[redacted]

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-: HAND WRITTEN NOTES:-

OF

CIVIL ENGINEERING

1

-: SUBJECT:-

RCC & PRESTRESSED CONCRETE

3

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S. No.	Date	Title	Page No.	Remarks
1		Working Stress Method		
		(i) Singly reinforced		
		(ii) Doubly reinforced		
		(iii) T- beam		
2.		Limit Stress Method		
		(i) Singly reinforced		
		(ii) Doubly		
		(iii) T- beam		

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WORKING STRESS METHOD

[redacted]

:→ Introduction

R.C.C ~~is made up of~~ has two main comp concrete and steel reinforcement

properties of concrete

① Young's modulus of elasticity $E_c = 5000\sqrt{f_{ck}}$

f_{ck} = characteristic strength of concrete

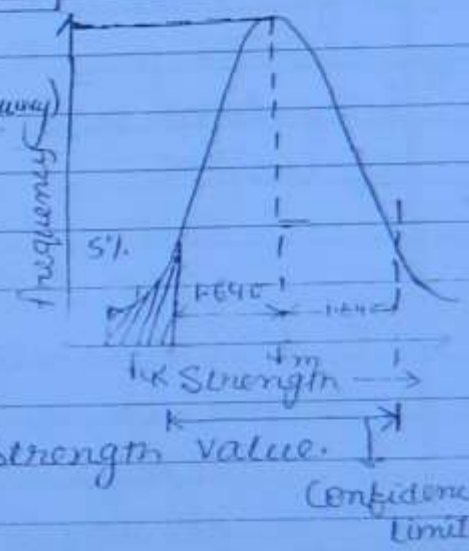
k → This is the value of strength below which not more than 5% of test results are expected to fall.

g:- If we take 100 no. of cubes to be tested then

17, 18.0 18.5 19.0 19.5, 20, 21, 22, ...

5% are fail — characteristic comp strength — 95%

M-20 Grade Concrete $\boxed{f_{ck} = 20 N/mm^2}$



f_m – mean strength value (having max frequency)

It is the average strength

$\boxed{f_m = f_{ck} + 1.64\sigma}$

σ = Standard deviation

for M20 & M25

M30 & above = $5 N/mm^2$

→ Design mix is done for target mean strength value.

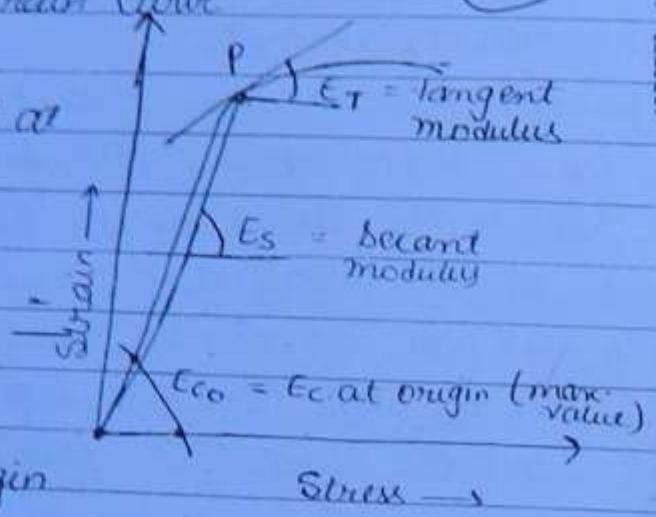
Confidence limit

Probability of a single test result is more of falling betn the limit $(f_m - 1.64\sigma)$ to $(f_m + 1.64\sigma)$. This limit is called confidence limit.

→ Probability % of lying a test result betn confidence limit is called confidence level.

→ If % of test result falling below a value is 50% the strength value f_m

Young's modulus of elasticity (E_c)
E_c is the Slope of Stress-Strain Curve



ex :- ① Young's modulus of elasticity at origin: $E_c = 5000\sqrt{f_{ck}}$
(Considered by IS:456)

② Secant modulus - Slope of line joining any point on, stress-strain curve with origin

③ Tangent modulus - Slope of tangent at any pt. on the curve

Long term modulus of elasticity (considering the effect of creep)

$$E_{ce} = \frac{E_c}{1+\theta} = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

θ creep coefficient

Here :-

As per age of testing	θ Creep Coefficient
7 days	2.2
28 days	1.6
1 de year	1.1

$$\text{Creep Coefficient} = \frac{\text{Creep Strain}}{\text{Elastic Strain}}$$

→ The value of Young's modulus of elasticity reduces 2 to 3 times due to the effect of creep.

Strength of Concrete
(a) Compressive strength

DATE:

Permissible stress in L.S.M

Grades	Direct Compression σ_{cc}	Bending Compression σ_{cbc}
M15	4.0	5.0
M20	5.0	7.0
M25	6.0	8.5
M30	8.0	10.0
M35	9.0	11.5
M40	10.0	13.0

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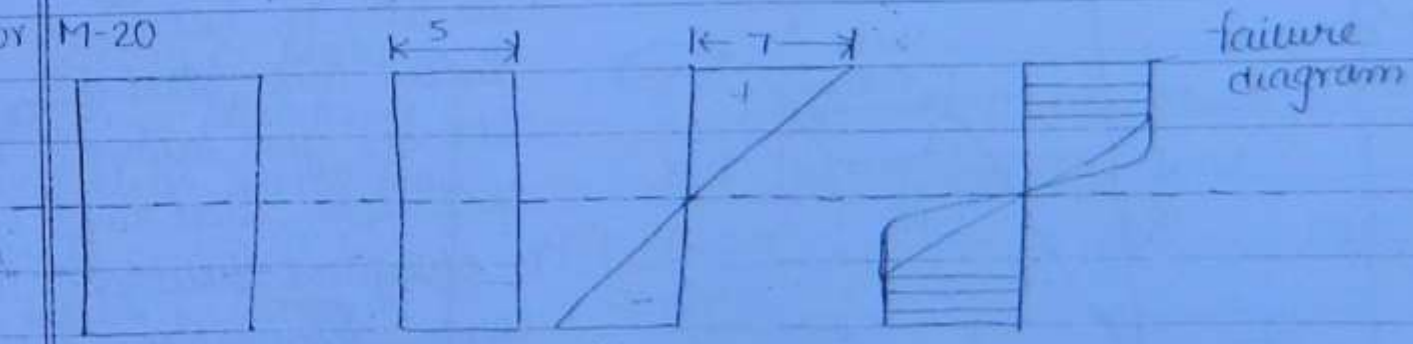
(b) Tensile stresses

flexural tensile strength (bending) of concrete

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Grades	Direct tensile strength	Bending tensile strength
M15	2.0	2.71
M20	2.8	3.13
M25	3.2	3.5
M30	3.6	3.83
M35	4.0	
M40	4.4	

for M-20



Direct Stress P/A

(All fibres are stressed to same value so chances of failure is more)

Bending Stress

(Only top & bottom fibre are at max. stress so permissible stress is more than that of direct stress value)

Grade Of Concrete (As per IS: 456) (Table-2/ Pg-No-16)

Ordinary Concrete	M10 to M20
Standard Concrete	M25 to M55
High Strength Concrete	M60 & above

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mix ← (M)(25) → fck value at 28 days strength

→ Min Recommended grade Of Concrete for is M 20 (IS:456 pg No-1)

Steel :- Types :-

① Mild steel – ↗ Ordinary
↘ Flat Rolled

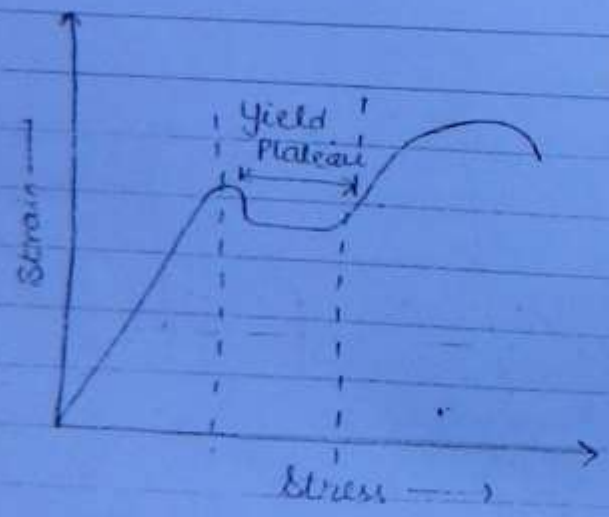
② Medium tensile – Steel ↗ Ordinary
↘ Hot Rolled

③ CTD bars (Cold twisted defined bars) or HYSD bars (High yield strength deformed bars)

④ TMT bars

Properties Of Hot Rolled mild steel bars --

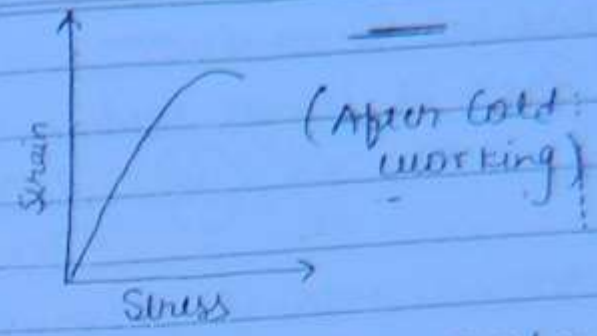
Hot Rolled
d Steel



(In case Of hot Rolled mild Steel if Stresses goes beyond yield limit, steel reinf is elongated resulting large deformation and Cracking (i.e undesirable))

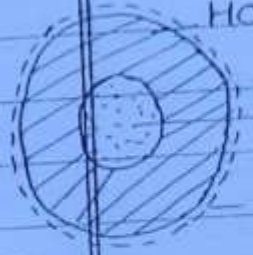
Remedy - (Cold working) → The Steel reinforcement is Stressed beyond yield limit either by Stretching or Twisting & then by untwisting it.

Cold twisted bars are made using cold working process



(9)

TMT bars -



Hard (strength)

Soft (ductility)

Anti rusting coating

Also called as thermo mechanically treated bars

Permissible stresses in steel (Table no- 22 / P. no - 82)

W.S.M

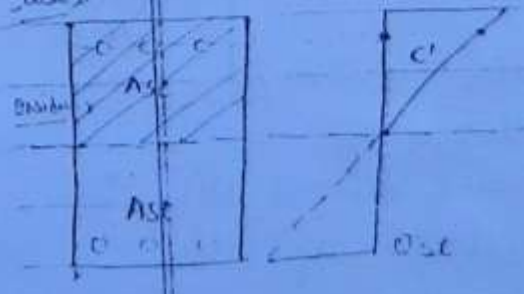
	Mild steel fe 250	Medium tensile steel	CTD bars or HYSD bars fe 415
① Tension (σ_{st} or σ_{sc})			
upto 20 mmϕ	140	190	230
> 20 mmϕ	130	=	
② Compression (σ_{sc})			
All steel	130		
③ Compression in bar in case of beam or slab			

Case I
When comp resistance of concrete is taking into account = 1.5 m·c'
c' = strain in concrete surrounding steel

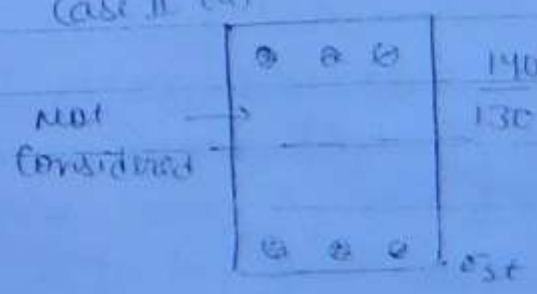
Case II
When comp resistance is not taking into account

upto 20mm	140	190	190
> 20mm	130		

Case I (3)



Case II (a)



SINGLY REINFORCED BEAM (W.S.M)

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(10)

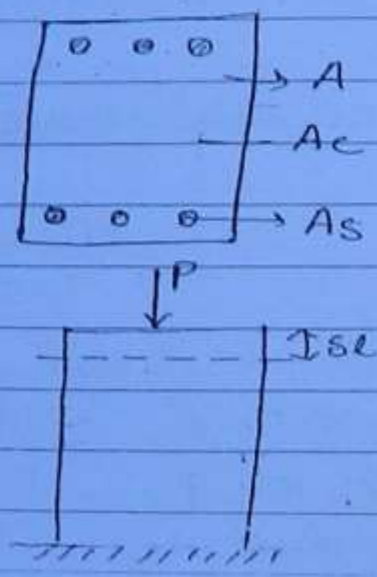
1 Modular Ratio (m)

Ratio of young's modulus of elasticity of Steel to Concrete

$$m = \frac{E_s}{E_c} = \frac{280}{3\sigma_{cbc}} \rightarrow \text{As per IS 456.}$$

	M15	M20	M25	M30	M35	M40
σ_{cbc}	5	7	8.5	10	11.5	13.0
m	19	13	11	9	8	7

Consider a Column having total Area A



$$A = A_c + A_s$$

If total load P

$$P = P_c + P_s$$

$$\boxed{P = A_s \cdot P_s + A_c \cdot P_c} \quad \text{— (i)}$$

Deformation δl Shall be same in Concrete & Steel

$$\delta l = \frac{P_s \cdot l}{A_s \cdot E_s} = \frac{P_c \cdot l}{A_c \cdot E_c}$$

$$\frac{P_s}{A_s \cdot E_s} = \frac{P_c}{A_c \cdot E_c} \Rightarrow \frac{P_s}{E_s} = \frac{P_c}{E_c}$$

$$\Rightarrow \frac{P_s}{P_c} = \frac{E_s}{E_c} = m$$

If strain value is same $p_s = m \cdot p_c$

$$t = m \cdot c \qquad \sigma_{st} = m\,\sigma_{cbc}$$

$$\boxed{c = t/m}$$

Value of p_s in (i) we get

$$P = A_s \cdot m\, p_c + A_c \cdot P_c$$

$$P = P_c (m A_s + A_c)$$

Total Equivalent area in terms of Concrete $= A_c + m A_s$

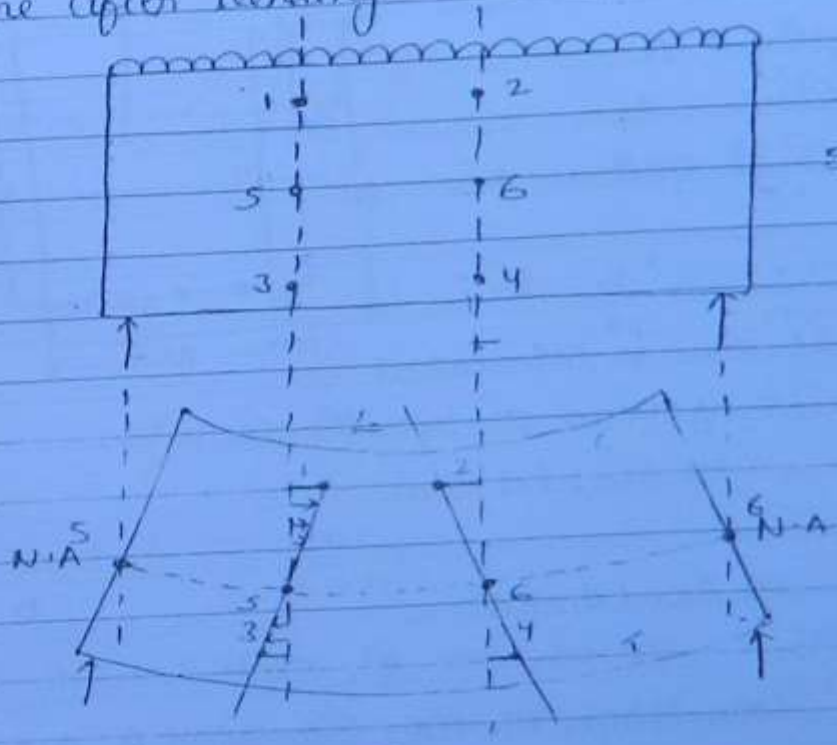
For A_s area of Steel, equivalent area in terms of Concrete $= m A_s$

Steel	Equivalent to Concrete
Ast	m·Ast
σst	σst/m

→ modular ratio value is used in working stress method & L.S.M method is also called modular ratio method.

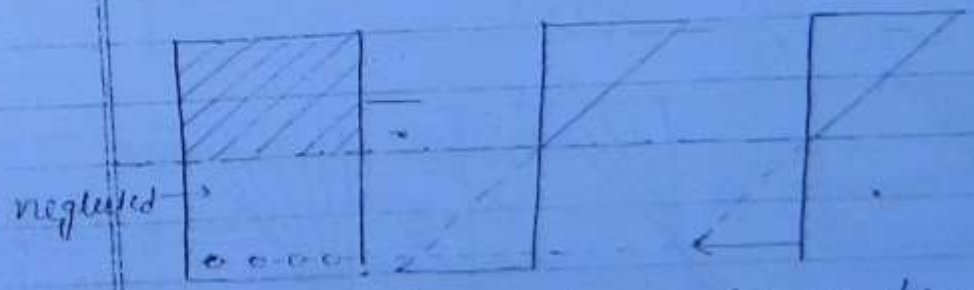
Assumptions :- (ANNEX - B) (~~B~~-1.3 P. no. 80)

1 At any cross-section plane section before bending remains plane after bending



1 & 2 → Compression
3 & 4 – tension
5 & 6 – Neutral axis

Result:- Strain diagram is linear



Strain dia Stress diagram

2. All tensile stresses are taken by steel only. Tensile strength of concrete is to be ignored.
Concrete area in tension zone is completely neglected

3. E_c value is constant
4. Stress-Strain relationship under working load is a straight line. (Stress diagram is linear)
5. modular ratio $m = \frac{280}{3\sigma_{cbc}}$

The exp given for m partially takes into account the effect of creep.

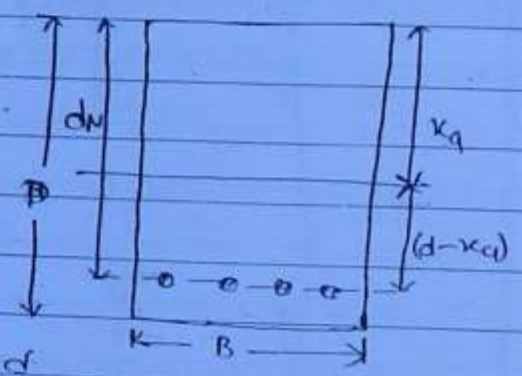
Actual depth of neutral axis

width B

Effective depth = d

Area of Steel = Ast

If depth of actual neutral axis = x_a



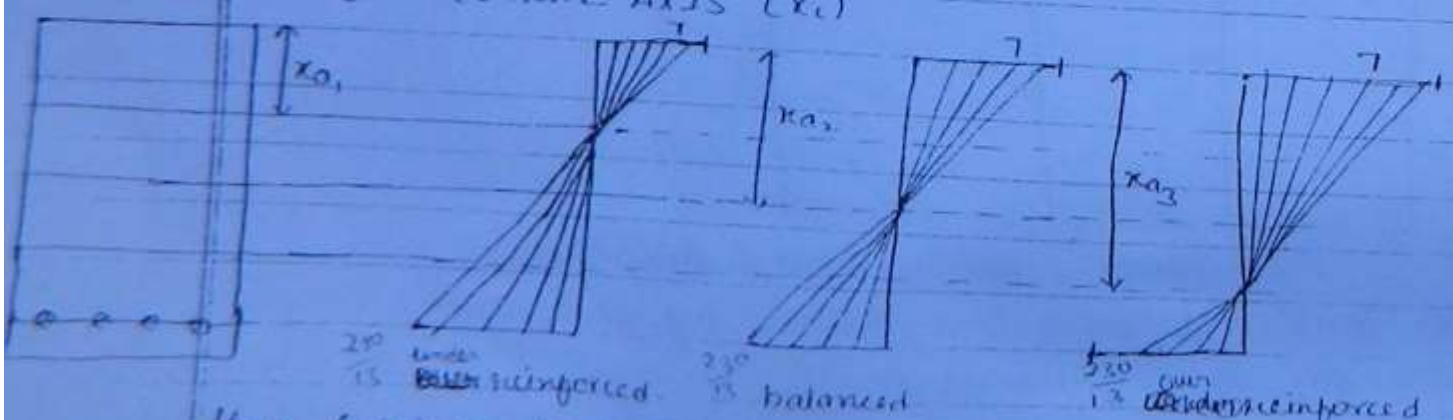
Depth of neutral axis is calculated equating moment of area on both the side of N.A

$$B \cdot x_a \cdot \frac{x_a}{2} = (m \cdot Ast) \cdot (d - x_a)$$

$$B \cdot \frac{x_a^2}{2} = m \cdot Ast\,(d - x_a)$$

For all purpose, calculation of MR, calculation of actual stress developed etc. x_a value has to be used

ITICAL DEPTH OF NEUTRAL AXIS (x_c)



Here critical depth of N.A = $x_{a_2} = x_c$

Critical depth of N.A is ~~such that~~ depth of N.A, so that stresses in Concrete & steel are attained to its max. permissible values at the same time at $x_a = x_c$ (Balanced section)

Stress in Concrete = σ_{cbc}

Stress in steel = σ_{st}

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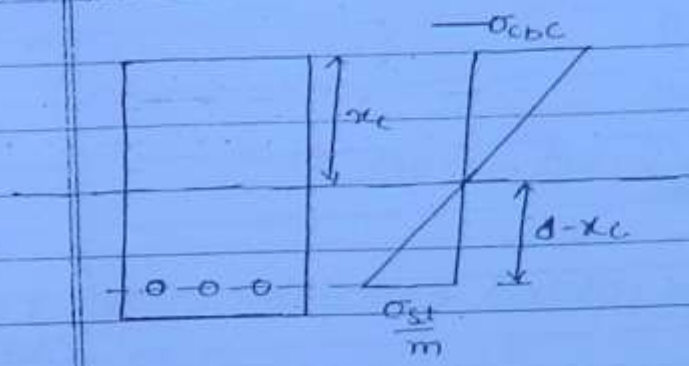
For balanced section (for x_c)



using similar Δ^{le}

$$\frac{\sigma_{cbc}}{x_c} = \frac{\sigma_{st}/m}{d-x_c}$$

$$\frac{c}{x_c} = \frac{t/m}{d-x_c}$$

$$\frac{d-x_c}{x_c} = \frac{t}{mc}$$

$$\Rightarrow \frac{d}{x_c} = \frac{t}{mc} + 1 \Rightarrow \frac{d}{x_c} = \frac{t+mc}{mc}$$

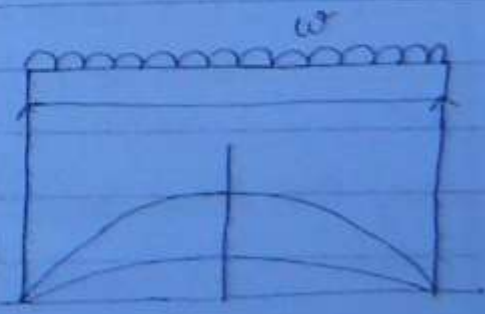
$$\frac{x_c}{d} = \frac{mc}{t+mc}$$

$$\frac{x_c}{d} = K = \frac{mc}{t+mc}$$

K = Critical depth of N.A factor $= \frac{mc}{t+mc}$

$$\boxed{x_c = Kd = \frac{mc}{t+mc} \times d}$$

Moment of Resistance.



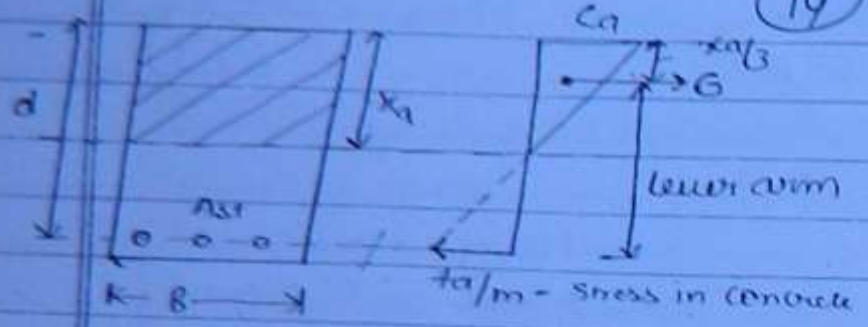
Bending moment is the value of moment developed due to load.

Moment of Resistance - It is the max. capacity of beam c/s to sustain moment

DATE:

For moment of Resistance any one material (concrete or steel) should be at max. permissible value.

(14)



Total Compressive force

C = Area x Avg. Stress

$B \cdot x_a \cdot \frac{C_a}{2}$

Total tensile force

T = Area x Avg. Stress

T = $A_{st} \times t_a$ = $\left(m \cdot A_{st} \times \frac{t_a}{m}\right)$

Lever arm = $d - \frac{x_a}{3}$

Moment of Resistance

M_R = C x lever arm

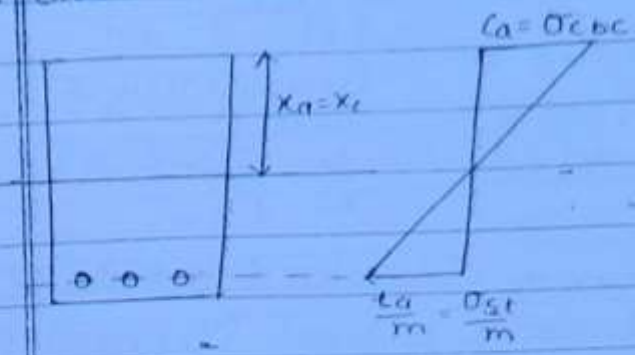
$B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$ — (i) (from compression)

M_R = T x lever arm

$A_{st} \times t_a \left(d - \frac{x_a}{3}\right)$ — (ii) (from tension)

Any one stress value should be at max. permissible stress value.

CASE I Balanced Section :- In this case :- $x_a = x_c$



$\frac{t_a}{m} = \frac{\sigma_{st}}{m}$

$C_a = \sigma_{cbc}$

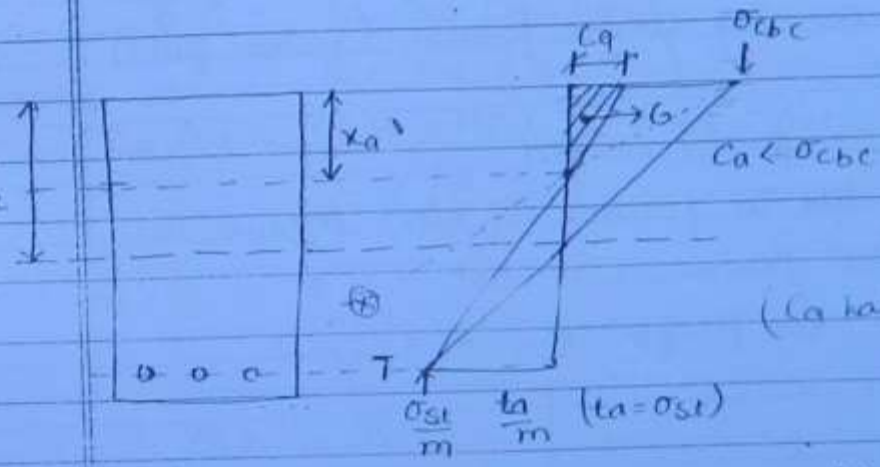
Now $M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$

$\therefore M_R = B \cdot x_c \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right)$ — (I)

again $M_R = A_{st} \cdot t_a \left(d - \frac{x_a}{3}\right)$

$\Rightarrow M_R = A_{st} \cdot \sigma_{st}\left(d - \frac{x_c}{3}\right)$ — (II)

CASE II Under reinforced Section :- $x_a < x_c$ (Area of steel is less than that req. for balanced sec.)



$C_a < \sigma_{cbc}$

$t_a = \sigma_{st}$

Moment Of Resistance

$M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$ — (1)

(C_a has to calculated by similar Δ)

$M_R = t_a \cdot A_{st}\left(d - \frac{x_a}{3}\right)$

$M_R = \sigma_{st} \cdot A_{st}\left(d - \frac{x_a}{3}\right)$ — (2)

direct formula

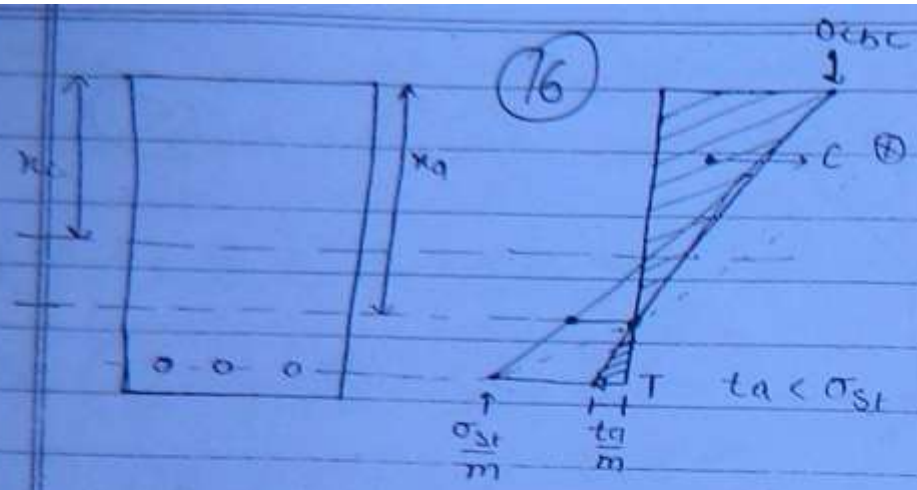
For (1) actual stress in Concrete C_a is to be calculated

$\frac{C_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a} \Rightarrow C_a = \left(\frac{x_a}{d - x_a}\right)\frac{\sigma_{st}}{m}$

CASE III Over reinforced Section :- $x_a > x_c$; $C_a = \sigma_{cbc}$; $t_a < \sigma_{st}$

$M_R = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right)$ — (1)

$M_R = t_a \cdot A_{st}\left(d - \frac{x_a}{3}\right)$ — (2)



$\sigma_{cbc} = C_a$

for actual stress in steel -

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$t_a < \sigma_{st}$ $\Rightarrow t_a = \frac{d - x_a}{x_a} m \cdot \sigma_{cbc}$

In case of over reinforced section

① Concrete is at its max. permissible stress value so Concrete fails first.

② Concrete failure is a brittle failure

③ It does not gives warning before failure (Sudden failure) It is to be avoided

In case of under reinforced section

① Failure is ductile due to failure of steel because in this steel is at max. permissible steel.

② This section should be preffered

Design formula

For a balanced section :- From Compression side

$$M_R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right)$$

$x_c = K d$

$$M_R = B \cdot x_c \frac{\sigma_{cbc}}{2}\left(d - \frac{x_c}{3}\right)$$

$$K = \frac{m \sigma_{cbc}}{\sigma_{st} + m \cdot \sigma_{cbc}}$$

put $x_c = Kd$; $\sigma_{cbc} = C$

$$M_R = B \cdot K d \frac{C}{2}\left(d - \frac{K d}{3}\right)$$

$$= B \cdot d^2 \cdot K \cdot \frac{C}{2}\left(1 - \frac{K}{3}\right)$$

put $1 - \frac{K}{3} \rightarrow j$ lever arm depth factor

$$M_R = \frac{1}{2} c \cdot j \cdot k\, Bd^2$$

$$M_R = Q\, Bd^2$$

[$Q = \frac{1}{2} c \cdot j \cdot k$ = Moment Coefficient

Where $k = \frac{mc}{t+mc}$

$j = 1 - k/3$

For design

(17)

STEP 1 Calculate load = w

STEP 2 Calculate bending moment

STEP 3 B.M = Moment of Resistance M_R

$B \cdot M = Q \cdot Bd^2$

depth $d = \sqrt{\frac{B \cdot M}{Q \cdot B}} = \sqrt{\frac{M_R}{Q \cdot B}}$

Area of Steel

$B \cdot M = \sigma_{st} \cdot A_{st} \left(d - \frac{x_c}{3}\right)$

$\Rightarrow A_{st} = \frac{(B \cdot M = M_R)}{\sigma_{st}\left(d - \frac{x_c}{3}\right)} \Rightarrow A_{st} = \frac{BM}{\sigma_{st}(d - kd/3)}$

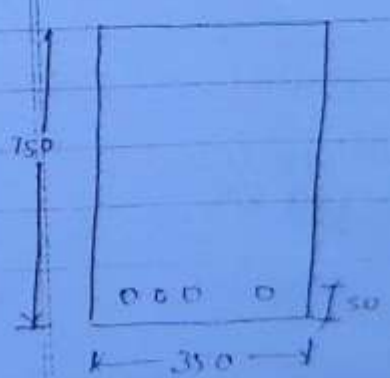
$= \frac{B \cdot M}{\sigma_{st}\left(1 - \frac{k}{3}\right) d}$

$$\boxed{A_{st} = \frac{B \cdot M}{\sigma_{st} \cdot j \cdot d}}$$

Ques:- 1 Calculate Moment of Resistance of a RCC Rectangular beam of size 350X750. fc 415 & M-20 is used

(1) A_{st} = 4 nos 16 mm ϕ bars

(2) A_{st} = 6 Nos 25 mm ϕ bars



Calculate M_R of balanced section & area of steel required for balanced section.

DATE:

Sol:- ① Critical depth of N.A x_c

$$x_c = \frac{m_c}{t+m_c} \times d$$

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$$= \frac{13 \times 7}{230 + 13 \times 7} \times 700 = 196.44 \text{ mm}$$

① Where area of steel = 4 nos - 16mm ϕ bar

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25 \text{ mm}^2$$

Actual depth of N.A

Equating moment of area on both side of N.A

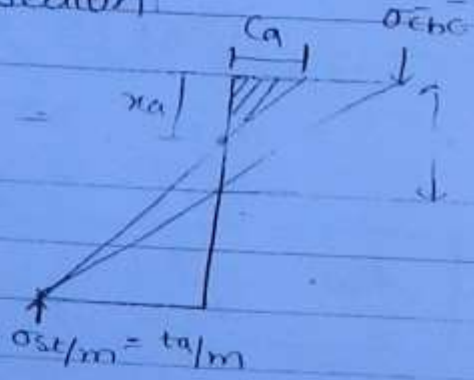
$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$350 \cdot \frac{x_a^2}{2} = 13 \times 804.25 (700 - x_a)$$

$$x_a = 176.80 \text{ mm}$$

$x_a < x_c$ → Under reinforced section

$C_a < \sigma_{cbc}$ & $t_a = \sigma_{st}$



tension side

$$M_R = \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right)$$

$$= 230 \times 804.25 \left(700 - \frac{176.80}{3}\right) \times \frac{1}{10^6}$$

$$M_R = 118.58 \text{ KN-m}$$

compression side

$$C_a = \frac{x_a}{d - x_a} \times \frac{\sigma_{st}}{m} = \frac{176.80}{700 - 176.80} \times \frac{230}{13}$$

$$C_a = 5.978 \text{ N/mm}^2$$

Now $M_R = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3}\right)$

$$= 350 \times 176.80 \times \frac{5.978}{2} \left(700 - \frac{176.80}{3}\right) \times \frac{1}{10^6}$$

$$= 118.57 \text{ KN-m}$$

② When area of steel = 6 nos - 25mm ϕ bar

$$6 \times \frac{\pi}{4} \times (25)^2 = 2945.24 \text{ mm}^2$$

DATE:

Actual depth of N.A → By equating moment on each side of N.A

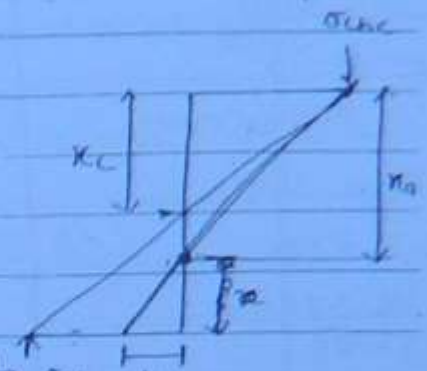
$B \cdot x_a^2 = m \cdot A_{st} (d - x_a)$

$350 \cdot \frac{x_a^2}{2} = 13 \times 2945.24 (700 - x_a)$ (19)

$x_a = 296.95$

$x_a > x_c$ → Over reinforced section

$t_a < \sigma_{st}$ & $C_a = \sigma_{cbc}$



from direct formula of compression side

$M_R = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_a}{3}\right)$

$= 350 \times 296.95 \times \frac{7}{2} \left(700 - \frac{296.95}{3}\right) \frac{1}{10^6}$ $\frac{t_a}{m} = \frac{\sigma_{st}}{m}$

$= 218.6$ kN-m

$t_a = \frac{d - x_a}{x_a} \sigma_{st} m$

$= \frac{700 - 296.95}{296.95} \times 7 \times 13$

$t_a = 123.51$

tension side $M_R = A_{st} \cdot t_a \left(d - \frac{x_a}{3}\right)$

$M_R = 2945.24 \times 123.51 \left(700 - \frac{296.95}{3}\right) \times \frac{1}{10^6}$

$M_R = 218.63$ kN-m

Balanced section $x_a = x_c = 198.44$ mm

$C_a = \sigma_{cbc} = 7$; $t_a = \sigma_{st} = 230$

$M_R = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3}\right) \Rightarrow 350 \times 198.44 \times \frac{7}{2} \left(700 - \frac{198.44}{3}\right) \frac{1}{10^6}$

$M_R = 154.08$ kN-m

tension side $M_R = \sigma_{st} \cdot A_{st} \left(d - \frac{x_a}{3}\right)$

$154.08 = 230 \cdot A_{st} \left(700 - \frac{198.44}{3}\right)$

$A_{st} = \frac{154.08 \times 10^6}{230 (700 - 198.44/3)}$

$A_{st} = 1056.9$ mm^2

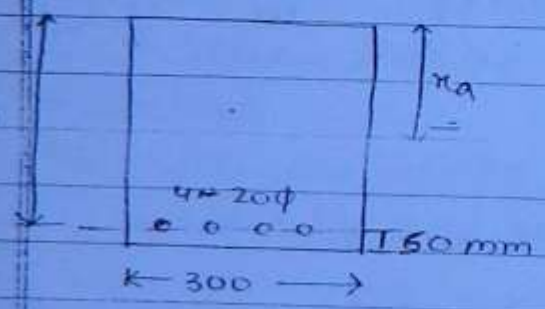
provide

Comparison (20)

Under Reinforced	Balanced	Overreinforced
B – 350 d = 700	Same	Same
A_{st} = 4–16 804.25 mm²	A_{st} = 1056.9 mm²	A_{st} = 2945.24 mm²
x_a = 5.978 t_a = 230 = 118.57 kN-m = 176.08	$C_a = \sigma_{cbc} = 7$ $t_a = \sigma_{st} = 230$ M_R = 154.08 kN-m x_a = 198.44	$C_a = \sigma_{cbc} = 7$ t_a = 123.5 M_R = 218.63 kN-m x_a = 296.95

Ques-2 A rectangular beam of size 300 x 650 mm is subjected to a U.D.L of 15 kN/m over a S.S. span of 5.6 m. (Load in excluding self wt.). Calculate stress developed in Concrete & Steel. A_{st} = 4 nos 20 mm ϕ bars. m = 15 mm Cover = 50 mm

Sol:-



① Actual depth of N.A

$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$300 \cdot \frac{x_a^2}{2} = 15 \times 4 \times \frac{\pi}{4} \times 20^2 (600 - x_a)$$

$$x_a = 218.85 \text{ mm}$$

② Load = 15 kN/m

Self wt = .3 x .65 x 1 x 25 = 4.875 kN/m

total load w = 19.875 kN/m

$$B.M = \frac{wl^2}{8} = \frac{19.875 \times 5.6^2}{8} = 77.91 \text{ kN-m}$$

DATE:

Now $B.M = \sigma_{st} \cdot A_{st}\left(d - \frac{x_a}{3}\right)$

$77.91 \times 10^6 = \sigma_{st} \cdot 4 \times \frac{\pi}{4} \times 20^2 \left(600 - \frac{218.85}{3}\right)$

(21)

$\sigma_{st} = 117.60\ N/mm^2$ [Also calculated from similar Q^n ~~bef~~]

$B.M = B \cdot x_a \cdot \frac{c_a}{2}\left(d - \frac{x_a}{3}\right)$

$77.91 \times 10^6 = 300 \times 218.85 \times \frac{\sigma_{cbc}}{2}\left(600 - \frac{218.85}{3}\right)$

$\sigma_{cbc} = 4.503\ N/mm^2$

A reinforced concrete beam of size 350 x 700 mm is to be designed for a B.M of (i) 100 kN-m (ii) 200 kN-m. Find out the steel reinforcement required for above B.M. M20, Fe415. What will be steel required for balanced section. Effective cover = 50mm

Sol^n MR of balanced section

$MR = QBd^2$

$Q = \frac{1}{2} \times 7 \times 0.905 \times 0.2835$

$= 0.898$

$MR = 0.898 \times 350 \times (550)^2 / 10^6$

$= 132.79\ kN\text{-}m$

$Q = \frac{1}{2} c \cdot j \cdot k$

$k = \frac{mc}{1+mc} = \frac{13 \times 7}{1+mc\ 250+13 \times 7}$

$k = 0.2835$

$j = \left(1 - \frac{k}{3}\right) = 0.905$

(i) $B.M = 100\ kN\text{-}m$

$BM < MR$ singly reinforced balanced section so we need under reinforced section.

$B.M = B \cdot x_a \cdot \frac{c_a}{2}\left[d - \frac{x_a}{3}\right]$ — (1)

$100 \times 10^6 = B\, x_a\, c_a/2 \left[d - x_a/3\right]$

$B.M = t_a \cdot A_{st}\left[d - \frac{x_a}{3}\right]$

DATE:

for an under reinforced section —

$C_a < \sigma_{cbc}$

$\varepsilon_{st} = \varepsilon_0$

22

So from eq^n (2)

$$100 \times 10^6 = 230 \cdot A_{st} \left(650 - \frac{x_a}{3}\right) \quad \text{--- (3)}$$

equating moment of area

$$\frac{B \cdot x_a^2}{2} = m A_{st} (d - x_a)$$

$$\frac{350 \times x_a^2}{2} = 13 \times A_{st} (650 - x_a)$$

$$A_{st} = \frac{350 x_a^2}{2 \times 13 (650 - x_a)}$$

$$A_{st} = \frac{13.46 x_a^2}{650 - x_a}$$

Putting the values of A_{st} in eq (3)

$$100 \times 10^6 = 230 \times \frac{13.46 x_a^2}{(650 - x_a)} \left(650 - \frac{x_a}{3}\right)$$

$$100 \times 10^6 = \frac{3095.8 x_a^2 (1950 - x_a)}{3(650 - x_a)}$$

$$100 \times 10^6 = \frac{1031.93 (1950 - x_a) x_a^2}{(650 - x_a)}$$

$$\frac{(1950 - x_a) x_a^2}{(650 - x_a)} = 96905.46$$

$$1950 x_a^3 - x_a^3 = 62988549 - 96905.46 x_a$$

$$x_a^3 - 1950 x_a^2 - 96905.46 x_a + 62988549 = 0$$

$$x_a = 162.5 mm$$

$$A_{st} = \frac{13.46 x_a^2}{650 - x_a}$$

$$A_{st} = 729.08 mm^2$$

Shortcut :- Only under-reinforced in beam

Considering a balanced section

(23)

$x_a = x_c = 0.2835d = 0.2835 \times 650$

$= 184.275$

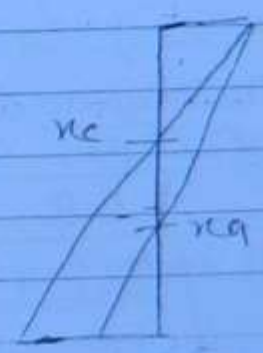
$A_{st} = \frac{B.M}{\sigma_{st} \cdot j \cdot d} = \frac{100 \times 10^6}{230 \times 0.905 \times 650} = 739.9 mm^2$

above formula can be used for slabs but not for beam.

(ii) B.M = 200 kN-m

B.M > MR lower reinforced singly or doubly

$200 \times 10^6 = B \cdot x_a \cdot \frac{c_a}{2} \left[d - \frac{x_a}{3} \right]$



$200 \times 10^6 = 350 \times x_a \cdot \frac{c_a}{2} \left[650 - \frac{x_a}{3} \right]$

$163265.31 = x_a \left[650 - \frac{x_a}{3} \right]$

$489795.9 = 1950 x_a - x_a^2$

$x_a = 296.14 mm$

From similar Δ^{le}

$\frac{c_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow t_a = \frac{(d - x_a) m \cdot c_a}{x_a}$

$t_a = \frac{(650 - 296.14) \times 13 \times 7}{296.14}$

$t_a = 108.74 N/mm^2$

$B.M = t_a \cdot A_{st} \left[d - \frac{x_a}{3} \right]$

$200 \times 10^6 = 108.74 \cdot A_{st} \left[650 - \frac{296.14}{3} \right]$

$A_{st} = 3336.3 mm^2$

Area of steel required for bal. sec

$A_{st} = \frac{MR}{\sigma_{st} j \cdot d} = \frac{132.79 \times 10^6}{230 \times .904 \times 650} = 982.5 mm^2$

DATE:

Ques. Design a rectangular R.C.C beam for a live load of 45 KN over a span of 7.50 m. use M-20 & fe 415. load is excluding self wt. of beam. Consider width = 400mm

SOLⁿ B = 400 mm Span = 7.50m (effective)

(24)

1. Load / B.M → L.L = 45 KN/m

Self wt = .4 X .75 X 1 X 25 = 7.50 KN/m

Assume depth = 750 mm = Span/10

total w = 45 + 7.50 = 52.50 KN/m

$B.M = \frac{wl^2}{8} = \frac{52.50 \times (7.5)^2}{8} = 369.14$ KN-m

Let us design for balanced section

$x_a = x_c$; $C_a = \sigma_{cbc}$; $t_a = \sigma_{st}$

(i) depth required —

$d = \sqrt{\frac{BM}{Q \cdot B}}$

$Q = \frac{1}{2} c \cdot j \cdot K$

$K = \frac{mc}{t + mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5}$

$K = 0.289$

$j = 1 - K/3 = 0.904$

$Q = 1.11$

$d = \sqrt{\frac{369.14}{1.11 \times 400}}$

d = 911.8

eff cover = 50

d = 961.80 > 750 mm

Let us consider d = 1000 mm

Self wt = .4 X 1.0 X 1 X 25 = 10 KN/m

total w = 45 + 10 = 55 KN/m

B.M = 386.76 KN-m

again $d = \sqrt{\frac{386.76}{1.11 \times 400}} = 933$ mm

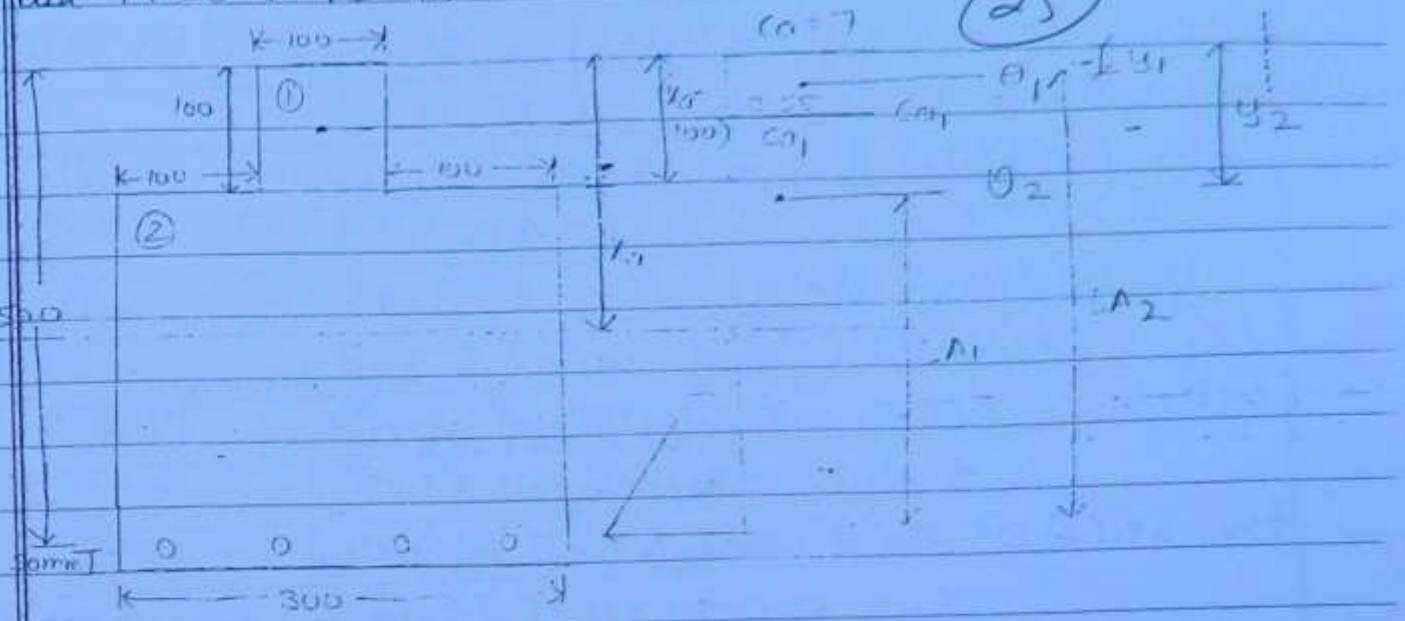
eff cover 50 ⇒ d = 983 mm < 1000 mm OK

$A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d} = \frac{386.72 \times 10^6}{230 \times .904 \times 950}$

$A_{st} = 1958 \text{ mm}^2$

Calculate Moment Of Resistance Of section shown in fig.
use M-20 & fe 415

(25)



Critical depth Of N.A $x_c = k.d$

$$x_c = \frac{mc}{t+mc} \times d = \frac{13\times7}{230+13\times7} \times 500 \Rightarrow 141.75\,mm$$

Actual depth Of N.A — Equating moment area on both side

assume $x_a > 100\,mm$

$$100 \times 100\,(x_a - 50) + 300\,(x_a - 100)\frac{x_a - 100}{2} = 300\,[x_a - 100]$$

$$66.67 \times (x_a - 50) + (x_a - 100)^2 = 69.70\,[500 - x_a]$$

$$x_a^2 - 200x_a + 10^4 + 66.67x_a - 3333.5 = 0$$

$x_a = 202.68 > 100\,mm$ (assumption is correct)

$x_a > x_c$; $C_a = \sigma_{cbc} = 7\,N/mm^2$

$t_a < \sigma_{st}$

Stress at junction

$$\frac{C_{a_1}}{x_a - 100} = \frac{C_a}{x_a}$$

$$C_{a_1} = \frac{(x_a - 100) \times 7}{x_a}$$

$C_{a_1} = 3.55\,N/mm^2$

DATE:

Moment of resistance

for sec ① C_1 = area × avg. stress

(26)

$C_1 = 100 \times 100 \times \left[7 + \frac{3.55}{2}\right] = 52750 N$

$\bar{y}_1 \left(\frac{a+2b}{a+b}\right)\frac{h}{3} = \frac{7+2\times3.55}{7+3.55} \times \frac{100}{3}$

$y_1 = 44.55 mm$

$LA_1 = (d - y_1) = 500 - 44.55 = 455.45 mm$

$MR_1 = C_1 \times LA_1 = 52750 \times 455.45 \times 10^{-6}$

$= 24.025 kN-m$

for sec ② $C_2 = 300 \times (x_a - 100) \times \frac{C_{a1}}{2}$

$= 54677.1 N$

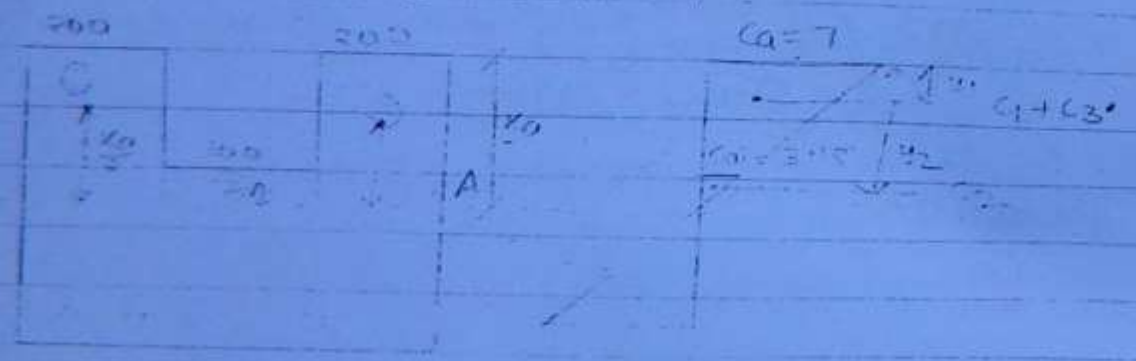
$y_2 = 100 + \left(\frac{x_a - 100}{3}\right) = 133.89 mm$

$LA_2 = d - y_2 = 366.11 mm$

$(MR)_2 = C_2 \times LA_2 = 20.02 kN-m$

Total M.R $= 24.025 + 20.02 = 44.04 kN-m$

A reinforced concrete beam has shown in fig there are 4 - 20ϕ bar. Calculate N.A & stress developed due to a B.M 85000 N-m



depth of N.A. equating moment of area

$2 \times 200 \times x_a \times \frac{x_a}{2} + 100 \times (-100 + x_a)\left(\frac{x_a - 100}{2}\right) = m \cdot A_{st}(d - x_a)$

$200 x_a^2 + 50(x_a - 100)^2 = 15 \times \frac{\pi}{4} \times (20)^2 (d - x_a)$

DATE:

$200x_a^2 + 50(x_a^2 + 100^2 - 2\times100x_a) = 4712.38(550 - x_a)$

$250x_a^2 + 500000 - 10000x_a = 2591809 - 4712.38x_a$

$250x_a^2 - 14712.38x_a + 2091809 = 0$

$x_a = 181.75$ mm > 100 (Assumption is correct)

$\frac{C_a}{x_a} = \frac{C_{a1}}{(x_a - 100)}$ (27)

$C_{a1} = \frac{C_a(x_a - 100)}{x_a} = \frac{C_a(81.75)}{181.75} = 0.45C_a$

$B.M = 85\times10^6$ N-mm

$C_1 + C_3 = 2$ (area x avg. stress)

$= 2(200\times x_a \times \frac{C_a}{2}) = 36350\,C_a$

$y_1 = \frac{x_a}{3} = 60.58$ mm

$LA_1 = 550 - y_1 = 489.416$

$(MR_1 + MR_3) = (C_1 + C_3)\times LA_1$

$= 36350\,C_a \times 489.416 = 17.79\,C_a$ KN-m

$C_2 = 100\times(x_a - 100)\times\frac{C_a}{2}$

$= 1839.37\,C_a$

$y_2 = 100 + (x_a - 100)$

$y_2 = 127.25$

$LA_2 = 550 - 127.25 = 422.75$ mm

$MR_2 = 0.77$ KN-m

Total $- MR_1 + MR_2 + MR_3 = 18.56\,C_a$

$18.56\,C_a = 85 \Rightarrow C_a = 4.58$ N/mm²

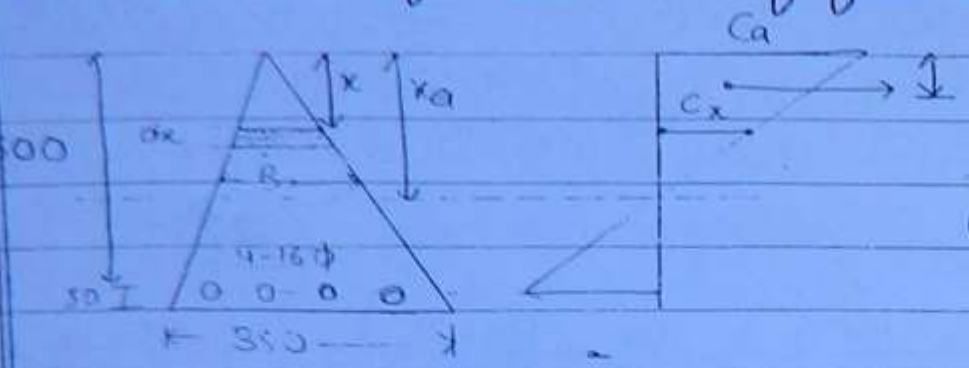
$C_{a1} = 0.45\times4.58 = 2.06$ N/mm²

$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow t_a = \frac{(d - x_a)(m\cdot C_a)}{x_a}$

$= \frac{(550 - 181.75)(15\times4.58)}{181.75}$

$t_a = 139.19$ N/mm²

Ques. Find Moment of resistance of following section



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Critical depth of N.A $\bar{x}_c = \frac{m_c}{t+m_c} \times d = \frac{13\times 7}{230+13\times 7} \times 500$

$x_c = 141.75\ mm$

Actual depth

$\frac{B_1}{x_a} = \frac{350}{550}$

$B_1 = \frac{7}{11} x_a$

$\frac{1}{2} \times B_1 \times x_a \cdot \frac{x_a}{3} = m \cdot A_{st} (d - x_a)$

$\frac{1}{2} \times \frac{7}{11} \times \frac{1}{3} \cdot x_a^3 = 13 \times 4 \times \pi/4 \times 16^2 (500 - x_a)$

$0.106 x_a^3 = 5227610.176 - 10455.22 x_a$

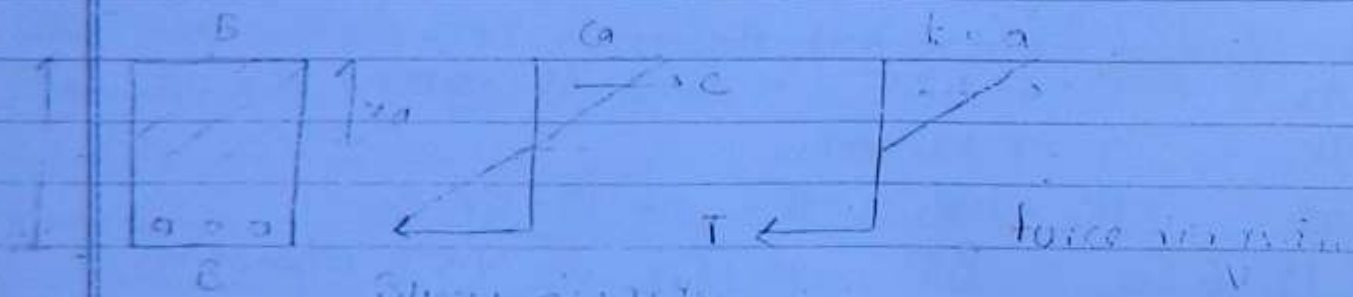
$.106 x_a^3 + 10455.22 x_a - 5227610.176 = 0$

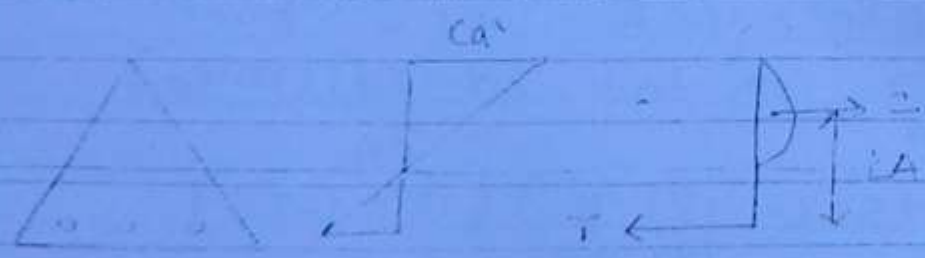
by trial & error $x_a = 279.2\ mm > x_c$

(over reinforced)

$C_a = C_{cc} = T$

$t_a < C_{st}$





Consider an elemental strip of dx thickness at distance x from top

$B_x = \frac{7}{11} x$ —(doubt)

$\frac{C_a}{x_a} = \frac{C_x}{(x_a - x)} \Rightarrow C_x = \frac{x_a - x}{x_a} C_a$

$\Rightarrow C_x \quad \frac{279.2 - x}{279.2} \times 7$

$C_x = \frac{279.2 - x}{39.88}$

If width of sec^n at N.A = B_1

$\frac{B_1}{x_a} = \frac{350}{550} = \frac{7}{11}$

$B_1 = \frac{7}{11} x_a$

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M.R of elemental strip $C_1 \times L.A$

= area × avg. stress × L.A

$MR = (B_x \cdot dx)(C_x)(d - x)$

$\frac{7}{11} x \cdot dx \left(\frac{279.2 - x}{39.88}\right)(500 - x)$

total M.R. →

$\int_0^{x_a} MR = \int_0^{x_a} \frac{7}{11} x \left(\frac{279.2 - x}{39.88}\right)(500 - x)\, dx$

$\frac{7}{11} \int \left(\frac{279.2 - x - x^2}{39.88}\right) \cdot (500 - x)\, dx$

$\frac{7}{11} \int \frac{279.2x}{39.88} \times 500 - \frac{279.2x^2}{39.88} - \frac{500x^2}{39.88} + \frac{x^3}{39.88}\, dx$

$\frac{7}{11} \int_0^{x_a} 3500 \cdot 50x - 7x^2 - 12.53x^2 + 0.025x^3\, dx$

$\frac{7}{11} \int_0^{x_a} 3500 \cdot 50x - 19.55x^2 + 0.025x^3\, dx$

$\frac{7}{11} \left[\frac{0.025x^4}{4} - 19.55\frac{x^3}{3} + \frac{3500 \cdot 50x^2}{2}\right]_0^{x_a}$

$MR = \frac{7}{11} \left[\frac{0.025x_a^4}{4} - \frac{19.55x_a^3}{3} + 1750.25x_a^2\right]$

$MR = \frac{7}{11} (0.00625x_a^4 - 6.52x_a^3 + 1750.25x_a^2)$

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DOUBLY REINFORCED SECTION

(31)

If Doubly reinforced section is needed if
1. Size of beam (width & depth) both are fixed
2. Beam has to sustain a higher B.M > M.R of singly reinforced balanced section

if size of beam = B×d

M.R of singly reinforced balanced section

$$M.R = Q.Bd^2$$

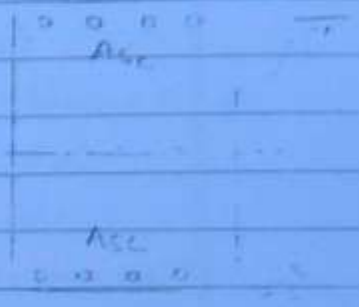
$$Q = \frac{1}{2} c.j.K$$

if B.M > MR

→ There are two option

(i) Over-reinforced section – avoided

(ii) Doubly-reinforced section – preffered



Permissible stress in Compression steel

$$= 1.5\,m.c'$$

c' = stress in Concrete at the level of Compression steel

Equivalent area per Asc area of steel in terms of concrete

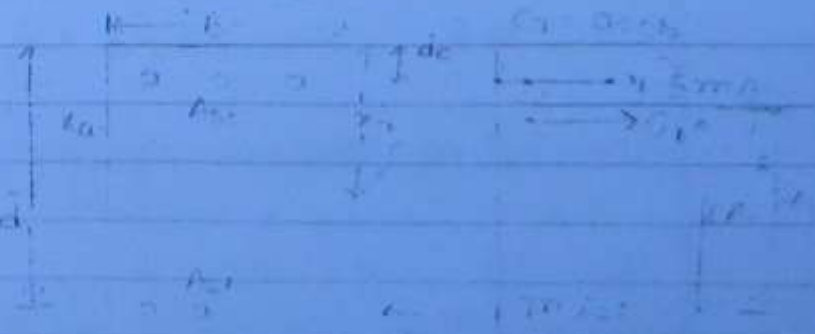
$$= 1.5\,m.Asc$$

1.5 m value is used for Compression steel, is due to Continued Compressive Strain for a long time (Creep.) Due to Creep effect actual stress in steel is much higher than stress developed due to Elastic Strain.

Analysis

1. Critical depth of N.A

$$x_c = \frac{mc}{t+mc} \times d$$



2. Actual depth of N.A

Equating moment of area on both side of N.A

$$B.x_a.\frac{x_a}{2} + 1.5m.Asc(x_a - d_c) - Asc(x_a - d_c) = m.Ast(d - x_a)$$

↑ concrete area not available

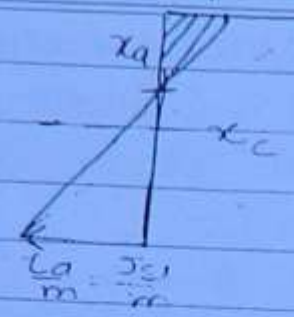
$$B.\frac{x_a^2}{2} + (1.5m - 1)A_{sc}(x_a - d_c) = m.Ast(d - x_a)$$

Compare —

if $x_a < x_c \rightarrow$ under reinforced

$C_a < \sigma_{cbc}$

$t_a = \sigma_{st}$ (32)



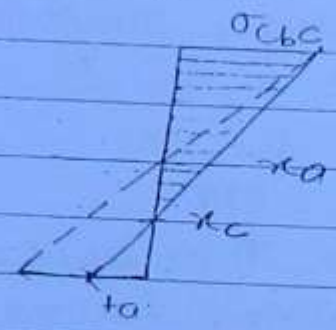
if $x_a = x_c \rightarrow$ balanced section

$C_a = \sigma_{cbc}$

$t_a = \sigma_{st}$

if $x_a > x_c \rightarrow$ Over reinforced

$C_a = \sigma_{cbc}$ $t_a < \sigma_{st}$



Moment of Resistance fig (i) (Compression side)

for Concrete $C_1 = B \cdot x_a \frac{C_a}{2}$

$LA_1 = \left(d - \frac{x_a}{3}\right)$

$MR_1 = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3}\right)$

Compression Steel $C_2 = 1.5m \cdot A_{sc} \cdot C' - A_{sc} \cdot C'$

$(1.5m - 1) A_{sc} \cdot C'$

lever arm $LA_2 = d - d_c$

$MR_2 = (1.5m - 1) A_{sc} \cdot C' (d - d_c)$

Total M.R $= B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc} \cdot C' (d - d_c)$

Moment of Resistance from tension side.

$C_R = C_1 + C_2$

$y_1 = \frac{x_a}{3}$; $y_2 = d_c$ & $\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$

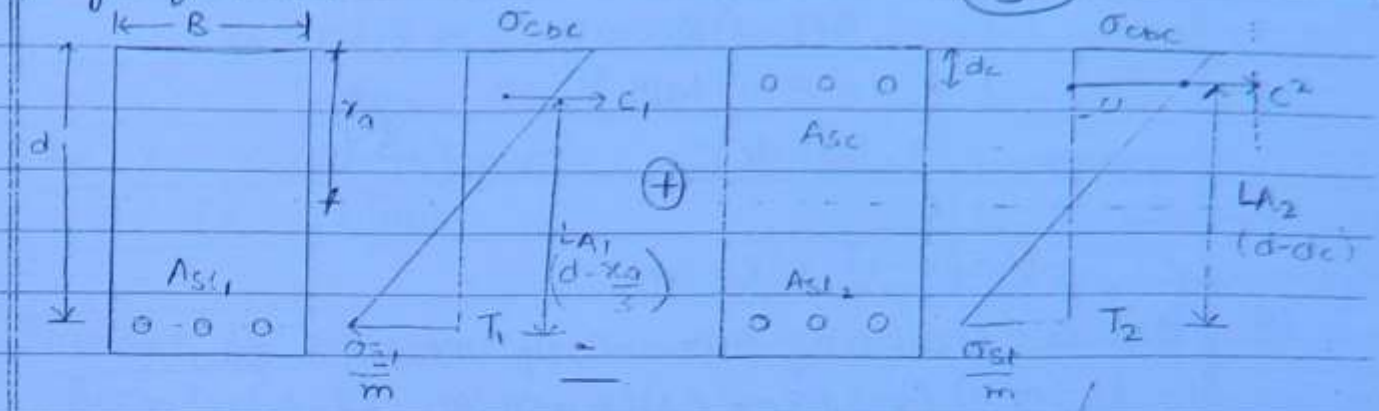
Net lever arm $= (d - \bar{y})$

M.R $= C_R (d - \bar{y})$

$= T \cdot (d - \bar{y})$

M.R $= A_{st} \cdot t_a (d - \bar{y})$

Design formula



$C_1 = B \cdot x_a \cdot \frac{C_a}{2}$

$LA_1 = (d - \frac{x_a}{3})$

$T_1 = t_a \cdot Ast_1$

$C_2 = (1.5m-1) Asc \cdot c'$

$LA_2 = (d - d_c)$

$T_2 = t_a \, Ast_2$

$M.R = C_1 \cdot LA_1 + C_2 \cdot LA_2$ (Compression)

$= B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3}\right) + (1.5m-1) Asc \cdot c' (d - d_c)$

for balanced section $M.R = (M.R_1)$ singly reinforced section

$= \underbrace{B \cdot x_c \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_c}{3}\right)}_{MR_1} + \underbrace{(1.5m-1) Asc \cdot c' (d - d_c)}_{MR_2}$

$Qbd^2 + (1.5m-1) Asc \cdot c' (d - d_c)$

$M.R = \underbrace{T_1 \cdot LA_1}_{MR_1} + \underbrace{T_2 \, LA_2}_{MR_2}$ (tension)

$= t_a \cdot Ast_1 \left(d - \frac{x_a}{3}\right) + t_a \cdot Ast_2 (d - d_c)$

Design steps :-

1. If beam has to be designed for a B.M = B.M

   $B.M > (MR_1 = Qbd^2)$

2. Check $MR_1 = Qbd^2$

   if $B.M > MR_1$ Then doubly reinforced section is required.

3. Calculate Ast_1 Equating MR_1 on both tension & compression side.

$$Ast_1 = \frac{MR_1 \quad Qbd^2}{t_a \cdot (d - x_a/3)} = \frac{MR_1}{\sigma_{st} (d - x_c/3)}$$

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4. ~~Remaing~~ Remaining B.M

$$MR_2 = BM - MR_1$$

$$Ast_2 = \frac{(MR_2 = B.M - MR_1)}{t_a \cdot (d - d_c)}$$

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5 $$A_{sc} = \frac{MR_2}{(1.5m - 1) \cdot c' (d - d_c)}$$

Or for A_{sc} equating moment of area for part 2

$$(1.5m - 1) A_{sc} \cdot (x_a - d_c) = m \cdot Ast_2 (d - x_a)$$

$$A_{sc} = \frac{m \cdot Ast_2 (d - x_a)}{(1.5m - 1)(x_a - d_c)}$$ ✓ (commonly used)

Generally M.R is calculated from compression side.

Ques:- A rectangular beam of 230 x 600 mm c. 50 mm eff. cover is reinforced 5-25 ϕ - on tension & 3-16 ϕ - compression. It is carrying an imposed load of 50 kN/m over an effective span of 7m. Check adequacy of section M-20 / fe 415.

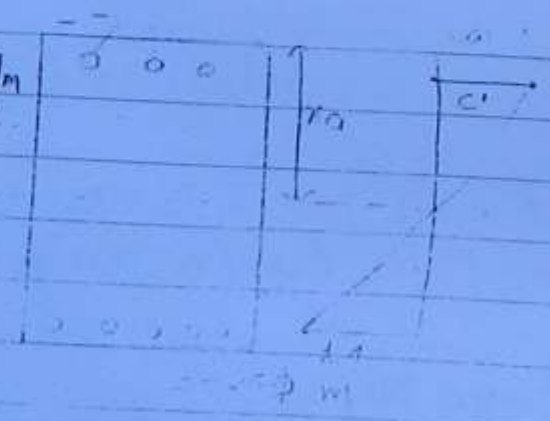
Sol:- Imposed load - 50 kN/m

Step I Self wt = 23 x .6 x 1 x 25 = 3.45 kN/m

total = 53.45 kN/m

$$B.M = \frac{wl^2}{8} = \frac{53.45 \times (7)^2}{8}$$

B.M = 321.38 kN-m



Moment of Resistance

Critical depth $x_c = \frac{mc}{t + mc} \times d = \frac{13 \times 7}{230 + 13 \times 7} \times 550$

= 155.92 mm

Actual depth

$$B \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = m \cdot Ast (d - x_a)$$

$$230 \cdot \frac{x_a^2}{2} + (1.5 \times 13 - 1) \cdot 603.20 (x_a - 50) = 13 \times 2454.4 (550 - x_a)$$

$$x_a^2 + 97.04x_a - 4851.83 = 152600 - 277.45x_a$$

$$x_a^2 + 374.49 - 157451.83 = 0$$

$$x_a = 251.3 \text{ mm}$$

$x_a > x_c \rightarrow$ Over reinforced $\quad C_a = \sigma_{cbc} \quad t_a < \sigma_{st}$

So $C_a = 7 N/mm^2$

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Moment Of Resistance

$$B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$$

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = \frac{7(251.3-50)}{251.3} = 5.607$$

$$MR = 230 \times 251.3 \times \frac{7}{2}\left(550 - \frac{251.3}{3}\right) + (1.5 \times 13 - 1) 603.20 \times 5.607 (550-50)$$

$M.R = 125.60$ KN-m

$\therefore$ B.M $\geq$ M.R So it is not safe

Method II - Calculate the stresses developed in the beam

$\rightarrow$ due to B.M 327.38 KN-m

1. Calculate x_a $\quad x_a = 251.31$ mm

If the stresses develop are C_a = in concrete & t_a = in steel

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = 0.8 C_a$$

Equating B.M & moment Of resistance we get

$$327.38 = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$$

$$327.38 \times 10^6 = 230 \times 251.31 \times \frac{C_a}{2}\left(550 - \frac{251.3}{3}\right) + (1.5 \times 13 - 1) 603.2 \times .8 C_a (550-50)$$

$C_a = 18.25 \ N/mm^2$

$C_a > 7 N/mm^2$ failed.

$C' = .8 \times 18.25 = 14.6 \ N/mm^2$

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Stress in compression steel

$= 1.5 m \times c' = 284.7 N/mm^2$

Stress in tension steel

Using similar Δ^{le} $\frac{c_a}{x_a} = \frac{t_a/m}{d - x_a}$

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$$t_a = \frac{(d - x_a) m \cdot c_a}{x_a} = \frac{(550 - 251.3) 13 \times 18.25}{251.31}$$

$$t_a = 281.9 N/mm^2 > 230 N/mm^2$$

(failed)

Ques:- Design a rectangular beam of size 300X750mm overall depth for a live load of 45 kN/m over a span of 7.5m use M-25 & fe 415

Sol. ① Live load = 45 kN/m

Self wt = $.3 \times .75 \times 1 \times 25 = 5.625$ kN/m

$B.M = \frac{wl^2}{8} = \frac{50.625 \times (7.5)^2}{8} = 355.95$ kN-m.

② Calculate M.R of singly reinforced balanced section

$MR_1 = QBd^2$

$Q = \frac{1}{2} c \cdot j \cdot k = 1.11$

$K = \frac{mc}{c + mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = .289$

$j = 1 - \frac{k}{3} = .904$

$Q = 1.11$

$MR_1 = 1.11 \times 230 \times 700^2$

$MR_1 = 163.17$ kN-m

$B.M > MR_1$

(doubly reinforced section)

③ $A_{st1} = \frac{MR_1}{\sigma_{st}(d - \frac{x_c}{3})} = \frac{MR_1}{\sigma_{st} j \cdot d}$

$= \frac{163.17 \times 10^6}{230 \times .904 \times 700}$ $1121.1 mm^2$

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(4) $MR_2 = \overline{B.M} - MR_1$

$= 355.96 - 163.17 = 192.79$ kN-m

(5) $A_{st_2} = \dfrac{192.79}{230(700-50)} = 1289.56 \text{ mm}^2$

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(6) $A_{sc} = \dfrac{m \cdot A_{st_2}(d - x_a)}{(1.5m-1)(x_a - d_c)} = \dfrac{13 \times 1289.56(700 - 0.289 \times 700)}{(1.5 \times 11 - 1)(.289 \times 700 - 50)}$

$= A_{sc} = 2990.6 \text{ mm}^2$

OR $A_{sc} = \dfrac{MR_2}{(1.5m-1)c'(d-d_c)} = \dfrac{192.79 \times 10^6}{(1.5 \times 11 - 1) \times 6.4(700-50)}$

$C_a = \sigma_{cbc} = 8.5 \text{ N/mm}^2$

$c' = \dfrac{x_a - d_c}{x_a} \times C_a = \dfrac{0.289 \times 700 - 50}{.289 \times 700} \times 8.5$

$c' = 6.4 \text{ N/mm}^2$

$\therefore A_{sc} = 2989.9 \text{ mm}^2$

Total $A_{st} = A_{st1} + A_{st2} = 1121.1 + 1289.56$

$= 2410.66 \text{ mm}^2$

T- BEAM

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→ By working stress Method.

Isolated beam

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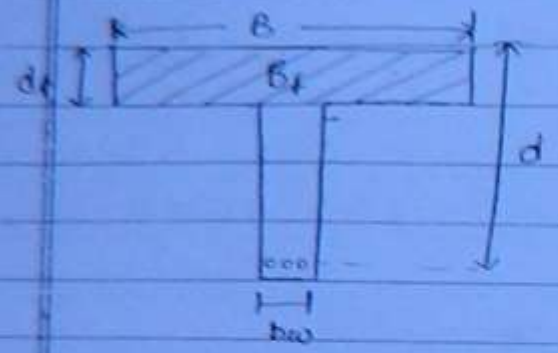


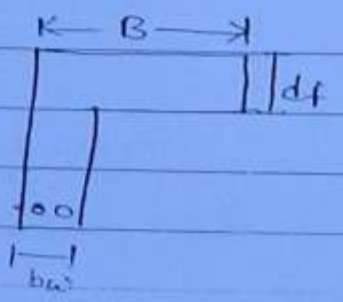
When width of flange is defined = B

Effective width of flange

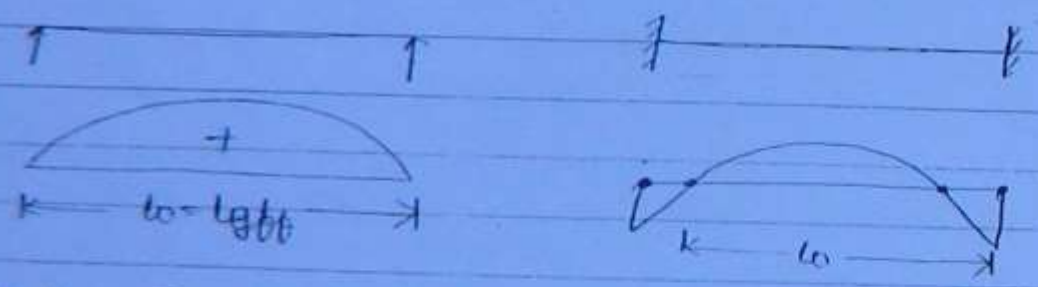
$$B_f = \frac{l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w \quad \text{for t-beam}$$

for L- beam

$$B_f = \frac{0.5\, l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$$



l_0 = distⁿ betⁿ point of zero moment of the beam



As per IS - 456

for fixed or continous beam

$l_0 = 0.7\, l_{eff}$

b_f = width of web flange

b_{eff} = width of web

slab + beam type (continous type)

Effective width of flange

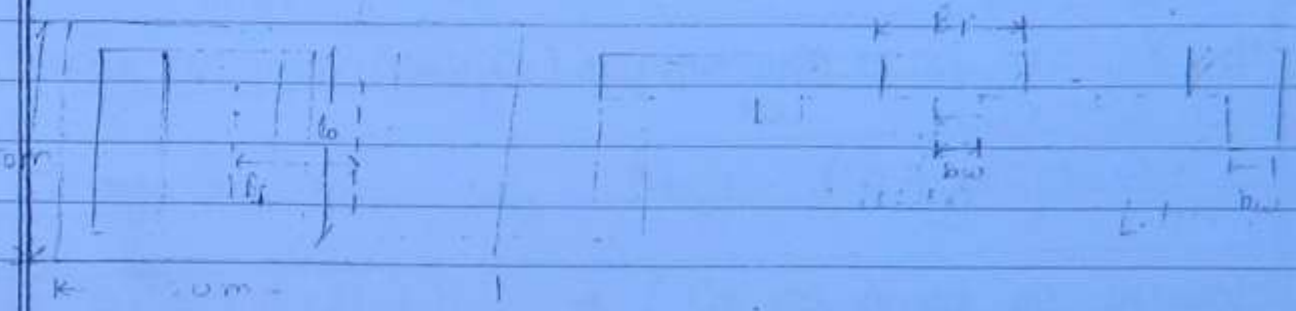
for T- beam $B_f = \frac{l_0}{6} + b_w + 6d_f$

for L- beams

$$B_f = \frac{l_0}{12} + b_w + 3d_f$$

OR for T-beam $B_f = b_w + \frac{l_1}{2} + \frac{l_2}{2}$

for L-beam $B_f = b_w + \frac{l_1}{2}$



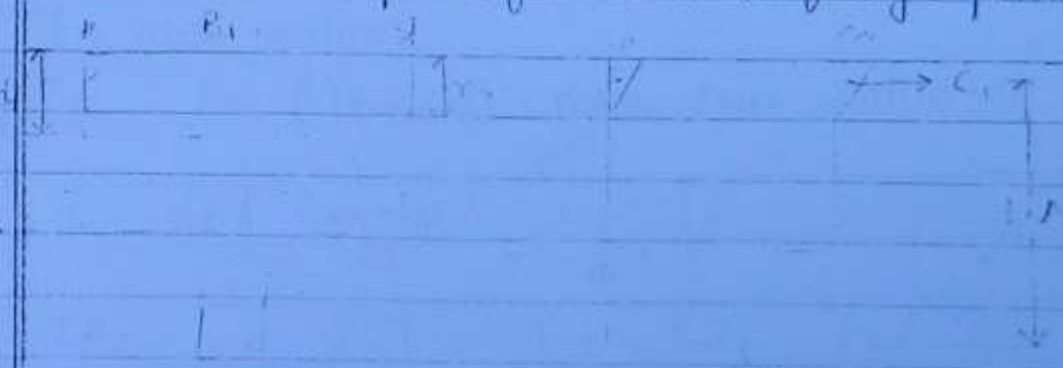
Effective width of flange is min of above two

Analysis of T- beam

① Critical depth of N.A

$$x_c = \frac{mc}{t + mc} \times d$$

Case I When actual depth of N.A is in flange portion ($x_a < d_f$)



Equating moment of area on both side of N.A

$$B_f \cdot x_a \cdot \frac{x_a}{2} = m \cdot A_{st} (d - x_a)$$

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$x_a = ?$$

Here $\boxed{x_a < d_f}$

(a) $x_a < x_c$ — under Reinforced Section

$C_a < \sigma_{cbc}$

$t_a = \sigma_{st}$

(b) if $x_a = x_c$ → Balanced Section

$C_a = \sigma_{cbc}$

$t_a = \sigma_{st}$

(c) if $x_a > x_c$ → over reinforced Section

$C_a = \sigma_{cbc}$

$t_a < \sigma_{st}$

Moment of Resistance

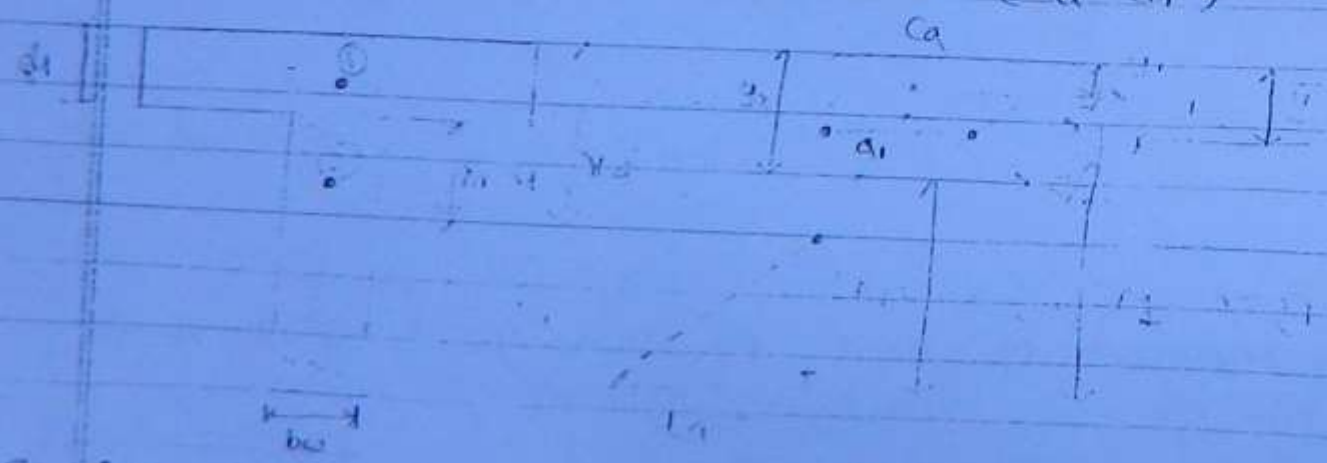
$MR = C_1 \cdot LA$

$= B_f \cdot x_a \cdot \frac{x_a}{2}\left(d - \frac{x_a}{3}\right)$ —— ①

$M_R = t_a \times A_{st} \left(d - \frac{x_a}{3}\right)$ —— ②

NOTE - Case I is same as that of rectangular beam of size $(B_f \times d)$

CASE II When N.A is within web area $(x_a > d_f)$



① Actual depth of N.A

Equating moment of area on both side of N.A

$$B_f \cdot d_f \cdot (x_a - d_f/2) + b_w (x_a - d_f)\left(\frac{x_a - d_f}{2}\right) = m A_{st} (d - x_a)$$

$$B_f \cdot d_f \cdot \left(x_a - \frac{d_f}{2}\right) + b_w \frac{(x_a - d_f)^2}{2} = m A_{st} (d - x_a)$$

(41)

2. Moment of Resistance

For flange portion - If stress a top fibre = c_a

$$\frac{c_1}{x_a - d_f} = \frac{c_a}{x_a}$$

Stress at junction $c_1 = \frac{x_a - d_f}{x_a} \times c_a$

Flange portion $c_1 = B_f \cdot D_f \left(\frac{c_a + c_1}{2}\right)$

$$y_1 = \left(\frac{a+2b}{a+b}\right)\frac{h}{3} \Rightarrow \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}$$

$LA_1 = d - y_1$

$$LA_1 = d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}$$

MR_1 (for flange) $= c_1 \cdot LA_1$

$$= B_f \times d_f \cdot \left(\frac{c_a + c_1}{2}\right) \times \left[d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}\right]$$

For web portion

$$C_2 = b_w (x_a - d_f)\frac{c_1}{2}$$

$$y_2 = d_f + \frac{(x_a - d_f)}{3}$$

$$LA_2 = d - y_2 = d - \left[d_f + \frac{(x_a - d_f)}{3}\right]$$

$MR_2 = C_2 \cdot LA_2$

$$= b_w (x_a - d_f)\frac{c_1}{2}\left[d - d_f - \left(\frac{x_a - d_f}{3}\right)\right]$$

Total Moment of Resistance $MR = MR_1 + MR_2$

$$= B_f d_f \cdot \left(\frac{c_a + c_1}{2}\right)\left[d - \left(\frac{c_a + 2c_1}{c_a + c_1}\right)\frac{d_f}{3}\right] + b_w (x_a - d_f)\frac{c_1}{2}$$

$$\left[d - d_f - \left(\frac{x_a - d_f}{3}\right)\right]$$

Moment Of Resistance from Tension Side

Total Resultant Comp. force $C_R = C_1 + C_2$

$$\bar{y} = \frac{C_1 Y_1 + C_2 Y_2}{C_1 + C_2}$$

Net lever arm = $(d - \bar{y})$

$$MR = C_R (d - \bar{y}) = T (d - \bar{y}) = f_a A_{st} (d - \bar{y})$$

Design formula

It is based on following assumption

① web area is neglected

② Lever arm = $0.9d$

$$MR = B_f \cdot d_f \cdot \left(\frac{C_a + C_1}{2}\right) \times 0.9d$$

$$= B_f \cdot d_f \left[\frac{C_a + \left(\frac{x_a - d_f}{x_a}\right) C_a}{2}\right] \times 0.9d$$

$$= B_f \, d_f \, C_a \left[\frac{2x_a - d_f}{x_a}\right] \times 0.45d$$

$$MR = B_f \cdot d_f \cdot \sigma_{cbc} \left[\frac{2kd - d_f}{kd}\right] \times 0.45d$$

$$MR = 0.45 \cdot B_f \cdot d_f \, \sigma_{cbc} \left[\frac{2kd - d_f}{k}\right]$$

Equating BM = MR we get value of d required

$$BM = f_a \cdot A_{st} \cdot 0.9d$$

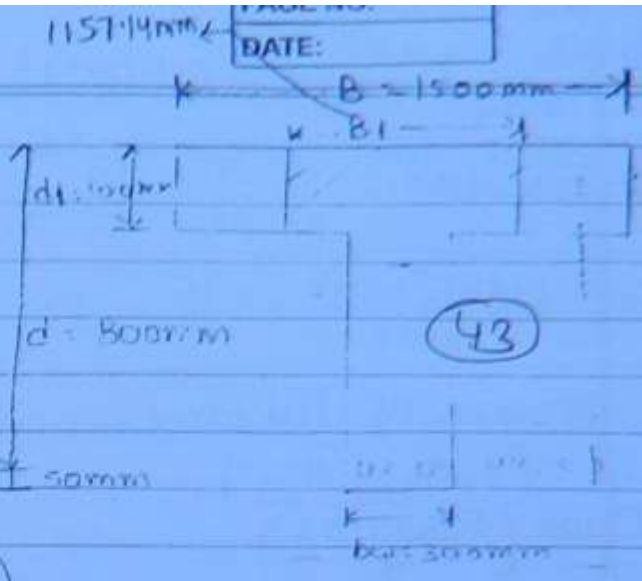
$$A_{st} = \frac{BM}{f_a \times 0.9d}$$

Calculate MR of an Isolated T beam supported over a simply supported span = $L_m = l_0$

① Eff width of flange (B_f)

$$B_f = \frac{l_0}{(l_0/B) + 4} + b_w$$

$$B_f = \frac{8000}{\frac{8000}{1500}+4} + 300 = 1157.14$$



Assuming N.A in flange area

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st}(d - x_a)$$

$$1157.14 \cdot \frac{x_a^2}{2} = 13 \times 4 \times \frac{\pi}{4}(25)^2(800 - x_a)$$

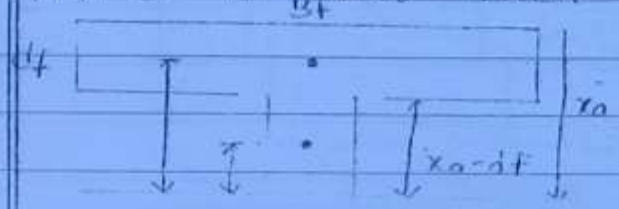
$$x_a^2 = 14768290.47(800 - x_a)$$

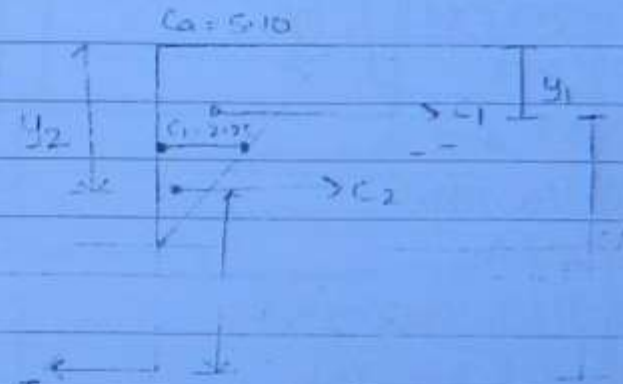
$$x_a^2 + 14768290.47\, x_a = 1.1814 \times 10^{10}$$

$$x_a = 167.09\ mm \quad > 100\ mm$$

$\therefore$ Assumption is not correct

$\rightarrow$ N.A is in web area





$$B_f \cdot d_f\left(x_a - \frac{d_f}{2}\right) + b_w\frac{(x_a - d_f)^2}{2} = m \cdot A_{st} \cdot (d - x_a)$$

$$1157.14 \times 100 \times \left(x_a - \frac{100}{2}\right) + 300\left(\frac{x_a - 100}{2}\right)^2 = 13 \times 4 \times \frac{\pi}{4}(25)^2(800 - x_a)$$

$$771.43\, x_a - 38571.67 + x_a^2 - 200\, x_a + 10^4 = 136135.68 - 170.17\, x_a$$

$$x_a^2 + 741.6\, x_a - 164707.35 = 0$$

$$x_a = 178.93\ mm$$

(2) Critical depth of N.A – $x_c = \frac{m c}{t_{mc} + m c} \times d = \frac{13 \times 1}{230 + 13 \times 1} \times 800$

$= 226.8\ mm$

$x_a < x_c$ $\therefore$ It is under reinforced case

C_a & σ_{cbc} & $t_c = \sigma_{st} = 230\ N/mm^2$

$x_a \quad d-x_a \quad d-x_a$

$$= \frac{178.93}{800-178.93} \times \frac{230}{13} = 5.10\ N/mm^2$$

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ess at junction

$$\frac{C_1}{x_a-d_f} = \frac{C_a}{x_a} \Rightarrow C_1 = \frac{x_a-d_f}{x_a} \times C_a$$

$$C_1 = \frac{178.93-100}{178.93} \times 5.10 = 2.25\ N/mm^2$$

3) Moment of Resistance

$$C_f - C_1 = B_f \cdot d_f \cdot \left(\frac{C_a+C_1}{2}\right)$$

$$1157 \times 100 \left(\frac{5.10+2.25}{2}\right)$$

$425248.95\ N$

$$\left(\frac{C_a+2C_1}{C_a+C_1}\right)\frac{d_f}{3}$$

$$\left[\frac{5.10+(2.25\times 2)}{5.10+2.25}\right] \times \frac{100}{3}$$

$= 43.537$

$= 756.46\ mm$

$= C_f \times LA_1$

$= 756.46 \times 425248.95$

$= 321.68\ kN-m$

→ web — $C_2 = b_w (x_a - d_f) \times C_1$

$$= 300 \times (178.93-100) \times \frac{2.25}{3}$$

$C_2 = 26638.875\ N$

$$y_2 = d_f + \frac{x_a-d_f}{3} = 100 + \left(\frac{178.93-100}{3}\right)$$

$= 126.31$

$LA_1 = (d-y_2) = 800 - 126.31 = 673.69\ mm$

$$MR_2 = C_2 \times LA_2 = 26638.875 \times \frac{673.69}{10^6}$$

$\Rightarrow MR_2 = 17.94\ kN-m$

Total $MR = MR_1 + MR_2$

$= 321.685 + 17.94$

$MR = 339.625\ kN\ m$

Q Design a T-beam for a commercial complex with reference to the data as stated below

Clear span of T-beam = 10 m

Spacing of T-beam = 2.5 m

Live load = 4 kN/m²

Thickness of slab = 15 cm = 0.15 m

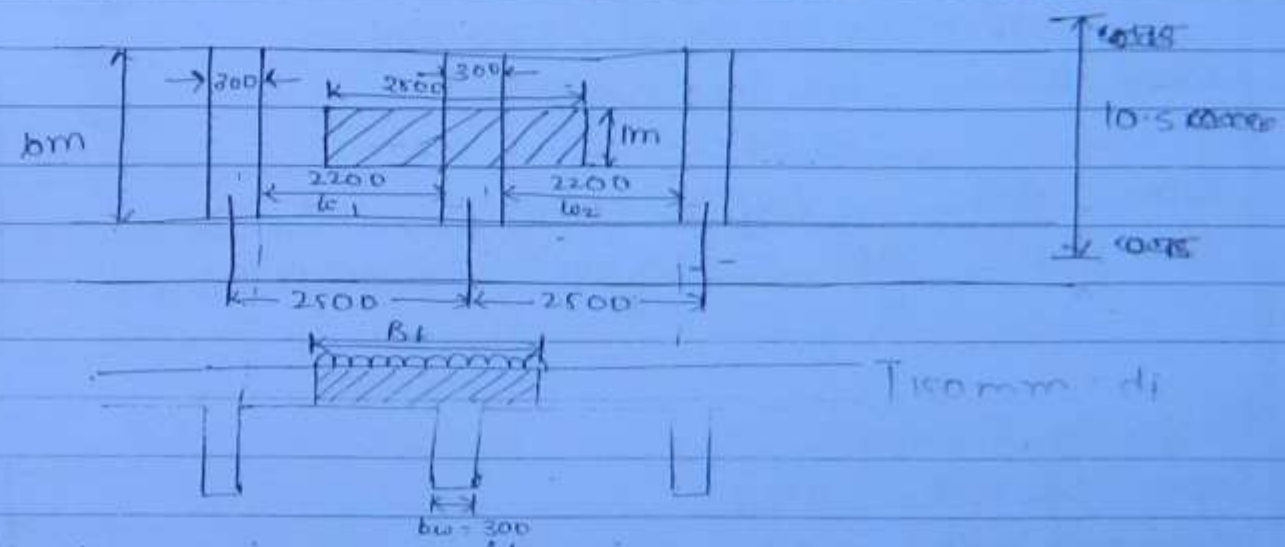
M 20 / Fe 415

Effective span of beam = 10.5 m

b_w = 300 mm

(45)

Sol:-



① Effective width of flange

for Cont Slab + beam type

$$B_f = \frac{l_0}{6} + b_w + 6d_f$$

$$= \frac{10500}{6} + 300 + 6 \times 150 = 2950 \text{ mm}$$

② Max B_f = c/c distance beam adjacent slab

$$= b_w + \frac{l_{01}}{2} + \frac{l_{02}}{2} = 300 + \frac{2200}{2} + \frac{2200}{2}$$

B_f = 2500 mm

③ Load calculation

over 1 beam per in length of beam

ⓐ dead

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Live load = 1 × 2.5 × 4 = 10 kN/m

Dead load of slab

= 0.15 × 1 × 2.5 × 25 = 9.37

(46)

Self wt. of web portion of slab

Assume depth of T-beam

$= \frac{Span}{20} = \frac{10000}{20} = 500$ mm

Slab thickness = 150 mm

Web depth = 350 mm

Dead load of web

= 0.30 × 0.35 × 1.0 × 25 = 2.625 kN/m

Total load = 10 + 9.37 + 2.625

= 21.995

~~Max B.M~~ Say 22 kN/m

Max B.M = $\frac{wl^2}{8} = \frac{22 \times 10.5^2}{8} = 303.2$ kN/m

Design of T beam

Depth of N.A = 0.9d

Neglecting web portion

B.M = MR = 0.45 · Bf · df · σcbc $\left(\frac{2kd - df}{k}\right)$

$303.2 \times 10^6 = 0.45 \times 2500 \times 7 \times 150 \left(\frac{2 \times 0.2835 \times d - 150}{0.2835}\right)$

d = 392.89

Say d = 400 mm

D = 400 + 50 = 450 mm

Area of steel $A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d}$

$\frac{303.2 \times 10^6}{230 \times 0.90 \times 400} = 3661.8$ mm²

No of 25 mm ϕ bars = $\frac{3661.8}{\pi/4 (25)^2}$ = 7.45 (Say 8 nos.)

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2nd of option

If deflection criteria is considered, Min effective depth required

$= \frac{Span}{20} = \frac{10500}{20} = 525\ mm$

47

Say 550 mm

+ 50 mm

Total = 600 mm

$A_{st} = \frac{303.2 \times 10^6}{230 \times 0.90 \times 550} = 2663\ mm^2$

No. of 25 mm ϕ bars $= \frac{2663}{\pi/4 \times 25^2} = 5.42$ (Say 6 Nos)

LIMIT STATE METHOD

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① Material Strength

(a) Characteristic Strength – The value of strength below which not more than 5% of test result are expected to fall

for Concrete → M25 $f_{ck} = 25\ N/mm^2$

for Steel – Fe 415 $f_y = 415\ N/mm^2$

(b) Design Strength

48

$$= \frac{\text{Characteristic Strength}}{\text{Partial factor of safety}}$$

① for Concrete $\gamma_{mc} = 1.50$

Permissible Stress in Concrete $= \frac{0.67 f_{ck}}{1.5}$ (As per IS-456)

$= 0.45 f_{ck}$

② for Steel : $\gamma_{ms} = 1.15$

Permissible Stress $= \frac{f_y}{1.15} = .87 f_y$

③ load

Characteristic load – the load value that has 95% probability of not being exceeded during its life time of structure. This value is called Characteristic load & used for design purpose.

Design load (In LSM)

$F_d = \gamma_f \times F$ F = Characteristic load

Value of γ_f

Ques Partial safety factor for loads (Table 18) P-68

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	D.L	L.L	W.L	D.L	L.L	W.L
D.L + L.L	1.5	1.5	–	1.0	1.0	–
D.L + (W.L or EQL)	1.5	–	1.5	1.0	–	1.0
D.L + L.L + (W.L or EQL)	1.2	1.2	1.2	1.0	0.8	0.8

Example - If DLM = 50kN-m
LLM = 40kN-m & Q CGLM = 20kN-m

① 1.5×50 + 1.5×40 = 135 kN-m — not Critical
② 1.5×50 + 1.5×20 = 105 kN-m
③ 1.2×50 + 1.2×40 + 1.2×20 = 132 kN-m

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NOTE ① 0.9G value is Consider when Stability against overturning is Critical
② WL & CQL are not Considered simultaneously.

Limit State Of Collapse	Limit State Of Serviceability
① Flexure	Deflection [DCVC]
② Shear	Cracking
③ Compression	Vibration
④ Torsion	Corrosion.

→ Examples

Working Stress Method	Limit State Method
1. Concrete	1. Concrete
factor of Safety for stress	Stress factor Of Safety
① M15 = $\frac{\text{Charac}}{\text{Permissible}} = \frac{15}{5} = 3$	= $\frac{\text{Charac. strength}}{\text{Permissible stress}} = \frac{f_{ck}}{0.45 f_{ck}} = 2.22$
M20 = $\frac{20}{7} = 2.86$	Load factor Of Safety = 1.50
M25 = 25/8.5 = 2.94	Net F.OS = 2.22×1.50 = 3.33
average = 3.0 times	which is higher than that
factor Of Safety for load	Of W.S.M
= 1.0	
Net factor Of Safety = 3.0×1.0	
3.0	

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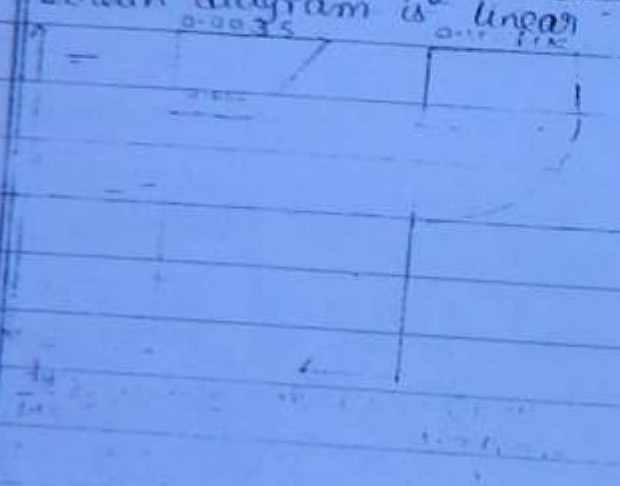
WSM	L.S.M
2. Steel Stress F.O.S (50) Mild Steel = $\frac{f_y}{\text{Permissible}}$ 250 = 250/140 = 1.78 415 = 415/250 = 1.6 Stress F.O.S = 1.6 Load F.O.S = 1.00 ult F.O.S = 1.60 × 1 = 1.60	Steel Stress F.O.S = $\frac{f_y}{.87 f_y}$ = 1.15 Load F.O.S = 1.50 Ult F.O.S = 1.15 × 1.50 = 1.725 less than that of W.S.M

Limit Method

SINGLY REINFORCED BEAM

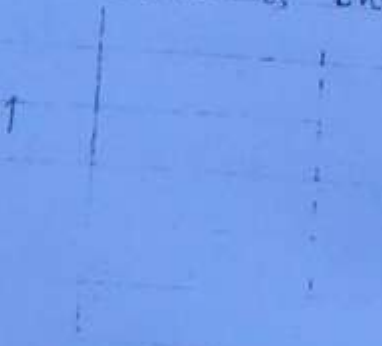
Assumptions (38.1 / Pg-69)

① Plane section before bending remains plane after bending
→ Strain diagram is linear



② The max. strain in concrete at outermost compression fibre is 0.0035

③ An acceptable stress-strain curve is shown in fig 01 for concrete →



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→ For design purpose the Comp. strength of Concrete shall be assumed to be 0.67 times characteristic strength. The partial F.O.S shall be applied in addition to it.

(57)

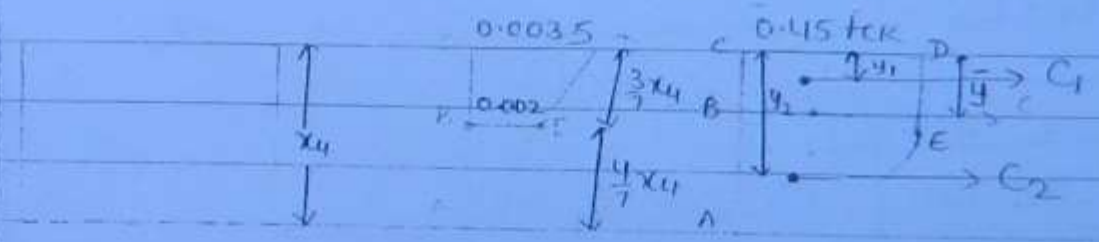
(4) Tensile strength of Concrete is ignored

(5) The stress in reinforcement are derived from stress-strain curve for the type of steel used. Curves are given in fig 23. For design purpose partial safety factor of 1.15 shall be applied

Permissible stress in steel $= \frac{f_y}{1.15} = 0.87 f_y$

(6) The max. strain in tension reinforcement in the section at the time of failure shall not be less than $\frac{f_y}{1.15 E_s} + 0.002$

Analysis of stress block parameter



$T = 0.87 f_y A_{st}$

K — B — K

If x_u = Depth of neutral axis (Actual depth of N.A)

$\frac{AB}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$

$AB = \frac{4}{7} AC = \frac{4}{7} x_u$

BC $3/7\, x_u$ (Depth of rectangular portion of stress diagram)

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Compressive force

C_1 = (width of beam × Area of stress diagram)

$= B \times 0.45 f_{ck} \times \frac{3}{7} x_u$

$= 0.192 B f_{ck} \cdot x_u \cdot B$

(52)

$y_1 = \frac{1}{2} \times \frac{3}{7} x_u = \frac{3}{14} x_u$

$C_2 = B \times \frac{2}{3} \times 0.45 f_{ck} \times \frac{4}{7} x_u$

$= 0.1714 f_{ck} \cdot x_u \cdot B$

$y_2 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u = 0.6429 x_u$

$C = C_1 + C_2$

$= .192 B f_{ck} \times x_u \cdot B + 0.1714 f_{ck} \cdot x_u \cdot B$

$= .3642 f_{ck} x_u B$

As per IS = $.36 f_{ck} x_u \cdot B$

$\bar{y} = \frac{C_1 \bar{y}_1 + C_2 y_2}{C_1 + C_2} = \frac{.192B \times \frac{3}{14} + .1714 \times .6429}{.3642} \times x_u$

$= .416 x_u$

$\bar{y} = .42 x_u$ As per IS 456

Total tensile force $= T = 0.87 f_y A_{st}$

lever arm

= distance betn C & T

$(d - 0.42 x_u)$

(7) Actual depth of Neutral axis (x_u)

Total Compressive force = Total tensile force

$C = T$

$0.36 f_{ck} x_u \cdot B = .87 f_y \cdot A_{st}$

$x_u = \frac{.87 f_y A_{st}}{.36 f_{ck} B}$ emp.

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② Moment Of Resistance.

M·R = Total force Of Comp. × lever arm

= ·36 fck x_u · B $(d - 0.42\, x_u)$ — imp

M·R = Total force Of Tension × lever arm

= ·87 fy Ast $(d - 0.42\, x_u)$ — imp.

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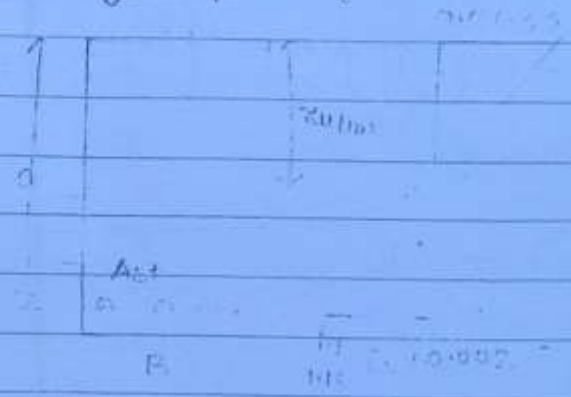
③ limiting depth Of Neutral axis (based on strain diagram)



At $x_{u\,lim}$ - value Of strain should be at max limiting value

from similiar Δ^{le}

$$\frac{x_{u\,lim}}{d - x_{u\,lim}} = \frac{0.0035}{\frac{fy}{1.15} E_s + 0.002}$$

$$\Rightarrow \frac{d - x_{u\,lim}}{x_{u\,lim}} = \frac{\frac{fy}{1.15} E_s + 0.002}{0.0035}$$

$$\frac{d}{x_{u.\,lim}} = \frac{.87 fy + 0.002 E_s}{0.0035 E_s} + 1$$

$$\frac{d}{x_{u.\,lim}} = \frac{.87 fy + 0.0055 E_s}{0.0035 E_s}$$

$$\frac{d}{x_{u.\,lim}} = \frac{0.87 fy + 0.0055 \times 2 \times 10^5}{0.0035 \times 2 \times 10^5}$$

$$\frac{d}{x_{u.\,lim}} = \frac{0.87 fy \times 1100}{700}$$

$$\frac{x_{u\,lim}}{d} = \frac{700}{.87 fy + 1100} = K$$

$x_{u\,lim}/d$	fy	$x_{u.\,lim}$
·53	fe 250	·53d
·48	fe 415	·48d
·46	fe 500	·46d

$x_u \leq x_{u\,lim}$ —

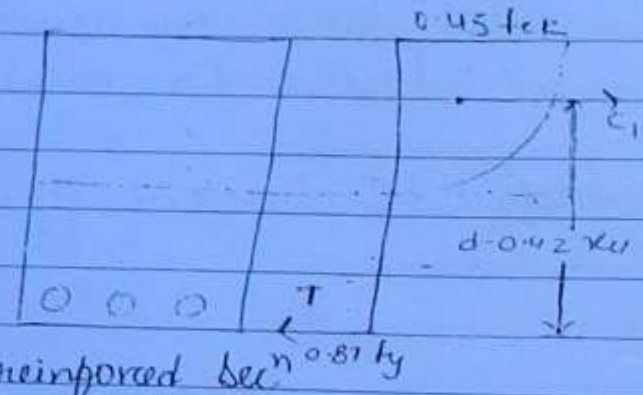
$x_u \not> x_{u\,lim}$ —

Moment of Resistance

1. under reinforced section

$$x_u = \frac{.87 f_y A_{st}}{.36 f_{ck} B}$$

$$x_{u\,lim} = \begin{cases} 0.53d \\ 0.48d \\ 0.46d \end{cases}$$



If $x_u < x_{u\,lim}$ - It is under reinforced sec^n

use x_u for M.R

$M.R = 0.36 f_{ck} B . x_u (d - 0.42 x_u)$ — ①

$= 0.87 f_y A_{st} (d - 0.42 x_u)$ — ②

limiting section

If $x_u = x_{u\,lim}$

use $x_{u\,lim}$ for M.R

$M.R = 0.36 f_{ck} B . x_{u\,lim} (d - 0.42 x_{u\,lim})$ — ①

$= 0.87 f_y A_{st} (d - 0.42 x_{u\,lim})$ — ②

Over reinforced section

If $\left(x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B} > x_{u\,lim} \right)$ It is an over reinforced sec^n

In this case x_u is limited to $x_{u\,lim}$

$M.R = 0.36 f_{ck} . B . x_{u\,lim} (d - 0.42 x_{u\,lim})$ — ①

Design formula

for a limiting sec^n

$BM = M.R = 0.36 f_{ck} . B . x_{u\,lim} (d - 0.42 x_{u\,lim})$

$x_{u\,lim} = K.d$

$BM = 0.36 f_{ck} B . K.d (d - 0.42 K.d)$

$B.M = [0.36\, f_{ck} \cdot B \cdot K \cdot (1 - 0.42K)]\, d^2 B$

$B.M = Q B d^2$

$d = \sqrt{\frac{BM}{Q \cdot B}}$

(55)

for Fe 415 $\quad Q = 0.36 \times f_{ck}\,(1 - 0.48 \times 0.42) \cdot 48$

$= .13B\, f_{ck}$

M20 $Q = 2.75$

M25 $Q = .13B \times 25 = 3.45$

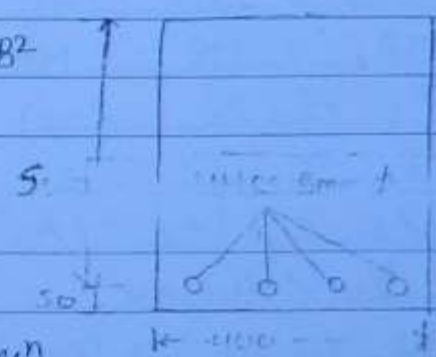
Ques- Calculate M.R of a beam size 400 X 600 mm (overall depth) Eff. Cover = 50mm, M20, Fe 415 is used. Area of steel is

(1) 4 Nos 18 mm ϕ bars (2) 8 Nos 25 mm ϕ bars

Calculate M.R of limiting section also & area of steel req for this section

Sol^n (i) $x_u = \frac{.87 f_y A_{st}}{.36 f_{ck} B} = \frac{.87 \times 415 \times 4 \times \pi/4 \times 18^2}{.36 \times 20 \times 400}$

$x_u = 127.61$ mm



$x_{u\,lim} = .48 \times d = .48 \times 550$

$= 264$ mm

$x_u < x_{u\,lim}$ – under reinforced sec^n

$\therefore x_u$ is used for the M.R

$MR = .36\, f_{ck} \cdot B \cdot x_u\,(d - 0.42\, x_u)$

$= .36 \times 20 \times 400 \times 127.61\,(550 - .42 \times 127.61)$

$= 182.44$ kN-m

(ii) Area of steel = 8 Nos 25 mm ϕ bars

$A_{st} = 8 \times \frac{\pi}{4} \times 25^2 = 3926.99$

$x_u = \frac{.87 \times 415 \times 3926.99}{.36 \times 20 \times 400} = 492.30$

$x_{u\,lim} = .48 \times d = 264$ mm

$x_u > x_{u\,lim}$ It is over reinforced sec^n

DATE:

$x_u = x_{u,lim}$

$M.R = .36 f_{ck} B \cdot x_{u,lim} (d - 0.42 x_{u,lim})$

$= .36 \times 20 \times 400 \times 264 (550 - 0.42 \times 264) \times \frac{1}{10^6}$

$= 333.87$ kN-m

(56)

M.R for limiting sec^n

$= 333.87$ kN-m

$333.87 \times 10^6 = .87 \times 415 \times A_{st} (550 - 0.42 \times 264)$

$A_{st} = \frac{333.87 \times 10^6}{158544.276}$

$A_{st} = 2105.85 \ mm^2$

W.S.M $x_{a1} < x_{a2} < x_{a3}$

$C_1 < C_2 < C_3$

$LA_1 > LA_2 > LA_3$

$MR_1 < MR_2 < MR_3$

L.S.M $C_1 < (C_2 = C_3)$

$T_1 < (T_2 = T_3)$

$LA_1 > (LA_2 = LA_3)$

$x_u > x_{umax}$

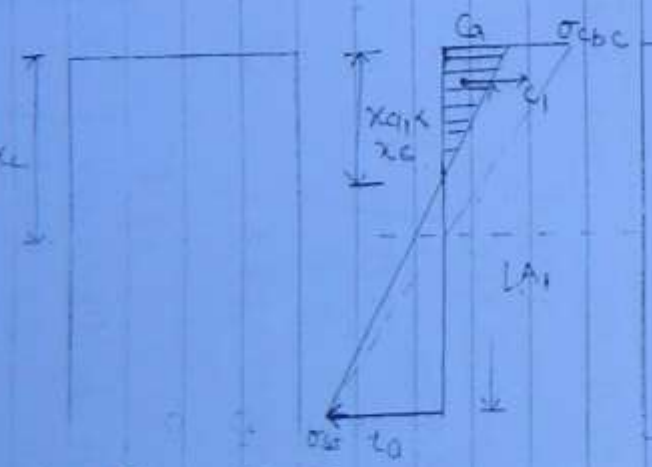
but x_u lim is considered

$MR_1 < (MR_2 = MR_3)$

Actual $(T_3 > T_2)$ but not considered

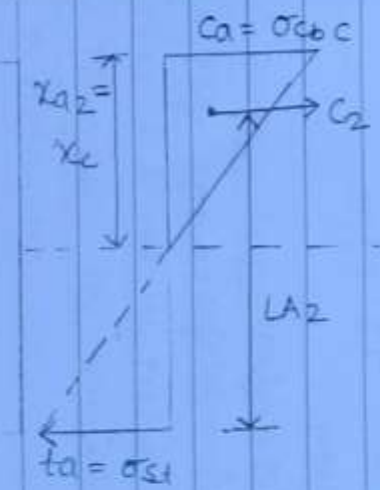
UNDER REINFORCED SECTION	BALANCED SECTION	OVER REINFORCED SECTION

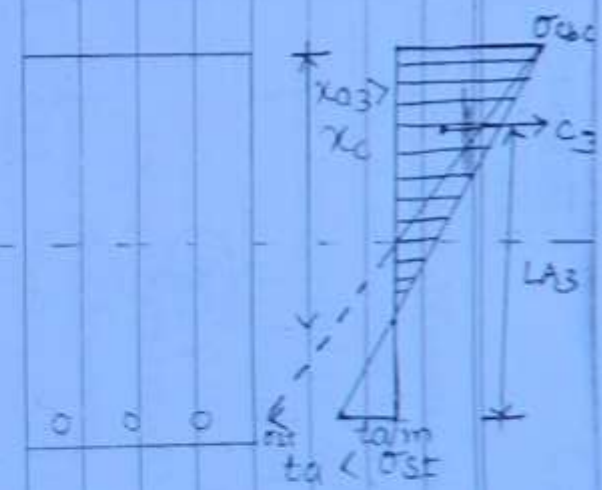
Working Stress Method



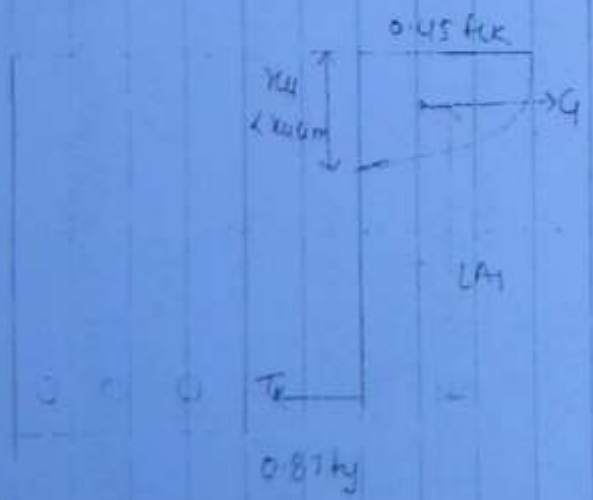
$C_a < \sigma_{cbc}$; $t_a = \sigma_{st}$

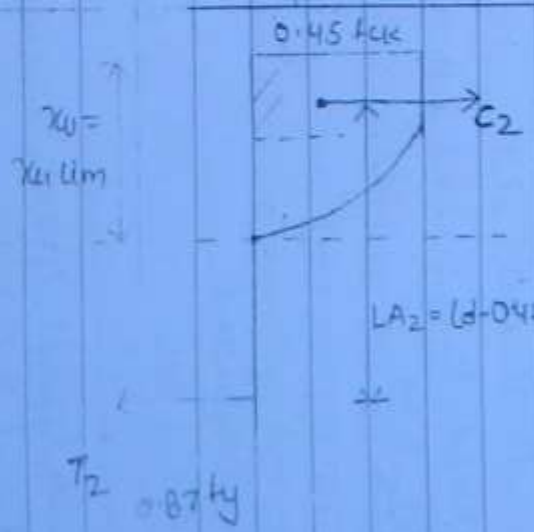
$MR_U < MR_B$

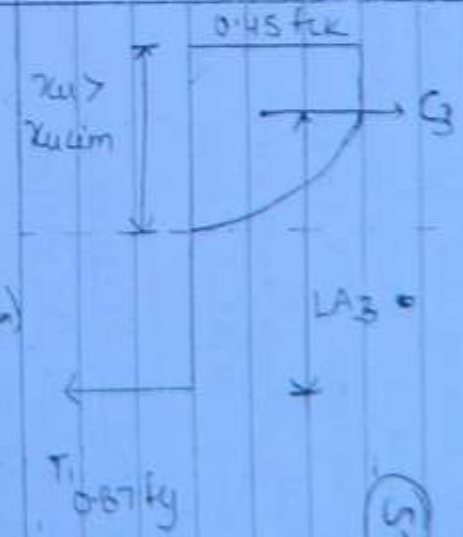




Limit State Method







$C_1 < C_2 = C_3$

$T_1 < T_2 = T_3$

$LA_1 > LA_2 = LA_3$

$MR_1 < MR_2 = MR_3$

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Ques - Use IS:456 to design a reinforced concrete rectangular beam having an eff. S.S. span of 6m. The beam is required to have live (service) load = 14 kN/m & superimposed load = 9.5 kN/m. Use Fe 415 & M 20.
Adopt $d/b = 2$
$P_{t\,lim} = 0.955\%$, unit wt. of concrete = 25 kN/m³

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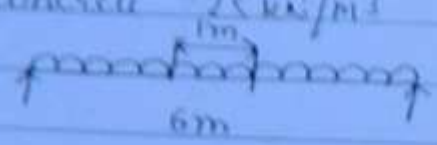
Sol^n 1. Load



Live load = 14 kN/m
S.I. D.L = 9.5 kN/m
Self wt of beam = $.6 \times .3 \times 1 \times 25 = 4.5$ kN/m
Total load $w = 28$ kN/m
Factored load = $1.5 \times 28 = 42$ kN/m
Max. B.M $= \frac{wl^2}{8} = \frac{42 \times 6^2}{8} = 189$ kN-m

(2) Depth required

$$d = \sqrt{\frac{BM}{R \cdot B}}$$

$R = .36 f_{ck} k(1 - 0.42k) = .138 \times f_{ck} = 2.75$

$$d = \sqrt{\frac{B \cdot M \times 2}{R \cdot d}}$$

$$d^2 = \frac{2BM}{R \cdot d}$$

$$d^3 = \frac{2 \times 189 \times 10^6}{2.75} = \boxed{516.08 \text{ mm} = d}$$

Say $d = 520$ mm

$b = \frac{d}{2} = \frac{520}{2} = 260$

$D = 520 + 50 = 570$

Load considered is slightly higher so ok.

Area of Steel Required

$$A_{st} = \frac{B.M}{\cdot 87 f_y (d - 0.42 x_{u,lim})}$$

(59)

$$= \frac{189 \times 10^6}{87 \times 415 (520 - 0.42 \times 0.48 \times 520)}$$

$$= 1261 \ mm^2$$

Check Limiting $A_{st} = \frac{\cdot 955}{100} \times 260 \times 520$

$$= 1291 \ mm^2$$

Ques A simply supported RCC rectangular beam is supported over a eff. span of 8.0 m. live load over beam = 30 kN/m M20 / Fe 415 is used. Design the beam by WSM & LSM & compare the results. Consider width of beam 350 mm in both case.

Sol^n (i) Working Stress Method

live load = 30 kN/m

Span = 8m ; Depth = 8000/10 = 800 mm

width = 350 mm

Self wt = $\cdot 35 \times \cdot 8 \times 1 \times 25$ = 7 kN/m

Total = 37 kN/m

$$BM = \frac{37 \times 8^2}{8} = 296 \ kN\text{-}m$$

$$d = \sqrt{\frac{B.M}{Q.B}}$$

$$Q = \frac{1}{2} \times 8.5 \times \cdot 289 \times \cdot 904$$

$$= 1.11$$

$$d = \sqrt{\frac{296 \times 10^6}{1.11 \times 350}} = 872.87 + 50$$

$$= 922 \ mm$$

$Q = \frac{1}{2} c j k$

$k = \frac{mc}{mc + t}$

$t = 230 \ m = 11$

$c = 8.5$

$k = \frac{11 \times 8.5}{11 \times 8.5 + 230}$

$= \cdot 289$

$j = \cdot 904$

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Consider $D = 950\,mm$ (60)

$d = 900\,mm$

Self wt = $.35 \times .95 \times 1 \times 25 = 8.3125\ kN/m$

total $w = 38.3125\ kN/m$

$B.M = \frac{wl^2}{8} = 306.5\ kN\text{-}m$

$d = \sqrt{\frac{306.5 \times 10^6}{1.11 \times 350}} = 888\ mm < 900\ mm$

OK.

Area of steel $= \frac{B.M}{\sigma_{st} \cdot j \cdot d}$

$= \frac{306.5 \times 10^6}{230 \times .904 \times 900}$

$= 1638\ mm^2$

Limit State method

Live load $30\ kN/m$

Self wt = $.35 \times .8 \times 1 \times 25 = 7\ kN/m$

$w = 37\ kN/m$

factored $= 1.5 \times 37 = 55.5\ kN/m$

$B.M = \frac{wl^2}{8} = \frac{55.5 \times 8^2}{8} = 444\ kN\text{-}m$

$d = \sqrt{\frac{B.M}{Q \cdot B}}$

$Q = .138 f_{ck} = .138 \times 25 = 3.45$

$d = \sqrt{\frac{444 \times 10^6}{3.45 \times 350}} = 606.3\ mm < 750\ mm.$

So consider $d = 600$

$D = 650\ mm$

Self wt = $.35 \times .65 \times 1 \times 25 = 5.6875$

$w = 1.5 \times 35.6875 = 53.53\ kN/m$

$BM = \frac{wl^2}{8} = 428.25$

$d = \sqrt{\frac{428.25 \times 10^6}{3.45 \times 350}} = 596 < 600\,mm$ ok

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$A_{st} = \frac{B.M}{.87 \times f_y \times (d - .42\, x_{u\,lim})}$

$= \frac{428.25 \times 10^6}{.87 \times 415 \times (600 - .42 \times .48 \times 600)}$

$= 2476\ mm^2$

(61)

Results :-

	W.S.M	L.S.M (Preferred because depth is less)
① Depth D	950 mm	650 mm
② Eff depth d	900 mm	600 mm
③ Area of Steel	1638 mm^2	2476 mm^2

Area of Steel

Reinforced section / (limiting section or under reinforced section) -

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck}\, Bd^2}}\right] B.d$$

eg. Size of beam = 400 x 700 mm & designed for 20 kN/m over a span of 7.5 m eff. using 25 / fe 415. Calculate area of steel

L.L = 20 kN/m

Self wt = .4 x .7 x 1 x 25 = 7 kN/m

$w = 1.5 \times 27 = 40.5$ kN/m

$B.M = \frac{40.5 \times 7.5^2}{8} = 284.77$ kN-m

Moment of Resistance of singly Reinforced balanced beam

$M_{u\,lim} = Q\, Bd^2$

$= .138 \times 25 \times 400 \times 650^2 / 10^6$

583.05 kN-m

B.M < $M_{u\,lim}$ — under reinforced section

B.M = M.R

$284.76 \times 10^6 = .36 f_{ck} B \cdot x_u (d - 0.42 x_u)$

$= .36 \times 25 \times 400 \, x_u (650 - 0.42 x_u)$

$0.42 x_u^2 - 650 x_u + 79100 = 0$

$x_u = 133.147$ mm

$A_{st} = \dfrac{B.M}{0.87 f_y (d - 0.42 x_u)}$

$= \dfrac{284.76 \times 10^6}{0.87 \times 415 (650 - 0.42 \times 133.147)}$

$= 1327.60 \text{ mm}^2$

2nd Method Using direct formula we get

$A_{st} = \dfrac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \dfrac{4.6 M_u}{f_{ck} B d^2}} \right] B d$

$= \dfrac{0.5 \times 25}{415} \left[1 - \sqrt{1 - \dfrac{4.6 \times 284.76 \times 10^6}{25 \times 400 \times 650^2}} \right] 400 \times 650$

$= 132.630 \text{ mm}^2$

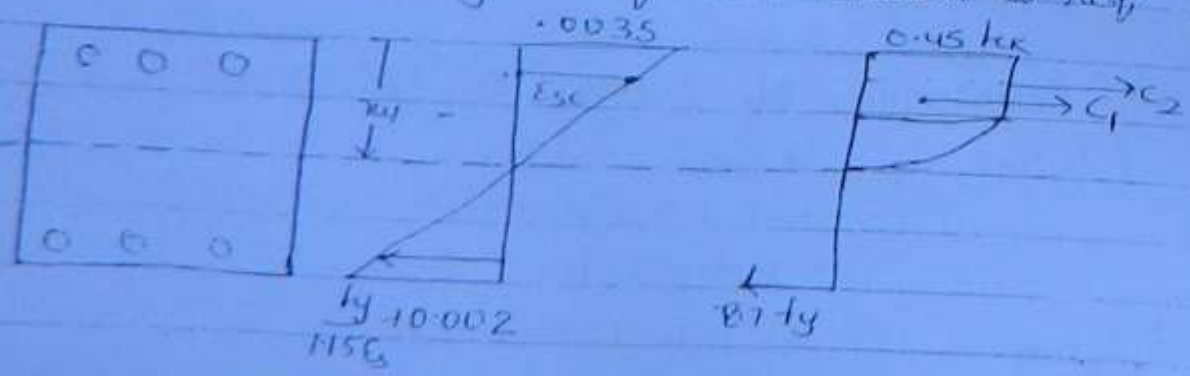
DOUBLY REINFORCED SECTION

If width & depth both are restricted and beam has to sustain B.M higher than moment of resistance of singly reinforced section. Doubly reinforced beam is required.

B and D/d are defined

$MR = QBd^2$ — for singly reinforced section

B.M > MR Doubly reinforced section is req.



Max. Permissible Stress

for Concrete = 0.45 fck

for tensile steel = 0.87 fy

for Comp. Steel

As per assumption IS 456-2000

Stress in reinforcement is derived from Stress-Strain Curve

If strain at Comp. Steel = ε_{sc}

$$\frac{\varepsilon_{sc}}{x_u - d_c} = \frac{0.0035}{x_u}$$

$$\varepsilon_{sc} = \left(\frac{x_u - d_c}{x_u}\right) \times 0.0035$$

As per value of ε_{sc} Stress in Comp. Steel is read from Stress-Strain Curve as per table.

Stress-Strain table

fe 415		fe 500	
Strain (ε_{sc})	Stress (f_{sc})	Strain	Stress
0.00144	288.7 (0.87 fy)	0.00174	347.8
0.00163	306.7	0.00195	369.6
0.00192	324.8	0.00226	391.3
0.00241	342.8	0.00277	413.0
0.00276	351.8	0.00312	423.9
0.00380	360.9	0.00417	434.8

Analysis of doubly reinforced Section

(1) Limiting depth of N.A

$x_{u\,lim}$ = 0.53 d — fe 250

= 0.48 d — fe 415

0.46 d — fe 500

(2) Actual depth of N.A

Equating total force of Comp. to total force of tension

$C = T$

$C_1 + C_2 = T$

Area of concrete not present

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$0.36 f_{ck} B \cdot x_u + A_{sc} \times f_{sc} - A_{sc} \times 0.45 f_{ck} = 0.87 f_y A_{st}$

Stress in conc

(64)

$0.36 f_{ck} B x_u + A_{sc}(f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}$

$x_u < x_{u\,lim}$ It is a under reinforced section

Calculate moment of resistance using x_u

Moment of resistance

$= C_1 \times LA_1 + C_2 \times LA_2$

$M_u = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

(ii) If $x_u = x_{u\,lim}$ ← limiting case

use $x_{u\,lim}$ in place of x_u

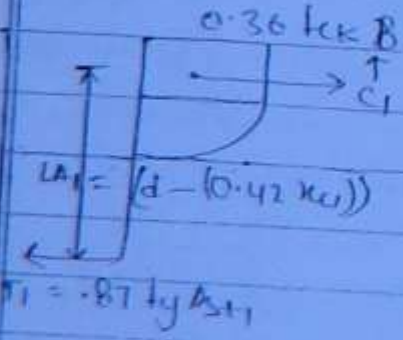
$M_u = 0.36 f_{ck} B \cdot x_{u\,lim} (d - 0.42 x_{u\,lim}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= QBD^2 + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

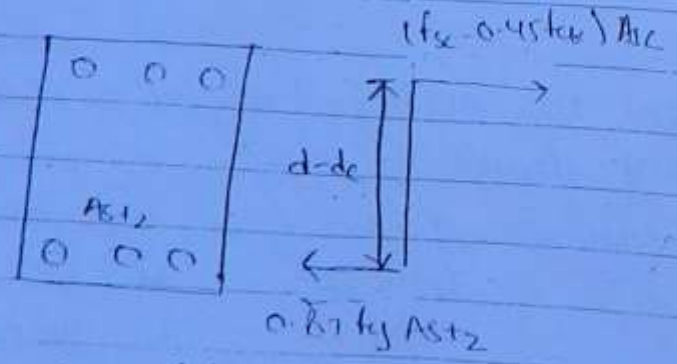
$= MR_1 + MR_2$

(iii) If $x_u > x_{u\,lim}$ use $x_u = x_{u\,lim}$ and moment of resistance shall be limited to M_{u1}

Part I



Part II



$C_1 = 0.36 f_{ck} B \cdot x_u$

$T_1 = 0.87 f_y A_{st1}$

$LA_1 = (d - 0.42 x_u)$

$C_2 = (f_{sc} - 0.45 f_{ck}) \cdot A_{sc}$

$T_2 = 0.87 f_y A_{st2}$

$LA_2 = (d - d_c)$

$M.R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= 0.87 f_y A_{st_1} (d - 0.42 x_u) + 0.7 f_y A_{st_2} (d - d_c)$

$MR_1 = (QBd^2)$ + $(MR_2 = BM - MR_1)$

Design formula

Step① The beam has to be designed for B.M. Calculate M.R of singly reinforced limiting section

$MR_1 = QBd^2 = 0.36 f_{ck} B \cdot x_{u\,lim} (d - 0.42 x_{u\,lim})$

If $B.M > MR_1$ — Then a doubly reinf. Sec^n is required

Step② A_{st1} for MR_1

$$A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{u\,lim})}$$

Step③ Remaining B.M

$MR_2 = B.M - MR_1$

Step④ Calculate A_{st2}

$$A_{st2} = \frac{(MR_2 = BM - MR_1)}{0.87 f_y \cdot (d - d_c)}$$

Step⑤ Find out ε_{sc}

$$\varepsilon_{sc} = \left(\frac{x_{u\,lim} - d_c}{x_{u\,lim}}\right) \times 0.0035$$

Step⑥ A_{sc} for MR_1

$$A_{sc} = \frac{MR_1}{(f_{sc} - 0.45 f_{ck})(d - d_c)}$$

Ques Determine ultimate moment capacity of a doubly reinf Sec^n

$B = 300$ mm ; $D = 600$ mm

$A_{st} = 2060$ mm^2 Eff. Cover = 50 mm

$A_{sc} = 804$ mm^2 M20 / Fe 415

Sol. $M_u = MR_1 + MR_2$

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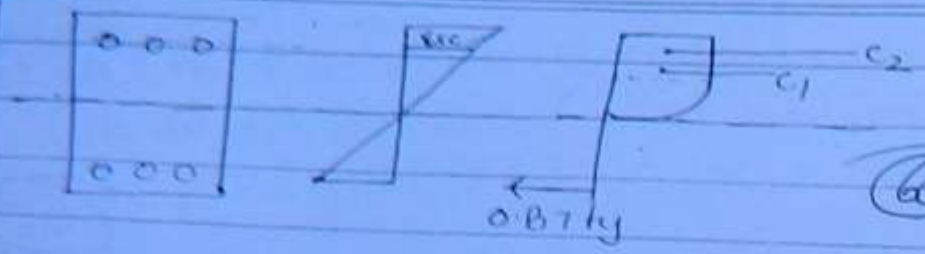

(66)

Limiting depth of N.A

$x_{u\,lim} = 0.48 \times d = 0.48 \times 550 = 264\,mm$

Actual depth of N.A

$C_1 + C_2 = T$

$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$0.36 \times 20 \times 300 \times x_u + (f_{sc} - 0.45 \times 20)\, 804 = 0.87 \times 415 \times 2060$

$2160\, x_u + 804 f_{sc} = 750999$

$2.68\, x_u + f_{sc} = 934.08$ —— (A)

rial (1) Use $f_{sc} = 350\ N/mm^2$

$x_u = 217.94$

$\varepsilon_{sc} = \left(\frac{x_u - d_c}{x_u}\right) \times 0.0035$

$= \left(\frac{217.94 - 50}{217.94}\right) \times 0.0035 = 0.00269$

$f_{sc} = 342 + \frac{351 - 342}{(.00276 - 0.00241)} \times (0.00269 - 0.00241)$

$= 349.46$

$x_u = 217.94$

$x_u < x_{u\,lim}$

So It is under reinforced sec^n

$M.R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

$= 0.36 \times 20 \times 300 \times 217.4 (550 - 0.42 \times 217.4) + (349.46 - 0.45 \times 20) \times 804 \times (550 - 50)]$

$M.R = 191.91 \times 10^6 + 136.86 \times 10^6 = 328.77 \times 10^6$

$M.R = 352.4\ kN\text{-}m$ Ans

The size of a R.C beam is restricted to 250 x 500 mm. It carries a S.I load of 25 kN/m over a span of 6m. Determine the reinf. by L.S.D method M20/Fe 415. Eff. Cover = 40mm.

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Stress	0.8 fyd	.85 fyd	.90 fyd	0.95 fyd	0.975 fyd	1.0 fyd
Strain	0.00144	0.00163	0.00192	0.00241	0.00276	0.00380
(fsc)	288.84	306.89	324.95	343	352.02	361.05

$f_{yd} = .87 f_y$

$= 0.87 \times 415 = 361.05$

Load and B.M

Super imposed load = 25 kN/m

Self wt = $.25 \times .50 \times 1 \times 25 = 3.152$ kN/m

$w = 28.152$ kN/m

factored load = $1.5 \times 28.152 = 42.2$ kN/m

Max. B.M = $\frac{wl^2}{8} = \frac{42.2 \times 6^2}{8}$

M.R of singly reinforced section

$MR_1 = QBd^2$

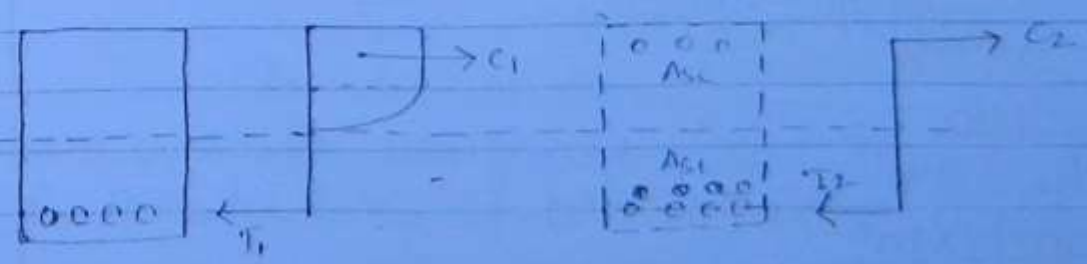
$= 0.36 f_{ck} \cdot B \cdot x_{u\,lim} (d - 0.42\, x_{u\,lim})$

$= 0.138 f_{ck} Bd^2$

$= 0.138 \times 20 \times 250 \times 450^2 \times \frac{1}{10^6}$

$MR_1 = 145.96$ kN-m

Req. $MR > MR_1$ so we have to use doubly reinf. section.



$MR_1 = 145.96$ + $MR_2 = 43.94$

$BM > MR_1$

$MR_2 = B.M - MR_1 = 189.9 - 145.96 = 43.94$ KN-m

Area Of Steel A_{st1} for part 1 (68)

$$A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{u\lim})}$$

$$= \frac{145.96 \times 10^6}{.87 \times 415 (460 - 0.42 \times 0.48 \times 460)}$$

$A_{st1} = 1100.75 \ mm^2$

Area Of Steel for part ② A_{st2}

$$A_{st2} = \frac{MR_2}{0.87 \times 415 \times (460 - 40)}$$

$$= \frac{43.94 \times 10^6}{0.87 \times 415 \times (460 - 40)} = 289.76 \ mm^2$$

Total area Of Steel $A_{st} = A_{st1} + A_{st2}$

$= 1100.75 + 289.76$

$1390.5 \ mm^2$

Value of ε_{sc}

$$\varepsilon_{sc} = \frac{x_{u\lim} - d_c}{x_{u\lim}} \times 0.0035$$

$$= \frac{0.48 \times 460 - 40}{.48 \times 460} \times 0.0035$$

$\varepsilon_{sc} = 0.00286$

$$f_{sc} = 352 + \frac{361 - 352}{(.00380 - .00276)} (.00286 - .00276)$$

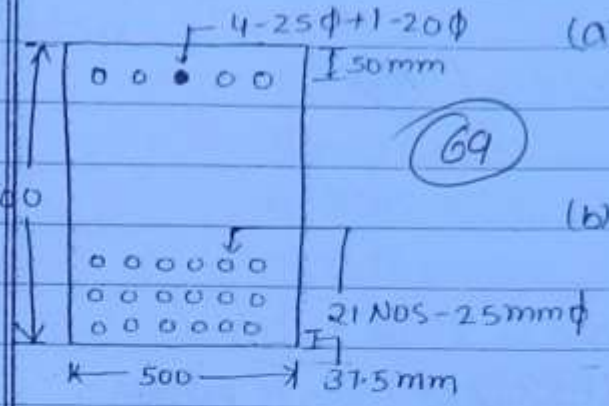
$f_{sc} = 352.8$

$$A_{sc} = \frac{MR_2}{(f_{sc} - 0.45 f_{ck})(d - d_c)}$$

$$= \frac{43.94 \times 10^6}{(352.8 - 0.45 \times 20) \times (460 - 40)}$$

$A_{sc} = 304.94 \ mm^2$

Ques A simply supported 18m effective span R.C.C rectangular beam 500x1500mm. M25/Fe-415



(a) Max. B.M at limit state of collapse

Stress in comp. rein - 0.9566 fyd

(b) What can be the super imposed load that can be applied over beam at working conditions.

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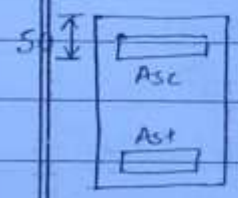
Soln $M_u = MR_1 + MR_2$



$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 + \frac{\pi}{4} \times 1 \times 20^2 = 2277 \, mm^2$

$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308.35 \, mm^2$

Eff. Cover on tension side

$= 37.5 + 25 + 25 + \frac{25}{2} = 100 \, mm$

$B = 500 \, mm$

$d = 1400 \, mm$

$A_{sc} = 2277 \, mm^2$

$A_{st} = 10308 \, mm^2$

① Limiting depth of $x_u = 0.48 \times 1400 = 672 \, mm$

② Actual depth of N.A $C_1 + C_2 = T$

$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$0.36 \times 25 \times 500 \times x_u + (345.20 - 0.45 \times 25) \times 2277 = 0.87 \times 415 \times 10308$

$x_u = 658.06 \, mm$

Beam is under reinforced ($x_u < x_{u\,lim}$)

$MR = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$

$= 0.36 \times 25 \times 500 \times 658.16 (1400 - 0.42 \times 658.16) + (345.20 - 0.45 \times 25) \times 2277 \times (1400 - 50)$

$M_u = 4354.25$ KN-m

(4) $B.M = \frac{W_u L^2}{8}$

(73)

$= \frac{8 \times 4354.25}{18^2} = W_u$

$W_u = 107.50$ KN/m

Service load / unfactored load = 71.66 KN/m /
deduct Self wt = $0.50 \times 1.5 \times 1 \times 25 = 18.75$ KN/m

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Service load that can be applied over beam = 52.91 KN/m.

Ques - A SSB of 5m eff. span is subjected to 24 KN/m live load $f_{ck} = 20$ N/mm², $f_y = 415$ N/mm². The overall depth of beam is 400 & width is 250 mm. Design the reinf^n of the beam $K = 0.138$; $j = 0.80$

Sol^n By limit state method we can done this Question

$j = \left(1 - 0.42 \frac{x_{umax}}{d}\right) = \left(1 - 0.42 \times 0.48\right)$
$= .7984 = $ Say $.80$

$Q = K.f_{ck}$

$Q = .138 \times 20 = 2.76$

1. Load & B.M

Live load = 24 KN/m

Self wt = $.4 \times .25 \times 1 \times 25 = 2.5$ KN/m

Total load $W = 26.5$ KN/m

$B.M = \frac{Wl^2}{8} = \frac{26 \times 1.5 \times 5 \times 5}{8}$

$= 124.22$ KN-m

MR of Singly reinforced limiting section

$MR_1 = Q.B.d^2$

$= .138 \times 20 \times 250 \times 350 \times 350 = 84.525$ KN-m

B.M > MR_1 So we have a doubly reinforced sections

3. Area of steel (A_{st})

(a) $A_{st1} = \dfrac{MR_1}{.87 f_y (d - 0.42 x_{u\,lim})}$

(71)

$= \dfrac{84.525 \times 10^6}{.87 \times 415 (350 - 0.42 \times 0.48 \times 350)} = \dfrac{84.525 \times 10^6}{100891.812}$

$= 837.78 \, mm^2$

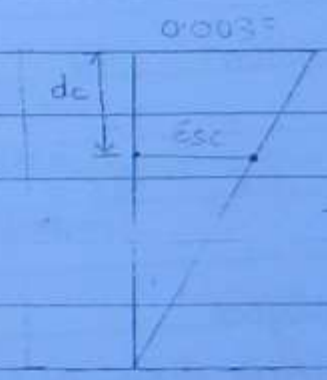
(b) A_{st2}

$A_{st2} = \dfrac{BM - MR_1}{.87 f_y (d - d_c)}$

$= \dfrac{39.695 \times 10^6}{.87 \times 415 (350 - 50)} = 366.48 \, mm^2$

Total $A_{st} = A_{st1} + A_{st2} = 837.78 + 366.48 = 1204.26 \, mm^2$

(c) $A_{sc} = \dfrac{B.M - MR_1}{(f_{sc} - 0.45 f_{ck}) \times (d - d_c)}$



value of f_{sc} depends upon ε_{sc}

$\varepsilon_{sc} = \dfrac{x_{u\,lim} - d_c}{x_{u\,lim}} \times 0.0035$

$= \dfrac{0.48 \times 350 - 50}{.48 \times 350} \times 0.0035 = 0.00246$

f_{sc} should be read for above values

Table is not given, So assuming $f_{sc} = 350 \, N/mm^2$

$A_{sc} = \dfrac{39.695 \times 10^6}{(350 - 0.45 \times 20)(350 - 50)}$

$= 388.02 \, mm^2$

T-BEAM / L-BEAM

DATE: 3/10/2011

Effective width of flange

1. Isolated T-beam / L-beam

T-beam $B_f = \dfrac{l_0}{\left(\frac{l_0}{B}\right)+4} + b_w$ (72)

L-beam $B_f = \dfrac{0.5 l_0}{\left(\frac{l_0}{B}\right)+4} + b_w$

2. Slab + Beam type

1. T-beam $B_f = \dfrac{l_0}{6} + b_w + 6d_f$

B_f = centre to centre betn adjacent slab $= b_w + \dfrac{l_1}{2} + \dfrac{l_2}{2}$

} Whichever is smaller

L-beam $B_f = \dfrac{l_0}{12} + b_w + 3d_f$

OR $B_f = b_w + \dfrac{l_1}{2}$

} Whichever is smaller

Analysis

1 Limiting depth of N.A

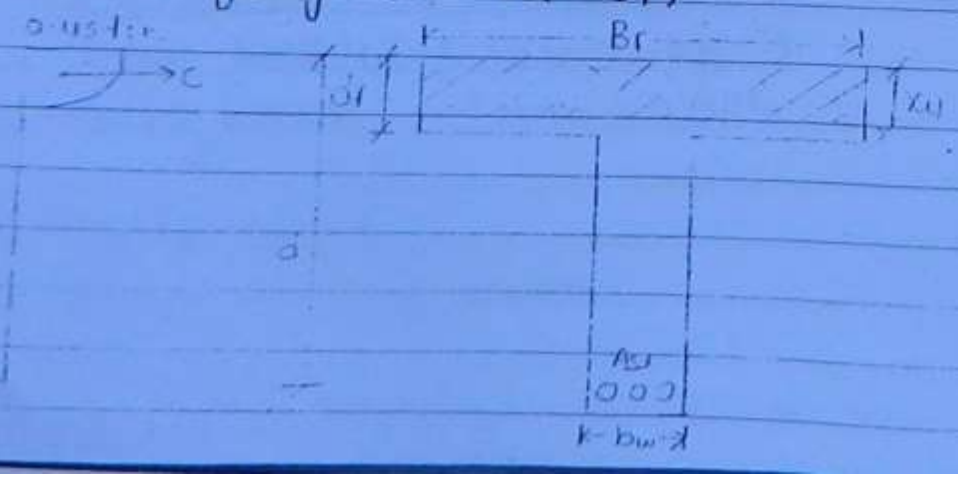
$x_{u\,lim} = 0.53d$ — fe 250

$0.48d$ — fe 415

$0.46d$ — fe 500

2. Actual depth of N.A & Moment of Resistance

CASE I When N.A lie within flange (i.e $x_u < d_f$)



$F_{ts} = 0.87 f_y A_{st}$ $\quad F_{tc} < F_{ts} \rightarrow$ will [illegible] the function

$F_{tc} = F_{ts} = N.A$ $\quad$ PAGE NO. DATE

$\overset{\shortparallel}{d_f}$

It is similar case as of a Rectangular section of $(B_f \times d)$

$C = 0.36 f_{ck} B_f \cdot x_u$

$T = 0.87 f_y A_{st}$

$L.A = (d - 0.42 x_u)$ $\quad$ (73)

Actual depth of N.A

$C = T$

$0.36 f_{ck} B_f \cdot x_u = 0.87 f_y A_{st}$

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 \cdot f_{ck} \cdot B_f}$$

Moment of Resistance

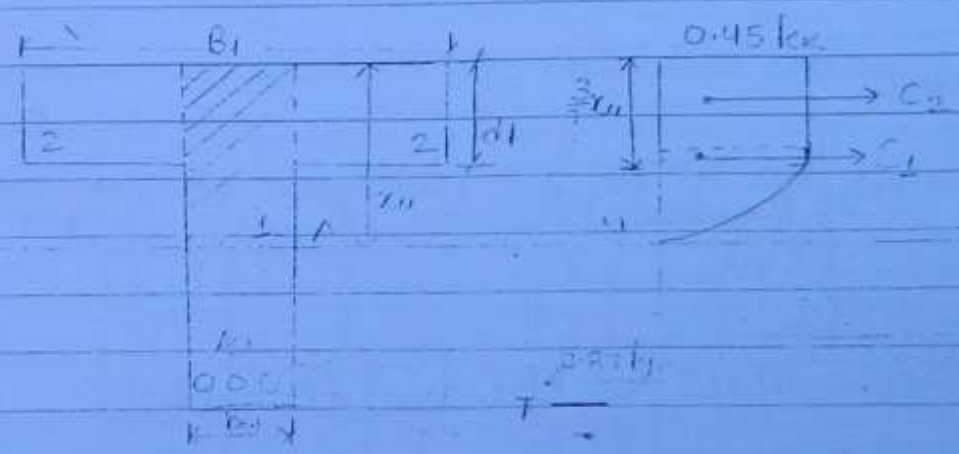
$MR = C \cdot L.A$ or $T \cdot LA$

$MR = 0.36 f_{ck} B_f \cdot x_u (d - 0.42 x_u)$ or $0.87 f_y A_{st} (d - 0.42 x_u)$

CASE II When N.A lie within web area

SUB: $x_u > d_f$ (N.A is within web area)

CASE I & $d_f < \frac{3}{7} x_u$ (Depth of flange is within rectangular portion of stress diagram)



Actual depth of N.A

$C_1 + C_2 = T$

$0.36 f_{ck} b_w \cdot x_u + (B_f - b_w) d_f \, 0.45 f_{ck} = 0.87 f_y \cdot A_{st}$

$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$

Find $x_u = ?$ $\quad$ — (A)

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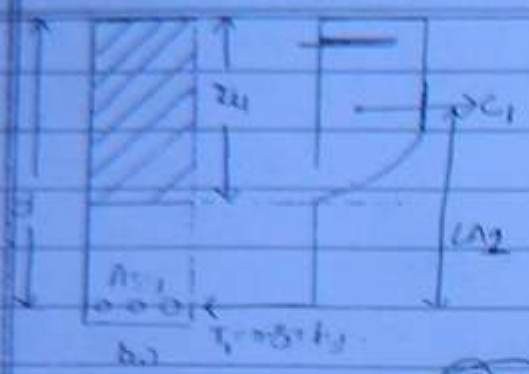
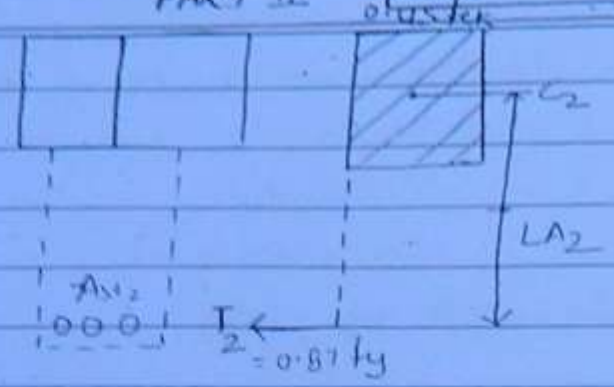
PART I



PART II



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$C_1 = 0.36 f_{ck} \cdot b_w \cdot x_u$

$T_1 = 0.87 f_y \cdot A_{st_1}$

$LA_1 = (d - 0.42 x_u)$

$C_1 = 0.45 f_{ck} (B_f - b_w) d_f$

$T_2 = 0.87 f_y A_{st_2}$

$LA_2 = \left(d - \frac{d_f}{2}\right)$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

Moment of Resistance.

m Compression side $MR = C_1 \cdot LA_1 + C_2 \cdot LA_2$

$MR = 0.36 f_{ck} \cdot b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

m tension side $MR = T_1 \cdot LA_1 + T_2 \cdot LA_2$

$MR = .87 f_y \cdot A_{st_1} (d - 0.42 x_u) + 0.87 f_y A_{st_2} \left(d - \frac{d_f}{2}\right)$

Design formula

$$C_1 = T_1$$

$$A_{st_1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{.87 f_y} \quad \text{--- } 1$$

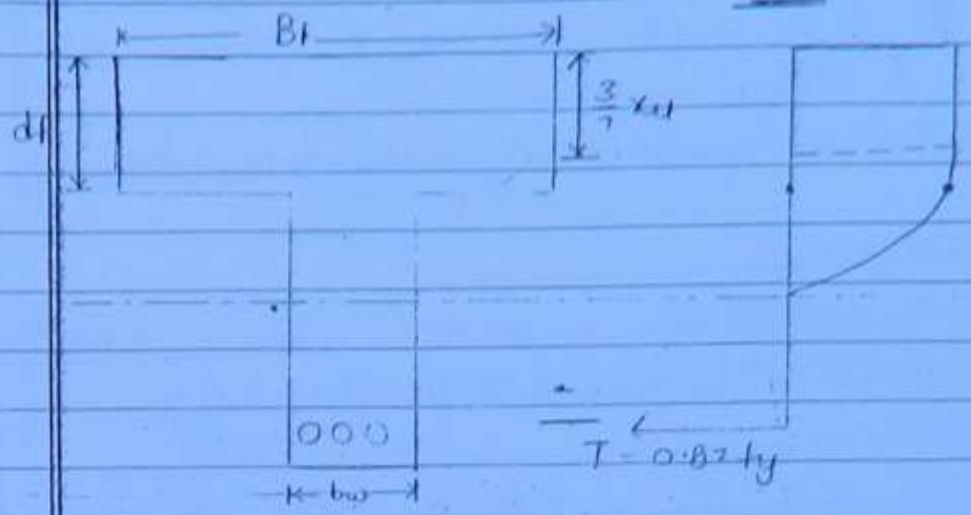
again $C_2 = T_2$

we get $A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w) d_f}{.87 f_y} \quad \text{--- } 2$

CASE II If $x_u > D_f$ (N.A is within web area)

but $d_f > \frac{3}{7} x_u$ (Depth of flange is more than the rectangular portion of stress diagram)

As per IS: 456:2000 ANNEX G P.g No-97

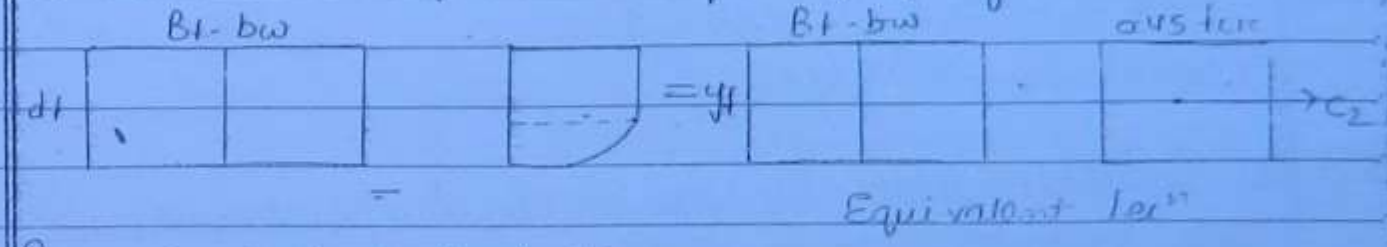


75

For flange area

Stress diagram is partly rectangular & partially parabolic.

In this case an equivalent depth of flange is considered

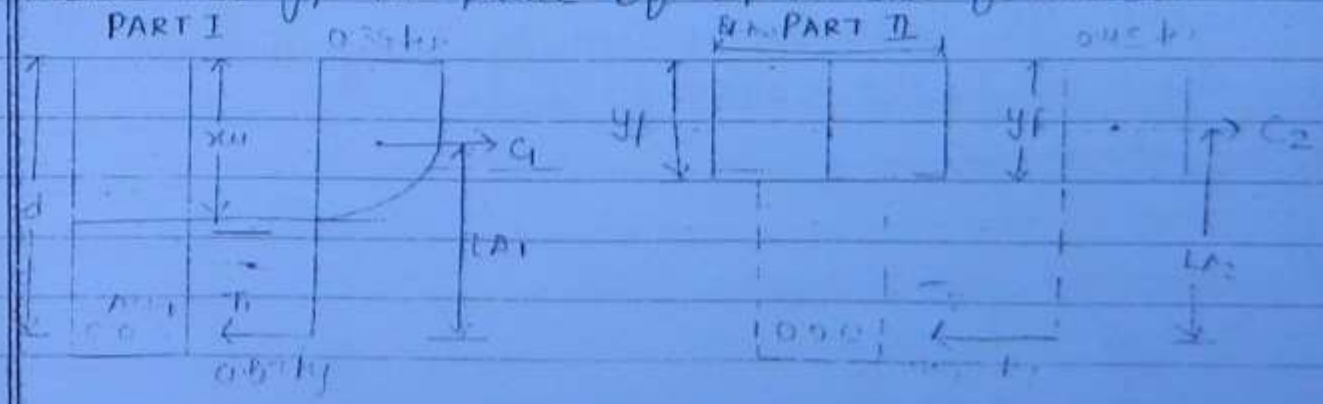


Depth of equivalent section

$$y_f = 0.15 x_u + 0.65 d_f \quad \text{— A}$$

Stress in overall depth of equivalent section is considered 0.45 fck

Now use y_f in place of d_f in all formula.



$C_1 = 0.36 f_{ck} b_w \cdot x_u$

$T_1 = 0.87 f_y A_{st1}$

$L.A_1 = (d - 0.42 x_u)$

$C_2 = 0.45 f_{ck} (B_f - b_w) y_f$

$T_2 = 0.87 f_y A_{st2}$

$LA_2 = (d - y_f/2)$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

$$0.36\, f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) y_f = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$$

Moment of Resistance

$$MR = C_1 \times LA_1 + C_2 \times LA_2$$

$$= 0.36 f_{ck}\, x_u \cdot b_w (d - 0.42\, x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d - y_f/2)$$

Design formula

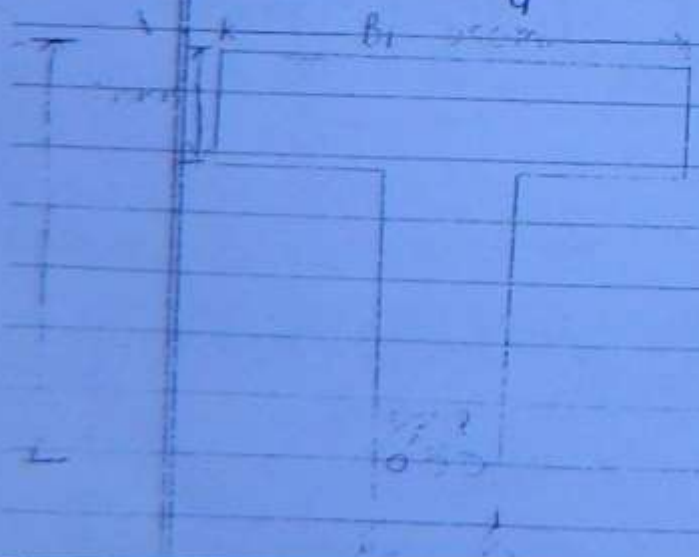
$$A_{st1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{0.87 f_y}$$

$$A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$$

Ques- An Isolated T-beam of span 6.0 m simply supported
Width of ~~beam~~ flange $= B_f = 1500$ mm
Overall depth $D = 800$ mm
Effective Cover = 50 mm
Area of steel = 5-25 ϕ
Width of web = 300 mm
Calculate MR of section if depth of flange is 100 mm

Use M-25
Fe415

Sol^n $A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37\ mm^2$



DATE:

1 Eff. width of flange $B_f = \dfrac{l_0}{\left(\frac{l_0}{B}\right)+4} + b_w$

$$B_f = \frac{6000}{\left(\frac{6000}{1500}\right)+4} + 300 = 1050\ mm$$

(77)

2 Given $b_w = 300mm$; $D = 800mm$; $d = 750mm$

$d_f = 100\ mm$

3 Limiting depth of N.A

$x_{u\,lim} = 0.48 \times d = 0.48 \times 750 = 360mm$

4 Actual depth of Neutral axis

CASE 1 - Assume that N.A lie within flange ($x_u < d_f$)

$0.36\, f_{ck}\, B_f \cdot x_u = 0.87\, f_y\, A_{st}$

$$x_u = \frac{0.87 \times 415 \times 2454.37}{0.36 \times 25 \times 1050} = 93.77\ mm$$

So assumption is correct

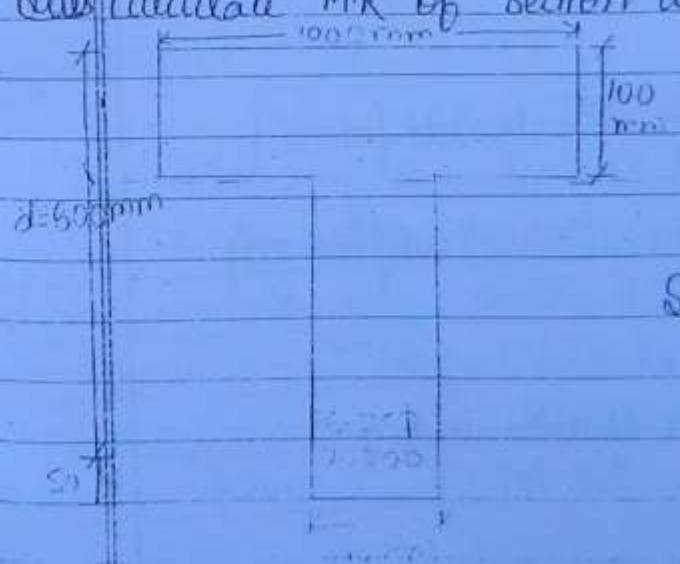
{ $x_u < x_{u\,lim}$ under reinforced section

$MR = 0.36 f_{ck}\, x_u \cdot B_f \times (d - 0.42\, x_u)$

$= 0.36 \times 25 \times 93.77 \times 1050\ (750 - 0.42 \times 93.77)$

$= 629.70\ KN\text{-}m$

Ques Calculate M.R of section as shown in fig.



Span of beam = 5.0 m

Dist beam point of zero moment in the beam = 4.0 m

Soln Eff. width of flange

$$B_f = \frac{l_0}{\frac{l_0}{B}+4} + b_w$$

$$= \frac{5000}{\left(\frac{5000}{1000}\right)+4} + 300$$

$= 800mm$

Limiting depth of N.A

$x_{u\,lim} = 0.42 \times d = 0.42 \times 800 = 384\,mm$

Actual depth of N.A

78

1 Assuming $x_u < d_f$

$= 0.36\, f_{ck} \cdot x_u \cdot B_f = 0.87\, f_y \cdot A_{st}$

$$x_u = \frac{0.87 \times 415 \times 6 \times \pi/4 \times 25^2}{0.36 \times 20 \times 800}$$

$= 184.61\ mm$

this assumption is not Correct

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$

using d_f value we get

$= 0.36\, f_{ck} \cdot b_w \cdot x_u + 0.45\, f_{ck} (B_f - b_w) d_f = 0.87\, f_y A_{st}$

$= 0.36 \times 20 \times 300 \cdot x_u + 0.45 \times 20 (800-300) 100 = 0.87 \times 415 \times \frac{\pi}{4} \times 25^2 \times 6$

$$x_u = \frac{613380.0259}{.36 \times 20 \times 300}$$

$x_u = 283.97\ mm$

$d_f < \frac{3}{7} \times 283.97$

$d_f < 121.70$ Its OK

$x_u < x_{u\,lim}$
x_u is used
(under reinforced Section)

So assumption is Correct

$MR = 0.36\, f_{ck} \cdot b_w \cdot x_u (d - 0.42 x_u) + 0.45\, f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

680.73 $\times 0.36 \times 20 \times 300 \times 283.97 (800 - 0.42 \times 283.97) + 0.45 \times 20 (500) 100 \left(800 - \frac{100}{2}\right)$

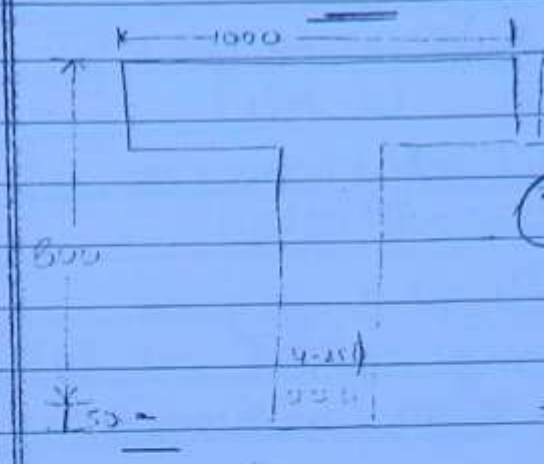
MR $= 755.04\ kN\text{-}m$

Ques- Calculate the MR of the section shown in fig.

use M 20 / fe 415

Span = 5m

$l_0 = 4m$



$$B_f = \frac{4000}{\frac{4000}{1000}+4} + 300 = 800\,mm$$

(79) $x_{u\,lim} = 0.42 \times d = 384\,mm$

Actual depth of N.A

1. Assuming $x_u < d_f$

$= 0.36\, f_{ck}\, x_u \cdot B = 0.87\, f_y \cdot A_{st}$

$$x_u = \frac{0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2}{0.36 \times 20 \times 800}$$

$x_u = 123.08$ c is > than 100

So assumption is incorrect.

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$

$= 0.36\, f_{ck}\, b_w \cdot x_u + 0.45\, f_{ck}\,(B_f - b_w)\, d_f = 0.87\, f_y\, A_{st}$

$x_u = 119.87\,mm$ → this is correct

$\frac{3}{7} x_u = 51.37\,mm$

$d_f > 51.37$ So assumption is incorrect

Now we know that $d_f > \frac{3}{7} x_u$ & also $x_u > d_f$

use $y_f = 0.15\, x_u + 0.65\, d_f = 0.15\, x_u + 65$

~~$y_f = 0.15 \times 119.87 + 65 = 82.98$~~

Actual depth of N.A

$C_1 + C_2 = T_1 + T_2$

$0.36\, f_{ck}\, b_w \cdot x_u + 0.45\, f_{ck}\,(B_f - b_w)\, y_f = 0.87\, f_y\, A_{st}$

$0.36 \times 20 \times 300 \times x_u + 0.45 \times 20\,(800 - 300)\,(0.15\, x_u + 65) = 0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2$

$2160\, x_u + 675\, x_u + 292500 = 708920.0172$

$x_u = 146.88\,mm > d_f$ OK

$\frac{3}{7} x_u = \frac{3}{7} \times 146.88 = 62.95 < d_f\ (100\,mm)$ OK

$y_f = 0.15 \times 146.88 + 65 = 87.032\,mm$

DATE:

$MR = 0.36\, f_{ck}\, x_u \cdot b_w (d - 0.42 x_u) + 0.45\, f_{ck} (B_f - b_w) y_f (d - y_f/2)$

$= 0.36 \times 20 \times 146.88 \times 300 (800 - 0.42 \times 146.88) + 0.45 \times 20 (800 - 300) 87.032 (800 - 87.032/2)$

$= 530.51$ KN-m

80

Ques In design section must be given or we can assume the section as balanced section.

Method- Step① If $x_u = d_f$, Calc MR of the section

$R = 0.36 f_{ck} \cdot B_f \cdot x_u (d - 0.42 x_u)$

$M_u = 0.36 f_{ck} B_f \cdot x_u (d - 0.42 d_f)$

If B.M < M_{u1}; the value of x_u is $0 < x_u < d_f$

equating B.M = MR

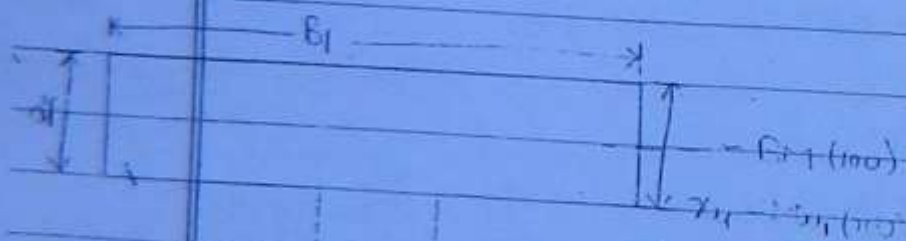
$= 0.36 f_{ck} \cdot B_f x_u (d - 0.42 x_u)$

find out $x_u = ?$

Calculate area of steel $A_{st} = \dfrac{BM}{0.87 f_y (d - 0.42 x_u)}$

Given - Size of beam B, B_f, b_w, d_f, d, D
Load or B.M
design B.M = B.M
Grade of Concrete & Steel
find out area of steel for req. B.M.

Step② If B.M > M_{u1} then $x_u = \frac{7}{3} d_f$ & Calculate MR for this is equal to M_{u2}



It is case 2nd So d_f is use

$d_f = \frac{3}{7} x_u$

$\overline{M.R} = Mu_2 = 0.36\, f_{ck}\, b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

Calculate Mu_{lim} $\boxed{\text{assuming } x_{u\,lim} > \frac{7}{3} d_f}$

(81)

So $d_f < \frac{3}{7} x_{u\,lim}$ use d_f formula

Calculate Mu_{lim} for $x_{u\,lim}$

$Mu_{lim} = 0.36\, f_{ck}\, b_w \cdot x_{u\,lim} (d - 0.42 x_{u\,lim}) + 0.45 f_{ck} (B_f - b_w) d_f (d - d_f/2)$

If B.M is less case 1 Mu_1 & Cas 2 Mu_2 then x_u is less x_u & $\frac{7}{3} d_f$ so $x_u < \frac{7}{3} d_f$

i.e $d_f > \frac{3}{7} x_u$ So use y_f formula.

If B.M is less Mu_2 & Mu_{lim} then x_u is less $\frac{7}{3} d_f$ & $x_{u\,lim}$ $x_u > \frac{7}{3} d_f$ $\boxed{\text{i.e } d_f < \frac{3}{7} x_u}$

So d_f will be Consider

p4 If B. $Mu_1 < BM < Mu_2$ $\longrightarrow$ $x_u < 7/3\, d_f$

So $d_f > \frac{3}{7} x_u$ So it is a y_f formula.

Equating B.M = MR

$B.M = 0.36 f_{ck}\, b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d_0 - y_f/2)$

$y_f = 0.15 x_u + 0.65 d_f$

Get x_u = ?

Now find Out Area Of Steel

$A_{st1} = \frac{0.36 f_{ck} \cdot b_w x_u}{0.87 f_y}$; $A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$

If $Mu_2 < B.M < Mu_{lim}$ $\longrightarrow$ $x_u > 7/3\, d_f$ $\boxed{\text{i.e } d_f < \frac{3}{7} x_u}$

So Df formula is used

Equating B.M = MR = $0.36 f_{ck}\, b_w\, x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f (d - d_f/2)$

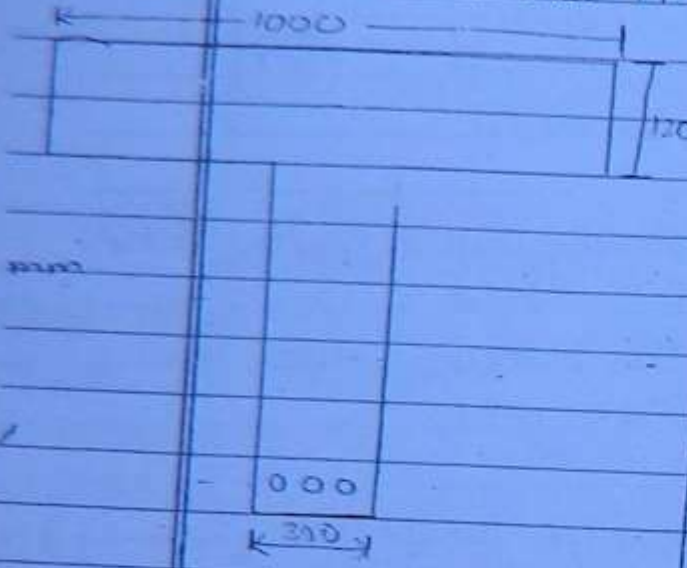
Get x_u = ?

& also find Area Of Steel A_{st1} & A_{st2}

DATE:

Ques- Design a T-beam of following cross-section. For bending moment due to service load ① 320 KN-m ② 430 KN-m. Use M-20 / fe-415

(82)



Given $B = 1000\ mm$

Span = 7.5 m

$l_0 = 0.8$ times Span $= 0.8 \times 7.5 = 6m$

$b_w = 300mm$

$d_f = 120mm$

$d = 600mm$

Effective width $B_f = \frac{6000}{\frac{6000}{1000} + 4} + 300 = 900\ mm$

$x_{u\,lim} = .48 \times 600 = 288mm$

Design B.M

① $BM_1 = 1.5 \times 320 = 480\ KN\text{-}m$

$BM_2 = 1.5 \times 430 = 645\ KN\text{-}m$

Step I When $x_u = d_f$ $\quad x_u = 120mm$

$MR = M_{u1} = .36\, f_{ck}\, B_f \cdot x_u (d - 0.42 x_u)$

$M_{u1} = .36 \times 20 \times 900 \times 120 (600 - 0.42 \times 120)$

$M_{u1} = 427.36$

$M_{u1} < BM_1$ So this is incorrect.

II When $x_u = \frac{7}{3} d_f = 280\ mm$

$d_f < 3/7\, x_u = d_f = 120 < \frac{3}{7} x_u = 120\ mm$

So use d_f formula

$M_{u2} = 0.36\, f_{ck}\, b_w \cdot x_u (d - 0.42 x_u) + 0.45\, f_{ck} (B_f - b_w) d_f (d - d_f/2$

$M_{u_2} = 0.36 \times 20 \times 300 \times 280 (600 - 0.42 \times 280) + 0.45 \times 20 (900-300)$
$120 (600 - 120/2)$

$= 641.67$ KN-m

$M_{u_2} < BM_1$

$M_{u_2} < BM_2$

83

III When $x_u = x_{u\,lim} = 288mm$

$\frac{3}{7} x_u = \frac{3}{7} \times 288 = 123.43mm$

$d_f < \frac{3}{7} x_u$ d_f formula is use

$M_{u\,lim} = 0.36 \times 20 \times 300 \times 288 (600 - 0.42 \times 288) + 0.45 \times 20$
$(900 - 300) \times 120 (600 - 120/2) \times 1/10^6$

$= 647.9$ KN-m

(1) for BM_1 it is less M_{u_1} & M_{u_2} So Calculate x_{u_1}

So $x_u < \frac{7}{3} d_f$ So $d_f > \frac{3}{7} x_u$ (y_f formula)

(2) for BM_2 it is less M_{u_2} & $M_{u\,lim}$ & Calculate x_{u_2}

$x_u > \frac{7}{3} d_f$ So $d_f < \frac{3}{7} x_u$ (d_f formula)

for (1) $B.M_1 = MR$

$BM_1 = 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f$
$(d - y_f/2)$

$y_f = 0.15 x_u + 0.65 d_f$

$480 \times 10^6 = 0.36 \times 20 \times 300 \times x_u (600 - 0.42 x_u) + 0.45 \times 20 (900-300)$
$(0.15 x_u + 0.65 \times 120) (600 - .075 x_u - 39)$

$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (.15 x_u + 78)(561 - 0.075 x_u)$

$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (.15 \times 561 x_u - .15 \times .075 x_u^2 + 43758 - 5.85 x_u)$

$129600 x_u - 907.2 x_u^2 + 454410 x_u - 60.75 x_u^2$
$+ 236293200 - 31590 x_u - 480 \times 10^6 = 0$

$-967.95 x_u^2 + 552420 x_u - 243706800 = 0$

$x_u = 155.38$

$y_f = 0.15 \times 155.38 + 18$

$= 101.307 \text{ mm}$

(84)

$A_{st1} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y} = \frac{.36 \times 20 \times 300 \times 155.38}{.87 \times 415}$

$A_{st1} = 929.57 \text{ mm}^2$

$A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y} = \frac{0.45 \times 20 (900-300) 101.307}{.87 \times 415}$

$A_{st2} = 1515.185 \text{ mm}^2$

For BM_2 (df formula)

$BM_2 = MR$

$= .36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2}\right)$

$= .36 \times 20 \times 300 \, x_u \times 600 - .36 \times 20 \times 300 \times 0.42 x_u^2 + 0.45 \times 20 (900-300) 120 \left(600 - \frac{120}{2}\right)$

$545 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 349920000 = 0$

$-907.2 x_u^2 + 1296000 x_u + 349920000 = 0$

~~$x_u = 232.24$~~

~~$A_{st1} = \frac{0.36 \times 20 \times 300 \times 232.24}{.87 \times 415}$~~

~~$A_{st2} = .45 \times 20 (900-300)$~~

$-907.2 x_u^2 + 1296000 x_u - 2950 8 \times 10^4 = 0$

$x_u = \frac{-1296000 \pm \sqrt{(1296000)^2 - 4(907.2)(2950 8 \times 10^4)}}{-2 \times 907.2}$

$= \frac{-1296000 \pm 780215.3975}{-1814.4}$

$= 284.24 \text{ mm}$

$A_{st1} = \frac{0.36 \times 20 \times 300 \times 284.24}{.87 \times 415}$

$= 1700.48 \text{ mm}^2$

$A_{st2} = \frac{.45 \times 20 (900-300) \times 120}{.87 \times 415} = 1794.77 \text{ mm}^2$

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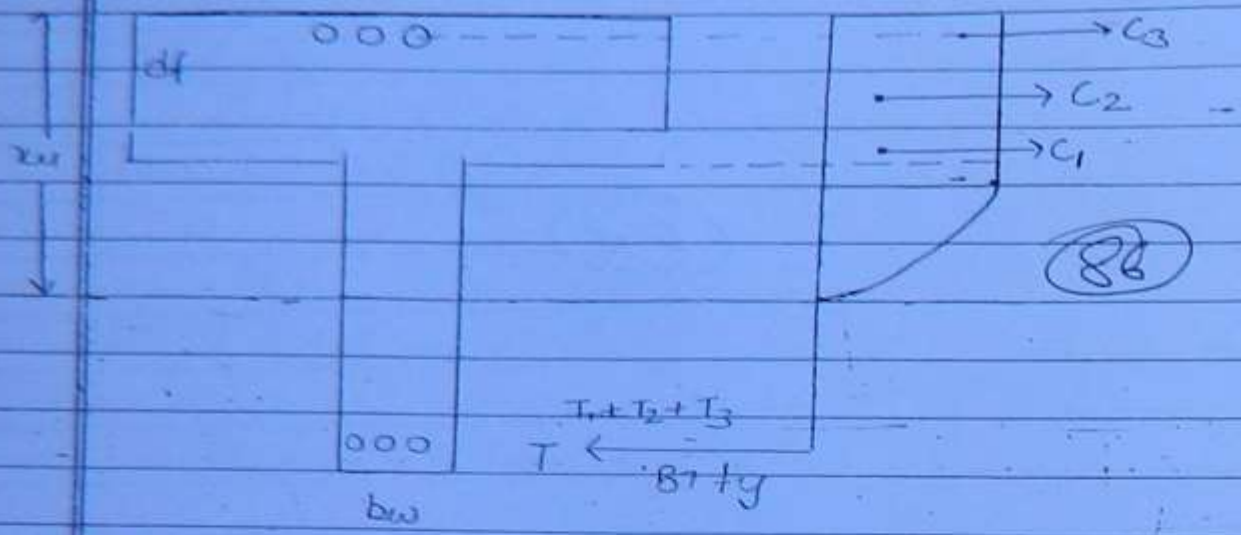
Doubly reinforced T-beam



When $x_u > d_f$

① $d_f < \frac{3}{7} x_u$ – d_f formula

② $d_f > \frac{3}{7} x_u$ – y_f formula

$y_f = 0.15 x_u + 0.65 d_f$

For actual depth of N.A

$C_1 + C_2 + C_3 = T_1 + T_2 + T_3$

$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w)(d_f \text{ or } y_f) + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st_1} + 0.87 f_y A_{st_2} + 0.87 f_y A_{st_2}$

Moment of Resistance

$MR = C_1 \times LA_1 + C_2 \times LA_2 + C_3 \times LA_3$

$= 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w)(d_f / y_f)\left(d - \frac{y_f \text{ or } d_f}{2}\right) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$

Area of Steel

$A_{st_1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{0.87 f_y}$

$A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w)(y_f \text{ or } d_f)}{0.87 f_y}$

$A_{st_3} = \frac{(f_{sc} - 0.45 f_{ck}) \cdot A_{sc}}{0.87 f_y}$

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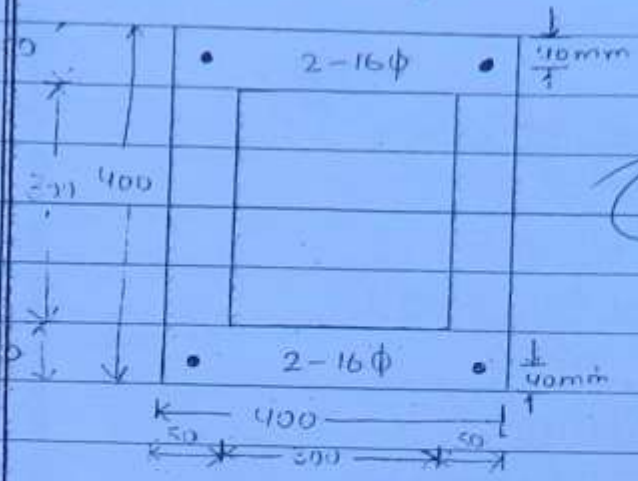
005 Doubly reinforced T-beam by working stress Method.

Ques Find the M.R of the hollow box section



Use M-20 | fe-415

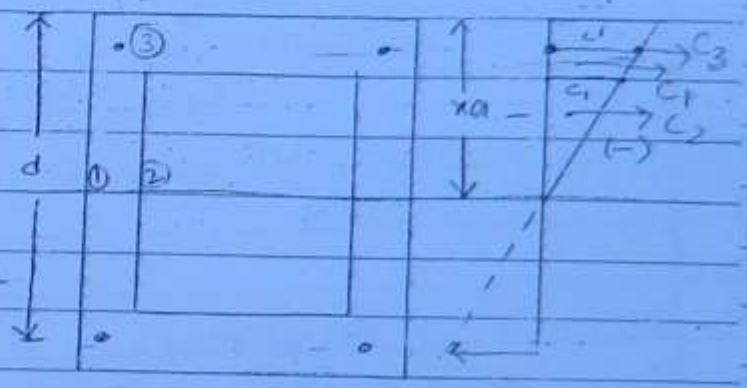
$\sigma_{cbc} = 7$

$\sigma_{st} = 230$

$m = 13.33$ in tension

$= 20$ in Compression

(87)



Sol^n $d = 400 - 40 = 360\,mm$

$A_{st} = A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12\,mm^2$

$x_c = \frac{m_c}{t + m_c} \times d = \frac{13.33 \times 7}{230 + 13.33 \times 7} \times 360$

$x_c = 103.9\,mm$

Actual depth of N.A - Assuming $x_a > d_f = 50\,mm$

Equating moment of area on both side of N.A

$B \times x_a \cdot \frac{x_a}{2}$

$400 \times x_a \cdot \frac{x_a}{2} - 300 \times \frac{(x_a - 50)^2}{2} + (1.5m - 1) \times A_{sc} (x_a - 40) = m A_{st} (d - x_a)$

$200 x_a^2 - 150 (x_a^2 - 100 x_a + 2500) + (1.5 \times 13.33 - 1) \times 402.12 (x_a - 40) = 13.33 \times 402.12 (360 - x_a)$

$1.333 x_a^2 - x_a^2 + 100 x_a - 2500 + 50.92 - 2036.87 + 35.73 x_a - 12864.62 = 0$

$x_a = 81.39\,mm > 50\,mm$

assumption is correct

Moment of Resistance $= C_1 LA_1 + C_2 LA_2 + C_3 LA_3$

MR $= C_1 \times LA_1 = 400 x_a$

DATE:

Stress – $x_a < x_c \rightarrow$ It is under-reinforced

$C_a < \sigma_{cbc}$

$t = \sigma_{st} = 230 N/mm^2$

(88)

$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} = \frac{C_a}{81.39} = \frac{230/13.33}{360-81.39}$

$C_a = 5.04 N/mm^2$

Lower $\frac{C_1}{x_a - 50} = \frac{C_a}{x_a} \Rightarrow C_1 = \frac{5.04(81.39-50)}{81.39}$

$C_1 = 1.94 N/mm^2$

Steel $C' = \frac{81.39-40}{81.39} \times 5.04 = 2.56 N/mm^2$

Moment Of Resistance

$C_1 = 400 \times x_a \times \frac{C_a}{2} = 400 \times 81.39 \times \frac{5.04}{2} = 82041.32 N$

$y_1 = \frac{x_a}{3} = \frac{81.39}{3} = 27.13$

$LA_1 = d - y_1 = 360 - 27.13 = 332.87$

$MR_1 = 82041.32 \times 332.87 \times \frac{1}{10^6}$

$= (+) 27.31 kN-m$

(-ve) $C_2 = 300 \times (x_a - 50) \times \frac{C_1}{2} = 300 \times (81.39-50) \times \frac{1.94}{2} = 9134.49$

$y_2 = \frac{x_a - 50}{3} + 50 = 10.46 + 5 = 60.46$

$LA_2 = d - y_2 = 360 - 60.46 = 299.54$

$MR_2 = (-) 9134.49 \times 299.54 \times \frac{1}{10^6} = (-) 2.74 kN-m$

$C_3 = (1.5m - 1) A_{sc} \times C'$

$(1.5 \times 13.33 - 1)(402.12) \times 2.56$

$= 19553.97 N$

$LA_3 = y_3 = 40$

$LA_3 = 360 - 40 = 320$ mm

$MR_3 = 1955.3 \times 320 \times \frac{1}{10^6} = 6.26$ kN-m

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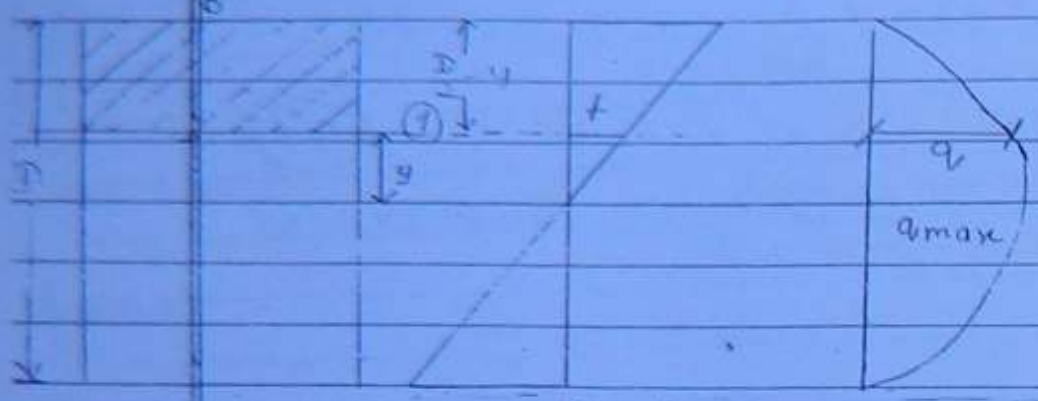
Total $MR = MR_1 + MR_2 + MR_3$

$= 27.31 - 2.74 + 6.26 = 30.83$ kN-m

SHEAR

Shear Stress in a homogenous Section



Bending Section

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \rightarrow \text{Bending}$$

bending stress $= f = \frac{M}{I} y$

Shear Stress at Secⁿ 1-1

$$q = \frac{V}{IB} (A\bar{y})$$

V = Shear force

I = M.I of Sec. abt x-x

B = Width of Section at 1-1

$A\bar{y}$ = Moment of area above 1-1 abt N.A

$$A\bar{y} = B\left(\frac{D}{2} - y\right)\left[y + \frac{1}{2}\left(\frac{D}{2} - y\right)\right]$$

$$= B\left(\frac{D}{2} - y\right) \times \left(y + \frac{D}{4} - \frac{y}{2}\right)$$

$$= B\left(\frac{D}{2} - y\right)\left(\frac{D}{4} + \frac{y}{2}\right)$$

$$= \frac{1}{2} B\left(\frac{D}{2} - y\right)\left(\frac{D}{2} + y\right)$$

$$= \frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

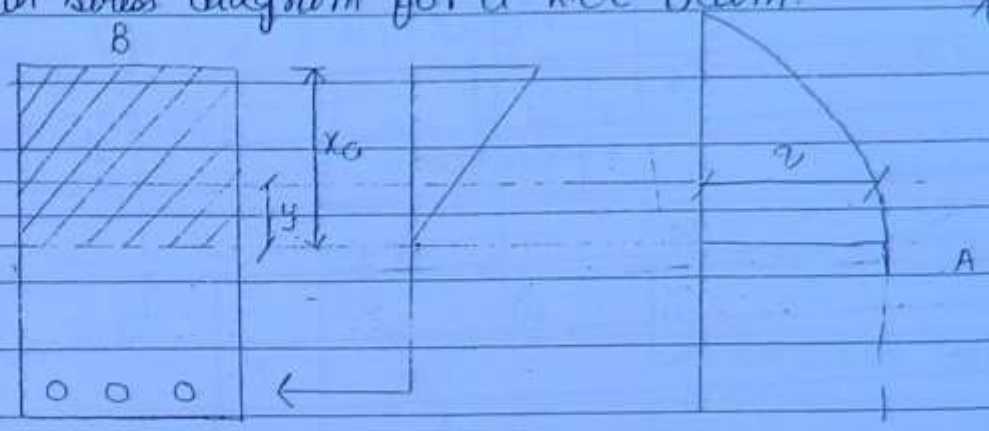
$$q = \frac{V}{IB} \frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

$$q = \frac{V}{2I}\left(\frac{D^2}{4} - y^2\right)$$

Max. q at $y = 0$ at N.A

$$q_{max} = \frac{V}{2I}\left(\frac{D^2}{4}\right)$$

Shear Stress diagram for a R.C.C Beam.



It is possible with zero shear stress at top & max at N.A

Shear stress at any section

$$q = \frac{V}{I_{eq} B}(A\bar{y})$$

Where $A\bar{y} = \frac{B}{2}(x_a^2 - y^2)$

$$q = \frac{V}{I_{eq} B} \cdot \frac{B}{2}(x_a^2 - y^2) \Rightarrow \boxed{q = \frac{V}{2I_{eq}}(x_a^2 - y^2)}$$

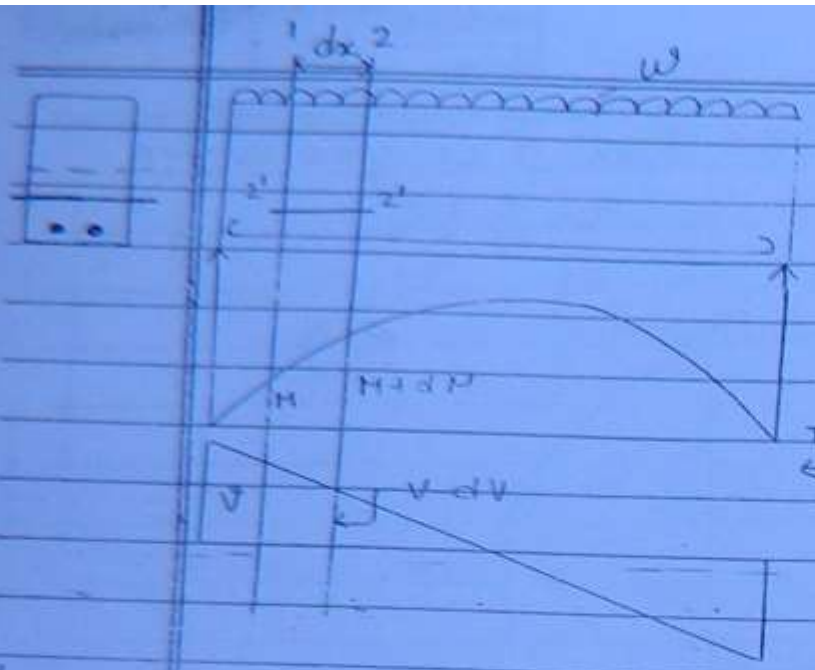
Shear Stress above N.A

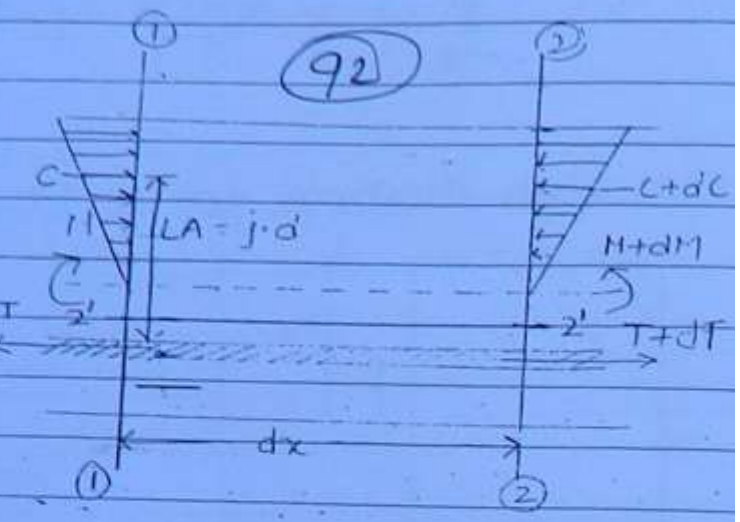
$$I_{eq} = \frac{B x_a^3}{3} + m A_{st}(d - x_a)^2$$

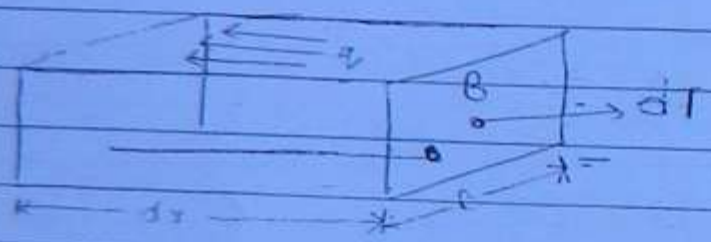
$$I_{base} = \frac{B x_a^3}{12} + B x_a \left(\frac{x_a}{2}\right)^2$$

$$= \frac{B x_a^3}{12} + \frac{B x_a^3}{4} = \frac{1+3}{12} B x_a^3$$

$$= \frac{B x_a^3}{3}$$







Consider two section 1-1 & 2-2 in the beam where moments are M & M+dM. At Section 1-1

Moment $M = C \cdot j \cdot d = T \cdot j \cdot d$ — 1

At section 2-2

Moment $M + dM = (C + dC)\, j \cdot d = (T + dT)\, j \cdot d$ — 2

(2) - (1) $dM = (T + dT)\, j \cdot d - T \cdot j \cdot d$

$dM = dT\, j \cdot d$

$dM = dT = \frac{dM}{jd}$ This unbalanced dT forces in the reinforcement causes

Shear stress in Concrete beam N.A & reinforcement

If q = Shear stress developed

Resulting force $= q \cdot B \cdot dx$ — (4)

Equating eqn (3) & (4)

$q \cdot B \cdot dx = dT = \frac{dM}{j \cdot d}$

DATE:

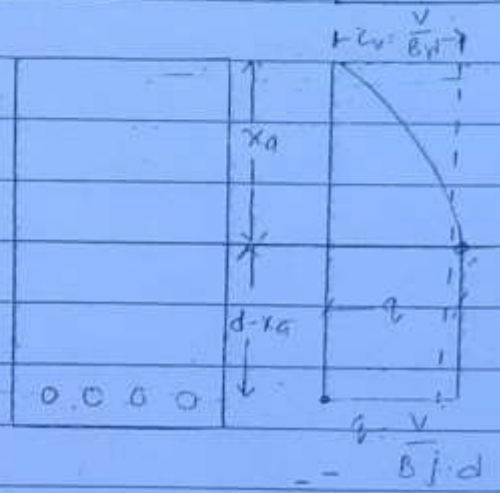
Shear Stress developed

$$q = \frac{dM}{B \cdot dx \cdot j \cdot d} = \frac{V}{B \cdot j \cdot d}$$

$$\boxed{q = \frac{V}{B \cdot j \cdot d}}$$ — I.S.

(93)

As per IS 456 $\boxed{q = \frac{V}{B \cdot d}}$



This shear stress $q = \frac{V}{B \cdot j \cdot d}$ is constant below N.A

Shear stress diagram is as shown in fig for a shear force = V

Shear force V = (width of section) x (Area of shear stress dia)

$$= B\left(\frac{2}{3} \times q \cdot x_a + q(d - x_a)\right)$$

$$= B \cdot q\left(\frac{2}{3} x_a + d - x_a\right)$$

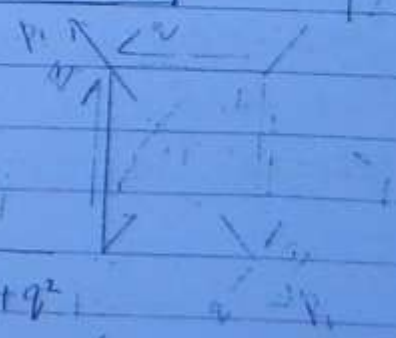
$$= B \cdot q\left(d - \frac{x_a}{3}\right) = B \cdot q \cdot j \cdot d$$

Shear value as per IS Code $\boxed{\tau_v = \frac{V}{B \cdot d}}$

Effect of Shear

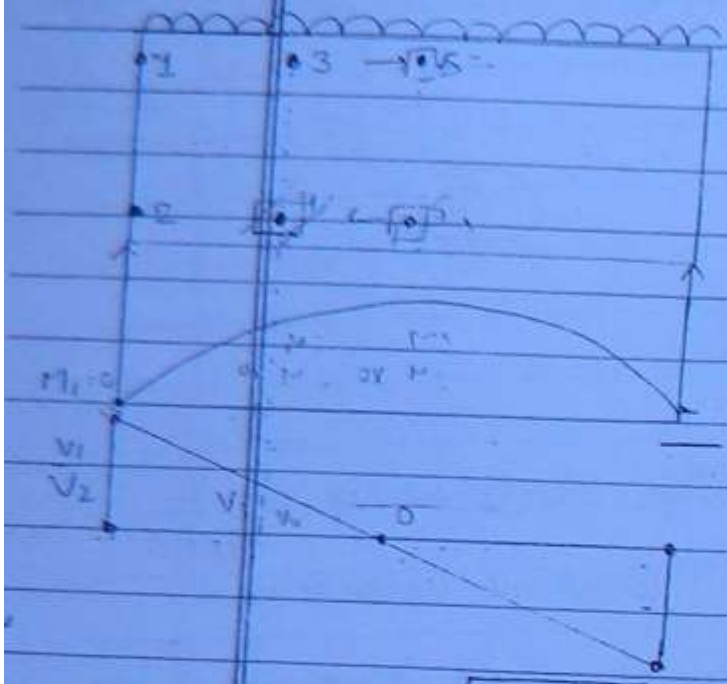
Point No. 4

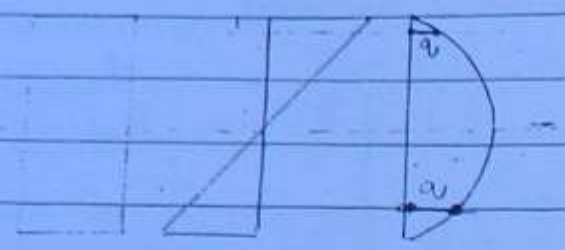
$$f = \frac{M_4}{I} y \quad \| \quad q = \frac{V_4}{I \cdot B}(A\bar{y})$$



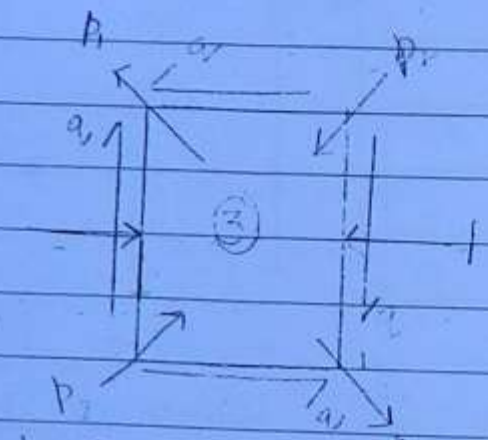
Principal Stresses $p_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$ (tensile +ve)

$p_2 = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$ (comp (-ve))





(94)



NO 3 $p_1 = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$

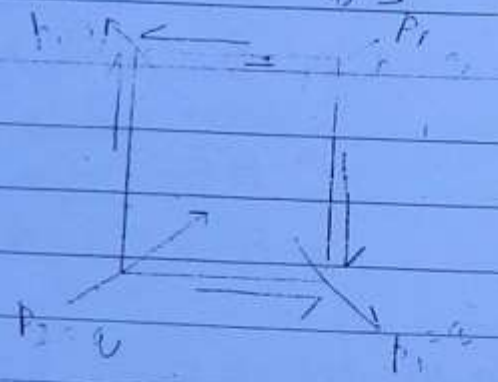
$p_2 = -\frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$

NO 2/1 $p_1 = q$

$p_2 = -q$

$\theta = 45^\circ$

is Called diagonal tension



Design Of beam for Shear (IS:456)

1. Shear force value is Calculated

Design S.F Should be Calculated at d distance from face Of Support

We can also design for max. S.F

DATE:

2. Nominal Shear Stress – It is the shear stress value developed in the section (τ_v)

$$\tau_v = \frac{V}{B \cdot d} \quad — 1$$

This $\boxed{\tau_v \neq \tau_{max}}$ in any case.

(95)

3. Design Shear Strength of Concrete

(a) Without Shear reinforcement (τ_c)

→ Shear Strength of Concrete without shear reinforcement (τ_c) depends upon

1. Grade of Concrete
2. % tension reinforcement

(TABLE - 23 – IS CODE - 456)

(b) With Shear reinforcement ($\tau_{c\,max}$)

(TABLE - 20, 24)

	M15	M20	M25	M30	M35	M40
WSM	1.6	1.8	1.9	2.2	2.3	2.5
LSM	2.5	2.8	3.1	3.5	3.7	4.0

In any case $\tau_v < \tau_{max}$

If $\tau_v > \tau_{cmax}$ → there is no other option then to revise the section, change the size.

Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{0.87 f_y}$$ (26.5.1.6 Page No - 48)

Design step

1. Calculate $\tau_v = \frac{V}{b \cdot d}$

2. Calculate % of Steel $p = \frac{A_{st}}{B \cdot d} \times 100$

Find out τ_c from table

DATE:

If $\tau_v < \tau_c$ → even ~~if~~ in this case - min shear reinforcement shall be provided

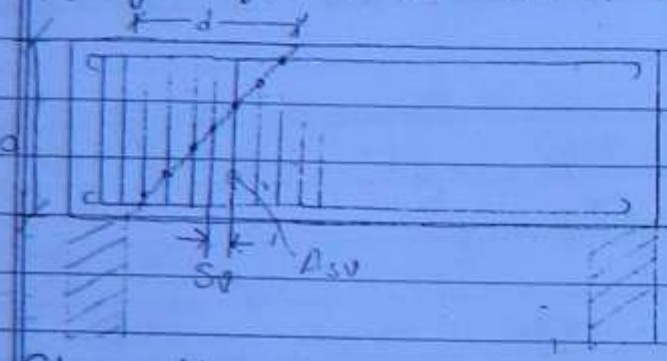
(96)

5. If $\tau_v > \tau_c$

Net shear force V_s for design of shear reinforcement

$$V_s = V - V_c$$
$$= V - \tau_c \cdot B \cdot d$$
$$= \tau_v B d - \tau_c \cdot B \cdot d$$
$$V_s = (\tau_v - \tau_c) B \cdot d$$

6 Design of vertical shear reinforcement


d
S_v
A_{sv}

Shear strength of reinforcement

$$V_s = n \times A_{sv} \times \sigma_{sv}$$
$$= \frac{d}{S_v} \times A_{sv} \times \sigma_{sv}$$

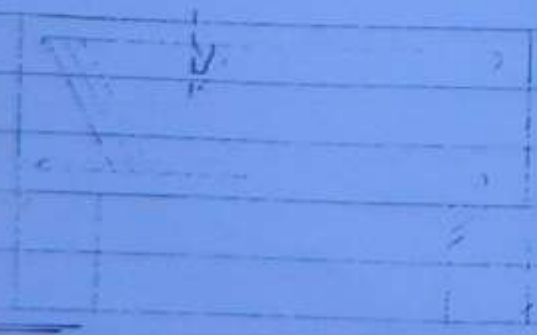
Spacing of V.S.R

$$\boxed{S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s}}$$ Working Stress Method

pacing V.S.R

$$\boxed{S_v = \frac{A_{sv}(0.87 f_y) d}{V_s}}$$ Limit State Method

7 Design of Inclined shear reinforcement


V

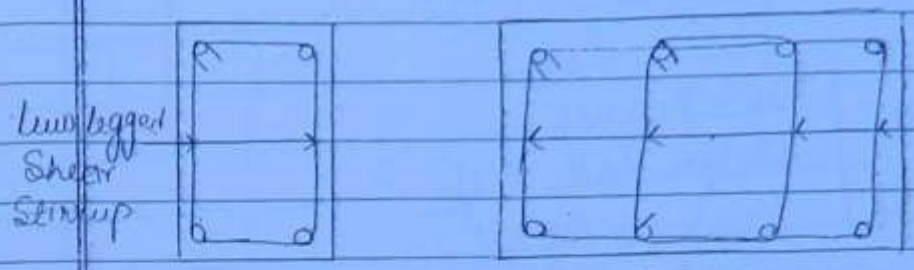
W.S.M

$$\boxed{S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V} (\sin\alpha + \cos\alpha)}$$

$$\boxed{S_v = \frac{A_{sv}(0.87 f_y) d}{V} (\sin\alpha + \cos\alpha)}$$

L.S.M

Area of steel (A_{sv}) at one section



$$A_{sv} = n \times \frac{\pi}{4} \times \phi^2$$

n = no. of legs

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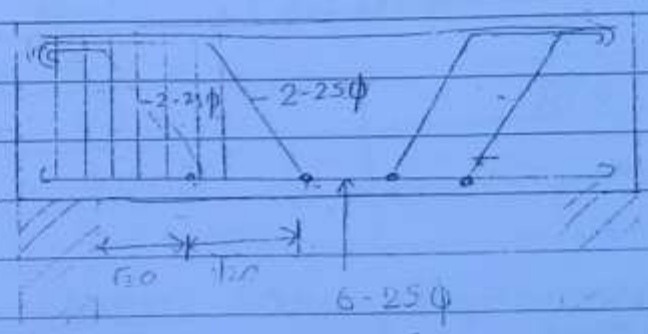
B Use of BENT UP BARS
(along ⊥ vertical shear reinforcement)



a = lever arm = $j \cdot d$

1st At location of Bent up bars = $\sqrt{2}a$
= $\sqrt{2} \cdot j \cdot d$ distance from support

2nd location at $\sqrt{2}\, j \cdot d$ distance from 1st location

Generally bars should not be bend beyond $\frac{\text{span}}{4}$ from support

Strength of bent up bars

$V_{sb} = A_{sb} \times \sigma_{sv} \sin\alpha$ → WSM

$V_{sb} = A_{sb} \times (.87 f_y) \sin\alpha$ — LSM

Contribution of bent up bars should be kept limited to $\frac{V_s}{2}$

Vertical shear reinf. should be designed for atleast
① $V_s/2$ ② $(V_s - V_{sb})$ } whichever is more

DATE:

Ex. $V = 400\,KN$

$\tau_c = ?$ $V_c = 100\,KN$

(98)

$V_s = 300\,KN$

$V_{sb} = 100\,KN$	$V_{sb} = 200\,KN$
Design vertical shear	Design V.S.R $V_s/2$
reinf. for $(V_s - V_{sb}) = 200\,KN$	$V_s - V_{sb} = 100\,KN$
	$V_s/2 = 150\,KN$ ✓

→ Maximum Spacing of shear reinforcement

(i) $0.75d$ — for vertical stirrups

(ii) d — for inclined stirrups at $45°$

(iii) $300\,mm$ — in any case

E-2004 Design shear reinforcement at support if τ_c

p	1.25	1.50	1.75
τ_c	0.70	0.74	0.78

Sol. $B = 500\,mm$

$D = 1500\,mm$

$d = 1400\,mm$

$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308\,mm^2$

$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 = 2277\,mm^2$

Service U.D.L $= 52.92\,KN/m$

Span $= 18\,m$

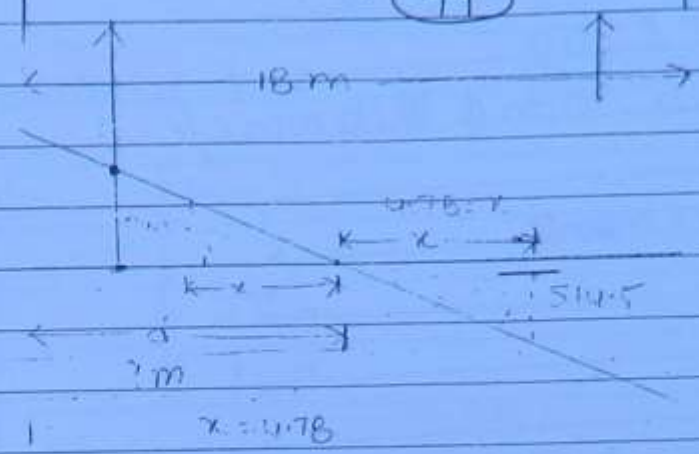
Soln Dead load $= 25 \times .5 \times 1.5 = 18.75\,KN/m$

Total load $= 52.92 + 18.75 = 71.67\,KN/m$

Factored load $= 1.5 \times 71.67 = 107.505\,KN/m$

$V = \frac{kll}{2} = \frac{107.505 \times 18}{2} = 967.545\,KN$

SFD $\frac{x}{9} = \frac{514.5}{967.5}$ $x = 4.78\,m$



In dotted area there is no requirement to provide Shear Reinforcement

→ Hence provide minimum Shear reinforcement.

Nominal Shear Stress $\tau_v = \frac{V}{B \cdot d} = \frac{967.50}{500 \times 1400} = 1.382 \text{ N/mm}^2$

% of Steel

$p = \frac{A_{st}}{Bd} \times 100$

$= \frac{10308}{500 \times 1400} \times 100 = 1.47$

τ_c (from table by interpolation)

$\tau_c = 0.7 + \left(\frac{0.74 - 0.7}{1.5 - 1.25}\right)(1.47 - 1.25)$

$= 0.7352 \text{ N/mm}^2$

Design Shear Strength of Concrete

$V_c = \tau_c \cdot B \cdot d = 0.735 \times 500 \times 1400$

$= 514.5 \text{ kN}$

Shear reinforcement is required for

$V_s = V - V_c$

$= 967.5 - 514.5$

$= 453 \text{ kN}$

Let us provide 2 legged - 8mm ϕ Stirrups

$A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53$

DATE:

$S_v = \frac{A_{sv} \times 0.87 f_y \times d}{V_s} = \frac{100.53 \times 0.87 \times 415 \times 1400}{453}$

$= 112.17$ mm

100

This value is betn 100 & 200

⇒ Hence provide 2-legged 8mm ϕ @ 110 mm c/c

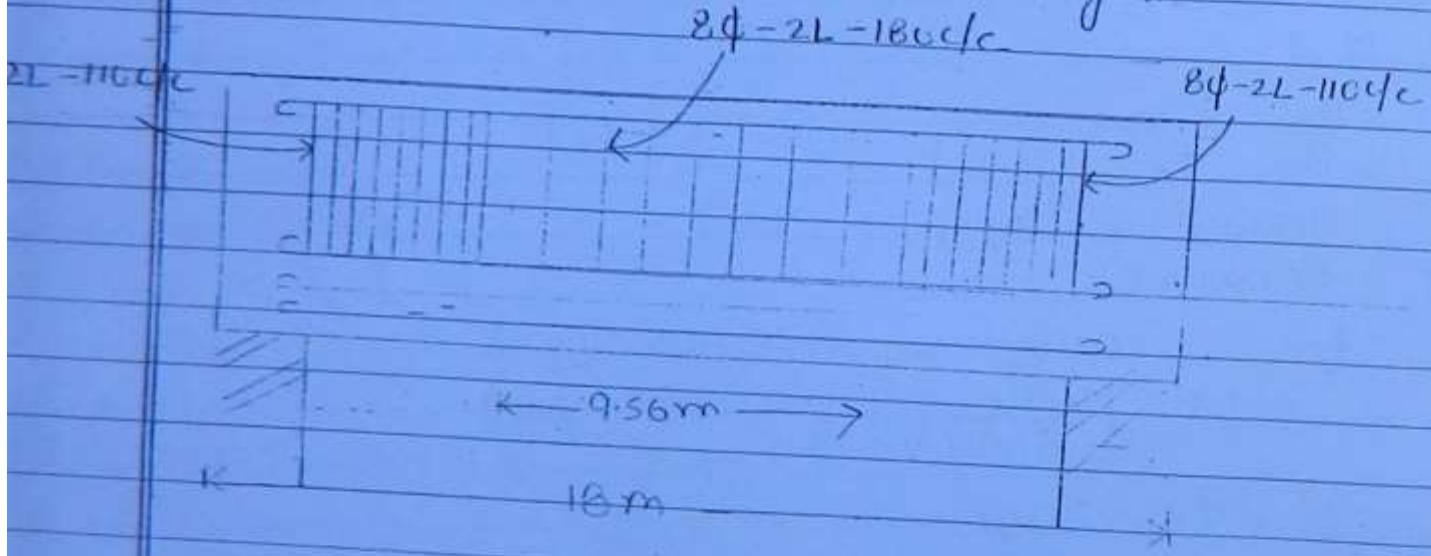
Check

Min. Shear reinforcement

$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{0.87 f_y}$

$S_v \leq 181.4$ mm

Provide 2 legged 8mm ϕ @ 180mm c/c in central $(4.78 \times 2) = 9.56$ m length



Maximum Spacing

1. For vertical stirrups – $0.75 \times 1400 = 1050$ mm
2. $= 300$ mm ⇒ Safe

A simply reinforced beam of size 300 x 750 mm eff. depth is supported over a clear span of 6m & subjected to a U.D.L of 65 kN/m including self wt. If 60 Nos of 25 mm ϕ has been provided as tension reinforcement Design shear reinforcement using bent up bars.

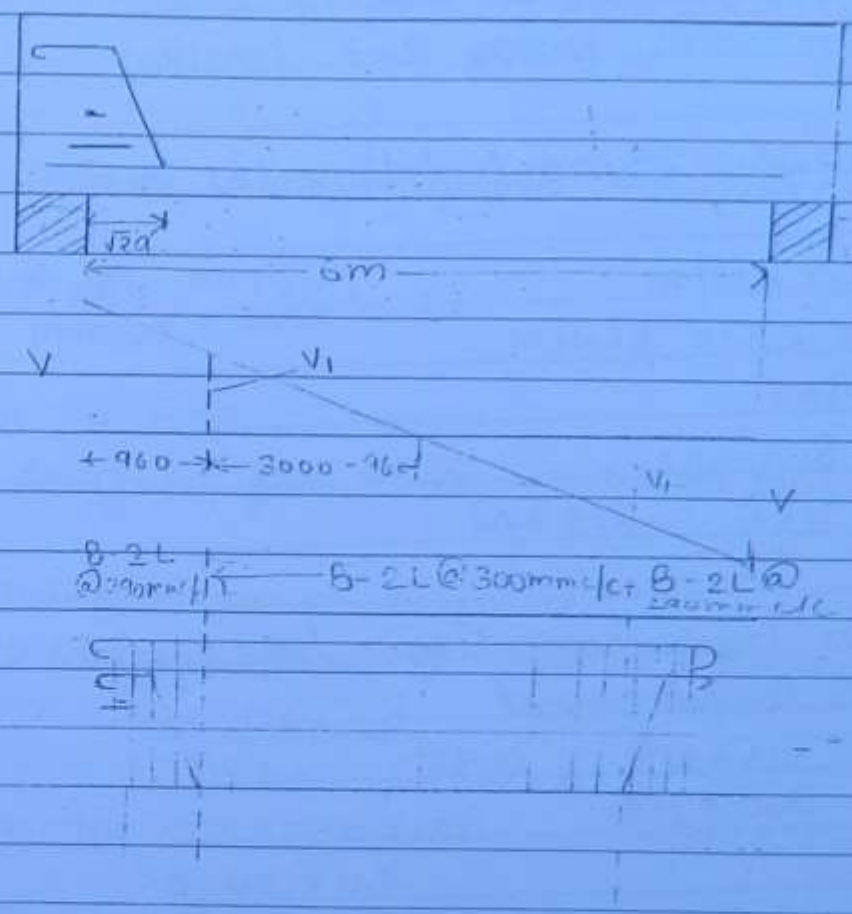
DATE:

Also at suitable location M-20/Fe-415 is used.

Total load = L.L + D.L

= 65 + 5.625 = 70.625 kN/m - (101)

factored load = 1.5 x 70.625 = 105.94 kN/m -



Location of bent up bars

1st location of bent up bars

$= \sqrt{2}a = \sqrt{2} \cdot j \cdot d$

$j = 1 - \frac{K}{3}$

$K = \frac{mc}{t + mc} = \frac{13 \times 7}{230 + 13 \times 7}$

$= 0.2835$

$j = 1 - \frac{K}{3} = 0.905$

$= \sqrt{2} \times .905 \times 700$

= 960 mm from Supports.

Ind location of bent up bars

$2\sqrt{2}\,a = 1920$ mm from support

$\frac{Span}{4} = \frac{6000}{4} = 1500$ mm

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We cannot use the 2nd bent up bars

⇒ Bars cannot be bent from 2nd location

Let us bent 2 nos. - 25 mm ϕ from 1st location

2. Load / SF

Live load = 65 kN/m

D.L = 8 x .3 x 25 = 6 kN/m

Total = 71 kN/m

750 mm - eff depth

Total = 800 mm

Max. S.F at Support

$V = \frac{wL}{2} = \frac{71 \times 6}{2} = 213$ kN

Design of Shear reinforcement with bent up bars

Nominal Shear Reinforcement

$\tau_v = \frac{V}{B \cdot d} = \frac{213 \times 10^3}{300 \times 750} = 0.95$

% of Steel

$p = \frac{6 \times \pi/4 \times 25^2}{300 \times 750} \times 100 = 1.31$

$\tau_c = 0.42 + \frac{0.45 - 0.42}{.25} \times [1.31 - 1.25]$

$\tau_c = 0.427$

$V_c = 0.427 \times 300 \times 750 = 96.12$ kN

Shear reinforcement designed for

$V_s = V - V_c$

$= 213 - 96.12$

$= 116.88 = 116.9$ kN

Strength of bent up bars

$V_{sb} = A_{sb} \times \sigma_{sb} \times \sin\alpha$

$= \left(2 \times 25^2 \times \frac{\pi}{4}\right)(230) \sin 45^0$

(T93)

$= \frac{.7070 \times .785 \times 2 \times 625 \times 230}{10^3}$

$= 159.56$ kN

In this case vertical shear reinforcement be designe for

$\frac{V_s}{2} = \frac{116.9}{2} = 58.45$ kN

$V_s - V_{sb} = -ve$

It is designed for $V_s/2 = 58.45$ kN

Shear reinforcement required.

use 2-legged 8mm ϕ vertical stirrups spacing

$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s/2}$

$= \frac{100.53 \times 230 \times 150}{58.45 \times 10^3} = 296.69$ mm

$\Rightarrow$ Provide 2-legged 8-mm dia @ 290 mm c/c

Shear force beyond bent up bars

$\frac{V_1}{V} = \frac{3000-960}{300} \Rightarrow V_1 = 144.84$ kN

Design for this shear force

$\tau_v = \frac{V_1}{B \cdot d} = \frac{144.84 \times 10^3}{300 \times 750}$

$= 0.64$

% of steel

$p = \frac{8 \times \pi/4 \times 25^2}{300 \times 750} \times 100 = 0.0174 \times 100 = 1.74\%$

τ_c for 1.74% = 0.47

$V_c = \tau_c \cdot B \cdot d = 0.47 \times 300 \times 750$ (104)

$= 105.75$ kN

$V_s = V - V_c = 144.84 - 105.75 = 39.09$ kN

Spacing of 2-legged 8mm dia

$$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s}$$

$$= \frac{100.53 \times 230 \times 750}{39.09 \times 10^3}$$

$= 443.63$ mm

Min Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

$= 302.47$ mm > 300 mm

Max. Spacing provided = 300mm

$S_v = 300$ mm.

In Rectangular R.C Beam grade M-15/Fe-415 size 250x500 effective. V = 60 kN. under service load Condition. Calculate Spacing of 8mm dia 2-legged stirrups. Steel - 3-16ϕ

P =	0.25	0.5	0.75	1.00
τ_c	0.35	0.45	0.54	0.64

Soln Load - 60 kN

D.L = 25 x .25 x .55 = 3.4375 kN

Total load = 63.4375 kN

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = 0.72$$

$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \pi/4 \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.46 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25) = 0.4512$$

DATE:

$V_c = \tau_c \times B \cdot d$

$= 0.4512 \times 250 \times 500$

$= 56.4$ kN

(105)

$V_s = V - V_c = 90 - 56.4 = 33.6$ kN

Spacing of 2 legged 8mm ϕ stirrups

$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$

$= 100.53$

$S_v = \frac{100.53 \times 0.87 \times 415 \times 500}{33600}$

$= 540.12$

Minimum Shear reinforcement

$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{0.87 f_y}$

$S_v = \frac{0.87 \times 415 \times 100.53}{0.4 \times 250}$

$= 362.96$ mm

Max. Spacing

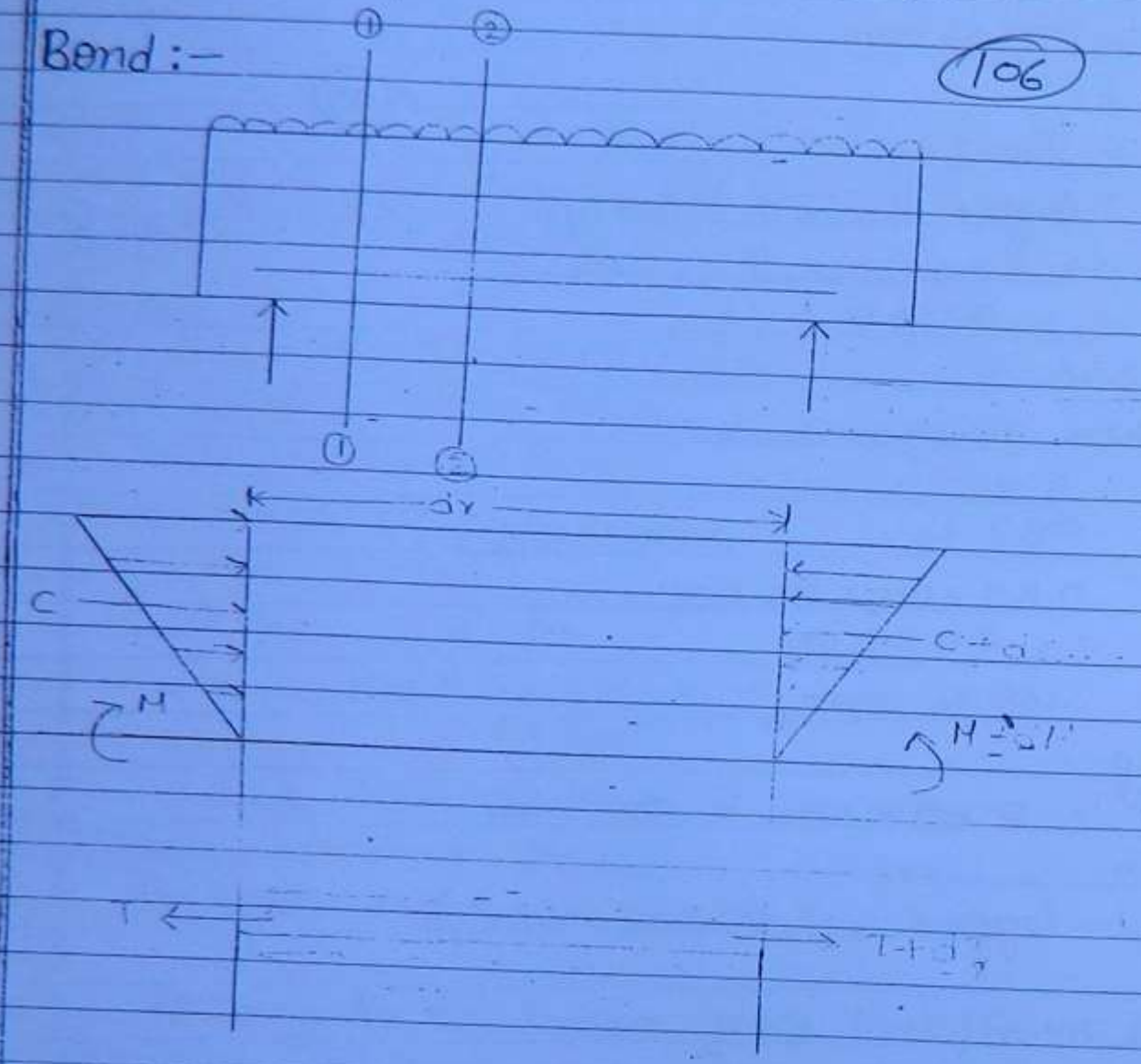
$0.75 \times d = 0.75 \times 500 = 375$ mm

300 mm

provide 2-legged 8ϕ @ 300 mm c/c

BOND / DEVELOPMENT LENGTH

Bond :-



In a simply supported beam / any other kind of beam subjected to B.M & S.F

Bond stress (longitudinal shear stress developed betn steel and concrete) is developed betn steel & concrete in longitudinal dirn due to unbalanced dT force as shown in fig.

Unbalanced force dT = Bond force

ΣO = Sum of all perimeter

$d \times \Sigma O \times \tau_{bd}$

$$\tau_{bd} = \frac{dT}{d \times \Sigma o} \quad \text{— (1)}$$

At section ①–①

$$M = C \cdot j \cdot d = T \cdot j \cdot d \quad \text{— (2)}$$

At section 2-2

$$M + dM = (C + dC) \cdot j \cdot d = (T + dT) \cdot j \cdot d$$

3-2

$$\Rightarrow dM = dT \cdot j \cdot d$$

$$dT = \frac{dM}{j \cdot d} \quad \text{— (4)}$$

Bond stress developed

$$\tau_{bd} = \frac{dM}{d \times \Sigma o \cdot j \cdot d} = \frac{V}{\Sigma o \cdot j \cdot d}$$

Here

$$\Sigma o = n \times \Lambda \times \phi$$

Developed value of bond stress due to shear force

CHECK

Permissible values of Bond stress → (Table-21 Page-80

For mild steel Reinf^n in tension	KISM	LSM → Page No-43
M-15	0.6	x
M20	0.8	1.2
M25	0.9	1.4
M30	1.0	1.5
M35	1.1	1.7
M40	1.2	1.9
M45	1.3	1.9
M50	1.4	1.9

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For HYSD bars ; values at given in table are increased by => 60% for both HSM & LSM.

For bars in Compression values in table are to be increased by 25% — HSM & LSM

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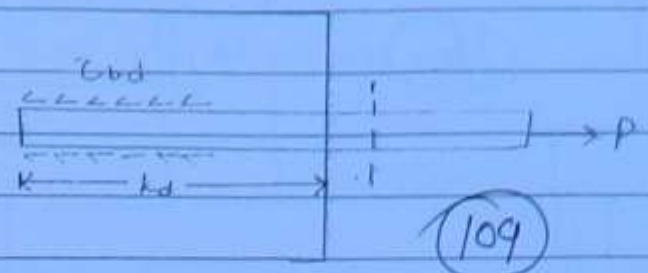
τ_{bd} Calculated $\neq$ τ_{bd} permissible.

Ex - τ_{bd} for M-30 Concrete for LSM for Fe-415 bars in Compression

$\tau_{bd} = 1.5 \times 1.6 \times 1.25 = 3$

Developement length — Min length of steel requirement required to be embedded in concrete so that strength of bond is equal to strength of reinforcement in tension



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$P = A_{st} \cdot \sigma_{st}$ = Bond strength

$A_{st} \cdot \sigma_{st} = \tau_{bd} \times \Sigma o \times L_d$

$\frac{\pi}{4} \times \phi^2 \times \sigma_{st} = \tau_{bd} \, \Sigma \times \pi \times \phi \times L_d$

$$\boxed{L_d = \frac{\phi \, \sigma_{st}}{4 \tau_{bd}}}$$ Working stress method.

$$\boxed{L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}}}$$ Limit state method.

Calculate developement length req. for a steel reinforcement fe-415 for tension and compression. use M-25 & use LSM.

Soln For tension

$\tau_{bd} = 1.4 \times 1.60 = 2.24$

$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}} = \frac{\phi (0.87 \times 415)}{4 \times 2.24} = 4.295 \phi$

In compression

$\tau_{bd} = 1.4 \times 1.6 \times 1.25 = 2.8 \, N/mm^2$

$L_d = \frac{\phi (.87 \times 415)}{4 \times 2.8} = 32.24 \phi$

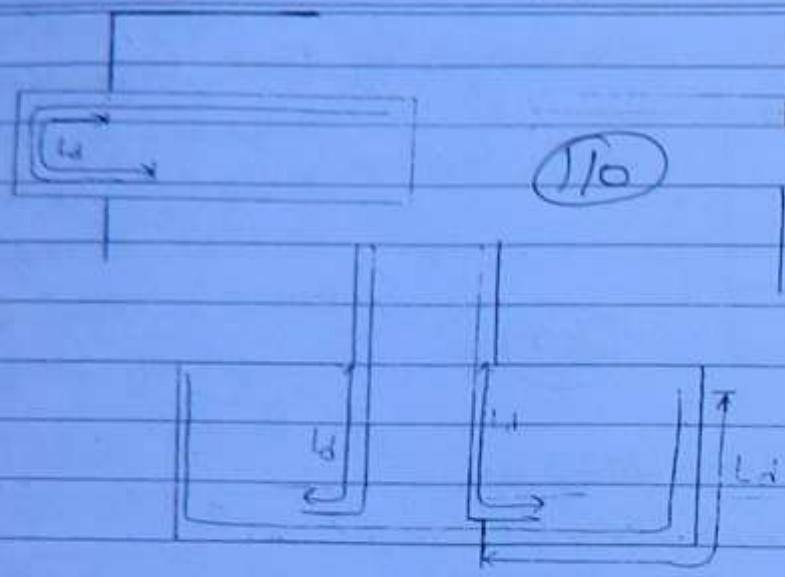
Use of developement length

1. For lap length of steel in slab / Beam / Column / foundation

   Lap length = L_d

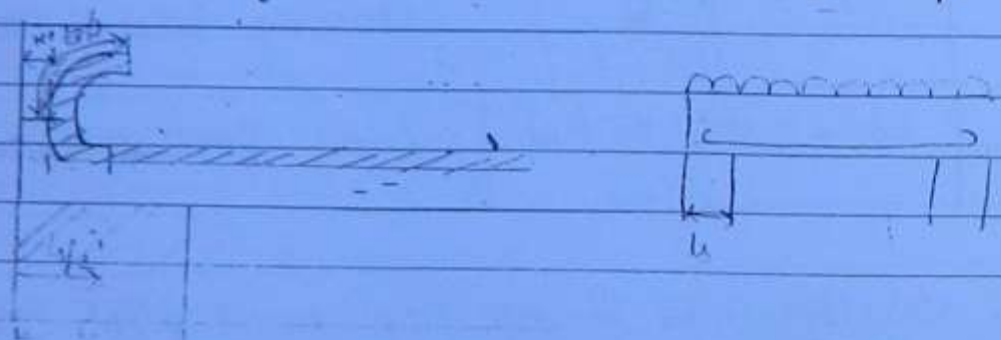
2. At corner in beam.

DATE:



Development length requirement At Simple Support

(Code 8.3.3)



positive moment reinforcement shall be limited to a dia such that

$$L_d \leq \frac{M_1}{V} + L_o$$

M_1 = Moment of resistance of section assuming all reinforcement stressed upto σ_{st} or $0.87 f_y$

$A_{st} \cdot \sigma_{st} \cdot (jd)$ — LISM

$A_{st} \cdot (0.87 f_y) j \cdot d$ — LSM

V = Shear force at support

L_o = length of reinf. beyond centre of support

$L_o = \frac{l_s}{2} - x' + 8\phi$ → (for mild steel)

$L_o = \frac{l_s}{2} - x'$ — for HYSD Bars

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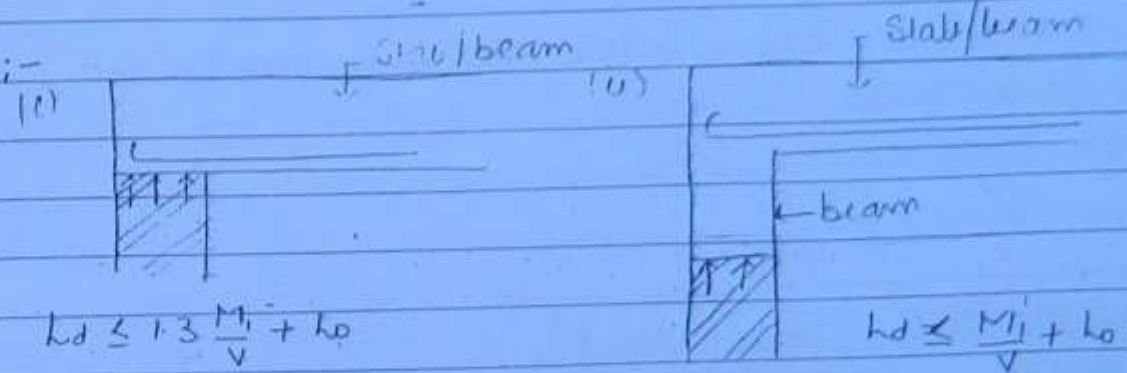
If the end of the reinforcement are confined by a compressive reaction.

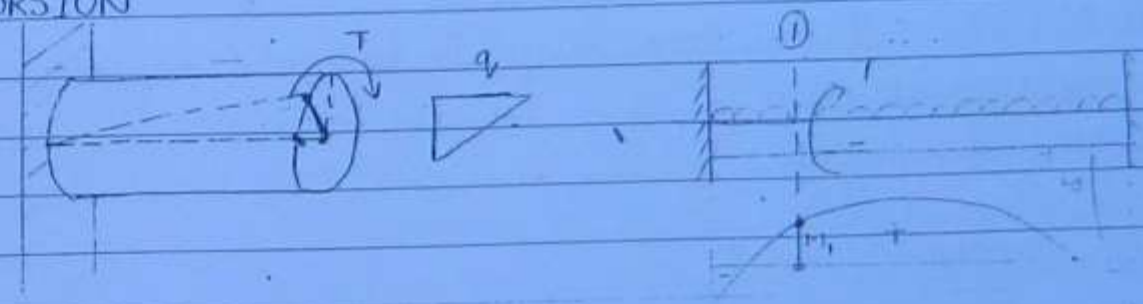
$$L_d \leq 1.3 \frac{M_1}{V} + L_o \quad \text{(iii)}$$

Example:-

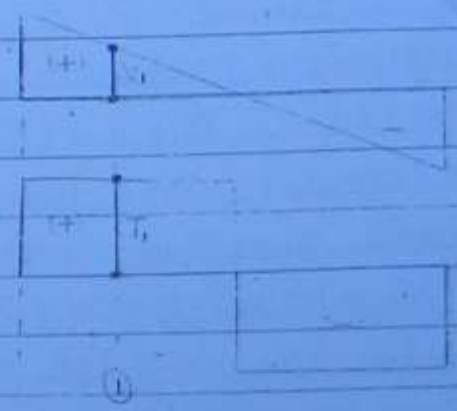


$$L_d \leq 1.3 \frac{M_1}{V} + L_o \qquad L_d \leq \frac{M_1}{V} + L_o$$

TORSION



If a beam is subjected to BM-M S.F = V & torsional moment T at a particular cross-section



Design
(By working stress method)

1. Equivalent shear

$$V_{eq} = V + \frac{1.6\,T}{B} \quad \text{— (i)}$$

Nominal shear stress

$$\tau_{veq} = \frac{V_{eq}}{B \cdot d} \not> \tau_{c\,max}$$

If $\tau_{veq} > \tau_{cmax}$ → change the size of beam

2. Equivalent Moment

$$M_{eq} = M + \underbrace{\frac{T}{1.7}\left(1+\frac{D}{B}\right)}_{M_T}$$

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$$M_{eq} = M + M_T$$

CASE I Generally ($M_T < M$) In this case design only tension reinforcement

depth required

$$d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$$

Area of Steel (tension reinf.)

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d} = \frac{M_{eq}}{\sigma_{st}(d - x_a/3)}$$

CASE II If $M_T > M$ - In this case design in tension & compression reinforcement both.

Check $d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d}$$

Compression reinforcement is also provided for a moment $M_{e_2} = (M_T - M)$

The moment M_{e_2} being taken as acting in the opposite sense of moment M.

If $M = (+)$ve B.M

Consider M_{e_2} as $(-)$ve BM

Compression Steel

$$A_{sc} = A_{s2} = \frac{M_{e_2}}{\sigma_{st} \cdot j \cdot d}$$

3. Design of Shear reinforcement

Spacing of Shear reinforcement

$$A_{sv} = \frac{T \cdot S_v}{b_1 d_1 (0.87 f_y)\sigma_{sv}} + \frac{V \cdot S_v}{2.5 d_1 (0.87 f_y)\sigma_{sv}}$$

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$(0.87 f_y)$

$$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d_1}{\left[\frac{T}{b_1} + \frac{V}{2.5}\right]} \quad (113)$$

Spacing of shear reinforcement

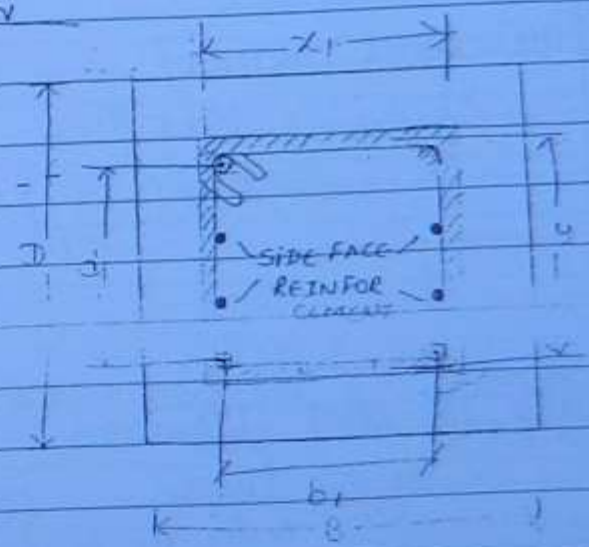
Max. Spacing of shear reinf^n (page No-48)

1. x_1
2. $\frac{x_1 + y_1}{4}$ — $x_1, y_1 \rightarrow$ short & long dim. of stirrups
3. 300mm

Min. Shear reinforcement in case of torsion -(Page No-47)
Total transverse reinforcement A_{sv} shall not be less than

$$A_{sv} \nless \frac{(\tau_{ve} - \tau_c) R \cdot S_v}{\sigma_{sv}}$$

$$\frac{A_{sv}}{B \cdot S_v} \nless \frac{(\tau_{ve} - \tau_c)}{\sigma_{sv}}$$



Side face Reinforcement
When depth of beam is more than
1. 750 mm – when beam is not subjected to torsion
2. 450mm – when subjected to torsion

Side face reinf^n shall be provided equal to 0.1% of web area & distributed on both face

$$A_{sf} = \frac{0.1}{100} \times B \cdot D$$

Max. Spacing betn Side face reinf^n = 300 mm or web thickness ≤ ever is less

DATE:

Limit state method

factored values

$B.M = M_u = 1.5M$ (114)

$T_u = 1.5T$

$V_u = 1.5V$

Equivalent Shear

$$V_{ueq} = V_u + \frac{1.6\,T_u}{B}$$

Nominal Shear Stress $\tau_{veq} = \frac{V_{ueq}}{B \cdot d} \not> \tau_{cmax}$

If $\tau_{veq} > \tau_{cmax}$ revise the section

Equivalent moment

$$M_{ueq} = M_u + M_{Tu}$$

$$= M_u + \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$$

$$d = \sqrt{\frac{M_{ueq}}{Q \cdot B}}$$

Area of Steel (tension)

$$A_{st} = \frac{M_{ueq}}{0.87 f_y \cdot j \cdot d}$$

$$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] B d$$

If $M_{Tu} > M_u \rightarrow$ provide comp reinf^n for M_{ue_2} =

$$M_{ue_2} = M_{Tu} - M_u$$

$$A_{sc} = A_{st_2} = \frac{M_{ue_2}}{0.87 f_y \cdot j \cdot d}$$

Shear reinf^n

$$S_v = \frac{A_{sv}(0.87 f_y) d_1}{\left[\frac{T_u}{b_1} + \frac{V_u}{2.5}\right]}$$

Min Shear reinf^n

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{uve} - \tau_c}{.87 f_y}$$

(115)

Max. Spacing

① x_1 ② $\frac{x_1 + y_1}{4}$ ③ 300mm

Side face reinf^n is same as L.S.M

Ques Design a beam subjected to a following at a particular section (use L.S.M)

1. B.M = 300 kN-m
2. S.F = 200 kN-m
3. T.M = 80 kN-m

use width of beam = 400mm

use M-25 / Fe-415

Sol. Factored values

M_u = 450 kN-m

T_u = 120 kN-m

V_u = 300 kN-m

Equivalent Shear

$$V_{ueq} = V_u + \frac{1.6 T_u}{B} = 300 + \frac{1.6 \times 120}{.400} = \cancel{300\ 48} \ 780 \text{ kN}$$

Assume depth = 600 mm (D) d = 550

Nominal Shear Stress = $\tau_{veq} = \frac{V_{ueq}}{B \cdot d}$

$$\Rightarrow \tau_{veq} = \frac{780 \times 10^3}{400 \times 550} = 3.54$$

τ_{cmax} M-25 = 3.10

$\tau_{veq} > \tau_{cmax}$ so revise the section

Assume depth = 900mm

$$\tau_{veq} = \frac{780 \times 10^3}{400 \times 850} = 2.29$$

Equivalent Moment $M_{ueq} = M_u + M_{tu} \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$

$$= 450 + \frac{120}{1.7}\left(1 + \frac{900}{400}\right) = 450 + \cancel{22}\ 229.41$$

DATE:

$M_{uea} = 679.41$ kN-m

$d = \sqrt{\frac{M_{uea}}{Q \cdot B}}$

$d = \sqrt{\frac{679.41 \times 10^6}{.138 \times 25 \times 400}}$ (116)

$d = 701.66$ mm

Say = D = 750 mm

~~D = 800 mm~~

$\tau_{vea} = \frac{V_{uea}}{B \cdot d} = \frac{780 \times 10^3}{400 \times 700} = 2.8$

Safe

$M_{uea} = 450 + \frac{120}{1.7}\left(1 + \frac{750}{400}\right)$

$= 450 + 202.94$

M_{uea} 652.9 kN-m

~~Area of~~ $d = \sqrt{\frac{652.9 \times 10^6}{.138 \times 25 \times 400}} = 688$ mm

< 700 mm provided

Size B = 400 mm

D = 750 mm

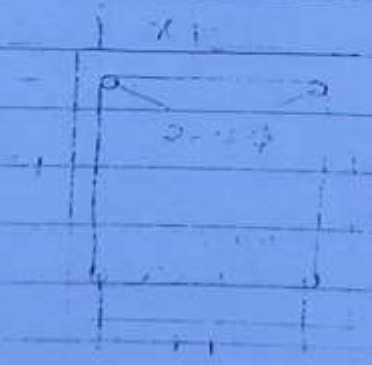
d = 700 mm

$A_{st} = \frac{M_{uea}}{87 f_y \cdot j \cdot d} = \frac{652.9 \times 10^6}{.87 \times 415 \times .80 \times 700}$

$= 3229.4$ mm^2 – 25mmϕ ≃ 7 NOS

Shear reinforcement

Spacing



$b_1 = 400 - 100 = 300$

$d_1 = 750 - 100 = 650$

$x_1 = 300 + 35 = 335$

$650 + 35 = 685$

$S_v = \frac{A_{sv}(0.87 \times f_y) d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{b_1}\right)}$

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$$= \frac{2 \times \frac{\pi}{4} \times 10^2 \times 0.87 \times 415 \times 650}{\left(\frac{300 \times 10^3}{2.5} + \frac{120 \times 10^6}{300}\right)} = 70.9 \text{ mm}$$

(117)

Using 2 legged 12 ϕ bars

$$= 70.89 \times \frac{12^2}{10^2} = 102.08 \text{ mm}$$

Use 2 legged @ 100 mm c/c

Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{uve} - \tau_c}{.87 f_y}$$

$$\frac{2 \times \pi/4 \times 12^2}{400 \times 100} \geq \frac{2.8 - 3.1}{.87 \times 415}$$

In a Rectangular R.C Beam grade M-15 / fe=415 size - 250 X 500 mm effective. V = 60 KN. under service load condition. Calculate spacing of 8mm dia 2-legged stirrups. Steel - 3-16ϕ

p	0.25	0.5	0.75	1.00
τ_c	0.35	0.45	0.54	0.64

Sol^n Load - 60 KN

D.L = 25 X .25 X .550 = 3.4375 KN

Total load = 63.4375 KN

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = 0.72$$

$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \pi/4 \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.46 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25)$$

$$= 0.4512$$

DATE:

Bending moment and Shear force Coefficient for a Continous beam

1. For bending moment

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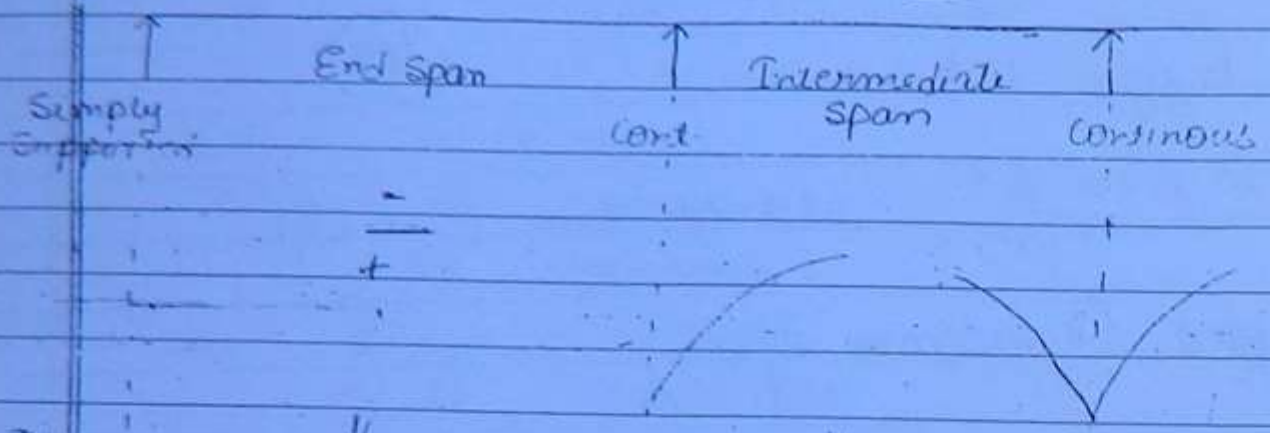
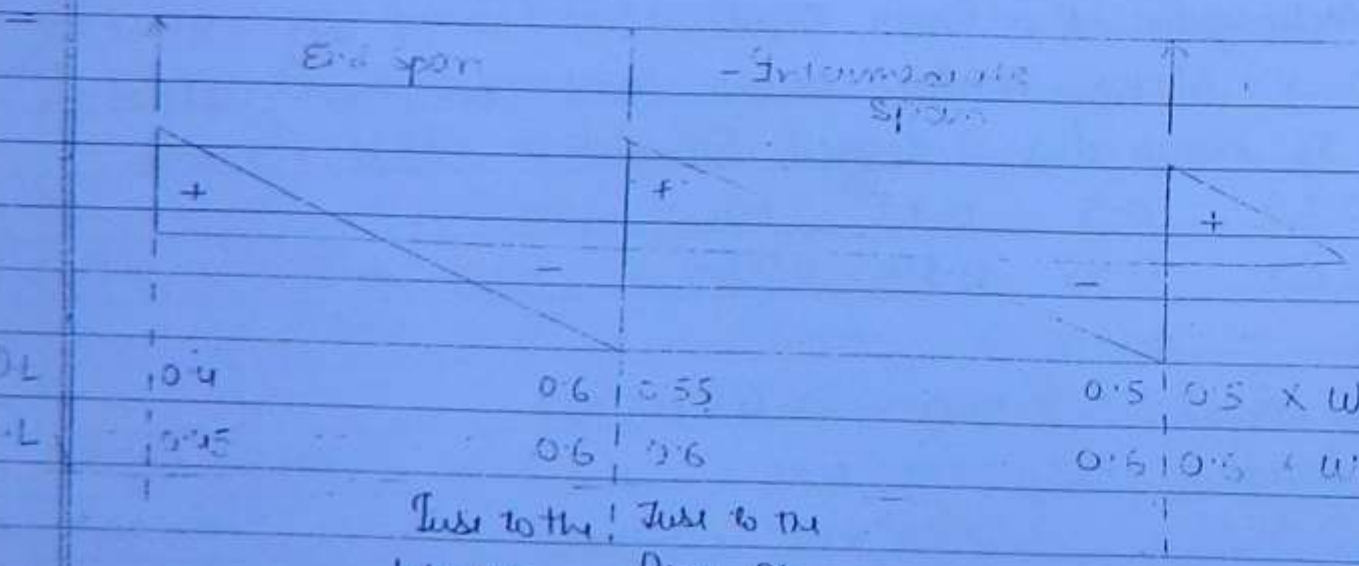


	End span	Cont	Intermediate span	Continous
D.L	1/12	-1/10	1/16	-1/12
L.L	1/10	-1/9	1/12	1/9

B.M at any location $= K_d w_d l^2 + K_l w_l l^2$

At Support $= -\frac{1}{10} w_d l^2 - \frac{1}{9} w_l l^2$

For Shear force (TABLE-13 IS: 456)



	End span	Left of Support (Outer Side)	Right of Support (Inner Side)	Intermediate span	
D.L	0.4	0.6	0.55	0.5	0.5 × w_d × l
L.L	0.45	0.6	0.6	0.5	0.5 × w_l × l

ES-1992

(a) 550 x 750 mm Overall depth

$M_u = BM = 150\ kN\text{-}m$

$T_u = 50\ kN\text{-}m$

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p_t	τ_c
0.25	0.35
0.50	0.46

Use M-15 | Fe-415

Determine longitudinal reinforcement.

(b) If $V_u = 130\ kN$

For the beam in Ques above Determine the transverse reinforcement

Solⁿ Equivalent Shear

$V_u = 0$; $T_u = 50\ kN\text{-}m$

$$V_{ue} = V_u + \frac{1.6 T_u}{B} = 0 + \frac{1.6 \times 50}{0.55} = 145.45\ kN$$

$$\tau_{ve} = \frac{V_{ue}}{Bd} = \frac{145.45 \times 10^3}{550 \times 700} = 0.38 < (\tau_{c\,max})$$

Safe

Equivalent moment

$$M_{ue} = M_u + M_{Tu} = M_u + \frac{T_u}{1.7}\left(1 + \frac{D}{B}\right)$$

$$= 150 + \frac{50}{1.7}\left(1 + \frac{750}{550}\right)$$

$$150 + 69.52 = 219.52\ kN\text{-}m$$

$\boxed{M_{Tu} < M_u}$ – Compression Steel not required

Depth $d = \sqrt{\dfrac{M_{ue}}{Q \cdot B}} = \sqrt{\dfrac{219.52 \times 10^6}{.138 \times 15 \times 550}}$

$= 439.12\ mm$

Available depth is = 700 mm OK

So We need under reinforced Section because depth available is more than required

(120)

Area Of Steel $A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_{ue}}{f_{ck} B d^2}}\right] Bd$

$A_{st} = \frac{0.5 \times 15}{415}\left[1-\sqrt{\frac{4.6 \times 219.52 \times 10^6}{15 \times 550 \times 700^2}}\right] 550 \times 700$

$A_{st} = 931.3 \ mm^2$

No. Of 16mm dia bar $= \frac{931.3}{\frac{\pi}{4} \times 16^2} = 4.63$

Say 5 No.

(ii) Soln If Shear force $V_u = 130 kN$

$T_u = 50 kN-m$

Equivalent Shear

$V_{ue} = 130 + \frac{1.6 \times 50}{.550} = 275.45$

$\tau_{vue} = \frac{V_{ue}}{bd} = \frac{275.45 \times 10^3}{550 \times 700} = .715 \ N/mm^2$

& τ_{cmax} (2.5) - M-15

OK (Safe)

Design Of Shear reinforcement

$S_v = \frac{A_{sv}(0.87 f_y) \cdot d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{b_1}\right)}$

Use 2-legged 8mm ϕ shear reinforcement

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

(121)

$$S_v = \frac{100.53 \times .87 \times 415 \times 450 \times 650}{\left(\frac{130 \times 10^3}{25} + \frac{50 \times 10^6}{450}\right)}$$

$S_v = 144.64$

provide 2-legged 8mm ϕ @ 140mm c/c

Check – Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{vue} - \tau_c}{.87 f_y}$$

% of steel $p = \frac{A_{st}}{B \cdot d} \times 100 = \frac{5 \times \pi/4 \times 16^2}{550 \times 700} \times 100$

$= .26\%$

$$\tau_c = .35 + \frac{0.46 - 0.35}{0.25} \times (0.26 - 0.25)$$

$= 0.3544$

$$\frac{100.53}{550 \times S_v} \geq \frac{(.715 - .3544)}{.87 \times 415}$$

$$S_v \leq \cancel{139.3678} \; \frac{.87 \times 415 \times 100.53}{139.3678 \; (550\,(.715 - .3544))} = 182\text{mm}$$

- Max. Spacing $x_1 = 450 + 16 + 8 = 474$

$y_1 = 650 + 16 + 8 = 674$

① $x_1 = 474$ mm

② $\frac{x_1 + y_1}{4} = \frac{474 + 674}{4} = 287$

③ 300 mm.

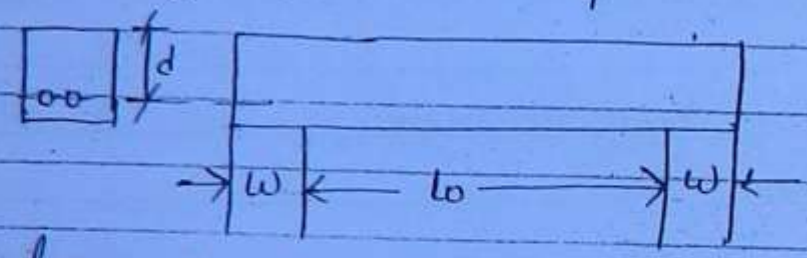
Provide 2 legged - 8mm ϕ @ 140 mm c/c

DESIGN OF BEAM AND SLAB (One way Slab)

Important IS Provision :—

① Effective Span

(a) Simply Supported beam/Slab (22.2 – Page No-34/35)

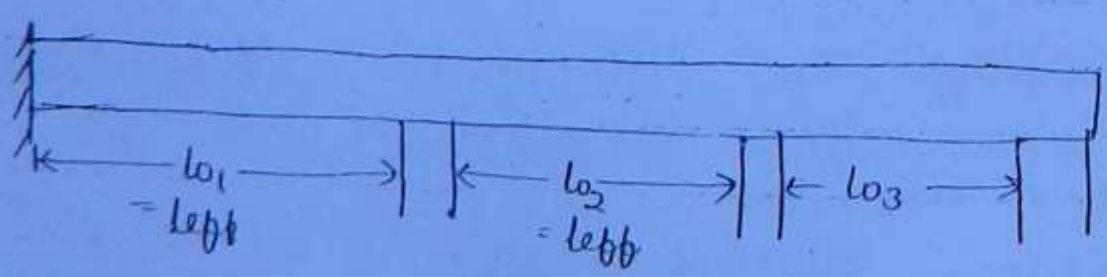


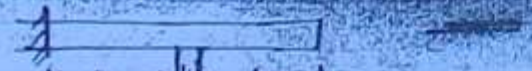
$$l_{eff} = l_o + d$$
$$= l_o + w$$
} whichever is less

(b) For a Continous beam or Slab

1. If width of support is less than $\frac{1}{12}$ of clear span $\left(w < \frac{l_o}{12}\right)$ then eff span is same as that of simply supported

2. If $\left(w > \frac{l_o}{12}\right)$ or $w = 600\,mm$ } then $\boxed{\text{Min}}$





$= l_{eff}$ $\quad l_{eff} = l_{o_2} + \frac{w}{2}$

(1) For one end fixed & other Continous or Both end fixed Continous (intermediate span)

l_{eff} = Clear span (123)

2. With one end Continous & other free (Simply supported)

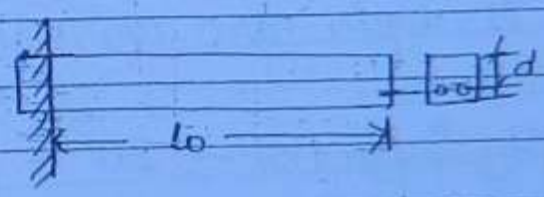
l_{eff} = Clear span + $\frac{L}{2}$ = $l_o + \frac{w}{2}$ } Which ever is small.

l_{eff} = Clear span + $d/2$ = $l_o + d/2$

(C)

Cantilever

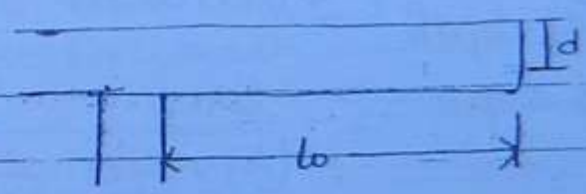
(i) with fixed end



$l_{eff} = l_o + \frac{d}{2}$

d = eff depth

(ii) with Continous end



$l_{eff} = l_o + \frac{L}{2}$

(d)

Frames – c/c distance between beam/Column.

Control of deflection

(Important Criteria of limit state of Serviceability)

(Page NO - 37)

The deflection shall generally be limited to following-

(a) The final defl^n of all load including the effect of temp, creep, & shrinkage & measured from the as-cast level of the support of floor, roof & all

horizontal member should not normaly exceed Span/250

(intermediate defl^n) (124)

(b) The deflection including the effect of Temp, Creep & Shrinkage occuring after erection of partition & the application of finishes should not normally exceed span/350 or 20mm whichever is less

Check for deflection

→ vertical defln limit may be generally be assumed satisfied if –

Span to depth ratio are not greater than

Cantilever – 7
Simply Supported – 20
Continous – 26

} (A) values (upto 10m span)

$\frac{\text{Span}}{\text{Eff depth}} \leq$ (A) values

$\text{Eff depth} \geq \frac{\text{Span}}{\text{(A) values}}$

Except for Cantilever if span is more than 10m the A Value should be multiplied by $\frac{10}{\text{Span in metre}}$

$\text{Eff depth} = \frac{\text{Span}}{\text{A value} \times \frac{10}{\text{Span (m)}}}$

eg – For a 20m span S.S.B eff. depth req will

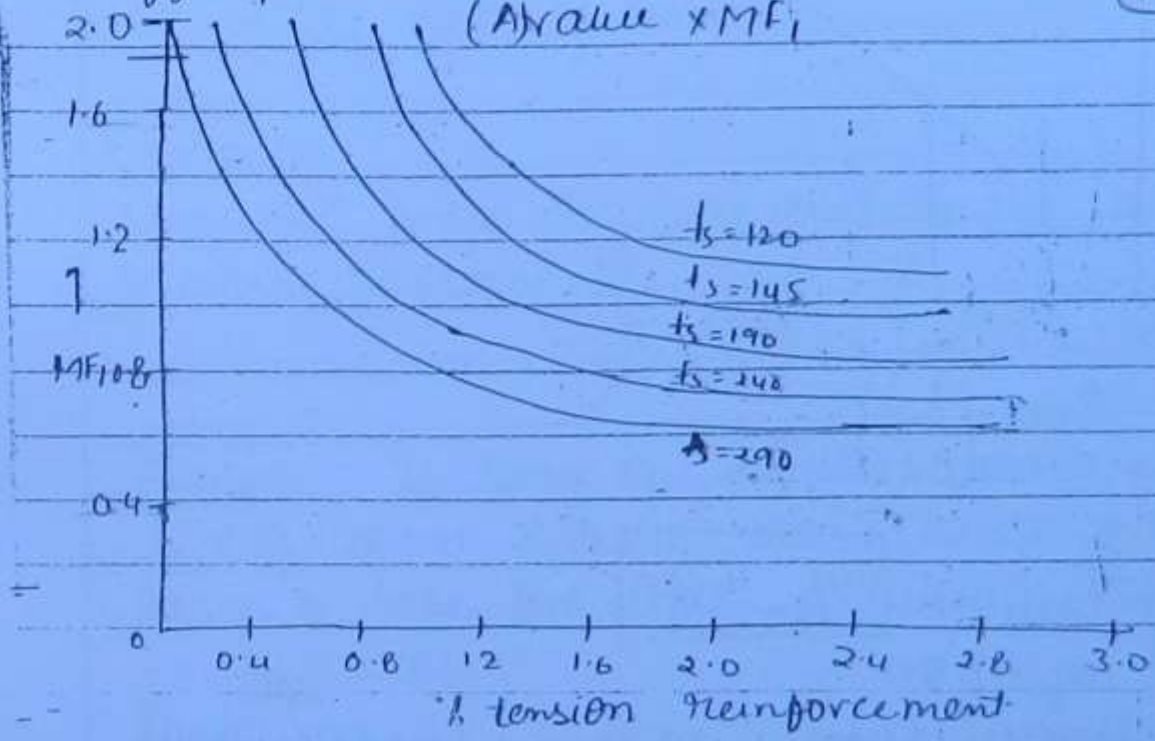
$\text{Eff} = \frac{20 \times 10^3}{20 \times \frac{10}{20}} = 2\text{m or } 2000\text{mm}$

Modification factor (MF_1) based on tension reinf^n

A value can be multiplied by a Modification factor MF_1 based on tension reinf provided.

$$\text{Eff depth} = \frac{\text{Span}}{(A)\text{value} \times MF_1}$$

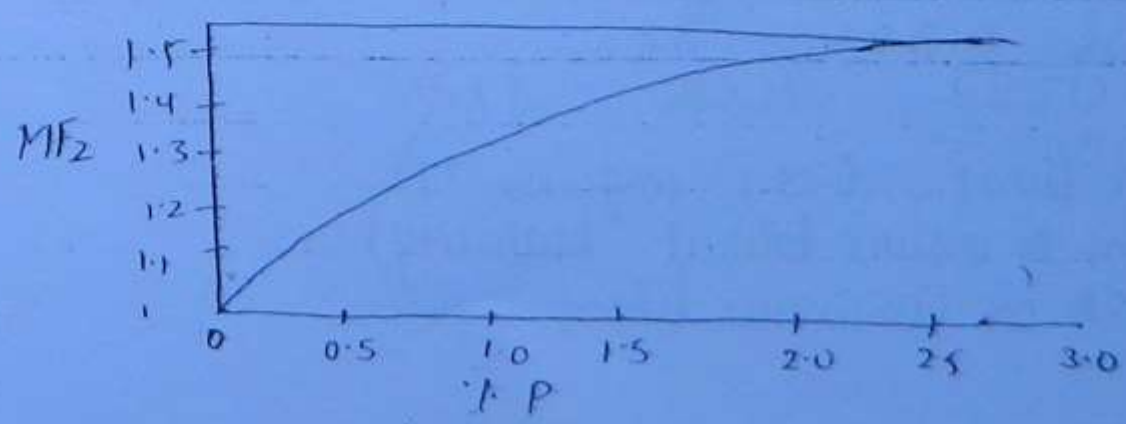
125



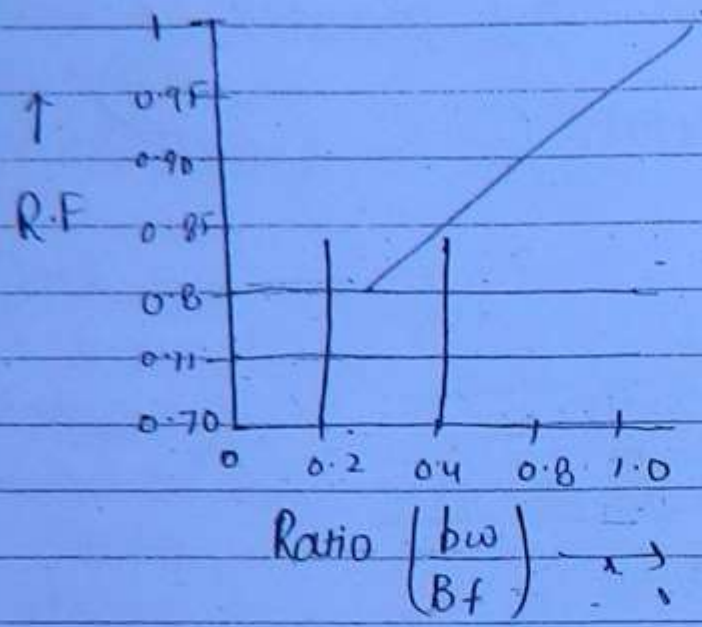
$$f_s = 0.58\, f_y \frac{A_{st}\ \text{req}}{A_{st}\ \text{provided}}$$

Modification factor (MF_2) based on Comp. reinforcement

$$\text{Eff depth} = \frac{\text{Span}}{(A)\text{value} \times MF_1 \times MF_2}$$



For flanged section (T-beam / L-beam)
Reduction factor is used.



Two way slab $\boxed{l_y/l_x < 2}$

Check for deflection

Only for shorter span < 3.5 m (upto load ≤ 3 kN/m²)

Span to overall depth ratio

Simply Supported = 35
Continous — 40 } for Mild steel

For HYSD Bars of grade fe 415 the values shall be multiplied by 0.6

S.S.B = 0.6 × 35 = 26

Continous - 0.6 × 40 = 32

Slenderness limit (23.3 - page No - 39)
(For beams to ensure lateral stability) -

① For S.S.B or Continous beam

Clear disⁿ betⁿ lateral restraint $\not> 60b$ } or
$\not> \frac{250b^2}{d}$ } (small)

b → width

d → eff. depth of beam

(127)

Cantilever

Clear disⁿ from the free end of the Cantilever to lateral restraint $\not> 25b$ } less
$\not> \frac{100b^2}{d}$ }

Example - If span of Cantilever beam is 5000 mm size of beam 250x450 mm. Check whether beam is safe for deflⁿ & lateral stability

→ two way

Deflⁿ →

Eff depth = $\frac{Span}{7} = \frac{5000}{7} = 714.29$ mm

> 450 mm

lateral stability = $\frac{100 b^2}{d}$

$= \frac{100 \times 250^2}{400} = 15625$ mm

(safe

$25 \times b = 25 \times 250 = 6250$ mm

> 5000 mm

OK

Steel Reinforcement (Beams)

Min tension reinforcement

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$$

128

Max tension reinf^n

= 4% of total Cross Section area

= 0.04 bD

Compression reinf^n

Max Comp reinf^n $\not>$ 0.04 bD

Slab

Min reinforcement

Mild Steel → 0.15% of total Cross Section

$$\frac{0.15}{100} \times BD$$

HYSD Bars → $\frac{0.12}{100} \times B \cdot D$

Max Spacing of bars

(i) for main reinf^n

= 3d or 300 mm

(ii) for distribution bars

= 5d or 450 mm

~~Max dia~~ Max dia of bars for slab

$\not>$ $\frac{\text{Depth}}{8} = \frac{1}{8} \times$ total thickness of slab.

If a slab is 75 mm thick

Max dia = $\frac{75}{8}$ = 9.375

Max 8mm ϕ bars is used.

130

DATE:

Design of One way slab ($l_y/l_x > 2$)

A slab is said one way only if :-

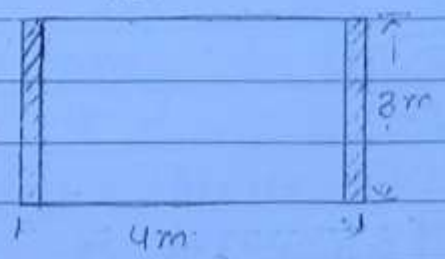
(i) If the slab is supported only on two support (opposite) (Span ratio is not to be seen) (fig i)

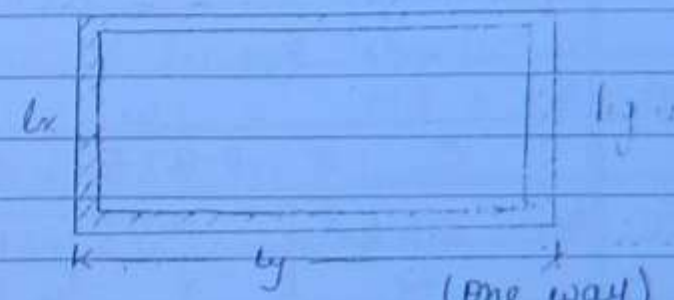
(ii) If the slab is supported on all four sides and span ratio $\frac{l_y}{l_x} > 2 \longrightarrow$ One way slab

(13)

$\frac{l_y}{l_x} \leq 2 \longrightarrow$ two way slab

fig (i)





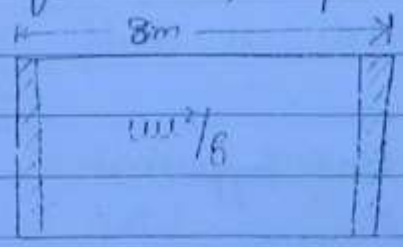
(One way) ($l_y/l_x > 2$)

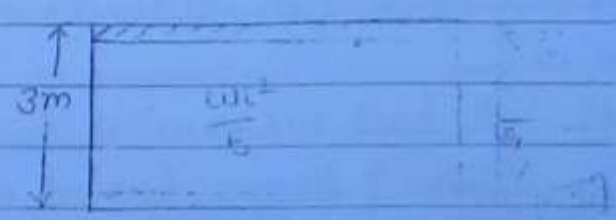
In fig (ii)

l_y = longer span & l_x = Shorter span

→ In this case slab is designed for shorter span only

→ In longer span dir^n only distribution bars (min. Steel reinforcement) is provided





Only shorter span is effective in design & design will be done for shorter span only due to above defl^n calculated. As we know moment is proportional to defl^n only so we in one way we design only for shorter span.

Design Steps

(1) Load over 1m x 1m of slab is calculated

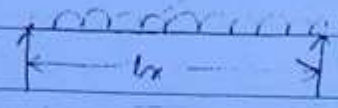
Dead load = $t \times 1 \times 1 \times 25 = w_d$

Live load = $w_l \times 1 \times 1 = w_l$

Floor finish = $t_f \times 1 \times 1 \times 24 = w_f$

Total load = w

(2) Bending Moment = $B.M = \frac{w \cdot l_x^2}{8}$



(3) Depth $d = \sqrt{\frac{B.M}{Q.B}}$ $B = 1000mm$

(4) $A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d} = \frac{M_u}{0.87 f_y \cdot j \cdot d}$ (132)

or $A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B \cdot d^2}}\right] \times B \cdot d$

(Useful for under reinforced section & also for limiting section)

(5) Check for shear

(6) Check for Bend

(7) Check for developement length

:-1984

Ques A hall 10x40m is covered by R.C.C slab supported over T-beam spaced at 4m interval

idth of beam =250mm

Floor finish = 60 kg/m²

Live load = 200 kg/m²

Use M-15 mild steel use

Design one of the intermediate slab

Solⁿ For one slab

Clear span $l_x = 3.75$ m

$l_y = 10$ m

Assume thickness of slab using deptⁿ criteria

Eff. depth = $\frac{Span}{26} = \frac{3750}{26} = 144mm$

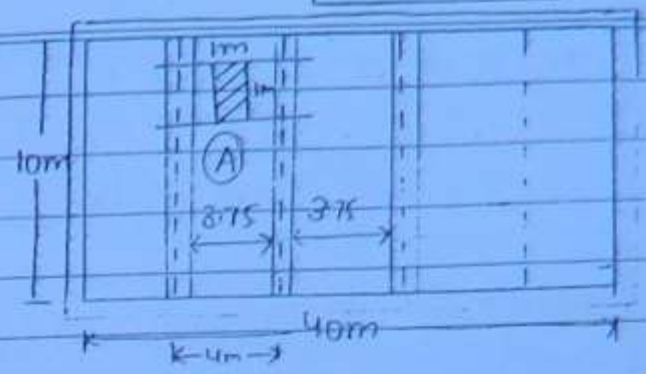
DATE:

Eff. Cover = 25 mm

= 169 mm

Say D = 170 mm

d = 145 mm



Effective span

Width = 1 = 250 mm

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$\frac{Span}{12} = \frac{3750}{12} = 312.5$ mm

$\left(1 < \frac{Span}{12}\right) \Rightarrow$ leff is same as simply supported

leff = $l_0 + d$ = 3.75 + 0.145 = 3.895 m } small

or $l_0 + w$ = 3.75 + .25 = 4.0 m

l_x = 3.895 m

Similarly l_y = 10.145 m

Span ratio $\frac{l_y}{l_x} = \frac{10.145}{3.895} = 2.6 > 2$

Hence it is one way slab

Slab shall be designed for l_x = 3.895 m

Load

Consider 1m width of slab

Dead load

Self wt = .17 × 1 × 1 × 25 = 4.25 kN/m²

Floor finish = 60 × 10 × 10^{-3} × 1 × 1 = 0.60 kN/m²

Live load

w_l = 2.0 × 1 × 1 = 2.0 kN/m²

Bending moment (Tom from Coefficient table)

$B.M = w_d \times \frac{1}{16} l_x^2 + w_l \times \frac{1}{12} \times l_x^2$

$M(+ve) = \frac{1}{16} \times 4.85 \times 3.895^2 + \frac{1}{12} \times 2 \times 3.895^2$

M(+ve) = 7.13 kN

(-ve) B.M $= -\frac{1}{10} W_d l_x^2 - \frac{1}{9} w_l l_x^2$

$= -\frac{1}{10} \times 4.85 \times 3.895^2 - \frac{1}{9} \times 2 \times 3.895^2$

(-ve) B.M = 10.73 kN

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Depth $d = \sqrt{\frac{B.M}{Q.B}}$

$d = \sqrt{\frac{10.73 \times 10^6}{0.87 \times 1000}}$

$d = 111\ mm < 145\ mm$

(safe)

$k = \frac{mc}{t + mc}$

$= \frac{19 \times 5}{140 + 19 \times 5}$

$= 0.404$

$j = 1 - \frac{k}{3} = 0.865$

$Q = \frac{1}{2} c \cdot j \cdot k$

$= \frac{1}{2} \times 5 \times .404 \times .865$

$= .87$

$A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d}$

(+ve) B.M $= \frac{7.13 \times 10^6}{140 \times 0.865 \times 145} = 406\ mm^2$

$\frac{a_{st}}{A_{st}} = S$

use 10mm ϕ bars spacing

$S = \frac{1000}{406} \times \frac{\pi}{4} \times (10)^2 = 193\ mm$

provide 10mm ϕ bars @ 190 mm c/c

top

(-ve) B.M $= \frac{10.73 \times 10^6}{140 \times 0.865 \times 145} = 611\ mm^2$

Spacing of 10 mm ϕ mo bars

$S = \frac{1000}{611} \times \frac{\pi}{4} \times 10^2 = 128\ mm$

provide 10mm ϕ bars @ 125 mm c/c

Check for shear

S.F $= 0.55 w_d \cdot l_x + 0.60 w_l \cdot l_x$

$= 0.55 \times 4.85 \times 3.895 + 0.60 \times 2 \times 3.895$

$V = 15.06$ kN

$\tau_v = \frac{V}{b \cdot d} = \frac{15.06 \times 10^3}{1000 \times 145} = 0.104$

$\tau_{c\,min} = 0.18$ $(\tau_v < \tau_c)$ safe

No shear reinforcement is required.

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Check for bond.

For (+ve) reinforcement 10mm ϕ bars @ 190 c/c

S.F at Support $V = 15.06$ kN

Bond stress developed

$\tau_{bd\,(developed)} = \frac{V}{\Sigma o \cdot j \cdot d} = \frac{15.06 \times 10^3}{\left(\frac{1000}{190}\right) \times \pi \times 10 \times 0.865 \times 145}$

n $\Sigma o = n \times \pi \phi$

$\tau_{bd} = 0.726$ N/mm²

$\tau_{bd\,(permissible)} = 0.6$

$\tau_{bd} > \tau_{bd\,(permi)}$ → failed

Change dia of bars provided

Use 8mm dia bars for +ve reinforcement

Spacing $= 190 \times \frac{8^2}{10^2} = 121.16$

Provide 8mm ϕ bars @ 120mm c/c

τ_{bd} developed $= \frac{15.06 \times 10^3}{\left(\frac{1000}{120}\right) \times \pi \times 8 \times 0.865 \times 145}$

$= 0.57 < 0.6$ (safe)

Distribution bars

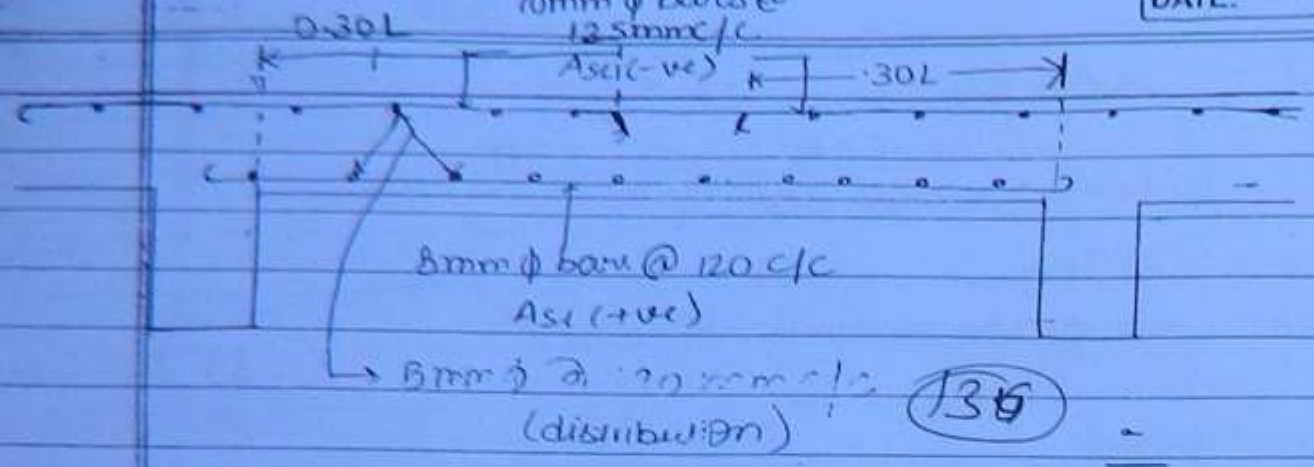
$= 0.15\%$ of total cross section area

$\geq \frac{0.15}{100} \times BD$

$\geq \frac{0.15}{100} \times 1000 \times 170 = 255$ mm²

Spacing of 8mm ϕ bars is $\frac{1000}{255} \times \frac{\pi}{4} \times 8^2 = 197.12$ mm

Provide 8mm ϕ bars @ 190 mm c/c

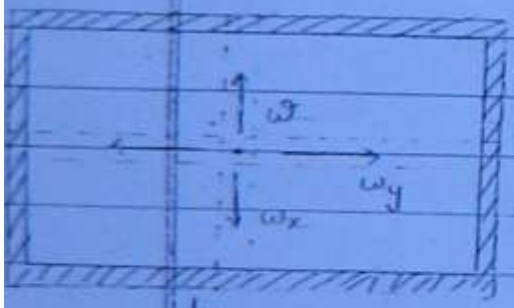


Design Of Two way Slab ($l_y/l_x \leq 2$)

1. A Slab is said to be two way only if -
1. Shall Slab should be supported on all four sides
2. Span Ratio ≤ 2

Design

1. Rankine Grashoff Method
(Simply Supported Slab)
With corner not held down position



(Corners are free to lift in upward dirn)

No negative B.M will generate (pure Simply Supported Case)

If w = total load

$w = w_x + w_y$ — 1

defl^n $\delta_x = \frac{5}{384} \frac{w_x l_x^4}{EI}$

$\delta_y = \frac{5}{384} \frac{w_y l_y^4}{EI}$

In every case $\delta_x = \delta_y$ $\frac{5}{384} \frac{w_x l_x^4}{EI}$, $\frac{5}{384} \frac{w_y l_y^4}{EI}$

DATE:

$$w_x l_x^4 = w_y l_y^4$$

$$\frac{w_x}{w_y} = \left(\frac{l_y}{l_x}\right)^4 = r^4$$

$$\boxed{w_x = r^4 w_y} \quad \text{—— (2)}$$

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Total load $w = r^4 . w_y + w_y$

$$(1+r^4) w_y$$

$$w_y = \left(\frac{1}{1+r^4}\right) w \quad \text{—— (3)}$$

$$w_x = r^4 w_y = \left(\frac{r^4}{1+r^4}\right) w \quad \text{—— (4)}$$

BM in x-dirn $\Rightarrow M_x = w_x \cdot \frac{l_x^2}{8} = \left(\frac{r^4}{1+r^4}\right) w \cdot \frac{l_x^2}{8}$

$$M_y = w_y \cdot \frac{l_y^2}{8} = \left(\frac{1}{1+r^4}\right) w \cdot \frac{l_y^2}{8}$$

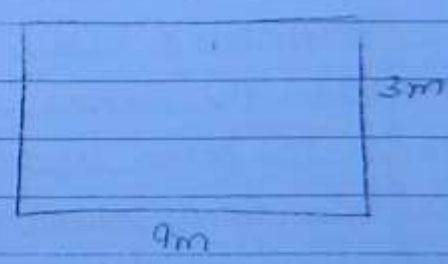
This is as per rankine grashoff theory

Eg. $w_x = \left(\frac{r^4}{1+r^4}\right) w$

$r = 3 = l_y / l_x$ (one way slab)

$w_x = \left(\frac{3^4}{1+3^4}\right) w = 0.99w$

$w_y = \left(\frac{1}{1+r^4}\right) w = 0.01 w$



2. Simply supported slab (when corners held down position)
(Negative B.M will develop at support and corner)
Method
(i) Pigeaud's method
(ii) Marcus Method
(iii) IS Code Method.

DATE:

1. Pigeaud's Method

$$M_x = \eta_x' \cdot \frac{W_x \cdot l_x^2}{B}$$

$$M_y = \eta_y' \frac{w_y \cdot l_y^2}{B}$$

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here η_x' & η_y' are Coefficients values are read from table.

2. Marcus Method. (Modification over Rankine grashoff Method)

Marcus Correction

$$C = \left(1 - \frac{5}{6}\frac{\eta^2}{1+\eta^4}\right)$$ is applied.

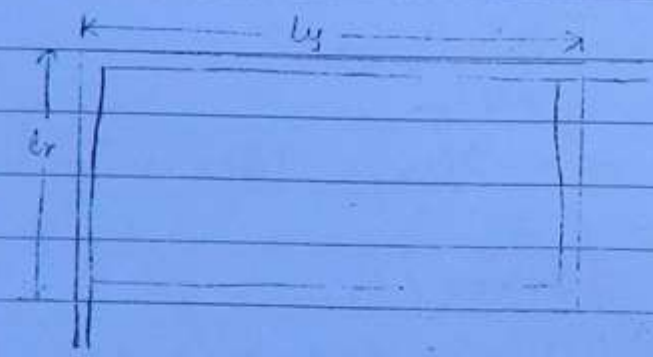
$$M_x = C \cdot w_x \cdot \frac{l_x^2}{B}$$

$$M_y = C \cdot w_y \cdot \frac{l_y^2}{B}$$

3. I.S Code Method (ANNEX D)

Restrained Slates

→ When the Corners of slab are prevented from lifting (Corners held down position)



M_x - B.M per unit width

$$M_x = \alpha_x \, w l_x^2$$

$$M_y = \alpha_y \, w l_x^2$$

α_x & α_y are Coefficient based on Support Condition. (TABLE-26)

l_x = Shorter span

w = total load on slab per m^2 area

Four B.M are found
$M_x(+ve)$, $M_x(-ve)$, $M_y(+ve)$
$M_y(-ve)$

4m	4	3	3	4
3m	2	1	3	2
4m	4	4	STAIR CASE	7
	6m		5m	

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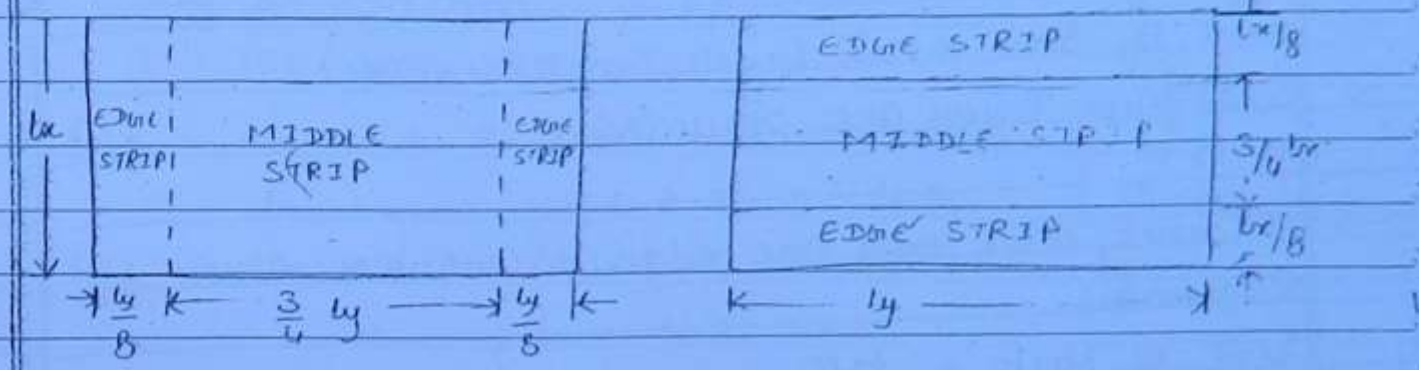
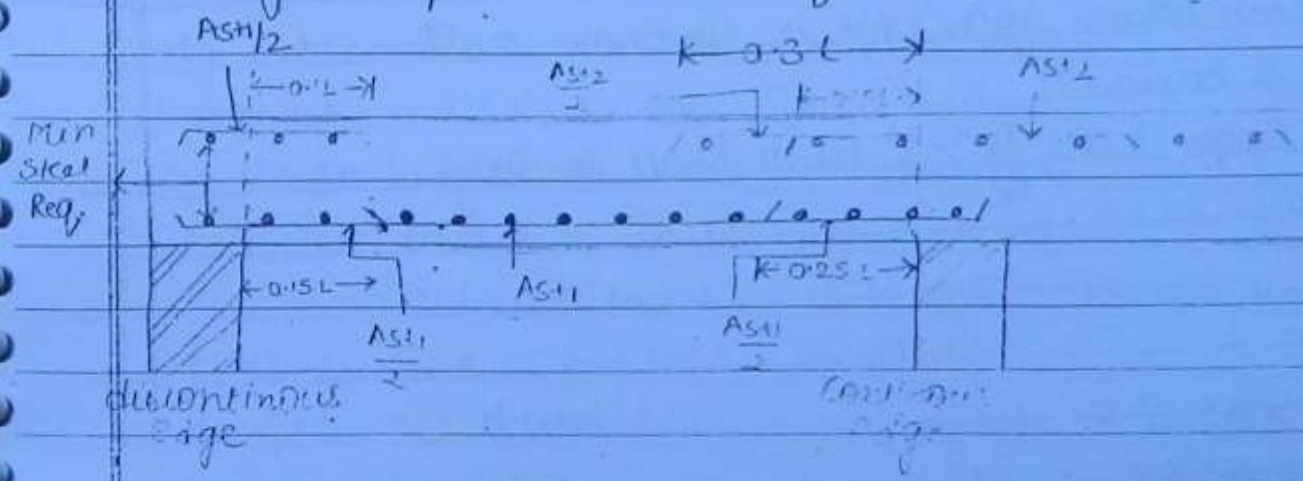
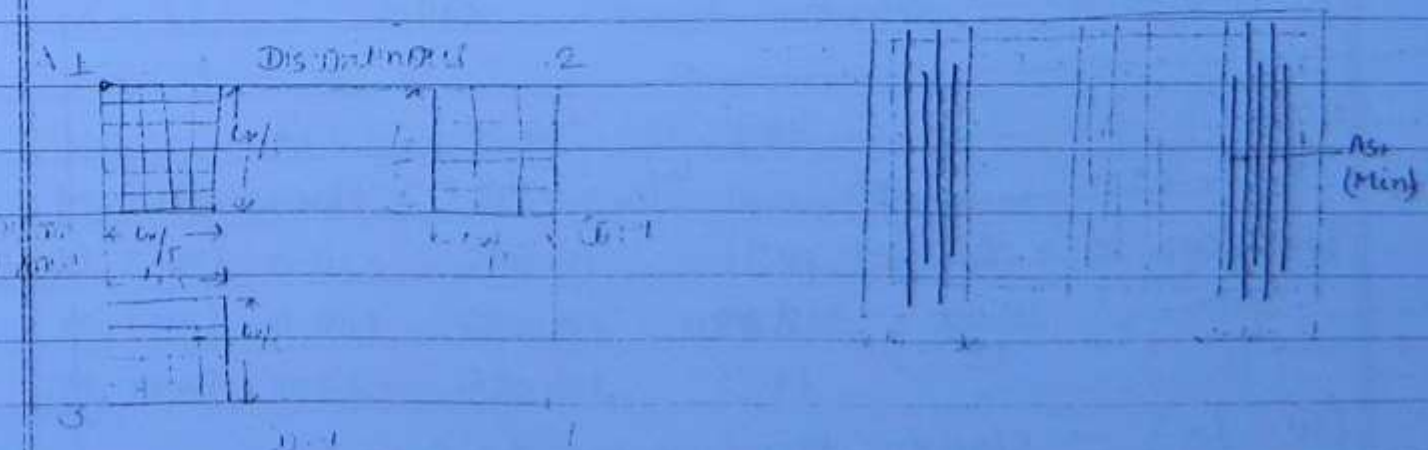
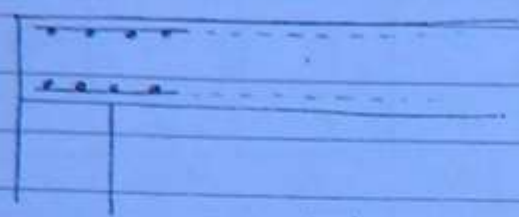


Moments M_x & M_y are applicable for middle strip only.
For edge strip min steel reinf^n required shall be provided.





→ Torsion reinforcement shall be provided at corner where two edges are discontinous (Simply supported)



(140)

Size of Mesh = $l_x/5$

Area of Steel in each layer = $\frac{3}{4} \times A_{stx}(+ve)$

→ Such four layers are provided

→ At corner when only one edge is continous other discontinous

Size of Mesh = $l_x/5$

Area of Steel = $\frac{3}{8} \times A_{stx}(+ve)$

→ At corner where both edge is continous no torsion reinf^n is required.

→ When span ratio $\frac{l_y}{l_x} > 2$ shall shall be designed as one way.

Ques [illegible]

Sol:- $l_x = 4.5\,m$; $l_y = 6$ $\quad l_y/l_x = 1.3 < 2$ (two way)

Eff span $l_1 = 300\,mm$

$\frac{Span}{12} = \frac{4500}{12} = 375\,mm$

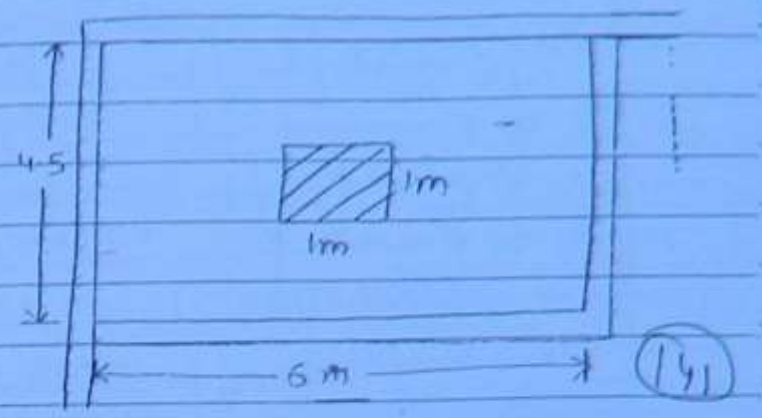
$\left(w < \frac{l_x}{12}\right)$ → Effective span as per S.S. case

DATE:

Assume thickness of slab as per deflection criteria

Continous = 0.8 X 40 = 32

Overall depth = $\frac{l_x}{32} = \frac{4500}{32}$



(141)

$D = 140.625$

Say $D = 150$ mm

$d = 150 - 30 = 120$ mm.

$l_{eff} = l_{x_0} + d = 4.50 + 0.12 = 4.62$ m

$l_{x_0} + u = 4.50 + .30 = 4.8$

l_{eff} 4.62 m is consider

$l_{eff} = l_{y_0} + d = 6.0 + 0.12 = 6.12$ m $= l_y$

$l_x = 4.62$ m

Span ratio $\frac{l_y}{l_x} = \frac{6.12}{4.62} = 1.32 < 2$ (two way slab)

Load - Calculation

Self wt = $.15 \times 1 \times 1 \times 25 = 3.75$ kN/m²

Floor finish = $1 \times 1 \times 1 \times$ [illegible] $= 2$ kN/m²

Live load = $1 \times 1 \times 4 = 4$ kN/m²

Total load = 8.75 kN/m² = w

factored load = $8.75 \times 1.5 = 13.125$ kN/m²

B.M Coefficient

$\alpha_x (+) = 0.050$ $\alpha_y (+) = \cancel{0.035}\ 0.035$

$\alpha_x (-) = 0.066$ $\alpha_y (-) = \cancel{0.047}\ 0.047$

~~$M_x (+)$~~

	α	$w \cdot l_x^2$	M	d Req.
M_x (+ve)	0.05	280.145	14.0	Provided 81.85
M_x (-ve)	0.066	280.145	18.49	Provide 120mm
M_y (-ve)	0.047	280.145	13.16	
M_y (+ve)	0.035	280.145	13.11	

Depth $d = \sqrt{\frac{M_u}{Q \cdot B}} = \sqrt{\frac{18.49 \times 10^6}{1.38 \times 20 \times 1000}} = 81.85 < 120$ mm (Safe)

DATE:

→ Ast

$$A_{st_1} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] \times B \cdot d$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 14.01 \times 10^6}{20 \times 1000 \times 120^2}}\right] 1000 \times 120$$

$$= 344.0 \ mm^2$$

$$A_{st_2} = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 18.4 \times 10^6}{20 \times 1000 \times 120^2}}\right] 1000 \times 120$$

$A_{st_2} = 464 mm$

$A_{st_3} = 236 mm$

$A_{st_4} = 322 mm$

142

Spacing of 10 mm ϕ bars

$$\frac{1000}{A_{st}} \times \frac{\pi}{4} \times 10^2 = \frac{78539.82}{A_{st}}$$

x (+) $S_1 \rightarrow 228 mm \rightarrow$ 10 mm ϕ @ 228 mm c/c

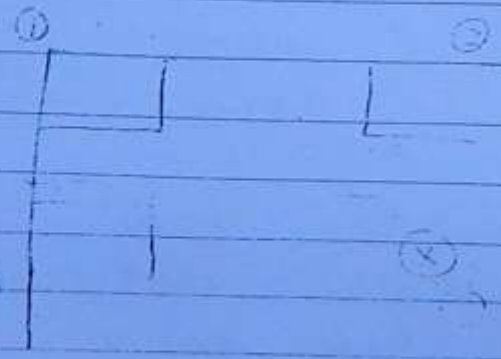
x (−) $S_2 \rightarrow 169 mm \rightarrow$ 10 mm ϕ @ 160 mm c/c

y (+) $S_3 \rightarrow$ 10 mm ϕ — 332; 8 mm ϕ — 213 → 8 mm ϕ @ 210 mm c/c

y (−) $S_4 \rightarrow$ 10 mm ϕ — 244; 8 mm ϕ — 156 → 8 mm ϕ @ 150 mm c/c

Torsion reinf^n

At Corner (1)



Size of Mesh $= \frac{l_x}{5} = \frac{4620}{5}$

$= 924 mm$

Area of Steel $= \frac{3}{4} \times A_{st_x}(+ve)$

$= \frac{3}{4} \times 344 = 258 mm^2$

Spacing (10 mm ϕ)

$$S = \frac{1000}{258} \times \frac{\pi}{4} \times 10^2 = 300 mm$$

8mm — Spacing = 194mm

Torsion reinf^n at Corner 1

$M_x(+)$ — Provide 10mm @ 220 c/c

$M_x(-)$ — 10mm ϕ @ 160 c/c

$M_y(+)$ — 8mm ϕ @ 105 c/c

$M_y(-)$ — 8mm ϕ @ 150 mm c/c

Corner 2/3

10 mm ϕ @ 220 mm c/c

10 mm ϕ @ 320 mm c/c

8 mm ϕ @ 210 mm c/c

8mm ϕ @ 300 mm c/c

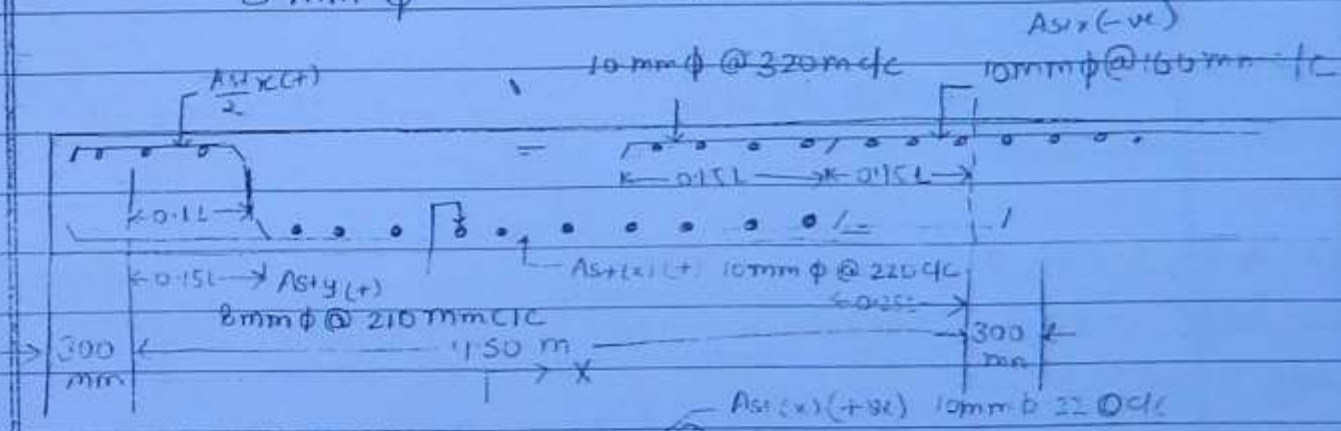
At Corner ②/③ (143)

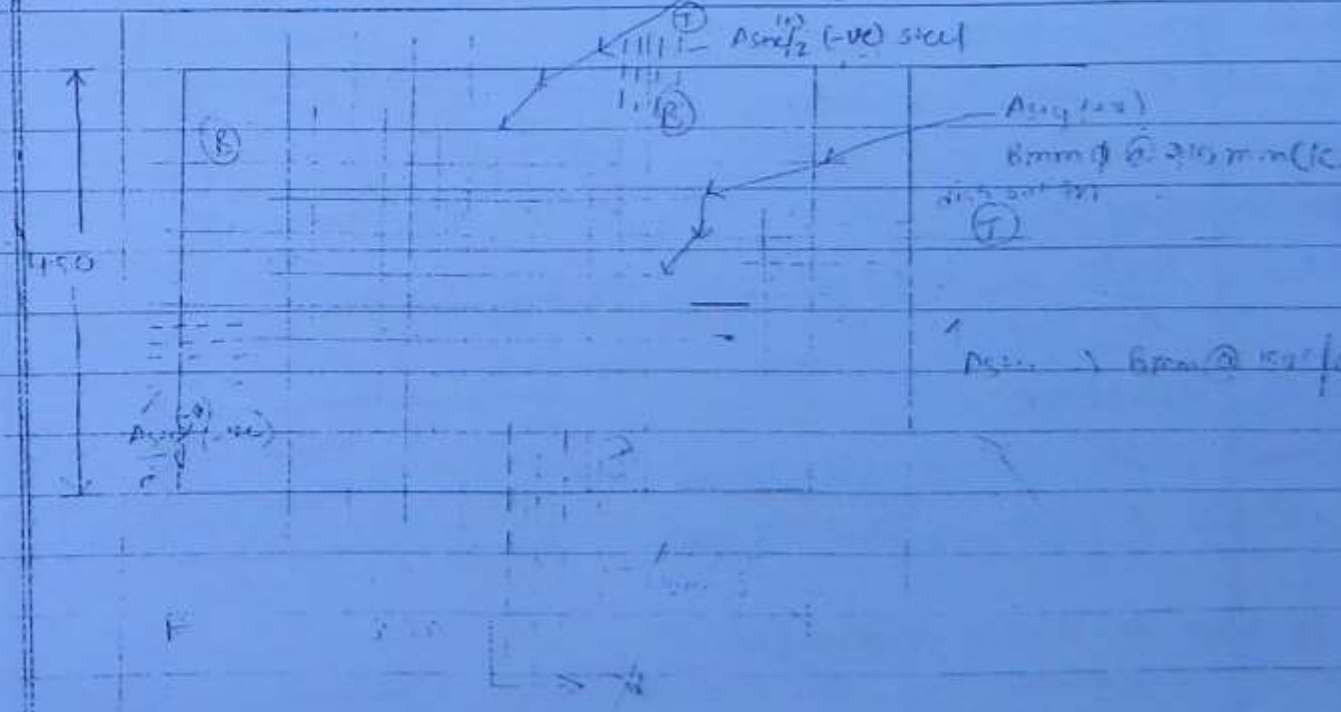
Size of Mesh = $l_x/5$ = 924 mm

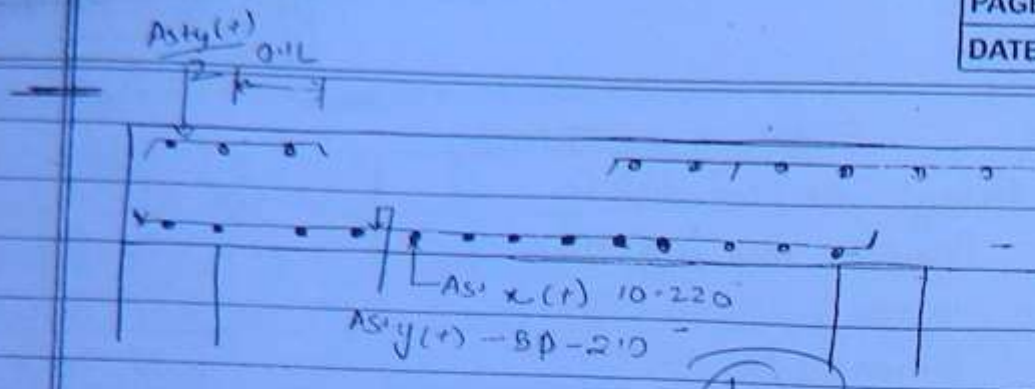
Area of Steel = $\frac{3}{8} \times A_{st\,x}(+ve) = \frac{3}{8} \times 344 = 129\ mm^2$

Spacing 10 mm ϕ = 600 mm

8 mm ϕ = 388 mm





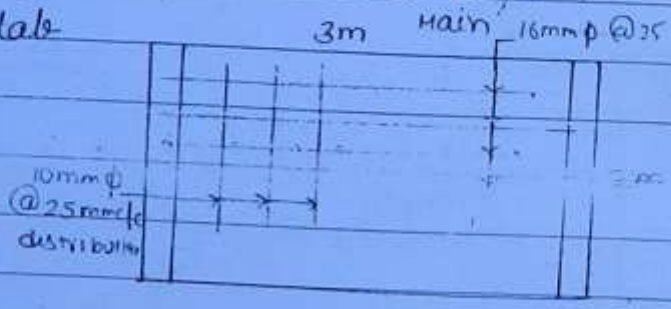


144

A rectangular R.C.C. slab 2m x 3m is simply supported along shorter edge such that clear distance between supports is 2.7m. The slab is 15 cm thick. Steel 16mm ϕ @ [illegible] c/c at cliff [illegible] 25mm along along longer edge [illegible] ϕ @ 25 mm c/c [illegible] 2 edge use M-15 / M-15 [illegible] is used. Permissible bond Bearing [illegible], shear [illegible] kg/m² [illegible] kg/m² [illegible] Calculate max load [illegible] load the slab can carry [illegible] the height.

Ans - Let us consider 1m width of slab

It is one way slab

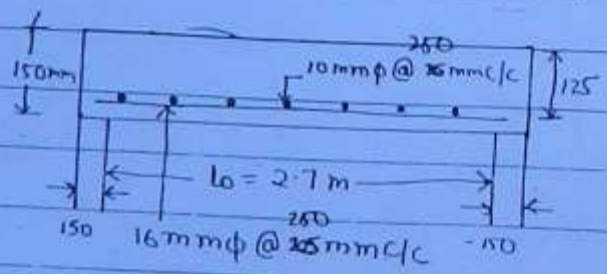


Eff. Span

$l_{eff} = l_0 + d = 2.7 + .125 = 2.825\,m$

$l_0 + d = 2.7 + .15 = 2.85\,m$

$l_{eff} = 2.825$

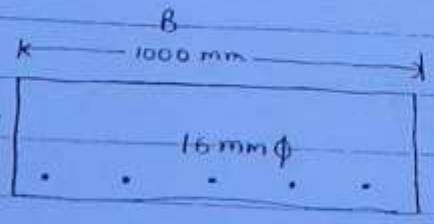


Consider w kN/m is total load i.e self wt. the beam can carry.

Check for bending :-

$$B.M = \frac{wl^2}{8} = \frac{w \times 2.825^2}{8} = .9976\,w \text{ kN-m}$$

Moment of Resistance



B = 1000 mm ; d = 125 mm ; D = 150 mm

$$A_{st} = \frac{1000}{250} \cdot \frac{\pi}{4} (16)^2 = 804.25\,mm^2$$

4 Nos of bars of 16mm ϕ is provided.

DATE:

$x_c = \frac{mc}{t+mc} \times d = \frac{19 \times 5}{140 + 19 \times 5} = 50.53\,mm$

$\frac{B \cdot x_a^2}{2} = m \cdot A_{st}(d - x_a) \Rightarrow \frac{1000}{2} \cdot 50 x_a^2 = 19 \times 804.25 (125 - x_a)$

$500 x_a^2 = 1910093.75 - 1910093.75 x_a$

$500 x_a^2 + 1910093.75 - 1910093.75 = 0$ (145)

$x_a = 48.38\,mm$

$x_a < x_c$ So it is under reinforced beam

$C_a < \sigma_{cbc}$ & $\sigma_{st} = t_a$

$M.R = t_a \cdot A_{st}\left(d - \frac{x_a}{3}\right) = 140 \times 804.25 \left(125 - \frac{48.38}{3}\right) \times \frac{1}{10^6}$

$= 12.26\,kN\text{-}m$

$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow C_a = \frac{x_a t_a}{(d - x_a) m} = \frac{48.38 \times 140}{(125 - 48.38) 19} = 4.653\,N/mm^2$

$M.R = B \cdot x_a \cdot \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) = 1000 \times 48.38 \times \frac{4.653}{2}\left(125 - \frac{48.38}{3}\right)\frac{1}{10^6}$

$= 12.26$

Equating B.M = MR

$0.9976 w = MR\ 12.26$

$w = 12.29\,kN/m$

Check for Shear

$V = \frac{wl}{2} = \frac{w \times 2.825}{2} = 1.4125 w$

$\tau_v = \frac{V}{Bd} = \frac{1.4125 \times 10^3 w}{1000 \times 125} = 0.0113 w$

$\tau_v = \tau_c \Rightarrow 0.0113 w = 0.30$

$w = 26.55\,kN/m$

Check for Bond

τ_{bd} (developed) $= \frac{V}{\Sigma o j d} = \frac{1.4125 w \times 10^3}{4 \times 16 \times \pi \times .865 \times 125}$

$= 0.06497\,w$

τ_{bd} permissible $= 0.6\,N/mm^2 \Rightarrow \tau_{bd}$ developed

τ_{bd} (developed) = τ_{bd} (permissible)

$0.06497 w = 0.6$

$w = 9.235$ kN/m

Self wt = $.15 \times 1 \times 1 \times 25 = 3.75$ kN/m

Live load that can be provided = 5.485 kN/m over the slab.

(146)

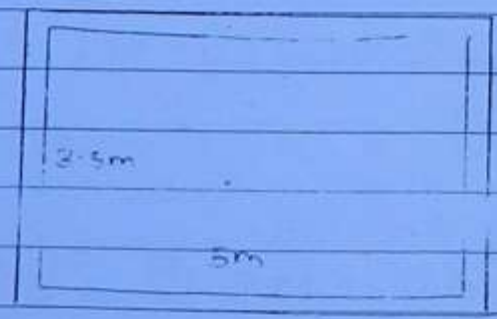
Ques Design a slab simply supported over a span of 3.5 m × 5 m clear span the slab is loaded c̄ live load of 4.0 kN/m² & floor finishes 50mm thick c-c flooring slab is prevented from lifting at corner & edges. use any method. use M-25/ fe 415 W.S.M

width of support = 250mm

Use Marcus Method

Sol^n $l_x = l_{xo} + d = 3.5 + .20 = 3.6$

$l_{xo} + l_y = 3.5 + .25 = 3.75$



Assume depth

Overall depth = $\frac{\text{Span}}{0.8 \times 35}$

$= \frac{3500}{0.8 \times 35} = 125$ mm

D = 125 mm

d = 100 mm = 0.10 m

Assume $l_x = 3.6$ m

$l_y = 5 + .1 = 5.1$ m

$\frac{l_y}{l_x} = 1.416 < 2$ two way

Load

Self wt = $.125 \times 1 \times 1 \times 25 = 3.125$

Live load = $4 \times 1 \times 1 = 4.00$ kN/m

Floor finish = $.05 \times 1 \times 1 \times 24 = 1.2$

8.325 kN/m

B.M =

$$M_x = \left(1 - \frac{5}{6}\frac{n^2}{1+n^4}\right)\left(w \cdot \frac{l_x^2}{8}\right)\left(\frac{n^4}{1+n^4}\right)$$

$$= \left(1 - \frac{5}{6}\frac{1.42^2}{1+1.42^4}\right)\left(\frac{1.42^4}{1+1.42^4}\right)\left(8.325 \times \frac{3.6^2}{8}\right)$$

$$= 7.23 \text{ kN-m}$$

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$$M_y = \left(1 - \frac{5}{6}\frac{n^2}{1+n^4}\right)\left(\frac{1}{1+n^4}\right)\left(w \cdot \frac{l_y^2}{8}\right)$$

$$= \left(1 - \frac{5}{6}\frac{1.42^2}{1+1.42^4}\right)\left(\frac{1}{1+1.42^4}\right)\left(8.325 \times \frac{5.1^2}{8}\right)$$

$$= 3.57 \text{ kN-m}$$

Depth of slab $d = \sqrt{\frac{M}{Q \cdot B}}$

$$d = \sqrt{\frac{7.23 \times 10^6}{1.11 \times 1000}} = 80.70 \text{ mm}$$

< 100 mm (Safe) OK

$K = \frac{mc}{t+mc} = \frac{11 \times 8.5}{230 + 11 \times 8.5}$

$= 0.289$

$j = 0.904$

$Q = \frac{1}{2} c \cdot j \cdot K$

$= 1.11$

Let us use $d = 100$ mm

$D = 125$ mm

Area of steel $A_{st(x)} = \frac{M}{\sigma_{st} \cdot j \cdot d} = \frac{7.23 \times 10^6}{230 \times .904 \times 100} = 348 \text{ mm}^2$

$A_{st(y)} = \frac{3.57 \times 10^6}{230 \times .904 \times 100} = 172 \text{ mm}$

Use 10 mm ϕ bars

Spacing (x) $= \frac{1000}{348} \times \frac{\pi}{4} \times 10^2 = 225$ mm

8mm ϕ Spacing $= \frac{1000}{348} \times \frac{\pi}{4} \times 8^2 = 144$ mm

Use 8mm ϕ @ 140 mm c/c

Spacing (y) $= \frac{1000}{172} \times \frac{\pi}{4} \times 8^2 = 292$ mm

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·12 % of BD

Min Steel $= \frac{.12}{100} \times 1000 \times 125$

$= 150\ mm^2$ Ok

Max Spacing = 300mm

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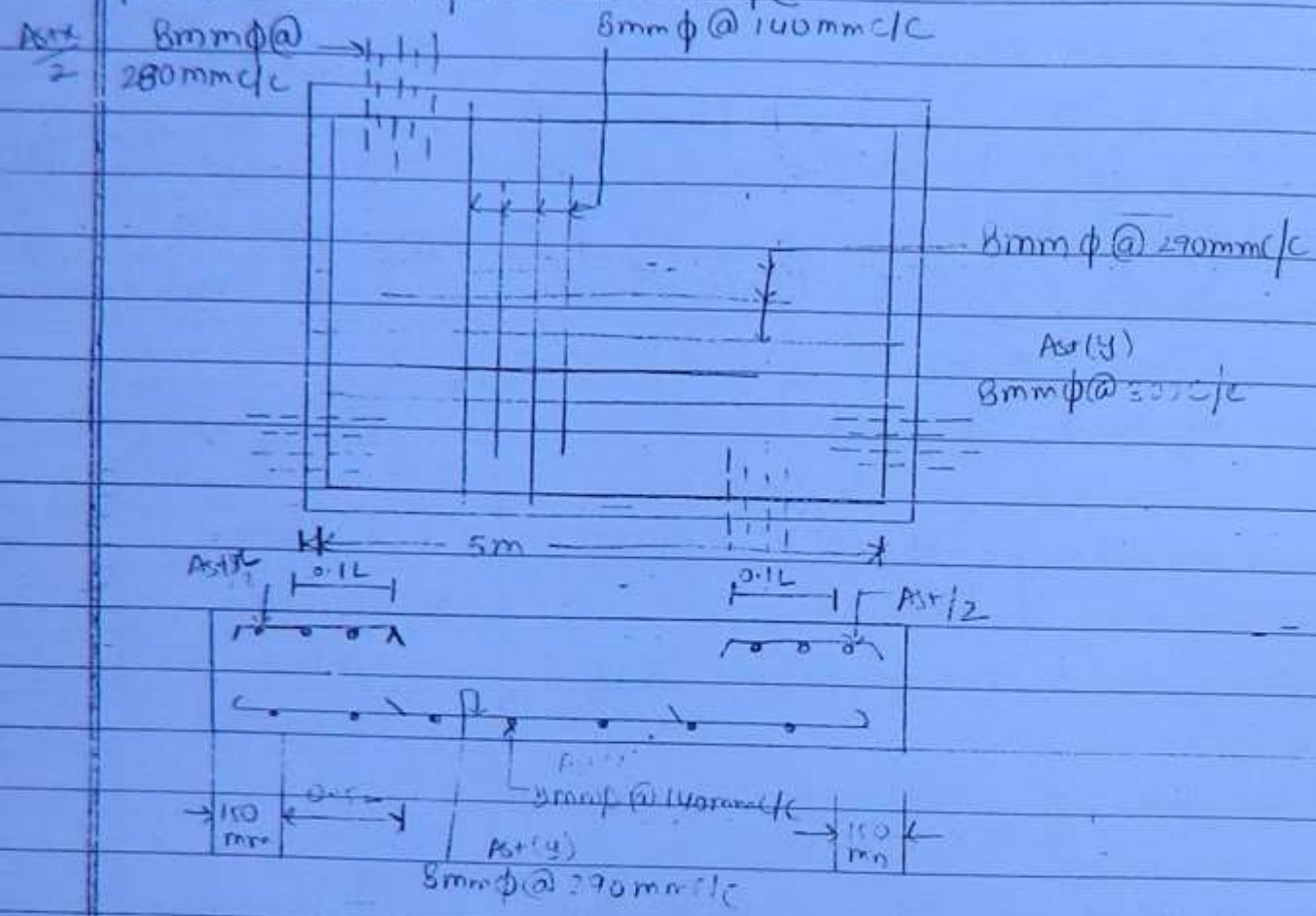
provide 8mm ϕ @ 290mm c/c



COLUMNS

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Working Stress Method

→ Long / Short Column

Slenderness Ratio = $\frac{\text{Eff. length}}{\text{least lateral dimension}}$

$\frac{l_{eff}}{B} < 12$ (Short Column)

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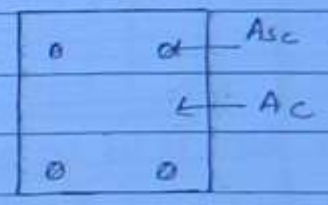
$\frac{l_{eff}}{B} > 12$ (Long Column)

— Load bearing Capacity of a Column

$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}$ — Short Column

$A_c = (A - A_{sc})$



For long Column

Load Carrying Capacity

$P = C_r \cdot [A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}]$

Where

$C_r = \left(1.25 - \frac{l_{eff}}{48B}\right)$ Reduction Coefficient

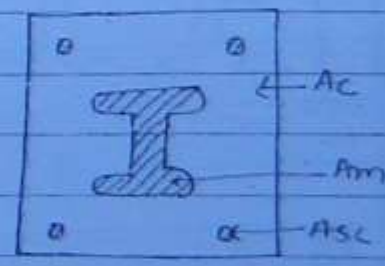
B = least lateral dimension

$C_r = \left(1.25 - \frac{l_{eff}}{160\, i_{min}}\right)$

i_{min} = least radius of gyration $= \sqrt{\frac{I}{A}}$

For a Composite Column

$A_c = (A - A_{sc} - A_m)$



Load Carrying Capacity

$P = C_r [A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc} + A_m \cdot \sigma_{mc}]$

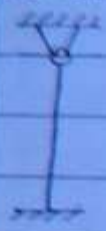
Effective length (TABLE 28)

Fixed – effectively held in position & restrained against Rotation

Theoritical = $0.50L$ – $L/2$

Recommended = $0.65L$

2 Effectively held in position but not restrained against Rotation (one end hinge & other end is fixed)



Theoritical – 0.70L $L/\sqrt{2}$

Recommended – 0.80L

150

3 Held in position but not restrained against Rotation (Both ends are hinge).

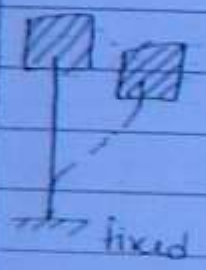
theoritical – 1.0 L

Recommended = 1.0 L

4 Not held in position but restrained against Rotation

Theoritical 1.0 L

Recommended 1.2 L

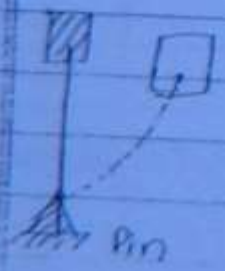


5 Not held in position & partially restrained against Rotation.

Theoritical – —

Recommended – 1.50L

6



Theoritical – 2.0L

Recommended – 2.0L

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Theoritical 2.0L
Recommended 2.0L

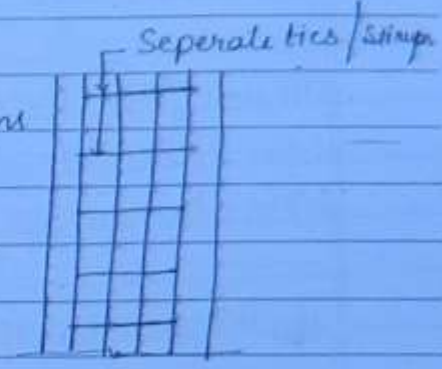
(15)

Design Of Circular Column

(a) (with Seperate Stirupps)

Design same as for simple R.C.C columns

$P = G_r (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$

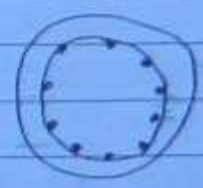


(b) With helical Reinforcement (Tie)

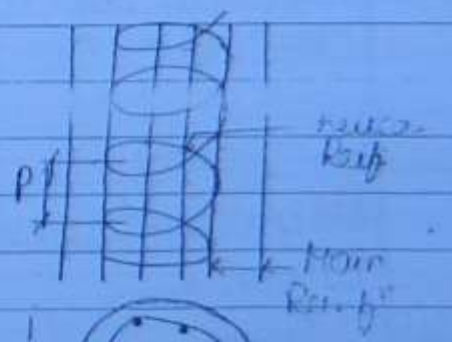
Load carrying capacity

$P = 1.05\ G_r (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$

For helical Reinforcement following criteria should be satisfied.



page No. 7 $0.36 \frac{f_{ck}}{f_y} \left[\frac{A_g}{A_c} - 1 \right] \leq \frac{V_h}{V_c}$ — A



Where

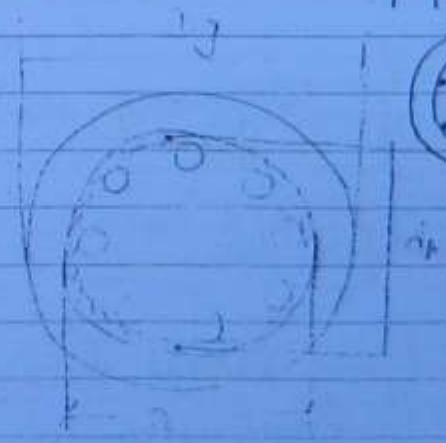
A_g = gross area = $\frac{\pi}{4} (D_g)^2$

D_g = gross diameter

A_c = Core area

$D_c = D_g = 2 \times$ Clear cover

$A_c = \frac{\pi}{4} D_c^2$



V_c = Volume of core area for 1 m length of column

$\frac{\pi}{4} (D_c)^2 \times 1000$

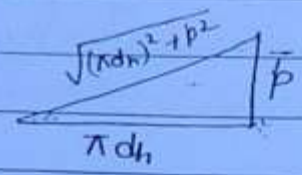
V_h = Volume of helical reinf^n in 1 unit length of column = (No. of turns) × length in 1 turn × c/s area of helix

$$V_h = \left(\frac{1000}{p}\right) \times (\pi d_h) \times \left(\frac{\pi}{4} \times \phi_h^2\right)$$

Where $d_h = d_c - \phi_h$

length in one turn

$= \sqrt{(\pi d_h)^2 + p^2}$



ϕ_h = diameter of helix.

152

Other I.S Recommendation

Min percentage of steel = 0.8% of total area

Max percentage of steel = 6% of total c/s area (If bars are not lapped)

If bars are lapped = 4.0% of total area

Minimum no. of bar – 4 – Rectangular

6 – Circular

Main Reinf) Min. dia of bars – 12mm dia of bars

Max^n dis^n betn main bars (i.e longitudinal) = 300 mm

Design of transverse reinforcement (Tie)

Diameter

(i) $\frac{\phi_{main}}{4}$

(ii) 6mm

} whichever is greater

ϕ_{main} for larger dia

Spacing – ① Least lateral dimension

② 16 ϕ_{main} (min dia)

③ 300 mm

Pitch – Pitch of helical reinforcement

① ≯ 75 mm

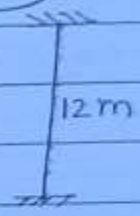
② ≯ $d_c/6$

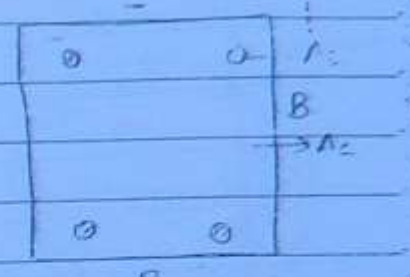
③ ≯ 3ϕ

Design a Column to carry a load of 1000 kN Column is 12m in length and restrained at both end use - M20 & Fe - 415 Design a Square Column.

(153)

$l = 12m$





Effective length of Column

$l_{eff} = 0.65 l$

$= 0.65 \times 12 = 7.80 m$

Assume steel 1% of Total C/s area

$A_{sc} = 0.01 A$

$A_c = A - A_{sc} = B^2 - 0.01 B^2 = \underline{0.99 B^2}$

Design of Column

Assuming Short Column

$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}$

$1000 \times 10^3 = 190 \cdot 0.01 B^2 + 0.99 B^2 \times 5$

$B = 382 mm$

Grade	σ_{cbc}	σ_{cc}
M-15	5	4
M-20	7	5
M-25	8.5	6
M-30	10	8
M-35	11.5	9
M-40	13	10
Steel	σ_{sc}	σ_{st}
Fe-250	130	140
Fe415	190	230

Slenderness Ratio

$\frac{l_{eff}}{B} = \frac{7800}{382} = 20.4 \not> 12$

It is a long Column

For long Column

$P = C_r (A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc})$

$1000 \times 10^3 = \left(1.25 - \frac{l_{eff}}{48B}\right)\left(0.01 B^2 \times 190 + 0.99 B^2 \times 5\right)$

$1000 \times 10^3 = \left(1.25 - \frac{7800}{18336}\right)\left(0.01 B^2 \times 190 + 0.99 B^2 \times 5\right)$

$= 6.85 (1.25 B^2 - 162.5 B)$

$1.25 B^2 - 162.5 B - 145985.4 = 0$

$B = 412 mm$

Use $B = 420 mm$

$A_{st} = 0.01 \times 4 B^2 \Rightarrow 0.01 \times 420^2 = 1764 mm^2$ ($412^2 = 1697.44$)

No. of 16 mm dia bars = ~~7 = 8.7~~

Use 4 - 20 mm ϕ = 1256.64

$4-12\phi = 452.37$

total = $1709.03\ mm^2$

Stirrups

(154)

① Diameter

$\frac{\phi_{main}}{4} = \frac{20}{4} = 5mm$

② 6mm

} use 6mm

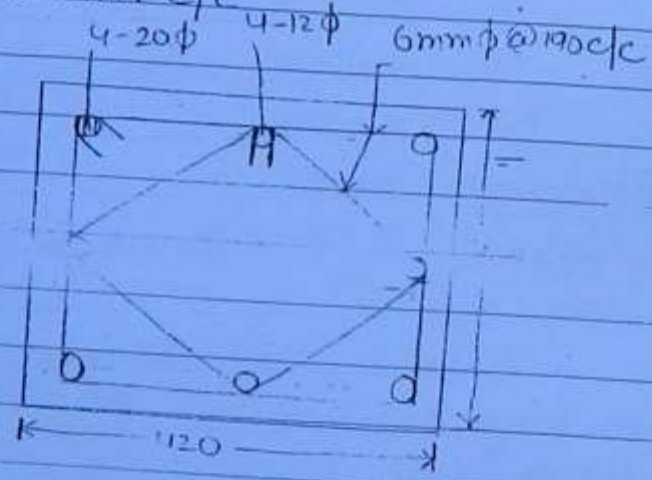
Spacing

① least lateral dimension = 420mm

② $16 \times \phi$ main (min dia) $= 16 \times 12 = 192mm$

③ 300mm

provide 6mm ϕ bars @ 190 mm c/c



Design a short circular column of 600 mm diameter for a load of 2500 kN using helical reinforcement use M-25 & Fe-415. Calculate main bars, size & pitch of helical ties.

① Design a short column — D = 600mm

$P = (A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc})\ 1.05$

$2500 \times 10^3 = 1.05\left(A_{sc} \cdot 190 + \left(\frac{\pi}{4} \times 600^2 - A_{sc}\right) 6\right)$

$2380952.381 = 190A_{sc} + 1696460.03 - 6A_{sc}$

$684492.348 = 184A_{sc}$

$A_{sc} = 3720\ mm^2$

DATE:

No. of 25 mm ϕ bars required

$$n = \frac{3720}{\frac{\pi}{4} \times 25^2} = 7.57 \text{ bay } 8 \text{ Nos.}$$

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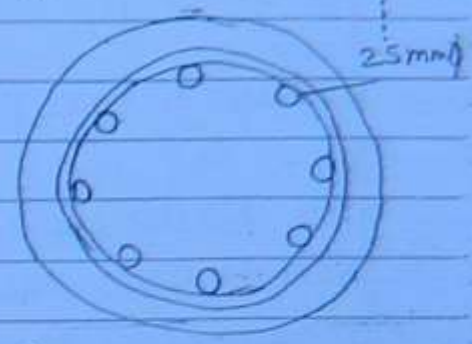


Design of helical reinforcement

$$0.36 \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \leq \frac{V_h}{V_c}$$

Where $D_g = 600mm$

$$A_g = \frac{\pi}{4}(600)^2 = 282743.33 \text{ mm}^2$$

Core diameter $D_c = D_g - 2 \times$ clear Cover

$$600 - 2 \times 40 = 520 \text{ mm}$$

$$A_c = \frac{\pi}{4} D_c^2 = \frac{\pi}{4} \times 520^2 = 213371.66 \text{ mm}^2$$

V_c for 1 m length

$$= A_c \times 1000$$

$$= 212371663.4 \text{ mm}^3$$

V_h = volume of helical reinf for 1m length of Column

Assuming dia of helical reinf^n

$$d_h = d_c - \phi_h = 520 - 8 = 512 \text{ mm}$$

$$V_h = \left(\frac{1000}{P}\right)\left(\pi \times \overset{d_h}{512}\right)\left(\frac{\pi}{4} \times \overset{\phi_h}{8^2}\right)$$

$$V_h = \frac{80851799.25}{P}$$

$$0.36 \times \frac{25}{415}(0.331) = \frac{80851799.25}{P \times 212371663.4}$$

$$7.186 \times 10^{-3} = \frac{0.3807}{P}$$

$$P = 52.98 \text{ mm}$$

DATE:

$p \not> 75$ mm OK

$p \not> \frac{d_c}{6} = \frac{520}{6} = 86.67$ mm

$p \not> 3 \times b_h = 3 \times 8 = 24$ mm

provide 8mm ϕ @ 50 mm c/c (156)

IS Recommendation

① Slenderness ratio (Page No - 42)

(a) unsupported length between end restraint $\not> 60$ times of least lateral dimension

Max. Slenderness ratio limit = 60

(b) If one end of column is unrestrained its unsupported length $l \not> \frac{100 B^2}{D}$

② Minimum eccentricity

All Columns shall be designed for a minimum eccentricity of = $\frac{\text{unsupported length of Column}}{500} + \frac{\text{lateral dimen.}}{30}$

Subjected to minimum of 20 mm

$e_{min} = \frac{l}{500} + \frac{B \text{ or } D}{30}$ or 20 mm

Whichever is greater.

→ Combined bending & direct Stress

P = Axial Compressive load

e_x = eccentricity abt Y-Y

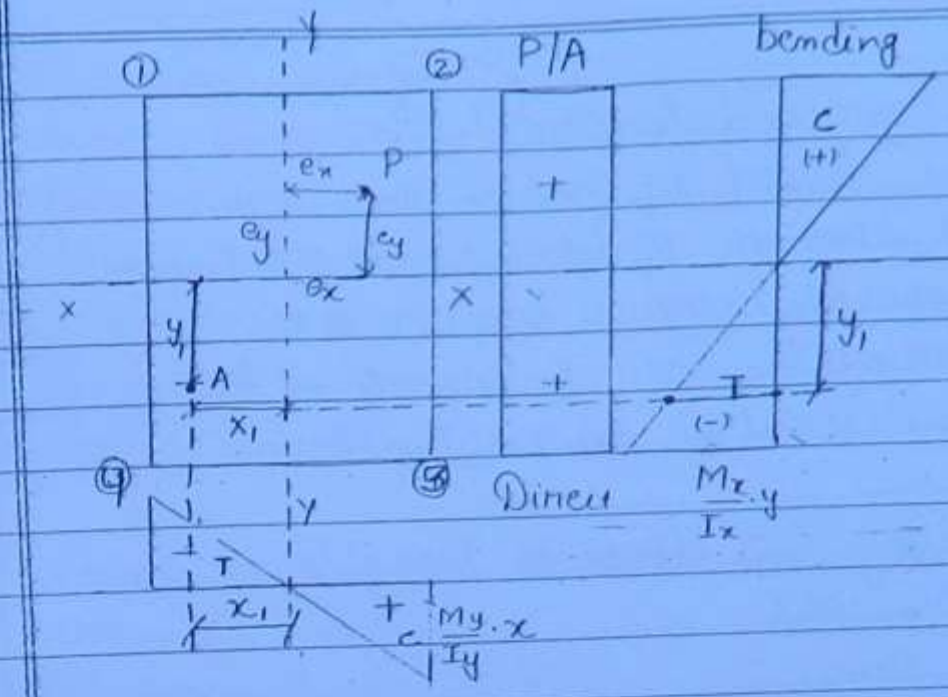
e_y = " " X-X

M_x = Moment abt X-X → $M_x = P.e_y$

M_y → $P.e_x$

Stresses

① Direct stress = $\sigma_0 = \frac{P}{A}$



(ii) Bending stress due to M_x

$$p_{bx} = \pm \frac{M_x}{I_x} y$$

(iii) Bending stress due to M_y -

$$p_{by} = \pm \frac{M_y}{I_y} x$$

Final stresses at any point on the column section -

$$p_0 + p_{bx} + p_{by}$$

$$= \pm \frac{P}{A} \pm \frac{M_x}{I_x} \cdot y \pm \frac{M_y}{I_y} \cdot x$$

At A point in fig. (x_1, y_1)

$$\text{final stress} = + \frac{P}{A} - \frac{M_x}{I_x} \cdot y_1 - \frac{M_y}{I_y} \cdot x_1$$

Stress at four corners of column

Corner 1 $= \frac{P}{A} + \frac{M_x}{I_x} \cdot \left(\frac{D}{2}\right) - \frac{M_y}{I_y} \cdot \left(\frac{B}{2}\right)$

2 $= \frac{P}{A} + \frac{M_x}{I_x} \left(\frac{D}{2}\right) + \frac{M_y}{I_y} \left(\frac{B}{2}\right)$

3 $= \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2}\right) + \frac{M_y}{I_y} \left(\frac{B}{2}\right)$

4 $= \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2}\right) - \frac{M_y}{I_y} \left(\frac{B}{2}\right)$

Design Criteria

1. Design based on uncracked Section (158)

→ If tensile Stresses develop in Concrete are within permissible tensile Strength of Concrete, Area in tension need not to be ignored. Design is based on total c/s area, without ignoring the area of Concrete in tension. This design principle is called design based on uncracked Section.

In this case following eqn should be satisfied.

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$$

if satisfied, column is safe as uncracked Section.

here $\sigma_{cc\,cal}$ = Direct Stress developed $= p_o = \frac{P}{A}$

$\sigma_{cbc\,cal}$ = Max Stress developed in Concrete in bending Compression at point no 2.

$$= \frac{M_x}{I_x}\left(\frac{D}{2}\right) + \frac{M_y}{I_y}\left(\frac{B}{2}\right)$$

6,8,9,10 σ_{cc} – Max. permissible stress in direct Compression

B5,10,11,8 σ_{cbc} – Max. permissible stress in bending Compression

Design based on Cracked Section

If the Criteria $\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$ is not satisfied

the Column Section should be checked as cracked secn

① Ignoring the area of Concrete in tension

② Tensile stresses are taken by steel alone.

Actual stresses developed in the Section are calculated. If the stresses are less than permissible limits Column is considered safe.

DATE:

Different Cases :-

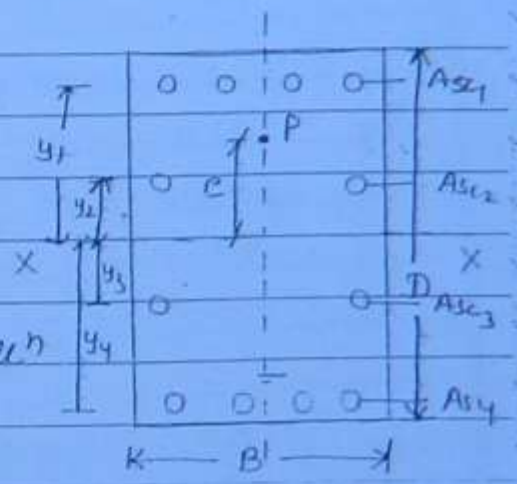
① Small eccentricity case $(e < D/4)$ (159)
Column is mostly designed as uncracked secⁿ

② Medium eccentricity case $(e > D/4)$ but $(e < 1.5D)$
(Complicated case.) but design acc. to cracked secⁿ

③ Long eccentricity case $(e > 1.5D)$
Column is design as cracked secⁿ due to high moment

→ Small eccentricity (Design based on uncracked secⁿ)

→ Uniaxial bending
(A symmetrical secⁿ)
Load = P
eccentricity abt X-X = e
Moment abt X-X = $M_x = P \cdot e$



If Column is safe as uncracked secⁿ

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc\,cal}}{\sigma_{cbc}} \leq 1$$

$$\sigma_{cc\,cal} = \frac{P}{A_{eq}} =$$

Where A_{eq} = Equivalent area of Column secⁿ

$$= A_c + (1.5m - 1)\, A_{sc}$$

$$A_{eq} = BD + (1.5m - 1)(A_{sc_1} + A_{sc_2} + A_{sc_3} + A_{sc_4})$$

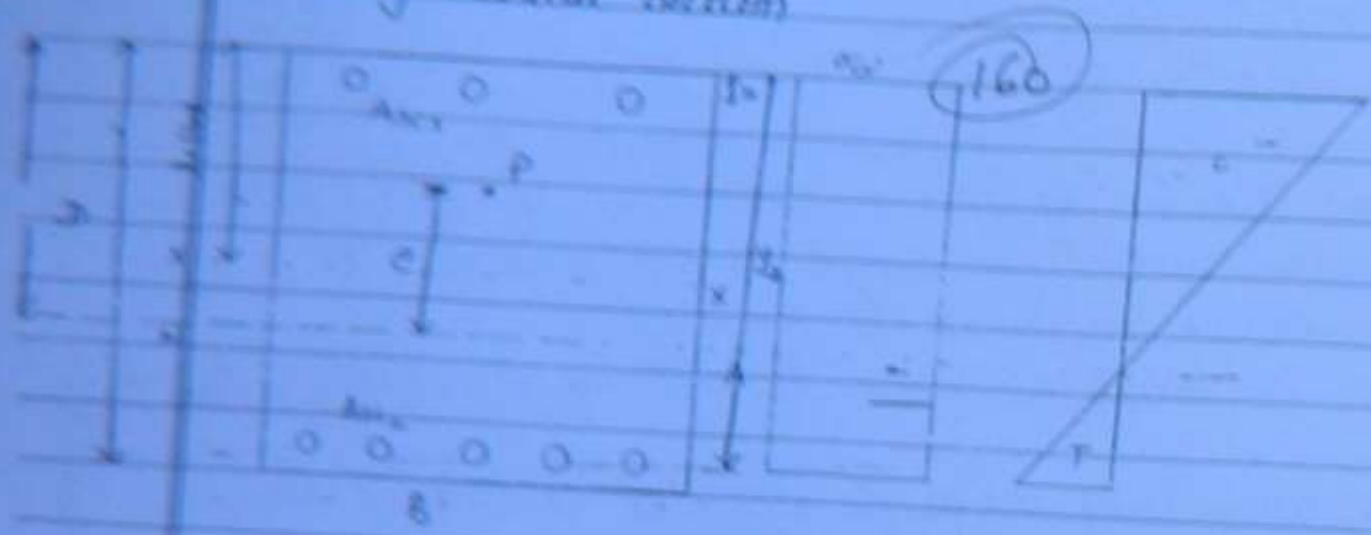
$$\sigma_{cbc\,cal} = \frac{M_x}{I_{eq}} \cdot \left(\frac{D}{2}\right) \quad \text{(at top stress)}$$

Where $I_{eq} = \frac{BD^3}{12} + (1.5m - 1)\left[A_{sc_1} \times y_1^2 + A_{sc_2} \times y_2^2 + A_{sc_3} \times y_3^2 + A_{sc_4} \times y_4^2\right]$
(Compute)

$$I_{eq} = \frac{BD^2}{12} + (1.5m - 1)\left[\Sigma A_{sc} \cdot y^2\right]$$

put these values in eqⁿ. If eqⁿ get satisfied then the column section is safe otherwise failed.

For unsymmetrical Section



If a load P is acting at an eccentricity e from x-x axis

$A_{eq} = B \times D + (1.5m - 1)(A_{sc_1} + A_{sc_2})$

C.G of Section (N.A) from Top

$$\bar{y} = \frac{B \cdot D \cdot \frac{D}{2} + (1.5m-1)(A_{sc_1} \cdot y_1 + A_{sc_2} \cdot y_2)}{A_{eq}}$$

$$= \frac{\frac{BD^2}{2} + (1.5m-1)(A_{sc_1} \cdot y_1 + A_{sc_2} \cdot y_2)}{B \cdot D + (1.5m-1)(A_{sc_1} + A_{sc_2})}$$

Net eccentricity $e_{net} = e + \left(\bar{y} - \frac{D}{2}\right)$

Moment $= P \cdot e_{net} = M$

Equivalent moment of Inertia

$$I_{eq} = \frac{BD^3}{12} + BD\left(\bar{y} - \frac{D}{2}\right)^2 + (1.5m-1)\left[A_{sc_1}(\bar{y} - y_1)^2 + A_{sc_2}(\bar{y} + y_2)^2\right]$$

$$\sigma_{c.c.\,cal} = \frac{P}{A_{eq}}$$

$$\sigma_{c.b.\,cal} = \frac{M}{I_{eq}} \bar{y} = \frac{P \cdot e_{net}}{I_{eq}} \times \bar{y}$$

Check $\frac{\sigma_{c.c.\,cal}}{\sigma_{c.c.\,all}} + \frac{\sigma_{c.b.\,cal}}{\sigma_{c.b.\,all}} \leq 1$

Hence Safe

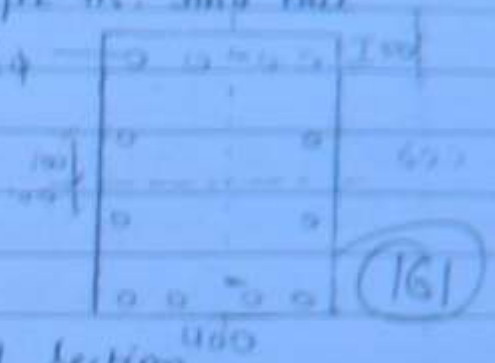
DATE:

A Column Secⁿ of size 400 x 600 mm is reinforced with 12 bars of 25 mm ϕ as shown in fig. Use M-25 & Fe-415. Find out

(i) max. eccentricity about X-X & a load can be applied so that their is no tension in the section

(ii) Max value of load that can be applied at eccentricity calculated above.

Column should be safe as unmasked section.



Solⁿ Uniaxial bending case

$A_{eq} = BD + (1.5m-1)[\Sigma A_{sc}]$

$= 400 \times 600 + (1.5 \times 11 - 1)(12 \times \frac{\pi}{4} \times 25^2)$

$= 331302.5 \ mm^2$

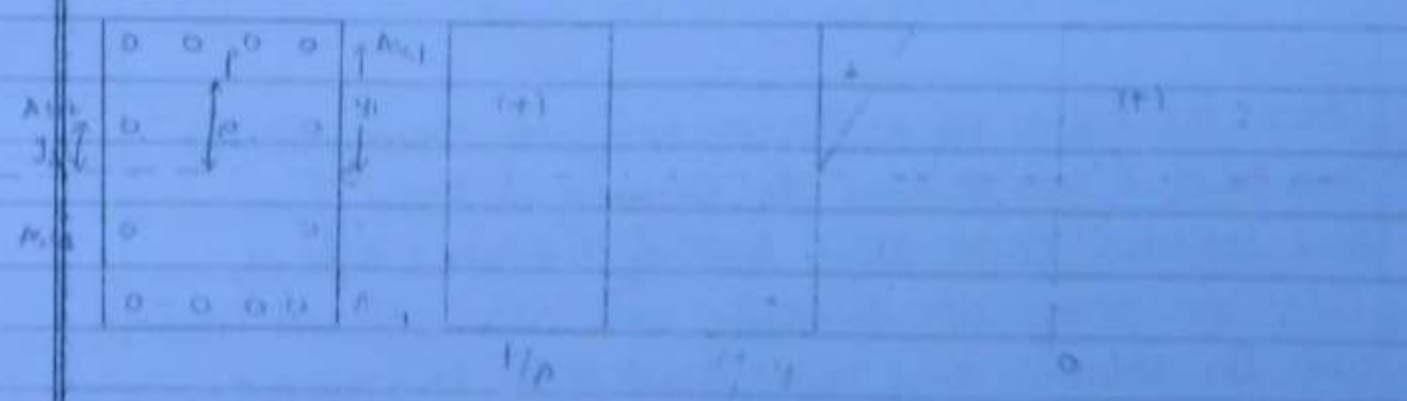
N.A is at AA

Moment of Inertia

$I_{eq} = \frac{BD^3}{12} + (1.5m-1)[A_{sc1} \cdot y_1^2 + A_{sc2} \cdot y_2^2]$

$= \frac{400 \times 600^3}{12} (1.5m-1)\left[2 \times 4 \times \frac{\pi}{4} \times 25^2 \times 250^2 + 2 \times 2 \times \frac{\pi}{4} \times 25^2 \times 100^2\right]$

$= 1.13086 \times 10^{10}$



If there is no tension, stress at bottom = 0

$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} \cdot y = 0$

$\frac{P}{A_{eq}} - \frac{P \cdot e}{I_{eq}} \cdot y = 0 \Rightarrow \frac{P}{A_{eq}} = \frac{P \cdot e \cdot y}{I_{eq}}$

DATE:

Max. eccentricity

$$e_{max} = \frac{I_{eq}}{A_{eq} \cdot y} = \frac{1.13086 \times 10^{10}}{331302.5 \times 300}$$

$$= 113.78 \text{ mm}$$

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Value of load.

Criteria req. for unbracked Sec^n

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbcc}} \leq 1$$

$$\sigma_{cc\,cal} = \frac{P}{A_{eq}} \quad \Big\| \quad \sigma_{cbc\,cal} = \frac{M}{I_{eq}}\bar{y} = \frac{P}{A_{eq}}$$

$$\sigma_{cc} = 6 \qquad \sigma_{cbc} = 8.5$$

$$\frac{P/A_{eq}}{6} + \frac{M/I_{eq} \cdot y}{8.5} = 1$$

$$\frac{P/A_{eq}}{6} + \frac{P/A_{eq}}{8.5} = 1$$

$$\frac{P}{A_{eq}}\left(\frac{1}{6} + \frac{1}{8.5}\right) = 1$$

$$P = \frac{A_{eq}}{\left(\frac{1}{6} + \frac{1}{8.5}\right)} = \frac{331302.5}{\frac{1}{6} + \frac{1}{8.5}}$$

$$= 1165.271 \text{ KN}$$

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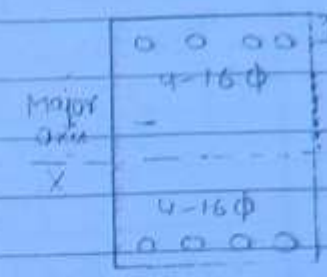
Ques Check the safety of a RCC rectangular Column 250x350 mm reinf c̄ 4-16 ϕ bars on each of its short face and subjected to an axial load of 450 KN & B.M of 15 kN-m acting abt its major axis

Can the Column any more load if so how much?

M20 Conc, H-19 . As per code the section is safe if $\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \leq 1$

DATE:

Sol: $P = 450\ KN$ & $M = 15\ KN\text{-}m$



$A_{eq} = BD + (1.5m - 1)\ \Sigma A_{sc}$

$= 250 \times 350 + (1.5 \times 19 - 1) \times 4 \times \frac{\pi}{4} \times 16^2$

$= 131733.6\ mm^2$

$I_{eq} = \frac{BD^3}{12} + (1.5m - 1)\left\{2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2\right\}$

$= \frac{250 \times 350^3}{12} + (1.5 \times 19 - 1)\left\{2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2\right\}$

$= 1584.38 \times 10^6\ mm^4$

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$\sigma_{cc}\ cal = \frac{P}{A_{eq}} = \frac{450 \times 10^3}{131733.6} = 3.416\ N/mm^2$

$\sigma_{cbc}\ cal = \frac{M}{I} \cdot y = \frac{15 \times 10^6}{1584.38 \times 10^6} \times \left(\frac{350}{2}\right)$

$= 1.657\ N/mm^2$

$\sigma_{cc} = 5$ || $\sigma_{cbc} = 7$

Check $\frac{\sigma_{cc}\ cal}{\sigma_{cc}} + \frac{\sigma_{cbc}\ cal}{\sigma_{cbc}} = \frac{3.416}{5} + \frac{1.657}{7}$

$0.92 < 1$

Column is safe

If moment is kept same

$\frac{\sigma_{cc}\ cal}{\sigma_{cc}} + \frac{\sigma_{cbc}\ cal}{\sigma_{cbc}} = 1$

$\frac{P / 131733.6}{5} + \frac{1.657}{7} = 1$

$P = 502752\ N = 502.75\ KN$

Extra load $= 502.752 - 450$

$= 52.75\ KN$

If eccentricity value is kept same $e_1 = \frac{M}{P} = \frac{15 \times 10^6}{450 \times 10^3}$

$= 33.33\ mm$

$\frac{\sigma_{cc}\ cal}{\sigma_{cc}} + \frac{\sigma_{cbc}\ cal}{\sigma_{cbc}} = 1$

DATE:

$$\frac{P}{A_{eq}\cdot\sigma_{cc}} + \frac{P\cdot e\cdot y}{I_{eq}\cdot\sigma_{cbc}} = 1$$

$$P\left[\frac{1}{131733.6\times5} + \frac{33.33\times350/2}{1584.38\times10^6\times7}\right] = 1$$

$$P = 489.21\,kN$$

Extra load = 39.21 kN

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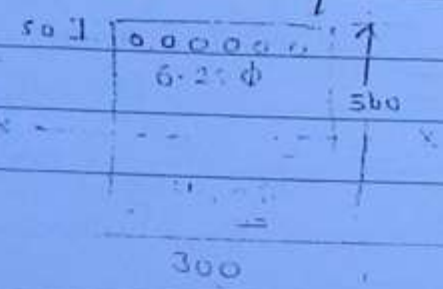
For a column section shown in fig.

(i) Find out max. eccentricity value at x-x so that no tension has developed in the section

(ii) Find out max. load value that can be applied on column at eccentricity as calculated above. Column should be safe as uncracked secn. use M-20/Fe-415

Secn properties



$= BD + (1.5m-1)\Sigma A_{sc}$

$= 300\times500 + (1.5\times13-1)\times\left[6\times\frac{\pi}{4}\times25^2 + 4\times\frac{\pi}{4}\times20^2\right]$

$= 227734.8\ mm^2$

$$\frac{30\cdot D/2 + (1.5m-1)\left[A_{sc1}\,y_1 + A_{sc2}\cdot y_2\right]}{A_{eq}}$$

$$= \frac{300\times500\times500/2 + (1.5+13-1)\left[2945.2\times50 + 1256.64\times450\right]}{227734.8}$$

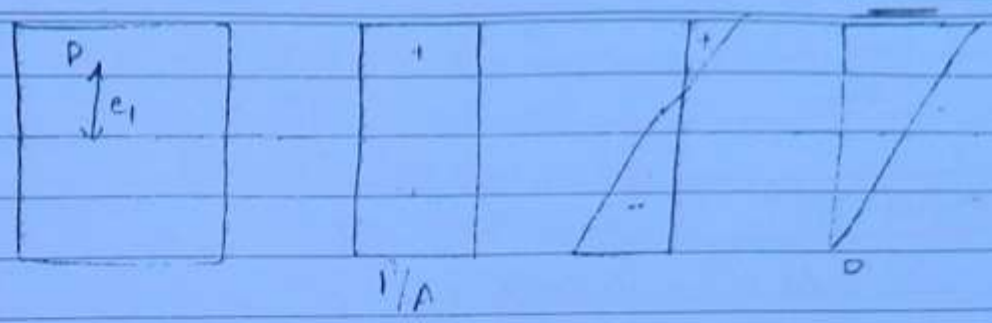
$= 222.56\ mm$

$$I_{eq} = \frac{BD^3}{12} + BD\left(\bar{y}-\frac{D}{2}\right)^2 + (1.5m-1)\left[A_{s1}(\bar{y}-y_1)^2 + A_{s2}(\bar{y}-y_2)^2\right]$$

$$= \frac{300\times500^3}{12} + 300\times500(222.56-250)^2 + (1.5\times13-1)\left[2945.2(222.56-50)^2 + 1256.64(222.56-450)^2\right]$$

$= 6062.9\times10^6\ mm^4$

CASE I



If e is above N.A stress at bottom = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_B = 0$$

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$$\frac{P}{A_{eq}} = \frac{P \cdot e_1}{I_{eq}} y_B \Rightarrow e_1 = \frac{I_{eq}}{A_{eq} \cdot y_B}$$

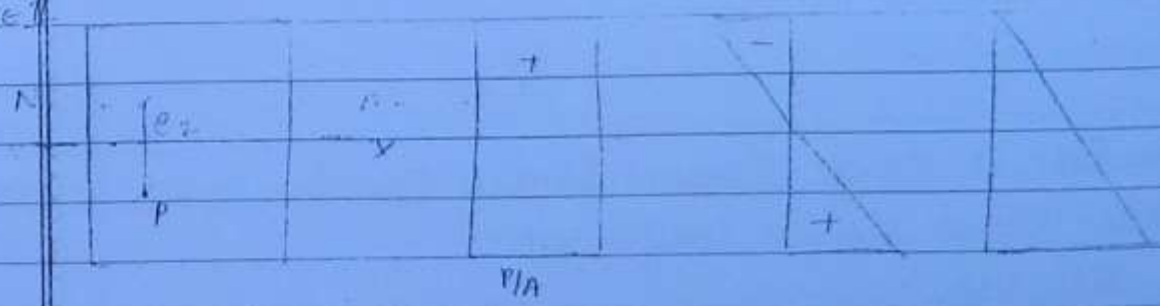
$$e_1 = \frac{6062.9 \times 10^6}{227734.78 \times (500 - 222.56)}$$

$$= 95.96 \text{ mm}$$

Eccentricity about X-X $= e_1 + \left(\frac{D}{2} - \bar{y}\right)$

$$= 95.96 + \left(\frac{500}{2} - 222.56\right) = 123.4 \text{ mm}$$

CASE II



If e is below N.A stress at top = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_t = 0$$

$$\frac{P}{A_{eq}} = \frac{P \cdot e_2}{I_{eq}} y_t \rightarrow e_2 = \frac{I_{eq}}{A_{eq} \cdot y_t}$$

$$e_2 = \frac{6062.9 \times 10^6}{227734.78 \times 222.56} = 119.62 \text{ mm}$$

Eccentricity abt X-X $= 119.62 - (250 - 222.56) = 92.18$ mm

In first case eccentricity about X-X is max.

$e_{max} = 123.4$ mm ; $e_{net} = 95.96$ mm

DATE:

$$\frac{\sigma_{cc\,cal}}{\sigma_{cc}} + \frac{\sigma_{bc\,cal}}{\sigma_{bc}} = 1 \Rightarrow \frac{P}{A_{eq}\cdot\sigma_{cc}} + \frac{P\cdot e_{net}\cdot y_t}{I_{eq}\cdot\sigma_{bc}} = 1$$

$$P\left[\frac{1}{227734.8\times 5} + \frac{95.56\times 222.56}{6062.9\times 10^4\times 7}\right] = 1$$

$$P = 724985\,N = 724.985\,kN$$

- LARGE ECCENTRICITY CASE (Design based on Cracked Section) $e > 1.5D$

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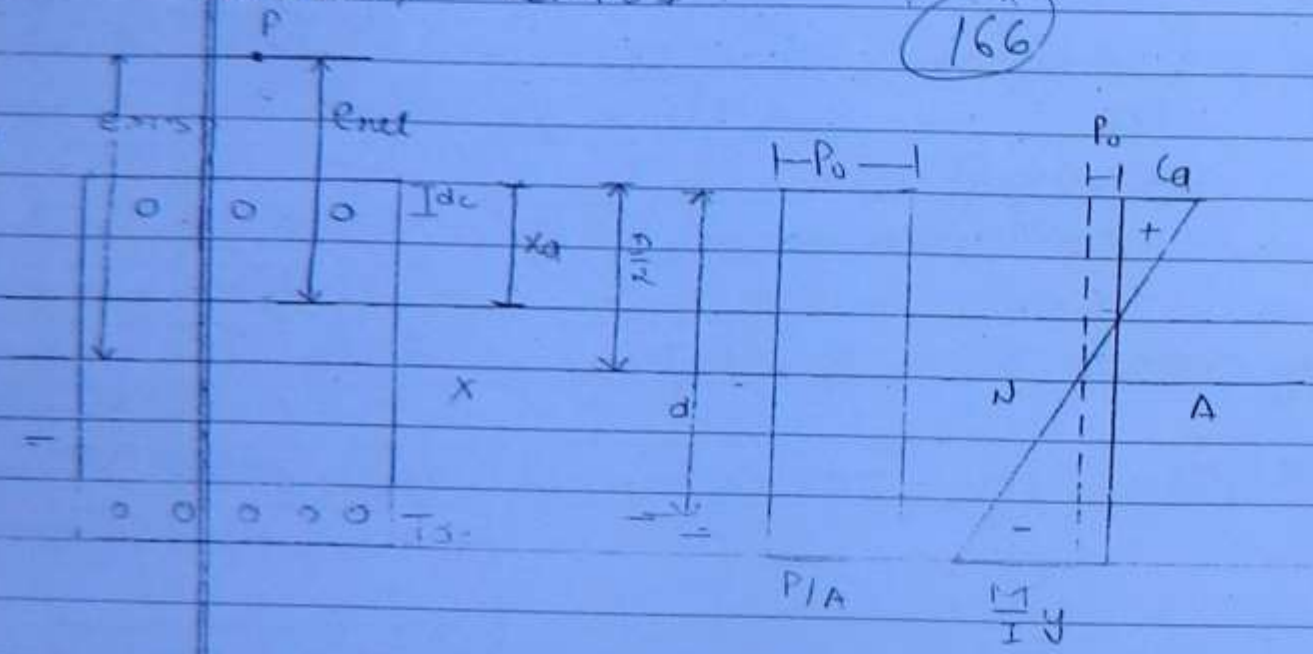

In this case a large tensile stresses are developed so area below N.A is neglected.

1. Depth of N.A

Equating moment of area on both side of N.A

$$B\cdot\frac{x_a^2}{2} + (1.5m-1)A_{sc}(x_a-d_c) = mA_{st}(d-x_a) \quad -(1)$$

2. Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a\right)$$

The column is subjected to a load P and moment $P\cdot e_{net}$

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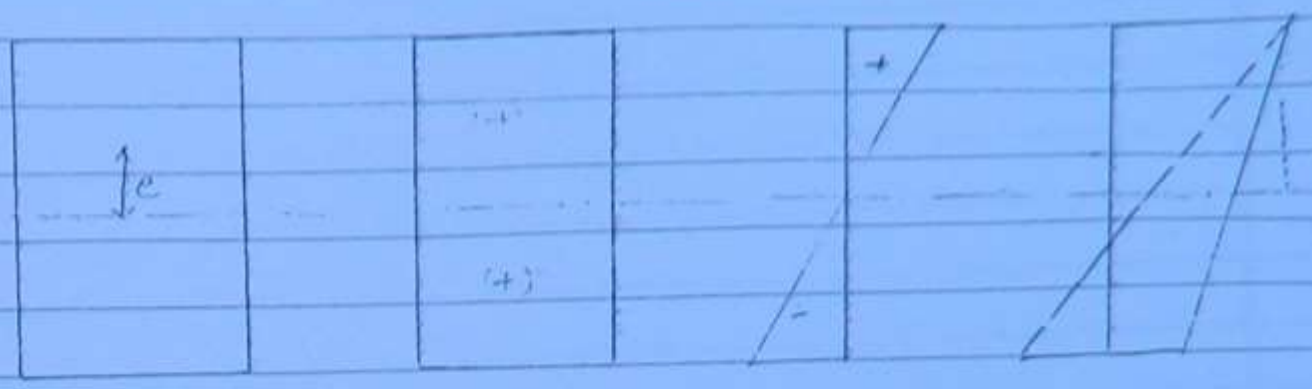

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Ques A Column Secⁿ 400x500mm size is subjected to a load of 80KN at an eccentricity of 900mm. Use M-20/Fe-415 m=13. Effective Cover = 50mm

Find out actual stress developed in the section. Check the safety of section.

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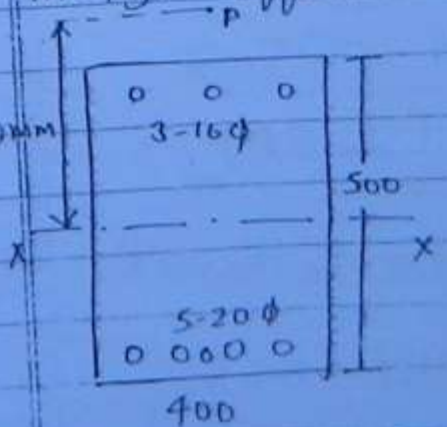

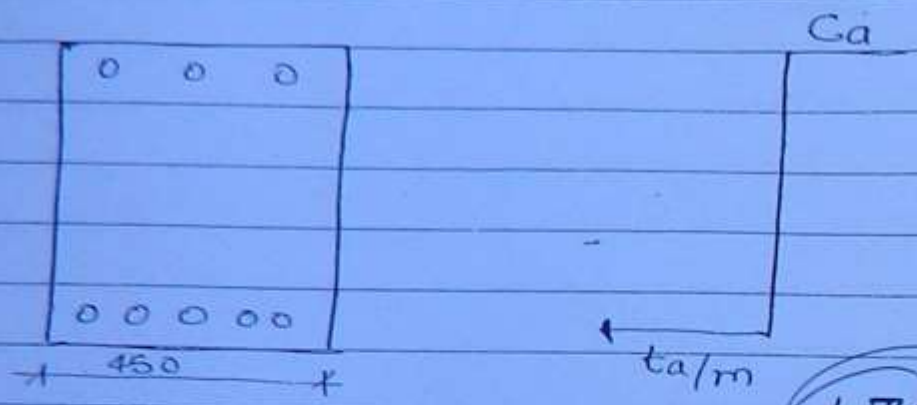

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$e = 900\,mm$

$1.5D = 1.5 \times 500 = 750\,mm$

$e > 1.5D$

~~170~~

This is a large eccentricity. Let us check the section as cracked section.

1. Depth of N.A.

$$B \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = m A_{st} \cdot (d - x_a)$$

$$900 \cdot \frac{x_a^2}{2} + (1.5 \times 13 - 1) \times$$

(2) Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a\right)$$

$$= 900 - (250 - 155.5) = 805.5\,mm$$

$(> 1.5D)$ OK.

(3). Moment

$$B.M. = P\, e_{net} = 80 \times \frac{805.5}{1000} = 64.44\ kN \cdot m$$

(4) If actual stress developed in concrete due to moment $P.e_{net} = C_a$

stress in con in comp^n steel C'

$$C' = \frac{x_a - d_c}{x_a} \times C_a = \frac{155.5 - 50}{155.5} \times C_a$$

$$= 0.678 C_a$$

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(5) Equating B.M. = M.R

$$84.44 \times 10^6 = B \cdot x_a \frac{C_a}{2}\left(d - \frac{x_a}{3}\right) + (1.5m - 1) A_{sc} \cdot c' (d - d_c)$$

$$= 400 \times 155.55 \times \frac{C_a}{2}\left(450 - \frac{155.5}{3}\right) + (1.5 \times 13 - 1) \times 603.2 \times 0.678 C_a \times (450 - 50)$$

$$C_a = 4.18 \text{ N/mm}^2$$

$$C' = 0.678 \times C_a = 2.84 \text{ N/mm}^2$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \quad \Big\| \quad t_a = \frac{(d - x_a) m C_a}{x_a}$$

$$= \frac{(450 - 155.5) \times 13 \times 4.18}{155.55}$$

$$t_a = 102.91 \text{ N/mm}^2$$

(6) Direct stresses :-

$$p_o = \frac{P}{A_{eqv.}}$$

$$= \frac{80\times10^{03}}{B.x_a + (1.5m-1)A_{sc} + mA_{st}}$$

$$= \frac{80\times10^{03}}{400\times155.5 + (1.5\times13-1)603.2 + 13\times1570.8}$$

$$= 0.85\ N/mm^2$$

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Final stresses

(i) In concrete $= p_o + c_a = 0.85 + 4.18$
$= 5.03\ N/mm^2$

$5.03 < 7\ N/mm^2$ (OK safe)

In steel (comp.)

$= 1.5m(p_o + c') = 1.5\times13\times(0.85 + 2.84) = 71.955$

$71.955 < \cancel{230}\ 190$

In tensile steel

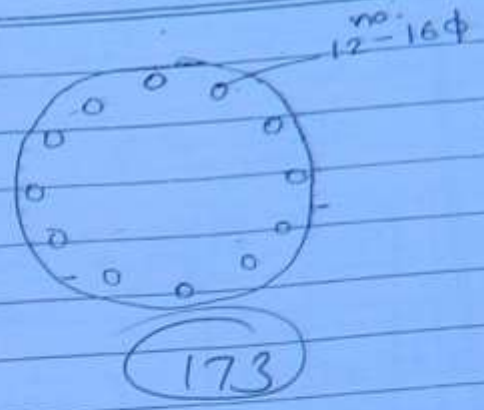
$t_a - mp_o = 102.91 - 13\times0.85 = 91.86 < 230$ OK

Question :- A circular column section of dia 600mm, has 12 nos. 16mm ϕ bars. It is subjected to a load of 1200 KN and B.M. of 150 KN-m. M25/ Fe 415.
Check whether column section is safe as uncracked section.

Sol^n :- Fire the section to be safe as uncracked section

DATE:

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \leq 1$$



(1) σ_{cc}' for $P = 1200\,kN$

$$\sigma_{cc}' = \frac{P}{A_{eqv.}}$$

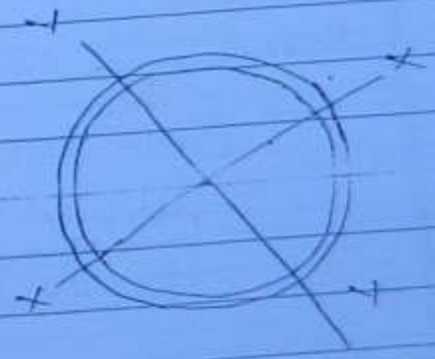
$$A_{eqv.} = \frac{\pi}{4}(D)^2 + (1.5m - 1)\,\Sigma A_{sc}$$

$$= \frac{\pi}{4}(600)^2 + (1.5m - 1) \times 12 \times \frac{\pi}{4} \times (16)^2$$

(11).

$$= 320140.85$$

$$\sigma_{cc}' = \frac{1200 \times 10^{03}}{320140.85} = 3.75\ N/mm^2$$



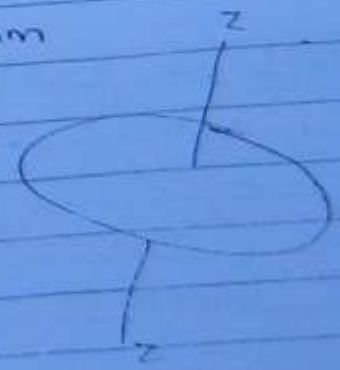
(2) $$\sigma_{cbc}' = \frac{M}{I_{eqv.}} \times y$$

For $I_{eqv.}$

For steel

Consider steel equally distributed over a circular ring.

$$R = 300 - 50 = 250\,mm$$



M.I about Z Z axis

$$I_{zz} = AR^2$$

$$= 18 \times \frac{\pi}{4} \times (16)^2 \times (250)^2$$

$$150796447.4$$

Applying perpendicular axis theorem

$I_{zz} = I_{xx} + I_{yy}$

$I_{zz} = 2 I_D$

$I_D = \frac{I_{zz}}{2} = \frac{150796447.4}{2} = 75398223.69 \, mm^4$

Total M.I. of column about Diameter

$= \frac{\pi}{64}(D^4) + (1.5m - 1) \times I_{steel}$ (74)

$= \frac{\pi}{64}(600)^4 + (1.5 \times 11 - 1) \times 75398223.69$

$= 7530.4 \times 10^{06} \, mm^4$

$\sigma'_{cbc} = \frac{M}{I_{eqv}} \times y$

$= \frac{150 \times 10^{06}}{7530.4 \times 10^{06}} \times 300$

$\sigma'_{cbc} = 5.97 \, N/mm^2$

Check

$\frac{\sigma'_{cc}}{\sigma_{cc}} + \frac{\sigma'_{cbc}}{\sigma_{cbc}} = \frac{3.75}{6} + \frac{5.97}{8.5}$

$= 1.33$ failed.

Question A column of size 300 × 400 mm² is subjected to ① Load P = 400 KN

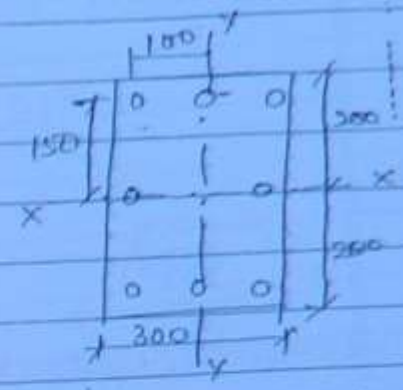
② BM_{xx} = 30 KN-m

③ BM_{yy} = 40 KN-m

8 Nos. 20mm ϕ Bars provided

Check wheather column is safe as uncracked section

M-25, Fe-415.



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Solu Symmetrical section

① $A_{eqv.} = 300 \times 400 + (1.5 \times m - 1)\, \Sigma A_{sc}$

$= 300 \times 400 + (1.5 \times 13 - 1) \times 8 \times \frac{\pi}{4} \times (20mm)^2$

$A_{eqv} = 166495.57\ mm^2$

② $I_{eqv\,xx} = \frac{BD^3}{12} + (1.5m - 1) \times \left(6 \times \frac{\pi}{4} \times 20^2 \times 150^2\right)$

$= \frac{300 \times 400^3}{12} + 784612765.2$

$= 2384.61 \times 10^{06}\ mm^4$

③ $I_{eqvyy} = \frac{400 \times 300^3}{12} + (1.5 \times 13 - 1) \times \left(6 \times \frac{\pi}{4} \times 20^2 \times 100^2\right)$

$= 1248.7 \times 10^{06}$

① $p_0 = \frac{P}{A_{eqv}} = \frac{400 \times 10^{03}}{166495.6} = 2.40\ N/mm^2$

② $\sigma_{cbc} = \frac{M_x}{I_{eqvxx}} \times y + \frac{M_y}{I_{eqvyy}} \times x$

DATE:

$$= \frac{30\times10^{06}}{2384.6\times10^{06}}\times200 + \frac{40\times10^{06}}{1248.7\times10^{6}}\times120$$

$$= 7.32\ N/mm^2$$

check.

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \leq 1$$

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$$\frac{2.40}{5} + \frac{7.32}{7.0} = 1.5 > 1 \quad \text{failed}$$

Column Design By LSM

(17) All column should be designed for min. eccentricity of

$$\left(\frac{l}{500} + \frac{D}{30}\right) \text{ or } 20\,mm \text{ whichever is less.}$$

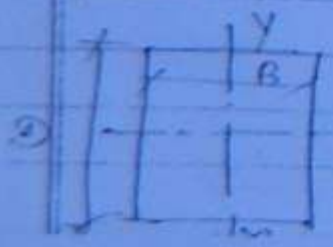
l = Unsupported length of column

D = lateral dimension.

About X-X

D = Lateral Dimension

About Y-Y



B = Lateral Dimension

(H) Assumption

All assumption for beam/ slab are also valed for Column.

(1) Max^m Comp. strain in concrete

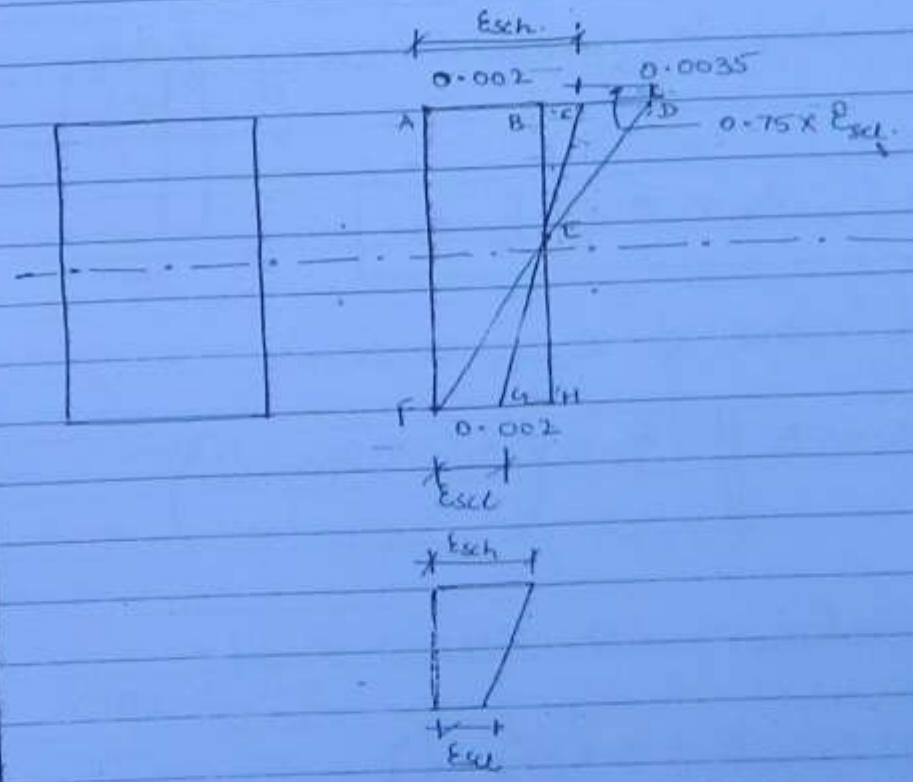
In axial comp. = 0.002

(In bending comp. = 0.0035)

(177)

(5) The max^m comp. strain at highly compressed fibre in concrete, subjected to axial comp. and when there is no tension in the section

= 0.0035 − 0.75 × strain at least comp fibre.



Ratio $\frac{BE}{EH} = \frac{BD}{FH} = \frac{0.0015}{0.002} = 0.75$

$\frac{BE}{EH} = \frac{EC}{EG} = \frac{CD}{FG} = 0.75$

$CD = 0.75\, FG$ or $0.75\, \varepsilon_{scl}$

So strain at highly comp. fibre

$$AC = 0.0035 - CD$$

$$\boxed{\varepsilon_{sch} = 0.0035 - 0.75 \times \varepsilon_{sc}}$$

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(ii) Design of Short Axially Loaded Column

→ All columns should be designed for min. eccentricity value.
except if $[e_{min} \leq 0.05D]$.
D = Lateral Dimension.

→ The column may be designed using following equation

$$\boxed{P_u = 0.4 f_{ck} \cdot A_c + 0.67 f_y \cdot A_{sc}}$$

The minimum value of $e_{min} = 20$ mm.

$$20 \leq 0.05D \quad \Big\| \quad D \geq \left(\frac{20}{0.05} = 450\,mm\right).$$

For Circular column with Helical reinf.

$$P_u = 1.05\left(0.4 f_{ck} \cdot A_c + 0.67 f_y A_{sc}\right)$$

and for helical Reinf

$$0.36 \frac{f_{ck}}{f_y}\left(\frac{A_g}{A_c} - 1\right) \leq \frac{V_h}{V_c}$$

$f_y \not> 415\ N/mm^2$

DATE:

Question
ES 2002

Design the
A reinforced concrete column of size 460×600 mm having effective length of 3.6m is to be designed using limit state method to support an axial service load of 2500 kN. Use M20/ Fe 415 steel.

Solu

$l_{eff} = 3.60m.$

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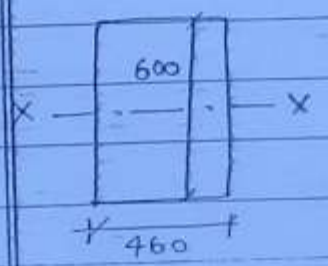
① Slenderness ratio $= \frac{l_{eff}}{B} = \frac{3600}{460} = 7.8 < 12$

It is a short column

② Min. eccentricity about XX



Lateral dimenension

$e_{min} = \frac{l}{500} + \frac{D}{30}$

$= \frac{3600}{500} + \frac{600}{30}$

> 20 mm

$0.05D = 0.05 \times 600 = 30$ mm.

$e_{min} < 0.05D.$

③ About Y-Y

$e_{min} = \frac{3600}{500} + \frac{460}{30}$

$= 22.53$ mm

> 20 mm

$0.05B = 0.05 \times 460 = 23$ mm

So we have $e_{min} \not> 0.05D.$

Now we can use the equation

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y \cdot A_{sc}$$

$$1.5 \times 2500 \times 10^3 = 0.4 \times 20 \times (A - A_{sc}) + 0.65 \times 415 \times A_{sc}$$

$$= 2208000 - 8 A_{sc} + 269.75 A_s$$

$A_{sc} = 5710 \, mm^2$ 180

No. of ~~20~~ 25 mm ϕ Bar $= \dfrac{5710}{\frac{\pi}{4} \times (25)^2} = 12$ Nos

Stirrups

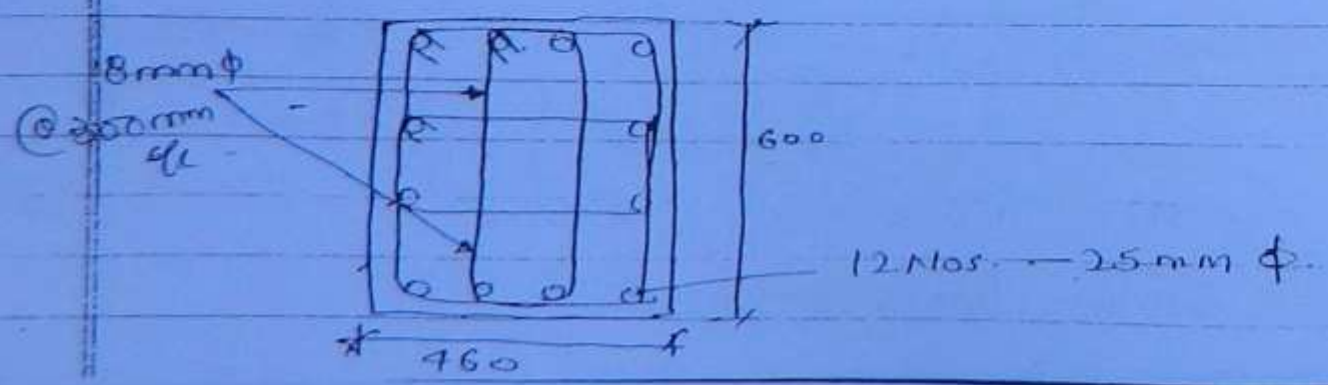
① Dia $= \dfrac{\phi_{max}}{4} = \dfrac{25}{4} = 6.25$ mm

use 8 mm ϕ Bars

② Spacing

1. LLD = 460 mm
2. $16 \phi_{main} = 16 \times 25 = 400$
3. 300 mm

Provide 8 mm ϕ @ 300 mm c/c



Design Of column Subjected to axial load + Bending (uniaxial bending) using interaction Chart

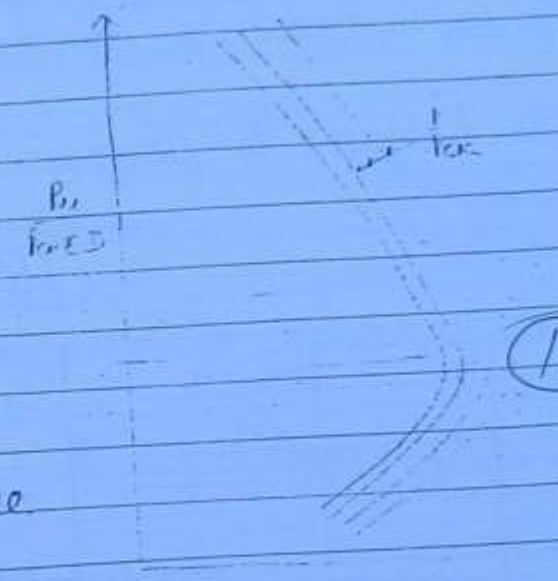
Load = P_u

Moment = M_u

Size Of Column = B×D

Find Out area Of Steel

A_{sc} = ?



Steps

1. Find Out $\frac{P_u}{f_{ck}BD}$ = y-value

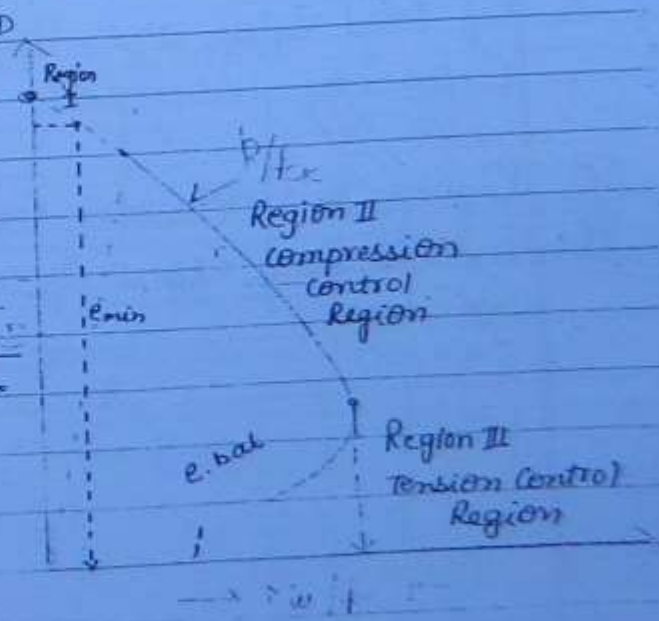
2. Find Out $\frac{M_u}{f_{ck}BD^2}$ = x-value

Based on x & y value read $\frac{P}{f_{ck}}$ from interaction Chart

$\frac{P}{f_{ck}} = K$

$P = K \cdot f_{ck} = \frac{A_{sc}}{BD} \times 100$

Area Of Steel $A_{sc} = \frac{P \times BD}{100}$



Stirrups – Same as previous

Dia – $\frac{\phi}{4}$ or 6mm

Spacing – 16D, 16φ, 300mm

For a column of size B×D & fixed area Of Steel (A_{sc})

P/f_{ck} = One value, we get One line On interaction chart

1. P_{uz} = Max. load Carrying capacity of column

$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$

Also called as ultimate load Carrying Capacity.

DATE:

② Region I – Min. eccentricity value. or region column is subjected to less than min eccentricity value.
Effect of P_u is more as compared to effect of M_u.

→ Column is subjected to compressive stress only.

③ Region II –

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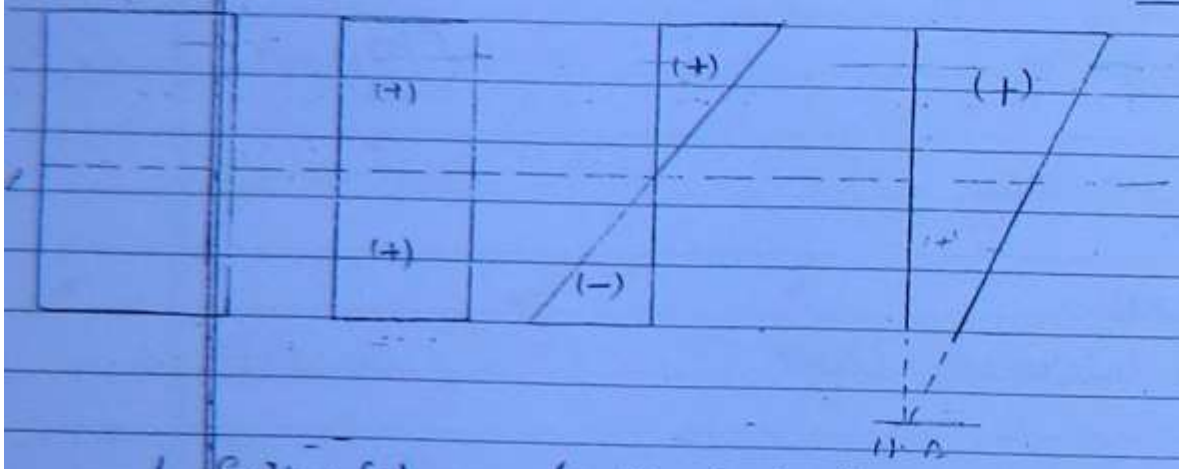
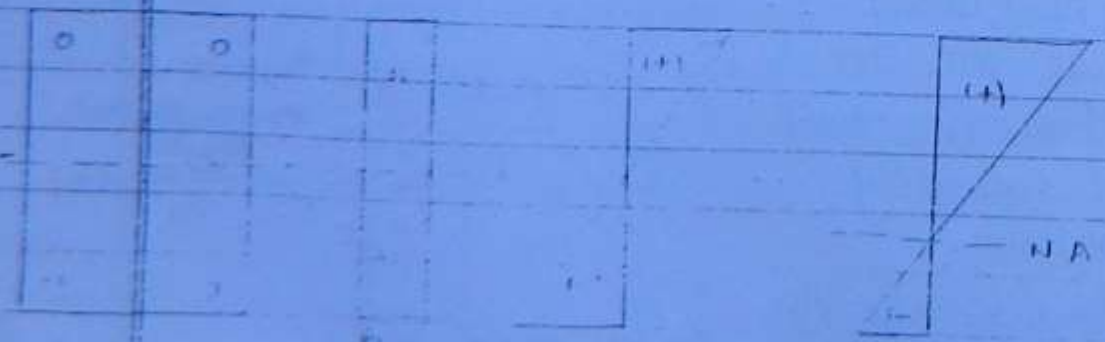


1. Entire column section is in compression
2. N.A lie outside the column section
3. $\varepsilon_{sc} = 0.0035 - 0.15\,\varepsilon_{sc}$
4. Effect of load & moment are equally high
5. If load value are ~~considered~~ reduced moment carrying capacity increased

④ Region III – Tension Control Region
(1) In this case effect of moment is more as load values are less



DATE:

2. Tensile stresses are developed in the section
3. With reduction of load even moment carrying capacity also reduced due to this reason curve takes a invalid direction
4. Maximum strain = 0.0035
5. N.A lies within the section

(183)

Design of column subjected to combined axial load & Biaxial bending.

If Column = B×D is to be designed for

1. Load → P_u
2. Moment → M_{ux} abt X-X
3. Moment → M_{uy} abt Y-Y
4. f_{ck} / f_y are used
5. Using interaction chart only for uniaxial bending.

Steps (i) - Consider a percentage of steel = p%

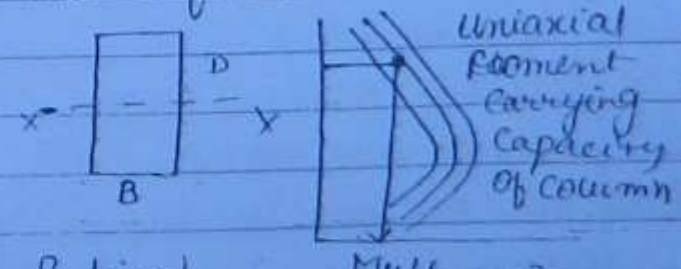
(ii) Find out $\frac{p}{f_{ck}}$ & $\frac{P_u}{f_{ck} B D}$ & $\frac{d'}{D}$

(About x-x)

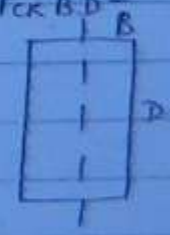
Read the appropriate chart No. & find out

$$\frac{M_u}{f_{ck} B D^2} = K_1$$

(i) $M_{ux1} = K_1 \cdot f_{ck} B D^2$



About Y-Y - Check for $\frac{d'}{D}$ & find out value from chart $\frac{p}{f_{ck}}$ & $\frac{P_u}{f_{ck} B \cdot D}$



& read values of $\frac{M_u}{f_{ck} \cdot D B^2} = K_2$

$M_{uy1} = K_2 f_{ck} D B^2$ - Uniaxial Moment carrying capacity of column abt Y-axis

The Column shall be safe if

$$\left[\frac{M_{ux}}{M_{ux_1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy_1}}\right]^{\alpha_n} \leq 1.0$$

Where α_n = an exponent & value depends upon P_u / P_{uz}

$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$

$\frac{P_u}{P_{uz}}$ = 0.2 to 0.8 the value of α_n = 1.0 to 2.0

for value less than 0.2 $\alpha_n = 1.0$

for value greater than 0.8 $\alpha_n = 2.0$

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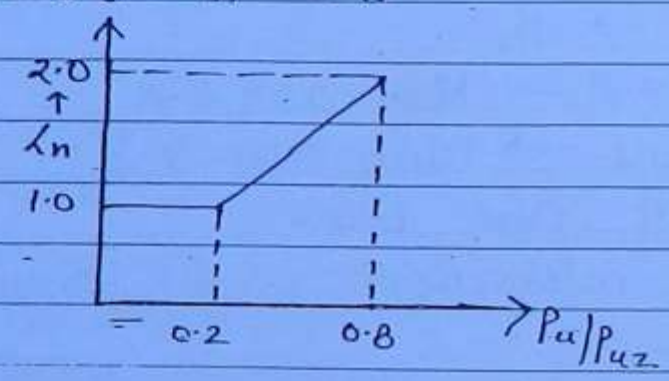

Design of Intermediate long Columns

If a column is subjected to

(i) Load P_u

(ii) Moment M_{ux}

(iii) Moment M_{uy}

(iv) Size of Column $B \times D$

(v) f_{ck} / f_y are used

(vi) To consider slenderness effect, some additional moment are added to above design moment.

$$M_{ax} = \frac{P_u D}{2000} \left\{ \frac{l_{eff} x}{D} \right\}^2$$

$$M_{ay} = \frac{P_u b}{2000} \left\{ \frac{l_{eff} y}{b} \right\}^2$$

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Now design the Column for

(i) P_u

(ii) $(M_{ux} + M_{ax})$

(iii) $(M_{uy} + M_{ay})$

Ques Design a Column of size 400x600 for following data

Concrete - M15 & Steel - fe415

Load $P_u = 1600$ KN

$M_{ux} = 120$ KN-m ; $M_{uy} - 90$ KN-m

Reinf^n are distributed equally on four sides.

Sol^n Moment due to min eccentricity (188)

$e_{min} = 20$ mm

$M_{umin} = P_u \cdot e_{min} = 1600 \times \frac{20}{1000} = 32$ KN

$M_{ux} > M_{umin}$ & $M_{uy} > M_{umin}$ Ok.

① Let us assume a percent of Steel

$p = 1.4\%$

$\Rightarrow \frac{p}{f_{ck}} = \frac{1.4}{15} = 0.093$

$\frac{P_u}{f_{ck} BD} = \frac{1600 \times 10^3}{15 \times 400 \times 600} = 0.444$

About X-X — $\frac{d'}{D} = \frac{50}{600} = 0.083$

use 0.10 & Chart no 44. SP-16

$\frac{M_{ux}}{f_{ck} BD^2} = \frac{120 \times 10^6}{15 \times 400 \times 600^2} = 0.1 = K_1$

$M_{ux1} = 0.1 f_{ck} BD^2$

$0.1 \times 15 \times 400 \times 600^2 = 216$ KN-m

About Y-Y — $\frac{d'}{B} = \frac{50}{400} =$

Chart no- 45 for value of $\frac{d'}{B} = 0.15$

DATE:

$$\frac{M_{uy}}{f_{ck} D B^2} = 0.10 = K_2$$

$$M_{uxy_1} = 0.10 \times 15 \times 400^2 \times 600$$

$$144 \text{ KN-m}$$

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_{ck} A_{sc}$$

$$= 0.45 \times 15 \times (400 \times 600 - \frac{1.4}{100} \times 400 \times 600) + 0.75 \times 15 \times \frac{1.4}{100} \times 400 \times 600$$

$$P_{uz} = 2643.12 \text{ KN}$$

$$\frac{P_u}{P_{uz}} = \frac{1600}{2643.12} = 0.605$$

P_u/P_{uz}	α_n
0.2	1.0
0.8	2.0

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for .605

$$\alpha_n = 1.0 + \frac{2-1}{0.8-0.2}(.605-0.2)$$

$$= 1.675$$

Check $\left(\frac{M_{ux}}{M_{uxy_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} \leq 1.0$

$$\left(\frac{120}{216}\right)^{1.675} + \left(\frac{90}{144}\right)^{1.675}$$

$$.3764 + 0.455$$

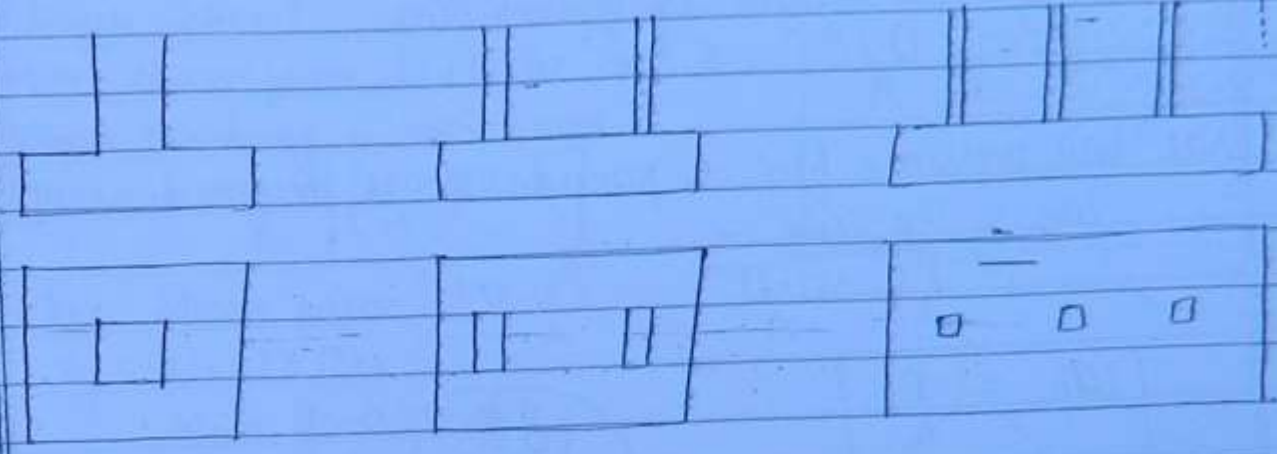
$$0.83 \leq 1.0 \quad \text{Safe}$$

Foundation

→ Types

(a) Isolated footing (b) Combined footing (c) Strip footing

imp



(d) Raft (e) Pile foundation

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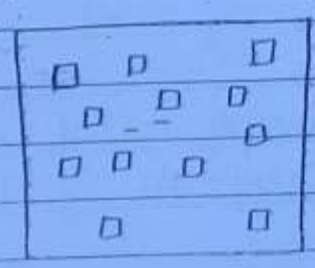
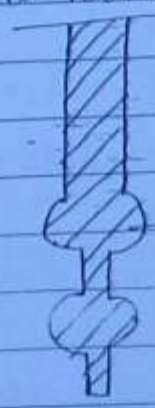




Isolated Footing Foundation

→ For a Rectangular footing of uniform thickness

Load from Column = P

Size of Column = a×b

Bearing Capacity of soil = q_0

Ques by are given

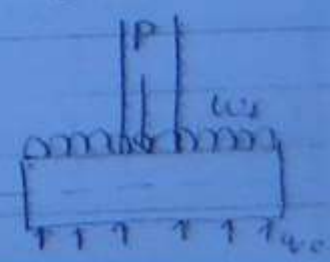
Step (1) Size of foundation

Load from Column = P

Weight of foundation = 0.10 P (actual 15-20%) (10% of load)

Total load P_T = 1.10 P

Area of foundation → $A = \frac{P_T}{q_0}$



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Find out length L & B

$A = L \times B$

Pressure due to weight of foundation

$$w_f = \frac{0.1P}{A}$$

Net soil pressure for c foundation is designed

$$w_o = q_o - w_f$$

$$= \frac{P_T}{A} - \frac{0.1P}{A} \qquad \frac{P_T - 0.1P}{A}$$

$$\boxed{w_o = \frac{P}{A}}$$

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For L.S.M it will be the factored.

$$\boxed{w_{uo} = \frac{1.5P}{A}}$$

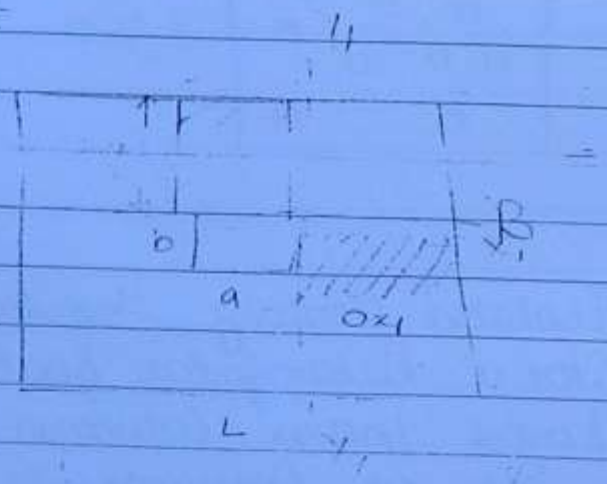
Step ② Check for Bending moment

Critical sec^n for B.M

→ At the face of column

Out $x_1 - x_1$

Consider 1m width of foundation



Overhang $oy_1 = \left(\frac{B-b}{2}\right)$

B.M at $x_1x_1 = w_o \times 1 \times \frac{oy_1^2}{2}$

$$= \frac{w_o}{2}\left(\frac{B-b}{2}\right)^2 = \boxed{\frac{w_o}{8}(B-b)^2}$$

B.M at $y_1y_1 = w_o \times 1 \times \frac{ox_1^2}{2}$

$$= \frac{w_o}{2}\left(\frac{L-a}{2}\right)^2 \qquad \boxed{\frac{w_o}{8}(L-a)^2}$$

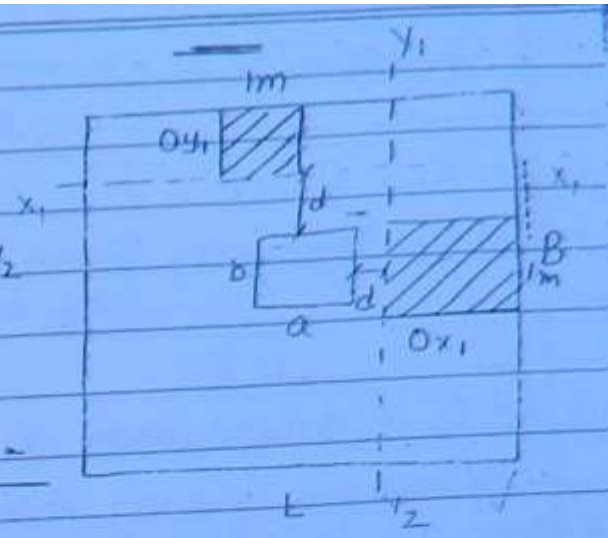
Depth of foundation $d = \sqrt{\frac{(M_x \text{ or } M_y)\ \text{greater}}{Q \cdot B}}$

Step 3 Check for one way Shear

Critical Secⁿ for one way Shear (Single Shear)

Max. Shear force shall be at Y_1-Y_2 where overhang is maximum.

Overhang $Ox_1 = \left[\left(\frac{L-a}{2}\right) - d\right]$



Max. Shear force

$V_y = \omega_0 \times 1 \times Ox_1$

$\omega_0 \times \left[\left(\frac{L-a}{2}\right) - d\right]$

Nominal Shear Stress

$\tau_v = \frac{V_y}{Bd} = \frac{V_y}{1000d} \leq \tau_{c\,min}$

$\leq K \cdot \tau_c$

(184)

K = factor depending on Slab thickness

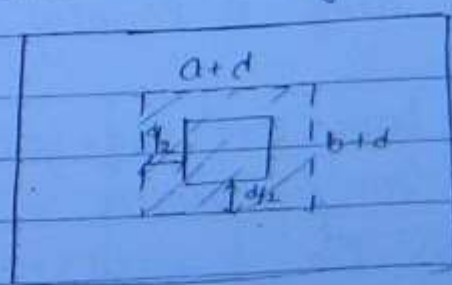
B.S.2.1.1 page No – 84

overall depth of slab	300 or more	275	250	225	200	175	150 or less
K	1.0	1.05	1.10	1.15	1.20	1.25	1.30

Step-4 Check for punching shear failure or two way shear

→ Net punching force:

$= P - \omega_0 \times (a+d) \times (b+d)$



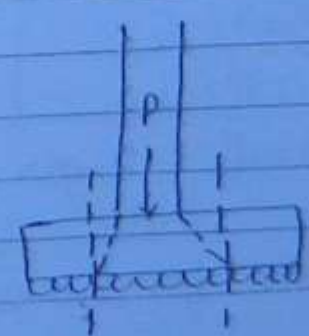
Net punching shear stress developed

$= \frac{\text{Net punching force}}{\text{Resisting area}}$

$= \frac{P_a - \omega_0 (a+b)(b+d)}{\text{Perimeter} \times \text{depth}}$

$= \frac{P - \omega_0 (a+d)(b+d)}{2[(a+d)+(b+d)] \times d}$

$\leq \tau_{vp}$ (permissible)



12% HSD Bar

Cover = 50mm

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Q.NO.59 τ_{vp} (permissible) $= 0.25\sqrt{f_{ck}} \times k_s \rightarrow$ L.S.M

$= 0.16\sqrt{f_{ck}} \times k_s$ K.L.S.M

$k_s = 0.5 + \beta_c \not> 1$

$\beta_c = \frac{b}{a} = \frac{\text{Short Side}}{\text{long Side}}$

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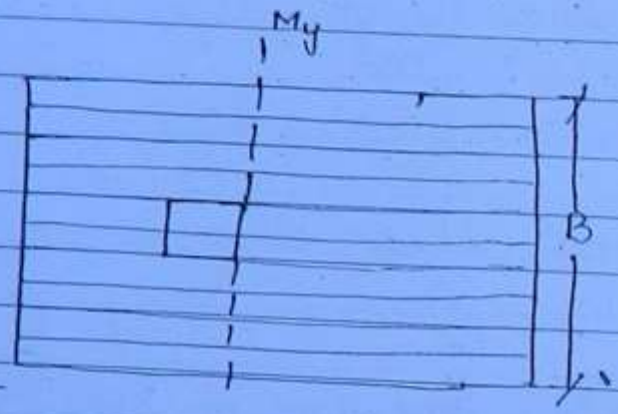
Step 5 Depth is decided, max. value required from above three criteria.

Area of Steel

(a) Longer bars are req. for M_y. For 1m width

$A_{st} = \frac{M_y}{\sigma_{st} \cdot j \cdot d}$ — K.L.S.M

$= \frac{M_y}{0.87 f_y \cdot j \cdot d}$ — L.S.M



$= \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_{uy}}{f_{ck} \cdot Bd^2}}\right] Bd$ — L.S.M

So for total B width area of steel = B·Ast

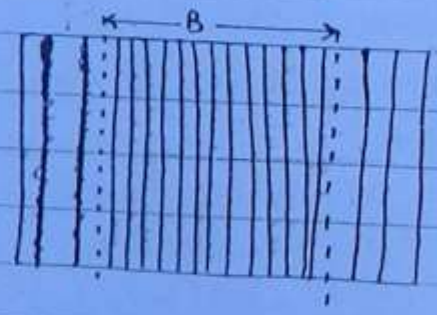
or No. of bars = B × (No. of bars req. for Ast)

This reinf^n is equally distributed over 'B' width.

(b) Short bars are req. for M_x. For L m width

$A_{st} = \frac{M_x}{\sigma_{st} \cdot j \cdot d}$ — L.S.M

$\sigma_{st} = 0.87 f_y$ — L.S.M



So for total L length area of steel = L·Ast

No. for bars for 1m width $n_1 = \frac{A_{st}}{\pi/4\, \phi^2}$

No. for bars for L width $n_L = L \times n_1$

No. of bars provided for in Central Band of width B is

$$n_c = n_T \left(\frac{2}{1 + L/B} \right)$$

Remaining bars are are distributed equally on two side bands.

(91)

ES-1993

Ques- A rectangular RC column of size 60X40 cm carries a load of 80 tonnes. Design a rectangular footing use M-15 / fc-250

$q_0 = 20 t/m^2$ $\quad a = 600mm$ $\quad 80T = \frac{800 \times 1000 \times 10}{1000} = 800 KN$

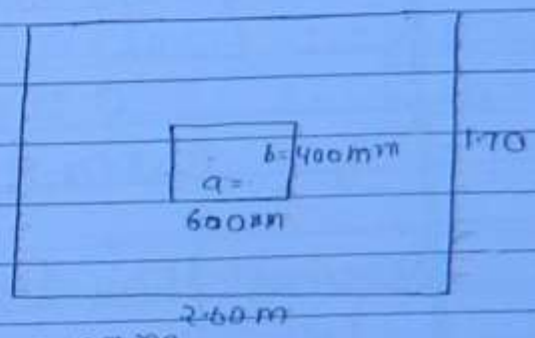
$P = 800 KN$ $\quad b = 400 mm$

$q_0 = 200 KN/m^2$ $(1.10 \times P)$

Total load $P_T = 880 KN$

Area of foundation $= \frac{880 \times 1.5}{200 \; 1.5}$

$= 4.4 m^2$



$\frac{L}{B} = \frac{a}{b} = 1.5$

Size of foundation (L X B) $= 1.5B^2 = 4.4$

$B = 1.71 m$ & $L = 2.57 m$

$B = 1.70 m$

$L = 2.60$

$A = 4.42 m^2$

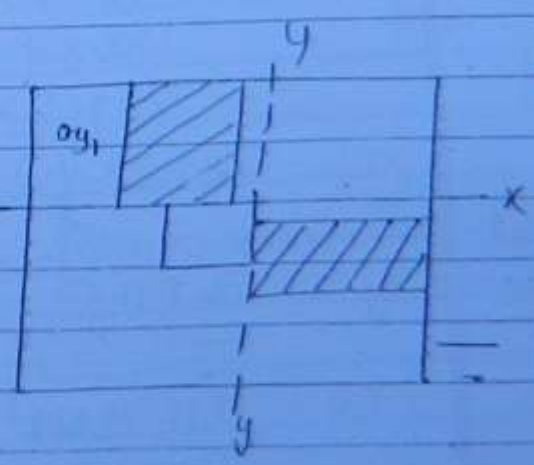
$W_0 = \frac{1.5P}{A}$ (Net soil pressure)

$= \frac{1.5 \times 800}{4.42} = 271.5 KN/m^2$

Check for B.M

At the face of column abt X_1-X_1

$O_{y_1} = \frac{1.70 - .4}{2} = 0.65$



B.M at x-x $= \frac{271.5 \times 0.65^2}{2}$

$57.35 KN \cdot m$

B.M at y-y $= \frac{271.5}{2} \times \left(\frac{2.60 - .6}{2} \right)^2$

$= 135.75 KN \cdot m$

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Depth of foundation $d = \sqrt{\frac{M_y}{Q \cdot B}}$

$$d = \sqrt{\frac{135.75 \times 10^6}{2.22 \times 1000}} = 247.29 \text{ mm}$$

Say = 250 mm

$$Q = 0.36 f_{ck} \frac{x_{u,lim}}{d}\left(1 - 0.42 \frac{x_{u,lim}}{d}\right)$$

$$= 0.36 \times 15 \times 0.53 \left(1 - 0.42 \times 0.53\right) = 2.22$$

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Check for One way Shear

Critical Secn

Max. S.F shall be at Y-Y where overhang is max

$$O_{x_1} = \left[\left(\frac{2.60 - 0.6}{2}\right) - .250\right]$$

$$= 0.75 \text{ m}$$

$$V_y = w_0 \times O_{x_1} \times 1 = 271.5 \times 0.75$$

$$= 203.625 \text{ kN}$$

Nominal Shear Stress

$$\tau_v = \frac{V_y}{Bd} = \frac{203.625 \times 10^3}{1000 \times 250} = 0.8145 > \tau_{min}$$

$\boxed{\text{So fail}}$

Depth req^n

$$0.28 = \frac{203.625 \times 10^3}{1000 \times d}$$

$$= 727.23 \quad \text{Say} =$$

$$d = \frac{727.23 + 250}{2} = 488 = \text{Say } 500$$

$$V_y = w_0 \times 0.5 = 271.5 \times 0.75$$

$$= 135.75 \text{ kN}$$

$$\tau_v = \frac{135.75 \times 10^3}{1000 \times 500} = 0.2715 < 0.28 \ (\tau_{c\,min})$$

Hence Safe.

Check for two way shear or punching

Net punching force $= P = w_0 (a + d)(b + d)$

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1.5×10^3

$= 800 - 271.5(.6+.5)(.4+.5)\times10^3$

$= 931215\ N/m^2$

Resisting area

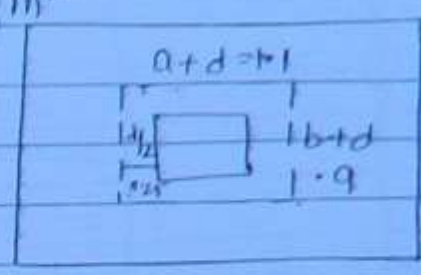
$= 2[(a+d)+(b+d)]\times d$

$= 2[1.1+.9]\times .5$

$= 0.99\ m^2 = 2\times10^6$

$= 0.990\times10^{-6}\ m^2$

τ_{vp} (developed) $= 0.465\ N/mm^2$



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τ_{vp}(permissible) $= 0.25\sqrt{f_{ck}}\times K_s$

$K_s = \left(0.5+\frac{b}{a}\right) = \left(0.5+\frac{400}{600}\right) = 1.67 > 1.0$

So take 1.0

τ_{vp} (permissible) $= 0.25\sqrt{15}\times1.0$

$= 0.968\ N/mm^2$

$> \tau_{vp}$ (developed) – safe

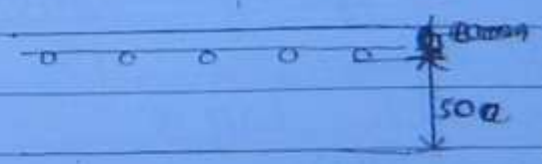
Depth required = 500 mm

Eff cover + 80 mm

Total depth = 580 mm

Eff cover = 50+16+8

= 74 mm



Area of steel

longer bar (M_y)

$A_{st} = \frac{0.5\times15}{250}\left[1-\sqrt{1-\frac{4.6\times135.75\times10^6}{15\times1000\times500^2}}\right]1000\times500$

$= 1305.75\ mm^2$

Use 16mmϕ bars. No. of bars

$= \frac{1305}{\pi/4\ 16^2} = 6.49$ No

Total bars req. for B width $n_1 = 6.49\times1.7$

$= 11.03$

Say 12 Nos. of 16mm ϕ bars

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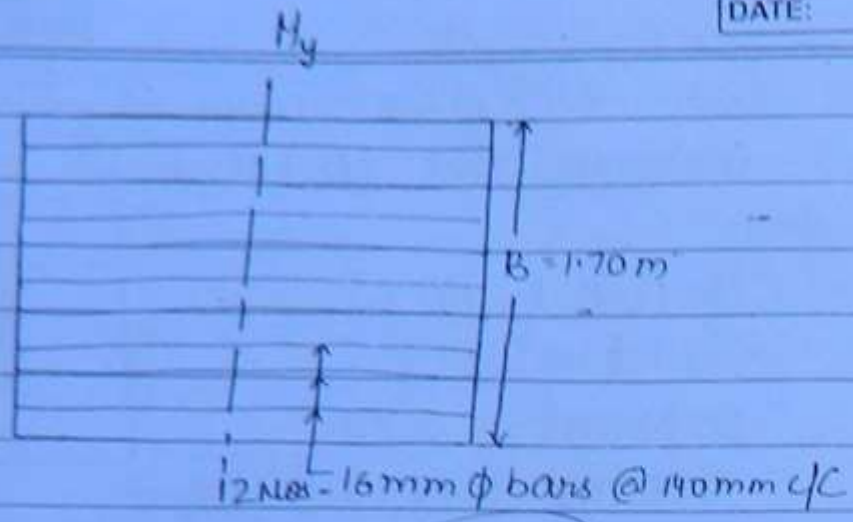
Spacing $= \frac{1700}{2}$
$= 141 mm$



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Shorter Side

$$A_{st} = \frac{0.5 \times 15}{250}\left[1 - \sqrt{1 - \frac{4.6 \times 57.35 \times 10^6}{15 \times 1000 \times 500}}\right] 1000 \times 500$$

$537.24 \ mm^2$

No. of bars 10mmϕ $= \frac{537.24}{\frac{\pi}{4} \times 10^2} = 6.84$

Total No $= 2.6 \times 6.84 = 17.784$

Say 18 Nos of 10 mm ϕ bars

~~Spacing $= \frac{2600}{18} = 144.44 mm$~~

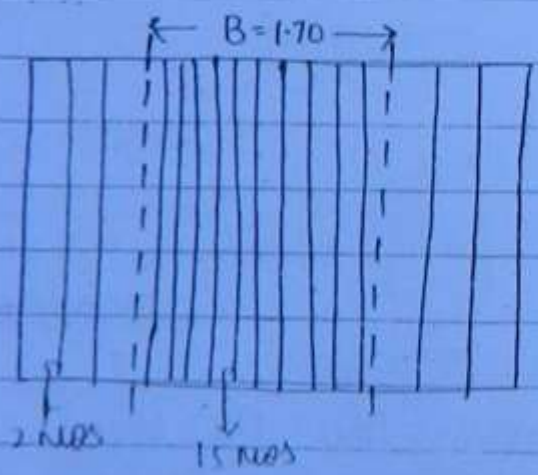
central band width

$n_c = 14.22$ Say 15

Spacing $= \frac{1700}{15} = 113$

No. of bars in side band

$= \frac{18-15}{2} = 1.5 = 2$ Nos.



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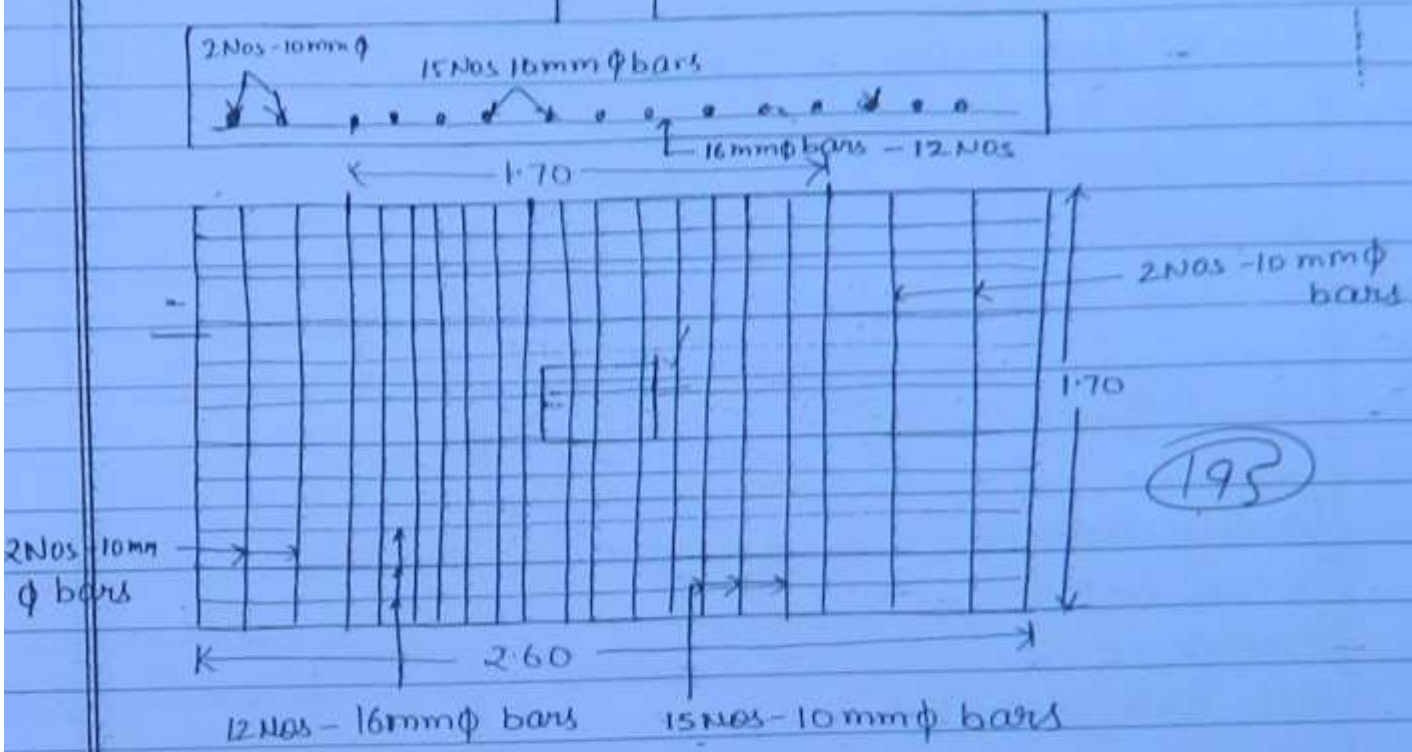

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Sloped Isolated footing

1. Size of foundation – Same

$$P + 0.10P = P_T$$

$$A = \frac{P_T}{q_0}$$

$$w_0 = \frac{P}{A}$$

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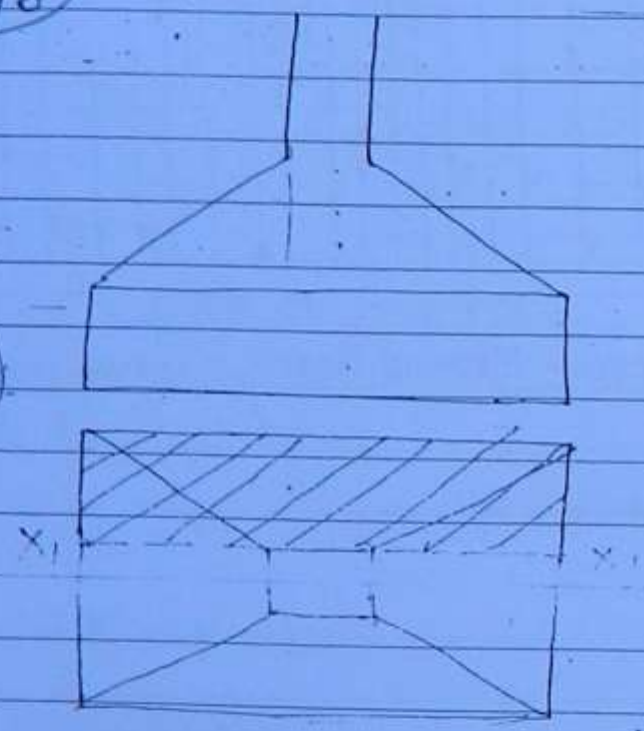
$$B = \sqrt{A}$$

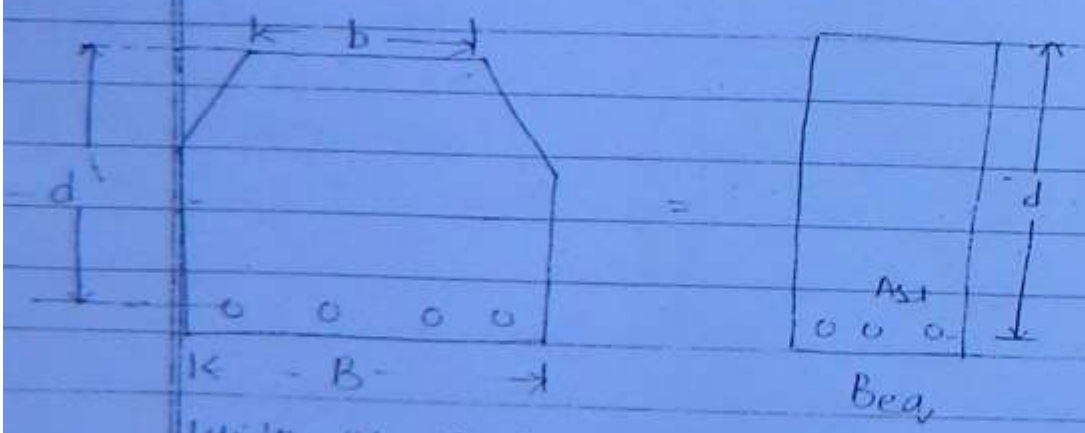
Bending moment

Critical sec^n at face of column

$$B.M = w_0 B \times \left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$$

$$= w_0 B \times \frac{(B-b)^2}{8}$$





Width of equivalent sec^n

$$B_{eq} = b + \frac{1}{2}(B-b)$$

Depth of sec^n req

$$d = \sqrt{\frac{BM}{Q.B_{eq}}}$$

Check for one way shear

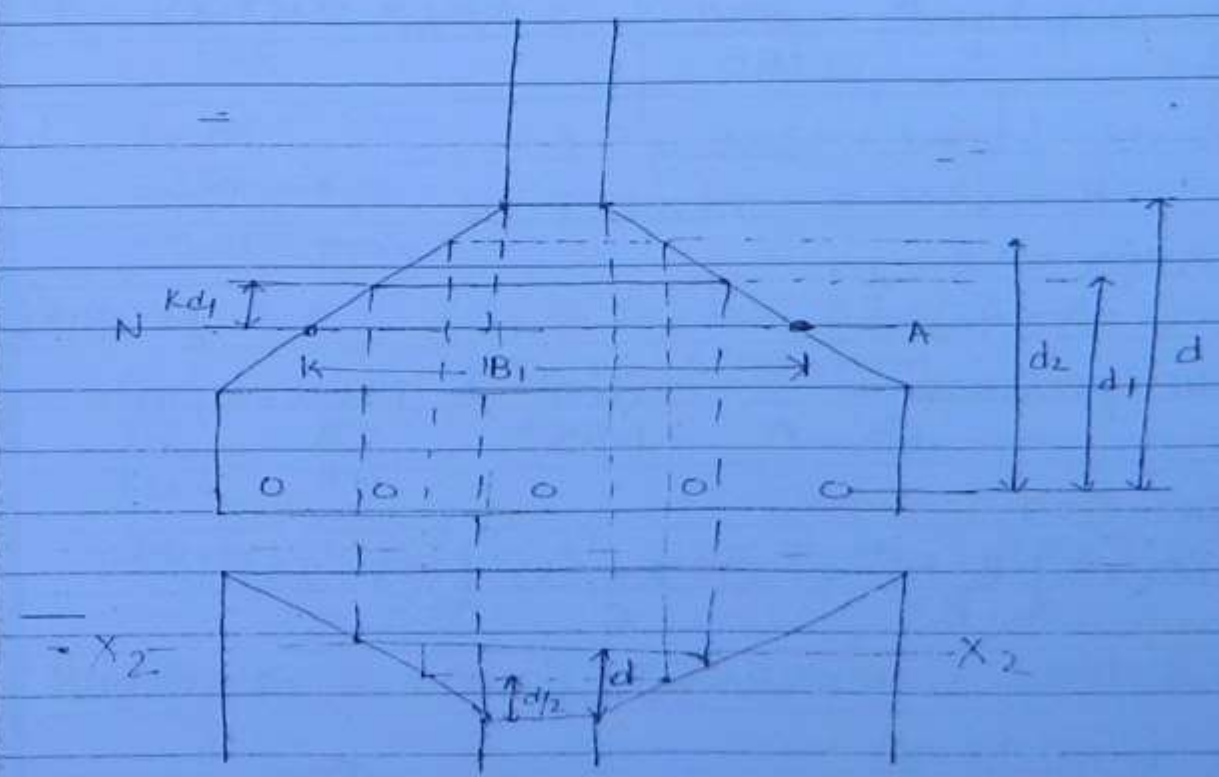
$$V = w_0 \times B \times \left[\frac{B-b}{2} - d\right]$$

Nominal shear stress $\tau_v = \frac{V}{B_1 d_1}$ (197)

here

B_1 = width of footing at critical sec^n or N.A

d_1 = depth of footing at critical sec^n.



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Punching Shear

Punching Shear force = $P - \omega_0 (a+d)(b+d)$

Punching Shear Stress = $\dfrac{P - \omega (a+d)(b+d)}{2[(a+d)+(b+d)] \times d_2} \leq \tau_{\varphi\ permissible}$

Area of Steel – Same.

2001

Ques Design a Sloped Square footing for RC Column, 300x300 mm Size, Carrying an axial load of 320 KN. The safe bearing capacity of soil is 150 KN/sqm. Use M-20/Fe-415 Use L.S.M

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1. Size of foundation

Load from Column = 320 KN

foundation weight = 32 KN

352 KN

Area required. = $\dfrac{352}{150} = 2.35\ m^2$

Size of Square footing :

$B = \sqrt{2.35} = 1.55 \times 1.55$

Area = 2.4 m

B = 1550 mm

Net Soil pressure $\omega_0 = \dfrac{P}{A} = \dfrac{320}{1.55^2} = 133.2\ KN/m^2$

Check for B.M.

$B.M = \omega_0 \times B \left(\dfrac{B-b}{8}\right)^2 = \dfrac{133.2 \times 1.55}{8}(1.55-0.3)^2$

$= 40.32\ KN\text{-}m$

$B_{eq} = b + \dfrac{1}{8}(B-b)$

$= 300 + \dfrac{1}{8}(1550-300) = 456.25\ mm$

Depth required $d = \sqrt{\dfrac{B.M}{f_c . B_{eq}}}$

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$$d = \sqrt{\frac{40.32 \times 10^6}{.898 \times 456.25}}$$

$d = 313.7$ mm

Say $d = 350$ mm

$K = \frac{mc}{t+mc} = \frac{13\times7}{230+13\times7} = 0.2835$

$j = 1 - K/3 = 0.9055$

$Q = \frac{1}{2} c \cdot j \cdot k = \frac{1}{2} \times 7 \times .9055 \times .2835$

$Q = .898$

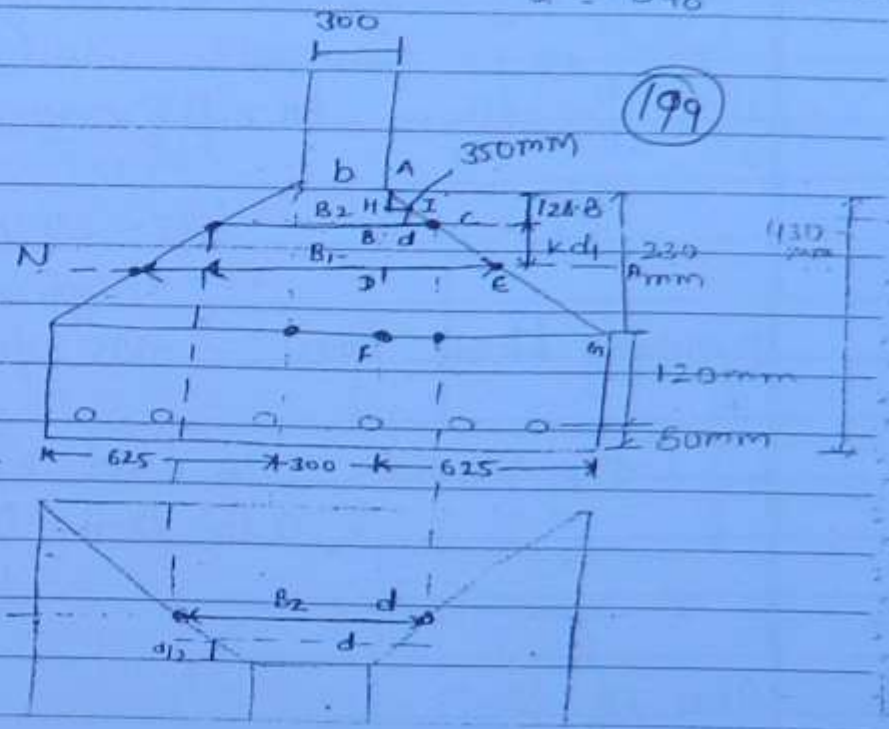
Check for one way shear

Total shear force

$= w_0 \cdot B \cdot \left[\frac{B-b}{2} - d\right]$

$= 133.2 \times 1.55 \left[\frac{1.55 - .3 - .35}{2}\right]$

$= 56.78$ kN



$\frac{AB}{350} = \frac{AE}{625} = 230$

$AB = \frac{230 \times 350}{625} = 128.8$

$d_1 = d - AB$

$= 350 - 128.8$

$= 221.2$ mm

Depth of N.A $\quad Kd_1 = 0.2835 \times 221.2$

$BD = 62.71$ mm

$\frac{DE}{FG} = \frac{AD}{AF}$

$DE = \frac{128.8 + 62.71}{230} \times 625 = 520.4$ mm

Width $B_1 = 300 + 2 \times 520.4$ mm $= 1340.8$ mm

Check for single shear

$\tau_v = \frac{V}{B_1 d_1} = \frac{56.78 \times 10^3}{1340.8 \times 221.2 \text{ mm}} = 0.19 \leq \tau_{c\,min} = 0.19$

(safe)

DATE:

Check for two way Shear (Punching Shear)

Punching Shear Stress $= \frac{\left[320 - 133.2\,(.3+.35)(.3+.35)\right] \times 10^3}{10^6 \times 2\left[(.3+.35)+(.3+.35)\right]}$ ·2856

$HI = \frac{d}{2} = \frac{350}{2} = 175\,mm$

200

$\frac{AH}{AF} = \frac{HI}{FG} = AH = \frac{175}{625} \times 230 = 64.4\,mm$

$d_2 = d - AH = 350 - 64.4\,mm$
$= 285.6\,mm$

punching Shear Stress $= .355\ N/mm^2 < \tau_{vp}$ permiss

τ_{vp} permissive $= 0.16\sqrt{f_{ck}} \cdot k_1$
$= 0.16\sqrt{20} = 0.7155$

Safe

Area Of Steel

$A_{st} = \frac{40.32 \times 10^6}{230 \times .9055 \times 350} = \frac{M}{\sigma_{st} \cdot j \cdot d}$

$= 553.14\ mm^2$

No. Of 10 mm ϕ bars $= \frac{553.14}{\pi/4 \times 10^2} = 7.04$

Provide 8 Nos 10 mm ϕ bars.

Circular footing

Ques. Design a circular Column Of 500mm dia having a load Of 1200 kN. Bearing Capacity Of Soil is 150 kN/m² Use M-25/Fe 415 Steel. use 1.5 M

Sol^n Size Of foundation

Load from Column = 1200 kN

Weight Of foundation = 120 kN

P_T = 1320 kN

DATE:

Area required $= \frac{1320}{150} = 8.8\ m^2$

Dia of footing $D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 8.8}{\pi}} = 3.35\ m$

Provide $D = 3400\ mm$ (201)

Radius $= 1700\ mm = 1.70$

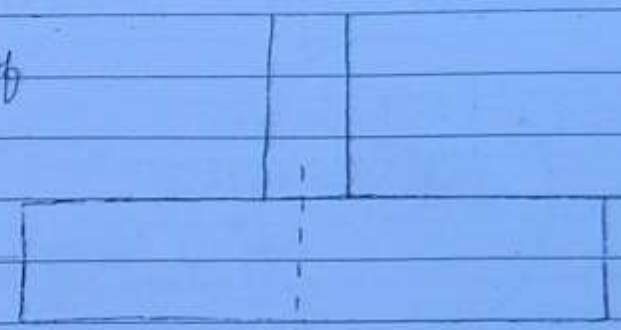
Net Soil pressure $= \frac{1200 \times 1.5}{\pi \times 1.7^2} = 198.26\ KN/m^2$

W_u

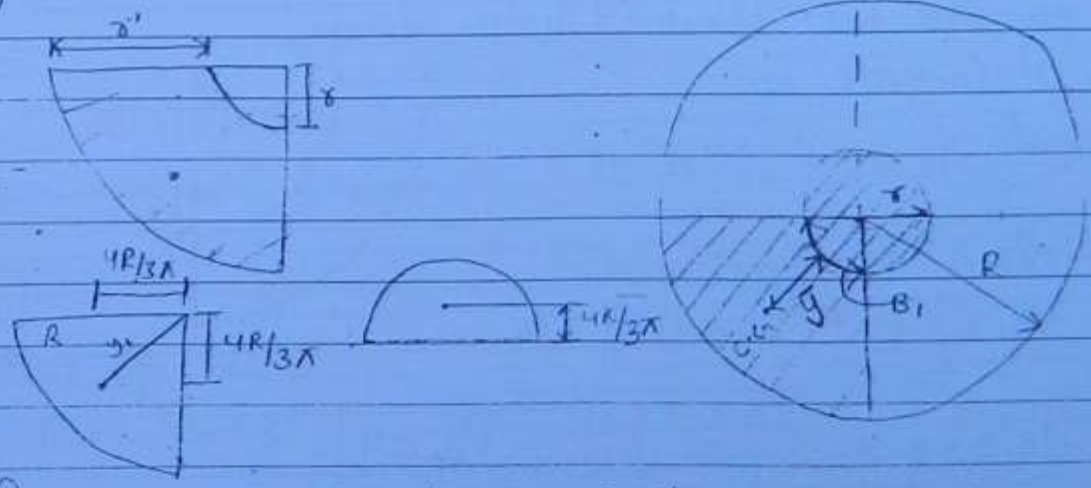
Check for B.M

Critical Section is at face of Column

$B.M = W_u \times \left(\frac{\pi R^2 - \pi r^2}{4}\right)\bar{y}$



$\bar{y}$ value



Distance of C.G from Centre

$y_1 = \sqrt{2} \times \frac{4R}{3\pi} = 0.6R$

Distance of C.G of Shaded portion from Centre

$= \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{\frac{\pi R^2}{4} \times 0.6R - \frac{\pi r^2}{4} \times 0.6r}{\frac{\pi}{4}R^2 - \frac{\pi}{4}r^2}$

$= \frac{0.6(R^3 - r^3)}{(R^2 - r^2)}$

Distance of C.G from face of column

$\bar{y} = \frac{0.6(R^3 - r^3)}{R^2 - r^2} - r$

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$$B.M = W_u \left(\frac{\pi R^2 - \pi r^2}{4}\right)\left[\frac{0.6(R^3 - r^3)}{R^2 - r^2} - r\right]$$

$$= 198.26 \times \frac{\pi}{4}(1.70^2 - 0.25^2) \times \left[\frac{0.6(1.70^3 - 0.25^3)}{1.70^2 - 0.25^2} - 0.25\right]$$

$$= 347.46 \text{ kN-m}$$

Depth req $= \sqrt{\frac{BM}{Q \cdot B_1}}$ (202)

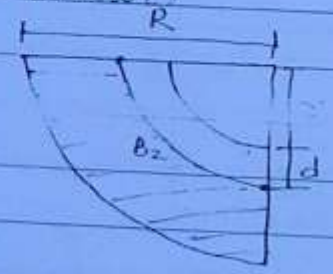
Width $B_1 = \frac{2\pi r}{4} = \frac{2 \times \pi \times 0.25}{4} = 392.7 \text{ mm}$

$$d = \sqrt{\frac{347.46 \times 10^6}{.138 \times 25 \times 392.7}} = 506.42 \text{ mm}$$

Say $= 510 \text{ mm} = d$

Check for one way shear - (Single shear)

Width at critical sec



$$B_2 = \frac{2\pi(r+d)}{4} = \frac{\pi(2 \times 760)}{2}$$

$$= 1193.8 \text{ mm} \cancel{596.9 \text{ mm}}$$

$$V_u = W_u \times \frac{\pi}{4}\left[R^2 - (r+d)^2\right]$$

$$= 198.25 \times \frac{\pi}{4}\left[1.70^2 - (.25 + 0.51)^2\right]$$

$$= 360.25 \text{ kN}$$

$$\tau_{vu} = \frac{V_u}{B_1 \cdot d} = \frac{360.25 \times 10^3}{1193.8 \times 510}$$

$0.59 > 0.24$ fail

Depth req.

$$d = \frac{V_u}{\tau_{cmax} \cdot B_1} = \frac{360.25 \times 10^3}{0.27 \times 1193.8} = 1040.6 \text{ mm}$$

Say d = 1040.6 + 75 = 1115.4 Say 1120 mm

$$V_u = 198.25 \times \frac{\pi}{4}\left[1.70^2 - (.25 + .8)^2\right]$$

$$= 278.32 \text{ kN}$$

$$B_2 = \frac{\pi(250 + 800)}{2} = 1649.34 \text{ mm}$$

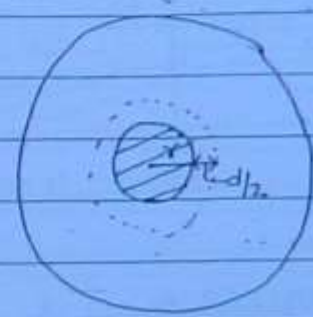
$$\tau_{vu} = \frac{278.32 \times 10^3}{1649.34 \times 800} = .21 < 0.29 \text{ (safe)}$$

(203)

Check for two way Shear (Punching Shear)

Net punching Shear Stress

$$\frac{P_u - W_u \pi \left(r + \frac{d}{2}\right)^2}{2\pi\left(r + \frac{d}{2}\right) \times d}$$



$$\frac{1.5 \times 1200 \times 10^3 - 198.25 \times \pi\left(.25 + \frac{.8}{2}\right)^2 \times 10^3}{2 \times \pi\left(250 + \frac{800}{2}\right) 800}$$

$$= 0.47 \text{ N/mm}^2$$

permissible τ_{vp} = $0.25\sqrt{f_{ck}}$ = $0.25\sqrt{25}$

$$= 1.25 \text{ N/mm}^2$$

Safe

Area Of Steel

$$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 347.46 \times 10^6}{25 \times 392.7 \times 800^2}}\right] 392.7 \times 800$$

B_1 B_1

$$= 1291.7 \text{ mm}^2$$

No. Of 12 mm ϕ bars $= \frac{1291.7 \times 4}{\pi \times 10^2} = 11.42$

Say 12 Nos.

provide 12 nos of 12mm ϕ bars.

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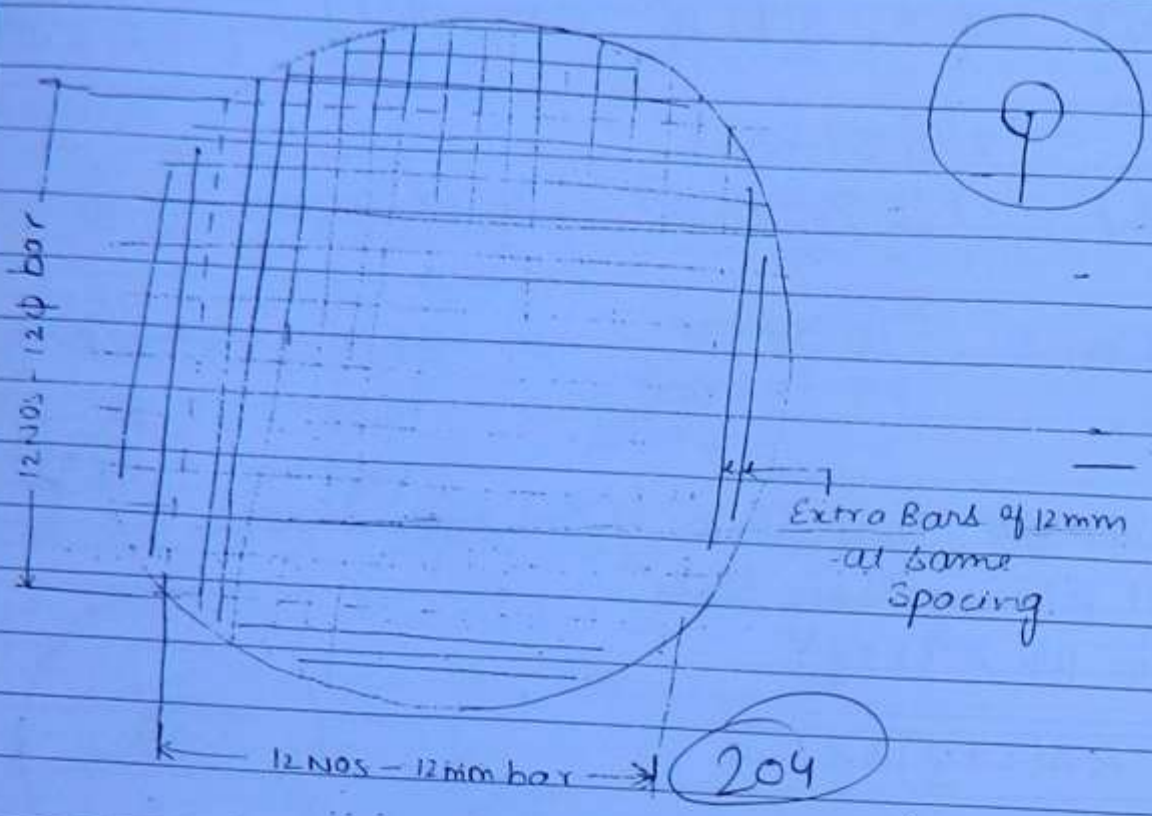


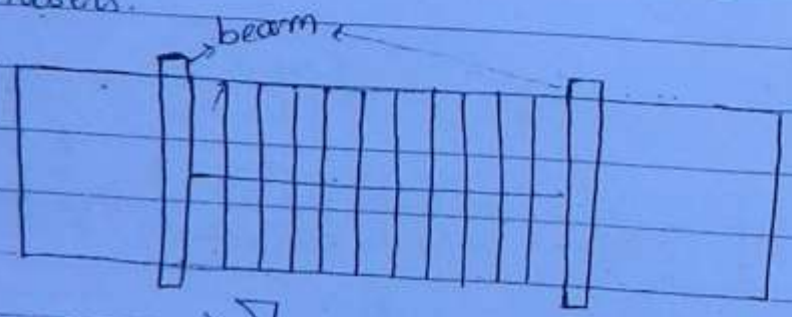
31/10/2011

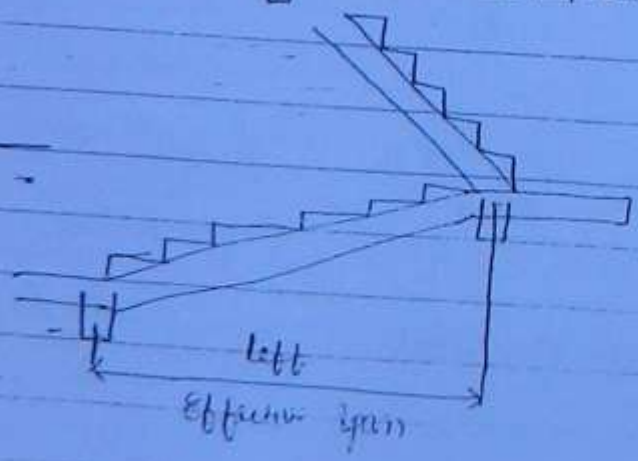
Staircase:-

IS Code provision:- (As per 33.1 / P-63)

Effective Span of Staircase

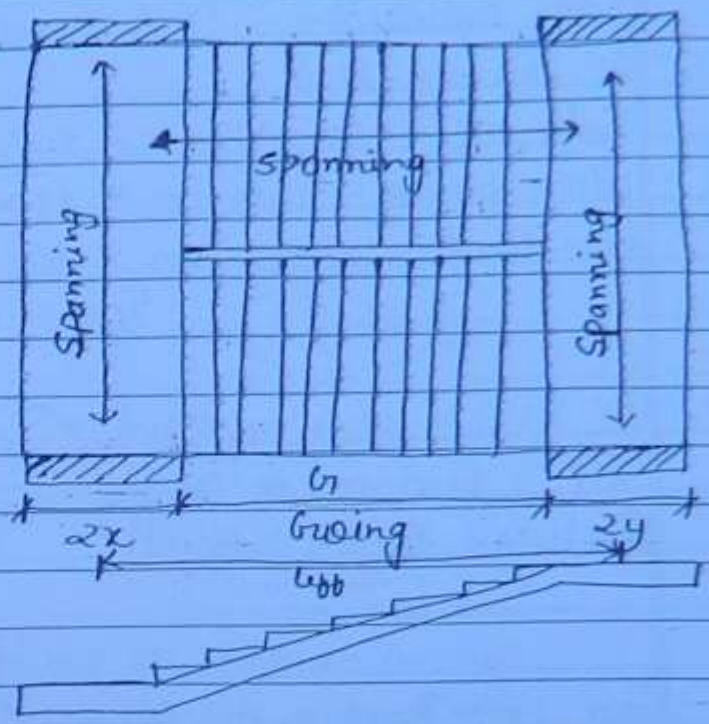
(a) When Supported at top and bottom risers spanning parallel with risers.





Effective Span = c/c distance betn beams

(b) Landings are supported on side ~~bar~~ walls and waist slab are supported on landings.



Eff. Span = l_{eff} = (205)
$G + x + y$

Where value of x or y ≯ 1 m

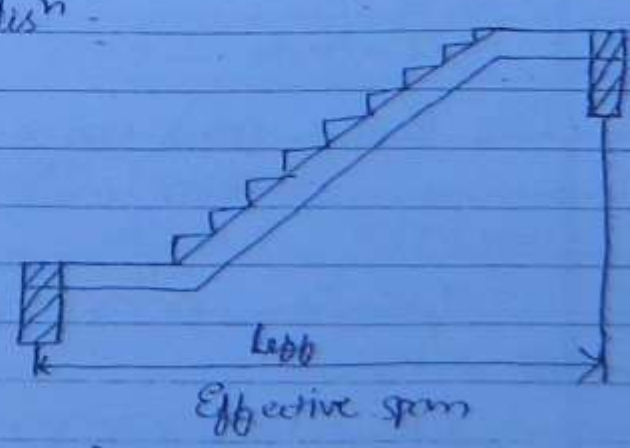
From Code.

x	y	Eff. Span
<1m	<1m	G+x+y
≥1m	<1m	G+1+y
<1m	≥1m	G+x+1
≥1m	≥1m	G+1+1

G = going (horizontal length of waist slab)

(c) When supported on two beams provided at end. Landing slab is also spanning in same direction as of waist slab.

Eff span = Centre to centre distⁿ betn beams.



Load Calculation:- For 1m width of Stair

1. At landing Slab

Self wt = $1m \times 1m \times t \times 25$ = ?

Floor finish = $1m \times 1m \times t_f \times 24$ = ?

Live load = $1m \times 1m \times w_L$ = ?

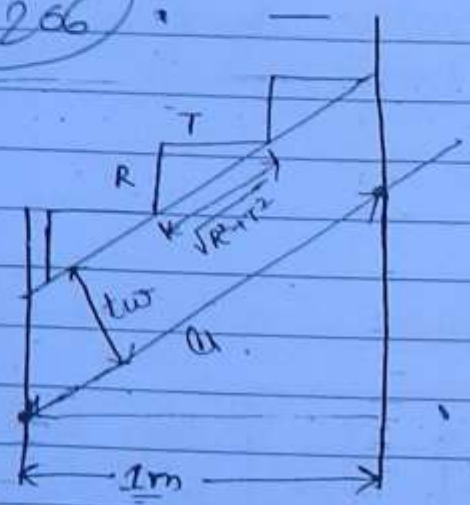
Total load = w_1

2. At waist slab portion (266)

For T horizontal length

Inclined length is

$\sqrt{R^2+T^2}$



For 1m horizontal length inclined length is

$$\frac{\sqrt{R^2+T^2}}{T} \times 1m = l_1$$

wt waist slab

$= l_1 \times 1m \times t_w \times 25$

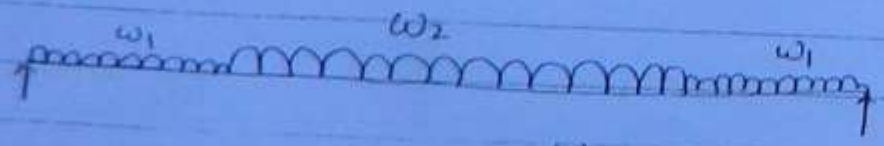
Self wt $= \frac{\sqrt{R^2+T^2}}{T} \times 1m \times 1m \times t_w \times 25$

Self wt. Steps $= \left(\frac{1}{T}\right) \times \frac{1}{2} \times R \times T \times 1m \times 25$

No. of steps if T is metre

Live load = $1m \times 1m \times w_L$

Total load = w_2



or

DATE:

$$B.M = \frac{w_2 \times l_{eff}^2}{8}$$

$$d = \sqrt{\frac{B.M}{Q.B}}$$

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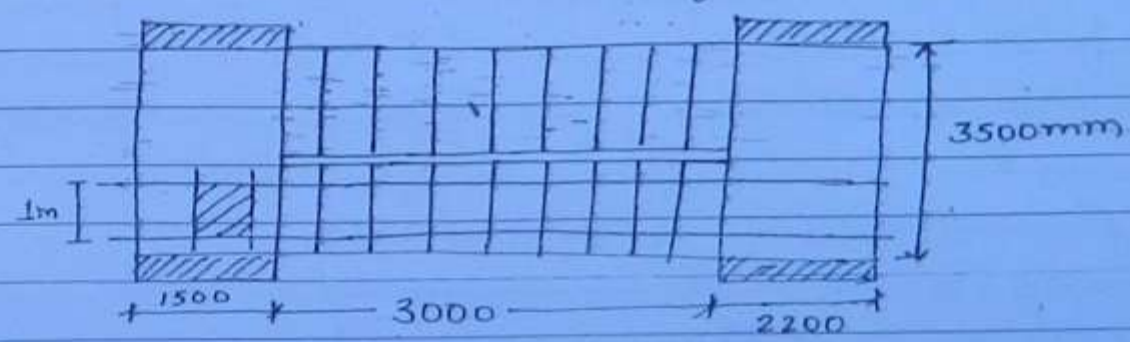
$$A_{st} = \frac{B.M}{0.87 f_y \cdot j \cdot d} = \frac{BM}{\sigma_{st} \cdot j \cdot d}$$

$$\text{or } A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] Bd$$

Ques Design a dog-legged staircase as shown in fig. Live load = 5kN/m² & Floor finish = 1kN/m²



Riser = 160mm
Treads = 300mm

Use - M-20 / Fe-415
Use L.S.M.

Sol^n Effective Span :- $l_{eff} = \frac{1500}{2} + 3000 + \frac{2200}{2} = 4850$

$$\overset{x}{0.75} + \overset{G}{3.0} + \overset{y}{1} = 4.75\,m$$

Load :-

Thickness of Slab (using defl^n Criteria)

thickness (Eff depth) $= \frac{\text{Span}}{20} = \frac{4750}{20} = 237.5\,mm$

$+ 30\,mm$

$267.5\,mm$

Use - 270 mm as thickness

DATE:

Landing Slab

Self wt - $1m \times 1m \times .27 \times 25 = 6.75$

Floor finish : $1 \times 1m \times 1 = 1.0$

live load = $1m \times 1m \times 5 = 5$

Total - 12.75 KN/m²

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Waist Slab

Self wt = $\frac{\sqrt{R^2+T^2}}{T} \times 1 \times 0.27 \times 25 =$

$1.13 \times 1 \times 0.27 \times 25 = 7.65$ KN/m²

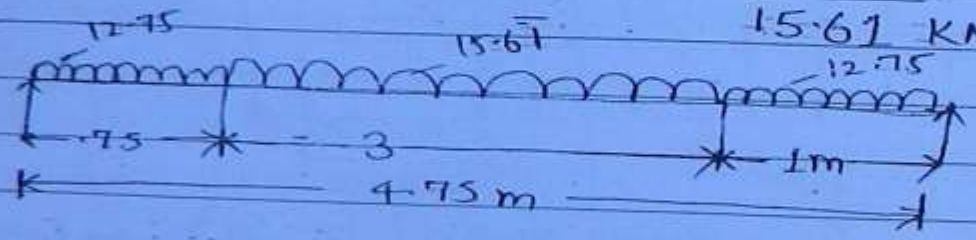
Self wt of Steps = $\left(\frac{1}{T}\right) \times \frac{1}{2} \times R \times T \times 1m \times 25$

$= \left(\frac{1000}{300}\right) \times \frac{1}{2} \times .16 \times .30 \times 1m \times 25 = 2.0$

Floor finish = $1 \times 1 \times 1 = 1.0$

Live load = $1 \times 1 \times 5 = 5.0$

15.61 KN/m²



Max. B.M (Considering same load on entire span)

$B.M = \frac{15.61 \times 4.75^2}{8} = 44.03$ KN-m

Factored = $1.5 \times B.M = 66.038$ KN-m

$d = \sqrt{\frac{B.M}{Q.B}} = \sqrt{\frac{66.04 \times 10^6}{.138 \times 20 \times 1000}}$

$d = 154.66$ mm $\nless$ 240 mm

Safe

Use D = 270 mm

d = 240 mm

DATE:

$$A_{st} = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 66.04 \times 10^6}{1000 \times 240^2 \times 20}}\right] 1000 \times 240^2$$

$= 820.75\ mm^2$

209

Use 16mm dia bars

$$\text{Spacing} = \frac{1000}{820.75} \times \frac{\pi}{4} \times 16^2 = 244.9\ mm$$

Use 12mm

$$\text{Spacing} = \frac{1000}{820.75} \times \frac{\pi}{4} \times 12^2 = 137\ mm$$

Use 12mm ϕ bars @ 130 mm c/c

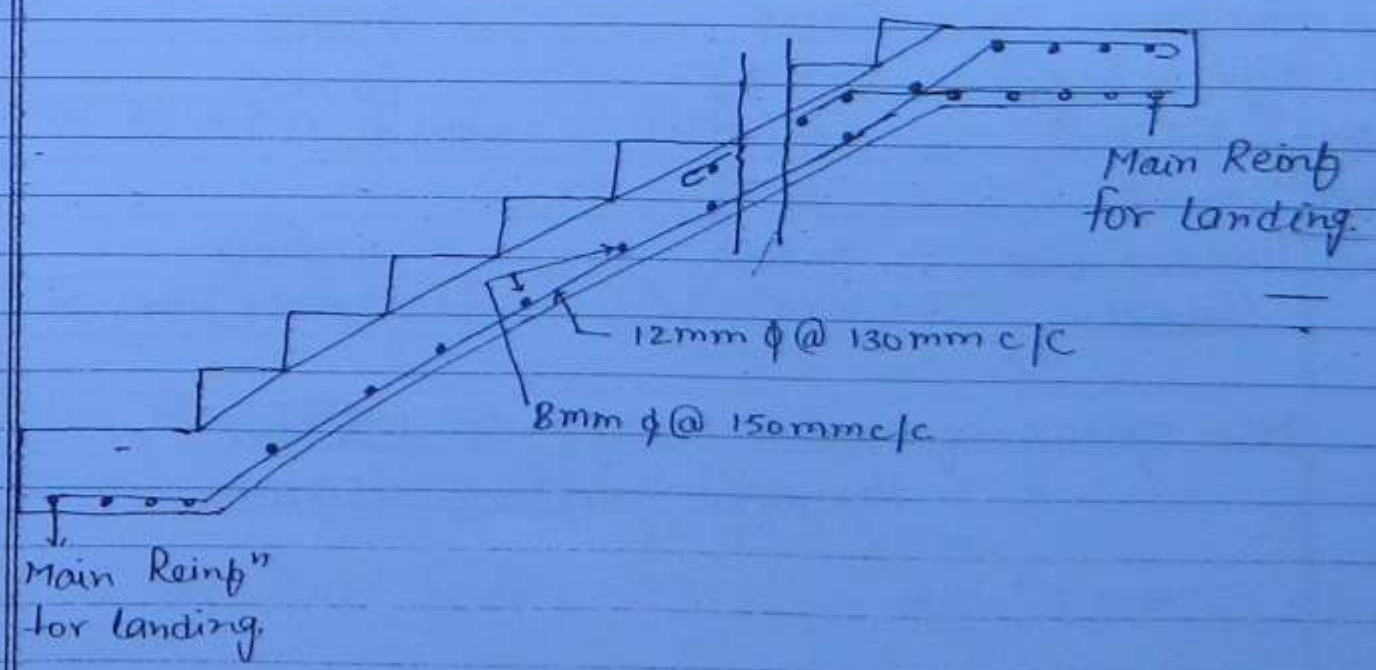
Distribution bars

$$= \frac{0.12}{100} \times 1000 \times 270 = 324\ mm^2$$

$$\text{Spacing of 8 mm } \phi \text{ bars} = \frac{1000}{324} \times \frac{\pi}{4} \times 8^2$$

$= 155\ mm$

Use 8mm ϕ bars @ 150 mm c/c



DATE:

→ Design of wall (As per 32 / P-61)
designed same as Column

32.3.3 → Max. eff. height to thickness ratio

$$\frac{H_{eff}}{t} \not> 30$$

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Effective height

(a) Restrained against rotation at both end
(i) By floor – 0.75 H_w
(ii) By intersecting walls = 0.75 L_1

(b) When not Restrained against rotation
(i) By floor = 1.0 H_w
By intersecting Walls = 1.0 L_1

Same

32.5 – Min Reinf^n required –
Horizontal & vertical bar reinf^n are provided.

	Vertical Reinf^n	Horizontal Reinf^n
(i) Deformed bars $\not>$ 16mm	0.0012 (0.12%)	0.0020 (0.20%)
(ii) For other type of bars (Mild Steel & greater than 16mm ϕ)	0.0015 (0.15%)	0.0025 (0.25%)
(iii) Welded wire fabrics	0.0012 (0.12%)	0.0020 (0.20%)

Max. Spacing of vertical reinf^n = 450 mm
Horizontal reinf^n = 450 mm

DATE:

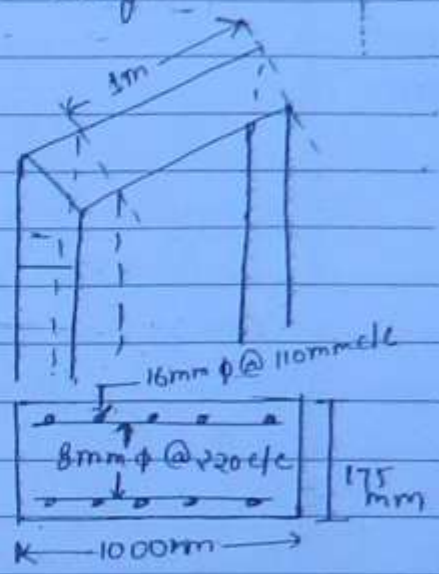
ES-1995

Ques A reinf Concrete wall 175 mm thickness & 3.2 m effective height is needed for a compressive load of 1000 kN/m. Design the wall. use - M20 / Mild Steel reinf.

P = 1000 kN for 1m length of wall

Area = 1000 × 175 mm

Asc =



(211)

$$\frac{H_{eff}}{t} = \frac{3200}{175} = 18.29 > 12$$

Design as long column.
Using L.S.M

$$P = C_r (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$$

$$\left(1.25 - \frac{l_{eff}}{48B}\right)\left[(1000 \times 175 - A_{sc}) \times 4 + A_{sc} \times 130\right]$$

$$= 1000 \times 10^3 = \left(1.25 - \frac{3200}{48 \times 175}\right)\left[1000 \times 175 \times 4 - 4A_{sc} + 130A_{sc}\right]$$

On each face. $A_{sc} = \frac{3576.86}{2} = 1788.43$

No. of 16mm ϕ bars $= \frac{1788.43}{\frac{\pi}{4} \times 16^2} = 8.89$ say 9 Nos.

Min vertical reinf $= \frac{0.5}{100} \times 1000 \times 175$
$= 262.5 \text{ mm}^2$ OK

Min Horizontal reinf $= \frac{0.25}{100} \times 1000 \times 175$
$= 437.5 \text{ mm}^2$

On each face $= \frac{437.5}{2} = 218.75 \text{ mm}^2$

Spacing of 8mm ϕ bars

$$= \frac{1000}{218.75} \times \frac{\pi}{4} \times 8^2 = 229mm$$

Tuesday
1/11/2011

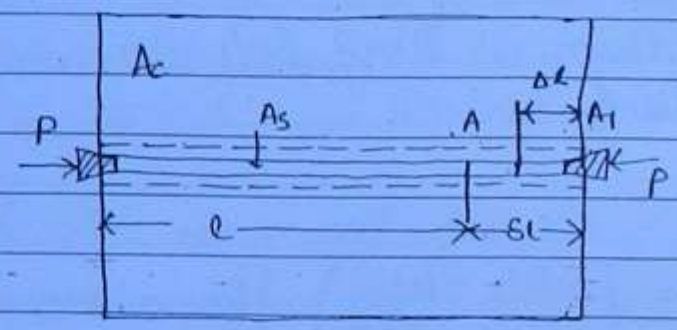
212

PRESTRESSED CONCRETE

Concept :- If a steel reinforcement provided in a concrete is applied a force and is anchored after getting an elongation of δl. Now force is removed

Strain in steel

$$\Rightarrow e = \frac{\delta l}{l}$$



Stress developed in steel

$$\Rightarrow p_s = e \cdot E_s$$

Then $p_s = \frac{\delta l}{l} E_s$

If cross-section area of steel = A_s

Force developed due to δl deformation

$$P_s = p_s \times A_s = \frac{\delta l}{l} E_s \cdot A_s$$

This force $P_s = P$ = Force applied on concrete

It is called prestressing force

P_s = Tensile in steel

P = Compressive in concrete

Stress developed in concrete $= p_c = \frac{P}{A_c}$

If due to any reason elongation provided is reduced, say by Δl. It will cause loss of prestress

If loss of elongation = Δl

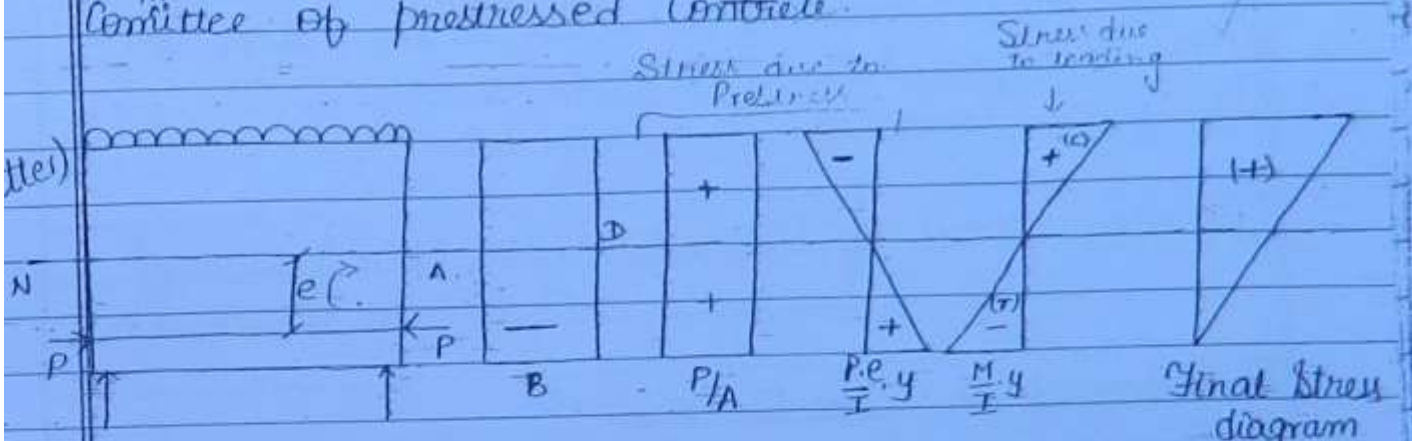
loss of strain $= \frac{\Delta l}{l}$

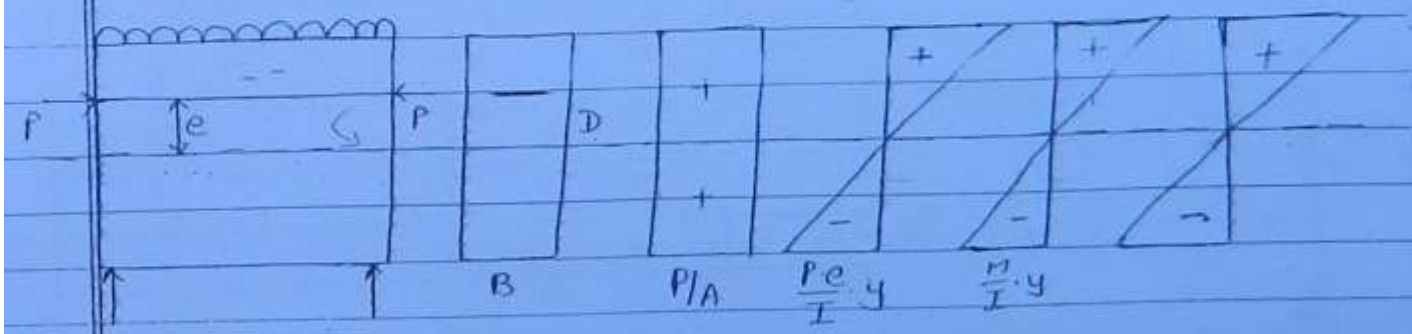
Loss of stress or prestress $= \frac{\Delta l}{l} E_s$

Net (Final) Stress available $= \frac{\delta l - \cancel{\Delta l}}{l} E_s$

(213)

Definition – "Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution, that stresses caused due to external loading are counterbalance upto a desired degree" As per ACI Committee of prestressed Concrete.





Necessity of high strength steel and concrete

(#) Steel – In case of steel, losses are very high. Only due to creep & shrinkage loss of strain = 0.0008.

loss of stress due to creep and shrinkage $= 0.0008 \times E_s$

$= 0.0008 \times 2 \times 10^5$

$= 160\ N/mm^2$

Total losses may not lie in the range of 200 to 300 N/mm^2

→ The losses should not be more that 10 to 20% of initial stresses provided in steel.

→ Initial ~~stress~~ required —

1000 N/mm² to 2000 N/mm²

Due to above reason we need a very high strength steel for PSC.

(214)

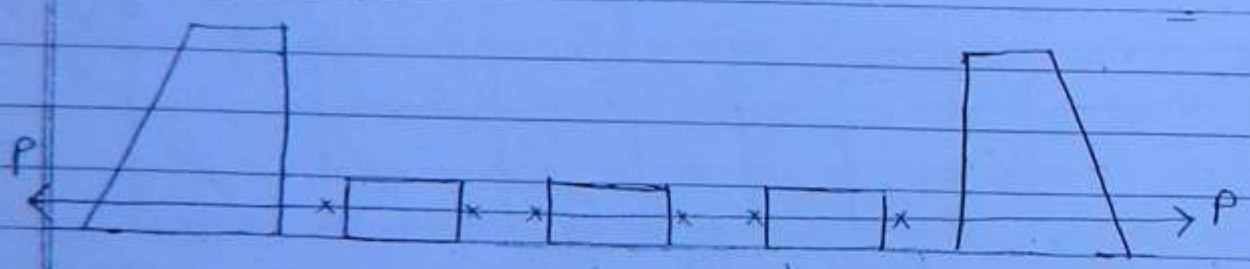
(ii) Concrete -

1. To bear high bearing stresses developed at ends.
2. In case of pre tensioned members to bear high bond stress developed betn steel and Concrete.
3. To reduce Creep and Shrinkage losses.

→ Minimum grade of Concrete - M-30 (Post tensioned)
- M-40 (Pre tensioned)

Pretensioned and post tensioned

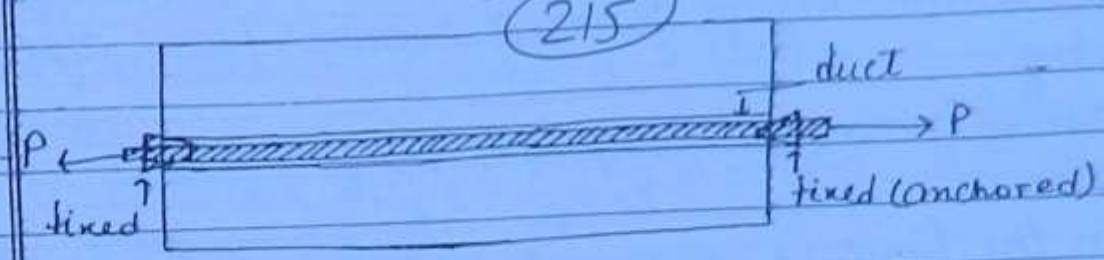
⇒ Pretensioned - (Hoyer Methor or long line Method)



Properties

(i) Steel is in direct Contact with Concrete
(ii) Steel is tensioned before Casting of Concrete
(iii) After Setting of Concrete steel reinf are Cut
(iv) Prestressing force is applied due to bond action betn steel and Concrete.

→ Post tensioned.



Properties

(i) Concrete is casted before prestressing of steel.

(ii) A duct is left in concrete during casting

(iii) After setting of concrete, steel reinf (tendons) are inserted in duct and tensioned either from one end or both end, and anchored

(iv) Steel and concrete are not in direct contact.

→ Advantage of using prestressed concrete

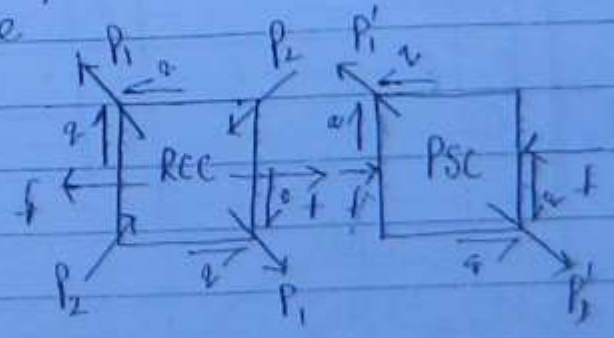
(i) Cross-section of concrete is utilised more effectively & entire cross-section is in compression (minor tensile stresses can be ignored)

(ii) In case of PSC, lighter c/s are required. Saving may be in supporting structure and foundation due to lighter weight of ~~suppo~~ superstructure.

(iii) Improve shearing resistance

$$p_1 = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$

$$p_1' = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$



Assumption

1. Plane section before bending remains plane after bending.
   → Strain diagram is uniform (216)

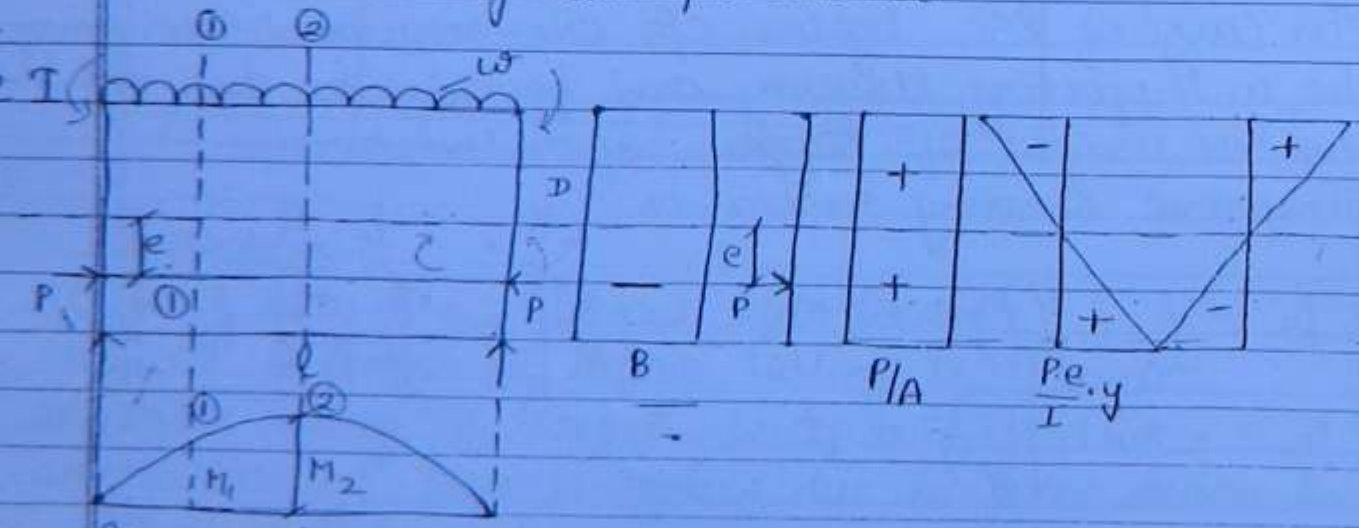
2. Within the limit of deformation hooke's law is valid. applicable to Concrete & Steel

3. The Stresses in reinforcement doesn't Changes along the length of reinforcement, Stress Changes takes place only for Concrete.
   Variation of Stress in reinf^n due to Change in external loading is ignored.

Basic Concepts

1. Stress Concept method
2. Strength Concept method
3. Load balancing Concept method

Case I



Stresses developed

① Direct stress due to P = ± P/A = P_0

2. Bending Stresses due to Pe = P_{b1} = ± $\frac{Pe}{I} \cdot y$

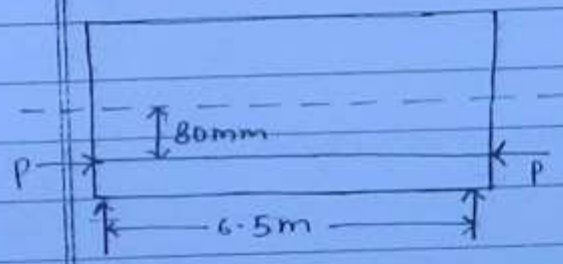
Moment due to load at Sec^n ① - ① $= \pm \frac{M_1 \cdot y}{I}$

→ Final stresses at top $= \frac{P}{A} - \frac{P.e.y}{I} + \frac{M \cdot y}{I}$

Final stresses at bottom $= \frac{P}{A} + \frac{P.e \cdot y}{I} - \frac{M y}{I}$ (217)

$$\boxed{\text{Final Stresses} = \frac{P}{A} \mp \frac{P.e \cdot y}{I} \pm \frac{M \cdot y}{I}}$$

A prestress Concrete beam 250mm x 300mm is prestress by 14 wires of 5 mmϕ (Straight Cable) provided at an eccentricity of 80mm. Initial prestress = 1500 N/m². If span is 6.5m & beam is subjected to live load of 8kN/m. Calculate max. stress developed in beam at mid span at end. ~~Losses~~ losses are Considered.



$P = A_s \times P_s = 14 \times \frac{\pi}{4} \times 5^2 \times 1500 = 412.33$ kN

$e = 80$ mm

Loads - D.L $= 0.25 \times 0.30 \times 1 \times 25$

L.L $= 8$ kN/m

total $= 9.875$ kN/m

Max. B.M $= \frac{wl^2}{8} = M_2 = \frac{9.875 \times 6.5^2}{8} = 52.15$ kN-m

Area $= 250 \times 300 = 75 \times 10^3$ mm²

$I = \frac{250 \times 300^3}{12} = 5625 \times 10^5$ mm⁴

$y = \frac{300}{2} = 150$

$y_t = 150$ mm

$y_B = 150$ mm

DATE:

1 At ends

Stress at top $= \frac{P}{A} - \frac{P.e}{I} \cdot y_t$

$\frac{412.33}{75\times10^3} - \frac{412.33\times80}{5625\times10^5}\times150$ (218)

$= -3.296\ N/mm^2$

Stress at bottom $= \frac{P}{A} + \frac{P.e}{I}\cdot y$

$\frac{412.33}{75\times10^3} + \frac{412.33\times80}{5625\times10^5}\times150 = 14.296\ N/mm^2$

Stress at mid span at top

$= \frac{P}{A} - \frac{P.e}{I}\cdot y_t + \frac{M_2}{I}\cdot y$

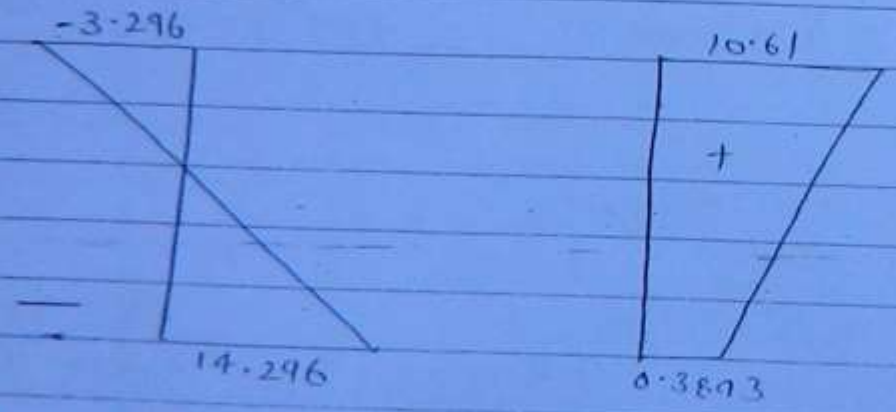
$-3.296 + \frac{52.15\times10^6}{5625\times10^5}\times150 = 10.61\ N/mm^2$

~~At bott~~ $+13.90$

At bottom $= P/A + \frac{P.e}{I}\cdot y - \frac{M_2}{I}\cdot y$

$14.296 - 13.90 = .3893$

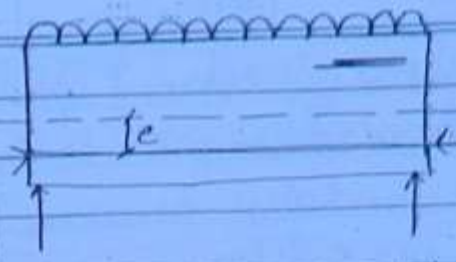
At end



(#) Effect of loading on stresses in tendons

Slope of beam at end due to U.DL = (i)

$\theta_1 = \frac{wl^3}{24EI}$



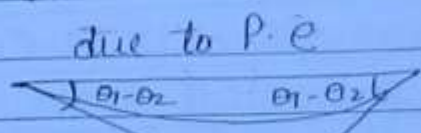
(219) Slope of beam at ends due to moment P.e

$$\theta_2 = \frac{P.e.l}{2EI}$$

(using mm1 area method)

due to Moment due to load

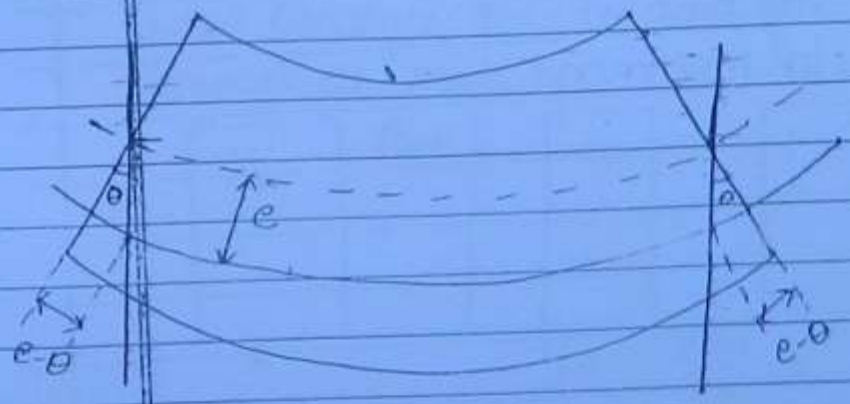
due to P.e



Net Slope $\theta = \theta_1 - \theta_2$

$$\theta = \frac{wl^3}{24E_cI} - \frac{P.e.l}{2E_cI}$$

Net Slope due to combined effect



total increase in length

$= 2e\theta$

Total increase in length of steel r/f (tendons)

$$\Delta l = (+)\ 2\cdot e\cdot\theta$$

gain in stress due to Δl increase in elongation

stress increases $= \frac{\Delta l}{l} \times E_s$

DATE:

PS 1990

Ques A PSC beam of span 6m has a width of 150 mm & depth = 300 mm. Initial stress = 1000 N/mm². Cable is at constant $e = 50$ mm. $A_s = 100$ mm². Find % increase in stress in wires if beam supports live load of 5 kN/m. Unit wt. of concrete = 24 kN/m³. $E_c = 36$ kN/mm², $E_s = 210$ kN/mm²

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Soln Load – D.L = 0.15 × 0.30 × 1 × 24 = 1.08

L.L = 5 kN/m

total = 6.08 kN/m

$P = A_s \times P_s = 100 \times 1000$

$e = 50$

Slope due to load $= \frac{wl^3}{24EI}$

$A = 150 \times 300 = 45 \times 10^3$ mm²

$I = \frac{150 \times 300^3}{12} =$

$\frac{wl^3}{24E_cI} =$

$= 0.0045$

Slope due to moment $= \frac{Pe \cdot l}{2E_cI}$

$= 1.2345 \times 10$

Net slope (Sagging) $\theta = \theta_1 - \theta_2$

$= 3.269 \times 10^{-3}$

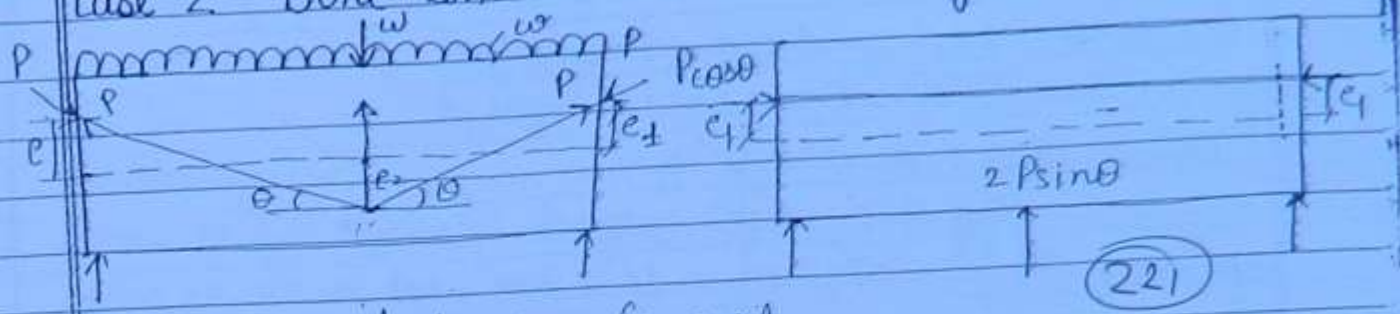
Gain in strength $= \frac{\Delta l}{l} \times E_s$

$= \frac{2 \cdot e \cdot \theta}{l} \times E_s$

$= 11.4415$ N/mm²

% increase $= \frac{11.44}{1000} \times 100 = 1.144$

Case-2. Bent tendons (Load balancing Concept method)



Using load balancing Concept.

1 Make the profile of Cable straight from the point where it is at end

2 upward balancing load due to profile of Cable

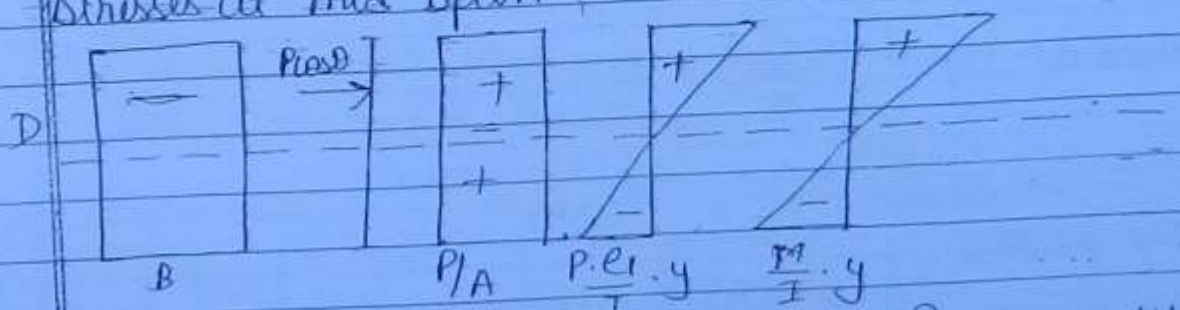
(Point load = $2P\sin\theta$ (↑)

downward load = W

Net point load = $(W - 2P\sin\theta)$

Now Analyse the beam for equivalent beam shown

Stresses at mid span ;

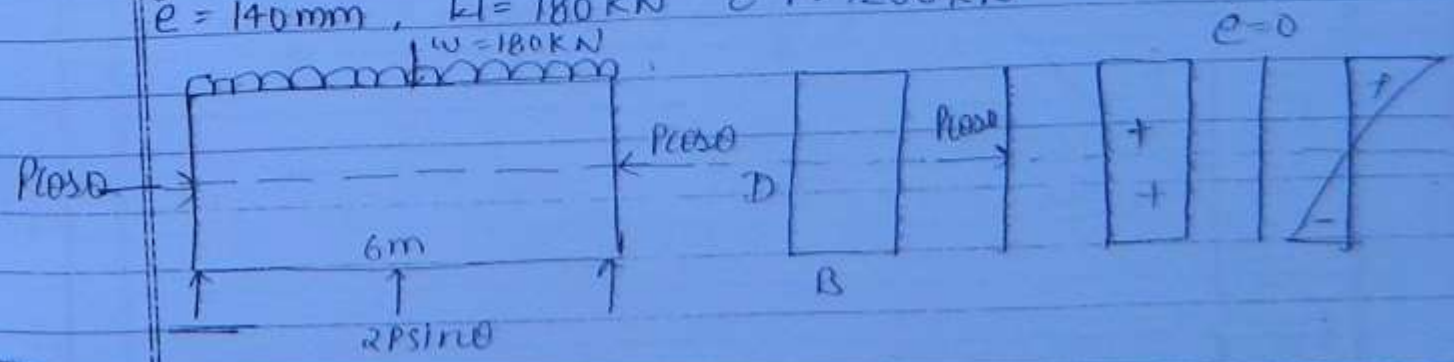


Where moment $M = \frac{W - 2P\sin\theta}{4} \cdot l + \frac{wl^2}{8}$

Final Stresses $= \frac{P}{A} \pm \frac{P \cdot e_1}{I} \cdot y \pm \frac{M}{I} \cdot y$

ES-2005 Ques A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at mid span considering self wt also.

$e = 140mm$, $W = 180kN$ & $P = 1200kN$



DATE:

$P = 1200\,kN$

$\theta = \tan\theta = \frac{140}{3000} = 0.0467$ radian

$\theta = 2^\circ 40' 18.71''$

$P\cos\theta = 1200 \times \cos 2^\circ 40' 18.71''$

$= 1198.99 \cong 1200\,kN$ 222

upward force $= 2P\sin\theta = 111.878\,kN$

net point load $= w - 2P\sin\theta = 180 - 111.88$

$= 68.12\,kN$

Self wt $= 0.4 \times 0.6 \times 1 \times 25 = 6\,kN/m = w$

B.M at mid span $= \frac{(w - 2P\sin\theta)}{4} l + \frac{wl^2}{8}$

$\frac{68.12}{4} \times 6 + \frac{6 \times 6^2}{8}$

$= 129.18\,kN\text{-}m$

$A = 400 \times 600 = 24 \times 10^4\,mm^2$

$I = \frac{400 \times 600^3}{12} = 72 \times 10^8\,mm^4$

Stresses at top $= \frac{P\cos\theta}{A} + 0 + \frac{M}{I} \cdot y$

$\frac{1200 \times 10^3}{24 \times 10^4} + 0 + \frac{129.18 \times 10^6}{72 \times 10^8} \times 300$

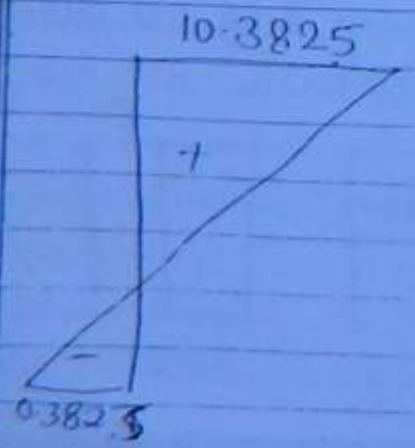
$5 + 5.3825$

$= 10.3825\,N/mm^2$

Stresses at bottom $= \frac{1200 \times 10^3}{24 \times 10^4} - \frac{M}{I} \cdot y$

$5 - 5.3825$

$= -0.3825\,N/mm^2$



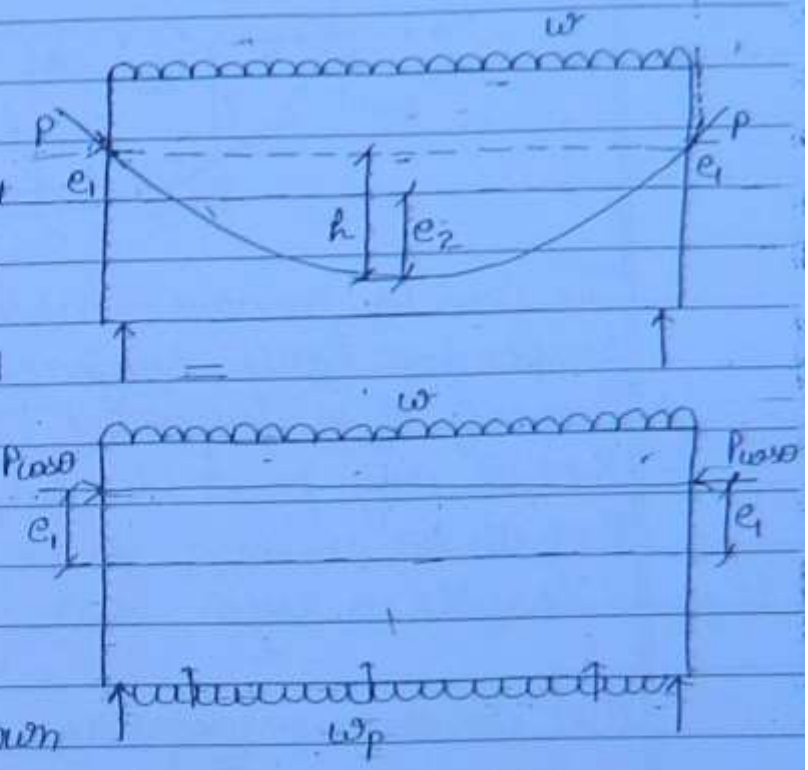
Tendon of parabolic profile

→ (load balancing concept)

In this case due to load balancing :-

1. Make profile of cable straight from pt. where the cable lie at ends

2. An upward balancing load (U.d.l) may be applied



upward $w_p = \frac{8Ph}{l^2}$
U.d.l

Now equivalent beam as shown in fig can be analysed.

Net U.d.l $= (w - w_p)$

$B.M = M = \frac{(w - w_p)l^2}{8}$

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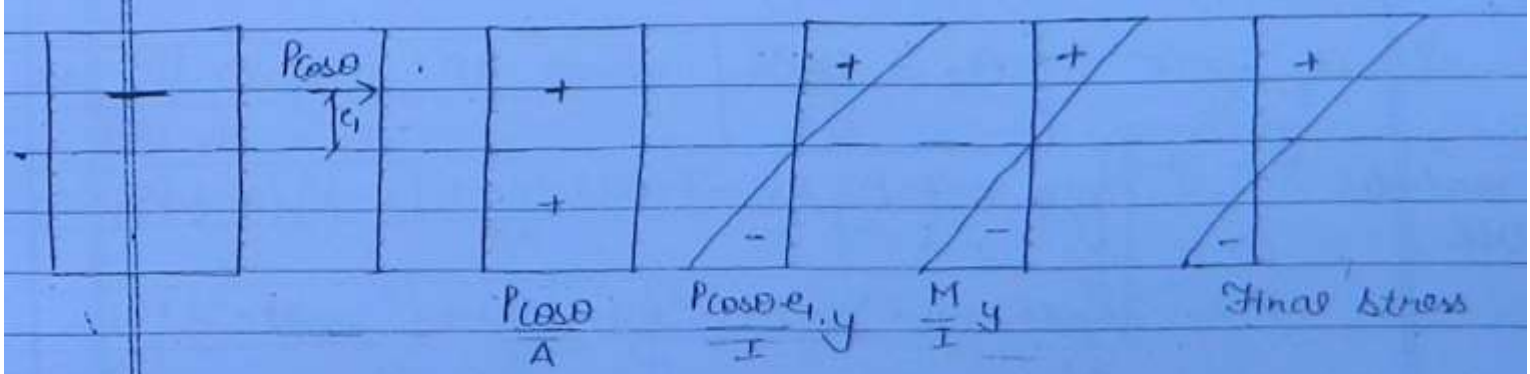
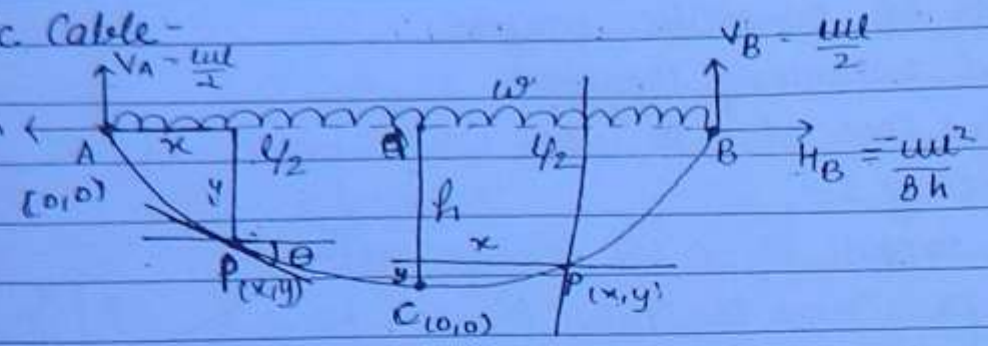


Final stresses

$$= \frac{P\cos\theta}{A} \pm \frac{P\cos\theta \cdot e_1}{I} y \pm \frac{M}{I} y$$

Parabolic Cable -



The Case of parabolic cable is similar to the Case of a suspended Cable, subjected to U.d.l (w)

→ Property of a suspended Cable.

:- B.M at any point in suspended Cable = 0.

B.M at C = 0

$V_A = V_B = \frac{wl}{2}$

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$$B.M \text{ at } C = \frac{wl^2}{8} + H_B \times h - V_B \times \frac{l}{2} = 0$$

$$\frac{wl^2}{8} + H_B \cdot h - \frac{wl^2}{4} = 0$$

$$H_B \cdot h = \frac{wl^2}{8}$$

$$\boxed{H_B = \frac{wl^2}{8h}}$$

For parabolic Cable

$H_B = P\cos\theta \simeq P$

$w = w_p$

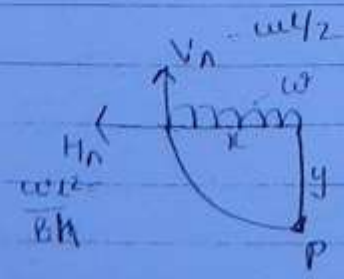
$P = \frac{w_p l^2}{8h}$

$$\boxed{w_p = \frac{8Ph}{l^2}}$$

Equation of parabolic tendon

Case 1 Origin is at ends.

B.M at P = 0



$$w \cdot x \cdot \frac{x}{2} + H_A \cdot y - V_A \cdot x = 0$$

$$\frac{wx^2}{2} + \frac{wl^2}{8h} y - \frac{wl}{2} x = 0$$

$$\frac{wl^2}{8h} y = \frac{wl}{2} x - \frac{wx^2}{2}$$

$$\frac{l^2}{4h} y = lx - x^2$$

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$$\frac{l^2}{4h} y = x(l-x)$$

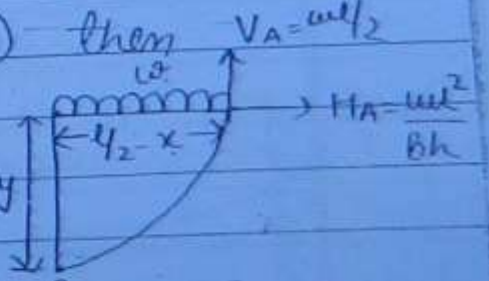
$$\boxed{y = \frac{4hx(l-x)}{l^2}}$$ Eq. of parabolic Cable (also called as deflection eq^n)

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$ Slope equation

Slope at ends at $x=0$ or $x=l$

$$\frac{dy}{dx} = \tan\theta = \frac{4h}{l} = \theta$$ Slope at end

If origin is at centre say C (0,0) then



B.M at P = 0

$$\frac{w}{2}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}-x\right) + \frac{wl^2}{8h}(h-y) - \frac{wl}{2}\left(\frac{l}{2}-x\right) = 0$$

$$\frac{w}{2}\left(\frac{l^2}{4} - lx + x^2\right) + \frac{wl^2}{8} - \frac{wl^2}{8h} y - \frac{wl^2}{4} + \frac{wlx}{2} = 0$$

$$\cancel{\frac{wl^2}{8}} - \cancel{\frac{wlx}{2}} + \frac{wx^2}{2} + \cancel{\frac{wl^2}{8}} - \frac{wl^2}{8h} y - \cancel{\frac{wl^2}{4}} + \cancel{\frac{wlx}{2}} = 0$$

$$\frac{wx^2}{2} = \frac{wl^2}{4 \cdot 8h} \cdot y$$

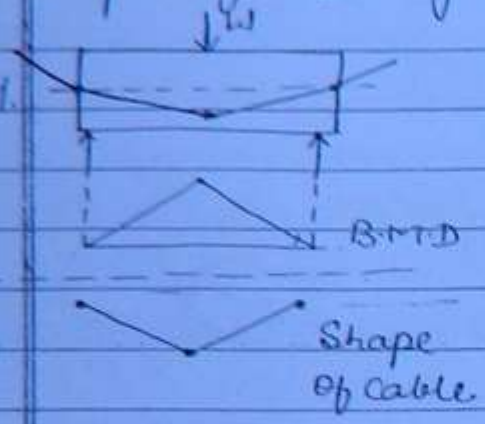
$$\boxed{\frac{4x^2 h}{l^2} = y}$$ Eq. of Cable if origin is at Centre

DATE:

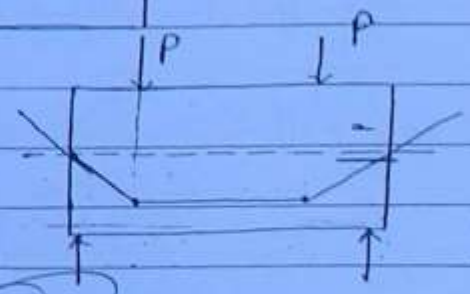
→ Shape of a load balancing Cable.

⇒ Shape of a load balancing Cable is similar to the Shape of bending moment diagram. (BMD)

Case 1.



$$W = 2P\sin\theta$$



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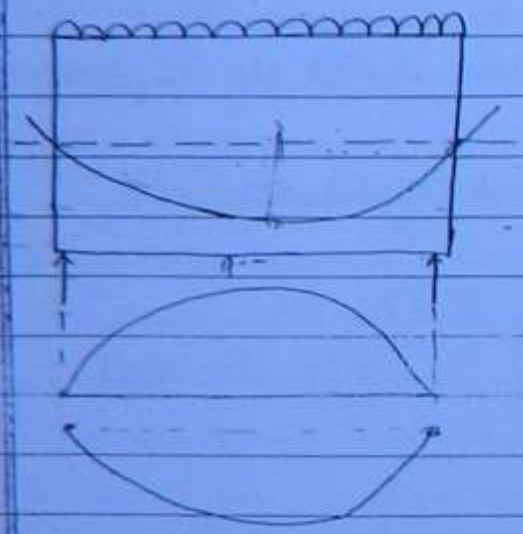
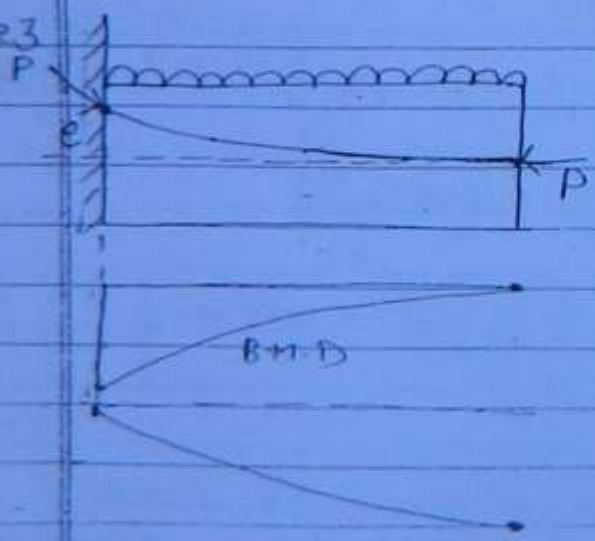
Case 2.

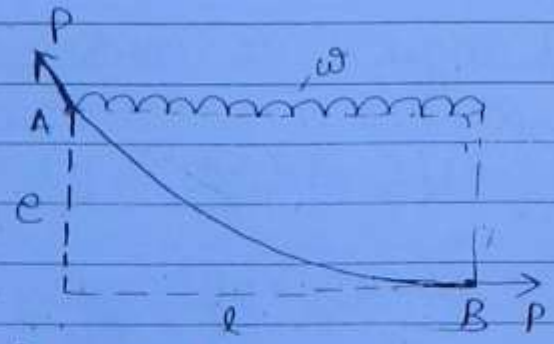


$$w_p = \frac{8Ph}{l^2}$$

$$h = \frac{w_p l^2}{8P}$$

Case 3





BM at A = 0

$$P \cdot e = \frac{wl \cdot l}{2}$$

$$e = \frac{wl^2}{2P}$$

DATE:

Ques Determine the profile of a load balancing cable for a cantilever beam of the span 4m carrying an U.D.L of 12 kN/m. Prestressing force is 1500 kN.

Sol. For a cantilever shape is shown on left hand side page. Shape of cable is mirror image of B.M.D as shown

eccentricity at support $= e = \frac{wl^2}{2P}$

$$e = \frac{12 \times 4^2}{2 \times 1500} = 0.064\ m$$

$$e = 64\ mm$$

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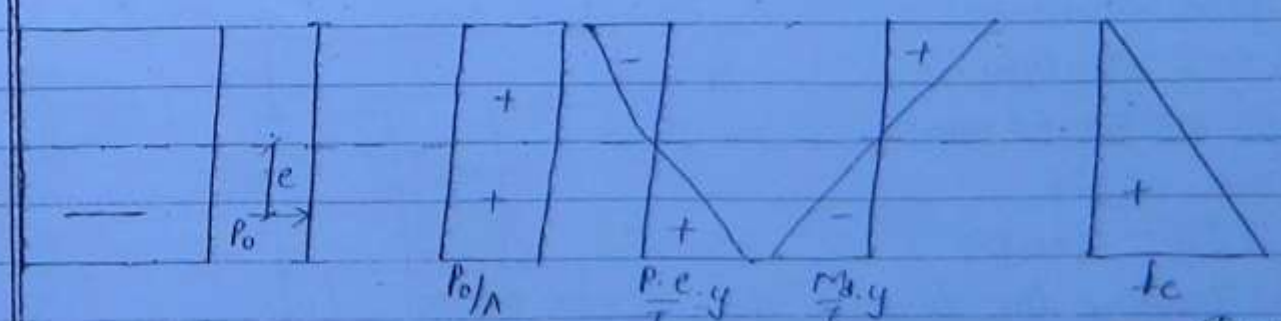
Stresses in a beam ~~of~~ at different stage of loading

1. At transfer
2. At final stage of loading

→ At transfer :- Just after transfer of prestressing force in the beam.

1. Losses of prestress are not considered.
2. Live load is not considered.
3. Dead load may be considered (if not supported)
4. Dead load not to be consider if beam is still on its prestressing bed.



Total stress diagram at transfer

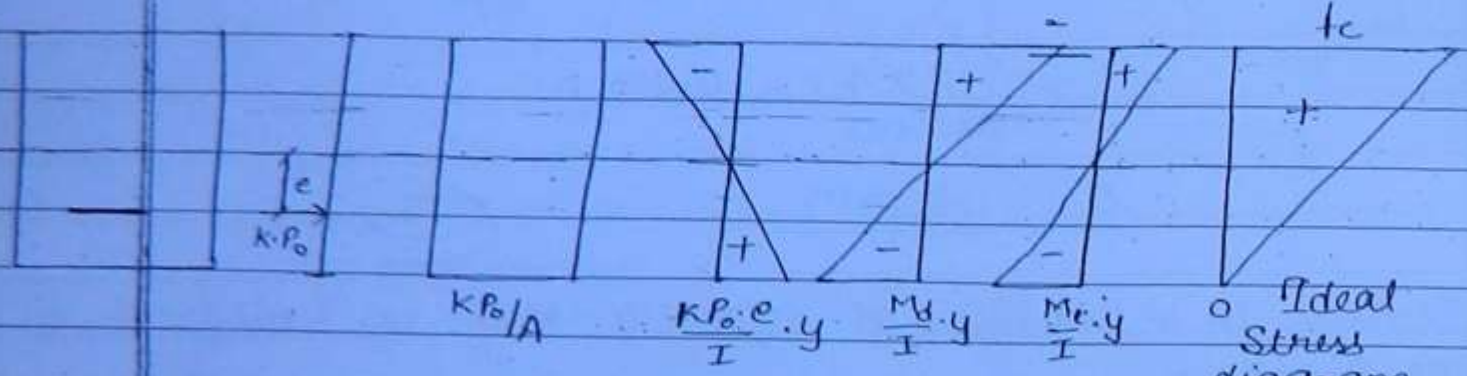
→ At final stage

1. Losses are to be considered

Final $P = KP_0$ || if loss = 20%

K = loss factor $(1 - \text{loss}\%)$ || $K = 0.80$

2. All ~~loss~~ loads are to be considered.



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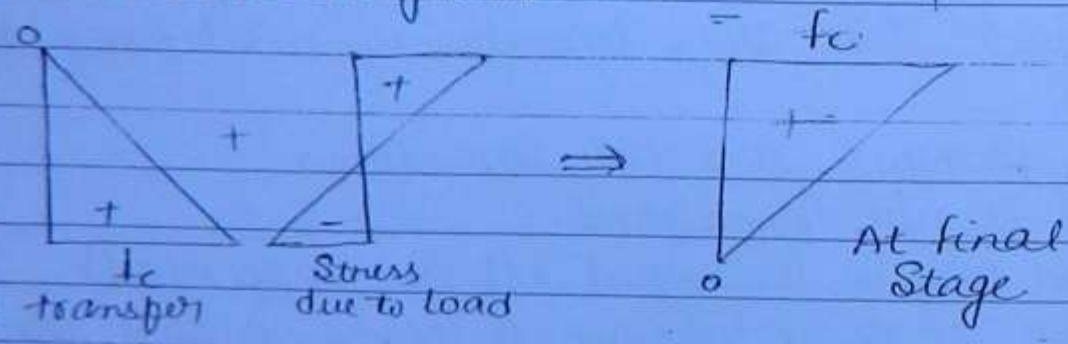
Ideal Stress diagram



Ques 1 A post tensioned prestress Concrete is as shown in fig.

Span = 20m

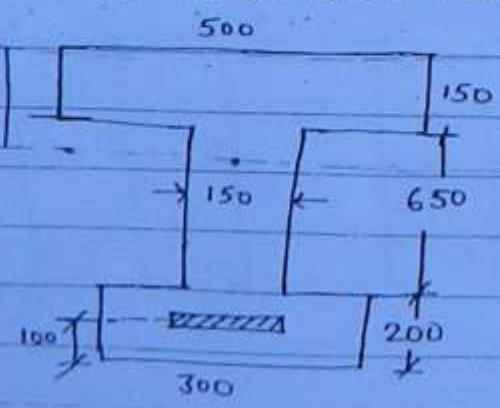
$P = 1100\ N/mm^2$

$A_s = 1385.44\ mm^2$

loss = 15%

U.d.l = 12 kN/m

Determine the stresses in beam at mid span at diff^t stage of loading



Sol. $A = 500\times150 + 650\times150 + 200\times300 = 232500\ mm^2$

$$\bar{y} = \frac{500\times150\times\overset{y_1}{75} + (150\times650\times475) + (300\times200\times900)}{232500}$$

$\bar{y} = y_t = 455.65\ mm$

$y_B = 1000 - 455.65 = 544.35\ mm$

$$I = \left[\frac{500\times150^3}{12} + 500\times150\times(\overset{\bar{y}}{455.65} - \overset{y_1}{75}) + \frac{150\times650^3}{12} + 150\times650(455.65-475)^2 + \frac{300\times200^3}{12} + 300\times200(455.65-900)^2\right]$$

$I = 2.6524\times10^{10}\ mm^4$

229

Dead load $= \frac{232500\times1\times25}{10^6} = 5.8125\ kN/m$

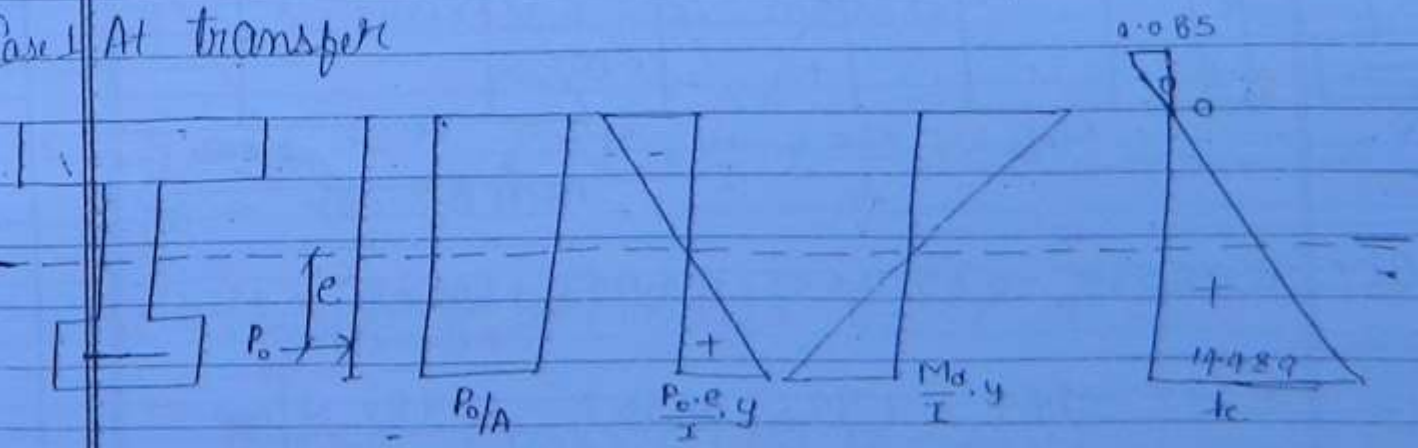
$M_d = \frac{w_d l^2}{8} = \frac{5.8125\times20^2}{8} = 290.625\ kN\text{-}m$

Live load $= 12\ kN/m$

$M_l = \frac{12\times20^2}{8} = 600\ kN\text{-}m$

$e = y_B - 100$
$= 444.35\ mm$

Case 1 At transfer



Prestressing force $P_o = A_s\times f_o = 1385.44\times1100$
$= 1523984\ N$

Stress at top

$$= \frac{P_0}{A} - \frac{P_0 \cdot e}{I} \cdot y_t + \frac{M_d}{I} \cdot y_t$$

$$= \frac{1523984}{232500} - \frac{1523984 \times 444.35}{2.6524 \times 10^{10}} \times 455.65 + \frac{290.625 \times 444.35}{2.6524 \times 10^{10}}$$

$$= 6.555 - 11.633 + 4.993 = -0.085 \ N/mm^2$$

Stress at bottom

~~6.555 + 11.633 - 4.993 = 600~~

$$= \frac{P_0}{A} + \frac{P_0 \cdot e}{I} \cdot y_B - \frac{M_d}{I} \cdot y_B$$

230

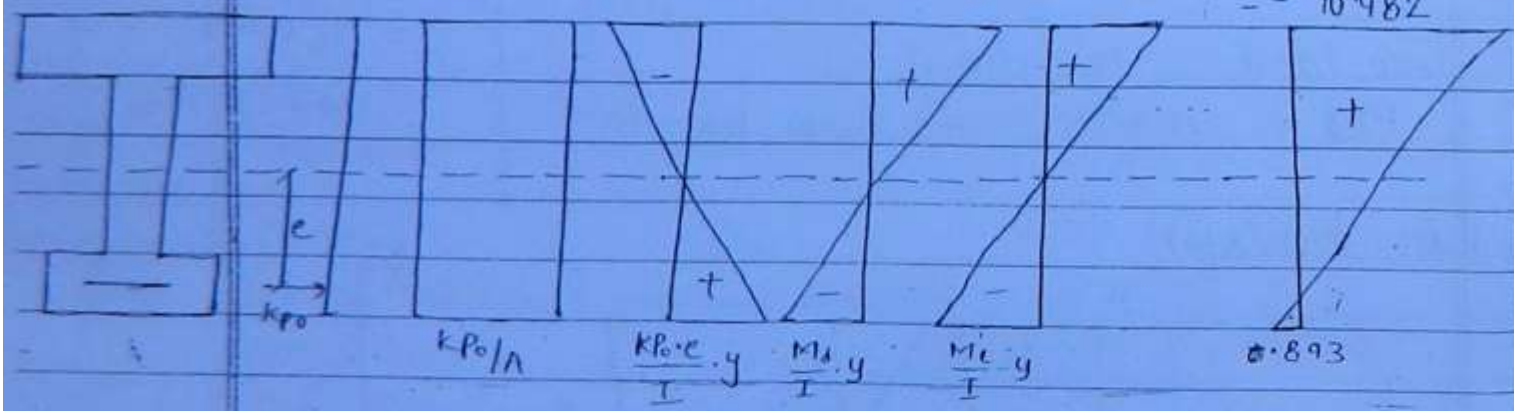
$$= 6.555 + 11.633 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65}$$

$$= 6.555 + 13.898 - 5.964 = 14.489 \ N/mm^2$$

Final Stage

Final $P = KP_0 = .85 \times 1523984 = 1295386.4$

10.982



tress at top = $.85 \times 6.555 - 0.85 \times 11.6333 + 4.993 + \frac{600 \times 10^6}{2.6524 \times 10^{10}} \times 455.65$

$= 5.572 - 9.89 + 4.993 + 10.307 = 10.982 \ N/mm^2$

tress at ottom = $5.572 + 9.89 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65} - 10.307 \times \frac{544.35}{455.65}$

$= -0.893 \ N/mm^2$

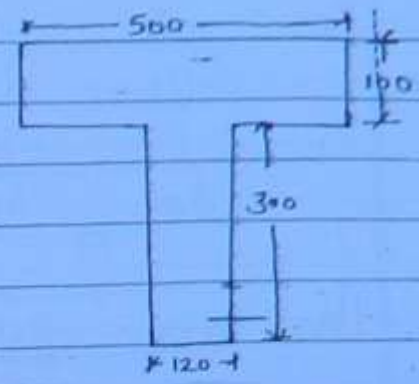
Ques A prestress concrete beam as shown in fig is to be designed for 11 kN/m live load, over a span of 6m. If stress in concrete should not exceed.

In compression = 15 N/mm²

In tension = 0

at any stage.

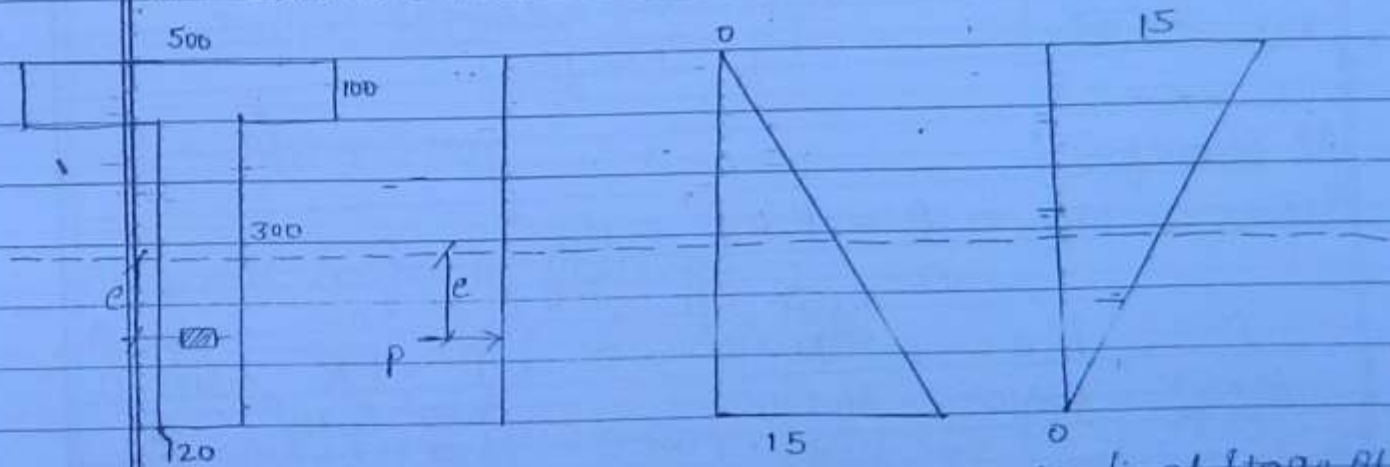
Find out prestressing force & eccentricity at which P should be applied.



231

Solⁿ Properties of Secⁿ

$A = 500 \times 100 + 300 \times 120 = 86 \times 10^3 \, mm^2$



At transfer

At final stage of loading

$$\bar{y} = \frac{500 \times 100 \times 50 + 300 \times 120 \times 250}{86 \times 10^3}$$

$\bar{y} = 133.72 = y_t$

$y_B = 266.28 \, mm$

$$I = \frac{500 \times 100^3}{12} + 500 \times 100 (133.72 - 50)^2 + \frac{120 \times 300^3}{12} + 120 \times 300 (133.72 - 250)^2$$

$= 1.15 \times 10^9 \, mm^4$

DATE:

$$\frac{M}{I}y = \frac{M}{Z}$$

$$Z_t = \frac{I}{y_t} = \frac{1.15 \times 10^9}{133.72} = 86 \times 10^5$$

$$Z_B = \frac{I}{y_B} = \frac{1.15 \times 10^9}{266.28} = 43.19 \times 10^5$$

$$\text{Dead load} = \frac{86 \times 10^3}{10^6} \times 1 \times 25 = 2.15 \text{ kN/m}$$

$$M_d = \frac{w_d l^2}{8} = \frac{2.15 \times 6^2}{8} = 9.675 \text{ kN.m}$$

live load = 11 (232)

$$M_l = \frac{11 \times 6^2}{8} = 49.5 \text{ kN.m}$$

At transfer t

Stress at top $= \frac{P}{A} - \frac{P.e}{I} y_t + \frac{M_d}{Z_t} = 0$

$$\frac{P}{86 \times 10^3} - \frac{P.e}{86 \times 10^5} + \frac{9.675 \times 10^6}{86 \times 10^5} = 0 \Rightarrow \frac{P}{86 \times 10^3} - \frac{P.e}{86 \times 10^5} + 1.125 = 0 \quad \text{— (1)}$$

Stress at bottom $\frac{P}{A} + \frac{P.e}{I} \cdot y - \frac{M_d}{Z} = 15$

$$\frac{P}{86 \times 10^3} + \frac{P.e}{86 \times 10^5} - \frac{9.675 \times 10^6}{43.19 \times 10^5} = \frac{P}{86 \times 10^3} + \frac{P.e}{86 \times 10^5} - 1.125 = 15$$

$$\frac{P}{86 \times 10^3} + \frac{P.e}{86 \times 10^5} = 16.125 \quad \text{— (2)}$$

Multiply eqn (1) by 86×10^5

$100P - P.e + 9.675 \times 10^6 = 0$

Multiply eqn (2)

$50.22P + P.e - 9.675 \times 10^6 = 15 \times 43.19 \times 10^5$

eqn (1) $100P - P.e + 9.675 \times 10^6$

eqn (2) $50.22P + P.e - 9.675 \times 10^6 = 15 \times 43.19 \times 10^5$

DATE:

eq ① → $\frac{P}{86\times10^3} - \frac{P.e}{86\times10^5} + 1.125 = 0$

eq ② → $\frac{P}{86\times10^3} + \frac{P.e}{86\times10^5} - 16.125 = 0$

Add both eqn

$\frac{P}{86\times10^3} + \frac{P}{86\times10^3} + 1.125 - 16.125 = 0$

$2.326\times10^{-5} P = 15$

$P = 431265\ N$

$e = \frac{100P + 9.675\times10^6}{P}$

$= \frac{100\times431265 + 9.675\times10^6}{431265}$ 233

$e = 122.43\ mm$

At final stage of loading

At top $= \frac{KP}{A} - \frac{KPe}{Z_t} + \frac{M_d + M_e}{Z_t}$

$= \frac{0.80\times431265}{86\times10^3} - \frac{0.80\times431265\times122.43}{86\times10^5} + \frac{(9.675+49.5)}{86\times10^5}$

$= 4.01 - 4.911 + 6.88 = 5.979 < 15$ OK

At bottom $= \frac{KP}{A} + \frac{K\cdot Pe}{Z_B} - \frac{M_d + M_e}{Z_t}$

$4.01 + \frac{0.80\times431265\times122.43}{43.19\times10^5} - \frac{(9.675+49.5)}{43.19\times10^5}$

$4.01 + 9.78 - 13.70 = 0.09 < 0$

Hence safe

$P = 431265\ N$

e 122.43 mm

DATE:

Cracking moment

The value of moment at which visible tensile cracks are developed in the section is called cracking moment.

If tensile stress developed due to B.M, increases the tensile strength of concrete, the corresponding B.M is called cracking moment.

If losses are not considered then

At bottom $\frac{P}{A} + \frac{P.e}{I}y - \frac{M}{I}y > (-f_t)$

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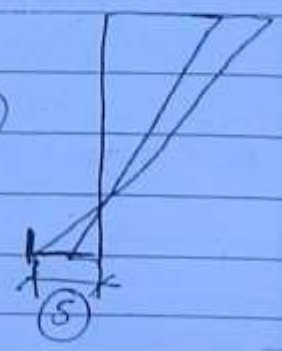
If losses are considered

$\frac{KP}{A} + \frac{KP.e}{I}y - \frac{M}{I}y > (-f_t)$



f_t is tensile strength of concrete

ES-1990 Ques A rectangular concrete beam $150 \times 300\ mm^2$ is prestressed by cable carrying $P = 225\ KN$ & $e = 50mm$

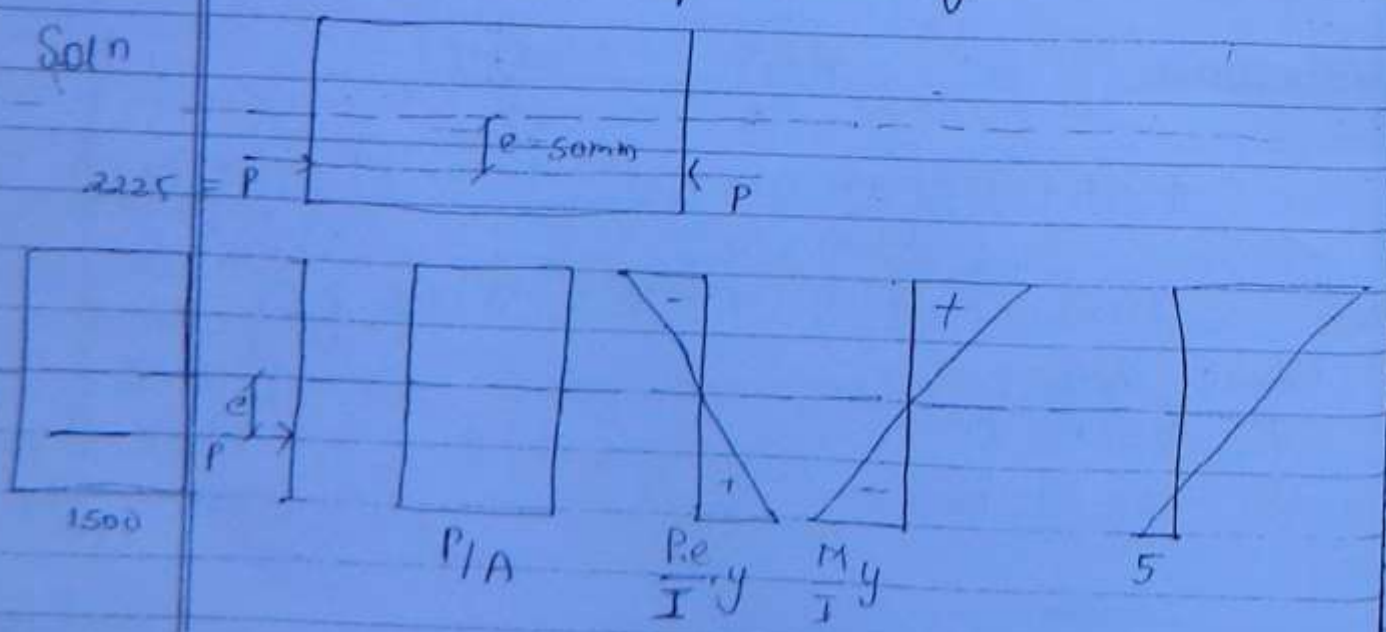
The beam supports a u.d.l 9.2 KN/m including (t) concentric self cut of beam

Span = 5m

If modulus of rupture = $5 N/mm^2$ (tensile stress)

Calculate load factor against cracking.

Soln



DATE:

Stress at bottom of beam

$$\frac{P}{A} + \frac{P.e}{I}y - \frac{M}{I}y = -5$$

$A = 45\times10^3$

$I = \frac{150\times300^3}{12} = 3375\times10^5$

$$\frac{225\times10^3}{45\times10^3} + \frac{225\times10^3\times50}{3375\times10^5}\times150 - \frac{M}{3375\times10^5}\times150 = -5$$

$$5 + 5 + 0.5 = \frac{M}{3375\times10^5}\times150$$

$M = 33.75\times10^6$ N-mm Cracking Moment

Cracking load $= \frac{w_c l^2}{8}$

$$33.75 = w_c \times \frac{5^2}{8}$$

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$w_c = 10.8$ kN/m

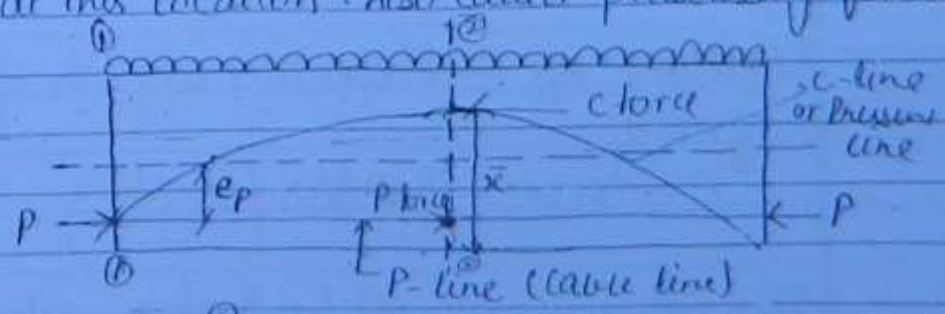
Service load or working load $= 7.2$ kN/m

load factor $= \frac{\text{cracking load}}{\text{working load}} = \frac{10.8}{7.2}$

$= 1.5$

imp # P-Line / C-Line
(Cable line) (Pressure line)

P-line – is nothing but location of Cable itself. Prestressing force is acting at this location. Also called prestressing force line.

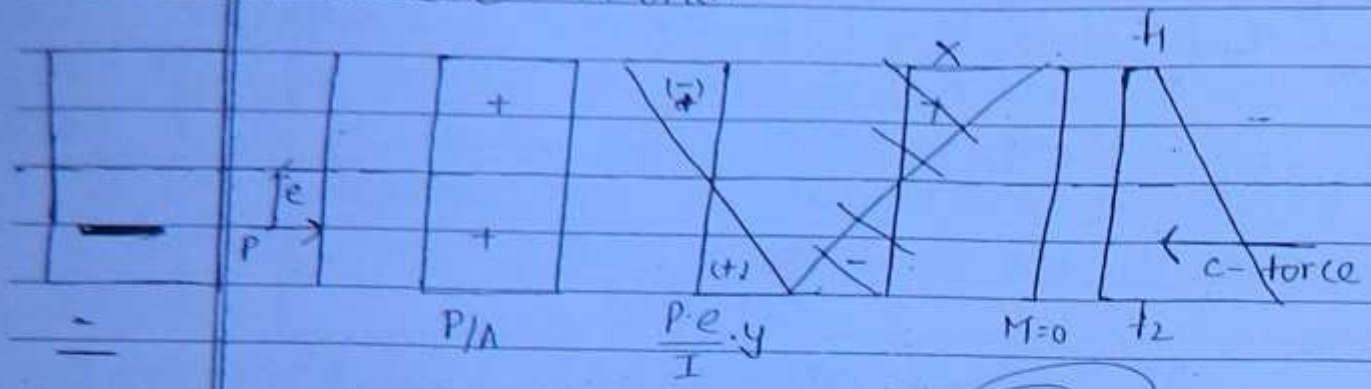


Prestressing force line

C-line – (Thrust line or Pressure line)
It is the location of Resultant thrust force due to final stress diagramme due to all loads. Pressure line is locus of Resultant force of stress diagramme on a beam.

DATE:

At Sec ①-① At end.



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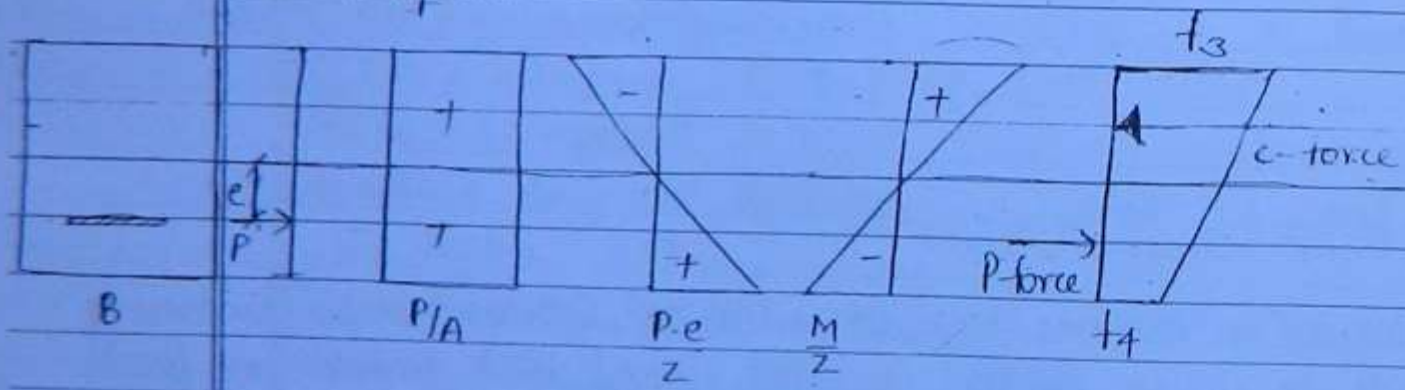
At end where M = 0

Resultant C-force always acts at same location of P-force

$$C\text{-force} = B \times D \times \left(\frac{f_1 + f_2}{2}\right) = P\text{-force}$$

At Mid span



Resultant of C-force act at C.G. of the final stress diagram.

$$C\text{-force} = B \times D \times \left(\frac{f_3 + f_4}{2}\right)$$

This value is equal to P-force always.

At all cross-section the value of c-force = P force

$\Rightarrow f_1 + f_2 \quad f_3 + f_4$

it means sum of stresses at top & bottom is always constant at all section

Shift of c-line w.r.t P-line ($\bar{x}$)

It is always equal to $\bar{x} = \frac{M}{\text{P-force / c-force}}$

$$\boxed{\bar{x} = \frac{M}{P}}$$

Shape of c-line is same as the shape of B.M.D.

At each c/s.

Moment $M = \text{c-force} \times \bar{x}$

$= \text{P-force} \times \bar{x}$

c-force or Pressure line follow the path similar the B.M.D.

At any cross-section eccentricity of c line

$e_c = \bar{x} - e_p$

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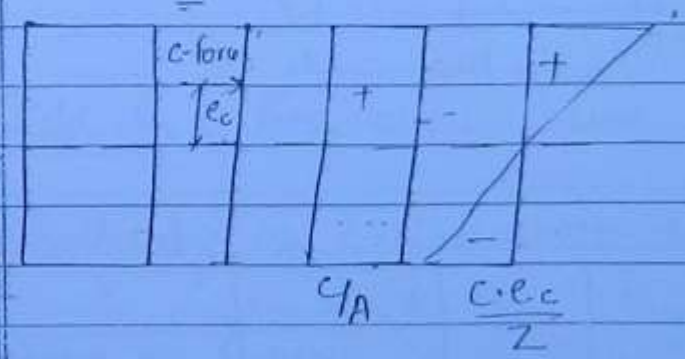
At Mid span



Final Stress $= \frac{c}{A} + \frac{c \cdot e_c}{Z}$ (top)

$= \frac{c}{A} - \frac{c \cdot e_c}{Z}$ Bottom

No moment is required to be considered.

(II) ~~Strength~~ Stress Concept Method. (P-force Method)

→ In this method stresses are found using actual location of prestressing force (P-force), without changing any value

PAGE NO.
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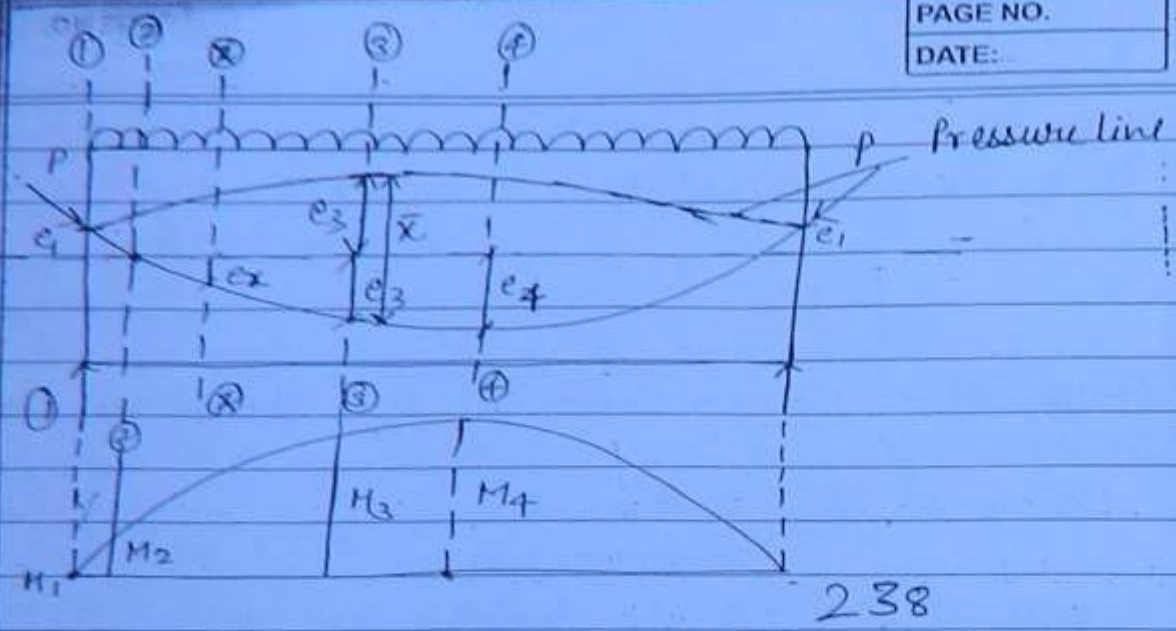

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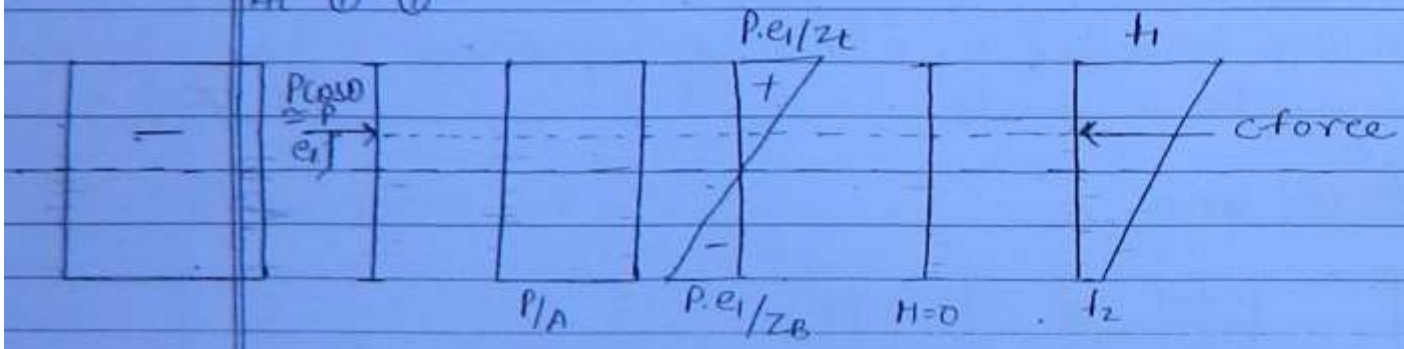
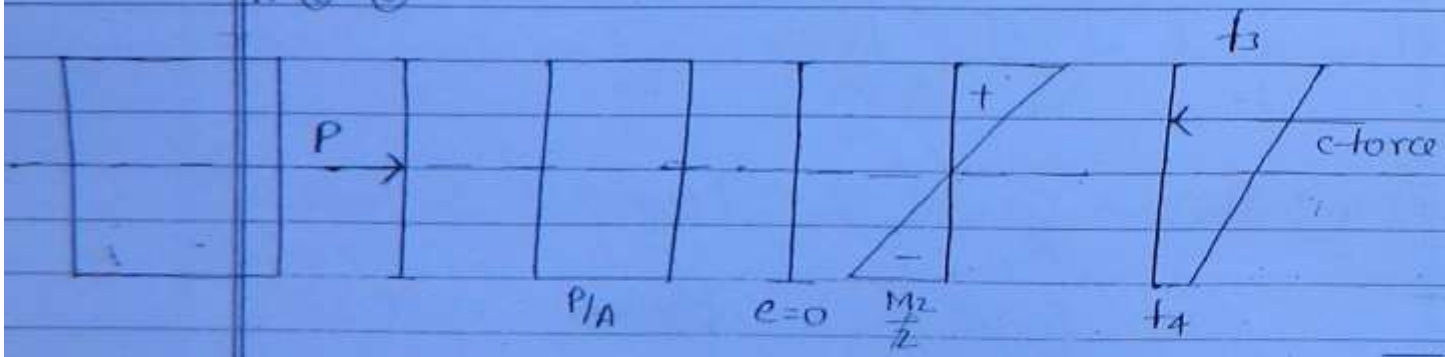
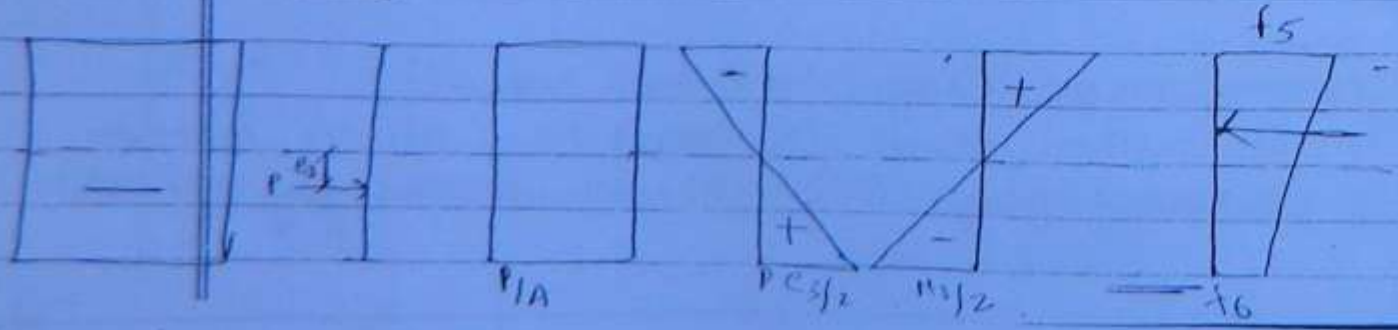
At ① - ①



A ② - ②



At ③ - B



Final Stress — $f_5 \| f_6$ $\frac{P}{A} \mp \frac{P \cdot e_3}{Z} \pm \frac{M_3}{Z}$

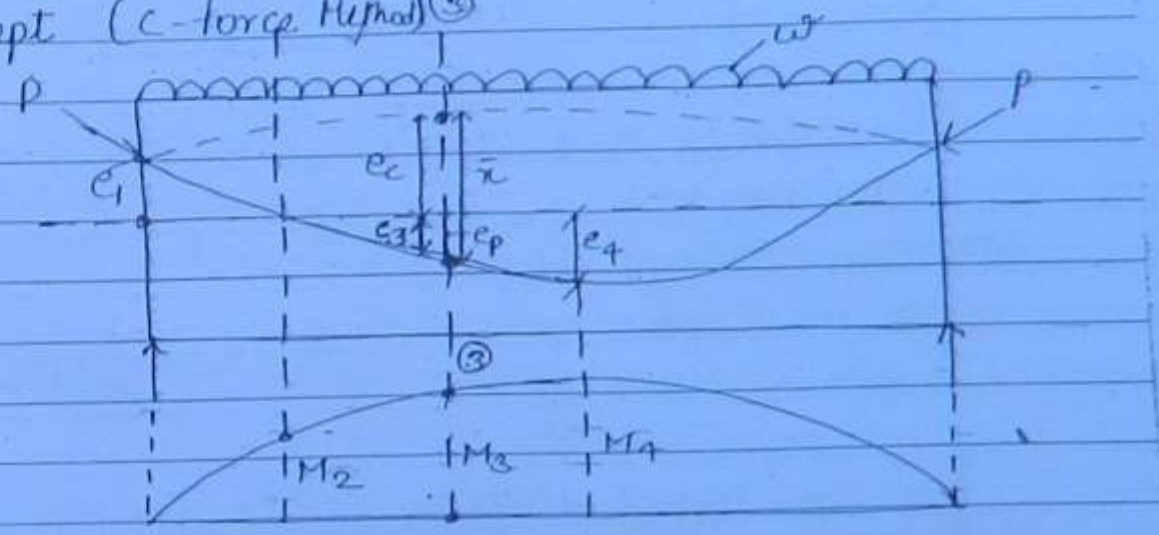
In this $\bar{x} = \frac{M_3}{P}$

$$\boxed{e_c = \bar{x} - e_3}$$

(239)

5-Nov-2011

Strength Concept (C-force Method) ③



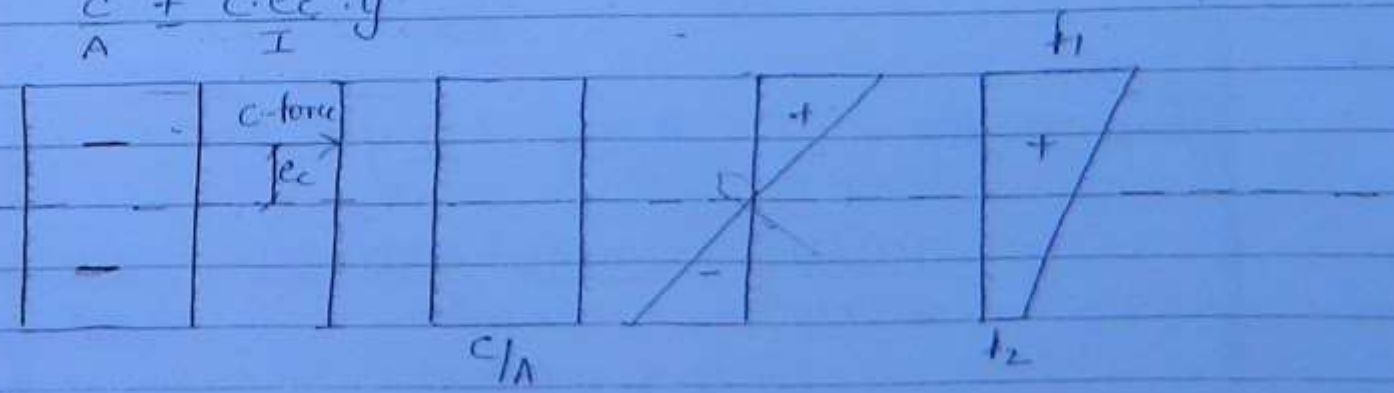
Shift of C line w.r.t P-line $\bar{x} = \frac{M_3}{P}$

Eccentricity of C-line w.r.t N.A

$$e_c = \bar{x} - e_p = \bar{x} - e_3$$

Stress in section

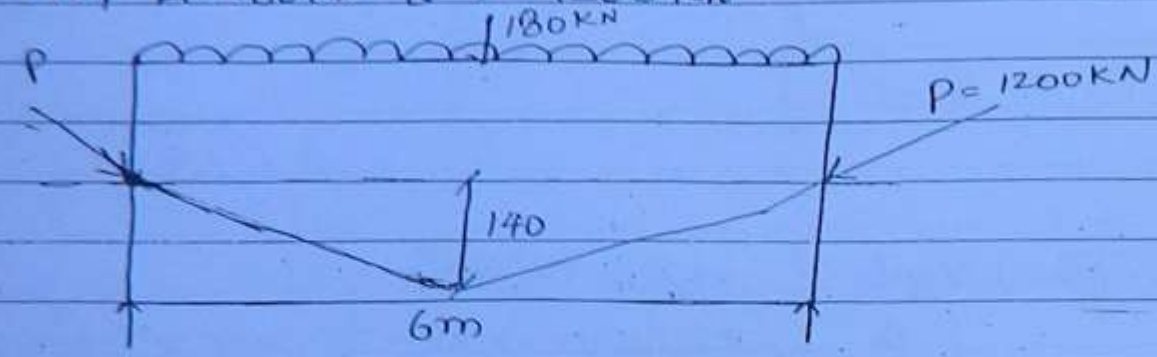
$$\frac{C}{A} \pm \frac{C \cdot e_c \cdot y}{I}$$



In this case, moment is not considered effect of moment is already considered in e_c

2005

Ques A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at midspan considering self wt. also.
$e = 140$, $w = 180$ kN & $P = 1200$ kN



Dead load $= 0.4 \times 0.6 \times 1 \times 25 = 6$ kN/m
$w = 180$ kN

(a) Stress Concept Method

At mid span

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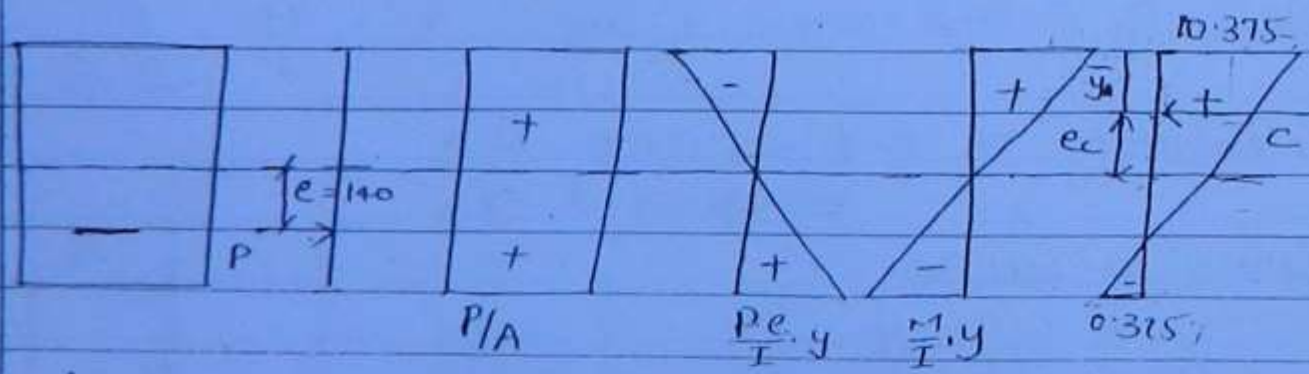
$$BM = \frac{wl}{4} + \frac{wl^2}{8}$$

$$= \frac{180 \times 6}{4} + \frac{6 \times 6^2}{8} = 297 \text{ kN-m}$$



Stress at top $= \frac{P}{A} - \frac{P.e.y}{I} + \frac{M}{I}y$

$$= \frac{1200 \times 10^3}{24 \times 10^4} - \frac{1200 \times 10^3 \times 140 \times 300}{72 \times 10^8} + \frac{297 \times 10^6 \times 300}{72 \times 10^8}$$

$$5 - 7 + 12.375$$

$$10.375$$

Stress at bottom $= 5 + 7 - 12.375$

$$= -0.375$$

DATE:

C-force

$C = 400 \times 600 \left(\frac{10.375 - 0.375}{2} \right)$

$C = 1200{,}000\,N \Rightarrow 1200\,kN$ = P force

$G_1 = \bar{y} = \frac{a+2b}{a+b} \times \frac{h}{3}$

$= \frac{10.375 + 2(-0.375)}{10.375 + (-0.375)} \times \frac{600}{3}$

$= 192.5\,mm$

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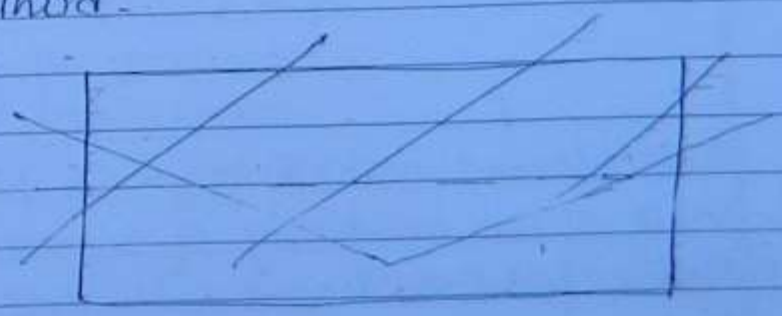
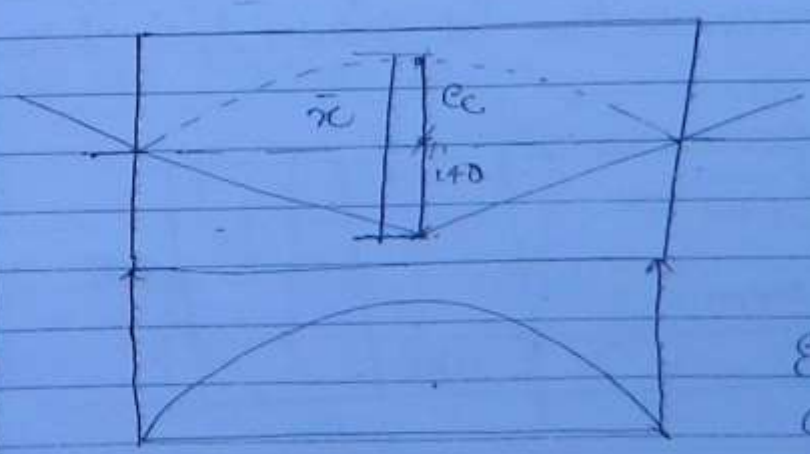
eccentricity of C-force

$e_c = 300 - 192.5 = 107.5$

Strength Concept Method.





Shift of c-line w.r.t to P line

$\bar{x} = \frac{M}{P} = \frac{297 \times 10^6}{1200 \times 10^3}$

$= 247.5$

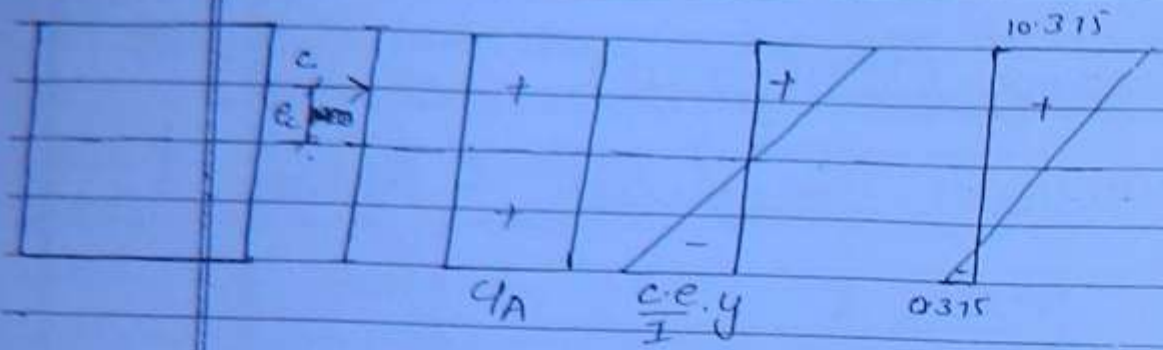
Eccentricity of C-line w.r.t to N.A

$e_c = \bar{x} - e_p$

$247.5 - 140$

$= 107.5\,mm$

(above N.A)



Stress $= \frac{C}{A} \pm \frac{c.e}{I}.y$ (242)

$= \frac{1200 \times 10^3}{24 \times 10^4} + \frac{1200 \times 10^3 \times 107.5 \times 300}{72 \times 10^8}$

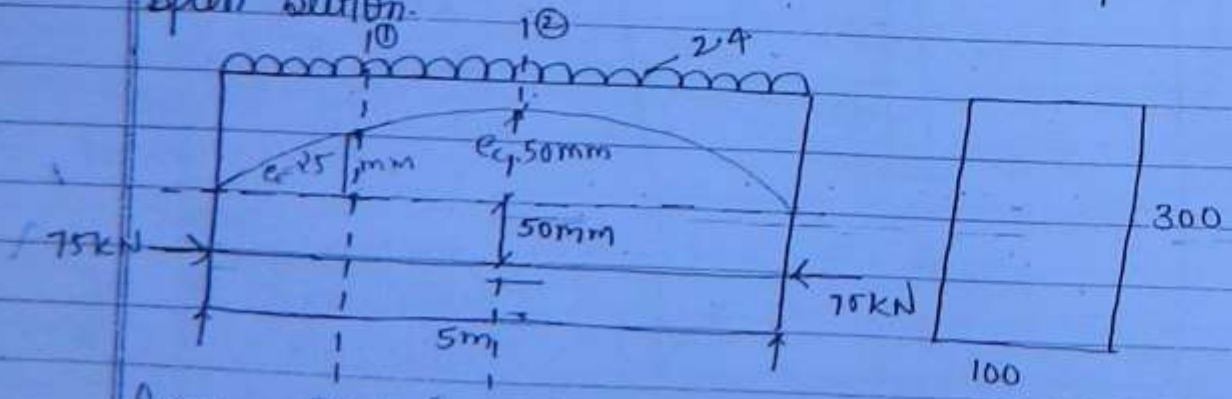
$5 - 5.375$

$= 10.375 \ N/mm^2$

at bottom $= 5 + 5.375$

$= -0.375 \ N/mm^2$

Es-1991 Ques A PSC beam $100 \times 300 \ mm^2$ is prestressed with a straight cable at an eccentricity of 50 mm, $P = 75 KN$, span $= 5m$, Total load on beam including self wt $= 2.4 \ kN/m$ Determine the pressure line at Quarter span & Mid span Sution.

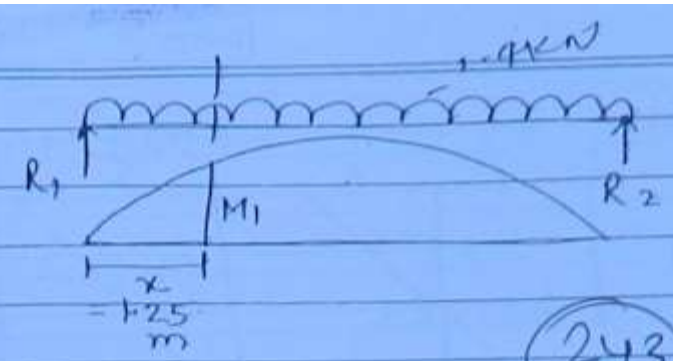


Area $= 3 \times 10^4 \ mm^2$

$I = 225 \times 10^6 \ mm^4$

$w = 2.4 \ kN/m$

B.M at mid span $= \frac{2.4 \times 5^2}{8} = 7.5 \ KN \cdot m$



$R_1 = R_2 = \frac{2.4 \times 5}{2} = 6\,kN$

$M_1 = R_1 \times 1.25 - \frac{w \times 1.25^2}{2}$

$= 6 \times 1.25 - \frac{2.4 \times 1.25^2}{2}$

$= 5.625$ kN·m

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Strength Concept Method

Shift of c-line w.r.t to P-force line

$\bar{x} = \frac{M \text{ at quarter span}}{P} = \frac{5.625 \times 10^6}{75 \times 10^3}$

$= 75$ mm

Eccentricity of c-line w.r.t to N.A

$e_c = \bar{x} - e_p$

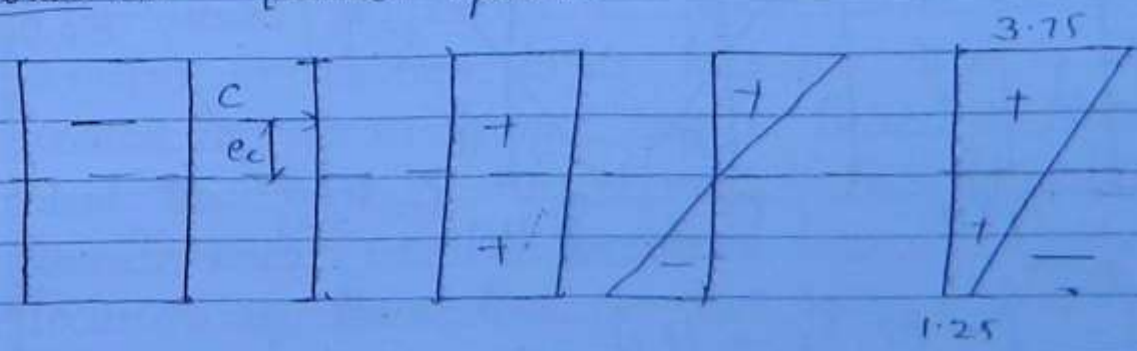
$= 75 - 50 = 25$ mm (above N.A)

Shift of c-line at mid span

$\bar{x} = \frac{M \text{ mid span}}{P} = \frac{7.5 \times 10^6}{75 \times 10^3} = 100$ mm

Eccentricity $= e_c = \bar{x} - e_p = 100 - 50 = 50$ mm

Stress at quarter span.



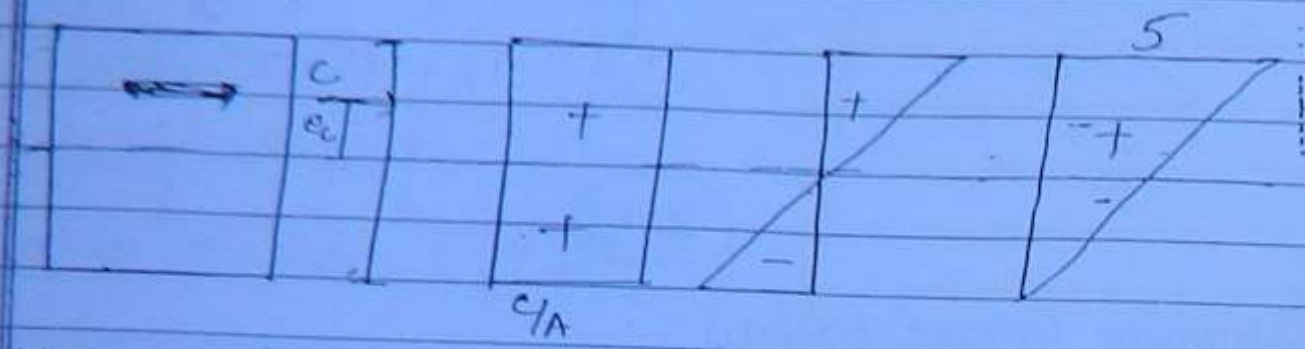
At top $= \frac{C}{A} + \frac{C e_c y}{I} = \frac{75 \times 10^3}{3 \times 10^4} + \frac{75 \times 10^3 \times 25}{225 \times 10^6} \times 150$

$= 2.5 + 1.25$

$= 3.75$

At bottom $= 2.5 - 1.25 = 1.25$

at mid span Stresses



at top $= \frac{C}{A} + \frac{C \cdot e_c \cdot y}{I}$

$= \frac{75\times10^3}{3\times10^4} + \frac{75\times10^3 \times 50}{225\times10^6} \times 150$

$= 2.5 + 2.5$

$= 5$

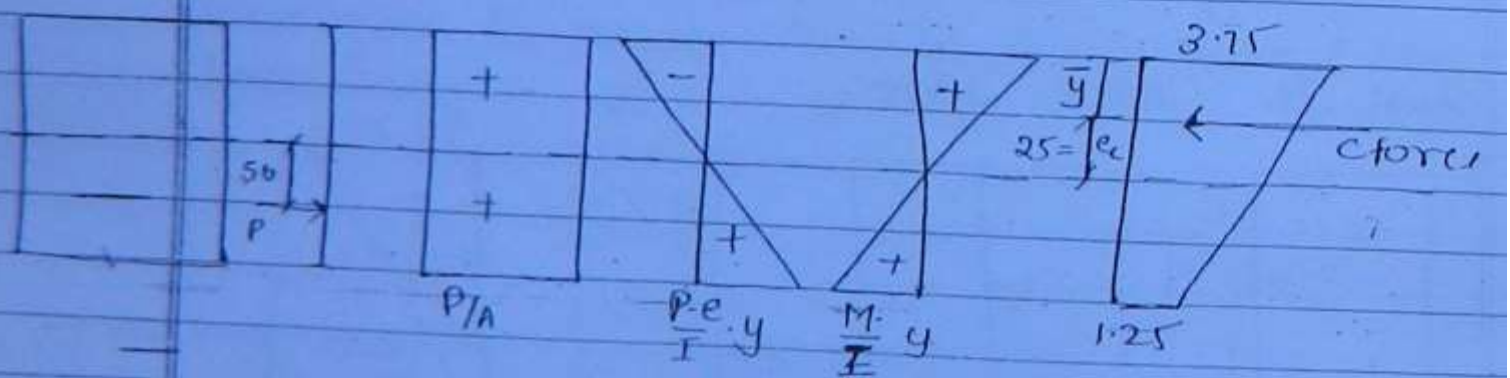
at bottom: $2.5 - 2.5$ (2481)

$= 0$

(b) Stress Concept Method

At Quarter Span

B.M = 5.625 KN-m



at top $= \frac{P}{A} - \frac{P \cdot e}{I} \cdot y + \frac{M}{I} y$

$= \frac{75\times10^3}{3\times10^4} - \frac{75\times10^3 \times 50}{225\times10^6} \times 150 + \frac{5.625\times10^6}{225\times10^6} \times 150$

$2.5 - 2.5 + 3.75$

$= 3.75\ N/mm^2$

DATE:

at bottom $\frac{P}{A} + \frac{P.e}{I} y - \frac{M}{I} y$

$= 2.5 + 2.5 - 3.75$

$= 1.25 \ N/mm^2$

Shift $\bar{x} = \frac{M}{P} = \frac{5.625 \times 10^6}{75 \times 10^3} = 75 mm$

$e_c = 75 - 50 = 25 mm$ (245)

$C.G = \frac{a+2b}{a+b} \times \frac{h}{3} = \frac{3.75 + 2(1.25)}{3.75 + 1.25} \times \frac{300}{3}$

$= 125 mm$

C-force $= \frac{B \times D \times (f_1 + f_2)}{2}$

$= 100 \times 300 \left(\frac{5}{2}\right)$

$= 75 KN = P$ force

$e_c = 150 - 125 = 25 mm$

2006-ES

Ques A PSC beam 300x600mm span = 12m carries a U.D.L 12 kN/m in addition to its self wt. Area of steel is 2000mm² located at 175mm from soffit. Cable is straight initially stress to a level of 800 N/mm² = P_0. It is bonded to concrete (i.e Pre tensioned beam). Determine the location of thrust line at mid span & end quarter span

$\frac{E_s}{E_c} = 6$

Sol. Prestress force $= P_0 \times A_s = 2000 \times 800 \times 10^{-3}$

$= 1600 KN$

B.M at Mid Span $= \frac{12 \times 12^2}{8} = \frac{wl^2}{8}$

$= 216$ KN-m

$A = 300 \times 600 = 18 \times 10^4$ mm^2

$I = \frac{300 \times 600^3}{12} = 54 \times 10^8$

At end Moment $= 0$

Shift of C-line w.r.t to P-line

$\bar{x} = \frac{M}{P} = 0$

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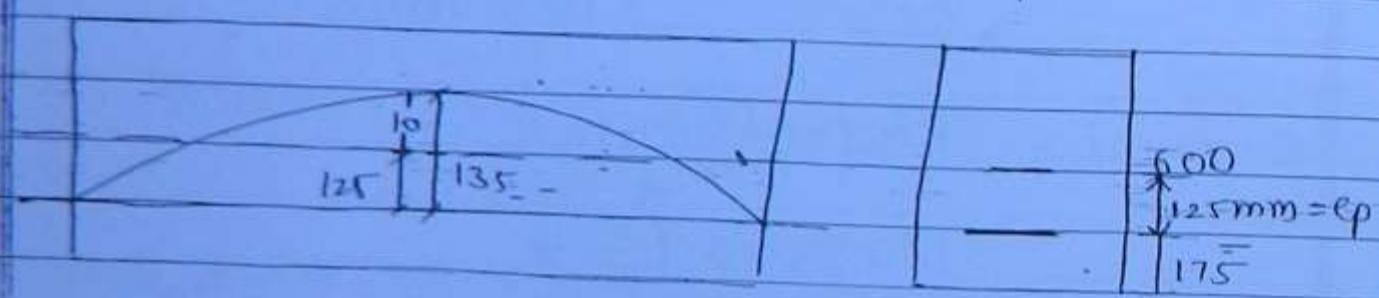
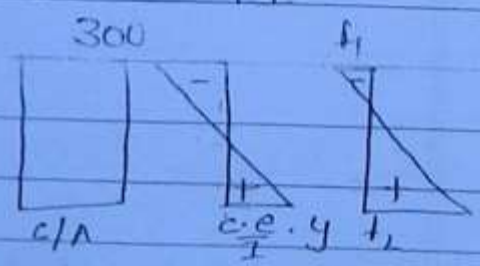
Eccentricity of C-line $= e_c = \bar{x} - e_p$



$e_c = 0 - 125$

$= -125$

125 mm below N.A



Stress $= \frac{C}{A} \mp \frac{C \cdot e_c \cdot y}{I}$

$= \frac{1600 \times 10^3}{18 \times 10^4} \mp \frac{1600 \times 10^3 \times 125}{54 \times 10^8} \times 300$

8.889 ∓ 11.11

At top $= -2.227$ N/mm^2

At bottom $= 19.999$ N/mm^2

$f_1 + f_2 = 17.78$ N/mm^2

b). At Mid Span

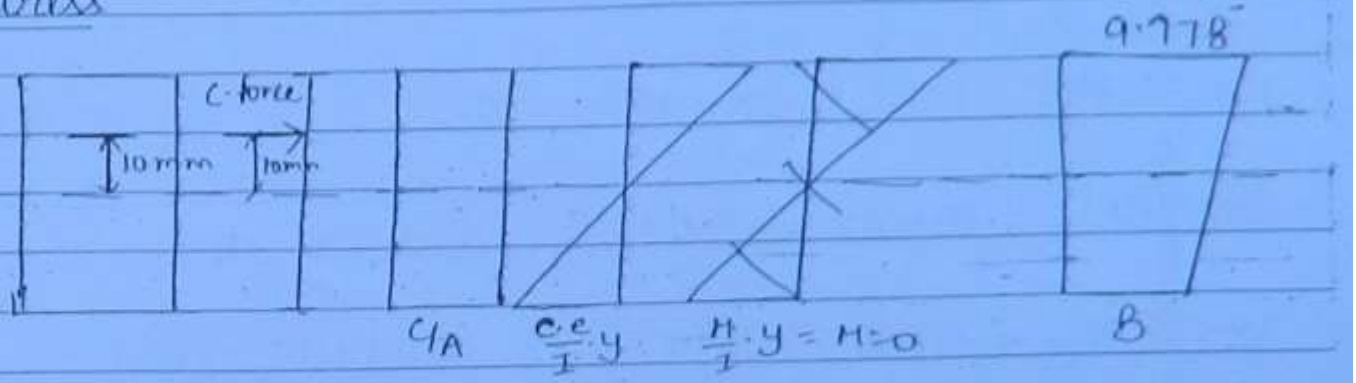
Shift $\bar{x} = \frac{M}{P} = 135$ mm

Eccentricity = $e_c = \bar{x} - e_p$

$= 135 - 125$

$e_c = 10$

Stress



Stresses $= \frac{C}{A} \pm \frac{c.e}{I} y \pm \frac{M}{I} \cdot y$

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$= 8.889 \pm \frac{1600 \times 10^3 \times 10}{54 \times 10^8} \times 300 \pm \frac{216 \times 10^6}{54 \times 10^8} \times 300$

$8.889 \pm 0.889 \pm 19$

at top = 9.778

at bottom = 8

Ques A PSC beam is as shown in fig.

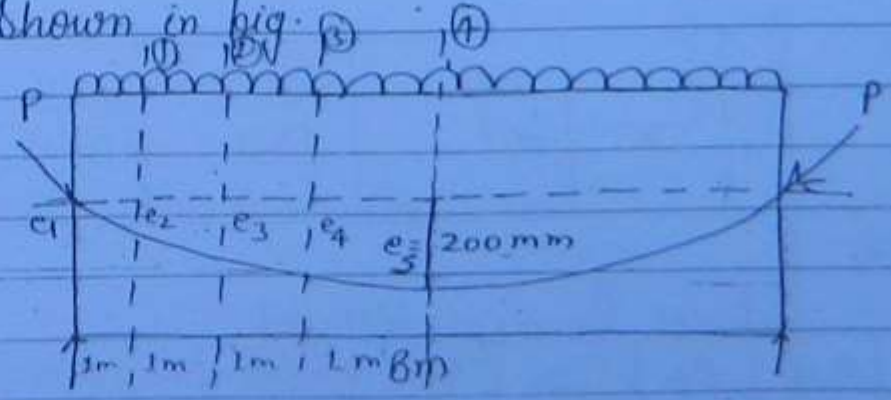
$w = 30$ kN/m

excluding self wt.

Size of beam = 400 × 750 mm

$e = 200$ mm

$P = 1500$ kN



Show the position of thrust line at every 1m distance & find out stresses at top & bottom.

Soln $A = 3 \times 10^5$ mm^2

$I = 1.41 \times 10^{10}$ mm^4

Self wt $= 24 \times .75 \times 1 \times 25 = 7.5$ kN/m

total $w = 37.5$ kN/m.

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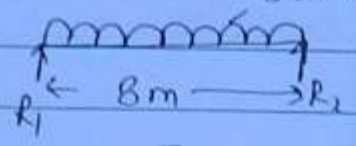
37.5 KN/m



$R_1 = R_2 = 150 = \frac{37.5 \times 8}{2}$

$B.M = 150 \times 1 - \frac{37.5 \times 1^2}{2}$

1m $B.M_1 = 131.25$ KN-m $= M_1$

2m $BM_2 = 150 \times 2 - \frac{37.5 \times 2^2}{2} = 225$ KN-m

3m $BM_3 = 150 \times 3 - \frac{37.5 \times 3^2}{2} = 281.25$ KN-m

4m $BM_4 = \frac{37.5 \times 8^2}{8} = 300$

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Eccentricity max eccentricity

$y = \frac{4b}{l^2} x(l-x)$

$= \frac{4 \times 200}{8000^2} x(8000-x) = \frac{x(8000-x)}{80000}$

y = 375

x	1m	2m	3m
	87.5	150	187.5

Stress Concept Method

$\bar{y}$ - 375

e_c

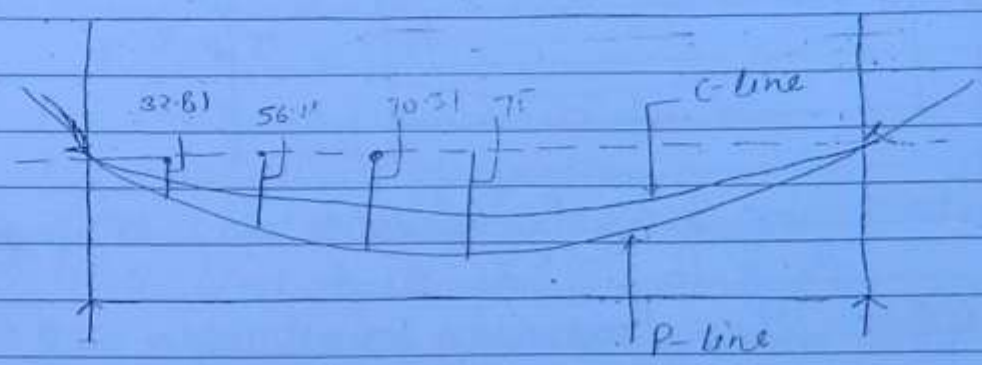
Point	B.M	e_p	P/A	$\frac{P.e}{I}.y$	$\frac{M}{I}.y$	Stress at top	Stress at Bottom	$\bar{y}$
0m	0	0	5	∓ 0	± 0	5	5	5625
1m	131.25	87.5	5	∓ 3.49	± 3.49	5	5	562.5
2m	225	150	5	∓ 5.98	± 6	5	5	562.5
3m	281.25	187.5	5	∓ 7.5	± 7.5	5	5	562.5
4m	300	200	5	∓ 8.0	± 8.0	5	5	562.5

$P = 2400\ kN$

PAGE NO.
DATE:

Point	BM	e_p	P/A	$\frac{P.e}{I}.y$	$\frac{M}{I}.y$	Stress at top	Stress at Bottom	$\bar{y}$	e_c
0 m	0	0	± 8	∓ 0	± 0	8	8	375	0
1 m	131.25	87.5	± 8	∓ 5.6	± 3.5	5.9	10.1	407.8	32.8
2 m	225	150	± 8	∓ 9.6	± 6	4.4	11.6	431.25	56.25
3 m	281.25	187.5	± 8	∓ 12.0	± 7.5	3.5	12.5	445.31	70.31
4 m	300	200	± 8	∓ 12.8	± 8	3.2	12.8	450	75

↓ below N.A



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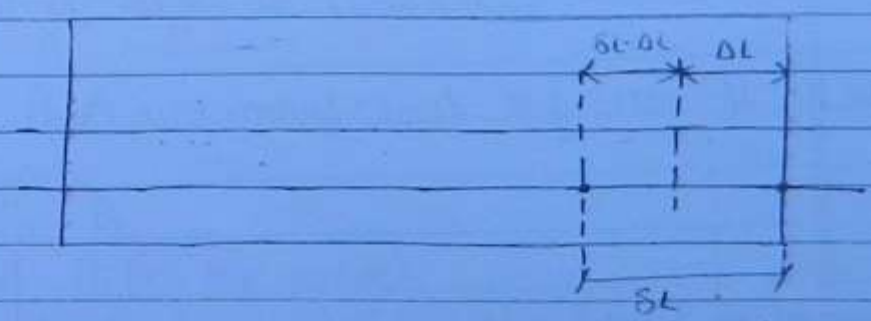
Losses (Loss of Prestress)



If an initial elongation of δl is provided

$$p_o = \frac{\delta l}{l} \times E_s$$

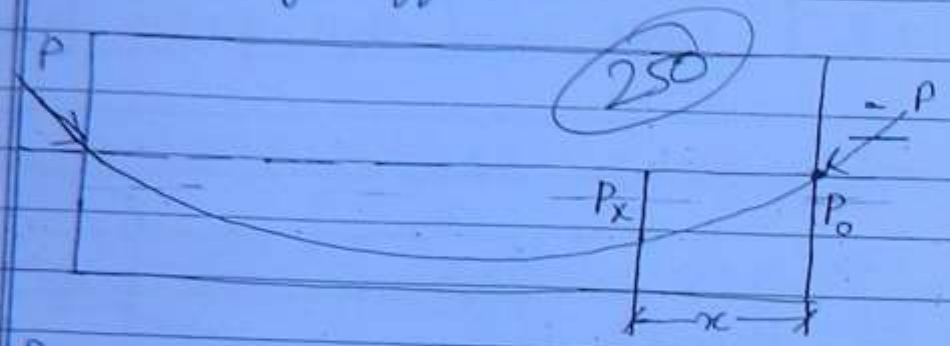
If Δl = loss of elongation provided (due to any reason)
Loss of Prestress

$$p_{loss} = \frac{\Delta l}{l} \times E_s$$

Final stress is due to $(\delta l - \Delta l)$ ~~if an initial elongation of δl is provided~~ $p_{final} = \frac{\delta l - \Delta l}{l} \times E_s$

Type of losses

1. Loss due to friction (During tensioning process)
(Due to combined effect of length & curvature).
→ Only for post tensioned beam or member
→ Due to length effect

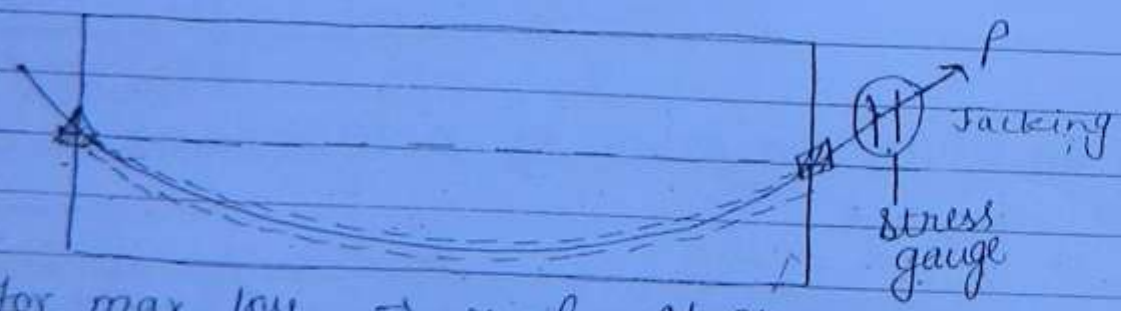


Prestressing force P_x available at x distⁿ from free end

$$P_x = P_o \cdot e^{-Kx}$$

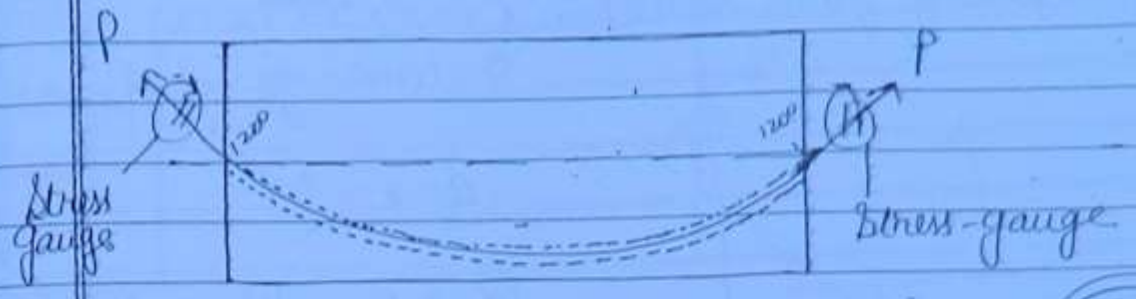
Where P_o = Prestressing force at free end.
x = distance
K = Wobble correction factor a constant
$= (15 \times 10^{-4}$ to $50 \times 10^{-4})$ per meter

Case 1 - If jacking is done from one end.



For max loss ⇒ $x = l$ at other end.

Case 2 When jacking is at both ends
For max loss $x = \frac{l}{2}$



→ Due to Curvature effect.

Prestressing force at x dis^n from free end.

$$P_x = P_0 e^{-\mu\alpha}$$

Where P_0 = Prestressing force at free end.

μ = Coefficient of friction

= 0.55 – Steel is moving over Concrete

⇒ 0.35 – Steel duct tendon is moving on steel duct.

⇒ 0.25 – Steel tendon are moving over lead.

α = Change in gradient betn two pt. of Consideration

→ Combined effect of length & Curvature

$$P_x = P_0 \cdot e^{-Kx-\mu\alpha}$$

$$P_x = P_0 (1-(Kx+\mu\alpha))$$

Loss of prestress.

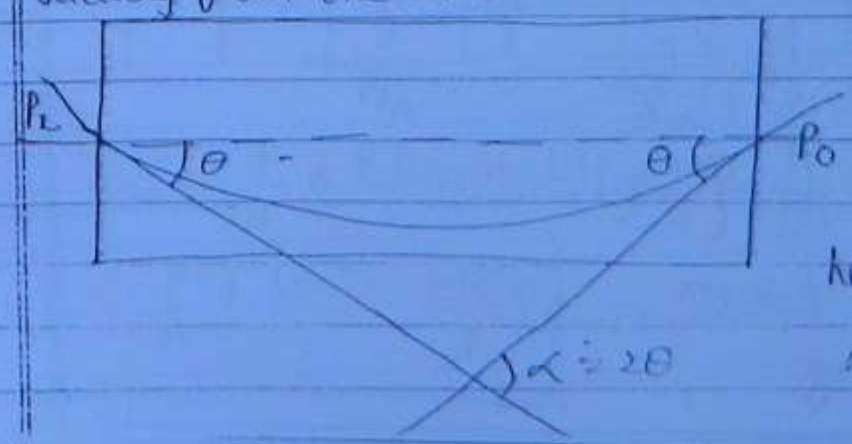
$$= P_0 - P_x$$

$$= P - P_0(1-Kx+\mu\alpha)$$

$$= P_0 (Kx+\mu\alpha)$$ – Same as recommended by IS code.

Value of α – Eq. of Parabolic Cable $= y = \frac{4h}{l^2}x(l-x)$

Jacking from one end.



Slope $= \frac{dy}{dx} = \frac{4h}{l}(l-2x)$

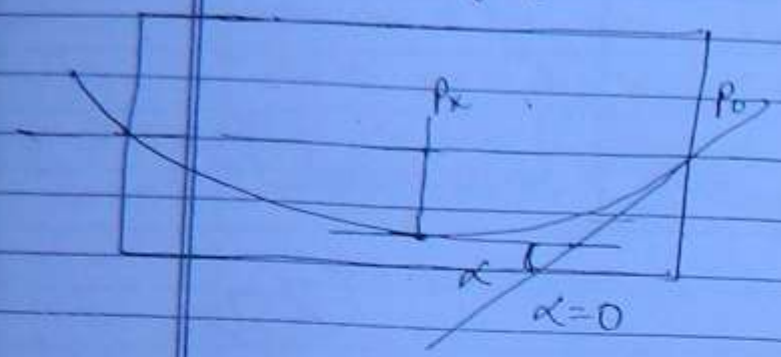
at $x = 0$

$\theta = \tan\theta = \frac{4h}{l}$ radian

here change in gradient

$\alpha = 2\theta = \frac{8h}{l}$ radian

When jacking from both end.



$$\theta = \tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

At $x = \frac{l}{2}$

$$\boxed{\alpha = \theta = \frac{4h}{l} \text{ radian}}$$

2. Loss due to Anchorage slip, deformation etc. (Post tensioned) (during Anchoring process)

If Δl = loss of elongation

loss $= \frac{\Delta l}{l} \times E_s$

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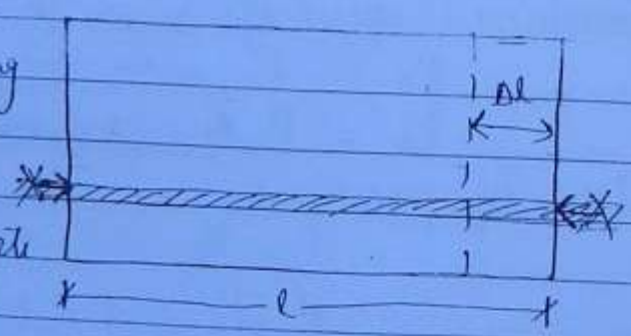
3. Loss occuring subsequently.

(a) Due to Elastic Shortening

→ For pretensioned Members.

(a) Just after Cutting the reinforcement prestressing force is applied on Concrete.



Elastic Shortening of Concrete due to P compressive force

$$\Delta l = \frac{Pl}{A_c E_c}$$

The same deformation will be in steel also

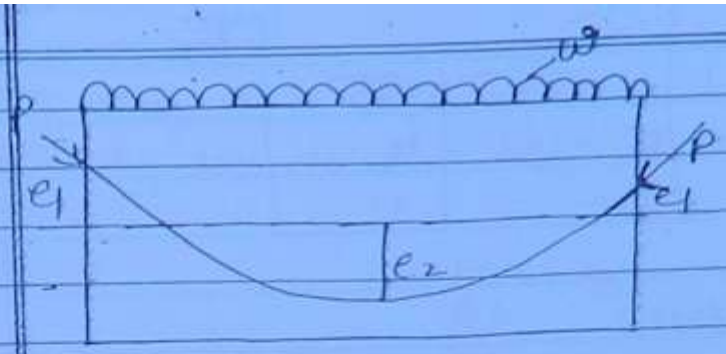
Loss of Prestress in steel $= \frac{\Delta l}{l} \times E_s$

$$= \frac{Pl}{A_c E_c \cdot l} E_s$$

$$= \frac{P}{A_c}\left(\frac{E_s}{E_c}\right)$$

loss of prestress due to Elastic shortening $= k \times m$

(b)



Stress in Concrete at the level of steel is found at end & at mid span.

→ At end.

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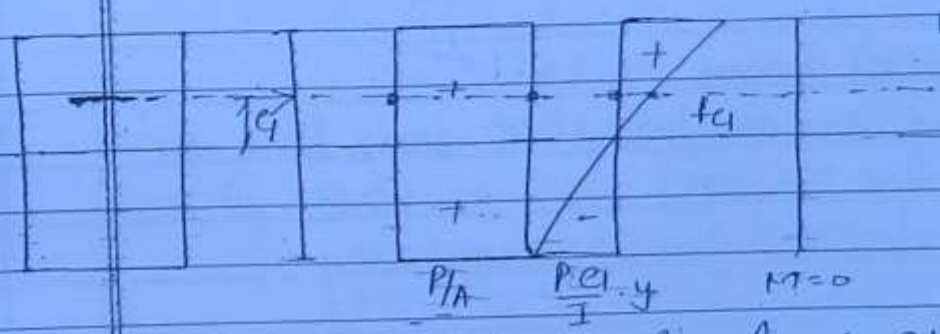
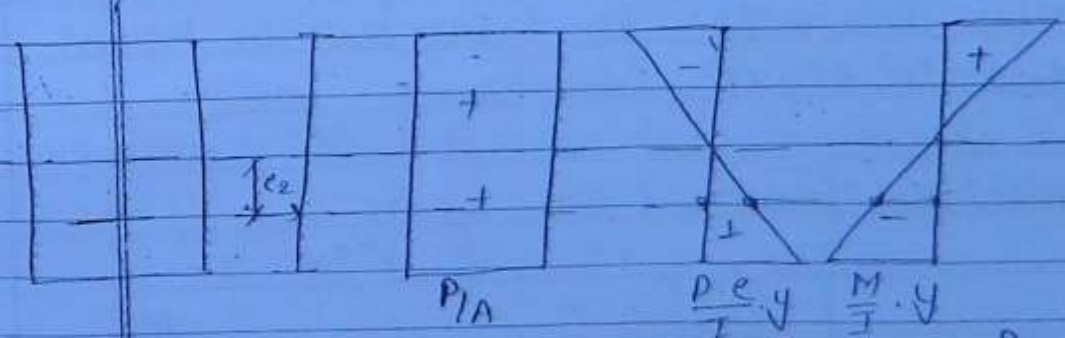


Stress in Concrete at the level of steel

$$f_{c_1} = \frac{P}{A} + \frac{P.e_1}{I}.y = \frac{P}{A} + \frac{P.e_1}{I}e_1$$

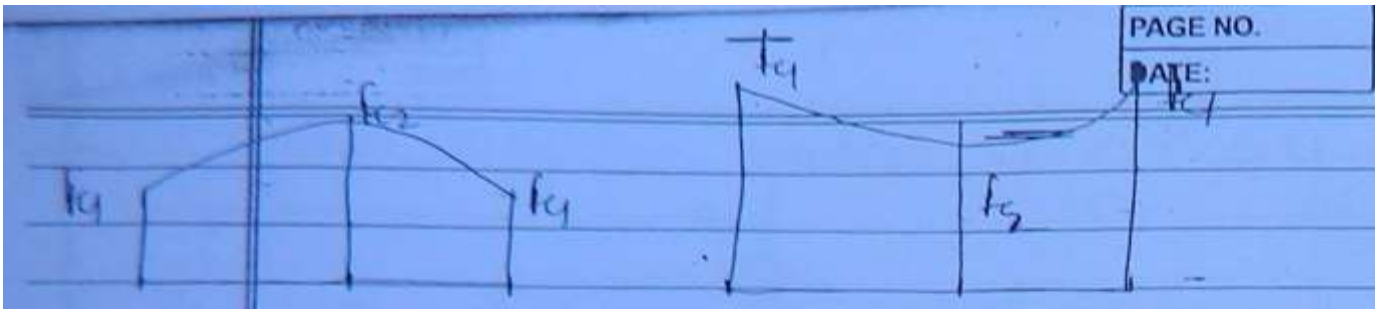
$$\boxed{f_{c_1} = \frac{P}{A} + \frac{P.e_1^2}{I}}$$

At Mid span



$$f_{c_1} = \frac{P}{A} \mp \frac{P.e}{I}y \pm \frac{M}{I}y \Rightarrow \frac{P}{A} + \frac{P.e_2}{I}y - \frac{M}{I}y$$

$$\boxed{f_{c_1} = \frac{P}{A} + \frac{Pe_2^2}{I} - \frac{Me_2}{I}}$$



→ Average f_c is Calculated.

① $f_c = \frac{f_{c1}+f_2}{2}$ ← B.C Punmia

② $f_c = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1})$ ← K. Raju

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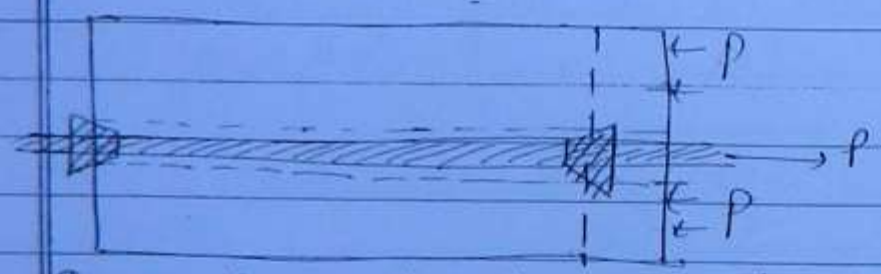
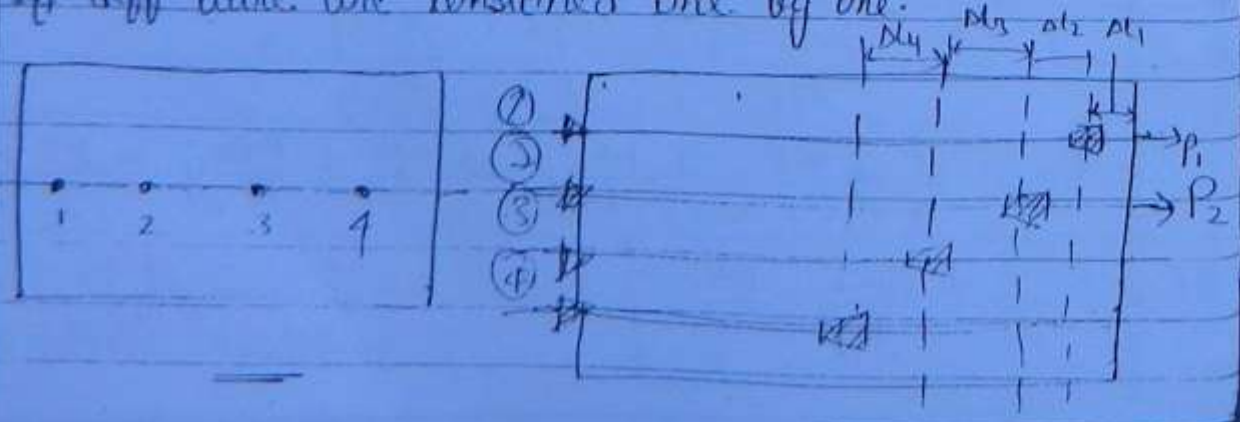
→ For post tensioned beam.

Case 1. If all tendons are tensioned at same time or There is only one tendon.
Loss of Stress due to elastic shortening = zero.



In this case elastic shortening take place before anchoring the reinforcement so there will be no further elastic shortening after anchoring. So there will be no loss in steel due to elastic shortening.

Case 2. If diff. wire are tensioned one by one.



If Reinf^n 1 is anchored 1 & 4 this is anchored last

Finally loss in steel. NO - 4 = 0

" " " - 3 = Due to prestressing force P_4

$$= \frac{\Delta l_4}{l} \times E_s$$

$$= \frac{P_4 \cdot l}{A_c E_c \times l} \cdot E_s = f_{c4} \cdot m$$

Loss of Stress in wire (2) - Due to prestressing force P_3 & P_4

$$\frac{\Delta l_3 + \Delta l_4}{l} \times E_s$$

$$\frac{P_3 + P_4}{A_c} \times m = (f_{c3} + f_{c4}) m$$

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Loss of stress in wire (1)

$$\frac{\Delta l_2 + \Delta l_3 + \Delta l_4}{l} \times E_s$$

$$= (f_{c2} + f_{c3} + f_{c4}) m$$

Loss due to Creep of Concrete

Loss of Stress = Creep Strain × E_s

$$\text{Creep Coefficient} = \frac{\text{Creep Strain}}{\text{Elastic Strain}}$$

$$\text{Creep Strain} = \text{Creep Coefficient} \times \text{Elastic Strain}$$

$$\phi \times \frac{\delta l}{l}$$

Loss of Stress due to Creep = $\phi \times \frac{\delta l}{l} \times E_s$

$$= \phi \times \frac{P \cdot l}{A_c \cdot E_c \cdot l} \times E_s$$

$$= \phi \left(\frac{P}{A_c}\right)\left(\frac{E_s}{E_c}\right)$$

$$= \phi \cdot m \cdot f_c$$

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Creep Coefficient

ϕ =	7days	28days	1 year
ϕ =	2.2	1.6	1.1

Loss due to Creep of Steel (Relaxation of steel)

loss = 1 to 5% of P_o

generally 2 to 3% of P_o is considered.

Loss due to Shrinkage of Concrete.

Loss of Stress = Shrinkage Strain $\times E_s$

(i) Pretensioned PSC Member

$= 3 \times 10^{-4} \times E_s$

(ii) Post tensioned PSC Member

$= \frac{2 \times 10^{-4}}{\log_{10}(T+2)} \times E_s$

T = Age of loading in days.

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TYPE OF LOSS	PRETENSIONED	POST-TENSIONED
→ During tensioning (due to friction) length & curvature	–	$P_o(Kx + \mu\alpha)$
→ During Anchoring due to Anchorage slip	–	$\frac{\Delta l}{l} \times E_s$
Due to elastic Shortening	$(m \times f_c)$	① If one / all tendon tensioned at same time = 0 ② Subsequent tensioning $= \frac{\Sigma \delta l}{l} \times E_s = \frac{\Sigma P}{A} \times m$
Due to Creep of Concrete	$\phi \cdot m \cdot f_c$	$\phi \cdot m \cdot f_c$
Due to Creep of Steel	1 to 5%	1 to 5%
Due to Shrinkage of Concrete	$3 \times 10^{-4} \times E_s$	$\frac{2 \times 10^{-4}}{\log_{10}(T+2)} \times E_s$

Total loss of Prestress

TYPE OF LOSS	PRETENSIONED	POST TENSIONED
1. Elastic Shortening	3%	1%
2. Creep of Concrete	6%	5%
3. Creep of Steel	2%	3%
4. Shrinkage of Concrete	7%	6%
	18%	15%

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ES-2001

Ques A Simply Supported Post tensionsed Concrete beam Span 15m has rectangular C/S 300X600 mm. The prestress force is 1150 kN & zero eccentricity at end & at mid span is 200 mm. Cable is parabolic. Friction Coeff^n = 0.15 per 100m (K & μ = 0.35. Determine the loss due to friction at centre of beam

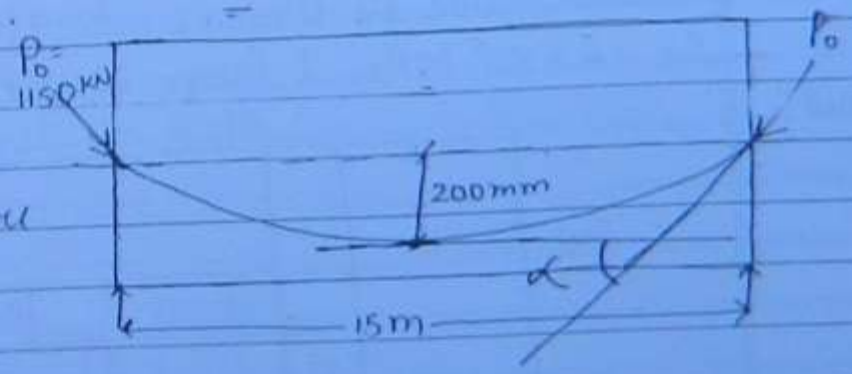
Sol^n P_0 = 1150 kN

$K = \frac{0.15}{100}$ per metre

$= 0.0015$ per metre

$\mu = 0.35$

$x = 7.5m$



Value of α

$\alpha = \theta$

Eq^n of Cable. $y = \frac{4h}{l^2} x(l-x)$

$$\frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

Slope at end at $x = 0$

$$\theta = \tan\theta = \frac{dy}{dx} = \frac{4h}{l}$$

$$\Rightarrow \alpha = \theta = \frac{4 \times 200}{15000} = 0.5333 \text{ radian.}$$

$\theta = 3^\circ 3^\circ 10.38''$

$= \frac{3^\circ 3^\circ 10.38 \times \pi}{180} = 0.05332\,B$ radians

Loss of prestress due to friction

$= P_0 (Kx + \mu\alpha)$

$= 1150 (0.0015 \times 7.5 + 0.35 \times 0.0533)$

$= 34.4\,KN$

$\% \text{ Loss} = \frac{34.4}{1150} \times 100 = 2.99\%$

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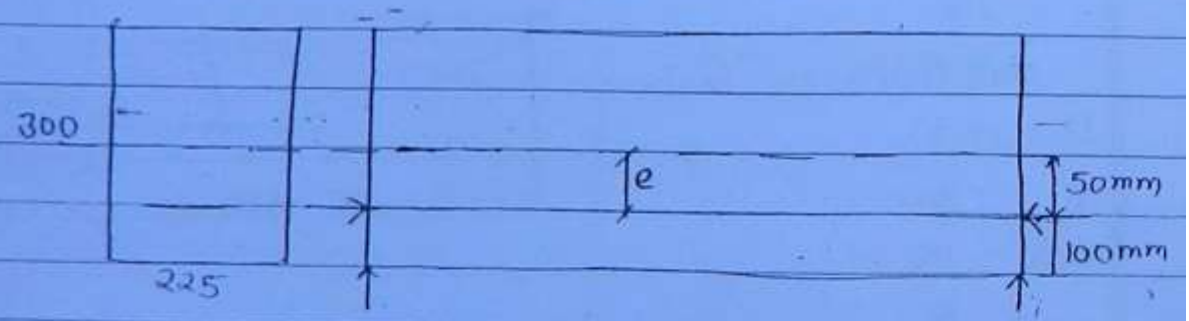
S-2003

Ques A Pretensioned beam of size 225 x 300mm is prestressed by 12 wires of 5mm ϕ initially prestressed at 1100 MPa. The Centroid of prestressing wire is located at 100mm from bottom. Estimate the loss of prestress due to all reason. Grade of Concrete - M-40, Relaxation of Steel = 5%. $E_s = 2 \times 10^5$ MPa & Creep Coefficient = 1.6

Sol.



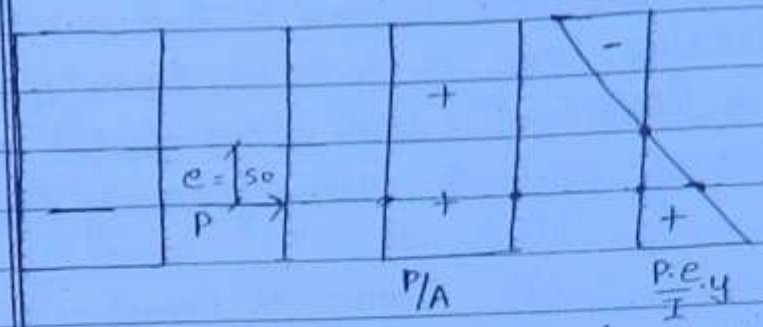
$A_s = \frac{12 \times \pi \times 5^2}{4} = 235.62\,mm^2$

$P_0 = A_s \times P_0 \Rightarrow 235.62 \times 1100$

$= 259.18\,KN$

$P_0 = 1100\,N/mm^2$ (initially Prestress)

1. Loss due to elastic shortening $= m \times f_c$



Stress in Concrete At the level of Steel -

$$= \frac{P}{A} + \frac{P.e}{I} y = \frac{259.18 \times 10^3}{300 \times 225} + \frac{259.18 \times 10^3 \times 50^2}{50625 \times 10^4}$$

$f_c = 3.84 + 1.28$

$f_c = 5.12 \ N/mm^2$

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loss $= m \times f_c$

$\frac{E_s}{E_c} \times f_c$

$\frac{2 \times 10^5}{5000\sqrt{40}} \times 5.12$

$= 32.38 \ N/mm^2$

% loss $= \frac{32.38}{1100} \times 100 = 2.94\%$

Creep of Concrete $= \phi \cdot m \cdot f_c$

$= 1.6 \times 32.38$

loss $= 51.81 \ N/mm^2$

Creep of Steel $= 5\%$ of P_0

$= \frac{5}{100} \times 1100 = 55 \ N/mm^2$

Shrinkage of Concrete $= 3 \times 10^{-4} \times E_s$

$= 3 \times 10^{-4} \times 2 \times 10^5$

$= 60 \ N/mm^2$

Total loss $= 32.38 + 51.81 + 55 + 60 = 199.19$

% loss $= \frac{199.19}{1100} \times 100 = 18.11\%$

PAGE NO.
DATE:

ES 1993 **Ques** A Post tensioned, Prestressed Concrete beam is tensioned initially by a Prestressing force of 1458 KN transfered at 28 days Strength of Concrete. The Cable is parabolic with an eccentricity of 520 mm at Centre. Using following data

$A_c = 2.42 \times 10^5\ mm^2$, $A_s = 1385\ mm^2$

$I = 5.3 \times 10^{10}\ mm^4$, $f_s = 1059\ N/mm^2$ at transfer

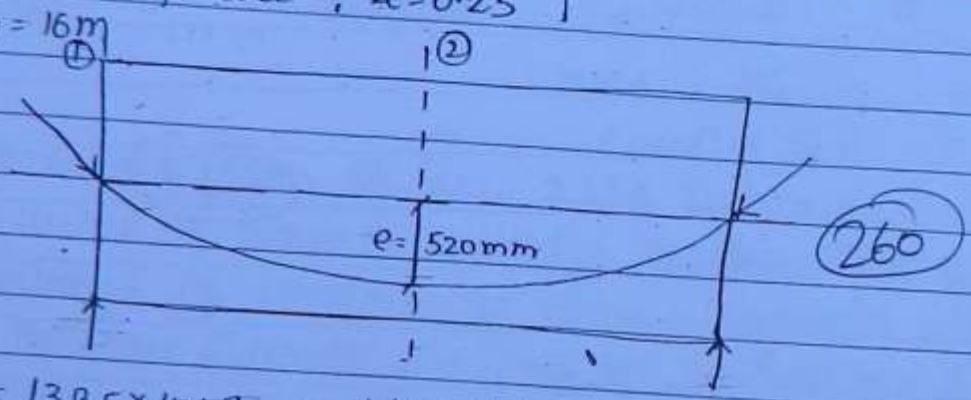
$E_s = 2.1 \times 10^5\ N/mm^2$, Anchorage Slip = 2.5mm

$E_c = 0.382 \times 10^5\ N/mm^2$

$k = 0.0015$ /metre, $\mu = 0.25$ | Calculate all losses.

Span = 16m

Soln



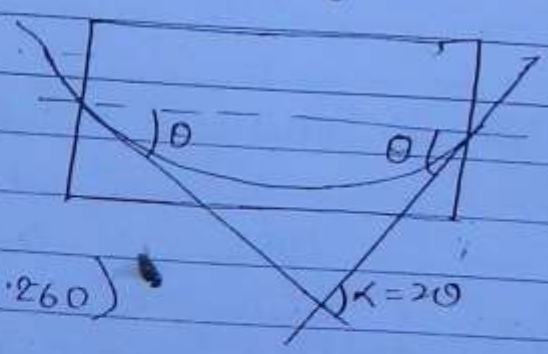
$P = 1385 \times 1059 = 1466.7\ KN.$

1. Loss due to friction $= P_0(Kx + \mu\alpha)$

Assuming jacking is done from one end only. Max loss will be at other end

$\alpha = 2\theta = \frac{8h}{l}$

$= \frac{8 \times 520}{16 \times 10^3} = .260$ radian



Loss $\Rightarrow$ 1059 $(0.0015 \times 16 + 0.25 \times .260)$

$= 94.251\ N/mm^2$

2. Loss due to Anchorage slip $= \frac{\Delta l}{l} \times E_s$

$= \frac{2.5}{16 \times 10^3} \times 2.1 \times 10^5$

$= 32.81\ N/mm^2$

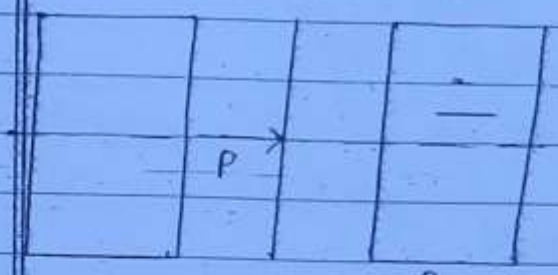
DATE:

3. Loss due to elastic shortening of Concrete = 0

4. Loss due to Creep of Concrete = $\phi \cdot m \cdot f_c$

Stress in Concrete at the level of steel.

At ends



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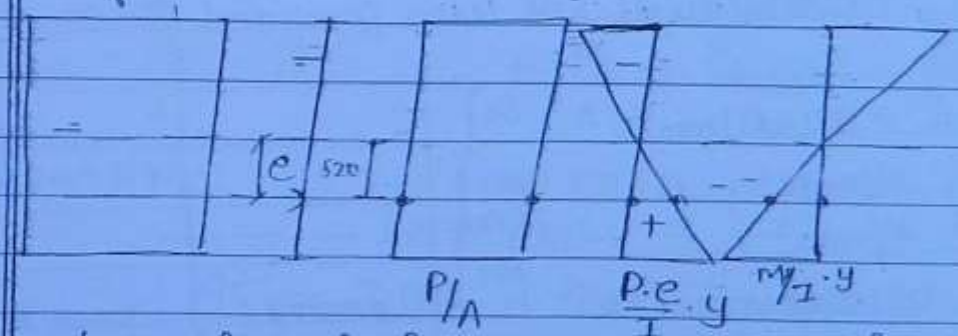
$$f_{c_1} = \frac{P}{A} = \frac{1466.7 \times 10^3}{2.42 \times 10^5} = 6.06 \text{ N/mm}^2$$

At Mid span

$$M_d = \frac{w_d \times l^2}{8}$$

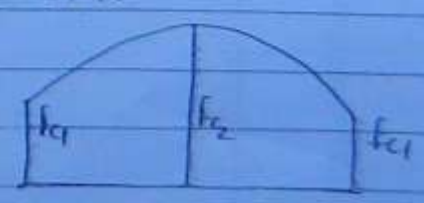
$$w_d = \frac{2.42 \times 10^5}{10^6} \times 1 \times 25 = 6.05$$

$$M_d = 193.6 \text{ KN-m}$$



$$f_{c_2} = \frac{P}{A} + \frac{P.e^2}{I} - \frac{M}{I}.y = \frac{1466.7 \times 10^3}{2.42 \times 10^5} + \frac{1466.7 \times 10^3 \times 520^2}{5.3 \times 10^{10}} - \frac{193.6 \times 10^6 \times 520}{5.3 \times 10^{10}}$$

$$f_{c_2} = 6.06 + 7.48 - 1.899 = 11.641$$



$$f_c = \frac{f_{c_1} + f_{c_2}}{2} = 8.85 \text{ N/mm}^2$$

$$\frac{E_s}{E_c} = m = 5.497$$

$$\text{Loss} = \phi \times m \times f_c = 1.6 \times 5.497 \times 8.85 = 77.84 \text{ N/mm}^2$$

ϕ = 1.6 at 28 days.

5. Loss due to Creep of Steel = 1 to 5% of P_0
= 3% of P_0
$= \frac{3}{100} \times 1059 = 31.77\ N/mm^2$

6. Loss due to Shrinkage $= \frac{2\times10^{-4}}{\log_{10}(t+2)} E_s$
$= \frac{2\times10^{-4}}{\log_{10}(28+2)} \times 2.1\times10^5$
$= 28.43\ N/mm^2$

Total loss = $265.101\ N/mm^2$

% loss $= \frac{265.101}{1059} \times 100$
$= 25\%$

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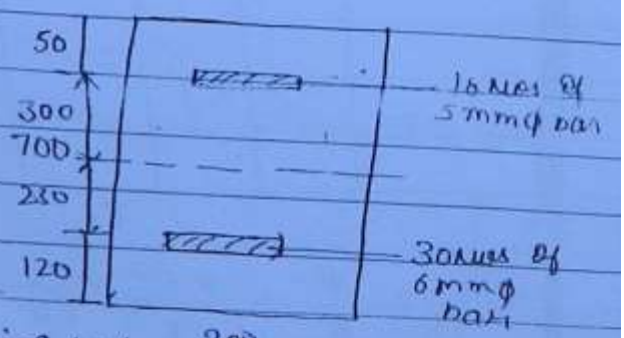
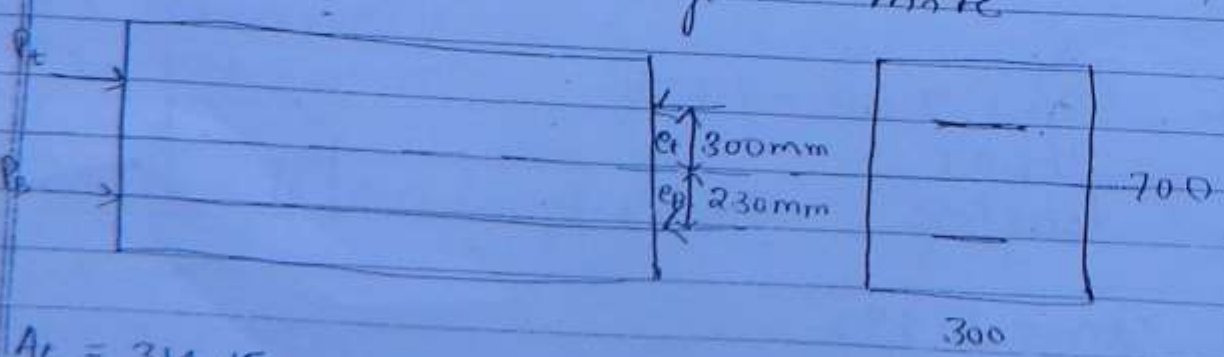
Ques A PSC beam (Pretensioned) has been provided tendons as shown in fig

Initial Prestress = $1200\ N/mm^2$
$E_s = 210\times10^3\ N/mm^2$
$E_c = 36\times10^3\ N/mm^2$
Calculate loss in wires due to elastic shortening. The member is still supported on its prestressing bed.

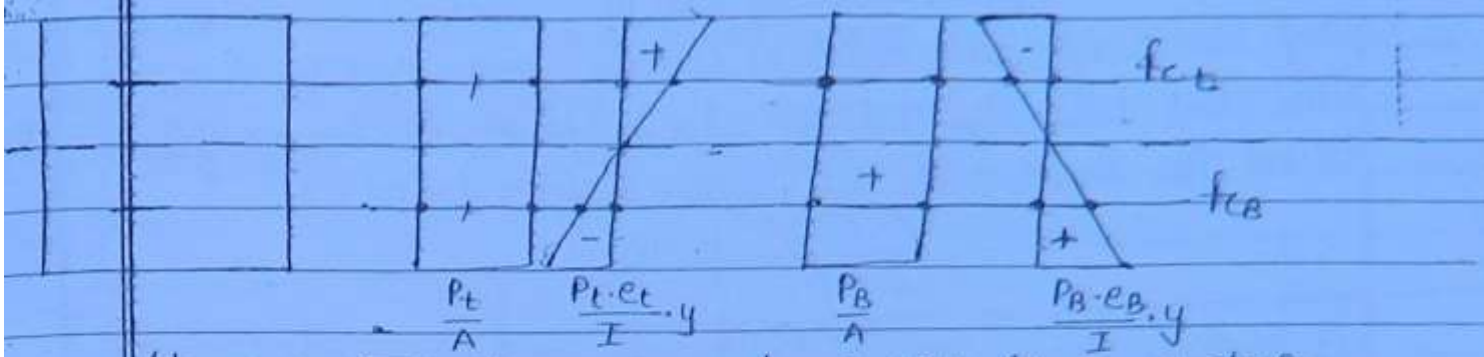


Sol Loss due elastic shortening $= m \times f_c$



$A_c = 314.16$
$P_t = 1200 \times 314.16 = 377\ kN$
$A_B = 30 \times \frac{\pi}{4} \times 6^2 = 848.23$

$P_B = 1017.9$ kN $\qquad I = 8575 \times 10^6$ mm^4



Stress in concrete at the level of steel at top

$$f_{ct} = \frac{P_t}{A} + \frac{P_t \cdot e_t}{I} \cdot y_{(et)} + \frac{P_B}{A} - \frac{P_B \cdot e_B}{I} \cdot y_{(et)}$$

$$= \frac{377 \times 10^3}{21 \times 10^4} + \frac{377 \times 10^3 \times 300^2}{8575 \times 10^6} + \frac{1017.9 \times 10^3}{21 \times 10^4} - \frac{1017.9 \times 10^3 \times 230 \times 38}{8575 \times 10^6}$$

$= 1.795 + 3.96 + 4.85 - 8.19$

$= 2.415$ N/mm^2

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$$f_{cB} = \frac{P_t}{A} - \frac{P_t e_t}{I} \cdot e_B + \frac{P_B}{A} + \frac{P_B \cdot e_B^2}{I}$$

$$= 1.795 - \frac{377 \times 10^3 \times 300 \times 230}{8575 \times 10^6} + 4.85 + \frac{1017.9 \times 10^3 \times 230^2}{8575 \times 10^6}$$

$= 1.795 - 3.03 + 4.85 + 6.28$

$= 9.895$ N/mm^2.

Loss of stress in top steel $= m \cdot f_{ct}$

$= \frac{E_s}{E_c} \times f_{ct}$

$= 5.83 \times 2.415 = 14.08$ N/mm^2

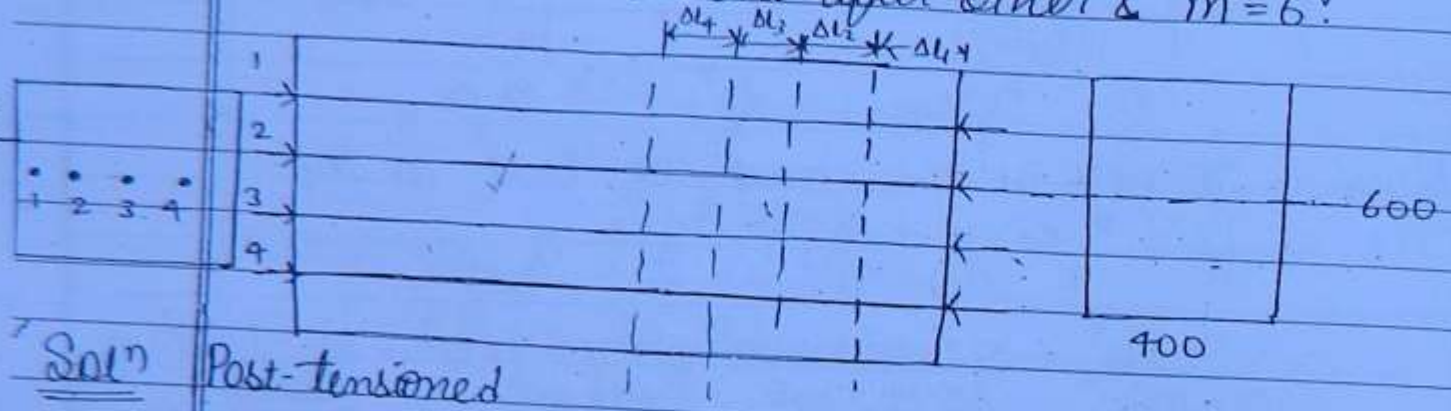
Loss of stress in bottom $= m \cdot f_{cB}$

$= 5.83 \times 9.895$

$= 57.68$ N/mm^2

DATE:

Ques (a) A Post-tensioned Concrete beam 20 m long has c/s 400X600 mm² is prestressed by 4-Cable of 300 mm² each provided without any eccentricity Each tendon is initially prestressed to 1800 N/mm². Find out loss of prestress due to elastic shortening of Concrete. Wires are tensioned one after other & m = 6.



Sol^n Post-tensioned

$A_s = 300 m^2$ each

$P_s = 1800 N/mm^2$

$E_s = 2 \times 10^5 N/mm^2$

$E_c =$

$P = 300 \times 1800 = 540 KN$

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$\Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta l_4$

$\Delta l = \frac{Pl}{A_c E_c} = \frac{540 \times 10^3 \times 20 \times 10^3 \times 6}{400 \times 600 \times 2 \times 10^5} = 1.35 mm$

Loss of stress due to one wire $= \Delta l / l \times E_s = \frac{1.35}{20 \times 10^3} \times 2 \times 10^5 = 13.5 N/mm^2$

loss in steel due to wire no 4 = 0

due to wire no. 3 $= \frac{\Delta l_4}{l} \times E_s$

$= 13.5 N/mm^2$

due to wire no. 2 $\frac{\Delta l_3 + \Delta l_4}{l} \times E_s$

$= \cancel{13.5 \times 2} \; \frac{1.35 \times 2}{20 \times 10^3} \times 2 \times 10^5$

$= 27 N/mm^2$

due to wire no 1 $= \frac{\Delta l_2 + \Delta l_3 + \Delta l_4}{l} \times E_s$

$= \frac{3 \times 1.35}{20 \times 10^3} \times 2 \times 10^5 = 40.5 N/mm^2$

(b) If it is desired that a final stress of 1800 N/mm² is maintained after losses how much initial stress should be given to diffⁿ wires.

Solⁿ Wire no - 4 → loss of stress = 0

Initial stress = 1800 N/mm²

" " 3 → loss = 13.5 N/mm²

$$\sigma_r = \frac{300 \times 1800}{400 \times 600} \times 6 = 13.5 \text{ N/mm}^2$$

Initial stress = 1800 + 13.5 = 1813.5 N/mm²

wire no. 2 → loss due to P_3 & P_4

$$\frac{P_3 + P_4}{A_c} \times m =$$

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$$= \frac{300 \times 1813.5 + 300 \times 1800}{400 \times 600} \times 6 = 27.10 \text{ N/mm}^2$$

Initial stress = 1800 + 27.10 = 1827.10 N/mm²

wire no. 1 — loss due to P_2 & P_3 & P_4

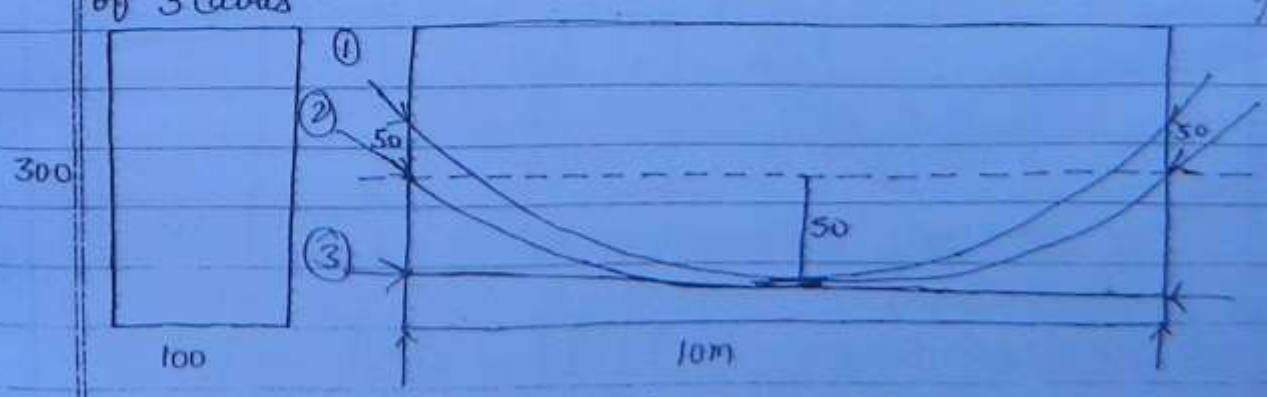
$$= \frac{P_2 + P_3 + P_4}{A_c} \times m$$

$$= \frac{(1827.10 + 1813.5 + 1800)300}{400 \times 600} \times 6$$

$$= 40.8 \text{ N/mm}^2$$

Initial stress = 1840.8 N/mm²

Ques A Post tensioned Concrete beam is stressed by successive tensioning of 3 cables $m = 6$



c/s of each cable = 200 mm², Initial stress in each cable = 1200 N/mm²

DATE:

Estimate % loss of stress in each cable due to plastic shortening only (Neglect self wt of & live load)

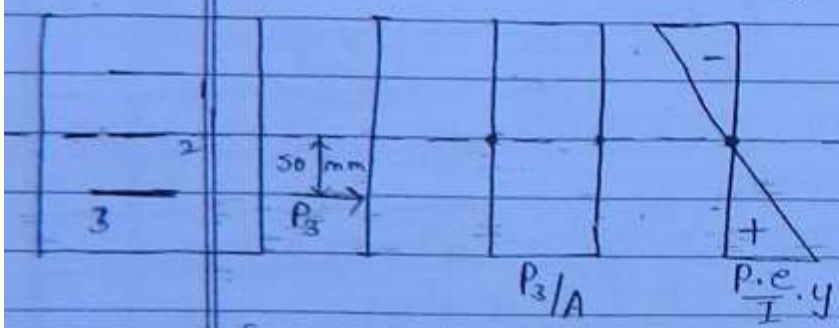
Sol^n wire no. 3 – loss of stress = 0

wire no. 2 – due to wire no. 3 loss will occur.

$$= \frac{\Delta l_3}{l} \times E_s = \frac{P_3 \cdot l}{A_c \cdot E_c \cdot l} E_s$$

$$= \frac{P_3}{A_c} \cdot \frac{E_s}{E_c} = f_{c_3} \times m$$

At end – stress in concrete at the location of wire no. 2 due to prestressing of wire no. 3



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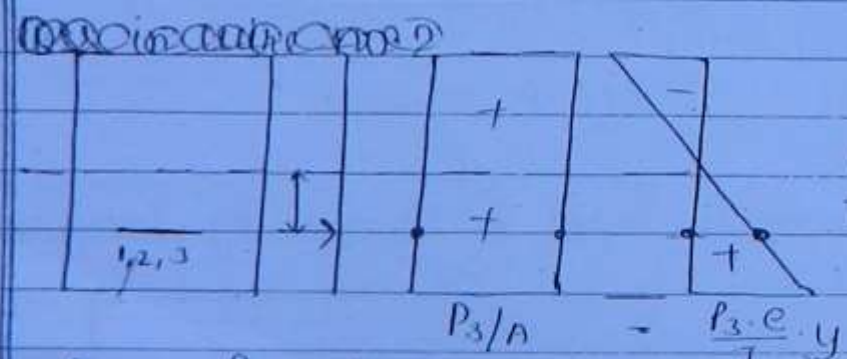
$$f_{c_2} = \frac{P_3}{A}$$

$$= \frac{200 \times 1200}{300 \times 100} = 8 \text{ N/mm}^2$$

Mid span



Stress in concrete at the level of wire no. 2 due to prestressing force P_3

$$f_{c_2} = \frac{P_3}{A} + \frac{P_3 \cdot e}{I} \cdot y$$

$$= \frac{200 \times 1200}{300 \times 100} + \frac{200 \times 1200 \times 50^2}{225 \times 10^6}$$

$$= 8 + 2.67$$

$$= 10.67 \text{ N/mm}^2$$

DATE:

loss = Average stress = $8 + 10.67 - f_{c_1} + \frac{2}{3}(f_{c_2} - f_{c_1})$

$= 8 + \frac{2}{3}(10.67 - 8)$

$f_{c_3} = 9.78\ N/mm^2$

(287)

loss of prestress = 9.78×6

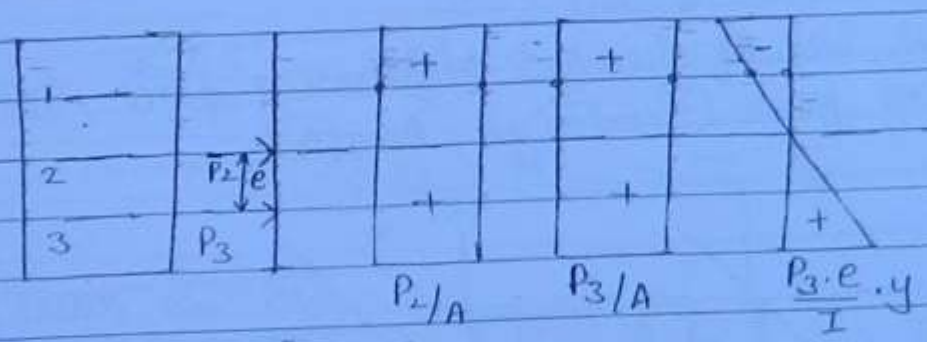
$= 58.68\ N/mm^2$

Wire No. 1 - due to wire no 2 & 3 loss will occur

loss = $(f_{c_3} + f_{c_2})\, m$

→ Stress in Concrete at the location of wire no. 1 due to force in wire no. 3 & 2

At end

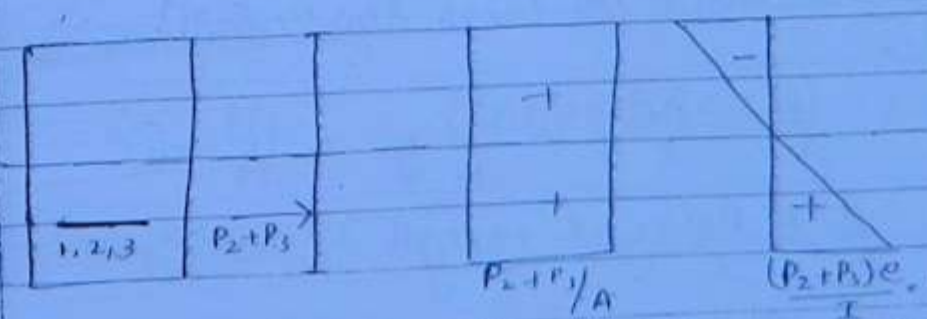


$f_{c_1} = \frac{P_2}{A} + \frac{P_3}{A} - \frac{P_3 \cdot e}{I} \cdot y$

$= \frac{200 \times 1200}{100 \times 300} + \frac{200 \times 1200}{300 \times 100} - \frac{200 \times 1200 \times 50}{225 \times 10^6} \times 50$

f_{c_1} $8 + 8 - 2.67 = 13.33$

At mid span



$f_{c_2} = \frac{P_2 + P_3}{A} + \frac{(P_2 + P_3)e}{I} \cdot y \Rightarrow \frac{2(200 \times 1200)}{100 \times 300} + \frac{2(200 \times 1200) \times 50}{225 \times 10^6}$

$= 8 \times 2 + 5.33 = 21.33\ N/mm^2$

DATE:

Average $= 13.33 + \frac{2}{3}(21.33 - 13.33)$

$= 18.66\ N/mm^2$

loss $= 18.66 \times 6$

$= 111.98\ N/mm^2$

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Design of Prestressed beam

Design of Rectangular beam (symmetrical secⁿ)

TYPE 1- Rectangular secⁿ:- for a given value of B.M

given values -

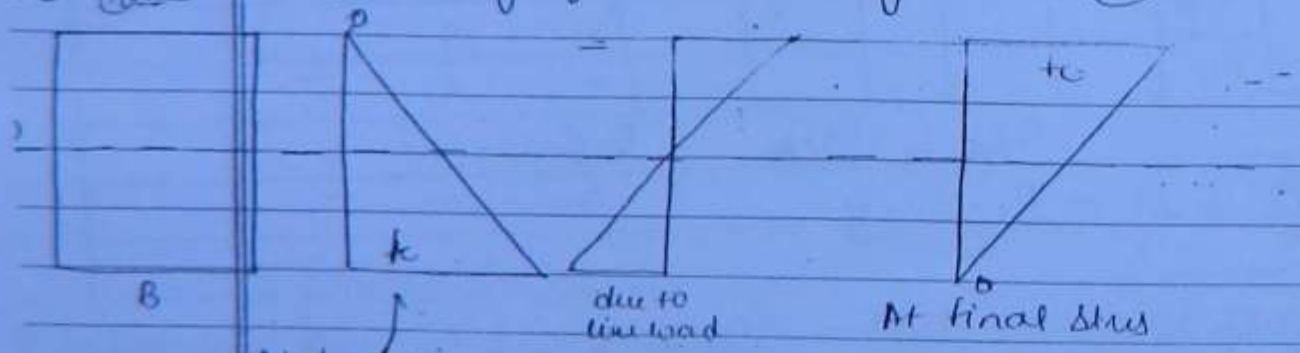
① Permissible stresses in concrete in compression f_c

in tension $= 0$

2. Permissible stress of steel $= f_s$

To find out size of beam, prestressing force, location of prestressing force (eccentricity), Area (A_s)

ss of stress = 0 in this case



Design formula are based on ideal stress diagram as shown in fig.

At transfer (top) $= \frac{P}{A} - \frac{P.e}{I}y + \frac{M_d}{I}y = 0$ — ①

Bottom $= \frac{P}{A} + \frac{P.e}{I}.y - \frac{M_d}{I}y = f_c$ — ②

At final stage (top) $= \frac{P}{A} - \frac{P.e}{Z} + \frac{(M_d + M_l)}{Z} = f_c$ — ③

$$\sigma_{bottom} = \frac{P}{A} + \frac{P \cdot e}{Z} - \frac{(M_d + M_l)}{Z} = 0 \quad \text{— (7)}$$

From (3) - (1)

$$\frac{M_l}{Z} = f_c \quad \text{— I}$$

(Section Modulus) $Z = M_l / f_c$

(264)

decide B & D so that $Z = \frac{BD^2}{6}$

$$A = B \times D$$

From (1) $\frac{Pe}{Z} = \frac{P}{A} + \frac{M_d}{Z}$

From (4) $\frac{Pe}{Z} = \frac{-P}{A} + \frac{M_d}{Z} + \frac{M_l}{Z}$

$$2\frac{Pe}{Z} = \frac{2M_d}{Z} + \frac{M_l}{Z}$$

$$e = \frac{2M_d + M_l}{2P} \quad \text{— II}$$

From (3) + (4)

$$\frac{2P}{A} = f_c \qquad \boxed{P = \frac{A \cdot f_c}{2}} \quad \text{— III}$$

Design Step

1. Find out live load Moment

$$M_l = \frac{w_l \times l^2}{8}$$

2. $Z = \frac{M_l}{f_c}$ (Section Modulus)

3. Decide size of beam so that $Z = \frac{BD^2}{6}$

4. Calculate Area $B \times D$

5. Calculate $P = \frac{A \times f_c}{2}$

6. Calculate Area Of Steel $= A_s = \frac{P}{f_s}$

7. Calculate dead load

$\Rightarrow w_d = B \times d \times 1 \times 25 =$

$M_d = \frac{w_d l^2}{8}$

8. Eccentricity $e = \frac{2M_d + M_l}{2P}$ (270)

S-199B

Ques Design a prestressed Concrete Slab Span in 12m, Carrying an imposed load Of $20 kN/m^2$, M-40 Concrete & Steel with ultimate Strength Of $1600 N/mm^2$ are available Permissible Stresses in Concrete in Comp $= 14 N/mm^2$ tension $= 0$. Neglect the loss in prestress. Cable Of 12 wires Of 5mm Φ are there Carrying an eff. prestress $= 225 kN$ each

Sol:-1 $w_l = 20 kN/m^2$

$M_l = \frac{w_l \cdot l^2}{8} = \frac{20 \times 12^2}{8} = 360 kN\text{-}m$

Section Modulus $Z = \frac{M_l}{f_c} = \frac{360 \times 10^6}{14}$

$Z = 25714285.71$

$\frac{BD^2}{6} = 25714285.71$

$BD^2 = 154285714.3$

Width Of Slab $= 1000 = B$

$D^2 = 154285.7143$

$D = 392.79 mm \simeq 392.8 mm$

Say $D = 400 mm$

DATE:

Area = $400 \times 1000 = 4 \times 10^5$ mm2

Prestressing force = $\frac{4 \times 10^5 \times 14}{2} = 2800$ KN

~~Area of Steel $A_s = \frac{2800 \times 10^3}{1600} = 1750$ mm^2~~

No. of Cables req (of 12 wires of 5mm dia)

$= \frac{2800}{225} = 12.44 \rightarrow$ 13 Nos. Cable

~~Total No. of wire — $13 \times 12 = 156$~~

~~Area of Steel $A_s = \frac{13 \times A}{4}$~~

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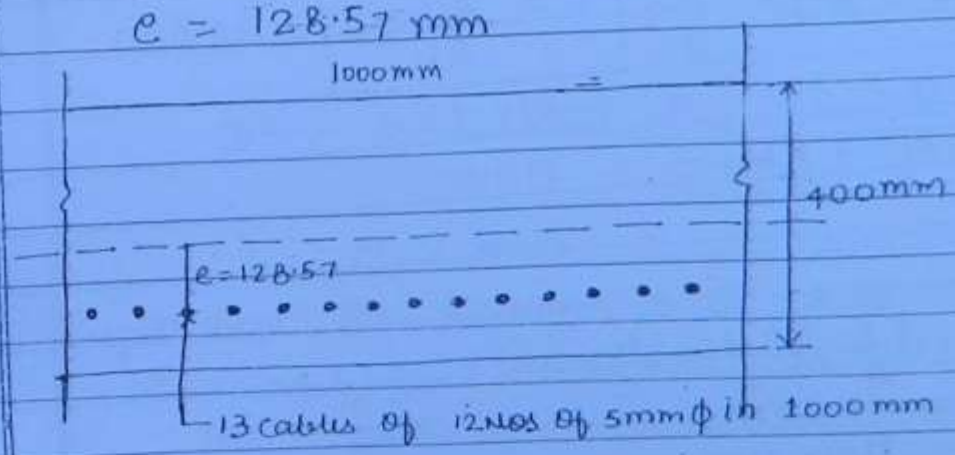
Dead load $= 1 \times .4 \times 1 \times 25 = 10$ KN/m^2

$M_d = \frac{10 \times 12^2}{8} = 180$ KN-m

$$e = \frac{2 \times 180 \times 10^6 + 360 \times 10^6}{2 \times 2800 \times 10^3}$$

$e = 128.57$ mm



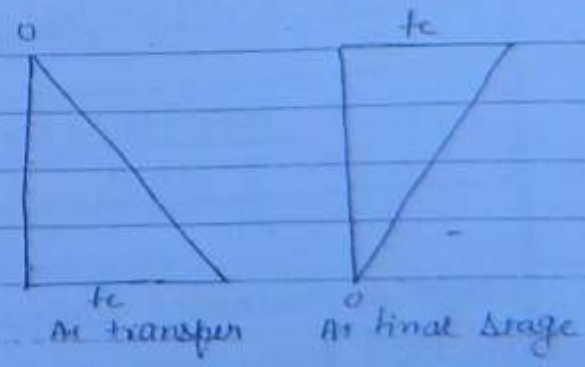
Case 2 If losses are Considered.

Permissible Stress in Concrete

in Compression = f_c

in Tension = 0

loss factor = K



At transfer

At top $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} = 0$ — (1)

At bottom $= \frac{P}{A} + \frac{P.e}{Z} - \frac{M_d}{Z} = f_c$ —— (2)

At final stage of loading

At top $= \frac{KP}{A} - \frac{KPe}{Z} + \frac{M_d}{Z} + \frac{M_l}{Z} = f_c$ —— (3)

At bottom $\frac{KP}{A} + \frac{KP.e}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} = 0$ —— (4)

From (3) - (1)K

$$\frac{M_d}{Z} + \frac{M_l}{Z} - \frac{K.M_d}{Z} = f_c$$

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$$\frac{(1-K)M_d}{Z} + M_l = f_c$$

Section Modulus $\boxed{\frac{(1-K)M_d + M_l}{f_c} = Z}$ —— (A)

From (3) + 4

$$\frac{2KP}{A} = f_c \quad \Bigg\| \quad \boxed{P = \frac{A.f_c}{2K}} \text{ —— (B)}$$

From (1) × K

$$\frac{K.Pe}{Z} = \frac{KP}{A} + \frac{KM_d}{Z}$$

From 4

$$\frac{K.Pe}{Z} = -\frac{KP}{A} + \frac{M_d}{Z} + \frac{M_l}{Z}$$

Adding these two we get

$$\frac{2KPe}{Z} = \frac{(1+K)M_d + M_l}{Z}$$

$$\boxed{e = \frac{(1+K)M_d + M_l}{2KP}} \text{ —— (C)}$$

Ques Design a prestressed concrete beam for a span of 20m, live load over beam is 35 kN/m. Permissible stress in con in compression = 18 N/mm² & in tension = 0

Permissible stress in steel = 1600 N/mm²
Loss of prestress = 15%.
Design a rectangular beam & show the prestressing force & its location.

Sol:- live load = 35 KN/m

$$M_l = \frac{35 \times 20^2}{8} = 1750 \text{ KN-m}$$

Let us assume c/s of rectangular beam
Size of beam 400 X 600 mm

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$$w_d = .4 \times .6 \times 1 \times 25 = 6 \text{ KN/m}$$

$$M_d = \frac{6 \times 20^2}{8} = 300 \text{ KN-m}$$

Now $Z = \frac{(1-.85)300 \times 10^6 + 1750 \times 10^6}{18}$

$Z = 99722222.22$

If B = 400

$$\frac{BD^2}{6} = Z$$

$$D^2 = \frac{6 \times 99722222.22}{400}$$

D = 1223 mm

Keep a size B = 600 mm & D = 1050 mm

$$w_d = .6 \times 1.05 \times 1 \times 25 = 15.75 \text{ KN/m}$$

$$M_d = \frac{15.75 \times 20^2}{8} = 787.5 \text{ KN-m}$$

$$Z = \frac{(1-.85)\,787.5 \times 10^6 + 1750 \times 10^6}{18}$$

Z = 103784722.2

D = 1018 mm & B = 600 mm

Size provided = 600 X 1050 mm

Area = 63×10^4 mm²

prestressing force $P = \frac{Af_c}{2K} = \frac{63 \times 10^4 \times 18}{2 \times .85} = 6670.59$ KN

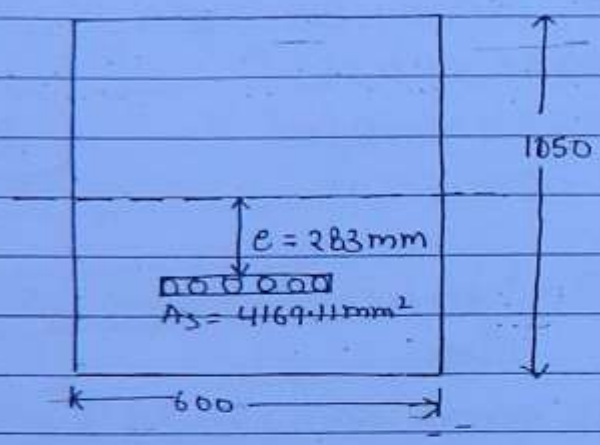
DATE:

Area of Steel $= \frac{P}{f_s} = \frac{6670.6 \times 10^3}{1600} = 4169.11$

$$e = \frac{(1+K)M_d + M_l}{2KP}$$

$$= \frac{(1+.85)787.5\times10^6 + 900\times10^6}{2\times.85\times6670.6\times10^3}$$

$e = 283\,mm$



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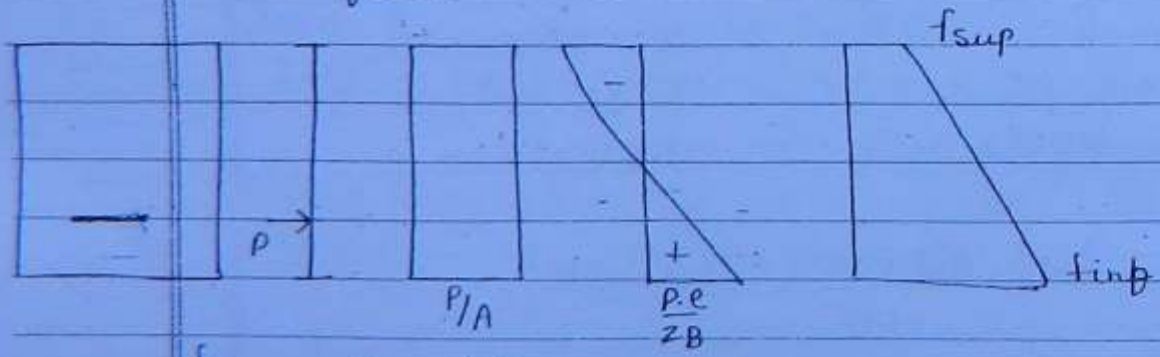
Case 3 If permissible stresses in Concrete
in compression $= f_c$
in tension $= -f_t$
loss factor $= K$



$$f_{sup} = \frac{P}{A} - \frac{P.e}{Z_t}$$

$$f_{inf} = \frac{P}{A} + \frac{P.e}{Z_B}$$

Design Steps

1. Assume a section
Calculate w_d & M_d

2. $M_e = \frac{w_e \cdot l^2}{8}$

3. Section modulus required
$$Z = \frac{(1-K)M_d + M_e}{(K \cdot f_c - f_t)}$$

4. $f_{sup} = \left(f_t - \frac{M_d}{Z_t}\right) = \left(f_t - \frac{M_d}{Z}\right)$

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5. $f_{inf} = \frac{f_t}{K} + \frac{M_d + M_e}{K Z_B}$

6. Prestressing force
$$P = \frac{A(f_{inf} \cdot Z_B + f_{sup} \cdot Z_t)}{Z_B + Z_t}$$

If $Z_B = Z_t = Z$
then $P = \frac{A(f_{inf} + f_{sup})}{2}$

7. Eccentricity $= e = \frac{Z_t \cdot Z_B (f_{inf} - f_{sup})}{A(f_{sup} Z_t + f_{inf} Z_B)}$

If $Z_t = Z_B = Z$
$$e = \frac{Z(f_{inf} - f_{sup})}{A(f_{sup} + f_{inf})}$$

ES-2007

Ques A Post-tensioned PSC Beam of rect. secn 240 mm wi is to be designed for a live load of 25 kN/m over an ef span of 12m. Permissible stresses in concrete in compr = 17 N/mm² & tensi n = 1.41 N/mm² loss = 15%. Calculate min possible depth of beam P & e

DATE:

Sol Assume size of beam

$B = 240mm$

$D = 600\ mm$

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dead load $= .24 \times .6 \times 1 \times 25 = 3.6\ kN/m$

$M_d = \frac{3.6 \times 12^2}{8} = 64.8\ kN\text{-}m$

$M_l = \frac{25 \times 12^2}{8} = 450\ kN\text{-}m$

$Z = \frac{(1-K) M_d + M_l}{(K f_c - f_t)}$

$= \frac{(1-.85)\ 64.8 \times 10^6 + 450 \times 10^6}{(.85 \times 17 + 1.4)}$

$= 29004416.4$

$\frac{BD^2}{6}$ $D = 851.53\ mm$

Assum $D = 860\ mm$

$w_d = .24 \times .86 \times 1 \times 25 = 5.16$

$M_d = 92.88\ kN\ m$

$Z = \frac{(1-.85)\ 92.88 \times 10^6 + 450 \times 10^6}{(.85 \times 17 + 1.4)}$

$Z = 29270157.73$

$D = 855\ mm$ ~~= Min depth req = 855~~

Say $= D = 860mm =$ Min depth req.

Size of beam $= 240 \times 860\ mm^2$

4. $f_{sup} = \left(f_t - \frac{M_d}{Z}\right)$ $Z = 29584 \times 10^3$

$= -1.4 - \frac{92.88 \times 10^6}{29270157.73}$ $\frac{240 \times 860^2}{6}$

$f_{sup} = -4.54\ N/mm^2$