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200

-:HAND WRITTEN NOTES:-

OF 200/

MECHANICAL ENGINEERING

1

-: SUBJECT:-

POWER PLANT ENGINEERING

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POWER PLANT ENGG

- 550°C — **Steam Power Plant**
  - Rankine Vapour Power cycle
  - Working fluid = Steam
  - 9000 : 10000 rpm
- 1300 K — **Gas Turbine Power Plant**
  - Joule or Brayton cycle
  - 25000 rpm
- 2500°C — **I/C Engine Power Plant**
  - Otto, Diesel, Dual
- **Hydroelectric Power Plant**

Working fluid = GAS

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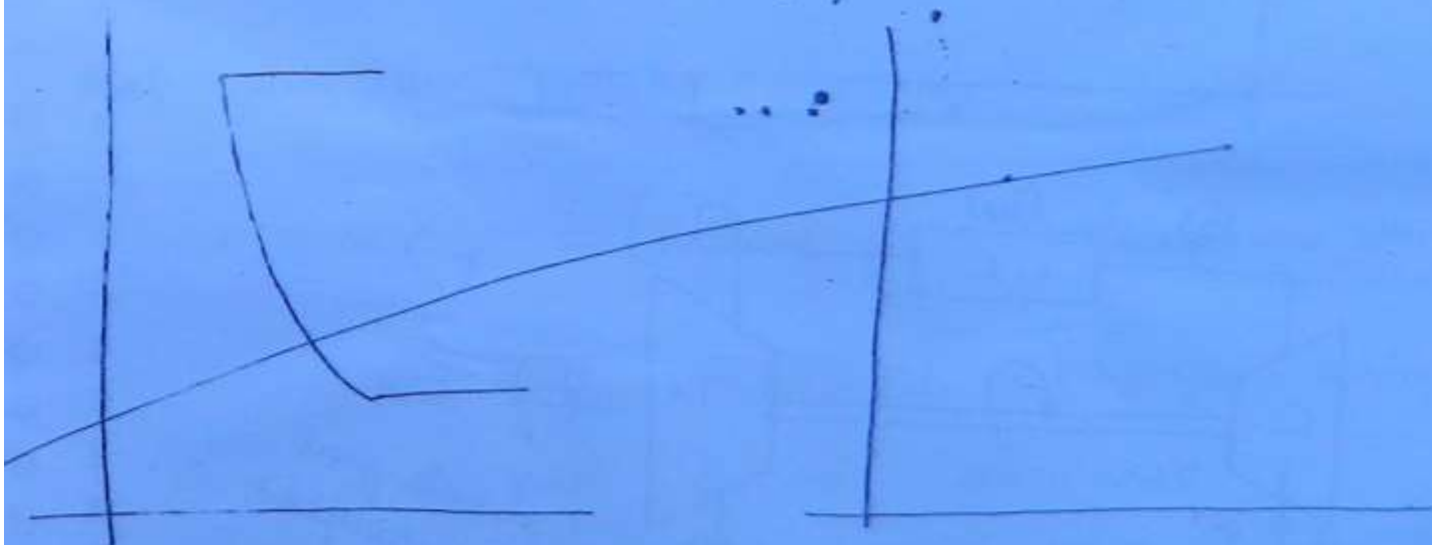
GAS POWER CYCLE

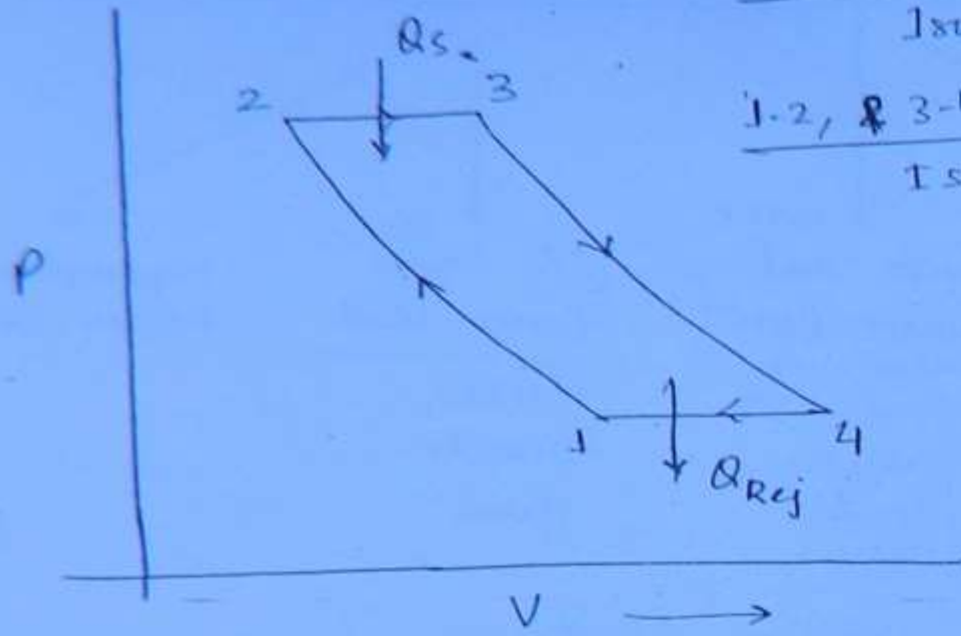
BRAYTON CYCLE or IDEAL JOULE CYCLE

(Turbo jets)



1-4 & 2-3,
Isobaric Process

1-2, & 3-4,
Isentropic Process
(Reversible adiabatic)



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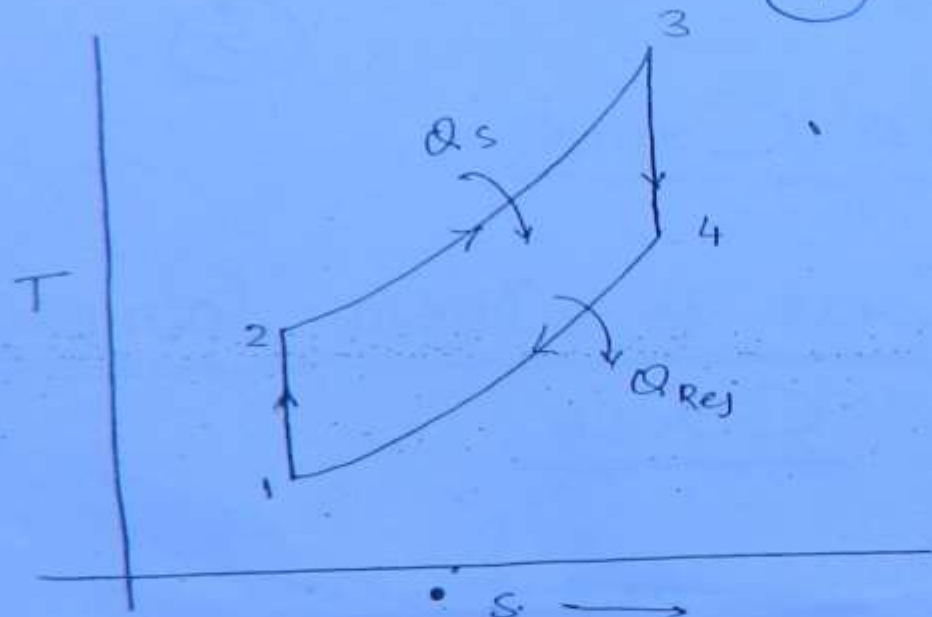

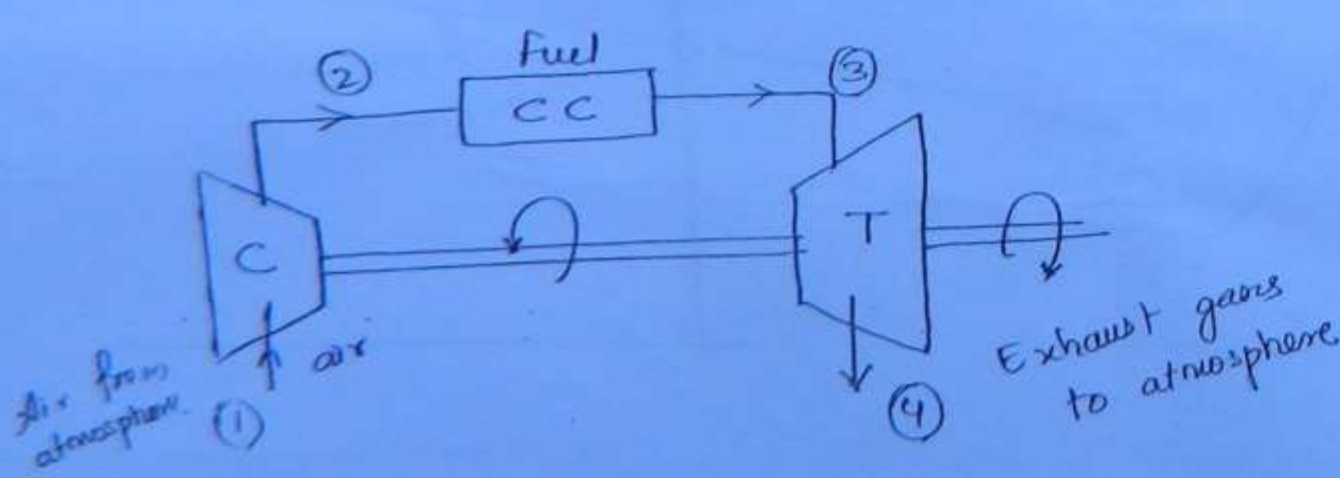

$\therefore$ From (1) & (2)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \boxed{\frac{T_4}{T_1} = \frac{T_3}{T_2}}$$

$$\eta_{th} = 1 - \frac{T_1\cancel{\left(\frac{T_4}{T_1}-1\right)}}{T_2\cancel{\left(\frac{T_3}{T_2}-1\right)}}$$ (as above)

$$\boxed{\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

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Note:-

As the pressure ratio of the Brayton Cycle, increases, the Ideal Air Standard Efficiency of the brayton cycle increases

Net work done per kg —

$$\boxed{\text{Net W.D}/kg = (\text{Turbine work}) - (\text{compressor Work})}$$

$$\text{Turbine Work} = \Delta h = (h_3 - h_4)\ KJ/kg$$

$\left[\because TdS = dh - vdP \right.$, if $ds = 0$ $\quad dh = vdP$

$\therefore \int dh = \int vdP \ \{ =$ W.D by a steady flo open system in rev. process

Compressor Work $= \Delta h$

For ideal gas, $\Delta h = C_p \cdot \Delta T$

$\therefore$ (TW) Turbine work $= \underline{C_{p_g}(T_3 - T_4)}$ KJ/kg.

(CW) Compressor Work $= \underline{C_{p_a}(T_2 - T_1)}$ KJ/kg

Net W.D/kg $=$ TW $-$ CW

$$\boxed{\text{Net W.D./kg} = C_{p_g}(T_3 - T_4) - C_{p_a}(T_2 - T_1)}$$

$C_{p_g} > C_{p_a} = 1.005$ KJ/kg K

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WORK RATIO (WR)

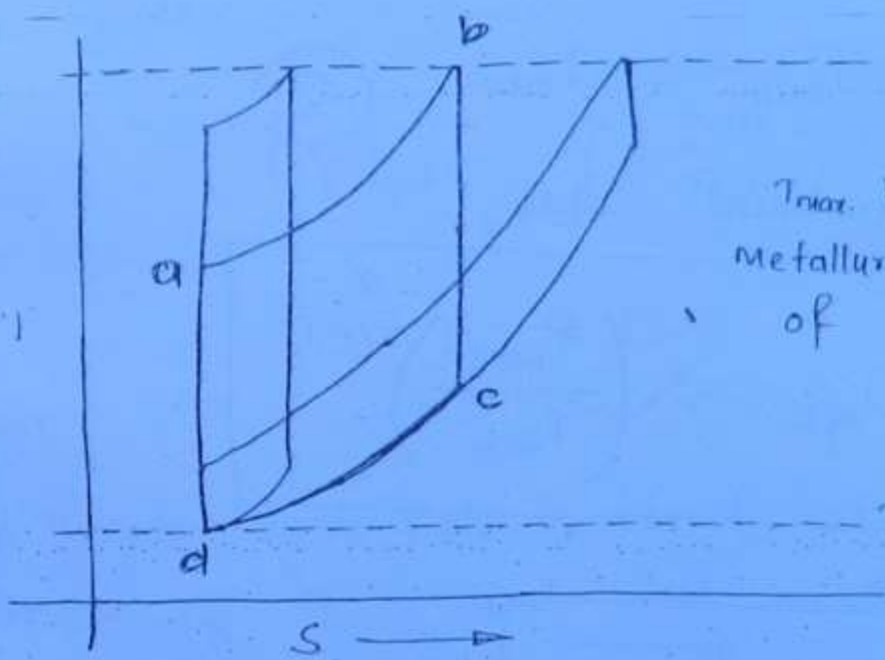
It is defined as net work done per kg and the turbine work done per kg

$$\boxed{WR = \frac{T.W. - C.W.}{T.W.}}$$

$$\boxed{WR = \frac{C_{p_g}(T_3 - T_4) - C_{p_a}(T_2 - T_1)}{C_{p_g}(T_3 - T_4)}}$$

Work ratio also depends on pressure ratio (r_p) of the cycle. As the pressure ratio increases work ratio also increase, reaches a maximum and then decreases

Effect of ~~Work~~ Pressure Ratio on net work done



$T_{max} = 1200$ K

T_{max} is based on Max. Metallurgical tolerable limit of Gas Turbines blades

$T_{min} = 300$ K

T_{min} = atm. temp

For a given maximum and minimum tempreatures of the cycle, there is a certain pressure ratio known as Optimum Pressure Ratio at which specific work output of cycle or its work ratio is maximum. Similarly, for a given T_{max}. and T_{min}, the thermal efficieny of the Ideal joule cycle, shall be maximum, when,

$$\boxed{r_p = (r_p)_{max.}}$$

$$\boxed{r_p = (r_p)_{max.} = \left(\frac{T_{max}}{T_{min.}}\right)^{\frac{\gamma}{\gamma-1}}}$$

But for a given $T_{max.}$ & $T_{min.}$ 8

W.R. shall be maximum, when,

$$\boxed{r_p = (r_p)_{opt.} = \left(\frac{T_{max.}}{T_{min.}}\right)^{\frac{\gamma}{2(\gamma-1)}}}$$

$$\boxed{(r_p)_{opt.} = \sqrt{(r_p)_{max.}}}$$

Note: Value of 'γ' —

for Helium — $\gamma = 1.67$

$\gamma_{diatomic} = 1.4$

$\gamma_{monoatomic} = 1.3$

if. $T_{max} = 1100$, $T_{min} = 300$, $\gamma = 1.4$

$$\therefore (r_p)_{max} = 9.439$$

and $$(r_p)_{opt} = 9.71$$

Gate 2000

Q.) In an ideal air standard, Gas Turbine cycle, the minimum and maximum tempreatures are respectively 310 K and 1100 K. Calc. optimal pressure ratio of the cycle.

Sol^n

$$(r_p)_{opt} = \sqrt{\left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}}} \quad (\gamma = 1.4)$$

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$$= 9.17$$

Gate 2001

Q.) A Brayton cycle (air standard) has a pressure ratio of 4 and inlet condition of 1 std. atm pressure and 27°c. Find the air flow rate required for 100 kW power output if the max. temp. in cycle is 1000°C. Assume $\gamma = 1.4$, $C_p = 1$ kJ/kg K

Sol^n

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = 4 \quad r_p = 4$$

$$\therefore T_2 = T_1 \cdot (r_p)^{\frac{\gamma-1}{\gamma}} =$$

Sol^n

$T_{max}\ (T_3) = 1000^\circ C = 1273\ K$

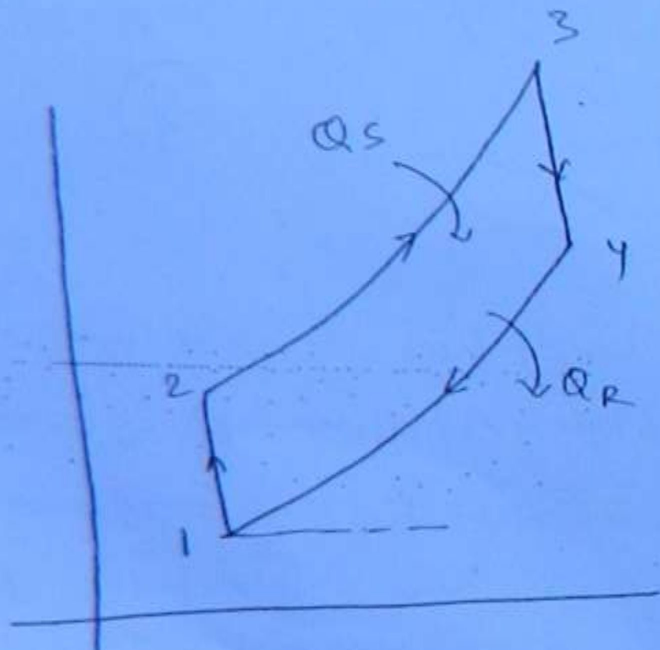
$T_{min}\ (T_1) = 27^\circ C = 300\ K$

$$T_2 = T_1\ (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$= 300\ (4)^{\frac{1.4-1}{4}}$$

$$= 445.7\ K$$

and $T_4 = 856.67\ K$



TW = 416.3

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Turbine Work $= C_{p_a}(T_3 - T_4)$

$= 416.33$ kJ/kg

Compressor Work $= C_{p_g}(T_2 - T_1)$

$= 145.7$ kJ/kg

$\therefore$ Net WD/kg $=$ TW $-$ CW

$= 270.6$ kJ/kg

$\therefore$ Mass flow rate

$\therefore$ Power Output $= \dot{m} \times$ Net. W.D./kg

$$\therefore \dot{m} = \frac{100}{270.6}$$

$$\boxed{\dot{m} = 0.369 \text{ kg/sec}}$$

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GATE 1999

Q.) An isentropic air turbine is used to supplied 0.1 kg/sec. of air at 0.1 MN/m^2 and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m^2. Determine the temperature at turbine inlet and the power developed.

Soln

$P_1 = P_4 = 400$ Pa. ,

$P_2 = P_3 = 100$ Pa , $T_3 = 285$ K

$$\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4} \quad \therefore T_4 = 423.5 \text{ K}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

The above given turbine is a part of air craft refrigeration system, running on Bel- Colemmon Cycle (Reversed brayton)

Now, $r_p = 0.25$

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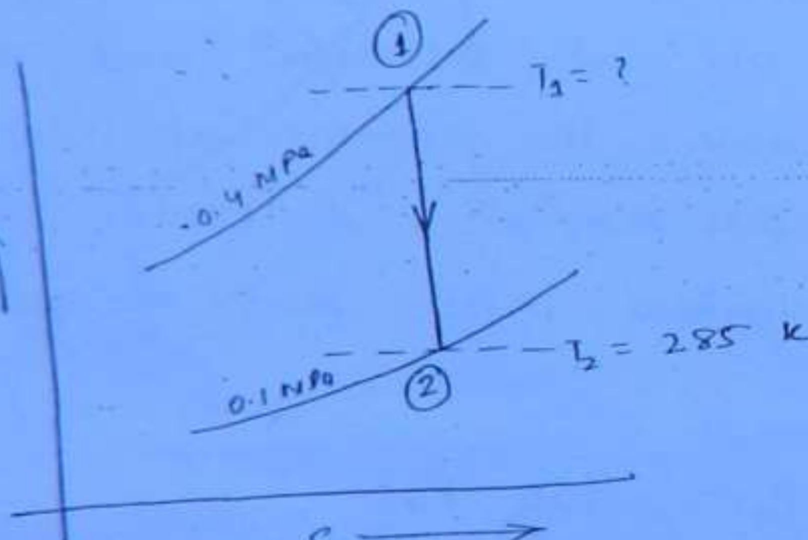
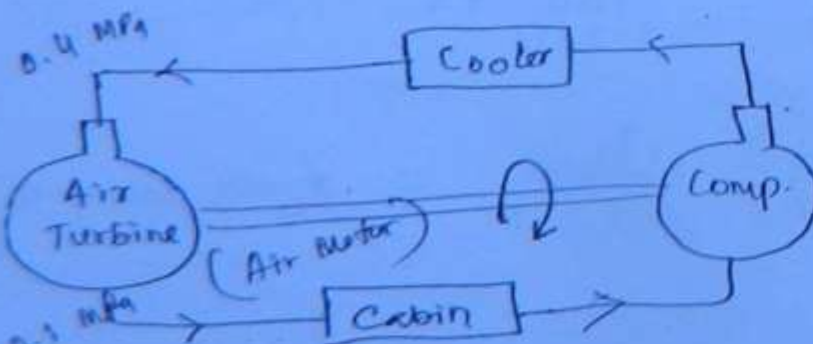
$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$





$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad (T_2 = 285\ K)$$

$$T_1 = T_2 \left(\frac{400}{100}\right)^{\frac{1.4-1}{1.4}}$$

$$= 423.7\ K$$

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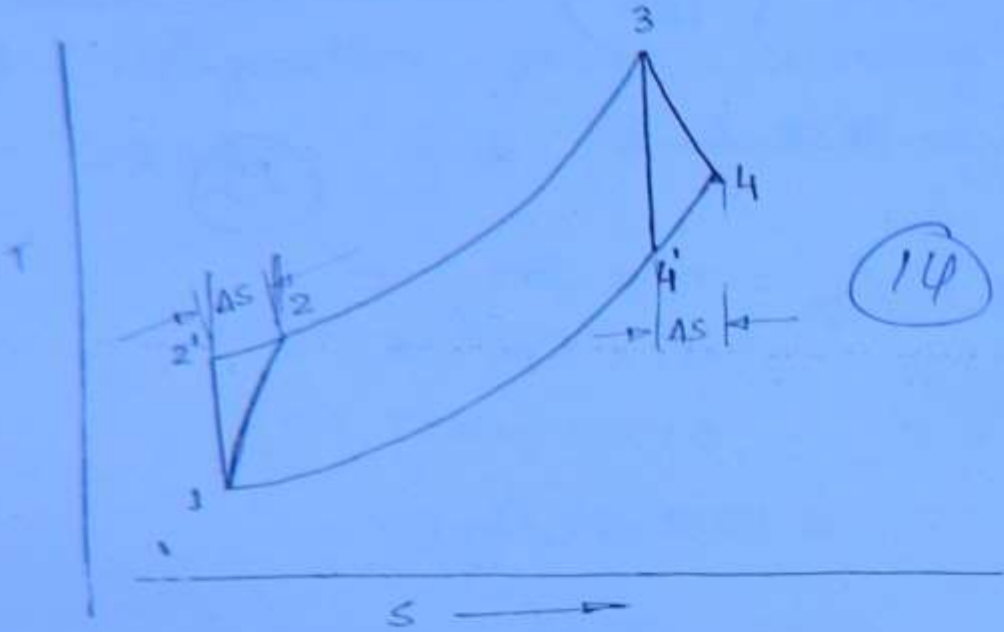
$\therefore$ Net WD/sec $= \dot{m} \times \Delta h$

$= \dot{m} \times C_p\,(T_1 - T_1)$

$= 0.1 \times 1\,(423.7 - 285)$

$= 13.85\ kJ/kg\ sec.$

Gas Turbine Cycle With Machine Efficiency



(14)

$1-2' \Rightarrow$ Reversible adiabatic or Isentropic Compression.

$1-2 \Rightarrow$ Actual compression with friction.

Isoentropic efficieny of the compressor is defined as the ratio between isoentropic work input to the compressor and actual work input

$$\eta_{(iso)\,compressor} = \frac{\text{Isentropic Work}}{\text{Actual work}}$$

$$\boxed{\eta_{\substack{ISO \\ (compressor)}} = \frac{\cancel{c_{p_a}}(T_2' - T_1)}{\cancel{c_{p_a}}(T_2 - T_1)}}$$ $(\simeq 80-85\%)$

Isentropic efficiency of the turbine is defined as the ratio between actual work output of the turbine with friction and the isentropic work output of the turbine without friction. (15)

$$\eta_{\substack{(ISO) \\ Turbine}} = \frac{\text{Actual Work}}{\text{Isentropic work output}} = \frac{\cancel{c_{p_g}}(T_3 - T_4)}{\cancel{c_{p_g}}(T_3 - T_4')}$$

$$\boxed{\eta_{\substack{(ISO) \\ turbine}} = \frac{T_3 - T_4}{T_3 - T_4'}}$$ $(\cong 80-85\%)$

Note

Steam Engine running on modified Rankine cycle

Oldest Engine

Gate 1998

Q) Find the required air fuel ratio in a gas turbine whos turbine and compressor efficiency are 85% and 80%. Max. cycle temp. is 875°. The working fluid can be taken as air. $C_p = 1$ kJ/kg K, $r = 1.4$, which enter the compressor at 1 atm and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in combustion chamber.

Soln

$T_3 = 875°$, $\eta_{comp} = 80\%$, $\eta_{turb} = 85\%$

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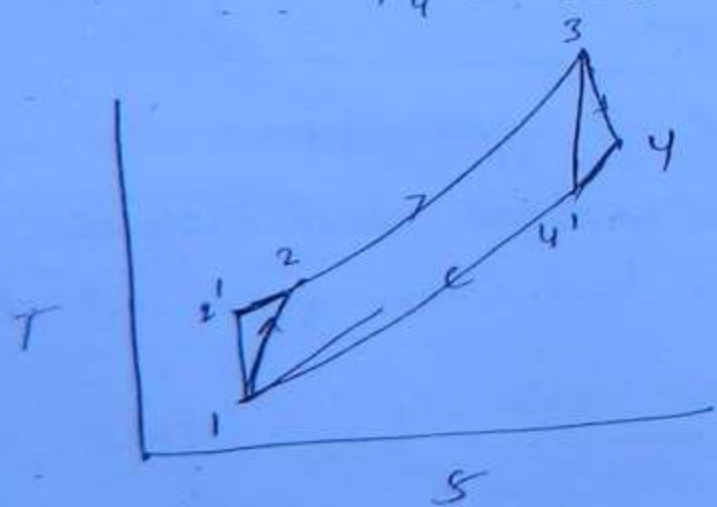
$$\frac{T_3}{T_4} = (r_p)^{\frac{r-1}{r}} \quad T_3 = 1148$$

$$T_4' = \cancel{588.8}\ 772.5 \text{ K}$$



$T_1 = 300\ K$

$$\therefore \frac{T_2'}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2' = 445.79\ K.$$

$$\eta_{comp.} = \frac{T_2' - T_1}{T_2 - T_1}$$

$$\therefore \boxed{T_2 = 482.25\,K}$$

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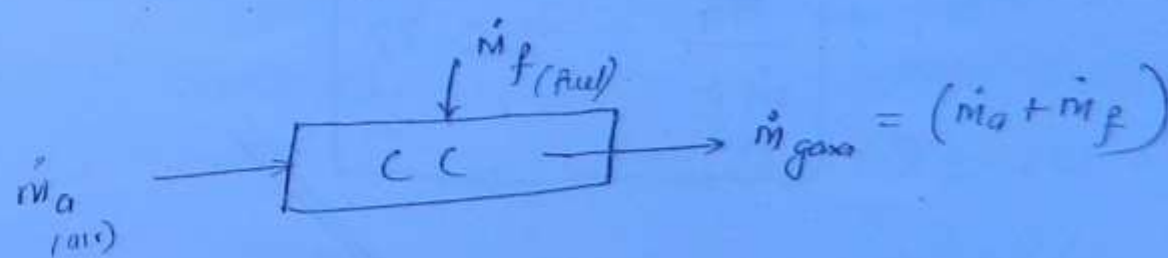
$$\eta_{turb} = \frac{T_3 - T_4}{T_3 - T_4'}$$

$$\therefore \boxed{T_4 = 828.83\,K}$$

for combustion champer (CC)



Energy balance

Heat liberated due to combustion of fuel

= Rate of change of Enthalpy of gases in CC.

$$\dot{m}_f \times L.C.V = (\dot{M}_a + \dot{m}_f)\, c_{pg} (T_3 - T_2)$$

$$L.C.V = \left(\frac{\dot{m}_a}{\dot{m}_f} + 1\right) c_{pg} (T_3 - T_2)$$

∴ Efficiency of combustion is 90% (due to 10% loss)

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$$\Rightarrow 0.9 \times LCV = \left(\frac{\dot{m}_a}{m_f} + 1\right) c_{pg} (T_3 - T_2)$$

$$\Rightarrow 0.9 \times 42000 = \left(\frac{\dot{m}_a}{\dot{m}_f} + 1\right) 1 (1148 - 482.25)$$

$$\therefore \boxed{\frac{\dot{m}_a}{\dot{m}_f} = 55.77}$$

Q.) Air enters the compressor of gas turbine plant operating on brayton cycle at 1 bar, 27°C. The pressure ratio in cycle is 6 if $w_T = 2.5\, w_c$ where, w_T & w_c are turbine & compressor work respectively. Calc the max. temperature and the cycle efficiency.

Soln

$$w_T = \cancel{6}\ 2.5\, w_c$$

$P_1 = 1$ bar
$= 100$ Pa

$T_1 = 300$ K

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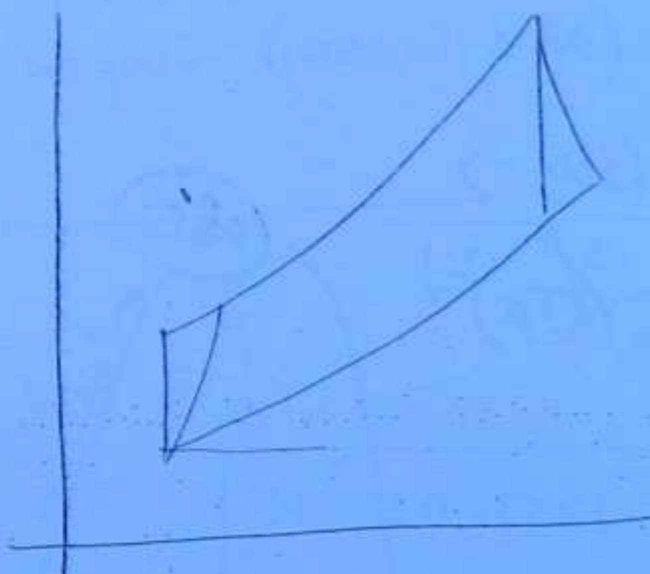

$$\frac{T_2}{T_1} = 6^{\frac{0.4}{1.4}} \Rightarrow T_2 = 500.55\ K$$

$$w_c = c_p (T_2 - T_1) = 200.55\ kJ$$

$$w_T = 501.38\ kJ/kg$$

$$r_p = \cancel{\sqrt{\left(\frac{T_{max}}{T_{min}}\right)^{\frac{r}{r-1}}}}$$

$$\eta = 1 - \frac{T_1}{T_2} \quad \text{or} \quad 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

$$\therefore \eta = 40\ \%$$

$$W_T = 2.5\, W_C$$

$$\cancel{C_p}(T_3 - T_4) = 2.5\,\cancel{C_p}(T_2 - T_1)$$

$$T_3\left(1 - \frac{T_4}{T_3}\right) = 2.5\,(500.5 - 300)$$

$$T_3 = \frac{2.5\,(200.5)}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

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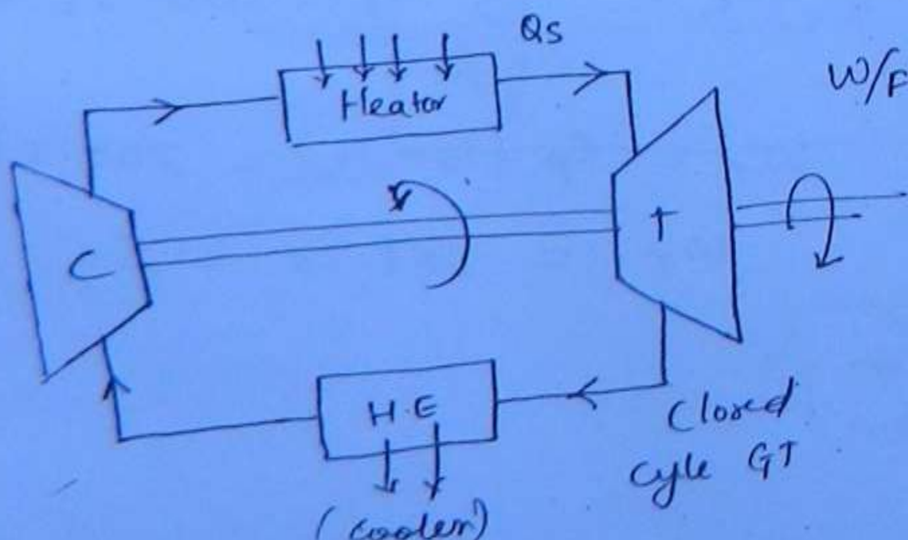
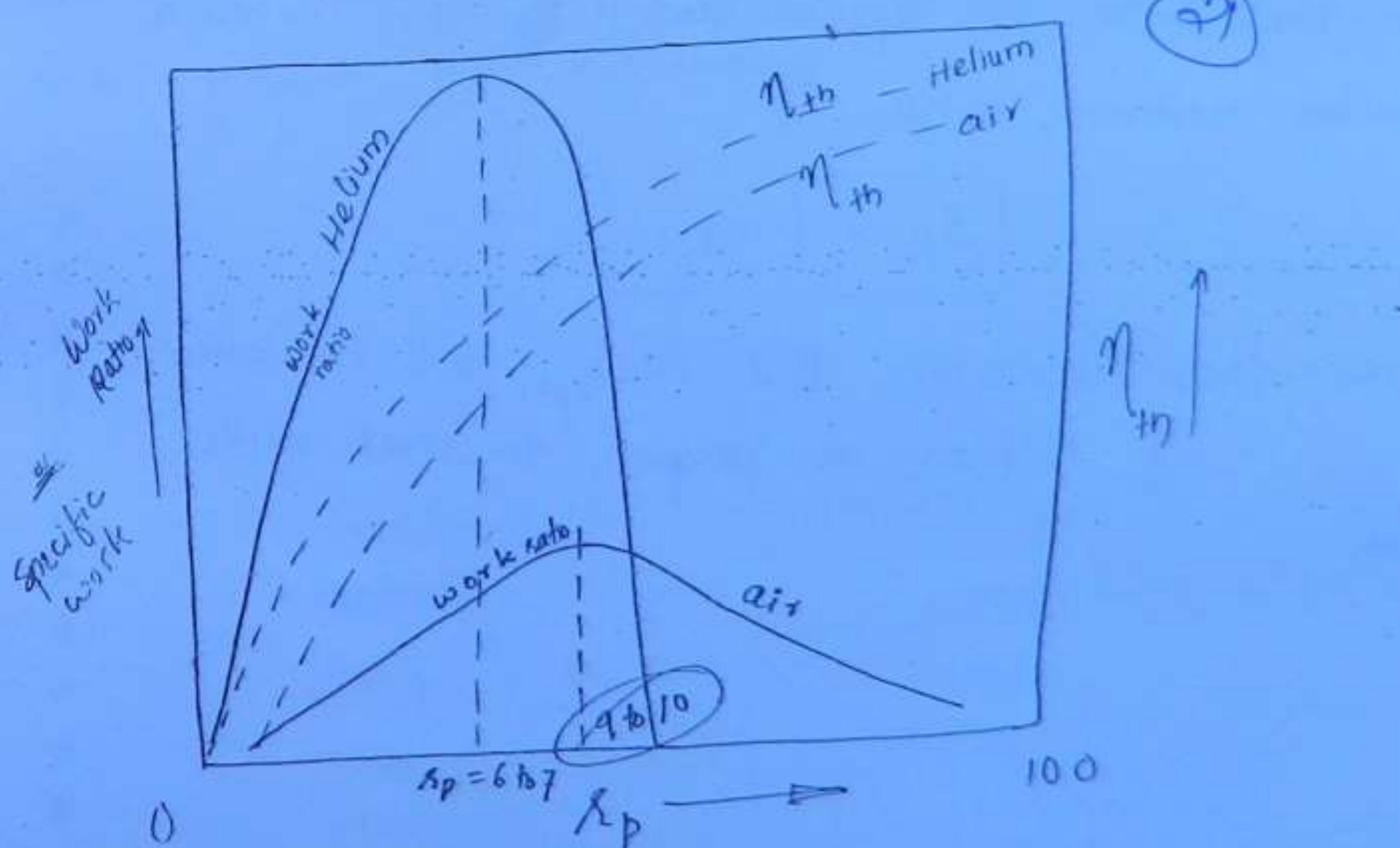
$$\boxed{T_3 = 1251.05\ \text{K}}$$



W/F = Helium
$\gamma = 1.66$

In closed cycle gas turbine power plant, the chemical composition of working fluid will not change because heat is supplied to it ~~from~~ in the heater, after having combustion of the fuel outside the heater (which means fuel) Also that, working fluid may be circulating in the system with higher pressure.

Since helium has better thermodynamic properties than air i.e. higher value 'γ', it can give higher thermal efficiencies.



(For given T_{max} & T_{min})
(1100 K & 300 K)

For a given $T_{min.}$ & $T_{max.}$ in the cycle, as the pressure ratio r_p increases, the thermal efficiency of the cycle increases either case of Helium or air, reaches a maximum, when,

$$\boxed{r_p = (r_p)_{max.}}$$

At any r_p (Pressure Ratio),

$$\boxed{\eta_{He} > \eta_{air}}$$

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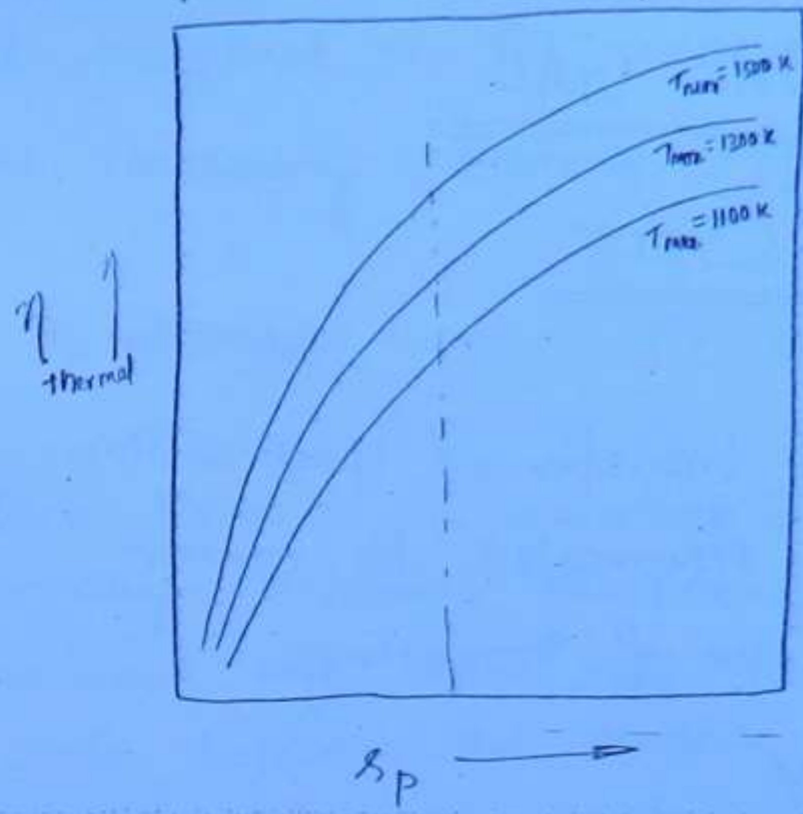
For work ratio, as r_p increases, it also increases, reaches maximum, when,

$$\boxed{r_p = (r_p)_{opt}}$$

and then decreases. But $(r_p)_{opt.}$ will be lesser in of helium as compare to that with air



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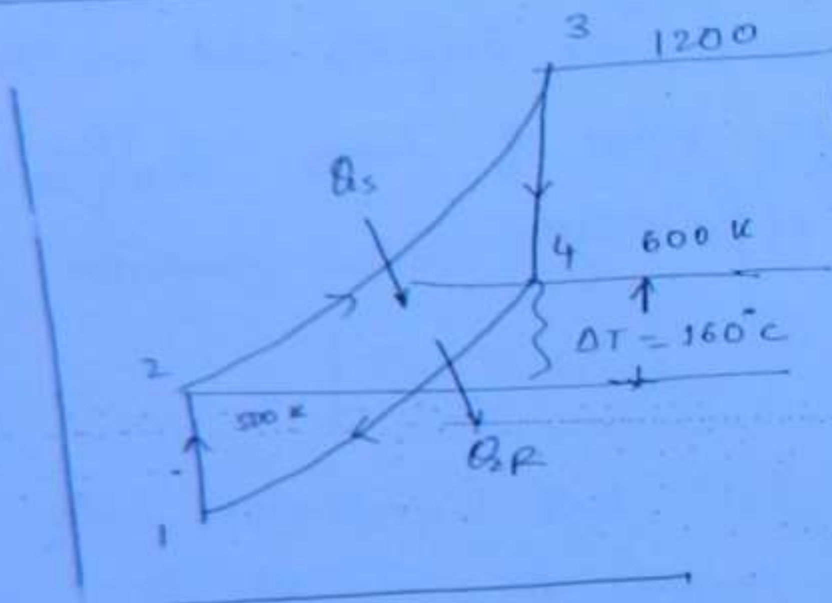
At any given pressure ratio improvided in the the cycle, higher the value of T_{max} in the cycle, greater the efficiency.

REGENERATION IN GAS TURBINE CYCLE

Regeneration means transfer of Heat within the cycle from Higher Temprature to lower temprature for the sake of increasing Thermal Efficiency.



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$r_p = 4$

$T_1 = 300\ K$

$T_3 = 1200\ K$

$T_2 = T_1 (4)^{\frac{\gamma-1}{\gamma}}$

$= \cancel{500.55}\ K$ 445.79 K

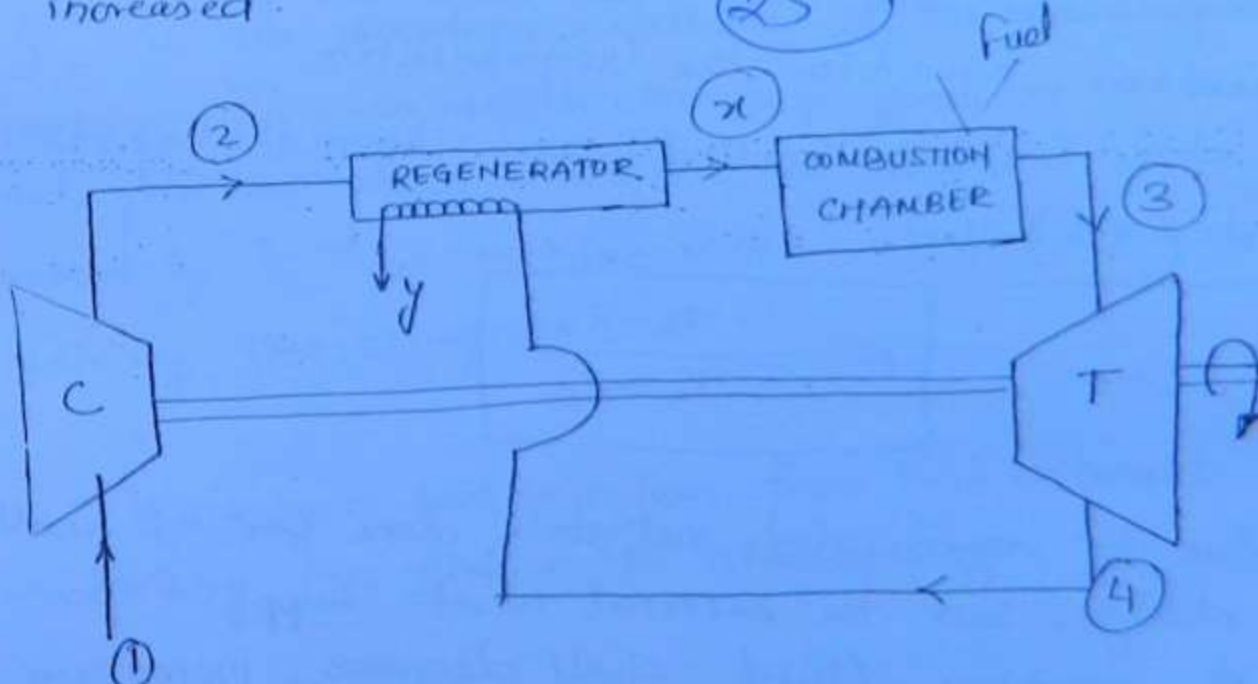
$T_4 = \dfrac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \cancel{740.25\ K}\ \cancel{659.77\ K}$ 807 K

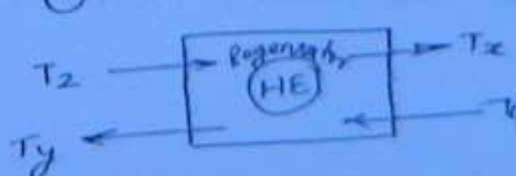
The Hot exhaust gases leaving the turbine are utilized for transferring the heat to the compressed air in a counter flow heat exchanger called **Regenerator**, there by reducing the external heat supplied in the combustion chamber, hence increasing thermal efficiency.

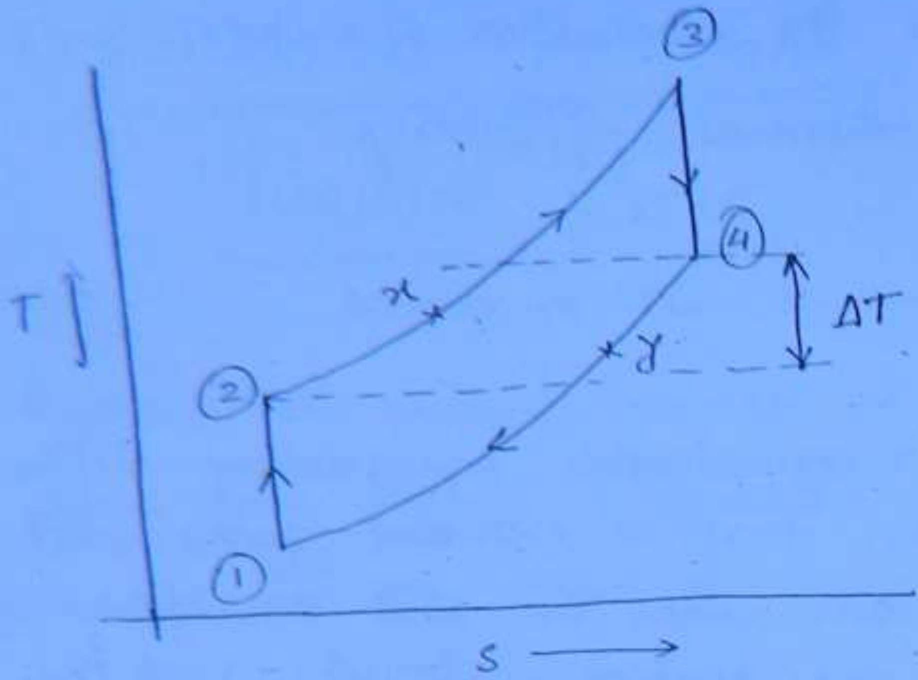
Note:

If r_p decreases $\longrightarrow$ ΔT increases

Since there is a considerable temperature difference existing between turbine exhaust gases and compressed air, there could be heat transfer possible between the turbine exhaust and the compressed air in a heat exchanger called <u>REGENERATOR</u>, thereby external heat supplied can be reduced & thermal efficiency can be increased.







T_x is the temprature of compressed air

The effectiveness of the regenerator is defined as the ratio between actual temperature rise of compressed air and the maximum possible temperature rise of air in the regenerator.

It is also called as Thermal Aspect Ratio (ϵ).

$$\boxed{\epsilon = \frac{T_x - T_2}{T_4 - T_2}} \simeq 0.7$$

Due to regeneration, network done per kg shall not change, but the external heat supplied per kg of working fluid shall decrease, increasing the thermal efficiency.

~~The compressor~~

* 1-2, is the isentropic compression of air from atmospheric pressure to combuction chamber pressure increasing the pressure and tempreature of air in the rotary compresso (centrifugal or axial)

* 2-3, is the isobaric heat addition to the air in the combustion chamber, due to the heat liberated by Burning of fuel which is sprayed directly into the air, rising the tempreature of the gases.

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* 3-4, is the isentropic expansion of the gases in the turbine from combustion chamber pressure to the exhaust pressure doing work on the turbine blade. Approximately 1/3rd of turbine work shall be used for driving the compressor.

* 4-1, is isobaric heat rejection from hot exhaust gases leaving the turbine to atmosphere

$$\text{Pressure Ratio } \boxed{(r_p) = \frac{P_2}{P_1} = \frac{P_3}{P_4}}$$

$$\boxed{\eta_{(th)\,\text{Ideal Brayton}} = 1 - \frac{Q_R}{Q_S}}$$

$$Q_{Rej} = C_p (T_4 - T_1) \quad KJ/kg$$

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$$Q_s = C_p (T_3 - T_2) \quad KJ/kg$$

$$\boxed{\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2}}$$

Now, 1-2 is, Isentropic Process,

$$\left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}} = \frac{T_2}{T_1} = \boxed{(r_p)^{\frac{r-1}{r}}} \quad \text{—— } (1)$$

3-4 is, Isentropic process,

$$\left(\frac{P_3}{P_4}\right)^{\frac{r-1}{r}} = \frac{T_3}{T_4} = \boxed{(r_p)^{r-1/r}} \quad \text{—— } (2)$$

Actual thermal efficiency of regenerative cycle is,

$$\eta_{(th)_{Reg.}} = \frac{\text{Net. Work Output/kg}}{Q_s \text{ /kg}} = \frac{W_T - W_C}{Q_s}$$

$$\therefore \eta_{th} = \frac{C_{pg}(T_3 - T_4) - C_{pa}(T_2 - T_1)}{\cancel{C_{pg}}(T_3 - T_x)}$$

$\because$ Mostly $C_{pg} \cong C_{pa}$

$$\therefore \boxed{\eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_x)}}$$

Hence, Regeneration in GT cycle results in,

(1) No change is W.D/kg

(2) [illegible] heat supply /kg get reduced.

(3) $\eta_{(th)_{cycle}}$ increases

(4) Specific fuel consumption gets ~~not~~ reduced
(Fuel Consumption required to produce 1KW Power)

Q.) A Gas Turbine Cycle operates with air having constt. specific heats. The inlet air is at 1 bar and 20°C. The max. cycle temp. is 1200 K. The compressor & turbine have same pressure ratio and have internal efficiency of 0.85. A regenarator with 85% effectiveness is consider. Cal^c all temp. around the cycle, η_{th}, heat added to produce 10MW power, cycle efficieny if no regenerator is used. Assume $r_p = 5$

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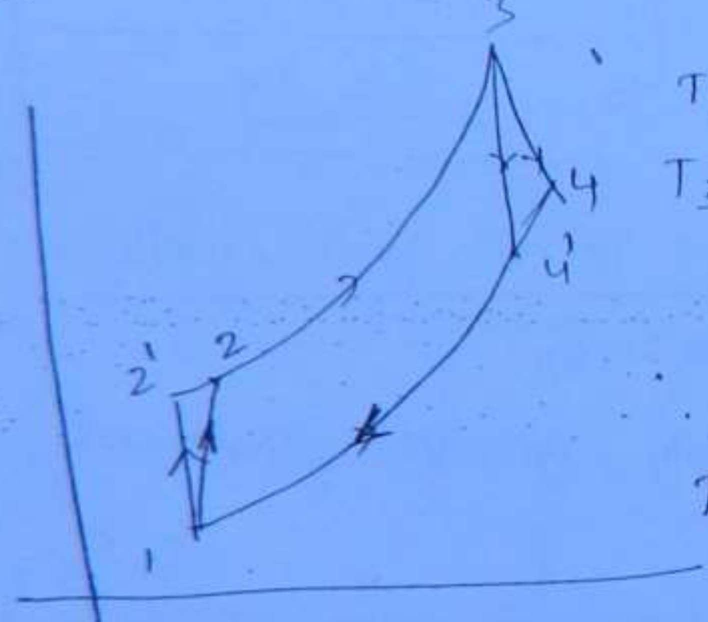
Sol^n



$T_1 = 293$ K

$T_3 = 1200$ K

$T_2' = 464.06$ K

$T_4' = 757.66$ K

$\eta_{cycle} = 1 - \frac{T_1}{T_2}$ $\therefore$ $0.85 - 1 = -\frac{T_1}{T_2}$

$T_2 = \frac{T_1}{1 - 0.85}$

or

$$\therefore \cancel{T_x = 713.62\ K}$$

$$\eta_{comp} = \frac{T_2' - T_1}{T_2 - T_1}$$

$$\therefore \cancel{T_2} = 0.85(T_2 - 293) = 464.06 - 29:$$

$$\boxed{T_2 = 494.2}\ K$$

$$\cancel{\eta_T = \frac{T_2' - T_1 \quad T_3 - T_4'}{T_3 - T_4}}$$

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$$\eta_T = \frac{T_3 - T_4}{T_3 - T_4'} \quad \therefore \boxed{T_4 = 824}\ K$$

$$\epsilon = \frac{T_x - T_2}{T_4 - T_2} \quad \therefore \boxed{T_x = 774.53\ K}$$

$$\therefore \eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_x}$$

$$= 0.41 = 41\%$$

Without regeneration,

$$\eta_{th} = \frac{(T_3 - T_4) - (T_2 - T_1)}{T_3 - T_2}$$

$$= 24.7\ \%$$

$$Q_s = \frac{\text{Power Supp.}}{\eta_{th}}$$

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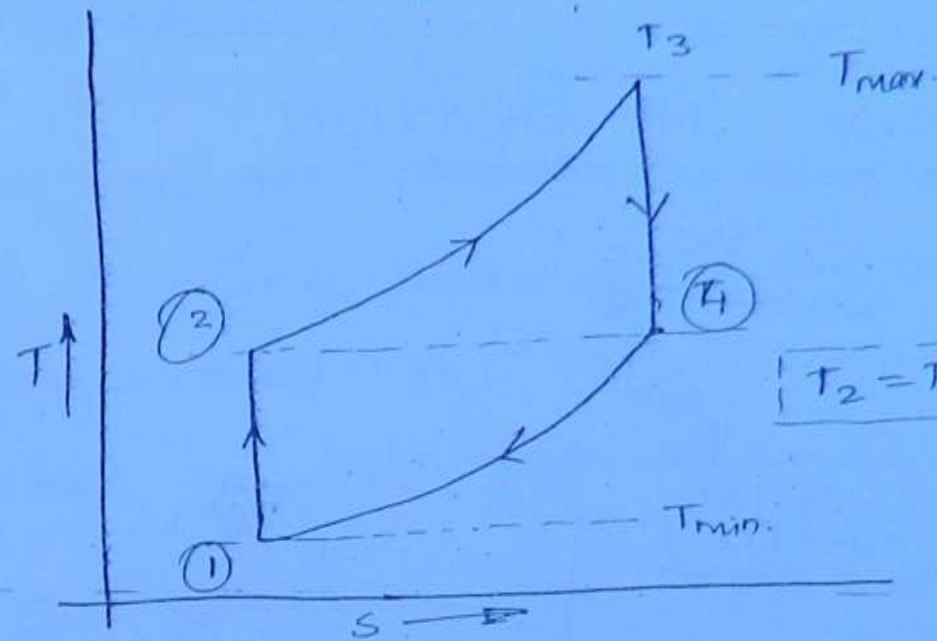
$$Q_s = \frac{10 \times 1000}{0.41} = 24.39\ MW$$

Note:

The effect of regeneration in increasing the efficiency is more pronounced at low air pressure ratio's, since there could greater "ΔT" between turbine exhaust gases and compressed air.

There is one particular pressure ratio called Critical Pressure ratio where, there is no chance for regeneration because at these pressure ratio, compressed air tempreature shall be equal to the turbine exhaust tempreature.

For a given T_{min} and T_{max}. —



$\boxed{T_2 = T_4}$

Here,
$$\frac{T_2}{T_1} = (r_p)_{critical}^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \frac{T_3}{T_4} = (r_p)_{critical}^{\frac{\gamma-1}{\gamma}}$$

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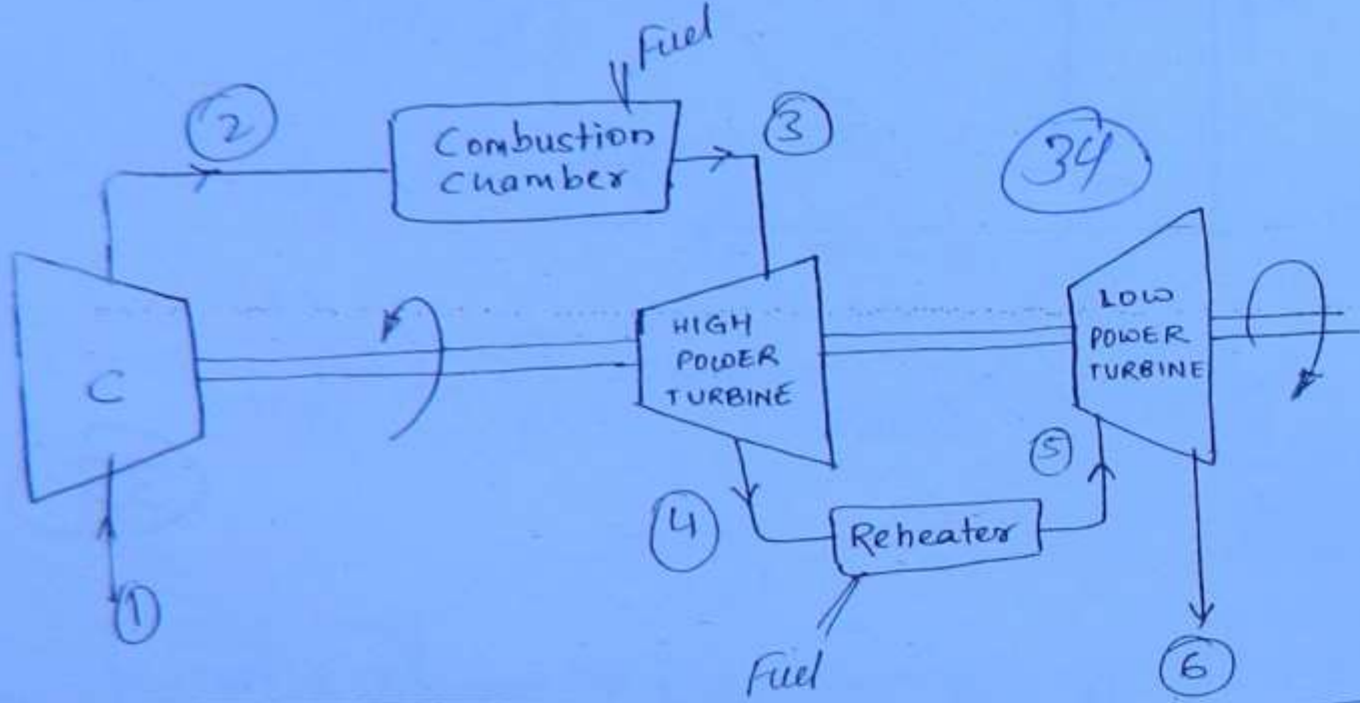
$$\boxed{\frac{T_2}{T_1} = \frac{T_3}{T_4}}$$

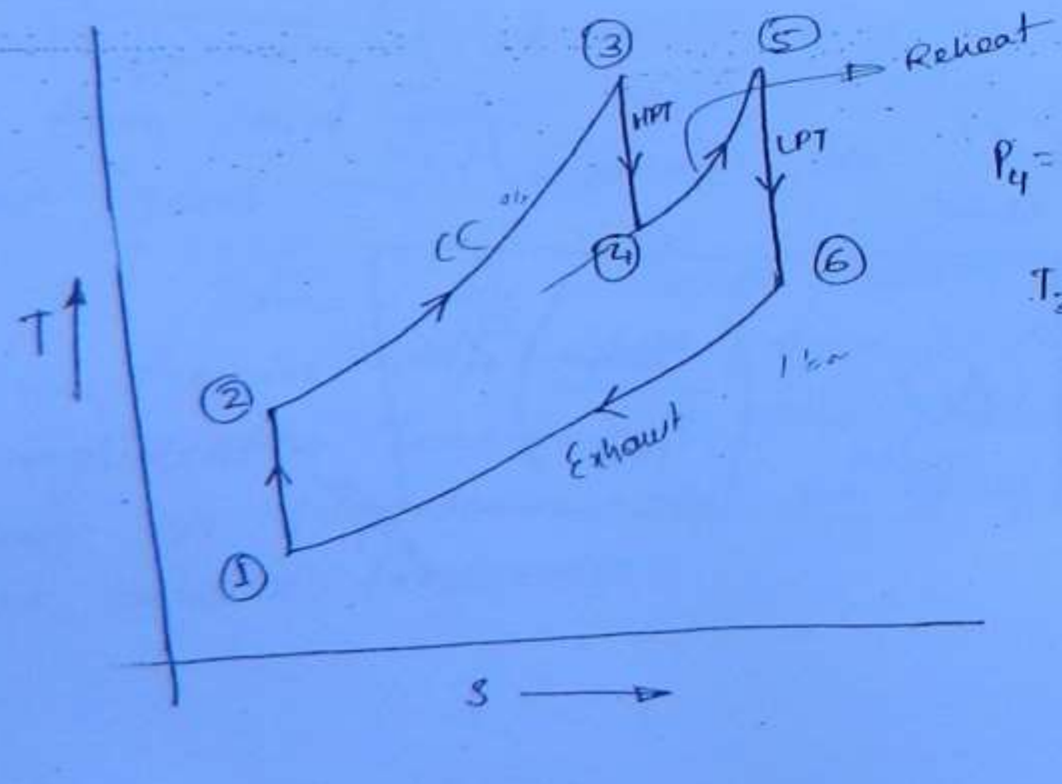
Put $T_2 = T_4 \Rightarrow T_2^2 = T_1 T_3 \Rightarrow \boxed{T_2 = \sqrt{T_1 T_3}}$

$$(r_p)_{Critical}^{\frac{\gamma-1}{\gamma}} = \frac{\sqrt{T_1 . T_3}}{T_1} = \sqrt{\frac{T_3}{T_1}}$$

$$\boxed{(r_p)_{Critical} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}}$$

PREHEATING WITH MULTISTAGE EXPANSION IN BRAYTON CYCLE





$P_4 = P_5$ = Reheater Pressure

$T_3 = T_5$

Gases generated in the combustion chamber, expand in the high pressure turbine from combustion chamber to some intermediate reheat temperature, where it is again reheated ~~to some~~ at constant pressure back to the same initial peak temperature. Then the gases expand in the low pressure turbine from reheat pressure to exhaust pressure.

For getting maximum total net work output from the cycle, when reheating is provided, the reheat pressure must be geometric mean of suction and delivery pressures i.e. pressure ratio across each stage of turbine, must be same

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$$\therefore \frac{P_3}{P_4} = \frac{P_5}{P_6} \quad \text{and} \quad P_4 = P_5$$

$$\therefore P_5 = \boxed{P_4 = \sqrt{P_3 P_6}}$$

Effects of Reheating :—

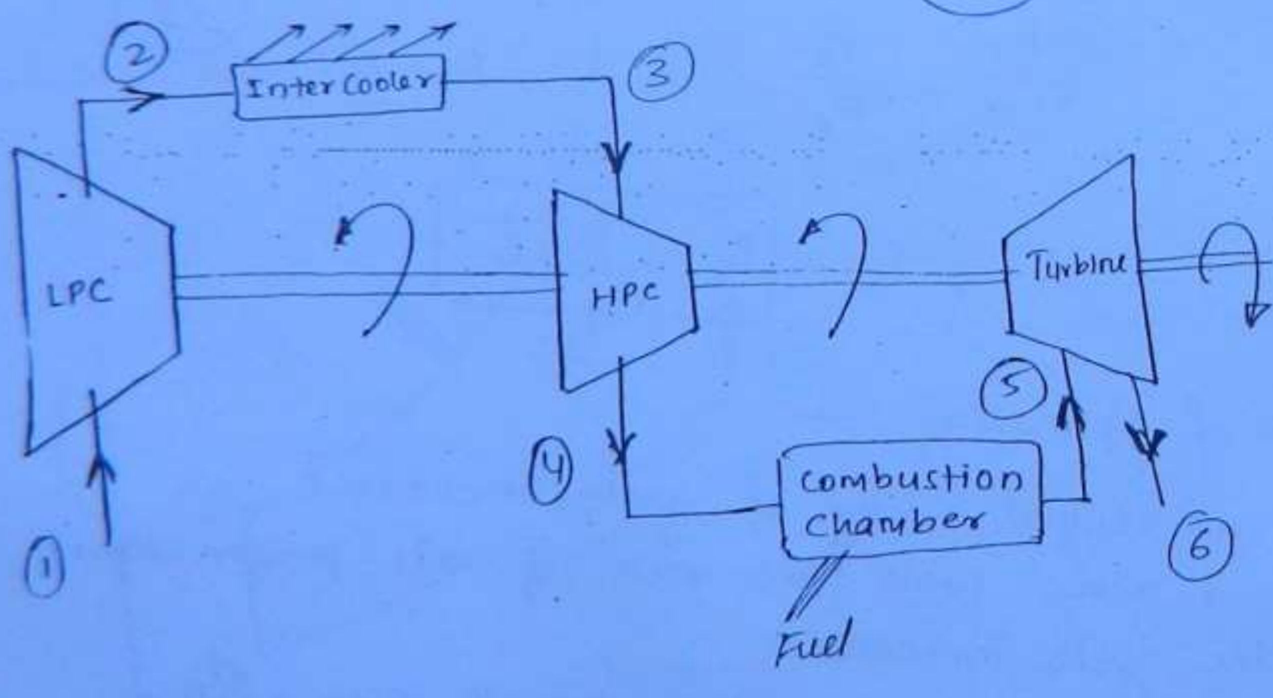
1. Net workoutput per kg of cycle increases.
2. For a given mass flow rate of air, power output of the cycle increases.
3. For a given power output, overall size of the plant becomes smaller. $(\because P = \underset{\text{decrease}}{\dot{m}_a} \times \underset{\text{increase}}{W.D/sec})$

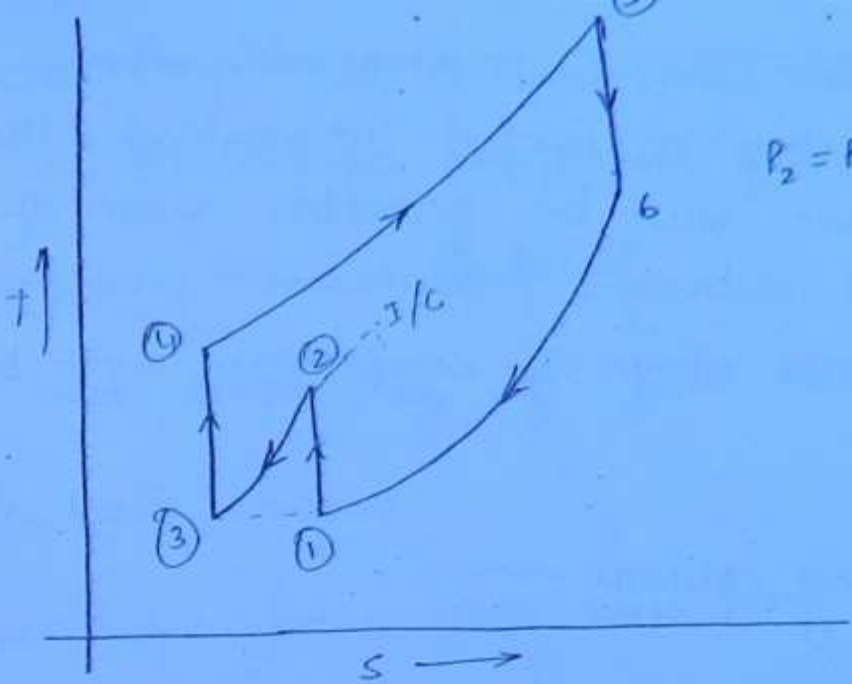
(4) Thermal efficiency marginally increases.

(5) Work Ratio increases.

INTERCOOLING WITH MULTISTAGE COMPRESSION IN BRAYTON CYCLE

For a given mass of air and for a given pressure ratio, work required in the multistage compression with intercooling shall always be lesser than work required for single stage compression.





$P_2 = P_3 =$ Intercooler Pressure

Air entering from the atmosphere into the compressor is compressed in the low pressure compressor from atmospheric pressure to some intermediate pressure called Intercooler pressure, where it is cooled at constant pressure (2) − (3) in the intercooler (37) back to the initial tempreature prior to L.P. compression. Then it is again compressed ~~the~~ in the H.P. compressor, from intercooler pressure to combustion chamber pressure.

Imp More the no. of stages of compression with intercooling, lesser the work input required. If infinite no. of stages of compression are provided, then it ideal most compression, which requires least work input i.e. isothermal compression is obtained.

For total minimum work input to the compressor, with ideal intercooling or perfect intercooling, the intercooler pressure must be geometric mean of the suction and delivery pressures i.e. pressure ratio across each stage of compression must be same.

Perfect Intercooling means —

① $P_2 = P_3$ (No pressure drop in intercooler)

② $T_3 = T_1$ (Temp. of air after intercooling = Temp of air prior to L.P. stage compression)

③ Then for total minimum work input,

$$\frac{P_2}{P_1} = \frac{P_4}{P_3}$$

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$$\Rightarrow P_2 = \boxed{P_3 = P_{I/c} = \sqrt{P_1 P_4}}$$

All 3 pressures (P_1, P_3, P_4) are in geometric progression

$\boxed{HPC} < \boxed{LPC}$ ($\because$ there is const m)

$\therefore \rho Q = C$

$\therefore V_3 < V_1$

$Q_3 < Q_1$ //

EFFECTS ~~OF MULTISTAGE COMPRESSION~~

Effects of Multistage Compressor with Intercooler —

(1) Compressor W_{input} Reduced.

(2) Net work output / kg of cycle increases

(3) Work Ratio increases.

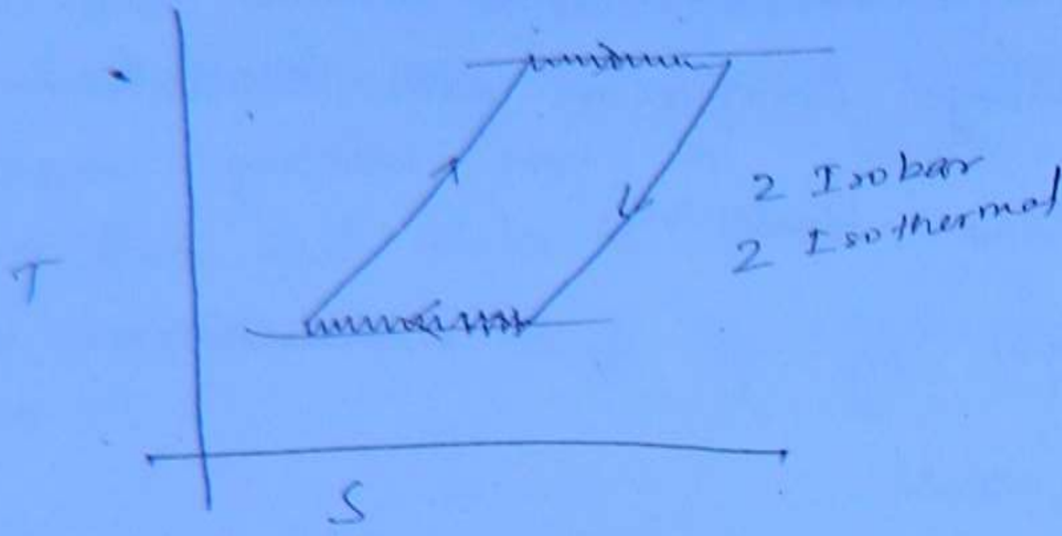
(4) Power Output for a given mass flow rate increases.

(5) η_{th} decreases!

The decrease in η_{th} because of multistage compressi with intercooling, can be compensated by coupling the regenerator into these system, because, there is a greater scope for regeneration in these cycle because ΔT between turbine exhaust and compressed air is higher here.

Q.) A gas turbine cycle with infinite no. of stages of multistage compression with intercooling and infinite no. of stages of multistage expansion with reheating, shall result in what cycle —

P.T.O



" ERICSSON CYCLE "

NOTE

$$\boxed{\eta_{Stirling} = \eta_{Carnot} = \eta_{ericsson}}$$

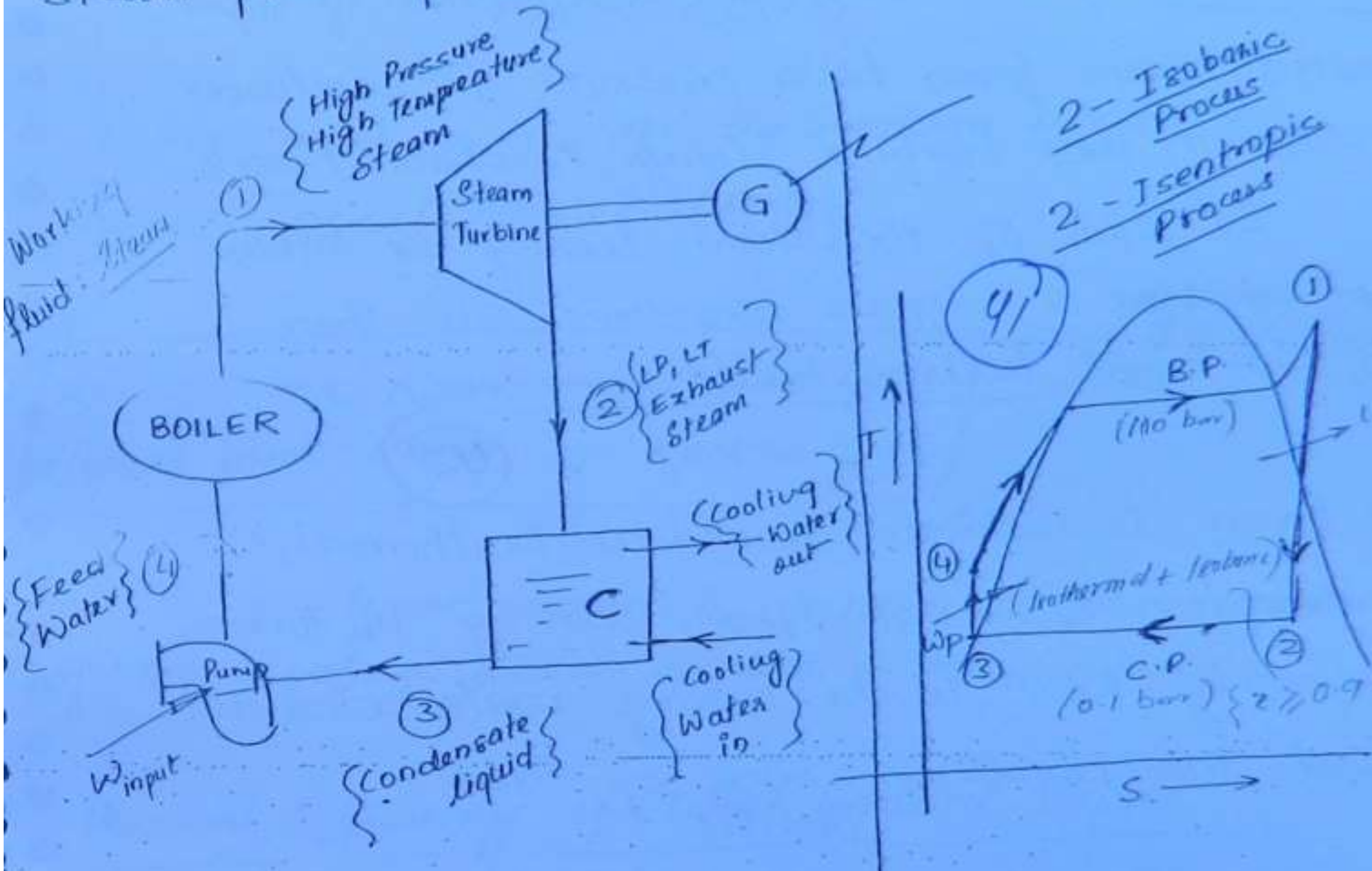
if regeneration is provided, then only

40

70% from ... in India

RANKINE VAPOUR POWER CYCLE

This is the basic thermodynamic cycle for steam power plant <u>or</u> thermal power plant



If $\boxed{x \leq 0.9} \rightarrow$ there will be erosion

$W_{Pump} <<< W_T$ (around 5% of W_T ~~200 times less~~)

<u>4-1</u> indicates isobaric heating of feed water, its change of phase from liquid to vapour and then its superheating in the boiler due to the heat transfer from the hot combustion flue gases to the feed water.

The state of the steam leaving the boiler is high pressure, high temperature, super heated steam.

1-2 Process, is isentropic expansion of super-heated steam from boiler pressure to condensor pressure in the turbine doing mechanical work.

The state of the exhaust steam leaving the turbine should not have a dryness fraction less than 90%, to avoid turbine blade erosion

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2-3 Process, is isobaric as well as isothermal, condensation of ~~s~~ wet steam leaving the turbine, by rejecting heat to the cooling water, circulated around the condensor bank of tubes (bundle)

The state of the water leaving the condensor should be saturated liquid corresponding to condensor pressure i.e. state (3).

Any undercooling ~~or~~ subcooling ~~in the~~ is underirable.

3-4 Process is the isentropic pumping of liquid condensate leaving the condensor, from condensor pressure to boiler pressure. The work required by the pump is supplied by turbine,

$$\boxed{W_P <<<< W_T}$$

work ratio of the rankine cycle is almost equal to 1 (unity)

Note:

$$\boxed{\begin{array}{l} \text{Lowest Work Ratio} \longrightarrow \text{Carnot Cycle} \\ \text{Highest Work Ratio} \longrightarrow \text{Rankine Cycle} \end{array}}$$

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The condensor pressure is decided by the tempera of the cooling water supplied to the condensor.

Thermal Efficiency of Ideal Rankine Vapour Power Cycle

$$\eta_{th} = \frac{\text{Net work done /cycle}}{Q_s}$$

$$= \frac{W_T - W_S}{(Q_s)_{\text{in boiler}}}$$

$\because Tds = dh - vdp$

$\because s = c \Rightarrow ds = 0$

$\therefore dh = v\,dP \Rightarrow \int dh = \int v\,dP$

$= W.D.$ by Turbine

Note:

$\int v\,dP$ indicates reversible work done on open system

$$\therefore W_T = (h_1 - h_2)\ kJ/kg$$

$W_{\text{input to pump}} = \int v\,dP$, But $v = C$ since liquid is Incompressible

(49)

$= v_{water}$ (Boiler pressure − Condenser pressure)

v_f

$= 0.001016\,(B.P - C.P) \cong 2$ to $10\ kJ/kg$

$\begin{pmatrix} B.P = 100\ bar \\ C.P = 0.1\ bar \end{pmatrix}$

$W_{pump} = (\Delta h)$ of water in pump

$= h_4 - h_3$

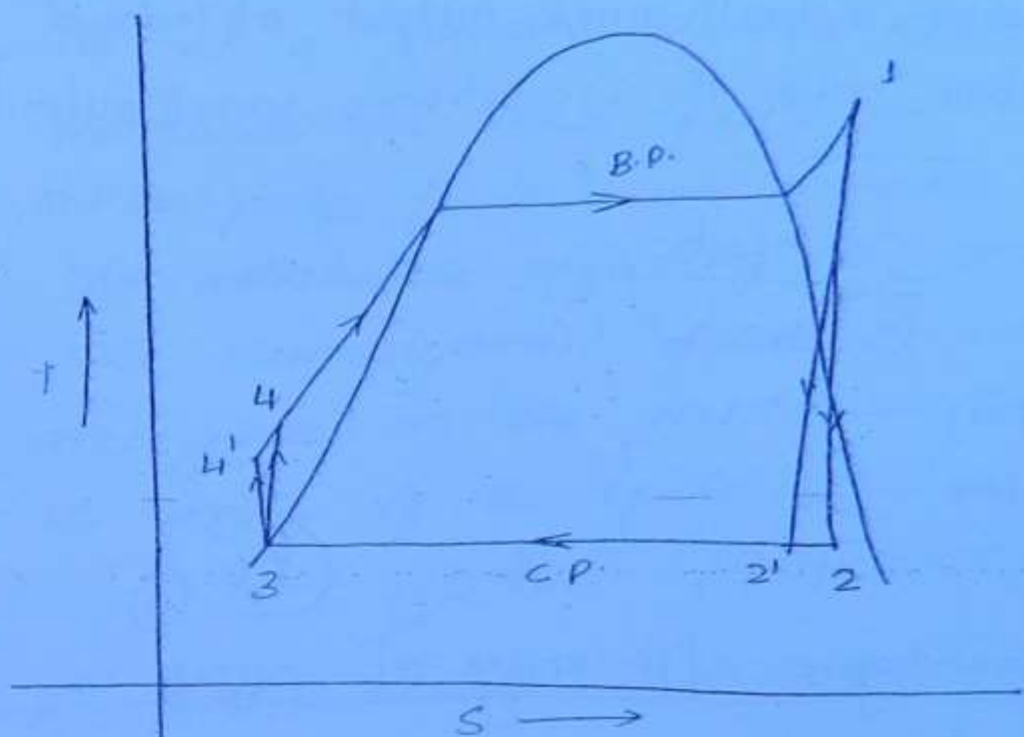
Since pump work is very small i.e. (2 to 6 kJ/kg)

$\Rightarrow h_4 \simeq h_3$

Neglecting Pump Work,

$$\boxed{\underset{\substack{\text{Ideal} \\ \text{Rankine Cycle}}}{\eta_{(th)}} = \frac{W_T}{Q_s} = \frac{h_1 - h_2}{h_1 - h_4} = \frac{h_1 - h_2}{h_1 - h_3}}$$

RANKINE CYCLES WITH MACHINE EFFICIENCY



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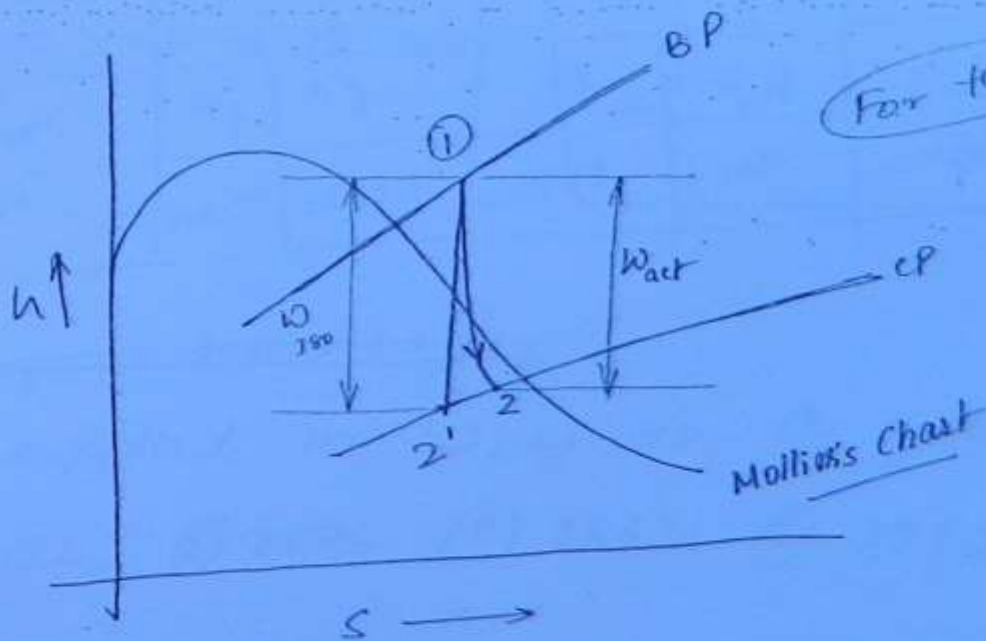
1-2' is isentropic expansion of steam in turbine
1-2 is actual expansion of steam with friction

For turbine



Iso entropic efficiency of turbine is defined as the ratio between actual work output obtained from the turbine and the isentropic work output

$$\eta_{(Iso)\,Turbine} = \frac{h_1 - h_2}{h_1 - h_2'} \cong 75 \text{ to } 80\%$$

(46)

Similarly isentropic efficiency of pump is defined as ratio between isentropic work input to the pump and actual work input to the pump

$$\eta_{(Iso)\,Pump} = \frac{h_4' - h_3}{h_4 - h_3} \cong 85 \text{ to } 90\%$$

In a steam power plant operating on the rankine cycle, steam enters the turbine at 4 MPa, 350°C and exits at a pressure of 15 KPa, then it enters the condenser and exits as saturated water, Next, ~~the~~ a pump feeds back water to the boiler. The adiabatic efficiency of turbine is 90%. The thermodynamic states of water and steam are given in the table — The net work output in KJ/kg of the cycle is ——

(a) [illegible] (b) 775 (c) 860 ✓ (d) 957

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Sol.
~~$P_1 = 4\,MPa = 4000\,Pa$ $T_1 = 623\,K$~~

~~$P_2 = 15\,KPa$~~

State	h (kJ/kg)		s (kJ/kg °C)		v m³/kg	
Steam : 4MPa 350°C	3092.5		6.5821		0.06645	
	h_f	h_g	s_f	s_g	v_f	v_g
Water at 15 KPa	225.94	2599.1	0.7549	8.0085	0.001014	10.02

~~$h_1 = 3092.5\ kJ/kg$~~

Heat supplied in kJ/kg to the cycle is ——

(a) 2372 (b) 2576 (c) 2863 (d) 3092

$$\eta_{\text{isen turbine}} = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$0.90 = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$6.5821 = \cancel{0.65}\ 0.7549 + x(8.0085 - 0.7549)$$

$$x = 0.803$$

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$$\therefore h_2' = 2132.42$$

$$\therefore \cancel{h_2 = 2228.43 \text{ kJ/kg}}$$

$$\therefore W_{(\text{isen}) \text{ turbine}} = h_1 - h_2'$$

$$\therefore \text{Actual W.D}_{\text{turbine}} = \eta \times W_{(\text{isen}) \text{ turbine}}$$

$$= 0.9 \times 960.08$$

$$= 864.072$$

$$\text{Net W.D/cycle} = 864.072 - \int v\,dP$$

$$= 864.072 - 0.001014(40 - 0.15) \times 100$$

$$= 864 - 4 = 860 \text{ kJ/kg}$$

$$Q_s = \frac{w_T - w_s}{\eta}$$

$$= \frac{h_4' - h_3}{h_4 - h_3}$$

$$h_3 = 225.94$$

$$\eta_{(th)\,cycle} = \frac{h_1 - h_2}{h_1 - h_3}$$

$$\therefore \eta_{(th)\,cycle}$$

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$\therefore$ Here, Pump work is $\int V\,dP = 0.001014(40 - 0.15)$
$= 4 \text{ kJ/kg}$

$$(Q_s)_{\text{in boiler}} = h_1 - h_4$$

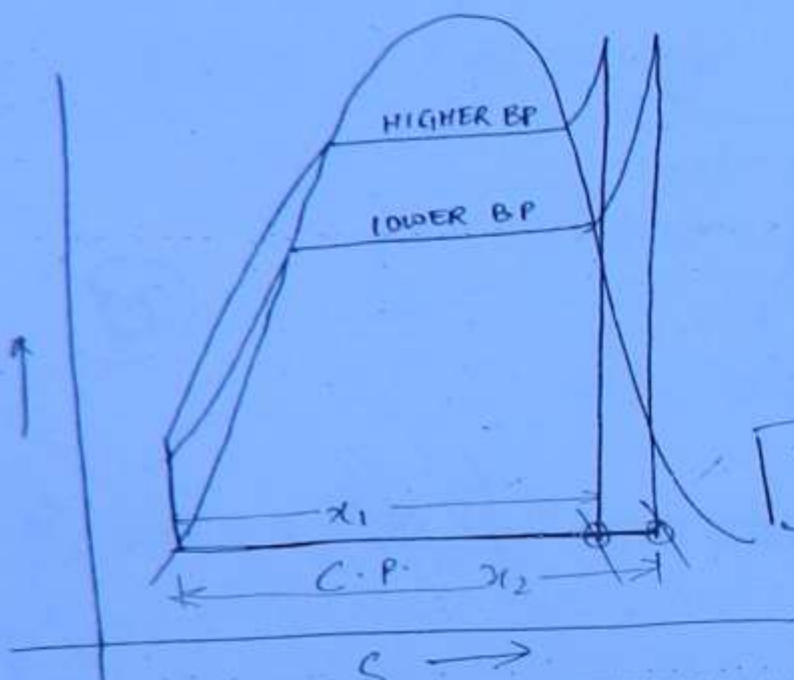
$$= h_1 - h_3$$

$$= 3092.5 - 225.94$$

$$= 2866.56 \text{ kJ/kg}$$

$$\eta_{th} = \frac{h_1 - h_2}{h_1 - h_3} = 0.301 = 30.1\%$$

Note: Reheating in the steam power plant is basically provided for the sake of increasing dryness fraction, ~~during~~ of steam during lateral stages of expansion particularly when higher boiler pressure are used.



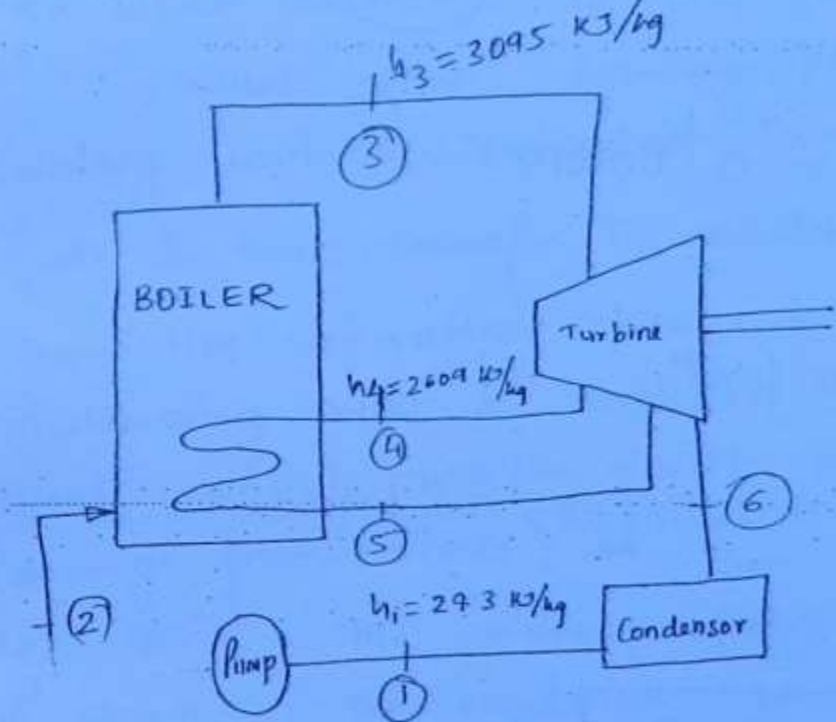
SO

$\boxed{x_1 < x_2}$

When higher boiler pressure are employed, the dryness fraction values of steam are low during later stages of expansion, causing blade erosion of turbine.

Reheating takes care of these matter (i.e. Reheating is done to avoid it)

Q.) Consider a steam power plant using a reheat cycle as shown. Steam ~~reach~~ leaves the boiler and enters the turbine at 4 MPa, 350°C ($h_3 = 3095$ kJ/kg). After expansion in the turbine, to 400 kPa ($h_4 = 2609$ kJ/kg). The steam is reheated to 350°C ($h_5 = 3170$ kJ/kg) and then expanded in a low pressure turbine to 10 kPa ($h_6 = 2165$ kJ/kg). The specific volume of the liquid handled by the pump can be assumed



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The thermal efficiency of the plant neglecting pump work is ——

(a) 15.8 % (b) 41.1 % (c) 48.5 % (d) 58.6 %

The enthalpy at the pump discharge (h_2) is ——

(a) 0.33 kJ/kg (b) 2.33 (c) 33.3 (d) 4.0

sol^n. $\eta_{th} = \dfrac{HPT + LPT - \text{Pump Work} - 0}{Q_s}$

$= \dfrac{(h_3 - h_4) + (h_5 - h_6)}{(h_3 - h_1) + (h_5 - h_4)}$

$= 41.1\ \%$

Pump work $= \int v\, dP$ 52

v_{water} is constt. $= 0.001014$

$h_2 - h_1 = v \int dP$

$h_2 - h_1 = 0.001014\,[40 - 0.1] \times 100$

$h_2 - 29.3 = 3.99$

$\therefore \boxed{h_2 = 33.3}$ KJ/kg

Q) ~~Assertion:~~ Condensor is an essential equipment in a steam power plant —

Reason: For same mass flow rate and for same pressure rise, a water pump requires substentially less power then a steam compressor

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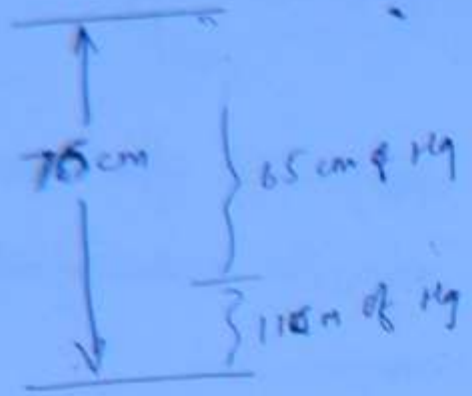
Note: Condensor is a shell and tube heat exchanger in which there is a transfer of heat from the exhaust steam leaving the turbine to the cooling water circulated in the condensor tubes (mode of heat transfer is conduction & convection)

Since the saturation tempreature of steam while condensing must be atleast 20°C greater then usuall cooling water inlet tempreature of 25°C (coming from river) & $T_{sat.}$ of steam is choosen as 45°C. The corresponding satoration pressure of steam in the condensor is 14 KPa or 0.14 Bar.

So, to maintain these low pressure of condensor, sufficient amount of cooling water supply must be ensured, also heat transfer must be appropriate

If the above conditions are not satisfied, condensor vacuum will ~~decrease~~ decrease, which increases the absolute pressure of steam in the condensor, there by decreasing the overall efficiency.



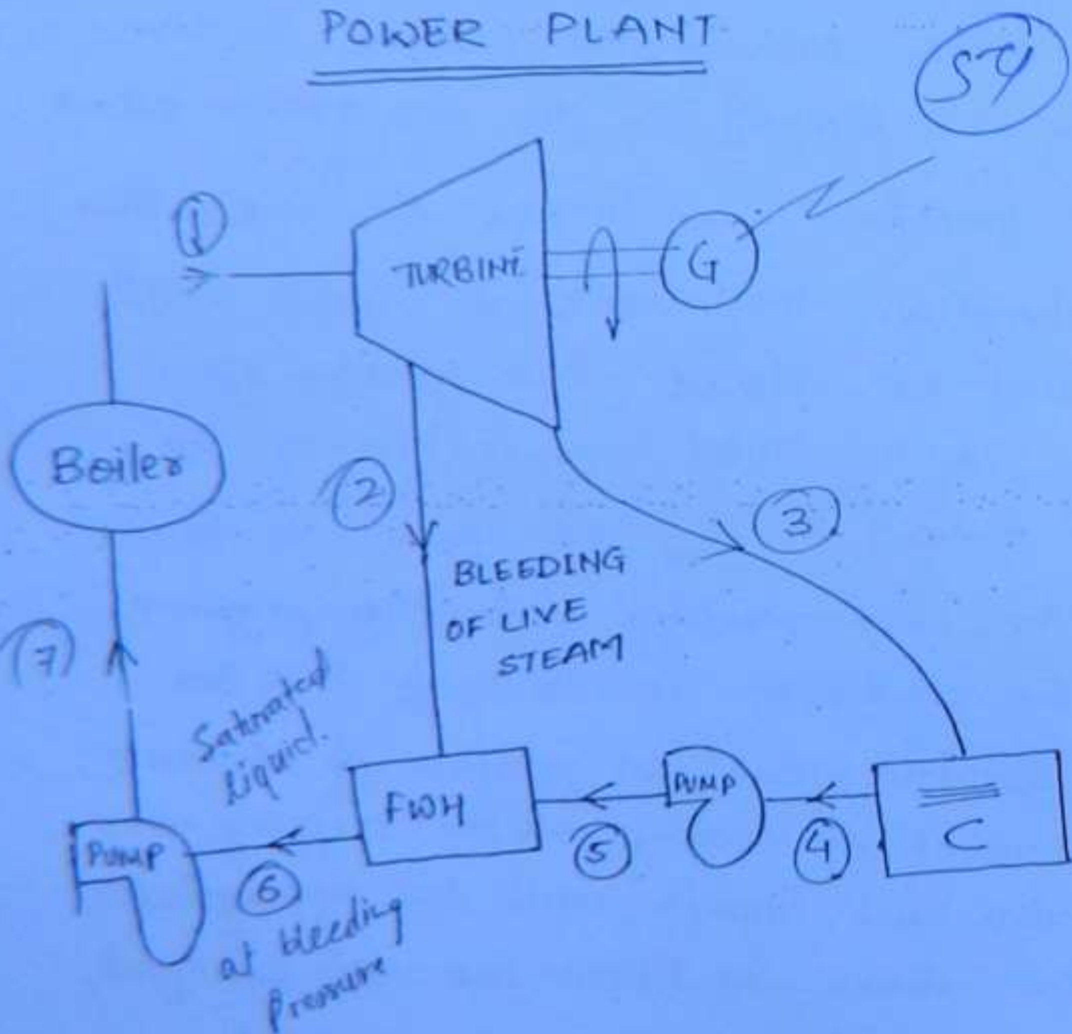
65 cm of Hg, Vaccum is maintained, in condensor,

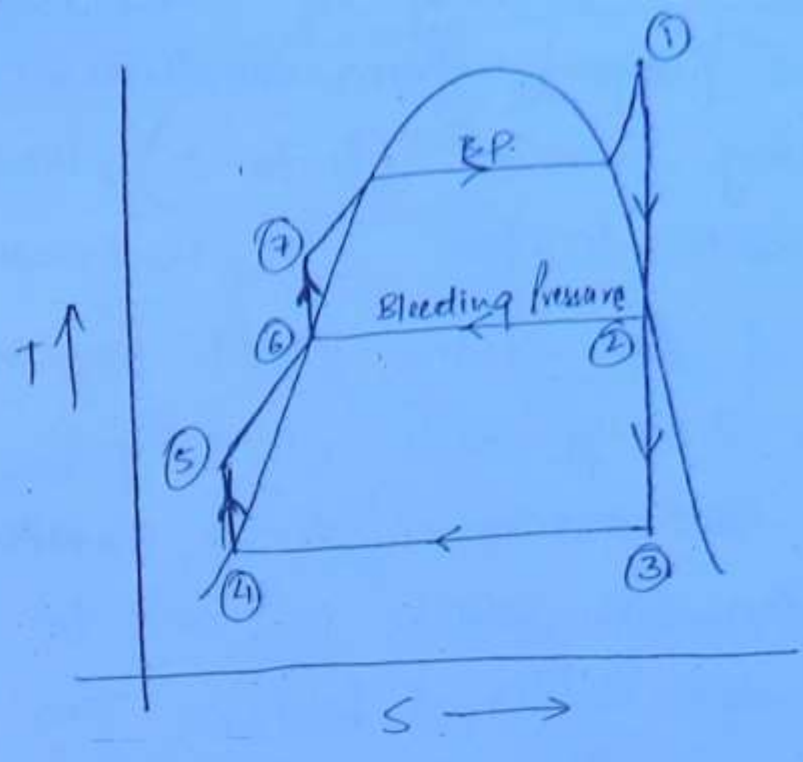
76 cm of Hg — 101.3 kPa

11 cm of Hg — $\frac{11}{76} \times 101.3$ kPa

=

REGENERATION OF RANKINE CYCLE STEAM POWER PLANT





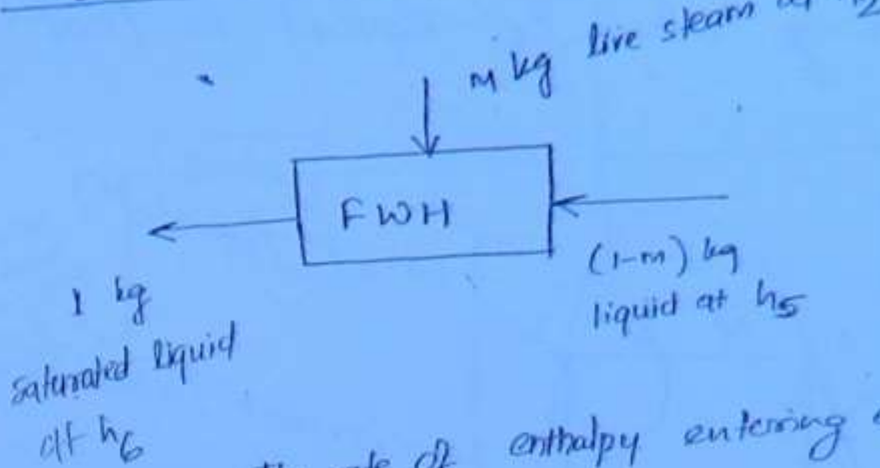
55

regeneration in rankine cycle shall result in increase in mean tempreature of heat addition which inturn results in increase of thermal efficieny of cycle

Let 1 kg of steam enters the turbine from the boiler which will expand from boiler pressure to bleeding pressure, doing work, then m kg of live steam is bleed from the turbine and remaining (1-m) kg continues to expand from bleeding pressure to condensor pressure doing work on the rotor. These (1-m) k of steam leaving the turbine condenses in the condensor at constant pressure becoming saturated liquid ~~and~~ at condensor pressure.

These liquid is pumped from condensor pressure to bleeding pressure (4 to 5), then enters the feed water heater. In feed water heater, 'm' kg of live steam bleed from the turbine is mixed directly with (1-m) kg of liquid water. The live steam condenses, liberating latent heat of condensation which is used to increase the tempreature of liquid heater. The resulting mixture being saturated liquid water at bleeding pressure (state 6). These liquid is pumped from bleeding pressure to boiler pressure and then supplied to the boiler.

Energy Balance for



The rate of enthalpy entering the FWH = Rate of enthalpy leaving FWH

$$(1-m)h_5 + (m \times h_2) = 1 \times h_6$$

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$$h_5 - mh_5 + mh_2 = h_6 \quad (h_6 = h_f \text{ at bleeding pressure})$$

$$\boxed{m = \left(\frac{h_6 - h_5}{h_2 - h_5}\right)}$$

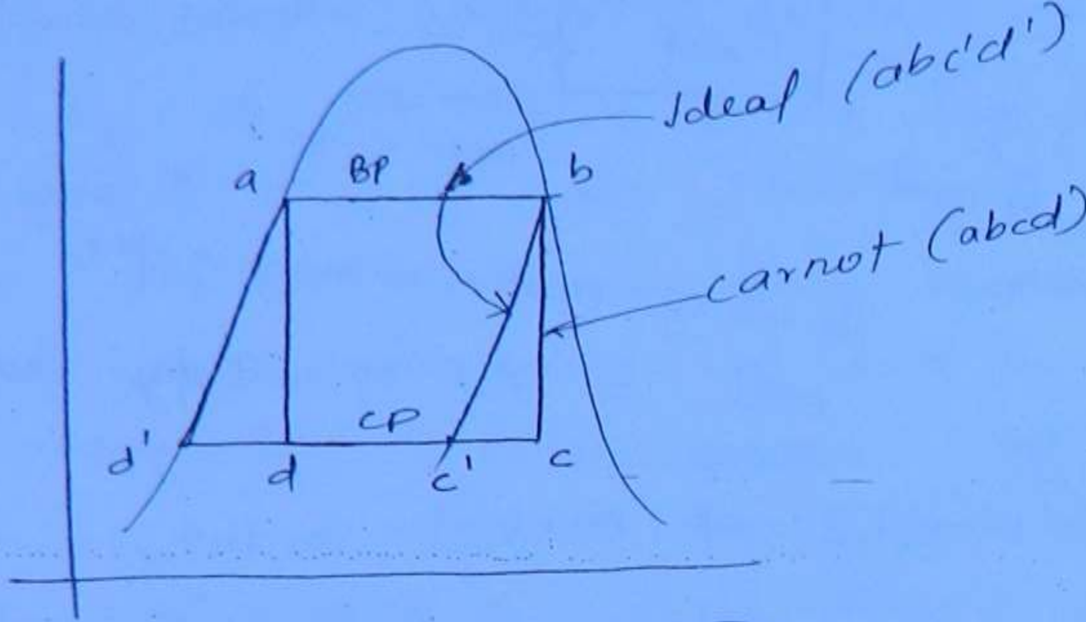
kg of live steam bleeds per 1 kg of steam entering turbine

Efficiency

$$\boxed{\eta_{th} = \frac{(h_1 - h_2) + (1-m)(h_2 - h_3)}{(h_1 - h_7 \approx h_6)}}$$

Higher the no. of bleeding points with feed water heaters, higher is the increasing in thermal efficiency. If infinite bleeding points are provided with infinite feed water heaters, then the cycle obtained is called <u>Ideal Regenerative Rankine Cycle</u>

whose thermal efficiency shall be equal to that of carnot cycle.



~~abcd~~

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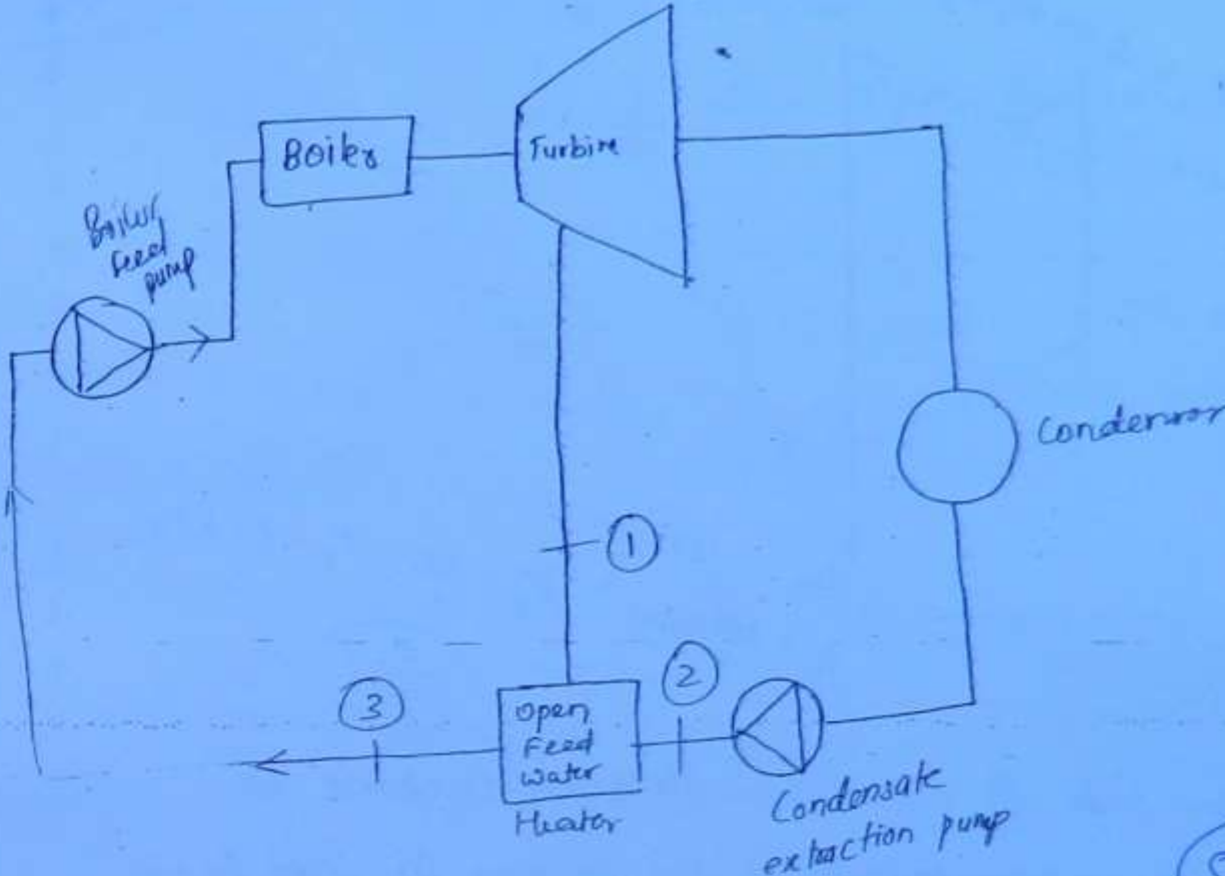
ate 08

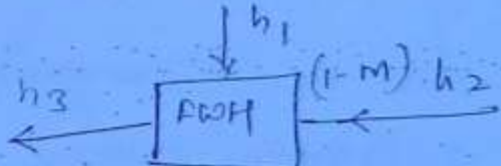
Q.) A thermal power plant operates on a regenerative cycle, with a single open feed water heater as shown in the figure. For the state points shown, the specific enthalpies are, $h_1 = 2800$ KJ/kg and $h_2 = 200$ KJ/kg. The bleed to the feed water heater is 20% of the boiler steam generation rate. The specific enthalpy at state 3 is

(a) 720 (b) 2280 (c) 1500 (d) 3000



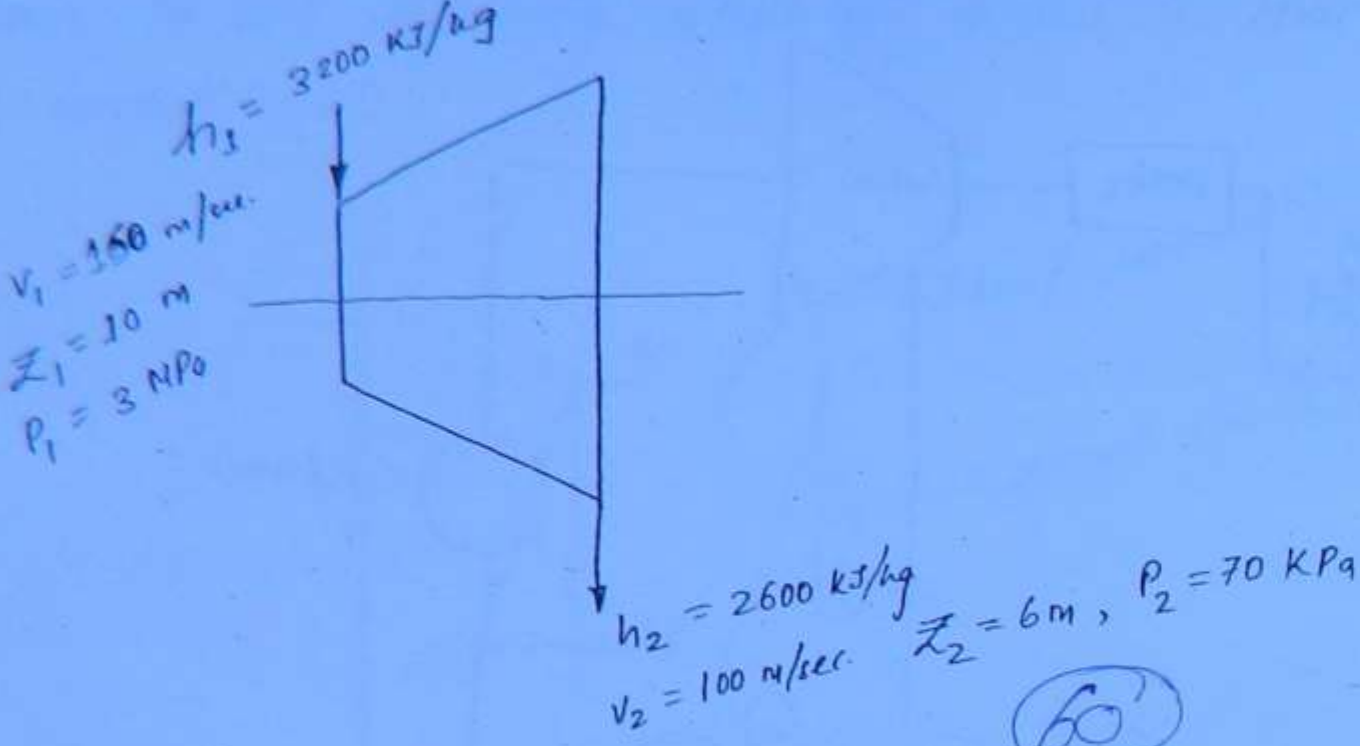
$$(1-0.2)h_2 + 0.2h_1 = 1\times h_3$$

$$\therefore h_3 = 720 \text{ kJ/kg}$$



Gate 09

The inlet and outlet conditions of steam for an adiabatic steam turbine are as indicated in the figure.



60

If mass flow rate of the steam through turbine is 20 kg/sec. The power output of the turbine in MW is —

(a) 12.157 (b) 12.941 (c) 165.001 (d) 168.785

Assume the above turbine to be part of a simple rankine cycle. The density of water at inlet to the pump is 1000 kg/m³. Ignoring Kinetic & Potential energy effects, the specific work in kJ/kg supplied to the pump is —

(a) 0.293 (b) 0.351 (c) 2.93 (d) 3.51

Solⁿ $P = \dot{m} \times WD/sec.$

Turbine is Steady flow Open System.

<u>SFEE:</u>

$$\cancel{q} - \omega = \Delta h + \Delta KE + \Delta PE$$

$=0$ ($\because$ adiabatic)

$$= (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2000}\right) + g\left(\frac{Z_2 - Z_1}{1000}\right)$$

$$= 607 \text{ kJ/kg}$$

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$$P = \dot{m} \times WD/sec$$

$$= 20 \times 607 = 12156.7 \text{ KW}$$

$$= 12.157 \text{ MW}$$

$$\text{Pump Work} = \int_{C.P}^{B.P} V\,dP = \frac{1}{1000}\left[B.P - C.P\right)_{\text{in kPa}}$$

$$= \frac{1}{1000}(3000 - 70)$$

$$= \frac{2930}{1000} = 2.93 \text{ kJ/kg}$$

Gate 07

Q) Which combination of following statements is correct: —

P) The incorporation of reheater in a steam power plant always increases the thermal efficiency of the plant.

Q) Always increases the dryness fraction of steam at condenser inlet

R.) Always increases the mean tempreature of heat addition

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S) Always increases the specific work output.

(a) P & S (b) Q & S (c) P, R & S (d) P, Q, R, S

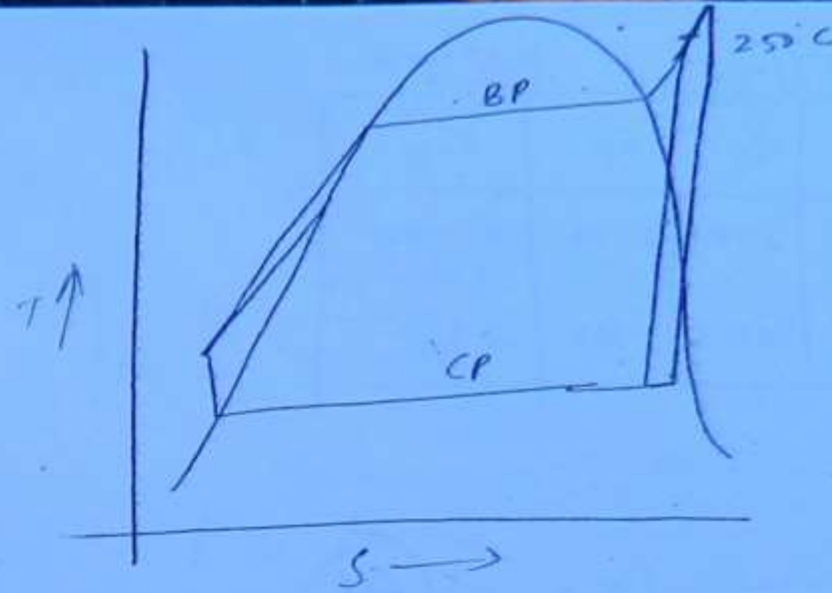
Q) In rankine cycle, regeneration results in higher thermal efficiency because —

(a) Pressure inside the boiler increases

(b) Heat is added before steam enters the low pressure turbine.

(c) The average tempreature of heat addition in the boiler increases.

(d) The work delivered by the turbine increase



Gate 2002

Q) The efficiency of supearheated rankine cycle is higher than that of simple rankine cycle because—

(a) Enthalpy of main steam is higher for superheat cycle

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(b) Mean temp of ~~reheat~~ heat addition ~~of~~ is higher for superheat cycle.

(c) Tempreature of steam in condensor is high

(d) Quality of steam in condensor is low

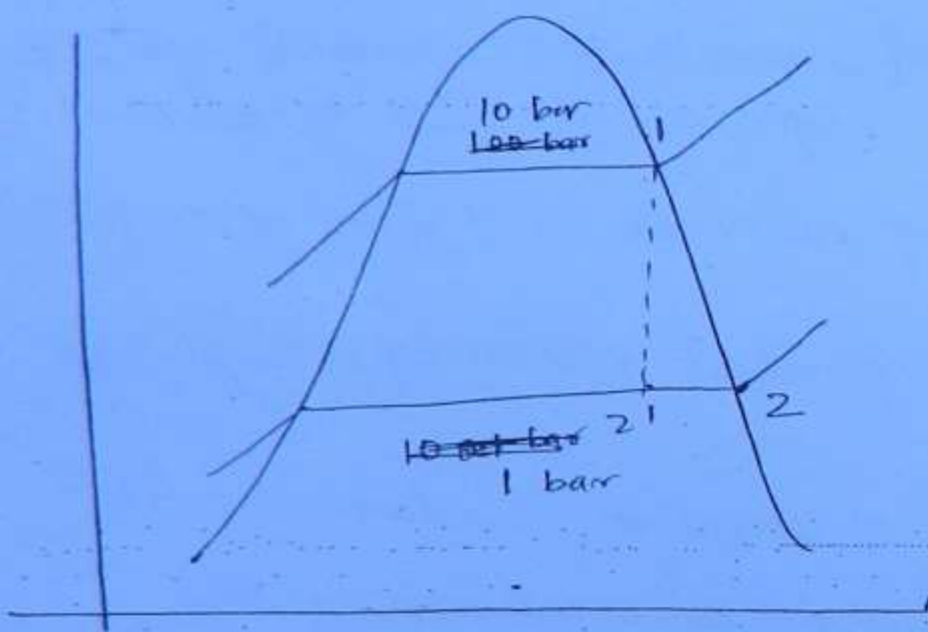
Gate 1999

Q.) An adiabatic steam turbine receives dry saturated steam at $1 MN/m^2$ and discharges it at $0.1 MN/m^2$. The steam flow rate is 3 kg/sec. and the moistur at exit is negligible. Calculate the power output and isentropic efficiency of turbine. The properti of steam are given as —

P MN/m²	$T_{sat.}$	h_f (KJ/kg)	h_g (KJ/kg)	s_f (KJ/kg K)	s_g (KJ/kg K)
1.0	179.9	762.8	2778.1	2.139	6.586
0.1	99.6	417.5	2675.5	1.303	7.359

Soln

$P.v = RT$



$$1\ MN/m^2 = 10\ bar$$

$$6.586 = 1.303 + x(7.359 - 1.303)$$

$$x = 0.872 = 87.2\ \%$$

$$h_2' = 417.5 + 0.872(2675.5 - 417.5)$$

$$= 2386.47\ KJ/kg$$

$h_1 = 2778.1 \text{ kJ/kg}$

$$\eta_{th} = \frac{h_1 - h_2}{h_1 - h_2'} = 26.2 \%$$

$W_{act} = h_1 - h_2$

$$P = \dot{m} W_{ac}$$

$$= \dot{m}\left[h_1 - h_2\right] = \cancel{1174.89 \text{ kJ/s}} \text{ kW}$$

$$= 307.8 \text{ kW} \qquad \cancel{= 1.175 \text{ MW}}$$

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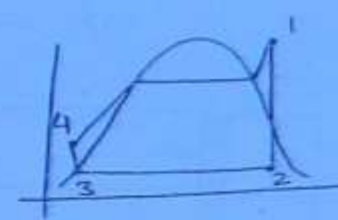
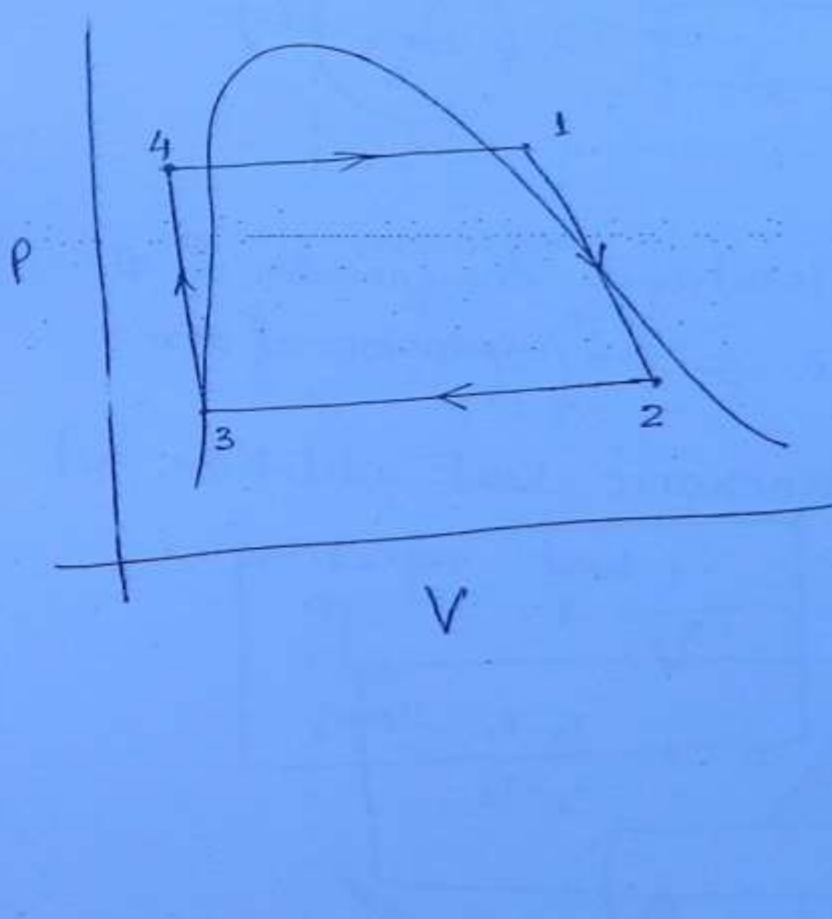
Draw Rankine cycle on PV - plane

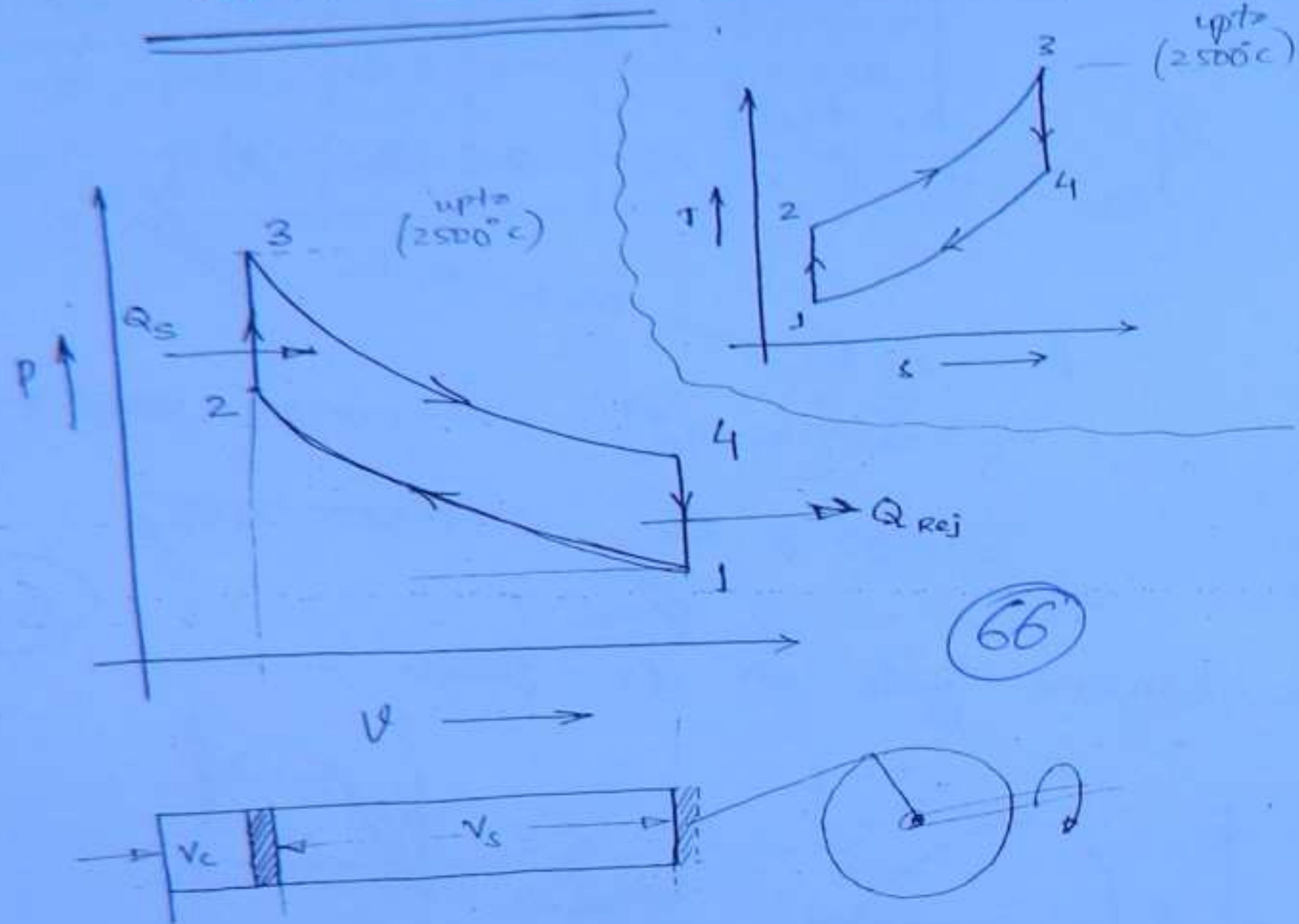


$\rho = 1000 \text{ kg/m}^3$



P.T.O

Otto Cycle (Air Standard Cycle)



1-2 & 3-4 → are isentropic compression of air and expansion of air

2-3 & 4-1 → are isochoric heat addition and heat rejection

$$\boxed{\eta_{th} = 1 - \frac{Q_R}{Q_s} = 1 - \frac{T_4 - T_1}{T_3 - T_2}}$$

$$\therefore \eta_{th\,(otto)} = 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}$$

$$\therefore \boxed{\eta_{th\,(otto)} = 1 - \frac{1}{\left(\frac{T_2}{T_1}\right)}}$$

Defining compression ratio of the cycle as the ratio betwe volume of the air at the begining of isentropic compressio and the volume of air at the end of the isentropic compression i.e.,

$$\boxed{r_0 = C.R. = \frac{V_1}{V_2}}$$

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For 1-2, it is isentropic,

$$\boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}} = r_0^{\gamma-1}$$

$$\boxed{\eta_{th\,(otto)} = 1 - \frac{1}{(r_0)^{\gamma-1}}}$$

$$r_0 = \frac{V_c + V_s}{V_c}$$

$$= 1 + \left(\frac{V_s}{V_c}\right)$$

$$\therefore \boxed{r_0 = 1 + \frac{\frac{\pi}{4} D^2 L}{V_c}}$$

Gate 2008

Which one of the following is NOT a necessary assumption for the air standard Otto cycle —

(a) All processes are both internally as well as externally reversible.

(b) Intake and exhaust processes are constt volume heat rejection processes.

(c) The combustion process is a constant volume heat addition process.

(d) The working fluid is an ideal gas with constant specific heat.

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2.) An engine working on Air Standard Otto Cycle has a cylinder diameter of 10 cm and stroke length of 15 cm. The ratio of specific heat for air 1.4. If clearance volume is 196.3 cc. and heat supplied per kg of air/cycle is 1800 KJ/kg, The work output per cycle/kg of air is —

(a) 879.1 ~~KJ~~ (b) 890.2 (c) 895.3 (d) 973.5

Soln

$$r = ~~8~~\ 1 + \frac{\frac{\pi}{4} D^2 L}{V_c} = 7$$

$$\eta_{th} = 1 - \frac{1}{r^{\gamma - 1}} = 0.5408$$

$$\cancel{\eta = 1 - \frac{Q_R}{Q_s}}$$

$$\eta_{th} = \frac{WD/kg}{Q_s/kg}$$

$$\therefore 0.54 \times 1800 = WD/kg$$

$$\therefore WD/kg = 973.6 \text{ kJ}$$

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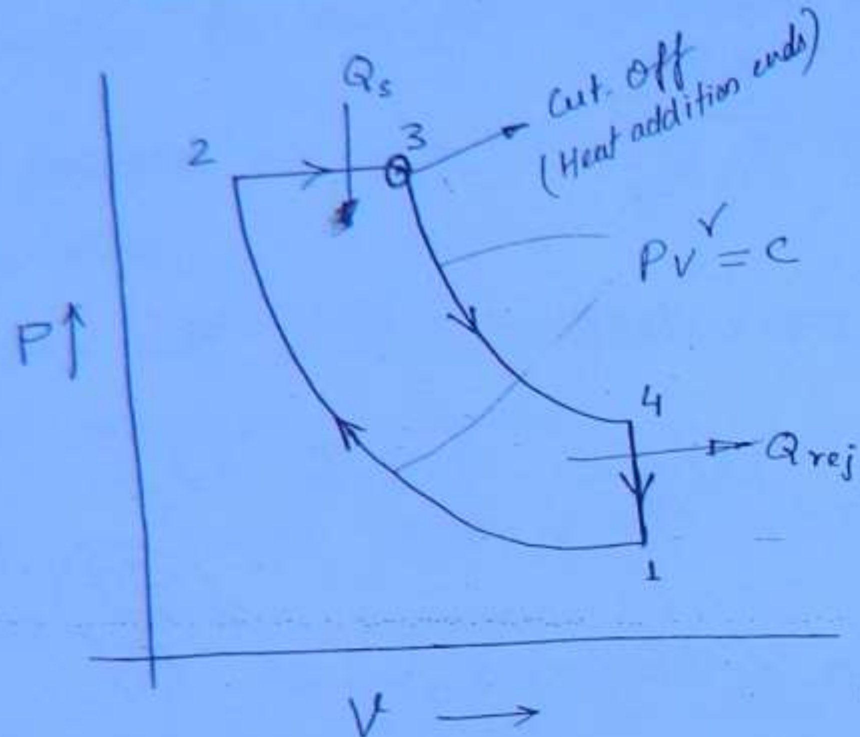
Note:

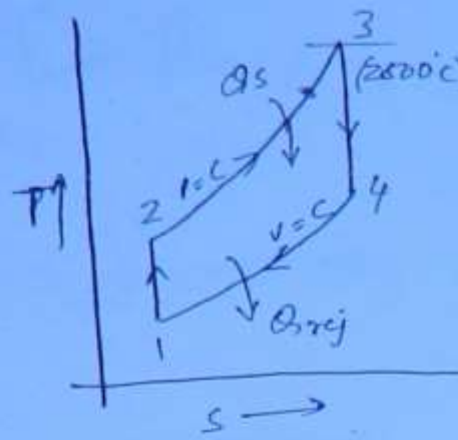
As the compression ratio (r) increases, Air Standard Thermal efficiency (η_{th}) of otto cycle also increases.

But in Carburetted S.I. engine the max. compression ratio is limited to 10 or 11 only

{Detonation or Knocking} tendency leds to increases with r increases

DIESEL CYCLE





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Compression Ratio $= r_c = \frac{V_1}{V_2}$

Cut off Ratio $= r_c = \dfrac{\text{Volume at which heat addition ceases}}{\text{Volume at which heat addition begins}}$

$$\boxed{r_c = \frac{V_3}{V_2}}$$

$$\boxed{\eta_{(th)\,diesel} = 1 - \frac{Q_R}{Q_s} = 1 - \frac{T_4 - T_1}{T_3 - T_2}}$$

$$\boxed{\eta_{th\,(diesel)} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right]}$$

As $r_c \uparrow \Rightarrow \eta_{th \, (diesel)} \uparrow$ (~~S.I.~~)

<u>In C.I. Engines</u> $r_c = 16 \text{ to } 22$

As $r_c \uparrow \Rightarrow \eta_{th} \downarrow$

When same compression ratio and same heat is supplied for both Otto & Diesel cycles, Thermal Efficiency of Otto Cycle is greater then that of diesel cycle.

$$\boxed{\eta_{th\,(otto)} > \eta_{th\,(diesel)}}$$

<u>For same "r_c"</u>

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$\because$ the value $\left[\dfrac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right]$ for diesel is greater then 1.

The more practical comparision is, for same peak temp and pressure in both Otto & Diesel cycles (ie for diff. compression ratios) and for the same heat supplied thermal efficiency of the diesel cycle is greater then that of Otto Cycle.

Imp <u>Note:</u> Practically C.I. engine are thermally more efficient then S.I. engines because they employee for greater compression ratio then S.I. Engines.

aught?

2S-CI , 4S-SI, 4S-CI, 2S-SI

Arrange in descending order of efficiency (thermal)

$$\boxed{4S\text{-}CI > 4S\text{-}SI > 2S\text{-}CI > 2S\text{-}SI}$$

due to compression ratio

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POWER - PLANT

73

[redacted]

* GAS TURBINE *

Advantage of Gas Turbine over I.C. engines ⇒

1. High speed can be obtained
2. Easy Balancing.
3. They are compact. $\left(\frac{W}{P}\right)$ [weight to power Ratio is less]
4. simple mechanism.

DISadvantage ⇒

1) As compressor is used in gas turbine system the work of compression is large and hence Network is less this Result in lower efficiency.
2) As gas turbine system are subjected to higher temp. continuously high heat resistant costly material is Required.
3) Additional Reduction gear box is required as the speed are very high

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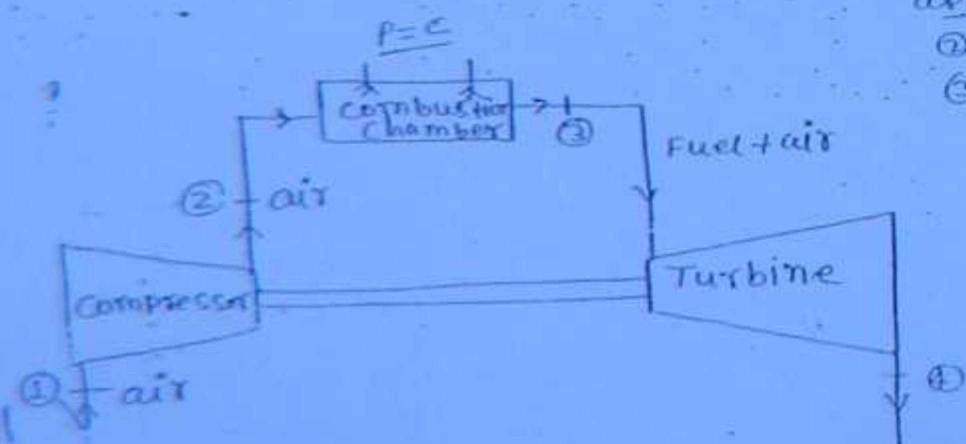
open cycle gas Turbine ⇒ works on (Joule / Brayton cycle)

application ⇒ ① aircraft.
② automotive (bus and truck)
③ industrial gas turbine installation.

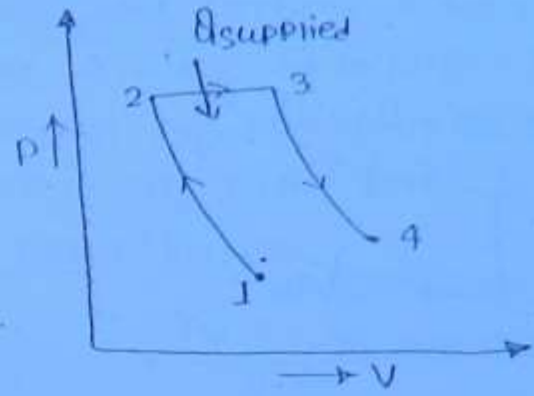


I.C. engine Called

1-2 → Reversible adiabatic compression (Isentropic)

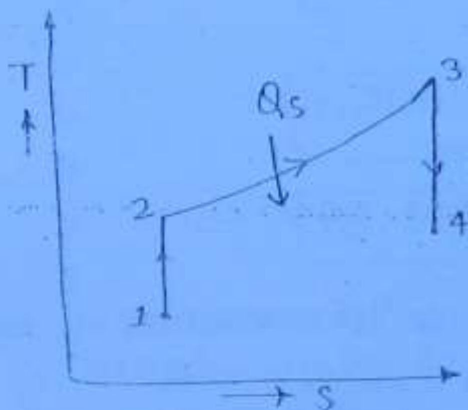
2-3 → Constant pressure heat addition

3-4 → Rev. adiabatic expansion. (Isentropic)



$\cancel{p}V = \cancel{m}\cancel{R}T$

$V \propto T$



<u>Note</u> ⇒ The Brayton cycle is the air standard cycle for the gas turbine power plant.

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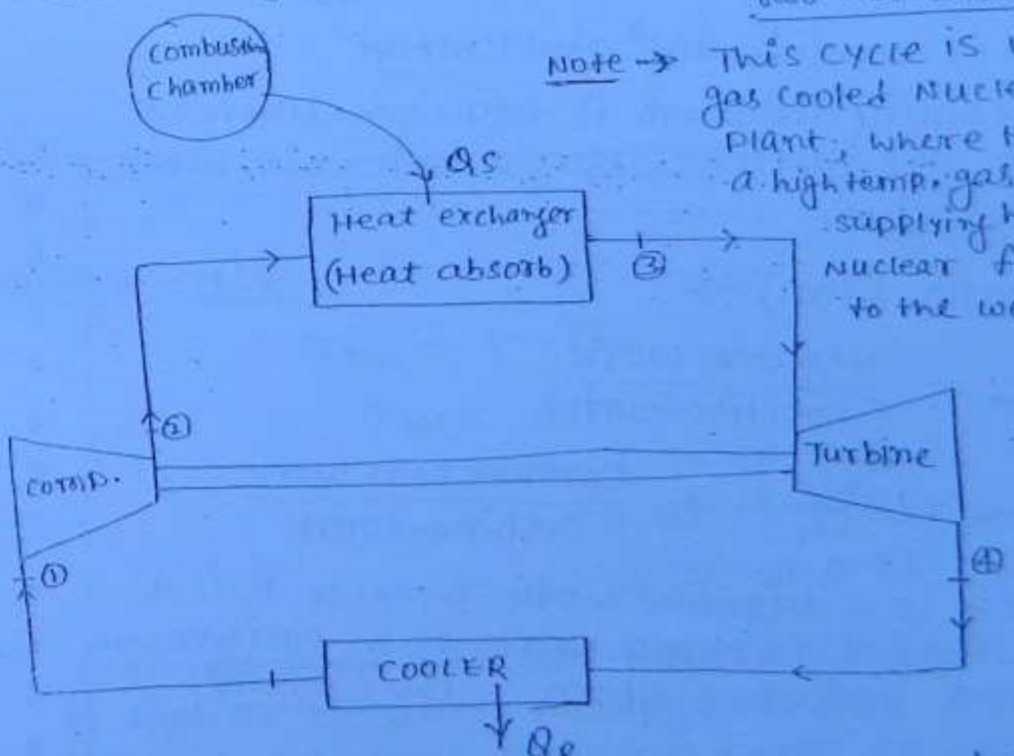
② <u>close cycle gas Turbine</u> → ⇒ <u>two Reversible isobars and two reversible adiabatic</u>

<u>Note</u> → This cycle is used in a gas cooled Nuclear reactor plant, where the source is a high temp. gas cooled Reactor supplying heat from Nuclear fission directly to the working fluid (a gas)



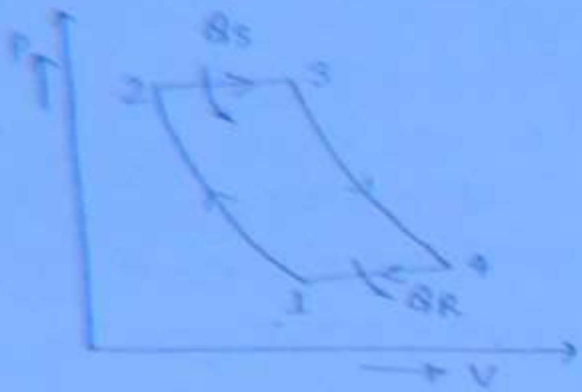
<u>External Combustion engine</u>

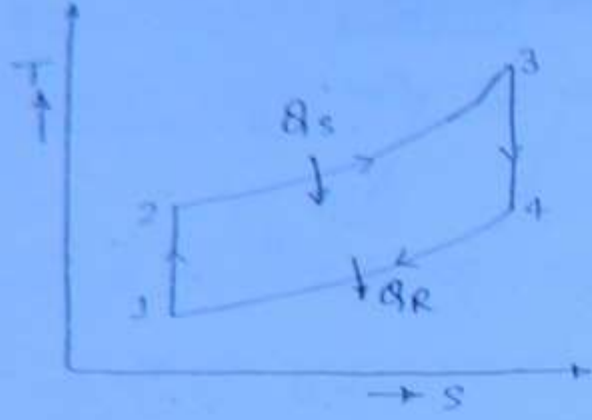
1 – 2 → Rev. adiabatic compression (Isentropic)

2 – 3 → Constant pressure heat addition (Isobar)

3 – 4 → Rev. adiabatic expansion (Isentropic)

.... heat rejection.





(BRAYTON CYCLE)

- Gas Turbine cycle work on Brayton cycle.

Advantage of closed cycle gas Turbine ⇒

1) It can work on any working fluid but open cycle gas turbine must work on air only.
2) any cheaper fuel can be used because the product of combustion do not come directly in contact with turbine blade.

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- Disadvantages ⇒

① The system is complicated and costly
② Additional cooling medium is required. whereas an open cycle gas turbine atmospheric air work as a cooling medium.

- Back work Ratio (r_{bw}) ⇒

$$r_{bw} = \frac{\text{Negative work}}{\text{positive work}}$$

$$r_{bw} = \frac{W_C}{W_T} \Rightarrow \frac{\text{compressor work}}{\text{Turbine work}}$$

compressor work is a Negative work because it is a open system whearas Turbine work is a positive work because this is a closed system $\boxed{W_c = -\int v\,dP}$

- It is the Ratio of Negative work to the positive work.

In case of gas Turbine power plant the Back work ratio are around 40 to 60%. Whearas in the case of Steam power plant (vapour power cycle / Rankine cycle) the Back work ratio are 1 to 2%.

(ii) Work Ratio (R_W) ⇒

$$r_W = \frac{\text{Net work}}{\text{Turbine work}} = \frac{W_{net}}{+\text{ve work}} \Rightarrow \frac{W_{net}}{W_T}$$

$$r_W = \frac{W_T - W_C}{W_T}$$

$$\boxed{r_W = 1 - \frac{W_C}{W_T}} \quad \text{because} \quad \boxed{r_{bw} = \frac{W_C}{W_T}}$$

$$\boxed{r_W = 1 - r_{bw}}$$

* The work Ratio of Rankine cycle (vapour power cycle) is almost unity (1).

77.

* Brayton cycle

$$r_{bw} = 0.4 \text{ to } 0.6$$

$$r_W = 1 - 0.4 \Rightarrow 0.6$$

$$= 1 - 0.6 \Rightarrow 0.4$$

$$\boxed{r_W = 0.4 \text{ to } 0.6}$$

* Rankine cycle

$$r_W = 1 - r_{bw}$$

$$r_{bw} = 1 \text{ to } 2\%$$

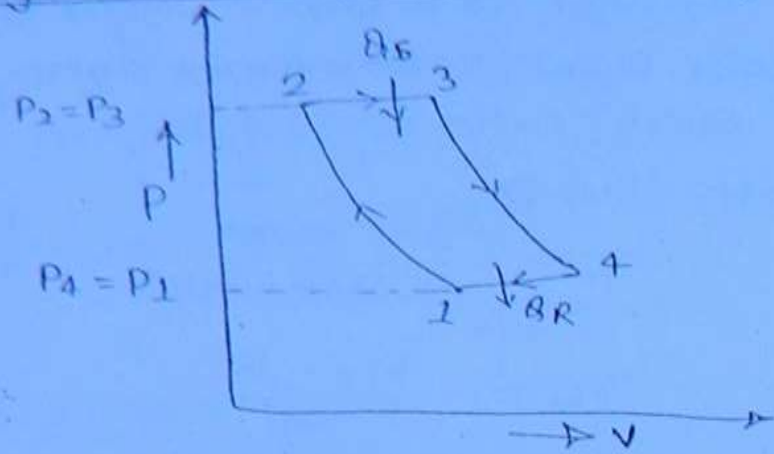
$$= 0.01 \text{ to } 0.02$$

$$r_W = 1 - 0.01 \Rightarrow 0.99$$

$$= 1 - 0.02 \Rightarrow 0.98$$

Note → Rankine cycle (vapour power cycle) work ratio is higher because pump are less work consume whearas Brayton cycle in compressor are more work is consume. (more power is consu-

Efficiency of a simple gas Turbine cycle on p-v diagram



$$\eta_T = \frac{W_{net}}{Q_s\,(\text{Heat supplied})}$$

$$\eta = \frac{Q_s - Q_R}{Q_s}$$

$$\eta = 1 - \frac{Q_R}{Q_s}$$

where $Q_R = h_4 - h_1 \Rightarrow C_p(T_4 - T_1)$

$Q_s = h_3 - h_2 \Rightarrow C_p(T_3 - T_2)$

$$\eta = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

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$$\eta = 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} \quad \text{——— (1)}$$

* Pressure Ratio $(r_p) = \frac{P_2}{P_1} = \frac{P_3}{P_4}$

1-2 → Reversible adiabatic compression

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{——— (2)}$$

3-4 → Reversible adiabatic expansion.

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{——— (3)}$$

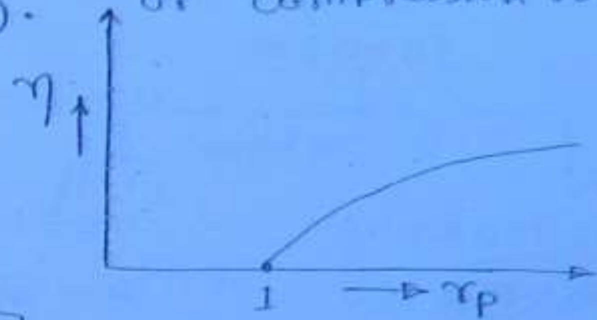
*** $\boxed{\frac{T_2}{T_1} = \frac{T_3}{T_4}}$ $\qquad$ $\boxed{\frac{T_4}{T_1} = \frac{T_3}{T_2}}$

$$\eta = 1 - \frac{T_1}{T_2}$$

$$\boxed{\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

Note → For the same compression ratio, the Brayton cycle efficiency is equal to the otto cycle efficiency.

Note ⇒ The efficiency of brayton cycle depend on the pressure ratio (r_p). or compression ratio.



* when $r_p = 1$, then $\eta = 0$, and r_p increases than η also increases.

$$\boxed{\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

Note ⇒
This equation of η is valid only when compression and expansion are Isentropic.

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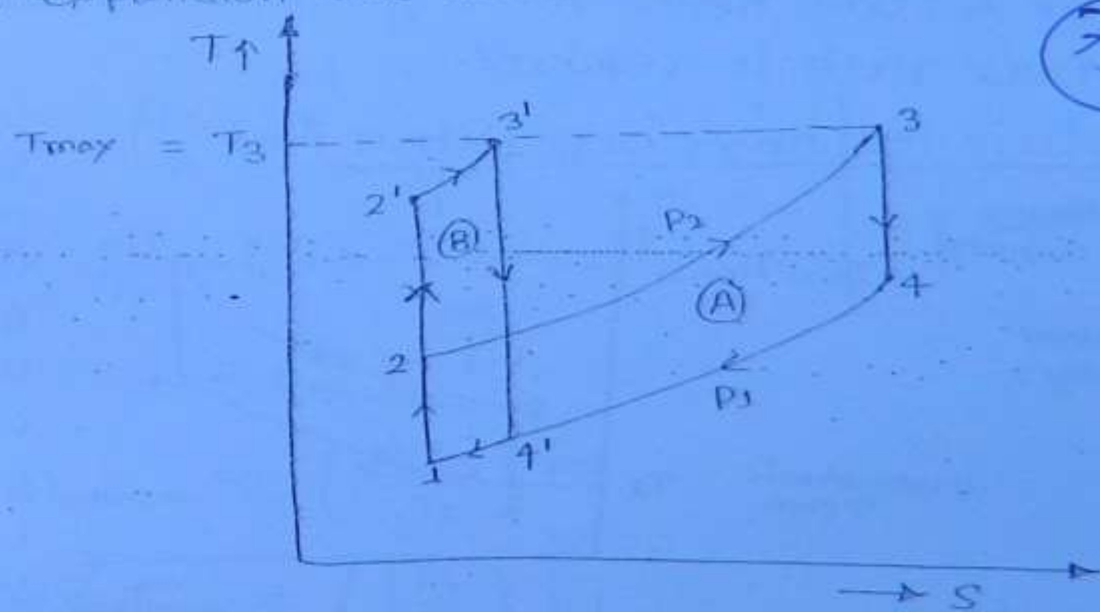

$$\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

$$\boxed{\eta_B > \eta_A}$$

⇒ It means efficiency depend on pressure ratio. because B cycle pressure ratio is more compare to cycle A.

$W_A > W_B$ → Area of A cycle is more compare to cycle B.
so when mass flow rate is more than area is much required then cost is high. So we are not take more pressure ratio.

case → SAME Capacity TO take.

$\boxed{\text{Power} = \text{mass} \times \text{network}}$

1500 KW

P = m Wnet

A

$W_A = 150\ kJ/kg$

$1500 = m_A \times 150$

$m_A = 10\ kJ/kg$

B

$W_B = 15\ kJ/kg$

$P = m_B \times 15$

$1500 = m_B \times 15$

$m_B = 100\ kJ/kg$

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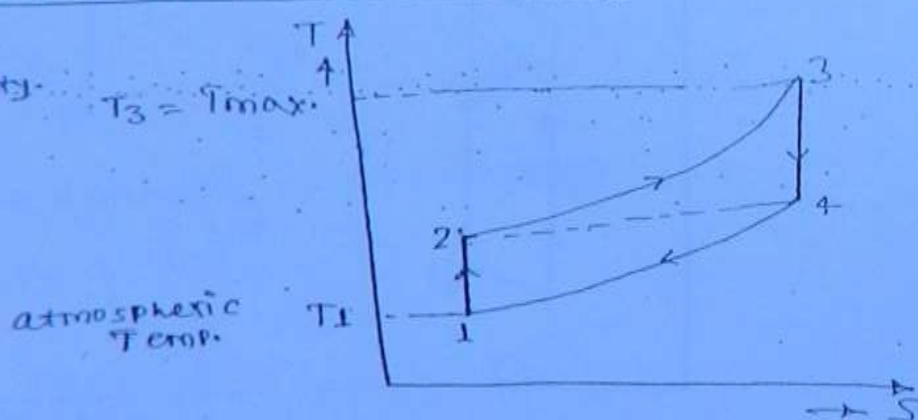
* Though the efficiency of $\eta_B > \eta_A$ the net work in B < cycle A and hence for a given capacity greater mass flow rate is required.

Optimum pressure for max. work output :→

where T_3 → Temp. constant because material property.

T_1, T_3 → Temp. constant (not change)



$$W_{net} = W_T - W_C$$

$$W_{net} = (h_3 - h_4) - (h_2 - h_1)$$

$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$W_{net} = C_p(T_3 - T_4 - T_2 + T_1)$$

$$= \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$T_4 = \frac{T_3 T_1}{\;}$$

$$W_{net} = C_p\left[T_3 - \frac{T_1 T_3}{T_2} - T_2 + T_1\right]$$

For max. W_{net} = $\frac{dW_{net}}{dT_2} = 0$

$$\frac{dW_{net}}{dT_2} = C_p\left[0 - T_1T_3\left(-\frac{1}{T_2^2}\right) - 1 + 0\right] = 0$$

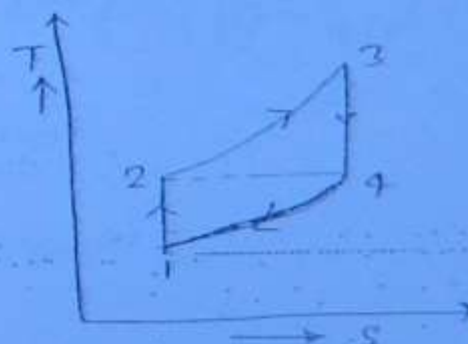
$$\frac{T_1T_3}{T_2^2} - 1 = 0$$

$$T_1T_3 = T_2^2 \Rightarrow T_2 = \sqrt{T_1T_3}$$

$$T_4 = \frac{T_1T_3}{\sqrt{T_1T_3}} \Rightarrow T_4 = \sqrt{T_1T_3}$$

** $$\boxed{T_2 = T_4 = \sqrt{T_1T_3}}$$

pressure ratio condition for max. work ⇒



$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

then $\frac{P_2}{P_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$

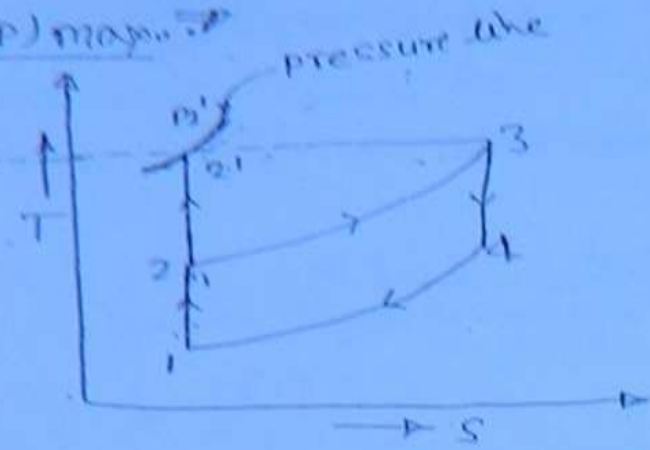
$$= \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$(r_p)_{optimum} = \left(\frac{\sqrt{T_1T_3}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$(r_p)_{optimum} = \left(\sqrt{\frac{T_3}{T_1}}\right)^{\frac{\gamma}{\gamma-1}}$$

** $$\boxed{(r_p)_{optimum} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}}$$ Condition for max. work

Maximum pressure ratio $(r_p)_{max}$:-



$W_C = W_T$

$W_{compression} = W_{expansion}$
then net work is zero

$\boxed{W_{net} = 0}$

$1-2 \rightarrow$ Rev. adiabatic process

$$(r_p)_{max} = \frac{P_2'}{P_1}$$

$$\frac{T_2'}{T_1} = \left(\frac{P_2'}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{T_3}{T_1}\right) = (r_p)_{max}^{\frac{\gamma-1}{\gamma}}$$

Because $\boxed{T_2' = T_3}$

$$\frac{P_2'}{P_1} = (r_p)_{max} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

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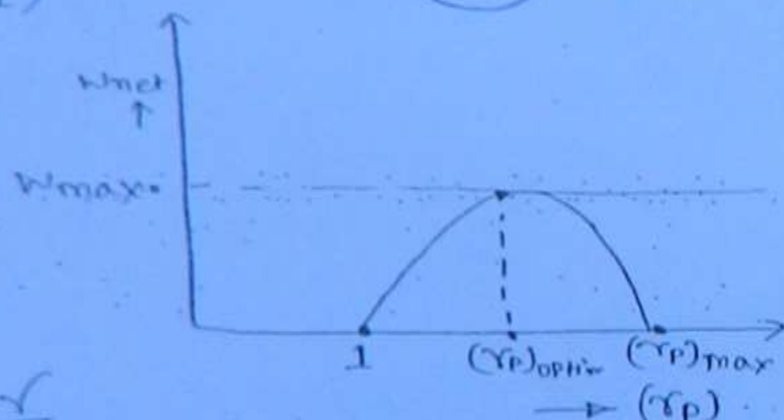


$$(r_p)_{max} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

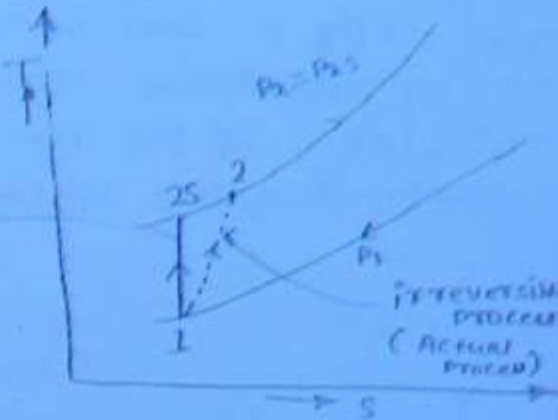
$$(r_p)_{optimum} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$(r_p)^2_{optimum} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{(r_p)^2_{optimum} = (r_p)_{max}}$$

Isentropic efficiency of a compressor ⇒

That means irreversible process in entropy increases.



(Isentropic line) Reversible adiabatic process

$$\eta_c = \frac{\text{Isentropic work}}{\text{Actual work}}$$

$$\eta_c = \frac{\text{compressor work} \rightarrow \text{less work}}{\cancel{\text{Turbine}}\ \text{work (Compressor Actual)} \rightarrow \text{more work}}$$

(~~Compressor work are less work because more power is required and Turbine Net work are more~~)

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{\text{Isentropic compression}}{\text{Actual compression}}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

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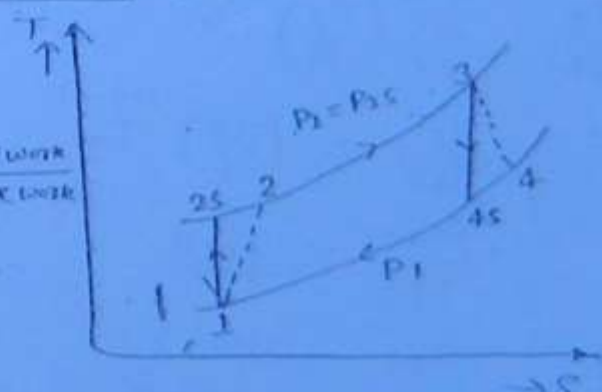
$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\left(\frac{T_{2s}}{T_1}\right) = (r_p)^{\frac{\gamma-1}{\gamma}}} \ *$$

Isentropic efficiency of a Turbine ⇒

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}}$$

$$\eta_T = \frac{\text{Turbine actual work} \rightarrow \text{less work}}{\text{Turbine isentropic work} \rightarrow \text{more work}}$$



$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

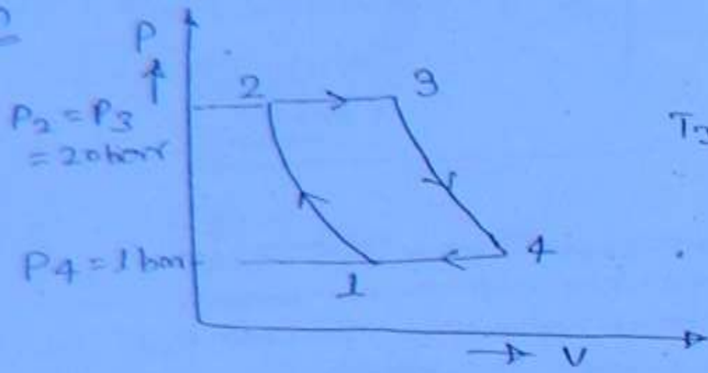
Note ⇒ Turbine work is opposite to compressor work.

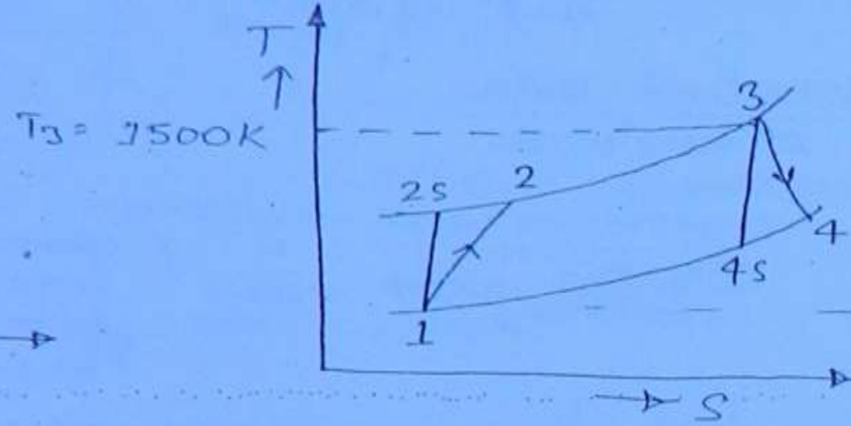
$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_{4s}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{T_3}{\ } = (r_p)^{\frac{\gamma-1}{\gamma}}} \ **$$

Q. In a gas turbine hot combustion product with $C_p = 0.98$ KJ/kg-K, and $C_v = 0.7538$ KJ/kg-K enter urbine at 20 bar, 1500K and leave at 1 bar. The sentropic efficiency of the turbine is 0.94 then find he work developed by the turbine per kg of gas.

Sol.





$C_p = 0.98$ KJ/kg-K

$C_v = 0.7538$ KJ/kg-K

$P_2 = P_3 = 20$ bar $\quad T_3 = 1500K$

$P_4 = 1$ bar $\quad \eta = 0.94\%$

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$W_T = h_3 - h_4$

$W_T = C_p(T_3 - T_4)$

$\gamma = \dfrac{C_p}{C_v}$

$\gamma = \dfrac{0.98}{0.7538}$

$$\frac{T_3}{T_{4s}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{20}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$\frac{1500}{T_{4s}} = \frac{1.99}{1}$$

$$1.99 \times T_{4s} = 1500 \times 1$$

$$T_{4s} = \frac{1500}{1.99}$$

$$\boxed{T_{4s} = 751.36\,K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$\eta_T (T_3 - T_{4s}) = T_3 - T_4$$

$$W_T = C_p (T_3 - T_4)$$

$$W_T = C_p \left[\eta_T \left[T_3 - T_{4s} \right] \right]$$

$$W_T = 0.98 \left[0.94 \left(1500 - 751.36 \right) \right]$$

$$\boxed{W_T = 689.8 \text{ kJ/kg}}$$

PROB-② air enter the compressor of a gas Turbine operating on Brayton cycle at $27°C$ ($T_1 = 273 + 27 = 300K$). The pressure ratio is $\left(\frac{P_2}{P_1} = 6\right)$. calculate the max. Temp. in the cycle and cycle efficiency. Assume Turbine work = 2 comp. work.

$$W_T = 2 W_C \quad , \quad \gamma = 1.4$$

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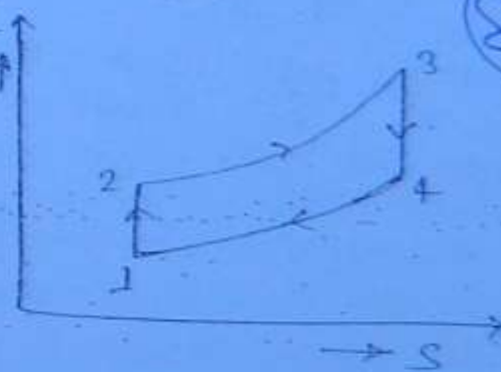
Soln

$\gamma = 1.4$

$W_T = 2.5\ W_C$

$\gamma_P = \frac{P_2}{P_1} = 6$

$T_1 = 300K$



$$\frac{T_2}{T_1} = (\gamma_P)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{300} = (6)^{\frac{1.4 - 1}{1.4}}$$

$$\boxed{T_2 = 500.5K}$$

$$W_T = 2.5\ W_C$$

$$(h_3 - h_4) = 2.5 (h_2 - h_1)$$

$$\cancel{C_p} (T_3 - T_4) = 2.5\ \cancel{C_p} (T_2 - T_1)$$

$$T_2 T_4 = T_1 T_3$$

$$\underline{T_1 T_3}$$

$$T_3 - \frac{T_1 T_3}{T_2} = 2.5\,(T_2 - T_1)$$

$$T_3\left(\frac{T_2 - T_1}{T_2}\right) = 2.5\,(T_2 - T_1)$$

$$\frac{T_3}{T_2} = 2.5$$

$$T_3 = 2.5 \times T_2$$

$$T_3 = 2.5 \times 500.5$$

$$\boxed{T_3 = 1251.\,K}$$

$$\eta_T = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

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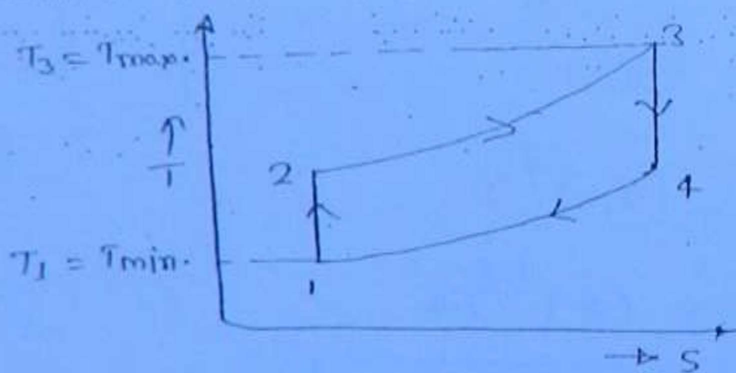
$$\eta_T = 1 - \frac{1}{(6)^{\frac{1.4-1}{1.4}}}$$

$$\boxed{\eta_T = 0.4 \Rightarrow 40\%}$$ Ans:

Q.③ In a simple gas Turbine power Plant T_1 is minimum Temp. and T_3 is the max. Temp. and what is the work ratio in terms of pressure ratio (r_p).

Sol.



$$r_w = \frac{W_{net}}{W_T}$$

$$r_w = \frac{W_T - W_C}{W_T}$$

$$r_w = 1 - \frac{W_C}{W_T}$$

$$r_w = 1 - \frac{(h_2 - h_1)}{(h_3 - h_4)}$$

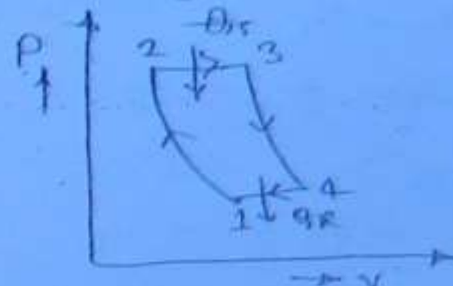
$$r_w = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)}$$

$$r_w = 1 - \frac{T_1\left(\frac{T_2}{T_1} - 1\right)}{T_3\left(1 - \frac{T_4}{T_3}\right)}$$

$$\eta_W = 1 - \left(\frac{T_1}{T_3}\left[\frac{(r_P)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}}\right]\right)$$

$$\boxed{r_W = 1 - \frac{T_1}{T_3}(r_P)^{\frac{\gamma-1}{\gamma}}}$$ Ans:

PROB-④ A simple gas Turbine power plant working on Brayton cycle. the power required to drive the compressor is 175 kW Heat supplied during constant pressure heat addition is 675 kW turbine output obtained during expansion is 425 kW. find the heat rejection.



Soln

$$W_C = 175\text{ kW}$$

$$W_{net} = Q_S - Q_R$$

$$W_T - W_C = Q_S - Q_R$$

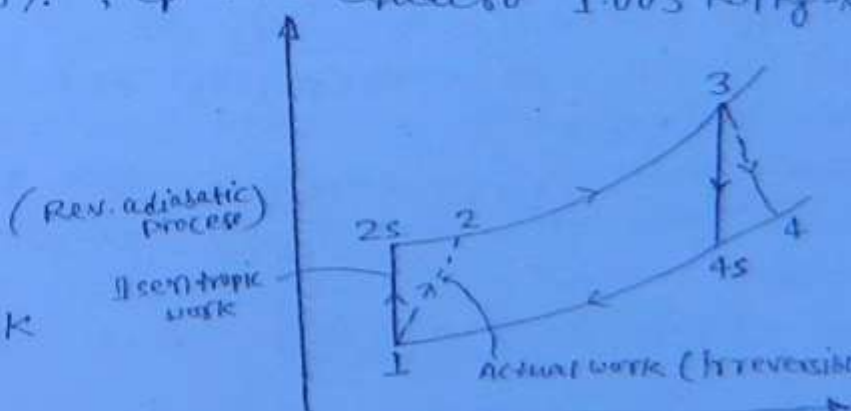
$$425 - 175 = 675 - Q_R$$

$$\boxed{Q_R = 425}\text{ kW}$$ Ans:

(82)

PROB-⑤ A closed cycle gas turbine unit develop a power of 1500 kW the pressure and temp. at the inlet to the compressor are 1 bar and 20°C. Pressure ratio is 6. the max. temp. to the inlet to the Turbine 800°C. find the isentropic efficiency of Turbine and mass flow rate of air if the isentropic efficiency of the compressor is 85%. and overall efficiency is 20%., C_p take equal to 1.005 kJ/kg-K $\gamma = 1.4$.

Soln.



$$P = 1500\text{ kW}$$

$$P_1 = 1\text{ bar}$$

$$T_1 = 273 + 20 = 293\text{ K}$$

$$r_P = 6 = \frac{P_2}{P_1}$$

$$T_3 = 273 + 800 = 1073\text{ K}$$

$$\eta_c = 0.85$$

$$\eta_o = 0.20$$

$$\eta_0 = \frac{\cdots\cdots}{Q_s}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_{2s} = 488.8\,K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \frac{\text{isentropic work}}{\text{Actual work}} \begin{matrix} \rightarrow \text{less} \\ \rightarrow \text{more compression} \end{matrix}$$

$$\eta_c = \frac{488.8 - 293}{T_2 - 293}$$

$$0.85 = \frac{488.8 - 293}{T_2 - 293}$$

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$$\boxed{T_2 = 523.4\,k}$$

2) $$\eta_0 = \frac{W_{net}}{Q_s} = \frac{W_T - W_c}{Q_s}$$

$$\eta_0 = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

$$\eta_0 = \frac{\cancel{C_p}(T_3 - T_4) - \cancel{C_p}(T_2 - T_1)}{\cancel{C_p}(T_3 - T_2)}$$

$$0.20 = \frac{(1073 - T_4) - (523.4 - 293)}{1073 - 523.4}$$

$$\boxed{T_4 = 732.6\,k}$$

$$\frac{T_3}{T_{4s}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1073}{T_{4s}} = (6)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_{4s} = 643.1\,K}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \begin{array}{l} \rightarrow \text{less work} \\ \rightarrow \text{more work (Because more expands decreased.)} \end{array}$$

$$\eta_T = \frac{1073 - 732.6}{1073 - 643.1}$$

$$\eta_T = 79\%$$

(3) $P = m \times W_{net}$

$P = m\,(\eta_0 \times q_s)$

Where $q_s = h_3 - h_2$

$q_s = C_p(T_3 - T_2)$

$= 1.005\,(1073 - 523.4)$

$$\boxed{q_s = 552.31\ KJ/Kg}$$

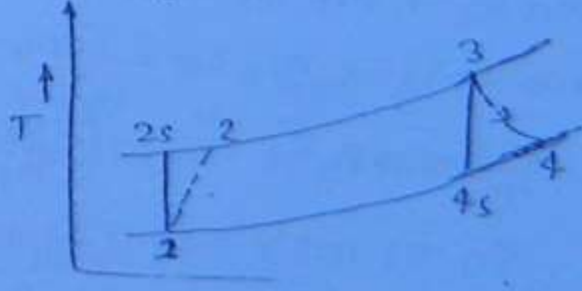
$P = m \times W_{net}$

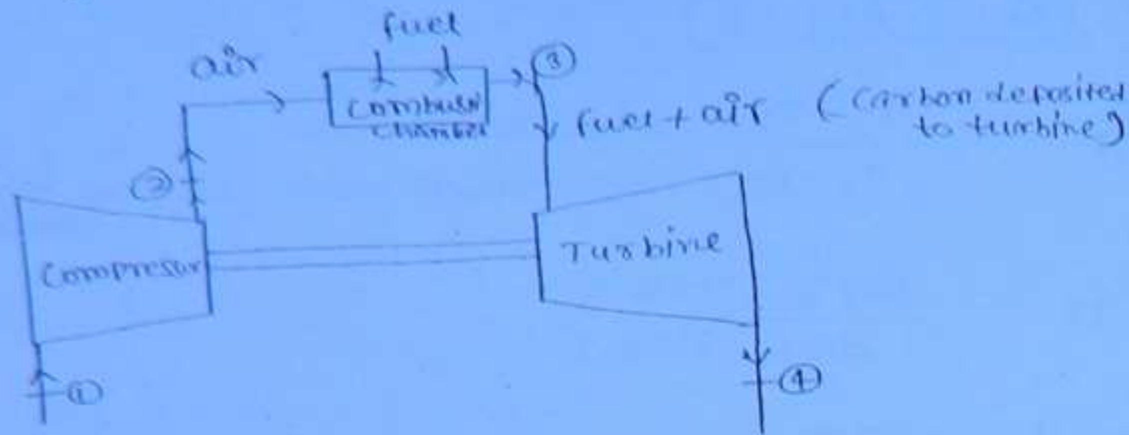
$1500 = m\,(552.31 \times 0.20)$

$$\boxed{m = 13.5\ Kg/sec}$$ * Ans:

PROB. (6) Air enters the compressor of a gas turbine at a bar and 293K the pressure after compression is 6 bar. Air fuel ratio 60 the isentropic efficiency of compressor and Turbine are 0.84 and 0.88 respectively. Calculate the power developed by the units & its overall efficiency Assume γ to be same for air and combustion product. take specific heat of gases = 1.1 KJ/Kg-K, mass of air 5Kg/sec C_v = 40000 KJ/Kg.

soln





$T_1 = 293$, $r_p = 6$

$\eta_c = 0.84$, $\eta_T = 0.88$ $\underline{\gamma = 1.4}$

$$\frac{T_{2s}}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

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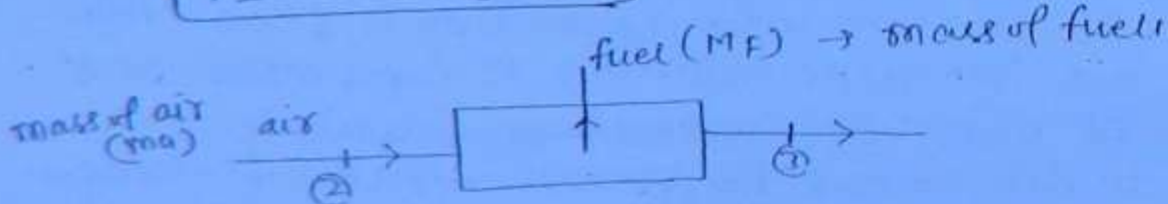
$$\boxed{T_{2s} = 488.87\,K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\text{isentropic work} \rightarrow \text{less}}{\text{Actual work} \rightarrow \text{more}}$$

$$0.84 = \frac{488.87 - 293}{T_2 - 293}$$

$$\boxed{T_2 = 526.18\,K}$$



Energy eqn.

$$m_a h_2 + m_F C_V = (m_a + m_F)\, h_3$$

$$m_a C_p T_2 + m_F C_V = (m_a + m_F)\, C_{p_{gas}} T_3$$

$$\hookrightarrow \frac{m_a}{m_F} C_p T_2 + C_V = \left(\frac{m_a}{m_F} + 1\right) C_{p_{gas}} \cdot T_3$$

$$60 \times 1.005 \times 526.18 + 40000 = (60+1) \times 1.1 \times T_3$$

$$\boxed{T_3 = 1050.3\,K}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_{4s}}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1059.3}{T_{4s}} = (6)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_{4s} = 635\ K}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$0.88 = \frac{1059.3 - T_4}{1059.3 - 635}$$

$$\boxed{T_4 = 685.8\ K}$$

* $W_c = m_a (h_2 - h_1)$ → Compressor work

$$= m_a \times C_p (T_2 - T_1)$$

$$= 5 \times 1.005 (526.18 - 293)$$

$$\boxed{W_c = 1171.72}\ kW$$

Turbine work $W_T = (m_a + m_F)(h_3 - h_4)$

$$= (m_a + m_F)\, C_{p_{gas}} (T_3 - T_4)$$

$$= \left(5 + \frac{5}{60}\right) \times 1.11 \times (1059.3 - 685.5)$$

$$\boxed{W_T = 2107\ kW}$$

$$W_{net} = W_T - W_c$$

$$= 2107 - 1171.72$$

$$\boxed{W_{net} = 935.7}$$

$$q_s = m_F \times C_V$$

$$\eta_0 = \frac{W_{net}}{q_s} \Rightarrow \frac{W_{net}}{m_F \times C_V} \Rightarrow \frac{935.7}{\frac{5}{60} \times 40000}$$

$$\eta_0 = 0.2807 \quad \text{(Ans:)}$$

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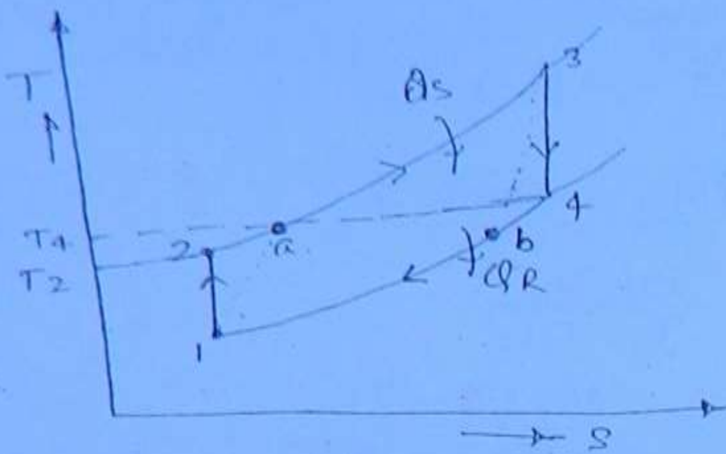
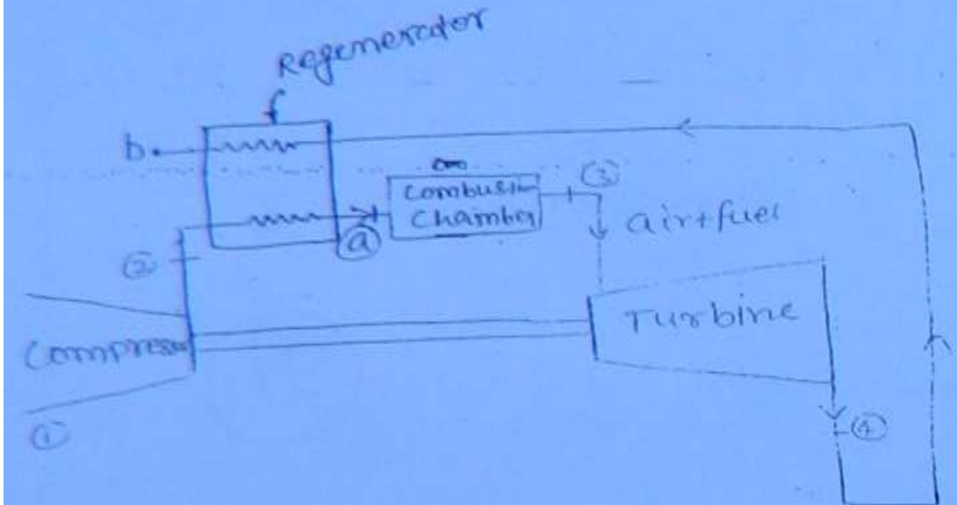
USE OF REGENRATOR IN GAS TURBINE ⇛





$$\eta = \frac{W_{net}}{Q_S} = \frac{W_T - W_C}{Q_S} \begin{matrix} \rightarrow \text{constant} \\ \downarrow \text{decreases} \end{matrix}$$

when ~~than~~ Q_S is decreases than efficiency of gas turbine increases.

effect of Regeneration ⇛

① No change in compressor work.
② No change in Turbine work.
③ No change in Net work.
④ Reduction in heat supplied.
⑤ Increase in Turbine efficiency.

$$\eta = \frac{W_T - W_C}{Q_S} \Rightarrow \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_a}$$

$$\eta = \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_a)}$$

$$\boxed{\eta = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_a}}$$

(#) effectiveness of Regenerator ⇒

It is the ratio of Actual rise in temp. to the ideal Rise in Temperature.

$$\epsilon = \frac{\text{Actual rise in temp.}}{\text{Ideal rise in temp.}}$$

$$\boxed{\epsilon = \frac{t_a - t_2}{t_4 - t_2}} \quad **$$

For effective utilisation of Regenerator the temp. differen between T_4 and T_2 should be large.

(#) USE OF REHEATING IN GAS TURBINE ⇒

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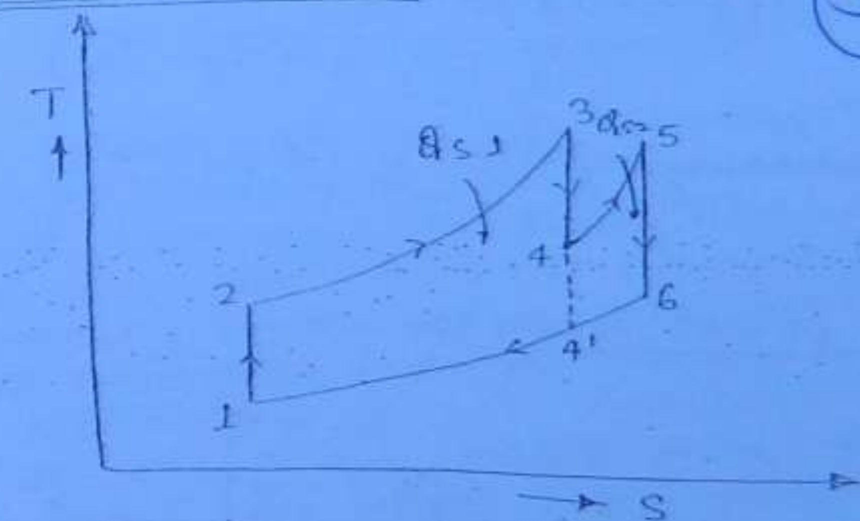

W_T (simple cycle) $= h_3 - h_4' = (h_3 - h_4) + (h_4 - h_4')$

W_T (Reheat cycle) $= (h_3 - h_4) + (h_5 - h_6)$

$W_{net} = W_T - W_C$ when W_C = constant

then W_T increase the network increa

As constant pressure line diverge

$$h_5 - h_6 > h_4 - h_4'$$

$\therefore$ W_T (reheat cycle) > W_T (without reheat)

$$\eta = \frac{W_{net} \uparrow}{Q_s \uparrow}$$ then efficiency will be decreases.

→ with reheating the Net work increases this does not mean an increase in efficiency of the cycle cause with reheating the reheat supplied is also increases with reheat because the temp $T_6 > T_4'$ and hence however reheating is used it is generally coupled with regeneration. Therefore Regeneration from efficiency increases and due to Reheating from Wnet increases.

$$\eta = \frac{W_T - W_C}{Q_s}$$

$$\eta = \frac{[(h_3 - h_4) + (h_5 - h_6)] - [h_2 - h_1]}{(h_3 - h_2) + (h_5 - h_4)}$$

$$\eta = \frac{[(T_3 - T_4) + (T_5 - T_6)] - [T_2 - T_1]}{(T_3 - T_2) + (T_5 - T_4)}$$

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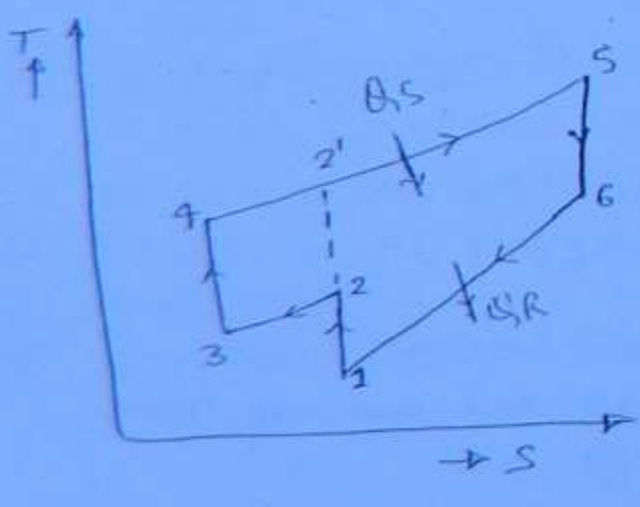
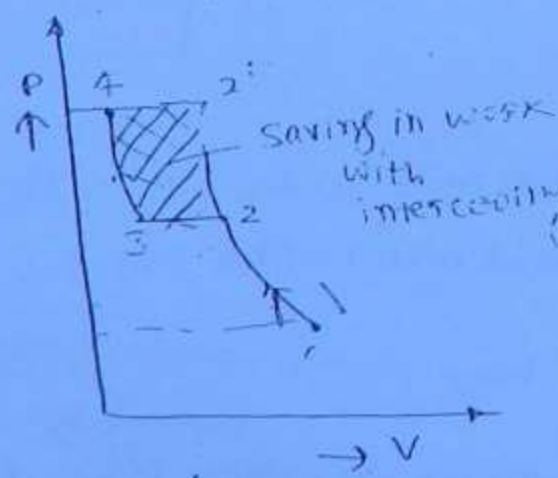
∴ Due to Reheating efficiency of the cycle decreases(↓)

Inter cooling in GAS Turbine ⇒





$W_{net} = W_T - W_C$

$$\downarrow \eta = \frac{W_{net} \uparrow}{Q_r \uparrow}$$

④ Intercooling in gas Turbine :→

In Inter cooling the work of compression decreases and this result in increase in Net work but this does not mean an increase in efficiency because the heat supplied also increases. but with inter cooling the scope of for Regeneration increases because $T_4 < T_2'$. and hence intercooling is generally coupled with regeneration. →

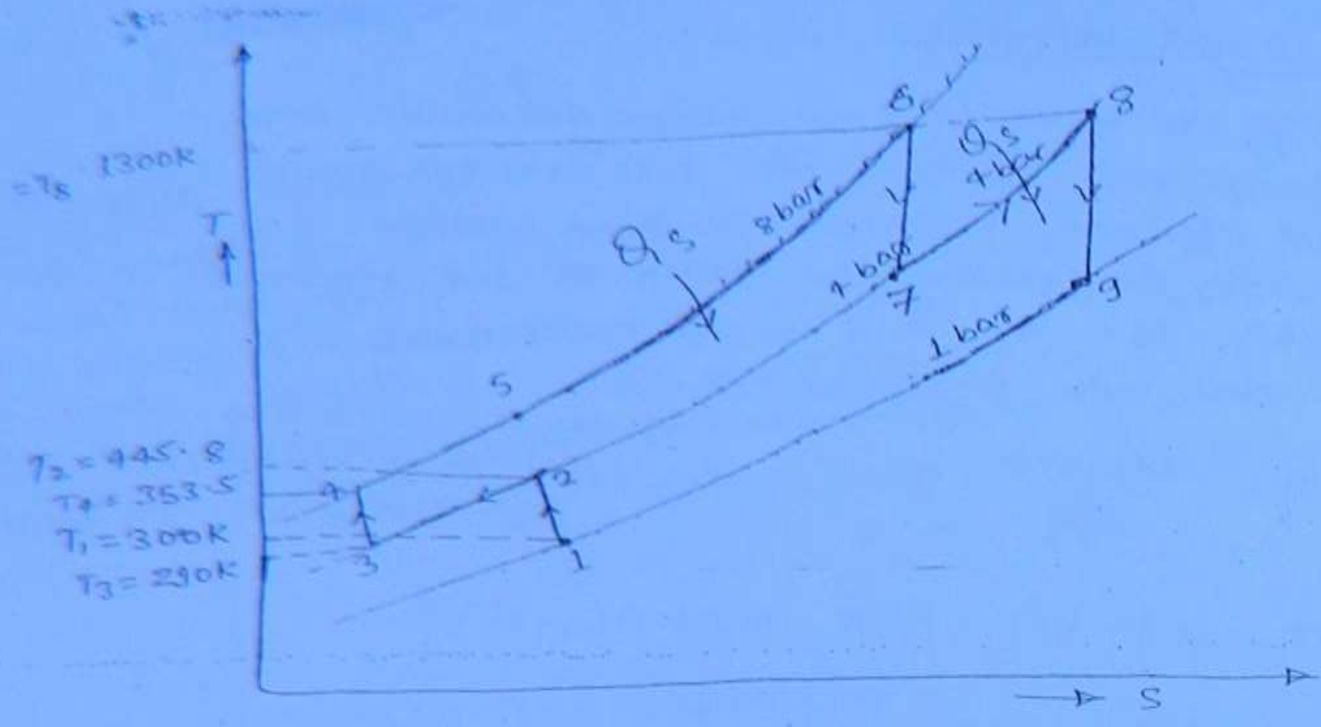
① due to inter cooling efficiency of cycle decreases (↓)

$$\eta = \frac{W_T - W_C}{Q_s}$$

$$\eta = \frac{(h_5 - h_6) - \left[(h_2 - h_1) + (h_4 - h_3)\right]}{h_5 - h_4}$$

$$\boxed{\eta = \frac{(T_5 - T_6) - \left[(T_2 - T_1) + (T_4 - T_3)\right]}{T_5 - T_4}}$$

PROB-① A Regenerative reheat cycle has air entering at 1 bar, 300K into the compressors, having intercooling in between two stage of compression air leaving first stage of compression is cooled upto 290K and 4 bar pressure in intercooler and subsequently compressed to 8 bar compressor air leaving second stage compressor is passed through a regenerator having an effectiveness of 0.8. subsequent combustion its 1300K at inlet to the turbine having expansion upto 4 bar and then heated upto 1300K before being expand upto 1 bar. exhaust from turbine is passed to regenerator. find the total turbine work and efficiency consider compression and expansion to the isentropic and air as the working fluid through out the cycle.



$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{300} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_2 = 445.8K}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

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$$\frac{T_4}{290} = \left(\frac{8}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_4 = 353.5K}$$

$$\frac{T_8}{T_9} = \left(\frac{P_8}{P_9}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1300}{T_9} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_9 = 874.83K}$$

$$\frac{T_6}{T_7} = \left(\frac{P_6}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{1300}{T_7} = \left(\frac{8}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_7 = 1066.4K}$$

$$\epsilon = \frac{T_5 - T_4}{T_9 - T_4}$$

$$0.8 = \frac{T_5 - 353.5}{874.83 - 353.5}$$

$$\boxed{T_5 = 770.6K}$$

$W_T = (h_6 - h_7) + (h_8 - h_9)$

$W_T = C_p(T_6 - T_7) + C_p(T_8 - T_9)$

$= 1.005(1300 - 1066.4) + 1.005(1300 - 874.8)$

$$\boxed{W_T = 662 \ \text{KJ/kg}}$$

$$\eta = \frac{W_T - W_C}{Q_s}$$

$W_C = (h_2 - h_1) + (h_4 - h_3)$

$= C_p(T_2 - T_1) + C_p(T_4 - T_3)$

$= 1.005(445.8 - 300) + 1.005(353.5 - 290)$

$$\boxed{W_C = 210 \ \text{KJ/kg}}$$

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$Q_s = (h_6 - h_5) + (h_8 - h_7)$

$Q_s = C_p(T_6 - T_5) + C_p(T_8 - T_7)$

$= 1.005(1300 - 770.5) + 1.005(1300 - 1066.4)$

$$\boxed{Q_s = 766.8}$$

$$\eta = \frac{W_T - W_C}{Q_s} \Rightarrow \frac{662 - 210}{766.8}$$

$$\eta = 0.589$$

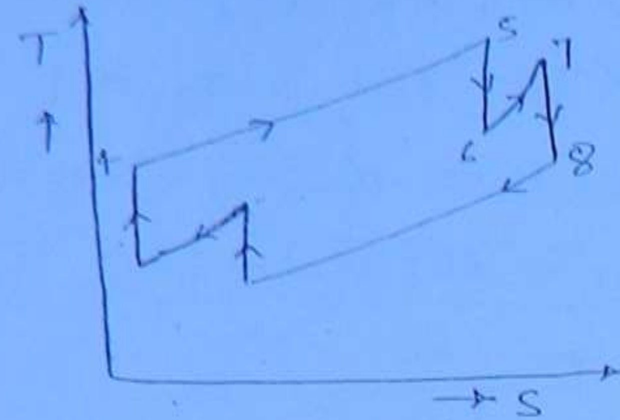
$$\boxed{\eta = 58.9\%}$$ Ans

I.E.S

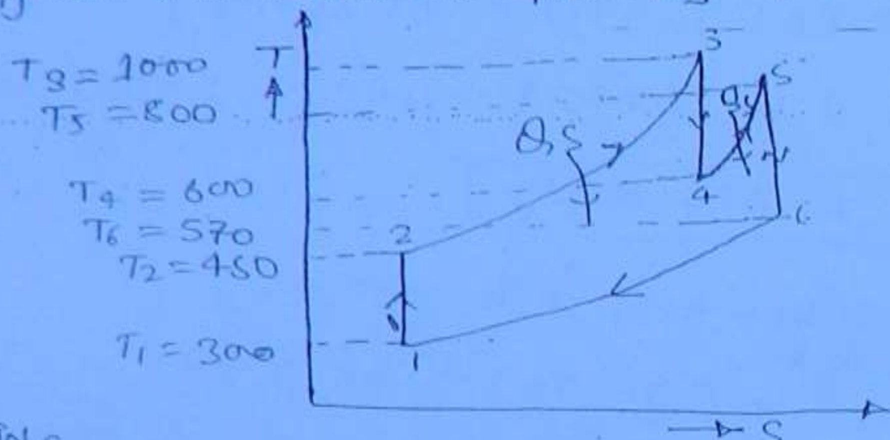
PROB-(2) The cycle shown in figure represent a gas turbine cycle with intercooling and reheating on T-S diagram.

match list (1) match list (2)

1. Inter cooler → 2-3
2. combustor → 4-5
3. Reheater → 6-7
4. High press. compressor → 3-4



RCB-③ The given figure shows a gas turbine cycle employing reheating on T-S diagram then find the efficiency of the cycle.



Soln

$$\eta = \frac{W_T - W_C}{Q_S}$$

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$$= \frac{[(h_3 - h_4) + (h_5 - h_6)] - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4)}$$

$$= \frac{(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)}{(T_3 - T_2) + (T_5 - T_4)}$$

$$\eta = \frac{(1000 - 600) + (800 - 570) - (450 - 300)}{(1000 - 450) + (800 - 600)}$$

$\eta = 0.64 = 64\%$ Ans

Ques-10 In a simple gas turbine power plant operating on the standard Brayton cycle 175 kW is required to drive the compressor heat supplied during constant pressure heat addition process is 675 kW. The turbine power is 425 kW then what is the heat rejection.

Soln

$$Q_s = 675$$
$$W_T = 425$$
$$W_C = 175$$

$Q_R = ?$ $\quad \Sigma Q = \Sigma \delta W$

$$Q_s - Q_R = W_T - W_C$$
$$675 - Q_R = 425 - 175$$
$$\boxed{Q_R = 425} \quad \text{Ans.}$$

Optimum pressure ratio for min. work input ⇒
(with perfect inter cooling)

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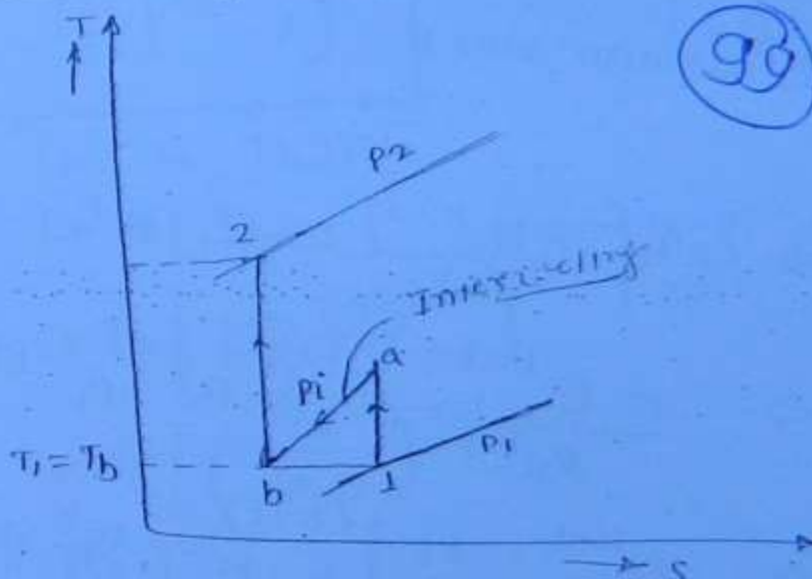

$$W_C = W_{C_1} + W_{C_2}$$
$$W_C = (h_a - h_1) + (h_2 - h_b)$$
$$W_C = C_p(T_a - T_1) + C_p(T_2 - T_b)$$
$$W_C = C_p\left[T_a - T_1 + T_2 - T_b\right]$$
$$W_C = C_p T_1\left[\frac{T_a}{T_1} - 1 + \frac{T_2}{T_1} - \frac{T_b}{T_1}\right]$$

because $\boxed{T_b = T_1}$

$$W_c = C_p T_1 \left[\frac{T_a}{T_1} - 1 + \frac{T_2}{T_1} - 1 \right]$$

$$W_c = C_p T_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 2 + \left(\frac{P_2}{P_i} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\because \quad \frac{T_2}{T_1} = \frac{T_2}{T_b} = \left(\frac{P_2}{P_i} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_a}{T_1} = \left(\frac{P_i}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Let $\frac{\gamma-1}{\gamma} = K$ constant

$$W_c = C_p T_1 \left[\left(\frac{P_i}{P_1} \right)^K - 2 + \left(\frac{P_2}{P_i} \right)^K \right]$$

$$W_c = C_p T_1 \left[\frac{P_i^K}{P_1^K} - 2 + \frac{P_2^K}{P_i^K} \right]$$

P_1 and P_2, C_p, T, K constant only variable P_i

For minimum work

100

$$\frac{dW}{dP_i} = 0$$

$$\Rightarrow C_p T_1 \left[\frac{K P_i^{K-1}}{P_1^K} - 0 + P_2^K \left(-K P_i^{-K-1} \right) \right]$$

$$\Rightarrow \frac{K P_i^{K-1}}{P_1^K} = K P_2^K P_i^{-(K+1)}$$

$$\Rightarrow P_i^{K-1} \cdot P_i^{+(K+1)} = P_2^K P_1^K$$

$$P_i^{2K} = (P_1 \cdot P_2)^K$$

$$P_i^2 = P_1 P_2$$

$$\boxed{P_i = \sqrt{P_1 P_2}}$$

* * * * *

Intermediate pressure = Geometric Mean of P_1 and P_2

$P_1 \rightarrow$ Inlet ~~compressor~~ pressure for compressor 1

$$\boxed{W_{c_1} = W_{c_2}}$$

$$W_c = W_{c_1} + W_{c_2} = 2W_{c_1}$$

$$\boxed{W_c = 2C_pT_1\left[\left(\sqrt{\frac{P_2}{P_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

For perfect inter cooling equal work input for each compressor will required.

$$W_{c_1} = h_a - h_1$$

$$W_{c_1} = C_p(T_a - T_1)$$

$$= C_pT_1\left[\frac{T_a}{T_1} - 1\right]$$

$$= C_pT_1\left[\left(\frac{P_i}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

where $P_i = \sqrt{P_1P_2}$

$$W_{c_1} = C_pT_1\left[\left(\frac{\sqrt{P_1}\sqrt{P_2}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\boxed{W_{c_1} = C_pT_1\left[\left(\sqrt{\frac{P_2}{P_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \quad (a)$$

$$W_{c_2} = h_2 - h_b = C_p(T_2 - T_b)$$

$$= C_pT_b\left[\frac{T_2}{T_b} - 1\right] \quad \text{where } T_b = T_1$$

$$= C_pT_1\left[\left(\frac{P_2}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= C_pT_1\left[\left(\frac{P_2}{\sqrt{P_1P_2}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$W_{c_2} = C_pT_2\left[\left(\sqrt{\frac{P_2}{P_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$W_{c_2} = W_{c_1}$$

$$W_c = W_{c_1} + W_{c_2}$$

$$W_c = 2W_{c_1}$$

$$\boxed{W_c = 2C_pT_1\left[\left(\sqrt{\frac{P_2}{P_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

Vapour power cycles (Rankine cycle)

Reason for using water as a working fluid in vapour power cycle.

1) It is cheap.
2) It is chemically stable.
3) It is harmless substance.

Specific steam consumption :→

$$SSC = \frac{\text{mass of steam}}{P_{net}} = \frac{m_s}{P_{net}} \rightarrow \frac{kg/s}{kJ/sec}$$

$$SSC = \frac{m_s}{m_s \times W_{net}} \quad \begin{matrix} m_s \rightarrow kg/s \\ W_{net} \rightarrow kJ/kg \end{matrix}$$

$$SSC = \frac{1}{W_{net}\ kJ/kg} \Rightarrow \frac{kg}{W_{net}\ kJ}$$

$$SSC = \frac{kg}{W_{net}\ \frac{kJ \times sec}{sec}}$$

$$SSC = \frac{kg}{W_{net}\ kW sec}$$

$\because$ 1 hr = 3600 sec

? = 1 sec

$\frac{1}{3600}$ hr = $\frac{1}{3600}$ sec

$$SSC = \frac{kg}{W_{net}\ kW \times \frac{1}{3600}\ hr}$$

$$SSC = \frac{3600\ kg}{W_{net}\ kW\ hr}$$

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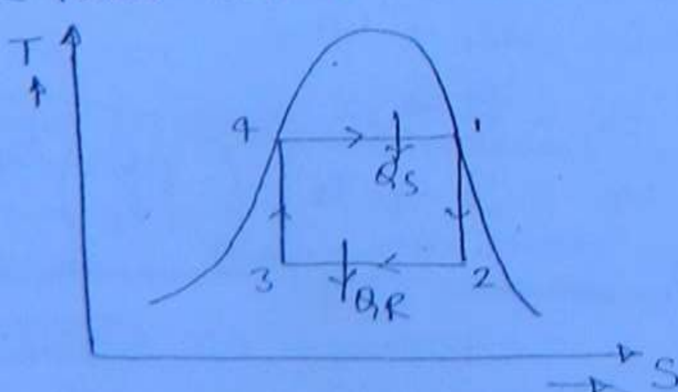
if the net work is more specific steam consumption is less and hence mass flow rate of steam will be less and hence the size of the plant will be small.

1) CARNOT CYCLES ⇒

$$\boxed{W_{net} = W_T - W_C}$$



(4) Drawback of Carnot cycle ⇒

(1) It is difficult to design and control partial condensor.

(2) The design of a compressor which handles a mixture of liquid and vapour is difficult.

(3) As compressor is used the compression work is large and hence the net work is less. $W_{net} = W_T - W_C$

(4) The saturated steam that enters the turbine at point (1) leaves at point (2) which in wet Region. The liquid droplets in high velocity steam cause erosive action on turbine blades.

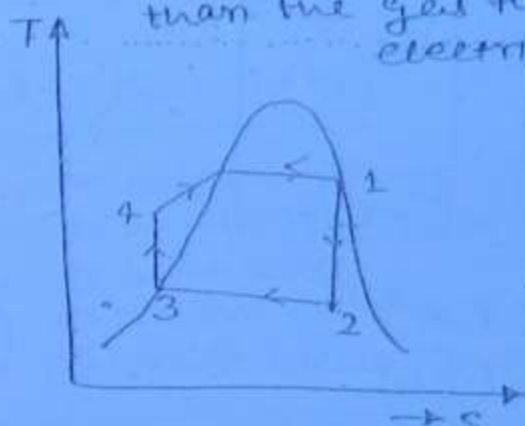
(4) Rankine cycle ⇒

Steam power plant are more popular than the gas turbine plant for electricity generation

$\boxed{W_T >> W_C}$

$W_{net} = W_T - W_P$

$W_P = -\int v\,dP$



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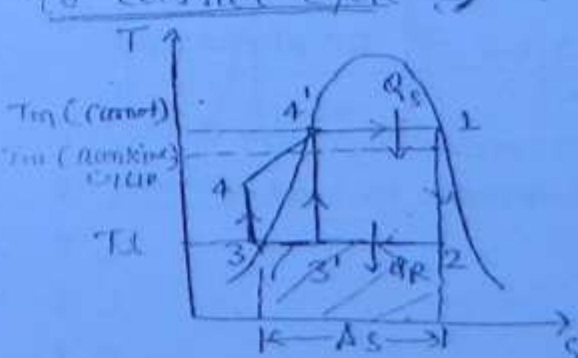
** Carnot cycle → $W_T - W_C\uparrow = W_{net}\downarrow$

Rankine cycle ⇒ $W_T - W_P\downarrow = W_{net}\uparrow$

$$\boxed{\eta = \frac{W_T - W_P}{Q_S} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}}$$

(4) Reason for lower efficiency of Rankine cycle compare to carnot cycle ⇒



$Q_R = T_l\,(\Delta S)$

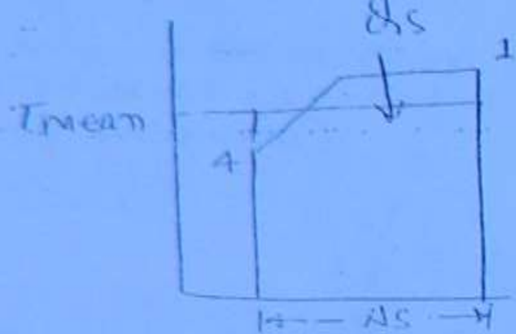
$$\eta = \frac{W_{net}}{Q_s}$$

$$\eta = \frac{Q_s - Q_R}{Q_s}$$

$$\eta = 1 - \frac{Q_R}{Q_s}$$

where $Q_R = T_l (\Delta S)$

and



$Q_s = h_1 - h_4$
$= T_{mean} (\Delta S)$
$Q_s = T_m (\Delta S)$

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$$\eta = 1 - \frac{T_l (\Delta S)}{T_m (\Delta S)}$$

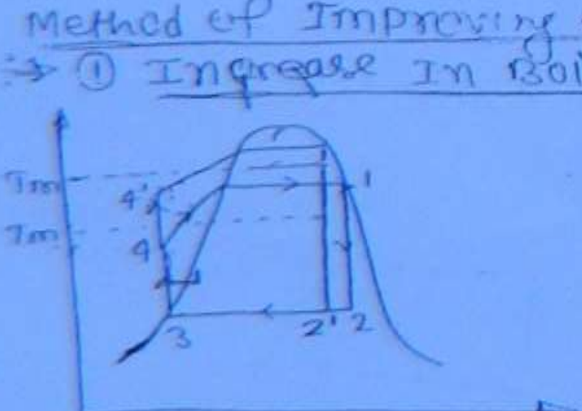
$$\boxed{\eta = 1 - \frac{T_l}{T_m}}$$

* $\boxed{(T_{mean})_{carnot} > (T_{mean})_{Rankine}}$
* $\boxed{\eta_{carnot} > \eta_{Rankine}}$

» Though the net work output of rankine cycle is greater than the net work output of carnot cycle the efficiency of rankine cycle is less because the mean temp. of heat addition in rankine cycle is lower than that in carnot cycle.

» Method of Improving the performance of Rankine cycle

⇒ ① Increase in Boiler Pressure ⇒

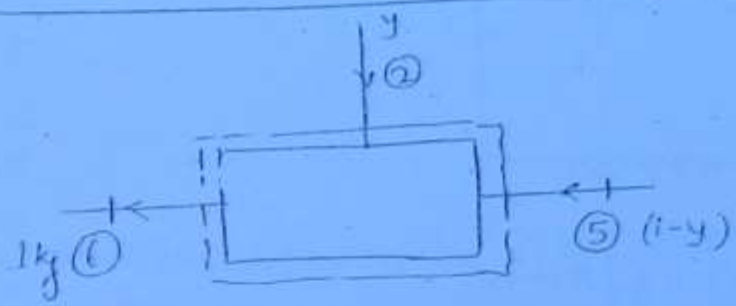


① $W_T \uparrow$ → Increase in Turbine work
② $Q_R \downarrow$ → decrease in Heat addition
③ $W_P \uparrow$
④ $T_m \uparrow$
⑤ $\eta \uparrow$
⑥ $x \downarrow$ dryness fraction is

$Q_s = 1(h_1 - h_7)$

$$\eta = \frac{W_T - W_P}{Q_s}$$

$$\boxed{\eta = \frac{\left[1(h_1 - h_2) + (1-y)(h_2 - h_3)\right] - \left[(1-y)(h_5 - h_4) + 1(h_7 - h_6)\right]}{1(h_1 - h_7)}}$$



$$\boxed{y \times h_2 + (1-y)h_5 = 1 \times h_6}$$ ** imp. eqn.

Prob-① A carnot steam cycle operate between a source temp of 311.06°C equal to 10 MPa and sink temp. of 32.88°C (P = 5KPa) find ① work ratio.
② Thermal efficiency.
③ specific steam consumption.

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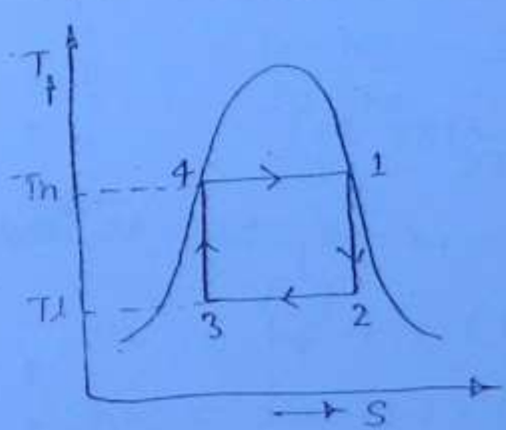
pressure	tsat °C	h_F kJ/kg	h_g (kJ/kg)	S_F (kJ/kg)	S_g (kJ/kg)
10 MPa	311.06	1407.56	2724.7	3.3596	5.6141
5 MPa	32.88	137.82	2561.2	0.4764	8.3951

Soln



$T_h = 311.06 + 273 = 584.06$

$T_l = 32.88 + 273 = 305.88$

$$\eta = \frac{T_h - T_l}{T_h}$$

$$\eta = 1 - \frac{T_l}{T_h}$$

$$\eta = 1 - \frac{305.88}{584.06} = 47.67\%$$

$$\eta = \frac{W_{net}}{Q_s}$$

$$Q_s = h_1 - h_4$$

$$= 2724.7 - 1407.56$$

$$Q_s = 1317.14$$

$$W_{net} = \eta \times Q_s$$

$$W_{net} = 0.476 \times 1317.14 = 627.8$$

$$SSC = \frac{3600}{627.8} = 5.739 \text{ Kg/Kw-hr}$$

$$\text{work Ratio} = \frac{W_{net}}{W_T}$$

where $W_T = h_1 - h_2$

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$$h_2 = hf + x\,(hg - hf)$$

$$hf = 137.82 \quad \text{at } 5\,\text{mpa}$$

$$hg = 2561.62 \quad \text{at } 5\,\text{mpa}$$

$$S_1 = S_2$$

$$S_2 = S_f + x\,(S_g - S_f)$$

$$5.6141 = 0.4764 + x\,(8.3951 - 0.4764)$$

$$\boxed{x = 0.648}$$

$$h_2 = 137.82 + 0.648 \times (2561.62 - 137.82)$$

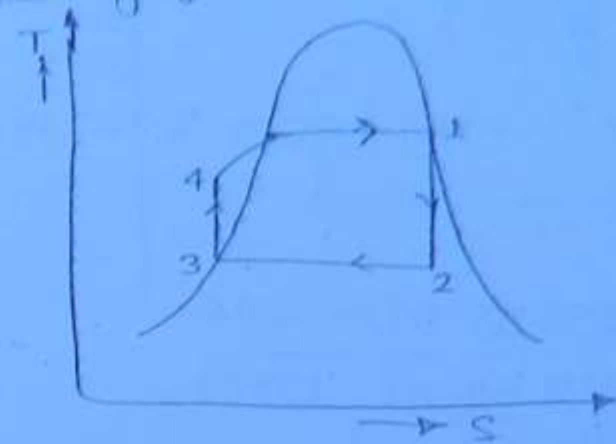
$$h_2 = 1708.3 \text{ KJ/kg}$$

$$W_T = 2724.7 - 1708.3 = 1016.3$$

$$r_w = \frac{627.8}{1016.3}$$

$$\boxed{r_w = 0.618} \quad \text{Ans:}$$

PROB-12 water is the working fluid in Rankine cycle saturated vapour enters the turbine at 2 mpa the condensor pressure is 10 kPa the pump work is Negligible.



Soli Heat supplied $Q_S = h_1 - h_4$ (because pump work is Negligible

$W_P = h_4 - h_3$ (but pump work is Negligible

so that $W_P = h_4 = h_3$

(02)

$$\eta = \frac{W_{net}}{Q_S}$$

P (kPa)	tsat °C	Vf (m³/kg)	Vg (m³/kg)	hf = (kJ/kg)	hg (kJ/kg)	Sf kJ/kg	Sg kJ/kg
2000	212.42	0.001177	0.09963	908.77	2799.51	2.447	6.3408
10	45.81	0.00101	14.6735	191.81	2584.63	0.6493	8.1501

$$\eta = \frac{W_T - W_P^{\to 0} \text{ (Negligible)}}{Q_S}$$

$$\eta = \frac{h_1 - h_2}{h_1 - h_4} \quad \text{—— (1)}$$

$h_1 = 2799.51$, ~~hg = 2584~~

10 kPa at $h_f = h_4 = h_3 = 191.81$, $h_g = 2584.63$

$h_2 = h_f' + x(h_g - h_f)$

$h_2 = 191.81 + x(2584.63 - 191.81)$

but $S_1 = S_2 = 6.3408$ at 2000 kPa

$S_2 = S_f + x(S_g - S_f)$

$$6.3408 = 0.6492 + x(8.15101 - 0.6492)$$

$$\boxed{x = 0.758}$$

$$h_2 = 191.81 + (0.758)[2584.63 - 191.81]$$

$$h_2 = 2006$$

now

$$\eta = \frac{2799.51 - 2006}{2799.51 - 191.81}$$

$$\eta = 30.4\% \text{ — } \underline{\text{Ans}}$$

QB-(3)
Consider a steam power plant using A Reheat ycle as shown in fig. steam leaves the boiller enter the urbine at 4 MPa, 350°C (h_3 = ~~3095~~ 3095 KJ/Kg). After xpansion in the turbine to 400 kPa (h_4 = 2609 KJ/Kg) he steam is Reheated to 350°C (h_5 = 3170 KJ/Kg) and hen expanded in a pressure Turbine to (100 kPa) h_6 = 2165 KJ/Kg find out. the enthalpy at the pump Discharge h_2 is. (ii) The efficiency of the plant is

(A) 19.3
(B) 16.3
(C) 22.3
(D) 33.3

$W_P = 0$
$h_2 - h_1 = 0$
$h_1 = h_2$

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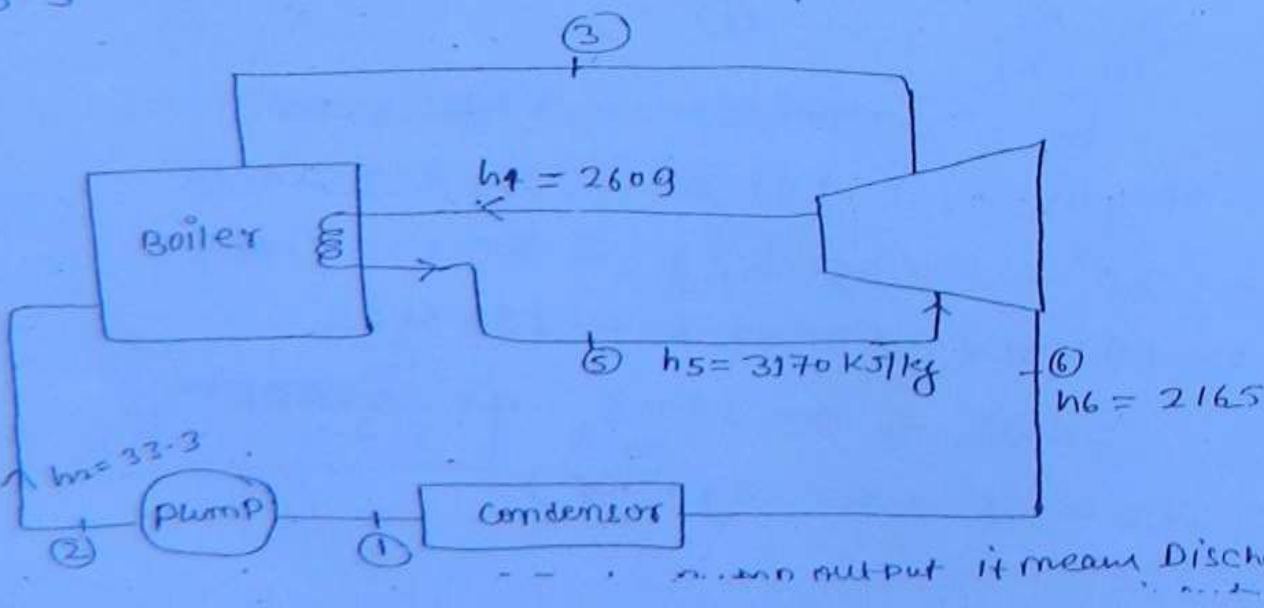
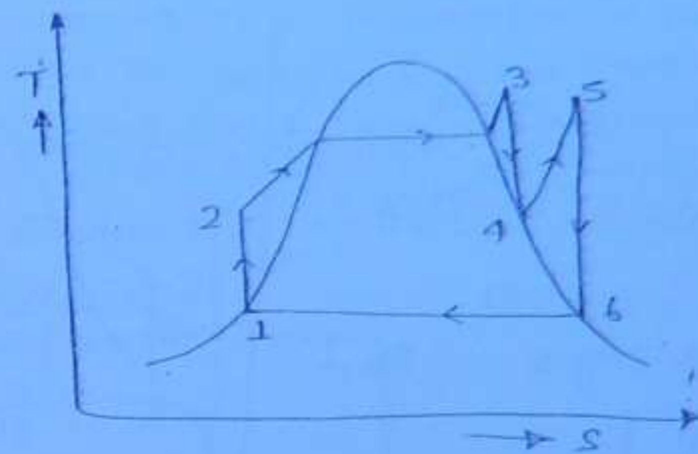


... no output it means Discharge



$$\eta = \frac{W_T - \cancel{W_P}^{\to 0}}{Q_S}$$

$$\eta = \frac{(h_3 - h_4) + (h_5 - h_6) - \cancel{(h_2 - h_1)}^{\,0}}{(h_3 - h_2) + (h_5 - h_4)}$$

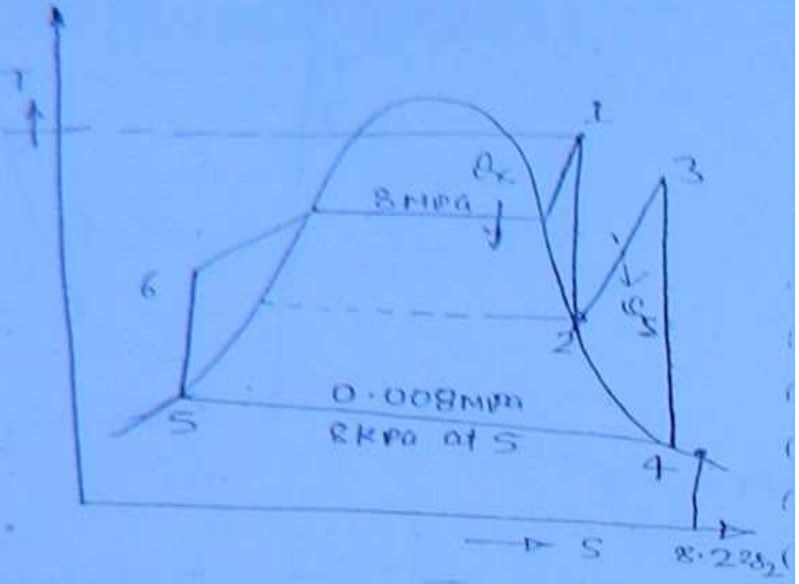
$$\eta = \frac{(3095 - 2609) + (3170 - 2165)}{(3095 - 29.31) + (3170 - 2609)}$$

(109)

$\eta = 41.08\%$ Ans.

PROB. (4) Steam is working fluid in Rankine cycle with superheat and Reheat steam enters the first stage of Turbine at 8 MPa, 480°C superheated state $h = 3548$ kJ/kg, $s = 6.6586$ kJ/kg-K and expanse to 0.7 MPa ($s_f = 1.9922$ kJ/kg-K) $s_g = 6.708$ kJ/kg, $h_f = 697.22$, $h_{fg} = 2066.3$ kJ/kg) it is then reheated to 440°C before entering the second stage turbine which is in superheat state at 0.7 MPa, 440°C ($h = 3353.3$; $s = 7.7571$) where it expand to condenser pressure of 0.008 MPa ($s_g = 0.5926$) $s_g = 8.2287$ $h_f = 173.88$, $h_{fg} = 2403.1$
the net power output is 100 Mwatt. Find the

1. efficiency of the cycle
2. mass flow rate of steam in kg/hr
3. take specific volume of saturated liquid at 0.008 MPa is 1.0085×10^{-3} m³/kg



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$s_2 = s_1 = 6.6586$ kJ/kg-K

$= 3353.8$ $\quad s_3 = 7.7571$

$g = 6.708$ kJ/kg K $> s_2$

$\boxed{= 2741.88}$

$= 2428.56$

$5 = 173.888$

oint 2 in wet Region

$s_2 = s_f + x_2 (s_g - s_f)$

$6586 = 1.9922 + x_2 (6.708 - 1.9922)$

$x_2 = 0.989$

$h_2 = h_f + x\, h_{fg} = 697.22 + 0.989\,(2066.3)$

$h_2 = 2741.88$

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$s_3 = s_4 = 7.7541$ kJ/kg-K

$< s_g\,(8.2287) \Rightarrow$ point 4 in wet Region.

$s_4 = s_f + x_4 (s_g - s_f)$

$7.7571 = 0.5926 + x_4 (8.2287 - 0.5926)$

$\boxed{x_4 = 0.938}$

$h_4 = h_f + x_4 (h_{fg}) \Rightarrow 173588 + 0.938\,(2403.1)$

$\boxed{h_4 = 2428.56 \text{ kJ/kg}}$

$W_P = -\int v\,dP \Rightarrow -1.0085 \times 10^{-3} \frac{m^3}{kg} (8000 - 8) \frac{kN}{m^2}$

$\Rightarrow -8.06$ kJ/kg

$W_P = h_6 - h_5 = 8.06$ kJ/kg

$= h_6 - 173.88 = 8.06 \qquad h_6 = 181.94$ kJ/kg

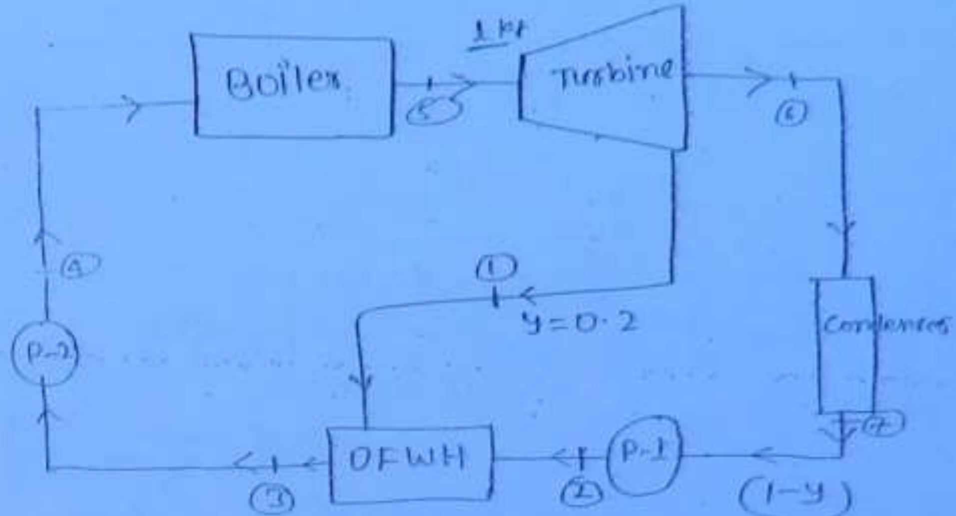
$\eta = \frac{W_{net}}{Q_S} \Rightarrow \frac{(h_1 - h_2) + (h_3 - h_4) - (W_P)}{(h_1 - h_6) + (h_3 - h_2)}$

$\eta = 0.4052 \Rightarrow 40.52\%$

(i) $Q_R = h_4 - h_5$ (ii) $W_{net} = (h_1 - h_2) + (h_3 - h_4) - W_T$

PROB: A thermal power plant operate on a regenerative cycle with open feed water heater As shown in fig for the state point shown enthalpy's at 1 and 2 are $h_1 = 2800$ kJ/kg $h_2 = 200$ kJ/kg. the bleed of feed water heater is 20% of steam generation rate then find the enthalpy at state 3

Soln:



$$y \times h_1 + (1-y)\, h_2 = y \times h_3$$

$$0.2 \times 2800 + (1-0.2) \times 200 = h_3 \times 1$$

$$\boxed{h_3 = 720 \text{ kJ/kg}}$$

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PROB 5: A steam Regenerative Rankine cycle at steam entering into Turbine 200 bar, 650°C superheated state and leaving at 0.05 bar considering the open feed water Heater into open type determine the efficiency of the plant where the feed water heater is operating at 8 bar.

At 8 bar $h_f = 721.11$, $h_{fg} = 2048$, $v_f = 0.001115$ m³/kg

At 200 bar, 650°C $h_1 = 3675.3$, $S_1 = 6.6582$

At 8 bar $S_f = 2.0462$ kJ/kg-K

$S_{fg} = 4.6166$ kJ/kg-K

Ans: $\eta = 41.5$[illegible]

Y = [illegible]

At 0.05 bar $h_f = 137.82$

$h_{fg} = 2423.7$

$v_f = \cancel{0.4764}$ 0.001005 m³/kg

$S_f = 0.4764$

S_{fg} = $\cancel{8.3951}$ 7.9187

$s_g - s_f = s_{fg}$

$s_g - s_f = 4.6166$

$s_g = s_f + 4.6166$

$= 2.0462 + 4.6166$

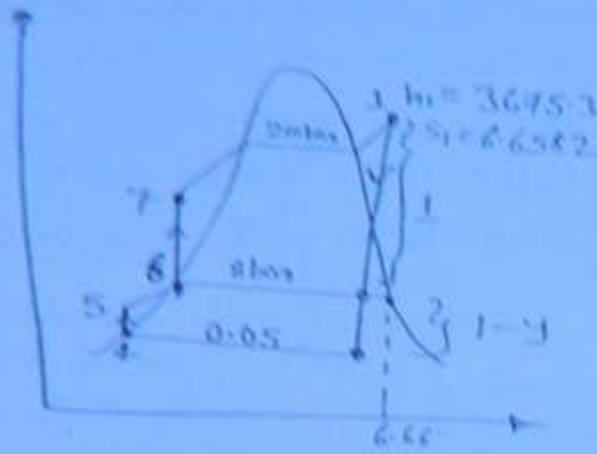
$s_g = 6.6628$

$h_2 = 2767.07$ KJ/kg

$h_3 = 2029.9$ KJ/kg

$h_4 = 137.82$ KJ/kg

$h_6 = 721.11$ KJ/kg.



Pump 5 — 4: 8 bar $= 8 \times 100$ kPa; 0.05 bar $= 0.05 \times 100$ kPa

$v = 0.001005$

when $W_p = V\,dp$ because already negative work

$W_p = 0.001005 \times (8 - 0.05) \times 100$

$W_p = 0.75375$ KJ/kg

$W_{P1} = h_5 - h_4$

$0.75375 = h_5 - 137.82$

$h_5 = 138.62$ KJ/kg

assume after open feed water leaves as saturated liquids

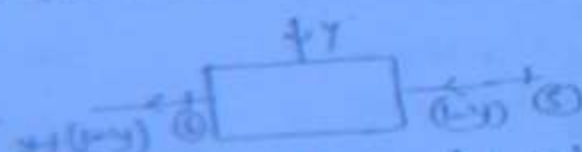
$\boxed{h_6 = 721.11 \text{ KJ/kg}}$

$W_{P2} = -\int V\,dp \Rightarrow 0.001115 \frac{m^3}{kg} \times (200 - 8) \times 100\ KN/m^2$

$W_{P2} = 21.408$ KJ/kg

$W_{P2} = h_7 - h_6$

$21.408 = h_7 - 721.11 \Rightarrow \boxed{h_7 = 742.518 \text{ KJ/kg}}$



$y \times h_2 + (1-y) \times h_5 = 1 \times h_6$

$y = 0.2216$

$W_T = 1 \times (h_1 - h_2) + (1-y)(h_2 - h_3)$

$\boxed{W_T = 1481}$ kg/m²

$W_P = (1-y) W_{P1} + 1 \times W_{P2}$ $\quad W_P = 22.07$ kg/m²

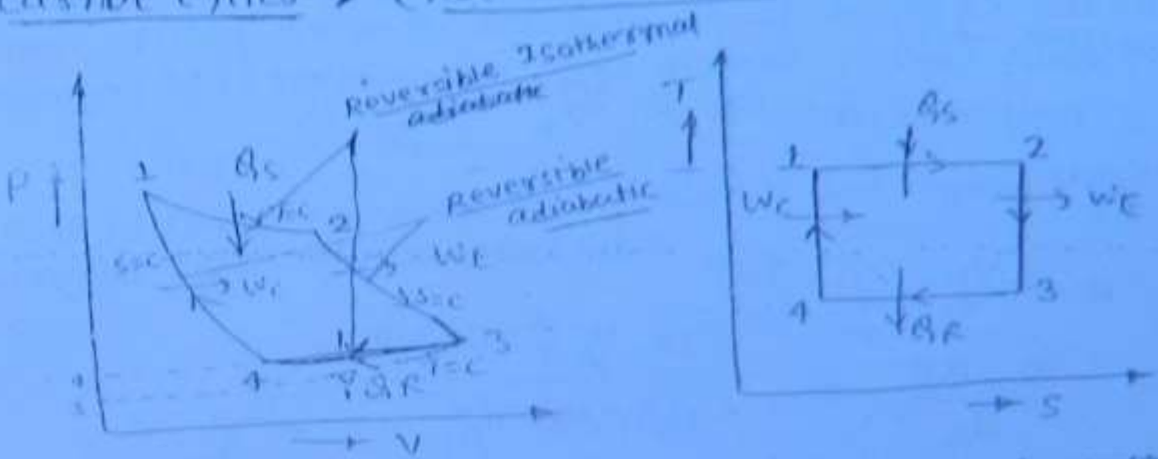
$Q_s = h_1 - h_7 \Rightarrow 2932.782$

$\eta = \frac{W_{net}}{Q_s} = 42.7\%$ Ans

Gas power cycles

gas power cycles are used in gas as a working fluid - it does not undergo any phase change - Engines operating on gas cycles may be either cyclic or non-cyclic. Hot air engine using air as the working fluid operate on a closed cycle. Internal combustion engine where the combustion of fuel take place inside the engine cylinder or non-cyclic heat engine

(1) Carnot cycles → (Reversible cycles) (1824)



1-2 → Isothermal heat addition (compression)

2→3 → Adiabatic expansion (expansion)

3-4 → Isothermal heat rejection

4-1 → Adiabatic compression.

- This cycle in two Reversible isothermal and two Reversible adiabatic.

→ If an ideal gas is assumed as the working fluid Then for 1 kg of gas

$Q_{1-2} = RT_1 \ln \frac{V_2}{V_1}$	$W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$
$Q_{2-3} = 0;$	$W_{2-3} = 0 - C_v (T_3 - T_2)$
$Q_{3-4} = RT_2 \ln \frac{V_4}{V_3}$	$W_{3-4} = RT_2 \ln \frac{V_4}{V_3}$
$Q_{4-1} = 0;$	$W_{4-1} = -C_v (T_1 - T_4)$.

$$\sum_{cycle} dQ = \sum_{cycle} dW$$

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now
$$\frac{V_2}{V_3} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

and
$$\frac{V_1}{V_4} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_2}{V_3} = \frac{V_1}{V_4} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Therefore
$$Q_S = \text{Heat added} = RT_1 \ln \frac{V_2}{V_1}$$
$$Q_R = \text{Heat rejected} = RT_2 \ln \frac{V_3}{V_4}$$

$$W_{net} = Q_S - Q_R$$
$$= R \ln \frac{V_2}{V_1} \cdot (T_1 - T_2)$$

$$\eta_{cycle} = \frac{Q_S - Q_R}{Q_S} = \frac{R \cancel{\ln \frac{V_2}{V_1}} (T_1 - T_2)}{R T_1 \cancel{\ln \frac{V_2}{V_1}}}$$

$$\boxed{\eta_{cycle} = \frac{T_1 - T_2}{T_1}} \Rightarrow \boxed{\frac{T_h - T_l}{T_h}}$$

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In a carnot gas cycle is big drawback because W_{4-1} process compressor work is more. so more power will consume then less power output.

1) The efficiency of all the Reversible engine working between same temp. limit is same.

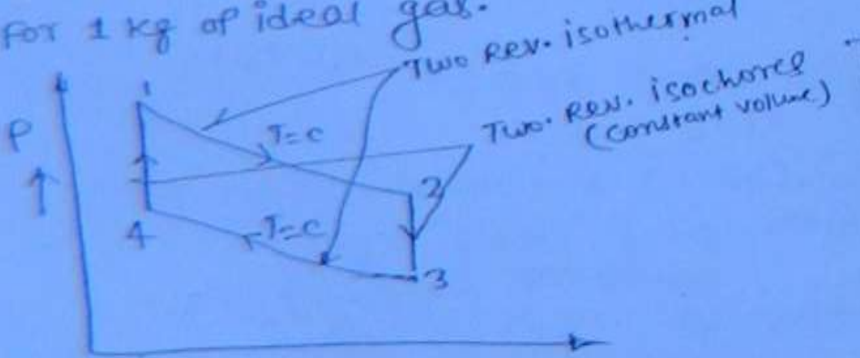
2) efficiency of a Reversible cycle depend only on Temp. limit.

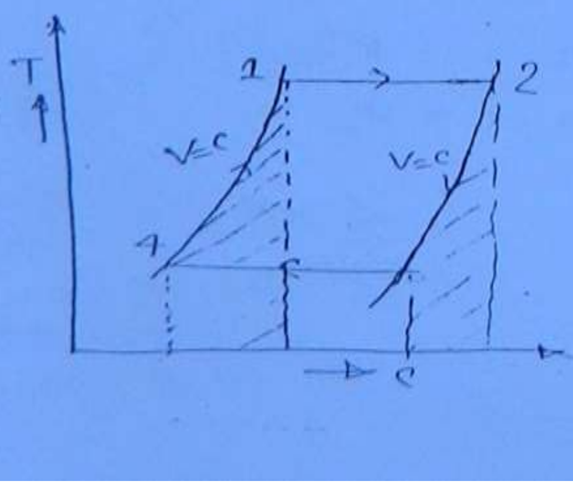
3) efficiency of a Reversible cycle independent of working fluid.

STIRLING CYCLE (1827) →

1) Two Reversible isothermal and two reversible isochores. (two Rev. constant volume process)

For 1 kg of ideal gas.





$$q_{1-2} = W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = -C_V(T_2 - T_1) \; ; \; W_{2-3} = 0$$

$$Q_{3-4} = W_{3-4} = RT_2 \ln \frac{V_3}{V_4}$$

$$Q_{4-1} = C_V(T_1 - T_2) \; ; \; W_{4-1} = 0$$

Due to heat transfer at constant volume processes, the efficiency of the stirling cycle is less than that of the carnot cycle. However if a regenrative arrangement is used such that

$$Q_{2-3} = Q_{4-1}$$

that is the area under 2-3 is equal to the area under 4-1, then the cycle efficiency becomes:

$$\eta = \frac{RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4}}{RT_1 \ln \frac{V_2}{V_1}}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}} \quad \text{Ans.}$$

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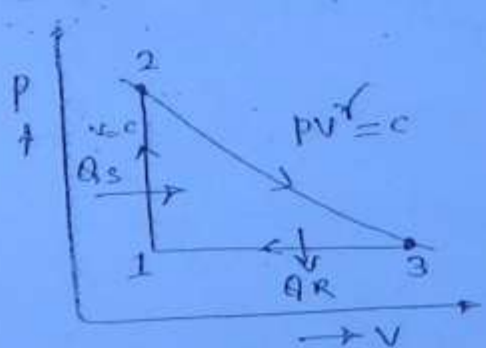
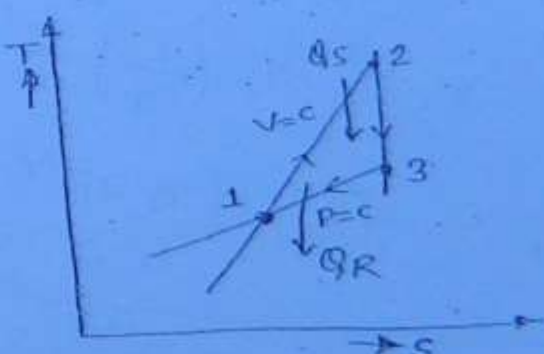
So the regenerative stirling cycle has the same efficiency as the carnot cycle.

LENOIR CYCLE → This cycle is applicable to pulse jet engine





1-2 → Constant volume heat addition.

2-3 → Reversible adiabatic expansion.

3-1 → Constant pressure heat rejection.

∴ by eqn

$$Q_1 = C_V(T_2 - T_1)$$

$$Q_2 = C_P(T_3 - T_1)$$

$$\eta_{cycle} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{C_P(T_3 - T_1)}{C_V(T_2 - T_1)}$$

where $\gamma = \frac{C_P}{C_V}$

$$\eta_{cycle} = 1 - \frac{\gamma(T_3 - T_1)}{(T_2 - T_1)}$$

using $r_P \rightarrow$ pressure ratio

$$r_P = \frac{P_2}{P_1}, \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore T_2 = T_1 \frac{P_2}{P_1}$$

$$\boxed{T_2 = r_P \cdot T_1}$$

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$$\frac{T_3}{T_2} = \left(\frac{P_2}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

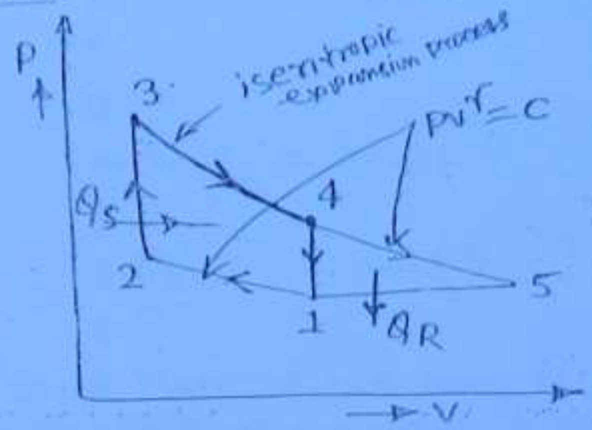
$$\therefore T_3 = T_2\left(\frac{1}{r_P}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_1 r_P \left(\frac{1}{r_P}\right)^{\frac{\gamma-1}{\gamma}}$$

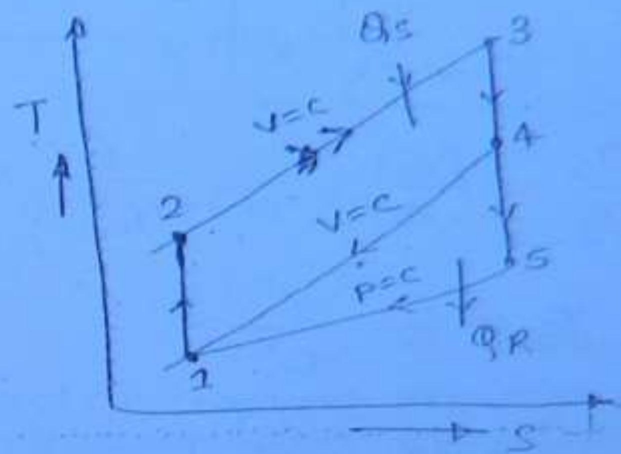
$$\eta_{Lenoir} = 1 - \frac{\gamma\left[\cancel{T_1} r_P^{1/\gamma} - T_1\right]}{\cancel{T_1} r_P - T_1}$$

$$\boxed{\eta_{Lenoir} = 1 - \gamma\left(\frac{r_P^{1/\gamma} - 1}{r_P - 1}\right)}$$

Thus the cycle efficiency depend on the pressure ratio and the specific heat ratio.

(1) ATKINSON CYCLE → Atkinson cycle is an ideal cycle for an otto engine exhausting to a gas turbine. In this cycle the isentropic expansion (3-4) process, of an otto cycle is allowed to further expand to the lowest cycle pressure (3-5) so as to increases the work output.





1-2-3-4-1 — otto cycle

1-2-3-5-1 — Atkinsion cycle

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For 1 kg gas, $Q_S = C_v(T_3 - T_2)$

$Q_R = C_p(T_5 - T_1)$

$$\eta_{cycle} = 1 - \frac{C_p(T_5 - T_1)}{C_v(T_3 - T_2)} \Rightarrow 1 - \frac{\gamma(T_5 - T_1)}{(T_3 - T_2)} \quad \text{—— (a)}$$

Let r_k → compression ratio $= \frac{V_1}{V_2}$

r_e → expansion ratio $= \frac{V_5}{V_3}$

$$\frac{T_2}{T_1} = \frac{V_1}{V_2} \qquad \therefore \; T_2 = T_1 r_k^{\gamma-1} \quad \text{—— (b)}$$

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = \frac{P_3}{P_5} \times \frac{P_5}{P_2} = \frac{P_3}{P_5} \times \frac{P_1}{P_2}$$

$$\frac{P_3}{P_5} = \left(\frac{V_5}{V_3}\right)^{\gamma} = r_e^{\gamma}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{1}{r^{\gamma}}$$

$$\therefore \quad T_3 = T_2 \cdot r_e^{\gamma} \cdot \frac{1}{r_k^{\gamma}}$$

$$= T_1 \cdot r_k^{\gamma-1} \cdot r_e^{\gamma} \cdot \frac{1}{r_k^{\gamma}}$$

$$\boxed{T_3 = T_1 \cdot \frac{r_e^{\gamma}}{r_k}} \quad \text{———} \quad (c)$$

$$\frac{T_5}{T_3} = \left(\frac{V_3}{V_5}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$T_5 = T_3 \cdot \frac{1}{r_e^{\gamma-1}}$$

$$= T_1 \cdot \frac{r_e^{\gamma}}{r_k} \cdot \frac{1}{r_e^{\gamma-1}}$$

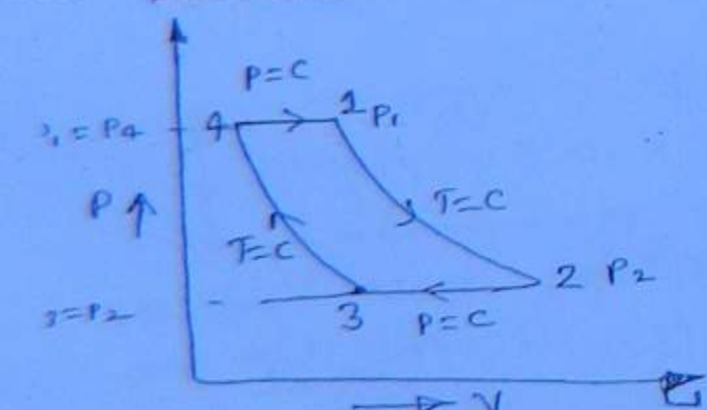
$$\boxed{T_5 = T_1 \cdot \frac{r_e}{r_k}} \quad \text{———} \quad (d)$$

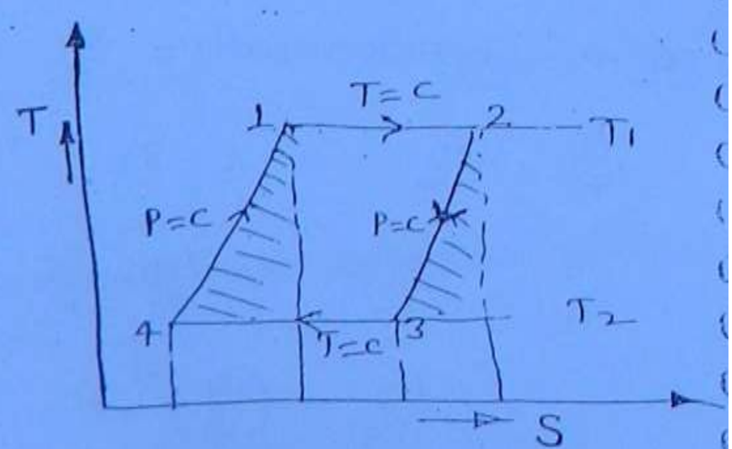
Substituting T_2, T_3, T_5 from eqn (b), (c) (d)

$$\eta_{Atkinson} = 1 - \gamma \frac{T_1 \cdot \frac{r_e}{r_k} - T_1}{T_1 \frac{r_e^{\gamma}}{r_k} - T_1 r_k^{\gamma-1}}$$ (11.8)

$$\boxed{\eta_{Atkinson} = 1 - \gamma \frac{r_e - r_k}{r_e^{\gamma} - r_k^{\gamma}}}$$

7) ERICSSON CYCLE →





Two Reversible isothermal and two Reversible isobars.

For 1 kg of ideal gas.

$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{P_1}{P_2}$$

$$Q_{2-3} = C_p(T_2 - T_1) \quad ; \quad W_{2-3} = P_2(V_3 - V_2) = R(T_2 - T_1)$$

because $T_2 = T_3$

$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{P_1}{P_2}$$

$$Q_{4-1} = C_p(T_1 - T_4) \quad ; \quad W_{4-1} = P_1(V_1 - V_4) = R(T_1 - T_2)$$

$T_4 = T_2$

since part of the heat is transferred at constant pressure and part at constant temp. the efficiency of the Ericssion cycle is less than that of the carnot cycle.

But with ideal regeneration $Q_{2-3} = Q_{4-1}$ so that all the heat is added from the external source at T_1 and all the heat is rejected to an external sink at T_2, the efficiency of the cycle becomes equal to the carnot cycle efficiency =

$$\eta = 1 - \frac{Q_2}{Q_1} \Rightarrow 1 - \frac{RT_2 \ln \frac{P_1}{P_2}}{RT_1 \ln \frac{P_1}{P_2}}$$

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$$\boxed{\eta \Rightarrow 1 - \frac{T_2}{T_1}}$$

The regenerative, stirling and Ericsson cycle have the same efficiency as the carnot cycle, but much less back work. Hot air engine working on these cycle have been successfully operated. But it is difficult to transfer heat to a gas at high rates. since the gas film has a very low thermal conductivity, so there has not been much progress in the development of hot air engines. However, since the cost of I.C. engine fuel is getting excessive, these may find a field of use in the near future.

Q. In a gas turbine plant air at pressure P_1 and Temp. T_1 is compressed to a pressure of $R P_1$ and then eated to a temp. T_3 at constant pressure. The air is then xpanded in a two stage Turbine such that pressure ratio s same in both stages. Air is Reheated between turbine age to Temp. T_3. Assuming that working fluid is perfect as having Ratio of specific heat γ and at that compressing nd both expansion to be isentropic. Show that net output er kg of working fluid will be maximum when R is equal o —

$$R = \left(\frac{T_3}{T_1}\right)^{\frac{2\gamma}{3(\gamma-1)}} = \left(\frac{P_2}{P_1}\right)$$

soln

$$W_{net} = (h_b - h_4) + (h_3 - h_a) - (h_2 - h_1)$$

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$$= C_p\left[(T_b - T_4) + (T_3 - T_a) - (T_2 - T_1)\right]$$

$$= C_p\left[T_4\left(\frac{T_b}{T_4} - 1\right) + T_3\left(1 - \frac{T_a}{T_3}\right) - (T_2 - T_1)\right]$$

$T_5 = T_3$

$$W_{net} = (h_5 - h_6) + (h_3 - h_4) - (h_2 - h_1)$$

$$= C_p (T_5 - T_6) + (T_3 - T_4) - (T_2 - T_1)$$

$$= C_p\left[T_5\left(1 - \frac{T_6}{T_5}\right) + T_3\left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)\right]$$

$$\frac{P_2}{P_i} = \frac{P_i}{P_1}$$ [equal pressure ratio in expansion of both Stages].

$$\Rightarrow \frac{RP_1}{P_i} = \frac{P_i}{P_1}$$

$$\Rightarrow P_i^2 = RP_1^2 \Rightarrow \boxed{P_i = \sqrt{R}\, P_1}$$

$T_3 = T_5$

$$W_{net} = C_p\left[\left\{T_3\left(1 - \left(\frac{P_1}{P_i}\right)^{\frac{\gamma-1}{\gamma}}\right\} + T_3\left\{1 - \left(\frac{P_i}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right\} - T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right]$$

Assume $\frac{\gamma-1}{\gamma} = K$

$$W_{net} = C_p\left[T_3\left\{1-\left(\frac{1}{\sqrt{R}}\right)^K\right\} + T_3\left\{1-\left(\frac{1}{\sqrt{R}}\right)^K\right\} - T_1\left\{R^K - 1\right\}\right.$$

For max. work

$$\frac{dW_{net}}{dR} = 0$$

$$W_{net} = C_p\left[-2T_3\left(-\frac{K}{2}\right)R^{-\frac{K}{2}-1} - T_1 \cdot K\, R^{K-1}\right.$$

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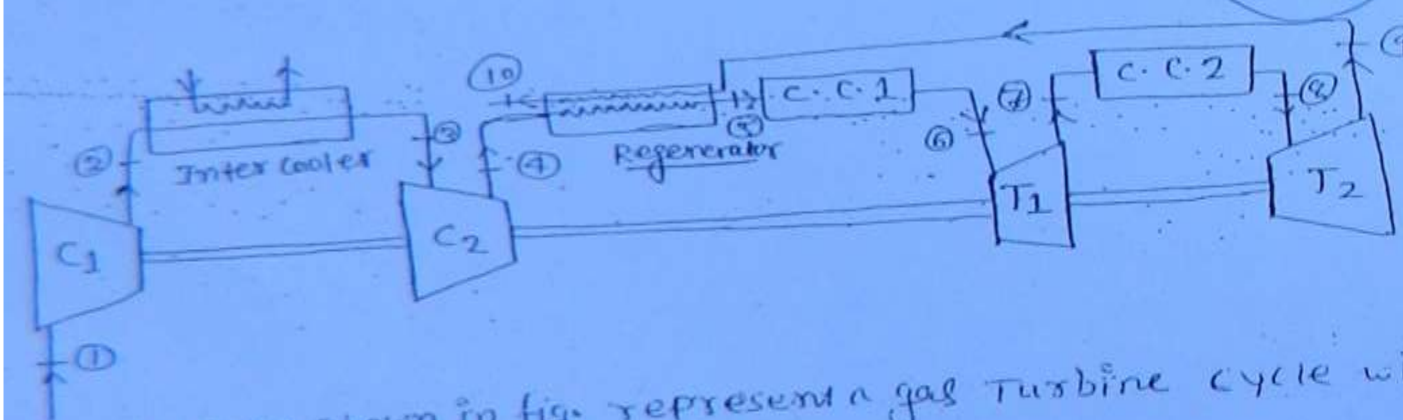
2 A Regenerative and reheat cycle have air entering at 1 bar, 300K into compresser, having intercooling between 2 stage of compression Air leaving first stage of compression is cooled to 290K, at 4 bar in intercooler and subsequential compressed to 8 bar. Compressed air leaving 2nd stage compresser is passed through a regenerated or having effectiveness of 0.8. subsequential combustion field 1300K at inlet to the turbine having expression upto 4 bar. and than reheated upto 1300K before being expanded upto 1 bar. Exhausted from Turbine is passed through regenerator before discharge out of cycle. For fuel having calorific value of 42000 KJ/kg find

1) Fuel-Air ratio in each combustion chamber.
2) Total turbine work.
3) Thermal efficiency.

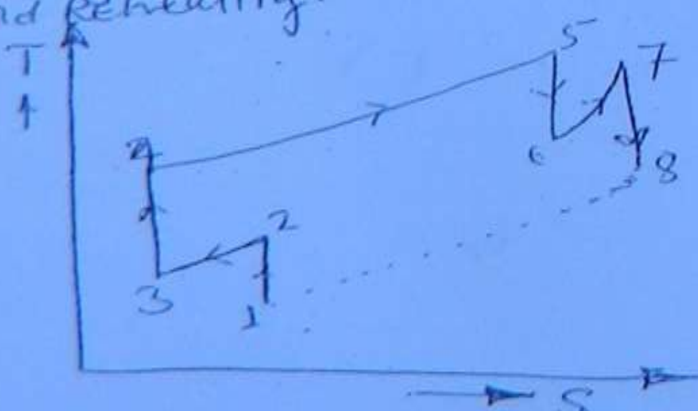
Consider compression and expansion to be and air is working fluid throughout the cycle. $C_{p\,air} = 1.005$ KJ/kg-K.

Soln <u>Al Ready Solved</u>

(124)



Q.3 The cycle shown in fig. represent a gas Turbine cycle with Intercooling and Reheating.



list 1		list II
(1) Inter cooler	$\Rightarrow$	2 - 3
(2) combustor	$\Rightarrow$	4 - 5
(3) Reheater	$\Rightarrow$	6 - 7
(4) High pressure compressor	$\Rightarrow$	3 - 4

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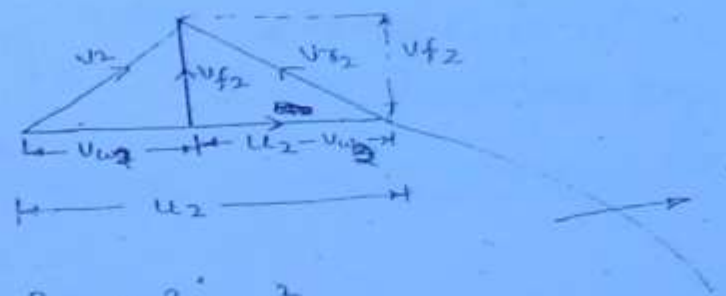

$$V_2^2 = V_{f_2}^2 + V_{w_2}^2$$

$$\Rightarrow V_{f_2}^2 = V_2^2 - V_{w_2}^2$$

$$V_{r_2}^2 = V_{f_2}^2 + (u_2 - V_{w_2})^2$$

$$V_{f_2}^2 = V_{r_2}^2 - (u_2 - V_{w_2})^2$$

$$\therefore V_2^2 - V_{w_2}^2 = V_{r_2}^2 - (u_2 - V_{w_2})^2$$

$$= V_{r_2}^2 - [u_2^2 + V_{w_2}^2 - 2u_2 V_{w_2}]$$

$$= V_{r_2}^2 - u_2^2 - V_{w_2}^2 + 2u_2 V_{w_2}$$

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$$\Rightarrow V_2^2 - V_{r_2}^2 + u_2^2 = 2u_2 V_{w_2}$$

$$\Rightarrow V_{w_2} u_2 = \frac{1}{2}[V_2^2 - V_{r_2}^2 + u_2^2]$$

similarly

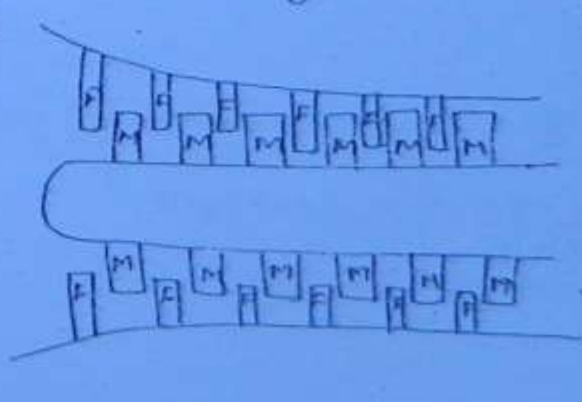
$$V_{w_1} u_1 = \frac{1}{2}[V_1^2 - V_{r_1}^2 + u_1^2]$$

$$\therefore \quad W = V_{w_2} u_1 - V_{w_1} u_1$$

$$\boxed{W = \frac{1}{2}\left[(V_2^2 - V_1^2) + (V_{r_1}^2 - V_{r_2}^2) + (u_2^2 - u_1^2)\right]}$$

Note ⇒

If the fluid is entering and leaving at the same diameter then $\boxed{u_1 = u_2}$



centrifugal compressor are NOT suitable for multi staging but axial flow compressor are suitable for multistaging.

~~supercharging~~ centrifugal comp. are used in superchasig. Axial flow compressor are used in gas turbine plants. in super charging.

) as entry and exit is occuring at the same diameter

$\boxed{u_1 = u_2}$. $\quad \because W = V_{w_2} u_2 - V_{w_1} u_1$

$$\boxed{W = (V_{w_2} - V_{w_1}) u_1} \qquad [u_1 = u_2 = u]$$

⇒ To Reduce the axial thrust on Bearings V_{f_1} is taken as equal to V_{f_2}

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work/stage $\quad W = \frac{1}{g}\left[(V_2^2 - V_1^2) + \frac{1}{g}(V_{r_1}^2 - V_{r_2}^2)\right]$

Fixed Blade (Absolute Velocity)

$$\boxed{\underbrace{(V_{w_2} - V_{w_1})\, u}_{\substack{m+F \\ \Downarrow \\ \text{work/stage}}} = \frac{1}{g}(V_2^2 - V_1^2) + \frac{1}{g}(V_{r_1}^2 - V_{r_2}^2)}$$

↳ moving Blade then (Relative velocity)

Degree of Reaction (R) ⇒

It is the ratio of enthalpy rise in moving blade to the enthalpy rise in stage.

$$R = \frac{(\Delta h)_m}{(\Delta h)_{stage}}$$

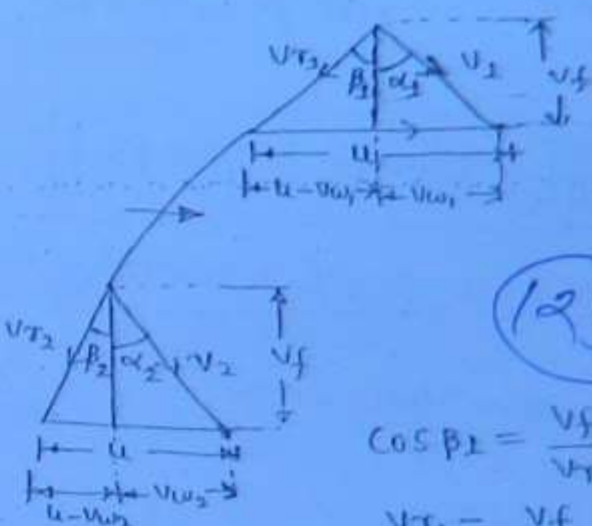
$$\boxed{R = \frac{(\Delta h)_m}{(\Delta h)_M + (\Delta h)_F}}$$

$$R = \frac{\frac{V_{r_1}^2 - V_{r_2}^2}{2}}{(V_{w_2} - V_{w_1})u} \Rightarrow \frac{V_{r_1}^2 - V_{r_2}^2}{2u(V_{w_2} - V_{w_1})}$$

V.V.Imp ***

$$\boxed{R = \frac{V_{r_1}^2 - V_{r_2}^2}{2u(V_{w_2} - V_{w_1})}}$$

(ii) Construct a velocity Triangle ⇒



$$W = (V_{w_2} - V_{w_1})u$$

$$\tan\beta_1 = \frac{u - V_{w_1}}{V_f}$$

$$V_f \tan\beta_1 = u - V_{w_1}$$

$$V_{w_1} = u - V_f \tan\beta_1$$

$$V_{w_2} = u - V_f \tan\beta_2$$

$$\cos\beta_1 = \frac{V_f}{V_{r_1}}$$

$$V_{r_1} = \frac{V_f}{\cos\beta_1}$$

$$\boxed{V_{r_1} = V_f \sec\beta_1}$$

Similarly $\boxed{V_{r_2} = V_f \sec\beta_2}$

***

$$W/\text{stage} = \left[(\cancel{u} - V_f \tan\beta_2) - (\cancel{u} - V_f \tan\beta_1)\right]u$$

$$W/\text{stage} = \left[V_f \tan\beta_1 - V_f \tan\beta_2\right]u$$

$$\boxed{W/\text{stage} = u V_f\left[\tan\beta_1 - \tan\beta_2\right]}$$

where β_1 and β_2 are blade angle at inlet and exit.

$$R = \frac{V_{r_1}^2 - V_{r_2}^2}{2\underbrace{u(V_{w_2} - V_{w_1})}_{uV_f(\tan\beta_1 - \tan\beta_2)}}$$

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$$R = \frac{V_f^2 \sec^2 \beta_1 - V_f^2 \sec^2 \beta_2}{2 u V_f (\tan \beta_1 - \tan \beta_2)}$$

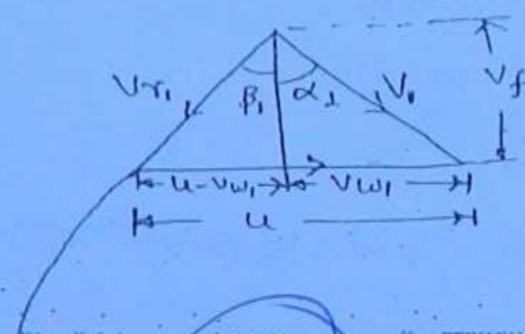
$$R = \frac{V_f^2 (\sec^2 \beta_1 - \sec^2 \beta_2)}{2 u V_f (\tan \beta_1 - \tan \beta_2)}$$

$$R = \frac{V_f}{2u} \left[\frac{\cancel{1} + \tan^2 \beta_1 - \cancel{1} - \tan^2 \beta_2}{\tan \beta_1 - \tan \beta_2} \right]$$

imp

$$\boxed{R = \frac{V_f}{2u} \left[\tan \beta_1 + \tan \beta_2 \right]}$$

Condition for 50% Reaction ⇒



$$\tan \beta_1 = \frac{u - V_{w_1}}{V_f}$$

$$\tan \alpha_1 = \frac{V_{w_1}}{V_f}$$

$$\tan \alpha_1 + \tan \beta_1 = \frac{u - V_{w_1}}{V_f} + \frac{V_{w_1}}{V_f}$$

$$\tan \alpha_1 + \tan \beta_1 = \frac{u}{V_f}$$

hen

$$R = \frac{V_f}{2u} \left[\tan \beta_1 + \tan \beta_2 \right]$$

$$R = \frac{V_f}{2u} \left[\tan \beta_1 + \tan \beta_2 + \tan \alpha_1 - \tan \alpha_1 \right]$$

$$R = \frac{V_f}{2u} \left[\frac{u}{V_f} + \tan \beta_2 - \tan \alpha_1 \right]$$

$$R = \frac{1}{2} + \frac{V_f}{2u} (\tan \beta_2 - \tan \alpha_1)$$

if $R = 50\% = 0.5 = 1/2$

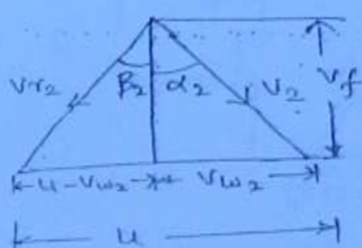
$$\frac{1}{2} = \frac{1}{2} + \frac{V_f}{2u}(\tan\beta_2 - \tan\alpha_1)$$

$$0 = \frac{V_f}{2u}(\tan\beta_2 - \tan\alpha_1)$$

$$\tan\beta_2 = \tan\alpha_1$$

$$\left.\begin{array}{l} \boxed{\beta_2 = \alpha_1} \\ \boxed{\beta_1 = \alpha_2} \end{array}\right\} \text{50\% Reaction.}$$

Similarly



$\alpha_1 = \beta_2$

$\cos\alpha_1 = \cos\beta_2$

$$\frac{V_f}{V_1} = \frac{V_f}{V_{r_2}}$$

v.v. imp

$$\boxed{V_1 = V_{r_2}}$$

$\alpha_2 = \beta_1$

$\cos\alpha_2 = \cos\beta_1$

$$\frac{V_f}{V_2} = \frac{V_f}{V_{r_1}}$$

v.v. imp

$$\boxed{V_2 = V_{r_1}}$$

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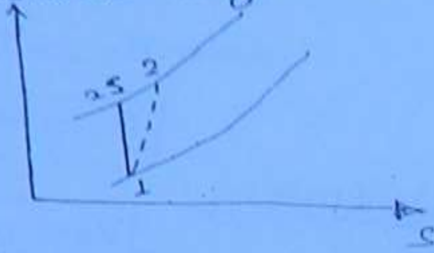
(#) Equation to be Remembered ⇒

1. $u_1 = u_2 = u$
2. $V_{f_1} = V_{f_2} = V_f$
3. $W/\text{stage} = u V_f(\tan\beta_1 - \tan\beta_2)$
4. $R = \frac{V_f}{2u}(\tan\beta_1 + \tan\beta_2)$
5. For 50% Reaction
   - $\alpha_1 = \beta_2$
   - $\beta_2 = \alpha_1$
   - $V_1 = V_{r_2}$
   - $V_2 = V_{r_1}$

IES 2002

1. A 10 stage Axial flow compressor provides an overall pressure ratio of 5 with an overall isentropic efficiency of 87%. The temp. of air inlet is 15°C. The work is equally divided between all stages. A 50% Reaction is used with a blade speed of 210 m/sec. and axial velocity of flow 170 m/sec. find the blade angle at inlet and exit.

$T_1 = 273 + 15 = \underline{288K}$

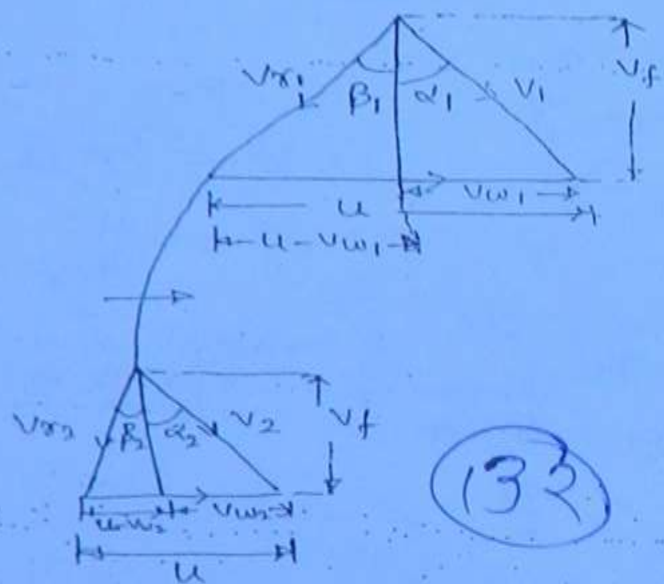


Ans.

$\eta_{isentropic} = 87\% = 0.87$

$u_1 = 210$ m/sec

$V_f = 170$ m/sec.



$W = \cancel{(V_{w_1} - V_{w_2})\, u}$

$= \cancel{(V_2 \sin\alpha_2 - V_1 \sin\alpha_1)\, u}$

$=$

(132)

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{288} = (5)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_{2s} = 456.1K}$$

$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.87 = \frac{456.1 - 288}{T_2 - 288}$$

$$\boxed{T_2 = 481.26K}$$

$$W = h_2 - h_1$$
$$= C_p (T_2 - T_1)$$
$$= 1.005\,(481.260 - 288)$$
$$\boxed{W = 194.2 \text{ KJ/Kg}}$$

$$\because W/\text{stage} = \frac{194.2}{10} = 19.42 \text{ KJ/Kg}$$

$$W/\text{stage} = \frac{u\,V_f\,(\tan\beta_1 - \tan\beta_2)}{1000}$$

$$19.42 = 210 \times 170\,(\tan\beta_1 - \tan\beta_2)$$

$$\tan\beta_1 - \tan\beta_2 = 0.544 \quad \text{--- (1)}$$

$$R = \frac{V_f}{2u}(\tan\beta_1 + \tan\beta_2)$$

$$\frac{1}{2} = \frac{V_f}{2u}(\tan\beta_1 + \tan\beta_2)$$

$$\text{or } \frac{V_f}{2u} = \tan\beta_1 + \tan\beta_2$$

$$\frac{210}{170} = \tan\beta_1 + \tan\beta_2$$

$$\tan\beta_1 + \tan\beta_2 = 1.235 \quad \text{--- (2)}$$

$$\boxed{\beta_1 = 41.65^\circ}$$
$$\boxed{\beta_2 = 19^\circ} \quad \text{Ans:}$$

Q.2

An axial flow air compressor of 50% Reaction has blades with inlet and outlet angles of 45° and 10° Respectively. compressor is to produce an overall pressure ratio of 6. with an overall isentropic efficiency of .85%. The inlet temp. is 37°C. The blade speed is 200 m/sec. Then find the Number of stages.

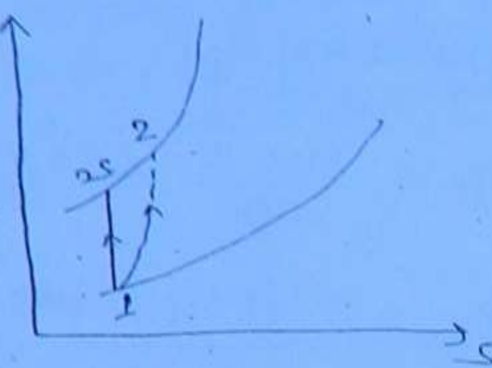
Sol:

$\beta_1 = 45°$, $\beta_2 = 10°$, $\frac{p_2}{p_1} = 6$, $T_1 = 273 + 37$

$T_1 = 310\,K$

$\eta = 85\% \Rightarrow 0.85$

$$\frac{T_{2s}}{T_1} = (6)^{\frac{1.4-1}{1.4}}$$



$$\frac{T_{2s}}{310} = (6)^{\frac{0.4}{1.4}}$$

$$T_{2s} = 517\,K$$

$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.85 = \frac{517 - 310}{T_2 - 310}$$

$$\boxed{T_2 = 553.8\,K}$$

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Total work, $W_T = c_p(T_2 - T_1)$

$= 1.005(553.8 - 310)$

$W_T = 245\,kJ/kg$

W/stage $= u V_f (\tan\beta_1 - \tan\beta_2)$

$$R = \frac{V_f}{2u}(\tan\beta_1 + \tan\beta_2)$$

$$\frac{1}{2} = \frac{V_f}{2 \times 200}(\tan 45° + \tan 10°)$$

$$\boxed{V_f = 170\ m/sec}$$

W/stage $= \frac{200 \times 170 [\tan 45° - \tan 10°]}{1000}$

$$\boxed{W/stage = 28.025\ kJ/kg}$$

No. of stage $= \frac{W_T}{W/\text{stage}} \Rightarrow \frac{245}{28.025}$

$\boxed{\text{No. of stage} = 8.71 \simeq 9 \text{ stage}}$ Ans.

Q.3 In an axial flow compressor producing a pressure ratio of 6 with air entering at 20°C. the Blade velocity in 200 m/sec The inlet and exit angle of moving blade are 45° and 10° For a 50% Reaction find. The No. of stage equal to 12. the isentropic efficiency of a Compressor. Ans. 58.58%

Soln

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

$$T_{2s} = 488.86 \text{ K}$$

$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$= \frac{488.86 - 293}{627.38 - 293}$$

$$\eta = 0.5857 \%$$

Ans

135

$u = 200$ m/sec
$\beta_1 = 45^\circ$
$\beta_2 = 10^\circ$
$N = 12$

$$R = \frac{V_f}{2u}\left(\tan\beta_1 + \tan\beta_2\right)$$

$$\frac{1}{2} = \frac{V_f}{2\times 200}\left[\tan 45^\circ + \tan 10^\circ\right]$$

$$\frac{1}{2} = \frac{V_f \times 1.176}{400}$$

$$V_f = 170 \text{ m/sec}$$

$$W/\text{stage} = \frac{u V_f(\tan\beta_1 - \tan\beta_2)}{1000}$$

$$= \frac{200\times 170\left(\tan 45^\circ - \tan 10^\circ\right)}{1000}$$

$W/\text{stage} = 28$ kN $\qquad W_{total} = 28 \times 12 = 336$

$$W_{total} = C_p\left(T_2 - T_1\right)$$

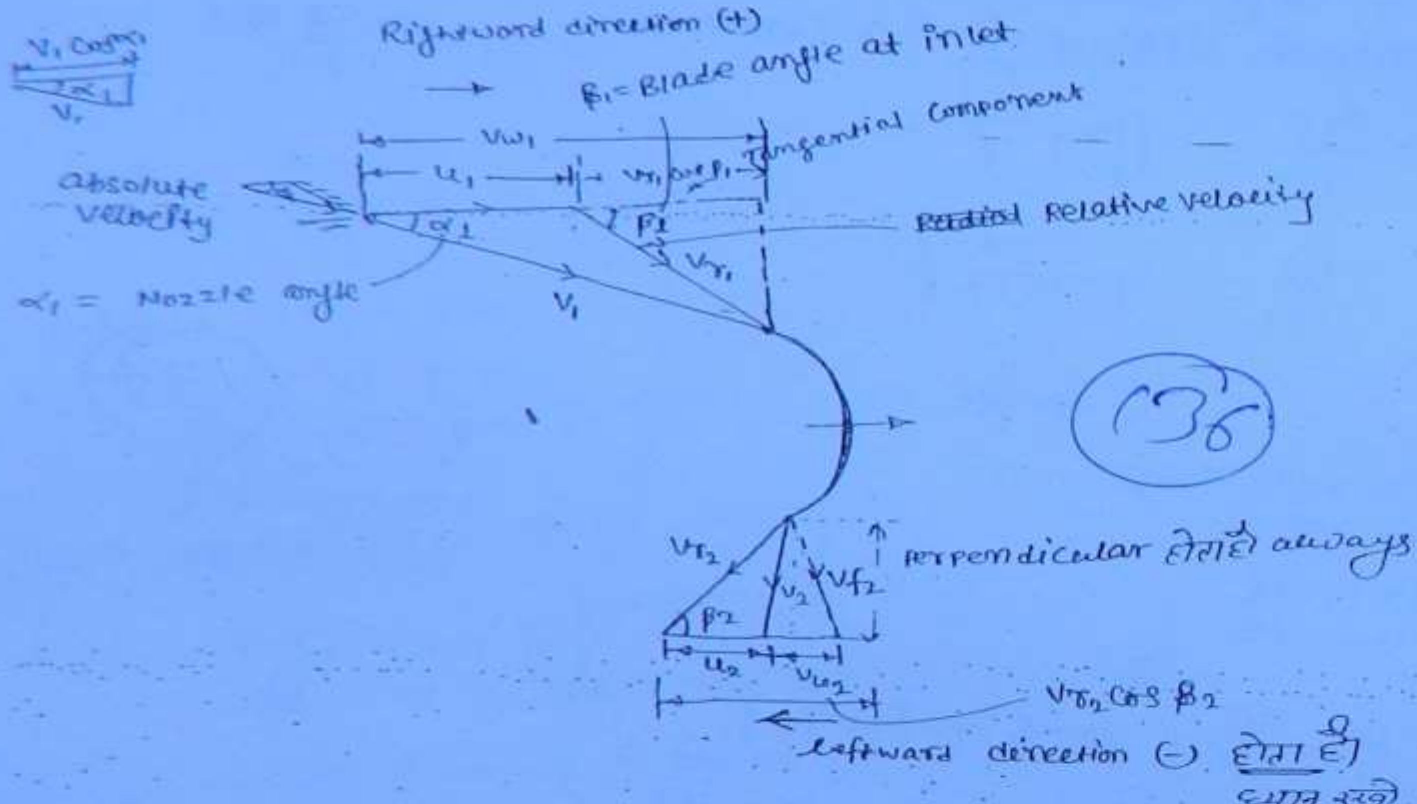
$$336 = 1.005\left(T_2 - 293\right)$$

627.38

STEAM TURBINES

IMPULSE Turbine ⇒ In impulse Turbine the fluid enter the turbine blade with K.E and while moving in the turbine blade, the Static pressure Remains constant and kinetic energy is used in developing power.

De. Leval is a simple Impulse Turbine.



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$$W = V_{w_1} u_1 - V_{w_2} u_2$$

$$W = (V_{w_1} - V_{w_2})\, u \qquad \because [u_1 = u_2 = u]$$

$$W = [V_{w_1} - (-V_{w_2})]\, u$$

$$W = (V_{w_1} + V_{w_2})\, u$$

$$\eta_d = \frac{\cancel{m}\,(V_{w_1} + V_{w_2})\,u}{\frac{1}{2}\,\cancel{m}\,V_1^2}$$

η_d → diagram efficiency or Blade efficiency.

$$\boxed{\eta_d = \frac{2\,(V_{w_1} + V_{w_2})\,u}{V_1^2}}$$

Condition for max. Efficiency ⇒ (Diagram or Blade efficiency (kramai))

1. The Blade are assumed to be symmetrical i.e. $\beta_1 = \beta_2$
2. The flow is assumed to be frictionless i.e. $V_{r_1} = V_{r_2}$

$$V_{r_1} = V_{r_2}$$

Blade velocity coefficient $\boxed{K = \frac{V_{r_2}}{V_{r_1}}}$

$$V_{w_1} = V_1 \cos\alpha_1$$

$$V_{w_2} = V_{r_2} \cos\beta_2 - u$$

$$V_{w_2} = V_{r_1} \cos\beta_1 - u$$

$$V_{w_2} = (V_{w_1} - u) - u$$

$$V_{w_2} = V_{w_1} - 2u$$

$$\Rightarrow \quad V_{w_2} = V_1 \cos\alpha_1 - 2u$$

$$\therefore \eta_d = \frac{2[V_1 \cos\alpha_1 + V_1 \cos\alpha_1 - 2u]u}{V_1^2}$$

$$\eta_d = \frac{2[2V_1 \cos\alpha_1 - 2u]u}{V_1^2}$$

$$\eta_d = \frac{4[V_1 u \cos\alpha_1 - u^2]}{V_1^2}$$

$$\boxed{\eta_d = 4\left[\frac{u}{V_1}\cos\alpha_1 - \frac{u^2}{V_1^2}\right]}$$

let $\frac{u_0}{V_1} = \rho$ = Blade speed ratio

$$\boxed{\eta_d = 4[\rho \cos\alpha_1 - \rho^2]}$$

For max. $\eta_d = \frac{d\eta_d}{d\rho} = 0$

$$= 4[\cos\alpha_1 - 2\rho] = 0$$

$$= \cos\alpha_1 = 2\rho$$

$$\Rightarrow \boxed{\rho = \frac{\cos\alpha_1}{2}}$$ Blade speed ratio for max. efficiency

ρ = Blade speed = $\frac{u_0}{V_1}$

$\rho = \frac{u_0}{V_1} = \frac{\cos\alpha_1}{2}$

$\boxed{u_0 = 0.5 V_1 \cos\alpha_1}$

Blade speed

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$$\eta_{max^d} = 4\left[\frac{\cos^2\alpha_1}{2} - \frac{\cos^2\alpha_1}{4}\right]$$

v.v.imp

*** $\boxed{\eta_{max.} = \cos^2\alpha_1}$ ⇒ max. efficiency of an Impulse Turbine.

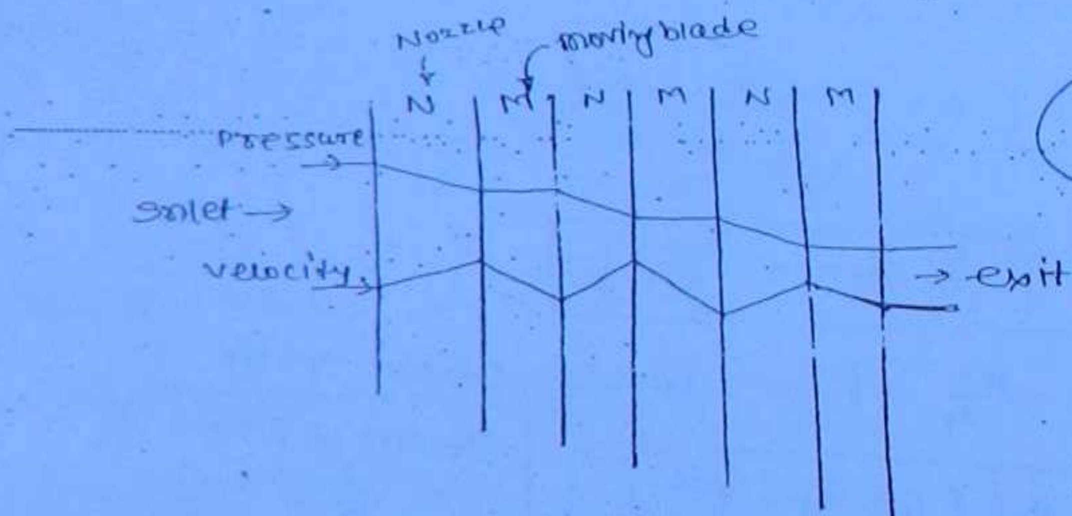
where $\alpha_1 \rightarrow$ Nozzle angle

This equation is valid when the blade are symmetrical and frictionless

⇒ Compounding in Impulse Turbine ⇒

In steam Turbine the steam enters the turbine at a pressure of about 170 bar and leaves at a Condenser pressure of about 0.035 bar thus the pressure ratio are about 4800. If steam is allowed to expand in a single stage the Rotor speed are very high (about 20,000 rpm). Therefore the speeds must be limited and to Reduce the Rotor speed compounding is used.

1. **Pressure compounding** ⇒ Pressure compounding to expand Ex. (Rateau Turbine)



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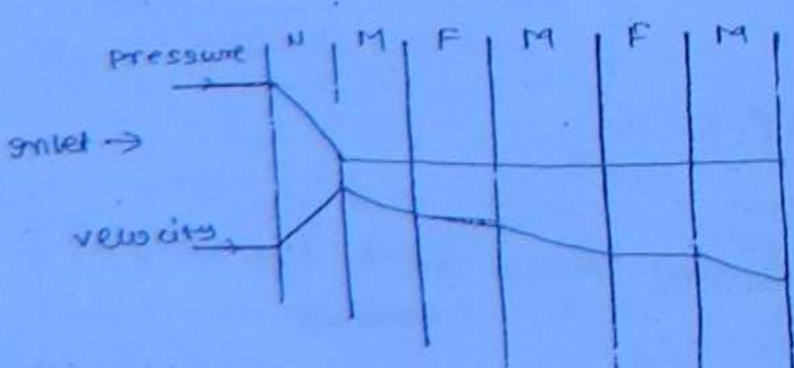
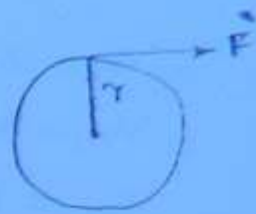
2. **velocity compounding** ⇒ (Curtis Turbine) ← Example



In velocity compounding velocity in Fixed Blade slightly decreases.

① Driving force on wheel



$$F = m\frac{(V-u)}{t}$$

$$F = \dot{m}\,[V-u]$$

$$F = \dot{m}\,[V_{w_1} - (-V_{w_2})]$$

$$\boxed{F = \dot{m}\,[V_{w_1} + V_{w_2}]} \Leftarrow \text{Driving force}$$

$$P = F \times u$$

$$\boxed{P = \dot{m}\,[V_{w_1} + V_{w_2}]\,u} \Leftarrow \text{Power}$$

$$\boxed{\text{Axial thrust} = \dot{m}\,[V_{f_1} - V_{f_2}]}$$

$$\boxed{K = \frac{V_{r_2}}{V_{r_1}}} \Rightarrow \text{Blade velocity Coefficient}$$

V. Imp

PROB-① The velocity of steam, leaving the Nozzle of an impulse turbine is 900 m/sec and Nozzle angle is 20° the Blade velocity is 300 m/sec (u). and Blade velocity coefficient is 0.7. Calculate for a mass flow rate of 1 kg/sec and symmetric Blading.

① Blade angle at inlet
② Driving force on wheel
③ Axial thrust
④ Power developed
⑤ Diagram efficiency.

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Soln.

given

$V_1 = 900$ m/sec

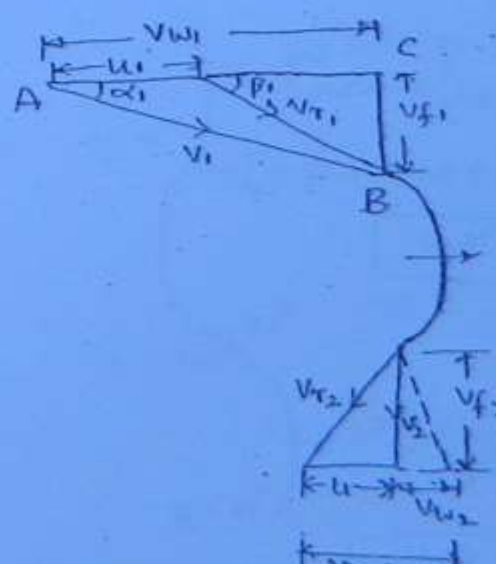
$\alpha_1 = 20°$

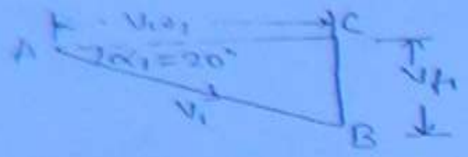
$u = 300$ m/sec

$K = 0.7$

$\dot{m} = 1$ kg/s

$\beta_1 = \beta_2$





$V_{w_1} = V_1 \cos\alpha_1$

$V_{w_1} = 900 \cos 20^\circ$

$V_{w_1} = 845.7$ m/sec

$V_{f_1} = V_1 \sin\alpha_1$

$= 900 \sin 20^\circ$

$V_{f_1} = 307.8$ m/sec

$$\tan\beta_1 = \frac{V_{f_1}}{V_{w_1} - u}$$

$$\tan\beta_1 = \frac{307.8}{845.7 - 300} \Rightarrow \boxed{\beta_1 = 29.4^\circ = \beta_2}$$

$$\sin\beta_1 = \frac{V_{f_1}}{V_{r_1}}$$

$$\sin 29.4^\circ = \frac{307.8}{V_{r_1}}$$

$$\boxed{V_{r_1} = 626.5 \text{ m/sec}}$$

$$k = \frac{V_{r_2}}{V_{r_1}} \Rightarrow 0.7 = \frac{V_{r_2}}{626.5}$$

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$$\boxed{V_{r_2} = 438.5 \text{ m/sec}}$$

$V_{w_2} = V_{r_2} \cos\beta_2 - u$

$= 438.5 \cos 29.4^\circ - 300$

$$\boxed{V_{w_2} = 82 \text{ m/sec}}$$

Driving force (F) $= \dot{m}\,[V_{w_1} + V_{w_2}]$

$= 1 \times [845.7 + 82]$

$$\boxed{F = 927.7 \text{ N}}$$

$$\sin\beta_2 = \frac{V_{f_2}}{V_{r_2}}$$

$$\sin 29.4 = \frac{V_{f_2}}{438.5}$$

$$\boxed{V_{f_2} = 215.47 \text{ m/sec}}$$

Thrust $= m(V_{f_1} - V_{f_2})$

$= 1(307.8 - 215.26]$

$= 92.34\,N$

Power developed (P) $= \dot{m}(V_{w_1} + V_{w_2})u$ $\quad \frac{kg}{s} \times \frac{m}{s} \times \frac{m}{s}$ $\quad N \times m/s, J/s$

$= 1 \times (845.7 + 82) \times 300$

$$\boxed{P = 278.3\,kW}$$

$$\eta_b = \frac{2(V_{w_1} + V_{w_2})u}{V_1^2} \Rightarrow \underline{0.687} \quad \underline{Ans:}$$

$$= \frac{2(845.7 + 82] \times 300}{900^2} \Rightarrow \underline{\underline{0.687}}$$

*****

PROB. ② The first stage of a turbine is a two Row velocity compounded impulse turbine. The velocity of steam at inlet is 600 m/sec. The blade speed is 120 m/sec. The blade velocity coefficient is 0.9 for all blades. The Nozzle angle is 16°. The exit angles for first Row of moving blades, fixed blades and second Row of moving blade are 18°, 21° and 35° Respectively. Calculate ① Blade inlet angle for each Row ② Driving force on each Row of moving Blade for a mass flow rate of 1 kg/sec. (14)

Ans: Ist Row $\beta_1 = 29.5°$

IInd row $\beta_1 = 34.38°$

Driving force on Ist

$F = 872.63\,N$

D.F on IInd

$F = 290.27\,N$

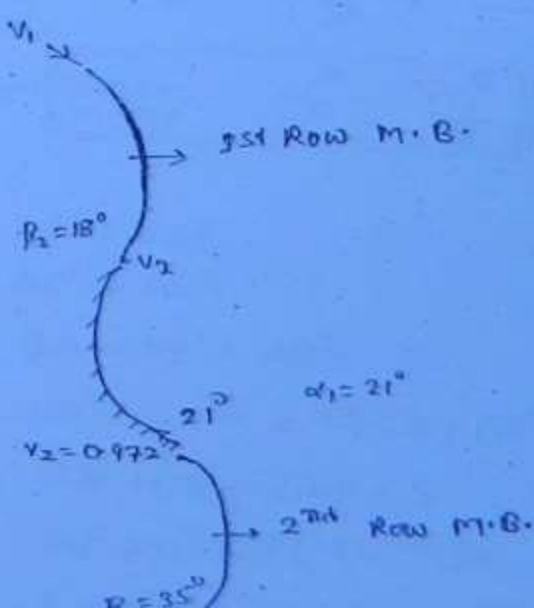
Sol:

$u = 120\,m/sec$

$\alpha_1 = 16°$

$k = 0.9$

$V_1 = 600\,m/sec$



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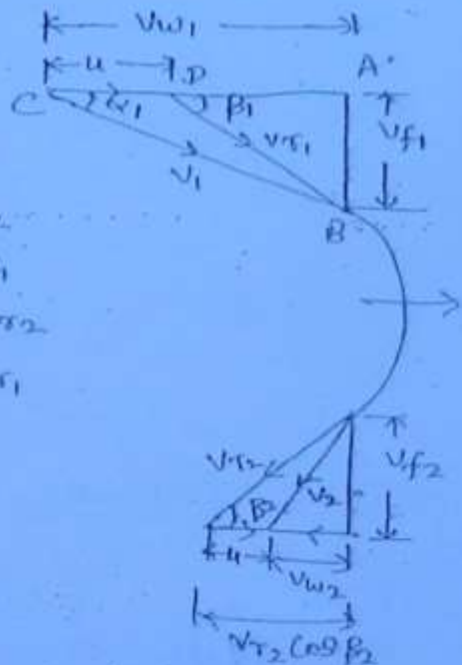
Reaction Turbine ## [redacted]

In case of Reaction Turbine the fluid enters with both kinetic and pressure energy therefore input energy is equal to

$$\text{Input energy/mass} = \underbrace{\frac{V_1^2}{2}}_{K.E.} + \underbrace{\frac{V_{r2}^2 - V_{r1}^2}{2}}_{\text{Pressure energy}}$$

Max. efficiency of parson's Reaction Turbine ⇒ (50% R)



$\alpha_1 = \beta_2$

$\alpha_2 = \beta_1$

$V_1 = V_{r2}$

$V_2 = V_{r1}$

$$W = V_{w1}u_1 - V_{w2}u_2$$

$$W = (V_{w1} + V_{w2})u$$

$$\eta_d = \frac{\text{output}}{\text{input}}$$

$$\eta_d = \frac{(V_{w1} + V_{w2})u}{\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}}$$

SES 2002

$$\boxed{V_{w1} = V_1\cos\alpha_1}$$

$$V_{w2} = V_{r2}\cos\beta_2 - u$$

$$\boxed{V_{w2} = V_1\cos\alpha_1 - u}$$

$$\underbrace{\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}}_{\text{Input}} \Rightarrow \frac{V_1^2}{2} + \frac{V_1^2 - V_{r1}^2}{2}$$

For Reaction Turbine $\boxed{\text{Input energy} = \frac{V_1^2 - \frac{V_{r1}^2}{2}}{}}$

From $\triangle ABD$

$$BD^2 = AD^2 + AB^2$$

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = (AD + CD)^2 + AB^2$$

$$BC^2 = AD^2 + CD^2 + 2AD\,CD + AB^2$$

$$BC^2 = BD^2 + CD^2 + 2AD\cdot CD$$

$$V_1^2 = V_{r1}^2 + u^2 + 2(V_{r1} - u)u$$

$$V_1^2 = V_{r1}^2 + u^2 + 2[V_1\cos\alpha_1 - u]u$$

$$V_1^2 = V_{r1}^2 + u^2 + 2uV_1\cos\alpha - 2u^2$$

$$V_1^2 = V_{r1}^2 - u^2 + 2uV_1\cos\alpha_1$$

$$\boxed{V_{r1}^2 = V_1^2 + u^2 - 2uV_1\cos\alpha_1}$$

$$\eta = \frac{(V_{w1} + V_{w2})u}{V_1^2 - \frac{V_{r1}^2}{2}}$$

$$\eta = \frac{(V_1\cos\alpha_1 + V_1\cos\alpha_1 - u)u}{V_1^2 - \frac{(V_1^2 + u^2 - 2uV_1\cos\alpha_1)}{2}}$$

$$\eta = \frac{(2V_1\cos\alpha_1 - u)u}{\frac{V_1^2}{2} - \frac{u^2}{2} + uV_1\cos\alpha_1}$$

$$\eta = \frac{(2uV_1\cos\alpha_1 - u^2)2}{V_1^2 - u^2 + 2uV_1\cos\alpha_1}$$

$\boxed{\rho = \frac{u}{V_1}}$ → Blade speed ratio

$$\eta = \frac{\frac{2[2uV_1\cos\alpha_1 - u^2]}{V_1^2}}{\frac{V_1^2 - u^2 + 2uV_1\cos\alpha_1}{V_1^2}}$$

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$$\boxed{\eta = \frac{2[2\rho\cos\alpha_1 - \rho^2]}{1 - \rho^2 + 2\rho\cos\alpha_1}}$$

$$\eta = \frac{2[1 + 2\rho\cos\alpha_1 - \rho^2 - 1]}{1 - \rho^2 + 2\rho\cos\alpha_1}$$

$$\eta = 2\left[1 - \frac{1}{1 - \rho^2 + 2\rho\cos\alpha_1}\right]$$

for max. efficiency η, $\dfrac{d\eta}{d\rho} = 0$

$$\frac{d\eta}{d\rho} = 2\left[0 - \frac{(-1)}{1-\rho^2+2\rho\cos\alpha_1}\, 2\left[0 - 2\rho + 2\cos\alpha_1\right]\right] = 0$$

$$= -2\rho + 2\cos\alpha_1 = 0$$

$$\boxed{\rho = \cos\alpha_1}$$

$$\eta_{max} = \frac{2\left[2\cos\alpha_1 \cdot \cos\alpha_1 - \cos^2\alpha_1\right]}{1 - \cos^2\alpha_1 + 2 \times \cos\alpha_1 \times \cos\alpha_1}$$

$$\boxed{\eta_{max} = \frac{2\left[\cos^2\alpha_1\right]}{1 + \cos^2\alpha_1}}$$

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Impulse	Reaction
$\beta_1 = \beta_2$	50% Reaction
$V_{r_1} = V_{r_2}$	$\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$
$\rho = \dfrac{\cos\alpha_1}{2}$	$V_1 = V_{r_2}$, $V_2 = V_{r_1}$
$\eta_{max} = \cos^2\alpha_1$	$\rho = \cos\alpha_1 = \dfrac{u}{V_1}$ → Blade speed ratio
	$\eta_{max} = \dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$

~~PROB~~ Note ⇒ $\left(w = V_{w_1}u_1 - V_{w_2}u_2 = \dfrac{V_1^2 - V_2^2}{2} + \dfrac{V_{r_2}^2 - V_{r_1}^2}{2}\right)$

(1) IN Reaction Turbine enthalpy drop in moving blade is equal to ⇒ $\dfrac{V_{r_2}^2 - V_{r_1}^2}{2}$

(2) $R = \dfrac{1}{2} = \dfrac{(\Delta h)_m}{(\Delta h)_m + (\Delta h)_F}$

$(\Delta h)_m + \Delta h_f = 2\Delta h_m$

$\Delta h_f = 2\Delta h_m - \Delta h_m$

$\Delta h_f = \Delta h_m$

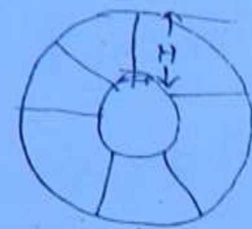
$\Delta h_s = \Delta h_m + \Delta h_f$

$\Delta h_s = \Delta h_m + \Delta h_m$

$\Delta h_s = 2\Delta h_m \Rightarrow \cancel{2}\left(\frac{V_{r_2}^2 - V_{r_1}^2}{\cancel{2}}\right)$

$$\boxed{\Delta h_s = 2\Delta h_m = V_{r_2}^2 - V_{r_1}^2}$$

Note → ③ where H → Height of Blade



Area of flow

$2\pi r\, dr$

$\pi D \cdot dr$

$A = \pi D H$

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$\dot{m} = \rho A V$

$\dot{m} = \rho \times \pi D H \times V_f$

Because $\boxed{\rho = \frac{1}{v}}$

$$\boxed{\dot{m} = \frac{1}{v} \cdot \pi D H V_f}$$

PROB-① A stage of a steam turbine with Parson's Blading delivers a dry saturated steam at 2.71 bar from fixed blades at 90 m/sec (V_1). The Blade height is 40 mm and the moving blade exit angle is $\beta_2 = 20°$. The flow component of velocity is 3/4th Blade velocity. ($V_f = \frac{3}{4}u$). Steam is supplied at the rate of 9000 kg/hr. The effect of Blade thickness can be Neglected.

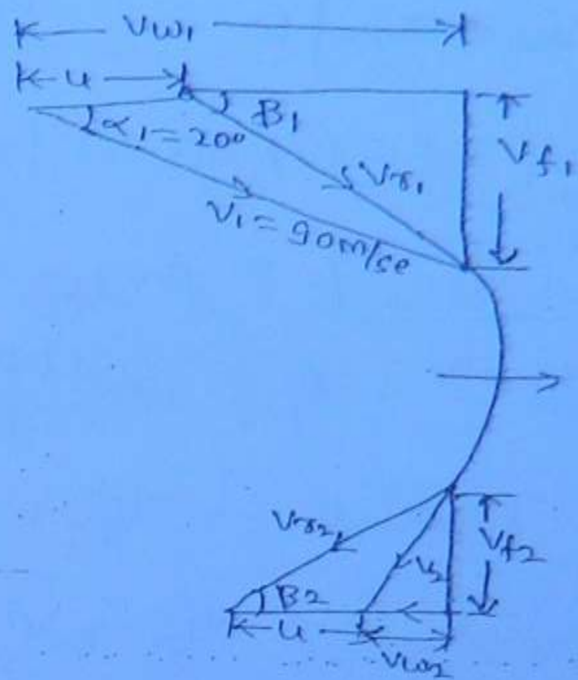
Calculate ① Rotational speed (N).
② Power developed.
③ efficiency.
④ enthalpy drop of steam in this stage.

Take specific volume of steam at 2.7 bar dry saturated = 0.6686 m³/kg.

$V_1 = 90\ m/sec$

$\beta_2 \rightarrow$ Blade exit angle $= \alpha_1$

$\beta_2 = 20^\circ = \alpha_1$



$$\sin\alpha_1 = \frac{V_{f_1}}{V_1}$$

$$V_{f_1} = V_1 \sin\alpha_1$$

$$V_{f_1} = 90 \sin 20^\circ$$

$$V_{f_1} = 30.78\ m/sec$$

$$V_f = \frac{3}{4} u$$

$$30.78 = \frac{3}{4} u$$

$$u = 41.04\ m/sec$$

$$u = \frac{\pi D N}{60}$$

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$$\dot{m} = \frac{\pi D H V_f}{v}$$

$$\frac{9000}{3600} = \frac{\pi D \times 40 \times 10^{-3} \times 30.78}{0.6686}$$

$$\boxed{D = 0.432\ m}$$

$$u = \frac{\pi D N}{60}$$

$$41.04 = \frac{\pi \times 0.432 \times N}{60}$$

$$\boxed{N = 1813\ rpm}$$

(2) power developed :$\Rightarrow$ $V_{w_1} = V_1 \cos\alpha_1$

$$= 90 \cos 20^\circ$$

$$V_{w_1} = 84.57\ m/sec.$$

$$V_{w_2} = V_{r_2} \cos\beta_2$$

$$V_{w_2} = (V_{r_2} \cdot \cos\beta_2 - u)$$

$$V_{w_2} = V_1 \cos\alpha_1 - \cancel{u}$$

$$= V_{w_1} - u$$

$$= 84.57 - 41.04$$

$P = \dot{m}\left[V_{w_1} + V_{w_2}\right]u$ $\frac{kg}{s} \times \frac{m}{s} \times \frac{m}{s}$

$P = \frac{9000}{3600}\left[84.57 + 43.57\right] \times 41.04$

$\boxed{P = 13.14 \text{ KW}}$ Ans.

3) Efficiency $\Rightarrow$ $\eta = \frac{\text{output}}{\text{input}}$

$\text{input} = \frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}$

$\frac{V_1^2}{2} + \frac{V_1^2}{2} - \frac{V_{r_1}^2}{2}$

$\text{input} = V_1^2 - \frac{V_{r_1}^2}{2}$ (148)

$\text{input} = \dot{m}\left[V_1^2 - \frac{V_{r_1}^2}{2}\right]$

$V_{r_1}^2 = u^2 + V_1^2 - 2uV_1\cos\alpha_1$

$V_{r_1}^2 = 41.04^2 + 90^2 - 2 \times 41.04 \times 90 \cdot \cos 20^\circ$

$\boxed{V_{r_1} = 53.3 \text{ m/sec}}$

$\text{input} = \frac{9000 \text{ kg}}{3600 \text{ sec}}\left[90^2 - \frac{53.3^2}{2}\right]$ $\frac{kg}{s} \times \frac{m^2}{s^2}$ $\frac{kgm}{s^2} \times \frac{m}{s}$

$\text{input} = 16.69 \text{ KW}$

$\eta = \frac{13.14}{16.69} = 0.786 \Rightarrow 78.6\%$

4) $= V_{r_2}^2 - V_{r_1}^2$

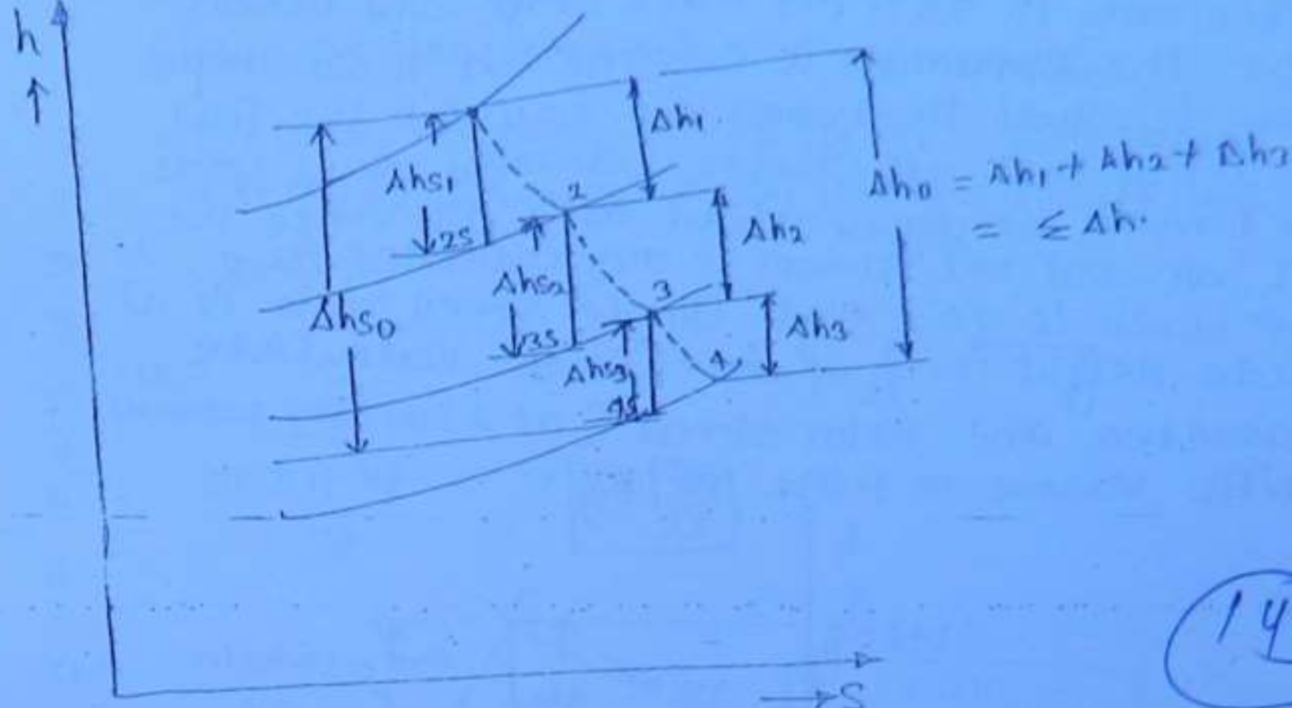
$= V_1^2 - 53.3^2$

$= 90^2 - 53.3^2$

$\Delta h) = 5259 \text{ J/kg}$

$(\Delta h)_{kg} = 5.259 \text{ kJ/kg}$

(11) REHEAT FACTOR $\Rightarrow$ (RF)



$$\eta_{s1} = \frac{\Delta h_1}{\Delta h_{s1}}$$

$$\boxed{\eta_{s1} = \eta_{s2} = \eta_{s3} = \eta_s}$$

$$\Delta h_1 = \eta_{s1} \Delta h_{s1}$$

$$\Delta h_2 = \eta_{s2} \Delta h_{s2}$$

$$\Delta h_3 = \eta_{s3} \Delta h_{s3}$$

$$\Delta h_1 + \Delta h_2 + \Delta h_3 = \eta_{s1} \Delta h_{s1} + \eta_{s2} \Delta h_{s2} + \eta_{s3} \Delta h_{s3}$$

$$\Delta h_0 = \eta_s [\Delta h_{s1} + \Delta h_{s2} + \Delta h_{s3}]$$

$$\frac{\Delta h_0}{\Delta h_{s0}} = \frac{\eta_s [\Delta h_{s1} + \Delta h_{s2} + \Delta h_{s3}]}{\Delta h_{s0}}$$

$$\boxed{\eta_0 = \eta_s [RF]}$$

$$\boxed{R.F > 1}$$

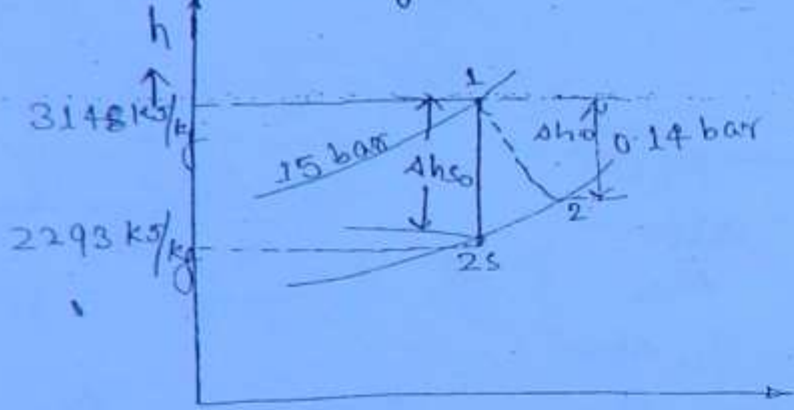
It is the ratio of summation of isentropic enthalpy drop in each stage to the ~~isen~~ overall isentropic enthalpy drop.

Steam at 15 bar and 350°C is expanded through 50% Reaction turbine through a pressure of 0.14 bar he stage efficiency is 75% for each stage and Reheat actor is 1.04. The expansion is carried out in 20 stages. and the power is equal to 12000 KW. Calculate the flow f steam assuming that all stages developed equal work.

IN the turbine mentioned about at 1 stage the pressure is 1 bar and the steam is dry saturated the exit angle of blade is 20°. and Blade speed ratio is 0.7. The Blade Height is $\frac{1}{12}$ of dia. of Blade. Calculate ① Blade diameter and Rotor speed (at 1 bar dry saturated steam specific volume = 1.694 m^3/kg.

$\beta_2 = \alpha_1$

$\beta_2 = 20° = \alpha_1$



$\eta_0 = \eta_s \times R.F$

$\eta_0 = 0.75 \times 1.04$

$\eta_0 = 0.78$

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$$\eta_0 = \frac{\Delta h_0}{\Delta h_{s_0}}$$

$$0.78 = \frac{\Delta h_0}{3148 - 2293}$$

$$\Delta h_0 = 666.9 \text{ KJ/kg}$$

$$\Delta h/\text{stage} = \frac{666.9 \text{ KJ/kg}}{\text{No. of stage}} \Rightarrow \frac{666.9}{20} \Rightarrow 33.35 \text{ KJ/kg}$$

$P = \dot{m} \times W$

$W = h_1 - h_2$

$$12000 \text{ KW} = \dot{m} \times 666.9 \text{ KJ/kg}$$

$$\dot{m} = 17.9 \text{ kg/sec.}$$

$\dot{W}$/stage $= \frac{(V_{w1}+V_{w2})u}{1000} = \underline{33.35}$

$= (V_1\cos\alpha_1 + V_1\cos\alpha_1 - u)u = 33.35 \times 1000$

$= (V_1\cos\alpha_1 + V_1\cos\alpha_1 - 0.7V_1)0.7V_1 = 33.35 \times 1000$

$= (V_1\cos 20^\circ + V_1\cos 20^\circ - 0.7V_1)0.7V_1 = 33.35 \times 1000$

$V_1 =$

$\frac{u}{V_1} = 0.7$

$u =$

$\dot{m} = \frac{\pi D H V_f}{v}$ $\boxed{H = \frac{D}{12}}$

$u = \frac{\pi D N}{60}$

Ans: $\boxed{\begin{array}{l} D = 1.3\,m \\ N = 2067\ rpm \end{array}}$

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(II) Compressible flow (Fluid)

→ Stagnation state ⇒ when a fluid is brought to zero velocity isentropically then that state is known as stagnation state.

$$\boxed{h_1 + \frac{V_1^2}{2} = h_{01}}$$

$c = \sqrt{\gamma RT}$, $M = \frac{V}{c}$
↳ Mach number

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_{01}$$

$$T_1 + \frac{V_1^2}{2C_p} = T_{01}$$

$$T_1 + \frac{V_1^2}{2 \times \frac{\gamma R}{\gamma - 1}} = T_{01}$$

$$T_1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{\gamma R} = T_{01} \Rightarrow T_1 + \frac{\gamma-1}{2} \cdot \frac{V_1^2 \times T_1}{\gamma R T_1} = T_{01}$$

$$T_1 \left[1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{C_1^2}\right] = T_{01}$$

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$$T_1 \left[1 + \frac{\gamma-1}{2} M_1^2\right] = T_{01}$$

$$\boxed{\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} \cdot M_1^2}$$

1 ———— 01
P_1, T_1 P_{01}, T_{01}

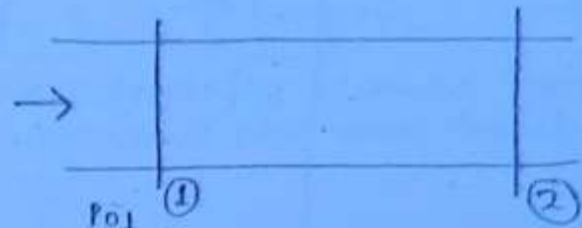
$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{\frac{P_{01}}{P_1} = \left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Air flow steadily through a pipe from 350 kPa., 60°C and 183 m/sec at the inlet state. The outlet condition are Mach No. = 1.3, stagnation pressure 385 kPa, stagnation Temp. 350 K, find the stagnation pressure and stagnation Temp. at inlet and static pressure and static temp. at outlet.

Sol^{n}



P_{01}
T_{01}

$P_1 = 350 kPa$
$T_1 = 333 K$
$V_1 = 183 m/sec$

$P_{02} = 385$
$T_{02} = 350$
$M_2 = 1.3$
$P_2 = ?$ $T_2 = ?$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$M_1 = \frac{V_1}{C_1}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$C = \sqrt{\gamma R T_1}$$

$$c^2 = \gamma R T_1 \cdot m^2/sec^2 \text{ or } J/kg$$

$$M_1 = \frac{183}{\sqrt{1.4 \times 287 \times 333}}$$

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$$M_1 = 0.5$$

$$\frac{T_{01}}{333} = 1 + \frac{1.4 - 1}{2} \times 0.5^2$$

$$T_{01} = 349.66 K$$

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{349.66}{333} = \left(\frac{P_{01}}{350}\right)^{\frac{1.4-1}{1.4}}$$

$$P_{01} = 415.17 kPa$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2$$

$$\frac{350}{T_2} = 1 + \frac{1.4 - 1}{2} (1.3)^2$$

$$T_2 = 261.58\ K$$

$$\frac{T_{02}}{T_2} = \left(\frac{P_{02}}{P_2}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{385}{P_2} = \left(\frac{350}{261.58}\right)^{\frac{1.4}{1.4 - 1}}$$

$$\boxed{P_2 = 138.9\ KPa}$$ Ans.

Fanno flow	Rayleigh flow
F ✓	F ✗
Q ✗	Q ✓

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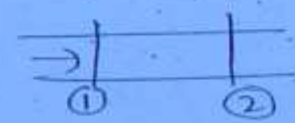
(Irreversible flow)

→ **FANNO FLOW** ⇒ Flow in constant area pipe with friction without Heat transfer is known as fanno flow.

→ Governing equation ⇒

① continuity equation ⇒

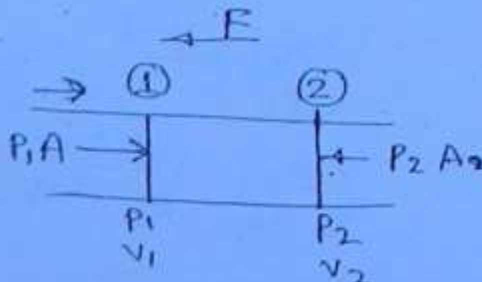


$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$A_1 = A_2$$

$$\boxed{\rho_1 V_1 = \rho_2 V_2}$$

② Momentum equation ⇒



$$F_{net} = m \cdot a$$

$$P_1 A - P_2 A - F = m \times acc^n$$

$$F_{net} = m\left(\frac{v - u}{t}\right)$$

$$F_{net} = \dot{m}(v - u)$$

$$P_1 A - P_2 A - F = \dot{m}\,(V_2 - V_1)$$

$$\boxed{(P_1 - P_2)\,A - F = \dot{m}\,(V_2 - V_1)}$$ Momentum equation.

③ <u>First Law of thermodynamics</u> ⇛

$$h_1 + \frac{V_1^2}{2} + \cancel{z_1 g} + \cancel{q} = h_2 + \frac{V_2^2}{2} + \cancel{z_2 g} + \cancel{w}$$

No heat transfer because fanno flow.
In Horizontal pipe work transfer is zero.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_{01} = h_{02}$$

$$\cancel{C_p} T_{01} = \cancel{C_p} T_{02}$$

v imp ** $\boxed{T_{01} = T_{02}}$

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Stagnation Temp. Remains constant in fannow flow.

④ <u>second Law of thermodynamics</u> ⇛

$$\boxed{T\,ds = dh - v\,dp}$$

⑤ <u>Equation of a state</u> :⇛

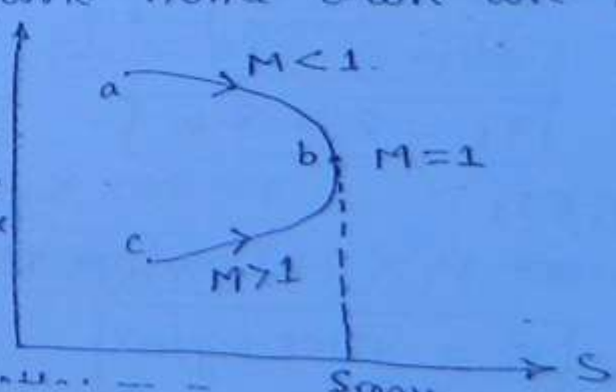
$$PV = mRT$$

$$P = \frac{m}{V} RT$$

$$\boxed{P = \rho RT}$$

⑥ $\boxed{dh = C_p\,dT}$

There are seven unknown (P, V, T, ρ, h, S, F) and there are six equation and hence there are infinite No. of solutions. if one Random Temp. is assumed then there are 6 unknown and there are 6 equations and we can find all these unknown corresponding to that temp. and this process is Repeated for other temp. an [illegible]



According to second law of thermodynamics

$(\Delta S)_{universe} \geq 0$

$(\Delta S)_{universe} = (\Delta S)_{sys} + \cancel{(\Delta S)}^{0}_{surrounding} > 0$

$(\Delta S)_{sys} > 0$

Because fanno flow is irreversible flow so no heat transfer in surrounding so it is zero;

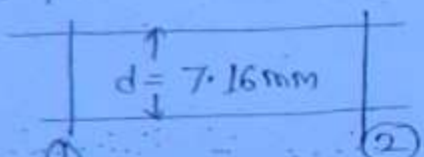
Fanno flow is possible either from A to B or C to B but not from B to A or B to C. Because this violates second law of thermodynamics.

208-② Air flow in an insulated pipe of 7.16 mm dia. the stagnation pressure and stagnation temp. at inlet are 101 kPa and 23°C Respectively. The static pressure is inlet is 98.5 kPa at a section ② located at some distance down stream in constant area pipe the temp. is 14°C. find ① mass flow rate

② ~~stagnation~~ static pressure at section ②

③ friction force on the wall ① and ②

Soln -



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$P_{01} = 101$ kPa $\qquad T_2 = 287$

$T_{01} = 296$ K. $\qquad T_{02} = 296$ K.

$P_1 = 98.5$ kPa

$\dot{m} = \rho_1 A V_1 \qquad T_1 = 293.8$ K.

$$\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{296}{T_1} = \left(\frac{101}{98.5}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_1 = 293.8 \text{ k}}$$

$P_1 = \rho_1 R T_1$

$98.5 = \rho_1 \times 0.287 \times 293.8$

$$\boxed{\rho_1 = 1.168 \text{ kg/m}^3}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{296}{293.8} = 1 + \frac{1.4-1}{2} M_1^2$$

$$\boxed{M_1 = 0.189}$$

$$M_1 = \frac{V_1}{C_1}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$0.189 = \frac{V_1}{\sqrt{1.4 \times 287 \times 293.8}}$$

$$\boxed{V_1 = 64.9 \text{ m/sec}}$$

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(1) mass flow rate $\Rightarrow$

$$\dot{m} = \rho_1 A_1 V_1$$

$$= 1.168 \times \frac{\pi}{4} (7.16 \times 10^{-3})^2 \times 64.9$$

$$\dot{m} = 3.05 \times 10^{-3} \text{ kg/sec}$$

(2) static pressure at section (2)

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$$

$$\frac{296}{287} = 1 + \frac{1.4-1}{2} M_2^2$$

$$M_2 = 0.396$$

$$M_2 = \frac{V_2}{C_2}$$

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}} \Rightarrow 0.396 = \frac{V_2}{\sqrt{1.4 \times 287 \times 287}}$$

$$\boxed{V_2 = 134.47 \text{ m/sec}}$$

$$\rho_1 V_1 = \rho_2 V_2$$

$$1.162 \times 64.9 = \rho_2 \times 134.47$$

$$\boxed{\rho_2 = 0.565 \text{ kg/m}^3}$$

$P_2 = \rho_2 R T_2$

$P_2 = 0.56 \times 0.287 \times 287$

$$\boxed{P_2 = 46.58\ kPa}$$

3) Friction force on the wall. (1) and (2)

$$(P_1 - P_2)A - F = \dot{m}(V_2 - V_1)$$

$\frac{kN}{m^2} \times m^2$ $\quad \frac{kg}{s} \cdot \left(\frac{m}{s}\right)$

$$[98.5 \times 10^3 - 46.58 \times 10^3] \frac{\pi}{4} \times (7.16 \times 10^{-3})^2 - F = 3.08 \times 10^{-3} \{134.4 - 64.$$

$$\boxed{F = 1.88\ N}$$ Ans:

Date 14/3/10

Rayleigh flow ⇒

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Flow in constant area pipe with heat transfer without friction is known as Rayleigh flow.

Governing equations ⇒

(1) continuity equation ⇒ $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$$\boxed{\rho_1 V_1 = \rho_2 V_2}$$

(2) Momentum equation ⇒ put $F = 0$ in fanno flow momentum eqn:

$$\boxed{(P_1 - P_2)A = \dot{m}(V_2 - V_1)}$$

(3) First Law of thermodynamics ⇒

$$h_1 + \frac{V_1^2}{2} + Q = h_2 + \frac{V_2^2}{2}$$

$$h_{01} + Q = h_{02}$$

$$Q = h_{02} - h_{01}$$

$$\boxed{Q = C_p(T_{02} - T_{01})}$$

(4) second Law of thermodynamics →

$$\boxed{Tds = dh - vdP}$$

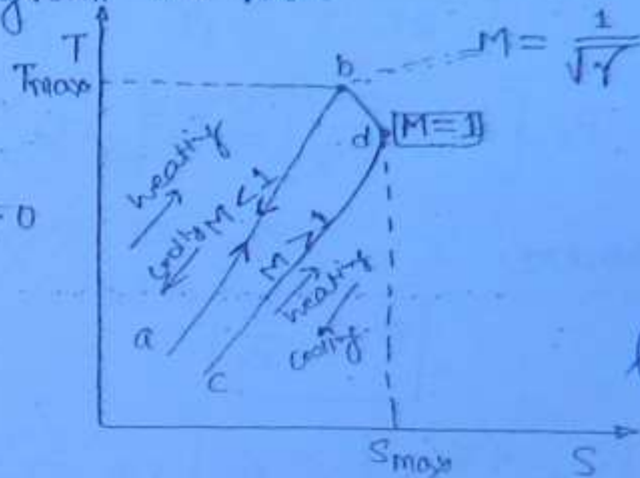
(5) equation of state → $PV = mRT$

$$P = \frac{m}{V} RT \qquad \boxed{\rho = \frac{m}{V}}$$

$$\boxed{P = \rho RT}$$

(6) $\boxed{dh = C_p\, dT}$

There are seven unknown (P, V, S, T, h, ρ, Q) and six equation. Therefore there are infinite no. of solution. Let us assume one value of temp. Now there are six unknown and six equation and hence find thes six values. If this process is repeated for other temp. and plotted on T-S diagram we have



Rayleigh: $Q \neq 0$

$(\Delta S)_{system} + (\Delta S)_{surroundings} > 0$

$T_{max} = \boxed{M \Rightarrow \frac{1}{\sqrt{\gamma}}}$

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In Rayleigh flow as there is a heat transfer there is entropy change for surroundings and hence for deciding the direction of flow system entropy and surrounding entropy must be taken into account. Therefore in Rayleigh flow the flow is possible either from a to b or b to a or c to d or d to c.

PROPERTY variation in Rayleigh flow ⇒

$A_1 = A_2 = A$

① ②

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$(P_1 - P_2) A = \dot{m}(V_2 - V_1)$$

$$(P_1 - P_2) A = \dot{m} V_2 - \dot{m} V_1$$

$$(P_1 - P_2) A = \rho_2 A_2 V_2 V_2 - \rho_1 A_1 V_1 \cdot V_1$$

$$(P_1 - P_2)\cancel{A} = \cancel{A}[\rho_2 V_2^2 - \rho_1 V_1^2]$$

$$P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$P_1 + \rho V^2 \rightarrow$ impulse function (F)

$F_1 = F_2 \rightarrow$ in Rayleigh flow, impulse function Remains Constant.

1) PRESSURE RATIO ⇒

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$P_1 = \rho_1 R T_1 \Rightarrow \rho_1 = \frac{P_1}{RT_1}$$

$$\rho_2 = \frac{P_2}{RT_2}$$

$$P_1 + \frac{P_1}{RT_1} \cdot V_1^2 = P_2 + \frac{P_2}{RT_2} \cdot V_2^2$$

$$P_1 \left[1 + \frac{V_1^2}{RT_1}\right] = P_2 \left[1 + \frac{V_2^2}{RT_2}\right]$$

where. $C_1 = \sqrt{\gamma RT}$

$$\boxed{M_1 = \frac{V_1}{C_1}}$$

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$$P_1 \left[1 + \frac{\gamma V_1^2}{\gamma RT_1}\right] = P_2 \left[1 + \frac{\gamma V_2^2}{\gamma RT_2}\right]$$

$$P_1 \left[1 + \frac{\gamma V_1^2}{C_1^2}\right] = P_2 \left[1 + \frac{\gamma V_2^2}{C_2^2}\right]$$

$$P_1 \left[1 + \gamma M_1^2\right] = P_2 \left[1 + \gamma M_2^2\right]$$

*** $$\boxed{\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}}$$

2) Temperature ratio ⇒

$$\rho_1 V_1 = \rho_2 V_2$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} \quad \text{—(1)}$$

where $M_1 = \frac{V_1}{C_1}$

$$V_1 = M_1 C_1$$

$$V_1 = M_1 \sqrt{\gamma R T_1}$$

$$V_2 = M_2 \sqrt{\gamma R T_2}$$

$$\frac{V_1}{V_2} = \frac{M_1 \sqrt{\gamma R T_1}}{M_2 \sqrt{\gamma R T_2}} \quad \text{—(2)} \Rightarrow \frac{V_1}{V_2} = \frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}} \quad \text{—(4)}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1\sqrt{T_1}}{M_2\sqrt{T_2}} \quad \text{——— (3)}$$

$$P_1 = \rho_1 R T_1 \rightarrow \rho_1 = \frac{P_1}{RT_1}$$

$$P_2 = \rho_2 R T_2 \rightarrow \rho_2 = \frac{P_2}{RT_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\frac{P_2}{RT_2}}{\frac{P_1}{RT_1}} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \quad \text{——— (4)}$$

from eqn (3) ~~and~~ Not equal to (4)

$$\frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}} = \frac{P_2}{P_1} \times \frac{\cancel{T_1}\sqrt{T_1}}{\cancel{T_2}\sqrt{T_2}}$$

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$$\frac{M_1}{M_2} = \frac{P_2}{P_1} \cdot \sqrt{\frac{T_1}{T_2}}$$

$$\frac{M_1^2}{M_2^2} = \frac{P_2^2}{P_1^2} \cdot \frac{T_1}{T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^2 \cdot \frac{M_2^2}{M_1^2}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \cdot \frac{M_2^2}{M_1^2}} \rightarrow \text{Temperature Ratio}$$

Density Ratio $\Rightarrow$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \sqrt{\frac{T_1}{T_2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \cdot \frac{1+\gamma M_2^2}{1+\gamma M_1^2} \cdot \frac{M_1}{M_2}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_1^2(1+\gamma M_2^2)}{M_2^2(1+\gamma M_1^2)}} \rightarrow \text{Density Ratio}$$

Velocity Ratio ⇒ $\rho_1 V_1 = \rho_2 V_2$

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{M_2^2 \cdot (1+\gamma M_1^2)}{M_1^2 \cdot (1+\gamma M_2^2)}}$$

) Stagnation Temperature Ratio ⇒

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}}{T_{01}} \cdot \frac{T_2}{T_1} \cdot \frac{T_1}{T_{01}}$$

where $\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$

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$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{T_{02}}{T_2}\right)\left(\frac{T_2}{T_1}\right) \frac{1}{\left(\frac{T_{01}}{T_1}\right)}$$

$$\boxed{\frac{T_{02}}{T_{01}} = \frac{1+\frac{\gamma-1}{2} M_2^2}{1+\frac{\gamma-1}{2} M_1^2} \times \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \cdot \frac{M_2^2}{M_1^2}}$$

Ex. ① Air flows with negligible friction in a constant area pipe at section ① the properties are $t_1 = 60°c$, $P_1 = 135$ kPa, $V_1 = 732$ m/sec. Heat is added b/w ① and ② and $M_2 = 1.2$ find P_2, t_2, V_2, ρ_2, Heat added (Q), entropy change and also sketch the process on T-s diagram.

Ans

$T_1 = 273 + 60 = 333$ K | $M_2 = 1.2$

$\gamma = 1.4$

$P_1 = 135$ kpa

$V_1 = 732$ m/sec

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$M_1 = \frac{732}{} \qquad \boxed{M_1 = 2}$$

$P_1 = \rho_1 R T_1$

$135 = \rho_1 \times 0.287 \times 333$

$$\boxed{\rho_1 = 1.413 \text{ kg/m}^3}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1^2 (1 + \gamma M_2^2)}{M_2^2 (1 + \gamma M_1^2)}$$

$$\frac{\rho_2}{1.413} = \frac{2^2}{1.2^2} \frac{(1 + 1.4 \times 1.2^2)}{(1 + 1.4 \times 2^2)}$$

$$\rho_2 = 1.799 \text{ kg/m}^3$$

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$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

$$\frac{P_2}{135} = \frac{1 + 1.4 \times 2^2}{1 + 1.4 \times 1.2^2}$$

$$P_2 = 295.42 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \cdot \frac{M_2^2}{M_1^2}$$

$$\frac{T_2}{333} = \left(\frac{1 + 1.4 \times 2^2}{1 + 1.4 \times 1.2^2}\right)^2 \cdot \frac{1.2^2}{2^2}$$

$$\boxed{T_2 = 574 \text{ K}}$$

$$\frac{V_2}{V_1} = \frac{M_2^2 \cdot (1 + \gamma M_1^2)}{M_1^2 \cdot (1 + \gamma M_2^2)}$$

$$\frac{V_2}{732} = \frac{1.2^2 (1 + 1.4 \times 2^2)}{2^2 (1 + 1.4 \times 1.2^2)}$$

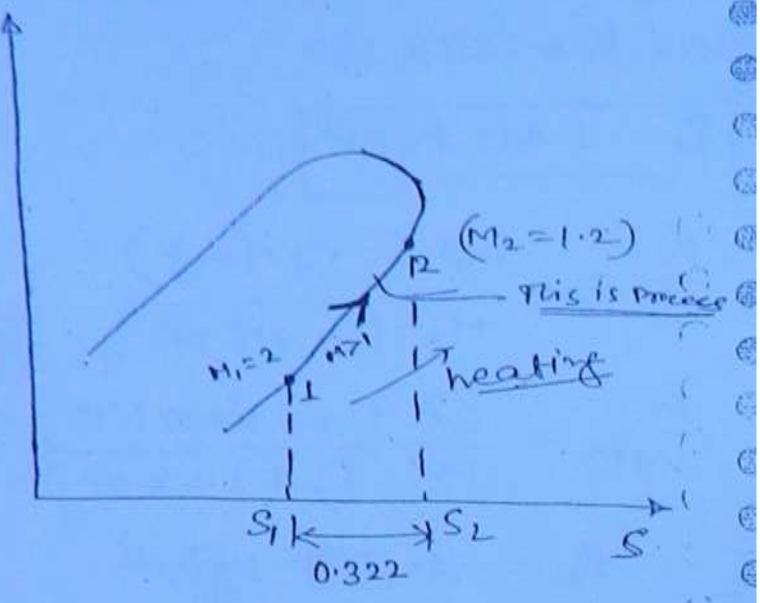
$$\boxed{V_2 = 576 \text{ m/s}}$$

$$Q = C_p\,(T_{02} - T_{01})$$

$$\frac{T_{02}}{T_1} = 1 + \frac{\gamma - 1}{2} M_2^2$$

$$\frac{T_{02}}{574} = 1 + \frac{1.4 - 1}{2}\, 1.2^2$$

$$\boxed{T_{02} = 739.41\,K}$$



$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{T_{01}}{333} = 1 + \frac{1.4 - 1}{2}\, 2^2$$

$$\boxed{T_{01} = 599.4\,K}$$

$$Q = C_p\,(T_{02} - T_{01})$$

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$$Q = 1.005\,(739.41 - 599.4)$$

$$\boxed{Q = 140.6\ \text{kJ/kg}}$$

$$T\,ds = dh - v\,dP$$

$$ds = \frac{dh}{T} - \frac{v}{T}\,dP$$

$$ds = C_p \frac{dT}{T} - \frac{R}{P}\,dP$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = 1.005 \ln\left(\frac{574}{333}\right) - 0.287 \ln\left(\frac{295.42}{135}\right)$$

$$\boxed{S_2 - S_1 = 0.322\ \text{kJ/kg-K}}$$

NORMAL SHOCK WAVES

These are highly irreversible discontinuity form in supersonic flows. the thickness of shock waves is around 0.2 um (0.2×10 ...
as shock waves are form in a faster manner $q=0$.
As the thickness of shock waves is very small friction $F=0$

Governing equation for shock waves ⇒

① continuity equation ⇒ $\boxed{\rho_1 V_1 = \rho_2 V_2}$

② momentum eqn ⇒

Thrust function are same on both side of Normal shock $\boxed{(P_1 - P_2) A = \dot{m}(V_2 - V_1)}$

③ First law of thermodynamics ⇒

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

stagnation enthalpy remains constant both side of shock wave $\boxed{h_{01} = h_{02}}$

$$C_p T_{01} = C_p T_{02}$$

∴ stagnation Temp. $\boxed{T_{01} = T_{02}}$

v.imp

** Stagnation Temp. Remains constant before and after the shock.

④ second law of thermodynamics

$$\boxed{T\,dS = dh - v\,dP}$$

⑤ equation of state ⇒ $\boxed{P = \rho R T}$

⑥ $\boxed{dh = C_p\, dT}$

The No. of unknown are six (6) and No. of equation are six (6) and hence there is a unique solution.

Property variation in shock waves ⇒

① Temperature Ratio ⇒ $\left(\frac{T_2}{T_1}\right)$

$$\frac{T_2}{T_1} = \frac{T_2}{T_{02}} \cdot \frac{T_{02}}{T_{01}} \cdot \frac{T_{01}}{T_1}$$

$$T_{01} = T_{02}$$

$$\frac{T_2}{T_1} = \frac{T_2}{T_{02}} \times \frac{T_{01}}{T_1}$$

$$\frac{T_2}{T_1} = \frac{\frac{T_{01}}{T_1}}{\frac{T_{02}}{T_2}} \Rightarrow \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

$$\boxed{\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}$$

2) Density Ratio $\Rightarrow$ We know that

$$\rho_1 V_1 = \rho_2 V_2$$

$$M_1 = \frac{V_1}{C_1} \Rightarrow V_1 = M_1 C_1$$

$$V_1 = M_1\sqrt{\gamma R T_1}$$

similarly

$$V_2 = M_2\sqrt{\gamma R T_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{M_1\sqrt{\gamma R T_1}}{M_2\sqrt{\gamma R T_2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\cdot\sqrt{\frac{T_1}{T_2}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\cdot\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{1/2}}$$

3) velocity Ratio $\Rightarrow$ $\rho_1 V_1 = \rho_2 V_2$

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{V_2}{V_1} = \frac{M_2}{M_1}\cdot\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2}}$$

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(4) PRESSURE RATIO $\Rightarrow$ we know that

$$P_1 = \rho_1 R T_2$$

$$P_2 = \rho_2 R T_2$$

$$\frac{P_2}{P_1} = \frac{\rho_2 R T_2}{\rho_1 R T_1}$$

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_2}{T_1}$$

$$\frac{P_2}{P_2} = \frac{M_1}{M_2} \cdot \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{1/2} \times \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

$$\boxed{\frac{P_2}{P_1} = \frac{M_1}{M_2} \cdot \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2}}$$

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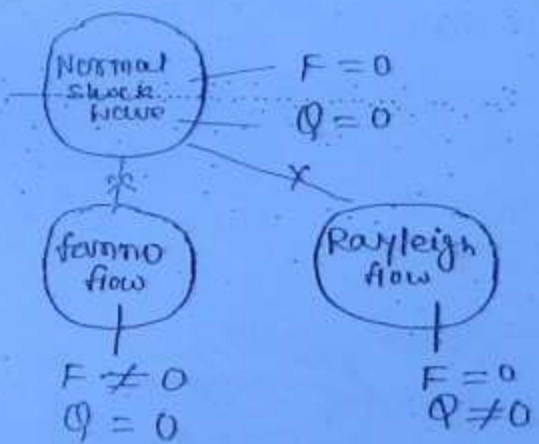
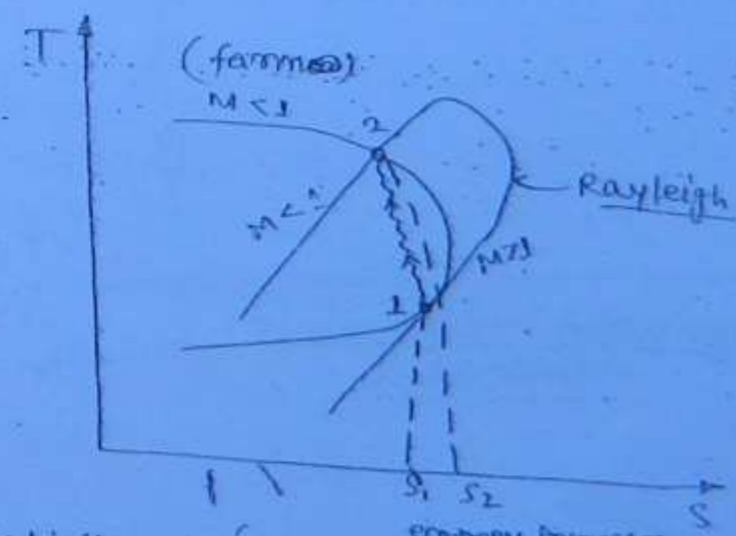
from the momentum eqn. the pressure ratio can also be derived as $\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$ (Refer Rayleigh flow)





** Normal shock wave must satisfies fanno and Rayleigh flow and hence it should lie on the intersection of fanno or Rayleigh flow.

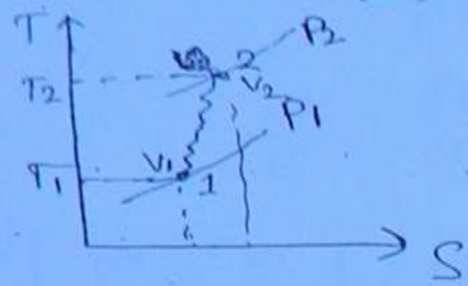
Shock waves are formed from 1 to 2 but not from 2 to 1 because if shocks are form 2 to 1. Universe entropy decreases and hence it violates 2nd law of thermodynamics. Therefore sho

After the shock waves, the flow change from supersonic to subsonic.

⇒ variation of properties ⇒

) stagnation Temp. Remains constant i.e.

$$\boxed{T_{01} = T_{02}}$$



increase in static Temp.

static pressure is increases

) Decrease in velocity ⇒

$$(P_1 - P_2)A = \dot{m}(V_2 - V_1)$$

$$P_2 > P_1$$

$$-ve = V_2 - V_1$$

Then velocity decr

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$V_1 = M_1\sqrt{\gamma R T_1}$$

$$V_2 = M_2\sqrt{\gamma R T_2}$$

$$\frac{V_2}{V_1} = \frac{M_2}{M_1}\frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}} \qquad M_2 < 1, \quad M_1 > 1$$

$$\frac{V_2}{V_1} = \frac{M_2}{M_1}\cdot\sqrt{\frac{T_2}{T_1}} \qquad \frac{M_2}{M_1} < 1$$

⇒ Density is increase ⇒ $\rho_1 V_1 = \rho_2 V_2$

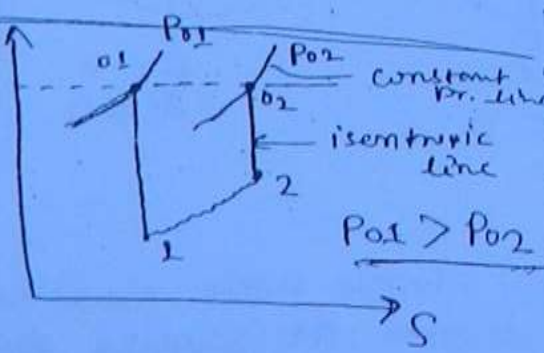
$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$$

$$\frac{\rho_2}{\rho_1} > 1$$

$$\rho_2 > \rho_1$$

⇒ Stagnation pressure is decreases.

$T_{01} = T_{02}$



$P_{01} > P_{02}$

* NOZZLE And DIFFUSER *

Nozzle is a device which is used for increasing velocity at the expense of pressure energy. (pressure energy decrease)

Diffuser is used for increasing pressure at the expense of kinetic energy. (velocity decrease)

Application of Nozzles:

1. Steam turbine
2. Gas turbine
3. Jet propulsion system.

Normal shock

Application of Diffuser ⇒ Diffuser are used in centrifugal compressor.

$$\left(\frac{\partial P}{\partial \rho}\right)_S = C^2$$

where $C \rightarrow$ sonic velocity or velocity of sound.

Mach NO. $M = \frac{V}{C}$

$M < 1 \Rightarrow$ subsonic.

$M = 1 \Rightarrow$ sonic.

$M > 1 \Rightarrow$ supersonic.

Nozzle flow Analysis ⇒

Assumptions:

1. Steady flow.
2. one dimensional flow.
3. Isentropic flow.

$$\boxed{\rho AV = \text{Constant}}$$ Continuity equation

$$d\rho \cdot AV + \rho V \cdot dA + \rho A dV = 0$$

$$\Rightarrow \frac{d\rho AV + \rho V dA + \rho A dV}{\rho AV} = \frac{0}{\rho AV}$$

$$\boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0} \quad \text{——— (1)}$$

For Bernoullies and Euler's equation, we have

$$\frac{dP}{\rho} + VdV + gdz = 0$$

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IF pressure energy changes negligible, $dz = 0$

hence $$\frac{dP}{\rho} + VdV = 0$$

$$\Rightarrow \frac{dP}{\rho} = -VdV$$

$$dP = -\rho V^2 \frac{dV}{V}$$

$$\Rightarrow \boxed{\frac{dP}{\rho V^2} = -\frac{dV}{V}} \longrightarrow (2)$$

from eqn. (1) $$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\frac{dA}{A} = -\frac{d\rho}{\rho} - \frac{dV}{V}$$

$$\Rightarrow \frac{dA}{A} = -\frac{d\rho}{\rho} + \frac{dP}{\rho V^2}$$

(170)

$$\Rightarrow \frac{dA}{A} = \frac{dP}{\rho V^2}\left[-\frac{V^2 d\rho}{dP} + 1\right]$$

$$= \frac{dP}{\rho V^2}\left[1 - \frac{V^2}{\frac{dP}{d\rho}}\right]$$

$$= \frac{dP}{\rho V^2}\left[1 - \frac{V^2}{C^2}\right]$$ where $M = \frac{V}{C}$, $M^2 = \frac{V^2}{C^2}$

$$= \frac{dP}{\rho V^2}\left[1 - M^2\right]$$

**** $$= -\frac{dV}{V}\left[1 - M^2\right]$$

$$\boxed{\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}}$$ — velocity

**. For Nozzle dV is always positive.

Case (1) $M < 1$ (subsonic)

ES 20—

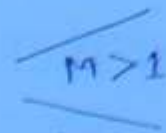
Converging Nozzle

$$\frac{dA}{A} = -\frac{dV}{V}$$

$$\frac{dA}{A} = -ve \text{ (Negative)}$$

i.e. area is decreasing.

Case-II $M > 1$ (supersonic)



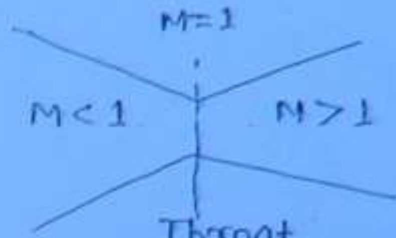
Divergent Nozzle

$$\frac{dA}{A} = +ve \text{ (positive)}$$

i.e. area is increasing.

Case-III $M = 1$ (sonic)

Convergent - Divergent Nozzle



(II) Diffuser ⇒

$$\frac{dA}{A} = (m^2 - 1)\frac{dv}{V}$$

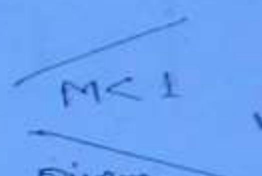
For diffuser (dv is always negative (-ve))

(17)

Case-1 When $M \not> 1$ (subsonic)

$$\frac{dA}{A} = +ve$$

∴ i.e. area is increasing

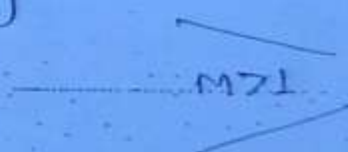


Divergent diffuser

Case-2 When $M > 1$ (supersonic)

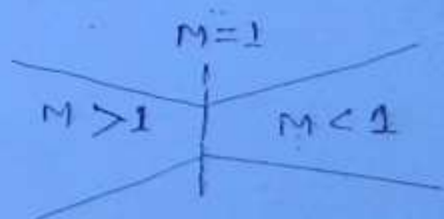
$$\frac{dA}{A} = -ve$$

∴ i.e. Area is decreasing.



Convergent diffu

Case-(3) When $M = 1$ (sonic)



Convergent - Divergent diffuser

or Reversible adiabatic

$$PV^{\gamma} = K \rightarrow \text{constant.}$$

$$\Rightarrow \frac{P}{\rho^{\gamma}} = K \qquad \text{because } \rho = \frac{m}{V}, \quad V \propto \frac{1}{\rho}$$

$$\Rightarrow P = K \cdot \rho^{\gamma}$$

$$\frac{dP}{d\rho} = K \cdot \gamma \cdot \rho^{\gamma - 1}$$

$$= \gamma K \cdot \frac{\rho^{\gamma}}{\rho}$$

$$= \frac{\gamma}{\rho} \times P \qquad \therefore P = \rho R T$$

$$= \gamma \times RT$$

$$c^2 = \gamma R T$$

$$\boxed{c = \sqrt{\gamma R T}} \quad ***$$

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or Nozzle ⇒

$$h_1 + \frac{V_1^2}{2} + \cancel{g z_1} + \cancel{q} = h_2 + \frac{V_2^2}{2} + \cancel{g z_2} + \cancel{w}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$V_1 << V_2$$

$$\Rightarrow h_1 = h_2 + \frac{V_2^2}{2}$$

$$\Rightarrow h_1 - h_2 = \frac{V_2^2}{2}$$

$$\Rightarrow V_2 = \sqrt{2(h_1 - h_2)}$$

$$V_2 = \sqrt{2 \times C_p (T_1 - T_2)}$$

$$V_2 = \sqrt{2 C_p T_1 \left(1 - \frac{T_2}{T_1}\right)}$$

for ideal gas $\quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$

$$\therefore V_2 = \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

As $\frac{P_1}{\rho_1^{\gamma}} = \frac{P_2}{\rho_2^{\gamma}}$

$$\Rightarrow \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \frac{P_2}{P_1}$$

$$\rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\dot{m} = \rho_2 A_2 V_2$$

※※※

$$\Rightarrow \boxed{\dot{m} = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} \cdot A_2 \cdot \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

⊞ critical pressure ratio :⇒

$$\dot{m} = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} \cdot A_2 \cdot \sqrt{2 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\dot{m} = \rho_1 A_2 \cdot \sqrt{2 C_p T_1 \left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{\dot{m}}{A_2} = \rho_1 \sqrt{2 C_p T_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

For max. mass flow rate per unit area.

$$\frac{d\left(\frac{\dot{m}}{A_2}\right)}{d\left(\frac{P_2}{P_1}\right)} = 0$$

$$\Rightarrow \frac{2}{\gamma}\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma}\left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}-1} = 0$$

$$\Rightarrow \frac{2}{\gamma}\left(\frac{P_2}{P_1}\right)^{\frac{2-\gamma}{\gamma}} = \frac{\gamma+1}{\gamma}\left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\Rightarrow \frac{2}{\gamma+1} = \frac{\left(\frac{P_2}{P_1}\right)^{1/\gamma}}{\left(\frac{P_2}{P_1}\right)^{\frac{2-\gamma}{\gamma}}}$$

$$\Rightarrow \frac{2}{\gamma+1} = \left(\frac{P_2}{P_1}\right)^{1/\gamma - \frac{2-\gamma}{\gamma}}$$

$$\Rightarrow \frac{2}{\gamma+1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}}$$ **** critical pressure ratio

$\frac{\dot{m}}{A_2}$ will be maximum, when A_2 is minimum. i.e. this occurs at the throat section.

Case-1 critical pressure ratio for air $\Rightarrow$

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}}$$

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$$\boxed{\frac{P_2}{P_1} = 0.528}$$ ** : PSU's

Case-② IF superheated steam expand then $n = 1.3$

$$\frac{P_2}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

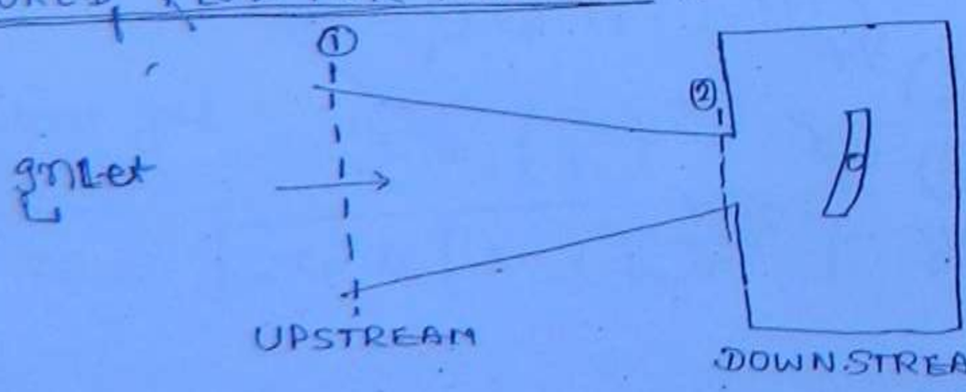
$$= \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{1.3-1}} \Rightarrow 0.545 \quad ***$$

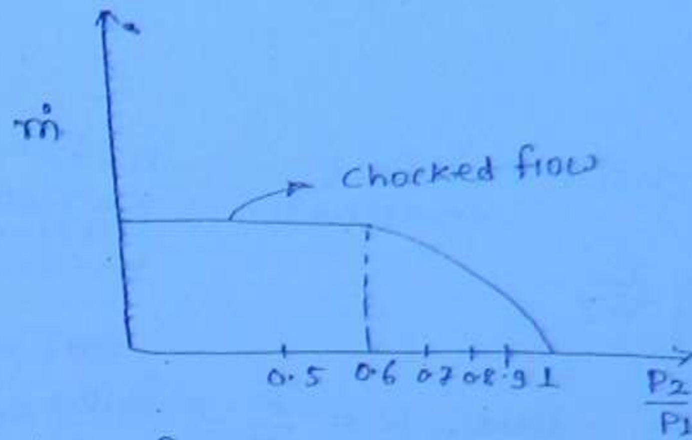
Case-③ IF wet steam then $n = 1.135$

$$\frac{P_2}{P_1} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}}$$

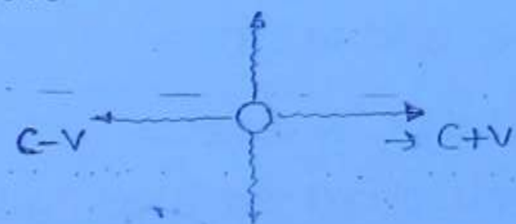
$$\frac{P_2}{P_1} = 0.577 \quad ***$$

Ⅲ CHOKED FLOW FOR NOZZLES $\Rightarrow$





maximum mass flow irrespective of pressure ratio is known as chocked flow.



$M = 1$

$V = C$

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$M < 1$

$\frac{V}{C} < 1$

$V < C$

once the critical condition are reached even by decreasing the pressure there will not be any change in mass flow rate. Because disturbance will not propogate in upstream direction and hence there will not be any change in mass flow rate, once critical condition are reached.

PROB ⑧ A fluid at 6.9 bar and 93°c enter a convergent nozzle with Negligible velocity. and expand isentropically into a space at 3.6 bar. calculate mass flow per m^2 of exit area.

① when fluid is He (Helium)

$C_p = 5.19$ kJ/kg-K, moleculer weight $= 4$

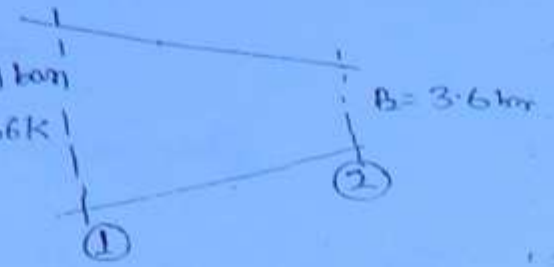
(2) when fluid is ethane

$C_p = 1.88$ kJ/kg-K, molecular weight $= 30$

Assume perfect gas behaviour in both cases.

Soln Case 1

First of all check critical pressure:-



$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$C_p = \frac{\gamma \cdot R}{\gamma - 1} \cdot \text{but } R = \frac{\bar{R}}{M} = \frac{8.314}{4} = 2.078$$

$$\Rightarrow 5.19 = \frac{\gamma \times 2.078}{\gamma - 1}$$

$$\Rightarrow \gamma = 1.668$$

$$\frac{P_2}{P_1} = \left(\frac{2}{1.668+1}\right)^{\frac{1.668}{1.668-1}}$$

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$$\frac{P_2}{P_1} = 0.486$$

$$\frac{P_2}{6.9} = 0.486$$

Critical pressure $\boxed{P_2 = 3.35 \text{ bar}}$ Ans.

As the exit pressure is greater than critical pressure critical condition are not reached and hence the flow is normal flow.

$$\dot{m} = \rho_2 A_2 V_2$$

$$\Rightarrow \frac{\dot{m}}{A_2} = \rho_2 V_2$$

$$\text{but} \quad h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000 (h_1 - h_2)}$$

$$= \sqrt{2000 C_p T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$V_2 = \sqrt{2000 \times 5.19 \times 366 \left[1 - \left(\frac{3.6}{6.9}\right)^{\frac{1.668-1}{1.668}}\right]}$$

$\boxed{V_2 = 922.8 \text{ m/s}}$ Ans.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \frac{T_2}{366} = \left(\frac{3.6}{6.9}\right)^{\frac{1.668-1}{1.668}}$$

$$\boxed{T_2 = 282.11\,K}$$

But $P_2 = \rho_2 R T_2$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{3.6 \times 100}{2.078 \times 282.11}$$

$$\rho_2 = 0.614\ kg/m^3$$

$$\therefore \frac{\dot{m}}{A_2} = \rho_2 V_2 = 0.614 \times 933.8$$

$$= 573.4\ kg/s/m^2$$

Case-II Check for critical pressure

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$R = \frac{\bar{R}}{M} = \frac{8.314}{30} \Rightarrow 0.277$$

$$C_p = \frac{\gamma R}{\gamma-1} \Rightarrow 1.88 = \frac{\gamma \times 0.277}{\gamma-1}$$

$$\gamma = 1.175$$

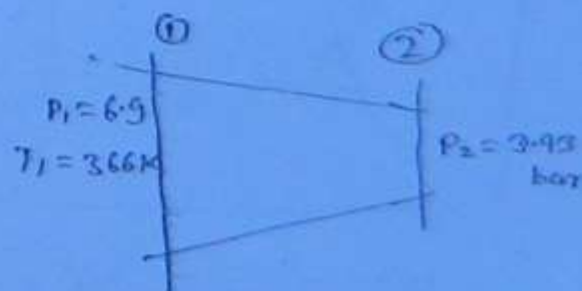
$$\frac{P_2}{P_1} = \left(\frac{2}{1.175+1}\right)^{\frac{1.175}{1.175-1}} = 0.569$$

critical pressure $P_2 = 0.569 \times 6.9$

$$\boxed{P_2 = 3.93\ bar}$$

As the exit pressure is less than critical pressure, critical condition are reached and hence the pressure at the exit of nozzle is 3.93 bar.

$$\frac{\dot{m}}{A_2} = \rho_2 V_2$$



$$V_2 = \sqrt{2000\, C_p T_1 \left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \sqrt{2000 \times 1.88 \times 366 \left(1-\left(\frac{3.93}{6.9}\right)^{\frac{1.775-1}{1.775}}\right)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_2}{366} = \left(\frac{3.93}{6.9}\right)^{\frac{1.775-1}{1.775}}$$

$V_2 =$

$T_2 =$

$$\rho_2 = \frac{P_2}{R T_2} = \frac{3.93}{0.277 \times}$$

$\rho_2 = ?$

$$\frac{\dot m}{A_2} = \rho_2 V_2 =$$

(178)

$$= 1394 \text{ kg/s/m}^2$$

***

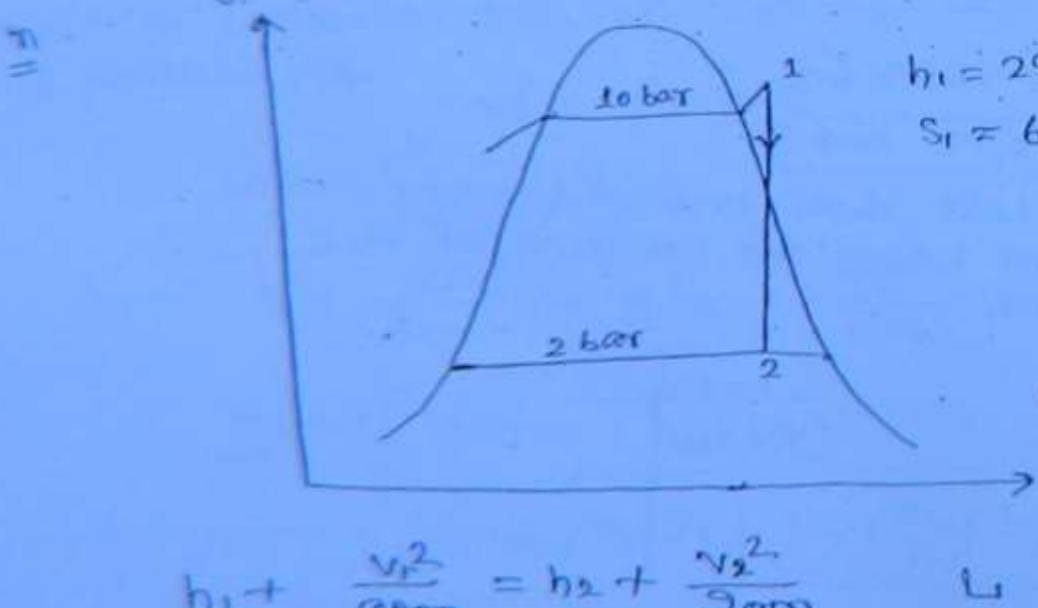
2018-(2) Steam at 10 bar and 250°C enters a Nozzle and is discharged at a pressure of 2 bar (wet condition). Initially the steam is in superheated state, find the final velocity of steam neglecting initially velocity and assume the flow to be isentropic.

At 10 bar, 250°C $\quad h = 2943$ KJ/kg

$S = 6.926$ KJ/kg-K

At 2 bar $\quad h_f = 504.7$ KJ/kg

$h_{fg} = 2201.6$, $\quad t_{sat} = 120.2°C$; $\quad S_f = 1.53$ KJ/kg-K.



$h_1 = 2943$ KJ/kg

$S_1 = 6.926$

$$h_1 + \frac{V_1^2}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

$h_2 = h_f + x \cdot h_{fg}$

$S_1 = S_2 = 6.926$

but $S_2 = S_f + x \cdot S_{fg}$

but $S_{fg} = \frac{h_{fg}}{T} = \frac{2201.6}{273+120.2} \Rightarrow 5.599$

$S_2 = 1.53 + x \times 5.599 = 6.926$

$6.926 = 1.53 + x \times 5.599$

$x = 0.963$

$h_2 = 504.7 + 0.963 \times 2201.6$

$\boxed{h_2 = 2625 \text{ kJ/kg}}$

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$V_2 = \sqrt{2000\,(h_1 - h_2)}$

$V_2 = \sqrt{2000\,(2943 - 2625)}$

$\boxed{V_2 = 795.73 \text{ m/s}}$ Ans

PROB ② A Nozzle of an impulse turbine of 1 MW of capacity has steam entering at 20 bar, 300°C (superheated state) and steam consumption is 8 kg/kWh. Steam leaves at 0.2 bar and 10% of heat drop is lost in over coming friction in diverging portion of Nozzle. If throat dia. of each Nozzle is 1 cm. find.

① No. of Nozzles required.

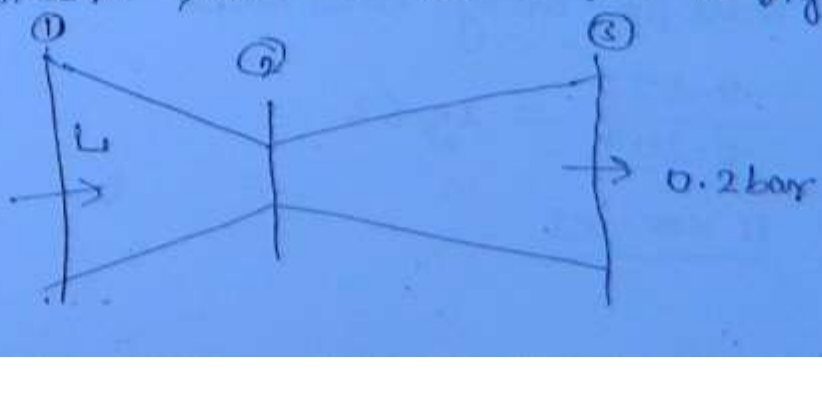
② Exit dia. of each Nozzle, take $n = 1.3$.

Isentropic enthalpy drop upto throat is equal to 142 kJ/kg

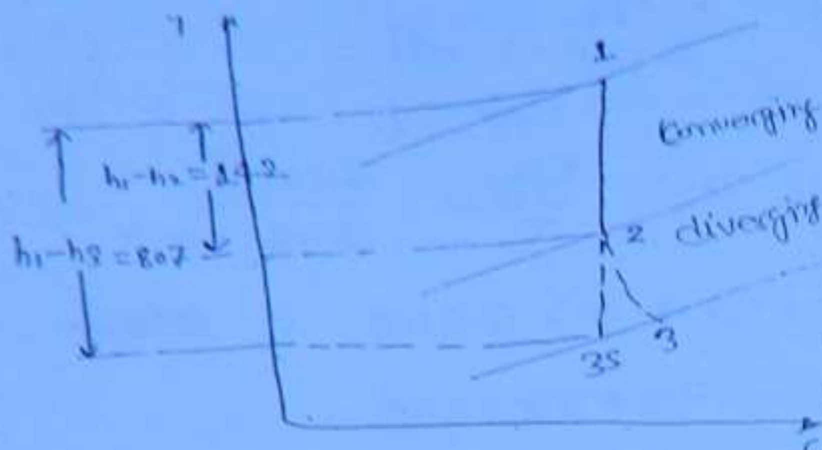
specific volume of steam at throat is $0.2 \text{ m}^3/\text{kg}$. Total isentropic enthalpy drop is equal to 807 kJ/kg.

specific volume at exit $= 7.2 \text{ m}^3/\text{kg}$.

sol:



$$SSC = \frac{m_s}{Power}$$



$$\Rightarrow \frac{8\,kg}{kWh} = \frac{m_s}{10^3\,kW}$$

$$m_s = \frac{10^3 \times 8\ kg}{3600\ sec}$$

$$m_s = 2.22\ kg/sec$$

<u>mass flow rate to each Nozzle</u> :-

$$\dot{m} = \rho_2 A_2 V_2$$

$$\text{But } \rho = \frac{m}{V} = \frac{1}{V/m} = \frac{1}{\nu}$$

$$\dot{m} = \frac{A_2 V_2}{\nu_2}$$

$$h_1 + \frac{\cancel{V_1^2}}{2000} = h_2 + \frac{V_2^2}{2000}$$

$$V_2 = \sqrt{2000\,(h_1 - h_2)}$$

$$V_2 = \sqrt{2000 \times 142} = 532.9\ m/s$$

$$\text{Now } \dot{m} = \frac{1}{0.2} \times \frac{\pi}{4} \times (0.01)^2 \times 532.9$$

$$\dot{m} = 0.209\ kg/sec$$

$$\text{No. of Nozzles} = \frac{\text{Total mass flow rate}}{\text{mass flow through each Nozzle}}$$

$$= \frac{2.22}{0.209} = 10.62$$

$$\simeq \underline{11\ \text{Nozzles}}$$

For exit diameter ⇒

$$\dot{m} = \rho_3 A_3 V_3 = \frac{A_3 V_3}{v_3}$$

but $\dot{m} = 0.209$, $v_3 = 7.2$

$$h_1 + \cancel{\frac{V_1^2}{2000}} = h_3 + \frac{V_3^2}{2000}$$

$$V_3 = \sqrt{2000\,(h_1 - h_3)}$$

$$h_2 - h_{3s} = 807 - 142 = 665 \text{ KJ/kg.}$$

$$h_2 - h_3 = 0.9\,(h_2 - h_{3s}) = 0.9 \times 665$$

$$h_2 - h_3 = 598.5 \text{ KJ/kg}$$

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$$\therefore h_1 - h_3 = (h_1 - h_2) + (h_2 - h_3)$$

$$= 142 + 598.5$$

$$h_1 - h_3 = 740.5 \text{ KJ/kg}$$

$$V_3 = \sqrt{2000 \times 740.5}$$

$$V_3 = 1216.9 \text{ m/s}$$

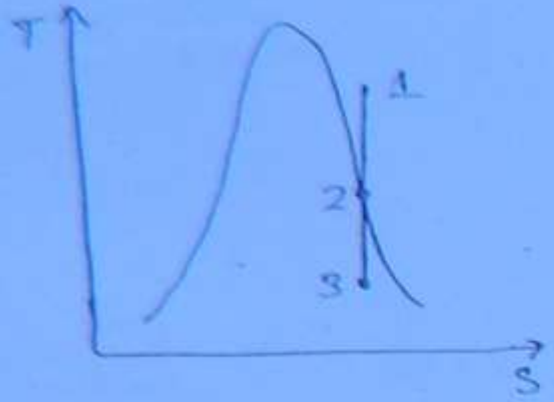
$$\dot{m} = \rho_3 A_3 V_3 = \frac{A_3 V_3}{v_3}$$

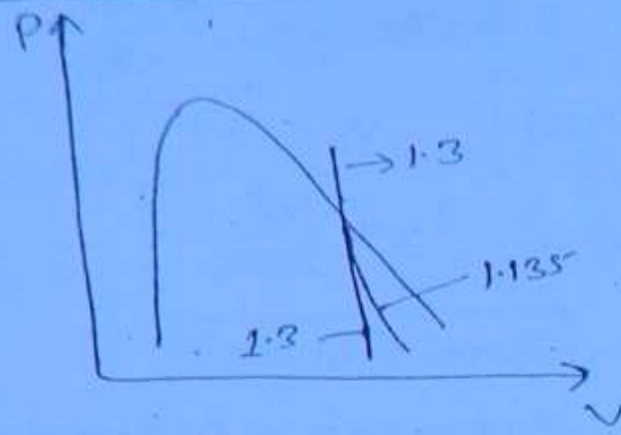
$$0.209 = \frac{\frac{\pi}{4}(D_3^2) \times 1216.9}{7.2}$$

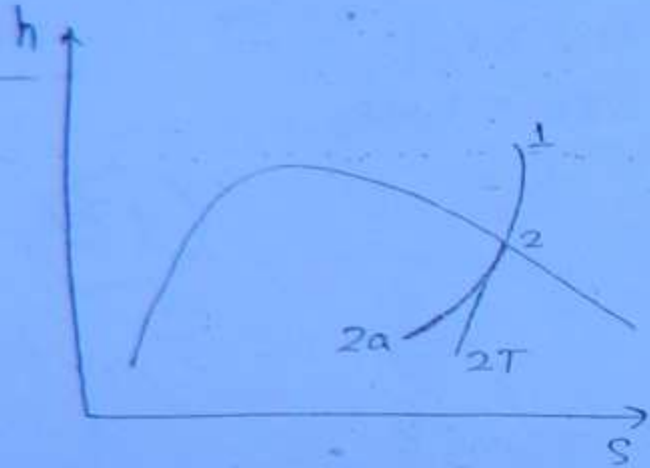
$$D_3 = 0.0396 \text{ m}$$

$$\boxed{D_3 = 3.96 \text{ cm}}$$ Ans:

Supersaturated flow / Metastable flow in steam Nozzle ⇒







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In steam Nozzle as steam expand from superheated condition condensation should begin once it reaches saturation vapour state. But due to very high velocities steam does not undergo condensation and condensation is delayed.

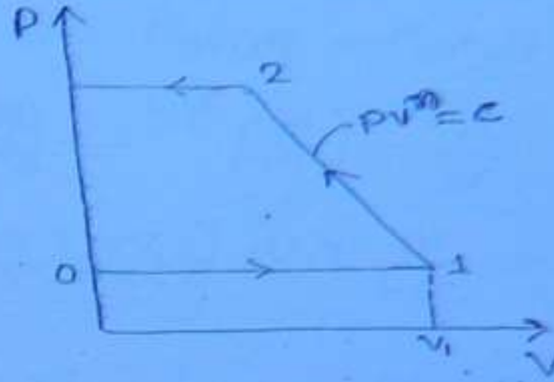
This type of flow is known as supersaturated flow or metastable flow in Nozzles.

Effect of super saturated flow ⇒

1. Reduction in specific volume.
2. Decrease in Enthalpy drop
3. Decrease in exit velocity (but effect is small)
4. Increase in mass flow rate.

(#) Reciprocating Compressor (#)

Work Input to the compressor without clearance volume.



$$W = \frac{n}{n-1}\left(P_1V_1 - P_2V_2\right)$$

$$= -\frac{n}{n-1}\left(P_2V_2 - P_1V_1\right)$$

$$W_{input} = \frac{n}{n-1}\left(P_2V_2 - P_1V_1\right)$$

$$= \frac{n}{n-1} P_1V_1 \left(\frac{P_2V_2}{P_1V_1} - 1\right)$$

But $P_1V_1^n = P_2V_2^n$

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$$\Rightarrow \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{-1/n}$$

$$\therefore W_{input} = \frac{n}{n-1} P_1V_1 \left[\frac{P_2}{P_1} \times \left(\frac{P_2}{P_1}\right)^{-1/n} - 1\right]$$

$$W_{input} = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

****

$$\boxed{W_{input} = \frac{n}{n-1} P_1 V_{act} \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

$$\boxed{W_{input} = \frac{n}{n-1} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]}$$ **

(#) **Work input to the compressor with clearance volume ⇒**

$$W_{input} = W_c - W_e$$

$$W_c = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_e = \frac{n}{n-1}\left(P_3V_3 - P_4V_4\right)$$

$$= \frac{n}{n-1} p_4 v_4 \left(\frac{p_3 v_3}{p_4 v_4} - 1 \right)$$

As $p_3 v_3^n = p_4 v_4^n$

$$\frac{v_3}{v_4} = \left(\frac{p_4}{p_3}\right)^{1/n} = \left(\frac{p_1}{p_2}\right)^{1/n} = \left(\frac{p_2}{p_1}\right)^{-1/n}$$

$$W_c = \frac{n}{n-1} p_1 v_4 \left[\frac{p_2}{p_1} \left(\frac{p_2}{p_1}\right)^{-1/n} - 1 \right]$$

$$= \frac{n}{n-1} p_1 v_4 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{input} = \frac{n}{n-1} p_1 (v_1 - v_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$\boxed{W_{input} = \frac{n}{n-1} p_1 V_{act} \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]}$$

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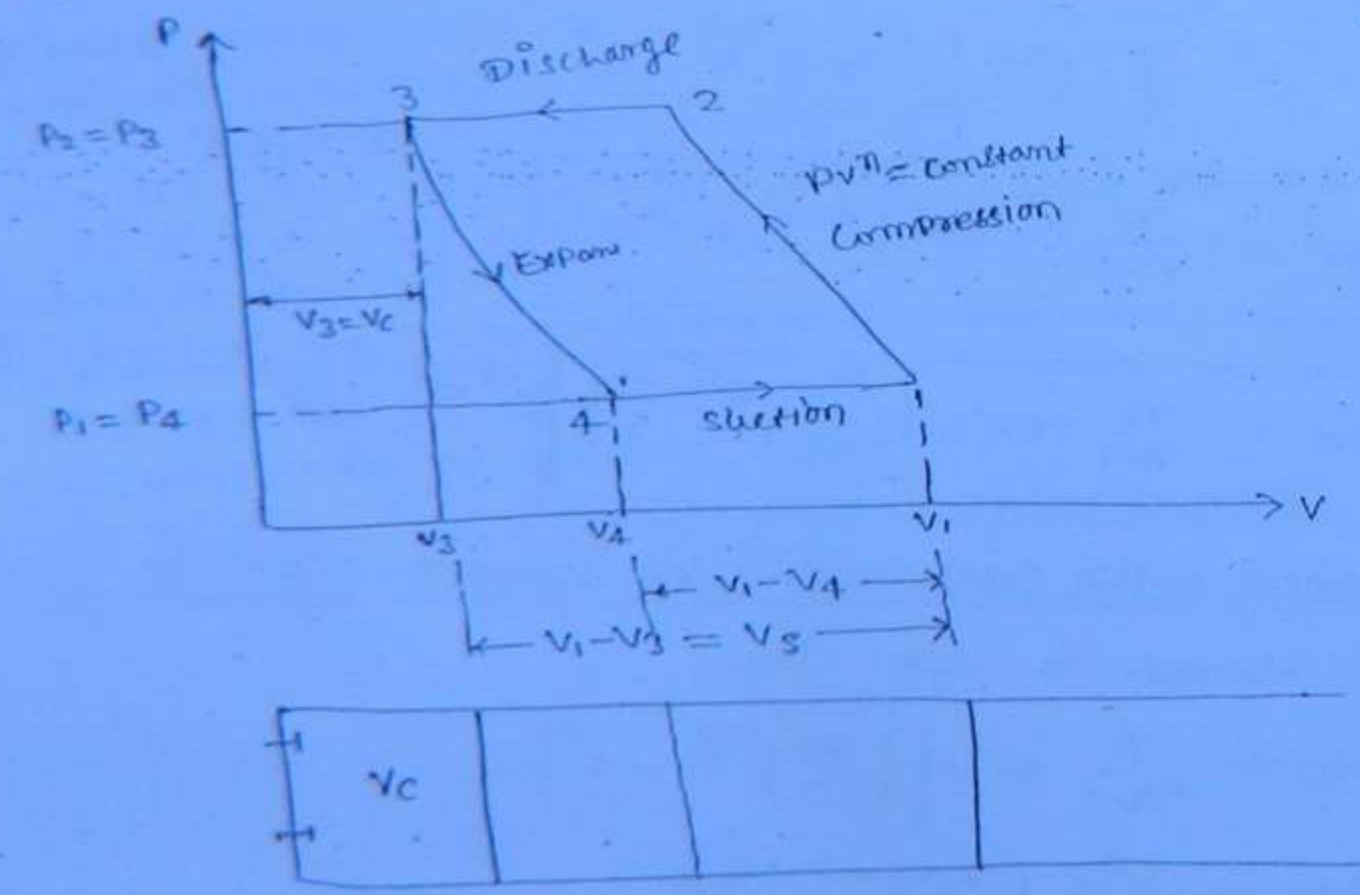
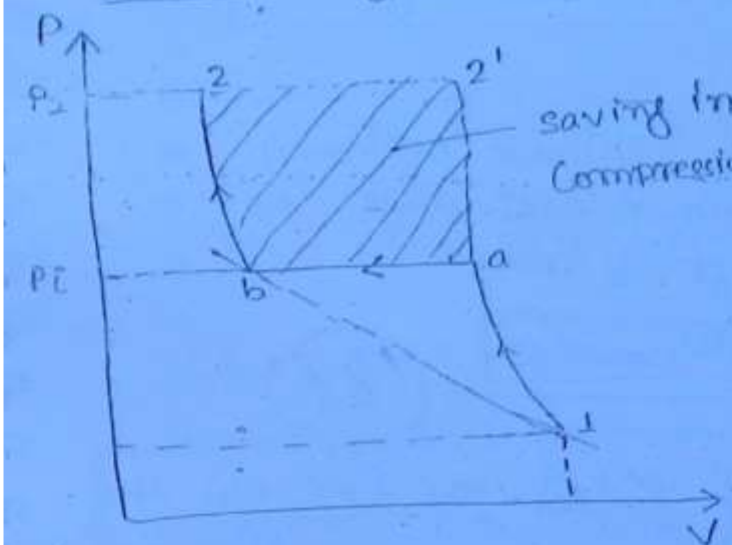


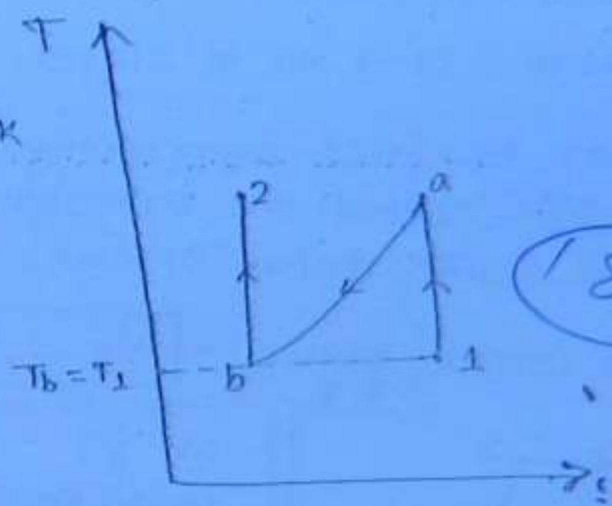
Note ⇒ Clearance volume has No effect on work input

*** $$\boxed{\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/n}}$$

Where C = Clearance factor $= \frac{V_c}{V_s}$

⊕ Inter Cooling in Reciprocating Compressor ⇒





Condition for min. work input with perfect inter cooling ⇒

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_1 = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_2 = \frac{n}{n-1} P_b V_b \left[\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 1\right]$$

But for perfect intercooling

$$P_1 V_1 = P_b V_b$$

$$\therefore W_2 = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 1\right]$$

For min. work input $P_i = \sqrt{P_1 P_2}$

$$\frac{P_i}{P_1} = \frac{\sqrt{P_1 P_2}}{P_1} = \sqrt{\frac{P_2}{P_1}},$$

$$\frac{P_2}{P_i} = \frac{P_2}{\sqrt{P_1 P_2}} = \sqrt{\frac{P_2}{P_1}}$$

overall Ratio $= \frac{P_2}{P_1}$

$$\Rightarrow \frac{P_2}{P_1} = \frac{P_2}{P_i} \times \frac{P_i}{P_1}$$

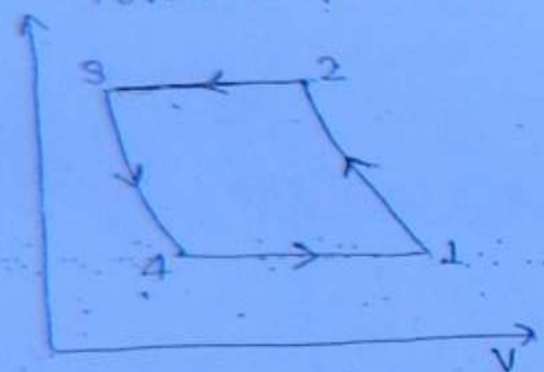
$$= \left(\frac{P_i}{P_1}\right)^2$$

verall pressure Ratio = $(\text{Pressure ratio in each stage})^N$

where $N \rightarrow$ No. of stages.

ote ⇒ For perfect intercooling with minimum (optimum) work input the pressure ratio in each stage is
me and work input in each stage is more.

Total work input → $\boxed{W_T = N \times W_{each\,stage}}$ 186



mass entering $= m_1 - m_4$

mass leaving $= m_2 - m_3$

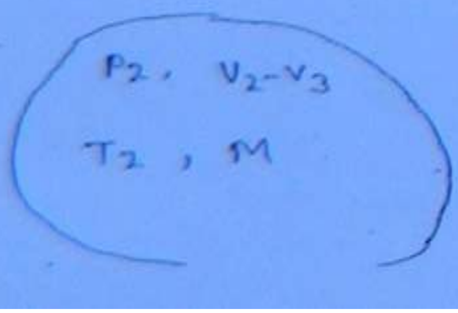
but $m_1 = m_2$, $m_3 = m_4$

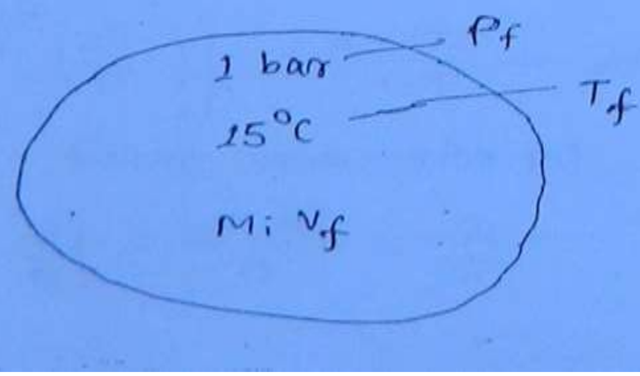
∴ mass entering = mass leaving

Because during compression and expansion no mass is entering or leaving.

⊛ FREE AIR DELIVERY (FAD) ⇒

The volume that is delivered i.e. $(V_2 - V_3)$, when reduced to 1 bar pressure 15°C temp. is known as free air delivery.





$$P_2(V_2 - V_3) = mRT_2 \qquad P_f V_f = mRT_f$$

$$m = \frac{P_2(V_2 - V_3)}{RT_2}, \qquad m = \frac{P_f V_f}{RT_f}$$

$$\Rightarrow \frac{P_2(V_2 - V_3)}{\cancel{R}T_2} = \frac{P_f V_f}{\cancel{R}T_f}$$

$$\Rightarrow \boxed{\frac{P_2(V_2 - V_3)}{T_2} = \frac{P_f V_f}{T_f}} \text{ — FAD}$$

At entry,

$$P_1(V_1 - V_4) = mRT_1$$

$$m = \frac{P_1(V_1 - V_4)}{RT_1} = \frac{P_2(V_2 - V_3)}{RT_2}$$

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$$\Rightarrow \frac{P_1(V_1 - V_4)}{\cancel{R}T_1} = \frac{P_2(V_2 - V_3)}{\cancel{R}T_2}$$

***

$$\boxed{\frac{P_1(V_1 - V_4)}{T_1} = \frac{P_2(V_2 - V_3)}{T_2} = \frac{P_f V_f}{T_f}}$$

PROB ① Determine the minimum No. of stages Required in an air compressor, which takes air at 1 bar, 27°c and deliver at 180 bar. The max. discharge temp. is limited to 150°c consider the index of polytropic compression as 1.25 and perfect and optimum intercooling in b/w stages.

$$T_1 = 273 + 27 = 300k$$

$$T_2 = 273 + 150 = 423k$$

Sol'n

$P_1 = 1$ bar $= 100$ kPa

$P_2 = 180$ bar $= 18000$ kPa

$T_1 = 300$ K, $T_2 = 423 = T_i$

overall pressure ratio $\frac{P_2}{P_1} = \frac{180}{1} \Rightarrow 180$

$$\frac{T_i}{T_1} = \left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{423}{300} = \left(\frac{P_i}{P_1}\right)^{\frac{1.25-1}{1.25}}$$

$$\frac{P_i}{P_1} = 5.57$$

As $\frac{P_2}{P_1} = \left(\frac{P_i}{P_1}\right)^N$

$$180 = (5.57)^N$$

N = No. of stages = 3.024.

$\boxed{N \simeq 3}$ Ans

Q- ② A single stage reciprocating has a bore of 200 mm and stroke of 300 mm. It runs at a speed of 500 rpm. clearance volume is 5% of swept volume and n = 1.3. Intake pressure and temp. are 97 KPa and 20°C. Delivery pressure is 550 KPa. If free air Condition are 101.325 KPa and 15°C. Then find.

① Free air delivery in m^3/min.

② volumetric efficiency.

③ Power required.

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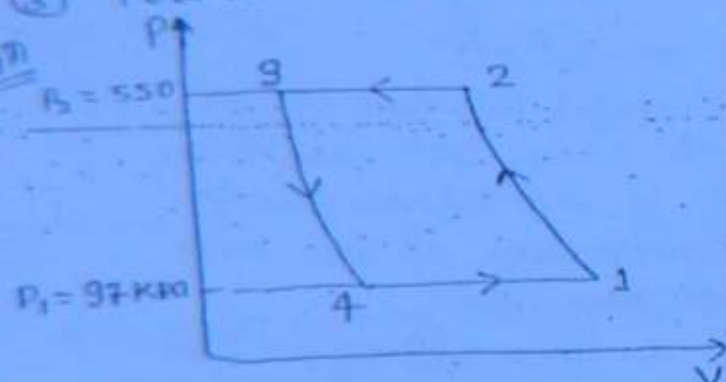
Ans



$T_1 = 273 + 20 = 293K$

$D_2 = 0.2m$, $L = 0.3m$

N = 500 rpm.

$P_f = 101.325$ KPa

$T_f = 273 + 15 = 288K$

$V_c = 0.05 V_s$

$n = 1.3$

$$\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/n}$$

where $C = \frac{V_c}{V_s} = \frac{0.05 V_s}{V_s} = 0.05$

$$\eta_{vol} = 1 + 0.05 - 0.05\left(\frac{550}{97}\right)^{1/1.3}$$

$= 0.86$

$= 86\%$

$$\frac{P_1(V_1-V_4)}{T_1} = \frac{P_2(V_2-V_3)}{T_2} = \frac{P_f V_f}{T_f}$$

but $\eta_{vol} = \frac{V_1-V_4}{V_s} = \frac{V_1-V_4}{\frac{\pi}{4}D^2L}$

$$= 0.86 = \frac{V_1-V_4}{\frac{\pi}{4}(0.2)^2 \times 0.3}$$

$$V_1 - V_4 = 8.1 \times 10^{-3}\ m^3$$

$$\Rightarrow \frac{97 \times (8.1 \times 10^{-3})}{293} = \frac{101.325 \times V_f}{288}$$

$$V_f = 7.62 \times 10^{-3}\ m^3/cycle$$

This is the free air delivery per cycle

1 cycle = 1 rev.

1 cycle/min = 1 rpm.

$$V_f = 7.62 \times 10^{-3} \frac{m^3}{cycle} \times 500 \frac{cycle}{min}$$

$$\boxed{V_f = 3.81\ m^3/min}$$ Ans:

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② Net work input :

$$W = \frac{n}{n-1} P_1 (V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W = \frac{1.3}{1.3-1} \times 97 \times (8.1 \times 10^{-3})\left[\left(\frac{550}{97}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$W = 1.67\ kJ/cycle$$

$$W = 1.67 \frac{kJ}{cycle} \times 500 \frac{cycle}{min}$$

$$W = 835\ kJ/min$$

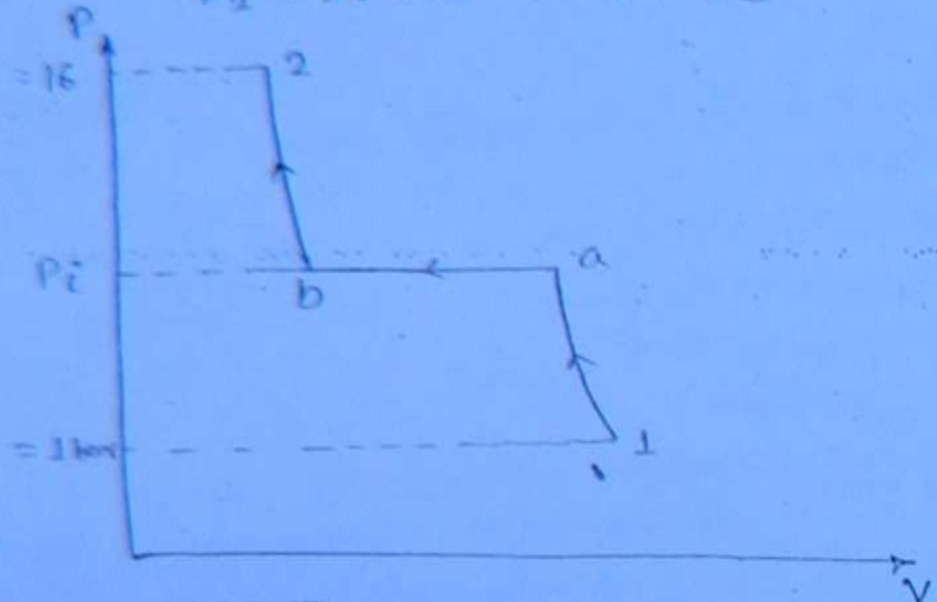
$$\boxed{W = 13.92\ kJ/sec}$$ Ans:

A two stage compressor with perfect intercooling delivers 5 kg/min of air at 16 bar. The intake condition are 1 bar and 300K. $n = 1.3$ the calculate power required, isothermal efficiency, heat transferred in each cylinder and also heat transfer in intercooler.

Sol:

$P_1 = 1$ bar $= 100$ KPa , $T_1 = 300$K

$P_2 = 16$ bar , $n = 1.3$



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$$P_i = \sqrt{P_1 P_2} = \sqrt{1 \times 16} = 4$$

$$W_1 = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{n-1} mRT_1 \left[\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3}{1.3-1} \times \frac{5}{60} \times 0.287 \times 300 \left[\left(\frac{4}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$\boxed{W_1 = 11.7 \text{ KW}}$$

$$W_T = N \times W_1 = 2 \times 11.7$$

$$\boxed{W_T = 23.4 \text{ KW}}$$

The work input in isothermal process is minimum and hence,

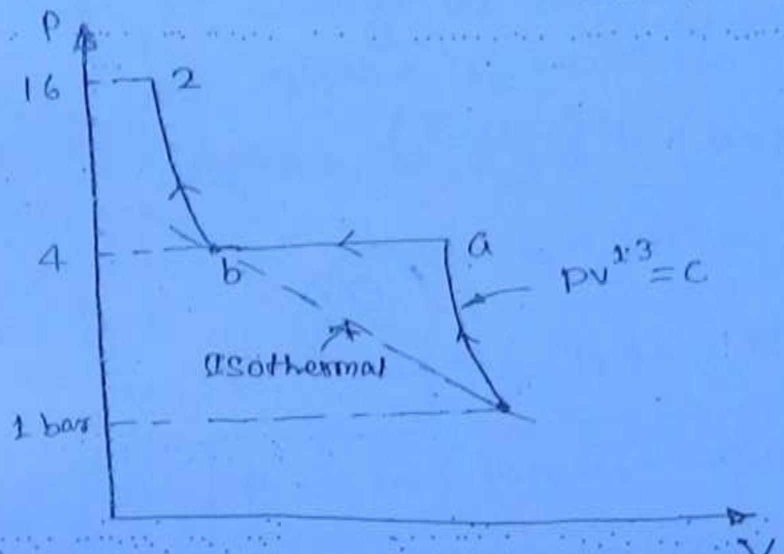
$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual work.}}$$

$$W_{isothermal} = mRT_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{P_1}{P_2}$$

$$= \frac{5}{60} \times 0.287 \times 300 \ln\left(\frac{1}{16}\right)$$

$$= -19.89 \text{ kW}$$

$$\eta_{isothermal} = \frac{19.89}{23.4} \times 100 = 85\%$$



(19)

$$\frac{T_a}{T_1} = \left(\frac{P_a}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{T_a}{300} = \left(\frac{4}{1}\right)^{\frac{1.3-1}{1.3}} \Rightarrow \boxed{T_a = 413.1 \text{ K}}$$

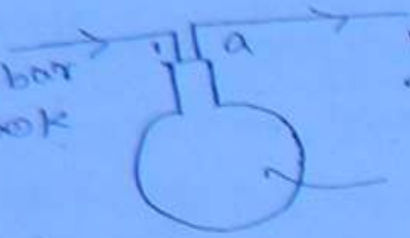
Heat rejected in intercooler $= mc_p(T_a - T_b)$

but $T_b = T_1 = 300\text{K}$

$$Q = \frac{5}{60} \times 1.005 \, (413.1 - 300)$$

$$\boxed{Q = 9.47 \text{ kJ/sec.}}$$

$P_a = 1\,\text{bar}$
$T_1 = 300\,K$



$P_a = 4$
$T_a = 413.1\,K$
$W = -11.7\ \text{kJ/sec.}$

$$h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gZ_1} + Q = h_a + \cancel{\frac{V_a^2}{2}} + \cancel{gZ_a} + W$$

$$h_1 + Q = h_a + W$$

$$Q = h_a - h_1 + W$$

$$Q = \dot{m}\,(h_a - h_1) + W$$

$$Q = \dot{m}\,C_p\,(T_a - T_1) + W$$

$$= \frac{5}{60} \times 1.005\,(413.1 - 300) + (-11.7)$$

$$\boxed{Q = 2.2\ \text{kJ/sec}}$$ Ans.

Principle of Rotating machines

Let the fluid enters the rotor at a radius r_1, then the rate of angular momentum at inlet is $\dot{m} V_{w_1} r_1$, where V_{w_1} is whirl component or tangential component of absolute velocity at Inlet. Similarly, if the fluid leaves the rotor at a radius r_2 then rate of angular momentum at exit is $\dot{m} V_{w_2} r_2$.

Rate of change of Angular momentum

$$= \dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1$$

But we know that

Rate of change of angular momentum = Torque

$$\Rightarrow \dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1 = T$$

$$\text{Power} = T \times w$$

$$P = (\dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1)\, w$$

$$P = \dot{m} \left[V_{w_2} r_2 w - V_{w_1} r_1 w \right]$$

$$\boxed{P = \dot{m} \left[V_{w_2} u_2 - V_{w_1} u_1 \right]}$$ **

euler's equation for compressor and pump.

**

$$\boxed{P = \dot{m} \left[V_{w_1} u_1 - V_{w_2} u_2 \right]}$$ For Turbine

When energy is transferred from fluid to rotor, then the device is turbine.

When energy is transferred from rotor to fluid then the device is compressor or pump.

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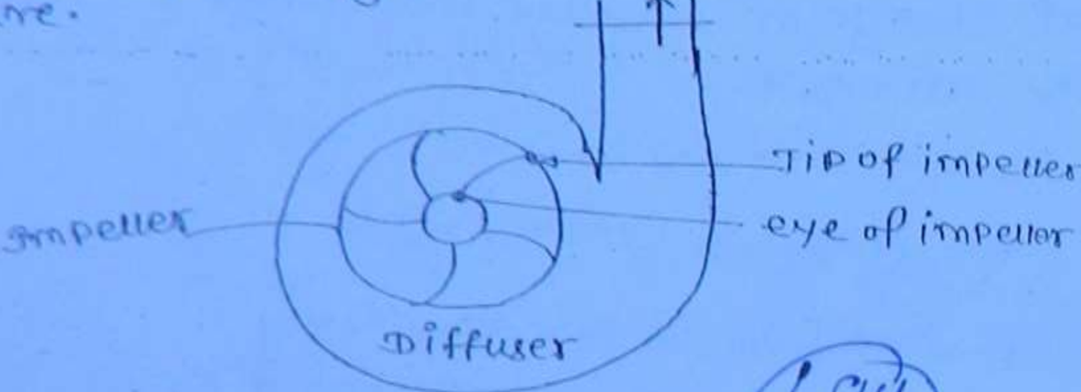
Centrifugal compressor

Principle of operation of a centrifugal compressor ⇒

In centrifugal compressor, the fluid enters axially at the eye of impeller and leaves radially at the tip of impeller. When the fluid moves radially outwards, due to centrifugal force the pressure increases. The impeller also imparts kinetic energy to the fluid and hence the fluid leaves the impeller at high kinetic energy and high pressure energy. The fluid after leaving the impeller enters diffuser, where K.E. of fluid is converted into pressure energy. Thus the fluid leaves at high pressure.



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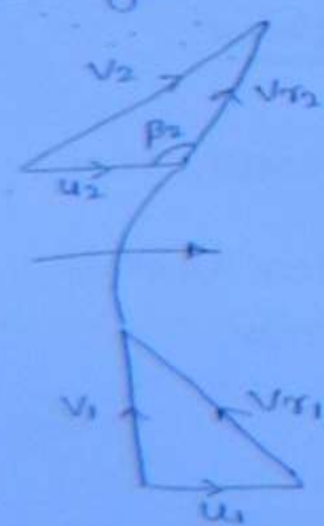
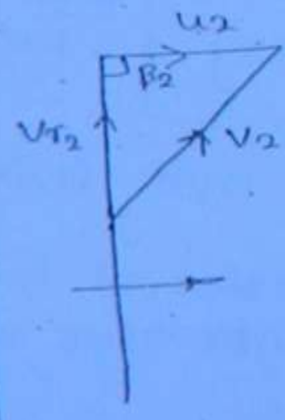
Blade Angle Triangles

Forward Angle Blade



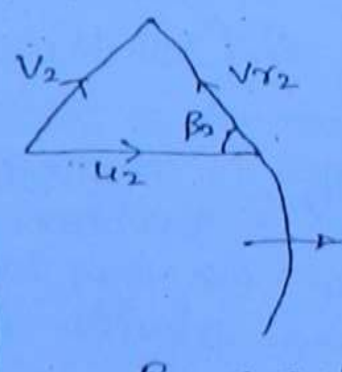
$\beta_2 > 90°$

Radial Blade



$\beta_2 = 90°$

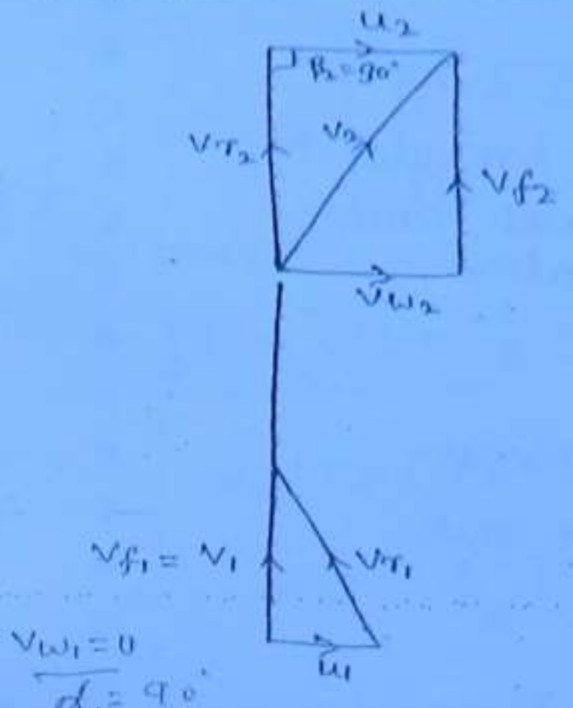
Backward angle blade



$\beta_2 < 90°$

If Nothing is mentioned then assume radial blade.

(#) Power Input for Radial Blades ⇒



$$P = \dot{m}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

But $V_{w_1} = 0$

$$P = \dot{m}\left[V_{w_2} u_2\right]$$

But $V_{w_2} = u_2$

$$\therefore \boxed{P = \dot{m} u_2^2}$$

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(#) Slip in centrifugal compressor : ⇒

Due to Inertia the fluid Particles are reluctant to move around the blade Passage and because of those there is reduction in V_{w_2}.

$$\text{Slip} = u_2 - V_{w_2}$$

$$\text{Slip factor } \sigma = \frac{V_{w_2}}{u_2}$$

$$P = \dot{m}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

for radial blade $V_{w_1} = 0$

$$P = \dot{m}\left[V_{w_2} u_2\right] \quad \text{but } V_{w_2} = \sigma u_2$$

$$\therefore \boxed{P = \dot{m} \sigma u_2^2}$$

(#) Power Input factor (ψ) ⇒

To account for frictional losses power input factor is introduced and it is the ratio of actual power input to the theoretical power input

$$\boxed{P = \dot{m} \sigma \psi u_2^2} \qquad \psi = 1.035 \text{ to } 1.05$$

→ If ψ and σ are not given in the problem, then take them unity.

USE OF SFEE IN Centrifugal compressor ⇒

Stagnation state : When the fluid is brought to zero velocity isentropically then that state reached is known as stagnation state.

$$h_{01} = h_1 + \frac{V_1^2}{2}$$

→ Isentropic process

$1 \longrightarrow 01$ ← stagnation state

$$h_1 + \frac{V_1^2}{2} + \cancel{z_1 g} + \cancel{Q} = h_2 + \frac{V_2^2}{2} + \cancel{g z_2} + W$$

$$\underline{h_1 + \frac{V_1^2}{2}} = \underline{h_2 + \frac{V_2^2}{2}} + W$$

$$h_{01} = h_{02} + W$$

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$$W = h_{01} - h_{02}$$

$$W = -(h_{02} - h_{01})$$

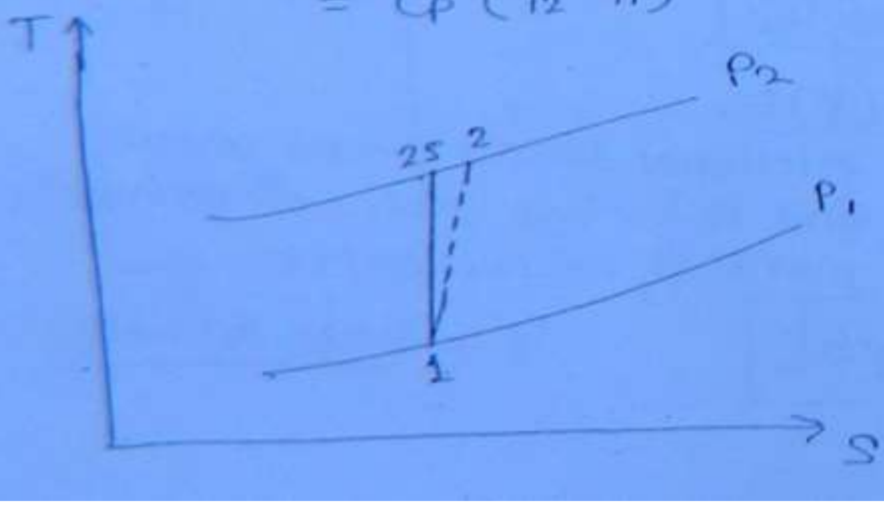
$$W_{input} = h_{02} - h_{01}$$

$$= C_p (T_{02} - T_{01})$$

If kinetic energy is neglected then

$$W_{input} = h_2 - h_1$$

$$= C_p (T_2 - T_1)$$



$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}}$$

**

We know:

$$\boxed{W = \sigma \psi u_2^2} \rightarrow \frac{m^2}{s^2} \rightarrow J/kg$$

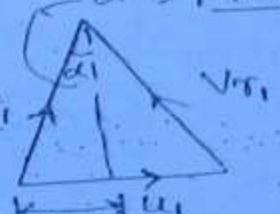
$$W = C_p (T_2 - T_1) \rightarrow kJ/kg$$

$$\text{But} \quad \frac{J}{kg} = \frac{Nm}{kg} = \frac{\cancel{kg}\, m \times m}{s^2\, \cancel{kg}} \Rightarrow \frac{m^2}{s^2}$$

Prewhirl in centrifugal compressor ⇒

To Reduce the Mach No. at the inlet prewhirl is given. Generally this prewhirl is mentioned with respect to axial direction.

$\alpha \rightarrow$ prewhirl angle



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Here $M = \frac{V_{r_1}}{C}$

for same u_1

** As α increases ↑ ⇒ V_{r_1} ↓

PROB: ①

A centrifugal compressor has an impeller dia of 0.5 m at the tip and running at 7000 rpm. Assume zero whirl at inlet and $T_1 = 290$ K, $\sigma = 1$, $\psi = 1$, and process of compression is isentropic. Determine the pressure ratio developed and specific work input.

Take $C_p = 1.005$ KJ/kg-K

Soln.

$D_2 = 0.5$ m , $N = 7000$ rpm

$\alpha = 0$, $T_1 = 290$K , $\sigma = 1$, $\psi = 1$

$$u_2 = \frac{\pi D_2 N_2}{60} = 183.25 \text{ m/s}$$

$W = \sigma \psi u_2^2 = 1 \times 1 \times (183.25)^2$

$= 33584 \ J/kg$

$\boxed{W = 33.584 \ KJ/kg}$

Also $W = h_2 - h_1 = C_p (T_2 - T_1)$

$33.584 = 1.005 (T_2 - 290)$

$\boxed{T_2 = 323.4 \ K}$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{323.4}{290}\right)^{\frac{1.4}{1.4-1}}$$

Pressure ratio $= 1.465$

PROB. ② Determine the isentropic efficiency of compressor which under test gave following result $N = 11500$ rpm, $T_1 = 21°C = 273 + 21 = 294 K$, pressure ratio $= 4$, Impeller tip dia. $D_2 = 0.75 m$, slip factor $\sigma = 0.92$

=

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.75 \times 11500}{60}$$

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$\boxed{u_2 = 451.6 \ m/s}$

$W = \sigma \psi u_2^2 = 0.92 \times 1 \times (451.6)^2$

$= 187630 \ J/kg$

$= 187.63 \ KJ/kg$

Also $W = C_p (T_2 - T_1)$

$187.63 = 1.005 (T_2 - 294)$

$\boxed{T_2 = 480.7 K}$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s}}{294} = (4)^{\frac{1.4-1}{1.4}}$$

$T_{2S} = 436.9K$

Isentropic efficiency $\eta_c = \dfrac{T_{2S} - T_1}{T_2 - T_1}$ $\overset{\text{"}}{3}$

$$\eta_c = \frac{436.9 - 294}{480.7 - 294}$$

$$\boxed{\eta_c = 76.5\%}$$

I.E.S. 2009

PROB. (3) A centrifugal compressor running at $N = 16000$ rpm takes air at $17°C$ and compressor it to a pressure ratio of 4 with an isentropic efficiency of 82%. At the exit the blade are radially inclined and slip factor is 0.85. Guide vanes at inlet give air a prewhirl angle of $20°$ to the axial direction. The dia of impeller eye is 200 mm and absolute velocity ~~impeller eye is 200 mm~~ of air at inlet is 120 m/sec find impeller tip dia.

Soln $N = 16000$ rpm, $T_1 = 17°C = 273 + 17 = 290K$,

$\frac{P_2}{P_1} = 4$, $\eta_c = 82\%$, $\sigma = 0.85$, $\alpha = 20°$

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$D_1 = 200mm = 0.2m$, $V_1 = 120$ m/sec.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60} = 167.55 \text{ m/s}$$

$$V_{w1} = V_1 \sin\alpha = 120 \times \sin 20°$$

$$V_{w1} = 41.04 \text{ m/s}$$

$$\frac{T_{2S}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{2S}}{290} = 1.486$$

$$\boxed{T_{2S} = 430.9K}$$

$$\eta_c = \frac{T_{2S} - T_1}{T_2 - T_1}$$

$$0.82 = \frac{430.9 - 290}{T_2 - 290}$$

$$\boxed{T_2 = 461.8K}$$

$C_p (T_2 - T_1)$

$= 1.005 (461.8 - 290)$

$W = 172.688 \text{ kJ/kg}$

But also $W = (V_{w_2} u_2 - V_{w_1} u_1)$

But $V_{w_2} = \sigma u_2$

$$10^3 \times 172.688 = [0.85 \times u_2^2 - 41.04 \times 167.55]$$

$$u_2^2 = 211252.6 \text{ m}^2/\text{s}^2$$

$$\boxed{u_2 = 459.6} \text{ m/s}$$

But $u_2 = \dfrac{\pi D_2 N}{60}$

$$459.6 = \frac{\pi \times D_2 \times 16000}{60}$$

$$D_2 = 0.549 \text{ m}$$

H.P dia. $\boxed{D_2 = 549 \text{ mm}}$ Ans.