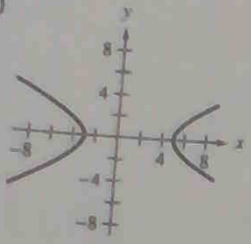
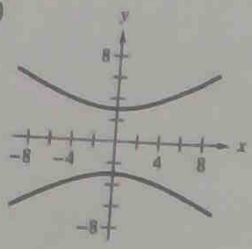
10.4 – Hyperbolas

In Exercises 1–4, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

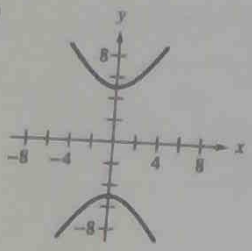
(a)



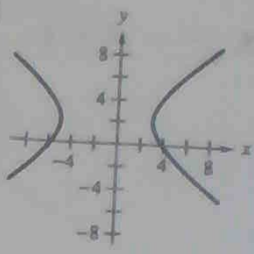
(b)



(c)



(d)



b **1.** $\frac{y^2}{9} - \frac{x^2}{25} = 1$

c **2.** $\frac{y^2}{25} - \frac{x^2}{9} = 1$

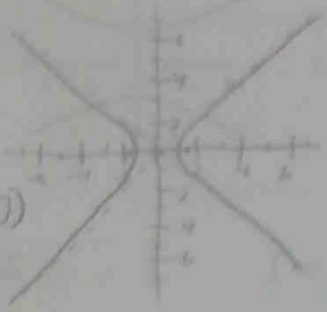
a **3.** $\frac{(x-1)^2}{16} - \frac{y^2}{4} = 1$

d **4.** $\frac{(x+1)^2}{16} - \frac{(y-2)^2}{9} = 1$

In Exercises 5–16, find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph using the asymptotes as an aid.

5. $x^2 - y^2 = 1$

$\frac{(x-0)^2}{1} - \frac{(y-0)^2}{1} = 1$



ce = (0,0) v = (±1, 0) f = (±√2, 0)

a = 1

b = 1

c = √2

$1^2 + 1^2 = c^2$

asympt =

$y = \pm x$

7. $\frac{y^2}{25} - \frac{x^2}{81} = 1$

$\frac{(y-0)^2}{25} - \frac{(x-0)^2}{81} = 1$

ce = (0,0) v = (0, ±5) f = (0, ±√106)

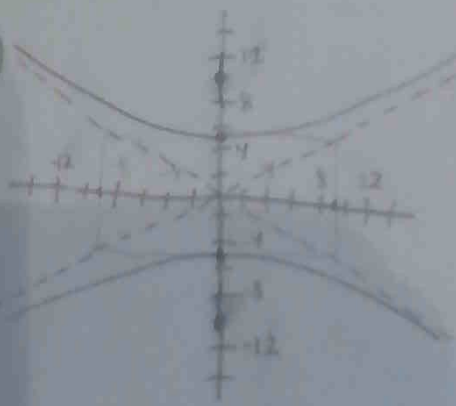
$a = \sqrt{25} = 5$

$b = \sqrt{81} = 9$

$c = \sqrt{106}$

asympt =

$y = \pm \frac{5}{9}x$



9. $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{1} = 1$

$cc = (1, -2)$

$a = \sqrt{4} = 2$

$b = \sqrt{1} = 1$

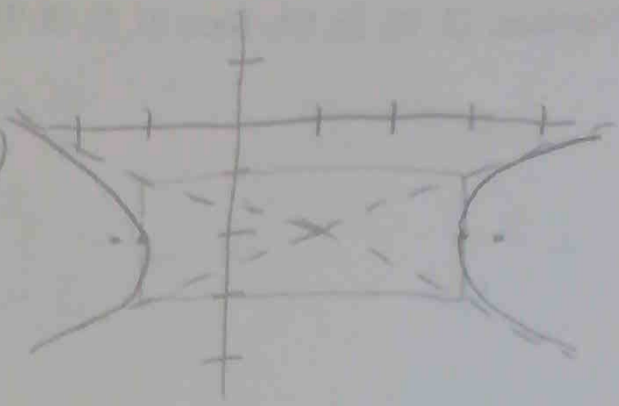
$c = \sqrt{5}$

asympt

$y + 2 = \pm \frac{1}{2}(x-1)$

$v = (3, -2)$
$(-1, -2)$

$f = (\sqrt{5}, -2)$
$(1-\sqrt{5}, -2)$



13. $9x^2 - y^2 - 36x - 6y + 18 = 0$

$9(x^2 - 4x + 4) - 1(y^2 + 6y + 9) = -18 + 36 - 9$

$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$

$cc = (2, -3)$

asympt

$y + 3 = \pm 3(x-2)$

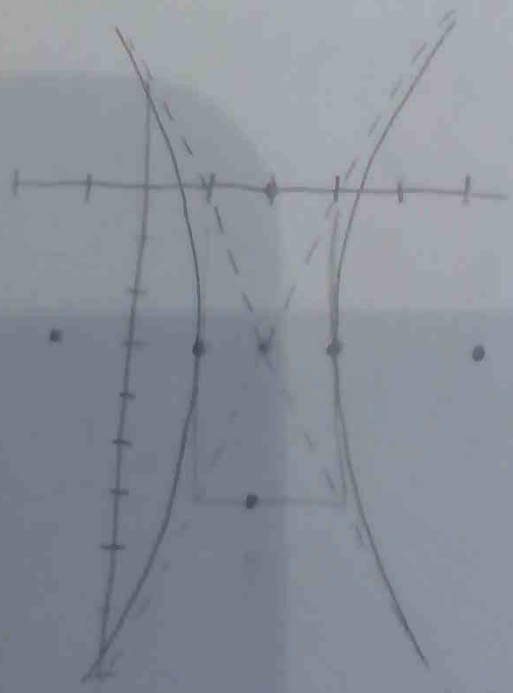
$a = \sqrt{1} = 1$

$b = \sqrt{9} = 3$

$c = \sqrt{10}$

$v = (1, -3), (3, -3)$

$f = (2\sqrt{10}, -3)(2-\sqrt{10}, -3)$



In Exercises 27–38, find the standard form of the equati of the hyperbola with the given characteristics.

27. Vertices: (2, 0), (6, 0); foci: (0, 0), (8, 0)

$ce = (4,0)$

$a = 2$

$c = 4$

$\frac{(x-4)^2}{4} - \frac{y^2}{12} = 1$

$c^2 - a^2 = b^2$

$16 - 4 = 12$

29. Vertices: (4, 1), (4, 9); foci: (4, 0), (4, 10)

$ce = (4,5)$

$a = 4$

$c = 5$

$\frac{(y-5)^2}{16} - \frac{(x-4)^2}{9} = 1$

$25 - 16 = 9$

34. Vertices: $(1, 2), (1, -2)$;
passes through the point $(0, \sqrt{5})$

$c = (1, 0)$

$a = 2$

$b = 2$

$\frac{y^2}{4} - \frac{(x-1)^2}{b^2} = 1$

$\frac{5}{4} - \frac{1}{b^2} = 1$

$-\frac{1}{b^2} = -\frac{1}{4}$

$b^2 = 4$

$b = 2$

$\frac{y^2}{4} - \frac{(x-1)^2}{4} = 1$

35. Vertices: $(1, 2), (3, 2)$;
asymptotes: $y = x,\ y = 4 - x$

$c = (2, 2)$

$a = 1$

$b = 1$

$\frac{(x-2)^2}{1} - \frac{(y-2)^2}{1}$