\right\} \text{cut off freq.}$$




······ Pole zero pattern.




........ Freq; Responce

Now, for $s \rightarrow j\omega$

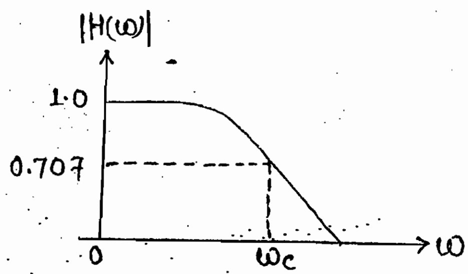
$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

(1.) $\omega < \omega_c$ ; $|H(\omega)| \triangleq 1$

(2.) $\omega = \omega_c$ ; $|H(\omega)| = \frac{1}{\sqrt{2}} = 0.707$

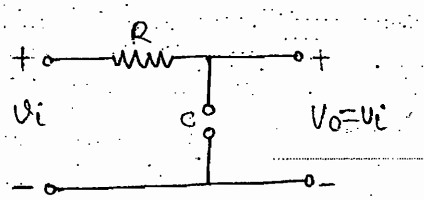
(3.) $\omega > \omega_c$ ; $|H(\omega)| = \frac{\omega}{\omega_c}$




* Justification →

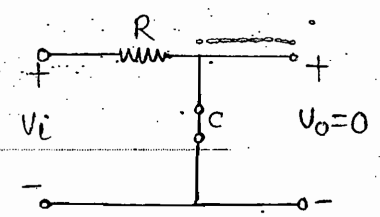
(1.) $\omega = 0$; $s = 0$

$$Z_c = \frac{1}{sc} = \infty \text{ (oc)}$$

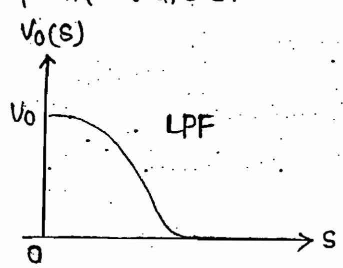



(2.) $\omega = \infty$, $s = \infty$,

$$Z_c = \frac{1}{sc} = 0 \text{ (short)}$$

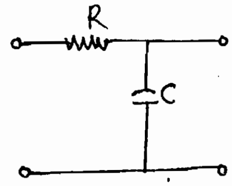


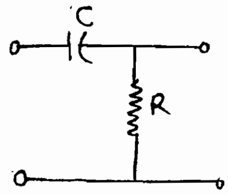

(3.) $s = s$ ; $V_o$ .... Finite value.



## 2) 1st order HPF →

| LPF | HPF | comments. |
|---|---|---|



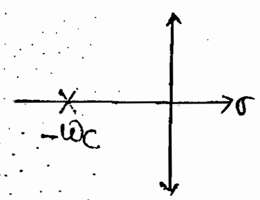

* Interchange the locn of R & C.

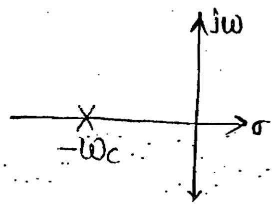
$$H(s) = \frac{1}{1+\left(\frac{s}{\omega_c}\right)} \qquad \frac{1}{1+\left(\frac{1}{s/\omega_c}\right)} = \frac{s/\omega_c}{1+\left(\frac{s}{\omega_c}\right)}$$

* Replace $\frac{s}{\omega_c}$ by $\frac{1}{(s/\omega_c)}$

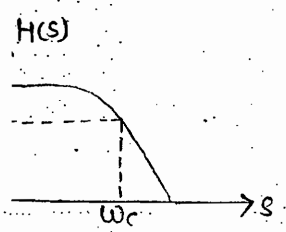
$$\omega_c = \frac{1}{RC} \qquad \omega_c = \frac{1}{RC}$$

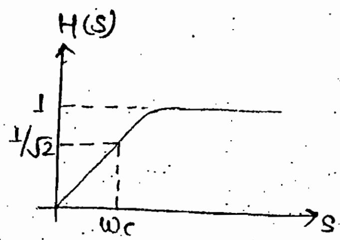
* Cut-off Freq.



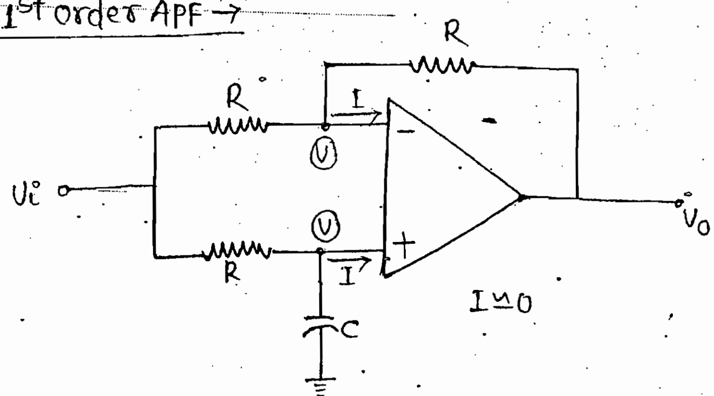

* Pole-zero plot




* Freq. Responce

## 1st order APF →




To find :-

(1) $TF = H(s) = \frac{V_o(s)}{V_i(s)}$

(2) Pole-zero plot

(3) magnitude responce, $|H(\omega)|$ vs $\omega$

Phase responce $\theta(\omega)$ vs $\omega$

The vol. at i/p terminal of OPAMP are nearly equal since the vol. gain in the differential mode is almost ∞.

The current drawn by the i/p terminal of the OPAmp are nearly zero. since the i/p impedance of ideal OPAmp is ∞.

* Always write KCL eqⁿ at the i/p terminals of OPAMP.
* Never write KCL eqⁿ at the o/p of OPAMP unless it is terminated by some Load resistor $R_L$.

KCL-(1) inverting i/p :-

$$\frac{V-V_i}{R}+\frac{V-V_o}{R}+0=0$$

$$\boxed{2V=V_i+V_o} \text{ ----(i)}$$

KCL(2) at NI i/p :-

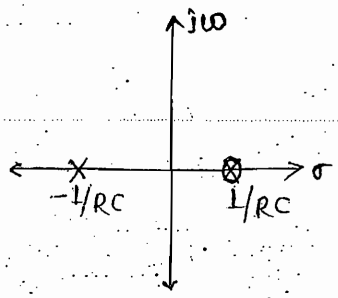
$$\frac{V-V_i}{R}+\frac{V-0}{1/sC}+0=0$$

$$V\left(\frac{1}{R}+sC\right)=\frac{V_i}{R}$$

$$\boxed{V=\frac{V_i}{1+RsC}} \text{ -----(ii)}$$

From eqⁿ (i) & (ii)

$$\frac{V_o(s)}{V_i(s)}=\boxed{H(s)=\frac{1-sRC}{1+sRC}}$$



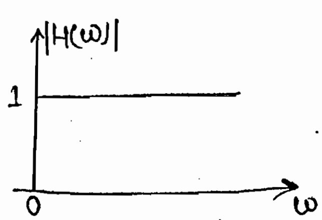

(Pole-zero Responce)

S→jω

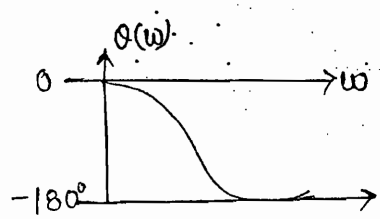
$$H(\omega)=\frac{1-j\omega RC}{1+j\omega RC}$$

$$|H(\omega)|=1$$

$$\theta(\omega)=-2\tan^{-1}\left(\frac{\omega RC}{1}\right)$$



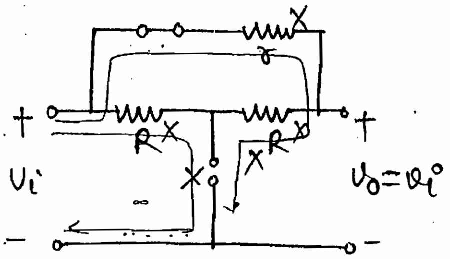

magnitude Responce




Phase Responce

(2/52) $S \to 0$

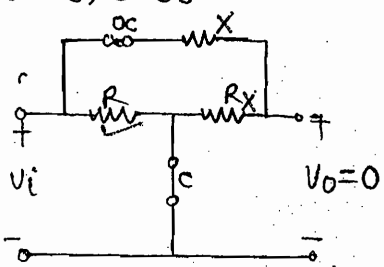
$C = OC$ ; $L = SC$

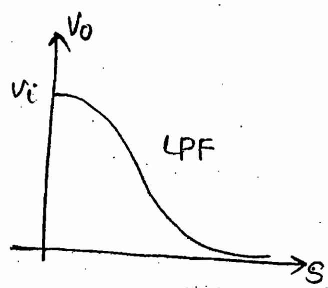



X→No current flow

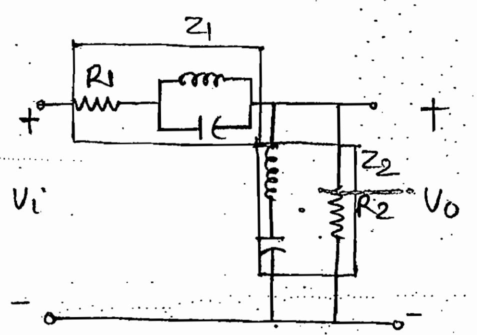
$S \to \infty \to$

$C = SC$, $L = OC$



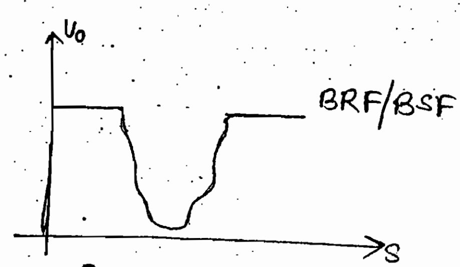

(3/52)




(1.) $S = 0$, $L \to SC$, $C \to OC$ $\quad V_o = \frac{R_2}{R_1 + R_2} V_i$ ...... finite

(2.) $S = \infty$, $L \to OC$, $C \to SC$ $\quad V_o = \frac{R_2}{R_1 + R_2} v_i$ ...... finite

(3.) $S = G$ $\quad V_o = \frac{Z_2}{Z_1 + Z_2} v_i$

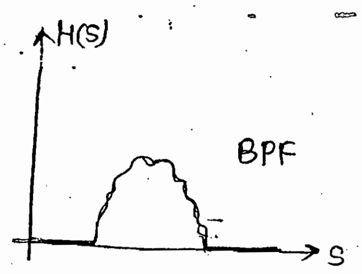



(0/52)

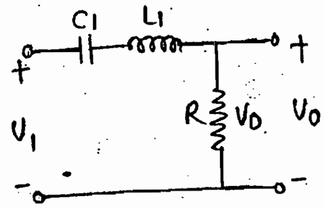
$$H(S) = \frac{10S}{S^2 + 10S + 100}$$

$$S = 0; \quad \frac{0}{0 + 0 + 100} = 0$$

$$S = \infty; \quad \frac{10/S}{\left(1 + \frac{10}{S} + \frac{100}{S^2}\right)} = 0$$

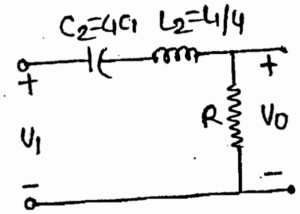


9/53




$B_1 = \Delta\omega_1 = \frac{R}{L}$

$\frac{B_1}{B_2} = ?$




$B_2 = \Delta\omega_2 = \frac{R}{L_2} = \frac{R}{L/4} = \frac{4R}{L} = 4B_1$

$\boxed{\frac{B_1}{B_2} = \frac{1}{4}}$

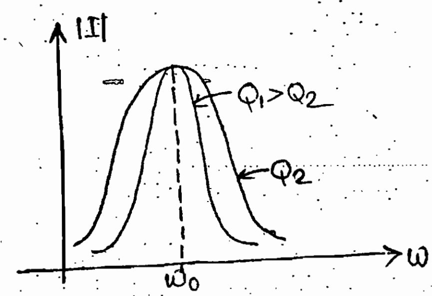
$\omega_0$ is same for above both n/w

$Q_0 = \frac{\omega_0}{B}$

$Q_0 \propto \frac{1}{B}$

$\left.\begin{matrix} B_1 \not> B_2 \\ Q_1 > Q_2 \end{matrix}\right\}$ ckt ① behaves as a better tuned ckt

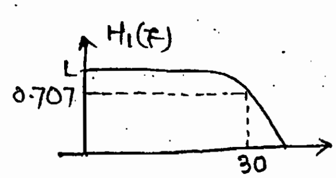
$B_1 > B_2$
$Q_2 > Q_1$




10/53 For cascaded filter;

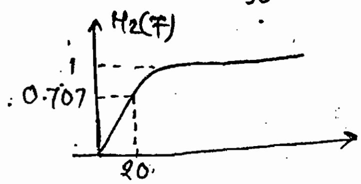
$H(s) = H_1(s) \cdot H_2(s)$

$H(f) = H_1(f) \cdot H_2(f)$




LPF

$BW = \Delta f = f_2 - \cancel{f_1 = 10Hz}$

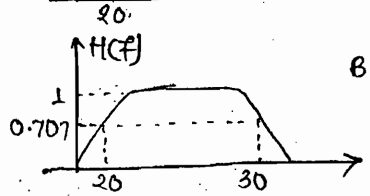



HPF

$f_0 = \sqrt{f_1 \cdot f_2}$

$= \sqrt{20 \times 30} = 10\sqrt{6}$ Hz.

$Q_0 = \frac{f_0}{\Delta f} = \frac{10\sqrt{6}}{10}$




BPF

$Q_0 = \sqrt{6}$

$$H(s) = H_1(s) \cdot H_2(s)$$

$$= \frac{1}{(1+sRC)^2}$$

$$= \frac{1/(RC)^2}{\underbrace{\left(s+\frac{1}{RC}\right)^2}_{s^2+2\xi\omega_n s+\omega_n^2}}$$

$\xi = 1$ ; criticaly damped

# Network Topology

........Graph theory

* Used to solve any ele. N/w.

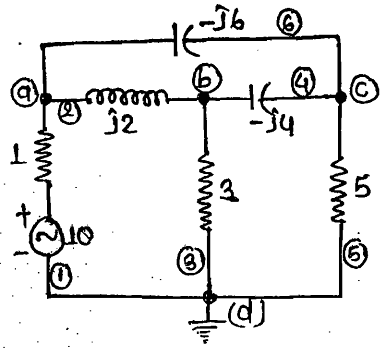
Example → Given: any ele. N/w

Find: (1.) Graph of N/w

(2.) Oriented graph

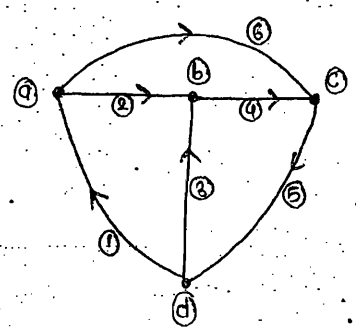
(3.) Tree of graph

(4.) Co-tree



Nodes(n) = 4

branches(b) = 6



Rank of graph $\boxed{r = (n-1)} = 3$

........ Oriented graph

...... Fully connected Graph (FC graph)

(all the nodes are directly conneted by branch to each other)

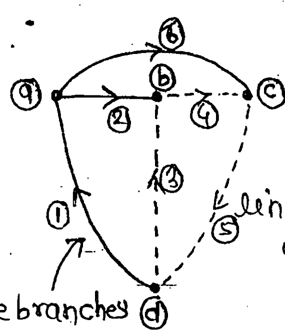
Graph

n: [a, b ..... d]

b: [1, 2 ..... 6]

no. of branches :- $\boxed{{}^{n}C_2 = \frac{n(n-1)}{2}}$ [only applicable for FC graph)

Tree(1) →



links (OR) chords

Tree branches (or) Twigs.

Tree: [1, 2, 6] ...... Tree branches (OR) Twigs

Co-tree [3, 4, 5] ...... links (or) chords

No. of tree branches $= n-1$

no. of links / chords $= b-(n-1)$

Total no. of trees $= n^{(n-2)}$ ....... only for FC graph.

$= \det[A \cdot A^T]$ ...... Applicable for both type (FC or NOT) (General result)

A ...... Reduced incidance matrix

* Branch voltages in terms of branch currents:-

$v_2 = j2 \cdot i_2$.

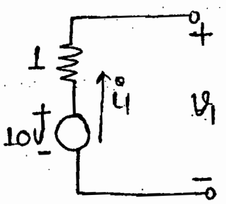
$v_3 = 3 \cdot i_3$

$v_4 = -4j \cdot i_4$

$v_5 = 5 \cdot i_5$

$v_6 = -j6 \cdot i_5$

$v_1 = 10 - i_1$




$v_1 - 10 + i_1 = 0$

$v_1 = 10 - i_1$.

ATE-05/12/14

The n/w topology is concern the manner in which various elements are connected b/n difference node irrespective of there values & types.

The tree of a graph →

) The tree of a graph contains all the nodes.

) There is no closed path & hence any tree is cktless.

) The tree of a graph is not unique.

) If the graph contains n node its tree will contain (n-1) branches.

The tree of a graph can be used to solve the given ele. n/w using :-

a.) Tie-set matrix.

b.) Cut-set matrix.

Any closed loop in a graph may not contain all the nodes of the graph.

In a fully connected graph each node is connected to every other node of the graph with a direct branch.

* The degree of a node represents the no. of branches which are meeting at the specified node.
* In a fully connected graph the degree of each node is same & represents the rank of the graph.
* When the graph is not fully connected degree of each node is diff. but the highest degree of node represents the rank of graph.

* main points →

(1.) No. of branches in a FC Graph $= {}^nC_2 = \frac{n(n-1)}{2}$

(2.) No. of trees of a graph $= n^{(n-2)}$ ....... FC Graph

$= \det[AA^T]$ ....... General Graph (weather FC or not)

A ..... Reduced incidance matrix.

(3.) No. of links/chords $= b-(n-1)$

= no. of KVL eq$^n$

= no. of fundamental loops $[l = b-(n-1)]$

= no. of fundamental tie sets

(4.) No. of tree branches/twigs $= n-1$

= no. of KCL eq$^n$

= no. of node pair voltages

= Rank of graph

= Degree of each node in a FC Graph.

= Max$^m$ degree of node when graph is not FC.

= No. of ~~fun~~ fundamental cut-sets.

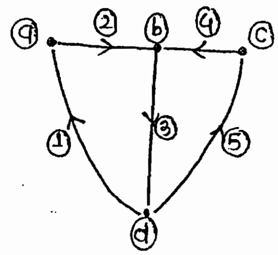
Example:- Given:-

Graph of a n/w

To find:-

(1) Incidance matrix

(2) Reduced incidance matrix [4]




* Not a FC Graph

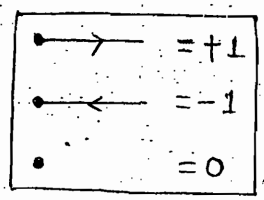
$$\begin{cases} n=4 \\ b=5 \end{cases}$$

* Incidance matrix →

The incidance matrix translates all the geometrical feature of the n/w into an algebric expn.

Every graph has an incidance matrix & vice-versa.

Each row of the matrix contains entries of +1, -1 (OR) 0




Each column of the matrix contains only 1 entry of +1 & only 1 entry of -1 so that the algebric sum of each column of matrix is zero.

The size of the matrix is given by [n x b]

n = no. of nodes
⇓
no. of rows

b = no of branches
⇓
No. of columns

2 graphs having same incidance matrix are called ISOMORPHIC graphs.

The determinant of incidance matrix of any closed loop is always = 0.

$$|\text{Incidance matrix}| = 0$$

## * Incidance matrix →

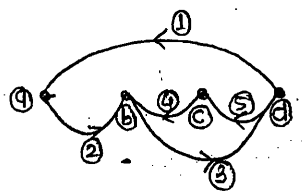
$$
\begin{array}{c|c}
\text{nodes} \downarrow & \text{Branches} \rightarrow \\
\end{array}
$$

$$
\begin{array}{c}
\\ a \\ b \\ c \\ d
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
-1 & +1 & 0 & 0 & 0 \\
0 & -1 & +1 & -1 & 0 \\
0 & 0 & 0 & +1 & -1 \\
+1 & 0 & -1 & 0 & +1
\end{bmatrix}
\end{array}
$$

$[n \times b] = [4 \times 5]$

### Obtaining Graph from matrix →

(using by column)




## * Reduced incidance matrix → [A]

* Any one node is taken as a ref. node.
* Delete the row corrosponding to the ref. node to obtain the reduced incidance matrix.
* The size of the matrix is $[(n-1) \times b]$

(b) ⇒ Ref. matrix

$[(n-1) \times b]$

$[3 \times 5]$

$$
[A] = \begin{array}{c}
\\ a \\ c \\ d
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
-1 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & +1 & -1 \\
+1 & 0 & -1 & 0 & +1
\end{bmatrix}
\end{array}
$$

$[(n-1) \times b] = [3 \times 5]$

no. of trees $= \det[AA^T]$

**Example →** Given:-

Tree of a graph

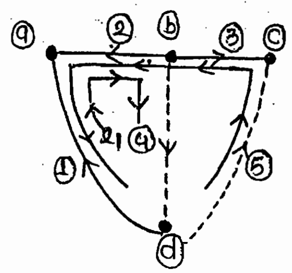
To Find :-

(1.) Fundamental tie set matrix

f- loop matrix

f- branch matrix $[B_f]$

(2.) Cut-set matrix $[Q_f]$




$n=4$
$b=5$

## * F-tie sets →

(1.) This represent optimum no. of tie sets with which the ckt can be solved completely.

(2.) Each F-tie set represents a group of branches to make a closed loop

(3.) Each F-loop contains one & only one link (or) chord, remaining are the tree branches.

(4.) Dirn of each F-tie set is same as the dirn of the link (or) the chord.

(5.) No. of F-tie sets = no. of links/chords.

$$= b-(n-1)$$

$$= 5-(4-1) = 2$$

(6.) The fundamental tie-set matrix can be used to write the branch currents in terms of loop currents.

## * F-tie sets matrix →

F-tie sets :-

$l_1$ :- [1, 2, $\boxed{4}$] → link/chord

$l_2$ :- [1, 2, 3, $\boxed{5}$] → link/chord

F-tie set matrix →

$$[B_F] = \begin{array}{c} \\ l_1 \\ l_2 \end{array} \begin{array}{c} \begin{array}{ccc|cc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccc|cc} +1 & -1 & 0 & +1 & 0 \\ -1 & +1 & -1 & 0 & +1 \end{array}\right] \end{array}$$

$B_t$ (tree) $\quad B_l = U$ (link)

$$\boxed{[B_F] = [B_t \mid U]}$$

## * Branch current →

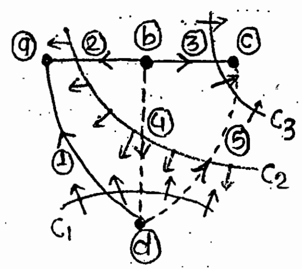
$$\left.\begin{aligned} i_1 &= +I_1 - I_2 \\ i_2 &= -I_1 + I_2 \\ i_3 &= -I_2 \\ i_4 &= +I_1 \\ i_5 &= +I_2 \end{aligned}\right\} \text{(I)}$$

## * f-cut set →

* The f-cut sets represent the optimum no. of cut sets with which we can solve the n/w completely.
* Each f-cut set represents a group of branches which must be cut so that the graph is devided into 2 parts.
* Each f-cut set contains one & only one tree branch remaining are the links (or) chords.
* The dir^n of each f-cut set is same as the dir^n of the tree branch.
* The no. of f-cut sets = no. of tree branches

$$= n-1$$
$$= 4-1 = 3$$

* The cut-set matrix can be used to represent the branch vol. in terms of nodal vol.




## * f-cut set →

$C_3$: [3, 5] → tree branch

$C_2$: [2, 4, 5] → tree branch.

$C_1$: [1, 4, 5] → tree branch.

## * f-cut set matrix →

$$[Q_f] = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccc|cc} +1 & 0 & 0 & -1 & +1 \\ 0 & +1 & 0 & +1 & -1 \\ 0 & 0 & +1 & 0 & +1 \end{array}\right] \end{array}$$

$Q_t = U$     $Q_l$

$$\boxed{[Q_f] = [U \mid Q_l]}$$

## *Branch voltages→

$$\left.\begin{aligned} v_1 &= V_1 \\ v_2 &= V_2 \\ v_3 &= V_3 \\ v_4 &= -V_1+V_2 \\ v_5 &= +V_1-V_2+V_3 \end{aligned}\right\} \text{(II)}$$

* The 3 set of eqn represent the equilibrium eqn. since this eqn represents any given n/w completely & uniquely.

***

$$\boxed{\begin{aligned}[B_t] &= -[Q_l^T] \\ [Q_l] &= -[B_t^T]\end{aligned}}$$

Hence f- tie set matrix corresponding to the tree branches is always equal to -ve of Xpose of f-cut set matrix corresponding to the links or the chords & vice-versa.

This result is always valid irrespective of the type of tree selected for a perticular graph of the corrosponding ele. n/w.

While designing any ele. n/w, when the value of any element in any branch the n/w is valid again & again then the analysis of n/w becomes simpli- using graph theory. since:-

1) Graph of n/w
2) Tree of graph.
3) F- tie set matrix
4) f-cut set matrix

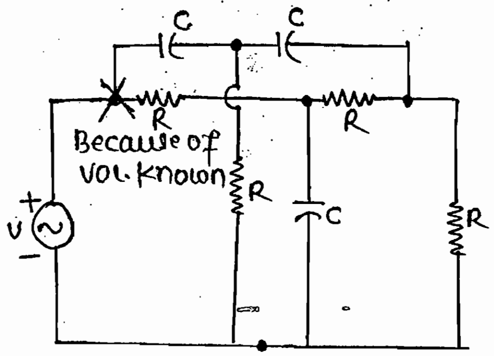
all will remain same. Only one eqn is modified where the branch vol has been represented in terms of branch current.

Under this condn the analysis of N/w using the KVL & KCL analysis becomes more complicated.

When all the element value in any ele. n/w remain fixed then the analysis of n/w using KVL & KCL eqn becomes simplify.

Under this condn the method of Graph theory becomes more typical.

6/22




Twin-T n/w

$\eta = 4$ ..... independant nodes

$b = 7$

No. of KVL = no. of f-tie sets $= b-(n-1) = 7-3 = 4$

no. of KCL = no. of f-cut sets $= n-1$

$= 4-1$

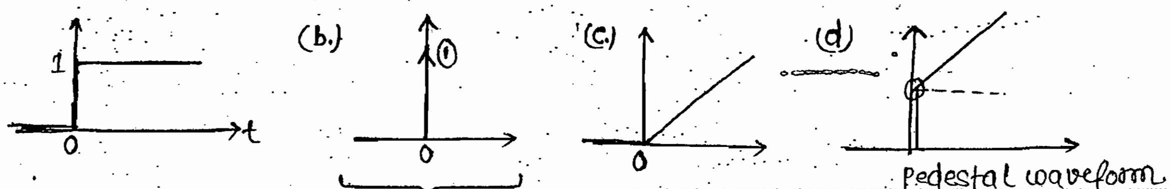
$= 3$

so min$^m$ no. of eq$^n$ will be ③

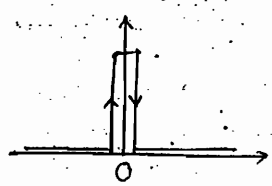
If we will make graph then 5th node will also included.

6/13

(a.) (b.) (c.) (d)




i(0+) is max$^m$ for δ(t)




i(0+) is max$^m$ through RL ckt

$$v(t) = i(t)\cdot R + L\frac{di(t)}{dt}$$

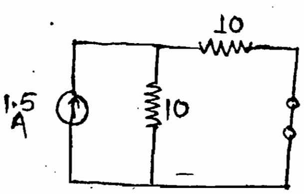
Because in impulse f$^n$ value of deno. is 1.

45/19

at $t=0^-$ →

S..... open

$L \Rightarrow SC$




$\downarrow i_L(0^-) = i_L(0^+)$

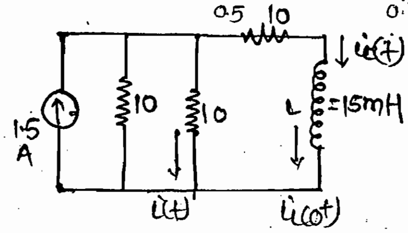
$= 0.75A$

(current division)

$t>0$ →

RL

$$i_L(t) = \underbrace{i_L(\infty)}_{0.5} - [\underbrace{i_L(\infty)}_{0.5} - \underbrace{i_L(0^+)}_{0.75}]e^{-t/T}$$



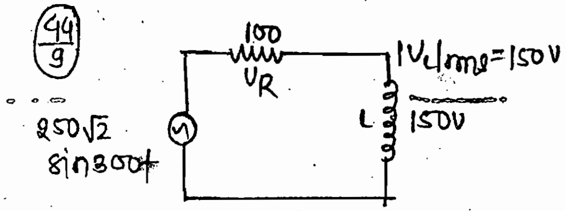

$T = \frac{L}{Req}\Big|_{1.5A \Rightarrow OC}$

$Req = (10 \| 10) + 10 = 15$

$T = \frac{15m}{15} = 1mS$

$$i(t) = \frac{v(t)}{10} = \frac{1}{10}\left[10 \cdot i_L(t) + L\frac{di_L(t)}{dt}\right]$$

(44/9)




$V_i = \sqrt{V_R^2 + V_L^2}$

$\uparrow$ 250 $\quad$ $\uparrow$ 150

$\boxed{V_R = 200V}$

$V_R = 100 I_{rms}$

$200 = 100 I_{rms}$

$\boxed{I_{rms} = 2A}$

$V_L = \omega L \cdot I_{rms}$.

$L = \frac{|V_L|}{\omega I_{rms}}$

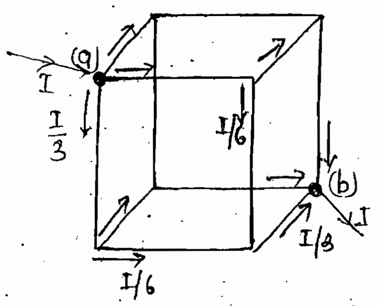
$= \frac{150}{300 \times 2}$

$\boxed{L = \frac{1}{4} H}$

(25/6)

$Q = \underbrace{\int_0^5 i(t)dt}_{(area)}$

(30/7)

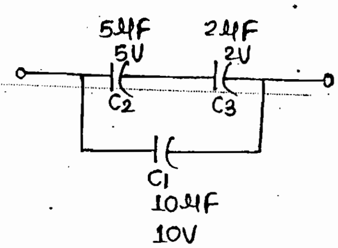



$R_{eq.} = \frac{V_{ab}}{I}$ (This is applicable only for symmetrical n/w)

$= \frac{1}{I}\left[\frac{I}{3} \times R + \frac{I}{6} R + \frac{I}{3} R\right]$

$= \frac{5}{6} \Omega$

(36/8)




$Q_2 = C_2 V_2 = 25\mu C$

$Q_3 = C_3 V_3 = 4\mu C$

$V_2 = \frac{Q_2}{C_2} = \frac{4}{5} = 0.8V$ $\quad$ $V_{ab} = 0.8 + 2 = 2.8V$

$V_3 = \frac{Q_3}{C_3} = \frac{4}{2} = 2V$.

$Q_1 = C_1 V_1 = 10 \times 2.8 = 28\mu C$

$Q_t = 4 + 28 = 32\mu C$.

(21/15)

$v(t) = 10(t - 0.01)e^{-100t}$

$V(s) = \frac{10}{(s+100)^2} - \frac{0.1}{(s+100)}$

denom. $\Rightarrow (s+100)^2$

so $\xi$ =

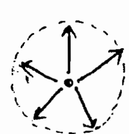
$s^2 + 2\xi\omega_n s + \omega_n^2$

$\omega_n = 10$,

$\boxed{\xi = 1}$

$|I| = \frac{V}{|Z_{in}|}$

$V_R = |I|R = \frac{VR}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$



(24/6)

$\omega = 0$, $C \Rightarrow oc$, $i_2 = 0$

(locus starts from origin)

$\omega = \infty$, $C \Rightarrow sc$; $|i_2| = \frac{E_m}{R_2}$

(Resistive)

V & i are in phase.

# Network Theorems

## * Superposition theorems →

* The responce of any element linear bilateral RLC n/w containing more than 1 independant vol. (or) current sources is the sum of responces produced by the sources each acting alone when:-

(1.) All other independant vol. sources are SC (OR) replaced by there internal impedances.

(2.) All other independant current sources are OC (OR) replaced by there internal impedances.

(3.) All depandant vol. (OR) current sources remain as they are & therefore this sources are neither SC nor OC.

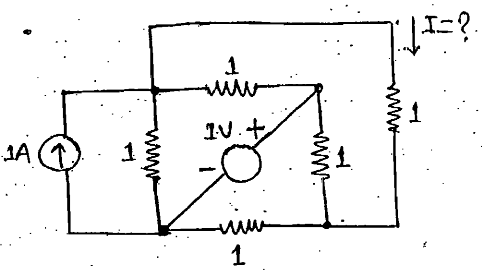
* The theorem is not applicable to the n/w containing:-

(1.) Non-linear elements

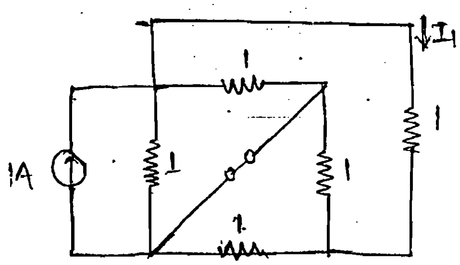
(2.) Unilateral elements. (such as p-n jun^n diode)

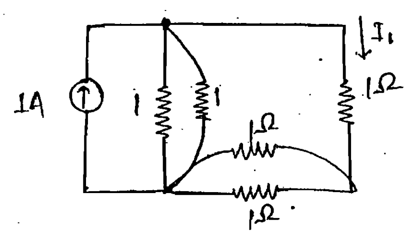
DATE-06/12/14

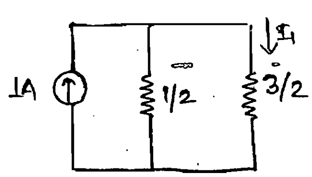
Que →




sol^n → $I_1$ (due to 1A source) →



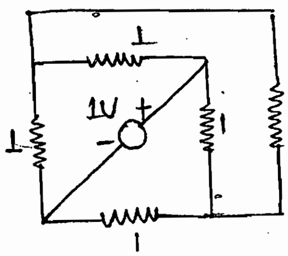




$$I_1 = 1 \times \frac{1/2}{1/2 + 3/2}$$

$$\boxed{I_1 = 1/4\,A}$$

$I_2$ (due to 1V) $\rightarrow$

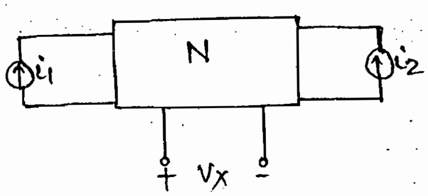



$\downarrow I_2 = 0$ (Balanced bridge)

$I = I_1 + I_2$

$\boxed{I = 1/4 A}$

Que:-




N:- Resistive & linear n/w

If $i_1 = 8A$, $i_2 = 12A$; $V_x = 80V$

$i_1 = -8A$, $i_2 = 4A$; $V_x = 0V$

To find $V_x$ if $i_1 = i_2 = 20A$

Sol$^n \rightarrow$

$i_1 = 0$, $V_{x_1} = i_2 R_1$

$i_2 = 0$, $V_{x_2} = i_1 R_2$

$V_x = V_{x_1} + V_{x_2}$

$\boxed{V_x = i_2 R_1 + i_1 R_2}$

$80 = 8R_1 + 12R_2$

$0 = -8R_1 + 4R_2$

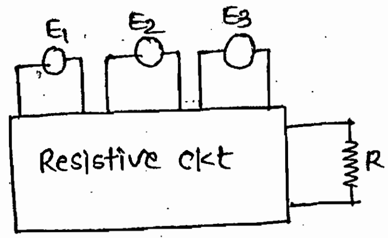
$R_1 = 5/2$, $R_2 = 5$

$V_x = i_2 R_1 + i_1 R_2$

$= 20 \times \frac{5}{2} + 20 \times 5$

$\boxed{V_x = 150V}$

Que:-




$P_1 = 18W$

$P_2 = 50W$

$P_3 = 98W$

$$\left.\begin{array}{l} P_{max} \neq P_1 + P_2 + P_3 \\ P_{min} \neq P_3 - P_1 - P_2 \end{array}\right\} \text{Not correct.}$$

Sol$^n$ →

$$P = I^2 R = \frac{V^2}{R}$$

$$V = \sqrt{P}\sqrt{R}$$

$$V_{max} = V_1 + V_2 + V_3$$

$$= (\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})\sqrt{R}$$

$$P_{max} = \frac{V_{max}^2}{R}$$

$$= \frac{(\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})^2 \times R}{R}$$

$$P_{max} = (\sqrt{P_1} + \sqrt{P_2} + \sqrt{P_3})^2$$

$$V_{min} = V_3 - V_2 - V_1$$

$$= (\sqrt{P_3} + \sqrt{P_2} - \sqrt{P_1})\sqrt{R}$$

$$P_{min} = \frac{V_{min}^2}{R}$$

$$= \frac{(\sqrt{P_3} - \sqrt{P_2} - \sqrt{P_1})^2 \times R}{R}$$

$$P_{min} = (\sqrt{P_3} - \sqrt{P_2} - \sqrt{P_1})^2$$

$$\boxed{\begin{array}{l} P_{max} = 450W \\ P_{min} = 50W \end{array}}$$

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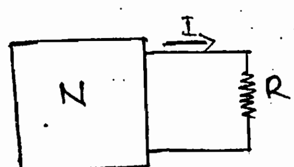
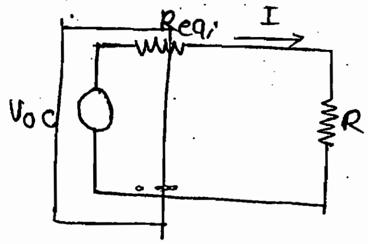



$R = 0, I = 3A$

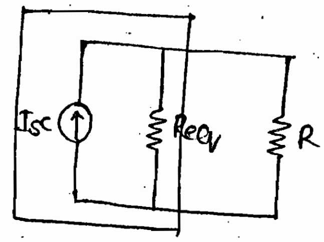
$R = 2; I = 1.5A$

$I = ?, R = ?$

N →
- Resistors
- Independant source
  - Vol. sour.
  - Current sources




Thevenin's model




Norton's eq. model.

$$\boxed{V_{oc} = I(R_{eq} + R)}$$

$$\boxed{I = \left(\frac{R_{eq}}{R_{eq}+R}\right) I_{sc}}$$

$$\begin{cases} V_{oc} = 3(R_{eq}+0) \\ V_{oc} = 1.5(R_{eq}+2) \end{cases}$$

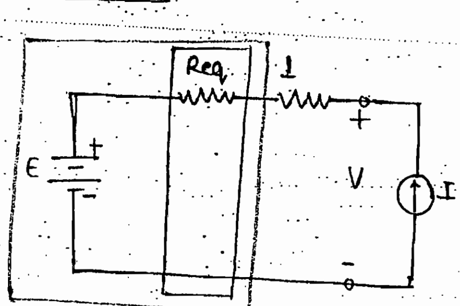
$V_{oc} = 6$, $R_{eq} = 2$

$V_{oc} = I(R_{eq}+R)$

$6 = I(2+1)$

$\boxed{I = 2A}$

9/0




Thevenin's model

(1.) $E = E_1$ ; $I = 0$, $V = 5V$

$\boxed{V = E_1 = E_1 = 5}$

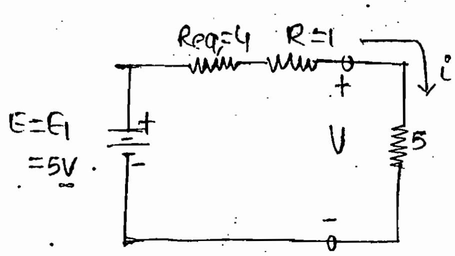
(2.) $E = 0$, $I = 1$, $V = 5$

$V = I(1 + R_{eq}) + \cancel{E}$

$\boxed{R_{eq} = 4}\,\Omega$

(3.) $E = E_1$, $I \Rightarrow$ Replaced by $5\Omega$

$V = ?$

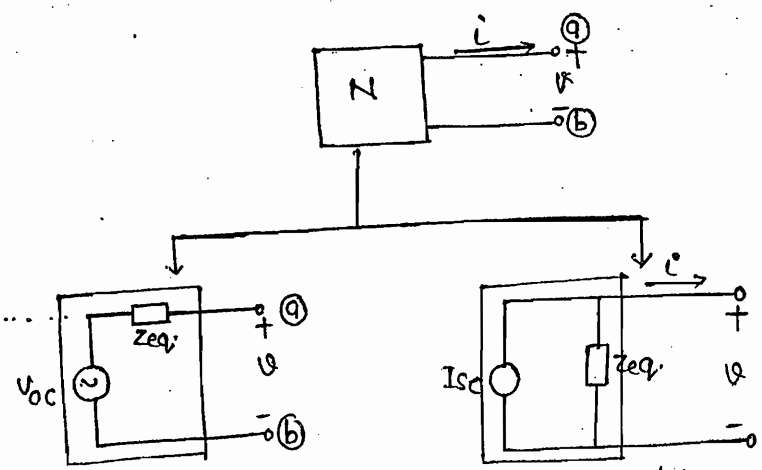



$i = \frac{5}{4+1+5} = 1/2 A$

$5i = V$

$\boxed{V = 5/2 A}$

## * Thevnin's & Norton's theorem →




Thevnin's model　　　　Nortons model

where; $V_{oc}$ = oc vol. b/n 2 terminal (a & b) when $i=0$

$I_{sc}$ = sc current b/n 2 terminal (a, & b) when $v=0$

$$\boxed{\frac{V_{oc}}{I_{sc}} = Z_{eq}}$$

* Thevnin's theorem → A linear active RLC n/w which contains one (or) more independant (or) depandant vol. (or) current sources can be replaced by a single vol. source $V_{oc}$ & series with eq. impedance between the terminals a & b when:-

* $Z_{eq}$ represents the eq. impedance b/n the terminals a & b when:-

(1.) All independant vol. sources are sc (or) replaced by there internal impedances.

(2.) All independant current sources are oc (or) replaced by there internal impedances.

(3.) All depandant vol. & current sources remain as they are & therefore these sources are neither sc nor oc

* The theorem is always applicable irrespective of:-

(1.) The type of the vol. & current sources wether dc (or) ac sources.

(2.) Nature of the elements wether reactive (OR) resistive.

* The theorem is not applicable for the N/w containing:-

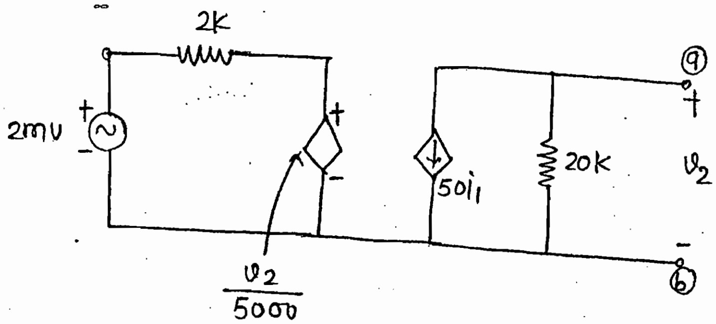
① Non linear elements

② Unilateral elements such as p-n jun diode.

## * Norton's theorem →

A linear, active RLC n/w which contains one (or) more independant (or) dependant vol. & current sources can be replaced by single current source Isc in parallel with an eq. impedance Zeq.

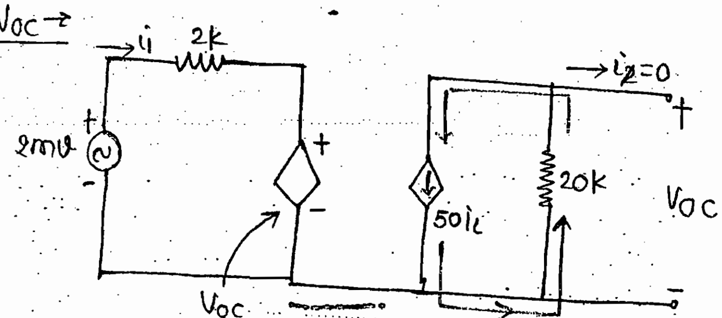
**Que.→**




Voc, Req ...... Thevenin's

Isc, Req ..... Norton's

**Sol^n → Voc →**

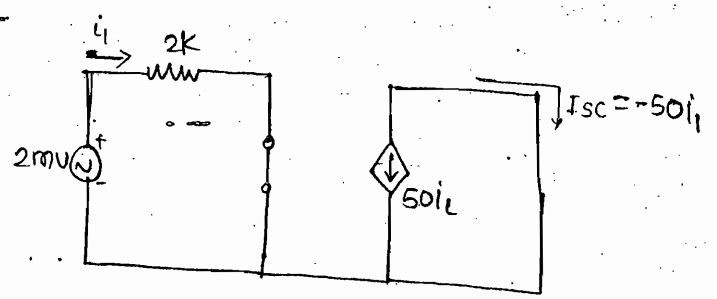



$$2\times10^{-3} = 2\times10^{3} i_1 + \frac{V_{oc}}{5000} \quad ----(i)$$

$$0 = 20\times10^{3}\times 50 i_1 + V_{oc} \quad ----(ii)$$

$$\boxed{V_{oc} = -\frac{10}{9} V}$$

**Isc →**




$$20\times10^{-3} = 20\times10^{+3} i_1$$

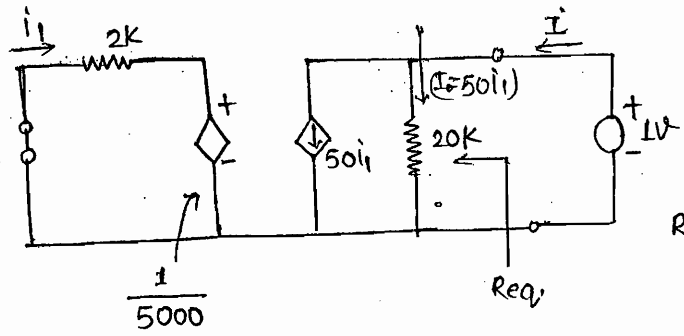
$$i_1 = 10^{-6}$$

$$I_{sc} = -50 i_1$$

$$= -50\times10^{-6}$$

$$\boxed{I_{sc} = -50 \mu A}$$

* Req:-




$Req = \frac{1}{I} = ?$

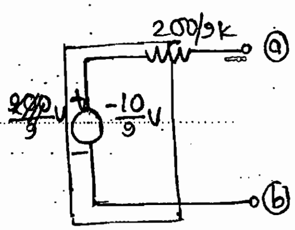
$$2\times10^3 i_1 + \frac{1}{5000} = 0 \quad \text{------ (i)}$$

$$1 = 20\times10^3 (I - 50 i_1) \quad \text{------ (ii)}$$

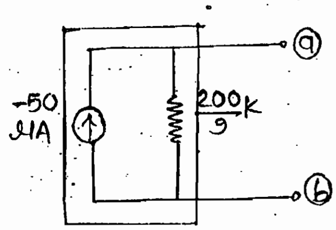
$$\boxed{\frac{1}{I} = Req = \frac{200}{9}K}$$

check →

$$\frac{V_{oc}}{I_{sc}} = \frac{-10/9}{-50\times10^{-6}} = \frac{200}{9}K = Req$$

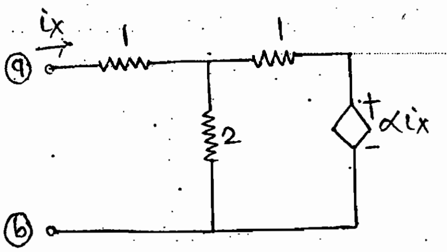



Thevenin's



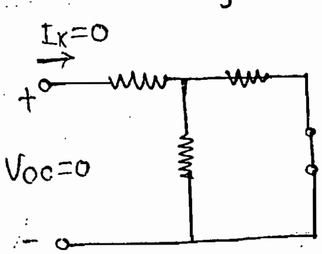

Norton's

(7/33)

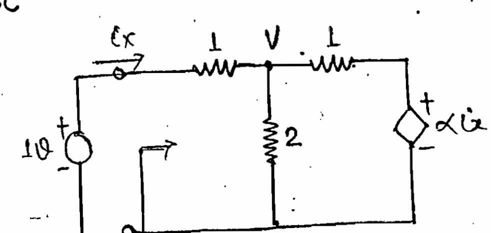



Voc } Thevnin's
Req }

Voc → $(i_x = 0)$; $\alpha i_x = 0$
voltage source ⇒ SC




Req ⇒



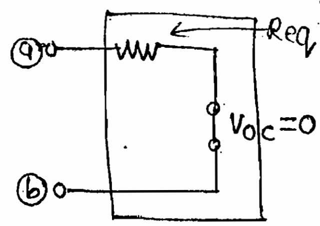

$Req = \frac{1}{i_x} = ?$

$$\frac{V-1}{1} + \frac{V-0}{2} + \frac{V-\alpha i_x}{1} = 0 \quad \text{------ (i)}$$

$$i_x = \frac{1-V}{1} \quad \text{------ (ii)}$$

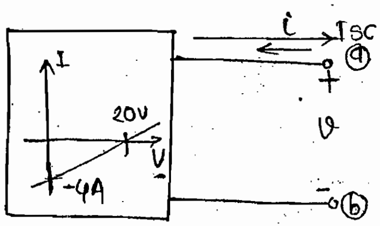
Eliminate v

Find $\frac{1}{i_x} = R_{eq}$




Thevnin's model

Que. →




Draw Thevnin's & Norton's model

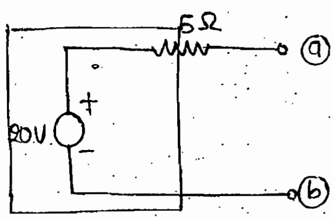
Soln →

$i = 0;\ v = V_{OC} = 20V$

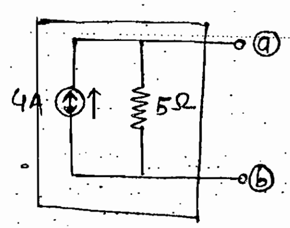
$v = 0;\ i = -4 \Rightarrow -I_{SC}$

$I_{SC} = 4A$

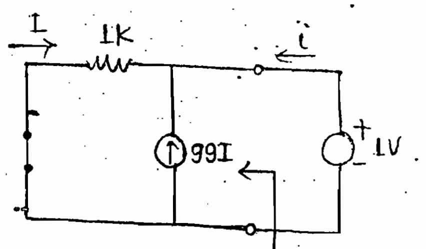
$\frac{V_{OC}}{I_{SC}} = R_{eq} = 5\Omega$




Thevnin's




Norton's




$R_{eq} = \frac{1}{i}$

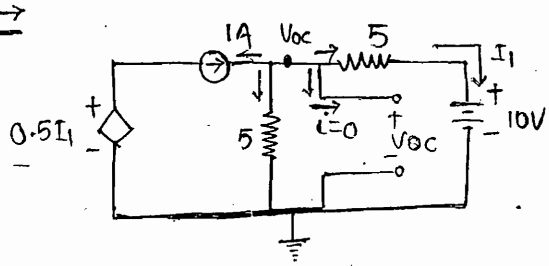
$I + 99I = -i$

$100I = -i$ -----(i)

$1 = -(1K)I$ -----(ii)

$R_{eq} = 0.01K = 10\Omega$

$V_{oc} \rightarrow$

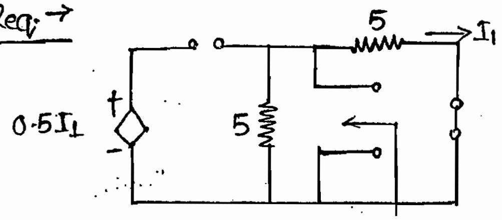


KCL →

$$-1+\frac{V_{oc}-0}{5}+0+\frac{V_{oc}-10}{5}=0$$

$$\boxed{V_{oc}=7.5V}$$

Req →




$Req. = 5\|5$
$= 2.5\Omega$

$$\boxed{Req = 2.5\Omega}$$

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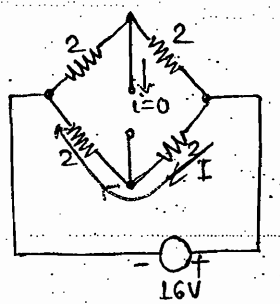
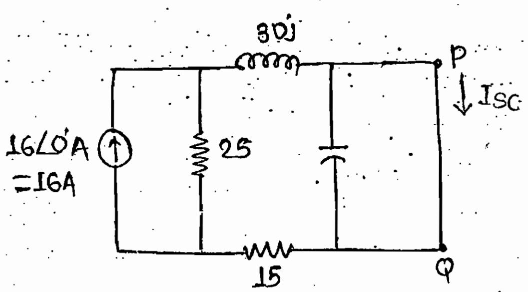


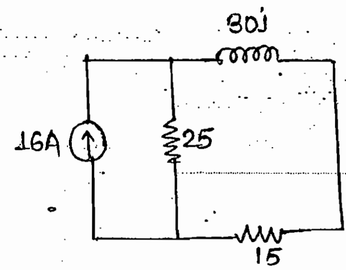

Balanced bridge

$$-16+(2+2)I=0$$

$$\boxed{I=4A}$$

45/31




$$I_{SC}=\frac{25}{25+30j+15}\times 16$$
$$=6.4-4.8j$$

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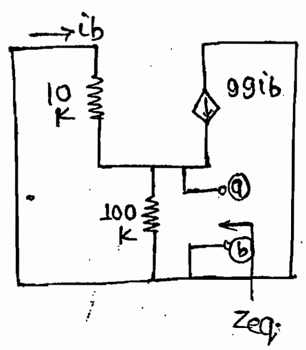
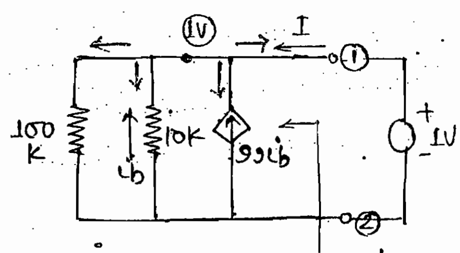



⇒



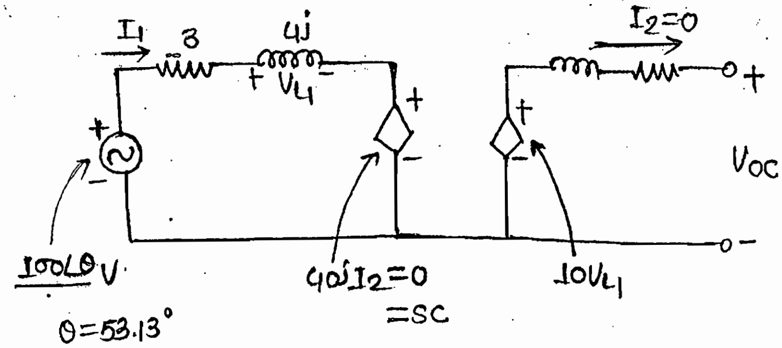

$Req = \frac{1}{I}$

$$\frac{1-0}{100k}+\frac{1-0}{10k}+(-99i_b)-I=0 \text{ ----- (i)}$$

$$-i_b=\frac{1-0}{10k} \text{ ----- (ii)}$$

$$\frac{1}{I}=Req.=0.1K=100\Omega$$

(49/32)




$V_{oc} = 10V_{L1} = 10 \times 4j I_1$

$= \frac{40j \times 100\angle\theta}{3+4j} = 5\angle\theta$

$= 5\angle\theta$

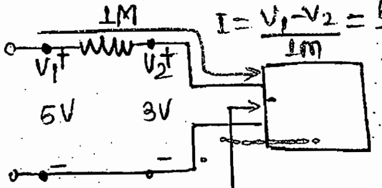
$\left[\theta = \tan^{-1}\frac{4}{3} = 53.13°\right]$

$= 800j$

$= 800\angle 90° \text{ ---- V}$

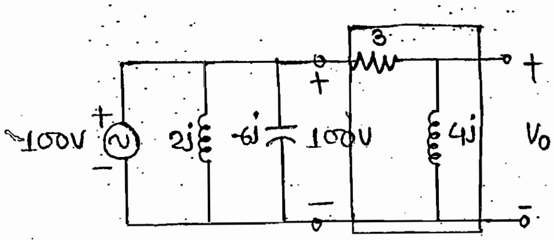
(26/28)

$I = \frac{V_1 - V_2}{1m} = \frac{5-3}{1m} = 2\mu A$




$Z_{in} = \frac{V_2}{I} = \frac{3}{2\mu} = 3/2\,M\Omega$

(7/25)

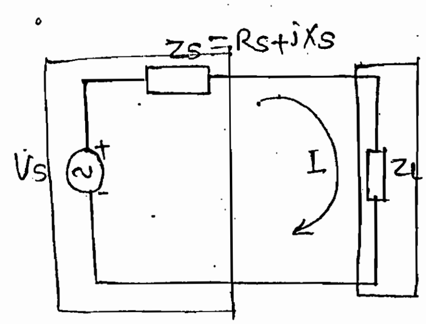



$V_o = \frac{4j}{3+4j} \times 100$

$= \frac{j400}{3+4j} \times \frac{3-4j}{3-4j}$

$= \frac{j400(3-4j)}{9+16}$

## * max^m power transfer theorem →




** $Z_L$ is variable

$Z_L = R_L + jX_L$

Thevenin's model

To find → Cond^n for max^m power Transfer from source to load.

$$I = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)} = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$P_L = I^2 Z_L$
$= I^2 R_L$
$= |I|^2 Z_L$
$= |I|^2 R_L$

$$P_L = \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \quad \cdots\cdots (i)$$

For $P_{L(max)}$

$$\frac{\partial}{\partial R_L}\left[R_L\right] = 0 \; ; \Rightarrow R_S = R_L$$

$$\frac{\partial}{\partial X_L}\left[P_L\right] = 0 \; ; \Rightarrow X_L = -X_S$$

$Z_L = R_L + jX_L$

$Z_L = R_S - jX_S$

But $Z_S = R_S + jX_S$;

$Z_S^* = R_S - jX_S$

$\boxed{Z_L = Z_S^* \; ; \; R_L = R_S \;, X_L = -X_S}$ cond$^n$ for max$^m$ power transfer

* To maximise the power Xfer from source to load the load impedance must be complex conjugate of the source impedance. Therefore the source & the load must have equal resistive part.
* They must also have equal reactive parts but opposite in sign. Therefore if the load is inductive the source must be capacitive & vice-versa.
* The max$^m$ power Xfered to the load is only 50% of the total power generated by the source.
* The results are valid only for variable load.
* To apply the max$^m$ power Xfer theorem any given ele. n/w has to be 1st Xform in the std. Thevnin's model.
* The theorem is always valid irrespective of :-
  (i) Nature of the source & the load impedances
  (ii) The nature of the vol. & the current sources.
* Hence the theorem is always valid for Ac as well as dc ckts.

DATE-07/12/14

$$P_L = \frac{V_s^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2} \quad \text{------(i)}$$

<u>Case(1) →</u> General case

$Z_s = R_s + jX_s$

$Z_L = R_L + jX_L$

$$\boxed{\begin{aligned} R_L &= R_s \\ X_L &= -X_s \\ Z_L &= Z_s^* \end{aligned}}$$

*** <u>case(2) →</u>

$Z_s = R_s + jX_s$

$Z_L = R_L$

$\frac{\partial}{\partial R_L}[P_L] = 0$

$R_L = |Z_s|$

$$\boxed{R_L = \sqrt{R_s^2 + X_s^2}}$$

<u>case(3) →</u>

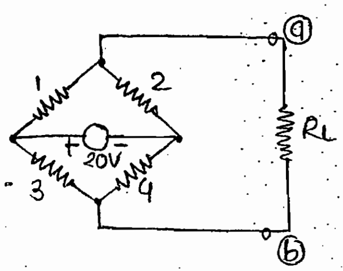
$Z_L = R_L$

$Z_s = R_s$ ..... dc ckt only

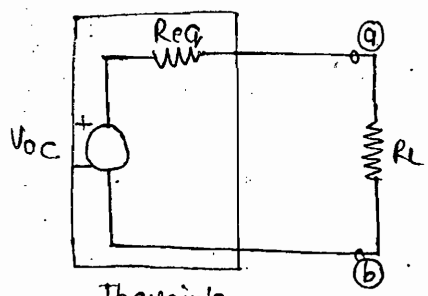
..... purely resistive

$$\boxed{R_L = R_s}$$

<u>ue →</u> In the ckt shown find the value of load resistance $R_L$ so that max^m power is Xfered to it. Hence cal. the max^m power Xfered to the load?

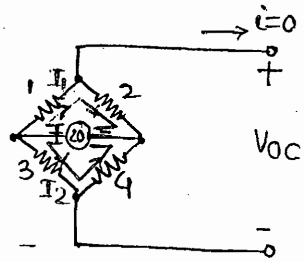



<u>sol^n →</u> for max^m Xfer convert the ckt in Thevnin's eq.




Thevnin's

<u>Voc →</u>




$I_1 + 2I_1 = 20$

$I_1 = 20/3 A$

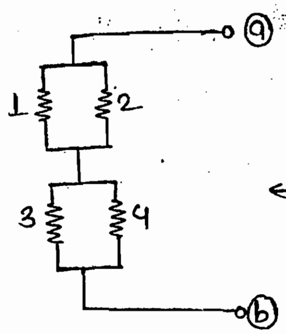
$3I_2 + 4I_2 = 20$

$I_2 = \frac{20}{7} A$

$V_{oc} = 2I_1 - 4I_2$

$\cancel{= 2\times} \frac{20}{3} - 4\times\frac{20}{7} = \frac{40}{3} - \frac{80}{7} = \frac{280-240}{21} = \frac{40}{21} V$

**Req →**




$$R_{eq} = (1 \| 2) + (3 \| 4)$$
$$= \frac{50}{21}\,\Omega$$

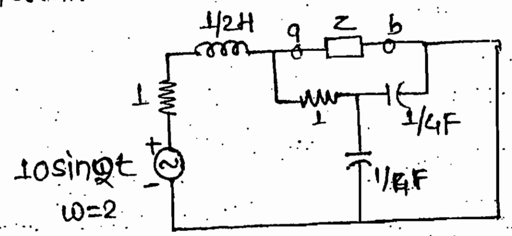
For Pmax ⇒

$$R_L = R_{eq} = \frac{50}{21}\,\Omega$$

$$P_{max} = I^2 R_L = \left(\frac{V_{oc}}{R_L + R_{eq}}\right)^2 R_L$$

$$= \frac{V_{oc}^2}{4R_L} = \frac{8}{21}\,W$$

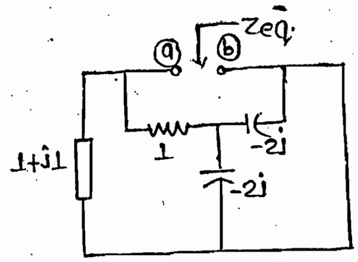
**Que:** In the circuit shown find the value of impe. z which must be connected b/n terminals a & b so that max^m power is transfered to it.

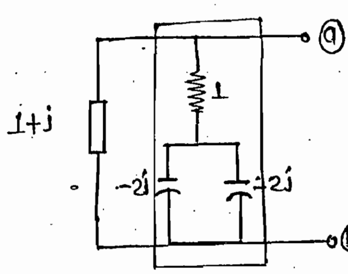



**Soln →**

$$\frac{1}{2}H \Rightarrow j\omega L = j \times 2 \times \frac{1}{2} = j\,\Omega$$

$$1/4F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j \times 2 \times 1/4} = -2j\,\Omega$$



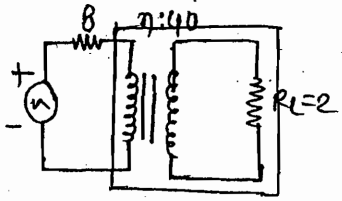

$$\frac{-2j \times -2j}{-2j - 2j} + 1$$

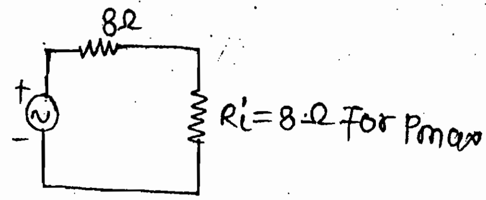
$$\frac{-4}{-4j} + 1 = 1 - j$$

$$Z_{eq} = \frac{(1+j)(1-j)}{(1+j)+(1-j)} = 1 \pm j0$$

For Pmax : $Z = Z_{eq}^* = (1 \pm j0)^* = 1\Omega$

(1/24)

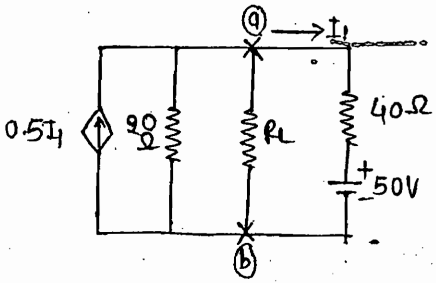


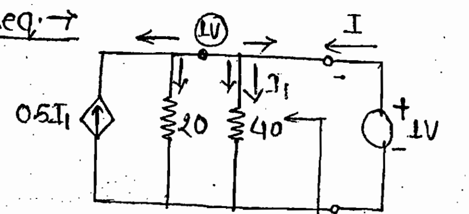

$$2\left(\frac{n}{40}\right)^2 \times \cancel{8} = 8$$

$$n = 80$$

(20/27)




Req →




$Req = \frac{1}{I} = ?$

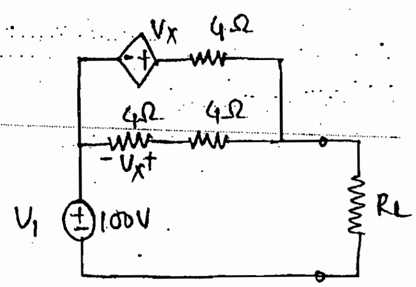
$$-0.5I_1 + \frac{1-0}{20} + \frac{1-0}{40} - I = 0 \quad \text{----- (i)}$$

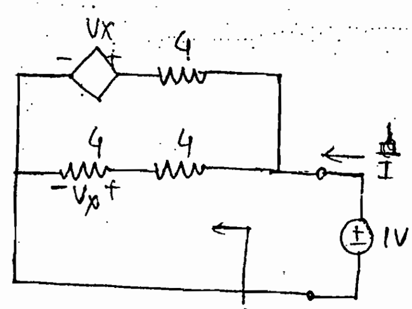
$$\therefore I_1 = \frac{1}{40} \quad \cdots \text{(ii)}$$

$$\frac{1}{I} = Req = 16$$

For Pmax-Pmax $R_L = Req = 16\,\Omega$

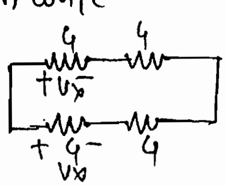
(10/30)




$Req. = \frac{1}{I}$

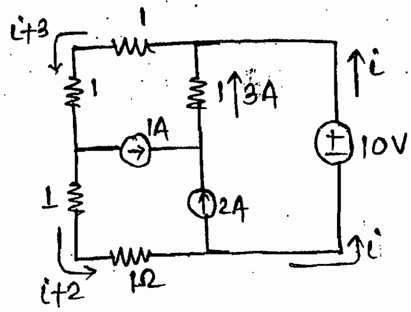
Shortcut → Because of same voltage across the given two parallel branches (Only applicable for same polarity vol. source) we can write




$$(4+4)\,||\,(4+4) = 4\,\Omega$$

For Pmax : $R_L = Req = 4\,\Omega$

44/31




Power supplied $= 10 \times i$

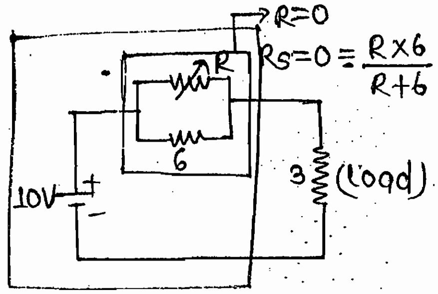
By KVL eqⁿ :-

$$-10 + (1+1)(i+3) + (i+2)(1+1) = 0$$

$$i = 0$$

Power supplied = 0W

46/31



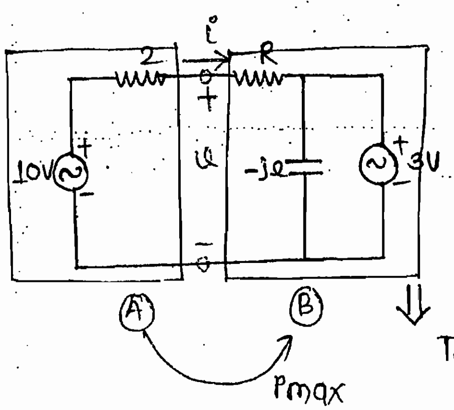

$\rightarrow R = 0$

$R_S = 0 = \dfrac{R \times 6}{R+6}$

$R_L \neq R_S$ for $P_{max}$

(Because of variable load max power will be delivered)

48/31




$P = v \cdot i$

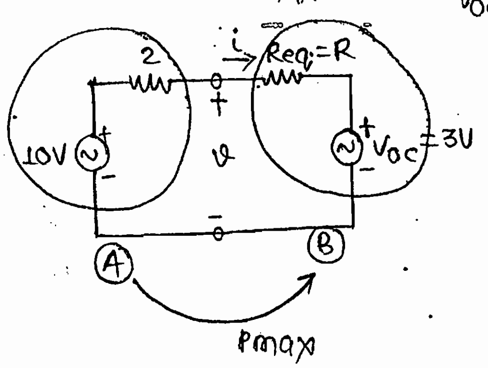
$\dfrac{\partial}{\partial R}[P] = 0$

Find R

Here the load is block B (passive elements included)

Theunin's:

$V_{oc} = 3V$, $R_{eq} = R$




$$i = \frac{10-3}{2+R} = \frac{7}{2+R}$$

$$v = iR + 3 = \frac{7R}{2+R} + 3 = \frac{10R+6}{2+R}$$

$$P = v \cdot i$$

$$= \frac{7(10R+6)}{(2+R)^2}$$

$$\frac{\partial}{\partial R}[P] = 0 \qquad \boxed{R = 0.8\Omega}$$

* Reciprocity Theorem → In a linear, bilateral single source n/w the ratio of the excitation to responce is constant when the positions of excitation & responce are interchanged.

The basic of theorem is the symmetry of a impedance & admittance matrices.

Impedance matrix :-

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$$\boxed{\begin{aligned} Z_{21} &= Z_{12} \\ Z_{31} &= Z_{13} \\ Z_{32} &= Z_{23} \end{aligned}}$$

....... For reciprocal n/w

....... Reciprocity theorem is Valid.

$[V] = [Z][I]$

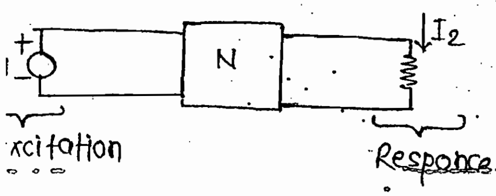
..... KVL

To verify the reciprocity theorem the basic configuration of the n/w remains same, only the external condn such as excitation & responce are interchanged.

The theorem is not applicable for the n/w containing :-

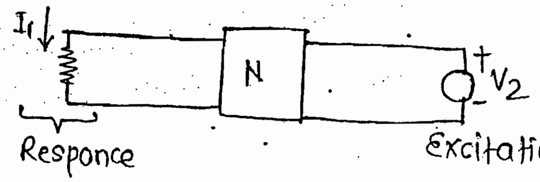
- Non-linear elements.
- Unilateral elements such as p-n junn diode.
- Multiple independant (or) depandant vol. (or) current sources.

Case(1) →




$$\frac{\text{Excitation}}{\text{Responce}} = \frac{V_1}{I_2} = Z_{12}$$

Case(2) →



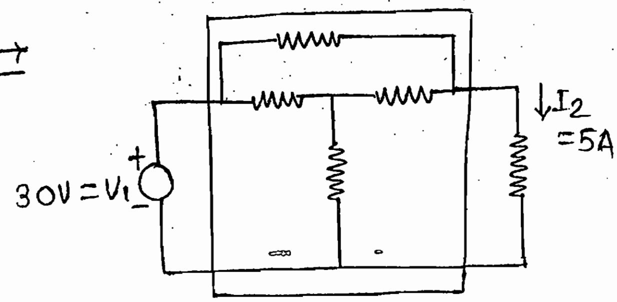

$$\frac{\text{Excitation}}{\text{Responce}} = \frac{V_2}{I_1} = Z_{21}$$

$$\boxed{\begin{aligned} \frac{V_1}{I_2} &= \frac{V_2}{I_1} \\ Z_{12} &= Z_{21} \end{aligned}}$$

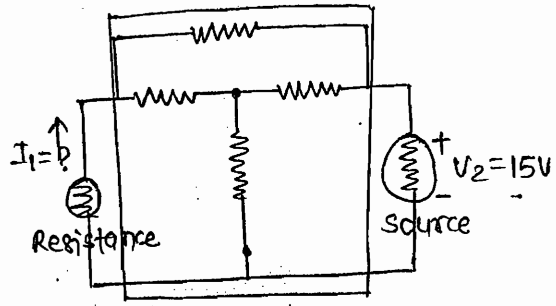
....... For reciprocal n/w

....... Reciprocity theorem is applicable

Que.→




..... Bridged-T n/w




Sol$^n$→

$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

Polarity of Vol. source be same.

$$\frac{30}{5} = \frac{15}{-I_1} \quad ; \; I_1 = -2.5A$$

Because of opposite current

* Tellegen's theorem →* In any n/w total instantaneous power consumed by various elements in different branches of n/w is always = 0.

* In any n/w total instantaneous power delivered by various active element must always be equal to the total instantaneous power consumed by various passive elements & different branches of N/w.

$$\boxed{\sum_{k=1}^{b} v_k \cdot i_k = 0}$$

$v_k$ ....... branch voltages

$i_k$ ........ Branch currents

b ........ No. of branch.

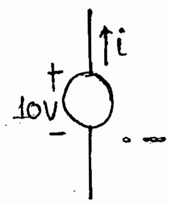
* The theorem is always valid irrespective of :-

(1.) The shape of n/w.

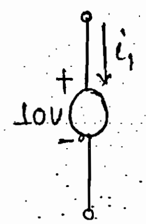
(2.) The type of elements contain in the n/w.

(3.) Value of each element in the n/w.

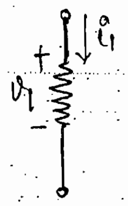
* The theorem is applicable so long as the KVL & KCL eqn are applicable to the given n/w.




Power delivered $= 10 \times i$

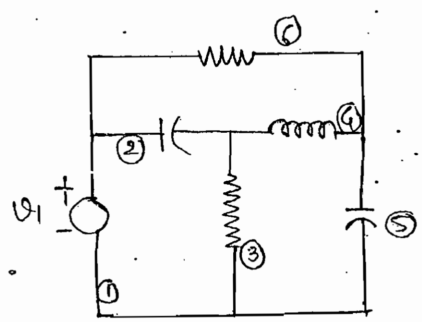



*** Power abosorb/ consumed $= 10 \times i_1$



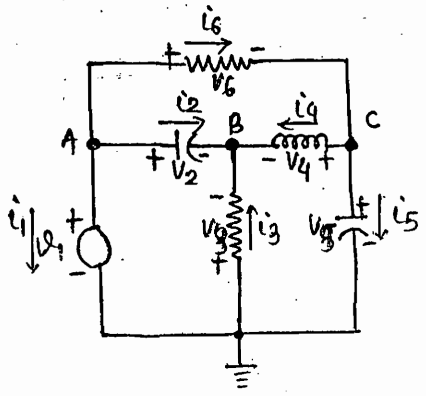

*** Power absorb/consumed $= v_1 i_1$

Que:- In the ckt shown find the missing branch vol. & branch current. Find the power absorb by each element in various branches on n/w. Hence calc. the total power absorb by all the elements in diff branches of n/w.




| | |
|---|---|
| $v_1 = 4V$ | $i_1 = 2A$ |
| $v_2 = 2V$ | $i_3 = 4A$ |
| $v_4 = 3V$ | $i_6 = 4A$ |

Solⁿ →



KVL →

$-V_1+V_2-V_3=0$ $\qquad$ $+V_3-V_4+V_5=0$ $\qquad$ $+V_6+V_4-V_2=0$

$-4+2-V_3=0$ $\qquad$ $-2-3+V_5=0$ $\qquad$ $V_6+3-2=0$

$\boxed{V_3=-2V}$ $\qquad$ $\boxed{V_5=5V}$ $\qquad$ $\boxed{V_6=-1V}$

KCL →

at node (A)

$i_1+i_2+i_6=0$

$2+i_2+4=0$

$\boxed{i_2=-6A}$

at node (B)

$i_2+i_3+i_4=0$

$-6+4+i_4=0$

$\boxed{i_4=2A}$

at node (C)

$i_4+i_5=i_6$

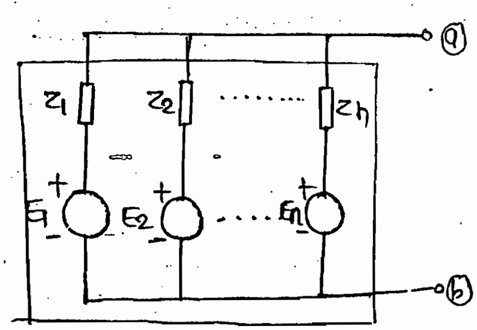
$2+i_5=4$

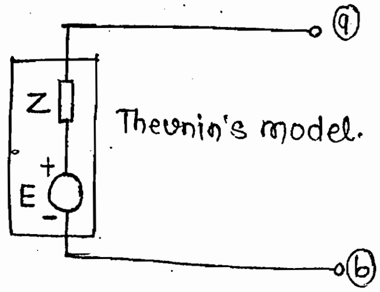
$\boxed{i_5=2A}$

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $V_k$ | 4 | 2 | -2 | 3 | 5 | -1 |
| $i_k$ | 2 | -6 | 4 | 2 | 2 | 4 |
| $V_k i_k$ | 8 | -12 | -8 | 6 | 10 | -4 |

$$\boxed{\sum_{k=1}^{6} V_k \cdot i_k = 0}$$

* Millman's theorem →



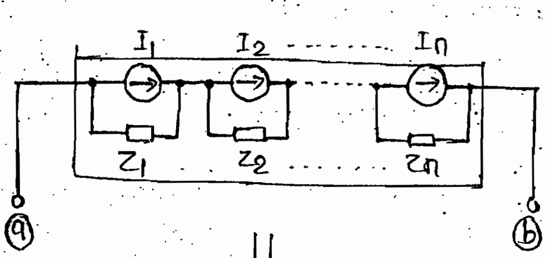


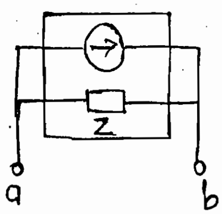

Thevnin's model.

$$\boxed{\begin{aligned} E &= \frac{\sum E_i Y_i}{\sum Y_i} \\ Z &= \frac{1}{\sum Y_i} \end{aligned}}$$

* This theorem is an extension of the Thevnin's theorem & is useful whenever no. of vol. sources are large in the given n/w.
* The theorem is applicable only when any given n/w can be rearranged in the std. format.

## * Dual millman's theorem →



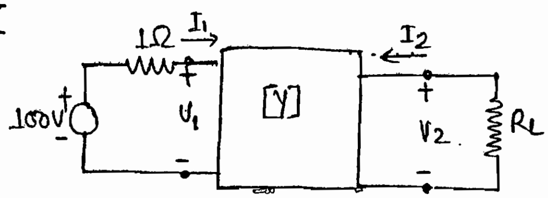

Norton's model

$$\boxed{\begin{aligned} I &= \frac{\sum I_i . Z_i}{\sum Z_i} \\ Z &= \sum Z_i \end{aligned}}$$

* The theorem is an extension of the Norton's theorem & is useful whenever there are large no. of current sources in the n/w

* The theorem is applicable only when any given n/w can be rearranged in the std. format.

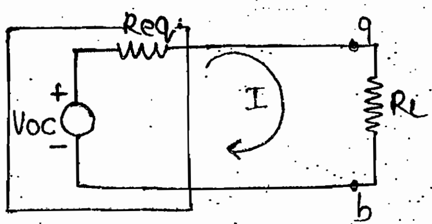
Que.→




$$[Y] = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} ; S$$

For the ckt shown cal. the value of RL for $max^m$ power Xfer. Hence cal. the $max^m$ power Xfered to the load.

Soln→




Thevnin's model

For Pmax; $R_L = R_{eq}$

$$P_{max} = \frac{V_{oc}^2}{4R_L}$$

$$\left.\begin{aligned} I_1 &= 4V_1 + 5V_2 \quad \cdots\cdots (1) \\ I_2 &= 5V_1 + 4V_2 \quad \cdots\cdots (2) \\ 100 &= I_1 + V_1 \quad \cdots\cdots (3) \end{aligned}\right\} \text{ from que. fig.}$$

**Voc →**

$I_2 = 0$

$V_2 = V_{oc}$

$$\begin{cases} I_1 = 4V_1 + 5V_{oc} \\ 0 = 5V_1 + 4V_{oc} \\ 100 = I_1 + V_1 \end{cases}$$

Find Voc

**Isc →**

$V_2 = 0$

$I_{sc} = -I_2$

$$\begin{cases} I_1 = 4V_1 + 5 \times 0 \\ -I_{sc} = 5V_1 + 4V_2 \times 0 \\ 100 = I_1 + V_1 \end{cases}$$

Find Isc

**Req →**

$V_2 = 1V$

$R_{eq} = \frac{1}{I_2}$

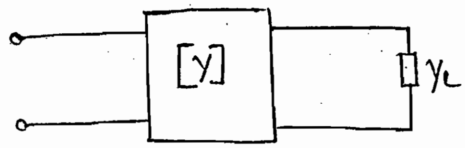
100V ⇒ SC

$$\begin{cases} I_1 = 4V_1 + 5 \\ I_2 = 5V_1 + 4 \\ I_1 + V_1 = 0 \end{cases}$$

Find $\frac{1}{I_2} = R_{eq}$

$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$

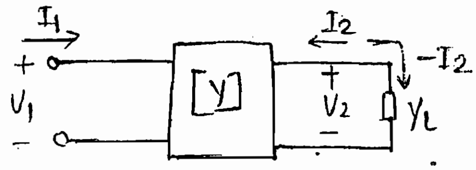
Que.→




To Find:-

$$G_{21} = \frac{V_2}{V_1}$$

Sol$^n$→




$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \text{......(i)}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \text{......(ii)}$$

$$-I_2 = V_2 Y_L \quad \text{------(iii)}$$

$$y_{21}V_1 + y_{22}V_2 = -V_2 y_L$$

$$V_2(y_{22} + y_L) = -y_{21}V_1$$

$$\frac{V_2}{V_1} = G_{21} = \frac{-y_{21}}{y_{22}+y_L}$$

$$\boxed{\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22}+y_L}}$$

DATE-08/12/19

# Network Synthesis

**Given:-**

Any N/w Fn in mathematical Form } $F(s)$ →
- $Z_{11}, Z_{22}; Y_{11}, Y_{22}$
- $Z_{12}, Z_{21}; Y_{12}, Y_{21}; G_{12}, G_{21}; \alpha_{12}, \alpha_{21}$

**To Find:-**

(1.) Shape of N/w.

(2.) Type of elements.

(3.) Value of each element.

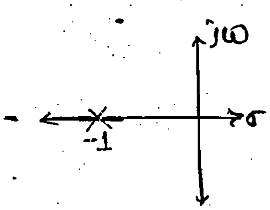
(1) Shape of N/w →
- Foster's Form →
  - type-I (series form)
  - type-II (Parallel form)
  - } ... using Partial Fraction expansion.
- Cauer's Form →
  - Type-I
  - Type-II
  - } ... ladder n/w } ... Continued Fraction expansion (C.F.E.)

(2) Type of elements →
- LC
- RC
- RL
- RLC

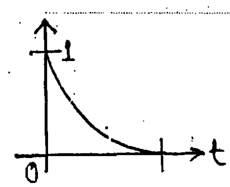
***Condition of any n/w to be:-**

(1.) Physicaly Realizable (causal).

(2.) Stable.




$$\frac{1}{(s+1)} \Rightarrow e^{-t}u(t)$$




*** For any ele. n/w to be stable n/w following condn must be satiesfied:-**

(1.) Network fns F(s) can't has pole in RHP.

(2.) N/w Fn F(s) can't have multiple pole along the jω axis including the origin.

(3.) The degree of numerator of n/w Fn F(s) can't exceed the degree of denominator by more than unity.

For any stable n/w the responce must be finite for finite excitation.

* For a physicaly relisable n/w (or) for a causal n/w the o/p can't exist before the i/p is applied.
* Any n/w fn $F(s)$ with poles in LHP has inverse laplace Xform which is 0 for $t<0$.

Therefore the stability emplies causility.
* Therefore any stable n/w is always a physicaly reliasable n/w but the reverse may not be true. Hence any physicaly reliasable n/w may (or) may not be stable n/w.

* Hurwith polynomial → P(s)

$$F(s) = \frac{\text{Num. poly}}{\boxed{\text{denom. poly}}\ P(s)}$$

...... Controls poles of n/w
........ must lie in LHP
........ n/w must be stable
........ P(s) must be Hurwith
........ The n/w must also be physicaly reliasable.

* Features of P(s) →

$$P(s) = 2s^6 + 2s^5 + 2s^4 + 6s^3 + 7s^2 + 9s + 5$$

Even parts
$P_e(s) = 2s^6 + 2s^4 + 7s^2 + 5$

Odd parts
$P_o(s) = 2s^5 + 6s^3 + 9s$

CFE of $\frac{P_e(s)}{P_o(s)}$

divisor ) dividend ( Quotient → ...... must be +ve

--------

Reminder

If $P(s) = s^6 + 2s^4 + 6s^2 + 9$ is given

then; $P'(s) = 6s^5 + 8s^3 + 12s$

CFE of $\frac{P(s)}{P'(s)}$

If $P(s) = P_1(s) \cdot P_2(s)$

Even :- $s^2+1$ , $s^3+4s$ (odd)
$s = \pm j$ $s = 0, \pm 2j$ (Both lying in $j\omega$)

* The Hurwiths poly. P(s) represent the deno. poly. of a physicaly realisable & stable n/w fn F(s).

* This poly. insures :-

(1.) No pole lies in RHP.

(2.) No multiple poles lie along the $j\omega$ axis including the origin.

* The poly P(s) is real for real value of s.

* All the coefficient of poly. must be real & +ve.

* No term must be missing in the Hurwiths poly. unless all the even term (or) all the odd terms are missing.

* The continued fraction expansion of the ratio of even to odd parts (or) odd to even parts must have all +ve coefficient term.

* The quotient term represent the numerical values of RLC compo. of N/W.

* If any poly is either even (or) odd then it is Hurwith if the CFE of $\frac{P(s)}{P'(s)}$ has all +ve quotient terms.

* If $P(s) = P_1(s) \cdot P_2(s)$ then the poly. P(s) is Hurwiths if the 2 poly. $P_1(s)$ & $P_2(s)$ are seprately Hurwiths.

* If any poly. is either even (or) odd then all its roots lie along the $j\omega$ axis only.

* If the deno. poly. of any n/w fn is Hurwiths then ~~all its~~ roots lie in LHP, the n/w will be stable & therefore physicaly relisable.

Que→ Find wether (or) not following poly. is Hurwith (OR).

Find weather (or) not following poly. represents the deno. poly. of stable & physicaly realiasable n/w.

$$P(s) = s^4+s^3+5s^2+3s+4$$

Soln→ $P_e(s) = s^4+2s^2+4$ ; $P_o(s) = s^3+3s$

CFE of $\dfrac{P_e(s)}{P_o(s)}$

Check:— (i) +ve coefficients
(ii) Imag. coefficients (or) not
(iii) missing term.

CFE→

$s^3+3s\,)\,s^4+5s^2+4\,(\,s$
$\quad s^4+3s^2$
$\quad 2s^2+4\,)\,s^3+3s\,(\,\frac{s}{2}$
$\quad\quad s^3+2s$
$\quad\quad s\,)\,2s^2+4\,(\,2s$
$\quad\quad\quad 2s^2$
$\quad\quad\quad 4\,)\,s\,(\,\frac{s}{4}$
$\quad\quad\quad\quad s$
$\quad\quad\quad\quad 0$

+ve quotient term
↓
P(s) is Hurwithz

CFE→ $$s+\cfrac{1}{\frac{s}{2}+\cfrac{1}{2s+\frac{s}{4}}}$$

Que→ $P(s) = s^4+4$

Soln→ Here $s^2$ is missing

∴ P(s) is not Hurwithz

Roots→ $s^4+4=0$

$s^4=-4$

$s^2=\pm j2$ ...... Because of multiple pole along $j\omega$ axis.

Que.→ $P(s) = s^3 + 2s^2 + 3s + 6$

Solⁿ→ $P_o(s) = s^3 + 3s$ ; $P_e(s) = 2s^2 + 6$

CFE → $\dfrac{P_o(s)}{P_e(s)}$

$$2s^2+6 \,\big)\, s^3 + \cancel{3s} \,\big(\tfrac{2s}{2}$$
$$\underline{-\cancel{s^3} \,\underline{+}\, 3s}$$
$$0$$

........ CFE has been terminated abruptively

let $\underbrace{s^3+3s}_{P_1(s)}$ is one factor of $P(s)$

To find :- 2nd Factor $P_2(s)$

$$P(s) = \underbrace{P_1(s)}_{\downarrow\, s^3+3s} ; P_2(s)$$

$$P_2(s) = \frac{P(s)}{P_1(s)}$$

$$s^3+3s \,\big)\, \cancel{s^3} + 2s^2 + \cancel{3s} + 6 \,\big(1 + \frac{2}{s} \Rightarrow 1 + \frac{1}{s/2} \Big\} P_2(s)$$
$$\underline{-\cancel{s^3} \quad \underline{+}\,\cancel{3s}}$$
$$2s^2 + \cancel{6}$$
$$\underline{-\cancel{2s^2} \,\underline{+}\, 6}$$
$$0$$

$$P(s) = P_1(s) \cdot P_2(s)$$
$$= \underbrace{(s^3+3s)}_{\downarrow\, s(s^2+3)} \underbrace{\left(1 + \frac{1}{s/2}\right)}_{\text{Hurwithz}}$$

roots :- $\underbrace{s=0, \pm j\sqrt{3}}$

$P_1(s)$ is Hurwitz

$$\therefore P(s) = \underbrace{\underbrace{P_1(s)}_{\text{Hurwithz}} \cdot \underbrace{P_2(s)}_{\text{Hurwithz}}}_{\text{Hurwitz}}$$

$\therefore$ $P(s)$ is Hurwithz

## * PRF (+ve Real fn) →

$$F(s) \to \begin{cases} z_{11}, z_{22} \\ y_{11}, y_{22} \end{cases} \qquad \left.\begin{matrix} z_{12}, z_{21} \\ y_{12}, y_{21} \\ g_{12}, g_{21} \\ \alpha_{12}, \alpha_{21} \end{matrix}\right\} \text{No study of this fns.}$$

* The +ve real fn represent physicaly realizable & stable passive driving point immitance fun$^n$.
* Any f$^n$ F(s) is said to be PRF if the following cond$^n$ is satisfied:-

$$\boxed{Re[F(s)]_{s=j\omega} \geq 0}$$

...... for all $\omega$

## * Features of PRF → F(s)

1.) If F(s) is +ve real real f$^n$ then $1/F(s)$ is also PRF.

2) The sum of 2 +ve real f$^n$ is also a PRF

$$F(s) = F_1(s) + F_2(s)$$

3.) All the zeros & poles lie in LHP.

4.) All the poles & zeros lie along the -ve real axis (or) along $j\omega$ axis (or) represent complex conjugate in nature.

5.) All the poles along $j\omega$ axis including the origin are simple in nature.

6.) The degree of numerator poly. can't exceed the degree of deno. by more than unity.

7.) The lowest power of s of numerator poly. can't exceed the lowest power of s of deno. poly. by more than unity.

* The necessary & sufficient cond$^n$ for any f$^n$ F(s) to be a PRF are:-

(1) The poles & zeros must lie in LHP.
Therefore the numerator & deno. poly. must be Hurwithz.

(2) The recidues of poles along the $j\omega$ axis must be real & +ve.
This is verified by making partial fraction expansion & checking the residues of only those pole which lie along $j\omega$ axis.

(3) $Re[F(s)]_{s=j\omega} \geq 0$ ....... for all $\omega$.

## (8) * Synthesis of ele. n/w containing specified element →

**Case(1) →** Driving point LC immettance Fns.

### * Features →

(1.) This immitance Fn is a ratio of even to odd (OR) odd to even poly.

(2.) The poles & zeros are simple; lie along the $j\omega$ axis & they interlace (alternate)

(3.) There must be either a pole (or) zero at the origin (or) at $\infty$.

(4.) Highest power of s of numerator & deno. poly. must differ at the most by unity.

(5.) The lowest power of s of numerator & deno. poly. must also differ at most by unity.

*

**Que.→** Find wether (or) not following Fn represent physicaly realisable & stable driving point LC immitance Fn.

(1.) $F_1(s) = \dfrac{s(s^2+4)}{(s^2+1)(s^2+3)}$  Zeross:- $s=0$,

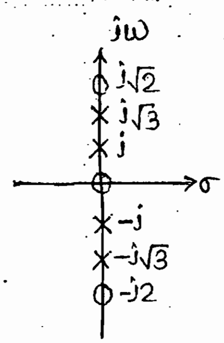
(2.) $F_2(s) = \dfrac{s^5+4s^3+5s}{3s^4+6s^2}$

(3.) $F_3(s) = \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)}$

| LC | RL(Y) RC(Z) | RL(Z) RC(Y) |
|---|---|---|
| $j\omega$ | -ve | -ve |
| 0 (p, z) | $0 \to P$ | $0 \to Z$ |
| $\infty$ (p, z) | $\infty \to Z$ | $\infty \to P$ |
| | Residue +ve | Residue -ve |
| | Alternate | |

**Soln →** *(1.) Zeros:- $s = 0, \pm j\sqrt{2}$

Poles:- $s = \pm j, \pm j\sqrt{3}$




Because of poles & zeros are not interlace/alternate; so this is not physicaly realisable.

*(2.) ~~Zeros:~~ (i) In the deno. $s^0$ is missing in $F_2(s)$

$$F_2(s) = \frac{s(s^4+4s^2+5)}{s^2(3s^2+6)}$$

(ii) Double pole exist

So it is also not realisable.

*(iii) Zeros :- $\pm j, \pm 3j$ poles:- $0, \pm j2$

By observation they are alternate.

All the cond$^n$ are satiesfied so it can be physicaly realisable.

Case(2) → Driving point

RL admittance fun$^n$ ($Y_{RL}$)

RC impedance fn ($Z_{RC}$)

* features:-

(1.) All the poles & zeros lie along the -ve real axis & they alternate.

2.) The recidues of poles must be real & +ve.

3.) The singularity nearest to (or) at the origin must be a pole where as the singularity nearest to (or) at $\infty$ must be a zero.

Que → Find weather following fn represents physicaly realisable & stable RC impedance fn (OR) RL admittance f$^n$

(1.) $F_1(s) = \dfrac{(s+1)(s+4)(s+8)}{s(s+2)(s+6)}$

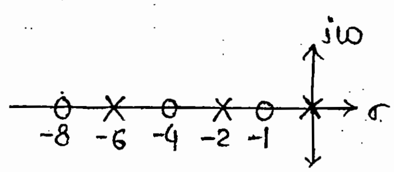
(2.) $F_2(s) = \dfrac{(s+1)(s+8)}{(s+2)(s+4)}$

(3.) $F_3(s) = \dfrac{(s+1)(s+2)}{s(s+3)}$

Sol$^n$ → (1.) Zeros:- $s=-1,-4,-8$

Poles:- $s=0,-2,-6$

...... Yes (physicaly Realisable)




For 3rd feature :- $F_1(s) \neq \dfrac{A}{s} + \dfrac{B}{(s+2)} + \dfrac{C}{(s+6)}$ (Because of equal power in Num. & deno. it can't here Partial fn$^n$)

(2.) Zeros:- $s=-1,-8$ * poles & zeros not alternate.

Poles:- $s=-2,-4.$ * nearest to origin is not pole.

..... No.

(3.) No, * they are not alternate (poles & zeros)

* Nearest to $\infty$ is not a zero.

* Case(3) → Driving point

* RL impedance $F^n$ ($Z_{RL}$)

* RC admittance $F^n$ ($Y_{RC}$)

* Features:-

(1.) The poles & zeros lie along the -ve real axis & they alternate.

(2.) The singularity nearest to (OR) at origin must be zero where as the singularity nearest to (OR) at $\infty$ must be a pole.

(3.) The residues of poles must be real & -ve.

Que → Find weather following $F^n$ represent physicaly realisable & stable driving point RL imp. $F^n$ (OR) RC admittance $F^n$:-

(1.) $F_1(s) = \dfrac{2(s+1)(s+3)}{(s+2)(s+6)}$ scale factor.

(2.) $F_2(s) = \dfrac{4(s+1)(s+2)}{(s+3)(s+6)}$

Sol$^n$ → (1.) Yes.

(2.) No, Because of not alternating (poles & zeros)

* Case(4) → Driving point immitance $F^n$

$Z_{RLC}$, $Y_{RLC}$

* Features → For RLC immitance $F^n$ no set rules are followed for loc$^n$ of the poles & zeros & therefore the poles & zeros are present any where in LHP.

Que.→ Following $f^n$ represents the driving point impedance $f^n$ for physicaly realisable & stable impedance $F^n$ for RLC N/w. Realize it in the cauer's type-(I) form

$$Z_{RLC} = \frac{s^2+2(s+2)}{s^2+s+1}$$

Soln →

Zeros ÷ $s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j1$

Poles → $s = \frac{-1 \pm \sqrt{1-4}}{2} = -0.5 \pm j0.866$

CFE of $\frac{s^2+2s+2}{s^2+s+1}$

$s^2+s+1)\ s^2+2s+2\ (1 \longrightarrow Z_1$ (series)

$\quad \cancel{s^2} + s + 1$

$s+1)\ s^2+\cancel{s}+1\ (s \longrightarrow Y_2$ (parallel)

$\quad \cancel{s^2} + s$

$1)\ s+1\ (\cancel{1/s}\, s \longrightarrow Z_3$ (series)

$\quad \cancel{s}+$

$1)\ 1\ (1 \longrightarrow Y_4$ (parallel)

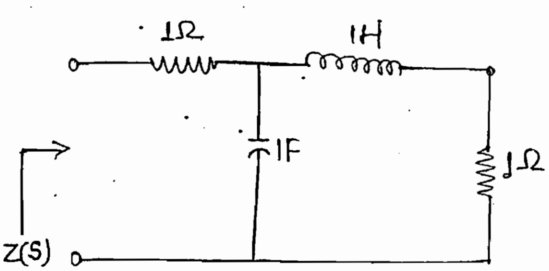
$\quad \frac{1}{0}$

$Z_1 = 1, = R_1$ , $R_1 = 1\Omega$ (series)

$Y_2 = s = sC_2$ ; $C_2 = 1F$ (parallel)

$Z_3 = s = sL_3$ ; $L_3 = 1H$ (series)

$Y_4 = 1 = \frac{1}{R_4}$ ; $R_4 = 1\Omega$ (prallel)




Couer's type-(I.) → ladder N/w

* If in option 4 ckt elements given then $\frac{s^2}{s^2} \rightarrow 1 \rightarrow R_1 = 1\Omega$ ......series (Because of z given)

* If que. is in the form of $\frac{s^3}{s^2} = s \rightarrow s4$, $z_1 = 4$ (series)

* If $\frac{s^4}{s^2} \rightarrow s^2$ [No form for compare with $s^2$, so not realizable]

* Cauer's type-I, L:-series, C:-shunt & for Cauer's-(II) vice-versa.

| Cauer's (I) | Cauer's-(II) |
|---|---|
| CFE of $\frac{s^2+2s+2}{s^2+s+1}$ | CFE of $\frac{2+2s+s^2}{1+s+s^2}$ |

* In Cauer's n/w & for RLC n/w total no. of elements in the ckt is equal to the algebric sum of the no. of finite zeros & poles.

* If the impedance fn is given then in the Cauer's form the 1st element represent an impedance element & represent a series element.

* In the Cauer's type-I n/w the indr is always a series element & capacitor is always a shunt elements.

* In Cauer's type-II n/w C is always a series element & an indr is always a shunt element.

* In Cauer's-I n/w CFE is found by arranging the num. & the deno. poly. in desending power of s.

* In Cauer's-II form the CFE is found by rearranging the num, & deno. poly. in ascending power of s.

* The Cauer's-I & II n/w ~~are~~ always indepandant of each other.

DATE - 09/12/14

Que. → The $F^n$ shows physicaly realisable & stable driving point LC immitance $F^n$. Realize it in :-

(1.) Foster's series Form (2.) Foster's parallel Form.

$$F(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Sol^n →

$$F_{LC}(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$F(S) = \frac{2s^4+20s^2+18}{s^3+4s}$$

$$s^3+4s)\ 2s^4+20s^2+18\ (2s$$
$$\underline{-2s^4 \pm 8s^2}$$
$$12s^2+18$$

$$= 2s + \underbrace{\boxed{\frac{12s^2+18}{s(s^2+4)}}}_{\frac{A}{s}+\frac{Bs+C}{s^2+4}}$$

$\frac{A}{s} + \frac{Bs+C}{s^2+4}$ ; $A = 9/2$, $B = 15/2$, $C = 0$

$$F(s) = 2s + \frac{9/2}{s} + \frac{(15/2)s}{s^2+4}$$

Case(1.) → Foster's Series Form (type-I)

$$F(s) = Z(s)$$

$$Z(s) = 2s + \frac{(9/2)}{s} + \frac{(15/2)s}{s^2+4}$$

$$= Z_1 + Z_2 + Z_3$$

$Z_1 = 2s = sL_1$ ; $L_1 = 2H$

$Z_2 = \frac{9/2}{s} = \frac{1}{(2/9)s} = \frac{1}{sC_2}$ ; $C_2 = \frac{2}{9}F$

$Z_3 = \frac{(15/2)s}{(s^2+4)} = \frac{1}{Y_3}$ ; $Y_3 = \frac{s^2+4}{(15/2)s} = \frac{2}{15}s + \frac{8}{15s} = Y_3' + Y_3''$

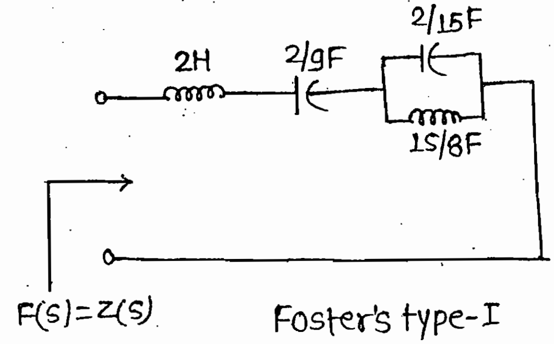
$Y_3' = \frac{2}{15}s = sC_3$ ; $C_3 = \frac{2}{15}F$

$Y_3'' = \frac{8}{15s} = \frac{1}{(15/8)s} = \frac{1}{L_3 s}$ ; $L_3 = \frac{15}{8}H$

$Z_1 = L_1 = 2H$

$Z_2 = C_2 = 2/9 F$

$Y_3' = C_3 = \frac{2}{15}F$ ; $Y_3'' = \frac{15}{8}H = L_3$




Foster's type-I

...... Series Form

## Case(2) Foster's type-(II)

...... Parallel Form

$$F(s) = Y(s)$$

$$Y(s) = 2s + \frac{9/2}{s} + \frac{(15/2)s}{s^2+4}$$

$$= Y_1 + Y_2 + Y_3$$

$$Y_1 = 2s = sC_1 \; ; \; C_1 = 2F$$

$$Y_2 = \frac{1}{\left(\frac{2}{9}\right)s} = \frac{1}{sL_2} \; ; \; L_2 = \frac{2}{9}H$$

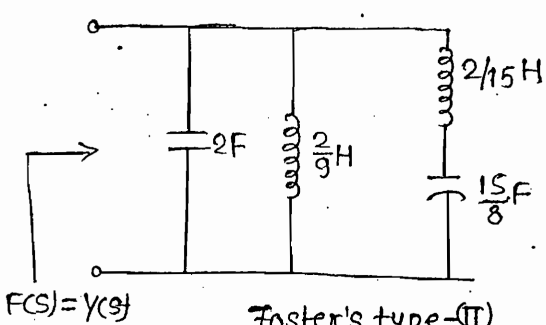
$$Y_3 = \frac{(15/2)\, s}{s^2+4} = \frac{1}{Z_3} \; ; \; Z_3 = \frac{s^2+4}{\left(\frac{15}{2}\right)s} = \left(\frac{2}{15}\right)s + \frac{1}{\left(\frac{15}{8}\right)s} = Z_3' + Z_3''$$

$$Z_3' = \left(\frac{2}{15}\right)s = L_3 s \; ; \; L_3 = \frac{2}{15}H$$

$$Z_3'' = \frac{1}{\left(\frac{15}{8}\right)s} = \frac{1}{C_3 s} \; ; \; C_3 = \frac{15}{8}H$$

$$Y_1 = C_1 = 2F, \quad Y_2 = L_2 = \frac{2}{9}H$$

$$Z_3' = L_3 = \frac{2}{15}H, \quad Z_3'' = C_3 = \frac{15}{8}H$$




Foster's type-(II)

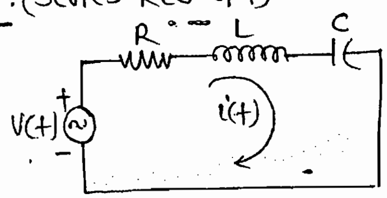
...... Parallel Form

Foster's type-I $\xleftrightarrow[\text{Networks}]{\text{Dual}}$ Foster's type-(II)

: The Foster's type-(I) & (II) N/w are dual of each other. This 2 n/w are not the equivalent n/w & therefore type-(I) n/w can't be replaced by type-(II) N/w.

: Duality →

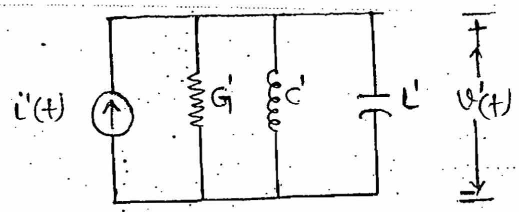
* $N_1$ : (series RLC ckt)




KVL :-

$$R\cdot i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int_0^t i(t)\cdot dt = v(t) \quad \text{-----(i)}$$

* $N_2$ :- (Parallel RLC ckt) ....... dual N/w




KCL →

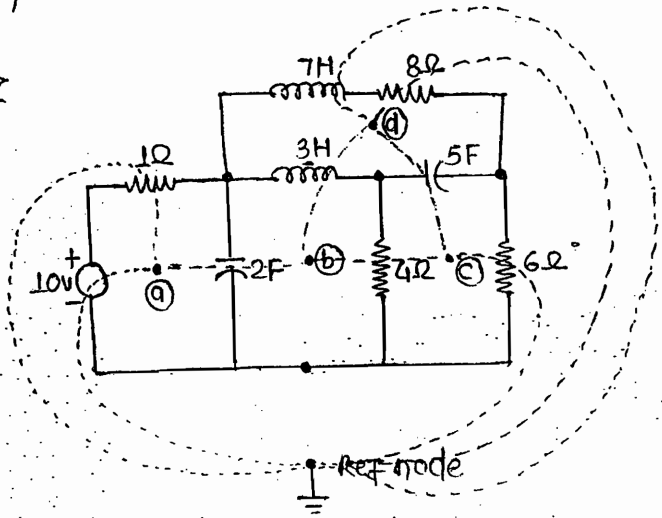
$$G'v'(t) + C'\frac{dv'(t)}{dt} + \frac{1}{L'}\int_0^t v'(t)\,dt = i'(t) \quad \text{-----(ii)}$$

| $N_1$ | dual N/w $N_2$ |
|---|---|
| Series ckt | parallel ckt |
| KVL | KCL |
| v(t) | i(t) |
| i(t) | v(t) |
| Vol. source | Current source |
| Loop | Node |
| R | G |
| L | C |
| C | L |
| closing switch | Opening switch |

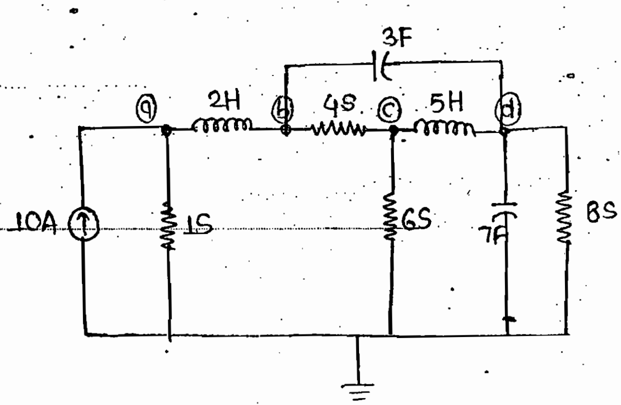
* The original N/w $N_1$ & $N_1$'s dual N/w are not the eq. N/w & therefore the original N/w can't be replaced by its dual N/w.
* If we know the responce of any series N/w in terms of current variable for specified excitation to the N/w then we can find the responce of its dual N/w in terms of the vol. variable for similar current excitation.

This responce can be written directly by inspection w/o actually solving it by using the transformation shown in table.

Que.→ calculate the duality for given N/w




solⁿ→

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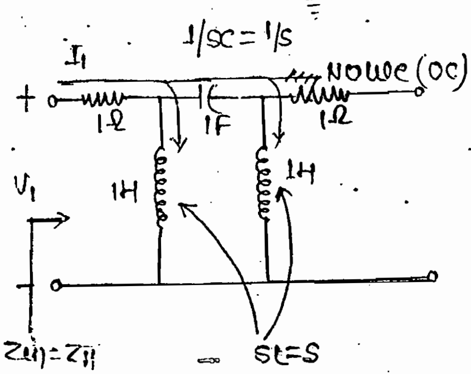


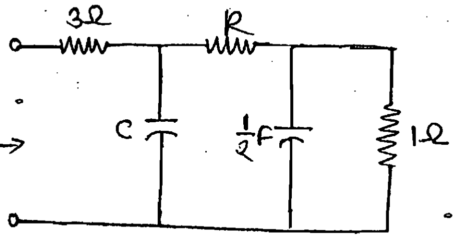

Cauer's-(II) N/w

$$Z_{in} = 1 + \frac{sx\left(s+\frac{1}{s}\right)}{s+\left(s+\frac{1}{s}\right)} = \frac{s^3+2s^2+s+1}{2s^2+1}$$

$$= \frac{1+s+2s^2+s^3}{1+2s^2}$$

$$\frac{s^3}{2s^2} = \frac{1}{2}s \Rightarrow sL$$

(4)
(42)

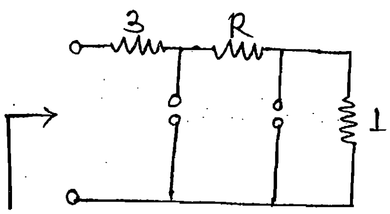



$$Z(s) = \frac{3(s^2+6s+8)}{s^2+4s+3}$$

1st method → By solving series parallel & cal. z(s) & then compare.

2nd method → $s=0$; C···· OC

$s=\infty$, C···· SC
C···· Can't be found out because of SC of C.



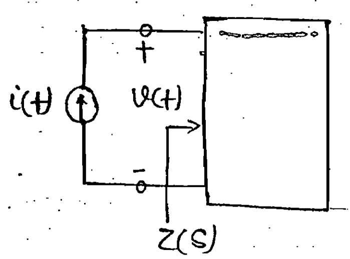

$$Z(0) = 3+R+1 = \frac{3(0+0+8)}{0+0+3}$$

$$\boxed{R=4\Omega}$$

3rd method → Because of C as shunt element it is in Cauer's -(I.)

CFE of $\frac{3s^2+18s+24}{s^2+4s+3}$ then find value of C

(5)
(43) -




$i(t) = u(t)$

$v(t) = 1+e^{-t/\tau}$

$I(s) = 1/s$

$V(s) = \frac{1}{s} + \frac{1}{s+(1/\tau)}$

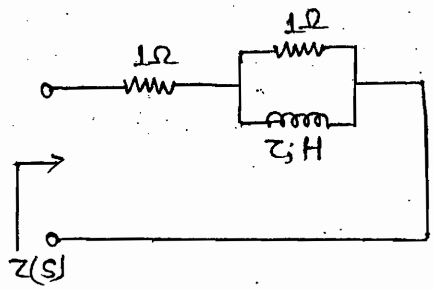
$$Z(s) = \frac{V(s)}{I(s)} = 1 + \frac{s}{s+1/\tau} = z_1+z_2$$

$Z_1 = 1; \Rightarrow R_1; R_1 = 1\Omega$

$Y_2 = \frac{1}{Z_2} = 1 + \frac{1}{s\tau} = Y_2' + Y_2''$

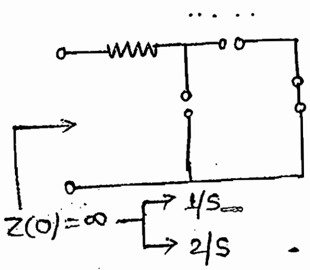
$Y_2' = 1 = \frac{1}{R_2}$ ; $R_2 = 1\Omega$

$Y_2'' = \frac{1}{s\tau} = \frac{1}{sL_2}$ ; $L_2 = \tau H$



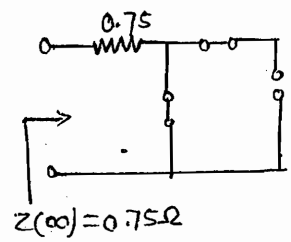

(8/43) $\underline{s=0}$

L = SC, C .... OC



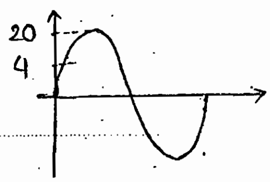

$\underline{s=\infty}$

L .... OC; C .... SC




(b) $Z(0) = 1/s$ ; $Z(\infty) = 0.75\Omega$

(d) $Z(0) = 2/s$ ; $Z(\infty) = 0.75\Omega$




$$V_{normalised} = \frac{V}{V_{max}} = \frac{4}{20}$$

so ans-(b) because of normalised is given

(10/43)

$$Z(s) = \frac{s+\alpha}{s+\beta} = \frac{V}{I} = \frac{j\omega+\alpha}{j\omega+\beta}$$

V leads I

I/p ...... sinusoidal

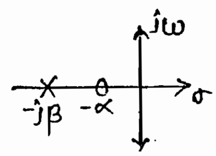
$$\theta = \tan^{-1}\frac{\omega}{\alpha} - \tan^{-1}\frac{\omega}{\beta} = +ve$$

$$\boxed{\alpha < \beta}$$

Another way;

RL ckt is given

$Z_{RL}$ $\boxed{\alpha < \beta}$



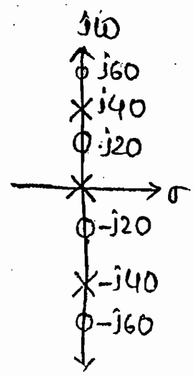
(13/44) ~ Reactive N/w $\rightarrow$ LC ckt

$Z_{LC}$

Zeros:- $\omega = 20, 60$

Poles:- $\omega = 0, 40, \infty$

$Z\big|_{\omega=10} = -j70\Omega$




$$Z(s) = \frac{K(s^2+20^2)(s^2+60^2)}{s(s^2+40^2)}$$

If we want to cal. the value of k then put $s = j\omega \rightarrow \omega = 10$ & compare $Z(s) = -j70\Omega$.

$$Z(s) = \frac{s^4}{s^3} = sL \quad (1^{st} \text{ term will L \& connected in series})$$

ans. (a)

(5/4)

$$\frac{V(s)}{I(s)} = Z(s) = \frac{(s+1)}{(s+\sqrt{2})(s+1/\sqrt{2})}$$

$i(t) = \underbrace{1}_{I=1} \sin \underbrace{t}_{\omega=1}$ $\qquad v(t) = v\sin t$

$|V| = ?$

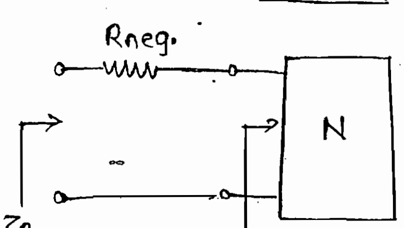
$s = j\omega = j1$

$$Z(j1) = \frac{(j1+1)}{(j1+\sqrt{2})(j1+\frac{1}{\sqrt{2}})} = \frac{V}{I=1}$$

$$|V| = \frac{\sqrt{2}}{\sqrt{1+2}\sqrt{1+1/2}}$$

$$\boxed{|V| = \frac{2}{3}}$$

(25/45)




$\underbrace{Z_2}$ must be PRF $\qquad Z_1 = \underbrace{r + jX}_{PRF}$

...... Physicaly realizable

$$Z_2 = r_n + Z_1 = r_n + (r \pm jx) = \underbrace{(r_n + r)} \underbrace{\pm jx}$$

$$|r_n| \leq r$$

$$|r_n| \leq Re\boxed{Z_1}$$

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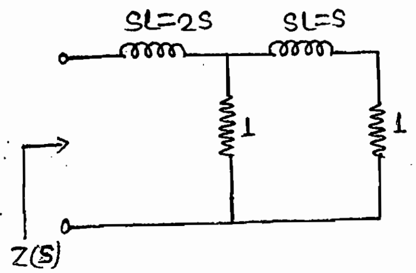


$$Z(S) = 2S + \frac{1 \times (S+1)}{1+(S+1)}$$

$S = -2, \infty$ ..... Poles

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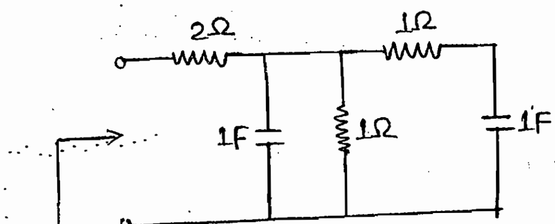
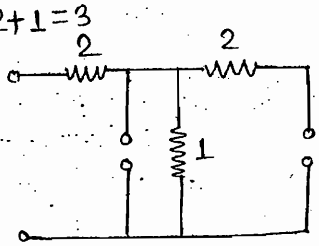



$$Z(S) = \frac{aS^2 + 7S + 3}{S^2 + 7S + b}$$

$\underline{S \to 0}$; $Z(0) = 2+1 = 3$




$$Z(0) = \frac{0+0+3}{0+0+b}$$

$$3 = \frac{3}{b}$$

$$\boxed{b=1}$$

$\underline{S \to \infty}$;

C.... (SC)

$$Z(\infty) = 2 = \frac{a + \cancel{(7/S)}^{0} + \cancel{(3/S^2)}^{0}}{1 + \cancel{(3/S)}^{0} + \cancel{(b/S^2)}^{0}}$$

$$\boxed{a=2}$$

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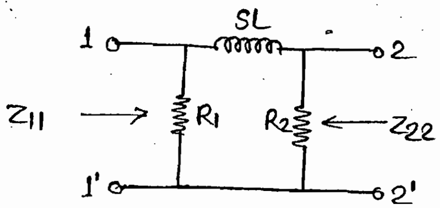



$$Z_{11} = k_1\left(\frac{S+3}{S+8}\right)$$

$$Z_{11} = \frac{R_1 \times (R_2 + SL)}{R_1 + R_2 + SL} = \frac{R_1 \cancel{L}}{\cancel{L}} \cdot \frac{\left(S + \frac{R_2}{L}\right)}{S + \left(\frac{R_1+R_2}{L}\right)}$$

$$k_1\left(\frac{S+3}{S+8}\right) = \frac{R_1\left(S+\frac{R_2}{L}\right)}{S+\left(\frac{R_1+R_2}{L}\right)}$$

$$R_1 = k_1 ; \quad \frac{R_2}{L} = 3, \quad \frac{R_1+R_2}{L} = 8$$

$$\frac{R_1}{L} = 5$$

$$Z_{22} = \frac{R_2 \times (R_1 + SL)}{(R_1+R_2+SL)} = \frac{R_2 \cancel{L} \times \left(S+\frac{R_1}{L}\right)}{\cancel{L} \times \left(S + \frac{R_1+R_2}{L}\right)}$$

$$= R_2 \times \frac{(S+5)}{(S+8)}$$

$$= k_2 \times \left(\frac{S+5}{S+8}\right)$$

ue.→ following $f^n$ represents physicaly realisable & stable driving point RLC impedance $f^n$. Realise it in Foster's-(II) Form.

$$Z(S) = \frac{(S+1)(S+4)}{(S+2)(S+3)} = Z_{RLC}(S)$$

soln→

$$Z(S) = \frac{(S+1)(S+4)}{(S+2)(S+3)} = Z_{RLC}(S)$$

...... Foster's type-(II) [Parallel Form]

$$\frac{1}{Z(S)} = Y(S) = \frac{(S+2)(S+3)}{(S+1)(S+4)} = \cancel{S^2+5S+6} \; 1 + \frac{2}{(S+1)(S+4)} = 1 + \frac{A}{(S+1)} + \frac{B}{(S+4)}$$

Because of ratio ~~is~~ of sq. is 2.

$$= 1 + \frac{(2/3)}{(S+1)} + \frac{(-2/3)}{(S+4)}$$

It can't be physicaly realisable because of -ve(B)

$$\frac{Y(s)}{s} = \frac{(s+2)(s+3)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+4)}$$

$$= \frac{3/2}{s} + \frac{(-2/3)}{(s+1)} + \frac{1/6}{s+4}$$

$$Y(s) = 3/2 + \frac{(-2/3)s}{(s+1)} + \frac{s(1/6)}{(s+4)}$$ → Need not to be devide because of +ve.

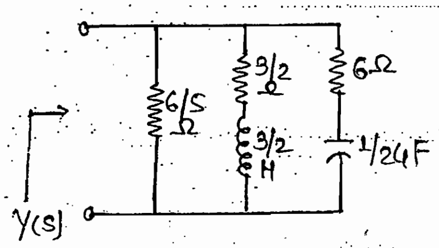
↓ same value of coefficient is -ve so we have to cal. +ve value so devide

$$s+1 \overline{) \; s \; (1} \quad \frac{-s+1}{-1}$$

$$Y(s) = \frac{3}{2} - \frac{2}{3}\left(1 - \frac{1}{s+1}\right) + \frac{1}{6}\cdot\frac{s}{s+4}$$

$$= \frac{5}{6} + \frac{2/3}{s+1} + \frac{1/6(s)}{(s+4)}$$

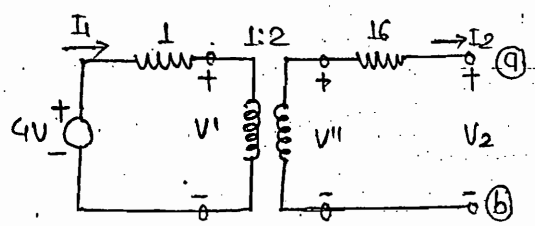
$$\boxed{Y(s) = Y_1 + Y_2 + Y_3}$$




Foster's type (II)

Parallel form

Que: →




Calcalate $V_{oc}$, ~~$I_{sc}$~~.

Sol$^n$ →

$4 = I_1 + V'$

$V_2 = -16I_2 + V''$

$\frac{V'}{V''} = \frac{1}{2}$ ; $V'' = 2V'$

$\frac{I_1}{I_2} = \frac{2}{1}$ ; $I_1 = 2I_2$

**Voc →**

$V_2 = V_{oc}$

$I_2 = 0$

**Isc →**

$I_2 = I_{sc}$

$V_2 = 0$

**Req →**

$\frac{V_{oc}}{I_{sc}}$

**Req →**

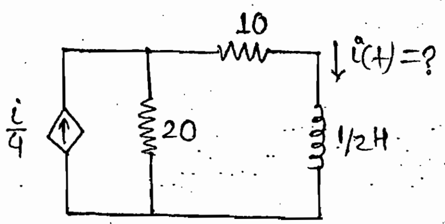
$V_2 = 1$

4V source ⇒ SC

$R_{eq} = \frac{1}{-I_2}$

$$\boxed{\begin{matrix} V_{oc} = 8V,\ R_{eq} = 20\Omega \\ I_{sc} = 2/5 A \end{matrix}}$$

**Que. →**



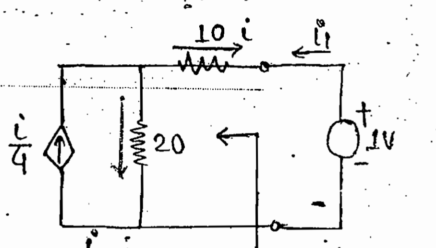

$i(0^+) = 10A$

**solⁿ →**

$$i(t) = \underbrace{i(0^+)}_{10A} e^{-t/T}$$

$$T = \frac{L}{R_{eq}}$$

**Req →**




$= \frac{i}{4} - i$

$= -3/4(i)$

$R_{eq} = \frac{1}{i_1} = \frac{1}{-i}$

**KVL →**

$1 = -10i + 20\left(\frac{-3}{4}\right)i$

$1 = -25i$

$\cancel{i = -1}$

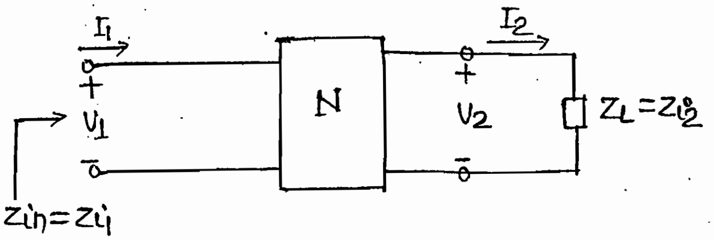
$\frac{1}{i} = -25$

$R_{eq} = 25\Omega$ ; $T = \frac{L}{R_{eq}} = \frac{1}{50}$

$i(t) = i(0^+) \cdot e^{-t/T}$

$$\boxed{\begin{matrix} i(t) = 10 \times e^{-50t} \ldots A \\ \text{for } t > 0 \end{matrix}}$$

## Image Impedances → $Z_{i_1}$ ; $Z_{i_2}$



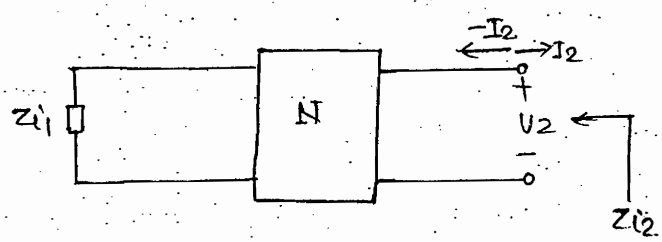

$$N : \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

$$V_2 = I_2 Z_L = I_2 Z_{i_2}$$

$$\frac{V_1}{I_1} = Z_{in} = Z_{i_1}$$




$Z_{i_1}, Z_{i_2}$ ...... Image impedances

$$\boxed{Z_{i_1} = \sqrt{\frac{AB}{CD}} \quad ; \quad Z_{i_2} = \sqrt{\frac{BD}{CA}}}$$

If n/w is symmetrical

$A = D$

$$\boxed{Z_{i_1} = Z_{i_2} = \sqrt{\frac{B}{C}}}$$