DUBLIN INSTITUTE OF TECHNOLOGY

DT211C BSc. (Honours) Degree in Computer Science (Infrastructure)

Year 1

SUMMER EXAMINATIONS 2015/2016

MATHEMATICS 1
[CMPU1018]

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WEDNESDAY 18TH MAY 9.30 A.M. – 11.30 A.M.

TWO HOURS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION 1** AND **TWO** OTHER QUESTIONS.

MATHEMATICAL TABLES AND GRAPH PAPER ARE AVAILABLE.
APPROVED CALCULATORS ALLOWED.

1.

a) Fifty people in a maths class recorded the results in the table below in their mid-term assessment. Using the data given in the table, draw a histogram to record the data.

Mark Range	Frequency
$0 \leq M < 40$	3
$40 \leq M < 50$	10
$50 \leq M < 60$	18
$60 \leq M < 70$	14
$M \geq 70$	5

(5 marks)

b) Use Euclid's algorithm to find $hcf(97020,34650)$.

(5 marks)

c) Let $f: \boldsymbol{N} \to \boldsymbol{N}$ be given by $f(x) = 2x^2 - 4$
Let $g: \boldsymbol{N} \to \boldsymbol{N}$ be given by $g(x) = \sqrt{x^2 + 2}$
Calculate:

i. $(fog)(5)$
ii. $(gof)(-1)$
iii. $(gog)(x)$

(5 marks)

d) Let $A = \{3, x, 4, y, 5\}, B = \{2, x, 7, y, z\}$. List the elements of the following sets:

i. $A \cup B$
ii. $A \cap B$

(5 marks)

e) Find the inverse of the matrix $\begin{pmatrix} 7 & 2 \\ 3 & -3 \end{pmatrix}$. Hence or otherwise, solve the following system of equations:

$$7x_1 + 2x_2 = 21$$
$$3x_1 - 3x_2 = 36$$

(5 marks)

f) Calculate the following modular operations

i. $(6 + 5) mod 8$
ii. $(9 \times 11) mod 15$

(5 marks)

g) Use the properties of logarithms to evaluate the following:

(i) $4log_5(\sqrt{125}) - \frac{1}{2}log_2(8^{\frac{2}{3}}) + log_3\left(\frac{1}{81}\right)$

(ii) $3log_2\left(\frac{1}{32}\right) - 4log_8\left(\frac{1}{512}\right) - 3log_{10}(0.1)$

(5 marks)

h) Find the mean, mode and variance of the following set of data:

$$13, 17, 25, 19, 17, 18, 27, 22$$

(5 marks)
[40 marks]

2.

a) Let $A = \{0, 1, 2, 3\}$ and let R be a binary relation on A given by $R = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$. Verify whether or not R is:

i) Reflexive
ii) Symmetric
iii) Transitive

(10 marks)

b) In a geometric progression (AP) the 3rd term is half the7th term and the sum of the 5th and 6th terms is 39. Determine the common difference, the first term and the sum of the first 12 terms.

(10 marks)

c) Two propositions are given as $\overline{(a \wedge b) \vee c}$ and $\overline{(a \wedge b)} \vee \bar{c}$.

i) Ascertain if the two propositions are logically equivalent by examining their truth tables.

(6 marks)

ii) Construct a combinatorial logic circuit for each proposition.

(4 marks)

[30 marks]

3.

a) Let $A = \begin{pmatrix} 10 & -3 \\ -3 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 & -4 \\ 2 & 8 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -3 & 10 \\ 3 & -6 \\ -4 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 9 & -3 & 7 \\ 12 & 3 & 5 \\ 14 & -5 & -3 \end{pmatrix}$.

Evaluate (if possible) the following:

i) $\frac{1}{3}AB$
ii) DB
iii) $C^T D$, where C^T denotes the transpose of C.

(6 marks)

b)

i) Use the Euclidean algorithm to find $d = hcf(194040, 102900)$.

(6 marks)

ii) Hence or otherwise, find integers s and t such that

$$102900s + 194040t = d.$$

(6 marks)

c) Write out the multiplication table for Z_{13}. Find the multiplicative inverse of $4\ (mod\ 13)$ using Fermat's Little Theorem i.e. in Z_{13}^{*}, and verify your answer from the multiplication table.

(12 marks)

[30 marks]

4.

a) A rectangle given in homogeneous coordinates is represented by the matrix

$$M = \begin{pmatrix} 20 & -10 & -10 & 20 \\ -10 & -10 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

A scaling can be represented by a scaling matrix given by $S = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find the image of the rectangle M given above, under a scaling S with factors $(4, 6)$.

(12 marks)

b) Construct the combinatorial circuit and truth table corresponding to the Boolean expression:

$$\overline{(a \wedge \bar{b})} \wedge (\overline{\bar{a} \vee b})$$

(8 marks)

c) Find Boolean expressions for the two combinatorial circuits given below in Figure 1 and Figure 2. Use a truth table to ascertain if they are equivalent circuits.

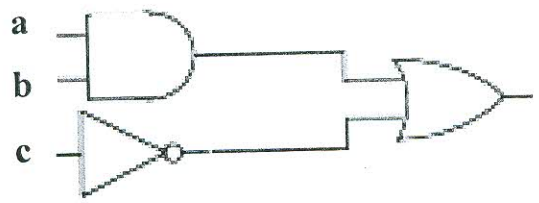

Figure 1

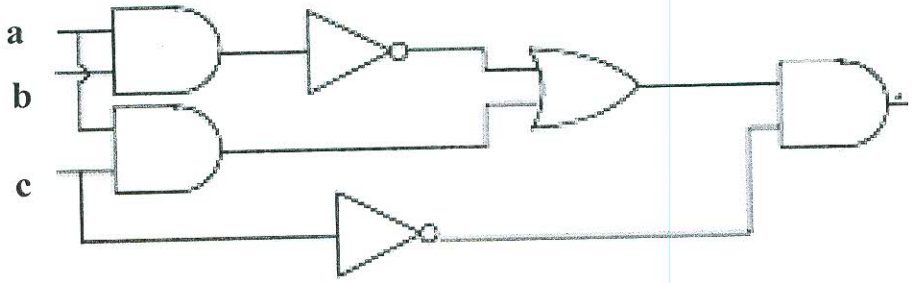

Figure 2

(10 marks)
[30 marks]