CMPU 1018

S211/102

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

BSc Computing

YEAR I

SUMMER EXAMINATIONS 2013/2014

MATHEMATICS 1

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Tuesday 20th May 2014 9.30 a.m. to 11.30 a.m.

Full marks for complete answers to **THREE** questions.
All Questions carry equal marks.

Mathematics Tables and graph paper provided

1 (a) In computer graphics the rotation of the plane counter clockwise about the origin (0,0) through an angle θ is given by the matrix

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Show that the inverse matrix R_θ^{-1} is given by

$$R_\theta^{-1} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

NOTE: $\cos^2\theta + \sin^2\theta = 1$

[10 marks]

(b) A triangle has vertices in homogenous coordinates

$$A = \begin{pmatrix} 14 \\ 10 \\ 1 \end{pmatrix} B = \begin{pmatrix} 20 \\ 10 \\ 1 \end{pmatrix} C = \begin{pmatrix} 17 \\ 5 \\ 1 \end{pmatrix}$$

using the information from above:

i. find the image of this rectangle under the rotation of the plane through an angle of 90° clockwise about the origin.

ii. find the image of this rectangle under the rotation of the plane through an angle of 120° counter clockwise about the origin.

[16 marks]

(c) By finding the inverse of the matrix

$$A = \begin{pmatrix} 2 & 4 \\ -3 & 1 \end{pmatrix}$$

solve the set of simultaneous equations:

$$2x + 4y = 2$$
$$-3x + y = 11$$

[7 marks]

2 (a) Use the Euclidean Algorithm to find the highest common factor of 22120 and 846, i.e. the $hcf(22120, 846)$ [5 marks]

(b) Calculate the mean, mode, median and standard deviation of the following set of numbers:

$$25, 36, 38, 41, 21, 73, 36$$

[11 marks]

(c) Using prime factorisation find the highest common factor and the lowest common multiple of the following pair

308 and 3234

[8 marks]

(d) Let $A = \{2, 3, 5, 7\}$ and $B = \{2, 5, 6, 7, 9, 11, 13\}$. Two relations from A to B are defined as:

aRb if and only if $3a < b$
aSb if and only if $a + 2 = b$
List the elements of R, S and $R \cup S$

[9 marks]

3 (a) Write out the operational tables for Z_7
Use Fermats Little Theorem to find the inverses of 3 and 5 modulo 7. Check your answers against the multiplication table for Z_7

[13 marks]

(b) Use the Euclidean Algorithm to find the multiplication inverse of 17 mod 67 i.e. in Z_{67}

[11 marks]

(c) i. Use Caesar's Shift algorithm with key $k = 9$ to encrypt the message:
"Enjoy the Summer holidays"

ii. Using the Caesar Shift with key $k = 4$, decrypt the message:
"XLMW MW E WIGVIX QIWWEKI"

[9 marks]

4 (a) Given $f(x) = x^2 + 3x + 2$ and $g(x) = 3x - 2$, find:

i. Calculate $f(4)$ and $g(10)$

[2 marks]

ii. the composite functions $f \circ g$ and $g \circ f$

[7 marks]

iii. the inverse function $g^{-1}(x)$

[3 marks]

(b) Calculate the following modular operations:

i.

$$(8 + 16) \bmod 3$$

ii.

$$(4 \times 13) \bmod 6$$

[6 marks]

(c) Using truth tables:

i. Prove De Morgans Law $\neg(P \wedge Q) \sim \neg P \vee \neg Q$

ii. Test whether $P \wedge Q \wedge R \rightarrow P \vee Q \vee R$ is a tautology

[15 marks]