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-: HAND WRITTEN NOTES:-

OF

CIVIL ENGINEERING

-: SUBJECT:-

STRENGTH OF MATERIAL

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[redacted]

STRENGTH OF MATERIALS.

- → Properties of material [Obj + Int.]
- → Simple and Thermal stresses [Obj + Conv.]
- Imp → Shear force and Bending moment [O+I+C]
- Imp → Principal Stresses [O+I+C]
- → Theory of Failure [O+I]
- → Deflection of beams [O+C]
- Imp → Torsion of Shafts [O+I+C]
- → Pressure vessels [O+C]
- → Column Theory [O]
- → Springs [O] GATE + IES
- → Principal axis & principal MOI [O+C]
- → Shear Center [O]

PROPERTIES OF METALS-

1. Why mild steel is most commonly used metal in mordern world in civil engg?

Ans- (i) Mild Steel is ductile.
(ii) Equally strong in tension & compression.
(iii) It's young modulus in tension and compression is equal.
(iv) Thermal coefficient (α) of M.S. is almost equal to thermal coefficient of concrete.

$\alpha_{conc} \approx \alpha_{steel} = 12 \times 10^{-6} / ^\circ C$

$\alpha_{cu} = 16 \times 10^{-6} / ^\circ C$, $\alpha_{Al} = 23 \times 10^{-6} / ^\circ C$

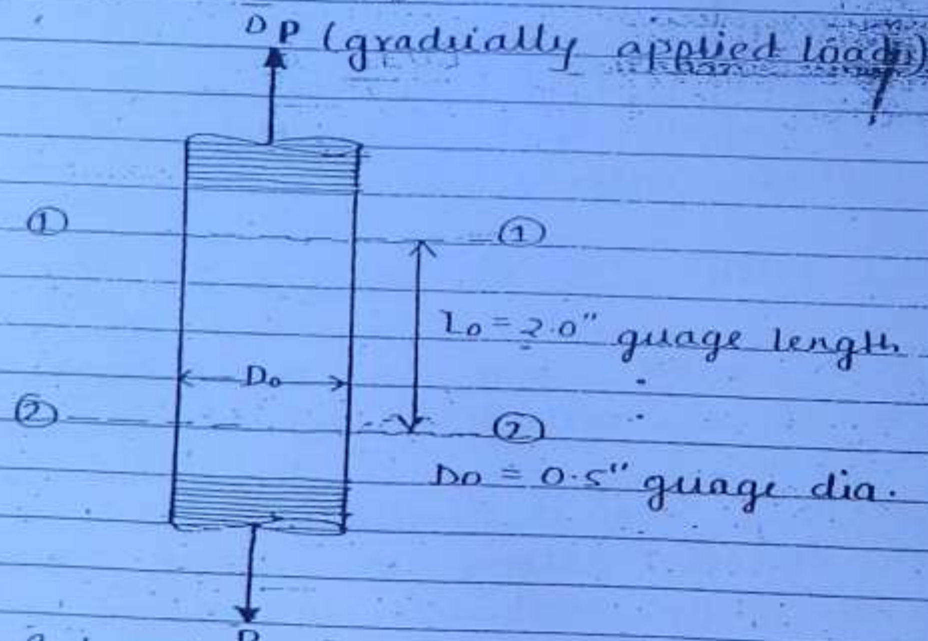
Equal 'α' will prevent failure of bond with thermal change.

(v) Recycable, weldable, cheaper & widely available

TENSION TEST IN MILD STEEL -

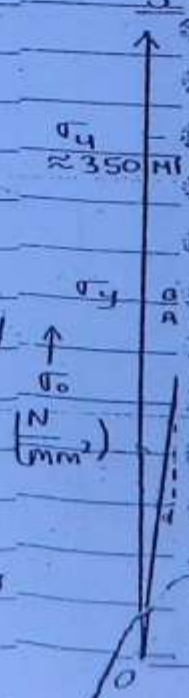
- Test is conducted in UTM.
- Test is standardised by ASTM.
- Standard specimen is a solid cylinder of length of guage length 'L' = 2.0" and guage dia. D = 0.5"

$L = 4.0$



STRESS - Internal resistance of metal offered against deformation which is force per unit area.

(Pressure is external force distributed on unit area and its unit is N/mm^2)



STRESS

- Engg/Nominal/conventional stress: $\sigma_0 = P/A_0 = P/\frac{\pi}{4}D_0^2$
- True/Actual stress: $\sigma_a = P/A_a = P/\frac{\pi}{4}D^2$

In tension test, actual stress is equal or greater than nominal stress. $\boxed{\sigma_a \geq \sigma_0}$

In Elastic Limit, both stresses are nearly equal bcoz change in area is insignificant

$\boxed{\text{STRAIN}}$

Nominal / Conventional or Engineering Strain.

$$\boxed{\epsilon_0 = \Delta L / L_0}$$

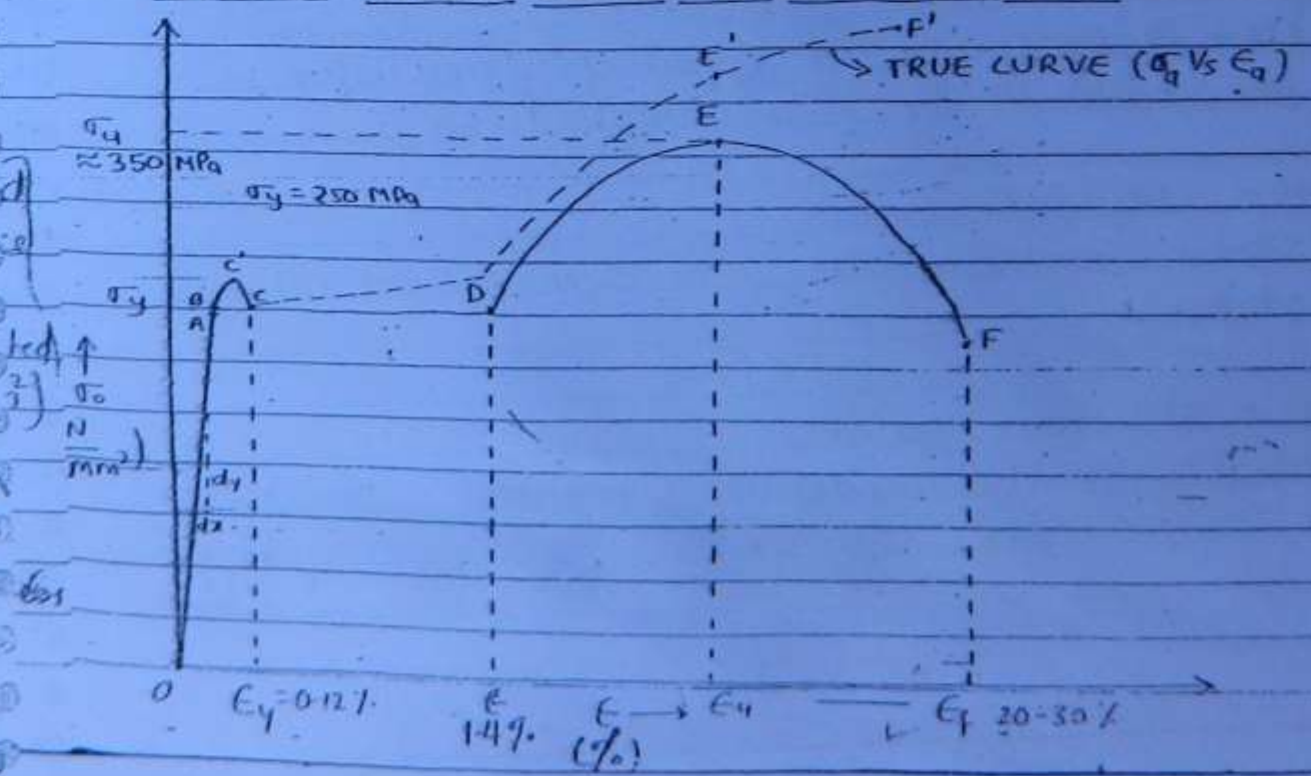
True / Actual Strain

$$\epsilon_a = \Delta L / L_a$$

$$\because L_a = L_0 \pm \Delta L$$

Stress - strain curve in tension test should be drawn either b/w engg. parameters or b/w true parameters.

STRESS - STRAIN CURVE IN TENSION TEST -



platue - state of little change following a period of progress.

A → proportional limit
OA is straight line
Slope OA is constant, $\frac{dy}{dx}$ = const. = E (Young's modulus)
Hookes law is valid in this zone.

B → Elastic limit
AB → Non linear

NOTE → for mild steel, B and A are so close but B is always greater than A

C' → Upper yield point
C → Lower yield point / Actual yield point

NOTE → C' → C decline in stress is due to - slipping of Carbon atoms in molecular structure of steel.

CD → Plastic zone or yield plateau. for mild steel C is close to B.

E → Ultimate point
Stress corresponding to E is called ultimate stress.

F → Fracture point
DE → Strain hardening region.
EF → Strain softening region or necking.

** Following points may be noticed -

1. Plastic strain (CD) is 10-15 times than that of Elastic strain (OB).

2. In mild steel, A, B, C are close to each other but $C > B > A$.

3. The fracture strain depends upon the % age of Carbon present in steel. With increase in % carbon, fracture strain reduces and ultimate stress increases.

4. The stress corresponding to fracture point is called fracture strength. And E_u correspond to E is called ultimate strength.

5. For mild steel, yield stress is nearly equal to elastic stress and within elastic limit curve is linearly straigth. Hence, it is linear elastic material.

Relation b/w σ_0 and σ_a -

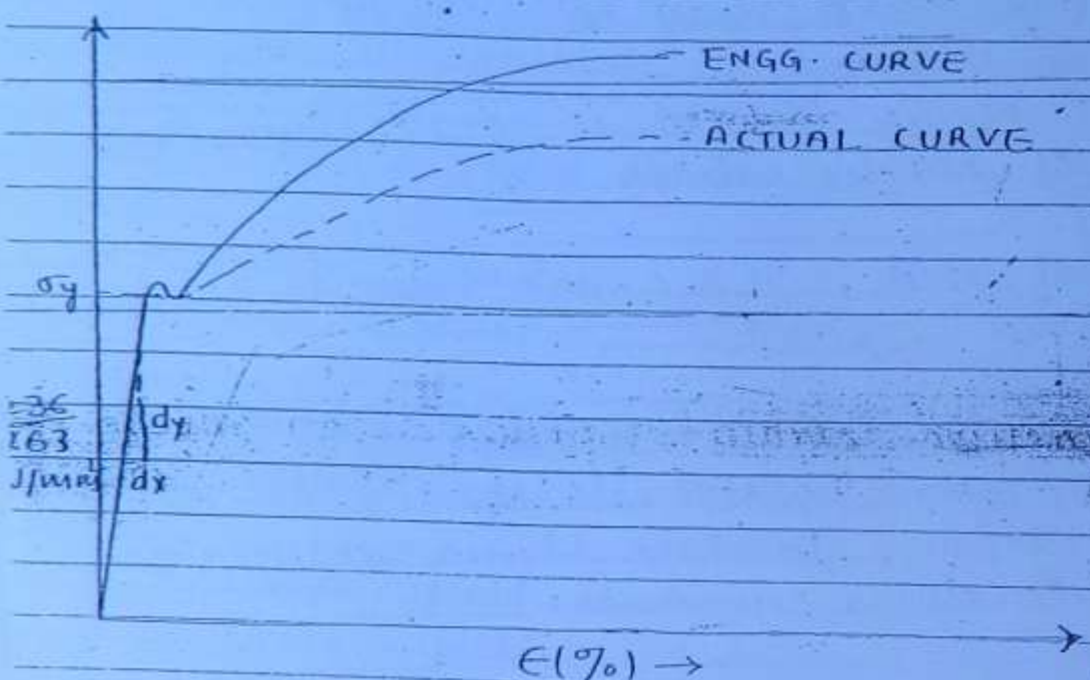
$$\boxed{\sigma_a = \sigma_0(1 \pm \epsilon_0)}$$

+ For tension
− for comp.

While deriving above eqn volume change is neglected which is true in plastic regio (non elastic region).

$\sigma_a < \sigma_o$

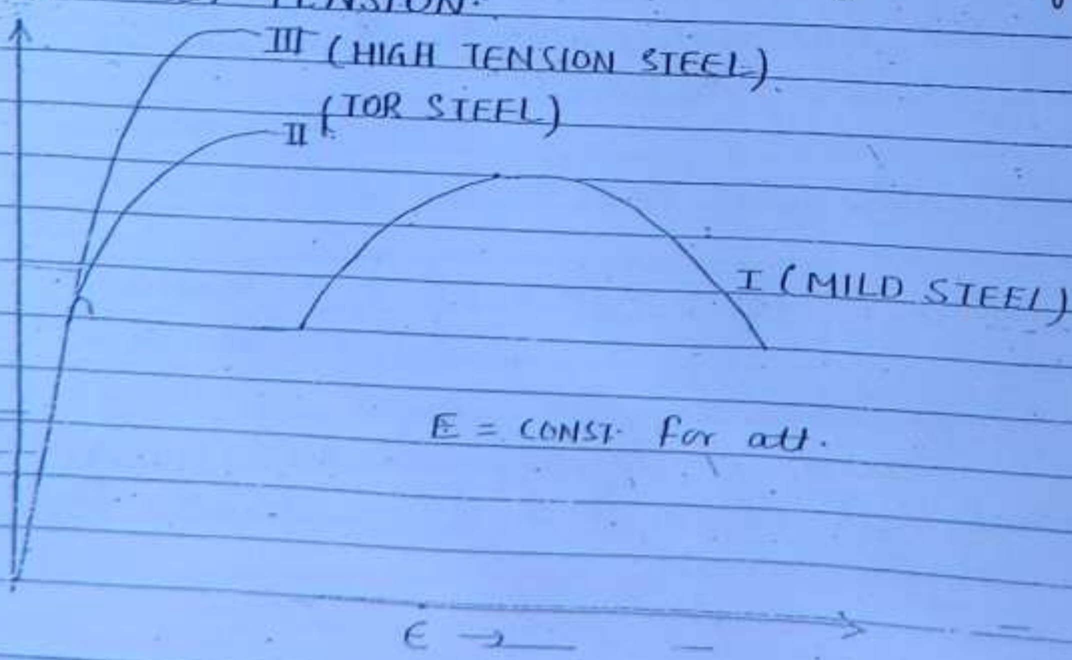
COMPRESSION CURVE FOR MILD STEEL →



$dy/dx = E = 2.1 \times 10^5\ N/mm^2 = E$ in comp. $= E$ in tension

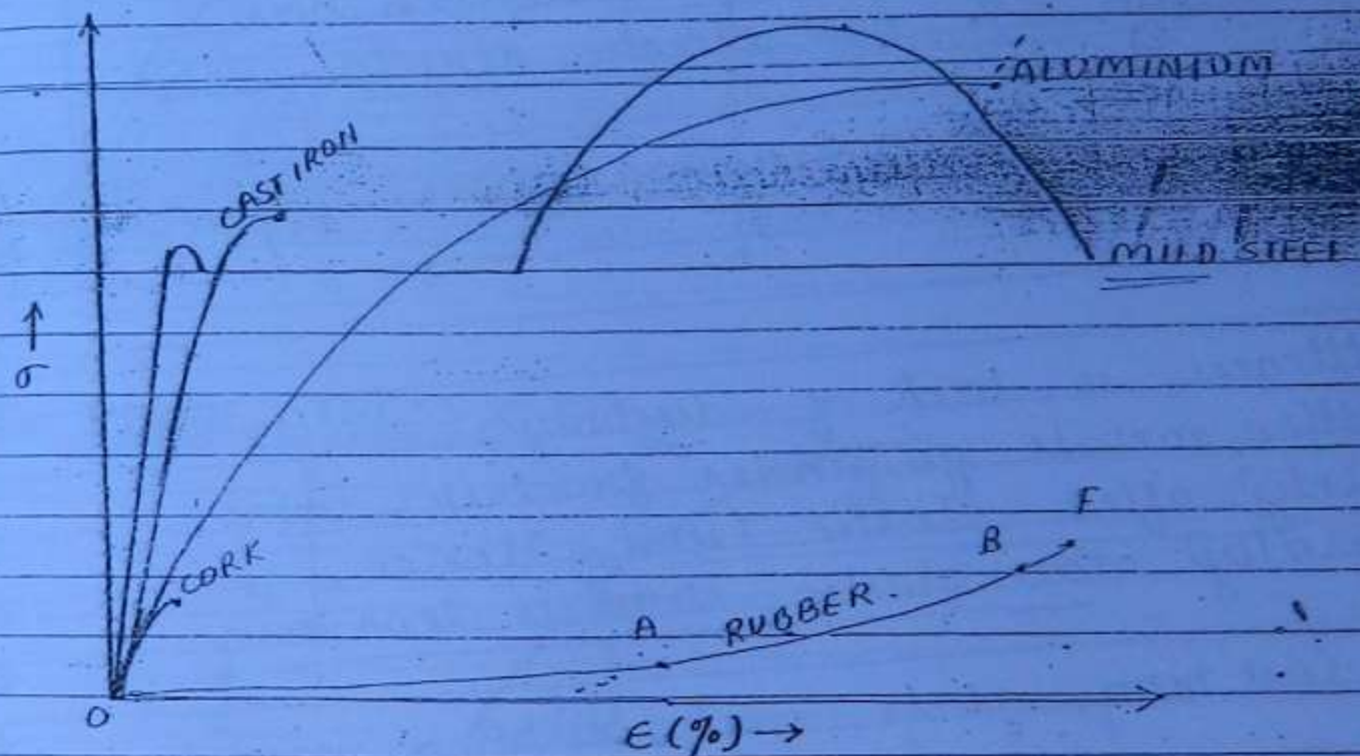
All grades of steel have same E.

Stress-Strain Curve for other grades of steel in TENSION.



Among the above high tension steel is more brittle and mild steel is more ductile.

STRESS-STRAIN CURVE IN TENSION FOR VARIOUS MATERIALS -



★ Steel is more elastic than rubber.

★ Rubber is brittle.

$$E_{CI} = \frac{1}{2} E_S$$

$$E_{Al} = \frac{1}{3} E_S$$

$$E_{WOOD} = \left(\frac{1}{10} \text{ to } \frac{1}{20}\right) E_S$$ Wood is brittle.

$$E_{CONC.} = \left(\frac{1}{10} \text{ to } \frac{1}{20}\right) E_S$$

DUCTILITY AND BRITTLENESS-

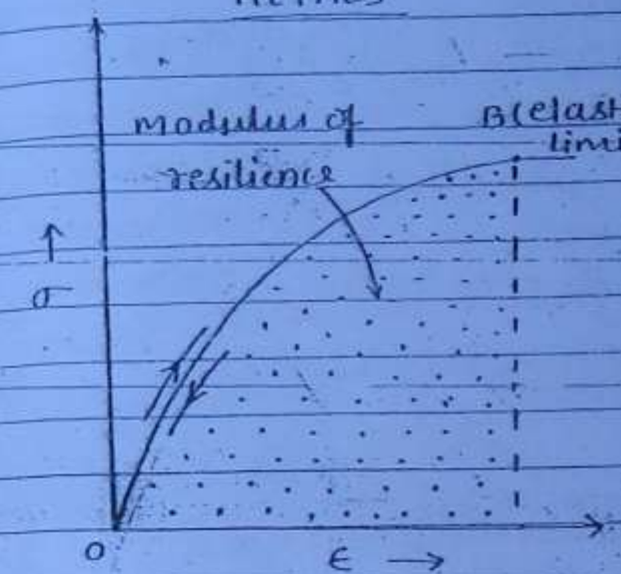
Ductility of the metal is that property due to which a metal sheet can be drawn into wire of thin section.
Those metals are ductile which has more than 5% post elastic strain (plastic strain)
Eg: Mild steel, Aluminium, copper, gold, silver, lead.

Brittleness is lack of ductility.
Brittler metals go under fracture immidiately after elastic limit. Hence, generally non elastic strain is less than 5%.
Eg. Cast Iron, glass, cork, rubber, wood etc.

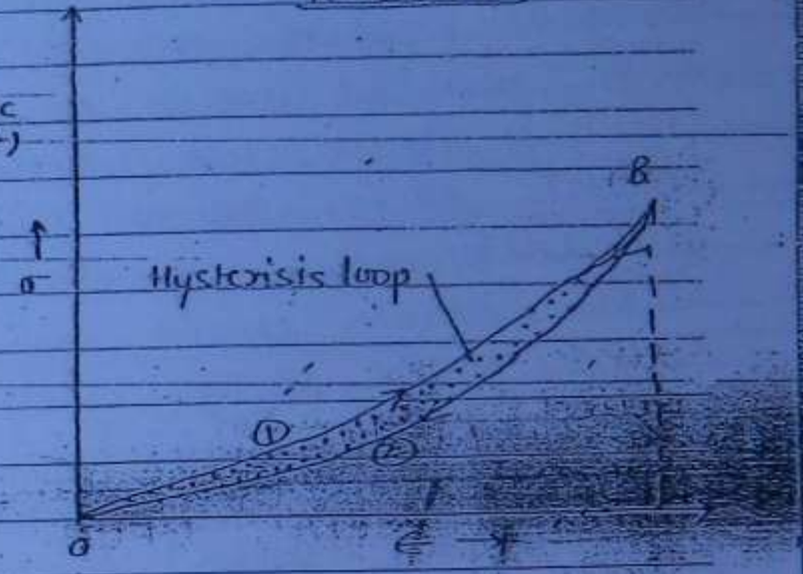
MALLEABILITY- It is that property of metal due to which a piece of metal can be converted into a thin sheet.

ELASTICITY- It is that property due to which original dimensions can be recovered after unloading.
Within elastic limit stress-strain curve may be linear or non-linear.

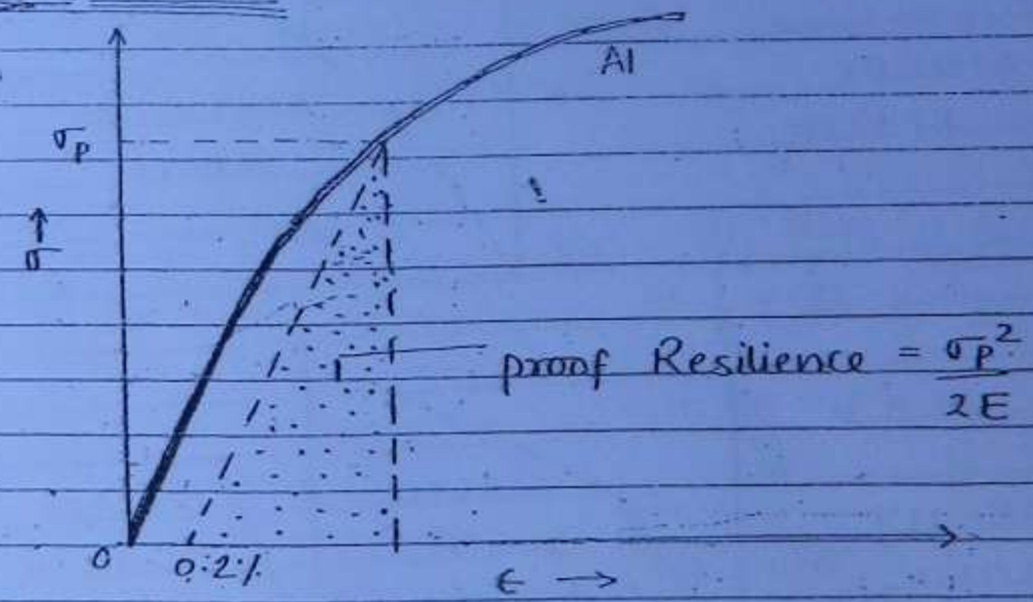
METALS



RUBBER



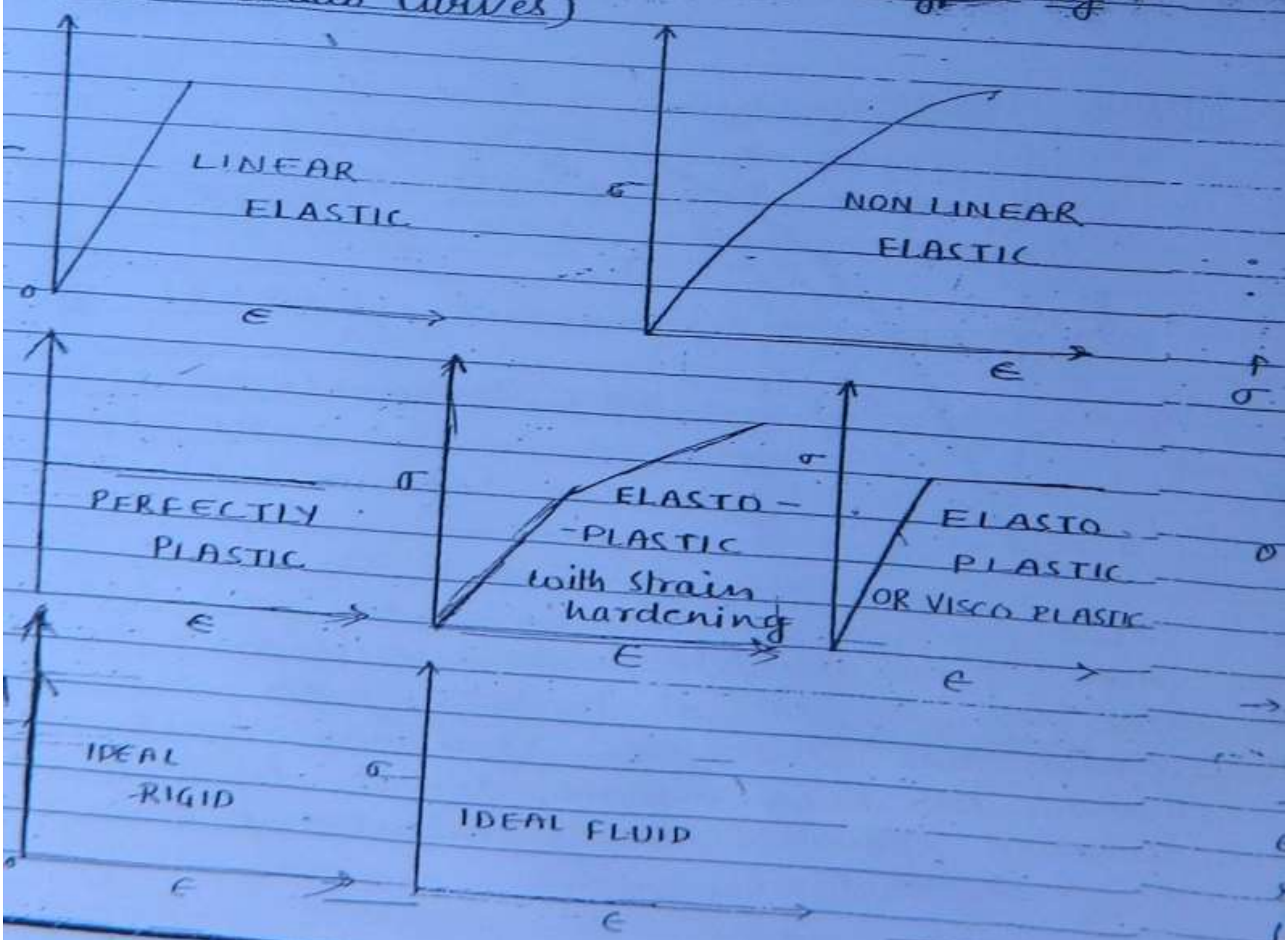
PROOF STRESS →



Some of the ductile metals such as Al, Cu and Iron do not show clear yield point in tension test. Therefore, σ_y is not clearly known. For such metals design stress which is called PROOF STRESS is calculated using offset method.

An offset of strain equal to given per-missible plastic strain (say 0.2%) for Al is marked on X-axis and a straight line is drawn which is parallel to initial stress-strain curve.
The intersection of straight line with stress strain curve gives design stress called proof stress.

TYPES OF METAL BEHAVIOURS (Types of Stress Strain Curves)

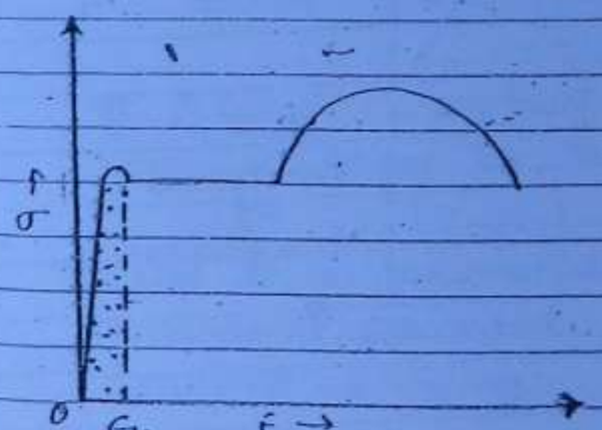


RESILIENCE → It is the total elastic strain energy which can be stored in the given volume of metal and can be released after unloading. It is also equal to area under load deflection curve within elastic limit.

Modulus of Resilience - (U_r):

- It is elastic strain energy per unit volume and it is equal to area under stress-strain curve within elastic limit.

For linear elastic metals, modulus of resilience is as follows-



$$U_r = \frac{1}{2} \times \sigma_e \times \epsilon_e.$$

$$\approx \frac{1}{2} \times \sigma_y \times \epsilon_y$$

$$\approx \frac{1}{2} \sigma_y \times \frac{\sigma_y}{E}$$

$$\therefore \boxed{U_r = \frac{\sigma_y^2}{2E}}$$

→ For suspension system, spring action and load absorption, a metal should be used with high resilience.

Greater the σ_y greater is the modulus of resilience. Hence, high ten. steel has greater U_r than mild steel.

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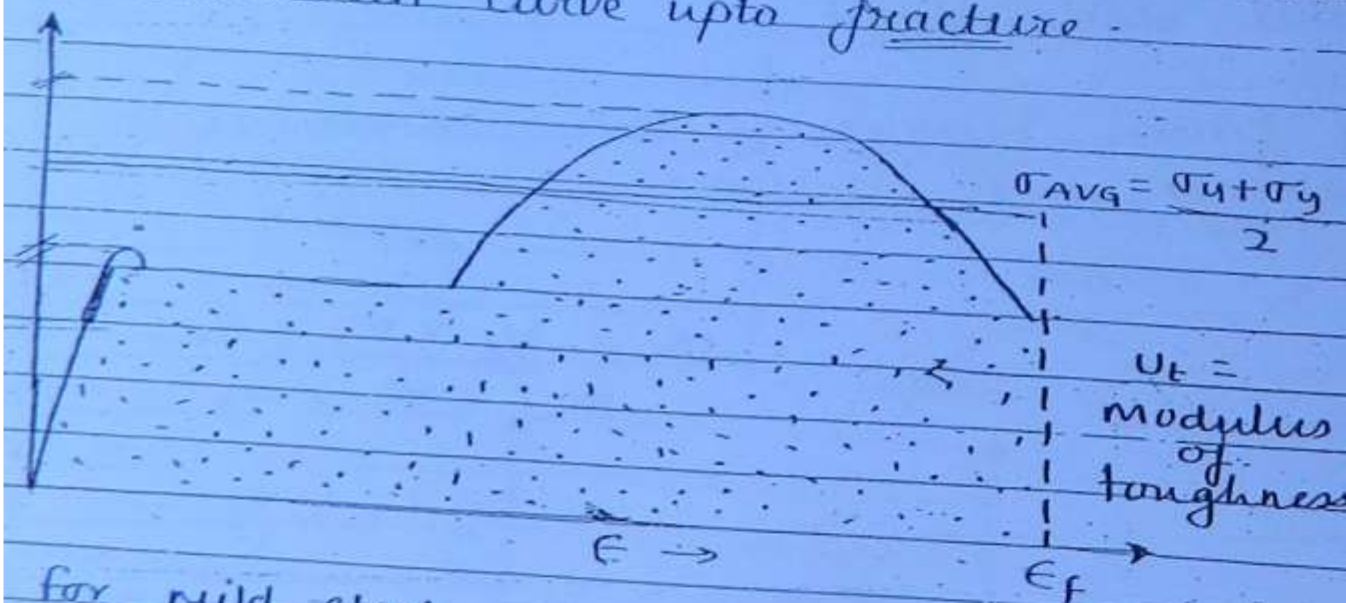
TOUGHNESS :- It is the resistance to the impact loading against fracture. If a metal is tough, then it has ability to store large strain energy before failure deflection curve upto fracture stage. Imp

Modulus of Toughness - (U_t) -

strain energy per unit volume upto fracture stage. It is total It is equal to area under stress strain curve upto fracture.



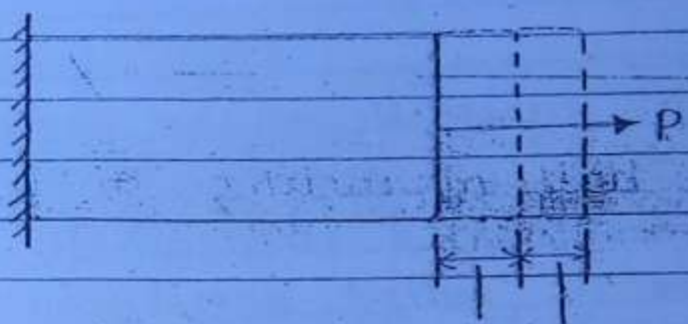
for mild steel,

Modulus of toughness, $\boxed{U_t = \left(\frac{\sigma_u + \sigma_y}{2}\right) \times \epsilon_f}$

Generally, greater ϵ_f gives greater toughness, which means mild steel is more tough than H.T.S.

HARDNESS - It is the resistance to the - scratch or abrasion.

Imp CREEP -



Δe Δc - creep deflection.

Δe = elastic deflection = $\frac{PL}{AE}$

P = Static Load

If P Load = constant for long period of time, then increase in Δ is creep deflection.

→ Creep is a permanent deformation which is ~~increased~~ recorded with passage of time at constant loading.

The total creep deflection continues to increase in time at asentotic rate.
↓
rate of deflect^n decreases with - time.

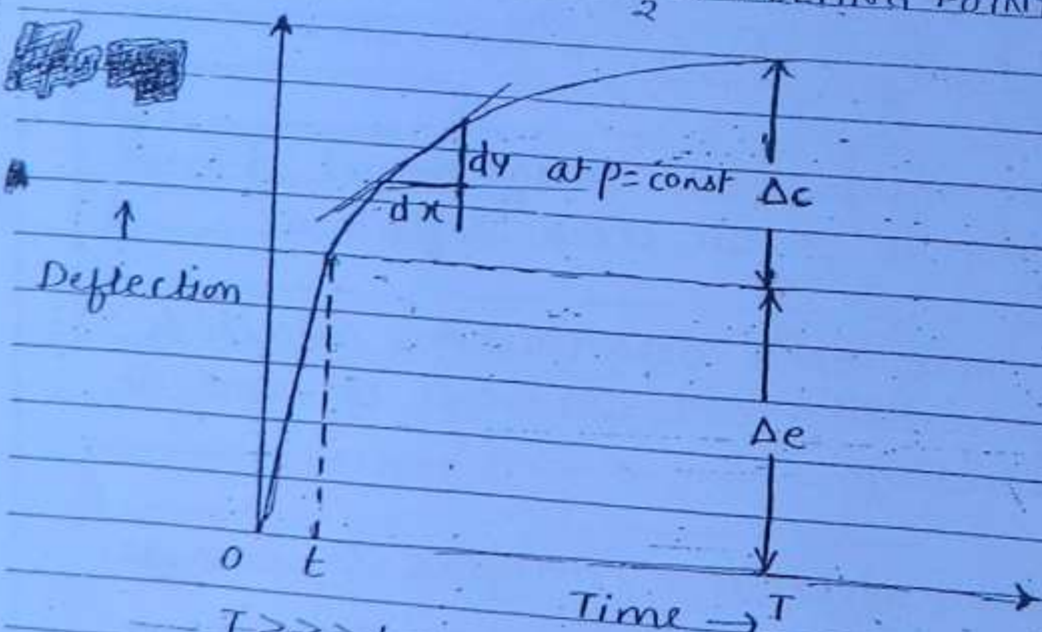
P.T.O.

* Factors Affecting Creep -

1. Magnitude of load.
2. Type of loading (static or dynamic)
3. Time or age
4. Temperature

At higher temp. creep is more and when temp. is half of melting point temperature, then creep is untolerable. Such temperature is called HOMOLOGUS temperature.

HOMOLOGUS TEMP. = $\frac{1}{2}$ x MELTING POINT TEMP.



T >>> t

t = Loading time.

$$\tan\theta = dy/dx = \text{Rate of creep}$$
$$= \frac{d\Delta}{dT} = \text{"}$$

With passage of time total creep deflectn increases but rate of creep deformation decreases.

Examples of creep failure –

1. Failure of pre stressed beam (due to stress relaxation caused by creep in wires.)

2. Sag in high tension electric wires.

FATIGUE → Due to cyclic and reverse cyclic loading fracture; failure may occur if total accumulated strain energy exceeds the toughness.

Fatigue causes rough fracture surface even in ductile material / metal.

The no. of load cycles required to initiate surface crack is called fatigue initiation life and additional no. of loads cycles required to propogate surface cracks is called fatigue propagation life.

To prevent fatigue failure the developed stress should be kept below endurance limit

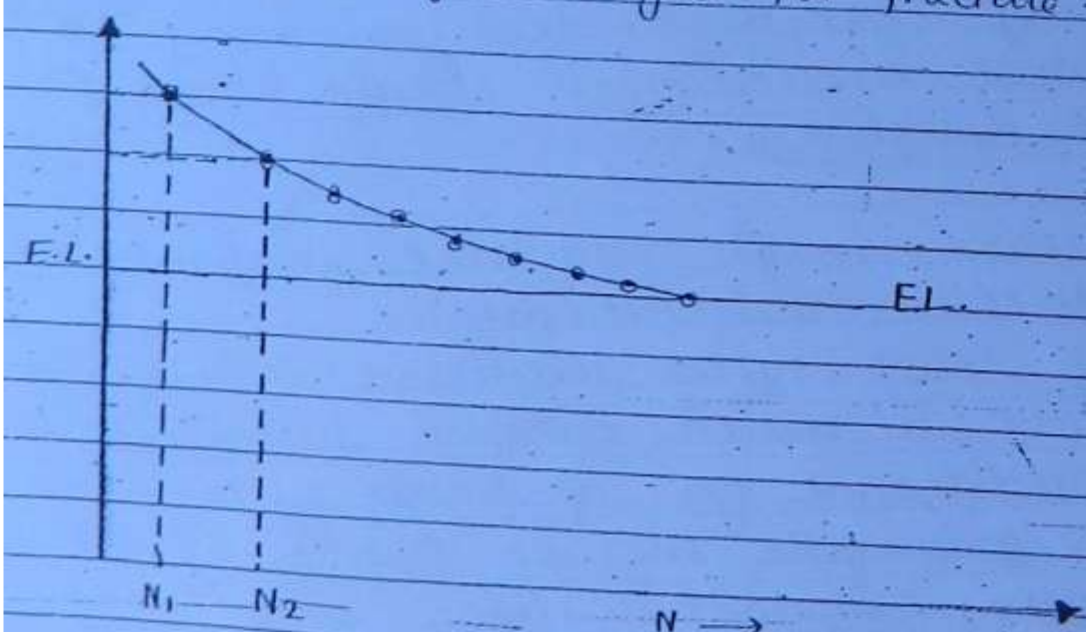
P.T.O.

Endurance limit – It is that stress level below which a material has a high probability of no failure even at infinite no. of load cycles.

For mild steel, endurance limit is 186 N
for Aluminium, E.L. = 131 N/mm²

→ Endurance limit is determined by S-N test.
Endurance limit is smaller than the proportional limit.
(determined by tension test)

S – N
Stress   no. of load cycles for fracture.



Eg. of fatigue failure –

1. Crashing of aircraft due to cracking in turbine blades.

2. Failure of fly wheels.
3. Breaking of wire due to cyclic bending.

Fatigue failure can occur early if there is surface cut or rough surface.

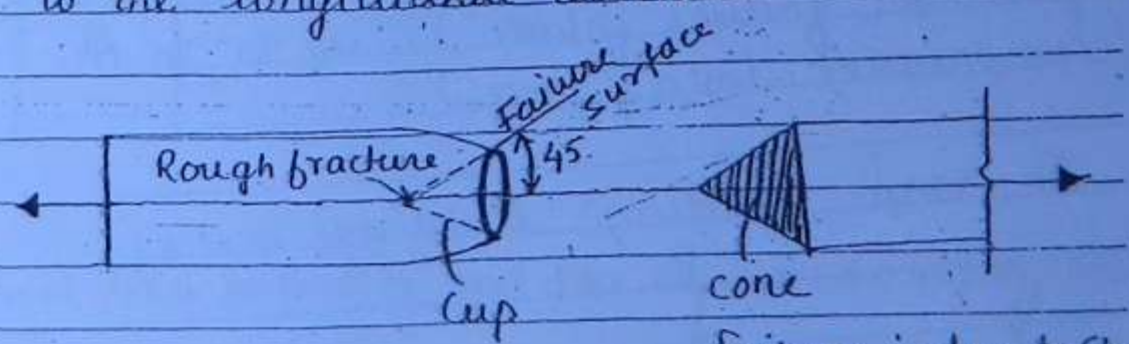
TYPES OF FAILURE IN TENSION AND COMPRESSION TEST:-

1. Ductile Metals In Tension Test:-

Ductile metals are weak in shear.

Shear strength < Tensile strength < Compression strength

Shear strength is 50-60% of tensile strength. In tension test, ductile metal shows cup-cone failure. The failure surface is at 45° to the longitudinal direction.



Failure is due to Sh

In ductile metal necking is formed before fracture i.e. large decrease in cross section area is recorded.

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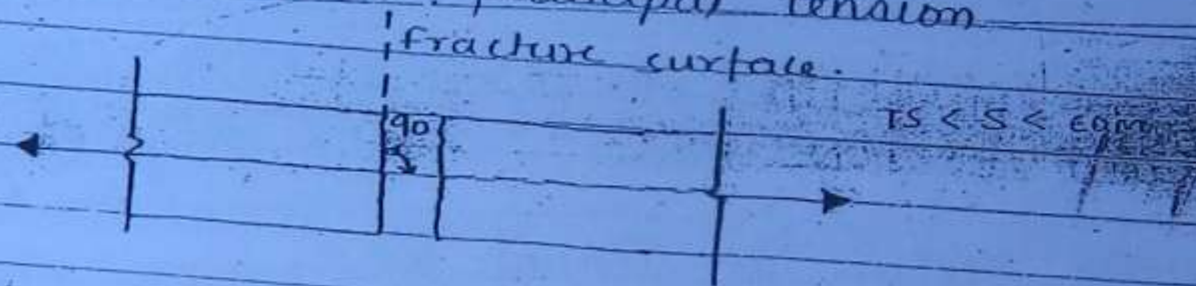
2. Brittle Metals In Tension Test -

Ans In Brittle metals, tensile strength is lower than shear strength which is lower than compressive strength. Hence, in brittle metal fracture occurs due to principal tension.

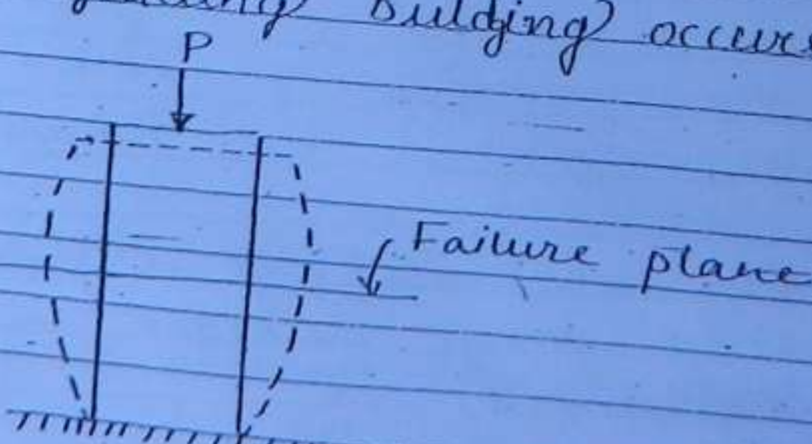


In brittle metals, failure surface is at 90° to the direction of load and it is rough fracture surface.
Failure is caused by principal tension.

3. Ductile metals in compression test -

Short comp. members fails in compression yielding, failure plane is at 90° to the compressive load / force.

At comp. yielding bulging occurs.



Stress System

Normal Stress (Acts ⊥ to the section) σ

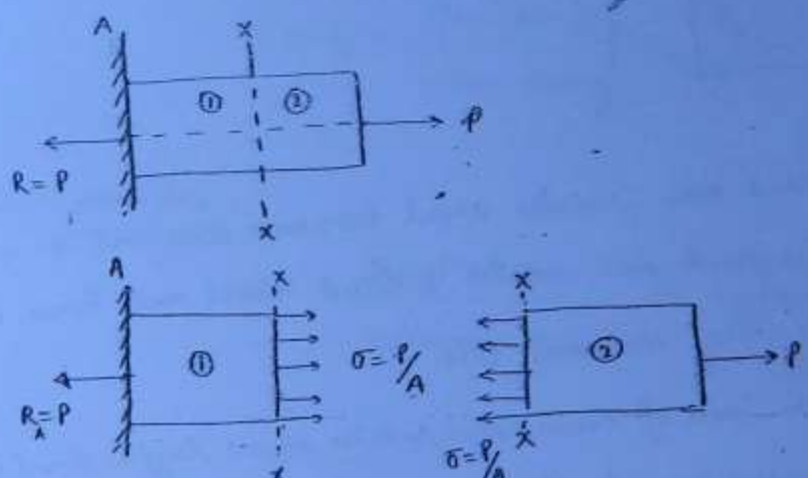
- (1) Direct stress (Due to axial force)
- (2) Bending stress (Due to Bending Moment)

Shear stress (tangential or $\parallel$ to the section) τ

- (3) Direct shear stress (Due to shear force)
- (4) Torsional shear stress (Due to torque)

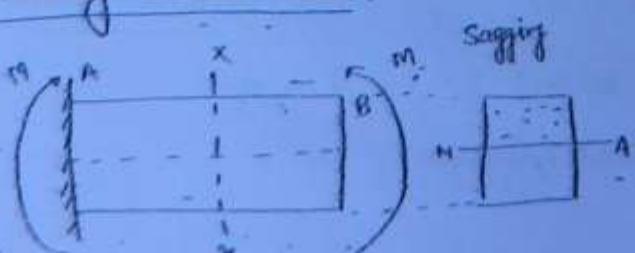
1) Direct Normal Stress :

- Tensile (+)
- Compressive (−)

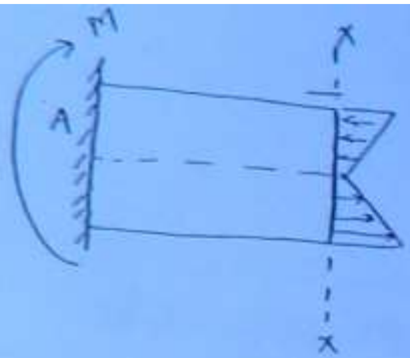


Direct Normal stresses produced by axial force are distributed uniformly across a section

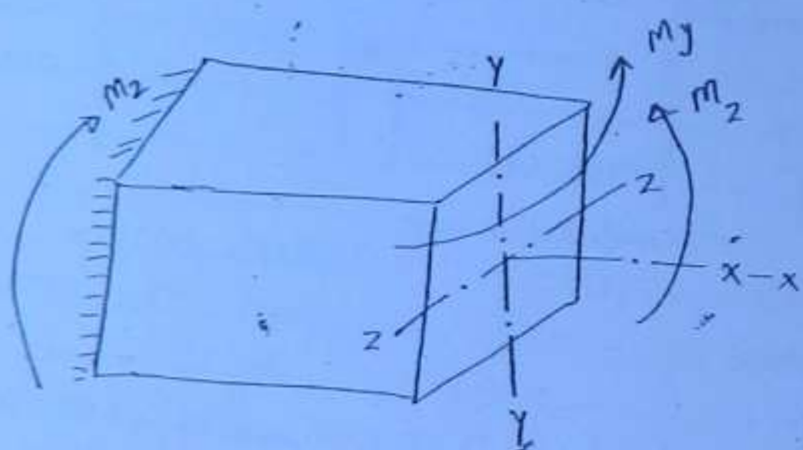
2) Bending Normal stress :



Neutral axis



Bending stresses are linearly distributed from 0 at neutral axis to maximum at surface, In case of Bending tension and Compression both are found.



In Bending, the cross sectional area rotates about transverse axis and the axis about which the cross sectional area rotates is called neutral axis hence in bending neutral axis is always transverse axis.

In torsion or twisting the area of cross section rotates about longitudinal axis or polar axis. In Bending Normal stresses are produced where as in twisting shear stresses are produced.

In above case M_y & M_z are B.M. and M_x is twisting moment.

are 21 indepent elastic constt.

Elastic Constants :-

1. Young's Modulus of Elasticity :-

$$E = \frac{\text{Normal stress}}{\text{Normal strain}} = \frac{\sigma}{\epsilon} \quad \text{in uniaxial loading}$$

$$= \frac{\text{long. Stress}}{\text{long. strain.}}$$

2. Shear Modulus / Modulus of Rigidity :-

$$C \text{ or } G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

3. Bulk Modulus :

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$ in hydrostatic loading [equal and alike stresses in all dirn].

$$= \frac{\sigma}{\epsilon_v}$$

· for an incompressible material K or Bulk modulus tends to ∞, it means Compressibility is inversly proportional to Bulk modulus.

4. Poisson Raitio :-

μ, or ν = − [Lateral strain]/[longitudinal strain] in uniaxial loading case

loading dir^n is longitudinal dir^n. lateral dir^n is transverse dir^n.
Generally If longitudinal strain is tensile (+) then lateral strain is Compressive (-ve).

NOTE :-

For Human tissue poission's ~~ratio~~ is ratio is -ve. $\boxed{-1 \le \mu \le 0.5}$.

For cork $\mu \rightarrow 0$

For glass $\mu \rightarrow 0.01$ to 0.05

For Concrete $\mu \rightarrow 0.1$ to 0.2

For elastic metals $\mu \rightarrow 0.25$ to $.42$

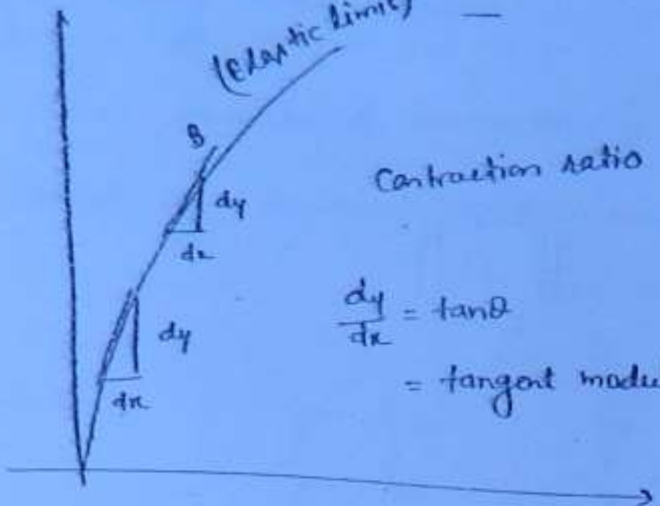
For plastic rubber $\rightarrow \mu \rightarrow 0.5$

Those materials for which Possion ratio is 0.5, there will be no volume change on loading.

NOTE :

1. The possion's ratio as the defined above is only valid with in elastic limit which is contt. beyond elastic limit the ratio of leteral strain to longitudinal strain is called contraction ratio which is not contt.

2. With in proporational limit the slope of stress strain is contt which is called young's modulus but in non-linear elastic materials with elastic limit slope of curve is not contt. hence slope of a tangent is called tangent modulus.



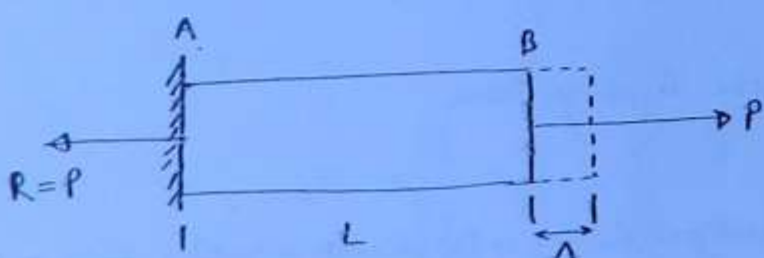
Contraction ratio

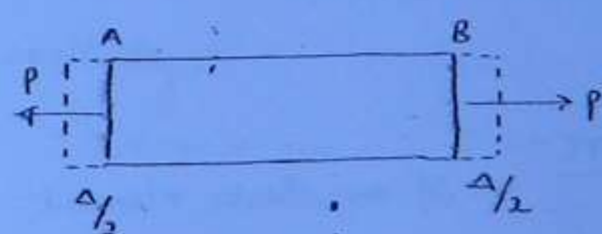
$$\frac{dy}{dx} = \tan\theta$$

$= \text{tangent modulus}$

Strain

- Normal or Linear strain (ϵ or e) (it is due to σ)
- Shear strain or Angular strain (ϕ or γ) (Due to τ)

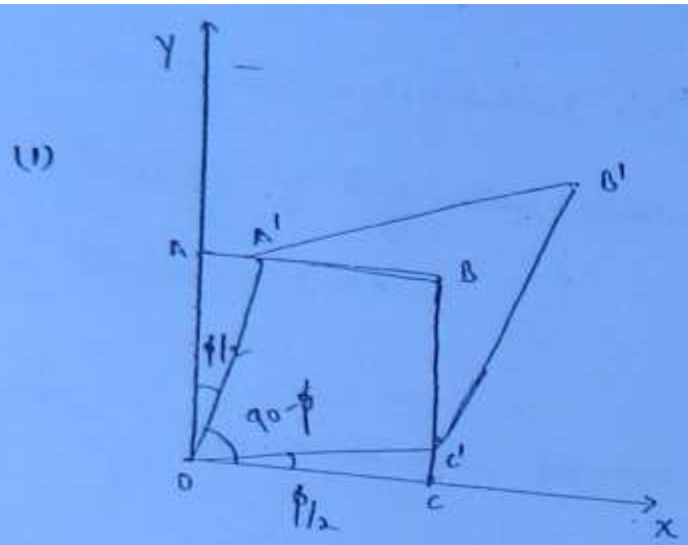




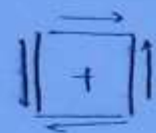
Total Defl^n = Δ ; $\epsilon = \Delta/L$

Shear strain = Total angular distortion.

(1)

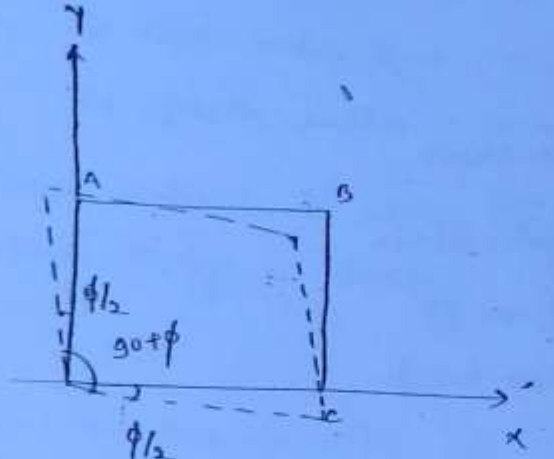


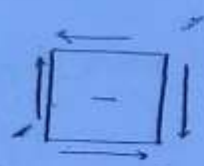
$\frac{\tau}{\phi} = G$



Total shear strain = ϕ

if the angle b/t xy face reduces then ϕ is taken (+ve)





-ve shear strain in $x-y$ plane

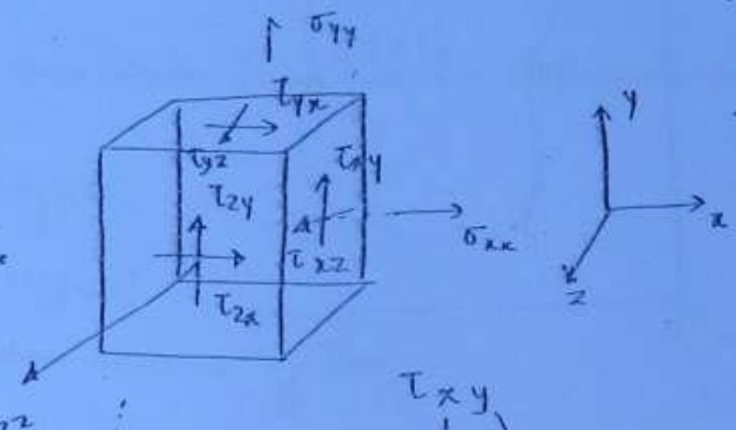
NOTE:-

If an elastic element is subjected to only direct stresses without shear then there will be no angular deformation but there may be change in volume. Similarly if only shear stresses acts then there will be angular deformation without change in volume.

If Both edges are free to deform then shear strain in a plane will be half from vertical dir^n and half from horizontal dir^n. But if one edge is fixed or a surface then entire deformation will be with respect to other edge.

Complimentary Shear stresses :—

Shear stresses on two mutually $\perp^r$ plane are always equal which can be proved by moment equilibrium cond^n.



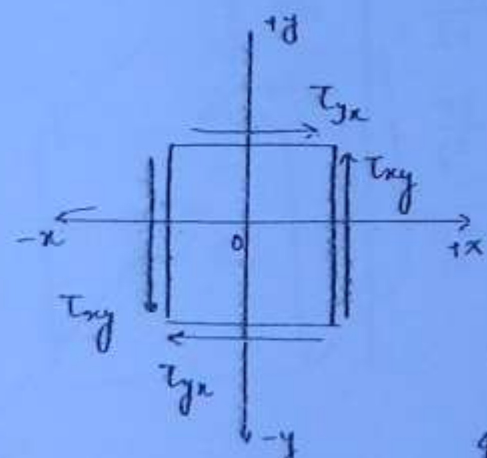
τ_{xy} → face plane, dir^n

m = scalar (zero order)

f_x = vector (1^{st} order)

τ_{xy} = Tensor (2^{nd} order)

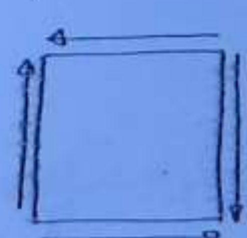
$$\left.\begin{array}{l}\tau_{xy} = \tau_{yx} \\ \tau_{xz} = \tau_{zx} \\ \tau_{yz} = \tau_{zy}\end{array}\right\} \text{Complimentary values.}$$



τ_{xy}

+ + ⇒ +ve

− − ⇒ +ve

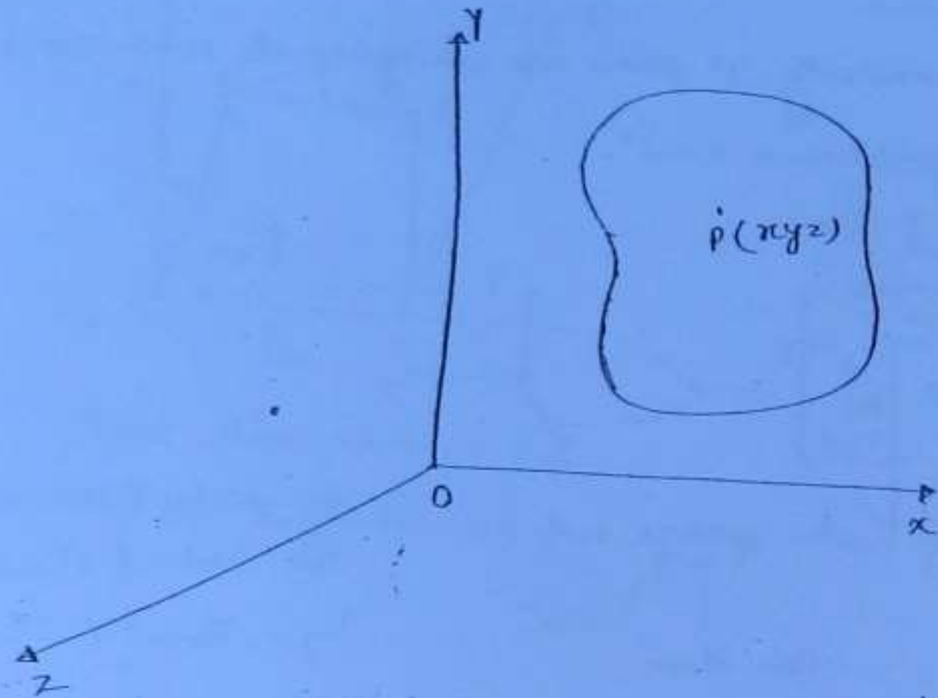


τ_{xy}

+ − ⇒ −ve

⇒ −ve

Differential form of strain :-



Before loading Position of Point $P(x,y,z)$ w.r.t origin.
If on loading Point P moves By u, v and w in $x, y,$ and z dir^n respectively then normal and shear strains at point P are given as

$$\epsilon_{xx} = \epsilon_x = \frac{\partial u}{\partial x}$$

$$\epsilon_{yy} = \epsilon_y = \frac{\partial v}{\partial y}$$

$$\epsilon_{zz} = \epsilon_z = \frac{\partial w}{\partial z}$$

Shear strain !-

1) In xy Plane

$$\phi_{xy} = \phi_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

ii) In x-z Plane

$$\phi_{xz} = \phi_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

iii) In y-z Plane

$$\phi_{yz} = \phi_{zy} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}$$

Q. In a plane strain situation in xy plane the displacements at point P is given as

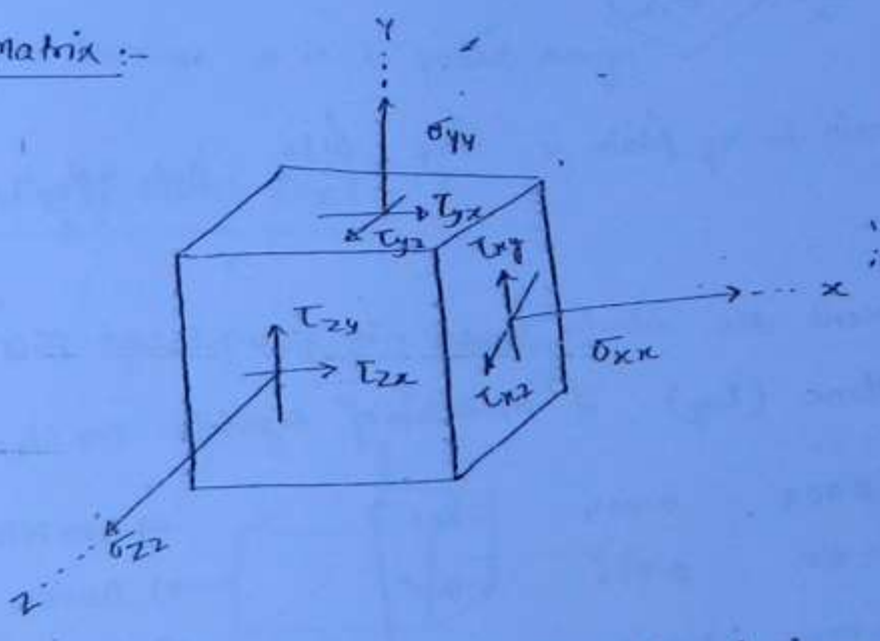
$u = (-2x + 8y) \times 10^{-6}$

$v = (-3x + 5y) \times 10^{-6}$

Find the shear strain in xy plane

a) 9×10 b) 7×10^{-6} c) 5×10^{-6} d) 3×10^{-6}

Stress Matrix :-



Total there are 9 stress components in a 3-D stress element out of these 9, 3 are normal components and 6 shear component.

All are presented in terms of matrix.—

$$[Stress] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

In a 2-D stress element there will be total 4 stress elements and 2x2. stress matrix will be generated

Strain Matrix :-

$$[Strain] = \begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$$

Total, shear strain in xy plain = $\frac{\phi_{xy}}{2} + \frac{\phi_{yx}}{2}$ But, $\frac{\phi_{xy}}{2} = \frac{\phi_{yx}}{2}$

xample:-

for a 3-D element the strain tensor is given below find shear stress in x-y Plane (τ_{xy}). if modulus of rigidity 100 GPa.

$$\begin{bmatrix} 0.002 & 0.004 & 0.006 \\ 0.004 & 0.006 & 0.000 \\ 0.006 & 0.000 & 0.008 \end{bmatrix}$$

a) 400 MPa
b) 800MPa
c) 1600 MPa
d) None of these

$\frac{\tau_{xy}}{\phi_{xy}} = G$ $\quad \tau_{xy} = 100 \times .008$ GPa $= 800$ MPa $\quad \frac{\phi_{xy}}{2} = .004$; $\phi_{xy} = .008$

Q. For a linear elastic metal which of the following statement is true?

a) $G = K$

b) $E = G$ ✗

c) $K = E$

d) $K = G = E$ ✗

$E = 3K(1-2\mu)$

If $E = K$.

$K = 3K(1-2\mu)$

$\mu = \frac{1}{3}$ O.K.

If $\Rightarrow G = E$

$E = 2G(1+\mu)$

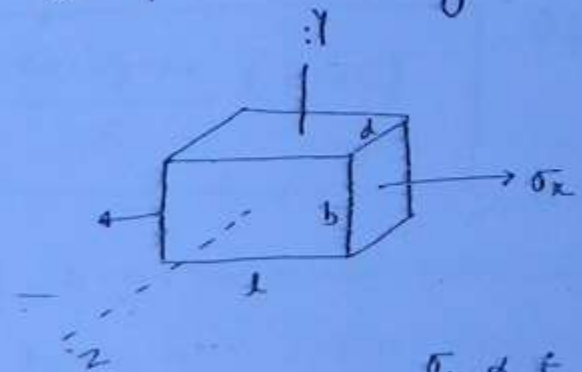
$\mu = -\frac{1}{2}$ ✗

$$\boxed{\mu = \frac{3K-2G}{6K+2G}}$$

$E = K$ because μ is in valid range.

Application of Hook's laws :-

Case I :- Effect of uniaxial loading :-



$\sigma_x \propto \epsilon$

$\sigma_x = E\,\epsilon_x$

$\boxed{\epsilon_x = \sigma_x / E}$ → longitudinal strain in x dir

Poisson's Ratio $= -\left[\text{lateral strain} / \text{longitudinal strain} \right]$

$$= -\epsilon_y / \epsilon_x = -\epsilon_z / \epsilon_x$$

ϵ_y & ϵ_z are leteral / transverse strain,

$$\epsilon_y = \epsilon_z = -\mu \epsilon_x$$

$$\boxed{\therefore \epsilon_y = \epsilon_z = -\mu \cdot \frac{\sigma_x}{E}}$$

$+ \longrightarrow$ expansion

$- \longrightarrow$ Contraction.

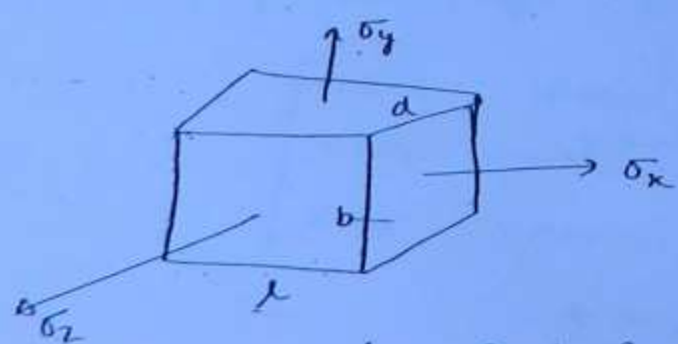
Imp

$\epsilon_x = \frac{\sigma_x}{E}$ — long strain in x dir.

$\epsilon_x = \frac{\delta l}{l} = \frac{\sigma_x}{E}$; $\epsilon_z = -\frac{\delta d}{d} = -\mu \cdot \frac{\sigma_x}{E}$;

$\epsilon_y = \frac{\delta b}{b} = -\mu \frac{\sigma_x}{E}$;

CASE II : Effect of triaxial loading :-



Long. strain in x dirn

$$\epsilon_x = \frac{\delta l}{l} = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E}.$$

$$\epsilon_y = \frac{\delta b}{b} = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\delta d}{d} = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

Volumetric strain :-

$$V = l \cdot b \cdot d$$

$$\delta v = \delta l \cdot b \cdot d + l\,\delta b \cdot d + l \cdot b \cdot \delta d$$

$$\text{Volu. strain} = \epsilon_v = \frac{\delta v}{v} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta d}{d}$$

$$\boxed{\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z}$$

Volumetric strain is equal to summation of linear strains in 3 mutually $\perp^r$ dir^n

$$\epsilon_v = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} + \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} + \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\boxed{\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)}$$ Imp.

NOTE: 1. If $\mu = 0.5$ for a metal then $\epsilon_v = 0$ i.e. metal is incompressible

2. If $\sigma_x + \sigma_y + \sigma_z = 0$ then resultant volumetric strain = 0 Hence if an elastic metal shows no change in volume then summation of stresses in three mutually $\perp^r$ dir^n must be zero.

3. If $\sigma_x = \sigma_y = \sigma_z = \sigma$ then $\epsilon_v = \frac{3\sigma}{E}(1-2\mu)$ —(i)

Hydrostatic loading

$$k = \frac{\sigma}{\epsilon_v}$$

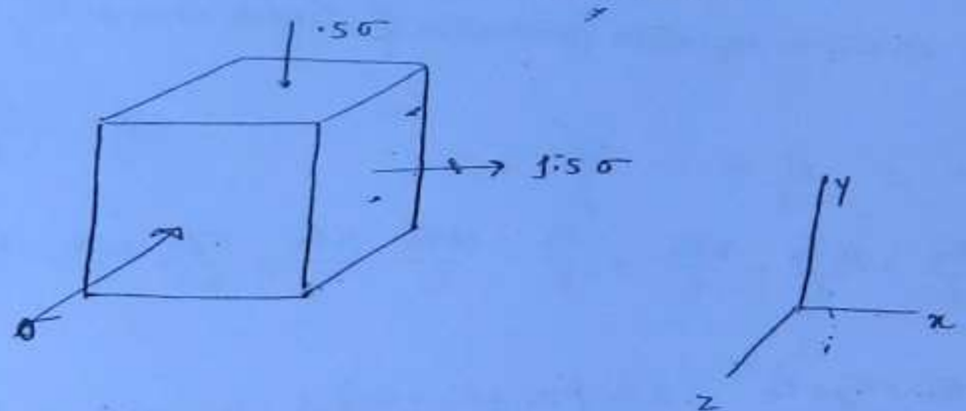
$$\epsilon_v = \frac{\sigma}{k} \quad \text{—(ii)}$$

from (i) & (ii)

$$\frac{\sigma}{K} = \frac{3\sigma}{E}(1-2\mu)$$

$$\Rightarrow \boxed{E = 3K(1-2\mu)}$$

ex. For the stress element shown in fig find linear strains in x, y and z dir^n and also find volumetric strain.



S^n:-

$$\sigma_x = 1{\cdot}5\sigma, \quad \sigma_y = -0{\cdot}5\sigma, \quad \sigma_z = -\sigma$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{1{\cdot}5\sigma}{E} + \frac{\mu(\cdot 5\sigma)}{E} + \frac{\mu\sigma}{E}$$

$$\epsilon_x = \frac{\sigma}{E}\left(1{\cdot}5 + \frac{\mu}{2} + \mu\right) = \frac{1{\cdot}5\sigma}{E}(1+\mu)$$

$$\boxed{\epsilon_x = \frac{3}{2}\frac{\sigma}{E}(1+\mu)}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$= -\frac{\sigma}{2E} - \frac{3\mu}{2}\frac{\sigma}{E} + \frac{\mu}{2}\frac{\sigma}{E}$$

$$= \frac{\sigma}{2E}\left(-1 + 3\mu + 2\mu\right)$$

$$= \frac{\sigma}{2E}\left(-1 - 2\mu\right) \Rightarrow -\frac{\sigma}{2E}(1+\mu)$$

$$\epsilon_z = -\frac{\sigma}{E}(1+\mu)$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

$$= \frac{1.5\sigma - .5\sigma - \sigma}{E}(1-2\mu)$$

$$= 0$$

No volume change

Conventional

Q:- Three metal Cubes A, B & C. of size 100 mm each are in direct contact with which are placed on a rigid surface. The cubes are confined in x dir^n. b/t two rigid plates. If the upper face of central cube (cube B) is subjected to uniform compressive stress of 0.5 KN/mm². Then compute for cube B?

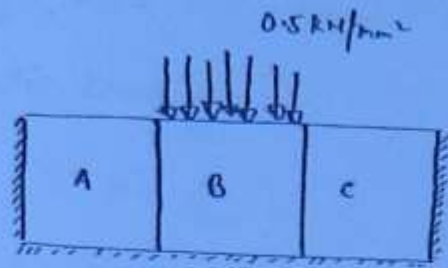
1) The direct stress in x- dir^n.
2) The direct strain in x, y, z dir^n.
3) Volumetric strain.

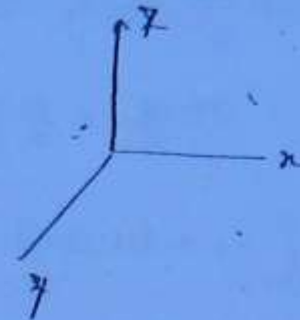
given that: for young's modulus for cube A & B

$E_A = E_C = 150$ KN/mm² — $\mu_A = \mu_C = 0.25$

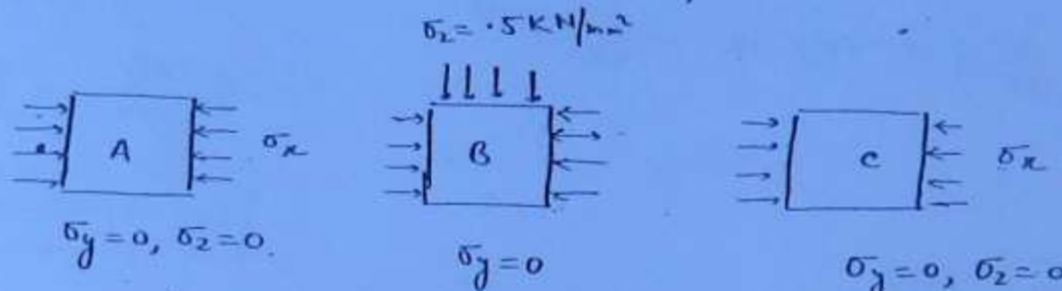
$E_B = 200$ KN/mm² $\mu_B = 0.30$

Sol^n:-





Cube A



Since cubes are confined in x dir^n

Hence

$$\Delta_{x_A} + \Delta_{x_B} + \Delta_{x_C} = 0$$

$$\frac{\Delta_{xA}}{L} + \frac{\Delta_{xB}}{L} + \frac{\Delta_{xC}}{L} = 0$$

$$\boxed{\epsilon_{xA} + \epsilon_{xB} + \epsilon_{xC} = 0} \quad \text{— (i)}$$

$$\left.\begin{aligned} \epsilon_{x_A} &= \frac{-\sigma_x}{E_A} \\ \epsilon_{x_B} &= \frac{-\sigma_x}{E_B} + \mu_B\left(\frac{0.5}{E_B}\right) \\ \epsilon_{x_C} &= \frac{-\sigma_x}{E_C} \end{aligned}\right\} \text{— (ii)}$$

Solving eq^n (i)

$$-\frac{\sigma_x}{E_A} - \frac{\sigma_x}{E_B} + \mu_B\left(\frac{.5}{E_B}\right) + \frac{\sigma_x}{E_C} = 0$$

$$\sigma_x\left(\frac{1}{E_A}+\frac{1}{E_B}+\frac{1}{E_C}\right)=\frac{\mu_B}{2E_B}$$

$$\sigma_x=\frac{\mu_B}{2\cancel{E_B}}\left(\frac{\cancel{E_B}E_cE_A}{E_BE_C+E_AE_c+E_AE_B}\right)$$

$$\sigma_x=0.041\ KN/mm^2$$

Strain for Cube B.

$$\epsilon_{x_B}=\circ-\frac{\sigma_x}{E_B}+\frac{\mu_B(\cdot 5)}{E_B}$$

$$=\frac{-\cdot 041}{200}+\frac{\cdot 3\times\cdot 5}{200}$$

$$\begin{array}{r}\cdot 150\\ -\cdot 041\\ \hline 109\end{array}$$

$$=\frac{\cancel{\cdot 191}}{200}=\frac{\cdot 109}{200}=0\cdot 5455\times 10^{-3}$$

$$\epsilon_{y_B}=\mu_B\frac{\sigma_x}{E_B}+\mu_B\frac{\sigma_z}{E_B}$$

$$=\frac{\cdot 3\times 0\cdot 041}{200}+\frac{0\cdot 3\times 0\cdot 5}{200}$$

$$=0\cdot 8114\times 10^{-3}$$

$$\epsilon_{z_B}=\frac{-\sigma_z}{E}+\frac{\mu_B\sigma_x}{E}$$

$$=\frac{-0\cdot 5+\cdot 3\times 0\cdot 041}{200}=\cancel{-\cdot 243\times 10^{-3}}\quad -2\cdot 43\times 10^{-3}$$

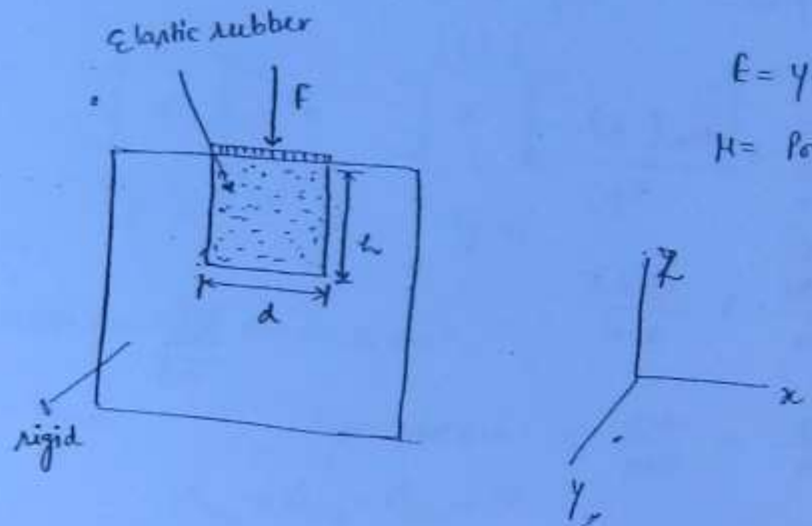
$$\epsilon_{v_B}=\epsilon_{x_B}+\epsilon_{y_B}+\epsilon_{z_B}$$

$$=(\cdot 5455+\cdot 8114\mp 2\cdot 43)\times 10^{-3}$$

$$\epsilon_{v_B}=-1\cdot 081\times 10^{-3}$$

A rubber block of elastic material has diameter d. and height h. rubber block is inserted inside a rigid block as shown in fig. If vertical compressible force F is applied through a rigid Plate on the rubber block, the applied force is axial, then find the pressure developed b/w rigid block and rubber block?

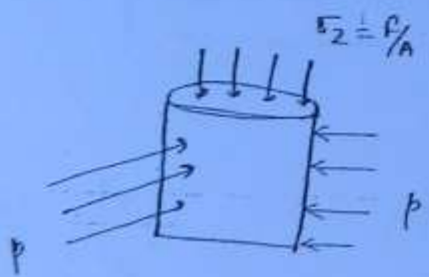
sol^n:



E = Young's modulus of rubber

μ = Poisson ratio.

Due to vertical compressive force F the lateral strains will be zero because rubber block is confined b/t rigid surfaces.



$\sigma_x = -p$

$\sigma_y = -p$

$\sigma_z = -F/A = \dfrac{-F}{\frac{\pi}{4} d^2}$

$= -\dfrac{4F}{\pi d^2}$

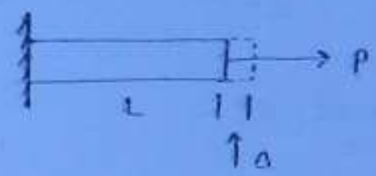
$\epsilon_x = \epsilon_y = 0$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E} = 0$$

$$= \frac{-p}{E} + \frac{\mu p}{E} + \frac{\mu 4F}{\pi d^2} = 0$$

$$\boxed{p = \frac{\mu}{1-\mu} \frac{4F}{\pi d^2}}$$

Axial deflection of Bars :→



$\Delta = P/K$

k = stiffness of spring

$f = 1/k$ = flexibility of sp.

$$\Delta = \frac{PL}{AE}$$

A = Constt.

$$\Delta = \frac{P}{\left(\frac{AE}{L}\right)} = P/K$$

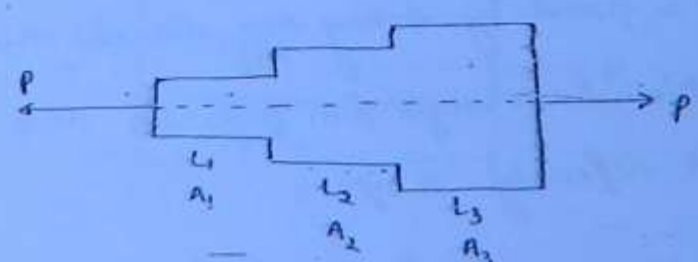
$\frac{AE}{L}$ = Axial stiffness of Bar.

AE = Axial rigidity

$\frac{L}{AE}$ = Axial flexibility.

CASE I :→

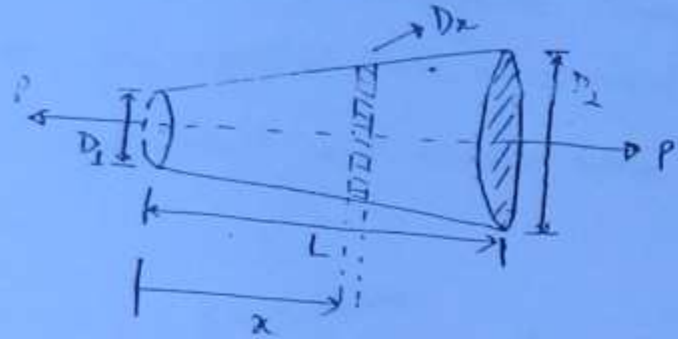
Deflection of a Stepped Bar :-



Total deflection $\Delta = \Delta_1 + \Delta_2 + \Delta_3$

$$\Delta = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

II) deflection of a circular tapering Bar :-



Dia at x;

$$D_x = D_1 + \left(\frac{D_2 - D_1}{L}\right) x.$$

Deflection of elemental length, ie $D_x\, dx$ is $d\Delta$.

$$d\Delta = \frac{P \cdot dx}{A_x \cdot E}$$

Total Defl^n $\Delta = \int d\Delta = \int_0^L \frac{P \cdot dx}{A_x E} = \int_0^L \frac{P\, dx}{\frac{\pi}{4} \cdot D_x^2 \cdot E}$

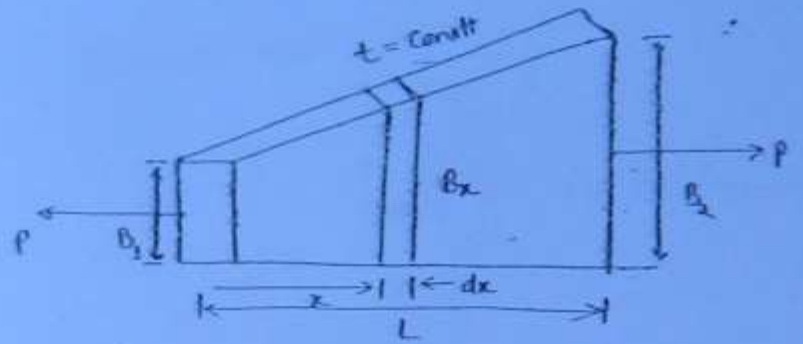
$$\Delta = \frac{4PL}{\pi E D_1 D_2}$$

NOTE :-

If Deflection of tapering bar is found by taking avg. diameter then result will be erroneous. If Diameter changes from D to 2D then actual defl^n and avg. defl^n will differ by 11.3%.

III> Rectangular tapering Bar :→

Rectangular tapering bar of constt. thickness and variable depth.



$$A_x = B_x * t$$

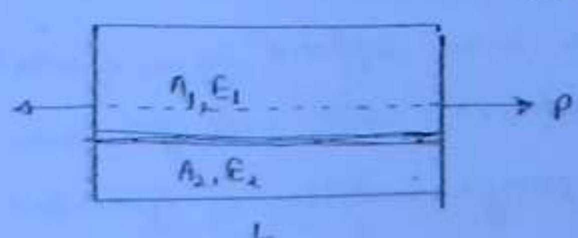
$$B_x = B_1 + \frac{B_2 - B_1}{L} \cdot x$$

$$\Delta = \int_0^L \frac{P\,dx}{A_x \cdot E}$$

$$\boxed{\Delta = \frac{PL \log_e \left(\frac{B_2}{B_1}\right)}{E \cdot t\,(B_2 - B_1)}}$$

VI) Deflection of a Composite Bar :→

Both metals are firmly jointed (face to face).



$$\Delta_1 = \Delta_2 = \Delta \quad \text{—— (i)}$$

Let P_1 & P_2 are forces developed in two metals

$$P_1 + P_2 = P \quad \text{—— (ii)}$$

$$\Delta_1 = \frac{P_1 L}{A_1 E_1}$$

$$\Delta_2 = \frac{P_2 L}{A_2 E_2}$$

$$\Delta_1 = \Delta_2$$

$$\frac{P_1}{P_2} = \frac{A_1 \cdot E_1}{A_2 \cdot E_2}$$

from eq^n (ii)

$$P_1 + P_1 \cdot \frac{A_2 E_2}{A_1 E_1} = P$$

$$P_1 = \frac{P \cdot A_1 E_1}{A_1 E_1 + A_2 E_2}$$

$$P_2 = \frac{P \cdot A_2 E_2}{A_1 E_1 + A_2 E_2}$$

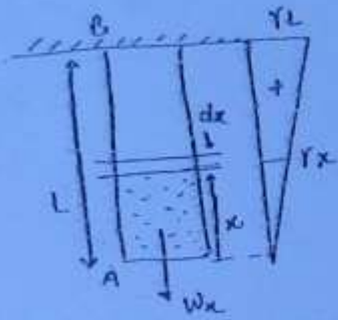
$$\boxed{\Delta = \frac{PL}{A_1 E_1 + A_2 E_2}}$$

NOTE:-

If above composite bare is to be replaced by an equivalent Homogeneous bar of total $(A_1 + A_2)$ which will give same defl^n under same load then. young's modulus of equivalent bar will be $\left(\frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}\right) = E_{eq}$.

Deflection Due to self weight :→

CASE I : Prismatic Bar



$\gamma \to$ wt unit wt.

$$\sigma_x = \frac{W_x}{A} = \frac{\text{wt. of Portion } x}{A}$$

$$= \frac{\gamma A \cdot x}{A}$$

$$\sigma_x = \gamma \cdot x$$

$$\sigma_A = 0;$$

$$\sigma_B = max = \gamma L$$

Deflection of elemental length dx

$$d\Delta = \frac{W_x \cdot dx}{A \cdot E}$$

Total deflection $\Delta = \int d\Delta = \int_0^L \frac{(\gamma A x) dx}{A E}$

$$\Delta = \int_0^L \frac{\gamma x \, dx}{E}$$

$$\boxed{\Delta = \frac{\gamma L^2}{2E}}$$

Deflection due to self wt in prismatic Bar is independent of area. If all dimensions are doubled then deflection will become 4 times

Let total wt. of the bar is W

$$W = \gamma \cdot AL$$

$$\gamma = \frac{W}{AL}$$

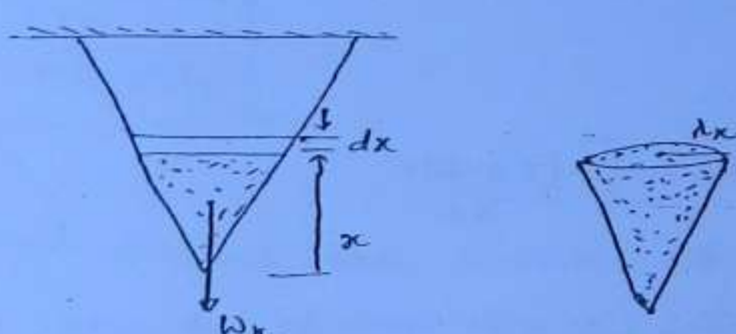
$$\Delta = \frac{\gamma L^2}{2E}$$

$$\Delta = \frac{\frac{W}{AL} \cdot L^2}{2E}$$

$$\boxed{\Delta = \frac{WL}{2AE}}$$

NOTE : Deflection due to self wt. W is equal to deflection caused by external load W/2 acting at free end. or concentrated load W acting at C.G.

Deflection of a conical bar due to self wt :—



$$W_x = \gamma \cdot \frac{1}{3} \pi \lambda_x^2 \cdot x$$

$$W_x = \gamma \cdot \frac{1}{3} A_x \cdot x$$

$$\sigma_x = \frac{W_x}{A_x} = \frac{\gamma \cdot A_x \cdot x}{3 \cdot A_x}$$

$$\sigma_x = \frac{\gamma x}{3}$$

Deflection of elemental lenth dx is $d\Delta$

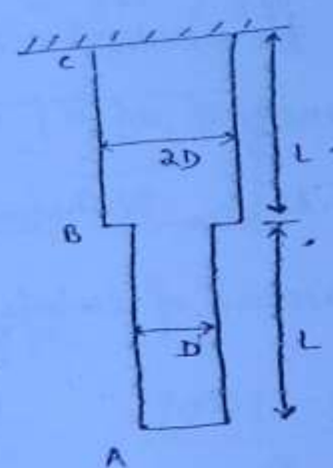
$$d\Delta = \frac{W_x}{A_x} \cdot \frac{dx}{E}$$

$$= \frac{\frac{\gamma \cdot \cancel{A_x} \cdot x}{3} \cdot dx}{\cancel{A_x} \cdot E}$$

$$d\Delta = \int_0^L \frac{\gamma x \cdot dx}{3E} = \frac{\gamma \cdot L^2}{6E} = \frac{1}{3} \cdot \frac{\gamma L^2}{2E}$$

Deflection of Conical bar due to self wt = $\frac{1}{3}$rd of deflection of Prismatic Bar of same lenth.

Q. find total deflection of a stepped bar due to self wt as shown in fig.



γ = unit wt.

deflection of AB due to self wt =

$$\Delta_1 = \frac{\gamma L^2}{2E}$$

deflection of BC due to self wt

$$\Delta_2 = \frac{\gamma L^2}{2E}$$

deflection of BC due wt of AB

$$\Delta_3 = \frac{W_{AB} \times L}{A_{BC}\, E}$$

$$\Delta_3 = \frac{W_n\ \gamma \cdot \cancel{\frac{\pi}{4}} D^2 \cdot L \cdot L}{\cancel{\frac{\pi}{4}} (2D)^2\ E}$$

$$\Delta_3 = \frac{\gamma L^2}{4E}$$

Total deflection $\Delta = \Delta_1 + \Delta_2 + \Delta_3$

$$= \frac{\gamma L^2}{2E} + \frac{\gamma L^2}{2E} + \frac{\gamma L^2}{4E}$$

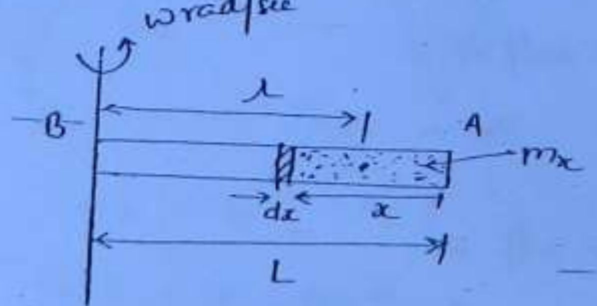
$$= \frac{5 \gamma L^2}{4E}$$

Ques:

A uniform cylindrical rod a metal of length l and cross-sectional area A is rotating in a horizontal plane about a vertical axis passing thru one end. The mass density of rod is ρ and it is rotating at constant angular vel. ω rad/sec in horizontal plane then proove that the elongation of rod on account of centrifugal action is

$$\Rightarrow \frac{\rho \omega^2 L^3}{3E}$$

Prooving :-



$$r = (L - x/2)$$

Centrifugal force at x-x

$$f_x = m_x \lambda \omega^2$$

$$= (\rho A x)\cdot\left(L - \frac{x}{2}\right)\cdot \omega^2$$

Defl^n of elemental length dx is $d\Delta$

$$d\Delta = \frac{f_x \cdot dx}{AE}$$

Total defl^n: $\Delta = \int_0^L \frac{f_x \cdot dx}{AE} = \int_0^L \frac{\rho \cancel{A} x\cdot\left(L-\frac{x}{2}\right)\omega^2}{\cancel{A}E} \cdot dx$

$$= \frac{\rho\omega^2}{E}\int_0^L \left(Lx - \frac{x^2}{2}\right) dx$$

$$= \frac{\rho\omega^2}{E}\left(L\cdot\frac{L^2}{2} - \frac{L^3}{6}\right)$$

$$\boxed{\Delta = \frac{\rho\omega^2 L^3}{3E}}$$

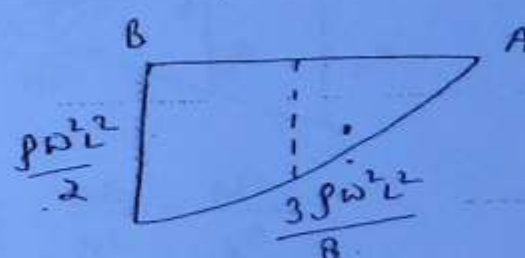
Stress at x-x

$$\sigma_x = \frac{f_x}{A} = \frac{(\rho \cancel{A} x)\left(L-\frac{x}{2}\right)\omega^2}{\cancel{A}}$$

$$\sigma_x = \rho x (L - x/2)\omega^2$$

$\sigma_A = 0$; $x = 0$

$\sigma_B = \frac{\rho\omega^2 L^2}{2}$; $x = L$



A rigid Bar ABCD is hinged at D and supported by two springs at A & B as shown in fig. The Bar carry a verticle load P. at C. The stiffness of spring A is 2K and that of spring B is K, then ratio of forces in spring A to that spring B will be ?

a) 1

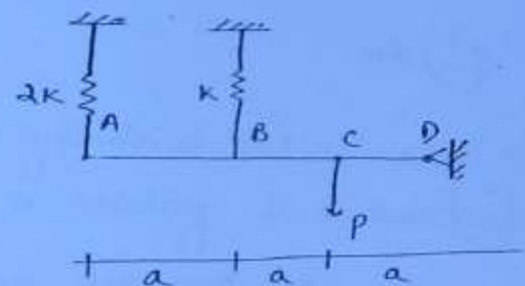
b) 2

c) 3

d) 4

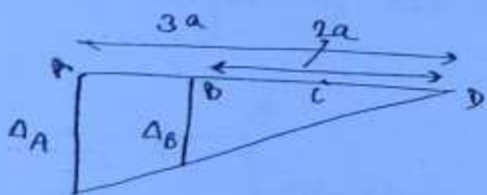
Sol:-

$(F = Kx)$ Δ



$\Sigma M_D = 0$

$P \times a = (2K x + K x)$



$F_A \times 3a + F_B \, a \, 2a + P \, a = 0$

$3F_A + 2F_B = -Pa$

$3F_A + 6F_A$

$11 F_B = -Pa$

$F_B = -\frac{1}{11} P$

F_A

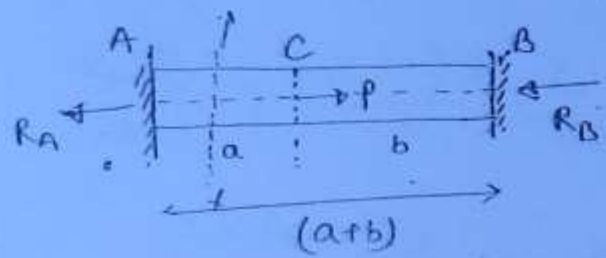
$$\frac{\Delta_A}{\Delta_B} = \frac{3a}{2a} = \frac{3}{2}$$

$$\Delta_A = \frac{F_A}{2K} \quad ; \quad \Delta_B = \frac{F_B}{K}$$

$$\frac{\frac{F_A}{2K}}{\frac{F_B}{K}} \Rightarrow \frac{F_A}{2F_B} = \frac{3}{2} \Rightarrow \frac{F_A}{F_B} = 3 \quad -(i)$$

Que. A prismatic bar AB is ~~hinged~~ Fixed at supports A and B and subjected to an axial force P. at C. then find the reaction at A and B. and movement of C w.r.t. A.?

Solⁿ:



assume R_A & R_B

$\Sigma F_x = 0$

$$R_A + R_B = P \quad \text{— (i)}$$

Equilibrium Condⁿ:

Compatibility condition.

$$\Delta_{AC} + \Delta_{CB} = 0 \quad \text{— (ii)}$$

From left side view.

$$\Delta_{AC} = \frac{F_{AC} \cdot a}{AE} = \frac{R_A \cdot a}{AE}, \qquad \Delta_{CB} = \frac{F_{CB} \cdot b}{AE}$$

$$\Delta_{CB} = \frac{(R_A - P) b}{AE};$$

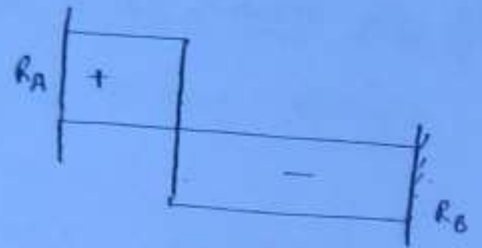
from eqⁿ (ii)

$$\frac{R_A \cdot a}{AE} + \frac{(R_A - P) b}{AE} = 0$$

$$R_A = \frac{P \cdot b}{(a+b)} = \frac{P \cdot b}{L}$$

$$\Delta_{AC} = \frac{P \cdot ab}{L \cdot AE}$$

$$R_B = \frac{P \cdot a}{a+b} = \frac{P \cdot a}{L}$$

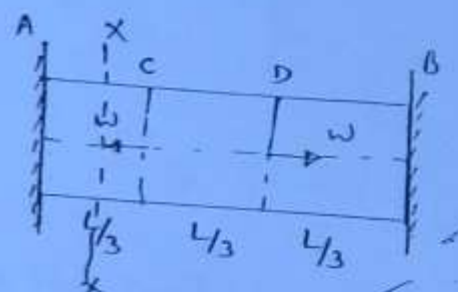


Axial force diagram.

$+ \rightarrow$ tension.
$- \rightarrow$ compress.

Q:- For the Prismatic bar shown in fig +

i) The magnitude of support reactions at A and B is



a) 0, 0, b) W, W. c) W/3, W/3. d) 2W/3, 2W/3.

The maximum axial force in the bar is

a) 0 b) W c) W/3 d) 2W/3

I → $\Sigma F_x = 0$, $R_A - W + W - R_B = 0 \Rightarrow R_A = R_B$ — (i)

Compatibility :→

$\Delta_{AC} + \Delta_{CD} + \Delta_{DB} = 0$

$$\Delta_{AC} = \frac{-R_A \cdot L/3}{AE}$$

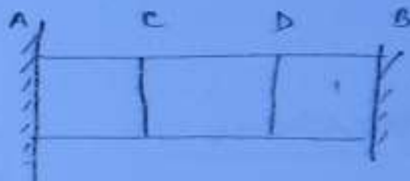
$$\Delta_{CD} = \frac{(W - R_A)\, 2L/3}{AE}$$

$$\Delta_{DB} = \frac{(-R_A + W - W)\, L/3}{AE}$$

from eq (ii)

$$\frac{-R_A\, L/3}{AE} + \frac{(-R_A + W)\, L/3}{AE} - \frac{R_A\, L/3}{AE} = 0$$

$$R_A = R_B = W/3$$

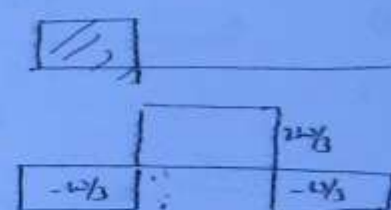


$\Rightarrow F_{AC} = -R_A = -W/3$

$\Rightarrow F_{CD} = -R_A + W = +\frac{2W}{3}$

$\Rightarrow F_{DB} = -R_A + W - W = -W/3$

Axial force diagram



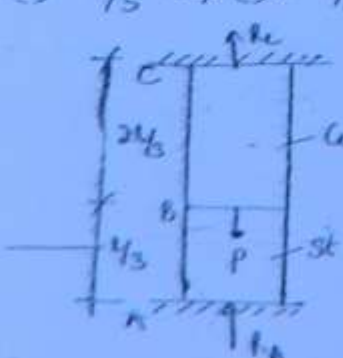
Q. A composit Bar of two metals has shown in fig has ratio of young's modulus of steel to that of Cu equal to 2 then find the force at Cu end support

(a) P/5 tens (b) P/5 Comp. (c) P/3 tension (d) P/3 Comp.

$\frac{St}{Cu} = 2$

$\frac{E_s}{E_c} = 2$

$E_s = 2E_c$



$R_A + R_c = P$ — (1)

Compatibility eq^n

$$\Delta_{AB} = \frac{-R_A L/3\ (-R_A + P) L/3}{AE_s}$$

$$\& \quad \Delta_{BC} = \frac{R_c \cdot 2L/3}{2A \cdot E_s/2}$$

$$\Delta_{BA} = \frac{(R_c - P)\, L/3}{A E_s}$$

$$\frac{R_c \cdot \cancel{4} L/3}{\cancel{2A} E_s} + \frac{(R_c - P)\cancel{L}/3}{A E_s} = 0$$

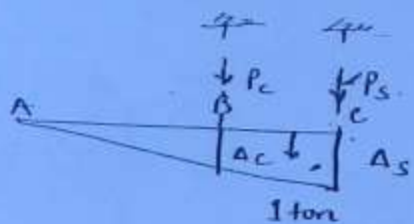
$$R_c \cdot 4 + R_c - P = 0$$

$$R_c = P/5 \quad \underline{\text{Tension}}$$

Temperature/stresses :-

Chapter : 2.

1)



~~As~~

$$\frac{\Delta_c}{\Delta_s} = 1/2$$

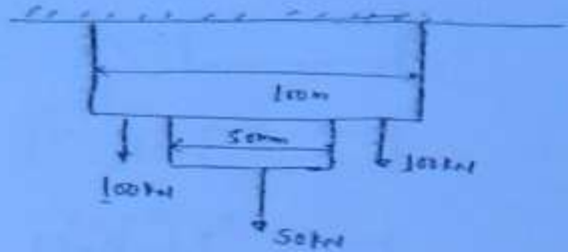
$$\Delta_c = \frac{P_c \cdot L_c}{A_c E_c}, \quad \Delta_s = \frac{P_s \cdot L_s}{A_s E_s}$$

$$\frac{P_c}{P_s}\left(\frac{L_c}{L_s}\right) \cdot \left(\frac{A_s}{A_c}\right) \times \frac{E_s}{E_c} = 1/2$$

$$\frac{P_c}{P_s} \cdot \left(\frac{8}{4}\right) \cdot \left(\frac{2}{1}\right) = 1/2$$

$$= .5$$

2)



$$= \frac{50\times10^3}{50\times50} = 20$$

$$\frac{250\times10^3}{100\times100} = 25$$

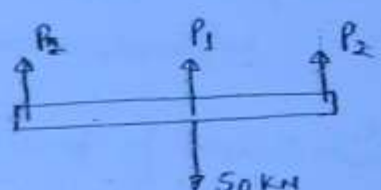
3) $U = \frac{1}{2} P \cdot \Delta$

$$= \frac{1}{2} \frac{AE}{L} \cdot \Delta \cdot \Delta \qquad ; \quad \Delta = \frac{PL}{AE}$$

$$= \frac{1}{2}\left(\frac{AE}{L}\right)\Delta^2 \qquad \boxed{P = \frac{AE}{L} \cdot \Delta}$$

$$= \frac{1}{2} K \Delta^2$$

4)



$$P_1 + 2P_2 = 50$$

$$\Delta_1 = \Delta_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2} \Rightarrow \frac{P_1}{P_2} = \frac{A_1 E_1}{A_2 E_2} \times \frac{L_2}{L_1} = \frac{3A}{2A} \cdot \frac{E \times 2L}{E \times 2L}$$

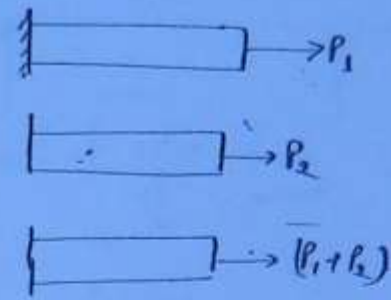
$$\frac{P_1}{P_2} = 3$$

5).

$$U = \frac{1}{2} P \cdot \Delta$$

$$= \frac{1}{2} P \cdot \frac{PL}{AE}$$

$$= \frac{1}{2} \frac{P^2 L}{AE}$$



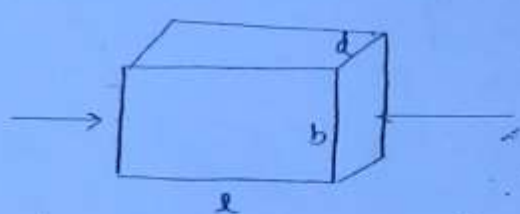
$$U_1 = \frac{P_1^2 L}{2AE}$$

$$U_2 = \frac{P_2^2 L}{2AE}$$

$$U_3 = \frac{(P_1+P_2)^2 L}{2AE}$$

$$\underline{U_3 > (U_1+U_2)}$$

).



$$\frac{\delta l}{l} = \epsilon_x = -e$$

$$V = lbd$$

$$\frac{-\epsilon_y}{\epsilon_x} = \mu$$

$$\frac{\delta b}{b} = \epsilon_y = \mu e$$

$$\frac{\delta d}{d} = \epsilon_z = \mu e$$

$$V = lbd$$

$$(V+\Delta V) = (l+\delta l)(b+\delta b)(d+\delta d)$$

$$= lbd\left[\left(1+\frac{\delta l}{l}\right)\left(1+\frac{\delta b}{b}\right)\left(1+\frac{\delta d}{d}\right)\right]$$

$$V' = V[1-e].[1+\mu e][1+\mu e]$$

$$= V[1-e][1+\mu e]^2$$

7) ✓

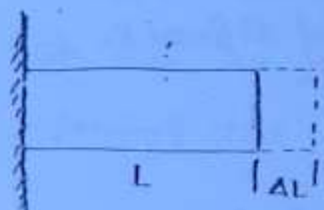
8)

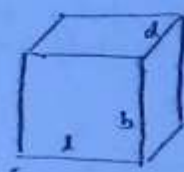
$\sigma_x = \sigma_y = \sigma$

$\sigma_z = 0$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

Temperature Stresses :-





$\Delta l = l\alpha T$

$\Delta b = b\alpha T$

$\Delta d = d\alpha T$

Theree are nothing but free expansion.

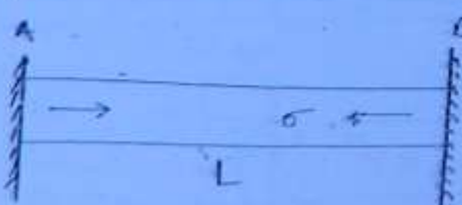
If Bar is free to expand along length width and depth then there will be no. stresses developed

α = Coeff. of thermal change

$\alpha_s = 12\times10^{-6}/°c$ $\quad$ $\alpha_{gunmetal} = 19\times10^{-6}/°c$

$\alpha_{cu} = 16\times10^{-6}/°c$ $\quad$ $\alpha_{Al} = 23\times10^{-6}/°c$

If free expansion in any direction is prevented then thermal stresses will be developed in that direction.



if T°c ↑

Net expansion of AB = 0

Free expansion − Contrⁿ due to σ = 0

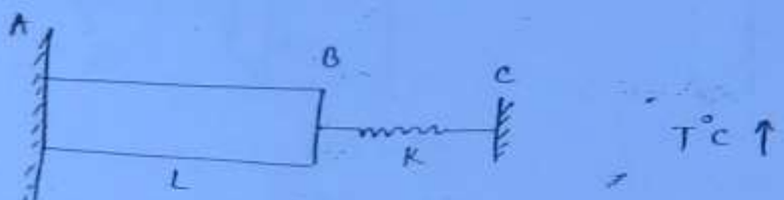
$$L\alpha T - \frac{\sigma}{E}\cdot L = 0$$

$$\Rightarrow \boxed{\sigma = E\cdot\alpha T} \rightarrow \text{Compr.}^n$$

Note that in a prismatic bar stresses are independent of L & A.

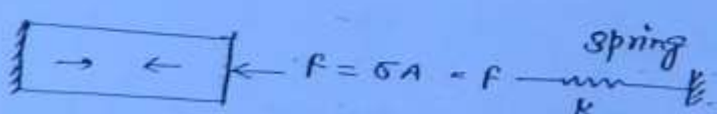
Special Case:

1. One end of the bar is rigid supported and other end is supported through a spring with coefficient of stiffness K. :—



If temp of Bar AB is raised by T°C then find the stresses developed?

Let the compressive stress developed in the bar is σ. and there will be equal comprⁿ force developed on the spring.



Net expansion of Bar AB = Contraction of spring

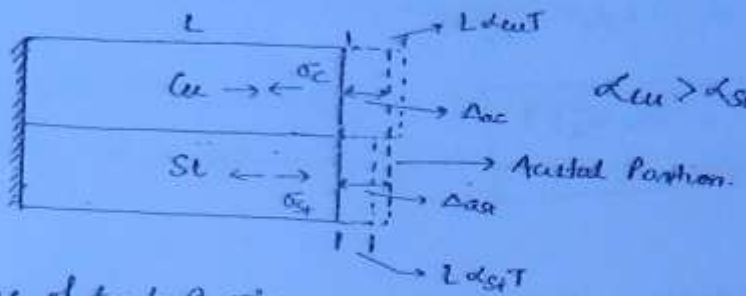
Free expansion − Contrⁿ due to σ = F/K

$$L\alpha T - \frac{\sigma}{E}\cdot L = F/K$$

Put $F = \sigma A$

$$\boxed{\sigma = \frac{E\alpha T}{\left[1 + \frac{AE/L}{K}\right]}} \Rightarrow \boxed{\sigma = \frac{E\alpha T}{1 + \frac{K_{Bar}}{K_{spr}}}}$$

Temperature stress in Composite Bar :→



$\alpha_{cu} > \alpha_{st}$

CASE I: Rise of temp By T°C :→

If Cu was free Due to rise of temp in Composite Bar free expansion of Cu → $L\alpha_c T$ will be greater than free exp. of steel ($L\alpha_s T$) But Since both Bars are jointed early, hence actual expansion of cupper is actual expansion of steel.

$$\Delta_{ac} = \Delta_{as}$$

Free expansion of cu − Contrⁿ due to σ_c = Free expⁿ of steel + extⁿ due to σ_s

$$L\alpha_c T - \frac{\sigma_c}{E_c} L = L\alpha_s T + \frac{\sigma_s}{E_s} L$$

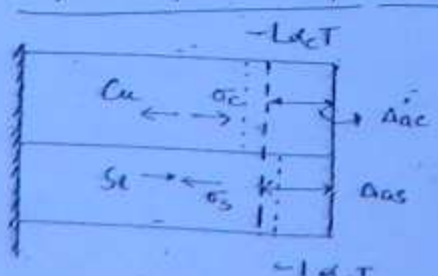
$$\left(\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s}\right) = (\alpha_c - \alpha_s)T \quad \text{— (i)}$$

Since there is no external force hence total compression force in cupper should be equal to total tension force in the steel.

$$\sigma_c \cdot A_c = \sigma_s A_s \quad \text{— (ii)}$$

From eq (i) & (ii) σ_c & σ_s can be Computed.

CASE II. If temperature of the assembly is lowered by T°C :



$$\Delta_{ac} = \Delta_{as}$$

$$\frac{\sigma_c}{E_c}T - L\alpha_c T = -L\alpha_s T - \frac{\sigma_s}{E_s}T$$

$$\boxed{\frac{\sigma_c}{E} + \frac{\sigma_s}{E_s} = (\alpha_c - \alpha_s)T} \quad \text{--- (i)}$$

Total Tension force in cu = Total C. force in St

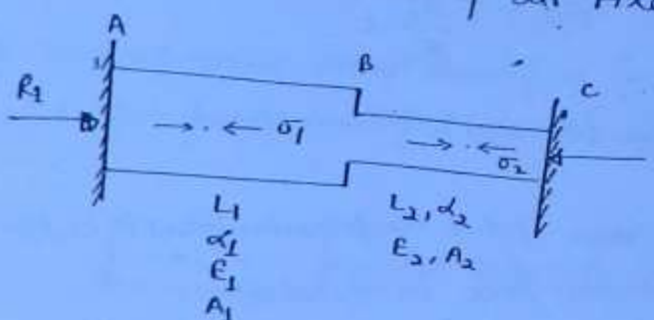
$$\sigma_c A_c = \sigma_s A_s \quad \text{--- (ii)}$$

NOTE :-

1. In composite bar due to rise of temp. that bar will be in compression which has greater α and due to fall of temp. that bar will be in compression which have smaller α.

2. If free expansion in any dir^n is permitted then there will be No stresses developed

CASE III : Series Combination of bar fixed b/w rigid support :→



$T°C \uparrow$

$$\Sigma F_x = 0$$

$$\Rightarrow R_1 = R_2$$

$$\sigma_1 A_1 = \sigma_2 A_2 \quad \text{--- (i)}$$

Net expansion of bar 1 + Net exp of bar 2 = 0

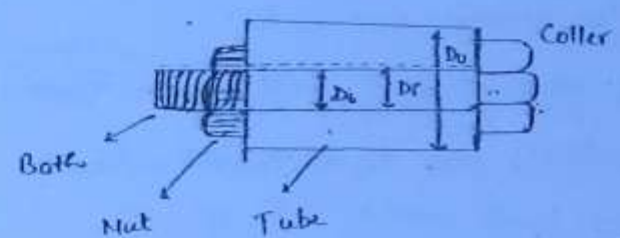
$$L_1\alpha_1 T - \frac{\sigma_1}{E_1}L_1 + L_2\alpha_2 T - \frac{\sigma_2}{E_2}L_2 = 0 \quad \text{--- (ii)}$$

From eq^n (i) & (ii) σ_1 & σ_2 are computed.

NOTE :-

If temp. of the above assembly is lowered by T°C then Both bars will be in tension. The above two equation remain valid.

Nut Bolt Assembly :-



Let

D_b = Dia of Bolt

D_i = inner dia of Tube

$D_b < D_i$,

D_o = outer Dia of Tube

p = Pitch of screw

Let Tube is made of Cu and Bolt is made of steel.

CASE I:

Effect of tightening of Nut :-

Let Nut is rotated by $\theta°$ (N turns) $\left(n = \theta/360°\right)$;

If N turn. Due to tightening, axial movement of Nut = np

Due to tightening of nut cu tube is in compression σ_{c_1} & Steel bolt is in tension (σ_{s_1})

Total compression force in copper tube should be equal to total tension force in steel.

$$\sigma_{c_1} \cdot A_c = \sigma_{s_1} \cdot A_s \qquad \text{— (i)}$$

Axial movement of nut due to tightening will be equal to contraction of cu tube + expansion of Bolt :

$$np = \left(\frac{\sigma_{c_1} L}{E_c}\right) + \left(\frac{\sigma_{s_1} L}{E_s}\right) \qquad \text{— (ii)} \qquad \left[\text{Summation of magnitude}\right]$$

Solving eqⁿ (i) & (ii) σ_{c_1} and σ_{s_1} will be found.

Temperature change :-

If temp. of nut bolt assembly is increased by $T°c$

Consider there are no stresses before temp change that is nut is lightly tighten by free hand, due to rise of temp free expansion of cu tube will be more than free expansion of steel bolt but free expansion of cu is not permitted due to presence of collar and nut.

Hence cu tube will be in compr. and st. bolt will be in tension. Let σ_{c_2} is compressive stress in cu. tube, and σ_{s_2} is tensile stress in steel bolt,

Total Compr. force in cu tube = Total tension force in st. bolt

$$\boxed{\sigma_{c_2}\cdot A_c = \sigma_{s_2}\cdot A_s} \quad \text{—— (i)}$$

The actual expansion of copper tube = Net exp. of steel bolt

$$\left(\alpha_{c}\cdot LT - \frac{\sigma_{c_2}}{E_c}\cdot L\right) = \left(\alpha_{s}\,LT + \frac{\sigma_{s_2}}{E_s}\,L\right)$$

$$\boxed{\left(\frac{\sigma_{c_2}}{E_c} + \frac{\sigma_{s_2}}{E_s}\right) = (\alpha_c - \alpha_s)T} \quad \text{—(ii)}$$

Solving eqⁿ (i) & (ii) σ_{c_2} and σ_{s_2} are found.

If Assembly is heated after tightening of Nut then final stresses will be algebric sum of both condⁿ:-

$$\sigma_c = \pm\sigma_{c_1} \pm \sigma_{c_2}$$

$$\sigma_s = \pm\sigma_{s_1} \pm \sigma_{s_2}$$

NOTE:-

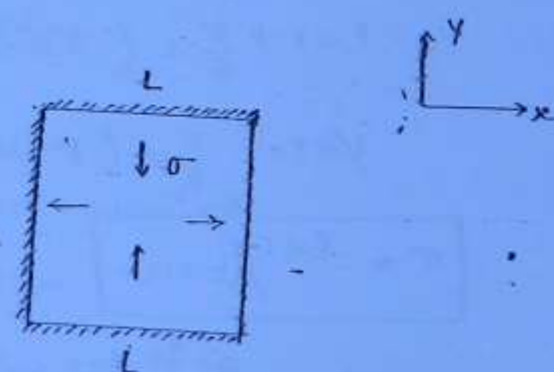
If there is no stress before temp change Nut just tightened. & then then if temp is lowered by T°c then free contrⁿ of cu tube will be greater than free contraction St. bolt. Hence the tube will become free inside collar and Nut, Hence No stresses will develop.

However Nut is tightened with stresses before temp. change and then temp is lowered then due to lowering of temp stresses will be reduced till a stage will occur when tube will become free.

example:-

ES'2000 A square plate of size (L×L) which is rigidly held at three edge and is free to expand along fourth edge. If temp is increased by T°c. & then final expansion along fourth edg will be.

a) $L\alpha T$,
b) $L\alpha T(1-\mu)$
c) $L\alpha T(1+\mu)$
d) $L\alpha T/(1-\mu)$



$\sigma = E\alpha T$ —(i)

final expansion in x dir

Δx = free expansion + effection of σ in x dir

$= L\alpha T + \mu \cdot \frac{\sigma}{E} L$ —(ii)

$= L\alpha T(1+\mu)$

Q:-

A cube of size L is constrained in all dir^n b/t rigid supports. If bar is heated uniformly by $T°c$ then stress developed in any dir^n is ~~EαT~~

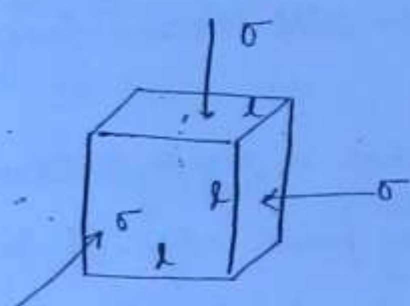
i) $E\alpha T$

ii) $E\alpha T(1+\mu)$

iii) $E\alpha T(1-2\mu)$

iv) $E\alpha T/(1-2\mu)$

$\frac{3\sigma}{E}(1-2\mu)$, $\frac{E\sigma_t}{E}$



The net deflection is any $dir^n = 0$

$$\Delta_x = 0$$

$$L\alpha T - \frac{\sigma}{E}L + \mu\frac{\sigma}{E}L + \mu\frac{\sigma}{E}L = 0$$

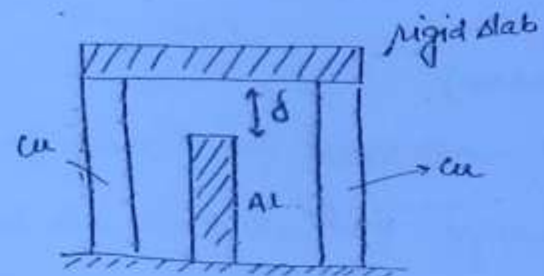
$$L\alpha T + \frac{\sigma}{E}L(2\mu - 1)$$

$$\cancel{L}\alpha T = \frac{\sigma}{E}\cancel{L}(1-2\mu)$$

$$\boxed{\sigma = E\alpha T/(1-2\mu)}$$

Q. An assembly is shown in fig in which there is gap of 0.18 mm b/t Al Bar and rigid slab. neglecting mass of slab calculate the stresses in each rod when temp of the assembly is increased by 85°c. For each cu, Bar area is 500 mm^2 and for Al. bar area is 400 mm^2 There is a band b/t cu bar and rigid slab, young's modulus of Al = 70 GPa , E_{cu} = 120 GPa

$\alpha_{cu} = 16.8 \times 10^{-6}/°c$, $\alpha_{Al} = 23.1 \times 10^{-6}/°c$ and length of cu bar 750 mm.



$749.82 \times 23.1 \times 85 \times 10^{-6}$

$\delta = \cdot 18$ mm

Free expansion of cu = $L_c \cdot \alpha_c T$

$= 750 \times 16.8 \times 10^{-6} \times 85$

$= 1.07$ mm

Free expansion of Al = $L_{al} \cdot \alpha_{al} \cdot T$

$= 749.82 \times 23.1 \times 10^{-6} \times 85$

$= 1.47$ mm

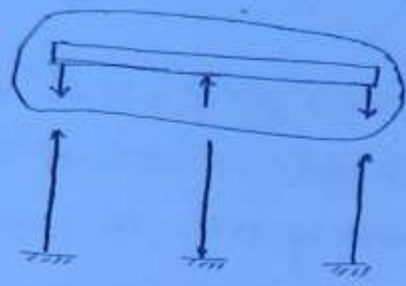
Free expansion of Al > δ + free exp. of Cu

$1.47 > 1.07 + 18 \Rightarrow 1.47 > 1.25$

Hence cu will be in tension and Al will be in Compression.

Let σ_A is compr. stress in Al.

& σ_C is tensile " " Cu.



Total compr. force in Al bar must be equal = Total tension force in both Cu bar.

$$\sigma_a \cdot A_a = \sigma_c A_c$$

$$\sigma_a \cdot 400 = \sigma_c (500+500)$$

$$\boxed{\sigma_c = .4\,\sigma_a} \quad \text{—— (i)}$$

Compatibility eqⁿ :-

The actual defl^n of Al bar = δ + actual deflection of Cu bar.

Free exp^n of Al + Contra^n due to σ_a = δ +

$$L_a \cdot \alpha_a \cdot T - \frac{\sigma_a}{E_a} \cdot L_a = .18 + L_c \alpha_c T + \frac{\sigma_c}{E_c} \cdot L_c$$

$$1.47 - \frac{\sigma_a}{E_a} \cdot L_a = .18 + 1.07 + \frac{\sigma_c}{E_c} \cdot L_c \quad \text{—— (ii)}$$

$$1.47 - \frac{\sigma_a}{} \times 749082 =$$

$\sigma_c = 6.52\ N/mm^2$ tension

$\sigma_{al} = 16.29\ N/mm^2$ —— compre.

find by How many °c temp should be increased to just fill the gap without Causing stress in the bar ?

Free exp. of Al = ·18 + free exp of cu

$$L_a \cdot \alpha_a \cdot T = \cdot 18 + L_c \cdot \alpha_c \cdot T$$

$$T\left\{ 400/ (749.82 \times 23.1\times10^{-6}) - (750\times 16\times10^{-6})\right\} = \cdot 18$$

$$(17320 - 841) -$$

$$T = 38.1\ ^\circ c$$

Q. A steel Bolt of dia of 10 mm passes thru a brass tube of internal dia 15 mm and external dia 25mm the bolt is tightening by a nut so that length of the tube is reduced by 0.15 mm. If Now temp. of Assembly is increased by 40 °c then estimate axial stresses developed in the bolt and tube. assuming length of Tube 150 mm.

$E_s = 2\times10^5\ N/mm^2$ - $E_{brass} = 1\times10^5\ N/mm^2$

$\alpha_s = 12\times10^{-6}\ /^\circ c$ $\alpha_b = 19\times10^{-6}\ /^\circ c$

Solⁿ:

Due to tightening of Nut, Tube is compressed

Compression in the Tube $= \Delta = \cdot 15mm$

Length of tube $= 150mm$

Compressive strain in tube $= \frac{\Delta}{L} = \frac{\cdot 15}{150} = 1\times10^{-3}$

Compressive stress in ~~bolt~~ tube $(\sigma_{b_1}) = $ Strain $\times E_b$

$= 1\times10^{-3}\times 1\times10^5$

$= 100\ N/mm^2$

Let σ_{s_1} — tension stress in Bolt

Total T·F in Bolt = T· Compⁿ in Tube .

$$\sigma_{b_1} A_b = \sigma_{s_1} A_s$$

$$100 \times \frac{\pi}{4}(25^2 - 15^2) = \sigma_{s_1} \times \frac{\pi}{4}(10)^2$$

$$\sigma_{s_1} = 400 \ N/mm^2 \quad \text{— (T)}$$

Since α of Brass $> \alpha_{st}$

Hence, on heating Brass tube will be in compression and Steel Bolt will be in tension.

Let σ_{b_2} is compressive stress in Brass tube and σ_{s_2} is tensile stress in steel Bolt.

Total compression force in Brass Tube = Total tension force in steel Bolt

$$\sigma_{b_2} \cdot A_b = \sigma_{s_2} A_s$$

$$\sigma_{b_2} \cdot \frac{\pi}{4}(25^2 - 15^2) = \sigma_{s_2} \times \frac{\pi}{4}(10)^2$$

$$\sigma_{s_2} = 4\sigma_{b_2} \quad \text{— (i)}$$

Net expanⁿ of Brass tube = Net expⁿ of steel Bolt

Free expⁿ of Brass − Contⁿ due to σ_{b_2}

= Free expⁿ of st + expⁿ due to σ_{s_2}

$$L\alpha_b \cdot T - \frac{\sigma_{b_2}}{E_b} \cdot L = L \cdot \alpha_s T + \frac{\sigma_{s_2}}{E_s} L$$

$$\left(\frac{\sigma_{b_2}}{E_b} + \frac{\sigma_{s_2}}{E_s}\right) = (\alpha_b - \alpha_s)T \quad \text{— (ii)}$$

Solving eqⁿ (i) & (ii), σ_{b_2} & σ_{s_2} will be known.

$$\sigma_{b_2} = 9.33 \ N/mm^2 \quad (C) \qquad \sigma_{s_2} = 37.33 \ N/mm^2 \quad (T)$$

Final stresses

$$\sigma_b = \sigma_{b_1} + \sigma_{b_2} = 100 + 9.33 = 109.33 \text{ N/mm}^2 \quad \text{(C)}$$

$$\sigma_s = \sigma_{s_1} + \sigma_{s_2} = 400 + 37.33 = 437.33 \text{ N/mm}^2 \quad \text{(T)}$$

The stresses developed in steel are greater than 250 N/mm², Hence if steel is mild steel then the data given are incompatible because the analysis is under the assumption of Hooke's law.

Ques: A Prismatic Bar AB is rigidly fixed betⁿ A and B. If temp of the bar is raised Non uniformly from 0°C At A to T°C At B having parabolic variation along the length then the temp stress developed in the bar will be.

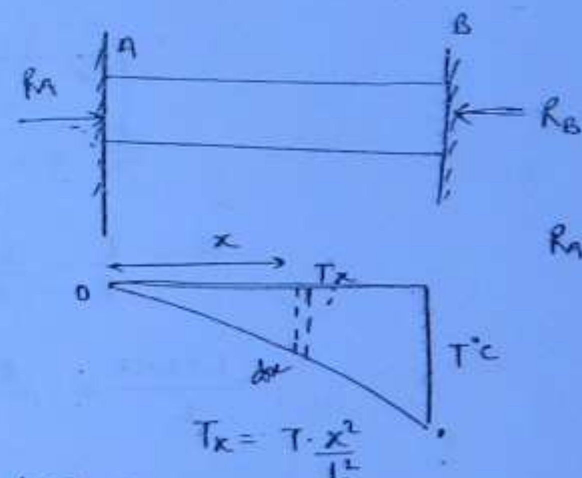
Gate

a) $E\alpha T$

b) $E\alpha T/2$

c) $E\alpha T/3$

d) N.O.T.



$R_A = R_B = \sigma A$

$T_x = T \cdot \frac{x^2}{L^2}$

Net deflⁿ in AB = 0

free expⁿ due to temp − Contrⁿ due to σ = 0

$$dx\,\alpha \int_0^L T_x - \frac{\sigma}{E} \cdot L = 0$$

$$= \int_0^L \alpha \cdot dx \cdot T_x$$

$$\alpha \cdot \frac{T}{L^2} \cdot \frac{L^3}{3}$$

$$\frac{L T \alpha}{3} = \frac{\sigma}{E} \cdot L \quad \therefore \quad \sigma = E\alpha T/3$$

5.

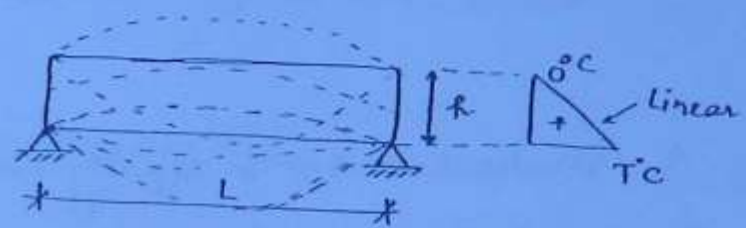
A simply supported Beam as shown in fig has length L & depth h. temp of the assembly is raised non uniformly. s.t. temp change at top chord is zero 0°C and at bottom cord is T°C. Temp variation along the depth is linear. due to heating the beam bends into a arc of a circle then central deflection of the bar neglecting higher orders will be.

a) $\frac{\alpha T h^2}{8L}$ ↓

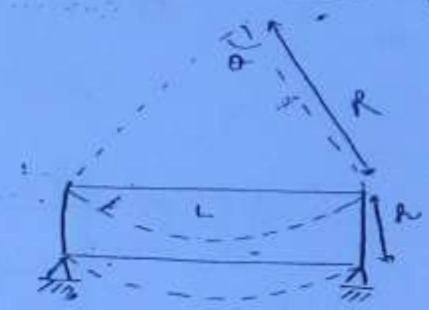
b) $\frac{\alpha T h^2}{8L}$ ↑

✓(c) $\frac{\alpha T L^2}{8h}$ ↓

d) $\frac{\alpha T L^2}{8h}$ ↑

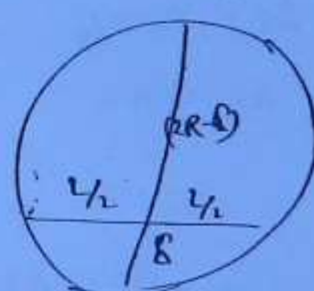


$\sigma = \alpha T L$



$\frac{\sigma n}{2} = \frac{wL^2}{8h}$

$$\theta = \frac{L}{R} = \frac{L+L\alpha T}{R+h}$$



$$\frac{L+L\alpha T}{L} = \frac{R+h}{R} \quad -(1)$$

$$\cancel{1}+\alpha T = \cancel{1}+h/R$$

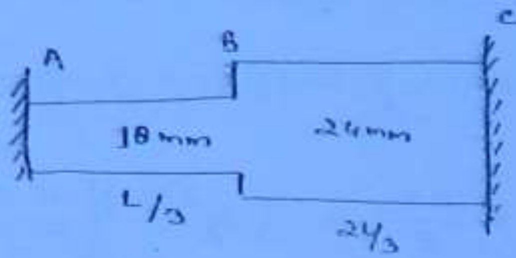
$$\boxed{\frac{1}{R} = \frac{\alpha T}{h}}$$

$$\frac{L}{2}\frac{L}{2} = \delta(2R-\delta)$$

$$\frac{L^2}{4} = 2R\delta - \cancel{\delta^2} \text{ neg.} \qquad \delta = \frac{L^2}{8R} = \frac{\alpha T L^2}{8h} \downarrow$$

Work Book :-

22.



$T^\circ c = 30^\circ c$

$\alpha = 12.5 \times 10^{-6}$

$E = 200 GN/m^2$

$\sigma_1 A_1 = \sigma_2 A_2$

$\sigma_1 \frac{\pi}{4} 18^2 = \sigma_2 \cdot \frac{\pi}{4} 24^2$

$\sigma_2 / \sigma_1 = 18^2 / 24^2 = 9/16$ — (i)

$\Delta_{AB} + \Delta_{BC} = 0$

Free contra^n of AB

$\left(-L_1 \alpha T + \frac{\sigma_1}{E} L_1\right) + \left(-L_2 \alpha T + \frac{\sigma_2}{E} L_2\right) = 0$

$-\frac{L}{3} \alpha T + \frac{\sigma_1}{E} \cdot \frac{L}{3} - \frac{2}{3} L \alpha T + \frac{\sigma_2}{E} \frac{2L}{3} = 0$

$-3L \alpha T + (\sigma_1 + 2\sigma_2) L = 0$

$+3 \times 12.5 \times 10^{-6} \times 30 = (\sigma_1 + 2\sigma_2)$

$37.5 \times 10^{-6} \times 30 = 34 \sigma_1$

$\sigma_1 = \frac{375 \times 3 \times 10^{-6} \times 2 \times 10^5}{34}$

$\simeq 105$ MPa

23 :-

$\epsilon_x = \frac{\sigma_x}{E} = 1.25 \times 10^{-3}$

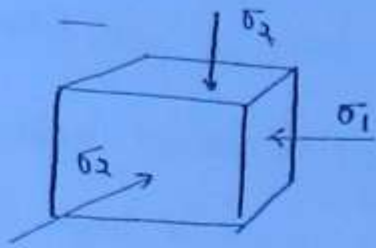
~~$\epsilon_v = 1.25$~~

$\epsilon_y = -\mu \epsilon_x = .3 \times 1.25 \times 10^{-3}$

$\epsilon_z = -\mu \epsilon_x = .3 \times 1.25 \times 10^{-3}$

$\epsilon_v = (1.25 - 2 \times .3 \times 1.25) \times 10^{-3}$

28.

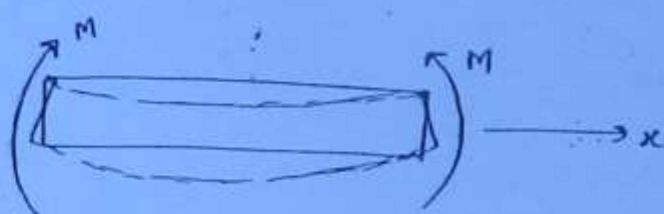


$$\epsilon_2 = -\frac{\sigma_2}{E} + \frac{\mu\sigma_1}{E} + \frac{\mu\sigma_2}{E} = \frac{1}{2}\epsilon_1' = \frac{1}{2}\mu\frac{\sigma_1}{E}$$

$$=$$

$$\sigma_2 = \frac{\mu}{1-\mu}\ \frac{\sigma_1}{2}$$

33.



$$\epsilon_x = -\frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = +\mu\frac{\sigma_x}{E}$$

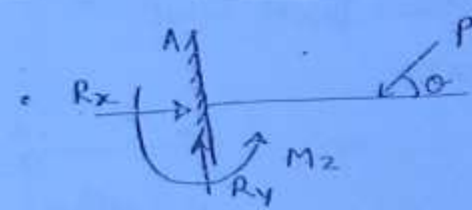
$$= \begin{pmatrix} -\frac{\sigma_x}{E} & 0 & 0 \\ 0 & +\frac{\mu\sigma_x}{E} & 0 \\ 0 & 0 & +\frac{\mu\sigma_x}{E} \end{pmatrix}$$

Shear Force and Bending Moment :→

Type of supports & Support Reactions :→

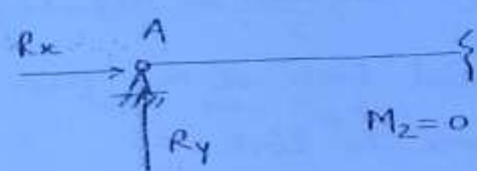
1) 2-D Supports :

i) Fixed support / Built in Support :—



At fix support
No. of Reaction = 3

ii) Hinge Support :—



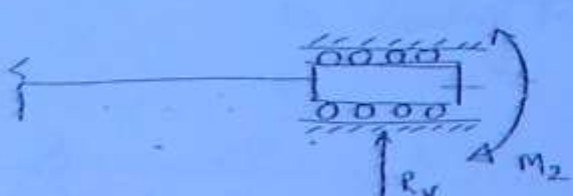
$M_z = 0$

$R = 2$

iii) Rollar support / Simple support

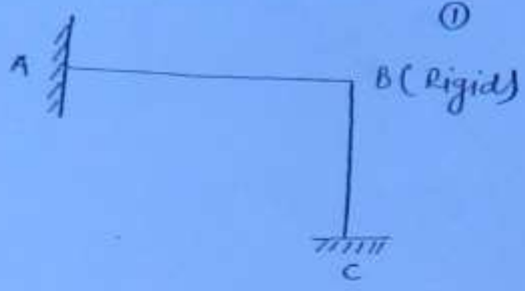
$R = 1$

iv) Double Rollar support :



$R = 2$

Internal Joints :-

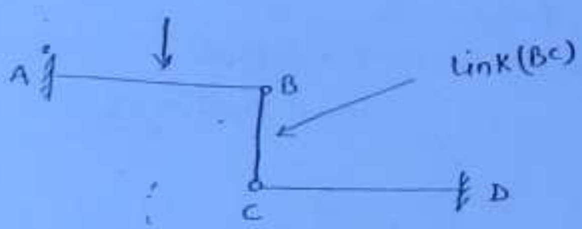
① 

② 

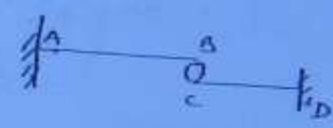
B.M. at Hinge = 0

$\therefore M_D = 0$.

③ Link



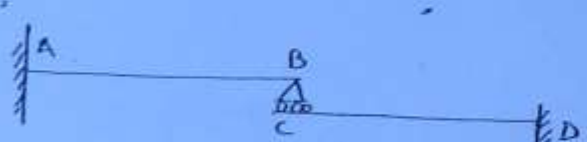
or



link is used to connect two members at diff. ht.

The link (BC) carry only axial force at and there will be no. Bending moment and shear force in link.

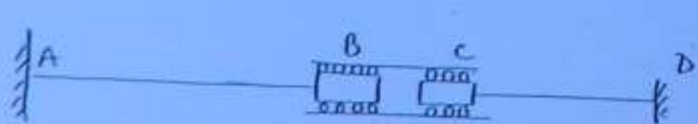
Internal Rollar :-



A.F. = 0

B.M. = 0

Double Rollar joint :-



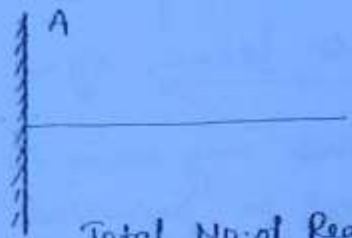
A.F = 0

or

Shear Force is zero at joint

3-D supports :-

1) fixed Support/Built in Support :-



Total No. of Reaction = 6 — R_x, R_y, R_z, M_x, M_y, M_z

2) Hinge support :-

Total. No. R. = 3 $\left.\begin{matrix} R_x \\ R_y \\ R_z \end{matrix}\right\}$

3) Rollar Support :-

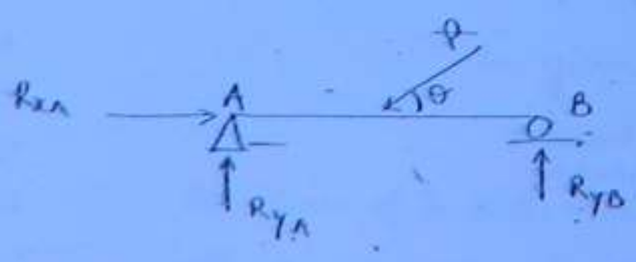
$R = 1$ i.e. R_y {For unguided Rollar}

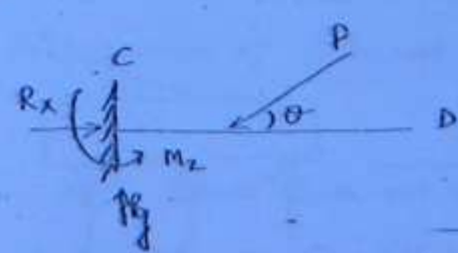
$R = 2$ i.e. R_y & R_z {For guided Rollar}

Stability :-

CASE-A 2D. loading :→

ie Loading in xy plane.





A body is stable if there is no rigid body displacement at supports it means there will not be large appreciable movement at the joints. In order to prevent movements, reactions are required to resist displacement.

- A Body is in stable cond^n if displacements $\Delta x=0$, $\Delta y=0$ & $\theta_z=0$

NOTE:

In elastic body small elastic displacements may occur in members but large rigid body displacement will not be permitted.

For stability, in 2-D Body following equilibrium cond^{n}s must Be satisfied.

$\Sigma F_x = 0$ — (i)

$\Sigma F_y = 0$ — (ii)

$\Sigma M_z = 0$ — (iii)

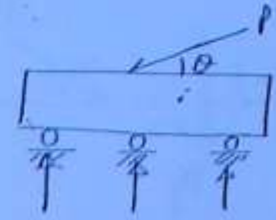
For static stability of a Plane structure following cond^n should be satisfied.

1.) Min^m No of Possible support reaction 3, i.e. total no. of reactions at all Support should be greater or equal to 3.

2) All the reactions (linear rx^n) should not be concurrent. if all the Rx^n all concurrent then angular un-stability will occur.

Rotation is occur about Pt of Concurrency.

3) All linear Rx^n should not be parallel. if all rex^n are $\|^l$ then linear unstability will occur.



unstable

4). There should be no Condⁿ of Mechanism. (mechanism is formed when there is partial collapse condⁿ due to presence of three collinear hinges in a row.

(Mechanism).

Partial collapse

5). For stability reactions should be non trivial (magnitude of Rⁿ should be large enough to prevent force).

Stable structure

Determinate

All reactions at supports Can be computed By Condition of equilibrium

No. of Reaction = No. of equili eqⁿ

$R = E$ or $D_i = (R-E)$

Indeterminate

No. of Reaction > No. of eqⁿ eqⁿ.

$R > E$

$D_i = R - E > 0$

NOTE :

To determine support reactions in case of determinate reactions eq Conditions are used.

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$ + Addition eqⁿ condition may be given if there is presence of internal hinge or Internal link.

In case of Indeterminate structure eqⁿ Condⁿ are not enough to find all reaction Hence additional compatibility condⁿ will be reqᵈ.

Compatibility condⁿ. are in terms of slope and deflection.

B.A. at fixed point slope $=0$, deflⁿ $=0$;

" Hing " deflⁿ $=0$;

NOTE :-

In case of determinate structures reaction are not dependent on properties of cross-section and metal (A, I, E) hence shear force, B.M. and axial force also be idependent of A, E, I).

In case indeterminate beams compatibility condⁿ are also needed, which depend on upon properties of cross-section and metal hence S.F, B.M. and A.F will also be function of properties of cross section.

3-D Case for stability :-

For static stability following condⁿ should be satisfied.—

1. No. of support reaction ≥ 6
2. All reaction should be non-coplainer.
3. All linear reaⁿ should be non-concurrent.
4. Raⁿ should be non-trivial.

There should be No. condⁿ of mechanism i.e. 3 collinear Hinges should not be present.

For, static stability of 3-D Body (space body) following 6 condⁿ of eqⁿ should be satisfied

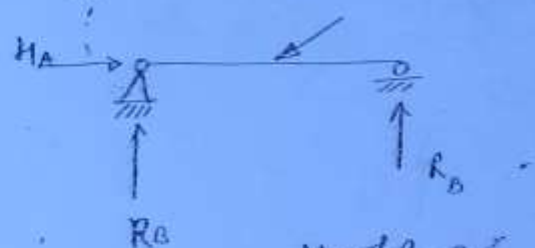
$$\Sigma F_x=0 \;;\; \Sigma M_x=0$$
$$\Sigma F_y=0 \;;\; \Sigma M_y=0$$
$$\Sigma F_z=0 \;;\; \Sigma M_z=0$$

If total No. of support Reaction are equal to total No. of equilibrium cond^n then Structure is stable and determinate provide cond^n of stability or met. and If total no. of support reactions are greater than total available eq^n cond^{n}s. then structure is called indeterminate.

If No. of support reactions are less than no. of eq^n cond^{n}s then structure is unstable.

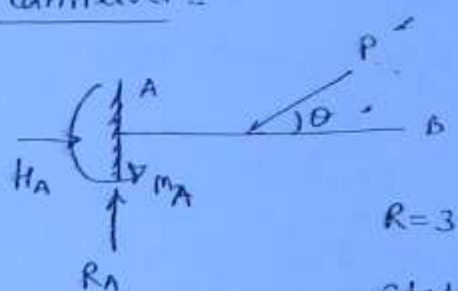
Type of Beams :→

I) Simply Supported Beam :–



No. of $R = 3$ Stable and determinate

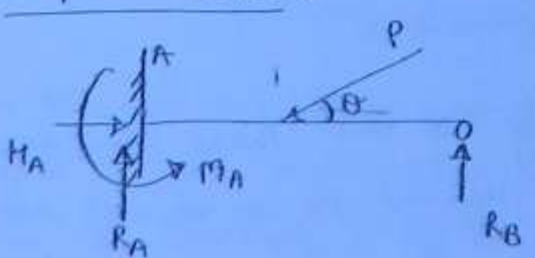
ii) Cantiliver :–



$R = 3$

Stable & Determinate

iii) Proped Cantiliver :–



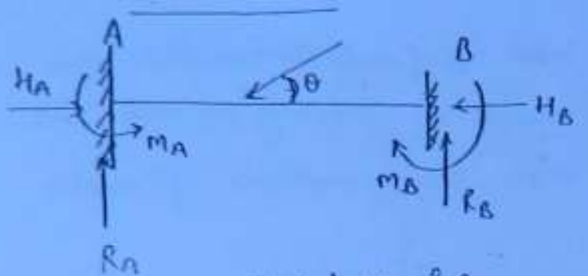
$R = 4$

$E = 3$

$D_i = 1$

Stable but indeterminate

* greater the No. of R^{n} greater is the stability.



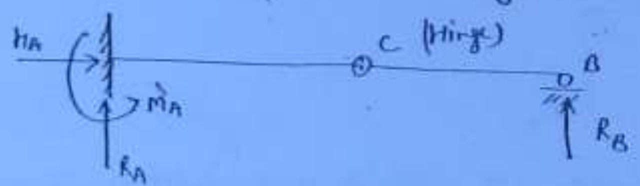
Total No. of $R_{x^n} = 6$

Total No. of $Eq^n = 3$

$Di = 6 - 3 = 3$

Stable but indeterminate.

Proped Cantilever with Internal Hinge :-



Total No. of Support $R_{x^n} = 4$

Total No. of $Eq^n = 3+1 \longrightarrow M_C = 0$

$\downarrow$

$\Sigma F_x = 0$

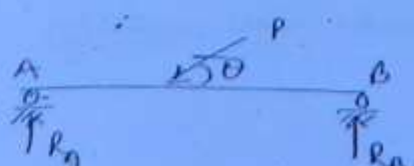
$\Sigma F_y = 0$

$\Sigma M_z = 0$

$Di = R - E = 0$

Stable and determinate.

. Beam supported on two roller has only two R_{x^n}s Hence, for a general loading in 2-D it will be unstable ; However for an special case of vertical loading it may become stable.

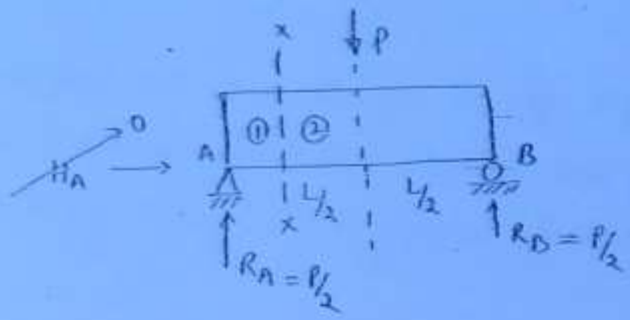


$R = 2$

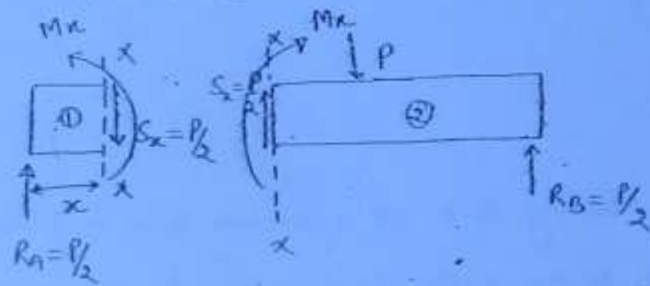
$E = 3$

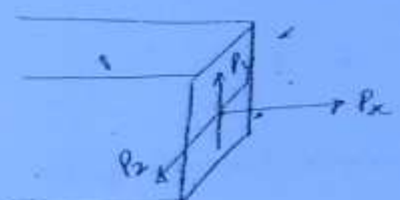
$Di = -1 < 0 \Rightarrow$ Unstable

Shear Force :-



F.B.D of ① & ②





P_x = Axial thrust
P_y = Vertical S. Force.
P_z = Horizontal S. Force.
} 3-D. Case.

* Shear force at any Section x-x is internal transverse force which is developed parallel to the cross-section to maintain free body eq^n of either left free body or Right free body.

(or).

Shear force at any section is resultant transverse force either to the left or to the right of that section.

In 2-D loading generally S.F is vertical transverse force (P_y) but in

Sign Convention of Shear Force :→

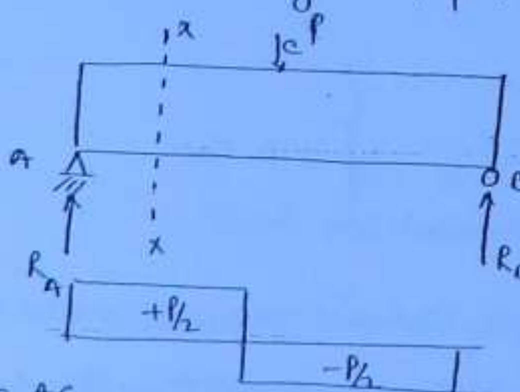
⇒ +ve

−ve

For verticle loading S.F at a section $\alpha-\alpha$ is +ve if resultant transverse force to the left of $x-x$ is upward or to the right of $\alpha-x$ is downward and vice versa.

In above beam S.F is at $\alpha\cdot\alpha'$ is $\pm \frac{P}{2}$.

S.F.D :– Shear force diagram represents variation of shear force at a crossection with the length.



S.F in AC

$$S_x\,(x \text{ is from } A) = +R_A$$
$$= +P/2$$

S.F in BC

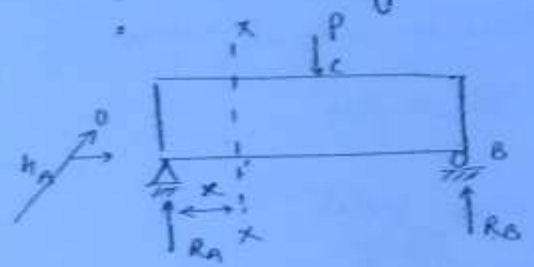
$$S_x\,(x \text{ from } A) = +R_A - P = \frac{+P}{2} - P = -P/2$$

S.F at C in AC = $+P/2$

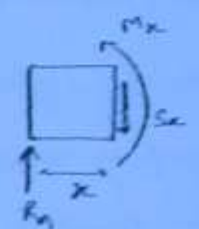
" " " " BC = $-P/2$

- Notice that S.F. have changes its sign at C but S.F is not zero at C
- If at a pt. a conc^n load or Reaction acts then shear force at that Point will Suddenly change by magnitude of that force and Reaction.

Shear force and Bending Moment :-



B.M. at any section x-x is moment of transverse component of force either from left or from right.



Sign Convention of B.M :

Sagging B.M. = +ve

Hogg Bending m = -ve

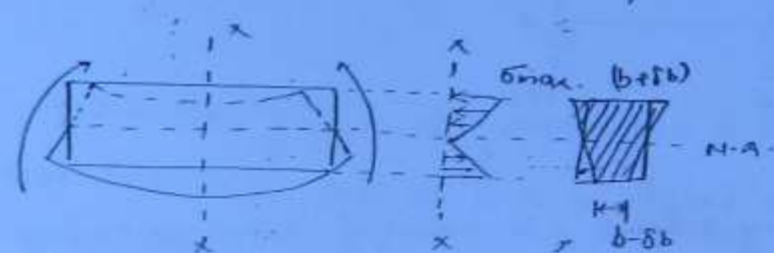
B.M. and S.F Convention of sign is with respect to a reference face. generally for beams top face is taken reference face and for closed frame reference face is taken outer face.

NOTE

- generally that face is taken reference face on which load acts. As far as possible reference face should be Continuous ~~phase~~ face

Effect of Sagging B.M:

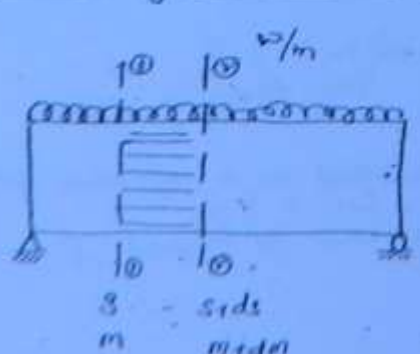
For the Beam with top face reference sagging moment causes Compressive stresses above the nutral axis and tensile stresses below nutral axis in longitudinal dir^n. therefore the length is compressed above the nutral axis and length is increased below nutral axis. Bending stresses are normal stress which vary from 0 at nutral axis to max^m at surface

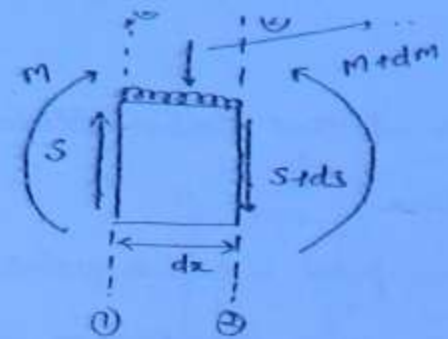


Before Bending Let section is rectangular, due to Bending (Sagging) length is compressed above the nutral axis hence width is increased above the nutral axis and length is expanded below nutral axis hence width is reduced below nutral axis. therefore a rectangular Section before Bending becomes trapozeodial after Bending.

Since δb is very small ($\delta b <<< b$) Hence for all practical purposes of Calculation change in width will be neglected.

Relation b/t Loading Rate and Shear force and Relation b/t S.F and B.m :-





For equilibrium:

$$\Sigma F_y = 0$$

$$S - w\cdot dx - (S + dS)$$

$$- w\cdot dx - dS = 0$$

$$\boxed{-\left(\frac{dS}{dx}\right) = w} \quad \text{--- (i)}$$

-ve slope of shear force curve represents downward loading rate or change of S.F. is equal to loading rate

$$\Sigma M_2 = 0$$

$$S\cdot dx + M - (w\cdot dx)\cdot\frac{dx}{2} - (M + dM) = 0$$

$$S\cdot dx + \cancel{M} - \cancel{\frac{w\,dx^2}{2}} - \cancel{M} - dM = 0 \quad \text{negligible}$$

$$\boxed{\frac{dM}{dx} = S} \quad \text{--- (ii)}$$

Imp

It means rate of change of B.M. at any section is equal to S.F. at that section or $\frac{dM}{dx}$ or slope of B.M. curve at any section will be equal to S.F. at that section

$$dM = S\,dx$$

$$\int_A^B dM = \int_A^B S\,dx$$

$$M_B - M_A = \int_A^B S\,dx$$

$$\boxed{M_B - M_A = \text{Area of S.F.D b/w A \& B}}$$

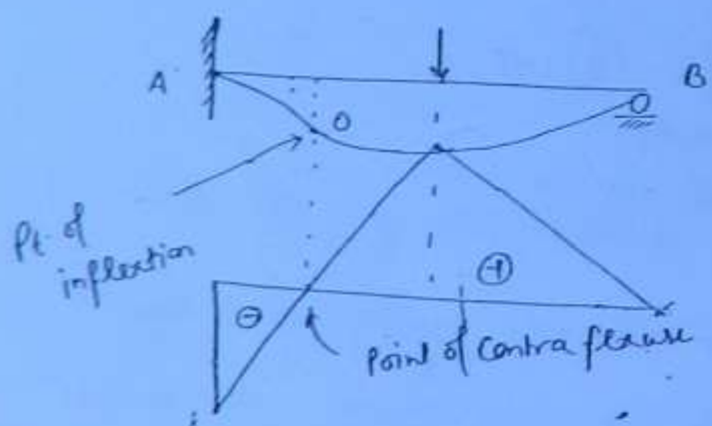
Change in B.M from A to B is equal to area of S.F.D b/t A & B

NOTE.

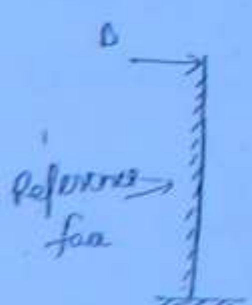
1. If S.F. changes sign at a section then B.M. at that section will be either max^m or min^m, note that reverse is not true

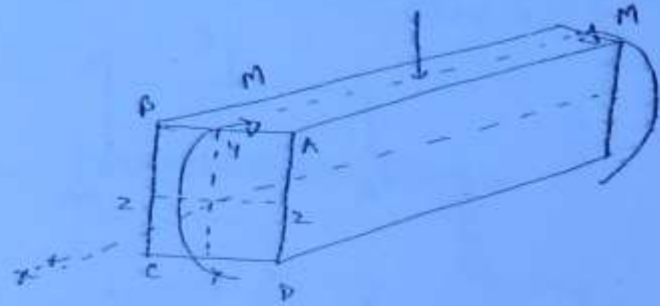
2. If B.M. changes sign at a section then such a point is called point of contraflexure. At the point of contraflexure curvature also change sign. (Concave to convex or convex to concave). Hence such a point is also called point of inflection.

3. S.F. curve is 1 degree higer than loading curve and Bending m. curve is 1 degree higher than shear force curve



Significance of Point of Contraflexure is that B.M. is opposite in nature Hence position of Reinforcement can be precisely determined





In Bending N.A is transverse axis about which the cross sectional area in Bending Rotates. If loading is in vertical plane (vertical Bending) then N.A. will horizontal transverse axis (z-z)

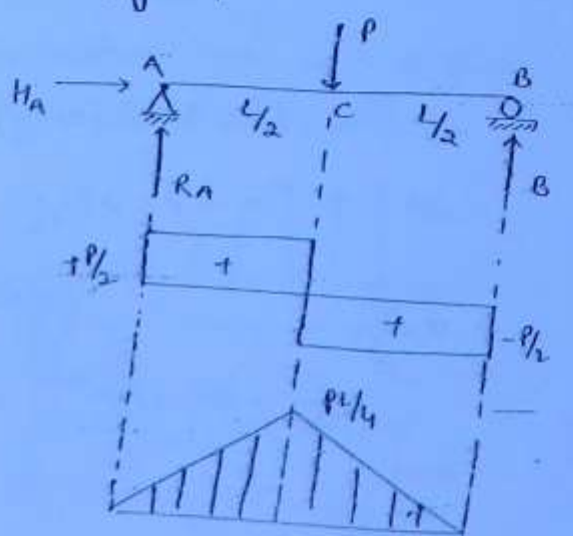
If Bending is in horizontal plane then neutral axis will be vertical transverse axis.

In pure Bending in elastic condⁿ, N.A will always pass thru centroid. However in inelastic B./Plastic Bending N.A. will pass thru equal area axis which may or may not be centroidal axis.

Note that if under moment ~~cr~~ Cross-Sectional area rotates about longitudinal axis then it will be twisting moment (Torque)

Q. For simply support Beam shown in fig draw S.F & B.M diagram?

Sol:



$\Sigma F_x = 0$

$H_A = 0$ —(i)

$\Sigma F_y = 0$

$R_A + R_B - P = 0$

$R_A + R_B = P$ —(ii)

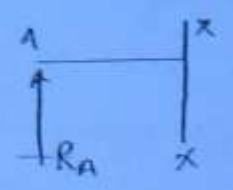
$\Sigma M_B = 0$

$R_A \times L - P \times L/2 = 0$

$R_A = P/2$

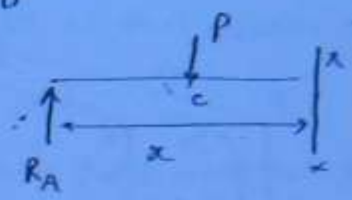
$R_B = P/$

S.F. of AC



$$S_x = R_A$$
$$= +\frac{P}{2}$$

S.F. in CD



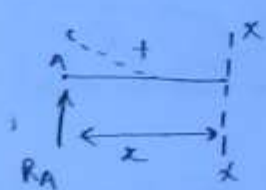
$$S_x = +R_A - P$$
$$= +P/2 - P$$
$$= -P/2$$

NOTE:-

If at a Point concentration load or Ra^n acts then ordinate of S.F. at that point will suddenly change by magnitude equal to that the Force

$$\text{S.F. at C in AC} = +P/2$$
$$\text{S.F. at C in CB} = -P/2$$

B.M in AC:

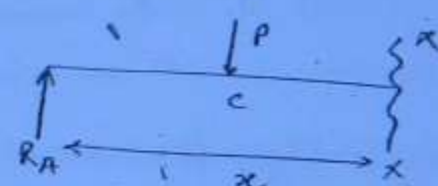


$$M_x = +R_A x$$
$$= \frac{P}{2}\cdot x \quad \left(0 \le x \le \frac{L}{2}\right)$$

$$M_A = 0$$

$$M_C = +\frac{P}{2}\cdot\frac{L}{2} = +\frac{PL}{4}.$$

B.M. in CB



$$M_x = +R_A\cdot x - P(x - L/2)$$
$$= \frac{P}{2}x - Px + PL/2$$

$$\left(L/2 < x < L\right)$$

$x = L/2$

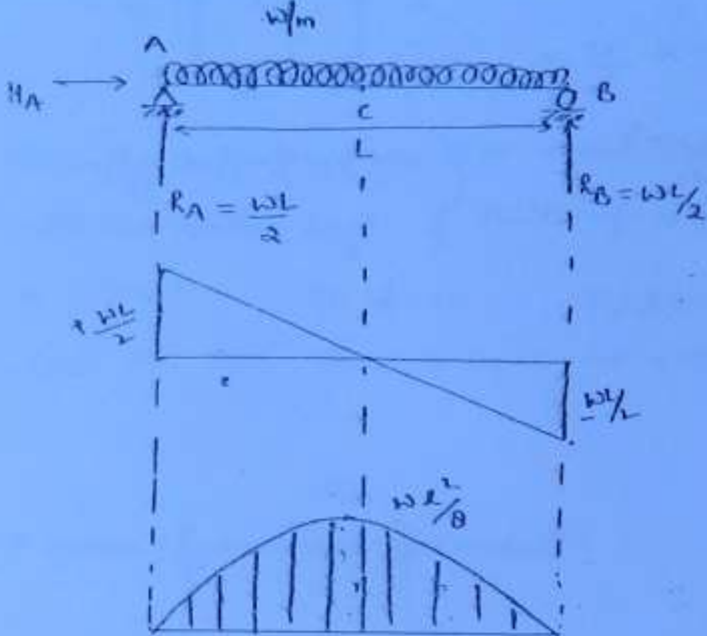
$$= \frac{PL}{4} - \frac{PL}{2} + PL/2$$

$x = L$

$M_B = 0$

Q. Draw S.F.D and B.M.D. for Simply supported B~~m~~ for udl ?

Solⁿ:



S.F in AB:-

$S_x = +R_A - wx$

$= w \cdot \frac{L}{2} - wx$

$S_A = wL/2 \quad (x=0)$

$S_B = -wL/2 \quad (x=L)$

$S_C = 0 \quad (x = L/2)$

B.M in AB.

$M_x = +R_A x - wx \frac{x}{2}$

$= \frac{wL}{2} \cdot x - \frac{wx^2}{2}$

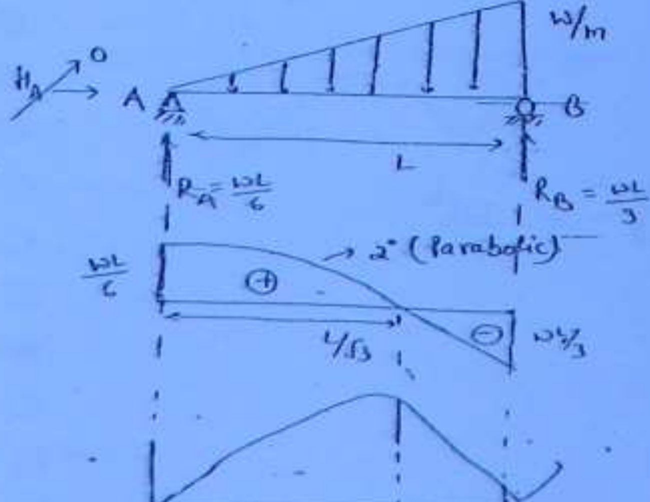
$M_A = 0 \quad (x=0)$

$M_B = +\frac{wL^2}{2} - \frac{wL^2}{0} = 0 \quad (x=L)$

$M_C = \frac{wL^2}{4} - \frac{wL^2}{8} \quad (x = L/2)$

$M_C = +\frac{wL^2}{8}$

Q. for Simply supported Beam with uniformly Variable load draw SFD & BMD.?



$\Sigma F_y = 0$

$R_A + R_B - \frac{1}{2} \times L \times w = 0$

$R_A + R_B = \frac{wL}{2} \quad -(i)$

$\Sigma M_B = 0$

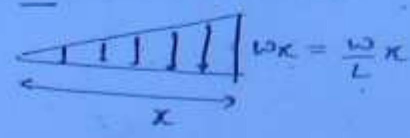
$R_A \times L - \left(\frac{1}{2} L \times w\right) \times \frac{L}{3} = 0$

$R_A = \frac{wL}{6} \quad -(ii)$

$R_A = wL$

S.F. in AB;

$$S_x = +R_A - \left(w \cdot \frac{x}{2}\right).$$



$$S_x = +R_A - \frac{1}{2}\cdot x \cdot \frac{w}{L}\cdot x$$

$$= \frac{wL}{6} - \frac{wx^2}{2L} \qquad (0 \le x \le L)$$

$$S_A = +\frac{wL}{6}$$

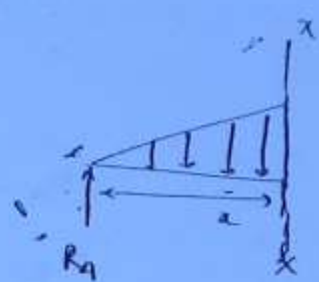
$$S_B = -\frac{wL}{3}$$

If S.F = 0

$$\frac{wL}{6} - \frac{wx^2}{2L} = 0$$

$$x = L/\sqrt{3}$$

B.M. in AB:



$$M_x = +R_A\cdot x - \left(\frac{1}{2}x\cdot\frac{w}{L}x\right)\cdot\frac{x}{3}$$

$$= \frac{wL}{6}\cdot x - \frac{wx^3}{6L}$$

$$M_A = 0$$

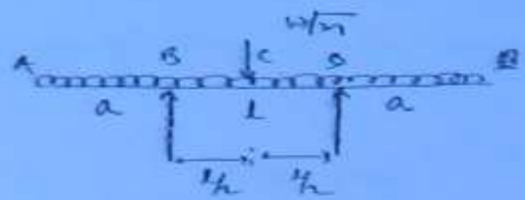
$$M_B = \frac{wL^2}{6} - \frac{wL^2}{6} = 0$$

$$M_c = \frac{wL}{6}\cdot\frac{L}{2} - \frac{wL^3}{48L}$$

$$= \frac{wL^2}{12} - \frac{wL^2}{48} = \frac{4wL^2 - wL^2}{48} = \frac{3wL^2}{48} = \frac{wL^2}{16}$$

M_{max} at $(x = L/\sqrt{3}) \Rightarrow M_{axm} = \frac{wL^2}{9\sqrt{3}}$

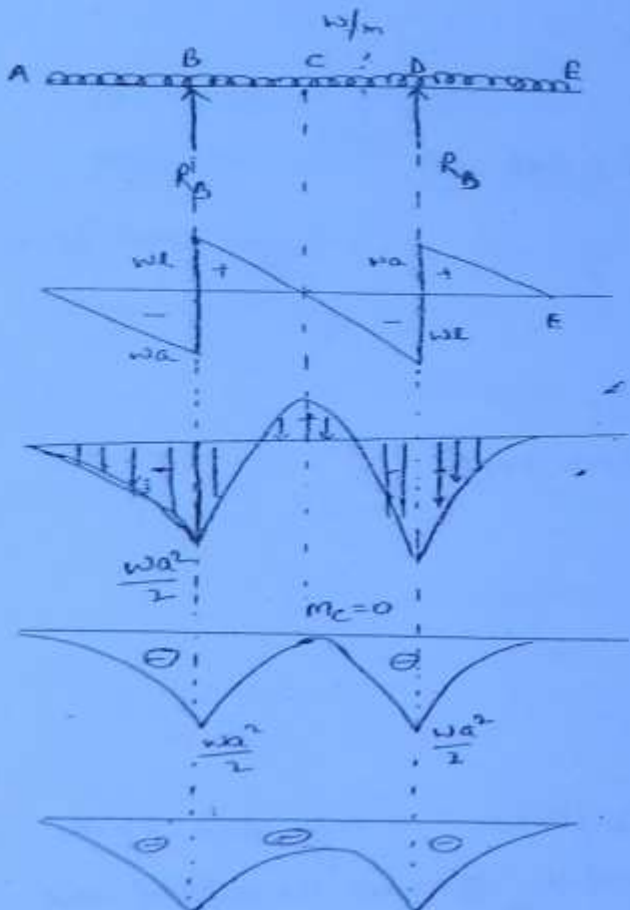
Q.



A simply supported Beam with equal overhang at either side as shown in fig carries udl over entire length if position of support is adjustable then draw S.F.D & B.M.D if

i) $l > 2a$ ii) $l = 2a$ iii) $l < 2a$, also find the Ratio of a/l for the Cond^n of Max^m B.M. to be min^m possible.

(i)



$$R = R_B = R_D = \frac{1}{2} w(l+2a)$$
$$= w\left(\frac{l}{2}+a\right)$$

S.F in AB

$$S_x = -(wx)$$
$$S_B = -wa$$

S.F in BCD,

$$S_x = -wx + R_B$$
$$= -wx + w\left(\frac{l}{2}+a\right)$$
$$S_B(\text{just to right of } B) = -wa + w\left(\frac{l}{2}+a\right)$$
$$= +\frac{wl}{2}$$
$$S_D = -\frac{wl}{2} \qquad x = (a+l)$$

S.F in DE.

$$S_x = -wx + R_B + R_D$$
$$= -wx + \frac{1}{2}w(l+2a)$$
$$S_D = +wa$$

Shape of S.F. curve will be same for all the ratio of (a/l).

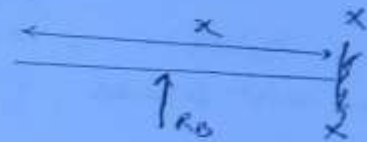
B.M. in AB.

$$B_x = -wx \cdot \frac{x}{2}$$

$$M_A = 0$$

$$M_B = -\frac{wa^2}{2}$$

B.M. for BCD



$$B_x = -wx \cdot \frac{x}{2} + R_B \cdot (x-a)$$

$$B_x = -\frac{wx^2}{2} + \cancel{\tfrac{1}{2}}\, w\left(a+\frac{l}{2}\right)(x-a) \quad \text{— (ii)}$$

$$M_c = \left(x = a+\tfrac{l}{2}\right) = -\frac{w\left(a+\frac{l}{2}\right)^2}{2} + w\left(a+\frac{l}{2}\right)\left(\cancel{a}+\frac{l}{2}-\cancel{a}\right)$$

$$= -$$

$$\boxed{M_c = \frac{w}{8}\left(l^2-4a^2\right)} \text{ S.p.}$$

Depending upon ratio $a:l$ there will be three cases

i) if $l>2a$, then $M_c = +ve$.

ii) if $l=2a$ " $M_c = 0$.

iii) if $l<2a$, $M_c = -ve$

NOTE I

i) There will be two points of contraflexure only when $l>2a$.

ii) The design B.M. will be greater of the B.M. at the support and B.M. at the centre.

iii) If B.M. at centre is zero then $l=2a$ hence %age overhange on each side is 25%.

iv) If supports are nearer to end A & E ordinate of B.M. at Centre will be much Higher. If supports are brought closer to each other then B.M. at centre reduces and B.M. at support increase.

It means cond^n of max^m B.M to be min will be achived when mag. of Sagging B.M at the centre is equal to Hogging Bending BM at support.

$$|M_C| = |M_B|$$

$$\frac{w}{8}(l^2 - 4a^2) = \frac{wa^2}{2}$$

$$\frac{w}{4}l^2 - wa^2 = wa^2$$

$$\frac{wl^2}{8} = wa^2$$

$$a = \frac{l}{2\sqrt{2}}$$

* If Position of supports is with respect to above cond^n then design of a prismatic cross. x cond^n for Bending will be most economical

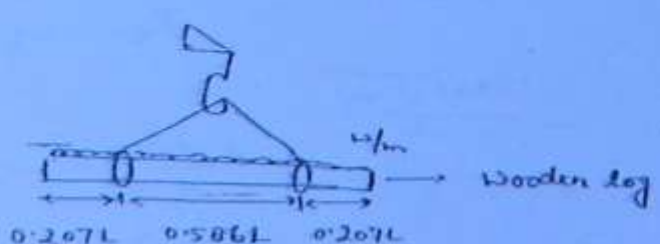
* If Total length of the Beam is $L = (l + 2a)$ then for most economic section %age overhang on each side will $(a/L \times 100)$

$$\% \text{ overhange} = \frac{a}{(l+2a)} \times 100$$

$$= \frac{a}{2\sqrt{2}a + 2a} \times 100$$

$$\% \text{ overhange} = 20.7\%$$

$$a = 0.207L$$

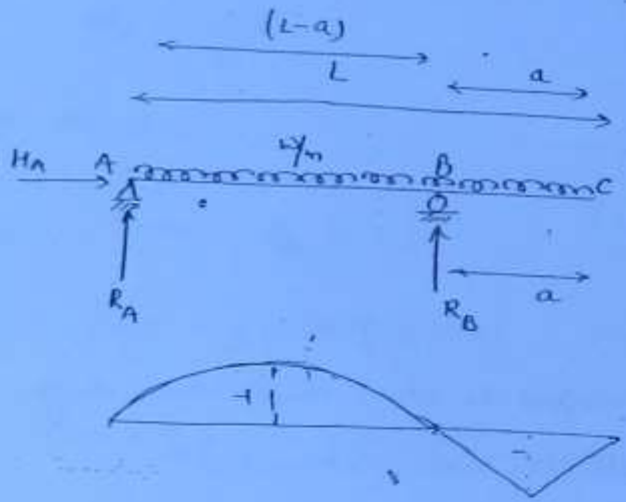


Safest Cond^n to lift because max^m B.M to be min^m

Q. — A Simply supported Beam which has one support at left and other support at a distance of a from right hand support as shown. If support B is movable then find the position of support B so that

Max^m B.M. to be min^m

Sol^n.



Cond^n:

1) Max^m B.M would be min^m when

Max^m sagging M at = max^m Hogging M at B

NOTE: Max^m sagging B.M. in AB at Point where S.F. changes sign.

$S_x = 0$.

~~S_x~~ Ans ⇒ $a = .293L$

$= 29.3\%$ of L.

S.F in AB.

$S_x = R_A$

S.F. in BC.

$S_x = R_A + R_B$

$\Sigma F_y = 0$

$R_A + R_B = wL$

$\Sigma M_B = 0$

$R_A(L-a) - wa \cdot a/_2 = 0$

~~$R_A L - R_a$~~ $R_A = \dfrac{wa^2}{2(L-a)}$ — (i)

$R_B = wL - \dfrac{wa^2}{2(L-a)}$ — (ii)

$\dfrac{-2a \pm \sqrt{b^2 - 4ac}}{2}$

$\dfrac{\cancel{2}\, w a^2}{\cancel{2}(L-a)} = wL$

$a^2 = (L-a)L$

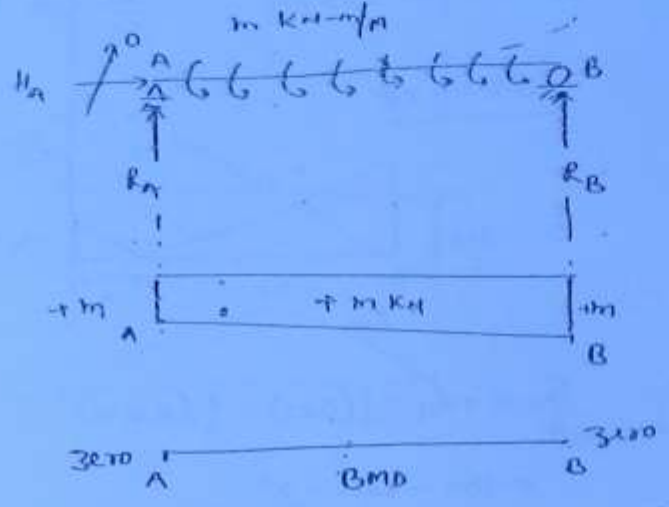
$a^2 = L^2 - aL$ $\quad$ $a^2 + aL - L^2 = 0$

$L^2 - aL - a^2 = 0$

$(L-a)(L+a)$

Q. For a simply supported Beam loaded with UDM as shown in fig. Draw S.F & B.M. diagram for simply supported Beam.

Sol:



$\Sigma F_y = 0$

$R_A + R_B = 0$ — (i)

$\Sigma M_B = 0$

$R_A \cdot L - mL = 0$

$R_A = +m$ kN

$R_B = -m$ kN

Shear Force in AB

$S_x = +R_A$

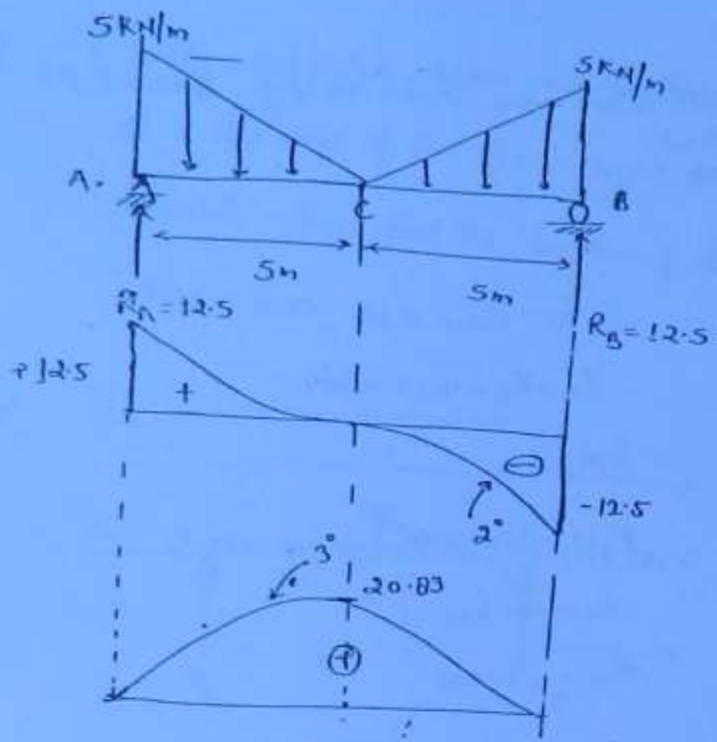
B.M. in AB

$M_x = +R_A \cdot x - mx$

$= mx - mx = 0$

Q. A simply supported Beam of length 10 m carry UVL for which load Intensity varies from a maximum of 5 kN/m at Both ends to zero at centre of the Beam. If it is desired to replace the above beam with another simply supported Beam which will be subjected to same maxⁿ B.M & Shear F. as in previous case then determine the length and rate of load for second to Beam. If U.DL is on entire length. Also Draw S.F.D and B.M.D for Both Cases.?

P.T.O



$R_A = R_B = \frac{1}{2} \times$ Total Downward Load

$= \frac{1}{2} \times 5 \times 5$

$= 12.5$ KN

S.F in CB

$S_x = +R_A - \frac{1}{2}(5\times5) - \frac{1}{2}(x \times x)$

$= 12.5 - 12.5 - \frac{x^2}{2}$

$S_x = -\frac{x^2}{2}$, 2° Parabolic curve

$S_C = 0$

$S_B = -12.5$

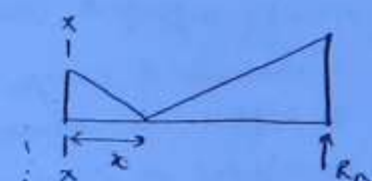
B.M. at centre ↦

$M_C = R_A \times 5 - \frac{1}{2}\times(5\times5)\times(\frac{2}{3}\times5)$

$= 20.83$ KN

S.F. in CA

S_x =



$S_x = -R_B + \frac{1}{2}(5\times5) + \frac{1}{2}x\cdot x$

$S_x = \frac{x^2}{2}$

$S_A = +12.5$

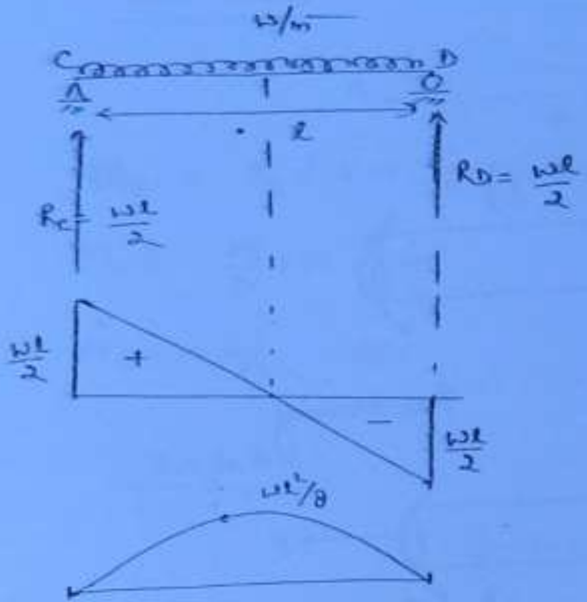
B.M. in CB

$B_x = R_A(5+x) - 12.5\left(\frac{10}{3} + x\right) - \frac{x^2}{2}\cdot\frac{x}{3}$

$= 62.5 + \cancel{12.5x}$

41.6

$\cancel{-12.5x} - \frac{125}{3} - \frac{x^3}{6}$



S_{max} in Beam AB = S_{max} in CD

$$12.5 = \frac{wl^2}{2}$$

$$wl = 25 \quad -(i)$$

Max. B.M in AB = Max B.M in CD

$$20.83 = \frac{wl^2}{8} \quad -(ii)$$

From (i) & (ii),

$$w = 3.75 \text{ kN/m}$$

$$l = 6.67 \text{ m}$$

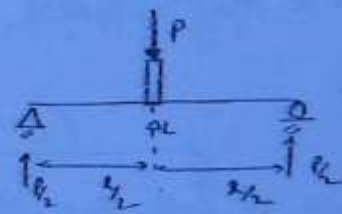
A simply supported Beam carry a concentrated load P appeied thru a column at the center of Beam. the column has width ΔL which is placed symmetrically about the centre. the S.F at the centre of Beam.

i) $P/2$

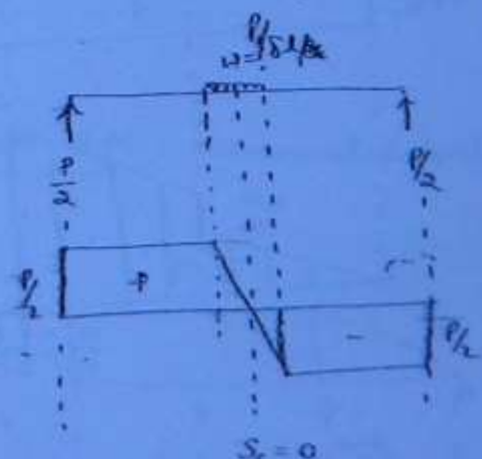
ii) 0

iii) P

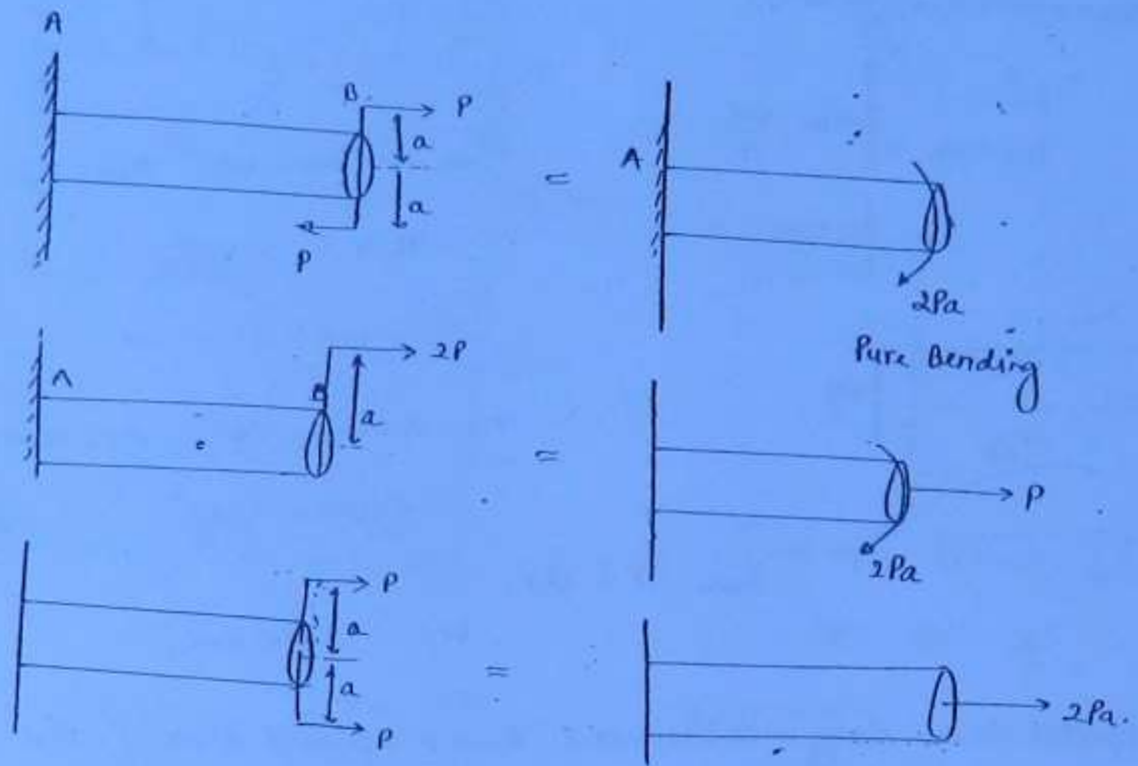
iv) 2P



Let. udl over length $\delta l = w = P/\delta l$

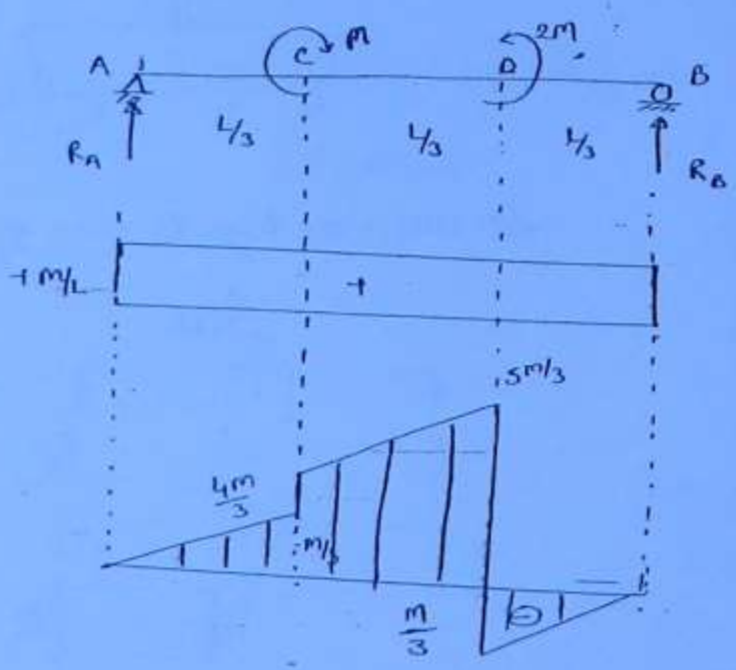


Effect of Couple :-



Q. For a sply supported Beam shown in fig draw SFD & BMD.

Sol.



$R_A + R_B = 0$ —(i)

$\Sigma M_B = 0$

$R_A \times L + M - 2M = 0$

$R_A = M/L$

$R_B = -M/L$

S.F. in AB

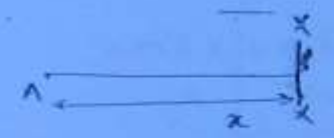
$S_x = +R_A = +M/L$

B.M in AC.

$M_x = +R_A \cdot x$

$M_A = 0$

B.M. in CD



$M_x = R_A x + M \quad \left(\frac{L}{3} \le x \le \frac{2L}{3}\right)$

$M_C = \frac{M}{3} + M = \frac{4M}{3}$

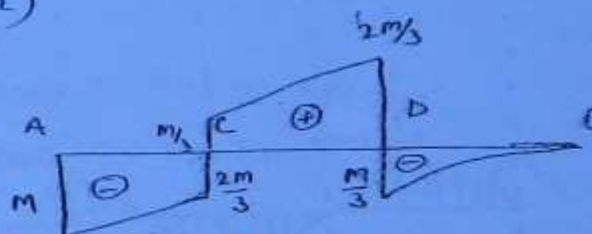
$M_D = 5M/3$

B.M. in DB

$M_x = R_A x + M - 2M$

$= \frac{M}{L} \cdot \frac{2L}{3} + M \qquad \left(\frac{2L}{3} \le x \le L\right)$

$M_D' = -M/3$



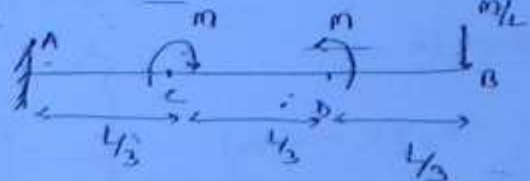
$M_B = \cancel{M} - \cancel{M} = 0$

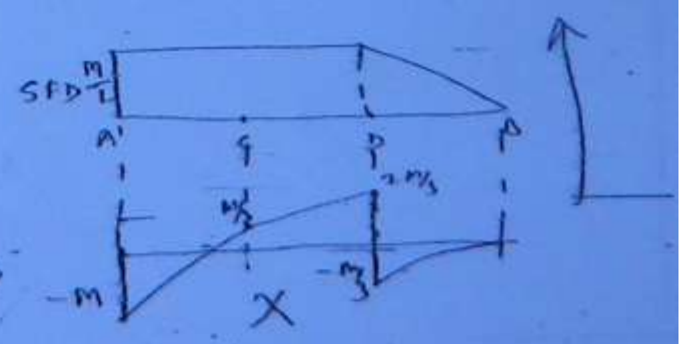
NOTE:-

If at a point a concentrated moment couple acts then at that point ordinate of B.M. suddenly changes by magnitude of that moment or couple.

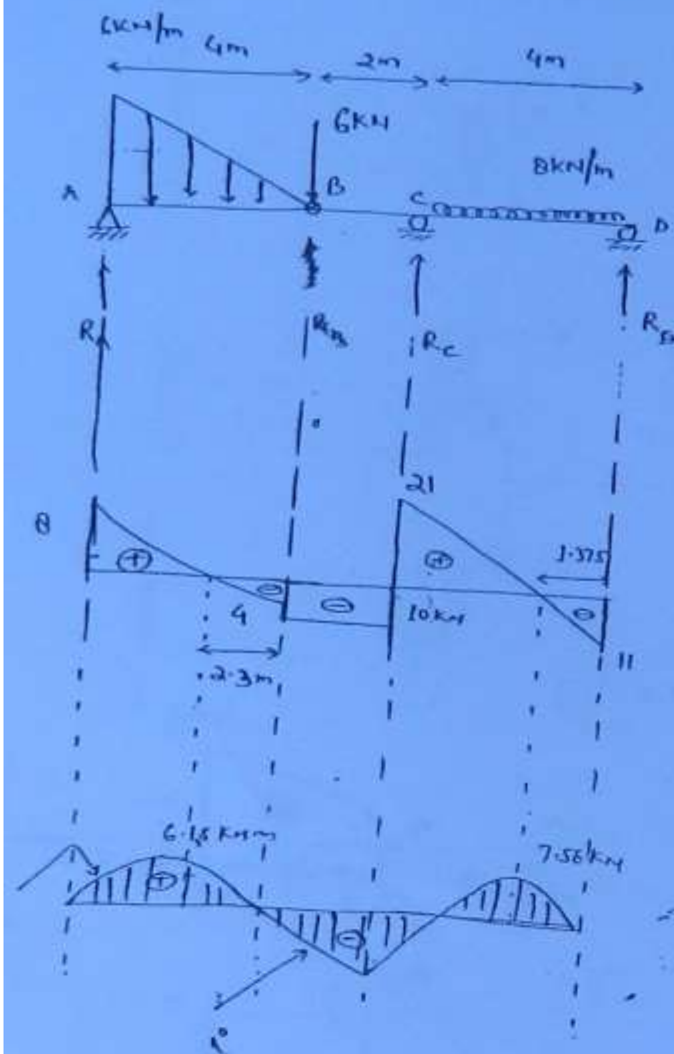
Ques.

Draw S.F and B.M.D and find Design B.M.. Concentrated load at B is M/L.





Q. For Loaded Beam shown in fig draw S.F.D and B.M.D



$\Sigma F_x = 0$

$H_A = 0$ — (i)

$\Sigma F_y = 0$.

$R_A + R_B + R_C + R_D = \frac{1}{2} \times 4 \times 6 + 6 + 8 \times 4$

$= 50\ kN$ — (ii)

$\Sigma M_B = 0$ (from left)

$R_A \times 4 - \frac{1}{2} \times 4 \times 6 \times \frac{2}{3} \times 4 = 0$

$R_A = 8\ kN$

$\Sigma M_D = 0$

$R_A \times 10 + R_C \times 4 + \frac{1}{2} \times 6 \times 4 \times \left(6 + \frac{2}{3} \times 4\right)$

$- 6 \times 6 - 8 \times 4 \times 2 = 0$

$R_C = 31\ kN$

$R_A = 8\ kN$

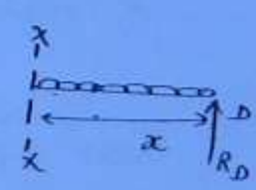
from eq (ii)

$R_D = 11\ kN$.

S.F in DC :

$S_x = -R_D + wx$

$= -11 + 8x$

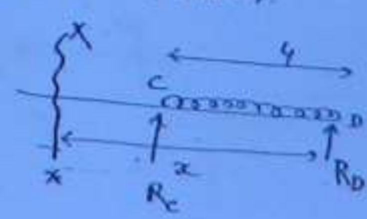


$S_D = -11\ kN$

$S_C = 21\ kN$

$S_x = 0 \Rightarrow 8x = 11 \Rightarrow 8 \cdot x = 1.375\ m$

S.F. in CB :



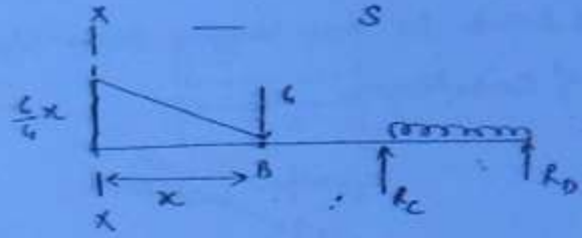
$S_x = -R_D + R_C + w(x_1)$

$= -11 + 32 - 31$

$= -10$

S

S.F. in BA.



$S_x = -R_D + 8\times4 - R_C + 6$

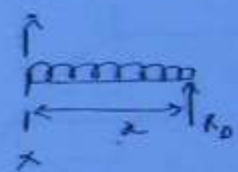
$+ \frac{1}{2}\times x \cdot \frac{6}{4}x$

$$\boxed{S_x = -4 + 0.75x^2}$$

$S_B = -4$

$S_A = +8$

B.M in DC:



Sagging

$M_x = +R_D x + wx\cdot\frac{x}{2}$

$= 11x - 4x^2$

$M_D = 0$

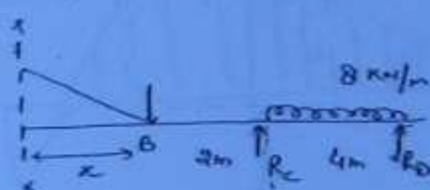
$M_C = 11\times4 - 4(4)^2$

$M_C = -20$ kN

$M_{1.375} = 11\times1.375 - 4(1.375)^2$

$= 7.56$ kN·m

B.M. in BA:



$M_x = +R_D(6+x) + R_C(2+x) - (8\times4)(4+x)$

$-6x - \frac{1}{2}x\cdot\frac{6}{4}x\cdot\frac{x}{3}$

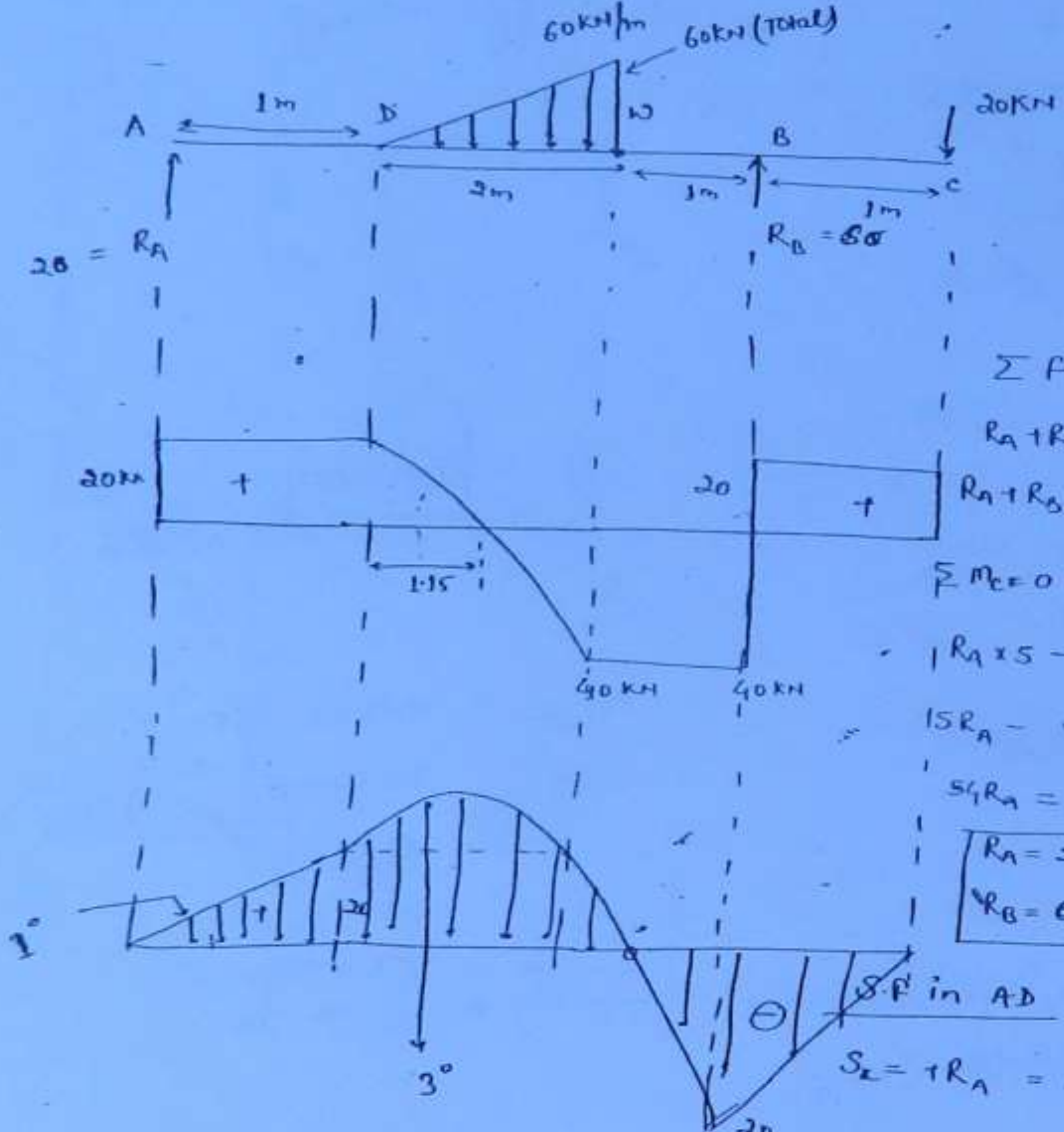
$M_x = 4x - \frac{x^3}{4}$

$M_B = 0$

$M_A = 16 - 16 = 0$

$M_{2.3} = 4\times2.3 - \frac{(2.3)^3}{4} = 6.15$ kN·m

1. Draw. B.M. and S.F.D. for over hanging beam shown in fig indicate significant values and point of contraflexure



$\frac{1}{2} \times 2 \times w = 60$

$w = 60$ KN/m

$\Sigma F_y = 0$

$R_A + R_B = 60 + 20$

$R_A + R_B = 80$ — (i)

$\Sigma M_C = 0$

$R_A \times 5 - 60 \times (2 + \frac{2}{3}) + R_B = 0$

$5R_A - 180 + R_B = 0$

$4R_A = 80$

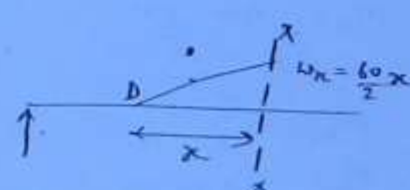
$\boxed{R_A = 20 \text{ KN} \\ R_B = 60 \text{ KN}}$

$R_A = 20$
$R_B = 60$

S.F in AD

$S_x = +R_A = 20.$

S.F. DE



$S_x = R_A - \frac{1}{2} \times x \times 30x$

$= 20 - 15x^2$

$S_D = 20$

$S_E = 20 - 15 \times 4$
$= -40$

B.M. in AD :-

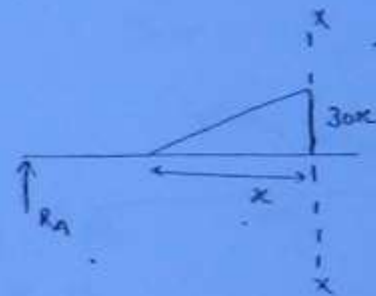
$M_x = R_A \cdot x$

$= 20x$

$M_A = 0$

$M_D = 20$ kN-m

B.M. DE :-



$M_x = R_A(1+x) - \frac{1}{2} x \cdot 30x \cdot \frac{x}{3}$

$= 20 + 20x - 5x^3$

$M_D = 20$

$M_E = 20 + 40 - 5 \times 8$

$M_{max} (x = 1.15) = 20 + 20 \times 1.15 - 5(1.15)^3$

$= +35.4$

B.M. in E

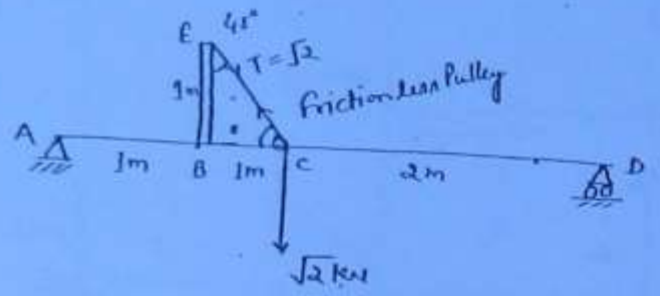
Design B.M = 35.4 KN

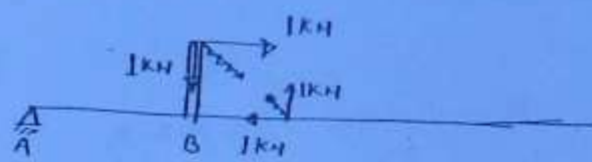
Design S.F = 40 KN

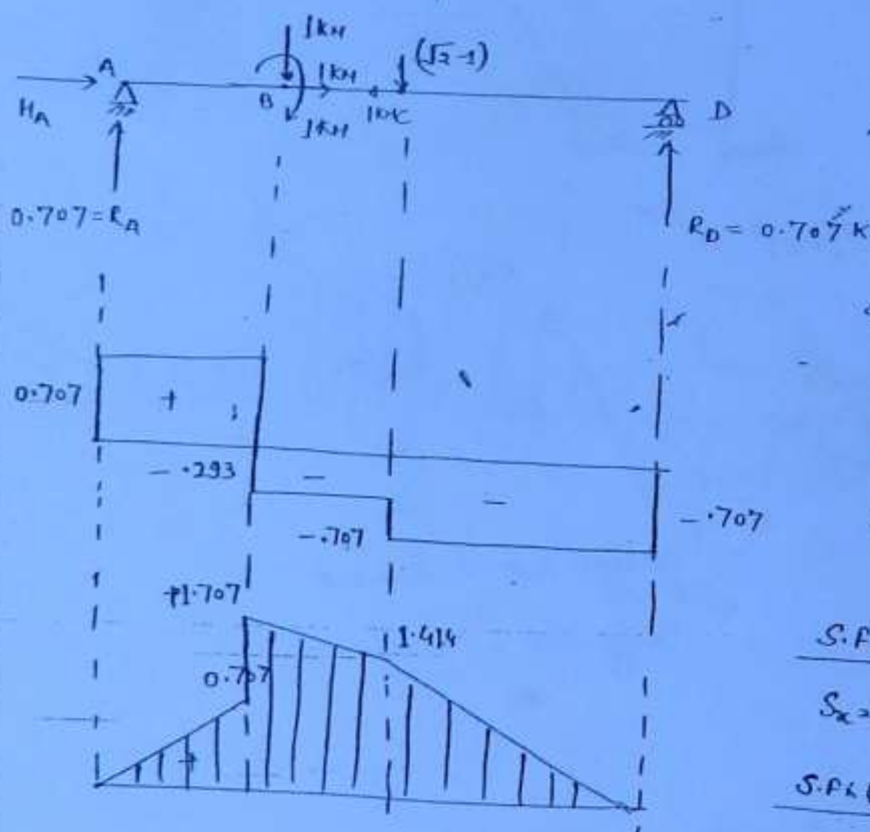
1

A Beam ABCD is hinged at A and simply supported at D a vertical strut BE which is 1 m long is fixed at B. AB is 1m and a frictionless pully is attached at C 2 m from A, A flexible string carry a load of $\sqrt{2}$ KN. and passes over the pully as show in fig. Draw, SFD, BMD and Axial thrust diagram?

d^n







$R_A + R_D - 1 - (\sqrt{2}-1)$

$R_A + R_D = \sqrt{2}$ — (i)

$\Sigma M_D = 0$

$R_A \times 4 + 1 - 1 \times 3 - (\sqrt{2}-1) \times 2 = 0$

$R_A = \frac{2\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$R_B = 1/\sqrt{2}$

S.F. in AB

$S_x = +R_A$

S.F. in B.C

$S_x = +R_A - 1 = 0.707 - 1 = -.293$

S.F in CD

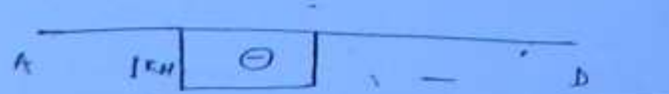
$S_x = R_A - 1 - (\sqrt{2}-1) = 0.707 - 1.414$

$= -0.707$

$x_1 = 0$

$F_{x_2} = -1$

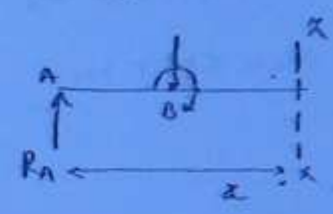
$F_{x_3} = 0$



B.m. in AB

$M_x = R_A x$

B.m. in BC



$M_x = R_A x + 1 - 1(x-1)$

$= 0.707x - x + 2$

$M_B = 1.0\ 1.707$

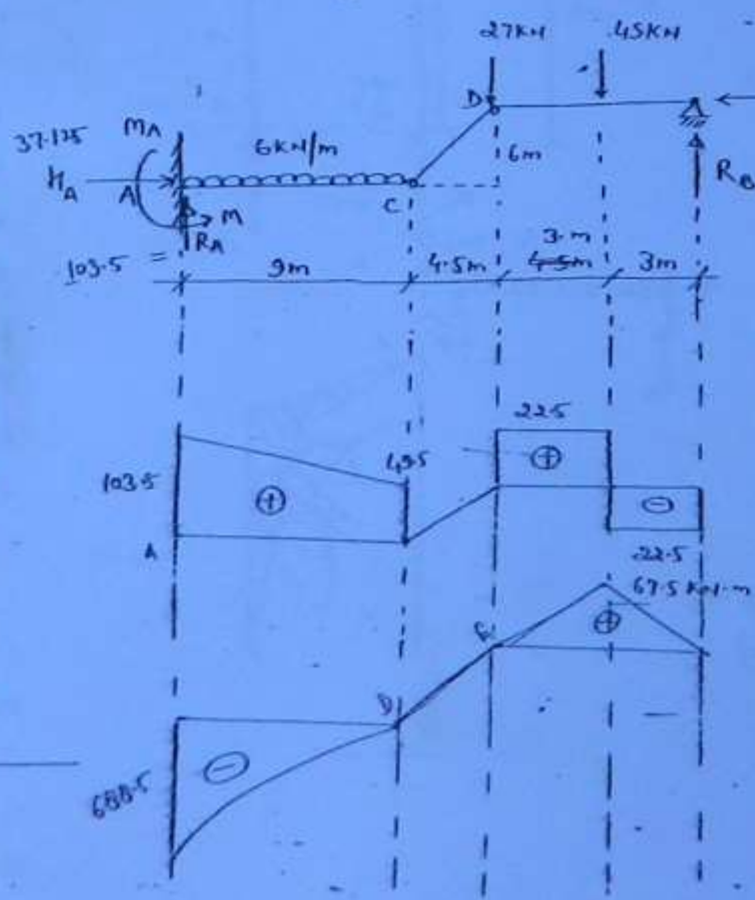
B.m in CD

$M_x = R_A x + 1 - 1(x-1) - (\sqrt{2}-1)(x-2)$

$M_D = .707 \times 4 + 1 - 1(3) - (\sqrt{2}-1)2$

$= 0$

Ques. ES Determine the R^{xn} at fixed support A and hinged support B, joint C and D are also hinged. ~~Determine the~~ Rxn comp. Also S.F.D and B.M.D.



$\Sigma F_x = 0$

$H_A - H_B = 0 \Rightarrow H_A = H_B$ —(i)

$\Sigma F_y = 0$

$R_A - 54 + 27 - 45 + R_B = 0$

$R_A + R_B = 126$ KN —(ii)

$\Sigma M_D = 0$ (from Right)

$R_B \times 6 - 45 \times 3 = 0$

$R_B = 22.5$ KN

$\Sigma M_c = 0$ (from Right)

$R_B \times 10.5 + H_B \times 6 - 45 \times 7.5 - 27 \times 4.5 = 0$

$\Sigma M_c = 0$ (from left)

$R_A \times 9 - M_A - 6 \times 9 \times 4.5 = 0$

$103.5 \times 9 - 6 \times 9 \times 4.5 = M_A$

$M_A = 688.5$ kN.

S.F in AC.

$S_x = +R_A - 6x$

$= 103.5 - 6x$

$S_A = 103.5$

$S_C = 103.5 - 6 \times 9$

$= +49.5$

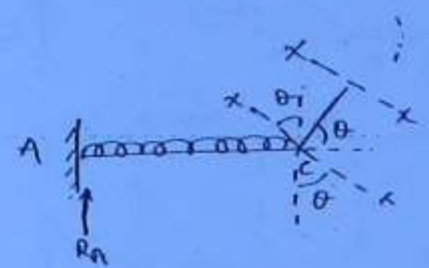
S.F. in DE.

$S_x = R_A - 6 \times 9 - 27 = 103.5 - 54 - 27 = 22.5$

S.F in EB:

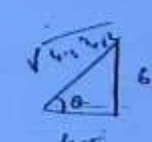
$S_x = R_A - 6 \times 9 - 27 - 45 = -22.5$

S.F in CD:



$S_x = R_A \cos\theta - 9 \times 6 \cos\theta - H_A \sin\theta$

$= (103.5 - 54)\cos\theta - 37.125 \sin\theta$



$= \dfrac{49.5 \times 4.5}{\sqrt{4.5^2+6^2}} - \dfrac{37.125 \times 6}{\sqrt{4.5^2+6^2}}$

$= \dfrac{1}{\sqrt{4.5^2+6^2}}\left(49.5 \times 4.5 - 37.125 \times 6\right)$

$= 0$

B.Moment eq^n for a the Beam :

B.m. in AC :-

$$M_x = +R_A x - M_A - 6x \cdot \frac{x}{2}$$

$$= 103.5x - 688.5 - 3x^2$$

$M_A = -688.5$

$M_C = 103.5\times9 - 688.5\times9 - 3\times9^2$

$M_C = 0$

B.m. in DE :-

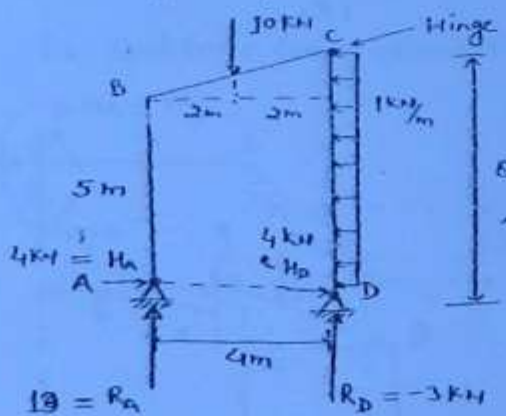
$$M_x\ (x \text{ from } B) = R_B \cdot x = 22.5x$$

$M_E = 22.5\times3 = 67.5$

$M_B = 0$

Ques:-> For the portal frame shown in fig, draw B.m.D. ?

Sol^n:->



$\Sigma F_x = 0$

$H_A + H_D = 1\times8$ — (i)

$\Sigma F_y = 0$

$R_A + R_D = 10$ — (ii)

$\Sigma M_D = 0$

$R_A\times4 - 10\times2 - (1\times8)\times4 = 0$

$R_A = 52/4 = 13$ KN

$R_D = -3$ kN [from eq^n (ii)]

$M_C = 0$

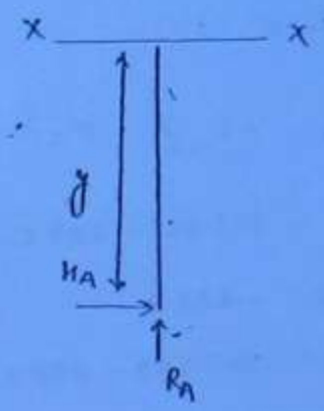
$R_A\times4 - H_A\times8 - 10\times2 = 0$

$H_A = 4$ KN

$H_D = 4$ KN [from eq (i)]

Consider outer face is reference face

B.M in AB :→



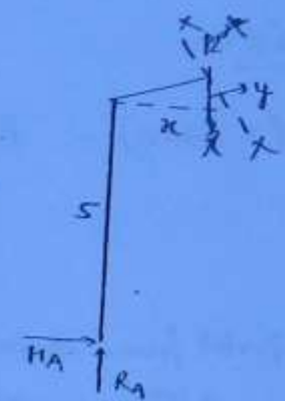
$M_x = -H_A \cdot y$

$M_A = 0$

$M_B = -H_A \times 5$

$= -4 \times 5 = -20$ kN-m

B.M. in BE,



$M_x = R_A \cdot x - H_A(5+y)$. [x & y are taken from B]

$= 13x - 4(5+y)$

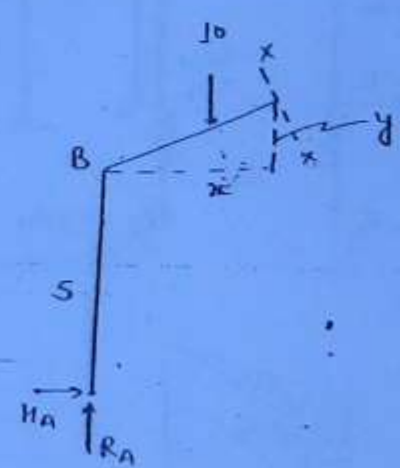
$M_B = -20$

$M_E = 13 \times 2 - \cancel{8}4(5+1.5)$

$= 26 - 20 - 6$

$= 0$

B.M in EC,

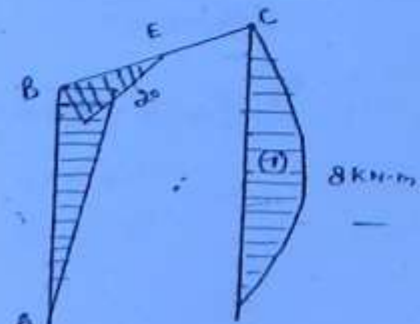


$M_x = R_A x - H_A(5+y) - 10(x-2)$

$= 13x - \cancel{20} - 4y - 10x + \cancel{20}$

$M_x = 3x - 4y$

$= 4y - 4y = 0$



B.M in DC.

$$M_x = +4y - 1\times y \cdot y/_2$$

$$= 4y - \frac{y^2}{2}$$

$$M_D = 0$$

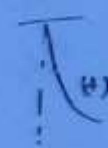
$$M_C = \cancel{16}\ 32 - \frac{8^2}{2}$$

$$= 0$$

If M is max

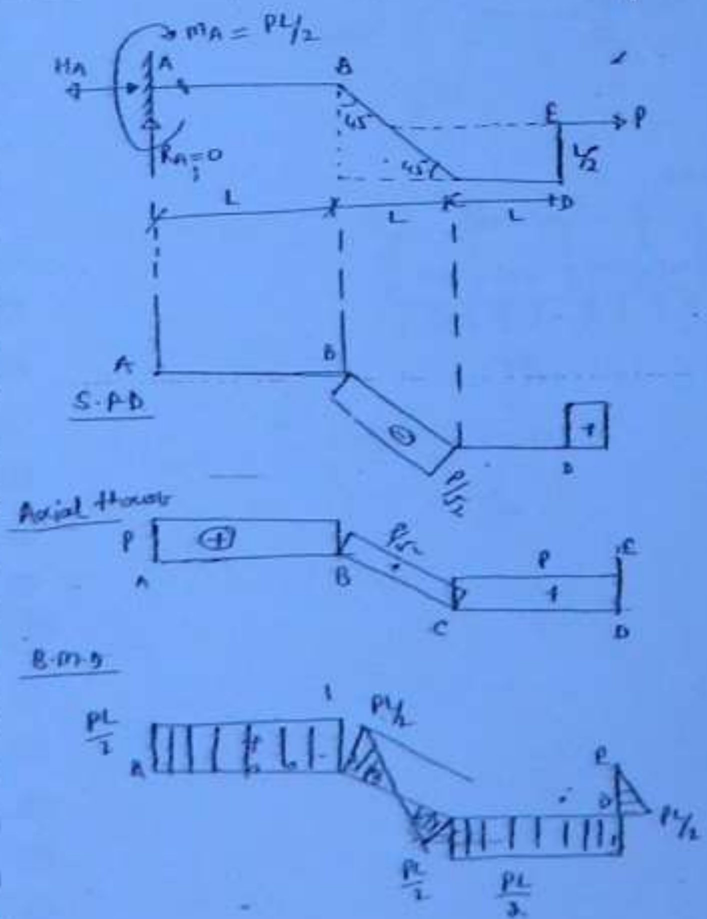
$$\frac{dM}{dx} = 0 \Rightarrow 4 - \frac{2y}{2} = 0$$

$$y = 4m$$



$$M_D = 16 - 8 = 8\ KN\text{-}m$$

Que:- For Cantilever frame shown in fig Draw S.F.D, B.M.D and axial thrust diagram ?



$$\Sigma F_x = 0$$

$$H_A = P \quad —(i)$$

$$\Sigma M_A = 0$$

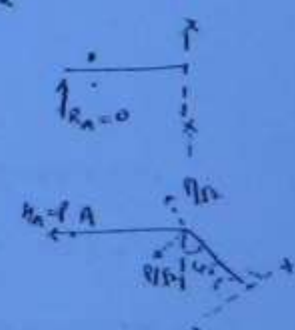
$$P\times L/_2 - M_A = 0$$

$$M_A = PL/_2$$

Consider top face as reference face

S.F in AB

$$S_x = 0$$



S.F in BC

$$S_x = -P/\sqrt{2}$$

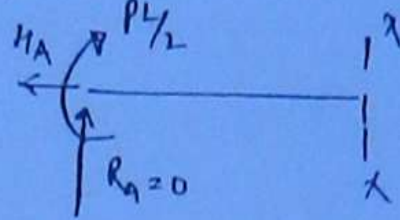
S.F in CD

$$S_x = 0$$

S.F in ED

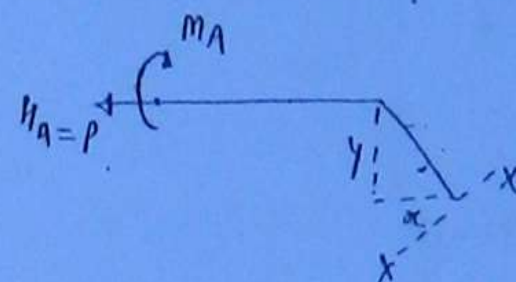
$$S = +P$$

B.M. in AB.



$$M_x = +\frac{PL}{2}$$

B.M in BC



$$M_x = -P_y + \frac{PL}{2}$$

$$= -P \cdot y + \frac{PL}{2}$$

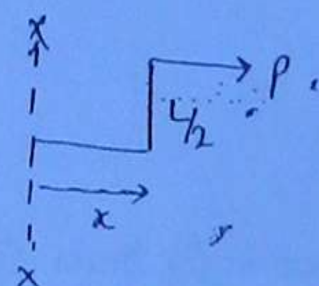
$$M_B = PL/2.$$

$$M_C = -PL/2$$

B

B.M. CD.

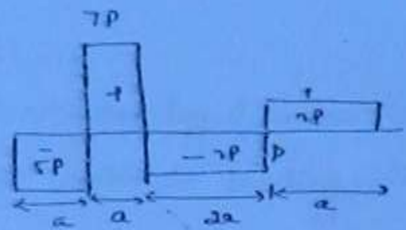
$$M_x = -\frac{PL}{2}$$

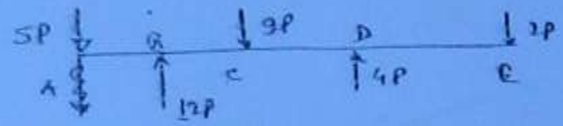


$$M_x = -\frac{PL}{2}$$

Pg. 30

13





$m_B = -5Pa$

$m_C = \cancel{-5Pa + 12P}$

$= -5P \times 2a + 12P \times a = +2Pa$

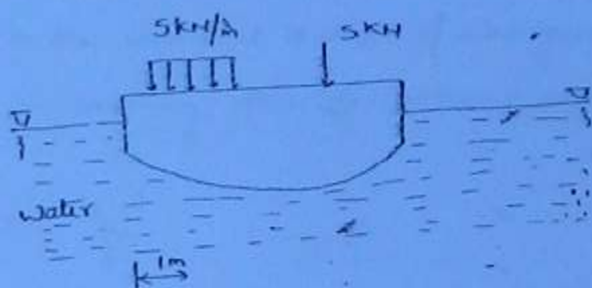
$m_D = -2Pa$

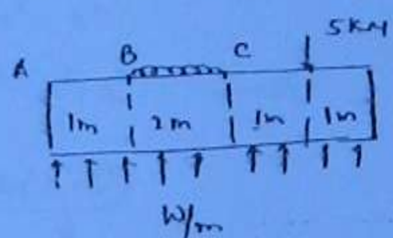
Pg. (20-27)

15

(A)

(17)





$W \times 5 = 5 \times 2 + 5$

$W = 3 \, KN/m$

$S_A = 0$

$S_B = 3 \times 1$

$S_C = 3 \times 3 - 5 \times 2 = -1$

$S_D = 3 \times 4 - 5 \times 2 = +2$

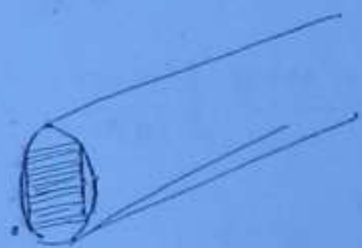
$S_{D(R)} = 3 \times 4 - 5 \times 2 - 5 = -3$

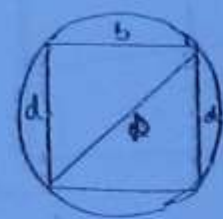
$S_E = 0$

(18)

ES 2011.

A circular log of timber has dia D. determine the dimensions of strongest rectang-ular section in Bending which can be derived from this log.





Let b is the width d is depth of strongest rectangular section if it is strongest in bending then its section modulus (Z) should be max^m

$$Z = \frac{bd^2}{6} \quad \text{— (i)}$$

$$b^2 + d^2 = D^2$$

$$d^2 = D^2 - b^2 \quad \text{— (ii)}$$

$$Z = \frac{b(D^2 - b^2)}{6}$$

$$Z = \frac{D^2 b - b^3}{6}$$

for Z_{max^m} ; $\frac{dZ}{db} = 0$.

$$\frac{D^2 \cdot b - 3b^2}{6} = 0$$

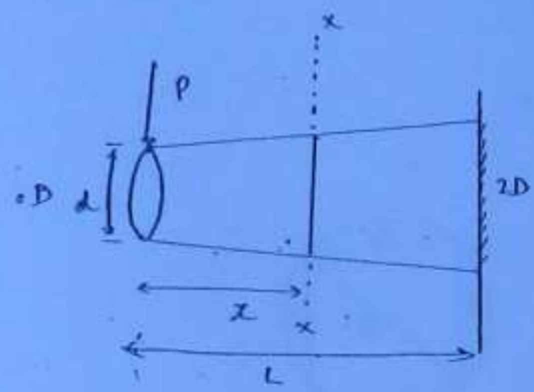
$$\boxed{b = \frac{D}{\sqrt{3}}}$$

$$d^2 = D^2 - \frac{D^2}{3} = \frac{2D^2}{3} \Rightarrow \boxed{d = \sqrt{\frac{2}{3}}\, D}$$

2.

A Cantilever beam of length L having circular tapering section of dia of D at free end and 2D at fixed end. A Conc^n load P is applied at the free end. Find at what distance from free end max^m bending stresses will occur. Also calculate magnitude of max^m bending stresses?

Sol^n:



$\sigma = M/Z$

Let at a distance x from a Bending stress are max^m

$M_x = -P \cdot x$

B. stress at Top of x-x.

$$\sigma = \frac{M}{Z} = \frac{P \cdot x}{\frac{\pi}{32} \cdot D_x^3} \quad -(i)$$

$$Z = I/y_{max} = \frac{\pi/64\, D^4}{D/2} = \frac{\pi}{32} D^3.$$

$$D_x = D + \left(\frac{2D - D}{L}\right) x$$

$$D_x = D + \frac{D}{L} \cdot x$$

$$D_x = \frac{D}{L}(L + x) \quad -(ii)$$

from eq^n (i)

$$\sigma = \frac{P \cdot x}{\frac{\pi}{32}\left[\frac{D}{L}(L+x)\right]^3}$$

$$\sigma = \frac{32PL^3}{\pi D^3}\left[\frac{x}{(L+x)^3}\right]$$

for σ_{max},

$$\frac{d\sigma}{dx} = 0$$

$$\frac{32PL^3}{\pi D^3}\left[\frac{(L+x)^3\cdot 1 - x\cdot 3(L+x)^2}{(L+x)^6}\right] = 0$$

$$x = L/2.$$

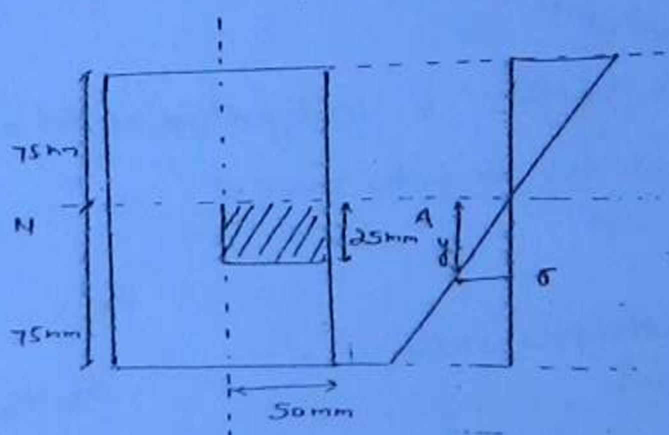
$$\sigma_{max} = \frac{32PL^3}{\pi D^3}\left[\frac{L/2}{27L^3/84}\right] \qquad \frac{L}{2}\times\frac{284}{27L^{3}2} \qquad 128$$

$$\sigma_{max} = \frac{128\,PL}{27\pi D^3}$$

Q.

A Beam with rectangular cross section given below is subjected to sagging B.M. causing compression at the top of 16 kNm. Find the tension force which will develope on the shaded area shown in fig.

Sol^n-



a) 5 KN

b) 8.9 KN

c) 17.8 KN

d) 10 KN

Bending stress at a distance y from N.A.

$$\sigma = \frac{M}{I}\cdot y$$

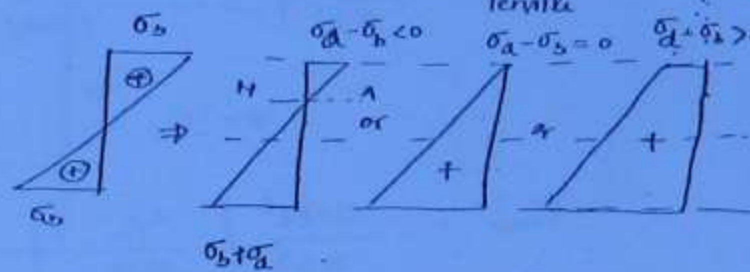
$$= \frac{16\times10^6}{\frac{100\times100^3}{12}}\times 25 \Rightarrow \sigma = 14.2\,N/mm^2$$

σ_{avg}

Combined Effect (Axial and Bending) :-

$\sigma_d = P/A$

Bending Stress $\sigma_b = \frac{M}{Z} = \frac{M}{bd^2/6}$



NOTE :

In pure bending with in elastic limit, nutral axis passes thru centroid. But under combined effect of axial force and Bending moment Neutral axis may shift its position. In above case N.A. shifts upward. which may lie inside or outside of the corr x. section depending upon mag of σ_b & σ_d.

In above case if σ_d (direct stress) $= +6 \text{ N/mm}^2$ & Bending stress at top and bottom $\sigma_b = 7 \text{ N/mm}^2$. Then nutral axis lie in the zone of.

i) at centroidal axis
ii) above " " within the section
iii) below " " " " "
iv) outer side the section

$\sigma_d - \sigma_b < 0$

Arrang

Arrange the following x-section in decreasing order of there flexural strength. If all have equal area of cross section and same metal.

1) circular Section
ii) Rectangular Section
iii) Square Section
iv) I Section

$Z_I > Z_R > Z_S > Z_C$

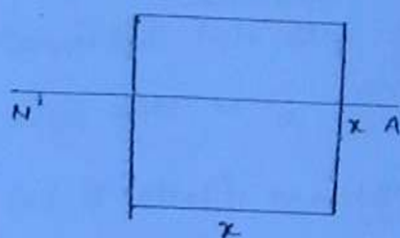
$\therefore$ Bending strgh $\propto Z$

iv) > ii > iii > i)

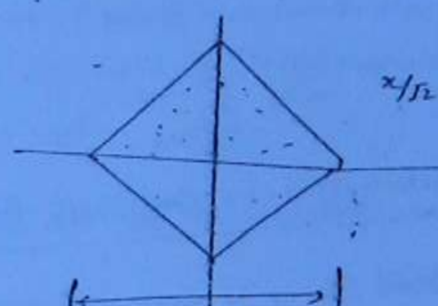
NOTE: In an I section & 80-90% B.M. is resisted by flanges and only 10-20% moment is resisted by web.

Q. Prove that for a given material the moment of resistance of a square section when placed with Horizontal & vertical sides is 41.4% greater that moment of resistance of the same beam when placed with horizontal an vertical diagonal ?

Sol:



(1)



$$Z_1 = \frac{bd^2}{6} = \frac{x^3}{6}$$

$$Z_2 = \frac{I_2}{y_{max}} \quad ; \quad I_2 = 2 \cdot \frac{bh^3}{12} \qquad b = x\sqrt{2}, \quad h = x/\sqrt{2}$$

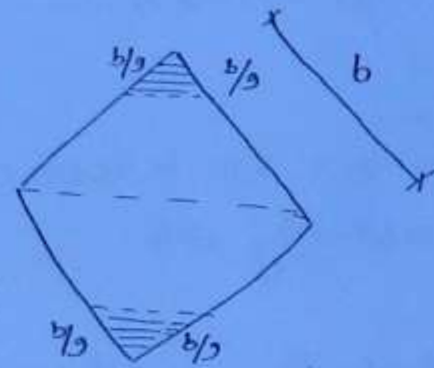
$$Z_2 = \frac{x^4}{12} \frac{\sqrt{2}}{x} = \frac{x^3}{6\sqrt{2}}$$

$$\frac{Z_1}{Z_2} = \frac{x^3/6}{x^3/6\sqrt{2}} = 1.414 \rightarrow \frac{Z_1}{Z_2} - 1 = 0.414$$

$$\% \text{ increase of } Z_1 = \frac{Z_1 - Z_2}{Z_2} \times 100 = 41.4\%$$

NOTE:

1) A Beam of square section placed with diagonals at top and bottom (Ver. & horizonta or diamond section) can be made more strong by cutting of top and bottom edges as shown in fig the max^m increase in bending strength (Z) will be 5.35% when the cutt-off portions are b/9 as shown.

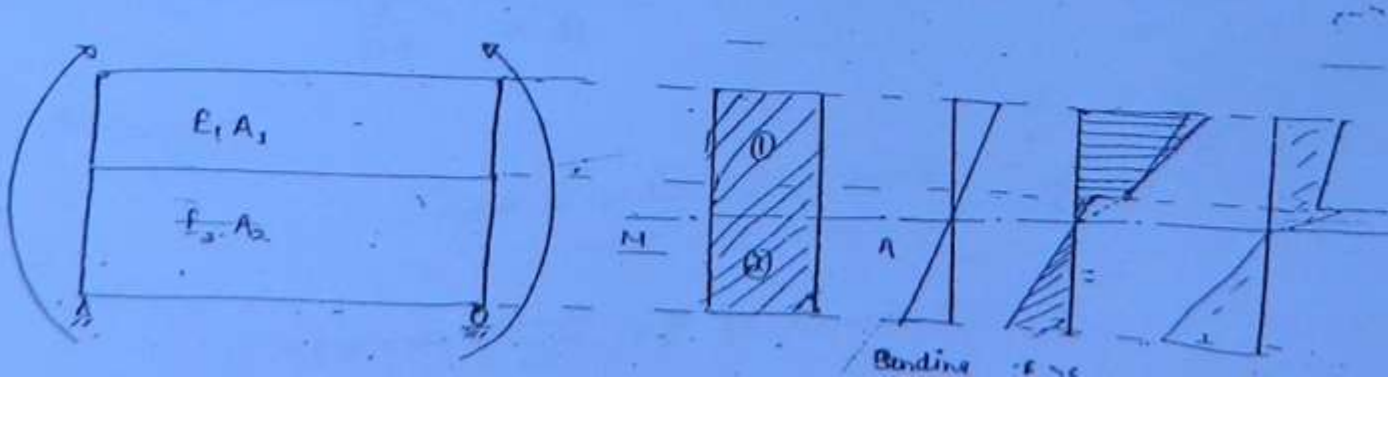


A circular section of diameter D can be made more strong by cutting of top and bottom corners the max^m increase in Z will be 0.7% when cutt-off portion is 0.017D $\delta = 0.011D$

$$Z = I/y_{max}$$

Bending Stress distribution in Composite Beam:→

A Composit beam is made of two or more metals jointed firmly to each other. there will be a common centriod and common N.A. about which beam will bend. The total moment of resistance will be summation of moment of resistance offered by both section



To find common centroid of combined Beam or N.A. use transformed area method. i.e. imagine an equivalent area made of one single metal either metal ① or ② s.t. it becomes homogeneous.

The equivalent section will have same m.o. resistance as composite section

$\sigma = \epsilon E$

$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = m$ modular ratio

In transformed area method normally depth 'd' is kept const. and width 'b' is changed.

Let E_s young's mod. of steel.

E_w " " " wood

Then modular ratio is $\frac{E_s}{E_w} = m$

if a composite beam is made of steel and wood then if wood is converted into steel by transformed area method then width of steel will be $\rightarrow$ $\frac{\text{width of wood}}{m}$

But if steel is converted into wood then equivalent wooden width $= mW_s$

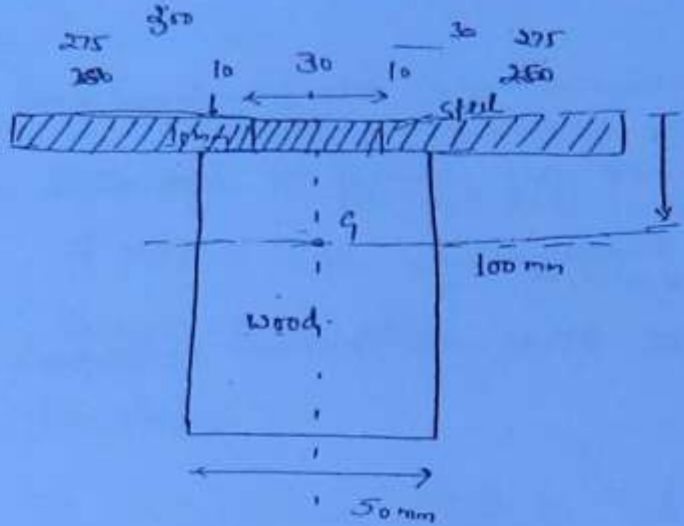
Q. A timber beam of rectangular x-section 100 mm x 50 mm as shown in fig has (30 x 10) mm steel securely fixed to the top of surface symmetrically about vertical axis. The depth of centroid of equivalent timber beam from the top surface will be. [given m = 20].

a) 5 mm

b) 30 mm

c) 15 mm

d) Can not be predicted



$mb = 600$

$$\frac{E_s}{E_w} = m = 20$$

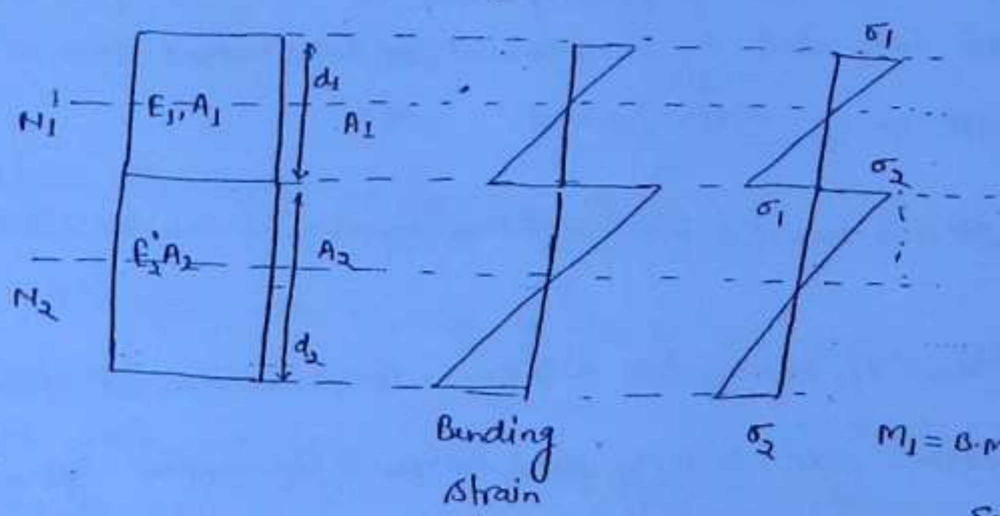
$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$= \frac{(10\times600)\times5 + 50\times100\times60}{600\times10 + 50\times100}$$

$$= 30\text{ mm}$$

NOTE :-

Bending stress distribution in Beams having more than one metals Placed one over the other without bond b/t them.



$\sigma_1 = M_1/Z_1$

$\sigma_2 = M_2/Z_2$

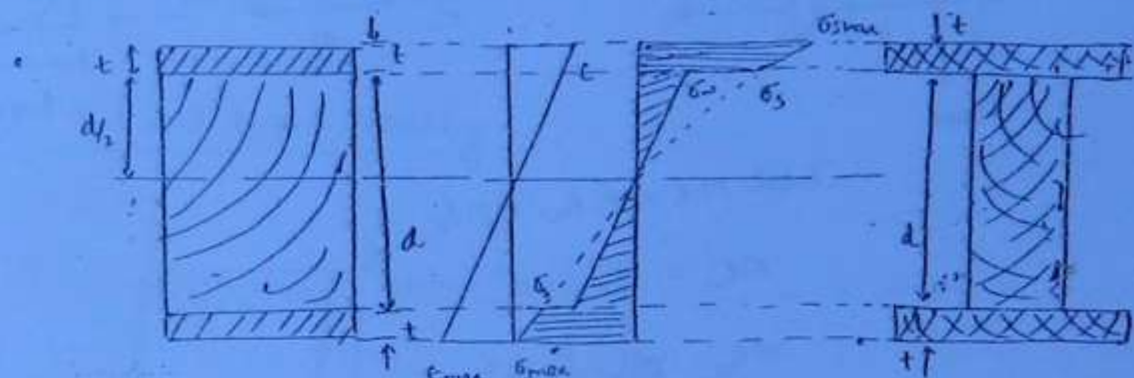
M_1 = B.M resisted by Section ①

M_2 = B.M resisted by Section ②

Flitched Beam:-

It is a composit Beam made of wood and metal jointed firmly to strengthen wooden section. the flitching is done symmetrically either on top and bottom or at sides.. flitching is required to avoid large section of wood for equal moment of resistance.

CASE (1) Top and bottom flitched Beam:→



$m = \frac{E_s}{E_w} = \boxed{10 \text{ to } 30}$

$\sigma_w = \epsilon E_w$

$\sigma_s = \epsilon E_s$

$B_{ew} = mb$

$\frac{\sigma_s}{\sigma_w} = \frac{E_s}{E_w} = m.$

Let σ_w is permissible stress in wood.

The moment of resistance of composite placed section is = moment of resi. of wooden section + " " " steel "

$$\boxed{\frac{\sigma_{Smax}}{\sigma_s} = \frac{d/2 + t}{t}}$$

$$\boxed{MR = MR_w + MR_s}$$

$$M \cdot R_w = \sigma_w \times \frac{b \cdot d^2}{6}$$

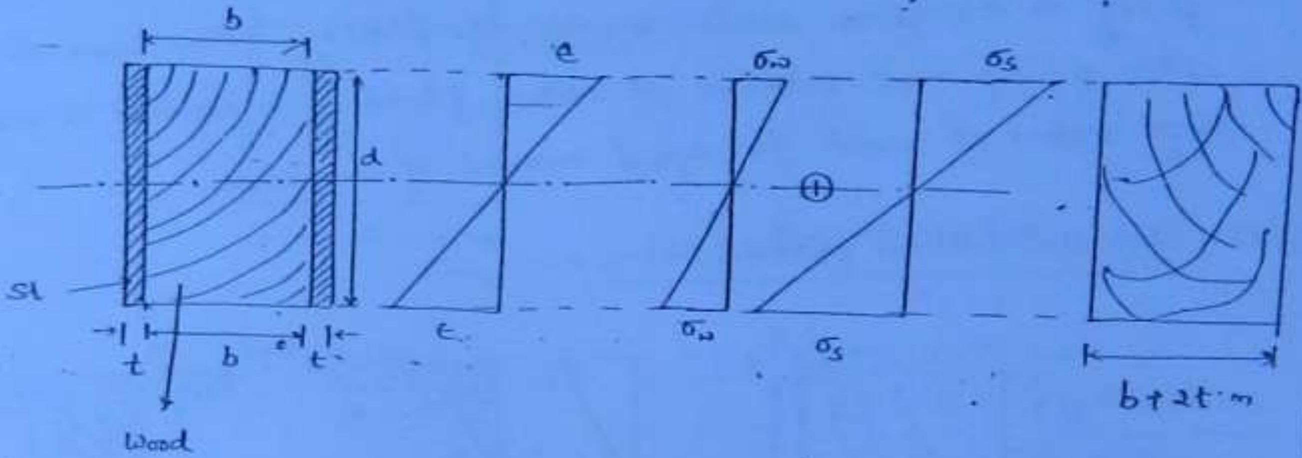
Moment of Res_{Steel} = m.R of steel of size $b \times (d+2t)$ − m.R. of steel of size $(b \times d)$

$$= \sigma_{Smax} \frac{b(d+2t)^2}{6} - \sigma_s \times \frac{bd^2}{6}$$

The equivalent homogeneous section made of either wood or of steel would have the shape of an I-Section shown below.

SE (i) Side Flitched Beam :→

Side Flitching :→



or

Total $M\cdot R = M\cdot R_w + MR_s$

$$MR_w = \sigma_w \times \frac{bd^2}{6}$$

$$MR_s = \sigma_s \times 2t\cdot\frac{d^2}{6}$$

$$MR = \sigma_w \times \frac{bd^2}{6} + \sigma_s\cdot 2t\cdot\frac{d^2}{6}$$

$$= \sigma_w\cdot\frac{d^2}{6}\left[b + \frac{\sigma_s}{\sigma_w}\cdot 2t\right] = \frac{\sigma_w d^2}{6}(b+2t\cdot m)$$

NOTE:

1. If a steel section is converted into a wood then equivalent wooden section will be made of rectangle having width $(b+2t\cdot m)$ and If equivalent section is made of steel then equit width ⇒ $(b/m + 2t)$.

NOTE:

Top and bottom flitched Beam is 3 to 5 times stronger than side ~~flitched~~ flitched beam. because in side flitched Beam the area of matl near the neutral axis is not fully stressed

Q. A timber beam 150 mm and 200 mm deep is to reinforced by bolting 2 steel flitches each of 150 mm wide and 12.5 mm thick. Find moment of resistance of the composite section when flitches are attached symmetrically

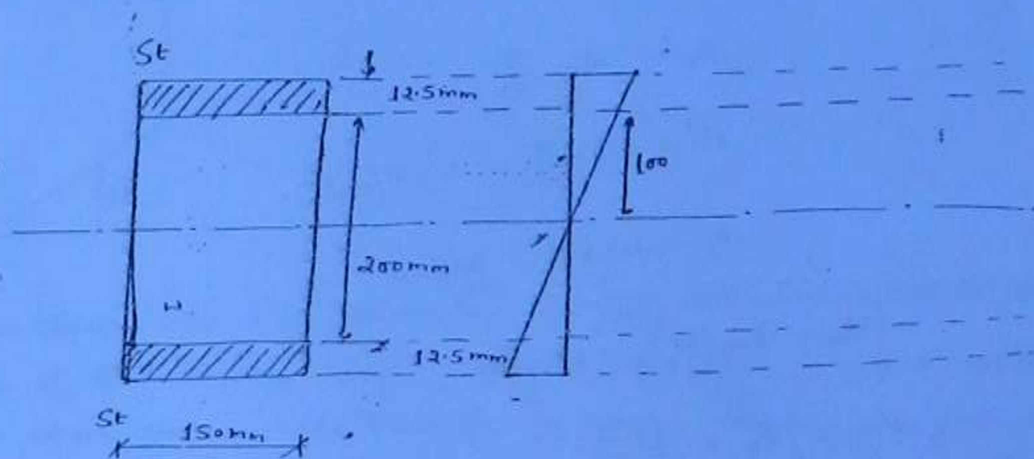
i) AT top and bottom.

ii) At sides.

given that max permissible stress in wood is 6 N/mm² and

$m = 20$ $\qquad \sigma_w = 6 \text{ N/mm}^2$

Solⁿ:- CASE(I) - Top and bottom flitching :-



$\sigma_w = 6$

$\sigma_s = m\sigma_w = 20 \times 6 = 120$

$\sigma_{s\,max} = \sigma_s \times \dfrac{112.5}{100} = 135 \text{ N/mm}^2$

$M.R_w = \sigma_w \dfrac{bd^2}{6} = 6 \times \dfrac{150 \times 200^2}{6} = 6 \times 10^6 \text{ N-mm}$

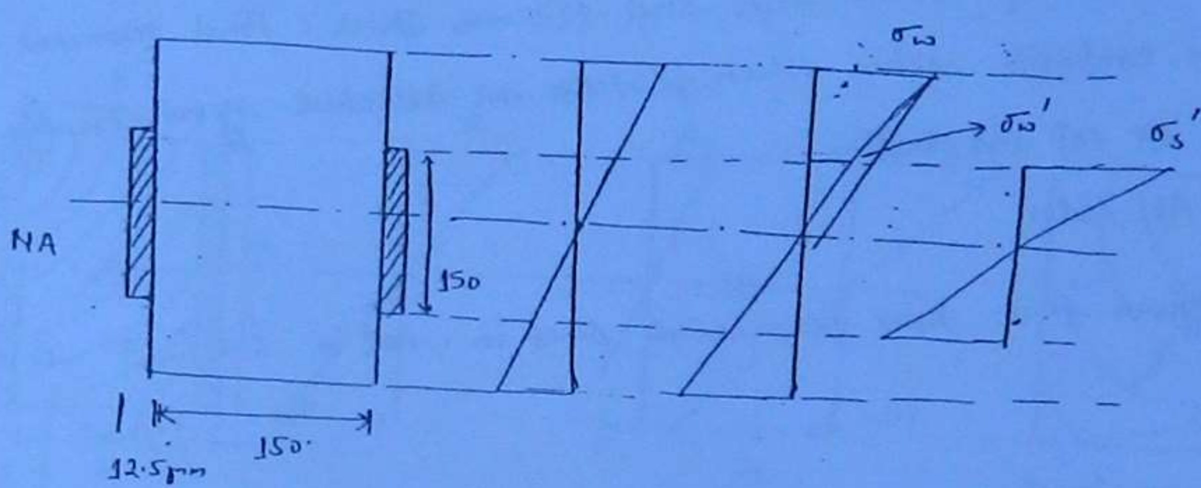
$= 6 \text{ KN-m}$

$MR_s = \left(\sigma_{s\,max} \dfrac{b(d+2t)^2}{6} - \sigma_s \times \dfrac{b(d)^2}{6} \right) = \dfrac{135 \times 150 \times (225)^2}{6} - \dfrac{120 \times 150 \times 200^2}{6}$

$= 50.86 \text{ KN-m}$

Total $M.R = 6 + 50.86 = 56.86$ KN.

2) Side flitched :-



$$\sigma_s' = m\,\sigma_w'$$

$$\sigma_w' = \sigma_w \cdot \frac{75}{100}$$

$$\sigma_w' = 6 \times \frac{75}{100} = 4.5\ N/mm^2$$

$$\sigma_s' = 20 \times 4.5 = \underline{\underline{90 N/mm^2}}$$

$$MR_w = \sigma_w \frac{bd^2}{6} = 6\ KN\text{-}m$$

$$MR_s = \sigma_s' \times \frac{2t \cdot d^2}{6}$$

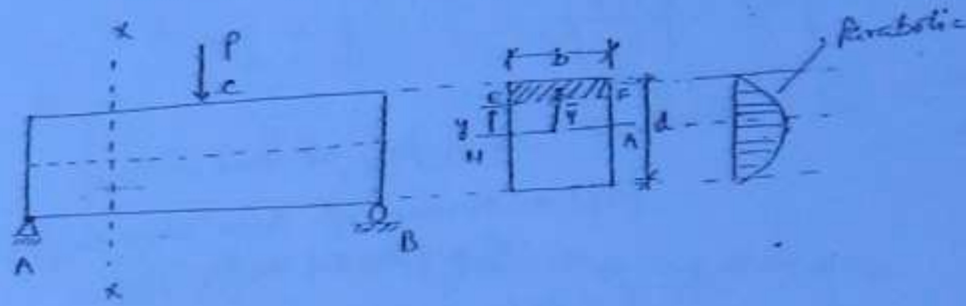
$$= 90 \times 2 \times 12.5 \times \frac{150^2}{6}$$

$$MR_s = 8.44\ KN\text{-}m$$

$$MR_{Total} = 8.44 + 6 = 14.44\ KN\text{-}m$$

$$\frac{MR.(\text{Top and bottom flitching})}{MR.(\text{Side flitching})} = \frac{56.86}{14.44} \approx 4$$

Shear stress distribution in Beams :-



S.F. at x-x = S

The shear stresses caused by shear force at any section are called direct shear stresses if B.M. is constt over the length then $\frac{dM}{dx} = 0$ Hence No shear force and No shear stresses will develop. the shear stresses are found '0' at top and bottom and max^m at interior which vary parabolically

Assumptions involved

1) Material is isotropic, homogeneous and linear elastic in which Hooke's law is valid.

2) The shear stresses at any distance y from nutral axis will be constt. along the width. It means shear stress is constt from E to F.

NOTE :- In real practice shear stresses at edges is '0' and max^m in interior from E to F but this variation is neglected.

3) Shear stresses vary with the depth and are found zero at top and bottom edge.

Shear stress at a distance y from nutral axis is given by:

$$\boxed{\tau = \frac{S(A\bar{y})}{I \cdot b}}$$

where $S \rightarrow$ Shear force at that section.

$(A\bar{y}) \rightarrow$ Moment of Area above the beyond the level of y (Shaded area in fig.).

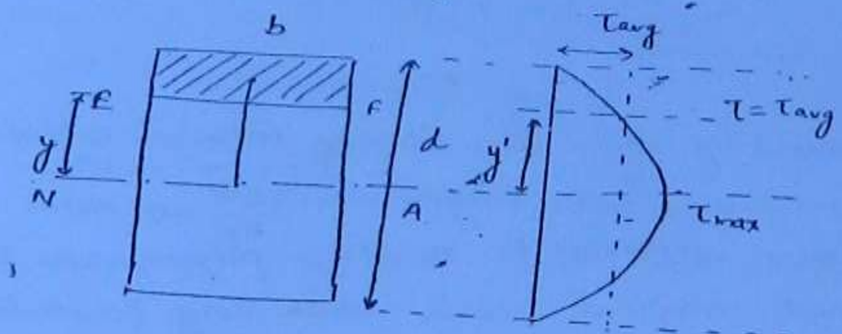
$\bar{y} \rightarrow$ distance of centroid of the shaded area from nutral axis.

$I \rightarrow$ moment of Inertia of Cross section about N.A

$b \rightarrow$ width of section at the level of y i.e $EF = b$

CASE (i).

Shear stress distribution in Rectangular Beam



$$\tau = \frac{S\left(b\cdot\left(\frac{d}{2}-y\right) \times \frac{y + d/2}{2}\right)}{\frac{bd^3}{12} \times b}$$

$$\boxed{\tau = \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)}$$

τ_{max} will occur if $\frac{d\tau}{dy} = 0$

$$\frac{6S}{bd^3}\left(0 - 2y\right) = 0$$

$$y = 0$$

In rectangular section τ_{max} occurs at neutral axis.

$$\tau_{max} = \frac{3}{2}\frac{S}{bd}$$

$$\tau_{avg} = \frac{S}{A} = \frac{S}{bd} \Rightarrow \frac{\text{Shear force}}{\text{Area}}$$

\# $$\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$$

Max^{m} shear stress is 50% greater than avg shear stress.

If at a distance y' from neutral axis actual shear stress is equal to avg. shear stress then $\tau = \tau_{avg}$

$$\frac{6S}{bd^3}\left(\frac{d^2}{4}-y'^2\right) = \frac{S}{bd}$$

\# $$\boxed{y' = \pm\frac{d}{2\sqrt{3}}}$$

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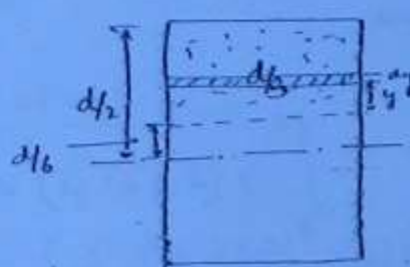
If a beam of Rectangular section is subjected to vertical S.F. F then S.F. resisted by top 1/3 of the cross-section will be.

i) $F/3$.

ii) $7F/27$

iii) $8F/27$

iv) $14F/27$.



$$dF = \tau \cdot dA$$

$$S.F = \int_{d/6}^{d/2} \frac{6S}{bd^3}\left(\frac{d^2}{4}-y^2\right)\cdot b\cdot dy$$

$$= \frac{6S}{bd^3}\cdot \frac{d^2}{4}\cdot y - \frac{y^3}{3}\Big|_{d/6}^{d/2}$$

$$\frac{d}{2}-\frac{d}{6}$$

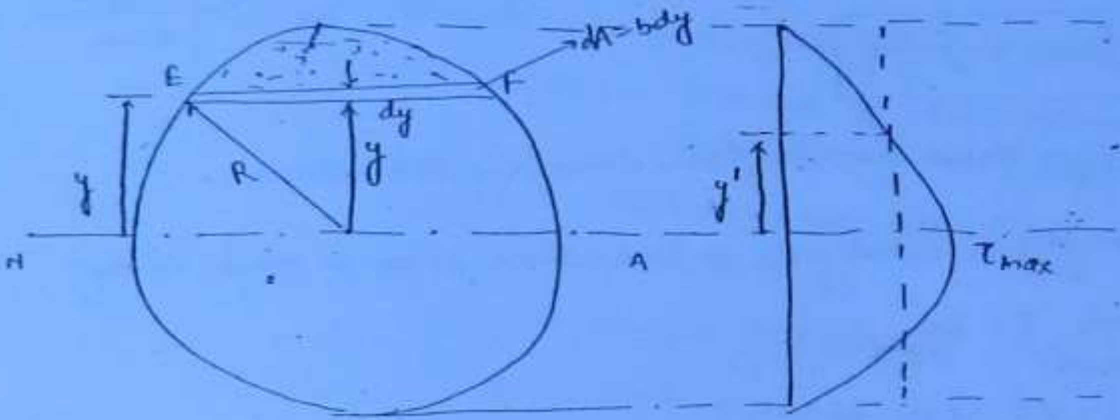
$$= \frac{6S}{bd^3}\left(\frac{d^3}{12} - \left(\frac{d^3}{24}-\frac{d^3}{216\times3}\right)\right)$$

$$\frac{6S}{}$$

$$\left(\frac{d^3}{12} - .04d^3\right)$$

S.F. resisted by central third strip is $\left(\frac{13}{27}\right)F$.

Shear stress Distribution in Circular Section :-



$$\tau = \frac{S(A\bar{y})}{I\,b}$$

b = width of Section EF

$$b = EF = 2\sqrt{R^2 - y^2}$$

$A\bar{y}$ = Mom. of area of section above EF about N.A.

$= \int$ Mom. of area dA abt. N.A.

$$A\bar{y} = \int_y^R y \cdot dA = \int_y^R y(b\,dy) = \int_y^R y\left(2\sqrt{R^2-y^2}\right)dy = \frac{2}{3}\left(R^2-y^2\right)^{3/2}$$

$$\tau = \frac{S\left[\frac{2}{3}\left(R^2-y^2\right)^{3/2}\right]}{\frac{\pi R^4}{4} \cdot 2 \times \sqrt{R^2-y^2}} \Rightarrow \boxed{\tau = \frac{4S}{3\pi R^4}\left(R^2-y^2\right)}$$

if $y = \pm R$ then $\tau = 0$

if $y = 0$, " $\tau = \tau_{max}$

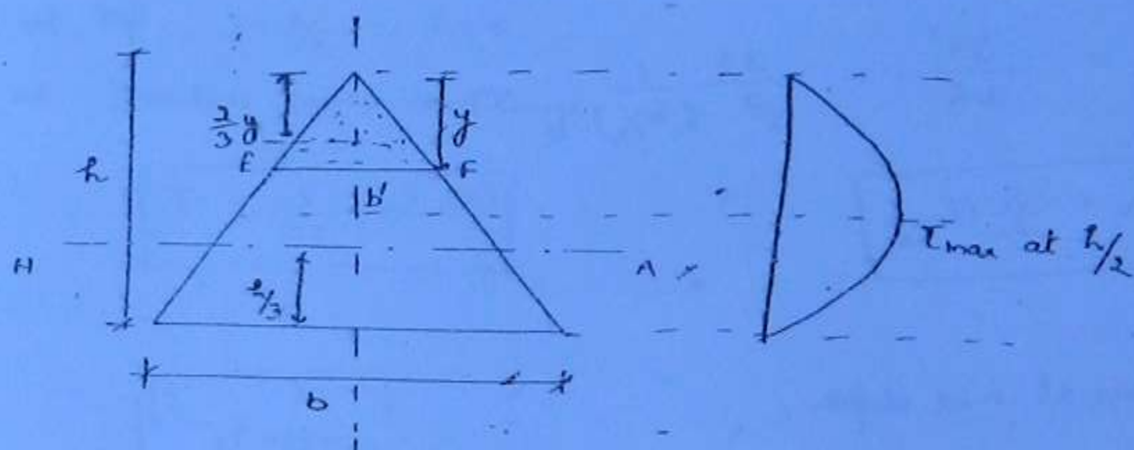
$$\boxed{\tau_{max} = \frac{4}{3}\frac{S}{\pi R^2} = \frac{4}{3}\frac{S}{A} = \frac{4}{3}\tau_{avg}}$$

At $y = y'$, $\tau = \tau_{avg}$

$$\frac{4S}{3\pi R^4}\left(R^2 - y'^2\right) = \frac{S}{\pi R^2}$$

$$\Rightarrow \boxed{y' = \pm R/2}$$

Shear stress distribution in Triangular Section:→



Consider y from vertex.

$$\frac{b'}{b} = \frac{y}{h}$$

$$b' = y \cdot \frac{b}{h}$$

$$\tau = \frac{S(A\bar{y})}{I \cdot b}$$

$$\tau = \frac{S \cdot \left(\frac{1}{2} \cdot \cancel{b'} \cdot y \times \left(\frac{2}{3}h - \frac{2}{3}y\right)\right)}{\frac{bh^3}{36} \times \cancel{b'}} = \frac{12S}{bh^3}\left(hy - y^2\right)$$

$$\boxed{\tau = \frac{12S}{bh^3}\left(hy - y^2\right)}$$

at $y = 0$, $\tau = 0$.

at $y = h$ $\tau = 0$.

For τ_{max},

$$\frac{d\tau}{dy} = 0$$

$$\Rightarrow \frac{12S}{bh^3}\left[h - 2y\right] = 0$$

at $y = h/2$

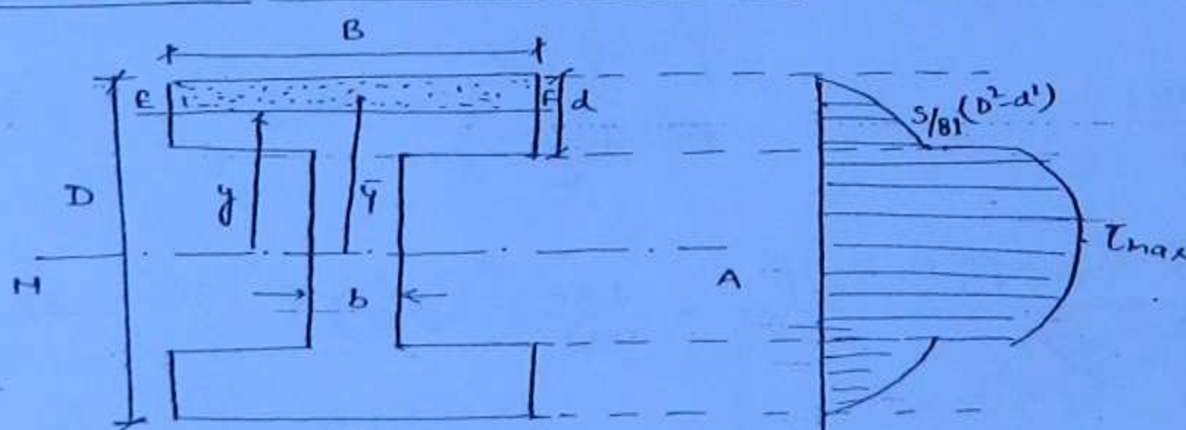
$$\tau_{max} = \frac{12S}{bh^3}\left[\frac{h^2}{2} - \frac{h^2}{4}\right] = \frac{12S}{bh^3} \cdot \frac{h^2}{4}$$

$$\tau_{max} = \frac{3S}{bh} = \frac{3S}{2} \cdot \frac{1}{(bh/2)}$$

$$\boxed{\tau_{max} = \frac{3}{2}\,\tau_{avg}}$$

τ_{max} occurs at mid depth.

Shear stress distribution in I-Section :-



Shear stress within flange at a distance y from N.A.

$$A = B\left(\frac{D}{2} - y\right)$$

$$\bar{y} = \frac{D/2 + y}{2}$$

$$\tau = \frac{S(A\bar{y})}{I \cdot B}$$

$$= \frac{S\left(\cancel{B}\cdot(D/2 - y)\left(\frac{D/2 + y}{2}\right)\right)}{I \cdot \cancel{B}}$$

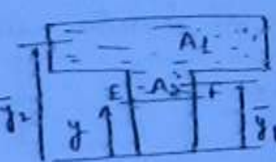
$$\tau = \frac{S}{2I}\cdot\left(\frac{D^2}{4} - y^2\right)$$

at top, $y = D/2$, $\tau = 0$

at Junction within the flange, $y = d/2$

$$\boxed{\tau = \frac{S}{8I}(D^2 - d^2)}$$

CASE B.



$$A\bar{y} = (A_1\bar{y}_1) + (A_2\bar{y}_2)$$

$$= \cancel{B \cdot d \times D}$$

$$= B\left(\frac{D}{2} - \frac{d}{2}\right)\cdot\left(\frac{\frac{D}{2} + d/2}{2}\right) + b\left(\frac{d}{2} - y\right)\left(\frac{\frac{d}{2} + y}{2}\right)$$

$$A\bar{y} = \frac{B}{2}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau = \frac{S(A\bar{y})}{I \cdot b}$$

$$= \frac{\frac{S}{2}\left(B\left(\frac{D^2 - d^2}{4}\right) + b\left(\frac{d^2 - y^2}{4}\right)\right)}{I \cdot b}$$

at $y = d/2$ Junction within web.

$$\tau = \frac{S}{8I} \cdot \frac{B}{b}\left(D^2 - d^2\right)$$

At Nutral axis $y = 0$

$$\tau = \tau_{max} = \frac{S}{8I} \cdot \frac{B}{b}\left(D^2 - d^2\right) + \frac{S}{8I} d^2$$

$$\boxed{\tau_{max} = \frac{S}{8I} \cdot \frac{B}{b} \cdot \left(D^2 - d^2\right) + \frac{S}{8I} d^2}$$

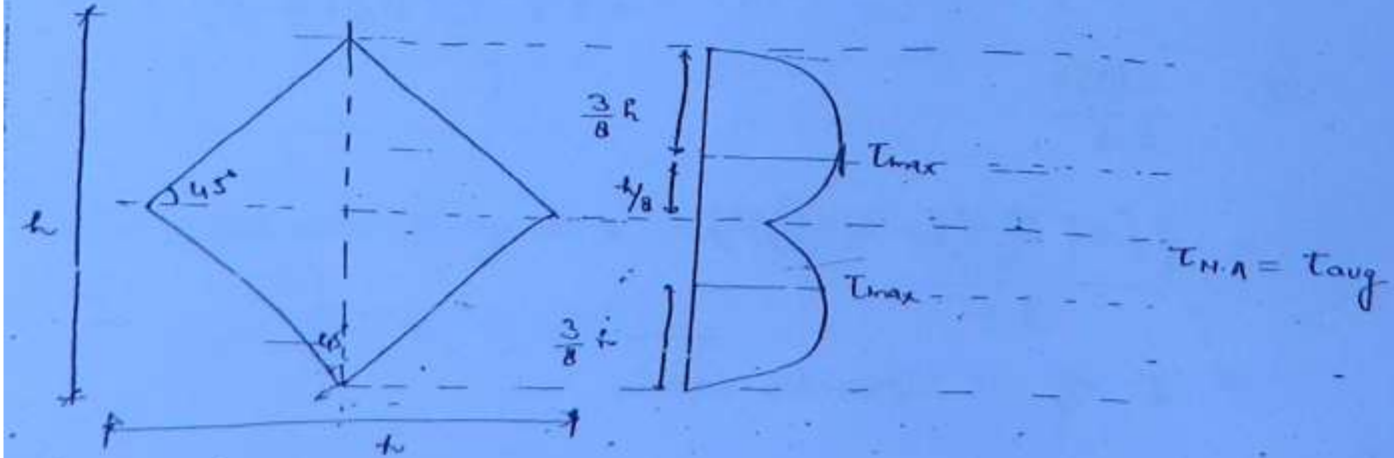
NOTE :-

1) In I. Section nearly 80 to 90%. Shear force is resisted by web. and only 10-20%. is resisted by flanges.

2) Shear stress is inversly related to width Hence at the junction width suddenly change hence shear stress suddenly change Note that if τ_1 is shear stress within web at junction where width is b_1 and τ_2 is " " " flange " " " " " b_2: then $\boxed{\frac{\tau_1}{\tau_2} = \frac{b_2}{b_1}}$ or $\boxed{\tau_1 b_1 = \tau_2 b_2}$

Width suddenly reduces and τ_2 suddenly increase and vice versa

Shear stress distribution in Diamond Section :-



τ_{max} occurs at $\frac{h}{8}$ from Nutral Axis

$$\boxed{\tau_{max} = \frac{9}{8}\tau_{avg}}$$

$$\tau_{avg} = \frac{S}{\frac{h^2}{2}} = \frac{2S}{h^2}$$

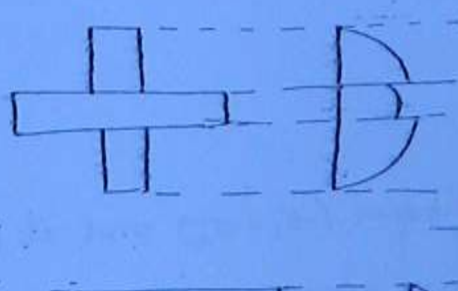
$$\boxed{\tau_{max} = \frac{9}{4}\frac{S}{h^2}}$$

* In diamond section max^m shear stress is 12.5% greater than avg. Shear stress.

Qualitative Shear stress diagram:→

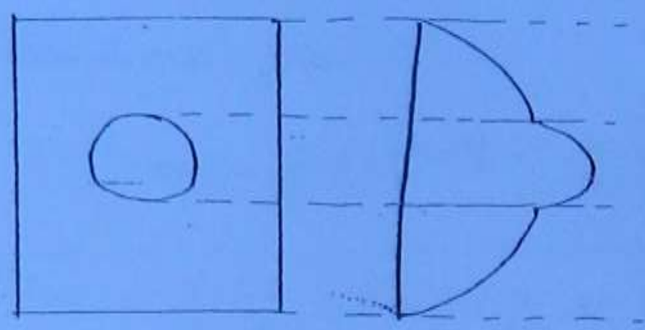
i) H Section:

ii) Cross Section:-



iii) T. Section:

iv) Hollow Rectangular Section :-



Principal Stresses :-

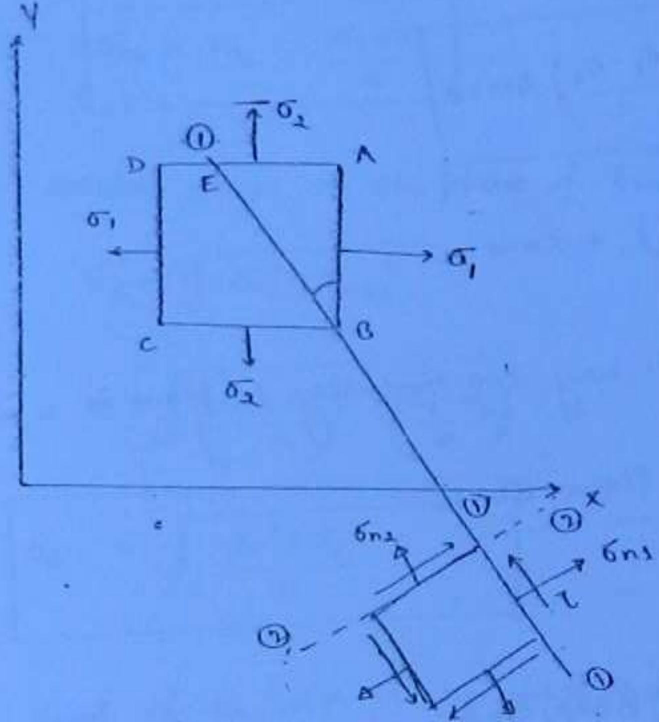
Principal stresses are max^m ~~and~~ or min^m normal stresses which develope on the plane of ~~shear~~ zero shear stresses. generally in ~~try~~ ~~an~~ triaxial loading there are three principal plane (mutually $\perp^r$). Methods of Stress transformation :-

i) Analytical Method or equili^m approach
ii) Graphical " or Mohar's circle method;

NOTE : Circle will be obtained in plane stress system and ellipsoid will be obtained in space stress system.

Analytical Method :-

CASE I : If principal stresses are known (σ_1 & σ_2) and it is reqrd to determine Normal and shear stresses at same point in any other plane which is inclined at an angle θ with vertical plane of σ_1.

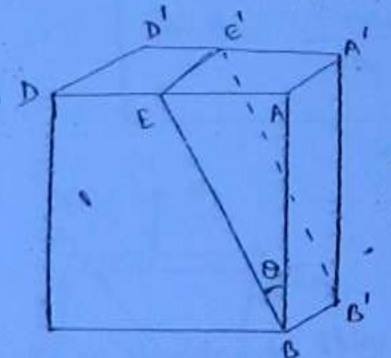


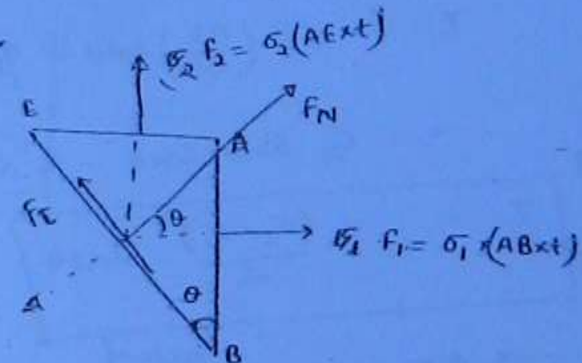
$\sigma_1 > \sigma_2 \rightarrow$ Minor pr. stress

Major pr. str.

$\sigma_{1,2,3}$: ... $\Rightarrow$ principle values.

No shear at principal Plane.





$$f_N = f_1 \cos\theta + f_2 \sin\theta$$

$$\sigma_n \times (BE \times t) = \sigma_1 (AB \times t) \cos\theta + \sigma_2 (AE \times t) \sin\theta$$

$$\sigma_n = \sigma_1 \cdot \frac{AB}{BE} \cos\theta + \sigma_2 \cdot \frac{AE}{BE} \sin\theta$$

$$\sigma_n = \sigma_{n_1} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$\boxed{\sigma_{n_1} = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta}$$

Normal stress on plane ②-② which is normal to the plane ①-① which is obtained by substituting $\theta = (\theta + 90)$ in above eqn.

$$\boxed{\sigma_{n_2} = \left(\frac{\sigma_1 + \sigma_2}{2}\right) - \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta}$$

* Note that $\sigma_{n_1} + \sigma_{n_2} = (\sigma_1 + \sigma_2) = \text{Constt}$

Summation of Normal stresses on any two mutually $\perp^r$ plane is a constt.

Tangential force or Shear force on Plane BE :-

$$F_T = F_2 \cos\theta - F_1 \sin\theta$$

$$\tau (BE \times t) = \sigma_2 \cdot (AE \times t) \cos\theta - \sigma_1 (AB \times t) \sin\theta$$

$$\tau = \sigma_2 \left(\frac{AE}{BE}\right) \cos\theta - \sigma_1 \left(\frac{AB}{BE}\right) \sin\theta$$

$$= \sigma_2 \sin\theta \cdot \cos\theta - \sigma_1 \sin\theta \cdot \cos\theta$$

$$\boxed{\tau = -\frac{(\sigma_1 - \sigma_2)}{2} \cdot \sin 2\theta}$$ Imp.

-ve sign means the actual dir^n of shear stress will be opposite to dir^n shown in fig.

Special CASE:

If $\theta = 45^\circ$ or 135° then

shear stress will be max^m.

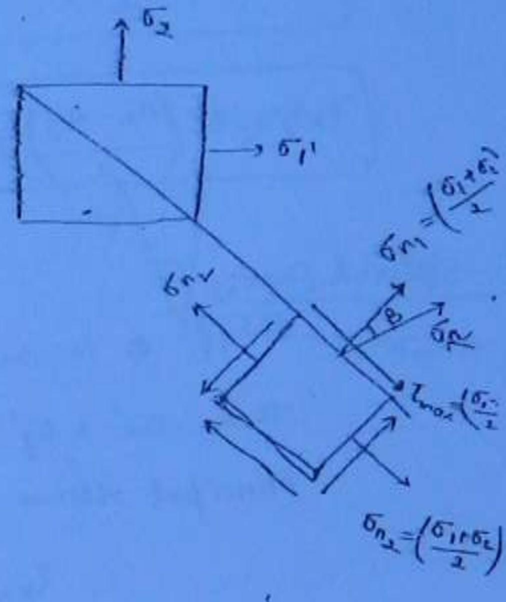
$$\tau_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

on the plane of τ_{max}, Normal stresses will not be zero but will be equal and alike

At $\theta = 45^\circ$ & 135°

$$\boxed{\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1+\sigma_2}{2}}.$$

The resultant stress on the plane of τ_{max}:



$$\sigma_R = \sqrt{\sigma_{n_1}^2 + \tau_{max}^2}$$

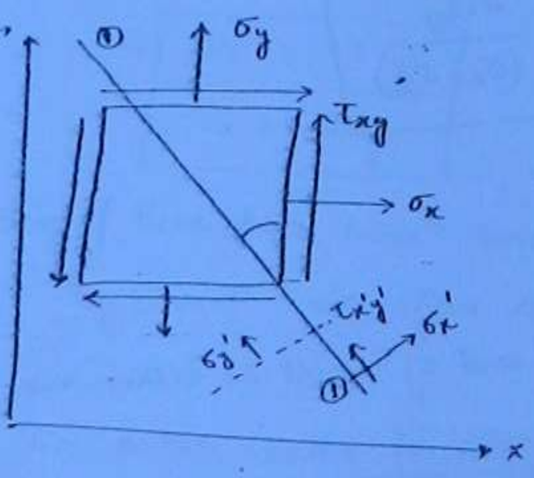
$$= \sqrt{\left(\frac{\sigma_1+\sigma_2}{2}\right)^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2}$$

$$\boxed{\sigma_R = \sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}}$$

Angle of σ_n with the normal of that Plane is given

$$\boxed{\tan\beta = \frac{\tau_{max}}{\sigma_n} = \frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}}$$

CASE-2:→



if σ_x and σ_y are normal stresses and τ_{xy} is shear stress in xy plane then find normal stresses and shear stresses on any plane 1-1 which is inclined at an angle θ with the plane of σ_x

$$\boxed{\sigma_x' = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$$

σ_y' can be found by putting $\theta = \theta + 90°$.

$$\boxed{\sigma_y' = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta}$$ Imp

$$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$$ Imp.

Special case:-

If θ is such that shear stress on plane 1-1. ($\tau_{x'y'}=0$)
Then σ_x' & σ_y' will be either max^m or min and will be called principal stress hence angle of principal plane is given by

$$\tau_{x'y'} = 0$$

$$-\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

$$* \quad \boxed{\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x-\sigma_y)}}$$

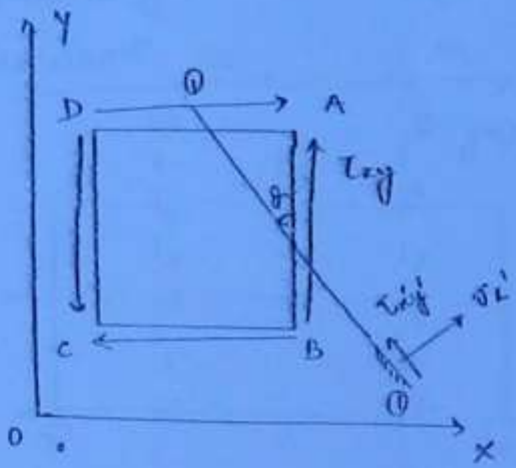
$$\theta_{P_1} = \theta$$

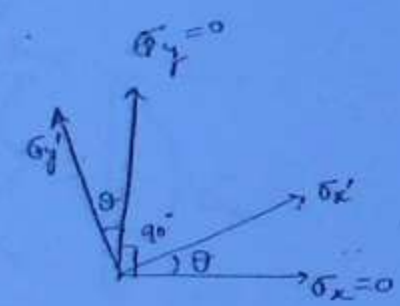
$$\theta_{P_2} = \theta \pm 90°$$

By substituting value of θ σ_x' and σ_y' will be either max^m or min^n
Hence principal stress

Imp $$\boxed{\sigma_1/\sigma_2 = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

CASE III : Case of pure shear :→





$\sigma_x' = \tau_{xy} \sin 2\theta$

$\sigma_y' = -\tau_{xy} \sin 2\theta$

$\tau_{x'y'} = \tau_{xy} \cos 2\theta$

For Principle Planes $\tau_{x'y'} = 0$, then $\theta = 45°$ or $135°$

$$\boxed{\begin{array}{l} \sigma_1 = +\tau_{xy} \\ \hline \sigma_2 = -\tau_{xy} \end{array}}$$

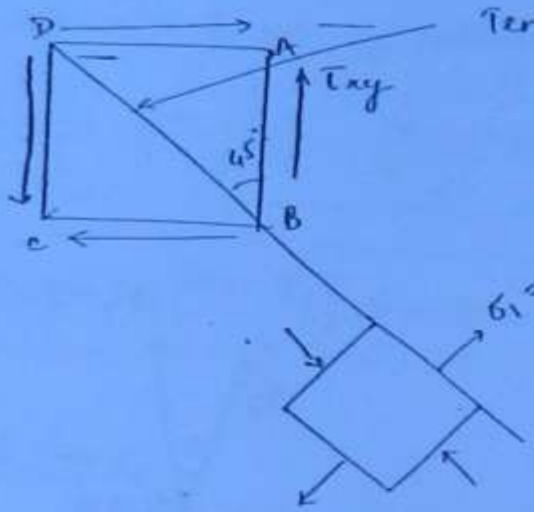
about principal Planes

In case of Pure shear max^m tensile stress ($\sigma_1 = +\tau_{xy}$) & max^m Compr. stress ($\sigma_2 = -\tau_{xy}$) and max^m shear stress $= \tau_{xy}$. [0]

NOTE :- For Brittle metals ~~tensile strength is less than~~

tensile strength < shear strng < Compr. Strg.

Hence in pure shear, failure will occur due to principal tension and failure plane will be plane of tensile stress which is at 45° from the plane of shear stress.



Tension failure Plane (Crack will occur along — Diagonal BD on which Tensile stress acts)

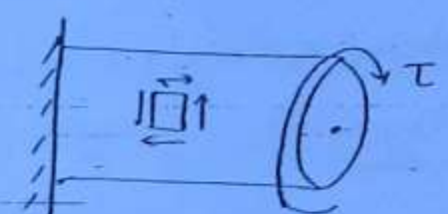
2). Ductile metals are weakest in shear.

Shear Strg < tensile Str ≤ Compressive Strgth.

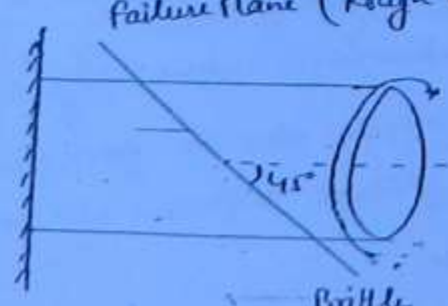
For ductile Metals shear strength is nearly 57% of tensile strength as per the test [Practical result]

Hence in pure shear state ductile metal will first fail in shear and failure plane will be plane of $\tau_{max} = \tau_{xy}$.

example of pure shear state is shaft subjected to torsion.

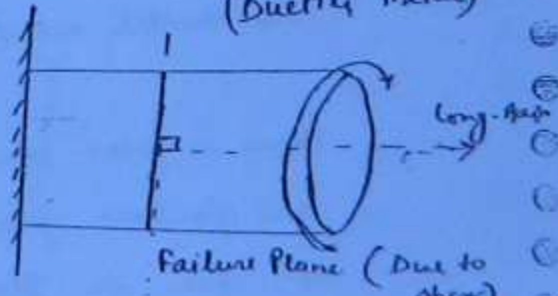


Failure Plane (Rough fracture Helical plane)



Brittle

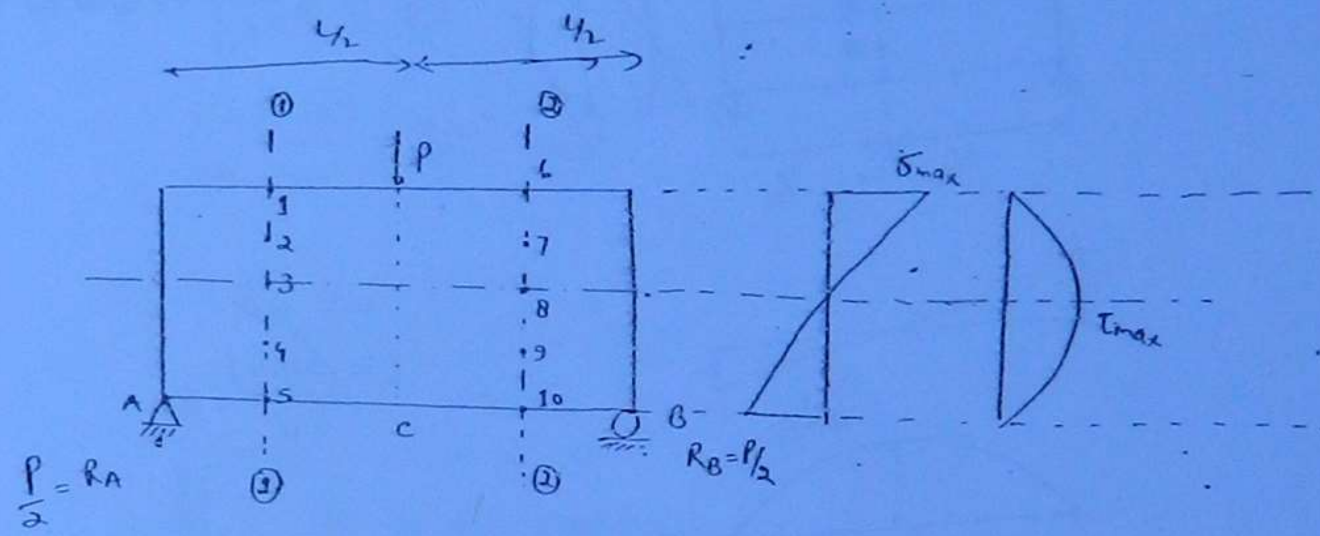
(Ductile Metal)

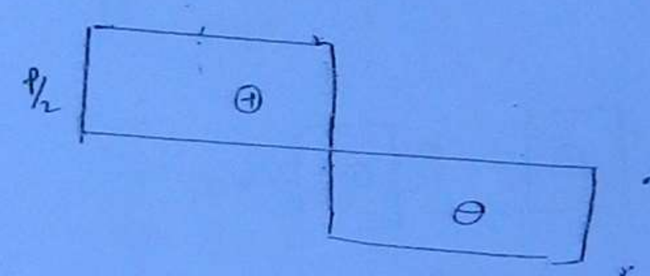


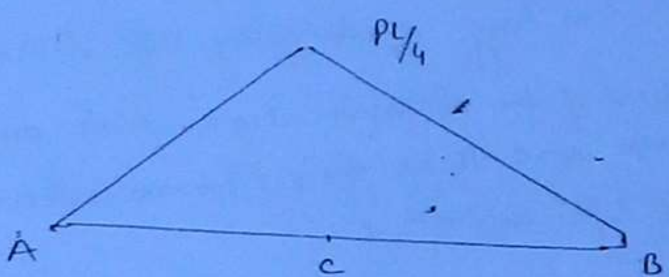
Failure Plane (Due to shear)

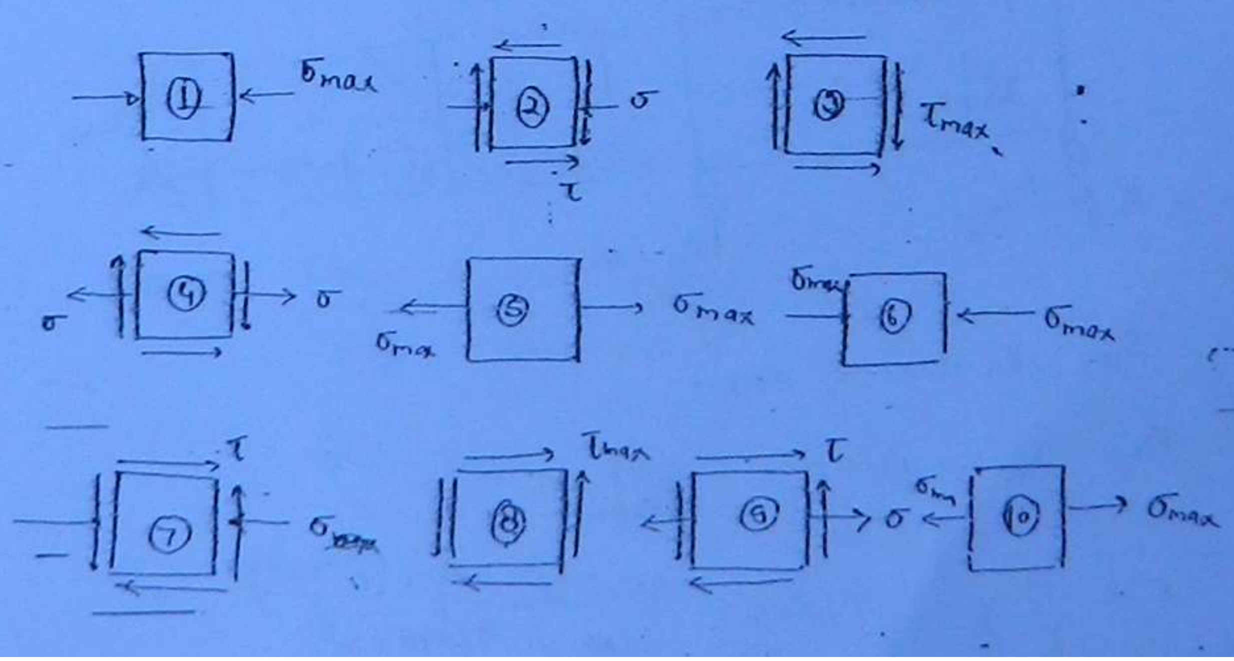
Due to torque max^m shear stress developed on in circumferential dir and in Complimentary dir^n.

For the beam shown in fig mark the stress element at points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

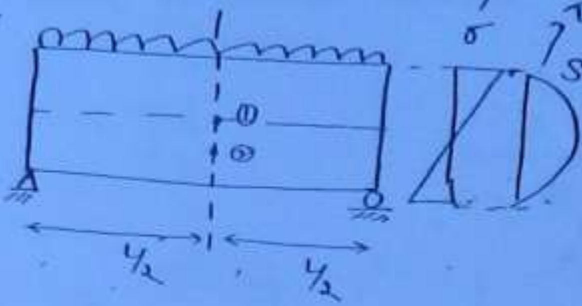




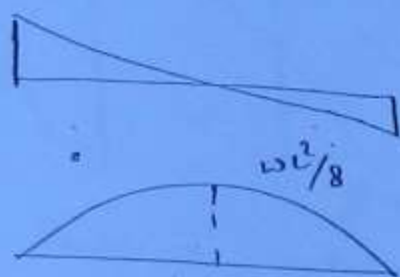




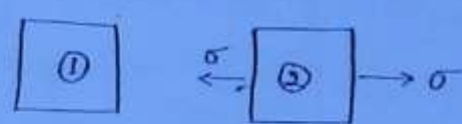
GATE 2011



Stress element at point ① ② ① is ?



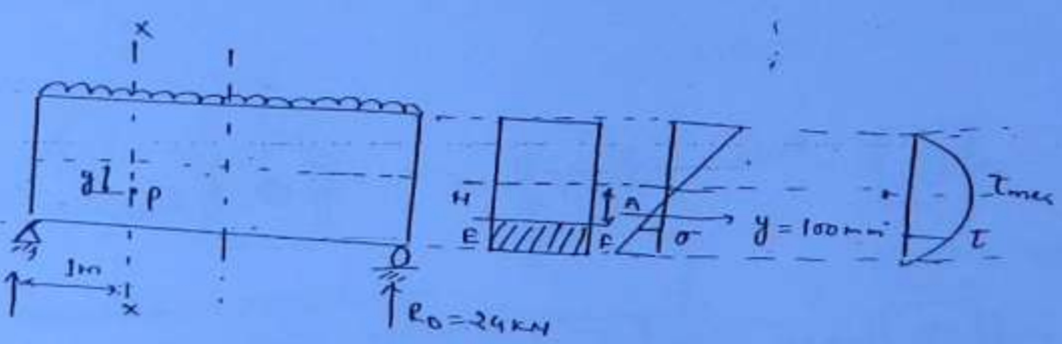
$$\tau = \frac{S(A\bar{y})}{Ib} = 0$$



ES'03

Q. A simply supported Beam a 4m long which carry UDL of 12 KN/m over the entire length. Cross section of the Beam is 300 mm wide and 400 mm deep. Find principal stresses and there dir^{n}s 100 mm below the Nutral axis. and 1m from left support ?

Sol^n:-



$24\ KN = \frac{wL}{2} = R_A$

$S_x = R_A - 12\times1 = 12KN$

$M_x = R_A \cdot x - \frac{12\times1\cdot1}{2} = 18KN\text{-}m$

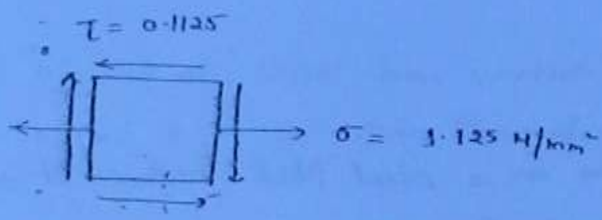
$$\sigma = \frac{M}{I}\cdot y = \frac{18\times10^6}{\frac{300\times400^3}{12}}\times100 = \frac{12\times18\times10^6}{400^3} = 1.125 N/mm^2$$

Shear Stress at y from N.A.

$$\tau = \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

$$= \frac{6 \times 12 \times 10^3}{300 \times 400^3}\left(\frac{400^2}{4} - 100^2\right)$$

$$\tau = 0.1125\ N/mm^2$$



$$\sigma_x = 1.125$$

$$\sigma_y = 0$$

$$\tau_{xy} = -0.1125$$

$$\sigma_1 / \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{1.125}{2} + \sqrt{\left(\frac{1.1}{2}\right)^2 + (.1125)^2}$$

$$\sigma_1 = +1.136\ N/mm^2$$

$$\sigma_2 = -0.011\ N/mm^2$$

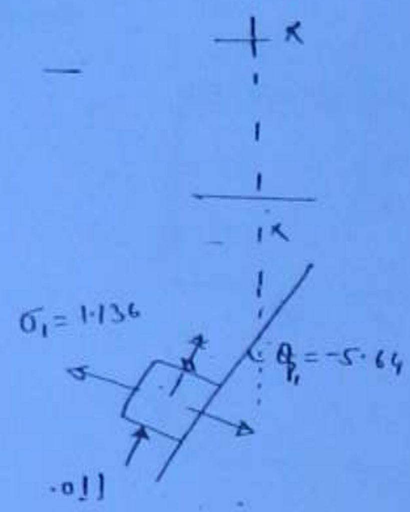
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times (-0.1125)}{1.125 - 0}$$

$$\theta = -5.64^\circ \quad \boxed{\circlearrowright}$$

$$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= +1.136\ N/mm^2$$

{θ with σ₁ - Plane}

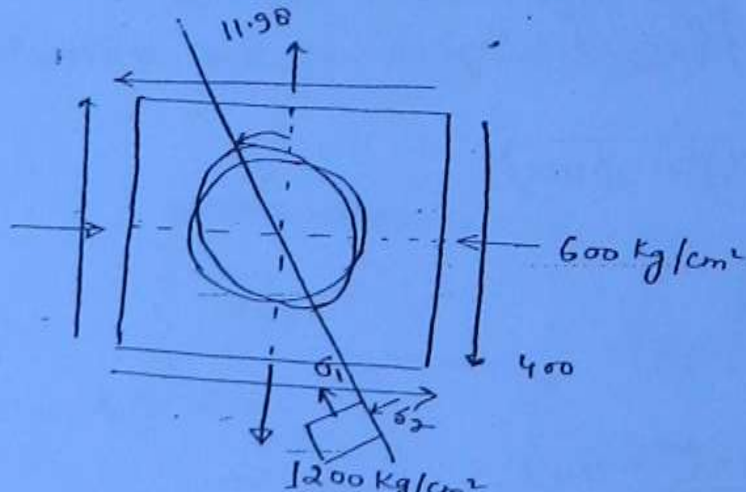


ES'

A circle of 100 mm dia. is drawn on a steel plate before it is loaded and then plate is loaded as shown in fig. After loading circle deforms into an ellipse. determine major and minor axis of ellipse and there dir^n. ?

given $E = 2.1 \times 10^6\ kg/cm^2$

$\mu = 0.28$



$\sigma_x = -600$

$\sigma_y = 1200$

$\tau_{xy} = -400$

$$\sigma_1 / \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 = 1284.88\ kg/cm^2$

$\sigma_2 = -684.88\ kg/cm^2$

$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} = \frac{2 \times -400}{-600 - 1200} = \frac{\cancel{2} \times 400}{9\cancel{18}00} = 4/9$$

$$\theta = 11.98$$

$$\sigma_{x'} = \frac{600}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{x'} = -604.88$$

Hence σ_2 acts at 11.98° from vertical plane.

and σ_1 " " 90° from Plane of σ_1.

$$\epsilon_1 = \frac{\sigma_1}{E_t} - \mu \frac{\sigma_2}{E}$$

$$= 6.42 \times 10^{-4} -$$

$$= 5.46 \times 10^{-4}$$

* In the dir^n of σ_1 major axis of ellipse will occur and in the dirc of σ_2 minor axis of ellipse will occure

Hence major axis of ellipse will be at 11.98° in anticlockwise dir^n from verti

Let.

δd_1 is change in dimention in direction of σ_1

$$\epsilon_1 = \frac{\delta d_1}{d} = \frac{\sigma_1}{E} + \mu \frac{\sigma_2}{2} = 1204.$$

$$\delta d_1 = 7.03 \times 10^{-4} \times 100$$

$$= .0703 \text{ mm}$$

major axis $= d + \delta d_1 = 100 + .0703 = 100.0703$ mm

$$\frac{\delta d_2}{d_2} = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E}$$

$$= \frac{-688.84 - .28 \times 1284.88}{2.1 \times 10^6}$$

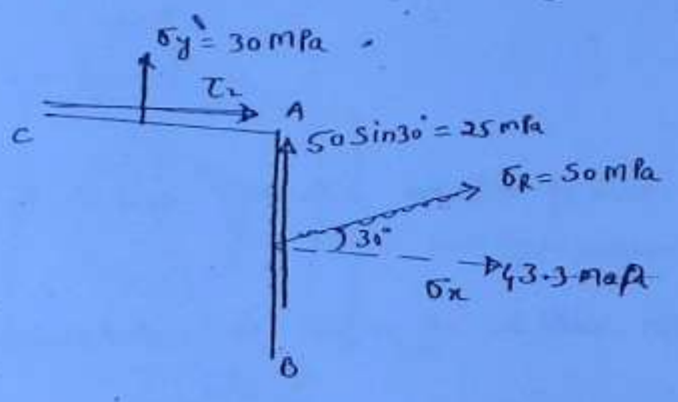
$$\frac{\delta d_2}{d_2} = -4.97 \times 10^{-4} \Rightarrow \delta d_2 = -0.0497 \text{ mm}$$

Length of minor axis of ellipse $= 100 - 0.0497 = 99.9503$

Ans

<u>Ques:</u> At a point in a material the intensity of resultant stress on a certain ~~pat~~ plane is 50 MPa as shown in fig which is inclined at 30° to the normal of that plane. The stress on the plane at $\perp^r$ angle to this plane has normal tensile component of 30 MPa. Find resultant stress on the second plane and also find principal stresses and principle plane?

<u>Sol^n:-</u>



$\tau_2 = 25$ MPa.

Resultant stress on Plane AC $= \sqrt{30^2 + 25^2} = 39.05$

$$\sigma_x = 43.3$$
$$\sigma_y = 30$$
$$\tau_{xy} = -25.$$

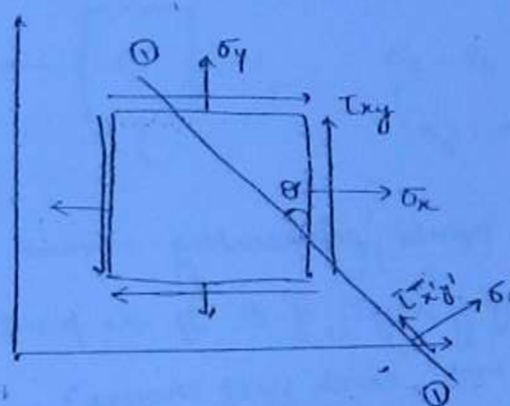
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta = 37.55^\circ$$

$$\sigma_1 / \sigma_2 = \frac{\sigma_x + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 664.2 \text{ MPa}$$

Mohr Circle Method :-



$$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \cos 2\theta + \tau_{xy} \sin$$

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 = \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin\right] \quad \text{—Ⓐ}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\left[\tau_{x'y'}\right]^2 = \left[-\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta\right]^2 \quad \text{—Ⓑ}$$

$$\left[\sigma_x' - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \left[\tau_{x'y'}\right]^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \cos^2 2\theta + \tau_{xy}^2 \sin^2 2\theta + \cancel{2 \times \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \times \tau_{xy}\sin 2\theta}$$

$$+ \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2 \cos^2 2\theta$$

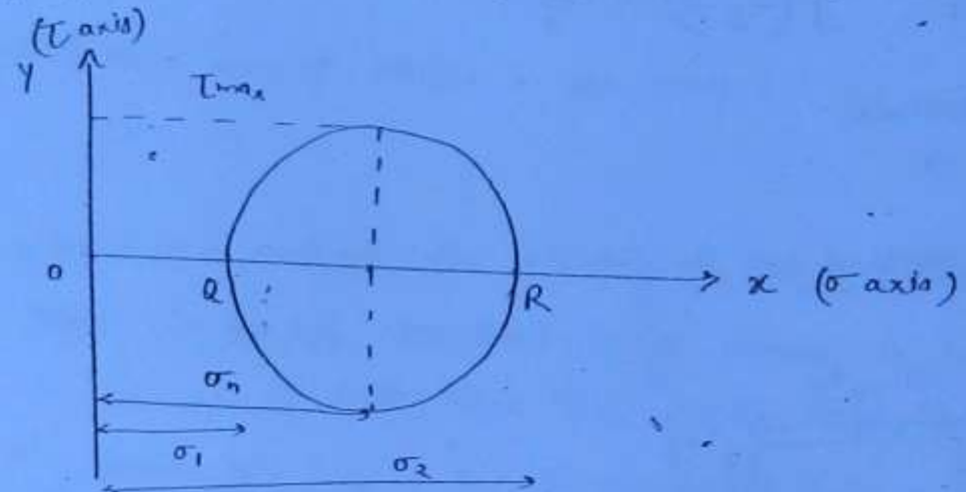
$$+ \cancel{2\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta\, \tau_{xy}\sin 2\theta}$$

$$= \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$[x-a]^2 + y^2 = r^2$$

$$r = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad x = \sigma_{x'} \qquad y = \tau_{x'y'}$$

eq^n of circle with centre $(a,0)$, radius $= r$.



Def:-

Mohr circle is the locus of points representing Normal and shear stresses at a plane with changing angle of θ. If the point or position in the beam is changed the mohr circle will change.

Properties of Mohr circle.

1) centre of the circle always lies on x axis. (σ axis) Hence circle is always symmetrical about σ axis.

2) circle cuts the σ axis at 2 points Q and R which represents major and minor stress.

$$OQ = \sigma_1 \ \& \ OR = \sigma_2.$$

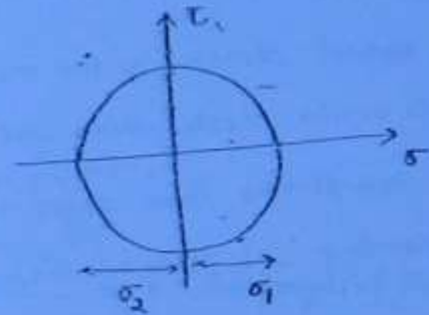
3) On the plane of τ_{max} normal stresses are niether max^n nor min^n but are equal and alike on 2 mutually $\perp^r$ planes of τ_{max}

4) Radius of circle $= \tau_{max} = \frac{\sigma_1 \mp \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

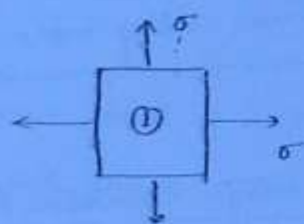
5) For the case of Pure shear mohr circle lies symmetrical about both axis with its centre and origin

In case of pure shear

$\Rightarrow$ $\boxed{\sigma_1 + \sigma_2 = 0}$

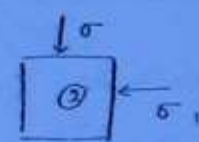


6) Mohr circle for the case of equal and alike stresses (Hydrostatic loading having no shear on the surface will be a point located on σ axis.



$\sigma_x = \sigma_y = +\sigma$

$\tau_{xy} = 0$



$\sigma_x = \sigma_y = -\sigma$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

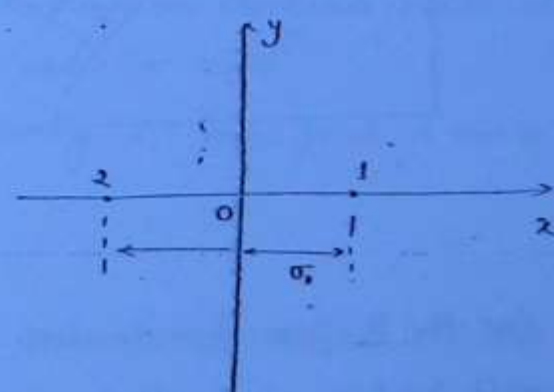
$= 0$

Centre $\Rightarrow (a, 0)$

$= \frac{\sigma_x + \sigma_y}{2}, 0$

$= \frac{\sigma + \sigma}{2}, 0$

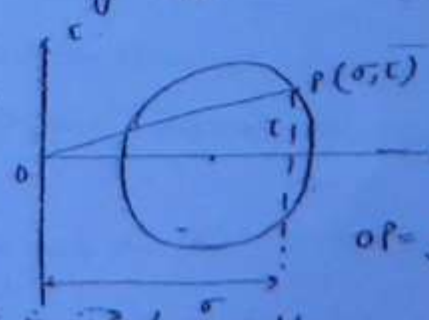
$= (\sigma, 0)$



* In such cases shear stress is zero in every plane, hence there can be infinite principle planes in hydrostatic loading case.

7). The distance of any point P on the circle from the origin is equal to resultant stress on a plane.

The angle of OP (β) with σ axis angle of obliquity of resultant stress with the normal ...



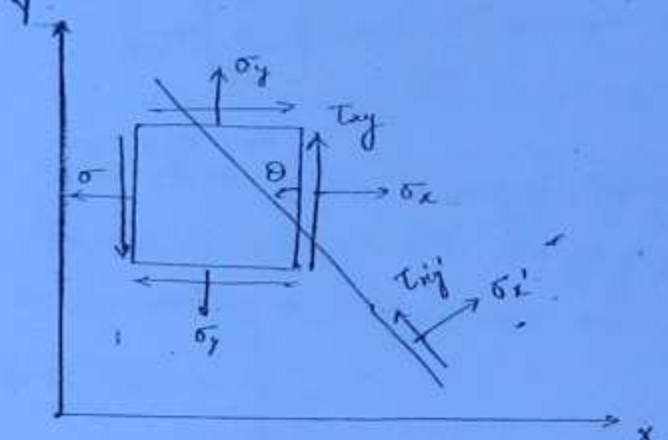
$OP = \sqrt{\sigma^2 + \tau^2}$

$$\boxed{\tan\beta = \tau/\sigma}$$

Q7 The normal stresses on two mutually $\perp^r$ plane will be represented by two points on the circle which will be diametrically opposite.

If two planes have angle b/t them θ then the point on the circle representing stresses on these plane will subtend angle at the centre $\underline{2\theta}$.

* Procedure to find Normal and Shear stresses on a plane ①-① which is inclined at an angle θ with the plane of σ_{max} using mohr circle method



Sign Convention for Mohr circle

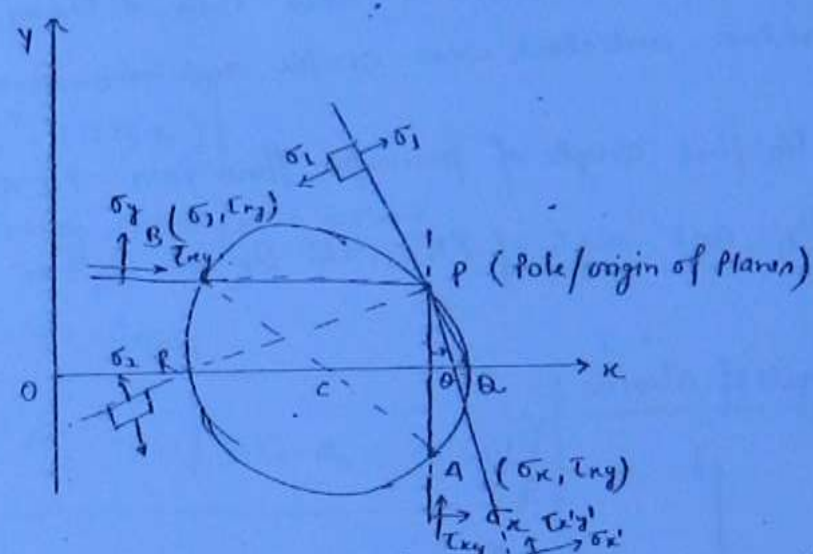
σ –
- (+) tensile (right side of origin)
- (–) compressive (left side of origin)

τ –
- (+) (above σ axis)
- (–) (below σ axis)

For the purpose of mohr circle those shear stress will be taken +ve which will produce clockwise couple on the centre of ed element & will be plotted above σ- axis.

step I :→

find the radius and centre of the circle and draw the circle



Mark point A on the circle representing Normal and shear stresses on vertical plane since shear stress on vertical plane produces anticlock-wise couple at centre hence pt A will lie below the circle.

Mark point B on the circle which represent σ_y and τ_{xy} in horizontal plane. Since shear on horizontal plane produces clockwise couple at the centre hence point B will lie above σ axis.

Since plane of σ_x and σ_y are mutually $\perp^r$ hence point A and B are diametrically opposite.

Step III:

Draw a vertical plane origi from A which will intersect the circle at P, P will be called pole or origin of Planes.

It means PA represents vertical Plane and PB horizontal Plane

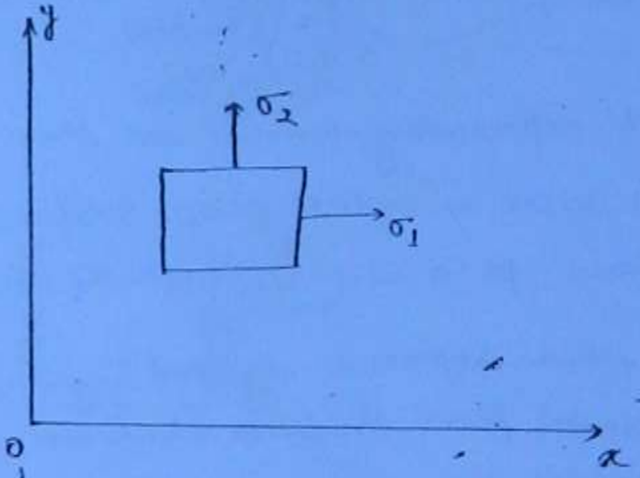
IV. A Plane which originates from P intersects to the circle at any other point say x then co-ordinate of x will represent normal and shear stress on plane Px.

To find normal and shear stress and on plane ①-① draw a plane

1-1 originating from P which Intersects the circle at point x. then co-ordinates of x will be, normal and shear stresses on Plane 1-1. if Point x lies below σ axis then +'Shear Stress on plane 1-1 will produce anticlock wise couple and vice-versa

V). To find angle of principle Plane join PQ and PR, angle of PQ will be θ_{P_1} and angle of PR will $\theta_{P_2} = \theta_{P_1} \pm 90^\circ$

Analysis of strains :-



Let σ_1 & σ_2 are acting in x & y dirⁿ resp.;

$$\epsilon_1 = \text{Principal strain} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \quad \text{— (i)}$$

$$\epsilon_2 = \text{Minor principal strain} = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \quad \text{— (ii)}$$

from (i) & (ii) —

$$\boxed{\sigma_1 = \frac{E}{(1-\mu^2)}(\epsilon_1 + \mu\epsilon_2)}$$

$$\boxed{\sigma_2 = \frac{E}{1-\mu^2}(\epsilon_2 + \mu\epsilon_1)}$$

Imp.

Total S.E stored in 2D case per unit volume.

$$U = \frac{1}{2}\sigma_1 \cdot \epsilon_1 + \frac{1}{2}\sigma_2 \cdot \epsilon_2$$

$$\boxed{U = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right)}$$

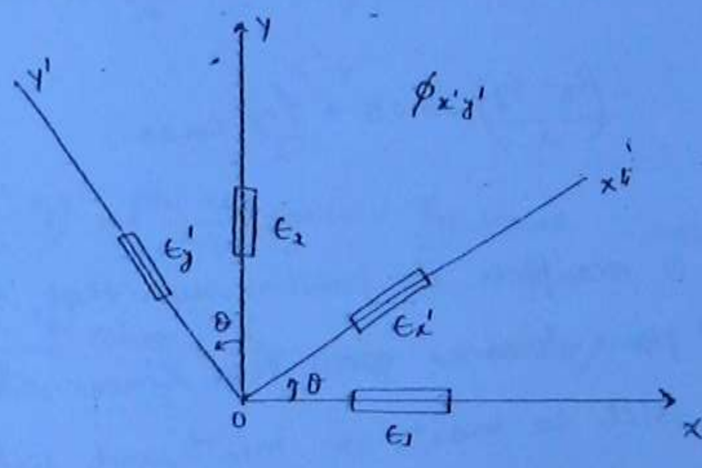
For 3D: Strain energy stored / unit volume

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2 + \frac{1}{2}\sigma_3\epsilon_3$$

$$\boxed{U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left[\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right]\right]}$$

Strain Transformation :-

CASE (I) when principal strains ϵ_1 & ϵ_2 are given in x and y dir^n. the find linear and shear strain in $x'y'$ plane



$$\epsilon_x' = \epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta$$

$$\epsilon_x' = \frac{\epsilon_1 + \epsilon_2}{2} + \left(\frac{\epsilon_1 - \epsilon_2}{2}\right)\cos 2\theta$$

$$\epsilon_y' = \left(\frac{\epsilon_1 + \epsilon_2}{2}\right) - \left(\frac{\epsilon_1 - \epsilon_2}{2}\right)\cos 2\theta$$

Note $\epsilon_x' + \epsilon_y' = \epsilon_1 + \epsilon_2 = $ const. Summation of linear strain in 2 ⊥^{r} dir^n.

Let shear strain in $x'y'$ plane is $\phi_{x'y'}$ then shear strain is given by

$$\frac{\phi_{x'y'}}{2} = -\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)\sin 2\theta$$

$$|\phi_{max}| = |\epsilon_1 - \epsilon_2|$$

Max^n shear strain is equal to difference of principle strain

CASE (II):-

Let ϵ_x and ϵ_y are linear strains in x and y dir^n and ϕ_{xy} is shear strain in xy plane then find linear strain in x', y' dir^n and shear strain in $x'y'$ plane

$$\epsilon_{x'} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

$$\epsilon_{y'} = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) - \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta - \frac{\phi_{xy}}{2}\sin 2\theta$$

$$\frac{\phi_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\phi_{xy}}{2}\cos 2\theta$$

Special case:

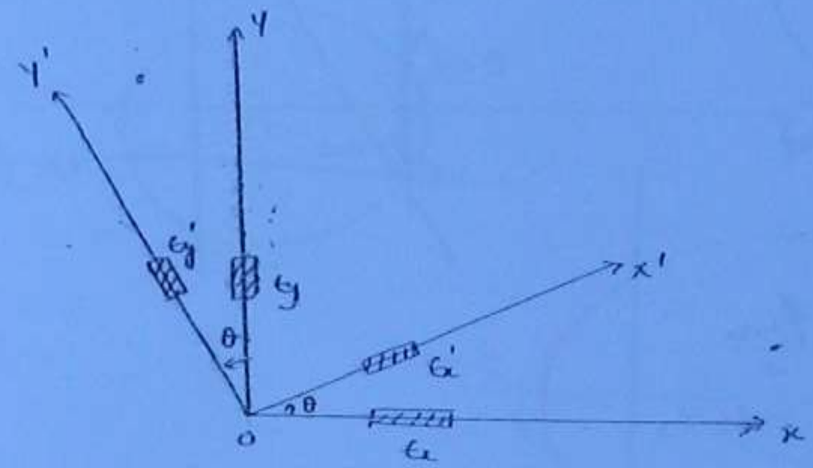
If θ occuppies its position such that shear strain in $x'y'$ plane becomes zero then linear strain in x' & y' dir^n will be max^n or min^n and will be called ~~max^n or min^n~~ principle strains, Hence angle of Pricipal strains from ϵ_x & ϵ_y is given by $\phi_{x'y'} = 0$

$$\boxed{\tan 2\theta = \frac{\phi_{xy}}{\epsilon_x - \epsilon_y}}$$

Principal strain will be

$$\boxed{\epsilon_1/\epsilon_2 = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\phi_{xy}}{2}\right)^2}}$$

Mohr circle for strain :-



ϕ_{xy} = shear strain in x-y plane

$\phi_{x'y'}$ = " " " x'-y' "

$$\epsilon_x' = \left(\frac{\epsilon_x+\epsilon_y}{2}\right) + \left(\frac{\epsilon_x-\epsilon_y}{2}\right)\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta \quad \text{—(i)}$$

$$\frac{\phi_{x'y'}}{2} = -\frac{(\epsilon_x-\epsilon_y)}{2}\sin 2\theta + \frac{\phi_{xy}}{2}\cos 2\theta \quad \text{—(ii)}$$

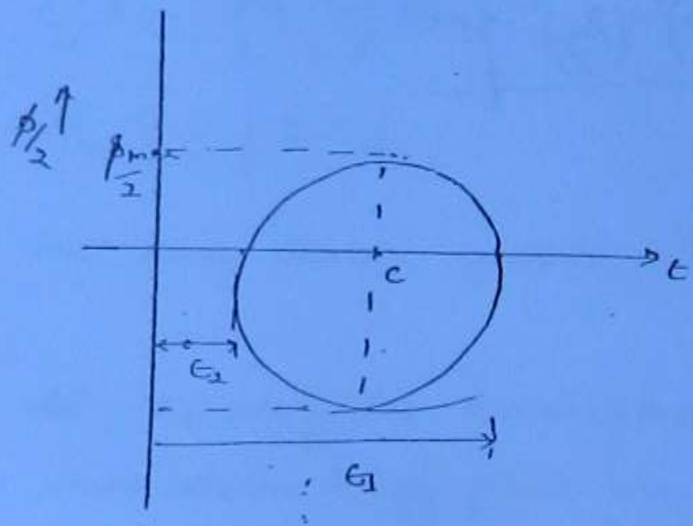
$$\left[\epsilon_x' - \left(\frac{\epsilon_x+\epsilon_y}{2}\right)\right]^2 + \left[\frac{\phi_{x'y'}}{2}\right]^2 = \left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2$$

$$[x-a]^2 + y^2 = r^2$$

Centre $(a, 0)$

$a = (\epsilon_x + \epsilon_y)$

Radius $r = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\left(\frac{\phi_{xy}}{2}\right)^2}$



Radius of circle $= r = \frac{\phi_{max}}{2}$

Dia of circle $= \phi_{max} = \sigma_1 - \sigma_2$

Centre is $= \frac{\sigma_1+\sigma_2}{2} = \frac{\sigma_x+\sigma_y}{2}$

Q. Using Mohr circle method find the stresses acting on a plane which is inclined at 22.5° with vertical axis as shown in fig. Also compute orientation of Principle Plain and mag. of Principal stresses.

Sol:-

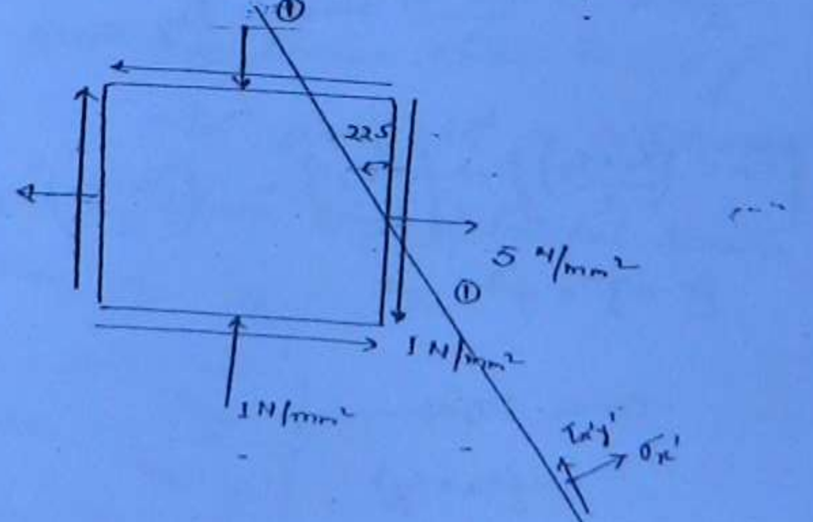
$\sigma_x = 5$

$\sigma_y = -1$

$\tau_{xy} = 1$

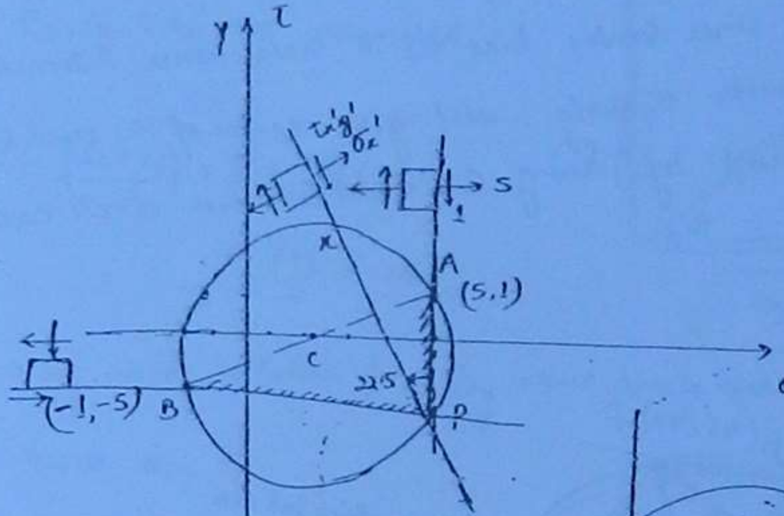
Centre = $(a, 0)$

$a = \frac{\sigma_x+\sigma_y}{2} = \frac{5-1}{2} = 2$

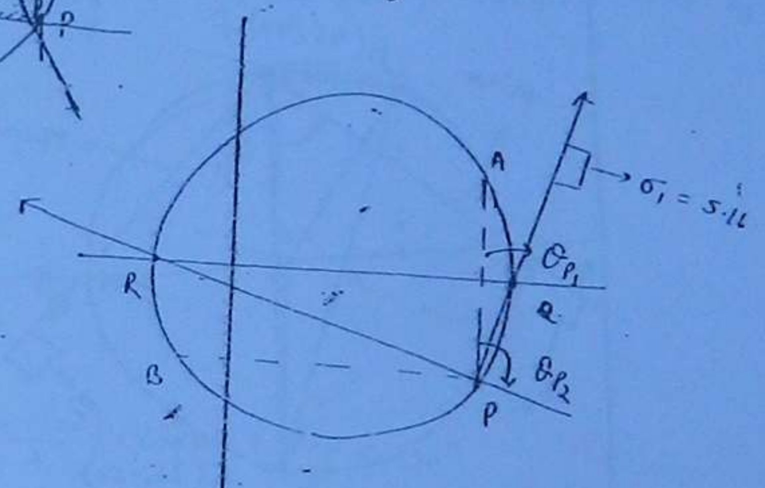


$$\text{Radius} = r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{5+1}{2}\right)^2 + 1^2} = 3.16$$



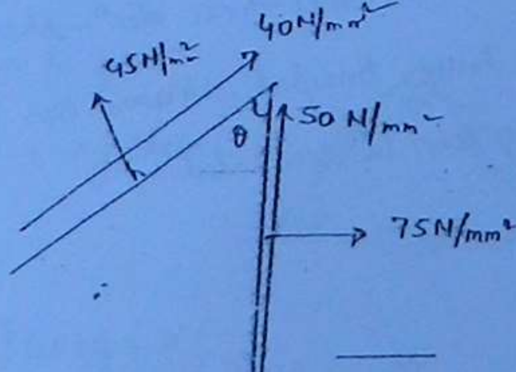
Co-ordinate of x are $\sigma_x' + \tau_{x'y}$



Point is right side of origin so tensile stress

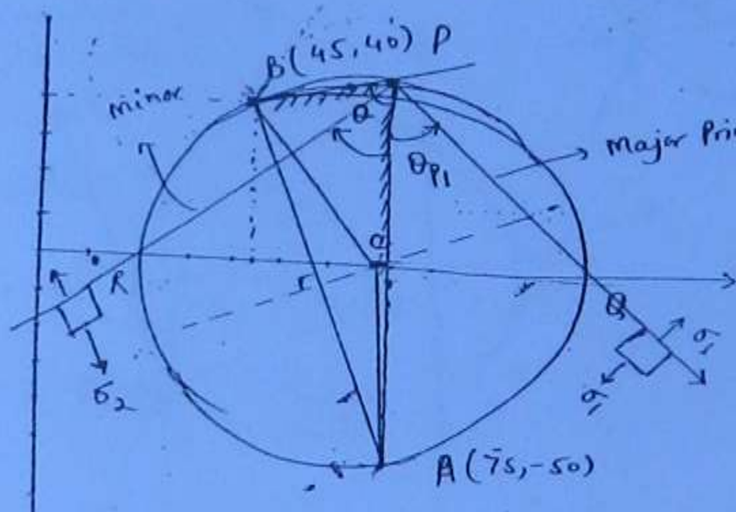
Q: At a point in material subjected to 2-D stresses the stresses on a certain plain are $75\ N/mm^2$ tensile & $50\ N/mm^2$ shear and on the other plain which is an angle θ. the stresses are $45\ N/mm^2$ tensile and $40\ N/mm^2$ shear as shown in fig. find. Principal stresses and angle b/t two Planes

Solⁿ:-



Let Point A represents Normal and shear stress on verticle Plane on the circle.

Let Pt. B represents Normal " " " " circle on inclined Plain. AB will be Chord of circle. Hence $\perp^r$ bisector of AB will pass thru the centre of circle and since Centre lies at σ axis hence Intersection of $\perp^r$ bisector of chord with σ axis will give centre of the circle (C). Hence radius can be found by joining CA and CB hence circle can be drawn.



Rossette :-

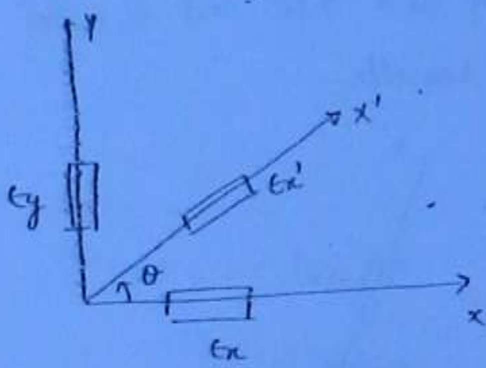
It is an arrangement of 3 linear strain gauges in which, linear strain is measured. in any three dir^n.

Using linear strain in the three dir^n shear strain can be computed and hence pricipal strains can be computed and principal streses can be computed.

special case I:

when two linear strain gauges are at 90° and third is at any angle θ. then it is called rectangular Rossette

ϵ_x, ϵ_y & ϵ_x' are measured.



$$\epsilon_x' = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

using above relation find ϕ_{xy} which can be used to compute θ ϵ_1 & ϵ_2 and hence σ_1, σ_2.

If $\epsilon_x = \epsilon_0$

$\epsilon_y = \epsilon_{90}$

$\epsilon_x' = \epsilon_{45}$ $\qquad \theta = 45°$

$$\epsilon_{45} = \left(\frac{\epsilon_0 + \epsilon_{90}}{2}\right) + \left(\frac{\epsilon_0 - \epsilon_{90}}{2}\right) \times \cos 90 + \frac{\phi_{xy}}{2}\sin 90°$$

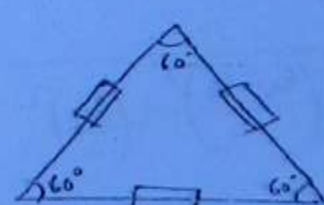
$$\boxed{\phi_{xy} = 2\epsilon_{45} - (\epsilon_0 + \epsilon_{90})}$$ obj

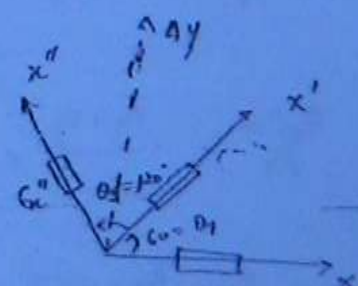
Sp. CASE II:

If ~~Rossettes~~ gauges are placed at 60° from each other then it is called Δ Rossette.

if ϵ_y is Linear strain in y dir^n

& ϕ_{xy} " shear " " xy plain





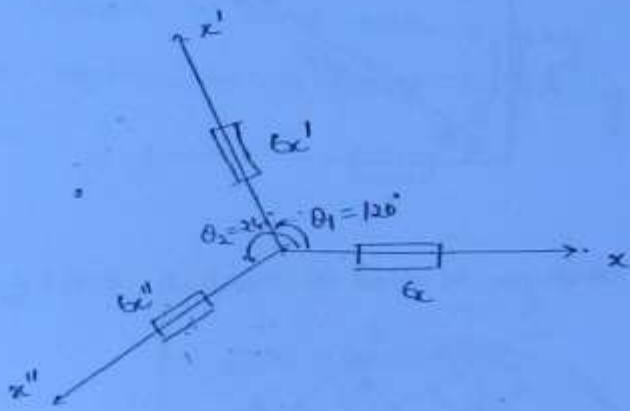
$$\epsilon_x' = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta_1 + \frac{\phi_{xy}}{2}\sin 2\theta_1 \quad \text{—(i)} \qquad \theta_1 = 60°$$

$$\epsilon_x'' = \left(\frac{\epsilon_x + \epsilon_y}{2}\right) + \left(\frac{\epsilon_x - \epsilon_y}{2}\right)\cos 2\theta_2 + \frac{\phi_{xy}}{2}\sin 2\theta_2 \quad \text{—(ii)} \qquad \theta_2 = 120°$$

from eq^n (i) & (ii) $-\epsilon_y$ and ϕ_{xy} can be calculated.

Special case (ii):

If $\theta_1 = 120°$ and $\theta_2 = 240°$ in above case then it is called star rosette.



Ques:-

The strain measurement from a rectangular rosette where $\epsilon_0 = 600\times10^{-6}$ $\epsilon_{45} = 500\times10^{-6}$ & $\epsilon_{90} = 200\times10^{-6}$, magnitude and dir^n of principal strain if $E = 2\times10^{5}\ N/mm^2$ & $\mu = 0.3$ then find principal stresses

sol^n

$$\phi_{xy} = 2\epsilon_{45°} - (\epsilon_0 + \epsilon_{90})$$

$$= 2\times500\times10^{-6} - (600+200)\times10^{-6}$$

$$\phi_{xy} = 200\times10^{-6}$$

$$\epsilon_1/\epsilon_2 = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= 400\times10^{-6} \pm \sqrt{\left(\frac{400\times10^{-6}}{2}\right)^2 + \left(\frac{200}{2}\times10^{-6}\right)^2}$$ 223.60×10^{-6}

$400\times10^{-6} \pm 282.84\times10^{-6}$ $= 682.84\times10^{-6}$ $= \epsilon_1 = 623.6\times10^{-6}$

$= 117.15\times10^{-6}$ $= \epsilon_2 = 176.39\times10^{-6}$

Angle of principal strain.

$$\tan 2\theta = \frac{\phi_{xy}}{\epsilon_x - \epsilon_y}$$

$$= \frac{200}{600-200}$$

$$\theta = 13.28$$

$$\sigma_1 = \frac{E}{1-\mu^2}(\epsilon_1 + \mu\epsilon_2)$$

$$= 148.69\ N/mm^2$$

$$\sigma_2 = \frac{E}{1-\mu^2}(\epsilon_2 + \mu\epsilon_1)$$

$$= 79.08\ N/mm^2$$

ES'2001, '15

Ques In a stressed member 2 strain gauges are fixed such that they are inclined at 30° to the known dir^n of principal stresses. the strains measured in these two gauges are +445 x 10^{-6} and -32×10^{-6} resp. if, $E = 2.1\times10^5\ N/mm^2$ and $\mu = 0.3$ then determine p magnitude of principal stresses.?

Sol^n. Consider principal strains are in x, y dir^n hence the measured strains will be at 30° from x & y dir^n.

Let measured strains are in x and y dir^n.

$$\epsilon_x' = 445\times10^{-6}$$

$$\epsilon_y' = -32\times10^{-6}$$

$$\epsilon_x' = \frac{\epsilon_1+\epsilon_2}{2} + \frac{\epsilon_1-\epsilon_2}{2}\cos 2\theta \quad ; \theta = 30°$$

$$3\epsilon_1 + \epsilon_2 = 1780\times10^{-6} \quad \text{--- (i)}$$

$$\epsilon_1 + \epsilon_2 = \epsilon_x' + \epsilon_y' = 413\times10^{-6} \quad \text{--- (ii)}$$

From (I) & (II)

$\epsilon_1 = 683.5 \times 10^{-6}$

$\epsilon_2 = -270.5 \times 10^{-6}$

$$\sigma_1 = \frac{E}{1-\mu^2}\left(\epsilon_1 + \mu\epsilon_2\right) = 139 \cdot N/mm^2$$

$$\sigma_2 = \frac{E}{1-\mu^2}\left(\epsilon_2 + \mu\epsilon_1\right) = -15.1\ N/mm^2$$

method:

Let given strains are linear strains in x and y dir^n and let ϕ_{xy} is Shear strain Hence angle of principal strain in 30° from x and y dir^n

Hence

$$\tan 2\theta = \frac{\phi_{xy}}{\epsilon_x - \epsilon_y}$$

$$\phi_{xy} = 477 \times 10^{-6} \times \sqrt{3}$$

$$= 826.18 \times 10^{-6}\ N/mm^2$$

$$\epsilon_1/\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$\epsilon_1 = 683.5 \times 10^{-6}$

$\epsilon_2 = -270.5 \times 10^{-6}$.

Theories of Failure :-

generally there are two modes of failure in metals depending on strength criteria

1) yeild failure or ductile failure
2) fracture failure or Brittle failure.

In theories of failure there is no consideration for stiffness criteria, Serviceability, local buckling., local crippling, function failure or fatigue Contingency failure.

The failure criteria is based on strength criteria.

1) Max^m Principal stress Theory / Rankine's Theory :-

- For no failure the max^m principal stress develope (σ_1) in a loaded Body Should be less than or equal to max yeild stress in uniaxial loading.

$$\sigma_1 \le \sigma_y$$

If F.O.S is taken then $\left[\text{permissible stress } \sigma_1 \le \left(\sigma_y / \text{fos}\right)\right]$ for design.

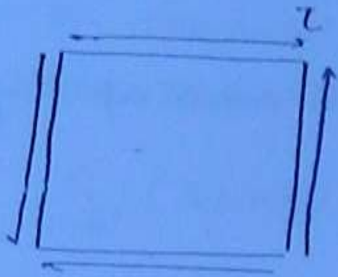
NOTE. If max^m principal stress is tensile then σ_y should be taken in tension and if σ_1 is Compressive then σ_y should be taken in Compression

Limitations:-

1) This theory is suitable for brittle metals but not suitable for ductile metal.
2) There is no consideration of other principal stress (σ_2 & σ_3)
3) It is not suitable for case of hydrostatic loading (equal and alike stress)
4) For the case of Pure shear in ductile metals, results are ~~unsfe~~ Unsafe

Ductile metals when tested in pure shear have shear strength nearly 57% of tensile strength (σ_y) but according to the above theory following

Results are found



(Pure shear)

$$\sigma_1 = +\tau$$
$$\sigma_2 = -\tau$$

According to theory for no failure

$$\sigma_1 \le \sigma_y$$
$$\tau \le \sigma_y$$

$$\boxed{\frac{\tau_{max}}{\sigma_y} \le 1}$$

Practical results $\boxed{\tau_{max} \le 0.57\,\sigma_y}$

Results of this theory are is unsafe for pure shear.

II). Max^m Principal strain theory or St. Vennant's Theory :→

Though It can be applied for Ductile and Brittle metals both but results of this theory are not accurate for any case.

Criteria.

According to this theory for no failure max^m Pricipal strain in the loaded body should be less than or equal to yeild strain at uniaxial loading

For No failure

$$\epsilon_1 \le \sigma_y/E$$

For Design

$$\epsilon_1 \le \left(\frac{\sigma_y/f.o.s}{E}\right)$$

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} \quad \text{for 3-D loading}$$

$$= \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \quad \text{for 2D - "}$$

Though there is consideration for σ_2 and σ_3 also but results are still unsafe for pure shear however better than previous

For pure shear.

$$\sigma_1 = +\tau$$

$$\sigma_2 = -\tau$$

$$\epsilon_1 \le \sigma_y/E$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \le \sigma_y/E$$

$$\sigma_1 - \mu\sigma_2 \le \sigma_y$$

$$\tau - \mu(-\tau) \le \sigma_y$$

$$\tau(1+\mu) \le \sigma_y$$

$$\tau \le \sigma_y/(1+\mu)$$

If $\mu = 0.3$

$$\tau \le \sigma_y/1.3$$

$$\boxed{\tau \le 0.77\,\sigma_y}$$

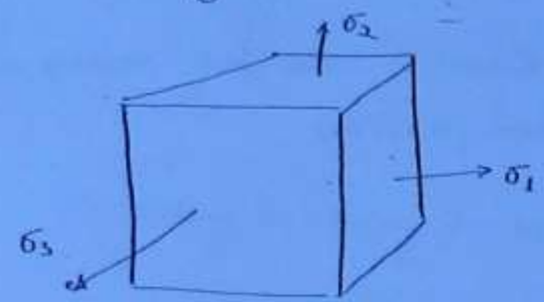
which is greater than $0.57\,\sigma_y$

Hence unsafe

iii) Max^m shear stress Thy / Tresca Th / J.J. Guest Thy :-

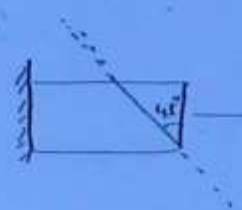
This is ~~bs~~ for ductile metal, Results of this theory are safest among all the theory.

- According to this theory for No failure max^m shear stress developed in a strained body should be less than or equal to max^m sh. stress in uniaxial loading at yield point.



For No failure; $\tau_{max} \le \sigma_y/2$ —— (i)

<u>For uniaxial.</u>



$\sigma_1 = \sigma_y$.

$$\tau_{max} = \frac{\sigma_1 - \cancel{\sigma_2}^{\,0}}{2} = \frac{\sigma_y}{2}$$

For Design.

$$\tau_{max} \le \frac{(\sigma_y / f.o.s)}{2}$$

In loaded body τ_{max} is greater of the following

$$\frac{\sigma_1 - \sigma_2}{2}, \ \frac{\sigma_2 - \sigma_3}{2}, \ \frac{\sigma_1 - \sigma_3}{2} \qquad \text{(For 3-D loading)}$$

For 2-D loading.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}.$$

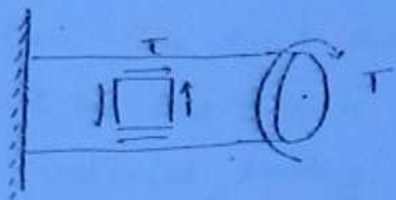
<u>Limitation:</u>

- This theory is not applicable when principal stresses are equal and alike.

2). For hydrostatic loading such as spherical pr. vessels this theory is not applicable.

3). for the case of pure shear Results of this theory are over safe

4).



$\sigma_1 = +\tau$

$\sigma_2 = -\tau$

According to this theory;

$\tau_{max} \le \sigma_y/2$

$\frac{\sigma_1 - \sigma_2}{2} \le \sigma_y/2$

$\sigma_1 - \sigma_2 \le \sigma_y$

$\tau - (-\tau) \le \sigma_y \Rightarrow 2\tau \le \sigma_y$

$\boxed{\tau \le \sigma_y/2}$. $\boxed{\tau \le 0.57\sigma_y}$ * Pract

Hence theoretical results are oversafe side

Max^m strain energy theory / Haigh's and Beltrami Theory:-

According to this theory for No failure the max^m strain energy developed in the loaded by should be less than or equal to max^m strain energy at yield point in uniaxial loading.

This theory is suitable for ductile metals.

For No. Failure $U \le \frac{\sigma_y^2}{2E}$

for - 3D $\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \le \sigma_y^2/2E$

For 2D. $\sigma_3 = 0$

$$\boxed{\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2)\right] \le \frac{\sigma_y^2}{2E}}$$

For design,

$$u \le \frac{\left(\sigma_y / f.o.s\right)^2}{2E}$$

NOTE:
1. This theory is not suitable for Brittle Metal.
2. For the case of pure shear in ductile " . results are little unsafe but better than principal stress theory and Principal strain theory.

Let

$$\sigma_1 = +\tau$$

$$\sigma_2 = -\tau$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\right] \le \frac{\sigma_y^2}{2E}$$

$$\frac{1}{2E}\left[\tau^2 + \tau^2 + 2\mu\tau^2\right] \le \sigma_y^2/2E$$

$$\Rightarrow \quad \tau \le \frac{\sigma_y^2}{\sqrt{2(1+\mu)}}$$

If $\mu = 0.3$

$$\tau \le \frac{\sigma_y^2}{\sqrt{2(1.3)}} \Rightarrow \boxed{\tau \le 0.62\sigma_y}$$

little unsafe

5) Max^m shear strain Energy Theory / Distortion Eng thy or Von Mises Hanky Th

It is most suitable theory for Ductile metal and is quite accurate for case of pure shear.

For no failure The total shear strain energy stored in a loaded body should be less than or equal to shear strain energy at yield point in uni axial loading.

$$U_s \le \frac{\sigma_y^2}{6G} \quad \text{.....} \quad \text{(for No failure)}$$

$$U_s \le \frac{(\sigma_y / f.o.s)^2}{6G} \quad \text{.....} \quad \text{(for design)}$$

U = Total strain energy per unit volume =

$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

Volumetric strain energy / vol. $= \frac{1}{2}\sigma_{avg} \times \epsilon_v$

$$U_v = \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right) \times \left(\frac{\sigma_1+\sigma_2+\sigma_3}{E}\right)(1-2\mu)$$

U_s = Shear strain energy / vol

$$U = U_s + U_v$$

$$U_s = U - U_v$$

$$U_s = \frac{\mu+1}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$$U_s = \frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

Espl. Case for 2D loading :- $\sigma_3 = 0$

$$U_s = \frac{1}{12G}\left(\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2+\sigma_2^2+\sigma_1^2\right)$$

$$U_s = \frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right]$$

Esp. case 2:

For uniaxial loading at yield

$$U_s = \frac{\sigma_y^2}{6G} \qquad \sigma_1 = \sigma_y, \quad \sigma_2 = 0.$$

Check for pure shear;

$$\sigma_1 = \tau, \quad \sigma_2 = -\tau, \quad \sigma_3 = 0.$$

$$\frac{1}{6G}\left[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right] \le \frac{1}{6G}\sigma_y^2$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \sigma_y^2$$

$$\tau^2 + \tau^2 + \tau^2 \le \sigma_y^2$$

$$\tau \le \sigma_y/\sqrt{3}.$$

$$\boxed{\tau \le .57\,\sigma_y}$$

which is same as test results.

:) Octahedral shear stress theory :→

Octahedral planes are those planes which are equally inclined to principal planes there are 8 such planes which construct an octahedral. The shear stress along the planes is called octahedral shear stress and the normal of such plane makes an angle θ from the normal of principal planes.

The θ is such that;

$$\cos\theta = 1/\sqrt{3}$$

The value of octahedral shear stress in a 3-D loaded body is given as

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

For 2-D case:

$$\tau_{oct} = \frac{\sqrt{2}}{3}\left[\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2\right]^{1/2}$$

for 1-D at yeild point

$$\sigma_1 = \sigma_y \quad (\sigma_2 = 0)$$

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_y$$

$$\boxed{\tau_{oct} = 0.47\sigma_y}$$

For No failure octahedral shear stress in a strained body should be less than or equal octahedral shear stress at yeild point in uniaxial loading.

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_y \longrightarrow \text{For No failure}$$

$$\tau_{oct} = \frac{\sqrt{2}}{3}\left(\sigma_y/f.o.s\right) \longrightarrow \text{For Design}$$

* This theory is also suitable for ductile metals and its results are accurate for case of pure shear

Refer Book for graph of theory.

Ques:

The principal stresses at a point in an elastic material are 1.5σ (tensile), σ (tensile) & 0.5σ (Compressive) the elastic limit in tension is 210 mPa and '$\mu = 0.3$, what should be the value of σ at failure when computed according to —

i) Tresca. Theory.

ii) Max^m. Strain energy Theory.

Sol^n:—

$\sigma_1 = +1.5\sigma$

$\sigma_2 = +\sigma$

$\sigma_3 = -0.5\sigma$

Max^m Shear Stress theory

$\tau_{max} \leq \sigma_y/2$

$\tau_{max} \Rightarrow \frac{1.5\sigma - \sigma}{2} = \frac{.5\sigma}{2}$ or $\frac{1.5\sigma + 0.5\sigma}{2} = \sigma$

$\tau_{max} = \sigma$

$\sigma \leq \frac{\sigma_y}{2}$

$\sigma \leq \frac{210}{2}$

$\boxed{\sigma \leq 105}$

Max^m Strain energy Th:

$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_3\sigma_1)\right] \leq \frac{\sigma_y^2}{2E}$

$(1.5\sigma)^2 + \sigma^2 + (.5\sigma)^2 - 2\times.3\left(1.5\sigma^2 - .5\sigma^2 + .75\sigma^2\right)$

$2.25\sigma^2 + \sigma^2 + .25\sigma^2 - .15\sigma^2$

$3.35\sigma^2 \leq \sigma_y^2$

$$\sigma \le \frac{\sigma_y}{\sqrt{3.35}}$$

$$\boxed{\sigma \le 114.735}$$

Ques A solid steel circular shaft is required to carry a Torque of 40 kN. and B.M. of 20 kN-m. Det^n. the suitable size of the shaft using

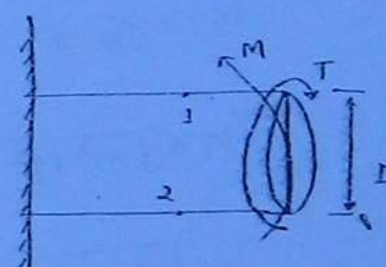
i) max^m Normal stress theory
ii) " Principal strain "

$F.O.S = 2.0$, $E = 2 \times 10^5 \ N/mm^2$. $\sigma_y = 250 \ N/mm^2$

$\mu = 0.3$,

1) Max^m Normal stress theory:

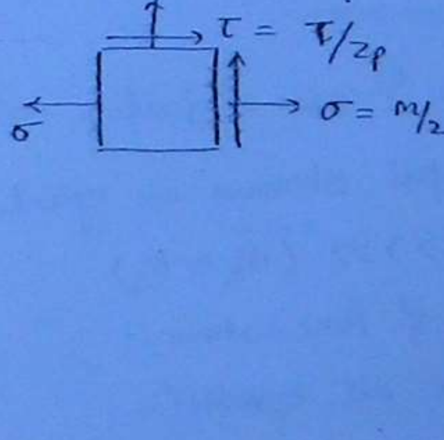
$$\boxed{\sigma_1 \le \sigma_y}$$



$$\tau_{max} = \frac{T}{Z_p} \quad \text{— (i)}$$

$$\sigma_{max} = \frac{M}{Z} \quad \text{— (ii)}$$

Consider Top surface element or Bottom surface element where Bending and shear stress will be max^m.



$$\sigma = \frac{M}{Z} = \frac{32M}{\pi D^3}$$

$$\tau = \frac{T}{Z_p} = \frac{16T}{\pi D^3}$$

$$\sigma_1/\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\sigma_1/\sigma_2 = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]}$$

According to max^m principal stress th^y:-

$$\sigma_1 \leq \sigma_y / F.o.s$$

$$\Rightarrow \frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] \leq \frac{\sigma_y}{2} \qquad F.o.s = 2$$

$$\Rightarrow \frac{16}{\pi D^3}\left[20 \times 10^6 + \sqrt{20^2+40^2} \times 10^6\right] \leq \frac{250}{2}$$

$$\cancel{D \geq 13.8} \quad \boxed{D \geq 138.5\ mm}$$

According to max^m principal strain theory:-

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \leq \frac{\sigma_y/fo.s}{E}$$

$$\sigma_1 - \mu\sigma_2 \leq \sigma_y / Fos$$

$$\frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] - \frac{\mu \cdot 16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] \leq 250/2$$

$$\boxed{D \geq 143.26\ mm}$$

NOTE :-

The size of a shaft according to max^m shear stress theory will be 153.8 mm hence results of this theory will be safest.

Q. All the Theories of failure will give nearly same result -

a) Principal stresses are equal in value
b) $\sigma_1 >>>> (\sigma_2 \text{ or } \sigma_3)$
c) Case of Pure shear.
d) Under all condition

Que: In a 2-D. stress system the 2. Principal stresses are $\sigma_1 = 180\ N/mm^2$ (tens) and σ_2 = Compr. If σ_y is = $240\ N/mm^2$ in tension and compr. both and poission ratio = 0.25, then according to max^n ~~shea~~ Normal ~~shear~~ strain theory. Find σ_2 ?

Sol^n

$$\frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} \leq \frac{\sigma_y}{E}$$

$$180 + \mu\sigma_2 \leq 240$$

$$\mu\sigma_2 \leq 60$$

$$\sigma_2 \leq 240$$

$-240 \leq 180$
420
$.25$

a) $240\ N/mm^2$ b) $135\ N/mm^2$ c) $180\ N/mm^2$ d) $165\ N/mm^2$

If max^n principal strain is tensile

$$\sigma_1 + \mu\sigma_2 \leq 240$$

$$180 + \mu\sigma_2 \leq 240$$

$$\sigma_2 \leq 240 \text{ (Compr)}$$

If max^n principal strain is compressive

$; -\frac{1}{3} \leq -\frac{1}{4}$

$+.33 \leq +.25$

$\geq$

$\geq$

$$-\frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} \leq -\frac{240}{E}$$

$$\sigma_2 \geq 240 - .25 \times 180$$

$$\sigma_2 \geq 135\ N/mm^2$$

Q. Failure of Ductile metal is best ?

a). Principal stress criteria

b) " strain "

c) Shear strain engy cr

Q. A cube is subjected to equal tensile stresses on all three faces and if the yield stress of material is σ_y, then based on strain engy theory the max^m tensile stress will be

a) $\dfrac{\sigma_y}{\sqrt{3(1-2\mu)}}$

c) $\dfrac{\sigma_y}{\sqrt{3(1+2\mu)}}$

b) $\dfrac{\sigma_y}{\sqrt{3(2-\mu)}}$

d) $\dfrac{\sigma_y}{\sqrt{3(1+\mu)}}$

Sol^n.

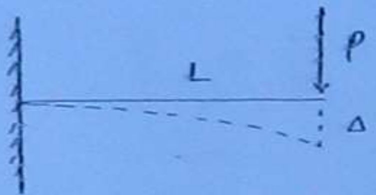
$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right] \le \frac{\sigma_y^2}{2E}$$

$$\sigma^2+\sigma^2+\sigma^2-2\mu(\sigma^2+\sigma^2+\sigma^2) \le \sigma_y^2$$

$$3\sigma^2(1-2\mu) \le \sigma_y^2$$

$$\boxed{\sigma \le \frac{\sigma_y}{\sqrt{3(1-2\mu)}}}$$

Deflection of Beams :-



$\Delta = \dfrac{PL^3}{3EI}$ (either due to Bending or shear)

EI = Flexural rigidity.

$\dfrac{EI}{L}$ = " . Stiffness

The transverse deflection in Beam may be caused by
1) Bending moment
2) Shear force.

The deflection caused by Bending moment is much greater than shear deflection, therfore for all practical resion of shear deflection is negligble.

Ratio of $\dfrac{\text{Bending def}^n}{\text{Shear def}^n} = K\left(\dfrac{L}{d}\right)^2$; L = Span or length of Beam.

d = depth of Beam

$(L >>> d)$

The max^m deflection developed in the finished Beam should be with in permissible limit and according to BIS.

$\Delta_{max} < L/325$

L is clear span b/t supports.

End Conditions at supports :-

i) 

Fixed end

$\delta_A = 0$ $\qquad$ $\delta_B \neq 0$

$\theta_A = 0$ $\qquad$ $\theta_B \neq 0$

ii)

$\delta_A = 0$ $\qquad\qquad$ $\delta_B = 0$

$\theta_A \neq 0$ $\qquad\qquad$ $\theta_B \neq 0$

ii)

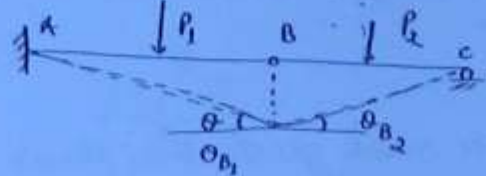
$\Delta x \neq 0$

$\Delta y = 0$

$\theta_B = 0$

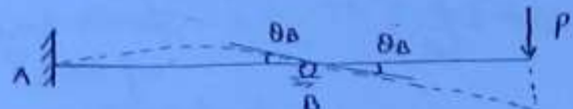
Double rollar support

i)



At internal Hinge there will be diff slope on either side.

i)



Internal rollar

$\Delta B = 0$

$\theta_B \neq 0$

$$\frac{dy}{dx} = \tan\theta \simeq \theta \quad (\text{if } \theta \text{ is small})$$

Std. Results for slope and deflection :→

$$\frac{M}{I} = \frac{E}{R} \Rightarrow M = \frac{EI}{R} \propto 1/R$$

$$\theta_B = \frac{ML}{EI}$$

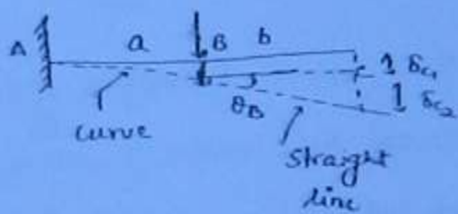
$$\delta_B = \frac{ML^2}{2EI}$$

CASE (ii)

$$\theta_B = \frac{PL^2}{2EI}$$

$$\delta_B = \frac{PL^3}{3EI}$$

CASE iii



$$\delta_c = \delta_{c_1} + \delta_{c_2}$$

$$\theta_B = \frac{P \cdot a^2}{2EI}$$

$$\delta_B = \frac{P \cdot a^3}{3EI}$$

& $\theta_c = \frac{P \cdot a^2}{2EI}$

$$\delta_c = \delta_{c_1} + \delta_{c_2}$$

$$= \frac{P a^3}{3EI} + \frac{P \cdot a^2 \cdot b}{2EI}$$

Special Case when $a = b = L/2$

$$\delta_B = \cancel{\frac{P\frac{L}{2}^3}{3EI}} \quad \frac{P \cdot L^3}{24 EI}, \qquad \delta_c = \frac{5}{48} \frac{PL^3}{EI}$$

$$\theta_B = \theta_c = \frac{PL^2}{8EI}$$

Case iv) :-

$$\theta_B = \frac{wL^3}{6EI}$$

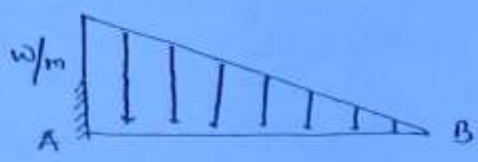
$$\delta_B = \frac{wL^4}{8EI}$$

(v).

$$\theta_B = \frac{wL^3}{6EI} - \frac{wa^3}{6EI}$$

$$\delta_B = \frac{wL^4}{8EI} - \left[\frac{wa^4}{8EI} + \frac{wa^3}{6EI} \cdot b\right]$$

CASE(vi):



$$\theta_B = \frac{wL^3}{24EI}$$

$$\delta_B = \frac{wL^4}{30EI}$$

(vii).

$$\theta_A = \theta_B = \frac{ML}{2EI}$$

$$\delta_c = \delta_{max} = \frac{ML^2}{8EI}$$

In case of supported Beam at Both ends max^m defl^n occurs at the point of zero slope Hence $\underline{\theta_C = 0}$.

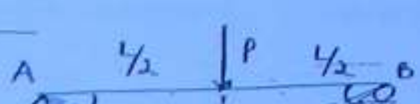
(viii)

$$\theta_A = \frac{ML}{6EI}$$

$$\theta_B = \frac{ML}{3EI}$$

$$\delta_{max} = \frac{ML^2}{9\sqrt{3}\,EI}$$ (at $L/\sqrt{3}$ from left support)

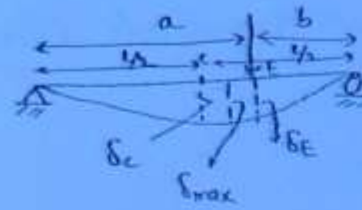
(ix)



$$\theta_A = \theta_B = \frac{PL^2}{16EI}$$

$$\delta_c = \delta_{max} = \frac{PL^3}{48EI}$$

(X)



In simply supported Beam with eccentric loading max^n defl^n occurs b/t centre of span and position of loading

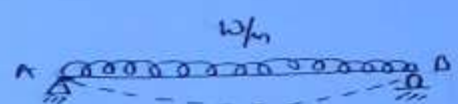
If load is very close to support B then δ_{max} will occur at $L/13$ from centre and when load & P moves toward A and load is closer to support A then δ_{max} occurs at $L/13$ to the left of centre moreover max^n def^n is only 2.5% greater than defl^n at centre when load is close to the support. It means that max^m deflection lies in the range of

$$x = \left(\frac{L}{2} \pm L/13\right) \text{ from support and}$$

δ_{max} is nearly equal to δ_c

The defl^n below the load is $\boxed{\delta_E = \frac{P \cdot a^2 b^2}{3EI \cdot L}}$

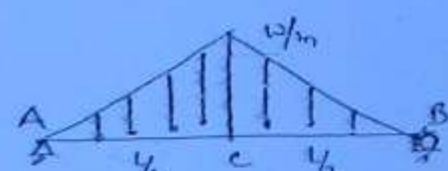
(XI).



$$\theta_A = \theta_B = \frac{wL^3}{24EI}$$

$$\delta_c = \delta_{max} = \frac{5}{384} \cdot \frac{wL^4}{EI}$$

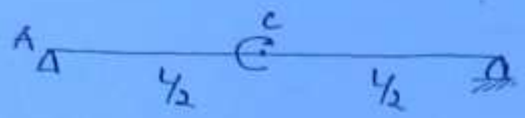
(XII)

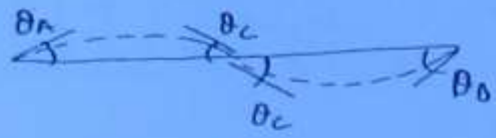


$$\theta_A = \theta_B = \frac{5}{192} \frac{wL^3}{EI}$$

$$\delta_c = \delta_{max} = \frac{wL^4}{120EI}$$

(XIII)

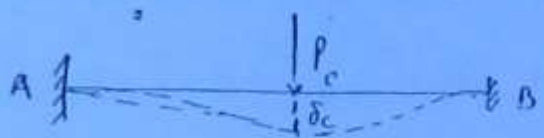




$$\theta_A = \theta_B = \frac{ML}{24EI} \qquad \theta_C = \frac{ML}{12EI}$$

$$\delta_A = \delta_B = \delta_C = 0$$

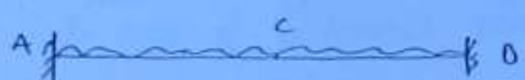
(XIV)



$$\theta_A = \theta_B = \theta_C = 0$$

$$\delta_C = \delta_{max} = \frac{PL^3}{192EI} = \frac{1}{4} \times \text{S.S defl}^n$$

(XV)



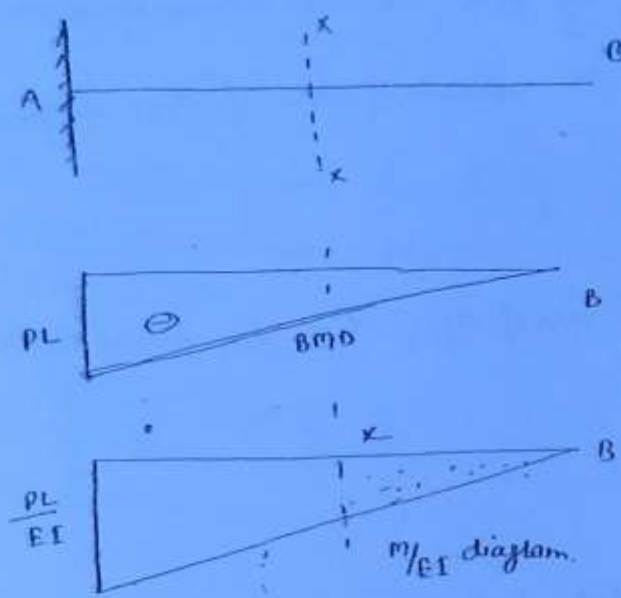
$$\theta_A = \theta_B = \theta_C = 0$$

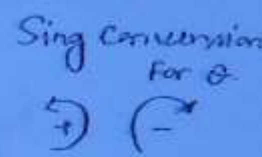
$$\delta_C = \delta_{max}$$

$$= \frac{wL^4}{384EI} = \frac{1}{5}\text{th of S.S defl}^n$$

Maxwell's Reciprocal Thm:-

Area Moment Theorem:-





Theorem-1:-

$$\boxed{\theta_B - \theta_x = \text{Area of } \frac{M}{EI} \text{ diagram b/t } x \text{ \& } B}$$

Special case:

If point of zero slope is known say fixed at A then this mtd. is more convenient. It means change in slope from A to B equal to area of M/EI diagram b/w A & B.

$$\theta_B - \theta_A = \text{Area of } M/EI \text{ diagram b/w A\&B.}$$

$$\theta_B - \cancel{\theta_A}^{0} = -\frac{1}{2} \times L \times \frac{PL}{EI}$$

$$\boxed{\theta_B = \frac{-PL^2}{2EI}}$$

area will be accounted with sign

$+ \rightarrow$ Sagg.

$- \rightarrow$ Hogg.

Theorem 2:→

~~Any~~ The deflection of any point B with respect to tangent at any point x, ~~of B~~ is equal to moment of area of M/EI diagram b/w x & B about B.

$\delta_{B/x}$ (δ_{B_3}) = Moment of area of M/EI dia b/w x & B about B.

$$\boxed{\delta_{B/x}(\delta_{B_3}) = \pm A \cdot \bar{x}}$$

NOTE:

If Reference point x is shifted at A then deflection of Beam with r.t. tangent at A will be equal to moment of area of M/EI diagram b/w A & B about B.

$\delta_{B/A}$ = Mom. of Area of M/EI, dia b/w A & B abt. B.

$$= -\frac{1}{2} \times L \times \frac{PL}{EI} \times \frac{2}{3}L = -\frac{PL^3}{3EI} \downarrow$$

Tangent to A is horizontal Hence total $\Delta_B = \delta_B/_A$.

Defl^n of B. w.r.t. horizontal is

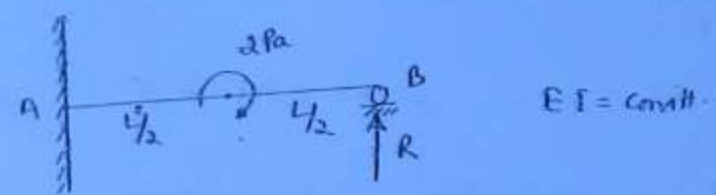
$$\Delta_B = \delta_{B_1} + \delta_{B_2} + \delta_{B_3}$$

$$= \delta_x + \theta_x \cdot L_{xB} + \delta_{B/x}$$

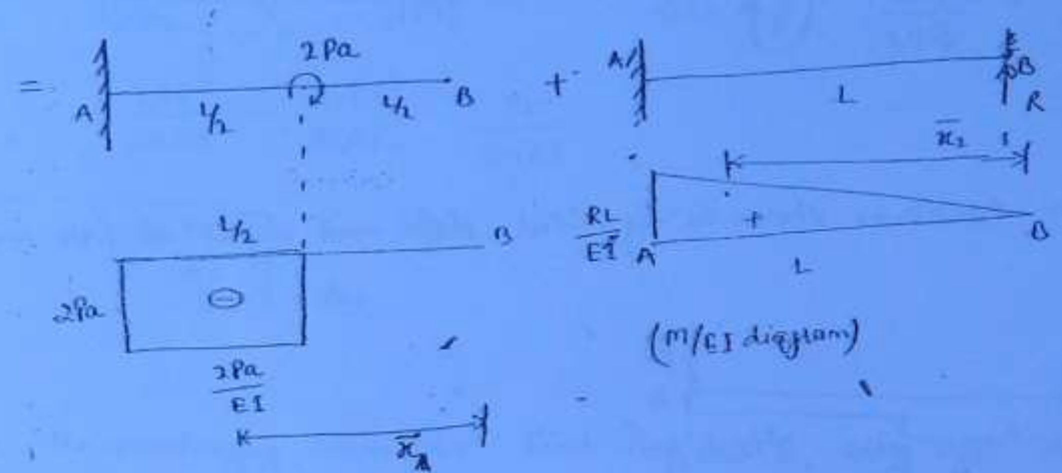
$$\boxed{\Delta_B = \delta_x + \theta_x \cdot L_{xB} + A\bar{x}}$$

* It means that this method is convinent when point of zero $\bar{\text{defl}}^n$ and zero slope is known.

Q. For the cantilever Beam shown in fig find prop. reaction R and slope at B using Area moment theorem.



$EI = \text{const}$.



(M/EI diagram)

defl^n of B w.r.t tangent of A is zero.

$$\delta_{B/A} = 0.$$

= Moment of area of M/EI dia b/w A & B about B.

$$= A_1\bar{x}_1 + A_2\bar{x}_2$$

$$\Rightarrow \left(-\frac{2Pa}{EI} \times \frac{L}{2}\right) \times \frac{3L}{4} + \frac{1}{2}\frac{RL}{EI} \times L \times \frac{2}{3}L = 0$$

$$\boxed{R = \frac{9}{4}\frac{Pa}{L}} \quad (\uparrow)$$

Change in slope from A to B.

= Area of M/EI diagram b/t A & B.

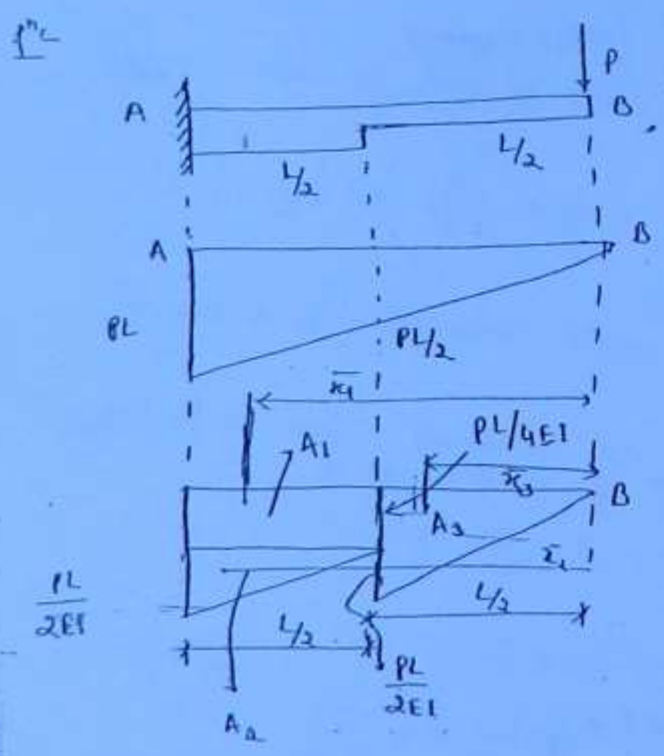
$$\theta_B - \cancel{\theta_A}^{0} = \frac{-2Pa}{EI} \times \frac{L}{2} + \frac{1}{2}\frac{RL}{EI} \times L$$

$$\theta_B = \frac{-\cancel{2}Pa \cdot L}{\cancel{2}EI} + \frac{9/4\, Pa}{2EI}$$

$$= \frac{-Pa \cdot L}{EI} + \frac{9}{8}\frac{Pa}{EI}$$

$$\theta_B = \frac{+Pa}{8EI}$$

Ques) For Cantilever shown in fig find slope and defl^n at free end.



$$A_1 = -\frac{PL}{4EI} \cdot \frac{L}{2} = -\frac{PL^2}{8EI}$$

$$\bar{x}_1 = \frac{L}{2} + \frac{L}{4} = \frac{3L}{4}$$

$$A_2 = -\left(\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI}\right) = \frac{-PL^2}{16EI}$$

$$\bar{x}_2 = \frac{L}{2} + \frac{2}{3} \cdot \frac{L}{2} = \frac{5}{6}L$$

$$A_3 = -\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI} = \frac{-PL^2}{8EI}$$

$$\bar{x}_3 = \frac{2}{3} \cdot \frac{L}{2} = L/3$$

Change in slope from A to B = Area of $\frac{M}{EI}$ dia b/t A & B

$$\theta_B - \cancel{\theta_A}^{\,0} = A_1 + A_2 + A_3$$

$$\boxed{\theta_B = \frac{-5}{16}\frac{PL^2}{EI}}$$

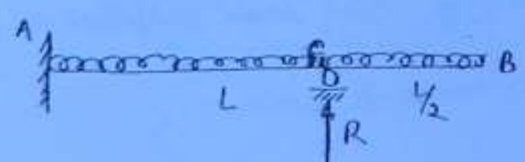
$$\delta_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

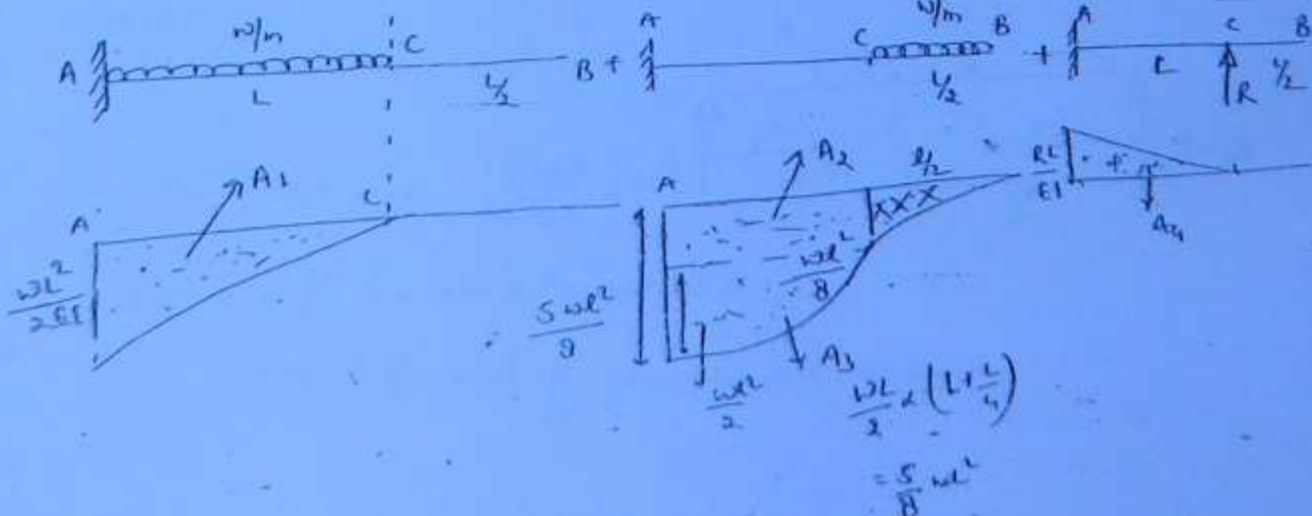
$$= -\frac{PL^2}{8EI} \times \frac{3L}{4} + \frac{PL^2}{16EI} \times \frac{5}{6}L - \frac{PL^2}{8EI} \times \frac{L}{3}$$

$$= -\frac{3PL^3}{32EI} - \frac{5PL^3}{96EI} - \frac{PL^3}{24EI}$$

$$\boxed{\delta_{B/A} = \frac{-3}{16}\frac{PL^3}{EI}}$$ Ans

Q. For the overhanging beam "AB" find Prop Reaction R, using area moment theorem, EI is const.?





Defl^n of c w.r.t tangent at A = 0.

$\delta_{C/A}$ = Mom. of area of M/EI dia b/t A & C about C = 0

$\Rightarrow$

$\Rightarrow \dfrac{-wL^4}{8EI} - \dfrac{wL^4}{16EI} - \dfrac{wL^4}{6EI} - \dfrac{RL^3}{3EI} = 0$

$A_1 = -\dfrac{1}{3} \times L \times \dfrac{wL^2}{2EI}$

$= \dfrac{-wL^3}{6EI}$, $\bar{x}_1 = \dfrac{3}{4}L$

$\dfrac{RL^3}{3EI} = \dfrac{30}{16\times 8 \times 3} + \dfrac{96+48+108}{16\times 8\times 2}\dfrac{wL^4}{EI}$

$A_2 = -\dfrac{wL^2}{8EI} \times L = \dfrac{-wL^3}{8EI}$

$x_2 = L/2$

$R = \dfrac{17}{16} wL$

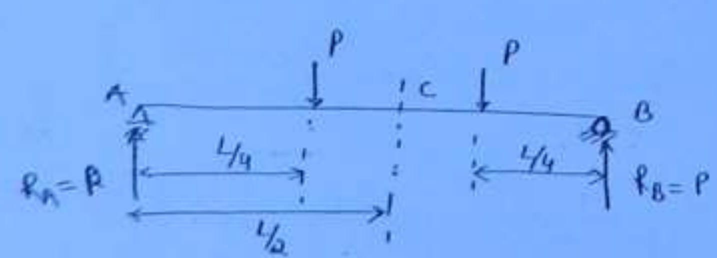
$A_3 = -\dfrac{1}{2} \times \dfrac{wL^2}{2} \times L = \dfrac{-wL^3}{4EI}$

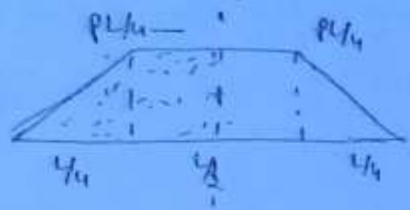
$\bar{x}_3 = 2L/3$

$A_4 = \dfrac{1}{2} \times \dfrac{RL}{EI} \cdot L = \dfrac{RL^2}{2EI}$

$\bar{x}_4 = \dfrac{2L}{3}$

9. For Simply supported beam shown in fig find slope at A & B and defl^n at centre C using Area moment theorem.





Change in slope from A to C. = Area of $\frac{M}{EI}$ dia. b/w A and C.

$$\cancel{\theta_C}^{0} - \theta_A = \left(\frac{1}{2} \times \frac{L}{4} \times \frac{PL}{4EI}\right) + \left(\frac{PL}{4EI} \times \frac{L}{4}\right)$$

$$= \frac{PL^2}{32EI} + \frac{PL^2}{16EI}$$

$$-\theta_A = \frac{3PL^2}{32EI}$$

$$\boxed{\theta_A = -\frac{3}{32}\frac{PL^2}{EI}}$$

$$\boxed{\theta_B = +\frac{3}{32}\frac{PL^2}{EI}}$$ from semmetry

Method I :-

Defl^n $cc' = cc'' - c'c''$

$$cc'' = \theta_A \times L/2 = \frac{3}{32}\frac{PL^3}{EI} \times \frac{L}{2} = \frac{3}{64}\frac{PL^3}{EI}$$

$c'c''$ = defl^n of c w.r.t. tangent at A.

= moment of area of $\frac{M}{EI}$ dia b/w A&C about c

$$c'c'' = \left(+\frac{1}{2} \times \frac{L}{4} \times \frac{PL}{4EI}\right) \times \left(\frac{L}{4} \times \frac{1}{3} \times \frac{L}{4}\right) + \frac{L}{4} \times \frac{PL}{4EI} \times L/8$$

$$= \frac{PL^2}{32EI} \times \frac{L}{3} + \frac{PL^3}{128EI}$$

$$= \frac{7}{384} \frac{PL^3}{EI}$$

Net Def^n = $cc'' - c'c''$

$$= \frac{3}{64}\frac{PL^3}{EI} - \frac{7}{384}\frac{PL^3}{EI}$$

$$\boxed{cc' = \frac{11}{384}\frac{PL^3}{EI}}$$

Method II :-

Defl^n of C i.e. $cc' = BB'$

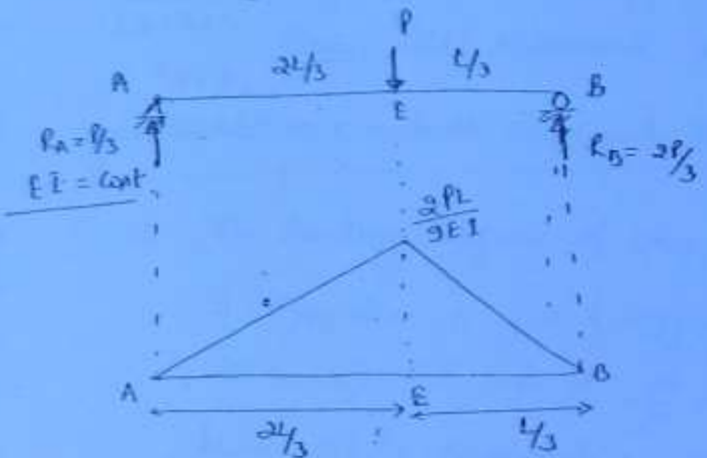
BB' = def^n of B w.r.t tangent at C

= moment of area of $\frac{M}{EI}$ diagram b/t C & B abt B.

$$BB' = \frac{PL}{4} \times \frac{L}{4} \cdot \left(\frac{L}{4} + \frac{L}{8}\right) + \frac{1}{2} \times \frac{L}{4} \times \frac{PL}{4EI} \times \frac{2}{3} \times \frac{L}{4}$$

$$= \frac{PL^3}{16EI}\left[\frac{3}{8} + \frac{1}{12}\right] = \frac{11}{384}\frac{PL^3}{EI}$$

Q: For simply supported beam shown in fig find slope at left support A and defl^n below load, EI = Cont.



$R_A + R_B = P$

$R_A \times L - P \times \frac{L}{3} = 0$

$R_A = P/3$

$R_B = 2P/3$

$\frac{M}{EI}$ diagram.

$BB' = \theta_A \cdot L$ = Defl^n of B' w.r.t. tangent at A.

= moment of area of $\frac{M}{EI}$ diagr b/t A & B about B.

$$\theta_A \cdot L = \frac{1}{2} \times \frac{2L}{3} \times \frac{2PL}{9EI} \times \left(L/3 + \frac{1}{3} \times \frac{2L}{3} \right) + \frac{1}{2} \times \frac{L}{3} \times \frac{2PL}{9EI} \times \frac{2}{3} \cdot L/3$$

$$\boxed{\theta_A = \frac{4PL^2}{81EI}}$$

Def^n of E = EE'

= EE'' − E'E''

= $\theta_A \cdot \frac{2L}{3}$ − (Def^n of E w.r. tangent at A).

$$= \frac{4PL^2}{81EI} \times \frac{2L}{3} - A\bar{x}$$

$$A\bar{x} = \frac{1}{2} \times \frac{2L}{3} \times \frac{2PL}{9EI} \times \frac{1}{3} \times \frac{2L}{3}$$

$$= \frac{4PL^3}{3 \times 81EI}$$

$$= \frac{4PL^3}{243EI}$$

$$EI' = \frac{8PL^3}{243EI} - \frac{4PL^3}{243EI}$$

$$EI' = \frac{4PL^3}{243PL^3}$$

Conjugate Beam Method (Mohr's theorem):

It is applicable for prismatic and non prismatic member both. It is suitable for determinate beam in which it is easy to draw $\frac{M}{EI}$ diagram.. It can be applied for the beam containing internal hinge.

Conjugate Beam is an imaginary beam for which loading diagram is $\frac{M}{EI}$ diagram of given real beam. If given beam is determinate and stable then conjugate beam is also determinate and stable and if given beam is indeterminate then conjugate beam is unstable and if ~~conjugate~~ given beam is unstable then conjugate is indeterminate.

Theorem I:

The slope ~~and~~ at any point on the given beam is equal to shear force at corresponding point in ~~conjgat~~ conjugate Beam. It means S.F.D of conjugate Beam will represent slope curve of real beam.

Theorem II.

Defl^n at any point in real beam is equal to B.m. at correspon-ding point in conjugate Beam. It means B.M.D. of conjugate Beam will represent ~~the~~ defl^n curve of real beam.

guidelines to construct Conjugate Beam :→

1. The loading diagram of Conjugate Beam is M/EI dia. of real Beam. If M/EI dia. is +ve (sagging) then loading in conjugate Beam will be upward and if $\frac{M}{EI}$ dia. is −ve then loading in Conjugate beam will be downward.

2. If shear force at any section in conjugate Beam is +ve then slope at corresponding section in real beam will be +ve (anticlock wise) and vice versa.

3. If B.M at any section in Conjugate Beam is +ve (sagging) then defl^n at corresponding Section in real beam will be also +ve (upwa and vice versa.

4. The end condi^n of conjugate Beam will be modified such that shear force in conjugate beam will correspond to slope in real Beam and Bending moment will correspond to defl^n.

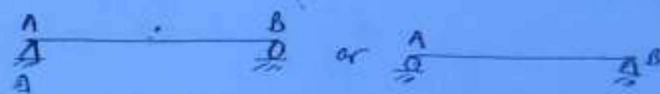
Real Beam Condition

Conjugate Beam Cond^n.

1)

$\theta_A \neq 0$ $\theta_B \neq 0$

$\delta_A = 0$ $\delta_B = 0$



or

$S_A \neq 0$ $S_B \neq 0$ $S_A \neq 0$ $S_B \neq 0$

$M_A = 0$ $M_B = 0$ $M_A = 0$ $M_B = 0$

means simply supported real beam gives simply supported conjugate beam and a hinge support may remain hinge or may become roller and vice versa.

Cantilever

$\delta_A = 0$ $\theta_B \neq 0$

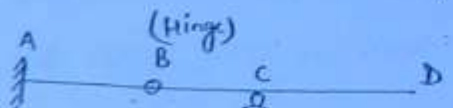
$\theta_A = 0$ $\delta_B \neq 0$

$S_A = 0$ $S_B \neq 0$

$M_A = 0$ $M_B \neq 0$

A cantilever real beam gives a cantilever conjugate beam but fixed end becomes free and free end becomes fixed.

(Hinge)



$\theta_B \to \theta_{B_1}$ (Just to left of B)

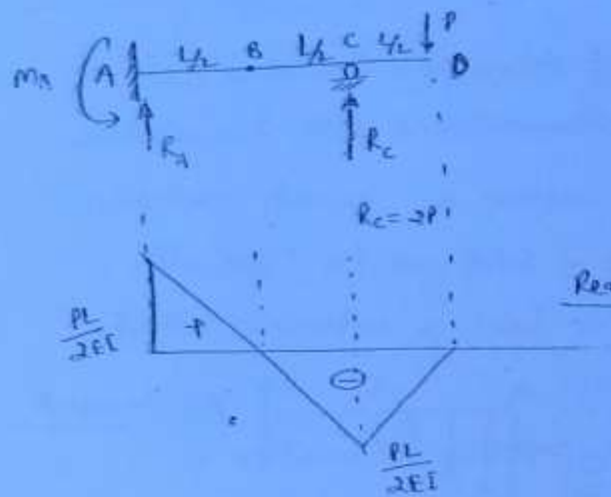
$\theta_B \to \theta_{B_2}$ (Just to right of B)

$S_B \to S_{B_1} \to$ S.F just to left of B

$S_B \to S_{B_2} \to$ S.F " " right of B.

Internal hinge becomes internal roller and internal roller " " hinge. Link also behaves like internal hinge hence link in reality becomes internal roller in conjugate beam.

Ques. for Beam shown in fig find slope at B and defⁿ at free end D.



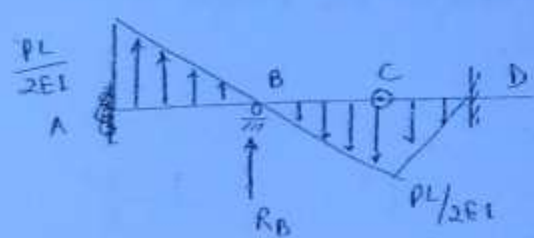
$R_A + R_C = P$

$M_A = R_A \times L/2$

~~$R_A \times L = P \cdot \frac{3L}{2} = 0$~~

Real Beam

Conjugate Beam :-



$M_C = 0$

$$R_B \times \frac{L}{2} + \left(\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI}\right)\left(\frac{L}{2} + \frac{2}{3} \cdot \frac{L}{2}\right)$$

$$- \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI} \times \frac{1}{3} \times \frac{L}{2} = 0$$

$$R_B = \frac{-PL^2}{6EI} \qquad = \frac{PL^2}{48EI}$$

θ_{B_1} = slope just to left of B

= S.F. just to left of B in Conjugate Beam

$$= \frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI} = +\frac{PL^2}{8EI}$$

θ_{B_2} = Slope just to Right of B = S.F just to right in Conju. Beam

$$= +\frac{PL^2}{8EI} + R_B = \frac{PL^2}{8EI} - \frac{PL^2}{6EI} = -\frac{PL^2}{24EI}$$

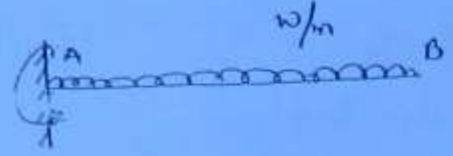
Deflection at D in real Beam = B.M. at D in Conjugate Beam

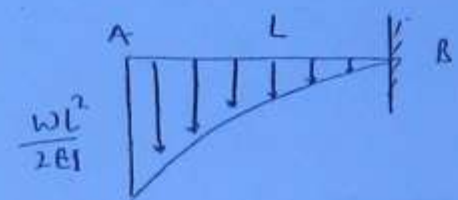
$$\Delta_D = \left(\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{2EI}\right) \times \left(L + \frac{2}{3} \times \frac{L}{2}\right) + R_B \times L - \frac{1}{2} \times L \times \frac{PL}{2EI} \times \frac{L}{2}$$

$$\boxed{\Delta_D = \frac{-PL^3}{8EI}} \downarrow$$

7. Find slope and defln at free end in cantilever loaded with u.d.l

Sol:-





θ_B in Real Beam = S_B in Conj. Beam.

$$= -\frac{1}{3} \times L \times \frac{wL^2}{2EI}$$

$$\theta_B = -\frac{wL^3}{6EI}$$

δ_B in real Beam = M_B in Conjugate Beam

$$= -\frac{1}{3} \times L \cdot \frac{wL^2}{2EI} \times \frac{3}{4} L$$

$$\boxed{\delta_B = -\frac{wL^4}{8EI} \downarrow}$$

Strain Energy Method (Castigliano's theorem):

This method is applicable for prismatic and non prismatic section and determinate and indeterminate cases. It is founde ~~&~~ more suitable for cantilever frames, or arches.

The defl^n at any point in the dir^n of ~~P~~ force P is equal to first partial derivative of total strain energy w.r.t that force

Assumptions:

1) Material is isotropic, homogeneous, linear elastic in which hook's is valid.
2) Temp is constt
3) Supports are unyielding.

$\rightarrow \Delta_B = \frac{\partial U}{\partial P}$

- Δ is in the direction of force

If Δ is +ve then it is in the dir^n of force

" " " -ve " " " opposite of to force

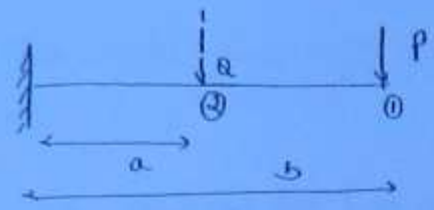
$\Delta = \frac{\partial U}{\partial Q}$

$\therefore \frac{\partial U}{\partial m} = \theta_B$

If deflection is desired at any point where force is not acting then apply a Pseudo load at that point in the dir^n of desired deflection and at the end put Pseudo load = 0

Let defl^n of pt ③ is desired to then apply a ~~psedo~~ pseudo load at ③

find the strain engy. in the term of given loading and applied pseudo load



$$U = f_n(P, Q)$$

$$\Delta_2 = \left(\frac{\partial U}{\partial Q}\right)_{Q=0}$$

Similarly at a point slope is required, say θ_1 is req^r then apply a pseudo moment at point 1, hence $\theta_1 = \left(\frac{\partial U}{\partial m}\right)_{m=0}$. $U = f_n(P, C, m)$

If the strain ~~stress~~ energy is computed due to B.M then $\left(\frac{\partial U}{\partial P}\right)$ will give Bending def^n.

In energy is computed due to shear force then $\left(\frac{\partial U}{\partial P}\right)$ will give shear def^n.

1) S.E due B.M.

$$\Rightarrow U = \int \frac{M_x^2 \cdot ds}{2EI}$$

2) S.E. due to Axial F.

$$\Rightarrow U = \int \frac{P_x^2 \cdot dx}{2AE}$$

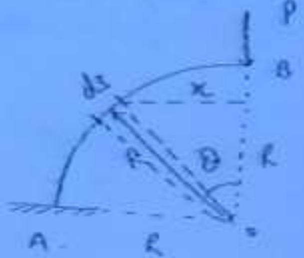
3) S.E due to Torque $\Rightarrow$ $U = \int \frac{T_x^2\, dx}{2G\, I_P}$ ($I_P \rightarrow$ polar m.I)

4). S.E. due to S.F

$$\Rightarrow U = \int \frac{S_x^2 \cdot dx}{2A_r \cdot G} \quad (A_r \rightarrow \text{shear area})$$

Ques :- Find vertical and Horizontal def^n at point B due to a vertical force P in a quarter circular cantilever frame with radius r and constant EI.

Sol^n:→



$$ds = R\,d\theta$$

$$M_x = -P\cdot x$$

$$= -PR\sin\theta$$

$$= \int \frac{M_x^2\,ds}{2EI}$$

$$= \int_0^{\pi/2} \frac{P^2 R^2 \sin^2\theta \cdot R\cdot d\theta}{2EI}$$

$$= \frac{P^2R^3}{2EI}\int_0^{\pi/2} \sin^2\theta\,d\theta$$

$$U = \frac{P^2R^3}{2EI}\left[\frac{\pi}{4}\right]$$

$$\Delta_B = \frac{\partial U}{\partial P}$$

$$\boxed{\Delta_{V_B} = \frac{PR^3}{2EI}\cdot\frac{\pi}{4}}$$

$$M_x = -(Px + Qy)$$

$$M_x = -\left[PR\sin\theta + QR(1-\cos\theta)\right]$$

$$U = \int_0^{\pi/2} \frac{\left\{-\left[PR\sin\theta + QR(1-\cos\theta)\right]\right\}^2 R\,d\theta}{2EI}$$

$$U = \frac{R^3}{2EI}\int_0^{\pi/2} \left(P\sin\theta + Q(1-\cos\theta)\right)^2 d\theta$$

$$\frac{\partial U}{\partial Q} = \frac{R^3}{2EI}\int_0^{\pi/2} 2\,(P\sin\theta + Q(1-\cos\theta)) \times (1-\cos\theta)\,.d\theta$$

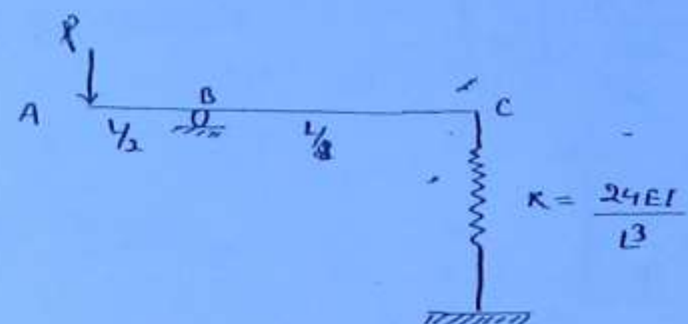
$$\left.\frac{dU}{\partial Q}\right|_{Q=0} = \frac{R^3}{\cancel{2}EI}\int_0^{\pi/2} \cancel{2}\, P\sin\theta\cdot(1-\cos\theta)\,d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}\left[\sin\theta - \frac{\sin 2\theta}{2}\right]d\theta$$

$$= \frac{PR^3}{EI}\left[-\cos\theta + \frac{\cos 2\theta}{4}\right]_0^{\pi/2} \qquad -1+\tfrac{1}{2}$$

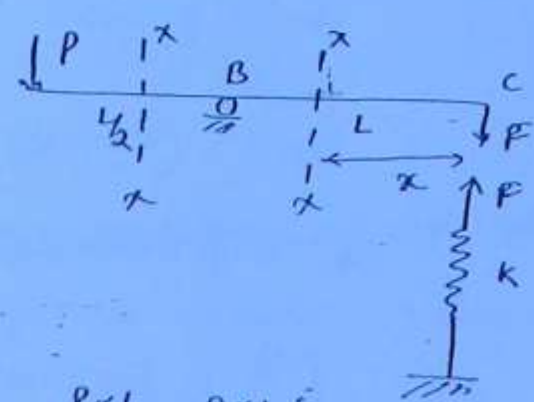
$$\boxed{\Delta_{h_D} \Rightarrow \frac{PR^3}{2EI}}$$

Ques:



Find defl^n below P ?

Sol^n:-



$$P \times \frac{L}{2} = F \times L$$

$F = P/2$. Force in spring.

Total Strain energy

$$U = U_{AB} + U_{BC} + U_{spring}$$

$$U_{spring} = \frac{F^2}{2K} = \frac{\left(\frac{P}{2}\right)^2}{2 \times \frac{24EI}{L^3}} = \frac{P^2}{4 \times 48EI} \times L^3$$

$$U_{spring} = \frac{P^2L^3}{192EI} \quad \text{—(i)}$$

$$U_{AB} = \int \frac{M_x^2 \cdot dx}{2EI} = \int \frac{(-Px)^2 dx}{2EI} = \int_0^{L/2} \frac{P^2x^2}{2E} dx$$

$$= \frac{P^2}{6EI}\left[x^3\right]_0^{L/2} \Rightarrow \frac{P^2L^3}{48EI}$$

$$U_{BC} = \int \frac{(-Fx)^2 dx}{2EI} = \frac{F^2}{6EI}\left[x^3\right]_0^L$$

$$U_{BC} = \frac{F^2L^3}{6EI} \Rightarrow \frac{P^2L^3}{24EI}.$$

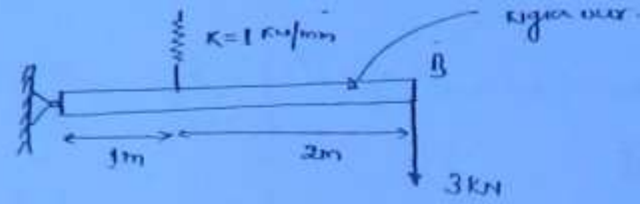
$$U = U_{AB} + U_{BC} + U_{spr.}$$

$$= \frac{P^2L^3}{48EI} + \frac{P^2L^3}{24EI} + \frac{P^2L^3}{192EI} =$$

Def^n below the load $\left(\frac{\partial U}{\partial P}\right) = \Delta_A = \frac{13}{96}\frac{PL^3}{EI}$

Q. A rigid bar AB is loaded as shown below what is the defl^n of point B under the effect of loading shown in fig-?

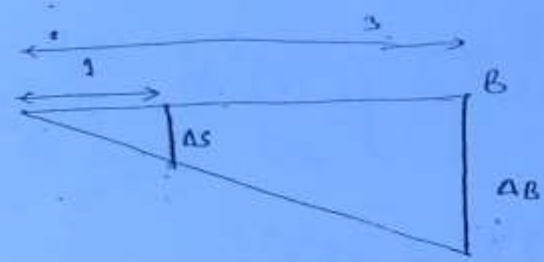
Q07

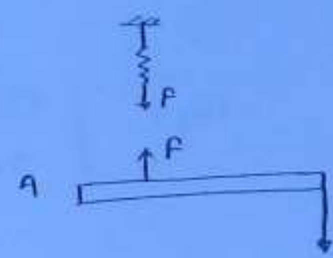


a) 1 mm d) 6 mm.

b) 9mm

c) 27 mm





$$\frac{\Delta_B}{\Delta_S} = \frac{3}{1}$$

$$\Delta_B = 3\Delta_S \quad \text{— (1)}$$

$$\Sigma M_A = 0$$

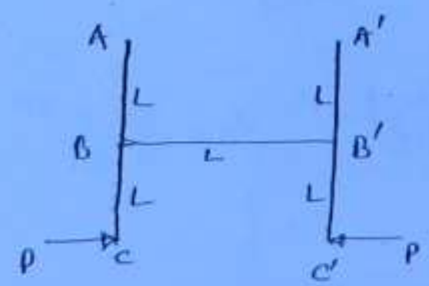
$$F \times 1 - 3 \times 3 = 0$$

$$F = 9\,kN.$$

$$\Delta_B = 3 \times 9 = 27\ mm.$$

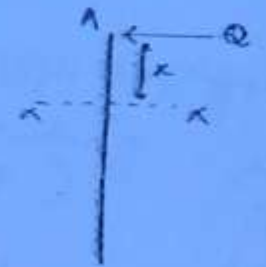
1. H frame is shown in fig having all memeber is flexible estimate movement of a a' away from each other considering B'B' axially rigid EI is contt for all members.

Sol:



apply pseudo load Q at A and A', so $\Delta_{AA'}$ (movement of AA') will be $\left(\frac{dU}{\partial Q}\right)_{Q=0}$.

$U = 2U_{AB} + 2U_{BC} + U_{BB'}$.

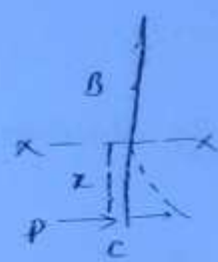


outer face is reference.

$$M_x = +Q \cdot x$$

$$U_{AB} = \int \frac{M_x^2\,dx}{2EI} = \int_0^L \frac{Q^2 x^2}{2EI}\,dx =$$

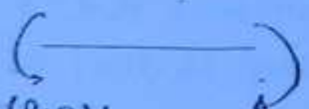
$$U_{AB} = \frac{Q^2 L^3}{6EI}$$



$$M_x = -P \cdot x$$

$$U_{BC} = \int \frac{M_x^2\,dx}{2EI} = \int_0^L \frac{P^2 x^2\,dx}{2EI} \rightarrow \frac{P^2 L^3}{6EI}$$

$$U_{BB'} = \int_0^L \frac{(P+Q)^2 L^2 \cdot dx}{2EI} \rightarrow \frac{(P+Q)^2 L^3}{2EI}$$



$(P+Q)L$

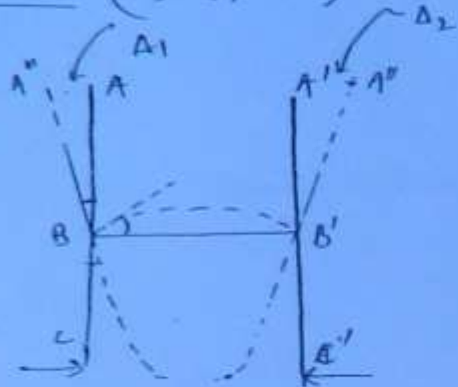
$$U_{Total} = 2U_{AB} + 2U_{BC} + U_{BB'}$$

$$= \frac{2Q^2L^3}{6EI} + \frac{2P^2L^3}{6EI} + \frac{(P+Q)^2L^3}{2EI}$$

$$\left.\frac{\partial U_T}{\partial Q}\right)_{Q=0} \Rightarrow \frac{4QL^3}{6EI} + 0 + \frac{\cancel{2}(P+Q) \cdot L^3}{\cancel{2}EI}$$

$$\boxed{\Delta_{AA'} = \frac{PL^3}{EI}}$$

II Method (obj approach).



$\Delta_1 = \Delta_2$.

Total movement of AA'

$= \Delta_1 + \Delta_2 \qquad (\Delta_1 = \Delta_2)$

$= 2\Delta_1$

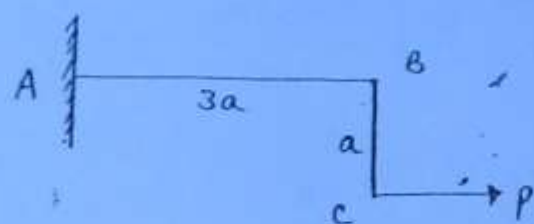
$= 2(\theta \cdot L)$

$= 2 \cdot \frac{PL^3}{2EI} = \frac{PL^3}{EI}$. Ans

$\theta = \frac{ML}{2EI}$

$= \frac{PL^2}{2EI}$.

Ques: what is vertical displacement at the point C of the structure shown in fig.



a) $\frac{9Pa^3}{2EI}$

b) $\frac{27Pa^3}{2EI}$

c) $\frac{27Pa^3}{8EI}$

d) $\frac{3Pa^3}{8EI}$

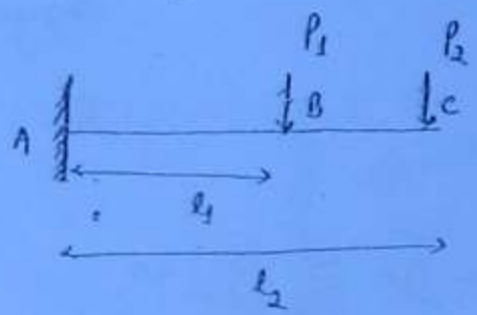
$\Delta_B = \frac{ML^2}{2EI}$

$= \frac{P \cdot a(3a)^2}{2EI}$

$= \frac{9P \cdot a^3}{2EI}$.

Superposition Theorem :-

In a linear elastic system the defl^n or slope due multiple loading is equal to effect of sum of each load ~~to~~ the individually.



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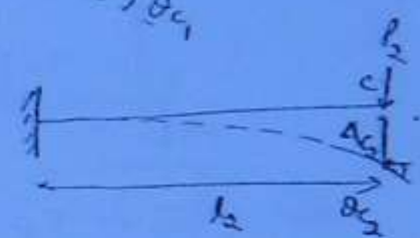


Find θ_c & Δ_c

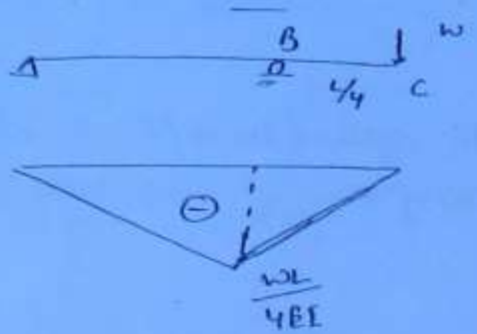
$$\text{Total } \theta_C = \theta_{C_1} + \theta_{C_2} = \frac{P_1 l_1^2}{2EI} + \frac{P_2 l_2^2}{2EI}$$

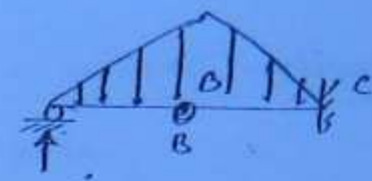
$$\text{"} \quad \Delta_C = \Delta_{C_1} + \Delta_{C_2} = \frac{P_1 l_1^3}{3EI} + \frac{P_2 l_2^3}{2EI}(l_2 - l_1) + \frac{P_2 l_2^3}{3EI}$$

Other Methods

i) Unit load Method

ii) Williot Mohr Diagram → (only for truss defl^n).





θ_A in real Beam = SF at A in Conjugate beam

$= R_A$

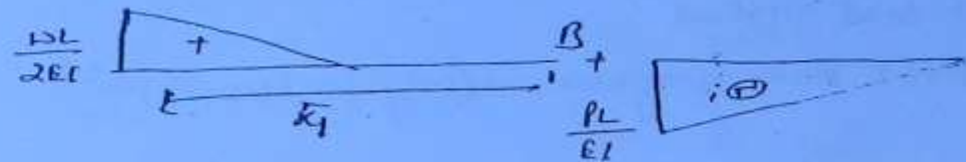
$M_B = 0$

$R_A \times L - \frac{1}{2} \times L \cdot \frac{wL}{4EI} \cdot \left(\frac{L}{3}\right) = 0$

$R_A = \frac{wL^2}{24EI}$

Pg-38

(15)



$\delta_{B/A} = 0$

$A_1\bar{x}_1 + A_2\bar{x}_2 =$

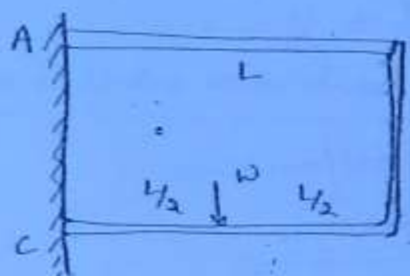
$= -\frac{1}{2} \times \frac{wL}{2EI} \times \frac{L}{2} \times \left(\frac{L}{2} + \frac{2}{3} \cdot \frac{L}{2}\right) + \frac{1}{2} \frac{PL}{EI} \times L \times \frac{2L}{3} = 0$

$w = \frac{16}{5} P$

01

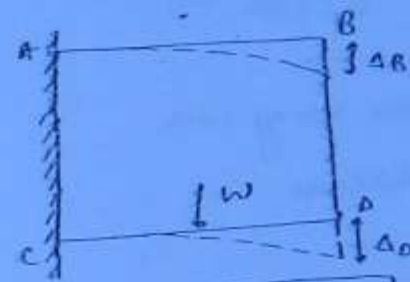
8. 2 Identical steel cantilever beams with constant EI and L long are built into a wall as shown in fig. If both cantilevers are connected at free end by a steel tie rod of dia. d. and length a. then find tension developed in the tie rod due to a conc^n load W. applied at the centre of CD.

Sol^n



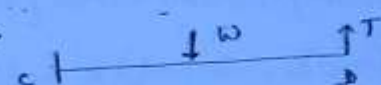
Tie rod
length = a
dia = d

$\Rightarrow$



$$\boxed{\Delta_D - \Delta_B = \text{Defl}^n \text{ in } BD} \quad \text{—(i)}$$

Let tension in tie rod is T



$$\Delta_B = \frac{TL^3}{3EI}$$

$$\Delta_D = \frac{5}{48}\frac{WL^3}{EI} - \frac{TL^3}{3EI}$$

Axial defl^n of BD

$$\frac{T \cdot a}{A \cdot E} = \frac{T \cdot a}{\frac{\pi}{4} d^2 \cdot E} = \frac{4Ta}{\pi d^2 E}$$

from eq^n (i).

$$\left(\frac{5WL^3}{48EI} - \frac{TL^3}{3EI}\right) - \frac{TL^3}{3EI} = \frac{4Ta}{\pi d^2 E}$$

$$= \frac{4Ta}{\pi d^2 E} - \frac{2TL^3}{3EI}$$

$T =$

Pressure Vessels :

Thick

$\frac{t}{D} > \frac{1}{10}$ to $\frac{1}{5}$

Stress varying with thickness.

Thin

$\frac{t}{D} < \frac{1}{10}$ to $1/5$

Stresses uniform across thickness

$t \rightarrow$ thickness

$D \rightarrow$ Inner or Internal dia

In closed vessel due to internal fluid pressure following type of stresses developed

1) Hoop stress / circumferential stress / Meridional stress $\rightarrow$ These are tensile. In thick ~~cells then~~ shells these vary to min^m at inner surface and max^m at outer surface while in thin shell these are assumed uniform.

2) longitudinal Stress:—

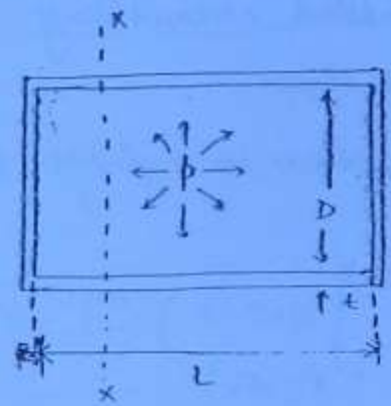
If ends of cylinder are closed then tensile longitudinal stresses are developed which are uniform across the thickness In thin and thick cylinder Both provided there is no bending If ends are open then longitudinal stresses will be zero.

3) Radial stress:

These are compressive and develope in the metal in the radial dir^n. In thick shell these vary from max^m at inner surface (equal to fluid pr.) to zero at outer surface. (equal to atm. pr.).

In thin shells radial stresses are assumed zero.

Thin cylinders with closed flat ends:→



Bore dia

D = internal Dia

t = thickness of wall

L = Inner length.

1) Longitudinal stresses:→

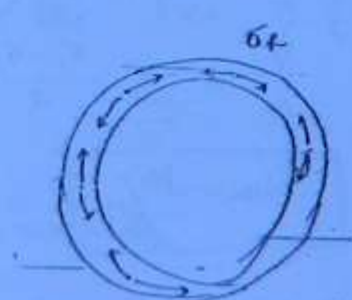
$$\Sigma F_x = 0$$

$$\sigma_L \times (\pi D \cdot t) - p \cdot \frac{\pi}{4} \cdot D^2 = 0$$

$$\boxed{\sigma_L = \frac{p \cdot D}{4t}}$$ Tensile

Since there is no shear stress in the plane of σ_L so σ_L is principal stress (minor principal stress σ_L).

2) Hoop stress or Circumferential stress:



$\Sigma F_y = 0$

$$\sigma_h (2t \times L) - P(D \times L) = 0$$

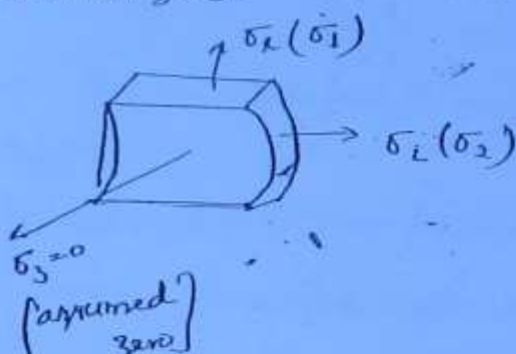
$$\boxed{\sigma_h = \frac{PD}{2t}} \quad \text{(Tensile)}$$

Since there is no shear on the plane of σ_h Hence σ_h is major principal stress (σ_1) (tensile)

NOTE:-

In thin shell these though radial stresses vary from p at inner surface and zero at outer surfaces.

But for all pratical purposes these are neglected Hence radial Stresses ares assumed zero.



$$\tau_{max} = \frac{\sigma_h - \sigma_L}{2} = \frac{\frac{PD}{2t} - \frac{PD}{4t}}{2} = \frac{PD}{8t}$$

at 45° from plan of σ_L to plane of σ_2.

τ_{max} in plane σ_1 & σ_3

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{PD}{2t}\Big/2 = \frac{PD}{4t}$$

$$\boxed{\tau_{max} = PD/4t} \Rightarrow \tau_{absolute\ max}$$

Hoop strain / Circumferential strain / Diametric strain :→

$$\epsilon_h = \epsilon_d = \frac{\delta D}{D}$$

$$\epsilon_h = \frac{\sigma_h}{E} - \frac{\mu \sigma_L}{E}$$

$$= \frac{pD}{2t \cdot E} - \frac{\mu p D}{4t \cdot E}$$

$$\boxed{\epsilon_h = \frac{pD}{4tE}(2-\mu)}$$ major principal strain

$$\epsilon_L = \frac{\sigma_L}{E} - \mu \frac{\sigma_h}{E}$$

$$\boxed{\epsilon_L = \frac{pD}{4tE}(1-2\mu)}$$

$$\boxed{\frac{\epsilon_h}{\epsilon_L} = \frac{(2-\mu)}{(1-2\mu)} = \frac{2m-1}{m-2}}$$ $\mu = 1/m$

Volume / capacity of cylinder :→

$$V = \frac{\pi}{4} D^2 L$$

$$(D^2 \delta L + L \cdot 2D \cdot \delta D)$$

$$e_v = \frac{\delta v}{v} = \frac{\pi}{4}\left(D^2 \cdot \delta L + L \cdot 2D \cdot \delta D\right)$$

$$\delta v = \frac{\pi}{4}\left(D^2 \delta L + L \cdot 2D \cdot \delta D\right)$$

$$\frac{\delta v}{v} = \frac{\delta L}{L} + \frac{2\delta D}{D}$$

$$\cancel{\epsilon_v = \epsilon_L + \epsilon_D}$$

$$\boxed{\epsilon_V = \epsilon_L + 2\epsilon_r}$$

$$= \frac{pd}{4tE}(1-2\mu) + \frac{pd}{4tE}(2-\mu)$$

$$\boxed{\epsilon_V = \frac{pD}{4tE}(5-4\mu)}$$

Unit change in capacity of cylinder is due to change in length and dia of cylinder. Here fluid is considered Incompressible. If fluid is Compressible & with its bulk modulus A then more fluid can be filled in, under pressure p.

Hence final volumetric strain will be.

$$\boxed{\epsilon_V = \frac{pD}{4tE}(5-4\mu) + \frac{p}{K}}$$ K → Bulk mod. of fluid

(Compressibility α 1/Bulk modulus)

For Incompressible fluid $K \to \infty$

Thin sphere :-

In ~~sphere~~ thin sphere longitudinal and circumferential stresses are equal. Hence ($\sigma_L = \sigma_c$)

$$\cancel{\frac{pD}{4tE}} \quad \sigma_L = \sigma_c = \frac{pD}{4t} \quad \text{(Tensile - -)}$$

$\tau_{max} = 0$

$$\tau_{abs.\,max} = \frac{\sigma_h - \sigma_r}{2} = \frac{pD}{8t}$$

$$\epsilon_h = \epsilon_l = \frac{\delta D}{D} = \frac{\sigma_h}{E} - \frac{\mu\,\sigma_L}{E}$$

$$\boxed{\epsilon_h = \frac{pD}{4tE}(1-\mu)}$$

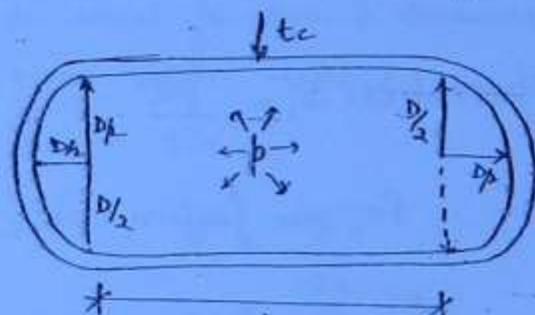
$$\text{Volumetric strain} = 3\,\frac{\delta D}{D}$$

$$\epsilon_v = 3\,\epsilon_h$$

$$\boxed{\epsilon_v = \frac{3\,pD}{4tE}(1-\mu)}$$

$$\boxed{\epsilon_v = \frac{3\,pD}{4tE}(1-\mu) + \frac{p}{K}}$$ → in compressible

Thin cylinder with hemisperical Ends :—



CASE I:

In order to develope equal Hoop stresses in cylindrical and sperical part at junction diff. thickess should be provided.

Let t_c is the thickness of cylindrical part

t_s " " " sphered " .

$$(\sigma_h)_c = (\sigma_h)_{spr}$$

$$\frac{pD}{2t_c} = \frac{pD}{4t_s}$$

$$\boxed{\frac{t_c}{t_s} = 2}$$

CASE II :

In order to prevent strain distortion at junction, hoop strain in cylindrical part should be equal to hoop strain in spherical part

$$(\epsilon_h)_c = (\epsilon_h)_{sp}$$

$$\frac{pD}{4t_c E}(2-\mu) = \frac{pD}{4t_s E}(1-\mu)$$

$$\boxed{\frac{t_c}{t_s} = \frac{2-\mu}{1-\mu}} > 1$$

~~Design of Thickness~~

Design of Thickness of cylinder and spherical vessels :→

Max^m pri stress $\sigma_1 = \frac{pd}{2t}$.

for no failure $\sigma_1 \le \sigma_y$

for design $\sigma_1 \le \sigma_y / f.o.s$

$$\sigma_1 \le f$$

$$\frac{pD}{2t} \le f$$

$$\boxed{t \ge \frac{PD}{2f}}$$

for sph:-

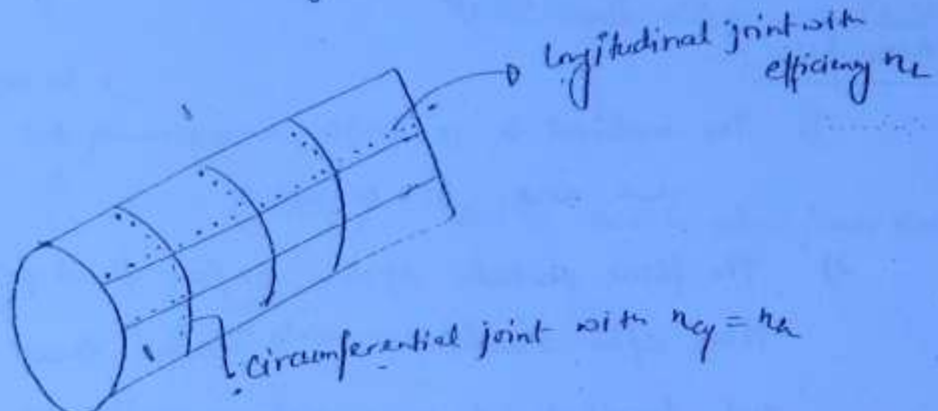
for design $\sigma_1 \le f$

$$\boxed{t \geq \frac{pD}{4f}}$$

Acc^n to max^m pri stresses theory

Rivetted Cylinders :→

Big cylindrical vessels s.as. Boilers are made of several curved metal plate jointed thru rivets or weilding having circumferential and Longitudinal joint



Since certain area is occupied by rivet holes and Hence net avilable area to resist pr. force is reduced therefore stresses developed are more which depends on s efficiency of joint.

$$\sigma_h = \frac{pD}{2t \cdot \eta_L} \qquad \sigma_L = \frac{pD}{4t \cdot \eta_c}$$

longi. joints will fail due hoop stress, and circumferential joint fail due to longitudinal stress.

Thick cylinders :→

In Thick cylinders longitudinal stresses are constant across thickness because there is no bending. but hoop stresses vary from max^n at inner at face to min^n at outer face, hyperbolically and are tensile and compressive radial stresses vary hyperbollically from f max^n at inner face to zero at outer face (atm. pr.).

The analysis of thick cylinder is done according to Lames theory.

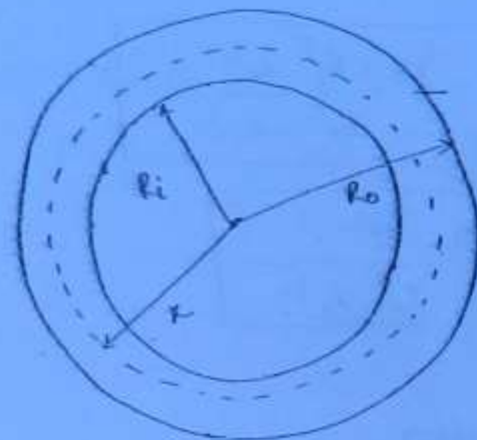
Assumption :-

1) The material is isotropic, homogeneous, and linear elastic in which hooks law is valid.
2) The plane vertical section before fluid press remains plane after fluid pr. It means there is no Bending and longitudinal ~~and~~ stress and strains are uniform.
3) Each fiber of material is free to contract and expand longi and laterally

Let

R_o = outer Radius

R_i = Inner radius



$R_i \le x \le R_o$

At any distance x the hoop stresses are given as

$$\sigma_x = \frac{B}{x^2} + A$$ - (Tensile) due to internal fluid pressure (will compr. due to ext. fluid pr.),

Radial stress at x.

$$p_x = \frac{B}{x^2} - A$$ --- (Comp.) due to ext & int. press Both.

A, B are Lame's Constant

A, & B are dimensional constant

$A \to N/mm^2$, $B \to N$. $\boxed{B >>> A}$

If fluid pr. is internal then A & B Both will be positive but if fluid pr. is external then A and B Both will be negative.

A & B can be found using end conditions for radial pressure

For ex:

At inner surface $x = R_i \Rightarrow p_x = p$,

At outer surface $x = R_o \Rightarrow p_x = 0$.

$$p = \frac{B}{R_i^2} - A$$

$$0 = \frac{B}{R_o^2} - A$$

$$\frac{B}{R_i^2} - \frac{B}{R_o^2} = p \Rightarrow \boxed{B = p \cdot \frac{R_i^2 \cdot R_o^2}{R_o^2 - R_i^2}}$$

$$\boxed{A = \frac{B}{R_o^2}} \boxed{= p \cdot \frac{R_i^2}{R_o^2 - R_i^2}}$$

$p_{max} = p$ at inner surface

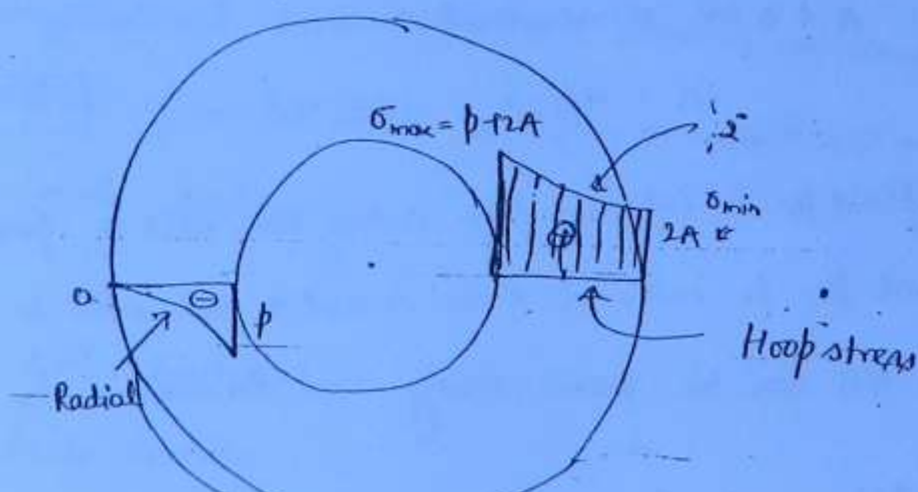
$p_{min} = 0$ at outer "

$$\boxed{\sigma_x - p_x = 2A} = \text{const}$$

$$\sigma_x = p_x + 2A.$$

$$\sigma_{max} = p_{max} + 2A = p + 2A$$

$$\sigma_{min} = \cancel{p_{min}}^{0} + 2A = 2A$$



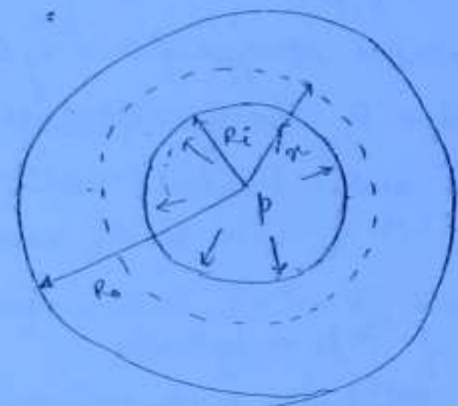
Longitudinal stress :-

Longitudinal stresses are tensile and uniform across the thickness if ends are closed

$$\sigma_L \times \pi\left(R_0^2 - R_i^2\right) = p \cdot \pi R_i^2$$

$$\boxed{\begin{aligned} \sigma_L &= p \cdot \frac{R_i^2}{R_0^2 - R_i^2} \\ \sigma_L &= A = \text{constant} \end{aligned}}$$ tensile

Thick sphere :-



$R_i \le x \le R_0$

hoop stresses and longitudinal stresses are equal and are given by.

$$\sigma_x = \frac{B}{x^3} + A \quad \text{-------} \quad (\text{Tensile})$$

Radial stresses are given by

$$p_x = \frac{2B}{x^3} - A \quad \text{-----} \quad (\text{Compressive})$$

A & B are Lame's constant

A & B are found using end conditions at $x = R_i$, $p_x = p$

& at $x = R_0$, $p_x = 0$

Following points may be noted:

1) Autofrittage: It is the process of prestressing thick cylinders before putting them into the service

Disruptive strength :-

Disruptive strength is tensile strength of a metal under equal and altile triaxial loading.

Compound cylinder :- If a cylinder (thick) is made of two or more metals then it is called compound cylinder, ex. bi gun barrel.

It is the case of autofrittage, compounding is done to improve circumferential strength of cylinder.

There is no effect on long. or strength or strain stress.

Wire winding : To improve circumferential strength of thin cylinder and pipes, a layer of wire is closely wrapped in the circumferential dirn of pipe. under iniatial tensile stress.

Wire Winding :→

Let D = inner diameter of Pipe or cylinder

t = thickness of the Pipe or cylinder

d_w = dia of wire

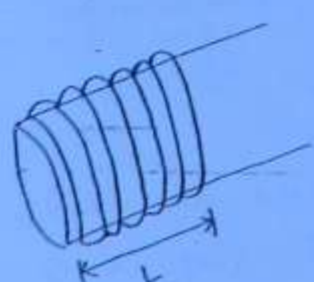
E_w = young's mod of wire

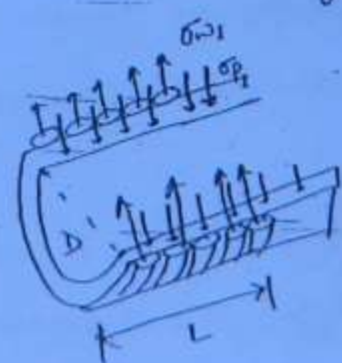
E_p = " " " pipe

CASE I. Effect of wire winding :

Let wires are wrapped closely over L length of pipe

$D >> t$ thin cylinder.





No. of wire = $\frac{L}{d_w} + \frac{L}{d_w}$

$\Sigma F_y = 0$;

σ_{w_1} × total no of wire section × Area of each wire section

= σ_{p_1} × Area of pipe metal over length L,

$$\sigma_{w_1} \times \frac{2L}{d_w} \times \frac{\pi}{4} d_w^2 = \sigma_{p_1} \times (2t \times L) \quad \text{— (i)}$$

$$\boxed{\sigma_{p_1} = \frac{\sigma_{w_1} \times \pi d_w}{4t}} \quad (\text{Comp.})$$

B. effect of fluid Pressure:

under fluid pres. p the stresses develope in pipe and wire in circumferential dir^n are σ_{p_2} & σ_{w_2} both tensile.

The Bursting force of press. in vertical dir^n should be equal to resisting force of wire and pipe

$$p \times (D \times L) = \sigma_{w_2} \times \frac{2L}{d_w} \times \frac{\pi}{4} d_w^2 + \sigma_{p_2} \times (2t \times L) \quad \text{..... (ii)}$$

Since wire and pipe are in contact before and after fluid pressure Hence circumferential strain in pipe should be equal to longitudinal strain in wire

$$\boxed{\frac{\sigma_{p_2}}{E_p} - \frac{\mu \sigma_L}{E_p} = \frac{\sigma_{w_2}}{E_w}} \quad \text{—— (iii)}$$

$\sigma_L = \frac{pD}{4t}$ if ends are closed

$\sigma_L = 0$ if ends are open

Solving eq^n (ii) & (iii) σ_{p_2} & σ_{w_2} are determined.

Final stresses:-

$\sigma_w = \sigma_{w_1}$ (C) + σ_{w_2} (T) → Tension

Ques:

A Cast iron pipe 300 mm internal dia and 12 mm thickness is closely wrapped by a layer of 5 mm dia steel wire under a tensile stress of 60 N/mm². If water is admitted under a pressure of 4 N/mm² then calculate final stresses set up in the pipe and wire assuming both ends are flat and closed.

$$E_p = 1\times10^5, \quad \mu_p = 0.3$$
$$E_w = 2\times10^5 \, N/mm^2$$

Sol:

$$\sigma_{w_1} = 60 \, N/mm^2$$

CASE I. Wire wrapping :-

Let σ_{p_1} is compressive stress developed in pipe.

$$\text{Compressive } \sigma_{p_1} = \frac{\sigma_{w_1} \cdot \pi \cdot d_w}{4t}$$

$$\sigma_{p_1} = \frac{60 \times \pi \times 5}{4 \times 12} = 19.63 \, N/mm^2 \text{ (Compress)}$$

CASE II: Fluid pressure :-

Let σ_{p_2} & σ_w are tensile stresses developed in pipe and wire.

$$p \times (D \times \cancel{L}) = \sigma_{w_2} \cdot \frac{2\cancel{L}}{d_w} \cdot \frac{\pi}{4} d_w^2 + \sigma_{p_2} \cdot (2t \times \cancel{L})$$

$$1200 = 7.85\,\sigma_w + 24\,\sigma_{p_2} \quad \text{— (i)}$$

$$\frac{\sigma_{p_2}}{1\times10^5} - \frac{.3\times\sigma_L}{1\times10^5} = \frac{\sigma_{w_2}}{2\times10^5}$$

$$2\sigma_{p_2} - .3\times 25\cancel{x} = \sigma_{w_2}$$

$$2\sigma_{p_2} - \sigma_{w_2} = 15 \quad \text{— (ii)}$$

$$\sigma_{p_2} = 33.19 \qquad \sigma_{w_2} = 51.38$$

Total $\sigma_w = 111.39$ ... $\sigma_p = 13.56$ (Tensile) $\rightarrow (-\sigma_{p_1} + \sigma_{p_2})$

Ques :-

A 1 mm dia steel wire is wrapped around a Cu d tube with external dia 140 mm and internal dia 120 mm to increase the strength of tube against internal fluid pr. find what initial tension must be given to the wire so that max^n allowable stress in the pipe/tube and wire reach 190 MPa and 200 MPa resp. and simultaneously assume μ for both metal = 0.3.

$$E_w = 2\times10^5 \text{ MPa}, \quad E_p = 1\times10^5$$

Consider both ends of pipe are open ✓

Sol^n :-

$$\sigma_w = 200 \text{ MPa} = \sigma_{w_1} + \sigma_{w_2} \quad \text{— (i)}$$

$$\sigma_p = 190 \text{ MPa} = -\sigma_{p_1} + \sigma_{p_2} \quad \text{— (ii)}$$

$$\sigma_{w_1} = (200 - \sigma_{w_2})$$

$$\sigma_{p_2} = (90 + \sigma_{p_1})$$

$$\sigma_{w_1} \times \frac{2L}{d_w} \cdot \frac{\pi}{4} \cdot d_w^2 = \sigma_{p_1} \times (2L \times t)$$

$$\frac{\sigma_{w_1}}{\sigma_{p_1}} = \frac{10}{.7} \; (?) \; 12.732 \Rightarrow \underline{\sigma_{w_1} = 12.732\, \sigma_{p_1}}$$

$$\boxed{\sigma_{p_1} = 0.0785\, \sigma_{w_1}} \quad \text{— (iii)}$$

Strain in pipe = Strain in wire

$$\frac{\sigma_{p_2}}{E_p} = \frac{\sigma_{w_2}}{E_w}$$

$$2\sigma_{p_2} = \sigma_{w_2}$$

$$2(90 + \sigma_{p_1}) = (200 - \sigma_{w_1}) \quad \text{— (iv)}$$

$$2\sigma_{p_1} + \sigma_{w_1} = 20 \quad \text{— (v)}$$

$\sigma_{p_1} = 1.356 \text{ N/mm}^2$ (c)

$\sigma_{w_1} = 17.28 \text{ N/mm}^2$ (T)

A Cu cylinder with 600 mm Bore dia and 12 mm thickness and 900 mm long Both ends are flat and closed. and water is gradually admitted into the cylinder until the pre. is reached to 0.8 mpa: find i) Strain energy Stored in the cylinder, (ii) pressure energy stored in the water,

$E_{cu} = 1 \times 10^5 \ N/mm^2$ & $\mu_{cu} = 0.35$, Bulk modulus for water $K = 2.13 \times 10^3 \ N/mm^2$

Sol^n:-

i) $\sigma_1 = \sigma_L = \dfrac{p \cdot D}{4t} = \dfrac{.8 \times 600}{4 \times 12} = 10 \ N/mm^2$

$\sigma_2 = \sigma_R = \dfrac{pD}{2t} = 20 \ N/mm^2$

$U =$ per unit vol.

$U = \dfrac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 \sigma_2)\right]$

$= \dfrac{1}{2 \times 10^5}\left[100 + 400 - 2 \times .35(50)\right]$

$= \dfrac{[500 - 140]}{2} \times 10^{-5}$

Total in whole volume :-

$= 180 \times 10^{-5} \times$ Volume

Volume $= (\pi D) t \times L$

$U = 36643.5 \ N\text{-}mm$

$U = 36.643 \ N\text{-}m$

$U = 36.643 \ J$

(ii) Pressure energy in water :-

$p = \dfrac{p^2}{2K} \times \dfrac{\pi}{4} D^2 \times L$

$= 38230085.5 \times 10^{-3}$

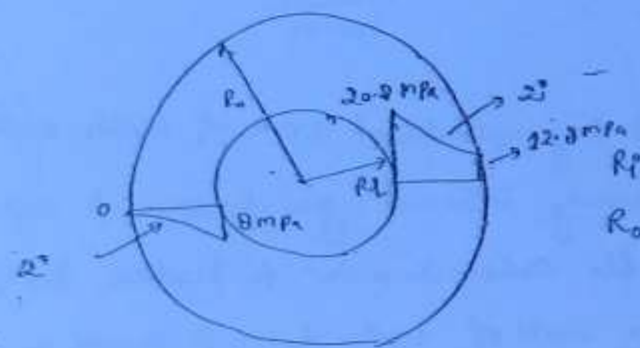
$= 382300 \ N\text{-}mm$

$p = 38.2 \ N\text{-}m = 38.2 \ J$

Q: A Cast iron pipe of 320 mm dia (inner dia) and 80 mm thick carry water under a pressure of 8 N/mm². Calculate max^m and min circumferential stresses and sketch the distribution of radial pr. and circumferential stresses across the thickness.

Sol^n:-

$$t/D = 80/320 = 1/4\,(1/4) \text{ thick.}$$



$R_i = 160$ mm

$R_o = 160 + 80 = 240$ mm

Hoop stress

$$\sigma_x = \frac{B}{x^2} + A$$

Radial stress

$$p_x = \frac{B}{x^2} - A$$

at $x = R_i$ $p_x = p$.

$$p = \frac{B}{R_i^2} - A$$

$$8 = \frac{B}{(160)^2} - A \quad \text{—(i)}$$

At $x = R_o$, $p_x = 0$

$$0 = \frac{B}{R_o^2} - A \text{ —(ii)}$$

from (i) & (ii)

$A = 6.4$ N/mm²

$B = 368640$ N/mm²

$\sigma_{max} = p + 2\lambda = 8 + 2 \times 6.4 = 20.8 \, N/mm^2$

$\sigma_{min} = 2\lambda = 12.8 \, N/mm^2$

$p_{max} = p = 8$

$p_{min} = 0$

ATE'06

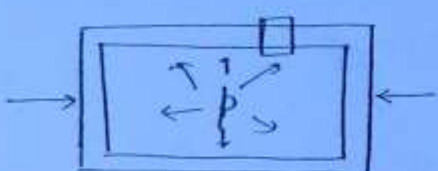
Que: A Thin walled long cylindrical tank of inside radius r is subjected to simultaneously internal gas pressure p and axial compressive force F at its ends. In order to produce ~~pu~~ pure shear stage of stress in the wall of cylinder F should be equal to..

a) $p \cdot \pi r^2$

b) $p \cdot 2\pi r^2$

c) $p \cdot 3\pi r^2$

d) $p \cdot 4\pi r^2$



$$\frac{p \cdot D}{4t} = \frac{F}{A}$$

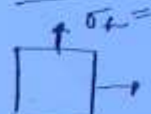
$$\frac{p \cancel{2} r}{\cancel{4} \cdot \cancel{t}} = \frac{F}{\pi \cdot D \cdot \cancel{t}}$$

$$F = p \cdot r \cdot \pi \cdot 2r$$

$$F = p \cdot 2\pi r^2$$

$$\sigma_h = \frac{pD}{}$$

$$\sigma_h = \sigma_1 = \frac{pD}{2t} = \frac{pr}{t}$$



$$\sigma_2 = \frac{pD}{4t} - \frac{F}{2\pi r \cdot t}$$

$F =$

pure shear

$$\sigma_1 + \sigma_2 = 0 \Rightarrow \frac{pr}{t} + \frac{pr}{2t} - \frac{F}{2\pi r t} = 0$$

$$F = p \cdot 3\pi r^2$$

Q.2). c   Q 4) d   Q5) b   Q6) b   Q7). $\sigma_1 \le \sigma_y$

Q 3). a.

Q 8) a   9). a   10). a   11). b $\frac{p \cdot D}{2t} \le 36m$

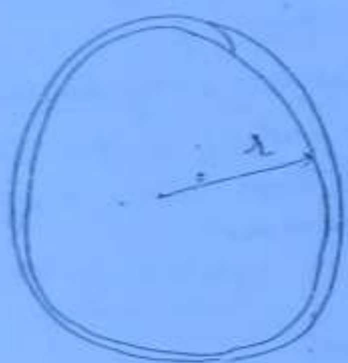
12) d   13) c

14) d   15) c.   16) c $p \le \frac{3600 \times 2 \times 3}{6}$

17) b)

$p \le 360 \, kg/cm^2$

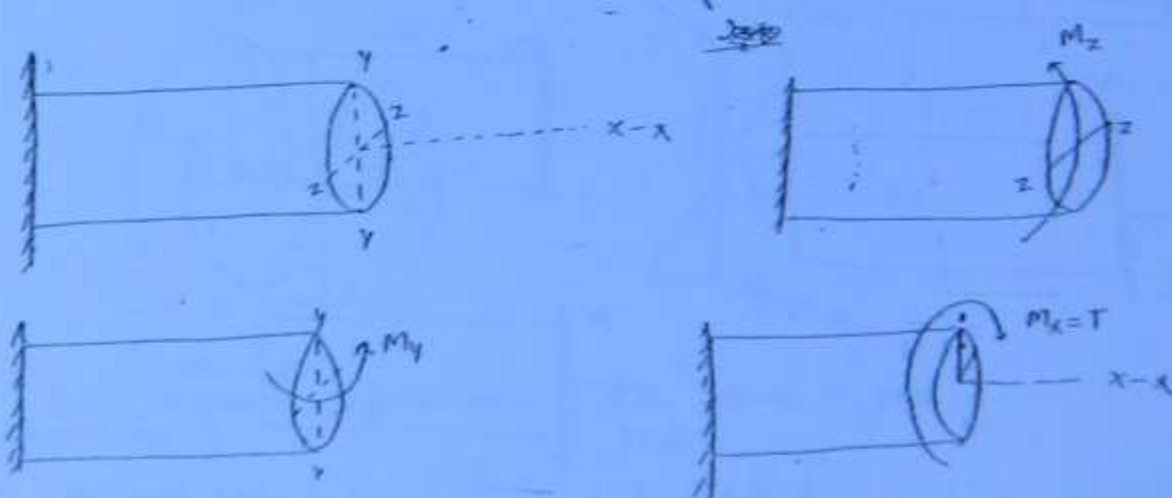
$\frac{(1-2\mu)}{(2-\mu)}$



Final radius = $r + u$

Circumferential strain = $\frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r}$

Torsion of shafts :-



Bending Moment

In Bending the plane of cross section rotates about transverse axis, i.e about Z-Z or yy

In Bending, stresses are developed normal to the cross section (Long. Normal stress).

B. Stresses vary linearly from zero at neutral axis to maximum at surface. If Bending in vertical Plane then N.A will horiz. transverse axis. (Z-Z) Hence max^n and min^n normal stresses will be at top and bottom fibers

Twisting Moment

1. The x-section rotates about longi axis or polar axis in fact radii rotates

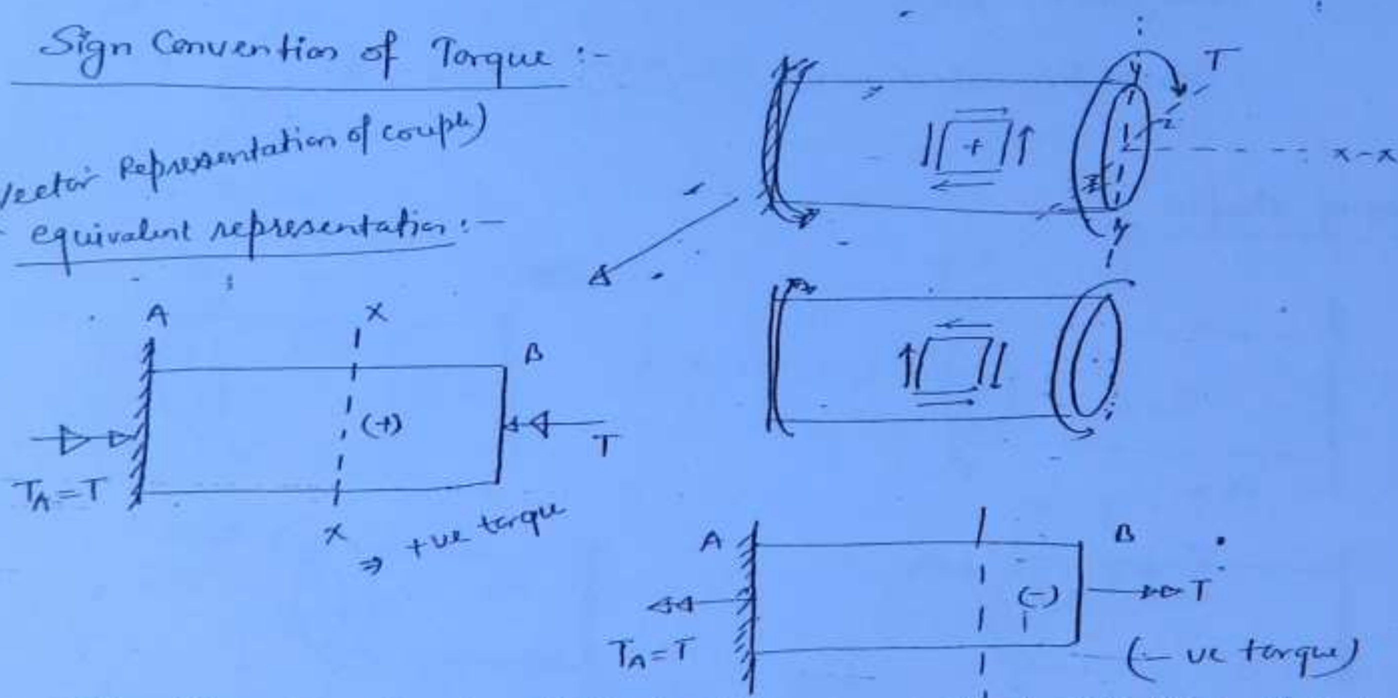
2. In twisting, shear stresses are developed which are tangential to the x-section.

3. Shear stresses vary from 0 at centre to max^n at surface. The variation of shear stress is symmetric in all radial dir^n.

Sign Convention of Torque :-

Vector Representation of couple)

equivalent representation :-



The sign convention is explained with the help of right hand thumb rule of Torque is acting in the dir^n of right hand ~~fig~~ finger then right hand thumb represents movement of nut on the cross section If there is tightening effect of nut i.e. thumb pointing towards the cross-section then such a Torque will be treated +ve and vice versa.

A +ve Torque will produce a +ve shear element on the shaft surface

Assumptions in theory of pure torsion →

1). The material is isotropic, homogeneous and linear elastic in which hook's law is valid.

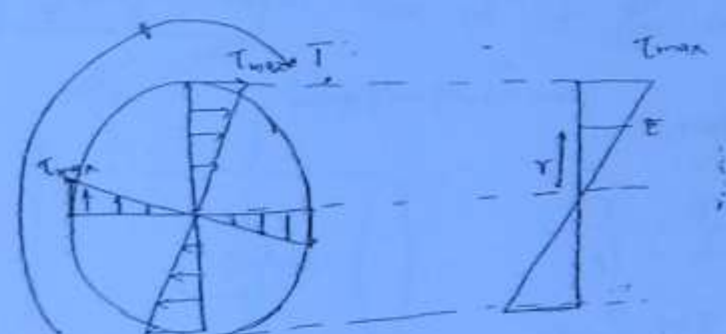
2). The plane section before twisting remains plane after twisting. it means the radii which are straight before twisting remain straight after twisting.

3). The shaft is circular which may be solid or hollow and shaft is prismatic i.e. I_p = const. along length.

4) Note: Non circular sections are torsionally weak and due to torque such section get warped i.e. plane section will not remain plane.

Effect of torsion :→

Due to torque shear stresses are produced and it is the case of pure shear, the shear stresses vary linearly in all radial dirⁿ from 0 at centre to maxᵐ at surface, the shear stresses at surface are tangential the shear stress produced are symmetrical.



At a distance r from centre shear stress is given by

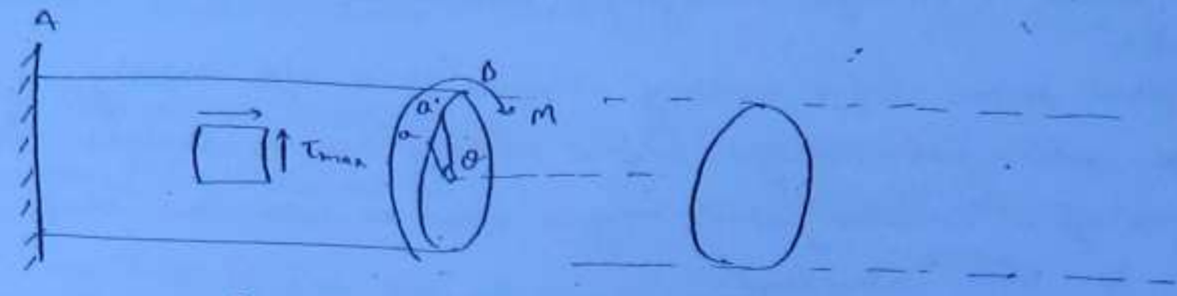
$$\boxed{\frac{\tau}{r} = \frac{T}{I_p} = \frac{G\theta}{L}}$$

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$

$$\tau = \frac{\tau_{max}}{R} \cdot r$$

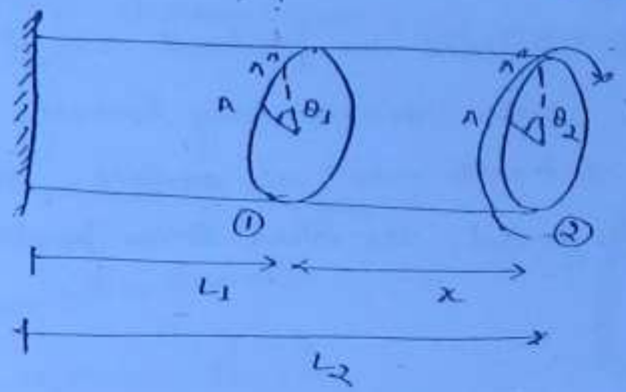
T is torque, I_p Polar moment of inertia $\left(\frac{\pi}{32} d^4 \to \text{solid}\right)$

θ angle of twist over length l.



θ = Angle of twist over length L under Torque T.

$$\theta = (\theta_B - \cancel{\theta_A}) = \frac{T \cdot L}{G \cdot I_p}$$



$$\theta = \frac{T \cdot x}{G \cdot I_p}$$

$$\boxed{\theta_2 - \theta_1 = \frac{T \cdot x}{G \cdot I_p}}$$

from above relation,

$$\frac{\tau}{r} = \frac{T}{I_p}$$

$$\tau = \frac{T}{I_p} \cdot r$$

$$\tau_{max} = \frac{T}{I_p} \cdot r_{max} = \frac{T}{I_p} \cdot R$$

$$\tau_{max} = \frac{T}{Z_p}$$

$$Z_p = \frac{I_p}{r_{max}} = \frac{I_p}{R}$$

$\boxed{Z_p = \text{Polar section Modulus}}$ or Torsional strength

$$\boxed{T = \tau_{max} \cdot Z_p}$$

A for a strongest shaft in torsion its Z_p should be max.

Hollow circular shaft > Solid circular shaft (strength)

Same area of x-section

* Due to torque

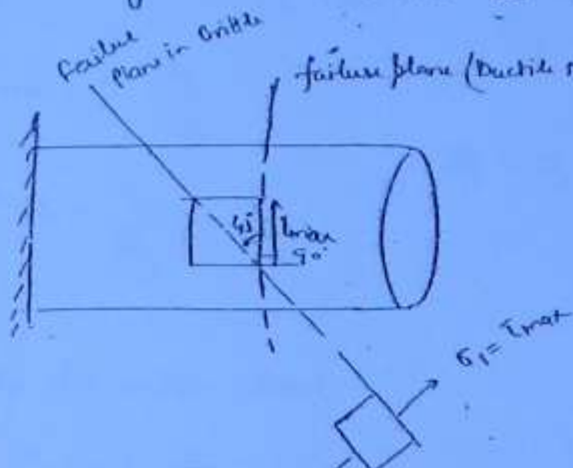
Shear stress on the surface are maximum and a state of pure shear is produced

Failure of Ductile Metal in torsion :-

- Since ductile metals are weakest in shear therefore shear failure occurs first, the failure plane is smooth plane at 90° to the longitudinal axis, it is the plane of τ_{max}

Failure of Brittle Metal in Torsion :-

Brittle metals are weakest in tension and in case of pure shear tensile stresses are principal stresses ($\sigma_1 = +\tau_{max}$) which acts at a plane 45° to the longitudinal axis hence failure plane is a rough helical plane at 45° from longi axis.



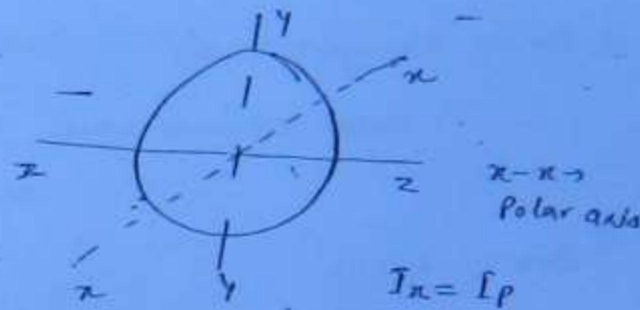
Shear stress distribution in different shafts :-

1) Solid circular shaft :-

$$\tau_{max} = T/Z_p$$

$$I_p = \frac{\pi}{32} D^4$$

$$Z_p = \frac{I_p}{R_{max}} = \frac{\pi}{16} D^3$$



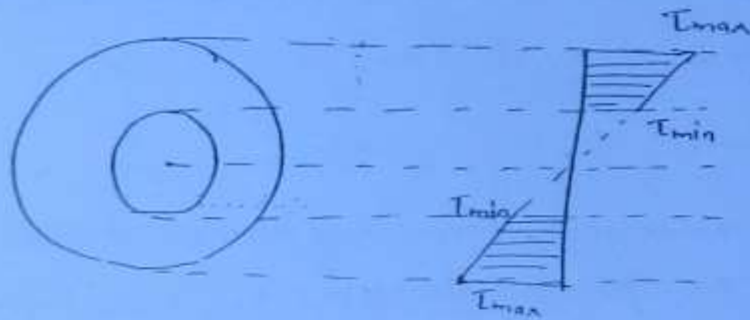
$$I_x = I_p$$

$$I_{zz} + I_{yy} = I_{xx}$$

$$= \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$$

$$\boxed{I_p = \frac{\pi}{32} D^4}$$

Hollow circular shaft:

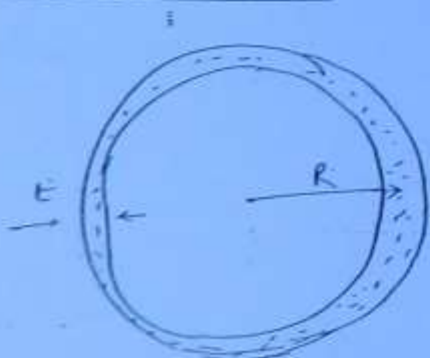


$$\tau_{max} = T/Z_p$$

$$I_p = \frac{\pi}{32} (D_o^4 - D_i^4)$$

$$Z_p = \frac{\pi}{16} \frac{(D_o^4 - D_i^4)}{D_o}$$

Thin circular Tubes:-



In thin tubes shear stress is assumed uniform across the thickness

The shear stress is given as $\tau_1 = T/Z_p$

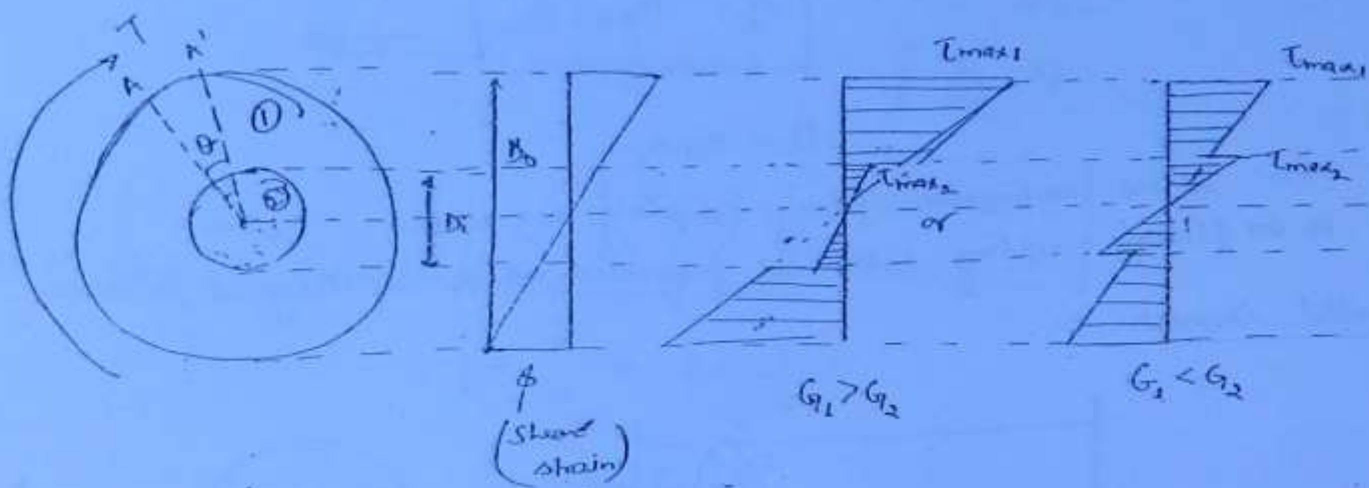
$$Z_p = 2\pi R^2 \cdot t$$

$$I_p = 2\pi R^3 \cdot t$$

$$Z_p = I_p / r$$

Shear stress distribution in composite shaft.

Let composite shaft is made of two metal. outer is metal 1 & Inner is metal 2, Both shafts are jointed firmly face to face Such that there is no relative motion. Hence shear strain at the junction in both shafts will be equal. If shear modulus (G) for both metals are different then there will be sudden change in shear stress at the junction.



$$\tau_{max_1} = \frac{T_1}{Z_{P_1}}$$

$$\tau_{max_2} = \frac{T_2}{Z_{P_2}}$$

$$\theta_1 = \theta_2 \Rightarrow \frac{T_1 \cdot L}{G_1 \cdot I_{P_1}} = \frac{T_2 \cdot L}{G_2 \cdot I_{P_2}}$$

$I_{P_1} = I_P$ for outer metal

$$= \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$

$I_{P_2} = I_P$ for Inner metal

$$\frac{\pi}{32}\left(D_i^4\right)$$

Total torque applied $= T$

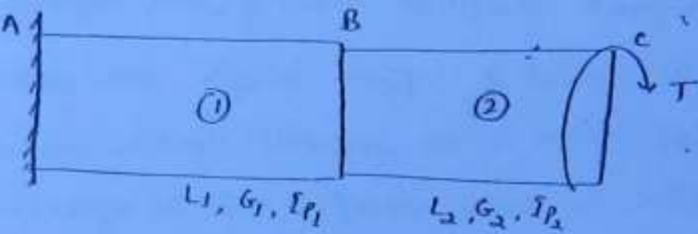
Torque resisted by outer shaft $= T_1$

" " " Inner " $= T_2$

$$\boxed{T = T_1 + T_2}$$

Shaft Connections :→

1) Series Connection :-



In series combination torque transmitted is constt. it means $\boxed{T_1 = T_2 = T}$ — (i)

The total angle of twist from A to C is equal to

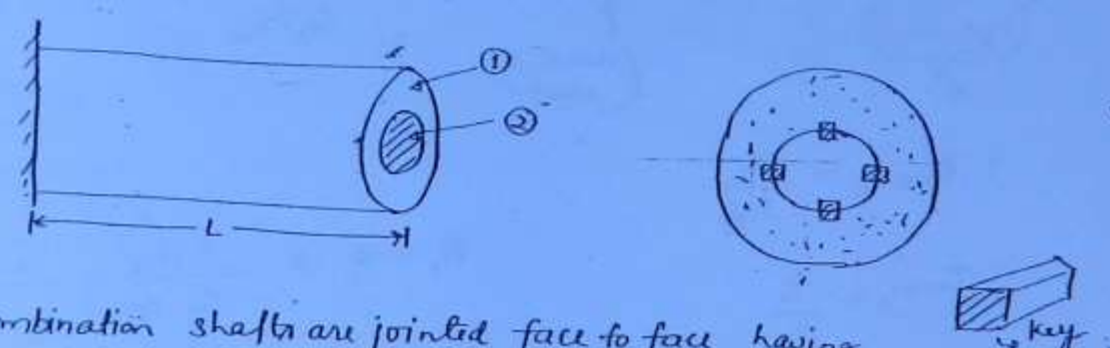
$$\boxed{\theta_{AC} = \theta_{AB} + \theta_{BC}} \quad \text{—— (ii)}$$

$$\theta_C = \theta_1 + \theta_2$$

For series combination flange coupling are used.

So on flange coupling bolts are provided which are designed for shear.

) Parallel Connection :-



A parallel combination shafts are jointed face to face having equal lengths. The relative motion b/t the shaft is prevented by keys or through welding. The total applied torque is transmitted.

Let T_1 is the Torque in outer shaft
T_2 " " " " inner shaft

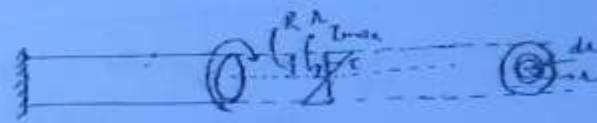
$$T = T_1 + T_2 \quad \text{—— (i)}$$

$$\theta_1 = \theta_2$$

$$\boxed{\frac{T_1 L}{G_1 I_{P_1}} = \frac{T_2 L}{G_2 I_{P_2}}} \quad \text{—— (ii)}$$

$\tau_{max_1} = T_1/2P_1$, $\tau_{max_2} = T_2/2P_2$

Strain Energy stored in the shaft :-



$dA = 2\pi r \cdot dr$

Let shear ~~shear~~ stress at a point is τ then strain energy per unit volume due to shear is $\frac{\tau^2}{2G}$ (shear resilience)

Strain Energy stored in elemental volume $dV = \frac{\tau^2}{2G} \cdot dV$

Total strain energy stored $U = \int \frac{\tau^2}{2G} \cdot dV$

$$\boxed{U = \int \frac{\tau^2}{2G} \cdot 2\pi r \cdot dr \cdot L}$$

CASE I: Strain energy in solid shaft in terms of Torque:

$$\frac{\tau}{r} = \frac{T}{I_p} \Rightarrow \tau = \frac{T}{I_p} \cdot r$$

$$U = \int \frac{\left(\frac{T}{I_p} r\right)^2}{2G} \cdot 2\pi r \cdot dr \cdot L$$

$$U = \frac{2\pi T^2 L}{2 I_p^2 G} \int_0^R r^3 \cdot dr$$

$$U = \frac{\pi T^2 L}{I_p^2 \cdot G} \left[\frac{R^4}{4}\right]$$

$I_p = \frac{\pi}{32} d^4$

$= \frac{\pi}{2} \cdot R^4$

$$= \frac{\pi T^2 L}{\left(\frac{\pi}{2} R^4\right)^2 G} \left[\frac{R^4}{4}\right]$$

$$= \frac{T^2 L}{2 G I_p^2} \left(\frac{\pi R^4}{2}\right) = \frac{T^2 L}{2G \cdot I_p^2} I_p = \frac{T^2 L}{2 G I_p}$$

$$\boxed{U = \frac{T^2 L}{2 G I_p}}$$

$$\boxed{U = T \cdot \frac{\theta}{2}} \quad \text{as} \quad \boxed{U = P \cdot \frac{\Delta}{2}}$$

If Torque or I_p is variable

$$\boxed{U = \int \frac{T_x^2 \cdot dx}{2G \cdot I_{p_x}}}$$

CASE I

Strain energy in solid shafts in terms of τ_{max}:-

$$\frac{\tau}{r} = \frac{\tau_{max}}{R}$$

$$\tau = \frac{\tau_{max}}{R} \cdot r$$

$$U = \int_0^R \frac{\tau^2}{2G} \cdot (2\pi r \cdot dr \cdot L)$$

$$U = \int_0^R \frac{\frac{\tau_{max}^2}{R^2} \cdot r^2 \cdot 2\pi r \cdot dr \cdot L}{2G} \Rightarrow \frac{\tau_{max}^2 \cdot \pi L}{G \cdot R^2} \int_0^R r^3 \cdot dr$$

$$= \frac{\tau_{max}^2 \cdot \pi L}{G \cdot R^2} \cdot \frac{R^4}{4} \Rightarrow \frac{\tau_{max}^2}{4G} (\pi R^2 \cdot L) \Rightarrow \frac{\tau_{max}^2}{4G} (\text{volume})$$

$$\boxed{U = \frac{\tau_{max}^2}{4G} \times \text{Volume}} \Rightarrow \boxed{\frac{U}{\text{Volume}} = u_s = \frac{\tau_{max}^2}{4G}}$$

(ii) Strain energy stored in Hollow shaft with internal Radius R_i and ext. rad. R_o (in terms of τ_{max}) :-

$$U = \int_{R_i}^{R_o} \frac{\tau^2}{2G} \cdot (2\pi r \cdot dr \cdot L) = \int_{R_i}^{R_o} \frac{\frac{\tau_{max}^2}{R_o^2} \cdot r^2 \cdot 2\pi r \cdot dr \cdot L}{2G}$$

$$= \frac{\tau_{max}^2 \cdot L \cdot 2\pi}{2G \cdot R_o^2} \int_{R_i}^{R_o} r^3 dr = \frac{\tau_{max}^2 \cdot \pi L}{G R_o^2} \left[\frac{R_o^4 - R_i^4}{4} \right]$$

$$= \frac{\tau_{max}^2}{4G} \cdot \pi (R_o^2 - R_i^2) \cdot L \cdot \left(\frac{R_o^2 + R_i^2}{R_o^2} \right)$$

$$U = \frac{\tau_{max}^2}{4G} \cdot (\text{volume}) \times \left(\frac{R_o^2 + R_i^2}{R_o^2} \right) \Rightarrow \boxed{U/\text{volume} = \frac{\tau_{max}^2}{4G} \times \frac{R_o^2 + R_i^2}{R_o^2}}$$

$$\boxed{U_H = \frac{\tau_{max}^2}{4G}\frac{(R_o^2+R_i^2)}{R_o^2} = \frac{\tau_{max}^2}{4G}\left(\frac{D_o^2+D_i^2}{D_o^2}\right)}$$

S.E. per unit volume

$$\frac{U_{Hollow}}{U_{Solid}} = \frac{R_o^2+R_i^2}{R_o^2} = \frac{D_o^2+D_i^2}{D_o^2} > 1$$

For same metal if max^m permissible shear stress is same in both hollow and solid shaft then Hollow shaft will store more energy than solid shaft per unit vol. Hence hollow shaft are more efficient.

Power transmitted By the Shaft :-

Let shaft is rotating at 'N' rpm or at ω rad/sec under mean torque T

Power transmitted $P = T \times \omega$

$$= T \cdot \frac{2\pi N}{60}$$

$$\boxed{P = \frac{2\pi NT}{60}}$$

if T is in KN-m, then P ⇒ KW

T " " N-m " P ⇒ Watt

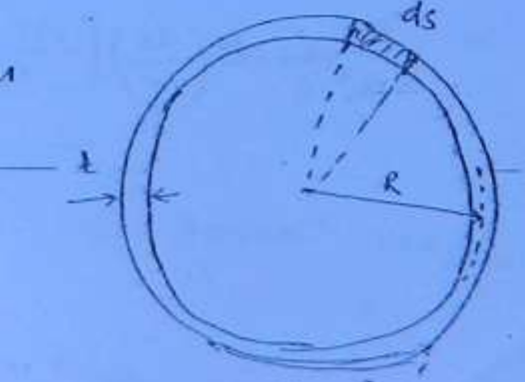
N ⇒ r.p.m.

If revolution frequency is given in Hertz then it represents no. of revolution per sec :

If shaft is rotating at 'f' Hz

$$\boxed{P = 2\pi f T}$$

Torsion in thin walled tubes :→



$R >>> t$

τ is constant across the thickness

$$\tau = \frac{T}{2\pi R^2 \cdot t}$$

$$\tau \cdot t = \frac{T}{2\pi R^2}$$

$$\tau t = \frac{T}{2\pi R}\ \frac{T}{2A_m} = \text{Const} = \text{Shear flow}$$

$$\boxed{\tau_1 t_1 = \tau_2 t_2}$$

In thin walled tube max^m shear stress is produced at the x-section of min^m thickness.

Strain Energy stored in tubular Section:-

Consider elemental length ds in fig given back and thickness t.

S.E. Stored per unit volume

$$= \frac{\tau^2}{2G}$$

S.E. Stored in elemental volume $dv = \frac{\tau^2}{2G} \cdot dv$

$$dU = \frac{\tau^2}{2G} \cdot (ds \cdot t) L.$$

$$U = \int dU = \int \frac{\tau^2}{2G} \, ds \cdot t \cdot L$$

$$\tau \cdot t = \frac{T}{2A_m} \Rightarrow \tau = \frac{T}{2A_m \cdot t} \Rightarrow U = \int \frac{T^2}{8A_m^2 t^2 G} \cdot t \cdot L \cdot ds$$

$$U = \frac{T^2 L}{8 \cdot A_m^2 \cdot G} \int \frac{ds}{t}$$

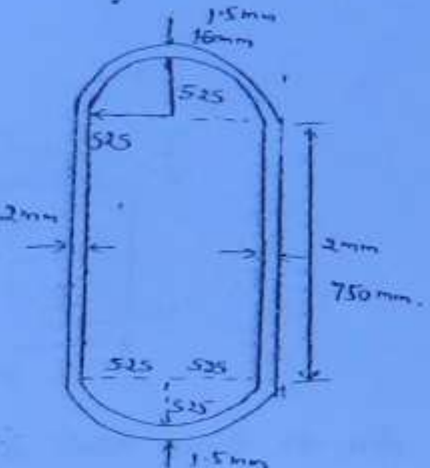
Imp
$$\boxed{U = \frac{T^2 L}{8 A_m^2 G} \Sigma \left(\frac{\Delta S}{t}\right)}$$

$$\theta = \frac{\partial U}{\partial T} = \frac{T \cdot L}{4 A_m^2 \cdot G} \Sigma \left(\frac{\Delta S}{t}\right)$$

A_m = Area bounded by median line

Q. Fig. shows a profile of x-section made of Al Alloy for which shear modulus $G = 27 \times 10^3$ N/mm² the thickness of metal structural varies as shown in fig. If max^m permissible shear stress in the metal is 80 N/mm² then det^n max^m permissible torque and angle of twist per unit length.

Sol^n:-



A_m = Area bounded by median lines

$$= \pi(525.75)^2 + 750 \times 1052$$

$$A_m = 165.73 \times 10^4 \text{ mm}^2$$

$$\tau_{max} \cdot t_{min} = \frac{T}{2A_m}$$

$$T = \tau_{max} \cdot t_{min} \times 2A_m$$

$$= 80 \times 1.5 \times 2 \times 165.73 \times 10^4$$

$$= 397.75 \times 10^6 \text{ N-m}$$

$$T = 397.75 \text{ KN-m}$$

$$\theta = \frac{TL}{4A_m^2 \cdot G} \Sigma\left(\frac{\Delta s}{t}\right)$$

$$\Sigma\left(\frac{\Delta s}{t}\right) = 2\left[\frac{\pi R}{t_1} + \frac{750}{t_2}\right]$$

$$= 2\left[\frac{\pi \times 525.75}{1.5} + \frac{750}{2}\right] = 2952.25$$

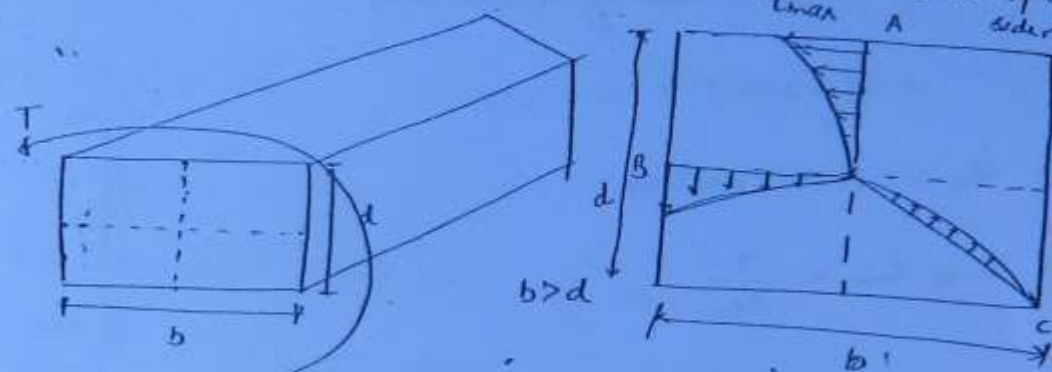
$$\frac{\theta}{L} = \frac{397.7 \times 10^6 \times 2952.25}{4 \times (165.7)^2 \times 27 \times 10^3} = 3.95 \times 10^{-6} \text{ rad/mm}$$

$$= 3.95 \times 10^{-3} \text{ rad/m length}$$

Torsion of non circular solid Section:-

Non circular sections are torsionally weak because shear stress is not symmetrical about the centre. Such section when subjected to torque get wraped and plane section doesn't remain plane after twisting

τ_{max} occurs at A which is centre of longer side



due to torque max^m shear stress is developed on the middle surface of longer side as shown in fig., Note that shear stress distribution is not linear.

Though distance of corner c is max^n from centre but shear stress at c is zero.

$$\tau_{max} = \frac{T}{\alpha \cdot bd^2} \qquad (b>d)$$

where α is a constant which depends on b/d ratio

if $b/d = 1$, then $\alpha = 0.208$

In square sections τ_{max} will occurs at middle surface of both sides.

The angle of twist θ is given as

$$\boxed{\theta = \frac{TL}{\beta \cdot G \cdot bd^3}}$$

β is a constt depends on b/d ratio

If $b/d = 1$; $\beta = \frac{1}{7.11}$

ES 11

A solid shaft transmits 250 kW at 100 rpm. If the shear stress is not to exceed 75 N/mm². what should be diameter of the shaft. If this shaft is to be replaced by a hollow shaft for which internal diameter is 0.6 times outer diameter then determine the size of Hollow shaft and percentage saving in the weight if maxⁿ permissible shear stress is same?

Solⁿ:-

$$P = \frac{2\pi N T}{60}$$

$P = 250$ kW

$N = 100$ rpm

$\tau_{max} = 75$ N/mm²

$$T = \frac{60 \times P}{2\pi \times N}$$

$$T = 23.87 \text{ kN-m}$$

$$T = \tau_{max} \cdot Z_P$$

$$23.87 \times 10^6 = 75 \times \frac{\pi}{16} D^3$$

$$\boxed{D = 117.47 \text{ mm}}$$

Let outer dia $= D_o$

Inner dia $= 0.6 D_o$

$$T = \tau_{max_1} \cdot Z_{P_1} = \tau_{max_2} Z_{P_2} \qquad \because \tau_{max_1} = \tau_{max_2}$$

$$\Rightarrow Z_{P_1} = Z_{P_2}$$

$$\Rightarrow \frac{\pi}{16} D^3 = \frac{\pi}{16} \frac{(D_o^4 - D_i^4)}{D_o}$$

$$(117.47)^3 = \frac{D_o^4 \left[1 - \left(\frac{D_i}{D_o}\right)^4\right]}{D_o} = D_o^3 \left[1 - (0.6)^4\right]$$

$D_o = 123.03$ mm

$D_i = 73.81$ mm.

$\pi D \cdot \pi r^2 \cdot \ell = \pi (D_o^2 - D_i^2) \cdot \ell$

$$\% \text{ Saving of weight} = \frac{W_{solid} - W_{hollow}}{W_{solid}} \times 100$$

$$= \left(1 - \frac{W_h}{W_s}\right) \times 100$$

$$= \left(1 - \frac{\gamma A_h L}{\gamma \cdot A_s L}\right) \times 100$$

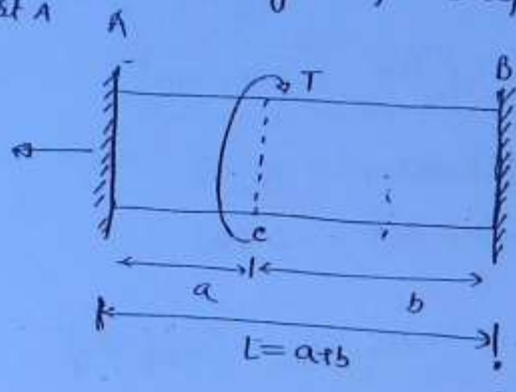
$$= \left(1 - \frac{A_h}{A_s}\right) \times 100 = \left[1 - \frac{\pi/4 (D_o^2 - D_i)^2}{\pi/4 \cdot D^2}\right] \times 100$$

$$= 29.78\ \%$$

Q:

A shaft is fixed at support A and B as shown in fig and subjected to a Torque T at C find Torsional reactions at A and B and twist angle at C. also draw torsional moment diagram for shaft AB.

Sol^n: Let T_A and T_B are Torsional reaction at A and B as shown in fig



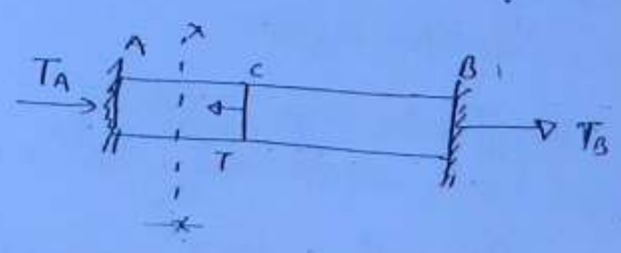
$\Sigma T = 0$

$$T_A + T_B - T_C = 0 \quad \text{—(i)}$$

$$\boxed{T_A + T_B = T_C}$$

$$\theta_{AC} + \theta_{CB} = 0 \quad \text{—(ii)}$$

$$\theta = \frac{T \cdot L}{G \cdot I_P}$$



$$\theta_{AC} = \frac{T_A \cdot a}{G \cdot I_P}$$

$$\theta_{BC} = \frac{(T_A - T)\ b}{G \cdot I_P}$$

$$\theta_{AC} + \theta_{CB} = \frac{T_A (a+b)}{G I_P} - \frac{T \cdot b}{G I_P} = 0$$

$$\boxed{T_A = \frac{T \cdot b}{L}}$$

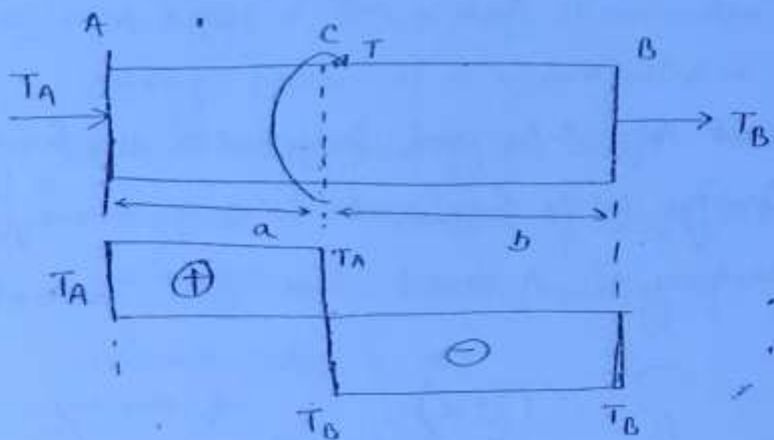
$$\boxed{T_B = \frac{T \cdot a}{l}}$$

— Twist angle at c

$$\theta_c = \theta_{AC} = \theta_c - \cancel{\theta_A}^{\,0}$$

$$= \frac{T_A \cdot a}{G \cdot I_p} = \frac{\frac{T \cdot b}{L} \cdot a}{G \cdot I_p}$$

$$\boxed{\theta_c = \frac{T \cdot a \cdot b}{L \cdot G \cdot I_p}}$$



Q: A shaft of Prismatic crosssection is fixed at A and free at B as shown in fig if shaft is subjected to uniformly distributed torque over entire length of Intensity q_0 KN-m/m then find angle of twist at free end B.

Solⁿ:

$$\Sigma T = 0$$

$$T_A - q_0 L = 0$$

$$T_A = q_0 L$$

— Twist of elemental length dx

$$d\theta = \frac{T_x \cdot dx}{G \cdot I_p}$$

$$\theta = \int_0^L \frac{T_x \cdot dx}{G \cdot I_p}$$

$$T_x = (T_A - q_0 \cdot x)$$

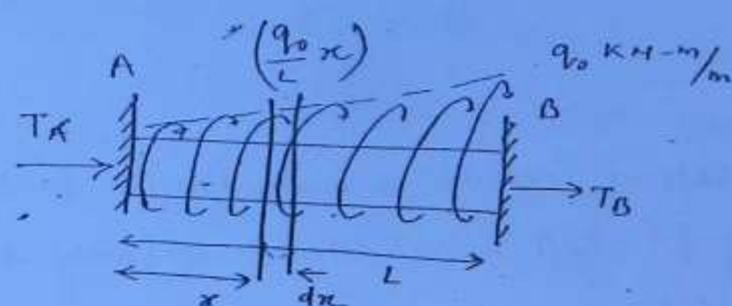
$$\theta = \int_0^L \frac{(T_A - q_0 x)\,dx}{G I_p} = \int_0^L \frac{(q_0 L - q_0 x)\,dx}{G I_p} = \frac{q_0}{G I_p}\int_0^L (L-x)\,dx$$

$$\theta = \frac{q_0}{G I_p}\left[Lx - \frac{x^2}{2}\right]_0^L \Rightarrow \theta = \frac{q_0}{G I_p}\left[L^2 - \frac{L^2}{2}\right].$$

$$\boxed{\theta = \frac{q_0 L^2}{2 G I_p}}$$

Q. A solid circular shaft is fixed at A and B and Subjected to uniformly Variable Torque for which intensity varies from 0 at A to q_0 kN-m/m at B. Then find torsional reactions at A and B and Plot Torsional Moment diagram.

Sol^n:



$\Sigma T = 0$

$T_A + T_B = \frac{1}{2} L \times q_0$

$$\boxed{T_A + T_B = \frac{q_0 L}{2}} \quad \text{—(i)}$$

$$T_x = \left(T_A - \frac{q_0}{L}x \times \frac{1}{2}x\right)$$

Angle of twist of elemental length dx is $\quad T_x = T_A - \frac{q_0 x^2}{2L}$

$$d\theta = \int \boxed{\frac{T_x \cdot dx}{G I_p} = d\theta}$$

$$\theta_B^0 - \cancel{\theta_A^0} = \int_0^L \frac{T_A - \frac{q_0 x^2}{2L}}{G I_p}\,dx = 0$$

$$T_A L - \frac{q_0 L^3}{6L} = 0 \qquad T_A = \frac{q_0 L}{6} \quad \text{—(ii)}$$

$\frac{q_0 L}{6} - \frac{q_0 L}{2}$

$-\frac{2-3}{6}$

$\frac{-2}{6} - \frac{1}{3}$ Sol

$T_B = \frac{q_0 L}{3}$

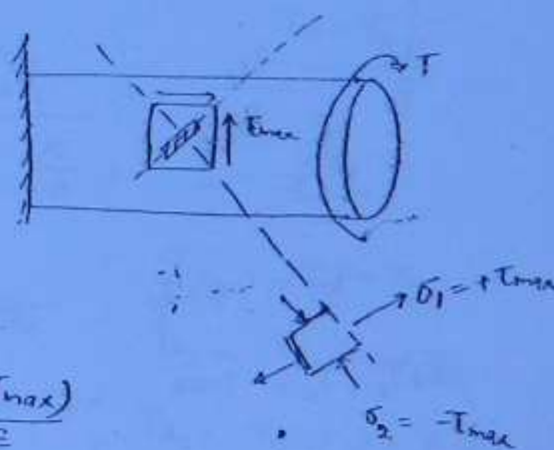
Not feasible

IES'ME

Ques:- A solid Bronze shaft 16 mm in dia is rotating at 800 rpm and transmitting power. If it is subjected to torsion only then determine the Power transmitted by the shaft and electric resistance strain gauge mounted on the surface of shaft with its axis at 45° to the shaft axis gives the strain reading of 3.98×10^{-4} the $E_B = 105$ GPa and Poisson's ratio is 0.3.

$d = 16$ mm

$N = 800$ rpm



At 45° from shaft axis σ_1 act as strain gauge in this dir^n will be ϵ_1

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{\tau_{max}}{E} - \mu\frac{(-\tau_{max})}{E}$$

$$\epsilon_1 = \frac{\tau_{max}}{E}(1+\mu)$$

$$\frac{3.98 \times 10^{-4} \times 1.05 \times 10^5}{(1+.3)} = \tau_{max} = 32.14 \text{ N/mm}^2$$

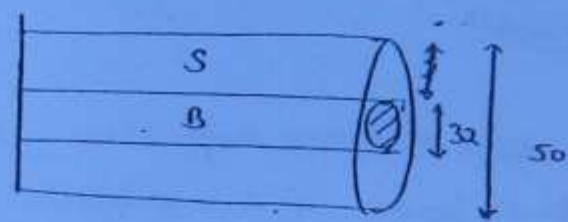
$$T = \tau_{max} \cdot Z_p = 32.14 \times \frac{\pi}{16} \times 60^3$$

$$T = 1.36 \text{ kN-m}$$

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 800 \times 1.36}{60} = 113.9 \text{ kW}$$ Ans

A composite shaft consist of a solid Brass rod 32 mm in dia which is ~~a cast~~ inside a steel tube of 50 mm eternal dia and 32 mm ~~exter~~ internal dia If there is a perfect Bond b/t both and combined assembly is subjected to pure torsion of 1000 N-m then evaluate max^n shear stress developed in the Brass and steel ?

$$G_S = 2\,G_B$$



$$T_B + T_S = 1000 \text{ N-m} \quad -(i)$$

$$\theta_B = \theta_S$$

$$\frac{T_B \cdot L}{G \cdot I_{P_B}} = \frac{T_S \cdot L}{G \cdot I_{P_S}}$$

$$\frac{T_B}{T_S} = \frac{I_{P_B}}{I_{P_S}} = \frac{\frac{\pi}{16} \cdot d^3}{\frac{\pi}{16} \cdot \left(\frac{D_o^4 - D_i^4}{D_o}\right)} = \frac{32^3}{\frac{50^4 - 32^4}{50}} =$$

$$= \cdot 1$$

$$T_B = \cdot\cancel{3}1\, T_S \quad -(ii) \qquad T_B = \cdot 1\, T_S$$

$$\cdot 31\, T_S + T_S = 1000$$

$$\cancel{T_S = 763.35 \text{ N-m}} \qquad T_S = 909.09 \text{ N-m}$$

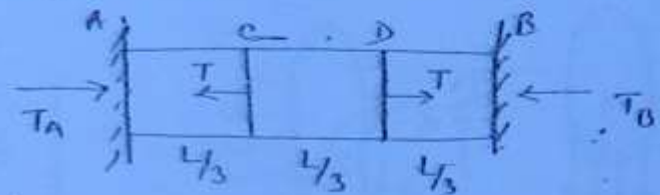
$$\cancel{T_B = 263\ 236.64 \text{ N-m}} \qquad T_B = 90.91 \text{ N-m}$$

$$\tau_{max} = \frac{T_S}{Z_{P_S}} = \frac{\cancel{763.35}}{} \quad \frac{909.09}{\frac{\pi}{16}(32)^3} = 141.4454$$

$$\tau_{max} = \frac{T_b}{Z_{P_B}} =$$

14.2341 N/mm²

A Prismatic shaft AB.



1) The magnitude of Torsional reacⁿ at A and B will be.

a) T, T

b) 0, 0

c) T/3, T/3

d) 2T/3, 2T/3

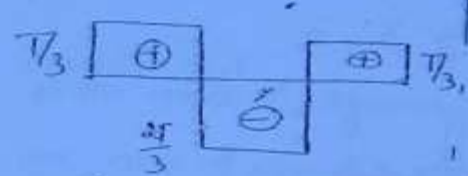
2) The maxⁿ torque in the shaft is

a) T

b) 0

c) T/3

d) 2T/3



$$T_A - T + T - T_B = 0$$

$$T_A = T_B \quad -(i)$$

$$\theta_{AC} + \theta_{CD} + \theta_{DB} = 0$$

$$\frac{T_A \cdot L/3}{G I_P} + \frac{(T_A - T) L/3}{G I_P} + \frac{T_A L/3}{G I_P} = 0$$

$$2 \cdot T_A \cdot \frac{L}{3} + T_A \cdot \frac{L}{3} = T \frac{L}{3}$$

$$3T_A = T$$

$$T_A = T/3$$

$$T_B = T/3$$

$\cancel{\tau_{max} = T / Z_P}$

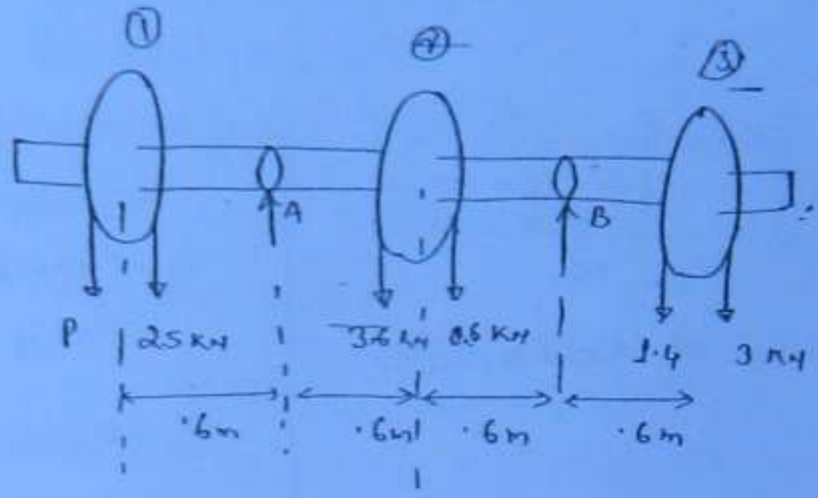
$$\tau_{x_1} = +T_A = T/3$$

$$\tau_{x_2} = T_A - T = -2T/3$$

$$\tau_{x_3} = T_A - T + T = +T/3$$

Ques:-

A solid circular shaft of constant cross section Carries 3 pulleys and belts which are all verticle calculate unknown tension P on the belt of first Pulley also determine the vertical reaction at bearing supports A & B and draw S.F., B.M. and Torsional moment diagram. the radius of first Pulley is 70 mm and radius of Second Pully is 100 mm and radius of third pulley is 80 mm.



$P = 0.0428$

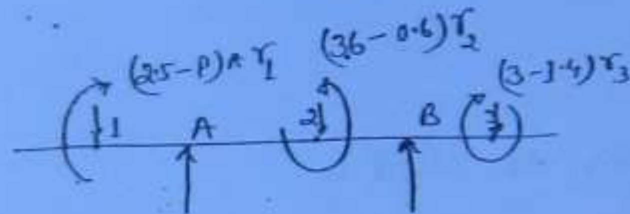
$P = ?$

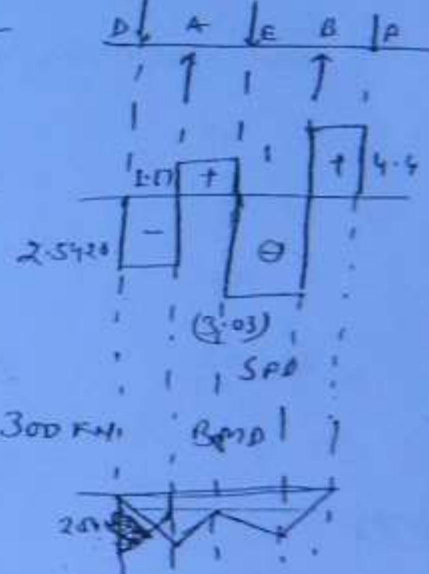
R_A & R_B = ?

$r_1 = 70, \quad r_2 = 100, \quad r_3 = 80$

$25 \times .07 - P r_1 =$

Let $P < 25$





Torque at Pulley 3, is. $T_3 = 128$ KN.

Torque at Pulley 2 is $T_2 = (3.6 - 0.6)\, r_2 = 300$ KN.

$\Sigma T = 0$

$T_1 = T_2 - T_3$

$(2.5 - P) \times 70 = 300 - 128 - 172$

$P = 0.0428$ KN.

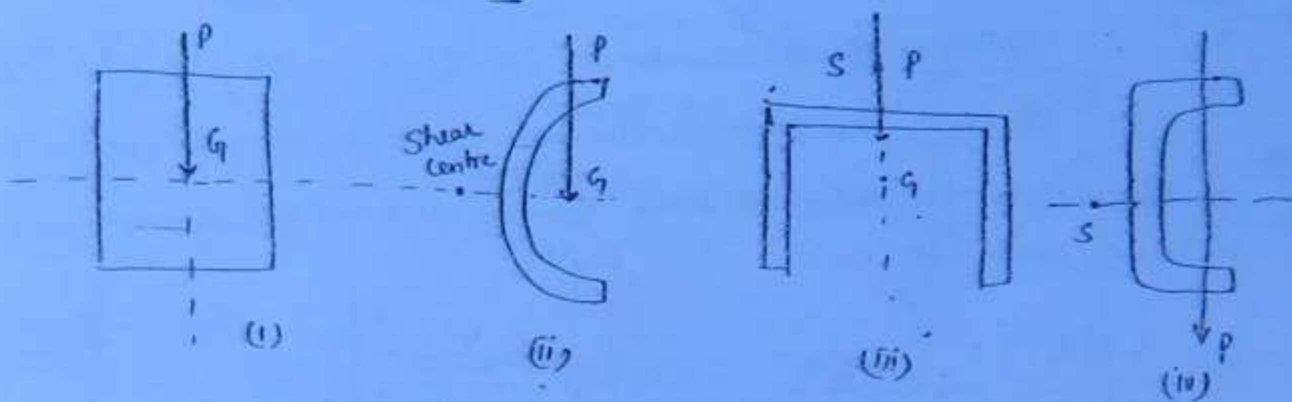
$\Sigma F_y = 0, \quad R_A + R_B = 11.428$ KN.

$\Sigma M = 0$

$R_A = ?$

$R_B = ?$

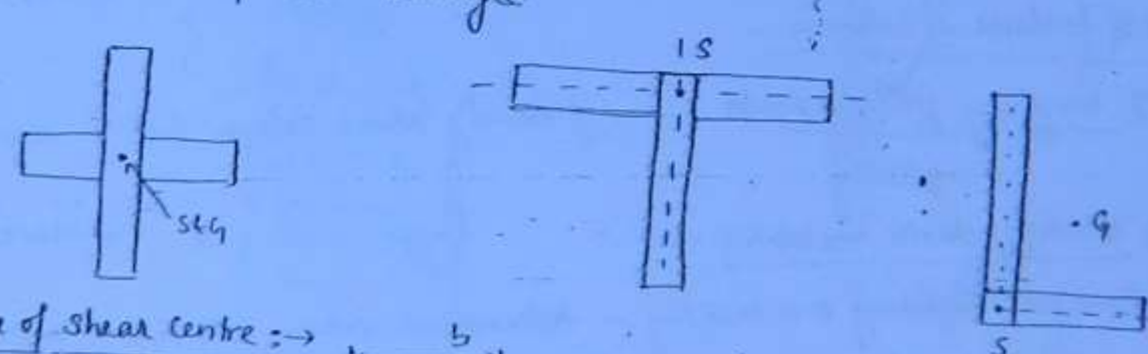
Shear Centre:-



(i) and (iii) are in Bending only
(ii) and (iv) are in Bending and twisting both.

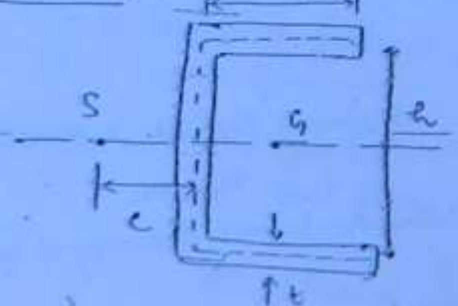
- Shear centre is that point through which if concentrated load passes then ter there will be no twisting and will be only bending it is that point through which resultant of shear passes and it is also called centre of flexure.

- Shear centre always lies on the axis of symmetry if exist
- If there are more than one axis of symmetry then shear centre will coincide with centroid i.e. C.G.
- If cross-section is made of two narrow rectangles then shear centre lies at the ~~jsaction~~ junction of both rectangle



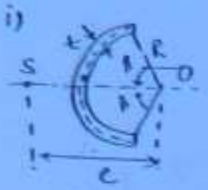
Distance of shear centre :→

(i)



$$e = \frac{b^2 h^2 t}{4I}$$

i)



$$e = 2R\left[\frac{\sin\beta - \beta\cos\beta}{\beta - \sin\beta\cos\beta}\right]$$

Sp. Case I :- For Semi Circular ring $(\beta = \pi/2) \Rightarrow e = 2R\left[\frac{1-0}{\pi/2 - 0}\right] = \frac{4R}{\pi} = 1.27R$

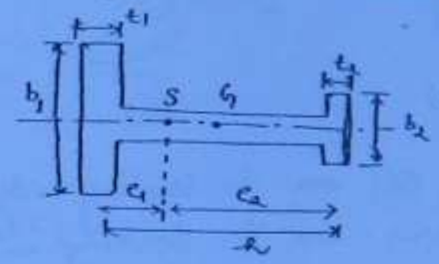
Sp. case II. For $\beta = \pi$. circular ring with open slit $\Rightarrow e = 2R\left[\frac{0+\pi}{\pi - 0}\right] = 2R$

NOTE:
If slit is closed then section will become symmetric about more than one axis hence shear centre will coincide with C.G.

ii)

$$e_1 = \frac{t_2 b_2^3 h}{t_1 b_1^3 + t_2 b_2^3}, \quad e_2 = \frac{t_1 b_1^3 h}{t_1 b_1^3 + t_2 b_2^3}$$

if $t_1 = t_2$ and $b_1 = b_2$ then $e_1 = e_2$.



Columns :-

Columns are axial compression members used in bldgs. Compression members are known by different name.

i) Pedestal :- It is a short compr. member normally used at base

ii) Post : It is compression member made of metallic section which doesn't fail in buckling but fail in crushing.

iii) Stanchions : These are compression members used in bridges made of steel

iv) Strut : , used in trusses.

v) Boom :- These are principal compression members used in cranes.

Modes of failure of coloumn :-

a) Crushing failure / yield — generaly short column fail in crushing or yielding.

b) Buckling / elastic unstability :- " long " " " Buckling

c) Combined Crushing and Buckling :- Intermediate column will fail in both.

Types of equilibrium :-

(i) Stable equilibrium :- Restoring Moment > overturning moment

$M_R > M_o$

under stable equlibrium the potential energy of system is minim

(ii) Nutral equlibrium :

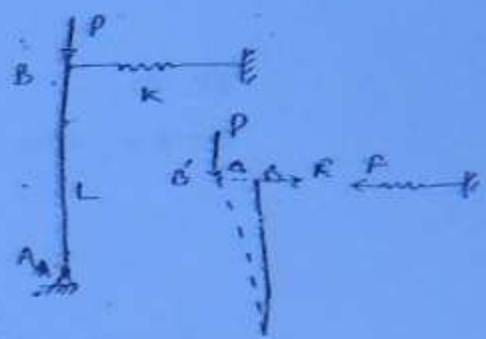
$M_R = M_o$

This stage is also called critical stage

... equilibrium:- $M_R < M_O$

A displaced body will further di

Critical load and elastic unstability :-



When load P is increased the Position displaced

Restoring Moment about A

$M_R = F \cdot L$

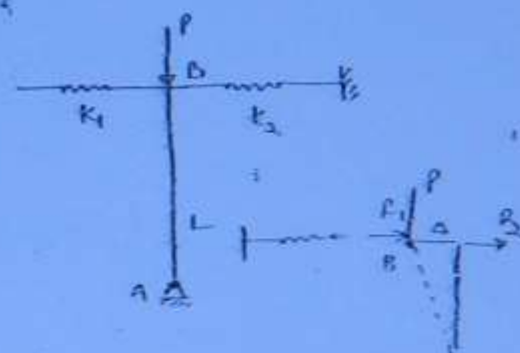
$= K \cdot \Delta \cdot L$

At N.E. or critical condⁿ

$M_R = M_O \Rightarrow K \cdot \Delta L = \boxed{KL}$ Imp.

If $P > P_{cr}$ it will cause elastic unstability

Qu. For the assembly shown in fig find critical



$M_R = F_1 L + F_2 L$, $M_O = P \cdot \Delta$

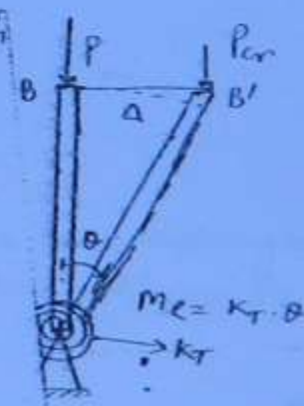
For critical

$M_R = M_O$

$(K_1 + K_2)\cancel{\Delta} \cdot L = P_{cr} \cancel{\Delta}$

$\boxed{P_{cr} = (K_1 + K_2) L}$

Ques: Find critical load for assembly shown in fig



$M_O = P \cdot \Delta$, $M_R = K_T \cdot \theta$

at critical condition.

$M_O = M_R$.

$P_{cr} \cdot \Delta = K_T \cdot \theta$

$P_{cr} \cdot \cancel{\Delta} = K_T \dfrac{\cancel{\Delta}}{L}$

$\boxed{P_{cr} = K_T / L}$.

K_T = Stiffness against rotation = rotational stiffness

Euler's Theory of Buckling failure :-

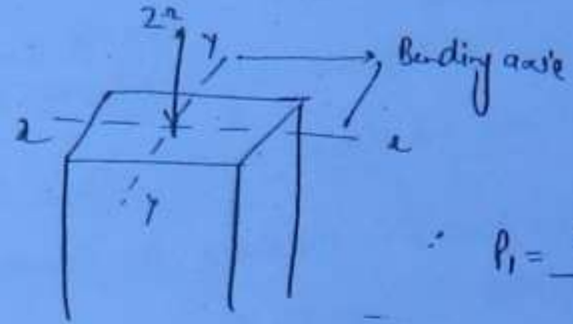
Assumption:-

i) axis of the column is perfectly when unloaded and load passes thru the axis

ii) Column is cylinder and long i.e. the buckling

(iii) Flexural rigidity (E.I) is const

(iv) Material is Homogeneous, Isotropic elastic in which hooks law is valid.



$\therefore P_1 = \dfrac{\pi^2 E I_x}{L_e^2}$

$P_2 = \dfrac{\pi^2 E I_y}{L_e^2}$

if $I_x < I_y$

then $P_1 < P_2$

So we take minm moment of Interia

$$EI\frac{d^2y}{dx^2} = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}\,y = 0$$

$$y = C_1\cos\left(x\sqrt{P/EI}\right) + C_2\sin\left(x\sqrt{P/EI}\right)$$

At $x=0,\ y=0$.

$$0 = C_1 + 0$$

$$C_1 = 0$$

at $x=L,\ y=0$

$$0 = \cancel{C_1}^{\,0}\cos\left(x\sqrt{P/EI}\right) + C_2\sin\left(L\sqrt{P/EI}\right)$$

$$C_2\sin\left(L\sqrt{P/EI}\right) = 0$$

Since $C \neq 0$

$$\sin\left(L\sqrt{P/EI}\right) = 0$$

$$L\sqrt{P/EI} = n\pi$$

$$\boxed{P = \frac{n^2\pi^2 EI}{L^2}}$$

$P \Rightarrow$ Euler's load/ Buckling load/ crippling load/ critical load

$I =$ minm moment of inertia about centroidal axis

$n =$ No of Buckling loop

For basic mode of failure $\Rightarrow n=1$

$L =$ maxm unsupported length

* If Both ends are ~~end~~ hinged then there is single Buckling loop and length b/w both hinged support is effective length

$$\left[P_e = \frac{\pi^2 EI}{L}\right]$$ Euler's Buckling load.

2) If end cond's are different and there is one buckling resp. the effective length should be used hence

euler's Buckling load $\left(P_e = \frac{\pi^2 E I}{L_e^2}\right)$

L_e = distance b/t two consequtive pt of zero Bending moment

$$P_e = \frac{\pi^2 \cdot E \cdot A \cdot r_{min}^2}{L_e^2}$$

r_{min} = radius of gyration min

$$P_e = \frac{\pi^2 E A}{\left(\frac{L_e}{r_{min}}\right)^2} = \frac{\pi^2 E A}{\lambda^2}$$

$$\boxed{\lambda = \frac{L_e}{r_{min}}}$$

$\Rightarrow$

$$\frac{P_e}{A} = \frac{\pi^2 E}{\lambda^2}$$

Imp $\boxed{\sigma = \frac{\pi^2 E}{\lambda^2}}$

Limitation of euler's theory :-

It is not valid for short column it means euler's thy will only valid when $\lambda \geq \lambda_c$ (λ_c = Critical selenderness ratio)

According to euler's theory column should fail in buckling not in crushing it means buckling stress (σ) should be less than or equal to σ_c $[\sigma \leq \sigma_c]$.

$$\sigma \leq \sigma_c$$

$$\frac{\pi^2 E}{\lambda_c^2} \leq \sigma_c$$

$$\lambda^2 \geq \frac{\pi^2 E}{\sigma_c}$$

$$\boxed{\lambda \geq \sqrt{\frac{\pi^2 E}{\sigma_c}}}$$

$\lambda_{min} = \lambda_c = \sqrt{\frac{\pi^2 E}{\sigma_c}}$ → critical Slenderness ratio

For mild steel.

$$E = 2\times10^5 \ N/mm^2$$

$$\sigma_c = \sigma_y = 250 \ N/mm^2$$

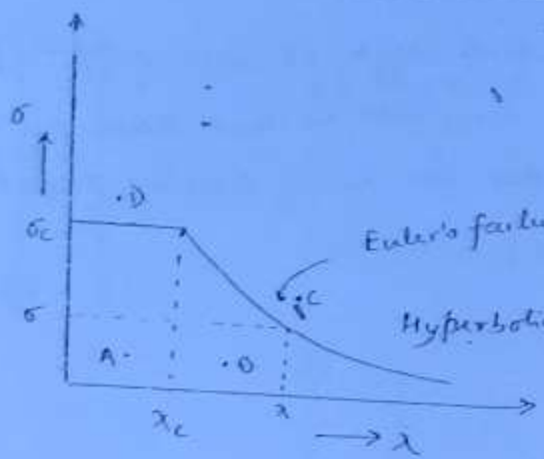
$$\lambda_c = \sqrt{\frac{\pi^2 \times 2\times10^5}{250}}$$

$$\lambda_c = 88.8$$

$$\boxed{\lambda_c = 89}$$

It means for mild steel euler's theory is valid only when ε slenderness ratio $\lambda > 89$;

Graph b/w failure stress and slenderness ratio :-



A → Short safe column

B → long Safe

C → long buckled

D → Short crushed

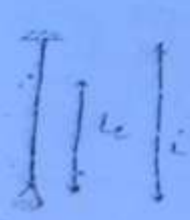
End condns

Effective length

1) Both ends hinged (or) Both supports are held in position But not restrained against rotation

$L_e = L$

2) One end is hinged other is fixed

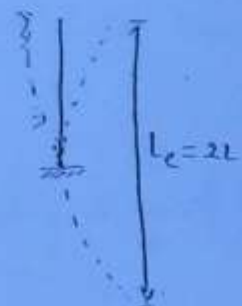


$L_e = L/\sqrt{2}$

3) Both ends are fixed

$L_e = L/2$

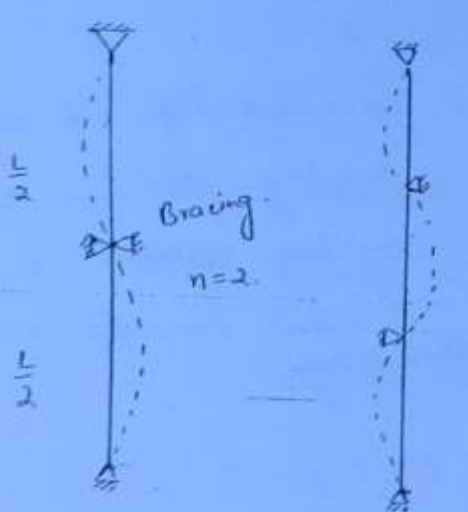
4) one end fixed other end is free



$L_e = 2L$

Effective length in Braced Column:-

when col. are supported at intermediate length at such support prevent movement of col. at that pt. then there will be more than one buckling loops in the failure, therefore either use no of Buckling loops to find euler's load or use effective length.



$$P_e = \frac{\pi^2 EI}{L_e^2} = \frac{4\pi^2 EI}{L^2}$$

take $L_e = 3L/4$ for min^m buckling load

$$P_e = \frac{\pi^2 EI}{\left(\frac{3}{4}L\right)^2}$$

$$L_e = \frac{1}{\sqrt{2}} \cdot \frac{2L}{3} = \frac{\sqrt{2}}{3}L$$

$L_e = L/2$

$L_e = \frac{1}{\sqrt{2}} \cdot \frac{L}{2}$

take $= L_e = L/2$.

Eff. length b/t in range of

$$\frac{L}{2} < L_e < L/\sqrt{2} \quad \text{depending upon } k.$$

k = rotational stiffness of spring.

$$\frac{L}{2} < L_e < L$$

Rankine's theory :—

The euler's theory is not valid for short column. Rankine's theory is applicable for ~~f~~ short and long column both, it assumes combined mode of failure (crushing and buckling both).

Let P_e is euler's buckling load and P_c is crushing load and let P is rankine's crippling load, then

$$\boxed{1/P = 1/P_e + 1/P_c}$$

$$P = \frac{P_e \cdot P_c}{P_e + P_c}$$

$$P = \frac{P_c}{1 + P_c/P_e} = \frac{\sigma_c A}{1 + \frac{\sigma_c \cdot \cancel{A}}{\frac{\pi^2 E \cancel{A}}{\lambda^2}}}$$

$$\boxed{P = \frac{\sigma_c A}{1 + \left(\frac{\sigma_c}{\pi^2 E}\right)\lambda^2} = \frac{\sigma_c A}{1 + \alpha \lambda^2}}$$

where $\alpha = \frac{\sigma_c}{\pi^2 E}$

= Rankine's const.

$\alpha \propto \left(\frac{\sigma_c}{E}\right)$ if $\frac{E}{\sigma_c} = K$

$\alpha \propto \left(1/K\right)$

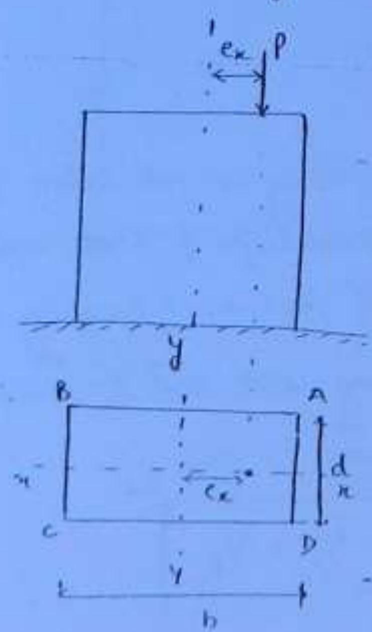
$\alpha = 1/9000$ for wrought iron

$= 1/7500$ for Mild steel

$= 1/1600$ for Cast iron.

Short columns loaded eccentrically :→

(Combined Axial and Bending Moment)



Due to eccentricity about y axis there will be 2 effects.

1) direct stresses $\sigma_d = P/A$ (compressive) which will be uniform across the cross section.

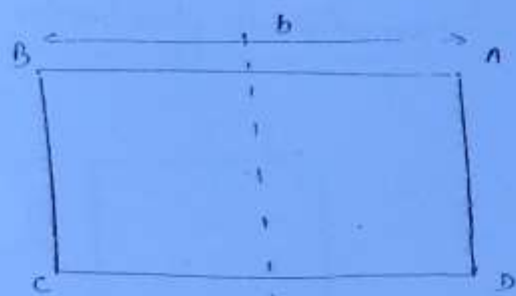
2) Bending stress $\sigma_b = \pm \frac{M_y}{I_y} x$, Bending stresses will vary from 0 at y-y to maximum at AD or BC.

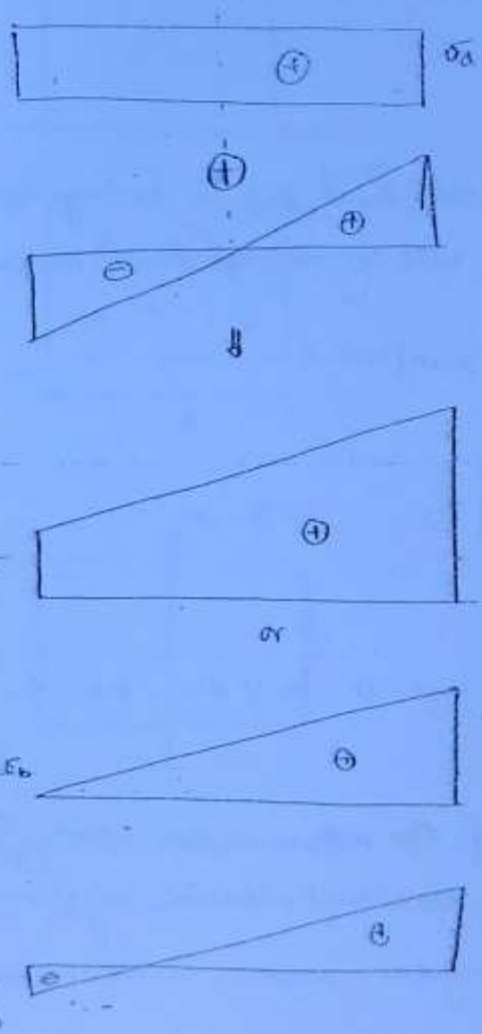
$M_y = P \cdot e_x$

For columns only

(+) Compression

(−) tension





$\sigma_d = P/A$

$\sigma_{max} = \sigma_b + \sigma_d$ (when $\sigma_d > \sigma_b$)

$\sigma_{min} = \sigma_d - \sigma_b > 0$

or

$\sigma_{max} = \sigma_d + \sigma_b = 2\sigma_d$ (when $\sigma_d = \sigma_b$)

$\sigma_{min} = \sigma_d - \sigma_b = 0$

~~$\sigma_{max} = \sigma_d + \sigma_b$~~ (when $\sigma_d < \sigma_b$)

$\sigma_{min} = \sigma_d - \sigma_b < 0$

In brittle columns failure may occur due to tensile stress. there fore large eccentricities are not permitted to avoid tensile stress. Hence for no tension condⁿ $(\sigma_{min} \geq 0)$

$$\sigma_{min} = \sigma_d - \sigma_b \geq 0$$

$$\frac{P}{bd} - \frac{M_y}{I_y} \cdot x_{max} \geq 0$$

$$\frac{P}{bd} - \frac{P\, e_x}{\frac{d\, b^3}{12}} \times \frac{b}{2} \geq 0$$

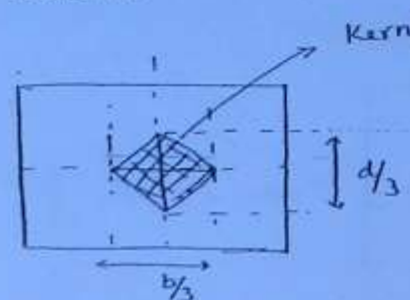
$$\boxed{e_x \leq b/6}$$

$e_{x\,max} = b/6$ for No tension in the column.

NOTE :-

Middle Third rule :

If load passes thru middle third strip in rectangular and square column then there will be no resultant tension any where on the column.



If eccentricity is about x axis i.e. in y dirⁿ then for no tension $e_{y\,max} \leq d/6$

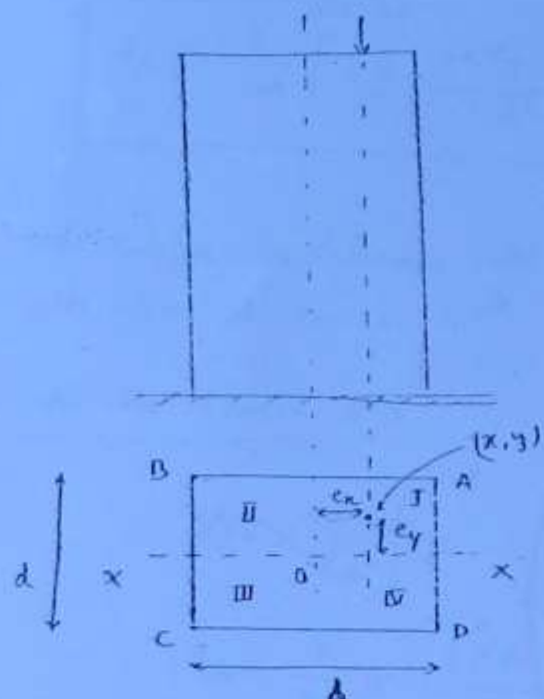
Kern or Core is that area of the column thru which if load passes then there will be no resultant tension anywhere on the column section. For rectangular section Kern is a rhombus

For hollow circle with external dia D and internal dia d and e should be less than equal to $\frac{D^2+d^2}{8D}$

$$e \leq \frac{D^2+d^2}{8D}$$

$$\text{Dia of Kern} = \frac{D^2+d^2}{4D}$$

Biaxial eccentricity in Columns :-



$$M_x = P\,e_y$$

$$M_y = P\,e_x$$

$$I_x = \frac{bd^3}{12}$$

$$I_y = \frac{db^3}{12}$$

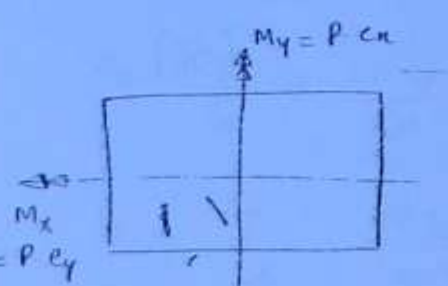
$$\sigma_1 = \sigma_d \pm \sigma_{bx} \pm \sigma_{by}$$

σ_d = direct stress = P/A

σ_{bx} = Bending stress due to B.M. abt. x-x $= \pm\frac{M}{I}\,y = \frac{M_x}{I_x}\cdot y$

σ_{by} =

$$= \pm\frac{M_y}{I_y}\cdot y$$



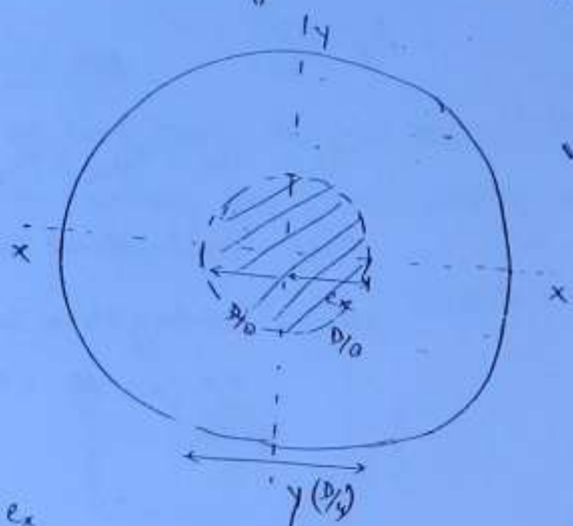
Due to M_x quadrant ① & ② in Compression and ③ & ④ will be in tension face AB will have max^m compressive stress and CD will have max^m tensile stress.

r of side length $= \sqrt{\frac{b^2+d^2}{6}}$

Shape of column	Shape of Kern
1. Rectangle	Rhombus
2. I Section	"
3. Square	Square
4. Solid Circle	Circle
5. Hollow "	"

Kern in circular Columns :-

Let eccentricity of the load is e_x



Kern is a circle of dia $(d/4)$

$M_y = P \cdot e_x$

$\sigma_{min} \geq 0$

$\sigma_d - \sigma_b \geq 0$

$$\frac{P}{\frac{\pi}{4} D^2} - \frac{P \cdot e_{max}}{\frac{\pi}{64} D^4} \cdot D/2 \geq 0$$

$$\boxed{e_{max} \leq D/8}$$

$$\boxed{e_{max} = D/8}$$

Due to M_y quadrant ① & ④ will be in compression and ② and ③ will be in tension. Max^m compressive stress on face AD and max^m tensile stress on face BC.

$$\sigma_A = \frac{P}{b \cdot d} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

$$\sigma_A = \frac{P}{bd} \pm \frac{P \cdot e_y}{\frac{bd^3}{12}} \cdot y \pm \frac{P \cdot e_x}{\frac{db^3}{12}} \cdot x$$

$$\boxed{\sigma_A = \frac{P}{bd}\left[1 \pm \frac{12 e_y \cdot y}{d^2} \pm \frac{12 \cdot e_x \cdot x}{b^2}\right]}$$

Max^m compressive stresses will occur at corner A and min^m compressive stress will occur at corner C

At the corner A :

$$\cancel{\sigma_A = \frac{P}{bd}\left[1 \pm\right.}$$

$$\sigma_A = \frac{P}{bd} + \frac{6P \cdot e_y}{bd^2} + \frac{6Pe_x}{\cancel{bd^2}\, db^2}$$

$$\sigma_B = \frac{P}{bd} + \frac{6P \cdot e_y}{bd^2} - \frac{6Pe_x}{db^2}$$

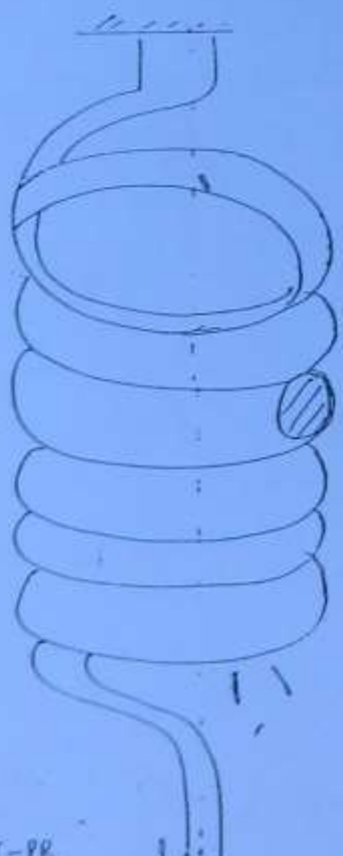
$$\sigma_C = \frac{P}{bd} - \frac{6P \cdot e_y}{bd^2} - \frac{6P \cdot e_x}{db^2}$$

$$\sigma_D = \frac{P}{bd} - \frac{6P \cdot e_y}{bd^2} + \frac{6Pe_x}{db^2}$$

Springs :-

- Closed helical springs (Angle of Helix < 10°)
  - Torsion spring
  - a. Railway suspension
- Open coiled Helical springs (Angle of Helix > 10°)
  - Torsion spring
  - Bike suspension
- Leaf springs
  - Bending Spring
  - Truck and Trolley suspension
- Spiral spring
  - Bending and Torsion Springs
  - clocks and watches etc

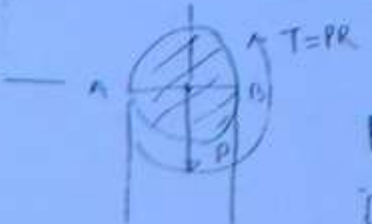
Closed coiled Helical spring :-



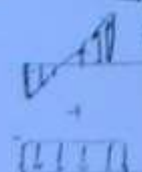
Let springs is made of a circular rod of dia melix d and length L let there are 'n' terms in the spring if mean Radius of spring is R then

$L = 2\pi R n$.

Due to axial load P on the spring there will be two effect on the section of rod.



i) Torque $T = PR$



$\tau_1 = \frac{T}{Z_p} = \frac{16T}{\pi d^3}$

ii) Shear force P.

$\tau_2 = \frac{P}{A} = \frac{4P}{\pi d^2}$

Due to torque shear stresses are produced which vary linearly from 0 at neutral axis through max^m at surface say τ_1. and due to shear force shear stresses are assumed uniform in view of small diameter say τ_2

Hence resultant stresses will be

i) At inner surface $(\tau_1+\tau_2)$

ii) " outer " $(\tau_1-\tau_2)$

It means max^m shear stresses are produced at inner surface of spring

$$\tau_{max} = \frac{16 \cdot PR}{\pi d^3} + \frac{4P}{\pi d^2}$$

$$\boxed{\tau_{max} = \frac{4PR}{\pi d^2}\left[\frac{4R}{d}+1\right]}$$

$$\frac{4R}{d} >> 1$$

$$\text{ie} \quad \tau_1 >> \tau_2$$

It means effect of shear force is negligible as compare to that of torque therefore this spring is called torsional spring.

Strain Energy Stored in above spring:-

Let P is torque on the spring Rod. neglect effect of S.F

$$U = \frac{T^2 L}{2G \cdot I_P}$$

$$= \frac{(P \cdot R)^2 \cdot 2\pi R n}{2 \cdot G \cdot \frac{\pi}{32} d^4}$$

objective

$$\boxed{U = \frac{32 P^2 R^3 n}{G d^4}}$$

Imp

Axial deflection of spring under force P —

$$\delta = \frac{\partial U}{\partial P} = \frac{64 P R^3 n}{G d^4}$$

$$\Delta = \boxed{\delta = \frac{64 P R^3 n}{G d^4}} \quad *$$

Coefficient of stiffness of spring

$$K = P/\Delta$$

$$= \frac{P \times G d^4}{64 P R^3 n}$$

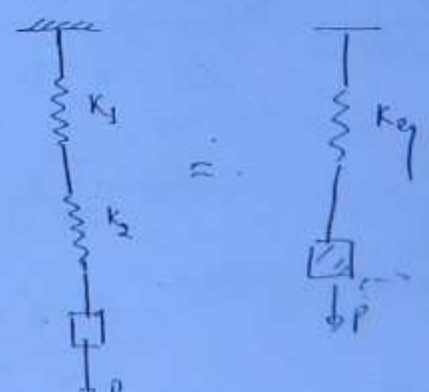
Imp $\quad * \quad \boxed{K = \frac{G d^4}{64 R^3 n}}$

$* \quad \boxed{K \propto 1/n}$

* It means if a spring of stiffness K and turn n is cut in two equal halfs then stiffness of each part will be 2K.

Combinations of springs :-

1) Series Combination :→



$$P_1 = P_2 = P \quad —(i)$$

$$\Delta = \Delta_1 + \Delta_2$$

$$\frac{P}{K_{eq}} = \frac{P}{K_1} + \frac{P}{K_2}$$

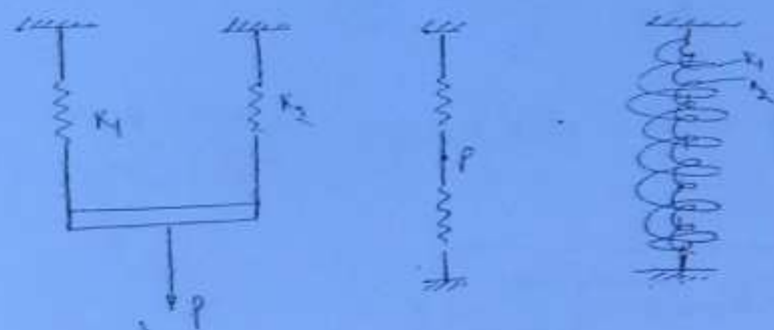
$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow \boxed{K_{eq} = \frac{K_1 \cdot K_2}{K_1 + K_2}}$$

If no. of springs are more than 2 then

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \cdots \frac{1}{K_n}$$

$f = 1/K$ = flexibility coefficient

<u>Parallel Connection</u>/ <u>Coaxial Springs</u> :→



In parallel connection.

$$\Delta_1 = \Delta_2$$

But forces are different

Total force ⇒ $P = P_1 + P_2$

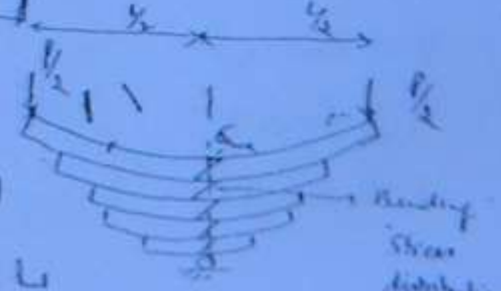
$$K_e \cdot \Delta = K_1 \Delta + K_2 \Delta$$

$$\boxed{K_e = K_1 + K_2}$$

$$\boxed{K_e = K_1 + K_2 + K_3 + \cdots + K_n}$$

$$\boxed{1/f_e = 1/f_1 + 1/f_2 + 1/f_3 + \cdots 1/f_n}$$

(ii) Leaf Spring or Laminated Spring :—



Leaf spring is made of flat curved Plates (b×t) size each

Let there are n plates at centre

M.I of each Plate = $\frac{bt^3}{12}$

Max^m B.M. occurs at centre = $\frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$

Hence B.M. resisted by n plates at centre = $PL/4$

B.M. " " " each plate = $\frac{PL}{4} / n$

Z of each plate = $\frac{I}{y_{max}} = \frac{\frac{bt^3}{12}}{t/2} = \frac{bt^2}{6}$

Max^m Bending stress in each plate = $\frac{M}{Z} = \frac{\frac{PL}{4n}}{\frac{bd^2}{6}} = \frac{3}{2}\frac{PL}{n \cdot b t^2}$

Imp.

$$\boxed{\sigma_{max} = \frac{3}{2}\frac{PL}{n \cdot b t^2}}$$

Max^m defl^n occurs at free end.

$$\boxed{\delta_{max} = \frac{3}{8}\frac{PL^3}{n \cdot b t^3 E}}$$

Q. If there are n plates having flexural stiffness of each spring = K if all the plates are jointed firmly then stiffness of spring will increase by

i) n time
ii) n^2 time. ✓
iii) $1/n$ times
iv) No change

$\boxed{\frac{K'}{K} = n^2}$

When all plates are not jointed the

$I = n \times I$ of each plate

$= n \cdot \frac{bt^3}{12}$

$K = \frac{E \cdot \frac{nbt^3}{12}}{L}$

When plates are jointed,

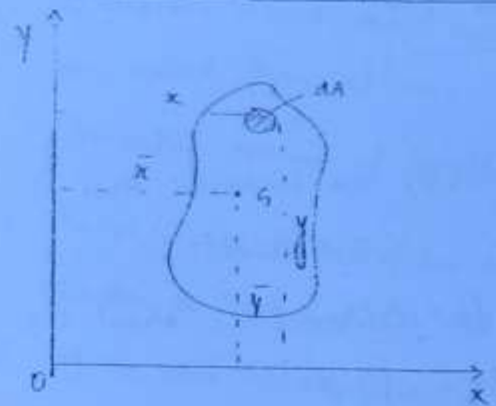
There will be ~~common~~ N.A

total depth at centre = nt

$I = \frac{b(nt)^3}{12}$

$K' = \frac{E\, n^3 t^3 b}{12 \cdot L}$

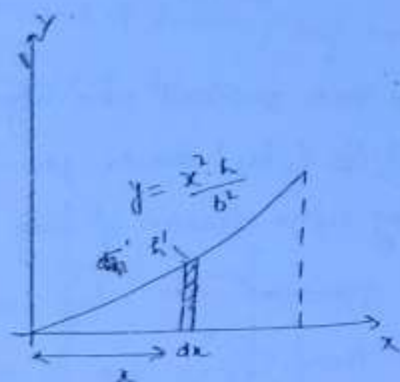
Centroid and principal Axis :-



$$\bar{x} = \frac{\int x\, dA}{\int dA} , \quad \bar{y} = \frac{\int y\, dA}{\int dA}$$

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} , \quad \bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + \cdots A_n\bar{x}_n}{A_1 + A_2 + \cdots A_n}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} , \quad \bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + \cdots + A_n\bar{y}_n}{A_1 + A_2 + \cdots A_n}$$



$x =$

$$h' = h\,\frac{x^2}{b^2}$$

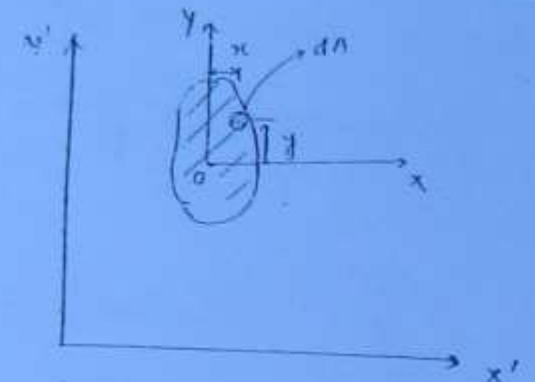
$$dA = h'\,dx$$

$$= \left(h \frac{x^2}{b^2}\, dx\right)$$

$$\bar{x} = \frac{\int_0^b x \cdot \frac{h \cdot x^2}{b^2} \cdot dx}{\int_0^b h \frac{x^2}{b^2} \cdot dx} = \frac{\frac{h}{b^2}\int_0^b x^3\, dx}{\frac{h}{b^2}\int_0^b x^2\, dx} \Rightarrow \frac{b^4}{4} \cdot \frac{3}{b^3} \cdot \frac{3b}{4}$$

$$\bar{y} = \int y \cdot h$$

Moment of Intertia and product of Intertia :→



Ox & Oy are Centroidal Axes

O → Centroid.

I_x = Centroidal M.I about x-x.

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$ = Centroidal M.I. about Y-Y.

$$I_{xy} = \int x \cdot y \, dA$$ = Centroidal product of Inertia about x & y axis.

M.I is second moment of area about any axis where as product of inertia is moment of area about Both axis.

NOTE:

1) The Moments of Inertia (I_x & I_y) are non zero and non negative quantities. where as product of Inertia (I_{xy}) can be zero. greater than zero or less than zero, depending upon location of area w.r.t origin.

2) If Area is located in 1^{st} or 3^{rd} quadrant then $I_{xy} > 0$ if Area is located in II or IV quadrant then $I_{xy} < 0$ but if area is located Symmetrically about x axis. about y axis or about both axis then $I_{xy} = 0$.

The self Product of Intertia (Product of Inertia about centroid axis) is zero for rectangle, square, circle, semicircle, equilateral triangle. I section. T section etc. Because these have one or more than one axis of symmetry

$I_{xx} = bd^3/12$

$I_{yy} = db^3/12$

$I_{xy} = 0$

3) If about any two mutually ⊥ˡ axis product of Inertia is zero then moment of inertia about these axis will be either max^m or min^m. Such axis are called principal axis and moments of Inertia are called principal moment of Inertia.

4) The M.I and Product of Inertia are also second order tensor quantities similar to that of stress and strain.

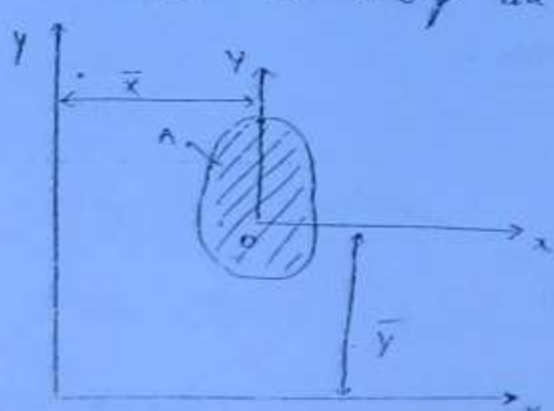
5) If an area has symmetrical centroidal axis then these must be principal axis however vice versa is not compulsory true.

* In case of circle every axis is symmetrical hence there are infinite principal axis in circle.

Parallel axis theorem :-

Let ox & oy are centroidal axis, Hence I_x & I_y are centroidal moment of Inertia / self moment of Inertia and I_{xy} is the centroidal product of Inertia or self Product of Inertia.

Let o'x' & o'y' are axis ∥ to ox & oy.



$I_x' =$ M.I about o'x'

$= I_{x\,self} + A\bar{y}^2$

$I_y' = I_{y\,self} + A\bar{x}^2$

$I_{x'y'} = I_{xy\,self} + A\bar{x}\bar{y}$.

$I_{x\,self} = \int y^2\,dA$. $I_y\ Self = \int x^2\,dA$

$I_{xy}\ self = \int xy\,dA \ \underline{=0}$ for circle, rectangle, square etc

If there are are several component of area then

$$I_{x'} = \sum I_{x\,self} + \sum A\bar{y}^2$$

$$I_{y'} = \sum I_{y\,self} + \sum A\bar{x}^2$$

$$I_{x'y'} = \sum I_{xy\,self} + \sum A\bar{x}\bar{y}.$$

Perpendicular axis theorem:-

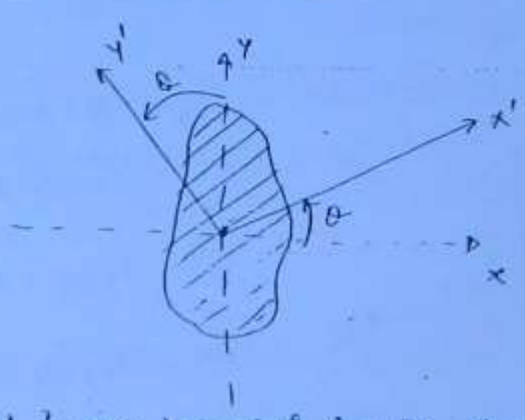
Let ox, oy and oz are centroidal axis, ox and oy are transverse axis. and oz is polar axis or longitudinal axis.

Let I_x and I_y are centroidal moment of ~~Interia~~ Inertia about Centroidal axis then

$$\boxed{I_z = I_x + I_y} = \text{Const} = I_P$$

$$\boxed{I_{z'} = I_{x'} + I_{y'}}$$

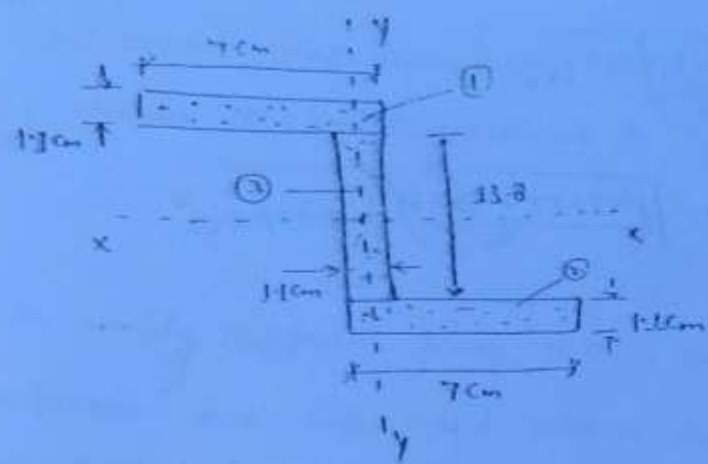
Transformation of Moment of Inertia :-



Let I_x and I_y are moment of inertia about centroidal axis x & y and let I_{xy} is product of Inertia about centroidal axis x & y,

Let $I_{x'}$ and $I_{y'}$ are moment of Inertia about transformed axis x' & y' and $I_{x'y'}$ is ~~transformed~~ product of inertia about transformed axis

Solⁿ -



$$I_x = \sum I_{x\,self} + \sum A\bar{y}^2$$

$$= \left[\frac{7\times 1.1^3}{12} + (7\times 1.1)\times 7.45^2\right]\times 2 + \frac{1.1\times 13.8^3}{12}$$

$$= 1097.19\ cm^4$$

$$I_y = \sum I_{y\,self} + \sum A\bar{x}^2$$

$$= \left[\frac{1.1\times 7^3}{12} + (7\times 1.7)\times 2.95^2\right]\times 2 + \frac{13.8\times 1.1^3}{12}$$

$$= 198.4\ cm^4$$

$$I_x = 1097.19$$
$$I_y = 198.4$$

$$I_{xy} = \sum \cancel{I_{xy\,self}}^{\,0} + \sum A\bar{x}\bar{y}$$

$$= A_1\bar{x}_1\bar{y}_1 + A_2\bar{x}_2\bar{y}_2 + A_3\bar{x}_3\bar{y}_3$$

$$= (7\times 1.1)\times(-(2.95)(7.45)) + (7\times 1.1)(2.95)(-7.45) + 1.1\times 13.8\times 0\times 0 = 0$$

$$I_{xy} = -338.45\ cm^4$$

Angle of Principal axis

$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y} = \frac{+2\times 338.45}{(1097.19 - 198.4)} \Rightarrow \boxed{\theta = 18.49^\circ}$$

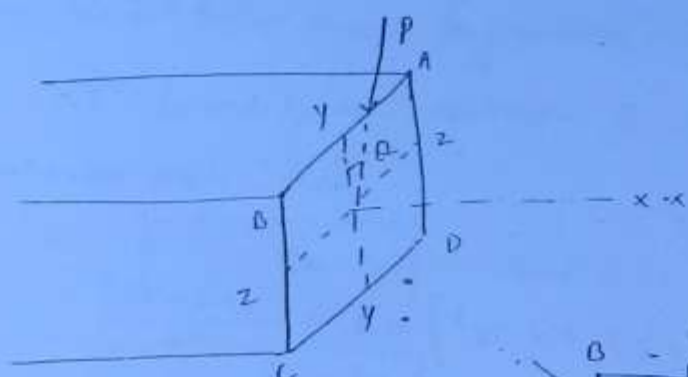
$$I_1/I_2 = \frac{I_x+I_y}{2} \pm \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2}$$

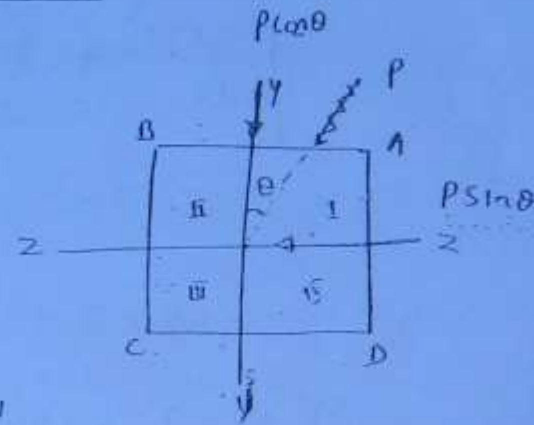
$$= \frac{1097.17+198.4}{2} \pm \sqrt{\left(\frac{1097.17-198.4}{2}\right)^2 + (-332.45)^2}$$

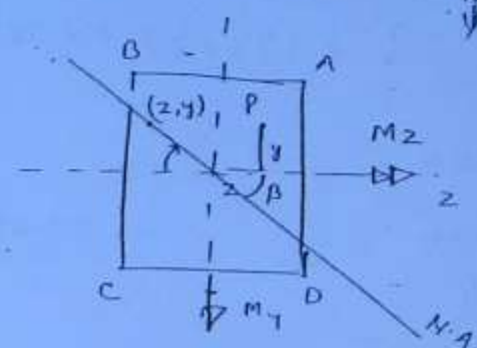
$I_1 = 1210.35\ cm^4$

$I_2 = 85.15\ cm^4$

Biaxial Bending in doubly Symmetric Beam :-







Resultant Bending stresses at any point P will be

$$\sigma_x = \pm \frac{M_z}{I_z}\cdot y \pm \frac{M_y}{I_y}\cdot z$$

for Beams : $+ \rightarrow$ tensile, $- \rightarrow$ Compression

Effect of M_z due to verticle component of force ($P_y = P\cos\theta$)

Bending will occur about horizontal transverse axis Z - Z

In cantilever due to downward verticle loading B.M is Hogging which will cause compression below N.A and tension above N.A 'zz'

Bending stress acts in longitudinal dir.

$$\sigma_x = \frac{M_z}{I_z} y$$

$M_z \rightarrow$ B.M. about zz

NOTE: In simply supported beam B.M is sagging hence above N.A will compression and below N.A will be tension.

On the face in which load acts (AB) stresses will be compressive in SSB

Effect of M_y:

Due to horizontal force B.M about vertical transverse axis y-y.

In cantilever tensile stresses will occur on face AD in which load acts and compressive stresses will occur on face BC. where as in simply supported Beam (SSB) it is reverse

Resultant Bending stresses in 1^{st} quadrant

$$\sigma_x = +\frac{M_z}{I_z} y + \frac{M_y}{I_y} z \qquad \text{(Tensile)}$$

At corner A $y = d/2, \; z = b/2$

$$\sigma_{x\,II} = +\frac{M_z}{I_z} y - \frac{M_y}{I_y} z$$

$$\sigma_{x\,III} = -\frac{M_z}{I_z} y - \frac{M_y}{I_y} z$$

$$\sigma_{x\,IV} = -\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

In first quadrant & III quad effect of M_y and M_z is similar hence resultant stress will not be zero but in II & IV quadrant effect of M_y and M_z is opposite hence N.A will pass thru II & IV quadrant and centroid

Let β is angle of N.A with ZZ.

$$\tan\beta = \left(\frac{y}{z}\right)$$

At N.A resultant Bending stress is zero.

$$\sigma_x = 0$$

$$\frac{M_z}{I_z} y - \frac{M_y}{I_y} z = 0$$

$$\frac{M_z}{I_z} \cdot y = \frac{M_y}{I_y} \cdot z$$

Imp

$$\boxed{\tan\beta = \frac{y}{z} = \left(\frac{M_y}{M_z}\right)\left(\frac{I_z}{I_y}\right)}$$

Spcl. Case

In above case with conc^n load,

$$M_y = P\sin\theta\, x$$

$$M_z = P\cos\theta\, x$$

$$\frac{M_y}{M_z} = \tan\theta$$

$$\boxed{\tan\beta = \frac{I_z}{I_y}\tan\theta}$$

If section is square or circular then $I_z = I_z$

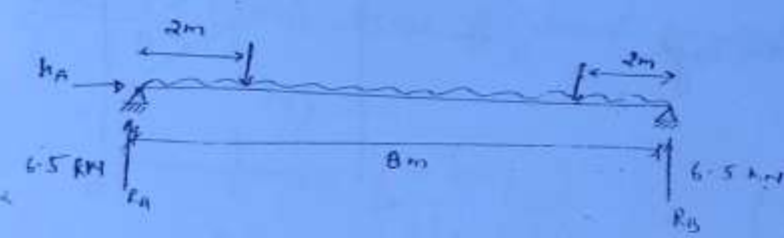
$$\tan\beta = \tan\theta \rightarrow \theta = \beta$$

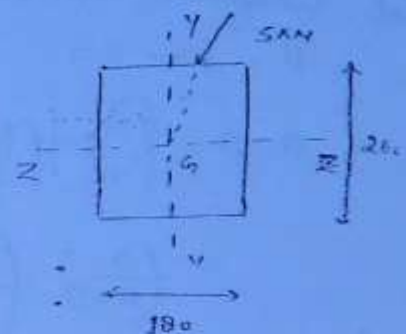
It means N A will $\perp^r$ to line of action of load

In general if load passes from I & III quadrant then N.A will pass thru II & IV quadrant and vice versa.

Ques: A rectangular beam 180mm wide and 260 deep in cross section is used as simply supported beam over a span of 8 m and is subjected to a u.d.l. of 1 kN/m and also a Conc^n load of 5 kN. udl is vertical over entire span but concentrated loads are inclined with vertical at 30° as shown in fig and acts at 2 m from each support determine Bending stresses. At the four corners A, B, C, D and at the centre of the span. Also find location of N.A.

Sol^n :-





$R_A + R_B = 2 \times 5 \sin\alpha$

The supports provide linear rx^n in y and z dir^n both.

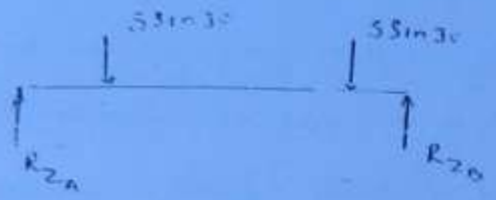
Consider effect of vertical loading.

$R_{Ay} = R_{By} = 8.33\ kN$

At the centre of span

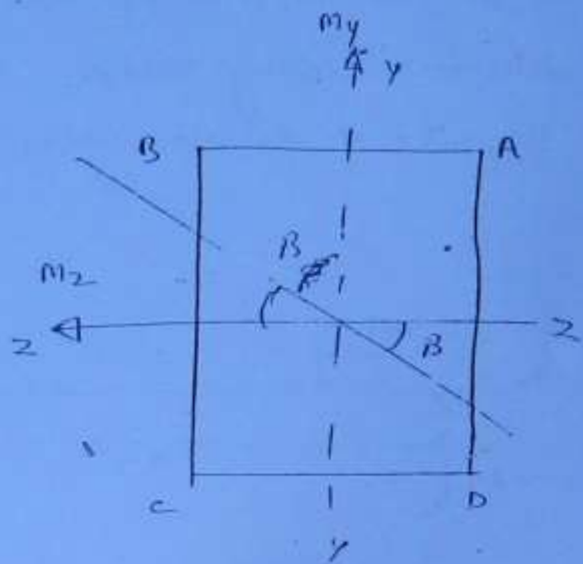
$$M_z = R_{Ay} \times 4 - 5\cos 30 \times 2 - 1 \times 4 \times 2$$

$$M_z = 16.67\ kN\text{-}m$$



$$R_{zA} = R_{zB} = 5 \sin 30 = 2.5$$

$$M_y = R_{zA} \times 4 - 5 \sin 30 \times 2$$
$$= 2.5 \times 4 - 2.5 \times 2$$
$$= 5 \text{ kN-m.}$$



Let location of N.A at an angle β, from z-z.

$$\tan\beta = \left(\frac{M_y}{M_z}\right)\left(\frac{I_z}{I_y}\right)$$

$$= \left(\frac{5}{16.67}\right) \cdot \left(\frac{180 \times 260^3/12}{260 \times 180^3/12}\right)$$

$$\boxed{\beta = 32.05^\circ}$$

Bending stresses at four corners :-

σ at corner A

$$\sigma_x = -\frac{M_z}{I_z} \cdot y - \frac{M_y}{I_y} \cdot z$$

$$= -\frac{16.67 \times 10^6}{180 \times 260^3/12} \times 130 - \frac{5 \times 10^6}{260 \times 180^3/12} \times 90 = -8.22 - 3.56$$

$$= -11.78 \text{ N/mm}^2$$

$$\boxed{I_{x'} = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos 2\theta - I_{xy}\sin 2\theta}$$

$$I_{y'} = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$\boxed{I_{x'y'} = \left(\frac{I_x - I_y}{2}\right)\sin 2\theta + I_{xy}\cos 2\theta}$$

NOTE

If Product of Inertia about x' and y' axis becomes zero then such axis will be called principal axis and $I_{x'}$ and $I_{y'}$ will be called principal moment of Inertia, Hence location of principal axis is given by

$$I_{x'y'} = 0$$

$$\tan 2\theta = \left(\frac{-2I_{xy}}{(I_x - I_y)}\right) \qquad \theta = \text{true}$$

If we substitute θ in above eqn

$$\boxed{I_{1/2} = \left(\frac{I_x + I_y}{2}\right) \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}}$$

Q: For the area shown in fig. Find product of inertia about x and y axis ?

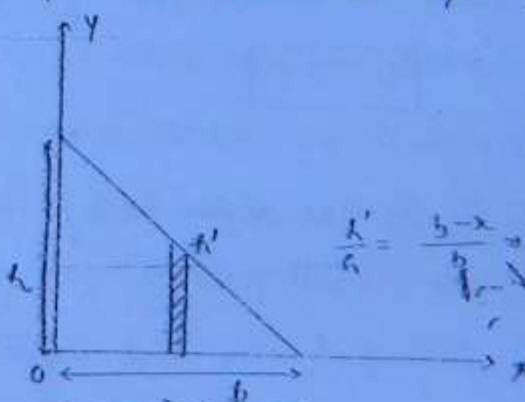
$$I_{xy} = \int x\, y\, dA$$

$$= \int x\left(\frac{h'}{2}\right)(h'dx)$$

$$= \int x \frac{h'^2}{2} dx$$

$$= \int x \frac{(b-x)^2}{2b^2} h^2\, dx$$

$$= \frac{h^2}{2b^2}\int_0^b (b^2x + x^3 - 2bx^2)\, dx$$



$$\frac{h'}{h} = \frac{b-x}{b} \Rightarrow h' = \left(\frac{b-x}{b}\right)h$$

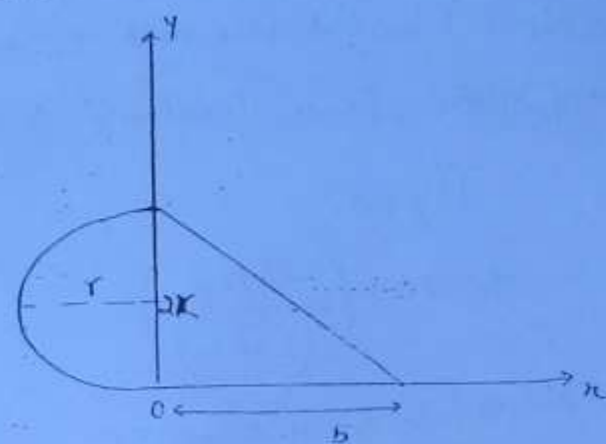
$$dA = h'dx$$

$$I_{xy} = \frac{b^2h^2}{24}$$

$$= \frac{h^2}{2b^2}\left[\frac{b^4}{2} + \frac{b^4}{4} - \frac{2}{3}b^4\right] \qquad \frac{3b^4}{4} - \frac{2}{3}b^4$$

$$I_{xy} = \frac{b^2h^2}{24}$$

Ques: find Relation b/w b and r if Product of Inertial about x and y axis shown in fig is zero.



$I_{xy} = 0$

$=$ I_{xy} for semicircle + I_{xy} for triangle

$$= \cancel{I_{xy\,self}} + A\bar{x}\bar{y} + \frac{b^2h^2}{24} = 0 \qquad I_{xy\,self} = 0 \quad (\text{Symmetric})$$

$$\frac{\pi r^2}{2}\left(-\frac{4r}{3\pi}\right)(r) = -\frac{b^2h^2}{24}$$

$$4r^4 = \frac{bh^2}{4} \qquad (h = 2r) \qquad 16r^4 = b\frac{(2r)^2}{4}$$

$$\boxed{b = 2r}$$

Imp

Q:- A Z Section shown in fig. det^n moment of inertia about x and y axis and product of Inertia about a xy axis, also find orientation of principal axis and magnitude of principal moments of Inertia?

σ at corner A

$$\sigma_x = -\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$= -8.22 + 3.56$$

$$= -4.66\ N/mm^2$$

σ at corner C

$$\sigma_x = +\frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$= +8.22 + 3.56$$

$$= 11.78\ N/mm^2$$

σ at D

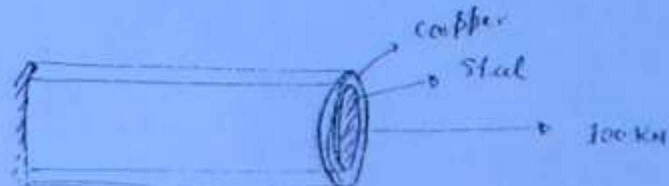
$$\sigma_x = +\frac{M_z}{I_z} y - \frac{M_y}{I_y} z$$

$$= +8.22 - 3.56$$

$$= 4.66\ N/mm^2$$

Q: A compound Bar consist of a circular rod of steel of dia 25 mm rigidly fitted into a copper tube of internal dia 25 mm and thickness 2.5 mm the Bar is subjected to an axial load of 100 KN. then find the stresses developed in two materials. given that $E_s = 2\times10^5\ N/mm^2$, $E_c = 1\times10^5\ N/mm^2$

Sol.



$A_c = 215.98 \text{ mm}^2$

$A_{steel} = \frac{\pi}{4} \cdot D^2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$

Let F_c is force in copper and F_s force in steel.

$F_c + F_s = 100$ — (i)

$A_{cu} = A_{st}$ — (ii)

$$\frac{F_c \cdot L}{A_{cu} E_c} = \frac{F_s \cdot L}{A_s \cdot E_s}$$

$$\frac{F_c}{F_s} = \frac{A_c}{A_s} \cdot \frac{E}{E_s}$$

$$\frac{F_c}{F_s} = \frac{215.98}{490.87} \times \frac{1.2}{2.0} = 0.264$$

$$F_s = \frac{100}{1.264} = 79.11 \text{ KN.}$$

$$F_c = 20.89 \text{ KN}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{79.11 \times 10^3}{490.87} = 161.16 \text{ N/mm}^2$$

$$\sigma_{cu} = \frac{F_c}{A_c} = \frac{20.89 \times 10^3}{215.98} = 96.72 \text{ N/mm}^2$$

Q. Draw the B.M.D & S.F.D for over hanging Beam shown in fig. showing significant values and point

(THE END)