CMPU 1018

W228/101

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

DT228 BSc Computer Science

YEAR I

SEMESTER I Examination 2013-14

Mathematics 1

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Wednesday 8th January 2014
4.00pm – 6.00pm

Answer Question 1 and any 2 other questions

Mathematical Tables and Graph paper are available

Q1

(a) Let A be the set of characters appearing in the string"$software$", B be the set of characters appearing in the string "$network$" and C be the set of characters in the string"$adobe$". List the elements of the following sets:

(i) $(A \cup C)$ (ii) $(A \cap B)$ (iii) $(A \cup C)\backslash B$ (iv) $(A \cup C) \cap (A \cup B)$

[5 marks]

(b) Use the properties of logarithms to evaluate the following:

(i) $-2log_3(243) + 2log_7\sqrt{7} + log_5\left(\frac{1}{25}\right)$

(ii) $log_6(7776) - 3log_5\sqrt{125} - log_2(1)$

[5 marks]

(c) Find the inverse of the matrix$\begin{pmatrix} 3 & 3 \\ 6 & -2 \end{pmatrix}$.

Hence or otherwise, solve the following system of equations:

$$3x + 3y = 6$$
$$6x - 2y = 28$$

[5 marks]

(d) Test the following binary relation R on the given set S for reflexivity, symmetry and transitivity

$$S = \{2, 3, 4, 5, 6, 7, 8, 9\}, \quad R = \{(a, b): a + 2b \geq 14\}$$

[5 marks]

(e) Find the mean, median and variance of the following set of data:

$$11, 14, 16, 18, 14, 13, 19, 22$$

[5 marks]

(f) Let $f: \boldsymbol{N} \rightarrow \boldsymbol{N}$ be given by$f(x) = 2x - 4$

Let $g: \boldsymbol{N} \rightarrow \boldsymbol{N}$ be given by $g(x) = 3x^2 + 2$

Calculate:

(i) $(f \circ g)(2)$

(ii) $(g \circ f)(x)$

(iii) $(f \circ f)(1)$

[5 marks]

(g) Let $U = \{1,2,3,4,5,6\}$ be the universal set. Represent the set $A = \{3,4,6\}$ with bit strings.

[5 marks]

(h) Use Euclid's Algorithm to find the hcf of 97,020 and 110,250.

[5 marks]

Q2

(a) Let $A = \{1, 2, 3\}, B = \{c, d\}$ and $C = \{2, 3, 4\}$ be sets. List the elements of the following sets:

(i) The symmetric difference of B and C, $B\Delta C$

(ii) The Cartesian product of A and B, $A \times B$.

(iii) The *power set of* $B, P(B)$

[10 marks]

(b) Let $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ be the universal set. Let $A = \{11, 14, 15, 16, 20\}$ and $B = \{10, 14, 15, 18, 19, 20\}$ be sets. Use bit string representation to find the following sets:

(i) $\bar{B}$

(ii) $A \cap B$

(iii) $A \cup B$

[10 marks]

(c) Use a truth table to verify if the following are equivalent formulas:

(i) $a \wedge (b \vee c) \sim (a \wedge b) \vee (a \wedge c)$

(ii) $\overline{(a \wedge b)} \sim \bar{a} \vee \bar{b}$ (De Morgan's Law)

[10 marks]

Q3

(a) Use ***prime factorisation*** to calculate $hcf(6930, 9900)$.

[8 marks]

(b) Write out the operational tables for $\boldsymbol{Z}_8$.
Use Fermat's Little Theorem to find the inverses of 5 and 7 modulo 8. Check your answers against the multiplication table for $\boldsymbol{Z}_8$.

[12 marks]

(c) Use the Euclidean Algorithm to find the multiplicative inverse of 23 in $\boldsymbol{Z}^*_{362}$.

[10 marks]

Q4

(a) Let $A = \begin{pmatrix} 3 & 3 & -1 \\ 2 & 0 & 3 \\ 4 & -1 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & 4 \\ -6 & 2 & 0 \end{pmatrix}, C = \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}$ and $D = \begin{pmatrix} 6 & 4 \\ 2 & 0 \\ -1 & 3 \end{pmatrix}$

Evaluate (if possible)

(i) $2AD$

(ii) $3BC$

(iii) $3CB$

(iv) C^{-1}

[6 marks]

(b) A square has vertices $p_1 = (1,1), p_2 = (2,1), p_3 = (2,2)$ and $p_4 = (1,2)$. Determine the image of this square when it is scaled about the origin by factors of 5 in the x-direction and 3 in the y-direction.

[12 marks]

(c) A rectangle having vertices A, B, C and Dgiven in homogenous coordinates

$$A = \begin{pmatrix} 15 \\ -5 \\ 1 \end{pmatrix}, B = \begin{pmatrix} -25 \\ -5 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -25 \\ 15 \\ 1 \end{pmatrix}, D = \begin{pmatrix} 15 \\ 15 \\ 1 \end{pmatrix}$$

is represented by the matrix

$$M = \begin{pmatrix} 15 & -25 & -25 & 15 \\ -5 & -5 & 15 & 15 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Find the image of this rectangle under the rotation of the plane through an angle of $\frac{3\pi}{4}$ radians clockwise about the origin.

[12 marks]