KINEMATICS AND DYNAMICS OF PLANAR MACHINERY

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to be found from Eqs. (1)–(3), using the value of e indicated in the table. Verification of these results is left as an exercise (Problems **1.37**-22, 23, and 24).

A rocker-crank mechanism can serve as a **quick-return mechanism**. To see this, note that the rocker tip C in Fig. 1 moves over path C_1CC_2 during the time that the crank tip D moves over path D_1DD_2. If the crank rotates counterclockwise at uniform speed ω, the time for this "forward motion" is

$$t_f = \frac{\theta_2 - \theta_1}{\omega} \tag{6a}$$

The "return motion" (from C_2 to C_1) of the rocker occurs in the interval

$$t_r = \frac{2\pi - (\theta_2 - \theta_1)}{\omega} \tag{6b}$$

The **time ratio** (TR) is therefore given by

$$\mathrm{TR} \equiv \frac{t_f}{t_r} = \frac{\theta_2 - \theta_1}{2\pi - (\theta_2 - \theta_1)} \tag{6c}$$

where θ_1 and θ_2 are found from Eqs. (4) and (5a).

1.33 *Change-Point Configurations*

A type of singularity known as a *change point* can arise when the equality form of Grashof's criterion, Eq. (**1.31**-1), is satisfied; i.e., $s + 1 = p + q$. By definition, *a change-point configuration occurs when all of the bars of the mechanism become collinear.*

Perhaps the most familiar type of *change-point mechanism* is the **parallelogram linkage**, formed by the horizontal coupling rod on locomotive wheels, as shown in Fig. 1(a) by $ABPQ$. When the rod moves into position $P'Q'$ (aligned with the frame AB), the motion becomes kinematically indeterminate. To see this, note that when point Q' on the forward wheel rotates to position Q'' the corresponding point P'' may remain on the parallelogram linkage as in Fig. 1(a), or else the rear wheel may change direction at the change point, so that the link $Q''P''$ "crosses" the frame as shown in Fig. 1(b). The linkage shown in Fig. 1(b) has been called the **antiparallel**

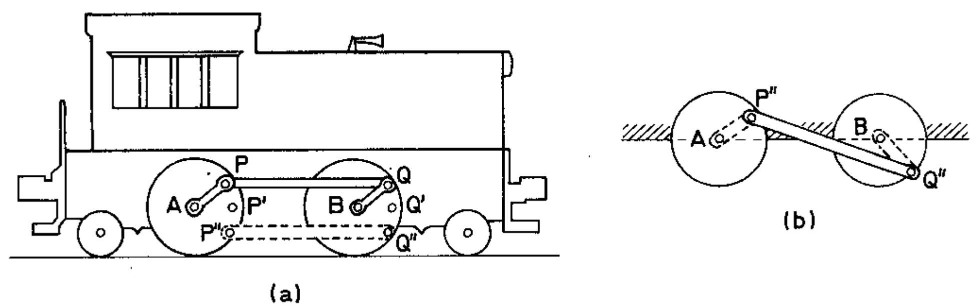

Figure 1.33-1. Change points in parallelogram linkage: (a) Gas turbine locomotive, circa 1954. [After R. T. Sawyer (1972)]; (b) Antiparallel crank (butterfly, bow-tie) linkage.

Verification of

crank linkage by Reuleaux [1876, p. 290], although the shorter terms **butterfly linkage** and **bow-tie linkage** are more descriptive. Whether the mechanism goes into the parallelogram or butterfly arrangement depends on how it is guided through the change point. For example, if the wheels are rotating at a reasonable speed, the *fly-wheel effect* (i.e., inertia torque) could carry it through in the parallelogram mode. On the other hand, a positive kinematic drive can be assured by adding a second connecting rod (perhaps on the other side of the locomotive) whose change point does not coincide with that of the first rod (see Fig. 2).

When a side rod broke on an old-time locomotive, the engineers knew that the opposite rod had to be demounted before the locomotive was to be restarted. Otherwise the good rod might start in the butterfly mode, and either the rod or one of the pins would break. Similar ideas were used to keep one crank of a multicylinder steam engine from starting in the wrong direction if one piston happened to be at dead center upon starting. (See Problems **1.37**-9 and -10 for further examples.)

Another interesting mechanism with change points is the **Galloway mechanism**[11] shown in Fig. 3. This so-called **isosceles** or **kite linkage** has one crank length equal to

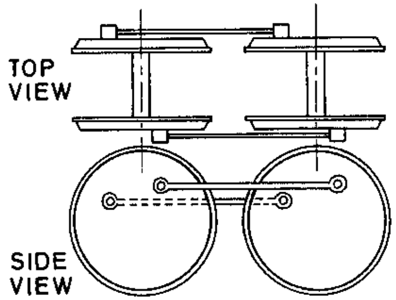

Figure 1.33-2. Guiding a linkage through change-point configuration.

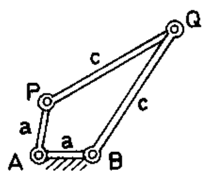

Figure 1.33-3. Galloway mechanism.

the frame length and the other equal to the coupler. The reader should verify that if $a < c$ and if link AP revolves continuously, link AP executes two revolutions for every one of link BQ.

It may be shown (Paul [1979], Dijksman [1976]) that a linkage can be a movable change-point mechanism if, and only if,

$$s + l = p + q \tag{1}$$

1.34 *Transmission Angle*

If AD is the input link in Fig. 1, the force applied to the output link BC is transmitted through the coupler link DC. For sufficiently slow motions (negligible inertia forces) the force in the coupler rod is pure tension or compression (negligible bending action) and is directed along DC. For a given force in the coupler rod, the torque communicated to the output bar (about point B) is a maximum when the angle μ between coupler bar DC and output bar BC is 90°. When μ, the **transmission angle**, deviates significantly from 90°, the torque on the output bar falls off and may not be

[11]Described by Reuleaux [1876, p. 197].