DUBLIN INSTITUTE OF TECHNOLOGY

DT211 BSc Hons Computer Science

Stage 1

SUMMER EXAMINATIONS 2017/2018

MATHEMATICS 1 [CMPU 1018]

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Thursday 10th May 9.30 am - 11.30 am

Answer **QUESTION 1** and **TWO** other question.
Question 1 carries 40 marks. All other questions carry 30 marks.

Approved calculators may be used

Mathematical tables are provided

New Cambridge Statistical Tables are NOT permitted

1. a) Simplify the following expression,

$$(10x^2)\left(\sqrt[5]{\frac{x^4}{(2x)^5}}\right).$$

(5)

b) Using prime factorisation, find the highest common factor (HCF) and lowest common multiple (LCM) for the following pair of numbers:

$$504 \text{ and } 540.$$

(5)

c) Let $A = \{x, y, z, w\}$, $B = \{v, y, z, w\}$ and $C = \{u, v, w, x\}$. Find:

i) $B \backslash A$,
ii) $A \cap C$,
iii) $(B \backslash A) \cup (A \cap C)$.

(5)

d) Consider the following function,

$$f: \mathbb{R} \to \mathbb{R}: f(x) = x^4 - 1.$$

i) Is the function *one-to-one*? Explain your answer.
ii) Is the function *onto*? Explain your answer.

(5)

e) Three numbers are in arithmetic progression. Their sum is 24 and product is 312. Determine the numbers.

(5)

f) Write down the truth table for the following:

$$\overline{p} \wedge (p \vee q).$$

(5)

g) Find the inverse of the matrix $\begin{pmatrix} 3 & 8 \\ 2 & 5 \end{pmatrix}$.
Hence or otherwise, solve the following system of equations:

$$3x + 8y = 13,$$
$$2x + 5y = 8.$$

(5)

h) Find the mean, mode and variance for the following set of data:

$$11, 15, 22, 18, 12, 16, 24, 18.$$

(5)

[40]

2. a) i) Use Euclid's Algorithm to calculate $\mathrm{HCF}(19,141)$.

(6)

ii) Hence, find integers s and t such that,

$$19s + 141t = \mathrm{HCF}(19,141).$$

(8)

iii) Hence, find the multiplicative inverse of 19 in $\mathbb{Z}_{141}$.

(3)

b) Calculate the following modular operations.

i) $(9 + 10) \bmod 12$,
ii) $(7 \times 10) \bmod 11$.

(3)

c) Let $U = \{1,2,3,4,5,6,7,8\}$ be the universal set. Let $A = \{2,4,6,8\}$ and $B = \{1,2,3,4\}$ be sets. Use **bit string notation** to find the following sets.

i) A,
ii) B,
iii) A^c,
iv) $\overline{B}$,
v) $A \cap B$,
vi) $\overline{A} \cup B$.

(10)
[30]

3. a) In computer graphics the rotation of the plane counter clockwise about the origin (0,0) through an angle θ is given by the matrix,

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Show that the inverse matrix, R_θ^{-1}, is given by

$$R_\theta^{-1} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

NOTE: $\cos^2\theta + \sin^2\theta = 1$

(10)

b) A triangle that has vertices in homogeneous coordinates,

$$A = \begin{pmatrix} 2 \\ -8 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} -30 \\ 22 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 16 \\ -20 \\ 1 \end{pmatrix},$$

is represented by the matrix,

$$M = \begin{pmatrix} 2 & -30 & 16 \\ -8 & 22 & -20 \\ 1 & 1 & 1 \end{pmatrix}.$$

Find the image of this triangle under rotation of the plane through an angle of 120° **counter clockwise** about the origin.

(10)

c) Let $A = \begin{pmatrix} 2 & 3 & -1 \\ 6 & 5 & 0 \end{pmatrix}, B = \begin{pmatrix} 8 & 0 \\ 10 & -7 \\ -3 & 5 \end{pmatrix},$

$C = \begin{pmatrix} -1 & 4 \\ 8 & 6 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 4 & 6 \\ 1 & 0 & 1 \\ 0 & 2 & 3 \end{pmatrix}.$

Evaluate the following expression if possible or explain why the calculation cannot be made,

i) $2AB + C^2$,

ii) $(DA)^T + 5B$, where ^{T} denotes the transpose of a matrix.

(10)

[30]

4. a) Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 5x + 3$ and let $g: \mathbb{R} \to \mathbb{R}$ be given by $g(x) = x^2 - 1$. Find:

   i) $(f \circ g)(x)$,
   ii) $(g \circ f)(x)$,
   iii) $(f \circ f)(x)$.

(10)

b) Let $A = \{1,2,3,4\}$ and let R be a relation on A given by,

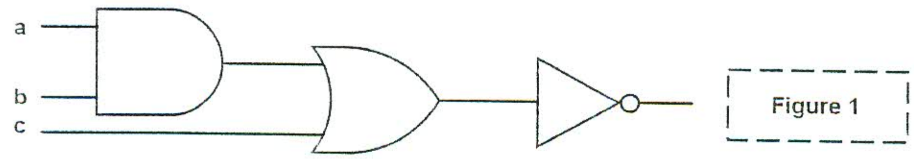
$$R = \{(1,1), (1,2), (2,2), (2,1), (3,3), (3,4), (4,3)\}.$$

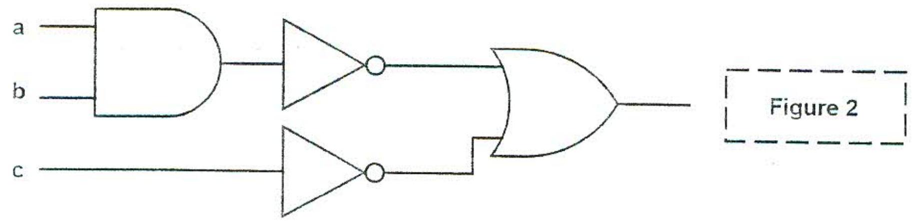
Verify whether or not R is:

   i) Reflexive,
   ii) Symmetric,
   iii) Transitive,
   iv) An Equivalence.

(8)

c) Find the Boolean expressions for the two combinatorial circuits given below in Figure 1 and Figure 2. Use truth table to ascertain if they are equivalent circuits.





(12)
[30]