DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

DT211 BSc Computing

YEAR I

Semester 2 Examinations 2014-15

Mathematics 1 (CMPU 1018)

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Date: Wednesday 20th May 2015 **Time:** 9.30 - 11.30 am

Full marks for complete answers to **Question 1** and any other **2** questions.
Question 1 carries 40 marks. All other questions carry 30 marks.

Mathematical Tables and Graph paper are available.

1.

(a) Let A be the set of characters appearing in the string "$domain$", B the set of characters appearing in the string"$memory$" and C be the set of characters appearing in the string "$mathematics$". List the elements of the following sets:

$$(i)\ A \cup B \quad (ii)\ A \cap C \quad (iii)\ A \backslash B \quad (iv)\ (A \cap B) \backslash (B \cap C)$$

[5 marks]

(b) Find the inverse of the matrix $= \begin{pmatrix} 7 & -5 \\ 6 & 2 \end{pmatrix}$. Hence or otherwise, solve the following system of equations:

$$7x_1 - 5x_2 = 1$$
$$6x_1 + 2x_2 = 26$$

[5 marks]

(c) Find the mean, mode and variance of the following set of data:

$$56, 23, 35, 48, 95, 32, 87, 23$$

[5 marks]

(d) Given the following matrices $E = \begin{pmatrix} 3 & -1 \\ 5 & -4 \end{pmatrix}$ and $F = \begin{pmatrix} 2 & 4 \\ 2 & 7 \end{pmatrix}$, evaluate (if possible) the following:

i. $(2E)^T$, where T denotes the transpose of a matrix.
ii. F^{-1}

[5 marks]

(e) Use Euclid's Algorithm to find $hcf(482, 914)$.

[5 marks]

(f) Let $f: \boldsymbol{N} \to \boldsymbol{N}$ be given by $f(x) = \sqrt{5x + 7}$.
Let $g: \boldsymbol{N} \to \boldsymbol{N}$ be given by $g(x) = x^2 - 2$.
Calculate:

i. $(fog)(2)$
ii. $(gof)(1)$
iii. $(gog)(x)$
iv. $(fof)(y)$

[5 marks]

(g) Calculate the following modular operations

i. $(9 + 2)mod7$
ii. $(5 \times 3)mod11$

[5 marks]

(h) Simplify

$$2x^3\sqrt{\frac{(3x^5)^3}{x}}$$

[5 marks]

2.

(a) Write out the operational tables for $\boldsymbol{Z}_7$.
Use Fermat's Little Theorem to find the inverses of 3 and 4 modulo 7. Check your answers against the multiplication table for $\boldsymbol{Z}_7$.

[12 marks]

(b)

i. Use Caesar's Shift algorithm with key $k = 7$ to encrypt the message "Tomorrow is another day".

ii. Using the Caesar shift with key $k = 5$, decrypt the message "YMJ BJFYMJW NX LWJFY".

[12 marks]

(c) Use *prime factorisation* to calculate $hcf(2120, 688)$.

[6 marks]

3.

(a) Let $A = \{3, 4, 5\}, B = \{x, y, z\}$ and $C = \{y, z, w\}$ be sets. List the elements of the following sets

(i) The *power set of* $A, P(A)$.
(ii) The symmetric difference of B and C, $B\Delta C$
(iii) The Cartesian product of A and B, $A \times B$.

[10 marks]

(b) Use truth tables to prove:

(i) $(A \wedge B) \wedge C$ is logically equivalent to $A \wedge (B \wedge C)$
(ii) $\overline{(F \wedge G)}$ is logically equivalent to $\bar{F} \vee \bar{G}$

[10 marks]

(c) Use the Euclidean Algorithm to find the multiplicative inverse of 39 in $\boldsymbol{Z}_{211}^*$ i.e. the inverse of 39 modulo 211.

[10 marks]

4.

(a) If $A = \begin{pmatrix} 2 & 6 & -7 \\ -3 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 5 & 0 \\ 1 & -6 & -4 \\ 8 & 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} -3 & 2 \\ -7 & 5 \end{pmatrix}$ evaluate (if possible)

i. $A + 2B$
ii. $B - A$
iii. C^2
iv. B^T

[4 marks]

(b) Find the image of the square which has vertices $(2,2), (4,2), (2,4), (4,4)$ after scaling about the origin by factors of 3 in the x-direction and 5 in the y-direction.

Note: The scaling matrix is given by $\begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

[12 marks]

(c) A rectangle has the following vertices A, B, C and D in homogeneous coordinates

$$A = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}, B = \begin{pmatrix} -10 \\ -1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -10 \\ 10 \\ 1 \end{pmatrix}, D = \begin{pmatrix} 5 \\ 10 \\ 1 \end{pmatrix}$$

Find the image of this rectangle under the rotation of the plane through an angle of $\frac{\pi}{3} rads = 60°$ counter-clockwise about the origin, given that the rotation of the plane counter-clockwise about the origin (0,0) through an angle θ radians is given by the matrix

$$R_\theta = \begin{pmatrix} Cos\theta & -Sin\theta & 0 \\ Sin\theta & Cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[14 marks]