W228/282/101C

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

CMPU 1018

DT228 BSc Computer Science
DT282 BSc Computer Science (International)

YEAR I

WINTER EXAMINATIONS 2014/2015

MATHEMATICS 1

MR. P. O'REILLY
DR. C. HILLS

Wednesday, 17th December 2014 9.30 - 11.30 am

Full marks for complete answers to **Question 1** and any other 2 Questions.
Question 1 carries 40 marks. All other questions carry 30 marks.

Mathematics Tables and graph paper provided

1 (a) Given that the inverse of the matrix

$$A = \begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix}$$

is

$$A^{-1} = \frac{1}{7}\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$$

solve the set of simultaneous equations:

$$3x - 2y = 11$$
$$2x + y = 12.$$

[5 marks]

(b) Simplify

i. $3\log_3 x - 2\log_3 x^2$.

ii. $2\ln(2x) - \ln(4x^2)$.

[5 marks]

(c) Let A be the set of characters appearing in the string "*binary*", B be the set of characters in the string "*mathematics*" and C be the set of characters in the string "*science*". List the elements of the following sets:
(i) $(A \cap C)$ (ii) $(A \cap C) \backslash B$ (iii) $(B \cup C) \cap (A \cup C)$.

[5 marks]

(d) Calculate the mean, mode and median of the following set of numbers:

$$13, 18, 13, 14, 13, 16, 14, 21, 13$$

[5 marks]

(e) Simplify

$$x^{-2}\sqrt{\frac{(x^2)^3}{x^4}}.$$

[5 marks]

(f) Calculate the following modular operations:

i. $(10 + 2) \bmod 5$.

ii. $(9 \times 14) \bmod 11$.

[5 marks]

(g) Solve the equations, giving your solutions to 2 decimal places:

i. $10^x = 8$.

ii. $e^y = 12$.

[5 marks]

(h) Given the matrices $\mathrm{B} = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$ and $\mathrm{C} = \begin{pmatrix} 3 & 5 \\ 2 & 4 \\ 1 & 7 \end{pmatrix}$

Evaluate (if possible):

i. $(CB)^T$.

ii. C^{-1}.

[5 marks]

2 (a) In computer graphics the rotation of the plane counter clockwise about the origin (0,0) through an angle θ is given by the matrix

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Show that the inverse matrix R_θ^{-1} is given by

$$R_\theta^{-1} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[12 marks]

(b) A triangle has vertices in homogenous coordinates

$$A = \begin{pmatrix} 16 \\ 8 \\ 1 \end{pmatrix} B = \begin{pmatrix} 22 \\ 8 \\ 1 \end{pmatrix} C = \begin{pmatrix} 22 \\ 5 \\ 1 \end{pmatrix} D = \begin{pmatrix} 16 \\ 5 \\ 1 \end{pmatrix}$$

Find the image of this rectangle under the rotation of the plane through an angle of 45° clockwise about the origin.

[12 marks]

(c) Use the Euclidean Algorithm to find the highest common factor, HCF, of 8435 and 720.

[6 marks]

3 (a) Let $A = \{1, 2, 5, 7\}$ and $B = \{2, 6, 7, 9, 10, 12\}$, Two relations from A to B, R and S, are defined as:

i. aRb if and only if $2a/b$

ii. aSb if and only if $a = b - 7$

List the elements of R and S, and find $\#S$ and $\#R$.

[11 marks]

(b) Given $f(x) = 7x^3 + 7x + 2$ and $g(x) = \dfrac{x+5}{7}$:

i. Calculate $f(-1)$, $g(9)$ and $g(x + 16)$.

ii. Find the composite function $g \circ f$.

iii. Find the inverse function $g^{-1}(x)$.

[9 marks]

(c) Using truth tables:

i. Show $(P \to Q) \vee (Q \to P)$ is a tautology.

ii. Test whether $\neg(A \vee B)$ and $\neg A \wedge \neg B$ are logically equivalent .

[10 marks]

4 (a) Write out the operational tables for Z_6.
Use Fermats Little Theorem to find the inverse of 5 modulo 6. Check your answers against the multiplication table for Z_6.

[10 marks]

(b) Use the Euclidean Algorithm to find the multiplication inverse of 23 mod 126 i.e. in Z_{126}.

[10 marks]

(c) i. Use Caesar's Shift algorithm with key $k = 3$ to encrypt the message: *"Hide this message"*.

ii. Using the Caesar Shift with key $k = 7$, decrypt the message: *"Ohcl h nvvk joypzathz"*.

[10 marks]

COLLEGE EXAMINATIONS

AMENDMENTS TO EXAMINATION QUESTION PAPER

COURSE REF: W228/282/101C **VENUE:** Basement 1

SUBJECT: Mathematics 1

DATE: 17/12/2014

TIME: 9.30 am – 11.30 am

SIGNED: Fiona Murray

INSTRUCTIONS:

2(b) A rectangle has vertices in homogenous coordinates

$$A = \begin{pmatrix} 16 \\ 8 \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} 22 \\ 8 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 22 \\ 5 \\ 1 \end{pmatrix} \quad D = \begin{pmatrix} 16 \\ 5 \\ 1 \end{pmatrix}$$

Find the image of this rectangle under the rotation of the plane through an angle of 45° clockwise about the origin.