BLADING IN TURBINES

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References:
Engineering Thermodynamics, Work & Heat Transfer By Rogers and Mayhew, 4th Edition, ISBN 0-582-04566-5.
Mechanics of Fluids, By Bernard Massey, Seventh Edition, ISBN 0-7487-4043-0.

Learning Outcomes

- Introduce the various types of fluid machines
- Understand the energy transfer in passages between blades
- Learn how to obtain the velocity diagrams for the flow between blades and how to use these diagrams to obtain the power output / input.
- Discuss the various types of axial turbine stages to highlight their performance characteristics, pressure, velocity and enthalpy variation, losses, efficiency and application.
    - Impulse stage
    - Reaction stage
    - Two row velocity compounded stages
- Overview of Computational Fluid Dynamics Capabilities for developing Turbines

Fluid Machines

A device for converting the energy held by a fluid into mechanical energy or vice versa.

Turbines

- A machine in which energy from the fluid is converted directly to the mechanical energy of a rotating shaft.

Pumps

- A machine where the energy is transferred from a rotating shaft or moving parts to the fluid.

Compressors

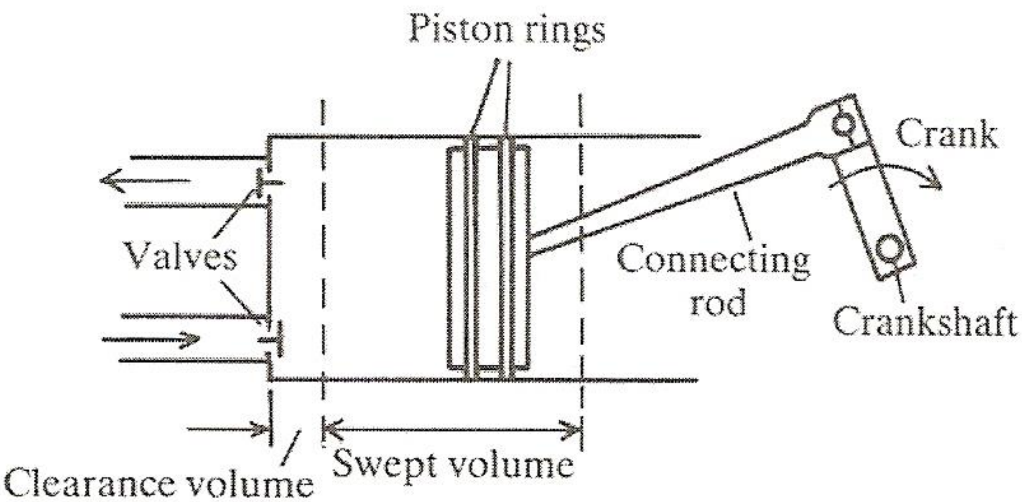
- When the fluid is gas and the primary objective is to increase the pressure of the gas, the machine is termed a compressor.

Fans

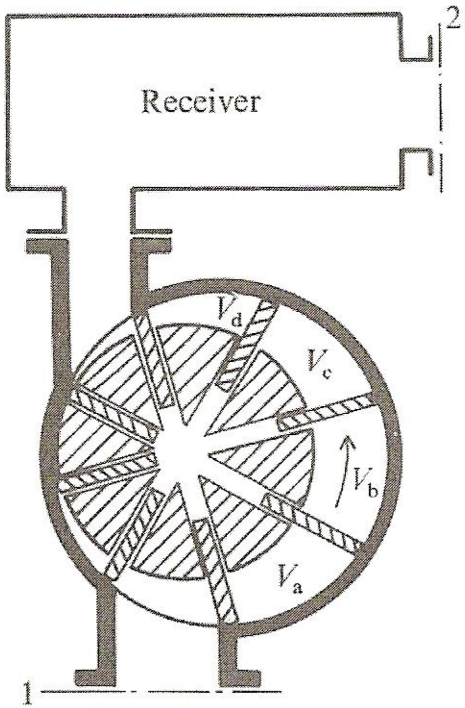
- A machine used primarily for causing the movement of a gas.

Fluid Machines

Positive –Displacement Machines such as reciprocating pumps and engines, diaphragm pumps and Gear pumps.



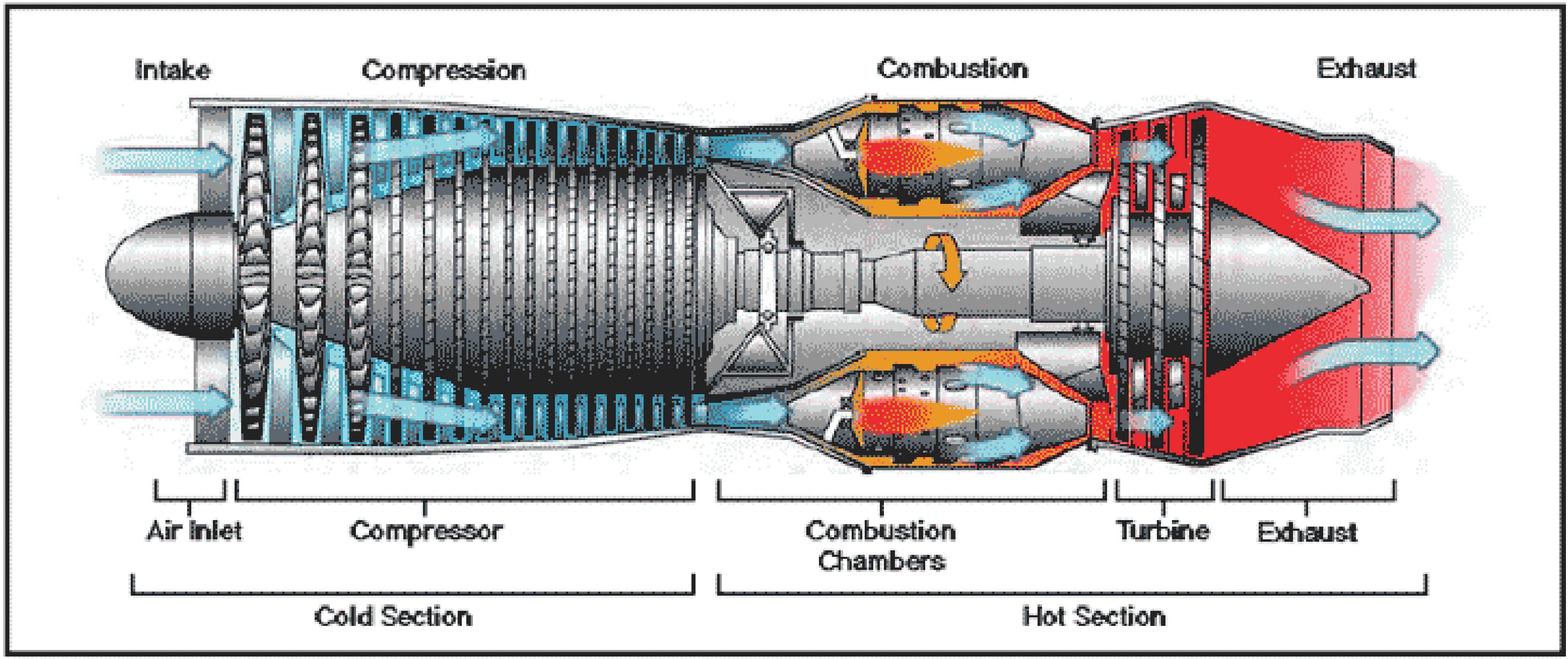
The Reciprocating Expander or Compressor



Rotary Vane Compressor

https://www.youtube.com/watch?v=4OJTN0M1DBk

Rotodynamic machines



All these machines have a rotating shaft to which blades are attached. These blades are generally known as the rotor. In the passages between the blades, the fluid flows with its velocity having a component tangential to the rotor and so a momentum component in the same direction. The rate at which this tangential momentum is changed corresponds to a tangential force on the rotor.

ADVANTAGES OF ROTODYNAMIC MACHINES OVER POSITIVE DISPLACEMENT ONES

•**Flow from most positive displacement machines is unsteady,** whereas for normal conditions of operation that from rotodynamic machines is essentially steady.

•**Most positive displacement machines require small clearances** between moving and stationary parts, and so are unsuited to handling fluids containing solid particles in general, rotodynamic machines are not restricted in this way.

•Dealing with a given overall rate of flow, a rotodynamic machine is usually less bulky than one of positive displacement type.

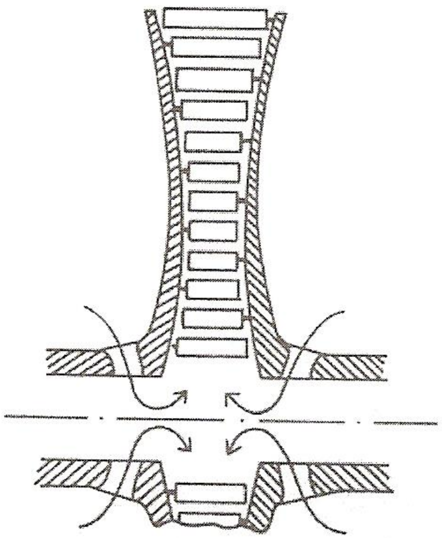
•While the operation of positive displacement machines depends **only on mechanical and hydrostatic principles,** the operation of rotodynamic machines depends on **principles of fluid mechanics**.

•Rotodynamic machines require relative motion between the fluid and the moving rotor of the machines.

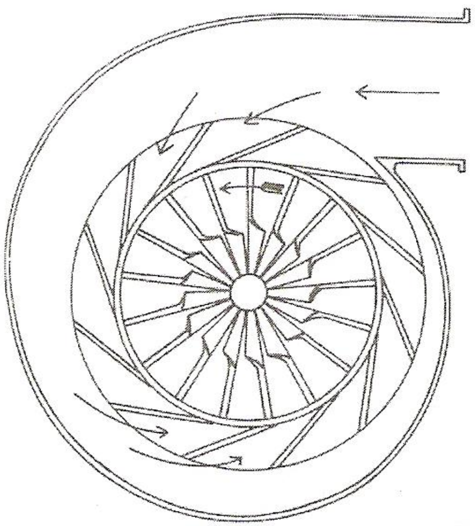
ROTODYNAMIC MACHINES

•**Radial Flow Machines**

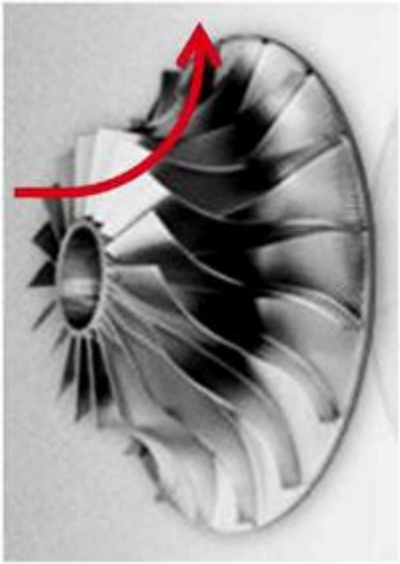
The fluid path is wholly or mainly in the plain of rotation; the fluid enters the rotor at one radius and leaves it at a different radius. Examples: Francis Turbine and Centrifugal pumps.



Outward Radial Machine



Inward Radial machine



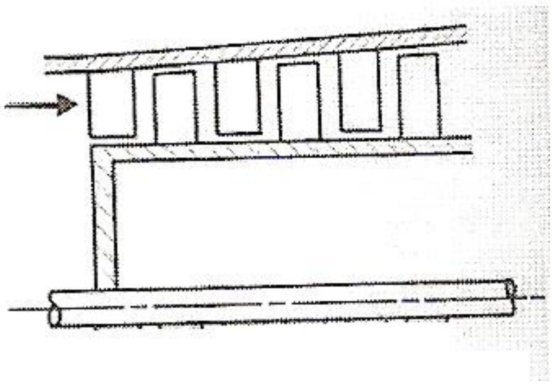
https://www.youtube.com/watch?v=Vhc-hEjh12I

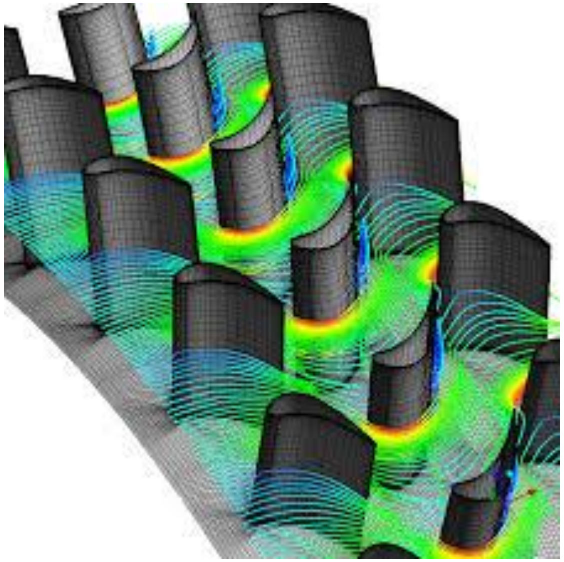
ROTODYNAMIC MACHINES

- Axial Flow Machines

The fluid main flow direction is parallel to the axis of rotation, so that any fluid particle passes through the rotor at a practically constant radius. Examples: Kaplan turbine, axial flow pumps.

Axial Flow Machines are most commonly used for turbines and compressors in Power Generation Plants.

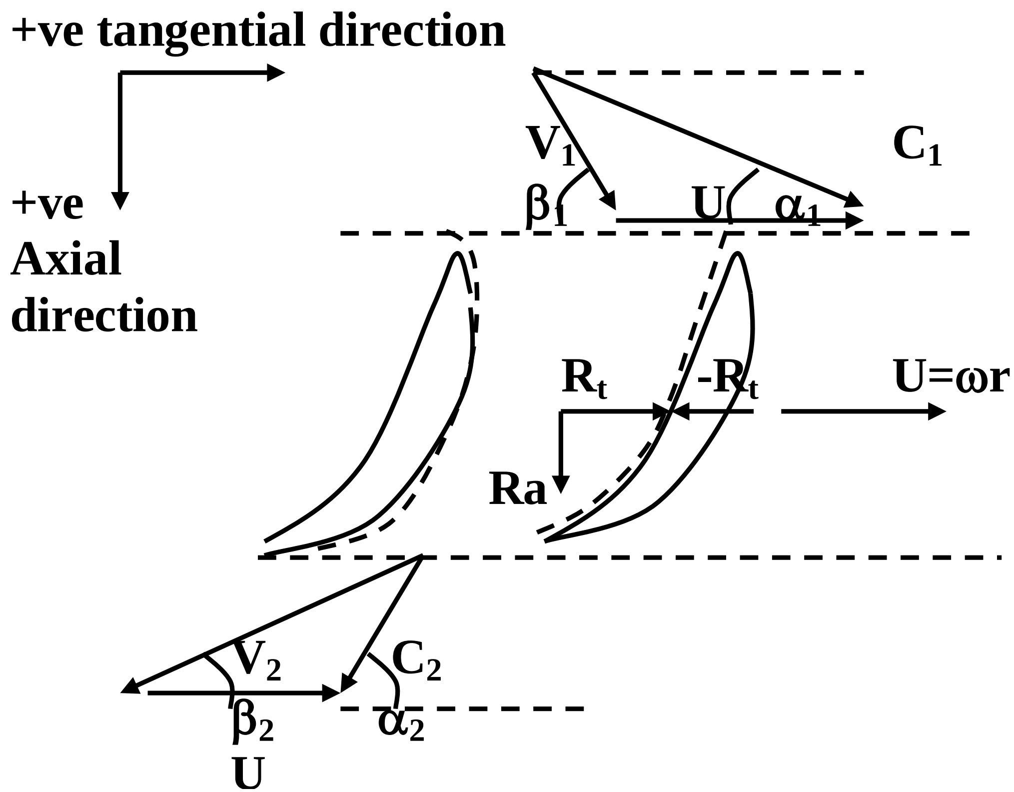




•Mixed Flow Machines

The fluid flow is partly radial and partly axial.

FLOW THROUGH BLADE PASSAGES



C: Absolute velocity: velocity detected by a stationary observer
V: Relative Velocity: velocity detected by an observer moving with the blades.
U: The velocity of the blades, U = ωr
ω : rotational speed of the blade (rev/sec), r = radius at mean height of the blade
α : angle between absolute velocity vector and the tangential direction
β : angle between relative velocity vector and the tangential direction

Conservation of Momentum Principle

For steady flow, the net force on the fluid in a control volume equals the net rate at which momentum flows out of the control volume, the force and momentum having the same direction. This is a statement of Newton's second law of motion applied to fluids. Momentum of fluid flow is equal to mass flow rate multiplied by its velocity.

Therefore for a control volume surrounding the fluid passing between two blades, the net force acting on the fluid is equal to the momentum of fluid leaving the control volume minus the momentum of fluid entering the control volume. This force could be analysed in tangential and axial directions. However, only the tangential component is important here as the reaction to this force component acts on the blades to produce power output.

Force on fluid in tangential direction = momentum of fluid leaving the control volume in the tangential direction – momentum of fluid entering the control volume in the tangential direction.

Conservation of Momentum Principle

In the tangential direction:

$$-R_t = \dot{m}(-C_2\cos\alpha_2 - C_1\cos\alpha_1)$$

$$R_t = \dot{m}(C_2\cos\alpha_2 + C_1\cos\alpha_1) = \dot{m}\Delta C_t$$

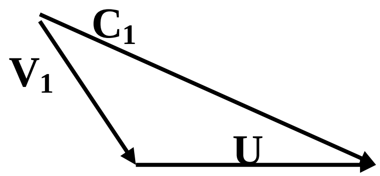
If R_t is positive then momentum is transferred from the fluid to the blades, this is the case of turbines.
If R_t is negative, then momentum is transferred from the blades to the fluid, this is the case of pumps and compressors.

The work output per unit time = Power = Force * Velocity

$$\boxed{\begin{aligned} Power &= R_t * U \\ \dot{W} &= \dot{m}U\Delta C_t \end{aligned}}$$

Velocity Triangles

Velocity triangles are used to relate velocity vectors at inlet and exit from the control volume. They facilitate vector addition and subtraction.



$$\vec{C}_1 = \vec{V}_1 + U \quad \text{or} \quad \vec{V}_1 = \vec{C}_1 - U$$

TURBINES

For any turbine, the energy held by the fluid is initially in the form of pressure.

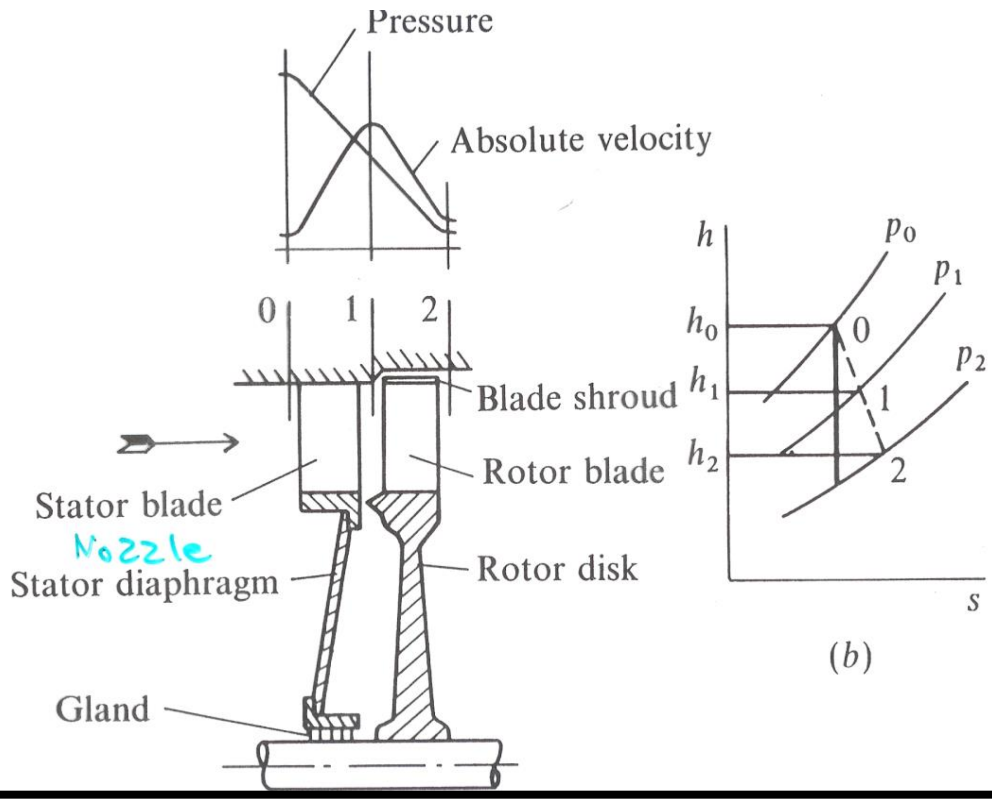
•In water turbines, the water comes from a high level reservoir and thus the pressure is due to the potential head of several meters.

•In steam turbines, pressure is produced by supplying heat in the boiler.

•In gas turbines, pressure is produced by fuel combustion.

AXIAL FLOW TURBINES

- Consists of one or more stages, each stage comprising one annulus of fixed stator blades (Nozzles) followed by one of moving blades.
- Usually the total pressure drop across the stage is divided between the stator and the rotor blades. However, the division is normally expressed in terms of the enthalpy drop and known as the degree of reaction Λ.

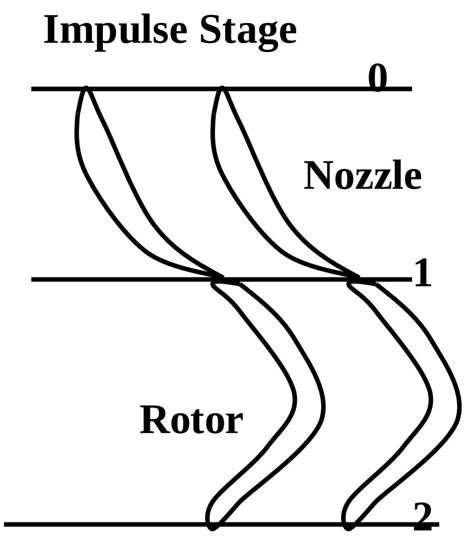


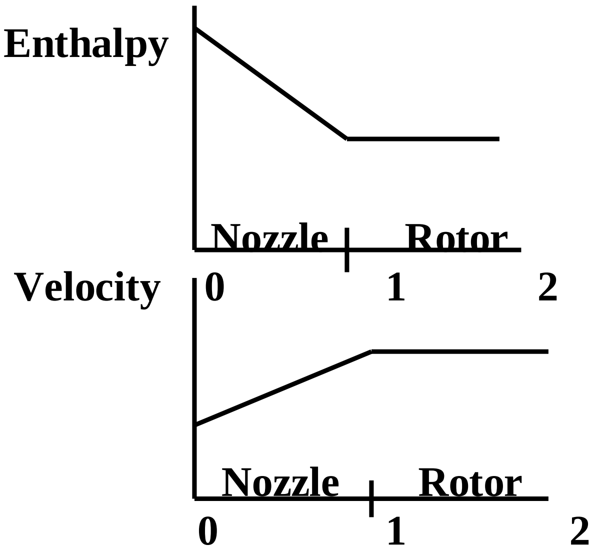
A stage in an axial flow turbine

Degree of Reaction

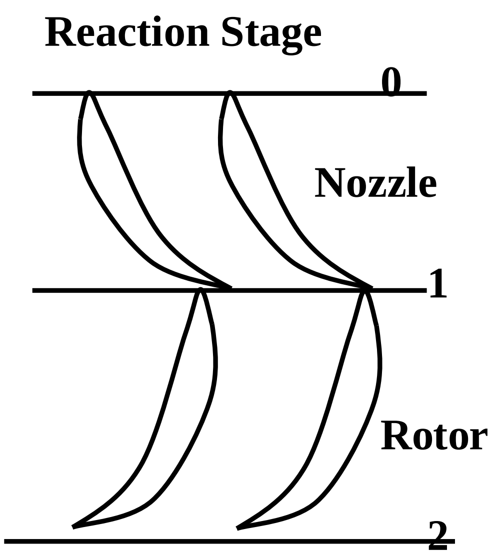
$$A = \frac{\text{Enthalpy drop in the rotor blades}}{\text{Enthalpy drop in the whole stage}} = \frac{h_1 - h_2}{h_0 - h_2}$$

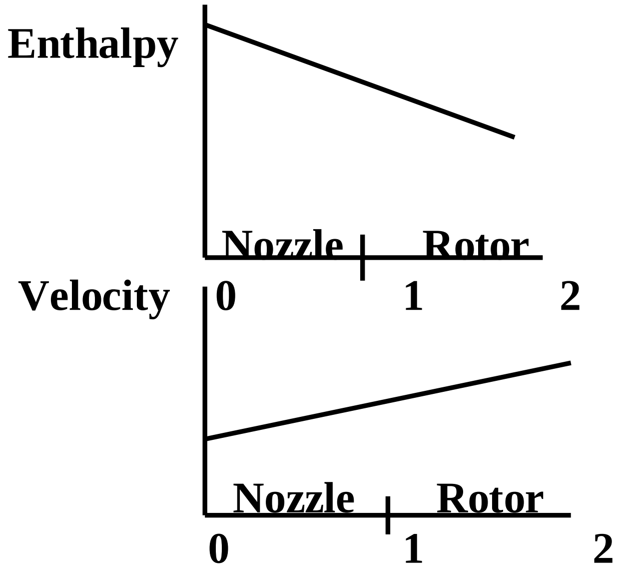
When A = 0, all the stage enthalpy and pressure drop occur in the nozzle blades and the stage is called an Impulse Stage where the fluid enters the rotor at maximum speed.





When A=50%, the pressure drop across the nozzle and the rotor blades are approximately equal. <u>This is referred simply as reaction stage.</u>





Rotor blades have approximately the same shape as nozzle blades. Acceleration of the fluid in the rotor blades makes an equal contribution to the pressure drop.

STAGE VELOCITY DIAGRAM

The fluid enters the stage (NOZZLE Blades) with velocity C_0, an angle α_0 and pressure P_0 and is expanded in the NOZZLE blades to P_1.

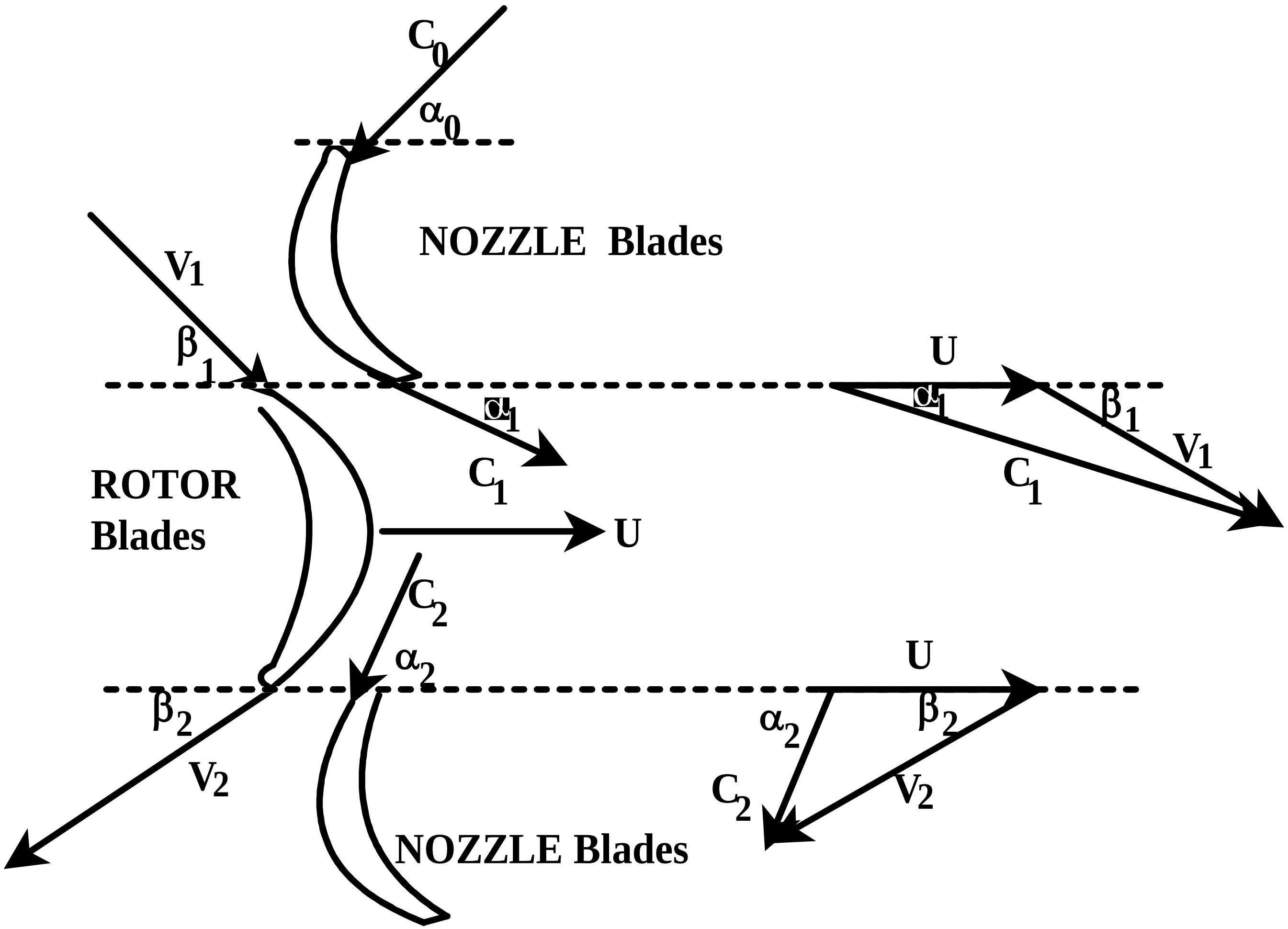
It leaves the NOZZLE Blades with a velocity C_1 making an angle α_1. The fluid velocity relative to the moving blades V_1 is determined by subtracting the blade velocity U from C_1 (Velocity Triangle). V_1 makes an angle β_1.

If the fluid is to flow smoothly from the NOZZLE Blades to the ROTOR Blades, the inlet angles of the ROTOR Blades must be made approximately equal to β_1. Also, if the angle of ROTOR Blades at outlet is β_2, then the direction of relative velocity V_2 is also β_2.

The effect of the ROTOR Blade is to change the velocity of the fluid from C_1 to C_2. Thus if the flow rate is m, the rate of change of momentum in the tangential direction is:

$$\dot{m}(C_{2t} - C_{1t})$$

STAGE VELOCITY DIAGRAM

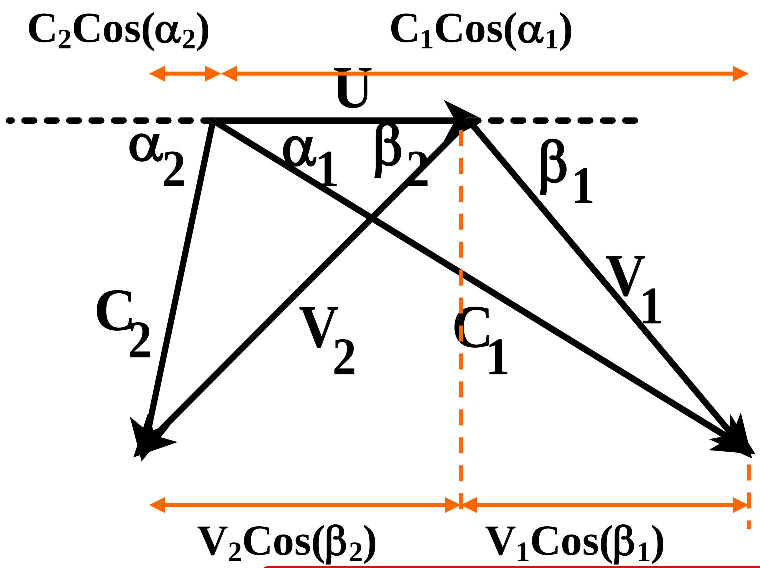


This will produce a force on the blades as: $$\dot{m}(C_{1t} - C_{2t})$$

The power produced is equal to:

$$\dot{W} = \dot{m}U(C_{1t} - C_{2t})$$

$$\dot{W} = \dot{m}U(C_1 \cos\alpha_1 + C_2 \cos\alpha_2)$$



Since vector U appears in both velocity triangles, we can combine them in one diagram as:

Using the velocity triangles

$$\mathrm{C_1}\cos\alpha_1 - \mathrm{U} = V_1 \cos\beta_1 \quad \text{and} \quad C_2 \cos\alpha_2 + U = V_2 \cos\beta_2$$

$$V_1 \cos\beta_1 + V_2 \cos\beta_2 = C_1 \cos\alpha_1 + C_2 \cos\alpha_2$$

$$\Delta V_t = \Delta C_t \qquad Thus \qquad \dot{W} = \dot{m}U\Delta C_t = \dot{m}U\Delta V_t$$

Steady Flow Energy Equation for the NOZZLE Blades results:

$$\boxed{h_0 - h_1 = \frac{C_1^2 - C_0^2}{2}}$$

(Q and W are Zero)

The NOZZLE isentropic efficiency is given by:

$$\boxed{\eta_{is} = \frac{\Delta h}{\Delta h_{is}} = \frac{h_0 - h_1}{(h_0 - h_1)_{is}}}$$

$C_{0,}$ the velocity at inlet to the Nozzle blades is small compared to C_1. Therefore, it can be neglected.

For the ROTOR Blades, we can apply the steady flow energy equation using the relative velocities (relative to the moving blades). The work output is still zero since no work is done relative to the moving blades:

$$\boxed{h_1 - h_2 = \frac{V_2^2 - V_1^2}{2}}$$

Or using the absolute velocity:

$$\boxed{\dot{W} = \dot{m}\left((h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} \right)}$$

TURBINE AS A WHOLE

$$\dot{W} = \dot{m} U \Delta C_t = \dot{m} \Delta h$$

The chief aim in the design of a turbine is to utilise the available enthalpy drop with the least possible friction losses and in the smallest number of stages.

The work done per stage increases with the blade speed U. However, the centrifugal stresses in the rotating parts limit the maximum permissible blade speed.

Stage Diagram Efficiency

A useful criterion which indicates the effectiveness of energy abstraction in a stage is the diagram efficiency (sometimes called utilisation factor):

$$\eta_d = \frac{\text{Work calculated from velocity diagram}}{\text{Energy available to the ROTOR}}$$

The diagram efficiency is an idealised term. In practice there will be friction and possibly leakage losses. Also a leaving loss occurs if $C^2/2$ is large for the last stage.

For an IMPULSE stage, the Energy available for the ROTOR Blades is $\dot{m}C_1^2/2$

For the REACTION Stage this is: $$\dot{m}\frac{C_1^2}{2} + \dot{m}\frac{V_2^2 - V_1^2}{2}$$

The second term is energy made available by the expansion in the ROTOR Blades.

IMPULSE STAGE

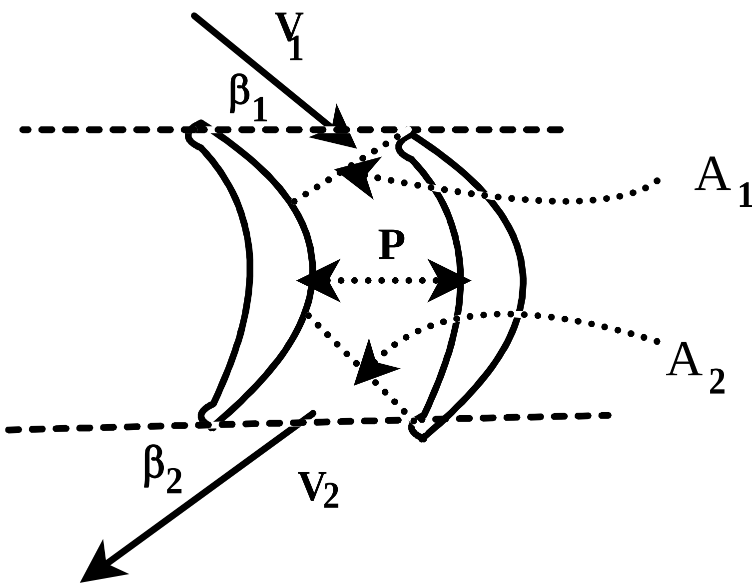
Nominally all enthalpy drop occurs in the NOZZLE Blade row.

With No enthalpy drop in the ROTOR Blades row, there would be no change in the relative velocity.

$$\boxed{\begin{aligned} h_1 - h_2 &= \frac{V_2^2 - V_1^2}{2} = 0.0 \\ Thus \qquad V_2 &= V_1 \end{aligned}}$$

--------------------Important

Also with the absence of friction losses (ideally) there would be no change in pressure or specific volume across the ROTOR Blades, and the Continuity equation then requires the area at right angles to the direction of flow to be the same at inlet and outlet.



ROTOR BLADES IN AN IMPULSE STAGE

At inlet $A_1 = L_1\, n\, p \sin\beta_1$ **At outlet $A_2 = L_2\, n\, p \sin\beta_2$**

where: n = no. of blades, p = pitch between the blades, L = height of the blades.

If the blades height is constant, then for A_1 to be equal to A_2, the inlet angle β_1 should be equal to the outlet angle β_2. This is why the ROTOR Blade is symmetrical.

Assuming $\beta_1 = \beta_2 = \beta$, The velocity diagram becomes:

$$\dot{W} = \dot{m}U(V_1\cos\beta_1 + V_2\cos\beta_2)$$

$$V_1 = V_2 \qquad \text{and} \qquad \beta_1 = \beta_2 = \beta$$

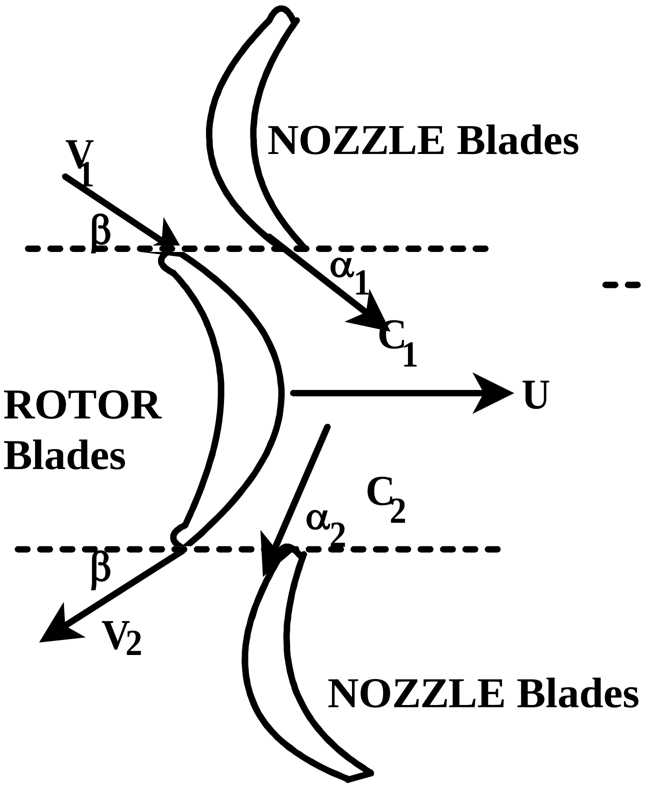
$$\dot{W} = 2\dot{m}UV_1\cos\beta$$

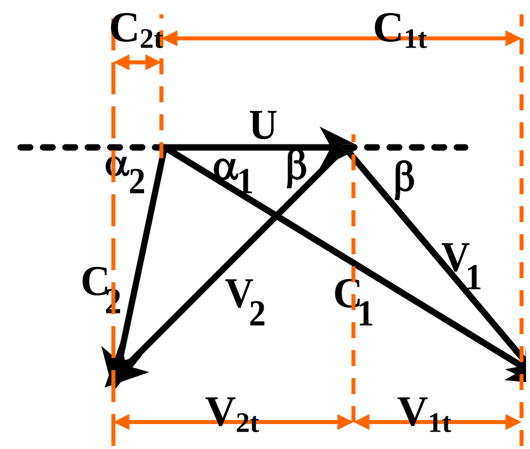
But

$$V_1\cos\beta = C_1\cos\alpha_1 - U$$

Thus

$$\dot{W} = 2\dot{m}U(C_1\cos\alpha_1 - U)$$





The diagram efficiency is

$$\boxed{\begin{aligned}\eta_d &= \frac{\dot{W}}{\frac{1}{2}\dot{m}C_1^2} = \frac{2\dot{m}U(C_1\cos\alpha_1 - U)}{\frac{1}{2}\dot{m}C_1^2} \\ \eta_d &= 4\frac{U}{C_1}[\cos\alpha_1 - \frac{U}{C_1}]\end{aligned}}$$

This is will be maximum when

$$\boxed{\begin{aligned}&\frac{\partial\eta_d}{\partial(U/C_1)} = 0.0 \\ &i.e \\ &\cos\alpha_1 - \frac{U}{C_1} + \frac{U}{C_1}(-1) = 0.0 \\ &\frac{U}{C_1} = \frac{1}{2}\cos\alpha_1\end{aligned}}$$

The corresponding diagram efficiency is $\boxed{\eta_d = \cos^2\alpha_1}$

and corresponding power output is $\boxed{\dot{W} = 2\dot{m}U^2}$

VELOCITY COMPOUNDED IMPULSE STAGE

Why ??

The pressure at inlet to a steam turbine can be very high (e.g. 160 bar). At this range of pressure, the density is very high. This will lead to problems with small flow areas i.e. short blade lengths especially for low demand range.

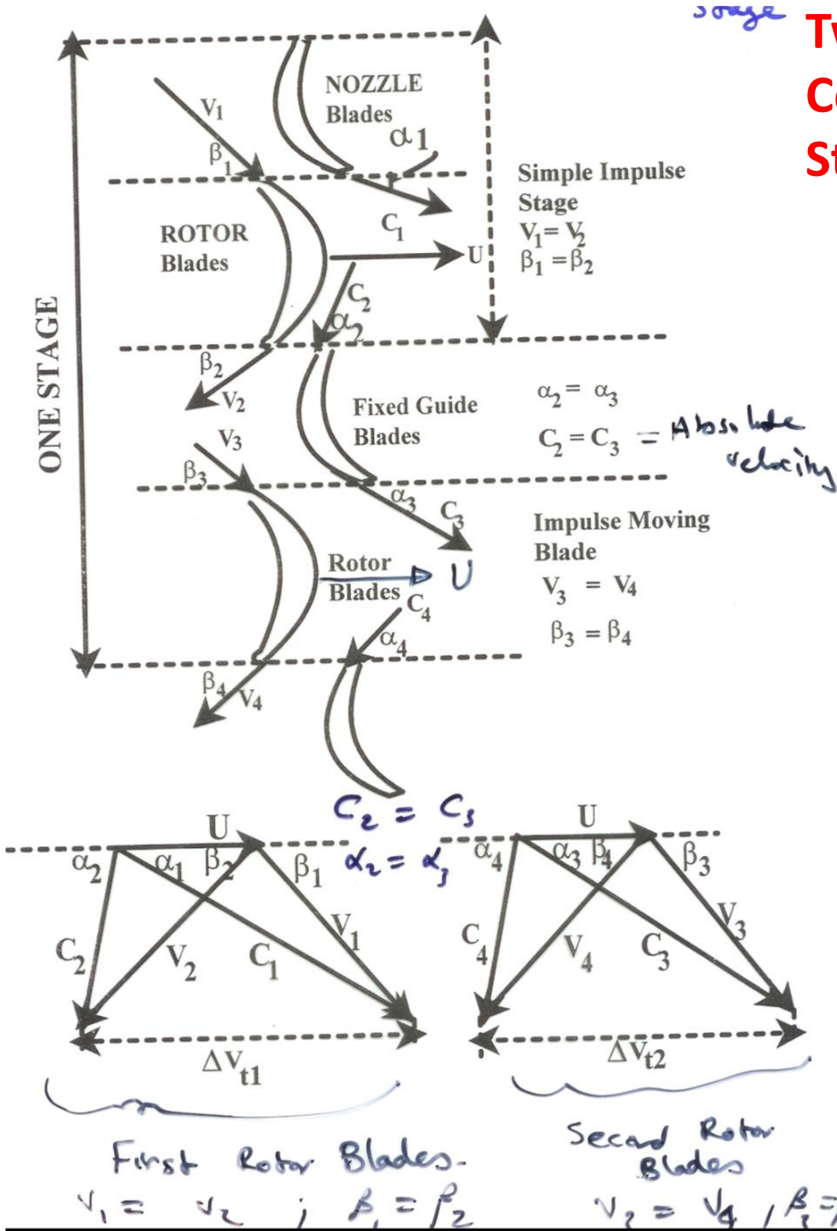
Also, sometimes it is desirable to carry out a large enthalpy drop in one stage, in which case the fluid velocities $C_1 \approx \sqrt{0.5(h_0 - h_1)}$ **are very high (1645m/s) and owing to the blade speed limitations, the U/C_1 ratio will be low and the work output is consequently a low fraction of the available enthalpy drop**

$$\eta_d = 4\frac{U}{C_1}(\cos\alpha_1 - \frac{U}{C_1})$$

The work output can be increased by velocity compounding even though still using a low blade speed ratio.

There is an advantage in having a large expansion in the first Nozzle, giving a lower temperature and higher specific volume when the flow meets the first row of turbine blades.

Two Row Velocity Compounded Impulse Stage



How it works??

The flow through the Nozzles and the first moving row is similar to the flow through the simple IMPULSE stage, except that the ratio U/C_1 is much smaller.

The fluid leaves the first row of ROTOR blades with a high velocity C_2 at an angle α_2 and enters a row of fixed guide blades having an inlet angle α_2. No further expansion takes place and these Blades merely turn the stream into the direction required for entry to a second row of ROTOR Blades. i.e. $C_3 = C_2$.

The guide blades are often made symmetrical and the outlet angle α_3 equal to the inlet angle α_2 and the blade height will be the same at inlet and outlet to give equality of areas.

The geometry of the velocity triangles then fixes the magnitude V_3 and direction β_3 of the relative velocity at inlet to the second moving blades row. If these blades are also symmetrical, β_4 is equal to β_3 and $V_4 = V_3$.

The work output is then given by: $$\boxed{\dot{W} = \dot{m}U(\Delta V_{t1} + \Delta V_{t2})}$$

It can be shown that, the diagram efficiency for the stage is maximum when: $$\boxed{\frac{U}{C_1} = \frac{1}{4}\cos\alpha_1}$$

at which the maximum efficiency is given by: $$\boxed{\eta_d = \cos^2\alpha_1}$$

same as the simple impulse stage.

and the maximum power is given by: $$\boxed{\dot{W} = 8\dot{m}U^2}$$

FOUR (NOT TWO) times that given by the simple impulse stage.

REACTION STAGE

Complex to analyse unless further assumptions are made. The following assumptions are particularly useful:

(a) Velocities at inlet and outlet of the stage are equal.

Thus: $C_0 = C_2$ **and** $\alpha_0 = \alpha_2$

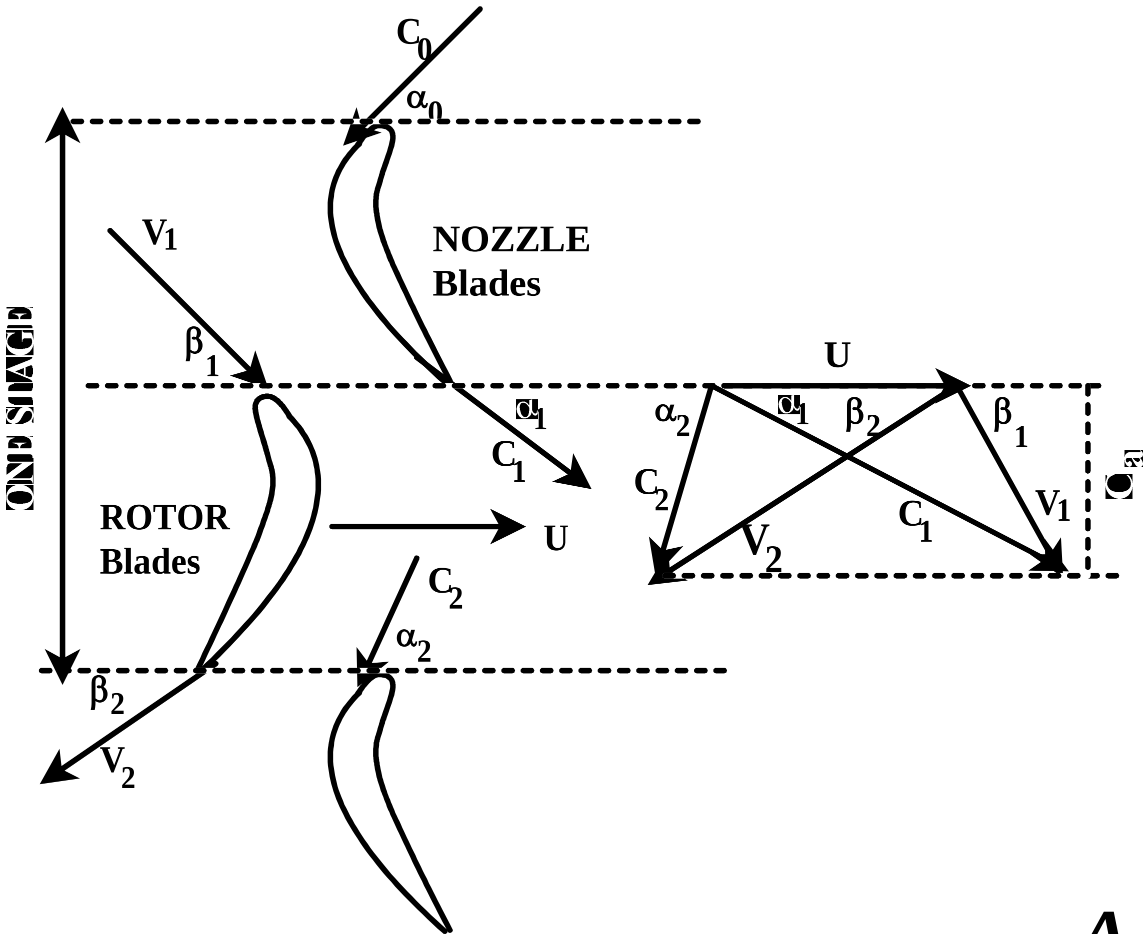
(b) Axial velocities throughout the stage remains constant.

$$C_a = C_0 \sin\alpha_0 = C_1 \sin\alpha_1 = C_2 \sin\alpha_2$$

$$= V_1 \sin\beta_1 = V_2 \sin\beta_2$$

Both of the above assumptions are commonly used in the design of reaction turbines because they enable the same blade shape to be used in successive stages.

REACTION STAGE



The general definition of degree of reaction is:

$$A = \frac{h_1 - h_2}{h_0 - h_2}$$

REACTION STAGE

$$\boxed{w = (h_0 - h_2) = U\Delta C_t = U\Delta V_t}$$

$$\boxed{\Delta V_t = \mathrm{C_a}(\cot an\beta_1 + \cot an\beta_2)}$$

Also

$$h_1 - h_2 = \frac{1}{2}(V_2^2 - V_1^2)$$

$$= \frac{1}{2}C_a^2[\cos ec^2\beta_2 - \cos ec^2\beta_1]$$

$$= \frac{1}{2}\mathrm{C_a^2}(\mathrm{cotan}^2\beta_2 \ \text{-}\ \mathrm{cotan}^2\beta_1)$$

Substituting in the definition of the degree of reaction results:

$$\boxed{A = \frac{C_a}{2U}(\cot an\beta_2 - \cot an\beta_1)}$$

For A = 0.0 i.e. Impulse Stage $\beta_1 = \beta_2$

For A = 0.5 we have reaction stage with:

$$\boxed{U / C_a = (\cot an\beta_2 - \cot an\beta_1)}$$

From the velocity triangles:

$$\boxed{\begin{aligned} U &= C_a (\cot an\alpha_1 - \cot an\beta_1) \\ &= C_a (\cot an\beta_2 - \cot an\alpha_2) \end{aligned}}$$

Comparing the two equations shows that:

$\alpha_1 = \beta_2$ **and** $\alpha_2 = \beta_1$

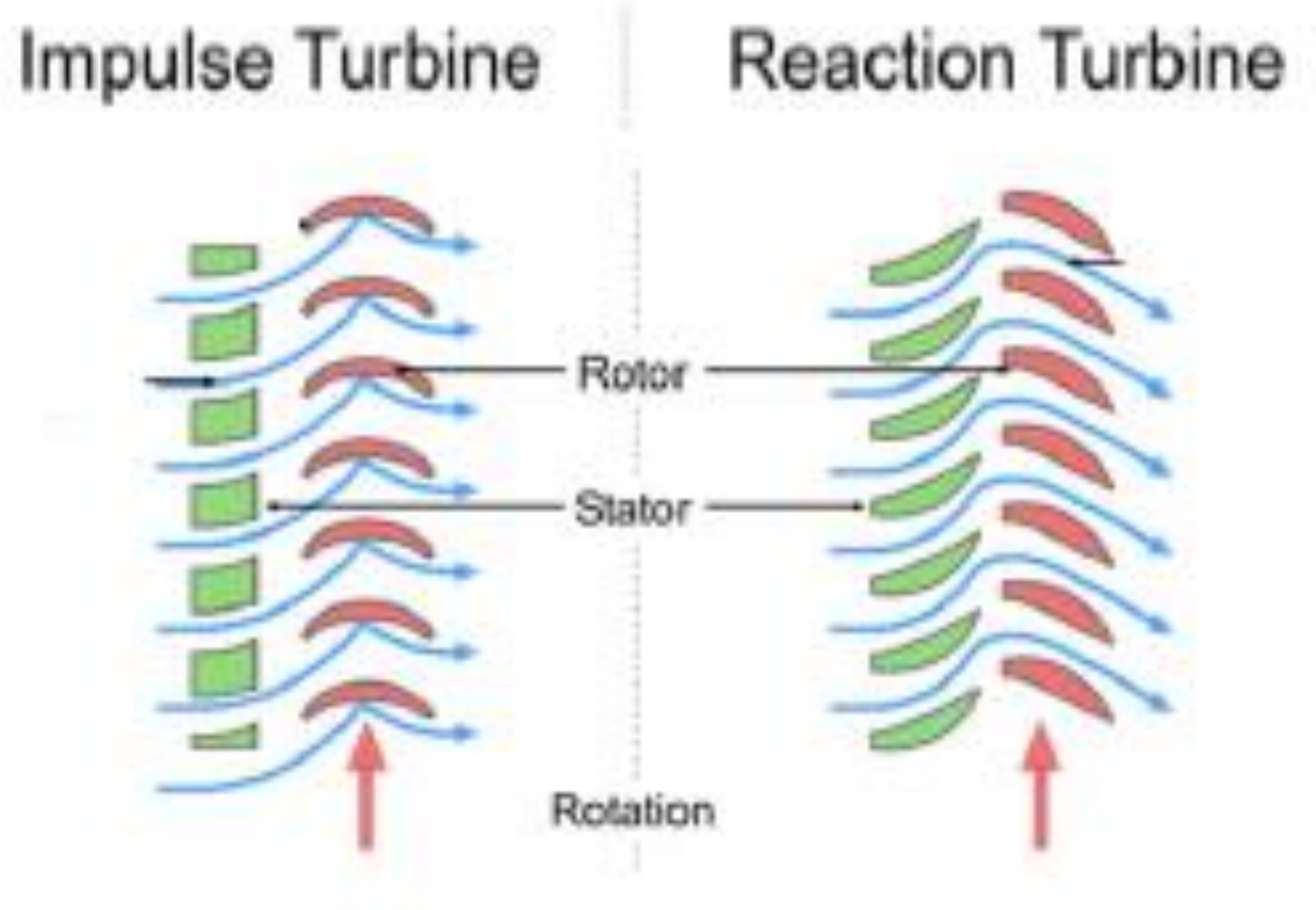
Hence $V_1 = C_2$ **and** $V_2 = C_1$

Also from initial assumptions: $\alpha_0 = \alpha_2$

And $C_0 = C_2$

Evidently for a 50% degree of reaction design, the NOZZLES Blades and the ROTOR Blades have the same shape and the velocity diagram is symmetrical.

Impulse Turbine Blade vs Reaction Turbine Blade



The optimum blade speed ratio for the reaction stage can be found as follows:

$$\dot{W} = \dot{m}U(\Delta V_t)$$

$$\dot{W} = \dot{m}U(V_1 \cos\beta_1 + V_2 \cos\beta_2)$$

$$But \quad V_1 \cos\beta_1 = C_1 \cos\alpha_1 - U$$

$$and \quad \mathrm{V}_2 = C_1 \quad and \quad \beta_2 = \alpha_1$$

$$Then \;\; \dot{W} = \dot{m}U(2C_1 \cos\alpha_1 - U)$$

The maximum energy available for the ROTOR blades is:

$$\dot{W}_{\max} = \dot{m}[\frac{C_1^2}{2} + \frac{V_2^2 - V_1^2}{2}]$$

$$Since \;\; V_2 = C_1 \text{ This becomes}$$

$$\dot{\mathrm{W}}_{\max} = \dot{m}[C_1^2 \; - \; \frac{1}{2} V_1^2]$$

$$Also \;\; \mathrm{V}_1^2 = C_1^2 + U^2 - 2UC_1 \cos\alpha_1$$

$$Then$$

$$\dot{W}_{\max} = \dot{m}[C_1^2 - U^2 + 2UC_1 \cos\alpha_1]$$

Substituting in the general definition of the diagram efficiency results:

$$\eta_d = \frac{\dot{m}U(2C_1\cos\alpha_1 - U)}{\frac{1}{2}\dot{m}(C_1^2 - U^2 + 2UC_1\cos\alpha_1)}$$

$$\eta_d = \frac{2\frac{U}{C_1}(2\cos\alpha_1 - U)}{1-(\frac{U}{C_1})^2 + 2(\frac{U}{C_1})\cos\alpha_1}$$

Differentiate and equate to zero to find the optimum blade speed ratio:

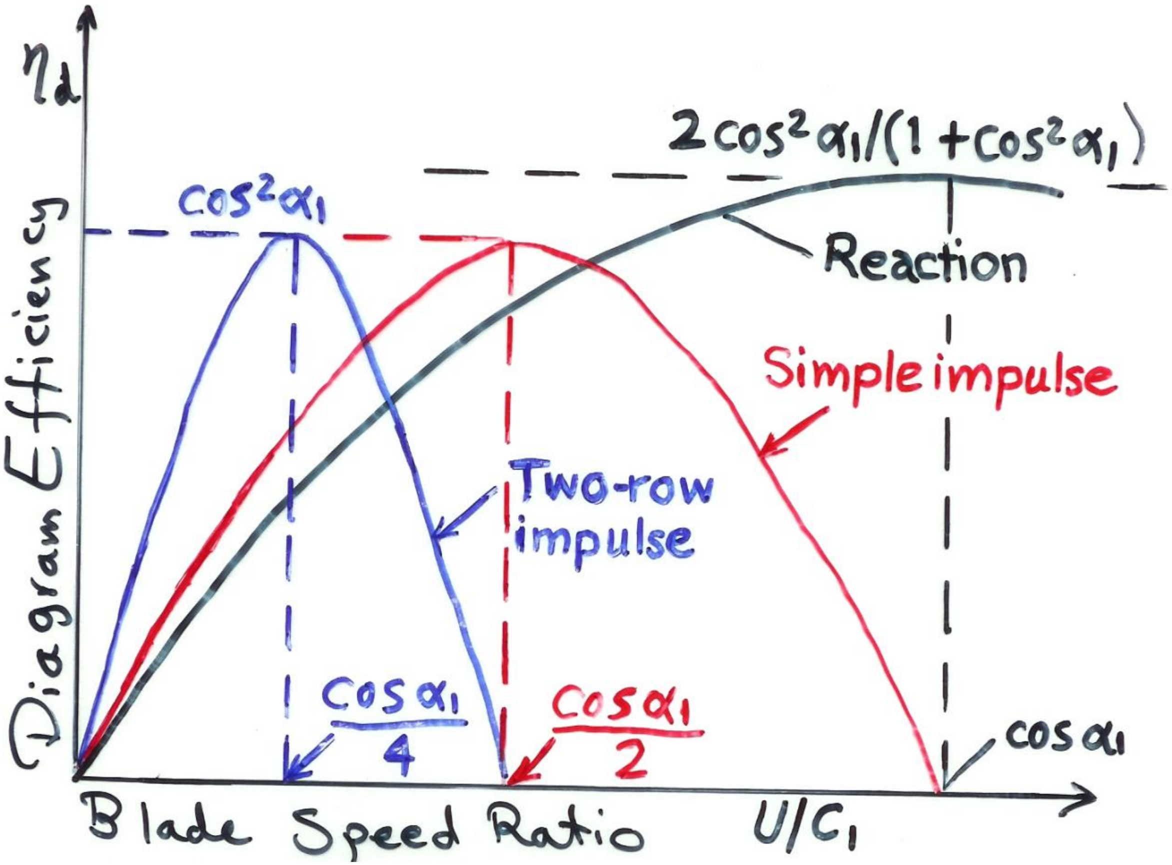
$$\frac{U}{C_1} = \cos\alpha_1$$

$$\dot{W} = \dot{m}U^2$$

$$\eta_{d\,\max} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$$

Summary

Turbine Type	Optimum Speed Ratio	Maximum diagram Eff.	Maximum power output
Impulse	$1/2 \cos\alpha_1$	$\cos^2\alpha_1$	$2mU^2$
Two Row Velocity Compounded Impulse	$1/4 \cos\alpha_1$	$\cos^2\alpha_1$	$8mU^2$
Reaction	$\cos\alpha_1$	$\dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$	mU^2



Two issues to consider:

•The range of U/C_1 which gives maximum efficiency for each type of stage.

When the optimum blade speed ratio is used, which type of turbine stage gives the maximum energy output.

Losses

External losses

(1) Bearing friction.

(2) The power required to drive auxiliaries.

These do not affect the selection of turbine blading.

Internal losses

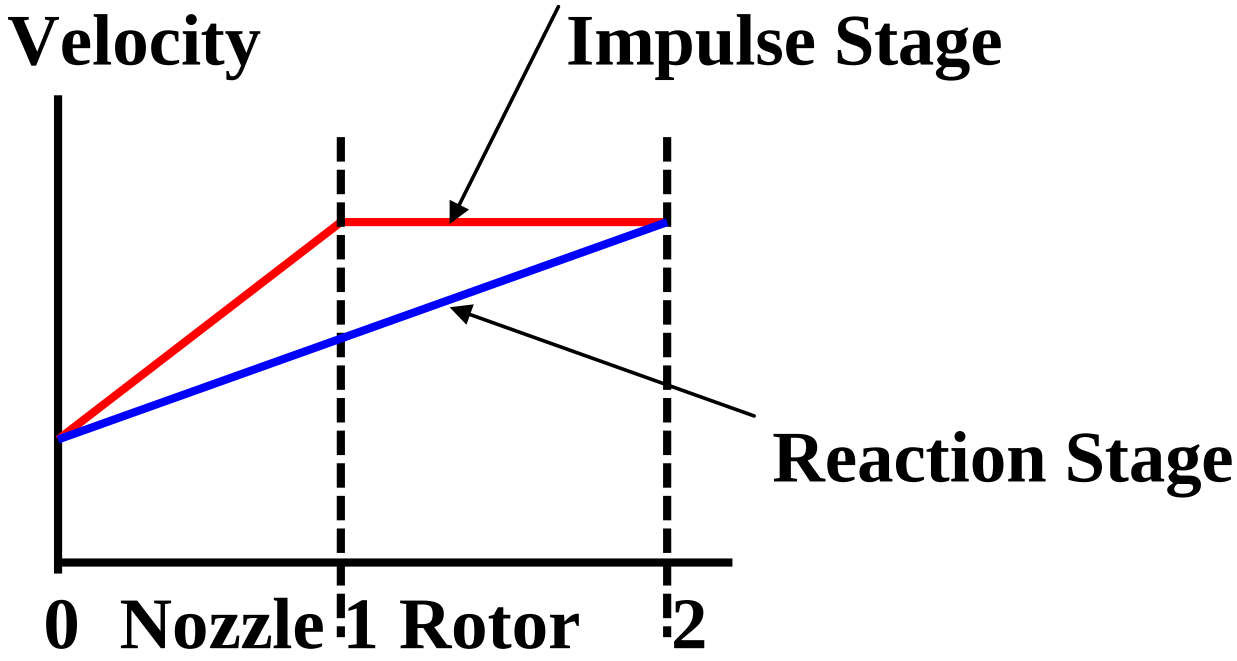
These do affect the selection of blading and they are Blading Friction Losses and Leakage Losses.

(1) Blading Friction Losses $\Delta P = \frac{4fL}{d}\frac{\rho V^2}{2}$

* Square of The Relative Velocity
* Size of Flow Passage
* Nature of Flow i.e. Laminar or Turbulent

(2) Leakage Losses

* Pressure Gradient
* Clearance / Construction Type



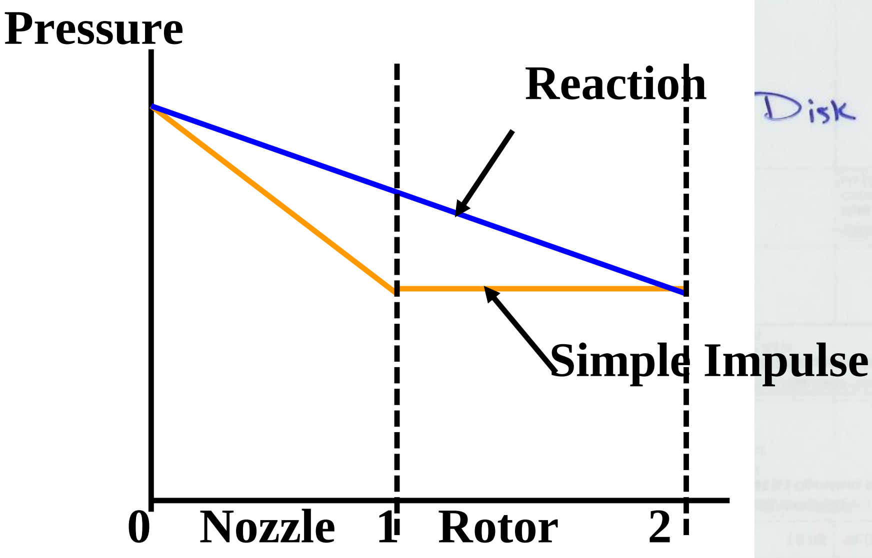
In Reaction Turbines, the average relative velocity is smaller than in the Impulse turbines.

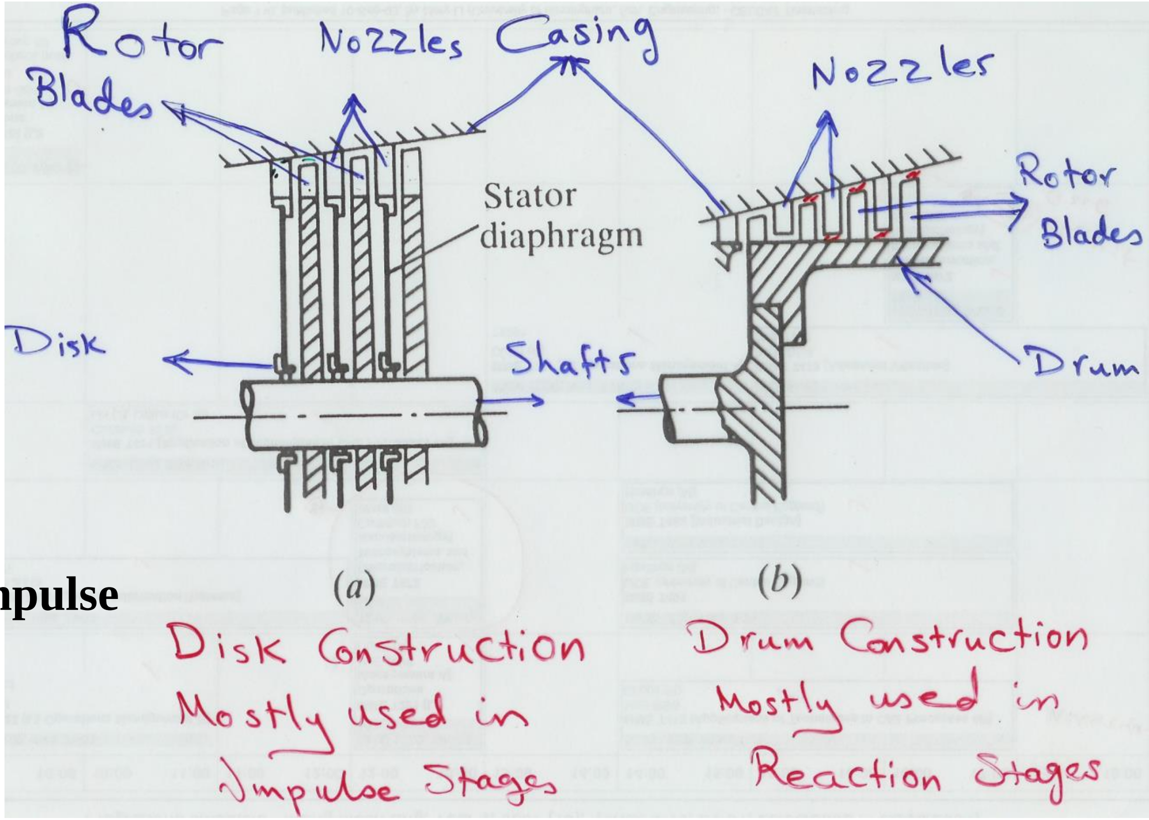
Also, the accelerating flow in Reaction turbines (due to pressure drop in the Rotor) is more stable and Laminar boundary layer develop over great part of the blade surface. This will result in lower friction coefficient.

Thus reaction turbines benefit from both lower average velocity and lower friction factor.

(b) Leakage Losses

Because clearance is needed between the moving and stationary parts, some fluid passes through the turbine without doing its full complement of work on the blading. <u>Since they involve a form of throttling, they contribute to the irreversible increase of entropy.</u>





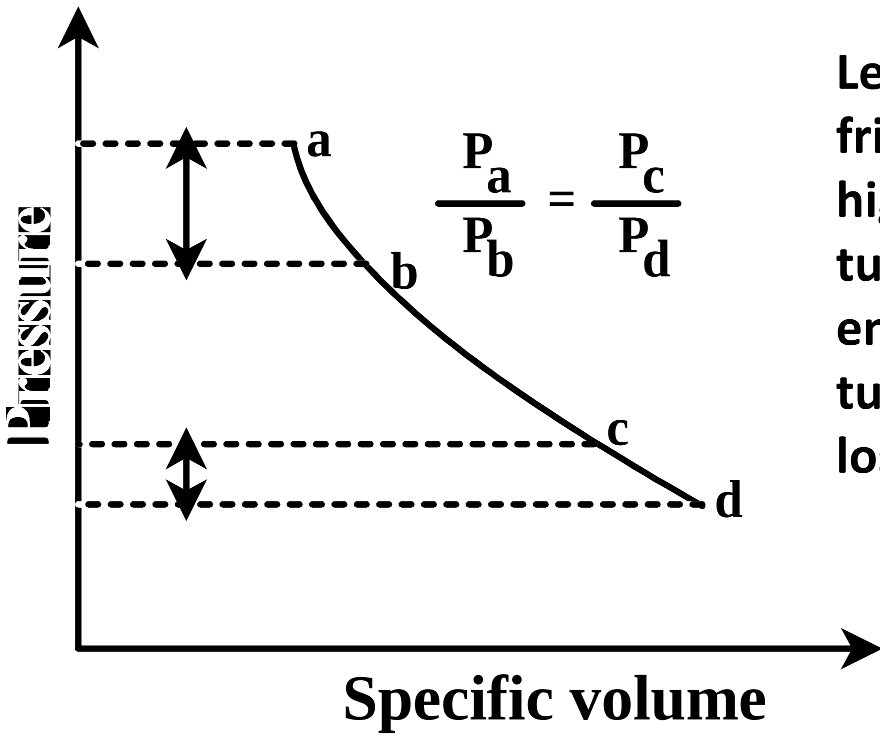
The leakage past the tips of the rotor blades will clearly be greater in a reaction stage, owing to the pressure drop which is absent in any impulse design.

In general, the total leakage losses are rather greater in a reaction design than in an impulse design. This is particularly true at the high-pressure end of a turbine where owing to the high density of fluid, the annulus area required is small and the tip clearance is a relatively large proportion of the blade height.

Summary

(a) Friction losses are least for a reaction design.
(b) Leakage losses are least for an Impulse design.

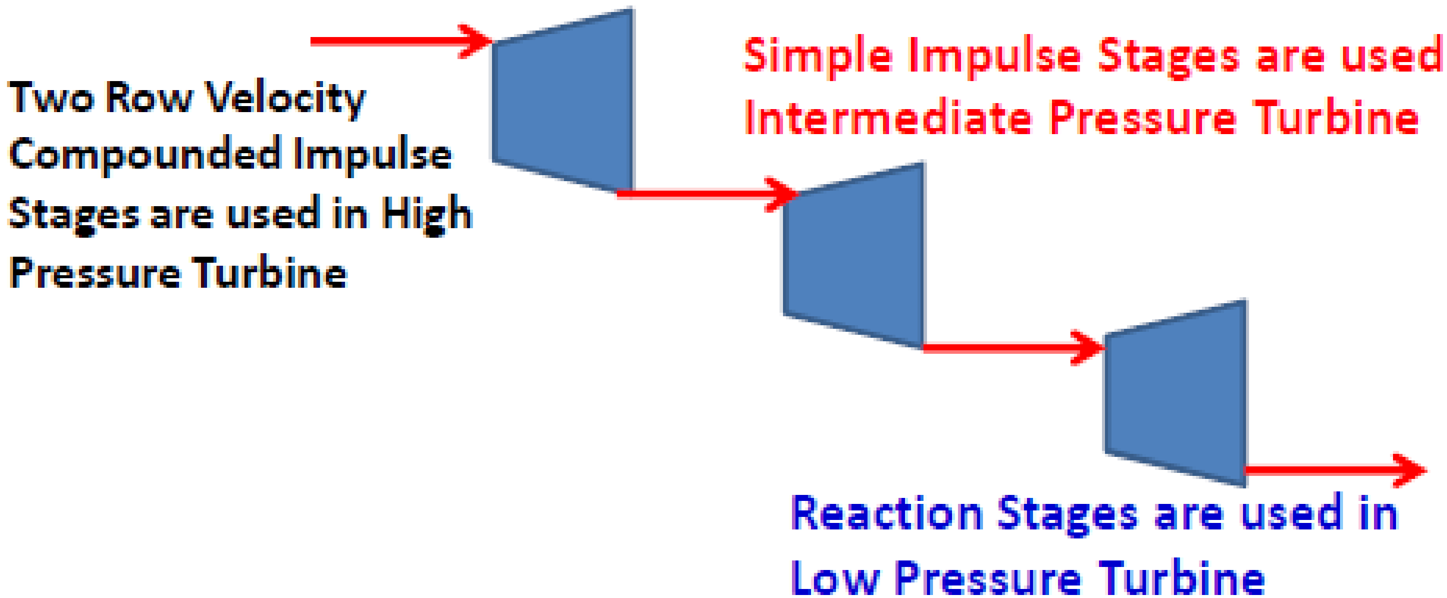
Pressure Differences



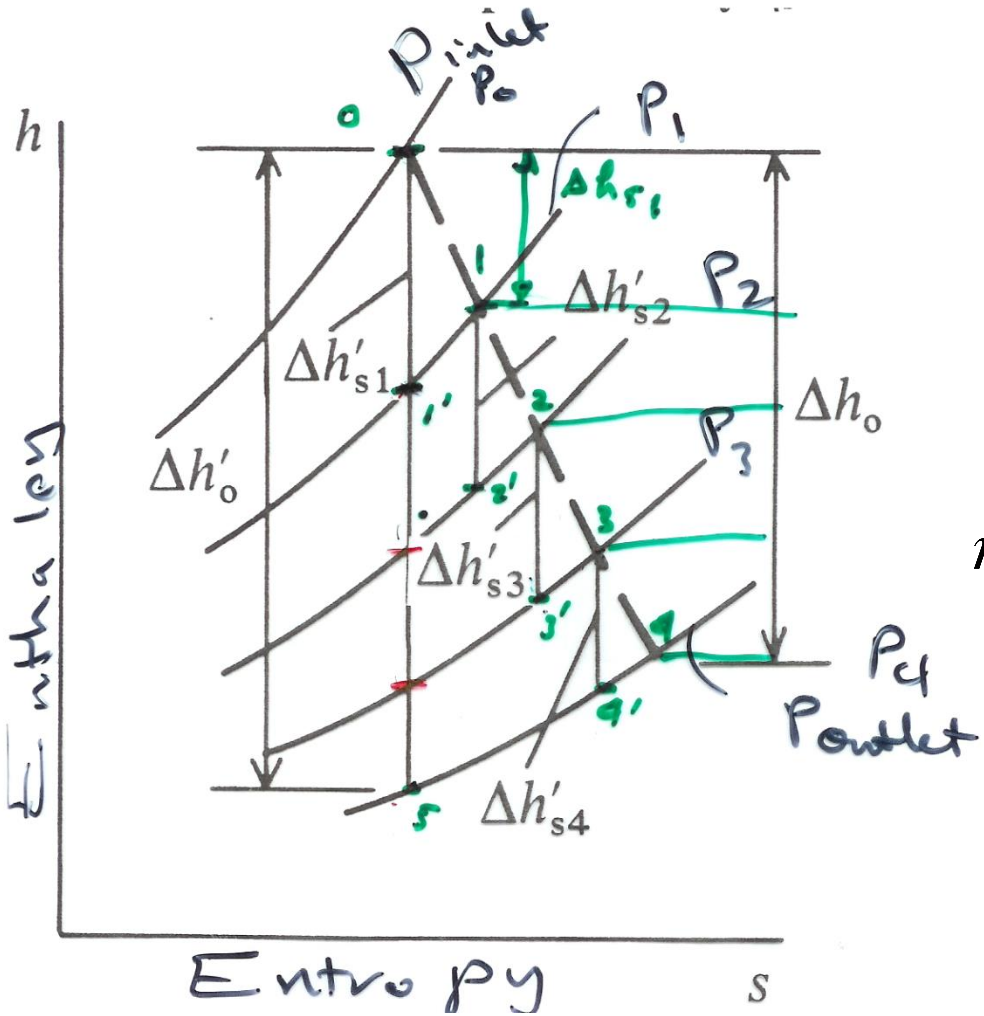
Leakage losses predominate over the friction losses at the high pressure end of high pressure ratio turbines (e.g steam turbines) whereas at the low pressure end, or throughout low pressure ratio turbines (e.g. gas turbines), the friction losses predominate.

Therefore, Reaction bladings are used in gas turbines. In steam turbines, Impulse stages are used especially at the high-pressure end.

In high-pressure ratios, it is usual to use at least two turbines in series; a high-pressure turbine with impulse stages followed by a low-pressure reaction turbine. The first impulse stage is commonly a two-row velocity compounded design, because with the pressure and superheat used in modern plants it is important to achieve a large enthalpy drop in the first row of Nozzles. This will avoid exposing the turbine casing and Rotor to the extreme conditions and also appreciably increase the volume flow before the steam reaches the first row of Rotor blades. Without this increase in volume, the blades might be too small for good efficiency.



Stage Isentropic Efficiency, Turbine Isentropic Efficiency and Reheat Factor



Consider a four stage turbine with each stage having the same isentropic efficiency η_s

$$\eta_s = \frac{\Delta h_s}{\Delta h_s^{'}} \qquad \text{and} \qquad \eta_o = \frac{\Delta h_o}{\Delta h_o^{'}}$$

The total work can be given as

$$\Delta h_o = w = \eta_o \Delta h_o'$$

$$or \quad as$$

$$\Delta h_o = w = \eta_s (\Delta h_{s1}' + \Delta h_{s2}' + \Delta h_{s3}' + \Delta h_{s4}')$$

$$= \eta_s \sum \Delta h_s'$$

$$But$$

$$\eta_o = \frac{\Delta h_o}{\Delta h_o'} = \frac{\eta_s \sum \Delta h_s'}{\Delta h_o'} = \eta_s R$$

$\sum \Delta h_s'$ = Cumulative enthalpy drop, is evidently greater than $\Delta h_o'$ because the vertical height between any pair of constant pressure lines increases with the increase of entropy.

R = called Reheat factor and is always greater than unity.

The physical interpretation of this result is that the internal losses in any stage (except the last) are partially recoverable in subsequent stages owing to the reheating effect of friction. It follows that it is important to use high efficiency stages at the low pressure end of a turbine than at the high pressure end.

The difference between η_o and η_s increases as the overall pressure ratio of the turbine is increased. Thus turbines of high pressure ratio tend to be more efficient than those of low pressure ratio.

$$\text{Reheat factor} = \frac{\text{Isentropic efficiency of turbine}}{\text{Stage Isentropic efficiency}}$$

Example

Steam enters an impulse turbine stage in which all processes are assumed to be reversible and adiabatic. The inlet pressure is 700 kPa and the inlet temperature is 400 ^oC. The exhaust pressure is 100 kPa. The steam leaves the nozzle and enters the turbine rotor at an angle of 20^o. The blade speed ratio is 0.5 and the rotor blade exit angle is 50^o. Determine the blade efficiency of this turbine.