

15/5/2019

09.30 - 11.30am

CMPU 1018 Mathematics 1

Basement 2, Kevin Street



**TECHNOLOGICAL UNIVERSITY DUBLIN – CITY CAMPUS**

**School of Mathematical Sciences**

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**DT211 BSc Computer Science (Infrastructure)**

**Year 1**

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**SUMMER EXAMINATIONS 2018/2019**

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**CMPU1018 MATHEMATICS 1**

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**DR C HILLS**

Answer question 1 and any two other questions

Question 1 carries 40 marks. All other questions carry 30 marks each.

Approved calculators may be used

Mathematical tables are provided

New Cambridge Statistical Tables are NOT permitted

1. a) Three numbers are in an arithmetic progression. Their sum is 21 and product is 280. Determine the numbers. (5 marks)

- b) Find the inverse of the matrix:

$$A = \begin{pmatrix} 4 & -6 \\ -1 & 4 \end{pmatrix}$$

(5 marks)

- c) Let  $A = \{2, 3, 4, 5, 6\}$ ,  $B = \{1, 4, 5, 9, 13\}$  and  $C = \{1, 2, 4, 9, 11, 15, 19\}$ . Find:

i)  $C \setminus A$ .

ii)  $(B \setminus A) \cup (B \cap C)$ . (5 marks)

- d) Use prime factorisation to find the Highest Common Factor (HCF) and the lowest common multiple (LCM) of the following pair of numbers:

66, 150 and 115, 500

(5 marks)

- e) Write down the truth table for the following Boolean expression:

$$(\bar{a} \wedge b) \vee \bar{c}.$$

(5 marks)

- f) Consider the following function,

$$f: R \rightarrow R : f(x) = x^2$$

- i) Is the function one-to-one? Explain your answer.

- ii) Is the function onto? Explain your answer. (5 marks)

- g) Find the mean, median and standard deviation for the following set of data, leaving your answers to 1 decimal place:

41, 19, 19, 20, 22, 29, 32, 33, 33, 33, 39, 40

(5 marks)

- h) Simplify the following expression using the properties of logarithms:

$$\log_4(1024) - \log_3\sqrt{27}.$$

(5 marks)

[40 marks]

2. a) i) Use the Euclidean Algorithm to find  $d = hcf(23, 217)$ . (6 marks)

ii) Hence or otherwise, find integers  $s$  and  $t$  such that

$$23s + 217t = d.$$

(7 marks)

iii) Hence, find the multiplicative inverse of 23 in  $Z_{217}$  i.e. mod 217.

(3 marks)

b) Calculate the following modular operations:

i)  $(11 + 13) \bmod 15$

ii)  $(9 \times 14) \bmod 19$

(4 marks)

c) Let  $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$  be the universal set. Let  $A = \{12, 14, 18, 20\}$  and  $B = \{11, 12, 13, 15, 16, 17, 18\}$  be sets. Use **bit string notation** to find the following sets:

i)  $A$ ,

ii)  $B$ ,

iii)  $A^c$ ,

iv)  $A \cup B$ ,

v)  $\overline{A} \cap B$ .

(10 marks)

[30 marks]

3. a) Let  $A = \{60, 61, 63\}$ ,  $B = \{x, y, z\}$  and  $C = \{a, y, z, b\}$ . List the elements of the following sets:

i) The *Power Set* of  $A$ ,  $P(A)$ .

ii) The *Cartesian Product* of  $A$  and  $C$ ,  $A \times C$ .

iii) The *Symmetric Difference* of  $B$  with  $C$ ,  $B \triangle C$ .

(10 marks)

b) A triangle that has vertices in homogenous coordinates,

$$A = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}, B = \begin{pmatrix} -28 \\ 30 \\ 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 19 \\ -26 \\ 1 \end{pmatrix}$$

is represented by the matrix

$$M = \begin{pmatrix} 3 & -28 & 19 \\ -4 & 30 & -26 \\ -1 & 1 & 1 \end{pmatrix}$$

Find the image of this triangle under rotation of the plane through an angle of  $135^\circ$  **counter clockwise** about the origin. Note: The rotation of a plane counter clockwise about the origin  $(0,0)$  through an angle  $\theta$  is given by the matrix:

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10 \text{ marks})$$

c) Let

$$A = \begin{pmatrix} 5 & 11 & -3 \\ -3 & 0 & 9 \end{pmatrix}, B = \begin{pmatrix} 12 & -3 \\ 6 & -11 \\ -12 & -1 \end{pmatrix}, C = \begin{pmatrix} 9 & 20 \\ -5 & 7 \end{pmatrix}, D = \begin{pmatrix} -8 & 2 & 5 \\ 3 & 11 & 0 \\ 0 & -5 & 10 \end{pmatrix}$$

Evaluate the following expression if possible or explain why the calculation cannot be made:

- i)  $4AB + C^2$
  - ii)  $(BC)^T - 12D$ , where  $T$  denotes the transpose of a matrix.
- (10 marks)

[30 marks]

4. a) Let  $f : R \rightarrow R$  be given by  $f(x) = 9x + 2$  and let  $g : R \rightarrow R$  be given by  $g(x) = 2x^2 - 1$ . Find

- i)  $(f \circ g)(x)$ ,
  - ii)  $(g \circ g)(x)$ ,
  - iii)  $(f \circ f)(2)$ .
- (10 marks)

b) The following binary relation  $R$  on the given set  $S$  is defined:

$$S = \{20, 40, 60, 80, 100\} \text{ and } R = \{(a, b) : a|b\}$$

Note:  $a|b$  means that  $a$  divides into  $b$  evenly.

- i) Write down the elements of  $R$ .
  - ii) Test  $R$  for reflexivity, symmetry and transitivity.
- (10 marks)

c) Use a truth table to ascertain if the following Boolean expressions are equivalent:

- i)  $\overline{(A \wedge B) \vee C}$ .
  - ii)  $\overline{(A \wedge B)} \vee \overline{C}$ .
- (10 marks)

[30 marks]