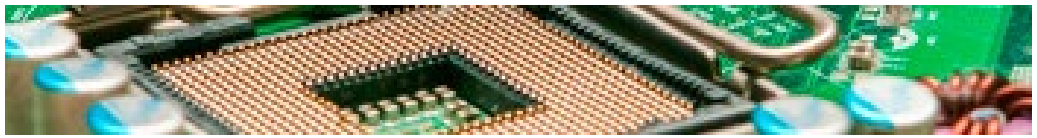
Unit 7

Set covering, set partitioning and set packing problems

Problemas de recubrimiento, particionamiento y empaquetamiento



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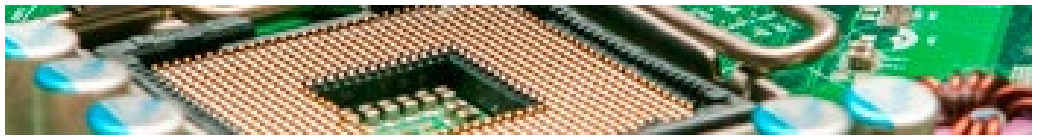
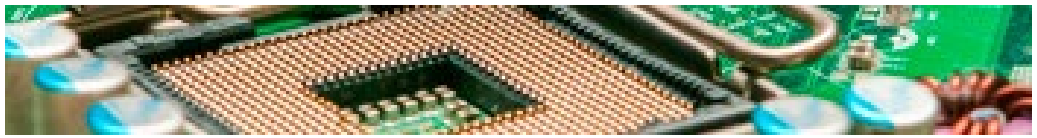


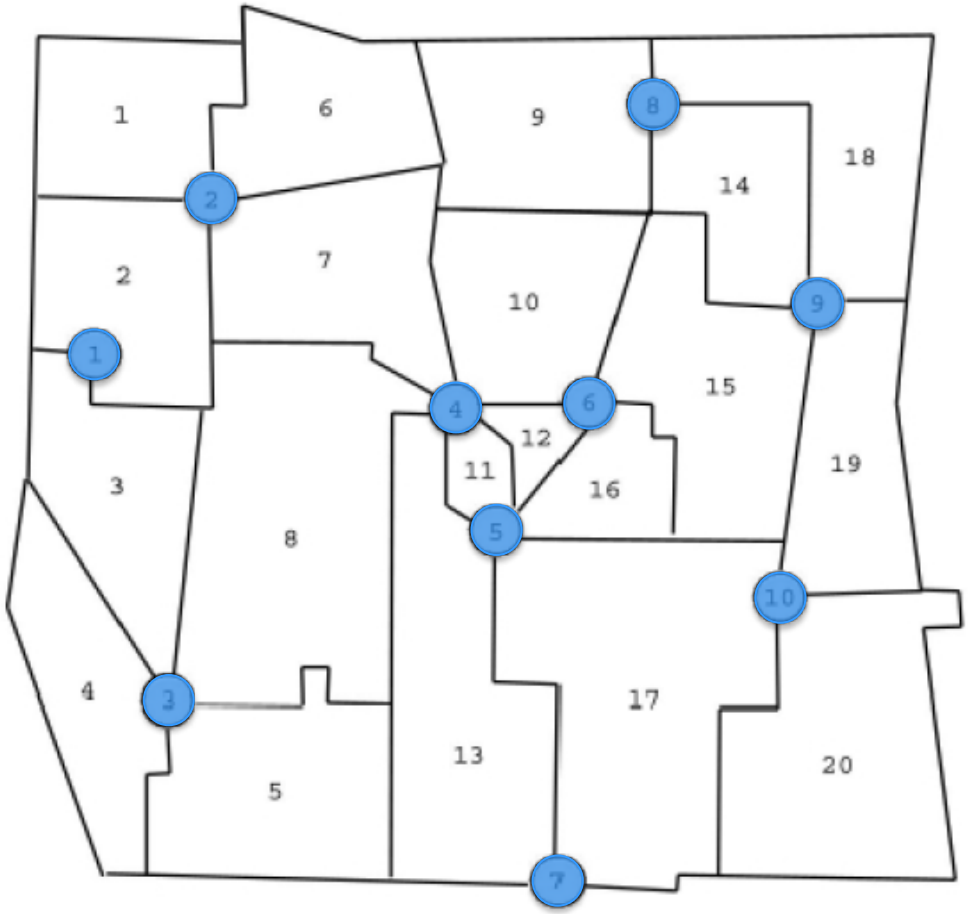
Motivation

- There are problems where we have to find the min number of sets that contain/cover a certain set of elements
  - resources (employees, machines, providers, vehicles, time periods, etc.) to be assigned to a set of tasks (jobs, places, goods, projects, etc.)
  - goal: to find the set of solutions that allows us to cover a second set of resources/necessities
  - usually, it's common to include costs - we call them *weighted* problems

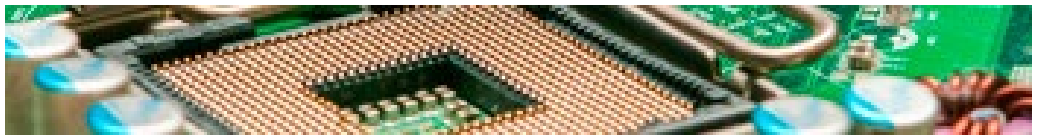


Examples

- A country is divided into **20 states** and there are **10 possible locations** to build new agencies to support all the states. Each agency has a building/maintenance cost and can support (*cover*) several states
- **Goal**: which is the best combination of agencies to cover all the states?



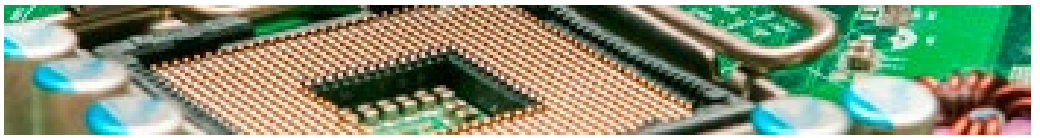
The same state can be covered by more than one agency



Examples

- We have a **project** with a **set of tasks**. Each task has a type and requires a fully qualified person
- There is a **list of candidates**, where each candidate is qualified for one or more task types
- **Goal**: which is the best (cheapest) team of candidates to be involved in the project?

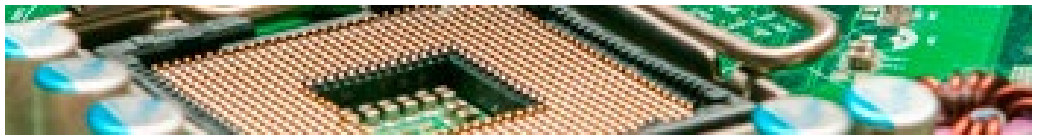
The same task can be covered by more than one candidate



Examples

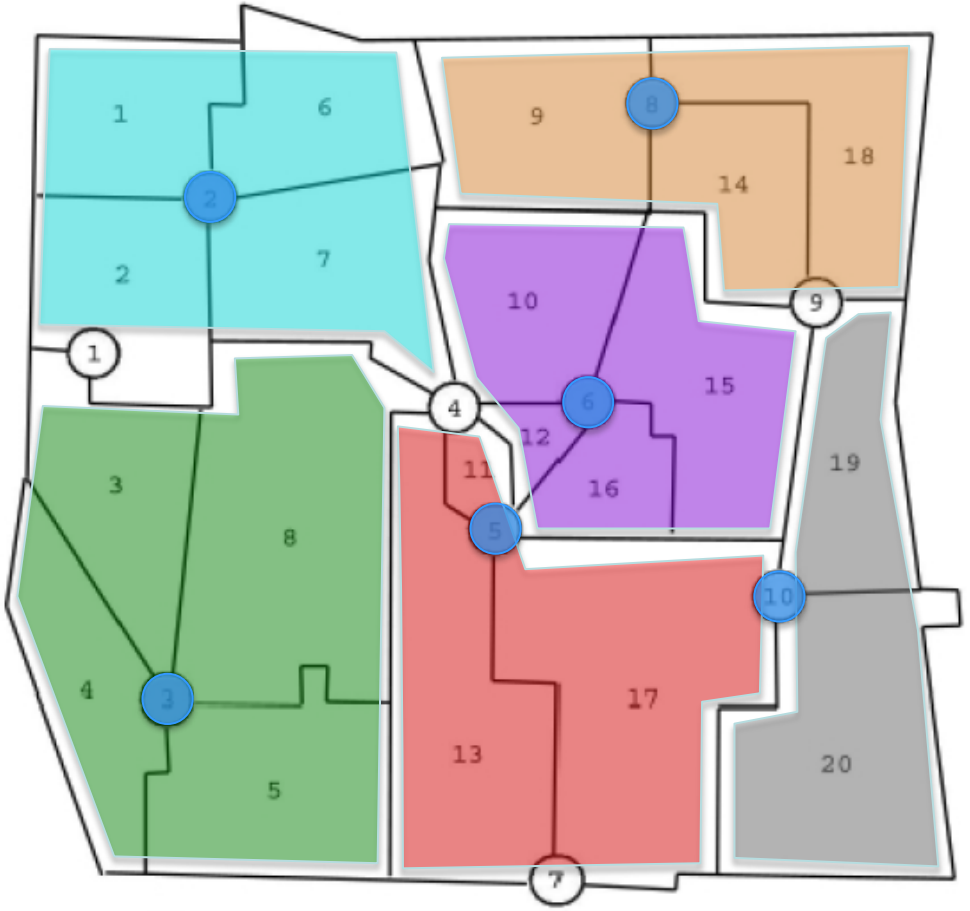
- We have an **assembly line** that needs some (raw) materials
- There are several **providers**, which can provide one, or more, materials. Same materials can be served by different providers at different prices
- **Goal**: which is the best combination of providers for the assembly line?

The same material can be provided by more than one provider

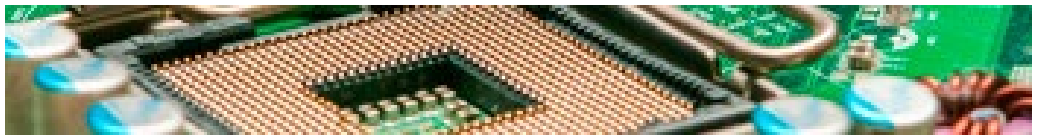


Motivation. A **new** problem/constraint

- In the previous example of **covering states by agencies**, what would happen if we want each state to be covered **just by one** agency?
- We face now a **partitioning problem**



Only one agency per state



Examples

- We have to deliver some orders/goods by using a set of trucks. The goal is to **choose which trucks** to use (we could also need to use all trucks)
  - Obviously, each order can **only** be delivered/transported by **one** truck
  - The order cannot be split into smaller suborders
  - We need to distribute (**partitioning**) all orders among the trucks

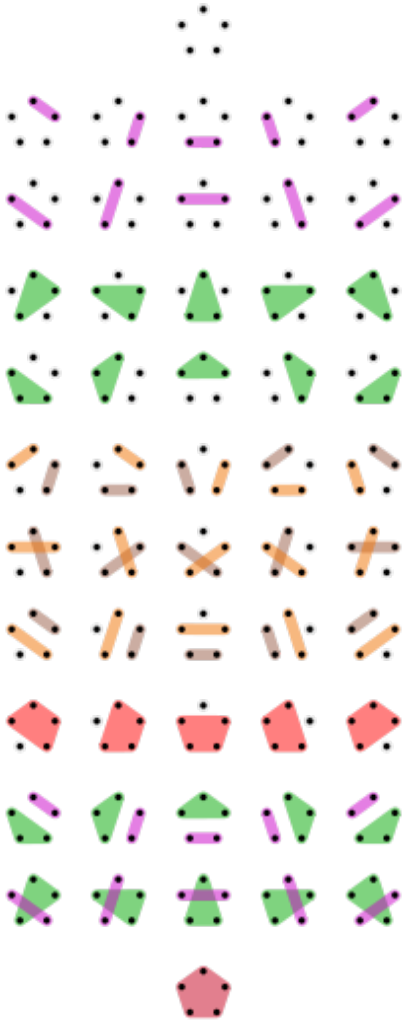
Each order in just one truck

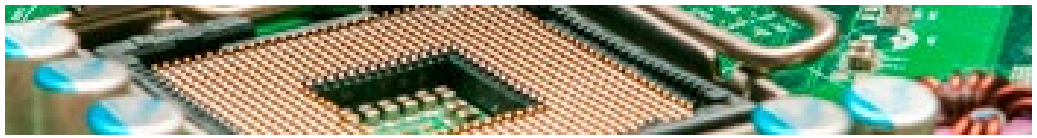
Examples

- We have a staff of **business agents** that have to make **visits** to a set of clients
- All agents have to make visits, but each client has to **receive only one** visit

Similar to the distribution of people among tables in an event (e.g. wedding)

One person cannot be in two partitions

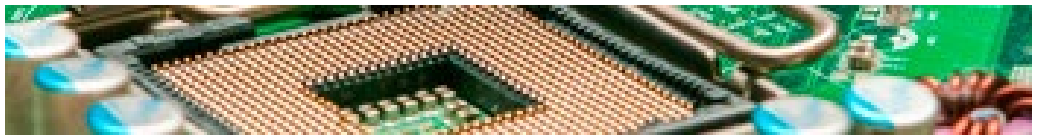




Motivation. A **new** problem/constraint

- There are several assembly lines for **different products**. Each product needs **different pieces** and the set of pieces is **limited**
- We want to assemble the best combination of products
- We face now a **packing problem**

Perhaps not all products are covered (i.e. there could be not enough pieces for all products)

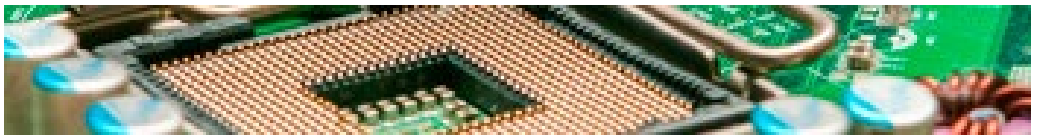


Examples

- We have a **limited capacity** of processing (delivering, manufacturing, raw material availability, etc.) and a set of tasks (orders, clients, servers, pieces, etc.) to support, which also give us a benefit
- **Goal**: find the processing combination that provides the highest benefit

Which is the best combination of items?

Not all the tasks need to be always fulfilled

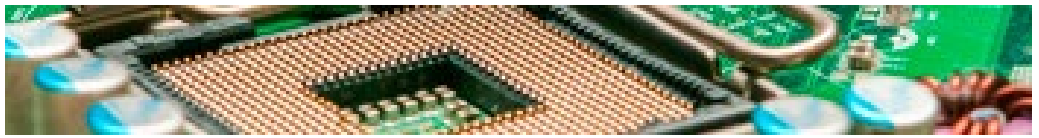


Examples

- We have a sheet metal roll with a **given size** and we have to cut some pieces with different shapes and benefits
- **Goal**: which is the max quantity of pieces that can be cut, and under which configuration, to minimise the metal waste?

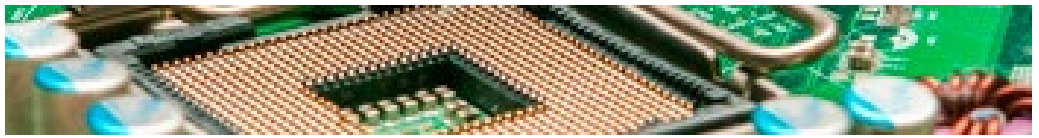
Plasma cutting process

Some pieces can remain uncut



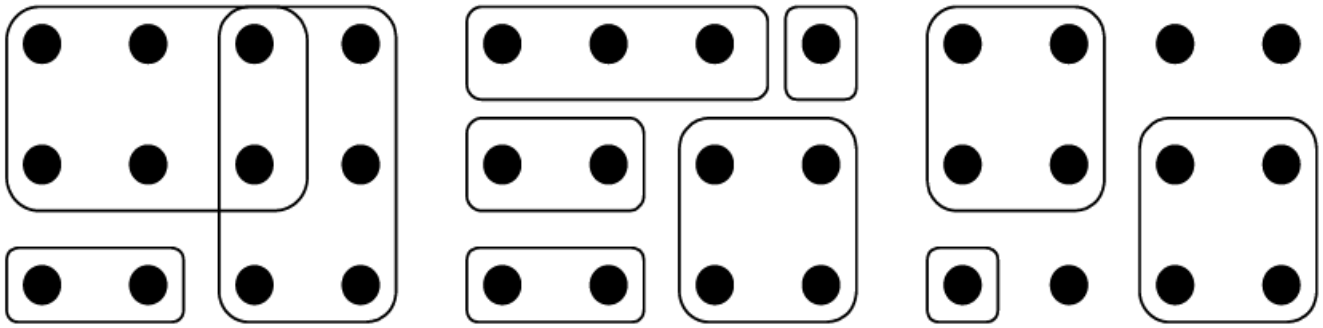
Examples

- In general, any problem to find the max benefit (or min waste) by combining a set of limited items, subject to some hard or soft constraints



In a problem of…

- Covering, all items have to be in **at least one** partition
- Partitioning, all items have to be in **one and only one** partition
- Packing, all the items have to be in **zero or one** partition (**one** partition **at most**)



References

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