Solutions Manual for

Fluid Mechanics: Fundamentals and Applications

Third Edition

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McGraw-Hill, 2013

Chapter 12
COMPRESSIBLE FLOW

Stagnation Properties

12-1C

Solution We are to discuss the temperature change from an airplane's nose to far away from the aircraft.

Analysis The temperature of the air **rises as it approaches the nose because of the stagnation process**.

Discussion In the frame of reference moving with the aircraft, the air decelerates from high speed to zero at the nose (stagnation point), and this causes the air temperature to rise.

12-2C

Solution We are to define dynamic temperature.

Analysis *Dynamic temperature* is **the temperature rise of a fluid during a stagnation process**.

Discussion When a gas decelerates from high speed to zero speed at a stagnation point, the temperature of the gas rises.

12-3C

Solution We are to discuss the measurement of flowing air temperature with a probe – is there significant error?

Analysis **No, there is not significant error**, because the velocities encountered in air-conditioning applications are very low, and thus the static and the stagnation temperatures are practically identical.

Discussion If the air stream were supersonic, however, the error would indeed be significant.

12-4

Solution Air flows through a device. The stagnation temperature and pressure of air and its velocity are specified. The static pressure and temperature of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at an anticipated average temperature of 600 K are c_p = 1.051 kJ/kg·K and k = 1.376.

Analysis The static temperature and pressure of air are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 673.2 - \frac{(570\text{ m/s})^2}{2\times 1.051\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 518.6\text{ K} \cong \mathbf{519\ K}$$

and

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{518.6\text{ K}}{673.2\text{ K}}\right)^{1.376/(1.376-1)} = \mathbf{0.231\ MPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

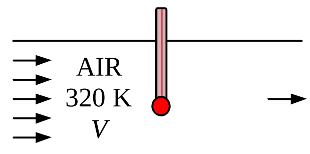
12-5

Solution Air at 320 K is flowing in a duct. The temperature that a stationary probe inserted into the duct will read is to be determined for different air velocities.

Assumptions The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg·K.

Analysis The air which strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature, T_0. It is determined from $T_0 = T + \frac{V^2}{2c_p}$. The results for each case are calculated below:



(*a*) $$T_0 = 320\text{ K} + \frac{(1\text{ m/s})^2}{2 \times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{320.0\text{ K}}$$

(*b*) $$T_0 = 320\text{ K} + \frac{(10\text{ m/s})^2}{2 \times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{320.1\text{ K}}$$

(*c*) $$T_0 = 320\text{ K} + \frac{(100\text{ m/s})^2}{2 \times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{325.0\text{ K}}$$

(*d*) $$T_0 = 320\text{ K} + \frac{(1000\text{ m/s})^2}{2 \times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{817.5\text{ K}}$$

Discussion Note that the stagnation temperature is nearly identical to the thermodynamic temperature at low velocities, but the difference between the two is significant at high velocities.

12-6

Solution The states of different substances and their velocities are specified. The stagnation temperature and stagnation pressures are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Helium and nitrogen are ideal gases.

Analysis (*a*) Helium can be treated as an ideal gas with c_p = 5.1926 kJ/kg·K and k = 1.667. Then the stagnation temperature and pressure of helium are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(240\text{ m/s})^2}{2 \times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{55.5°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.25\text{ MPa})\left(\frac{328.7\text{ K}}{323.2\text{ K}}\right)^{1.667/(1.667-1)} = \mathbf{0.261\,MPa}$$

(*b*) Nitrogen can be treated as an ideal gas with c_p = 1.039 kJ/kg·K and k =1.400. Then the stagnation temperature and pressure of nitrogen are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50°\text{C} + \frac{(300\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{93.3°C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.15\text{ MPa})\left(\frac{366.5\text{ K}}{323.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{0.233\ MPa}$$

(*c*) Steam can be treated as an ideal gas with c_p = 1.865 kJ/kg·K and k =1.329. Then the stagnation temperature and pressure of steam are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 350°\text{C} + \frac{(480\text{ m/s})^2}{2 \times 1.865\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{411.8°C} = \mathbf{685\ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.1\text{ MPa})\left(\frac{685\text{ K}}{623.2\text{ K}}\right)^{1.329/(1.329-1)} = \mathbf{0.147\ MPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

12-7

Solution The state of air and its velocity are specified. The stagnation temperature and stagnation pressure of air are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.

Analysis The stagnation temperature of air is determined from

$$T_0 = T + \frac{V^2}{2c_p} = 238\text{ K} + \frac{(325\text{ m/s})^2}{2 \times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 290.5 \cong \mathbf{291\,K}$$

Other stagnation properties at the specified state are determined by considering an isentropic process between the specified state and the stagnation state,

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (36\text{ kPa})\left(\frac{290.5\text{ K}}{238\text{ K}}\right)^{1.4/(1.4-1)} = 72.37\text{ kPa} \cong \mathbf{72.4\,kPa}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

12-8E

Solution Steam flows through a device. The stagnation temperature and pressure of steam and its velocity are specified. The static pressure and temperature of the steam are to be determined.

Assumptions **1** The stagnation process is isentropic. **2** Steam is an ideal gas.

Properties Steam can be treated as an ideal gas with c_p = 0.4455 Btu/lbm·R and k =1.329.

Analysis The static temperature and pressure of steam are determined from

$$T = T_0 - \frac{V^2}{2c_p} = 700°\text{F} - \frac{(900\ \text{ft/s})^2}{2 \times 0.4455\ \text{Btu/lbm} \cdot °\text{F}}\left(\frac{1\ \text{Btu/lbm}}{25{,}037\ \text{ft}^2/\text{s}^2}\right) = \mathbf{663.7°F}$$

$$P = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (120\ \text{psia})\left(\frac{1123.7\ \text{R}}{1160\ \text{R}}\right)^{1.329/(1.329-1)} = \mathbf{105.5\ psia}$$

Discussion Note that the stagnation properties can be significantly different than thermodynamic properties.

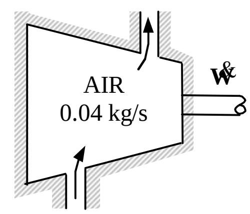
12-9

Solution The inlet stagnation temperature and pressure and the exit stagnation pressure of air flowing through a compressor are specified. The power input to the compressor is to be determined.

Assumptions **1** The compressor is isentropic. **2** Air is an ideal gas.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4.

Analysis The exit stagnation temperature of air T_{02} is determined from



$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (308.2\ \text{K})\left(\frac{900}{100}\right)^{(1.4-1)/1.4} = 577.4\ \text{K}$$

From the energy balance on the compressor,

$$\dot{W}_{\text{in}} = \dot{m}(h_{20} - h_{01})$$

or,

$$\dot{W}_{\text{in}} = \dot{m}c_p(T_{02} - T_{01}) = (0.04\ \text{kg/s})(1.005\ \text{kJ/kg}\cdot\text{K})(577.4 - 308.2)\text{K} = \mathbf{10.8\ kW}$$

Discussion Note that the stagnation properties can be used conveniently in the energy equation.

12-10

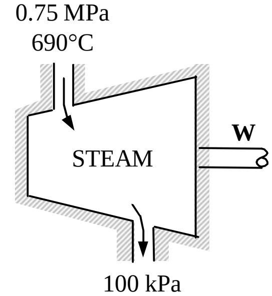
Solution The inlet stagnation temperature and pressure and the exit stagnation pressure of products of combustion flowing through a gas turbine are specified. The power output of the turbine is to be determined.

Assumptions **1** The expansion process is isentropic. **2** Products of combustion are ideal gases.

Properties The properties of products of combustion are c_p = 1.157 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.33.

Analysis The exit stagnation temperature T_{02} is determined to be

$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (963.2\text{ K})\left(\frac{0.1}{0.75}\right)^{(1.33-1)/1.33} = 584.2\text{ K}$$



Also,

$$c_p = kc_v = k\left(c_p - R\right) \longrightarrow c_p = \frac{kR}{k-1} = \frac{1.33\left(0.287\text{ kJ/kg}\cdot\text{K}\right)}{1.33-1} = 1.157\text{ kJ/kg}\cdot\text{K}$$

From the energy balance on the turbine,

$$-w_{\text{out}} = (h_{20} - h_{01})$$

or, $$w_{\text{out}} = c_p(T_{01} - T_{02}) = (1.157\text{ kJ/kg}\cdot\text{K})(963.2 - 584.2)\text{K} = 438.5\text{ kJ/kg} \cong \mathbf{439\ kJ/kg}$$

Discussion Note that the stagnation properties can be used conveniently in the energy equation.

One Dimensional Isentropic Flow

12-11C

Solution We are to determine if it is possible to accelerate a gas to supersonic velocity in a converging nozzle.

Analysis **No**, it is not possible.

Discussion The only way to do it is to have first a converging nozzle, and then a diverging nozzle.

12-12C

Solution We are to discuss what happens to several variables when a subsonic gas enters a diverging duct.

Analysis (*a*) The **velocity decreases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid increase**.

Discussion The velocity decrease is opposite to what happens in supersonic flow.

12-13C

Solution We are to discuss the pressure at the throats of two different converging-diverging nozzles.

Analysis **The pressures at the two throats are identical**.

Discussion Since the gas has the same stagnation conditions, it also has the same sonic conditions at the throat.

12-14C

Solution We are to discuss what happens to several variables when a supersonic gas enters a converging duct.

Analysis (*a*) The **velocity decreases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid increase**.

Discussion The velocity decrease is opposite to what happens in subsonic flow.

12-15C

Solution We are to discuss what happens to several variables when a supersonic gas enters a diverging duct.

Analysis (*a*) The **velocity increases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid decrease**.

Discussion The velocity increase is opposite to what happens in subsonic flow.

12-16C

Solution We are to discuss what happens to the exit velocity and mass flow rate through a converging nozzle at sonic exit conditions when the nozzle exit area is reduced.

Analysis (*a*) The **exit velocity remains constant at sonic speed**, (*b*) the **mass flow rate through the nozzle decreases because of the reduced flow area**.

Discussion Without a diverging portion of the nozzle, a converging nozzle is limited to sonic velocity at the exit.

12-17C

Solution We are to discuss what happens to several variables when a subsonic gas enters a converging duct.

Analysis (*a*) The **velocity increases**. (*b*), (*c*), (*d*) The **temperature, pressure, and density of the fluid decrease**.

Discussion The velocity increase is opposite to what happens in supersonic flow.

12-18

Solution Helium enters a converging-diverging nozzle at specified conditions. The lowest temperature and pressure that can be obtained at the throat of the nozzle are to be determined.

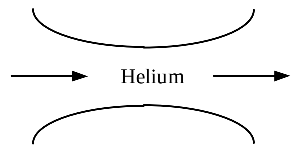
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are k = 1.667 and c_p = 5.1926 kJ/kg·K.

Analysis The lowest temperature and pressure that can be obtained at the throat are the critical temperature T^* and critical pressure P^*. First we determine the stagnation temperature T_0 and stagnation pressure P_0,

$$T_0 = T + \frac{V^2}{2c_p} = 800\text{ K} + \frac{(100\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 801\text{ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.7\text{ MPa})\left(\frac{801\text{ K}}{800\text{ K}}\right)^{1.667/(1.667-1)} = 0.702\text{ MPa}$$



Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (801\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{601\text{ K}}$$

and

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.702\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.342\text{ MPa}}$$

Discussion These are the temperature and pressure that will occur at the throat when the flow past the throat is supersonic.

12-19

Solution The speed of an airplane and the air temperature are give. It is to be determined if the speed of this airplane is subsonic or supersonic.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis The temperature is -50 + 273.15 = 223.15 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(223.15\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}\left(\frac{3.6\text{ km/h}}{1\text{ m/s}}\right) = 1077.97\text{ km/h}$$

and

$$\text{Ma} = \frac{V}{c} = \frac{1050\text{ km/h}}{1077.97\text{ km/h}} = 0.9741\text{ km/h} \cong \mathbf{0.974}$$

The speed of the airplane is **subsonic** since the Mach number is less than 1.

Discussion Subsonic airplanes stay sufficiently far from the Mach number of 1 to avoid the instabilities associated with transonic flights.

12-20

Solution The critical temperature, pressure, and density of air and helium are to be determined at specified conditions.

Assumptions Air and Helium are ideal gases with constant specific heats at room temperature.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, k = 1.4, and c_p = 1.005 kJ/kg·K. The properties of helium at room temperature are R = 2.0769 kJ/kg·K, k = 1.667, and c_p = 5.1926 kJ/kg·K.

Analysis (*a*) Before we calculate the critical temperature T^*, pressure P^*, and density ρ^*, we need to determine the stagnation temperature T_0, pressure P_0, and density ρ_0.

$$T_0 = 100°\text{C} + \frac{V^2}{2c_p} = 100 + \frac{(250\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot°\text{C}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 131.1°\text{C}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{404.3\ \text{K}}{373.2\ \text{K}}\right)^{1.4/(1.4-1)} = 264.7\ \text{kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{264.7\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(404.3\ \text{K})} = 2.281\ \text{kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (404.3\ \text{K})\left(\frac{2}{1.4+1}\right) = \mathbf{337\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (264.7\ \text{kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{140\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (2.281\ \text{kg/m}^3)\left(\frac{2}{1.4+1}\right)^{1/(1.4-1)} = \mathbf{1.45\ kg/m^3}$$

(*b*) For helium, $T_0 = T + \frac{V^2}{2c_p} = 40 + \frac{(300\ \text{m/s})^2}{2\times 5.1926\ \text{kJ/kg}\cdot°\text{C}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 48.7°\text{C}$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{321.9\ \text{K}}{313.2\ \text{K}}\right)^{1.667/(1.667-1)} = 214.2\ \text{kPa}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{214.2\ \text{kPa}}{(2.0769\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(321.9\ \text{K})} = 0.320\ \text{kg/m}^3$$

Thus,

$$T^* = T_0\left(\frac{2}{k+1}\right) = (321.9\ \text{K})\left(\frac{2}{1.667+1}\right) = \mathbf{241\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{97.4\ kPa}$$

$$\rho^* = \rho_0\left(\frac{2}{k+1}\right)^{1/(k-1)} = (0.320\ \text{kg/m}^3)\left(\frac{2}{1.667+1}\right)^{1/(1.667-1)} = \mathbf{0.208\ kg/m^3}$$

Discussion These are the temperature, pressure, and density values that will occur at the throat when the flow past the throat is supersonic.

12-21E

Solution Air flows through a duct at a specified state and Mach number. The velocity and the stagnation pressure, temperature, and density of the air are to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R and k = 1.4.

Analysis First, T = 320 + 459.67 = 779.67 K. The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.06855\ \text{Btu/1bm}\cdot\text{R})(779.67\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/1bm}}\right)} = 1368.72\ \text{ft/s}$$

Thus,

$$V = \text{Ma} \times c = (0.7)(1368.72\ \text{ft/s}) = 958.10 \cong \mathbf{958\ \ ft/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{25\ \text{psia}}{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(779.67\ \text{R})} = 0.086568\ \text{1bm/ft}^3$$

Then the stagnation properties are determined from

$$T_0 = T\left(1 + \frac{(k-1)\text{Ma}^2}{2}\right) = (779.67\ \text{R})\left(1 + \frac{(1.4\text{-}1)(0.7)^2}{2}\right) = 856.08\ \text{R} \cong \mathbf{856\ \ R}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (25\ \text{psia})\left(\frac{856.08\ \text{R}}{779.67\ \text{R}}\right)^{1.4/(1.4-1)} = 34.678\ \text{psia} \cong \mathbf{34.7\ \ psia}$$

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (0.08656\ \ \text{1bm/ft}^3)\left(\frac{856.08\ \text{R}}{779.67\ \text{R}}\right)^{1/(1.4-1)} = 0.10936\ \text{lbm/ft}^3 \cong \mathbf{0.109\ \ lbm/ft^3}$$

Discussion Note that the temperature, pressure, and density of a gas increases during a stagnation process.

12-22

Solution Air enters a converging-diverging nozzle at specified conditions. The lowest pressure that can be obtained at the throat of the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air at room temperature is k = 1.4.

Analysis The lowest pressure that can be obtained at the throat is the critical pressure P*, which is determined from

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1200\ \text{kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = \mathbf{634\ kPa}$$

Discussion This is the pressure that occurs at the throat when the flow past the throat is supersonic.

12-23

Solution The Mach number of scramjet and the air temperature are given. The speed of the engine is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.287 kJ/kg·K. Its specific heat ratio at room temperature is k = 1.4.

Analysis The temperature is -20 + 273.15 = 253.15 K. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(253.15\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 318.93\ \text{m/s}$$

and

$$V = c\text{Ma} = (318.93\ \text{m/s})(7)\left(\frac{3.6\ \text{km/h}}{1\ \text{m/s}}\right) = 8037\ \text{km/h} \cong \mathbf{8040\ km/h}$$

Discussion Note that extremely high speed can be achieved with scramjet engines. We cannot justify more than three significant digits in a problem like this.

12-24E

Solution The Mach number of scramjet and the air temperature are given. The speed of the engine is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The gas constant of air is R = 0.06855 Btu/lbm·R. Its specific heat ratio at room temperature is k = 1.4 .

Analysis The temperature is 0 + 459.67 = 459.67 R. The speed of sound is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.06855\ \text{Btu/lbm}\cdot\text{R})(459.67\ \text{R})\left(\frac{25{,}037\ \text{ft}^2/\text{s}^2}{1\ \text{Btu/lbm}}\right)} = 1050.95\ \text{ft/s}$$

and

$$V = c\text{Ma} = (1050.95\ \text{ft/s})(7)\left(\frac{1\ \text{mi/h}}{1.46667\ \text{ft/s}}\right) = 5015.9\ \text{mi/h} \cong \mathbf{5020\ \ mi/h}$$

Discussion Note that extremely high speed can be achieved with scramjet engines. We cannot justify more than three significant digits in a problem like this.

12-25

Solution Air flows through a duct. The state of the air and its Mach number are specified. The velocity and the stagnation pressure, temperature, and density of the air are to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air at room temperature are R = 0.287 kPa.m^3/kg.K and k = 1.4.

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(373.2\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 387.2\ \text{m/s}$$

Thus,

$$V = \text{Ma} \times c = (0.8)(387.2\ \text{m/s}) = \mathbf{310\ \ m/s}$$

Also,

$$\rho = \frac{P}{RT} = \frac{200\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(373.2\ \text{K})} = 1.867\ \text{kg/m}^3$$

Then the stagnation properties are determined from

$$T_0 = T\left(1 + \frac{(k-1)\text{Ma}^2}{2}\right) = (373.2\ \text{K})\left(1 + \frac{(1.4\text{-}1)(0.8)^2}{2}\right) = \mathbf{421\ \ K}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (200\ \text{kPa})\left(\frac{421.0\ \text{K}}{373.2\ \text{K}}\right)^{1.4/(1.4-1)} = \mathbf{305\ \ kPa}$$

$$\rho_0 = \rho\left(\frac{T_0}{T}\right)^{1/(k-1)} = (1.867\ \text{kg/m}^3)\left(\frac{421.0\ \text{K}}{373.2\ \text{K}}\right)^{1/(1.4-1)} = \mathbf{2.52\ \ kg/m^3}$$

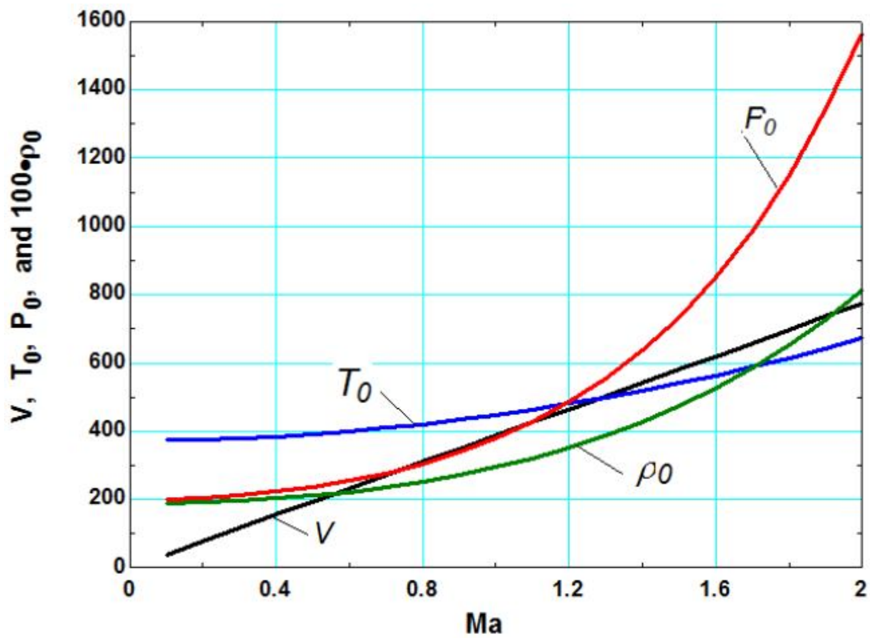
Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

12-26

Solution Problem 12-41 is reconsidered. The effect of Mach number on the velocity and stagnation properties as the Ma is varied from 0.1 to 2 are to be investigated, and the results are to be plotted.

Analysis The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
P=200
T=100+273.15
R=0.287
k=1.4
c=SQRT(k*R*T*1000)
Ma=V/c
rho=P/(R*T)

"Stagnation properties"
T0=T*(1+(k-1)*Ma^2/2)
P0=P*(T0/T)^(k/(k-1))
rho0=rho*(T0/T)^(1/(k-1))
```



Mach num. Ma	Velocity, V, m/s	Stag. Temp, T_0, K	Stag. Press, P_0, kPa	Stag. Density, ρ_0, kg/m^3
0.1	38.7	373.9	201.4	1.877
0.2	77.4	376.1	205.7	1.905
0.3	116.2	379.9	212.9	1.953
0.4	154.9	385.1	223.3	2.021
0.5	193.6	391.8	237.2	2.110
0.6	232.3	400.0	255.1	2.222
0.7	271.0	409.7	277.4	2.359
0.8	309.8	420.9	304.9	2.524
0.9	348.5	433.6	338.3	2.718
1.0	387.2	447.8	378.6	2.946
1.1	425.9	463.5	427.0	3.210
1.2	464.7	480.6	485.0	3.516
1.3	503.4	499.3	554.1	3.867
1.4	542.1	519.4	636.5	4.269
1.5	580.8	541.1	734.2	4.728
1.6	619.5	564.2	850.1	5.250
1.7	658.3	588.8	987.2	5.842
1.8	697.0	615.0	1149.2	6.511
1.9	735.7	642.6	1340.1	7.267
2.0	774.4	671.7	1564.9	8.118

Discussion Note that as Mach number increases, so does the flow velocity and stagnation temperature, pressure, and density.

12-27

Solution An aircraft is designed to cruise at a given Mach number, elevation, and the atmospheric temperature. The stagnation temperature on the leading edge of the wing is to be determined.

Assumptions Air is an ideal gas with constant specific heats at room temperature.

Properties The properties of air are R = 0.287 kPa.m^3/kg.K, c_p = 1.005 kJ/kg·K, and $k = 1.4$.

Analysis The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(236.15\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 308.0\ \text{m/s}$$

Thus,

$$V = \text{Ma} \times c = (1.1)(308.0\ \text{m/s}) = 338.8\ \text{m/s}$$

Then,

$$T_0 = T + \frac{V^2}{2c_p} = 236.15 + \frac{(338.8\ \text{m/s})^2}{2 \times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{293\ K}$$

Discussion Note that the temperature of a gas increases during a stagnation process as the kinetic energy is converted to enthalpy.

12-28

Solution Quiescent carbon dioxide at a given state is accelerated isentropically to a specified Mach number. The temperature and pressure of the carbon dioxide after acceleration are to be determined.

Assumptions Carbon dioxide is an ideal gas with constant specific heats at room temperature.

Properties The specific heat ratio of the carbon dioxide at room temperature is $k = 1.288$.

Analysis The inlet temperature and pressure in this case is equivalent to the stagnation temperature and pressure since the inlet velocity of the carbon dioxide is said to be negligible. That is, $T_0 = T_i$ = 400 K and $P_0 = P_i$ = 1200 kPa. Then,

$$T = T_0\left(\frac{2}{2+(k-1)\text{Ma}^2}\right) = (600\ \text{K})\left(\frac{2}{2+(1.288\text{-}1)(0.6)^2}\right) = 570.43\ \text{K} \cong \mathbf{570\ K}$$

and

$$P = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (1200\ \text{kPa})\left(\frac{570.43\ \text{K}}{600\ \text{K}}\right)^{1.288/(1.288-1)} = 957.23\ \text{K} \cong \mathbf{957\ \ kPa}$$

Discussion Note that both the pressure and temperature drop as the gas is accelerated as part of the internal energy of the gas is converted to kinetic energy.

Isentropic Flow Through Nozzles

12-29C

Solution We are to analyze if it is possible to accelerate a fluid to supersonic speeds with a velocity that is not sonic at the throat.

Analysis **No, if the flow in the throat is subsonic**. If the velocity at the throat is subsonic, the diverging section would act like a diffuser and decelerate the flow. **Yes, if the flow in the throat is already supersonic**, the diverging section would accelerate the flow to even higher Mach number.

Discussion In duct flow, the latter situation is not possible unless a second converging-diverging portion of the duct is located upstream, and there is sufficient pressure difference to choke the flow in the upstream throat.

12-30C

Solution We are to discuss what would happen if we add a diverging section to supersonic flow in a duct.

Analysis The fluid would **accelerate even further**, as desired.

Discussion This is the opposite of what would happen in subsonic flow.

12-31C

Solution We are to discuss the difference between Ma* and Ma.

Analysis Ma* is the **local velocity non-dimensionalized with respect to the sonic speed at the throat**, whereas Ma is the **local velocity non-dimensionalized with respect to the local sonic speed**.

Discussion The two are identical at the throat when the flow is choked.

12-32C

Solution We are to consider subsonic flow through a converging nozzle with critical pressure at the exit, and analyze the effect of lowering back pressure below the critical pressure.

Analysis (*a*) **No effect on velocity**. (*b*) **No effect on pressure**. (*c*) **No effect on mass flow rate**.

Discussion In this situation, the flow is already choked initially, so further lowering of the back pressure does not change anything upstream of the nozzle exit plane.

12-33C

Solution We are to compare the mass flow rates through two identical converging nozzles, but with one having a diverging section.

Analysis **If the back pressure is low enough so that sonic conditions exist at the throats, the mass flow rates in the two nozzles would be identical**. However, **if the flow is not sonic at the throat, the mass flow rate through the nozzle with the diverging section would be greater**, because it acts like a subsonic diffuser.

Discussion Once the flow is choked at the throat, whatever happens downstream is irrelevant to the flow upstream of the throat.

12-34C

Solution We are to discuss the hypothetical situation of hypersonic flow at the outlet of a converging nozzle.

Analysis Maximum flow rate through a converging nozzle is achieved when Ma = 1 at the exit of a nozzle. For all other Ma values the mass flow rate decreases. Therefore, **the mass flow rate would decrease if hypersonic velocities were achieved at the throat of a converging nozzle**.

Discussion Note that this is not possible unless the flow upstream of the converging nozzle is already hypersonic.

12-35C

Solution We are to consider subsonic flow through a converging nozzle, and analyze the effect of setting back pressure to critical pressure for a converging nozzle.

Analysis (*a*) The **exit velocity reaches the sonic speed**, (*b*) the **exit pressure equals the critical pressure**, and (*c*) the **mass flow rate reaches the maximum value**.

Discussion In such a case, we say that the flow is *choked*.

12-36C

Solution We are to discuss what happens to several variables in the diverging section of a subsonic converging-diverging nozzle.

Analysis (*a*) The **velocity decreases**, (*b*) the **pressure increases**, and (*c*) the **mass flow rate remains the same**.

Discussion Qualitatively, this is the same as what we are used to (in previous chapters) for incompressible flow.

12-37C

Solution We are to discuss what would happen if we add a diverging section to supersonic flow in a duct.

Analysis The fluid would **accelerate even further** instead of decelerating.

Discussion This is the opposite of what would happen in subsonic flow.

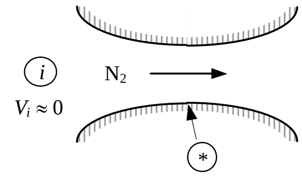
12-38

Solution Nitrogen enters a converging-diverging nozzle at a given pressure. The critical velocity, pressure, temperature, and density in the nozzle are to be determined.

Assumptions **1** Nitrogen is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of nitrogen are k = 1.4 and R = 0.2968 kJ/kg·K.

Analysis The stagnation pressure in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle,



$$P_0 = P_i = 700 \text{ kPa}$$

$$T_0 = T_i = 400 \text{ K}$$

$$\rho_0 = \frac{P_0}{RT_0} = \frac{700 \text{ kPa}}{(0.2968 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(400 \text{ K})} = 5.896 \text{ kg/m}^3$$

Critical properties are those at a location where the Mach number is Ma = 1. From Table A-13 at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283, and ρ/ρ_0 = 0.6339. Then the critical properties become

$$T^* = 0.8333T_0 = 0.8333(400 \text{ K}) = \mathbf{333 \text{ K}}$$

$$P^* = 0.5283P_0 = 0.5283(700 \text{ kPa}) = \mathbf{370 \text{ MPa}}$$

$$\rho^* = 0.6339\rho_0 = 0.6339(5.896 \text{ kg/m}^3) = \mathbf{3.74 \text{ kg/m}^3}$$

Also,

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.4)(0.2968 \text{ kJ/kg}\cdot\text{K})(333 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{372 \text{ m/s}}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-39

Solution For an ideal gas, an expression is to be obtained for the ratio of the speed of sound where Ma = 1 to the speed of sound based on the stagnation temperature, c^*/c_0.

Analysis For an ideal gas the speed of sound is expressed as $c = \sqrt{kRT}$. Thus,

$$\frac{c^*}{c_0} = \frac{\sqrt{kRT^*}}{\sqrt{kRT_0}} = \left(\frac{T^*}{T_0}\right)^{1/2} = \left(\frac{\mathbf{2}}{\boldsymbol{k}+\mathbf{1}}\right)^{\mathbf{1/2}}$$

Discussion Note that a speed of sound changes the flow as the temperature changes.

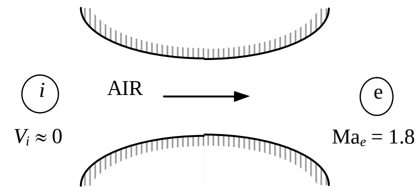
12-40

Solution Air enters a converging-diverging nozzle at a specified pressure. The back pressure that will result in a specified exit Mach number is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis The stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. It remains constant throughout the nozzle since the flow is isentropic,



$$P_0 = P_i = 1.2 \text{ MPa}$$

From Table A-13 at $Ma_e = 1.8$, we read $P_e/P_0 = 0.1740$.

Thus, $P = 0.1740P_0 = 0.1740(1.2 \text{ MPa}) =$ **0.209 MPa** = **209 kPa**

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

12-41E

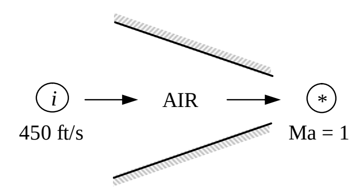
Solution Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are $k = 1.4$ and $c_p = 0.240$ Btu/lbm·R (Table A-2Ea).

Analysis The properties of the fluid at the location where Ma =1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T + \frac{V_i^2}{2c_p} = 630\text{ R} + \frac{(450\text{ ft/s})^2}{2 \times 0.240\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/1bm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 646.9\text{ R}$$



$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (30\text{ psia})\left(\frac{646.9\text{ K}}{630\text{ K}}\right)^{1.4/(1.4-1)} = 32.9\text{ psia}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma =1, we read $T/T_0 = 0.8333$, $P/P_0 = 0.5283$. Thus,

$T = 0.8333T_0 = 0.8333(646.9\text{ R}) =$ **539 R** and $P = 0.5283P_0 = 0.5283(32.9\text{ psia}) =$ **17.4 psia**

Also,

$$c_i = \sqrt{kRT}_i = \sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(630\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1230\text{ ft/s} \quad \text{and}$$

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{450\text{ ft/s}}{1230\text{ ft/s}} = 0.3657$$

From Table A-13 at this Mach number we read $A_i/A^* = 1.7426$. Thus the ratio of the throat area to the nozzle inlet area is

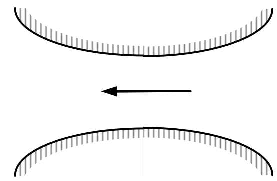
$$\frac{A^*}{A_i} = \frac{1}{1.7426} = \mathbf{0.574}$$

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

12-42

Solution For subsonic flow at the inlet, the variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.

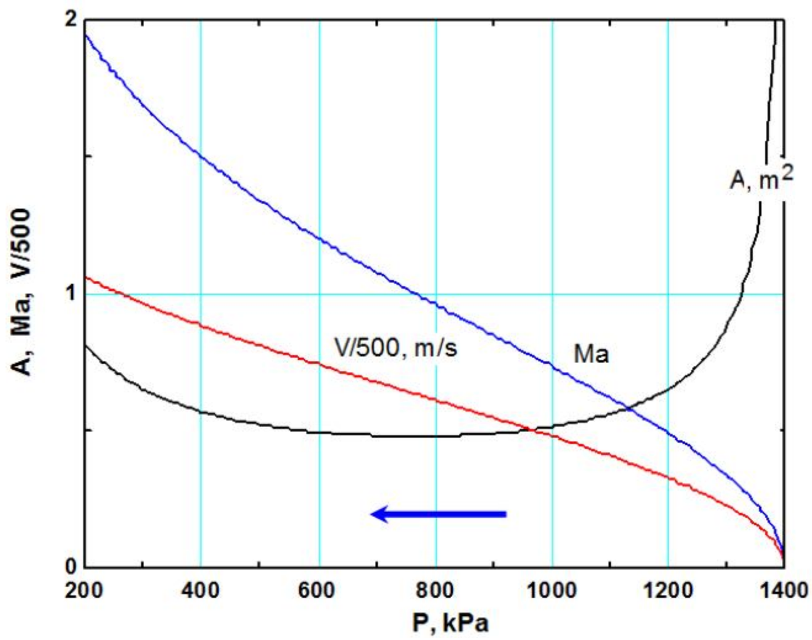
Assumptions **1** The gas is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The flow is choked at the throat.



$Ma_i < 1$

Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure drops from a stagnation value of 1400 kPa to 200 kPa. Note that the curve for *A* is related to the shape of the nozzle, with horizontal axis serving as the centerline. The EES equation window and the plot are shown below.

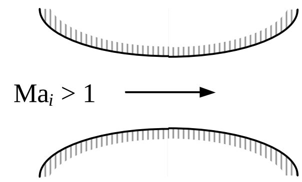
```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```



Discussion We are assuming that the back pressure is sufficiently low that the flow is choked at the throat, and the flow downstream of the throat is supersonic without any shock waves. Mach number and velocity continue to rise right through the throat into the diverging portion of the nozzle, since the flow becomes supersonic.

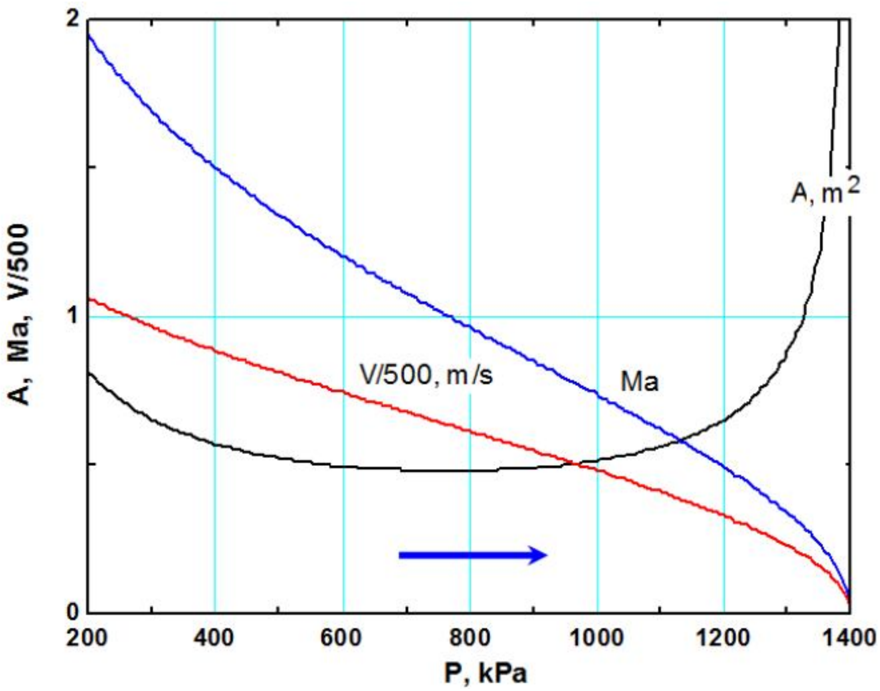
12-43

Solution We repeat the previous problem, but for supersonic flow at the inlet. The variation of pressure, velocity, and Mach number along the length of the nozzle are to be sketched for an ideal gas under specified conditions.



Analysis Using EES and CO_2 as the gas, we calculate and plot flow area *A*, velocity *V*, and Mach number Ma as the pressure rises from 200 kPa at a very high velocity to the stagnation value of 1400 kPa. Note that the curve for *A* is related to the shape of the nozzle, with horizontal axis serving as the centerline.

```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
P0=1400 "kPa"

T0=473 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
rho_norm=rho/rho_0 "Normalized density"
T=T0*(P/P0)^((k-1)/k)
Tnorm=T/T0  "Normalized temperature"
V=SQRT(2*Cp*(T0-T)*1000)
V_norm=V/500
A=m/(rho*V)*500
C=SQRT(k*R*T*1000)
Ma=V/C
```



Discussion Note that this problem is identical to the proceeding one, except the flow direction is reversed. In fact, when plotted like this, the plots are identical.

12-44

Solution It is to be explained why the maximum flow rate per unit area for a given ideal gas depends only on $P_0/\sqrt{T_0}$. Also for an ideal gas, a relation is to be obtained for the constant a in $\dot{m}_{max}/A^* = a\left(P_0/\sqrt{T_0}\right)$.

Properties The properties of the ideal gas considered are R = 0.287 kPa.m^3/kg·K and k = 1.4.

Analysis The maximum flow rate is given by

$$\dot{m}_{max} = A^* P_0 \sqrt{k/RT_0}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \quad \text{or} \quad \dot{m}_{max}/A^* = \left(P_0/\sqrt{T_0}\right)\sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

For a given gas, k and R are fixed, and thus the mass flow rate depends on the parameter $P_0/\sqrt{T_0}$. Thus, $\dot{m}_{max}/A^*$ can be expressed as $\dot{m}_{max}/A^* = a\left(P_0/\sqrt{T_0}\right)$ where

$$a = \sqrt{k/R}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} = \sqrt{\frac{1.4}{(0.287\text{ kJ/kg.K})\left(\dfrac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}}\left(\frac{2}{1.4+1}\right)^{2.4/0.8} = \mathbf{0.0404\ (m/s)\sqrt{K}}$$

Discussion Note that when sonic conditions exist at a throat of known cross-sectional area, the mass flow rate is fixed by the stagnation conditions.

12-45

Solution An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 1.8 is specified. The flow area where Ma = 0.9 is to be determined.

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio is given to be k = 1.4.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where Ma_2 = 0.9 is determined using A/A^* data from Table A-13 to be

$$\text{Ma}_1 = 1.8: \quad \frac{A_1}{A^*} = 1.4390 \longrightarrow A^* = \frac{A_1}{1.4390} = \frac{36\text{ cm}^2}{1.4390} = 25.02\text{ cm}^2$$

$$\text{Ma}_2 = 0.9: \quad \frac{A_2}{A^*} = 1.0089 \longrightarrow A_2 = (1.0089)A^* = (1.0089)(25.02\text{ cm}^2) = \mathbf{25.2\ cm^2}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

12-46

Solution An ideal gas is flowing through a nozzle. The flow area at a location where Ma = 1.8 is specified. The flow area where Ma = 0.9 is to be determined.

Assumptions Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The flow is assumed to be isentropic, and thus the stagnation and critical properties remain constant throughout the nozzle. The flow area at a location where $Ma_2 = 0.9$ is determined using the A/A^* relation,

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left\{\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right\}^{(k+1)/2(k-1)}$$

For $k = 1.33$ and $Ma_1 = 1.8$:

$$\frac{A_1}{A^*} = \frac{1}{1.8}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}1.8^2\right)\right\}^{2.33/2\times0.33} = 1.4696$$

and, $$A^* = \frac{A_1}{2.570} = \frac{36\text{ cm}^2}{1.4696} = 24.50\text{ cm}^2$$

For $k = 1.33$ and $Ma_2 = 0.9$:

$$\frac{A_2}{A^*} = \frac{1}{0.9}\left\{\left(\frac{2}{1.33+1}\right)\left(1+\frac{1.33-1}{2}0.9^2\right)\right\}^{2.33/2\times0.33} = 1.0091$$

and $$A_2 = (1.0091)A^* = (1.0091)(24.50\text{ cm}^2) = \mathbf{24.7\ cm^2}$$

Discussion Note that the compressible flow functions in Table A-13 are prepared for k = 1.4, and thus they cannot be used to solve this problem.

12-47E

Solution Air enters a converging-diverging nozzle at a specified temperature and pressure with low velocity. The pressure, temperature, velocity, and mass flow rate are to be calculated in the specified test section.

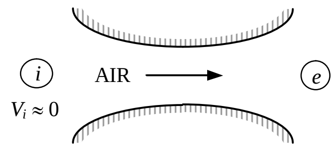
Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4 and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

$P_0 = P_i = 150$ psia and $T_0 = T_i = 100°\text{F} \approx 560$ R

Then,



$$T_e = T_0\left(\frac{2}{2+(k-1)\text{Ma}^2}\right) = (560\text{ R})\left(\frac{2}{2+(1.4\text{-}1)2^2}\right) = \mathbf{311 R}$$

$$P_e = P_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = (150\text{ psia})\left(\frac{311}{560}\right)^{1.4/0.4} = \mathbf{19.1\ psia}$$

$$\rho_e = \frac{P_e}{RT_e} = \frac{19.1\text{ psia}}{(0.3704\text{ psia.ft}^3/\text{1bm}\cdot\text{R})(311\text{ R})} = 0.166\text{ 1bm/ft}^3$$

The nozzle exit velocity can be determined from $V_e = \text{Ma}_e c_e$, where c_e is the speed of sound at the exit conditions,

$$V_e = \text{Ma}_e c_e = \text{Ma}_e\sqrt{kRT_e} = (2)\sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(311\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1729\text{ ft/s} \cong \mathbf{1730\ \ ft/s}$$

Finally,

$$\dot{m} = \rho_e A_e V_e = (0.\ 166\text{ 1bm/ft}^3)(5\text{ ft}^2)(1729\text{ ft/s}) = 1435\text{ lbm/s} \cong \mathbf{1440\ lbm/s}$$

Discussion Air must be very dry in this application because the exit temperature of air is extremely low, and any moisture in the air will turn to ice particles.

12-48

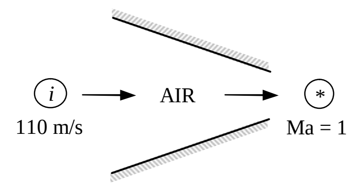
Solution Air enters a nozzle at specified temperature, pressure, and velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air are k = 1.4 and c_p = 1.005 kJ/kg·K.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 420\ \text{K} + \frac{(110\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 426.02$$



and

$$P_0 = P\left(\frac{T_0}{T}\right)^{k/(k-1)} = (0.5\ \text{MPa})\left(\frac{426.02\ \text{K}}{420\ \text{K}}\right)^{1.4/(1.4-1)} = 0.52554\ \text{MPa}$$

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma = 1, we read T/T_0 = 0.8333, P/P_0 = 0.5283. Thus,

$$T = 0.8333T_0 = 0.8333(426.02\ \text{K}) = 355.00\ \text{K} \approx \mathbf{355\ K}$$

and

$$P = 0.5283P_0 = 0.5283(0.52554\ \text{MPa}) = 0.27764\ \text{MPa} \approx \mathbf{0.278\ MPa} = \mathbf{278\ kPa}$$

Also,

$$c_i = \sqrt{kRT}_i = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(420\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 410.799\ \text{m/s}$$

and

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{110\ \text{m/s}}{410.799\ \text{m/s}} = 0.2678$$

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{150\ \text{m/s}}{410.799\ \text{m/s}} = 0.3651$$

From Table A-13 at this Mach number we read A_i/A^* = 2.3343. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A} = \frac{1}{2.3343} = 0.42839 \cong \mathbf{0.428}$$

Discussion We can also solve this problem using the relations for compressible isentropic flow. The results would be identical.

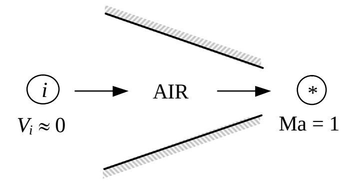
12-49

Solution Air enters a nozzle at specified temperature and pressure with low velocity. The exit pressure, exit temperature, and exit-to-inlet area ratio are to be determined for a Mach number of Ma = 1 at the exit.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.



$T_0 = T_i = 420$ K and $P_0 = P_i = 0.5$ MPa

From Table A-13 (or from Eqs. 12-18 and 12-19) at Ma =1, we read T/T_0 =0.8333, P/P_0 = 0.5283. Thus,

$T = 0.8333T_0 = 0.8333(420\text{ K}) = \mathbf{350\ K}$ and $P = 0.5283P_0 = 0.5283(0.5\text{ MPa}) = \mathbf{0.264\ MPa}$

The Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. From Table A-13 at this Mach number we read $A_i/A^* = \infty$. Thus the ratio of the throat area to the nozzle inlet area is $\dfrac{A^*}{A_i} = \dfrac{1}{\infty} = \mathbf{0}$.

Discussion If we solve this problem using the relations for compressible isentropic flow, the results would be identical.

12-50

Solution Air enters a converging nozzle at a specified temperature and pressure with low velocity. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure are to be calculated and plotted.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

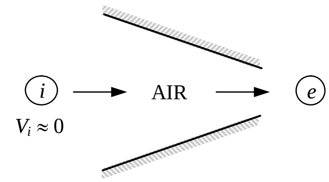
Properties The properties of air are $k = 1.4$, $R = 0.287$ kJ/kg·K, and $c_p = 1.005$ kJ/kg·K.

Analysis The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.,

$$P_0 = P_i = 900 \text{ kPa}$$

$$T_0 = T_i = 400 \text{ K}$$

The critical pressure is determined to be



$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (900\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 475.5\text{ kPa}$$

Then the pressure at the exit plane (throat) will be

$$P_e = P_b \qquad \text{for} \qquad P_b \geq 475.5 \text{ kPa}$$

$$P_e = P^* = 475.5 \text{ kPa} \qquad \text{for} \qquad P_b < 475.5 \text{ kPa (choked flow)}$$

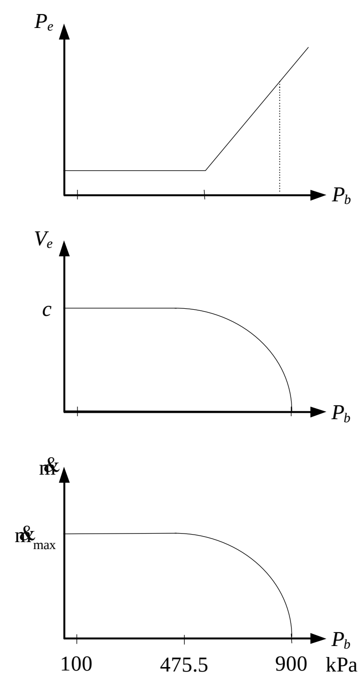
Thus the back pressure will not affect the flow when $100 < P_b < 475.5$ kPa. For a specified exit pressure P_e, the temperature, the velocity and the mass flow rate can be determined from

Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (400\text{ K})\left(\frac{P_e}{900}\right)^{0.4/1.4}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005\text{ kJ/kg}\cdot\text{K})(400 - T_e)\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287\text{ kPa}\cdot\text{m}^3/kg\cdot K)T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001\text{ m}^2)$$



The results of the calculations are tabulated as

P_b, kPa	P_e, kPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	$\dot{m}$, kg / s
900	900	400	0	7.840	0
800	800	386.8	162.9	7.206	1.174
700	700	372.3	236.0	6.551	1.546
600	600	356.2	296.7	5.869	1.741
500	500	338.2	352.4	5.151	1.815
475.5	475.5	333.3	366.2	4.971	1.820
400	475.5	333.3	366.2	4.971	1.820
300	475.5	333.3	366.2	4.971	1.820
200	475.5	333.3	366.2	4.971	1,820
100	475.5	333.3	366.2	4.971	1.820

Discussion We see from the plots that once the flow is choked at a back pressure of 475.5 kPa, the mass flow rate remains constant regardless of how low the back pressure gets.

12-51

Solution We are to reconsider the previous problem. Using EES (or other) software, we are to solve the problem for the inlet conditions of 0.8 MPa and 1200 K.

Analysis Air at 800 kPa, 1200 K enters a converging nozzle with a negligible velocity. The throat area of the nozzle is 10 cm2. Assuming isentropic flow, calculate and plot the exit pressure, the exit velocity, and the mass flow rate versus the back pressure P_b for 0.8>= P_b >=0.1 MPa.

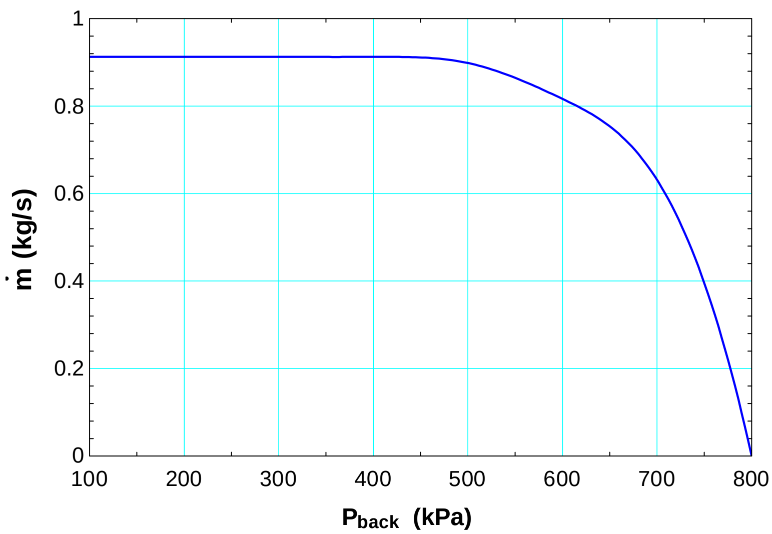
```
Procedure ExitPress(P_back,P_crit : P_exit, Condition$)
If  (P_back>=P_crit)  then
    P_exit:=P_back                              "Unchoked Flow Condition"
    Condition$:='unchoked'
else
    P_exit:=P_crit                              "Choked Flow Condition"
    Condition$:='choked'
Endif
End

Gas$='Air'
A_cm2=10  "Throat area, cm2"
P_inlet =800"kPa"
T_inlet= 1200"K"
"P_back =422.7"  "kPa"

A_exit = A_cm2*Convert(cm^2,m^2)
C_p=specheat(Gas$,T=T_inlet)
C_p-C_v=R
k=C_p/C_v
M=MOLARMASS(Gas$)                               "Molar mass of  Gas$"
R= 8.314/M                                      "Gas constant for Gas$"

"Since the inlet velocity is negligible, the stagnation temperature = T_inlet;
and, since the nozzle is isentropic, the stagnation pressure = P_inlet."

P_o=P_inlet                                     "Stagnation pressure"
T_o=T_inlet                                     "Stagnation temperature"
P_crit /P_o=(2/(k+1))^(k/(k-1))                 "Critical pressure from Eq. 16-22"
Call ExitPress(P_back,P_crit : P_exit, Condition$)

T_exit /T_o=(P_exit/P_o)^((k-1)/k)              "Exit temperature for isentopic flow, K"

V_exit ^2/2=C_p*(T_o-T_exit)*1000               "Exit velocity, m/s"

Rho_exit=P_exit/(R*T_exit)                      "Exit density, kg/m3"

m_dot=Rho_exit*V_exit*A_exit                    "Nozzle mass flow rate, kg/s"

"If you wish to redo the plots, hide the diagram window and remove the { } from
the first 4 variables just under the procedure.  Next set the desired range of
back pressure in the parametric table.  Finally, solve the table (F3).  "
```

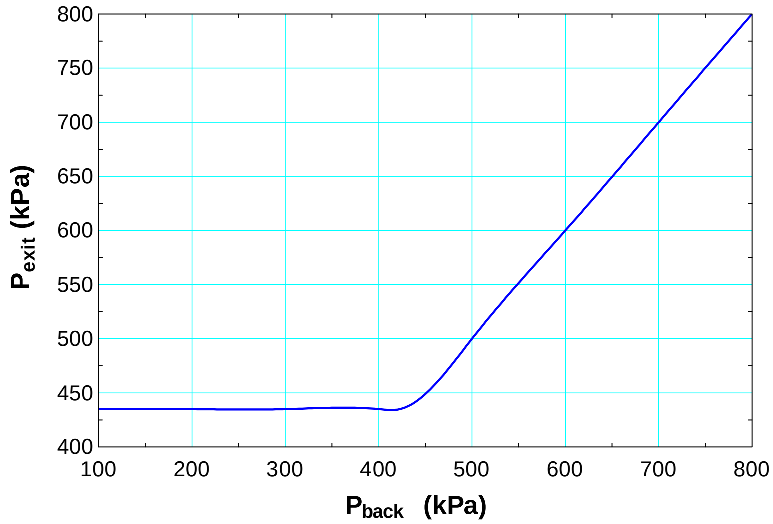
The table of results and the corresponding plot are provided below.

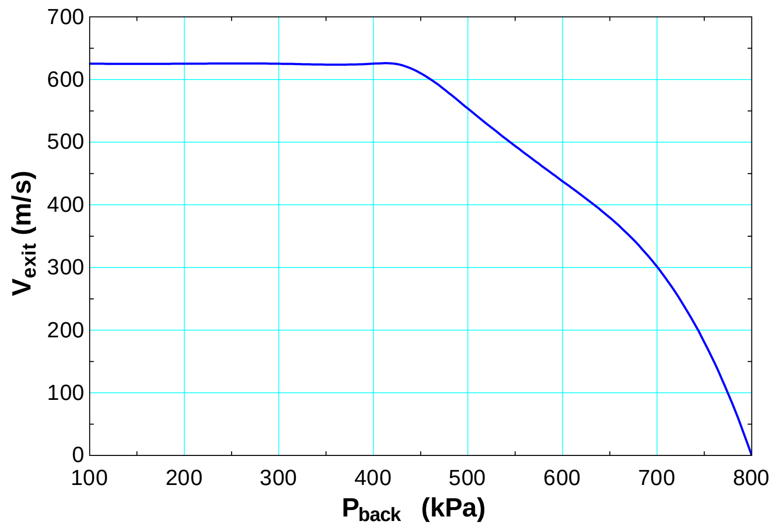
EES SOLUTION

A_cm2=10
A_exit=0.001
Condition$='choked'
C_p=1.208
C_v=0.9211
Gas$='Air'
k=1.312
M=28.97
m_dot=0.9124
P_back=422.7
P_crit=434.9
P_exit=434.9
P_inlet=800
P_o=800
R=0.287
Rho_exit=1.459
T_exit=1038
T_inlet=1200
T_o=1200
V_exit=625.2

P_{back}[kPa]	P_{exit} [kPa]	V_{exit} [m/s]	m [kg/s]	T_{exit} [K]	ρ_{exit} [kg/m^3]
100	434.9	625.2	0.9124	1038	1.459
200	434.9	625.2	0.9124	1038	1.459
300	434.9	625.2	0.9124	1038	1.459
400	434.9	625.2	0.9124	1038	1.459
422.7	434.9	625.2	0.9124	1038	1.459
500	500	553.5	0.8984	1073	1.623
600	600	437.7	0.8164	1121	1.865
700	700	300.9	0.6313	1163	2.098
800	800	0.001523	0.000003538	1200	2.323







Discussion We see from the plot that once the flow is choked at a back pressure of 422.7 kPa, the mass flow rate remains constant regardless of how low the back pressure gets.

Shock Waves and Expansion Waves

12-52C

Solution We are to discuss the applicability of the isentropic flow relations across shocks and expansion waves.

Analysis The isentropic relations of ideal gases are ***not*** **applicable for flows across (*a*) normal shock waves and (*b*) oblique shock waves**, but they ***are*** **applicable for flows across (*c*) Prandtl-Meyer expansion waves**.

Discussion Flow across any kind of shock wave involves irreversible losses – hence, it cannot be isentropic.

12-53C

Solution We are to discuss the states on the Fanno and Rayleigh lines.

Analysis The *Fanno line* represents the **states that satisfy the conservation of mass and energy equations**. The *Rayleigh line* represents the **states that satisfy the conservation of mass and momentum equations**. The *intersections* points of these lines represent the **states that satisfy the conservation of mass, energy, and momentum equations**.

Discussion *T-s* diagrams are quite helpful in understanding these kinds of flows.

12-54C

Solution We are to analyze a claim about oblique shock analysis.

Analysis **Yes, the claim is correct**. Conversely, normal shocks can be thought of as special oblique shocks in which the shock angle is $\beta = \pi/2$, or 90^{o}.

Discussion The component of flow in the direction normal to the oblique shock acts exactly like a normal shock. We can think of the flow parallel to the oblique shock as "going along for the ride" – it does not affect anything.

12-55C

Solution We are to discuss the effect of a normal shock wave on several properties.

Analysis (*a*) **velocity decreases**, (*b*) **static temperature increases**, (*c*) **stagnation temperature remains the same**, (*d*) **static pressure increases**, and (*e*) **stagnation pressure decreases**.

Discussion In addition, the Mach number goes from supersonic (Ma > 1) to subsonic (Ma < 1).

12-56C

Solution We are to discuss the formation of oblique shocks and how they differ from normal shocks.

Analysis *Oblique shocks* occur when **a gas flowing at supersonic speeds strikes a flat or inclined surface**. Normal shock waves are perpendicular to flow whereas inclined shock waves, as the name implies, are typically **inclined relative to the flow direction**. Also, normal shocks form a straight line whereas **oblique shocks can be straight or curved**, depending on the surface geometry.

Discussion In addition, while a normal shock must go from supersonic (Ma > 1) to subsonic (Ma < 1), the Mach number downstream of an oblique shock can be either supersonic or subsonic.

12-57C

Solution We are to discuss whether the flow upstream and downstream of an oblique shock needs to be supersonic.

Analysis **Yes**, the *upstream flow* has to be supersonic for an oblique shock to occur. **No**, the flow *downstream* of an oblique shock can be subsonic, sonic, and even supersonic.

Discussion The latter is not true for normal shocks. For a normal shock, the flow must always go from supersonic (Ma > 1) to subsonic (Ma < 1).

12-58C

Solution We are to determine if Ma downstream of a normal shock can be supersonic.

Analysis **No**, the second law of thermodynamics requires the flow after the shock to be subsonic.

Discussion A normal shock wave always goes from supersonic to subsonic in the flow direction.

12-59C

Solution We are to discuss shock detachment at the nose of a 2-D wedge-shaped body.

Analysis **When the wedge half-angle δ is greater than the maximum deflection angle θ_{max}**, the shock becomes curved and detaches from the nose of the wedge, forming what is called a *detached oblique shock* or a *bow wave*. The numerical value of the shock angle at the nose is $\beta = 90^o$.

Discussion When δ is less than θ_{max}, the oblique shock is attached to the nose.

12-60C

Solution We are to discuss the shock at the nose of a rounded body in supersonic flow.

Analysis When supersonic flow impinges on a blunt body like the rounded nose of an aircraft, the wedge half-angle δ at the nose is 90^o, and an attached oblique shock cannot exist, regardless of Mach number. Therefore, **a detached oblique shock must occur in front of *all* such blunt-nosed bodies**, whether two-dimensional, axisymmetric, or fully three-dimensional.

Discussion Since $\delta = 90^o$ at the nose, δ is always greater than θ_{max}, regardless of Ma or the shape of the rest of the body.

12-61C

Solution We are to discuss if a shock wave can develop in the converging section of a C-V nozzle.

Analysis **No**, because the flow must be supersonic before a shock wave can occur. The flow in the converging section of a nozzle is always subsonic.

Discussion A normal shock (if it is to occur) would occur in the supersonic (diverging) section of the nozzle.

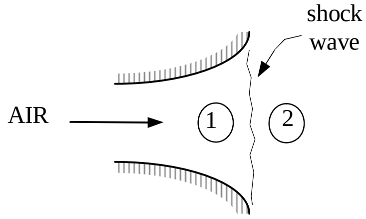
12-62

Solution Air flowing through a nozzle experiences a normal shock. Various properties are to be calculated before and after the shock.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are k= 1.4, R = 0.287 kJ/kg·K, and c_p = 1.005 kJ/kg·K.

Analysis The stagnation temperature and pressure before the shock are



$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 230 + \frac{(815\ \text{m/s})^2}{2(1.005\ \text{kJ/kg}\cdot\text{K})}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 560.5\ \text{K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (26\ \text{kPa})\left(\frac{560.5\ \text{K}}{230\ \text{K}}\right)^{1.4/(1.4-1)} = 587.3\ \text{kPa}$$

The velocity and the Mach number before the shock are determined from

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(230\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{304.0\ m/s}$$

and

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{815\ \text{m/s}}{304.0\ \text{m/s}} = \mathbf{2.681}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For Ma_1 = 2.681 we read

$$\text{Ma}_2 = \mathbf{0.4972}, \quad \frac{P_{02}}{P_1} = 9.7330, \quad \frac{P_2}{P_1} = 8.2208, \quad \text{and} \quad \frac{T_2}{T_1} = 2.3230$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

P_{02} = 9.7330P_1 = (9.7330)(26 kPa) = **253.1 kPa**

P_2 = 8.2208P_1 = (8.2208)(26 kPa) = **213.7 kPa**

T_2 = 2.3230T_1 = (2.3230)(230 K) = **534.3 K**

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.4972)\sqrt{(1.4)(0.287\ \text{kJ/kg.K})(534.3\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{230.4\ m/s}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

12-63

Solution Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K and c_p = 1.005 kJ/kg·K.

Analysis The entropy change across the shock is determined to be

$$\begin{aligned} s_2 - s_1 &= c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \\ &= (1.005\ \text{kJ/kg}\cdot\text{K})\ln(2.3230) - (0.287\ \text{kJ/kg}\cdot\text{K})\ln(8.2208) \\ &= \mathbf{0.242\ kJ/kg\cdot K} \end{aligned}$$

Discussion A shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

12-64

Solution For an ideal gas flowing through a normal shock, a relation for V_2/V_1 in terms of k, Ma_1, and Ma_2 is to be developed.

Analysis The conservation of mass relation across the shock is $\rho_1 V_1 = \rho_2 V_2$ and it can be expressed as

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{P_1/RT_1}{P_2/RT_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)$$

From Eqs. 12-35 and 12-38,

$$\frac{V_2}{V_1} = \left(\frac{1+k\text{Ma}_2^2}{1+k\text{Ma}_1^2}\right)\left(\frac{1+\text{Ma}_1^2(k-1)/2}{1+\text{Ma}_2^2(k-1)/2}\right)$$

Discussion This is an important relation as it enables us to determine the velocity ratio across a normal shock when the Mach numbers before and after the shock are known.

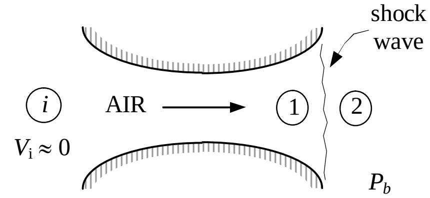
12-65

Solution Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{01} = P_i = 2 \text{ MPa}$$



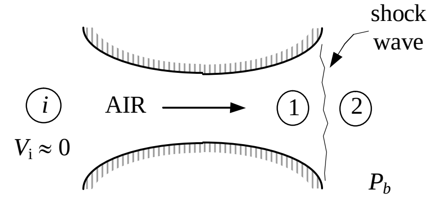
It is specified that A/A^* =3.5. From Table A-13, Mach number and the pressure ratio which corresponds to this area ratio are the Ma_1 =2.80 and P_1/P_{01} = 0.0368. The pressure ratio across the shock for this Ma_1 value is, from Table A-14, P_2/P_1 = 8.98. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 8.98P_1 = 8.98\times0.0368P_{01} = 8.98\times0.0368\times(2 \text{ MPa}) = \mathbf{0.661\ MPa}$$

Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

12-66

Solution Air enters a converging-diverging nozzle at a specified state. The required back pressure that produces a normal shock at the exit plane is to be determined for the specified nozzle geometry.



Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Analysis The inlet stagnation pressure in this case is identical to the inlet pressure since the inlet velocity is negligible. Since the flow before the shock to be isentropic,

$$P_{0x} = P_i = 2 \text{ MPa}$$

It is specified that A/A^* = 2. From Table A-13, the Mach number and the pressure ratio which corresponds to this area ratio are the Ma_1 =2.20 and P_1/P_{01} = 0.0935. The pressure ratio across the shock for this M_1 value is, from Table A-14, P_2/P_1 = 5.48. Thus the back pressure, which is equal to the static pressure at the nozzle exit, must be

$$P_2 = 5.48P_1 = 5.48\times0.0935P_{01} = 5.48\times0.0935\times(2 \text{ MPa}) = \mathbf{1.02\ MPa}$$

Discussion We can also solve this problem using the relations for compressible flow and normal shock functions. The results would be identical.

12-67E

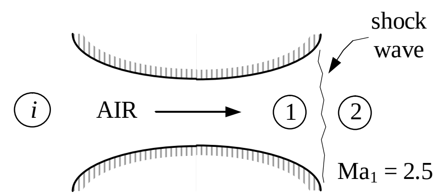
Solution Air flowing through a nozzle experiences a normal shock. Effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are k = 1.4 and R = 0.06855 Btu/lbm·R, and the properties of helium are k = 1.667 and R = 0.4961 Btu/lbm·R.

Analysis The air properties upstream the shock are

Ma_1 = 2.5, P_1 = 10 psia, and T_1 = 440.5 R



Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For Ma_1 = 2.5,

$$\text{Ma}_2 = \mathbf{0.513},\ \frac{P_{02}}{P_1} = 8.5262,\ \frac{P_2}{P_1} = 7.125,\ \text{and}\ \frac{T_2}{T_1} = 2.1375$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 8.5262P_1 = (8.5262)(10\text{ psia}) = \mathbf{85.3\ psia}$$

$$P_2 = 7.125P_1 = (7.125)(10\text{ psia}) = \mathbf{71.3\ psia}$$

$$T_2 = 2.1375T_1 = (2.1375)(440.5\text{ R}) = \mathbf{942\ R}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.513)\sqrt{(1.4)(0.06855\text{ Btu/1bm}\cdot\text{R})(941.6\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = \mathbf{772\ \ ft/s}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-14 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k/(k-1) - 1}\right)^{1/2} = \left(\frac{2.5^2 + 2/(1.667-1)}{2\times 2.5^2 \times 1.667/(1.667-1) - 1}\right)^{1/2} = \mathbf{0.553}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{1 + 1.667\times 2.5^2}{1 + 1.667\times 0.553^2} = 7.5632$$

$$\frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2} = \frac{1 + 2.5^2(1.667-1)/2}{1 + 0.553^2(1.667-1)/2} = 2.7989$$

$$\frac{P_{02}}{P_1} = \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2}\right)\left(1 + (k-1)\text{Ma}_2^2/2\right)^{k/(k-1)}$$

$$= \left(\frac{1 + 1.667\times 2.5^2}{1 + 1.667\times 0.553^2}\right)\left(1 + (1.667-1)\times 0.553^2/2\right)^{1.667/0.667} = 9.641$$

Thus, $P_{02} = 11.546P_1 = (11.546)(10\text{ psia}) = \mathbf{115\ psia}$

$$P_2 = 7.5632P_1 = (7.5632)(10\text{ psia}) = \mathbf{75.6\ psia}$$

$$T_2 = 2.7989T_1 = (2.7989)(440.5\text{ R}) = \mathbf{1233\ R}$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.553)\sqrt{(1.667)(0.4961\text{ Btu/1bm.R})(1232.9\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = \mathbf{2794\ \ ft/s}$$

Discussion This problem could also be solved using the relations for compressible flow and normal shock functions. The results would be identical.

12-68E

Solution We are to reconsider Prob. 12-67E. Using EES (or other) software, we are to study the effects of both air and helium flowing steadily in a nozzle when there is a normal shock at a Mach number in the range 2 < Mx < 3.5. In addition to the required information, we are to calculate the entropy change of the air and helium across the normal shock, and tabulate the results in a parametric table.

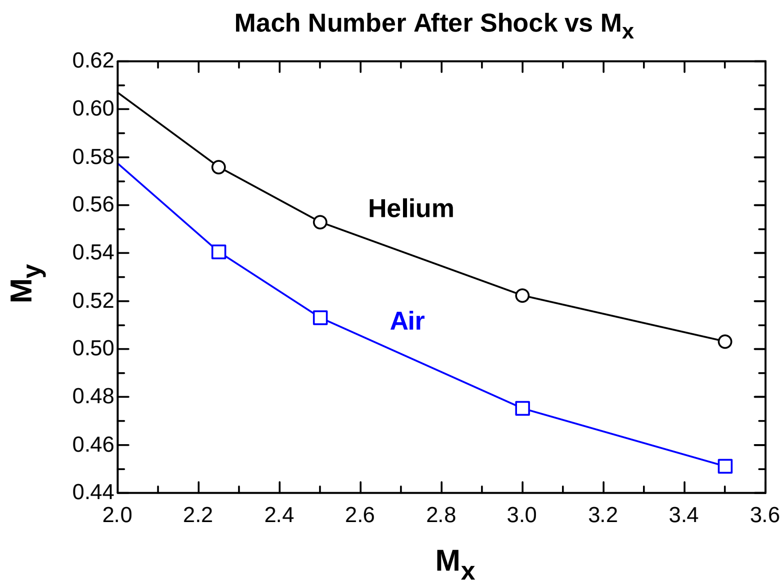
Analysis We use EES to calculate the entropy change of the air and helium across the normal shock. The results are given in the Parametric Table for 2 < M_x < 3.5.

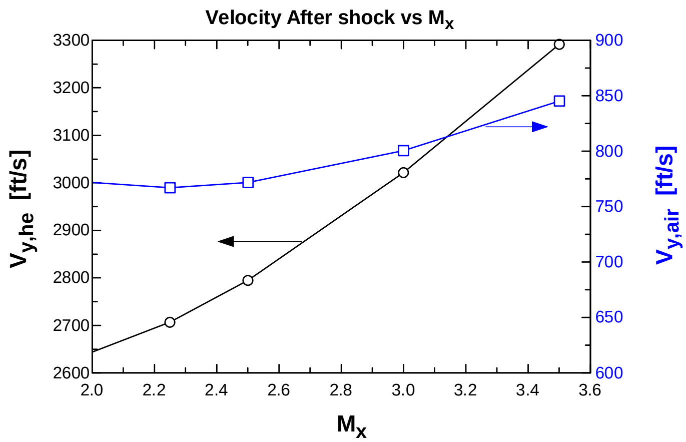
```
Procedure NormalShock(M_x,k:M_y,PyOPx, TyOTx,RhoyORhox, PoyOPox, PoyOPx)
    If M_x < 1 Then
            M_y = -1000;PyOPx=-1000;TyOTx=-1000;RhoyORhox=-1000
            PoyOPox=-1000;PoyOPx=-1000
    else
            M_y=sqrt( (M_x^2+2/(k-1)) / (2*M_x^2*k/(k-1)-1) )
            PyOPx=(1+k*M_x^2)/(1+k*M_y^2)
            TyOTx=( 1+M_x^2*(k-1)/2  )/(1+M_y^2*(k-1)/2  )
            RhoyORhox=PyOPx/TyOTx
            PoyOPox=M_x/M_y*(  (1+M_y^2*(k-1)/2)/ (1+M_x^2*(k-1)/2)   )^((k+1)/(2*(k-1)))
            PoyOPx=(1+k*M_x^2)*(1+M_y^2*(k-1)/2)^(k/(k-1))/(1+k*M_y^2)
    Endif
End

Function ExitPress(P_back,P_crit)
If P_back>=P_crit then ExitPress:=P_back     "Unchoked Flow Condition"
If P_back<P_crit then ExitPress:=P_crit      "Choked Flow Condition"
End

Procedure GetProp(Gas$:Cp,k,R) "Cp and k data are from Text Table A.2E"
   M=MOLARMASS(Gas$)             "Molar mass of  Gas$"
   R= 1545/M                     "Particular gas constant for Gas$, ft-lbf/lbm-R"
                                 "k = Ratio of Cp to Cv"
                                 "Cp = Specific heat at constant pressure"
   if Gas$='Air' then
                                 Cp=0.24"Btu/lbm-R"; k=1.4
   endif
   if Gas$='CO2' then
                                 Cp=0.203"Btu/lbm_R"; k=1.289
   endif
   if Gas$='Helium' then
                                 Cp=1.25"Btu/lbm-R"; k=1.667
   endif
End

"Variable Definitions:"
"M = flow Mach Number"
"P_ratio = P/P_o  for compressible, isentropic flow"
"T_ratio = T/T_o for compressible, isentropic flow"
"Rho_ratio= Rho/Rho_o for compressible, isentropic flow"
"A_ratio=A/A* for compressible, isentropic flow"
"Fluid properties before the shock are denoted with a subscript x"
"Fluid properties after the shock are denoted with a subscript y"
"M_y = Mach Number down stream of normal shock"
"PyOverPx= P_y/P_x  Pressue ratio across normal shock"
"TyOverTx =T_y/T_x  Temperature ratio across normal shock"
"RhoyOverRhox=Rho_y/Rho_x Density ratio across normal shock"
"PoyOverPox = P_oy/P_ox  Stagantion pressure ratio across normal shock"
"PoyOverPx = P_oy/P_x  Stagnation pressure after normal shock ratioed to pressure before shock"

"Input Data"
{P_x = 10 "psia"}                "Values of P_x, T_x, and M_x are set in the Parametric Table"
{T_x = 440.5 "R"}
```

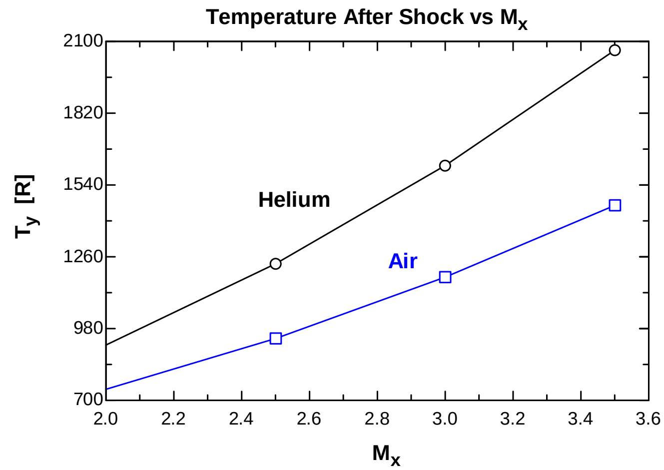
```
{M_x = 2.5}
Gas$='Air'  "This program has been written for the gases Air, CO2, and Helium"
Call GetProp(Gas$:Cp,k,R)
Call NormalShock(M_x,k:M_y,PyOverPx, TyOverTx,RhoyOverRhox, PoyOverPox, PoyOverPx)
P_oy_air=P_x*PoyOverPx              "Stagnation pressure after the shock"
P_y_air=P_x*PyOverPx                "Pressure after the shock"
T_y_air=T_x*TyOverTx                "Temperature after the shock"
M_y_air=M_y                         "Mach number after the shock"

"The velocity after the shock can be found from the product of the Mach number and
speed of sound after the shock."
C_y_air = sqrt(k*R"ft-lbf/lbm_R"*T_y_air"R"*32.2 "lbm-ft/lbf-s^2")
V_y_air=M_y_air*C_y_air
DELTAs_air=entropy(air,T=T_y_air, P=P_y_air) -entropy(air,T=T_x,P=P_x)

Gas2$='Helium'  "Gas2$ can be either Helium or CO2"
Call GetProp(Gas2$:Cp_2,k_2,R_2)
Call NormalShock(M_x,k_2:M_y2,PyOverPx2, TyOverTx2,RhoyOverRhox2, PoyOverPox2, PoyOverPx2)
P_oy_he=P_x*PoyOverPx2              "Stagnation pressure after the shock"
P_y_he=P_x*PyOverPx2                "Pressure after the shock"
T_y_he=T_x*TyOverTx2                "Temperature after the shock"
M_y_he=M_y2                         "Mach number after the shock"

"The velocity after the shock can be found from the product of the Mach number and
speed of sound after the shock."
C_y_he = sqrt(k_2*R_2"ft-lbf/lbm_R"*T_y_he"R"*32.2 "lbm-ft/lbf-s^2")
V_y_he=M_y_he*C_y_he
DELTAs_he=entropy(helium,T=T_y_he, P=P_y_he) -entropy(helium,T=T_x,P=P_x)
```

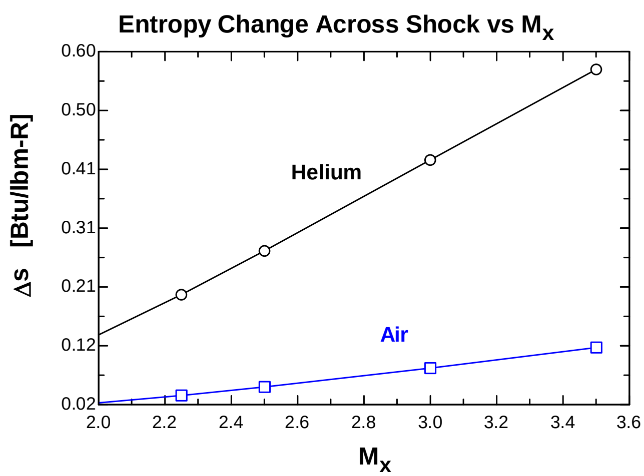
The parametric table and the corresponding plots are shown below.

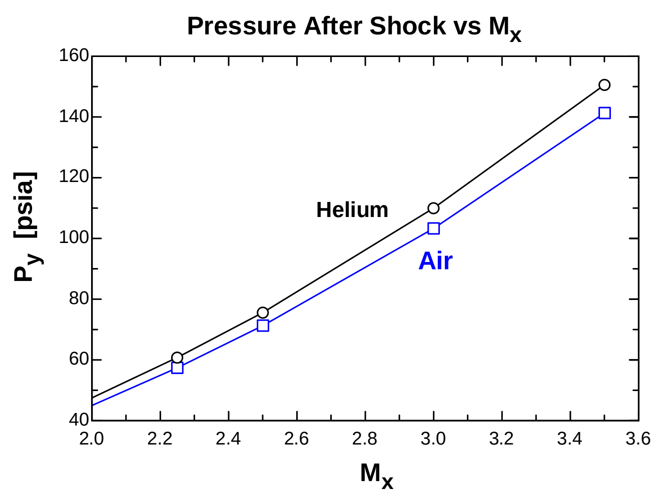
$V_{y,he}$ [ft/s]	$V_{y,air}$ [ft/s]	$T_{y,he}$ [R]	$T_{y,air}$ [R]	T_x [R]	$P_{y,he}$ [psia]	$P_{y,air}$ [psia]	P_x [psia]	$P_{oy,he}$ [psia]	$P_{oy,air}$ [psia]	$M_{y,he}$	$M_{y,air}$	M_x	Δs_{he} [Btu/lbm-R]	Δs_{air} [Btu/lbm-R]
2644	771.9	915.6	743.3	440.5	47.5	45	10	63.46	56.4	0.607	0.5774	2	0.1345	0.0228
2707	767.1	1066	837.6	440.5	60.79	57.4	10	79.01	70.02	0.5759	0.5406	2.25	0.2011	0.0351
2795	771.9	1233	941.6	440.5	75.63	71.25	10	96.41	85.26	0.553	0.513	2.5	0.2728	0.04899
3022	800.4	1616	1180	440.5	110	103.3	10	136.7	120.6	0.5223	0.4752	3	0.4223	0.08
3292	845.4	2066	1460	440.5	150.6	141.3	10	184.5	162.4	0.5032	0.4512	3.5	0.5711	0.1136











Discussion In all cases, regardless of the fluid or the Mach number, entropy increases across a shock wave. This is because a shock wave involves irreversibilities.

12-69

Solution Air flowing through a converging-diverging nozzle experiences a normal shock at the exit. The effect of the shock wave on various properties is to be determined.

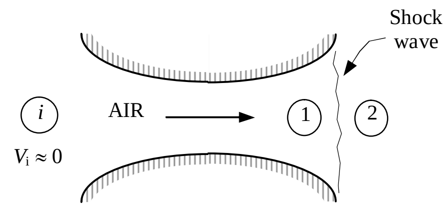
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Then,

$$P_{01} = P_i = 1 \text{ MPa}$$

$$T_{01} = T_i = 300 \text{ K}$$



Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (300\text{ K})\left(\frac{2}{2+(1.4-1)2.4^2}\right) = 139.4\text{ K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_0}\right)^{k/(k-1)} = (1\text{ MPa})\left(\frac{139.4}{300}\right)^{1.4/0.4} = 0.06840\text{ MPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 2.4$ we read

$$\text{Ma}_2 = 0.5231 \cong \mathbf{0.523}, \quad \frac{P_{02}}{P_{01}} = 0.5401, \quad \frac{P_2}{P_1} = 6.5533, \quad \text{and} \quad \frac{T_2}{T_1} = 2.0403$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.5401 P_{01} = (0.5401)(1.0\text{ MPa}) = \mathbf{0.540\text{ MPa} = 540\text{ kPa}}$$

$$P_2 = 6.5533 P_1 = (6.5533)(0.06840\text{ MPa}) = \mathbf{0.448\text{ MPa} = 448\text{ kPa}}$$

$$T_2 = 2.0403 T_1 = (2.0403)(139.4\text{ K}) = \mathbf{284\text{ K}}$$

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5231)\sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(284\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{177\ \ m/s}$$

Discussion We can also solve this problem using the relations for normal shock functions. The results would be identical.

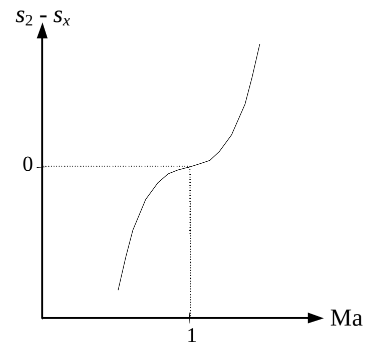
12-70

Solution The entropy change of air across the shock for upstream Mach numbers between 0.5 and 1.5 is to be determined and plotted.

Assumptions **1** Air is an ideal gas. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are k = 1.4, R = 0.287 kJ/kg·K, and c_p = 1.005 kJ/kg·K.

Analysis The entropy change across the shock is determined to be



$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

where

$$\mathrm{Ma}_2 = \left(\frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2\mathrm{Ma}_1^2 k/(k-1) - 1}\right)^{1/2}, \quad \frac{P_2}{P_1} = \frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}, \text{ and } \frac{T_2}{T_1} = \frac{1+\mathrm{Ma}_1^2(k-1)/2}{1+\mathrm{Ma}_2^2(k-1)/2}$$

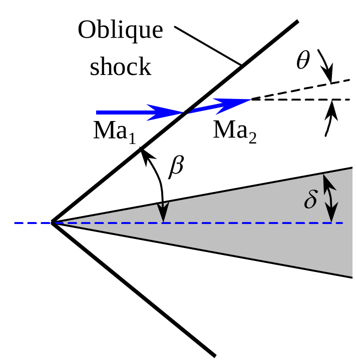
The results of the calculations can be tabulated as

Ma_1	Ma_2	T_2/T_1	P_2/P_1	$s_2 - s_1$
0.5	2.6458	0.1250	0.4375	-1.853
0.6	1.8778	0.2533	0.6287	-1.247
0.7	1.5031	0.4050	0.7563	-0.828
0.8	1.2731	0.5800	0.8519	-0.501
0.9	1.1154	0.7783	0.9305	-0.231
1.0	1.0000	1.0000	1.0000	0.0
1.1	0.9118	1.0649	1.2450	0.0003
1.2	0.8422	1.1280	1.5133	0.0021
1.3	0.7860	1.1909	1.8050	0.0061
1.4	0.7397	1.2547	2.1200	0.0124
1.5	0.7011	1.3202	2.4583	0.0210

Discussion The total entropy change is negative for upstream Mach numbers Ma_1 less than unity. Therefore, normal shocks cannot occur when $\mathrm{Ma}_1 < 1$.

12-71

Solution Supersonic airflow approaches the nose of a two-dimensional wedge and undergoes a straight oblique shock. For a specified Mach number, the minimum shock angle and the maximum deflection angle are to be determined.



Assumptions Air is an ideal gas with a constant specific heat ratio of k = 1.4 (so that Fig. 12-41 is applicable).

Analysis For Ma = 5, we read from Fig. 12-41

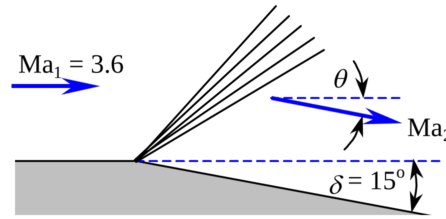
Minimum shock (or wave) angle: $\beta_{\min} = \mathbf{12°}$

Maximum deflection (or turning) angle: $\theta_{\max} = \mathbf{41.5°}$

Discussion Note that the minimum shock angle decreases and the maximum deflection angle increases with increasing Mach number Ma_1.

12-72

Solution Air flowing at a specified supersonic Mach number undergoes an expansion turn. The Mach number, pressure, and temperature downstream of the sudden expansion along a wall are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is k = 1.4.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15^{\circ}$. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right)$$

Upstream:

$$\nu(\mathrm{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(3.6^2-1)}\right) - \tan^{-1}\left(\sqrt{3.6^2-1}\right) = 60.09°$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\mathrm{Ma}_2) = \theta + \nu(\mathrm{Ma}_1) = 15° + 60.09° = 75.09°$$

Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

Downstream: $$\nu(\mathrm{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}\mathrm{Ma}_2^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}_2^2-1}\right) = 75.09°$$

Solution of this implicit equation gives Ma_2 = **4.81**. Then the downstream pressure and temperature are determined from the isentropic flow relations:

$$P_2 = \frac{P_2/P_0}{P_1/P_0}P_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-k/(k-1)}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-k/(k-1)}}P_1 = \frac{[1+4.81^2(1.4-1)/2]^{-1.4/0.4}}{[1+3.6^2(1.4-1)/2]^{-1.4/0.4}}(32\text{ kPa}) = \mathbf{6.65\ kPa}$$

$$T_2 = \frac{T_2/T_0}{T_1/T_0}T_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-1}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-1}}T_1 = \frac{[1+4.81^2(1.4-1)/2]^{-1}}{[1+3.6^2(1.4-1)/2]^{-1}}(240\text{ K}) = \mathbf{153\ K}$$

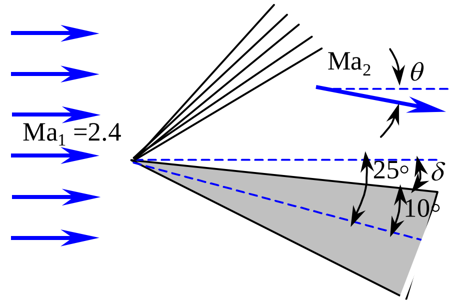
Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html .

12-73

Solution Air flowing at a specified supersonic Mach number undergoes an expansion turn over a tilted wedge. The Mach number, pressure, and temperature downstream of the sudden expansion above the wedge are to be determined.

Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.



Properties The specific heat ratio of air is k = 1.4.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta$ = 25° - 10° = 15°. Then the upstream and downstream Prandtl-Meyer functions are determined to be

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right)$$

Upstream:

$$\nu(\mathrm{Ma}_1) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(2.4^2-1)}\right) - \tan^{-1}\left(\sqrt{2.4^2-1}\right) = 36.75°$$

Then the downstream Prandtl-Meyer function becomes

$$\nu(\mathrm{Ma}_2) = \theta + \nu(\mathrm{Ma}_1) = 15° + 36.75° = 51.75°$$

Now Ma_2 is found from the Prandtl-Meyer relation, which is now implicit:

Downstream: $$\nu(\mathrm{Ma}_2) = \sqrt{\frac{1.4+1}{1.4-1}}\tan^{-1}\left(\sqrt{\frac{1.4-1}{1.4+1}(\mathrm{Ma}_2^2-1)}\right) - \tan^{-1}\left(\sqrt{\mathrm{Ma}_2^2-1}\right) = 51.75°$$

It gives Ma_2 = **3.105**. Then the downstream pressure and temperature are determined from the isentropic flow relations

$$P_2 = \frac{P_2/P_0}{P_1/P_0}P_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-k/(k-1)}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-k/(k-1)}}P_1 = \frac{[1+3.105^2(1.4-1)/2]^{-1.4/0.4}}{[1+2.4^2(1.4-1)/2]^{-1.4/0.4}}(70\text{ kPa}) = \mathbf{23.8\ kPa}$$

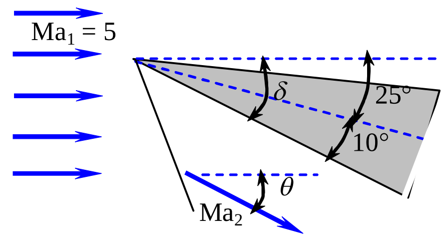
$$T_2 = \frac{T_2/T_0}{T_1/T_0}T_1 = \frac{[1+\mathrm{Ma}_2^2(k-1)/2]^{-1}}{[1+\mathrm{Ma}_1^2(k-1)/2]^{-1}}T_1 = \frac{[1+3.105^2(1.4-1)/2]^{-1}}{[1+2.4^2(1.4-1)/2]^{-1}}(260\text{ K}) = \mathbf{191\ K}$$

Note that this is an expansion, and Mach number increases while pressure and temperature decrease, as expected.

Discussion There are compressible flow calculators on the Internet that solve these implicit equations that arise in the analysis of compressible flow, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html .

12-74

Solution Air flowing at a specified supersonic Mach number undergoes a compression turn (an oblique shock) over a tilted wedge. The Mach number, pressure, and temperature downstream of the shock below the wedge are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, the deflection angle is determined to be $\theta \approx \delta = 25° + 10° = 35°$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\text{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\text{Ma}_1^2(k+\cos 2\beta)+2} \qquad \rightarrow \qquad \tan 12° = \frac{2(3.4^2 \sin^2\beta - 1)/\tan\beta}{3.4^2(1.4+\cos 2\beta)+2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\text{weak}} = 49.86°$ and $\beta_{\text{strong}} = 77.66°$. Then for the case of strong oblique shock, the upstream "normal" Mach number $\text{Ma}_{1,n}$ becomes

$$\text{Ma}_{1,n} = \text{Ma}_1 \sin\beta = 5\sin 77.66° = 4.884$$

Also, the downstream normal Mach numbers $\text{Ma}_{2,n}$ become

$$\text{Ma}_{2,n} = \sqrt{\frac{(k-1)\text{Ma}_{1,n}^2 + 2}{2k\text{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(4.884)^2 + 2}{2(1.4)(4.884)^2 - 1.4 + 1}} = 0.4169$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\text{Ma}_{1,n}^2 - k + 1}{k+1} = (70\text{ kPa})\frac{2(1.4)(4.884)^2 - 1.4 + 1}{1.4+1} = \mathbf{1940\ kPa}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2+(k-1)\text{Ma}_{1,n}^2}{(k+1)\text{Ma}_{1,n}^2} = (260\text{ K})\frac{1940\text{ kPia}}{70\text{ kPa}}\frac{2+(1.4-1)(4.884)^2}{(1.4+1)(4.884)^2} = \mathbf{1450\ K}$$

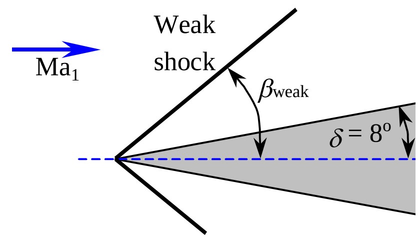
The downstream Mach number is determined to be

$$\text{Ma}_2 = \frac{\text{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.4169}{\sin(77.66° - 35°)} = \mathbf{0.615}$$

Discussion Note that $\text{Ma}_{1,n}$ is supersonic and $\text{Ma}_{2,n}$ and Ma_2 are subsonic. Also note the huge rise in temperature and pressure across the strong oblique shock, and the challenges they present for spacecraft during reentering the earth's atmosphere.

12-75E

Solution Air flowing at a specified supersonic Mach number is forced to turn upward by a ramp, and weak oblique shock forms. The wave angle, Mach number, pressure, and temperature after the shock are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the ramp, i.e., $\theta \approx \delta = 8^{\circ}$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \qquad \rightarrow \qquad \tan 8^{\circ} = \frac{2(2^2 \sin^2\beta - 1)/\tan\beta}{2^2(1.4 + \cos 2\beta) + 2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\boldsymbol{\beta_{\text{weak}}}$ **= 37.21°** and $\beta_{\text{strong}} = 85.05^{\circ}$. Then for the case of weak oblique shock, the upstream "normal" Mach number $\mathrm{Ma}_{1,\mathrm{n}}$ becomes

$$\mathrm{Ma}_{1,\mathrm{n}} = \mathrm{Ma}_1 \sin\beta = 2\sin 37.21^{\circ} = 1.209$$

Also, the downstream normal Mach numbers $\mathrm{Ma}_{2,\mathrm{n}}$ become

$$\mathrm{Ma}_{2,\mathrm{n}} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,\mathrm{n}}^2 + 2}{2k\mathrm{Ma}_{1,\mathrm{n}}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.209)^2 + 2}{2(1.4)(1.209)^2 - 1.4 + 1}} = 0.8363$$

The downstream pressure and temperature are determined to be

$$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,\mathrm{n}}^2 - k + 1}{k + 1} = (12\ \text{psia})\frac{2(1.4)(1.209)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{18.5\ psia}$$

$$T_2 = T_1 \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1 \frac{P_2}{P_1}\frac{2 + (k-1)\mathrm{Ma}_{1,\mathrm{n}}^2}{(k+1)\mathrm{Ma}_{1,\mathrm{n}}^2} = (490\ \text{R})\frac{18.5\ \text{psia}}{12\ \text{psia}}\frac{2 + (1.4-1)(1.209)^2}{(1.4+1)(1.209)^2} = \mathbf{556\ R}$$

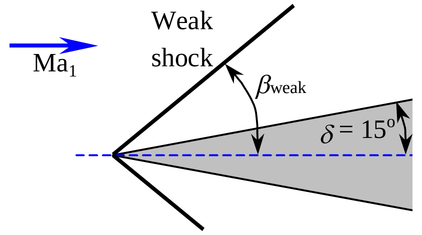
The downstream Mach number is determined to be

$$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,\mathrm{n}}}{\sin(\beta - \theta)} = \frac{0.8363}{\sin(37.21^{\circ} - 8^{\circ})} = \mathbf{1.71}$$

Discussion Note that $\mathrm{Ma}_{1,\mathrm{n}}$ is supersonic and $\mathrm{Ma}_{2,\mathrm{n}}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock (it is *subsonic* across the strong oblique shock).

12-76E

Solution Air flowing at a specified supersonic Mach number is forced to undergo a compression turn (an oblique shock)., The Mach number, pressure, and temperature downstream of the oblique shock are to be determined.

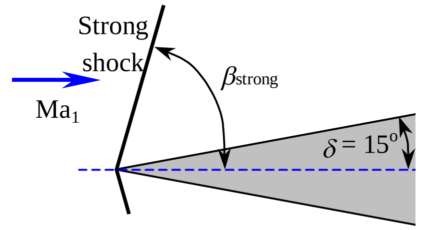


Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is $k = 1.4$.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta = 15^{\circ}$. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k+\cos 2\beta)+2} \quad \rightarrow \quad \tan 15^{\circ} = \frac{2(2^2 \sin^2\beta - 1)/\tan\beta}{2^2(1.4+\cos 2\beta)+2}$$



which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\beta_{\mathrm{weak}} = 45.34^{\circ}$ and $\beta_{\mathrm{strong}} = 79.83^{\circ}$. Then the upstream "normal" Mach number $\mathrm{Ma}_{1,n}$ becomes

Weak shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 45.34^{\circ} = 1.423$

Strong shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 2\sin 79.83^{\circ} = 1.969$

Also, the downstream normal Mach numbers $\mathrm{Ma}_{2,n}$ become

Weak shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}} = \sqrt{\frac{(1.4-1)(1.423)^2+2}{2(1.4)(1.423)^2-1.4+1}} = 0.7304$$

Strong shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2+2}{2k\mathrm{Ma}_{1,n}^2-k+1}} = \sqrt{\frac{(1.4-1)(1.969)^2+2}{2(1.4)(1.969)^2-1.4+1}} = 0.5828$$

The downstream pressure and temperature for each case are determined to be

Weak shock: $$P_2 = P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1} = (6\text{ psia})\frac{2(1.4)(1.423)^2-1.4+1}{1.4+1} = 17.57 \cong \mathbf{17.6\ psia}$$

$$T_2 = T_1\frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1\frac{P_2}{P_1}\frac{2+(k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (480\text{ R})\frac{17.57\text{ psia}}{8\text{ psia}}\frac{2+(1.4-1)(1.423)^2}{(1.4+1)(1.423)^2} = 609.5\text{ R} \cong \mathbf{610\ R}$$

Strong shock: $$P_2 = P_1\frac{2k\mathrm{Ma}_{1,n}^2-k+1}{k+1} = (8\text{ psia})\frac{2(1.4)(1.969)^2-1.4+1}{1.4+1} = 34.85 \cong \mathbf{34.9\ psia}$$

$$T_2 = T_1\frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = T_1\frac{P_2}{P_1}\frac{2+(k-1)\mathrm{Ma}_{1,n}^2}{(k+1)\mathrm{Ma}_{1,n}^2} = (480\text{ R})\frac{34.85\text{ psia}}{8\text{ psia}}\frac{2+(1.4-1)(1.969)^2}{(1.4+1)(1.969)^2} = 797.9\text{ R} \cong \mathbf{798\ R}$$

The downstream Mach number is determined to be

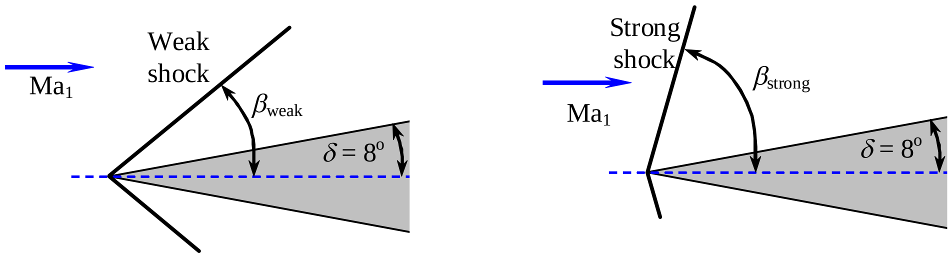
Weak shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.7304}{\sin(45.34^{\circ}-15^{\circ})} = \mathbf{1.45}$$

Strong shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta-\theta)} = \frac{0.5828}{\sin(79.83^{\circ}-15^{\circ})} = \mathbf{0.644}$$

Discussion Note that the change in Mach number, pressure, temperature across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock cases, $\mathrm{Ma}_{1,n}$ is supersonic and $\mathrm{Ma}_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.

12-77

Solution Air flowing at a specified supersonic Mach number impinges on a two-dimensional wedge, The shock angle, Mach number, and pressure downstream of the weak and strong oblique shock formed by a wedge are to be determined.



Assumptions **1** The flow is steady. **2** The boundary layer on the wedge is very thin. **3** Air is an ideal gas with constant specific heats.

Properties The specific heat ratio of air is k = 1.4.

Analysis On the basis of Assumption #2, we take the deflection angle as equal to the wedge half-angle, i.e., $\theta \approx \delta$ = 8^o. Then the two values of oblique shock angle β are determined from

$$\tan\theta = \frac{2(\mathrm{Ma}_1^2 \sin^2\beta - 1)/\tan\beta}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2} \quad \rightarrow \quad \tan 8° = \frac{2(3.4^2 \sin^2\beta - 1)/\tan\beta}{3.4^2(1.4 + \cos 2\beta) + 2}$$

which is implicit in β. Therefore, we solve it by an iterative approach or with an equation solver such as EES. It gives $\boldsymbol{\beta_{weak} = 23.15^o}$ and $\boldsymbol{\beta_{strong} = 87.45^o}$. Then the upstream "normal" Mach number $Ma_{1,n}$ becomes

Weak shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 3.4\sin 23.15° = 1.336$

Strong shock: $\mathrm{Ma}_{1,n} = \mathrm{Ma}_1 \sin\beta = 3.4\sin 87.45° = 3.397$

Also, the downstream normal Mach numbers $Ma_{2,n}$ become

Weak shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(1.336)^2 + 2}{2(1.4)(1.336)^2 - 1.4 + 1}} = 0.7681$$

Strong shock: $$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}} = \sqrt{\frac{(1.4-1)(3.397)^2 + 2}{2(1.4)(3.397)^2 - 1.4 + 1}} = 0.4553$$

The downstream pressure for each case is determined to be

Weak shock: $$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (60\ \mathrm{kPa})\frac{2(1.4)(1.336)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{115.0\ kPa}$$

Strong shock: $$P_2 = P_1 \frac{2k\mathrm{Ma}_{1,n}^2 - k + 1}{k+1} = (60\ \mathrm{kPa})\frac{2(1.4)(3.397)^2 - 1.4 + 1}{1.4 + 1} = \mathbf{797.6\ kPa}$$

The downstream Mach number is determined to be

Weak shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.7681}{\sin(23.15° - 8°)} = \mathbf{2.94}$$

Strong shock: $$\mathrm{Ma}_2 = \frac{\mathrm{Ma}_{2,n}}{\sin(\beta - \theta)} = \frac{0.4553}{\sin(87.45° - 8°)} = \mathbf{0.463}$$

Discussion Note that the change in Mach number and pressure across the *strong shock* are much greater than the changes across the *weak shock*, as expected. For both the weak and strong oblique shock cases, $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock.

12-78

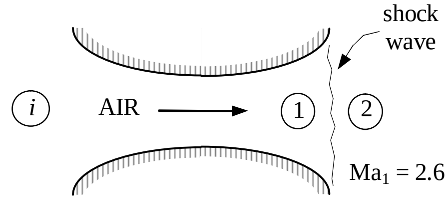
Solution Air flowing through a nozzle experiences a normal shock. The effect of the shock wave on various properties is to be determined. Analysis is to be repeated for helium under the same conditions.

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are $k = 1.4$ and $R = 0.287$ kJ/kg·K, and the properties of helium are $k = 1.667$ and R = 2.0769 kJ/kg·K.

Analysis The air properties upstream the shock are

$\text{Ma}_1 = 2.6$, $P_1 = 58$ kPa, and $T_1 = 270$ K



Fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions in Table A-14. For $\text{Ma}_1 = 2.6$,

$$\text{Ma}_2 = \mathbf{0.5039},\ \frac{P_{02}}{P_1} = 9.1813,\ \frac{P_2}{P_1} = 7.7200,\ \text{and}\ \frac{T_2}{T_1} = 2.2383$$

We obtained these values using analytical relations in Table 14. Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$P_{02} = 9.1813P_1 = (9.1813)(58 \text{ kPa}) =$ **532.5 kPa**

$P_2 = 7.7200P_1 = (7.7200)(58 \text{ kPa}) =$ **447.8 kPa**

$T_2 = 2.9220T_1 = (2.2383)(270 \text{ K}) =$ **604.3 K**

The air velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5039)\sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(604.3\text{K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{248.3\ m/s}$$

We now repeat the analysis for helium. This time we cannot use the tabulated values in Table A-14 since k is not 1.4. Therefore, we have to calculate the desired quantities using the analytical relations,

$$\text{Ma}_2 = \left(\frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k/(k-1) - 1}\right)^{1/2} = \left(\frac{2.6^2 + 2/(1.667-1)}{2\times 2.6^2 \times 1.667/(1.667-1) - 1}\right)^{1/2} = \mathbf{0.5455}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{1 + 1.667\times 2.6^2}{1 + 1.667\times 0.5455^2} = 8.2009$$

$$\frac{T_2}{T_1} = \frac{1 + \text{Ma}_1^2(k-1)/2}{1 + \text{Ma}_2^2(k-1)/2} = \frac{1 + 2.6^2(1.667-1)/2}{1 + 0.5455^2(1.667-1)/2} = 2.9606$$

$$\frac{P_{02}}{P_1} = \left(\frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2}\right)\left(1 + (k-1)\text{Ma}_2^2/2\right)^{k/(k-1)}$$

$$= \left(\frac{1 + 1.667\times 2.6^2}{1 + 1.667\times 0.5455^2}\right)\left(1 + (1.667-1)\times 0.5455^2/2\right)^{1.667/0.667} = 10.389$$

Thus, $P_{02} = 10.389P_1 = (10.389)(58 \text{ kPa}) =$ **602.5 kPa**

$P_2 = 8.2009P_1 = (8.2009)(58 \text{ kPa}) =$ **475.7 kPa**

$T_2 = 2.9606T_1 = (2.9606)(270 \text{ K}) =$ **799.4 K**

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.5455)\sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(799.4\text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = \mathbf{907.5\ m/s}$$

Discussion The velocity and Mach number are higher for helium than for air due to the different values of k and R.

12-79

Solution Air flowing through a nozzle experiences a normal shock. The entropy change of air across the normal shock wave is to be determined.

Assumptions **1** Air and helium are ideal gases with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs.

Properties The properties of air are R = 0.287 kJ/kg·K and c_p = 1.005 kJ/kg·K, and the properties of helium are R = 2.0769 kJ/kg·K and c_p = 5.1926 kJ/kg·K.

Analysis For air, the entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (1.005\text{ kJ/kg}\cdot\text{K})\ln(2.2383) - (0.287\text{ kJ/kg}\cdot\text{K})\ln(7.7200) = \mathbf{0.223\ kJ/kg\cdot K}$$

For helium, the entropy change across the shock is determined to be

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (5.1926\text{ kJ/kg}\cdot\text{K})\ln(2.9606) - (2.0769\text{ kJ/kg}\cdot\text{K})\ln(8.2009) = \mathbf{1.27\ kJ/kg\cdot K}$$

Discussion Note that shock wave is a highly dissipative process, and the entropy generation is large during shock waves.

Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)

12-80C

Solution We are to discuss the effect of heating on the flow velocity in subsonic Rayleigh flow.

Analysis Heating the fluid **increases the flow velocity in subsonic Rayleigh flow**, but **decreases the flow velocity in supersonic Rayleigh flow**.

Discussion These results are not necessarily intuitive, but must be true in order to satisfy the conservation laws.

12-81C

Solution We are to discuss what the points on a *T*-*s* diagram of Rayleigh flow represent.

Analysis The points on the Rayleigh line represent the **states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations for a given state**. Therefore, for a given inlet state, the fluid cannot exist at any downstream state outside the Rayleigh line on a *T*-*s* diagram.

Discussion The *T*-*s* diagram is quite useful, since any downstream state must lie on the Rayleigh line.

12-82C

Solution We are to discuss the effect of heat gain and heat loss on entropy during Rayleigh flow.

Analysis In Rayleigh flow, **the effect of heat gain is to increase the entropy** of the fluid, and **the effect of heat loss is to decrease the entropy**.

Discussion You should recall from thermodynamics that the entropy of a system can be lowered by removing heat.

12-83C

Solution We are to discuss how temperature and stagnation temperature change in subsonic Rayleigh flow.

Analysis In Rayleigh flow, the **stagnation temperature T_0 always increases with heat transfer to the fluid**, but the temperature T decreases with heat transfer in the Mach number range of $0.845 < \text{Ma} < 1$ for air. Therefore, the **temperature in this case will decrease**.

Discussion This at first seems counterintuitive, but if heat were *not* added, the temperature would drop even *more* if the air were accelerated isentropically from Ma = 0.92 to 0.95.

12-84C

Solution We are to discuss the characteristic aspect of Rayleigh flow, and its main assumptions.

Analysis The characteristic aspect of Rayleigh flow is **its involvement of heat transfer**. The main assumptions associated with Rayleigh flow are: the flow is **steady**, **one-dimensional**, and **frictionless** through a constant-area duct, and the fluid is an **ideal gas with constant specific heats**.

Discussion Of course, there is no such thing as frictionless flow. It is better to say that frictional effects are negligible compared to the heating effects.

12-85C

Solution We are to examine the Mach number at the end of a choked duct in Rayleigh flow when more heat is added.

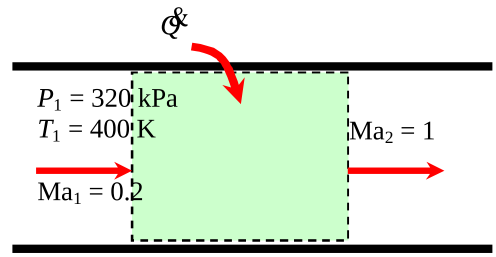
Analysis The flow is choked, and thus the flow at the duct exit **remains sonic**.

Discussion There is no mechanism for the flow to become supersonic in this case.

12-86

Solution Argon flowing at subsonic velocity in a constant-diameter duct is accelerated by heating. The highest rate of heat transfer without reducing the mass flow rate is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.



Properties We take the properties of argon to be k = 1.667, c_p = 0.5203 kJ/kg·K, and R = 0.2081 kJ/kg·K.

Analysis Heat transfer stops when the flow is choked, and thus Ma$_2$ = V_2/c_2 = 1. The inlet stagnation temperature is

$$T_{01}=T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)=(400\text{ K})\left(1+\frac{1.667\text{-}1}{2}0.2^2\right)=405.3\text{ K}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are

$T_{02}/T_0^* = 1$ (since Ma$_2$ = 1)

$$\frac{T_{01}}{T_0^*}=\frac{(k+1)\mathrm{Ma}_1^2[2+(k-1)\mathrm{Ma}_1^2]}{(1+k\mathrm{Ma}_1^2)^2}=\frac{(1.667+1)0.2^2[2+(1.667-1)0.2^2]}{(1+1.667\times 0.2^2)^2}=0.1900$$ Therefore,

$$\frac{T_{02}}{T_{01}}=\frac{T_{02}/T_0^*}{T_{01}/T_0^*}=\frac{1}{0.1900} \qquad \rightarrow \quad T_{02}=T_{01}/0.1900=(405.3\text{ K})/0.1900=2133\text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q}=\dot{m}_{\text{air}}c_p(T_{02}-T_{01})=(1.2\text{ kg/s})(0.5203\text{ kJ/kg}\cdot\text{K})(2133-400)\text{ K}=\mathbf{1080\ kW}$$

Discussion It can also be shown that T_2 = 1600 K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-15 since they are based on k = 1.4.

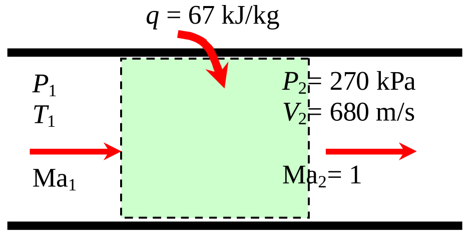
12-87

Solution Air is heated in a duct during subsonic flow until it is choked. For specified pressure and velocity at the exit, the temperature, pressure, and velocity at the inlet are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Noting that sonic conditions exist at the exit, the exit temperature is



$$c_2 = V_2/\mathrm{Ma}_2 = (680\ \mathrm{m/s})/1 = 680\ \mathrm{m/s}$$

$$c_2 = \sqrt{kRT_2} \rightarrow \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 680\ \mathrm{m/s}$$

It gives $T_2 = 1151$ K. Then the exit stagnation temperature becomes

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 1151\ \mathrm{K} + \frac{(680\ \mathrm{m/s})^2}{2\times 1.005\ \mathrm{kJ/kg\cdot K}}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 1381\ \mathrm{K}$$

The inlet stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{01} = T_{02} - \frac{q}{c_p} = 1381\ \mathrm{K} - \frac{67\ \mathrm{kJ/kg}}{1.005\ \mathrm{kJ/kg\cdot K}} = 1314\ \mathrm{K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value in this case is T_{02} since the flow is choked. Therefore, $T_0^* = T_{02} = 1381$ K. Then the stagnation temperature ratio at the inlet, and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{01}}{T_0^*} = \frac{1314\ \mathrm{K}}{1381\ \mathrm{K}} = 0.9516 \quad \rightarrow \quad \mathrm{Ma}_1 = 0.7779 \cong \mathbf{0.778}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.7779$: $T_1/T^* = 1.018$, $P_1/P^* = 1.301$, $V_1/V^* = 0.7852$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $V_2/V^* = 1$

Then the inlet temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{1.018} \quad \rightarrow \quad T_1 = 1.018T_2 = 1.018(1151\ \mathrm{K}) = \mathbf{1172\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.301} \quad \rightarrow \quad P_1 = 1.301P_2 = 1.301(270\ \mathrm{kPa}) = \mathbf{351.3\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1}{0.7852} \quad \rightarrow \quad V_1 = 0.7852V_2 = 0.7852(680\ \mathrm{m/s}) = \mathbf{533.9\ m/s}$$

Discussion Note that the temperature and pressure decreases with heating during this subsonic Rayleigh flow while velocity increases. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

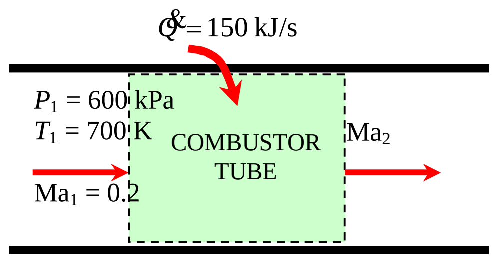
12-88

Solution Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are



$$T_{01} = T_1\left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right) = (700\text{ K})\left(1 + \frac{1.4-1}{2}0.2^2\right) = 705.6\text{ K}$$

$$P_{01} = P_1\left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\text{ kPa})\left(1 + \frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4}$$
$$= 617.0\text{ kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) \quad \rightarrow \quad 150\text{ kJ/s} = (0.3\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(T_{02} - 705.6)\text{ K}$$

It gives

$$T_{02} = 1203\text{ K}$$

At $Ma_1 = 0.2$ we read from $T_{01}/T_0^* = 0.1736$ (Table A-15). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{705.6\text{ K}}{0.1736} = 4064.5\text{ K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-15)

$$\frac{T_{02}}{T_0^*} = \frac{1203\text{ K}}{4064.5\text{ K}} = 0.2960 \qquad \rightarrow \qquad \mathrm{Ma}_2 = 0.2706 \cong \mathbf{0.271}$$

Also,

$Ma_1 = 0.2$ $\rightarrow$ $P_{01}/P_0^* = 1.2346$

$Ma_2 = 0.2706$ $\rightarrow$ $P_{02}/P_0^* = 1.2091$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.2091}{1.2346} = 0.9794 \quad \rightarrow \quad P_{02} = 0.9794 P_{01} = 0.9794(617\text{ kPa}) = 604.3\text{ kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 604.3 = \mathbf{12.7\text{ kPa}}$$

Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

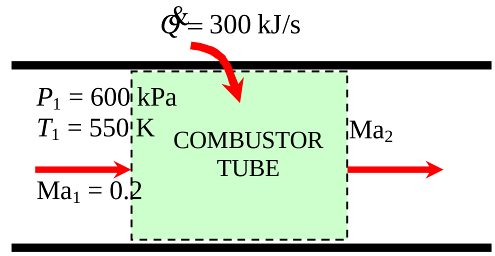
12-89

Solution Air enters the combustion chamber of a gas turbine at a subsonic velocity. For a specified rate of heat transfer, the Mach number at the exit and the loss in stagnation pressure to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The cross-sectional area of the combustion chamber is constant. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet stagnation temperature and pressure are



$$T_{01} = T_1\left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right) = (700\ \mathrm{K})\left(1 + \frac{1.4-1}{2}0.2^2\right) = 705.6\ \mathrm{K}$$

$$P_{01} = P_1\left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right)^{k/(k-1)} = (600\ \mathrm{kPa})\left(1 + \frac{1.4\text{-}1}{2}0.2^2\right)^{1.4/0.4}$$

$$= 617.0\ \mathrm{kPa}$$

The exit stagnation temperature is determined from

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p (T_{02} - T_{01}) \quad \rightarrow \quad 300\ \mathrm{kJ/s} = (0.3\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg\cdot K})(T_{02} - 705.6)\ \mathrm{K}$$

It gives

$$T_{02} = 1701\ \mathrm{K}$$

At $\mathrm{Ma}_1 = 0.2$ we read from $T_{01}/T_0^* = 0.1736$ (Table A-15). Therefore,

$$T_0^* = \frac{T_{01}}{0.1736} = \frac{705.6\ \mathrm{K}}{0.1736} = 4064.5\ \mathrm{K}$$

Then the stagnation temperature ratio at the exit and the Mach number corresponding to it are (Table A-15)

$$\frac{T_{02}}{T_0^*} = \frac{1701\ \mathrm{K}}{4064.5\ \mathrm{K}} = 0.4185 \qquad \rightarrow \qquad \mathrm{Ma}_2 = 0.3393 \cong \mathbf{0.339}$$

Also,

$$\mathrm{Ma}_1 = 0.2 \qquad \rightarrow \quad P_{01}/P_0^* = 1.2346$$
$$\mathrm{Ma}_2 = 0.3393 \qquad \rightarrow \quad P_{02}/P_0^* = 1.1820$$

Then the stagnation pressure at the exit and the pressure drop become

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}/P_0^*}{P_{01}/P_0^*} = \frac{1.1820}{1.2346} = 0.9574 \quad \rightarrow \quad P_{02} = 0.9574 P_{01} = 0.9574(617\ \mathrm{kPa}) = 590.7\ \mathrm{kPa}$$

and

$$\Delta P_0 = P_{01} - P_{02} = 617.0 - 590.7 = \mathbf{26.3\ kPa}$$

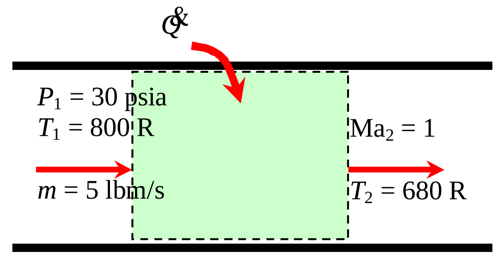
Discussion This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-90E

Solution Air flowing with a subsonic velocity in a round duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the pressure drop are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be k = 1.4, c_p = 0.2400 Btu/lbm·R, and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.



Analysis The inlet density and velocity of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{30\text{ psia}}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(800\text{ R})} = 0.1012\text{ lbm/ft}^3$$

$$V_1 = \frac{\dot{m}_{air}}{\rho_1 A_{c1}} = \frac{5\ \text{lbm/s}}{(0.1012\ \text{lbm/ft}^3)[\pi(4/12\text{ ft})^2/4]} = 565.9\text{ ft/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 800\text{ R} + \frac{(565.9\text{ ft/s})^2}{2\times 0.2400\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/lbm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 826.7\text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(800\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/lbm}}\right)} = 1386\text{ ft/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{565.9\text{ ft/s}}{1386\ \text{ft/s}} = 0.4082$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 0.4082$: $T_1/T^* = 0.6310$, $P_1/P^* = 1.946$, $T_{01}/T_0^* = 0.5434$

$\mathrm{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6310} \quad\rightarrow\quad T_2 = T_1/0.6310 = (800\text{ R})/0.6310 = 1268\text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{1.946} \quad\rightarrow\quad P_2 = P_1/2.272 = (30\text{ psia})/1.946 = 15.4\text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.5434} \quad\rightarrow\quad T_{02} = T_{01}/0.1743 = (826.7\text{ R})/0.5434 = 1521\text{ R}$$

Then the rate of heat transfer and the pressure drop become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (5\text{ lbm/s})(0.2400\text{ Btu/lbm}\cdot\text{R})(1521 - 826.7)\text{ R} = \mathbf{834\ \ Btu/s}$$

$$\Delta P = P_1 - P_2 = 30 - 15.4 = \mathbf{14.6\ psia}$$

Discussion Note that the entropy of air increases during this heating process, as expected.

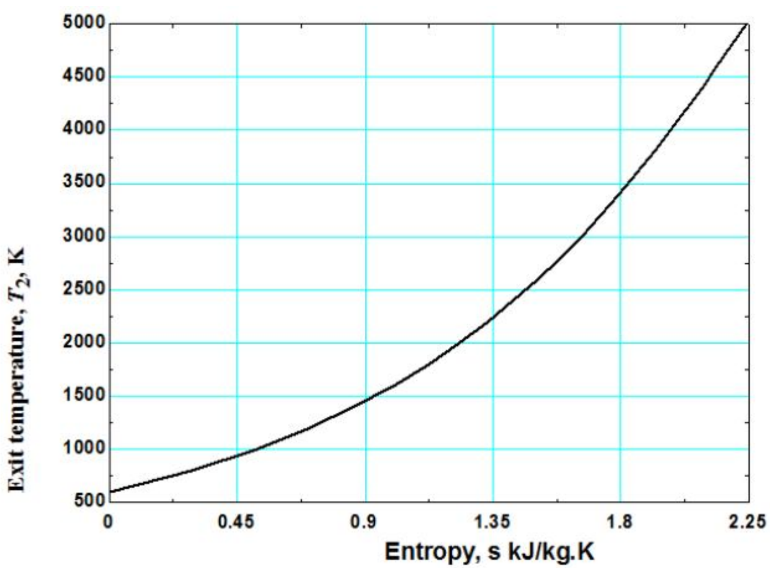
12-91

Solution Air flowing with a subsonic velocity in a duct. The variation of entropy with temperature is to be investigated as the exit temperature varies from 600 K to 5000 K in increments of 200 K. The results are to be tabulated and plotted.

Analysis We solve this problem using EES making use of Rayleigh functions. The EES *Equations* window is printed below, along with the tabulated and plotted results.

```
k=1.4
cp=1.005
R=0.287
P1=350
T1=600
V1=70
C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T1*(1+0.5*(k-1)*Ma1^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))
F1=1+0.5*(k-1)*Ma1^2
T01Ts=2*(k+1)*Ma1^2*F1/(1+k*Ma1^2)^2
P01Ps=((1+k)/(1+k*Ma1^2))*(2*F1/(k+1))^(k/(k-1))
T1Ts=(Ma1*((1+k)/(1+k*Ma1^2)))^2
P1Ps=(1+k)/(1+k*Ma1^2)
V1Vs=Ma1^2*(1+k)/(1+k*Ma1^2)
F2=1+0.5*(k-1)*Ma2^2
T02Ts=2*(k+1)*Ma2^2*F2/(1+k*Ma2^2)^2
P02Ps=((1+k)/(1+k*Ma2^2))*(2*F2/(k+1))^(k/(k-1))
T2Ts=(Ma2*((1+k)/(1+k*Ma2^2)))^2
P2Ps=(1+k)/(1+k*Ma2^2)
V2Vs=Ma2^2*(1+k)/(1+k*Ma2^2)
T02=T02Ts/T01Ts*T01
P02=P02Ps/P01Ps*P01
T2=T2Ts/T1Ts*T1
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
Delta_s=cp*ln(T2/T1)-R*ln(P2/P1)
```

Exit temperature T_2, K	Exit Mach number, Ma_2	Exit entropy relative to inlet, s_2, kJ/kg·K
600	0.143	0.000
800	0.166	0.292
1000	0.188	0.519
1200	0.208	0.705
1400	0.227	0.863
1600	0.245	1.001
1800	0.263	1.123
2000	0.281	1.232
2200	0.299	1.331
2400	0.316	1.423
2600	0.333	1.507
2800	0.351	1.586
3000	0.369	1.660
3200	0.387	1.729
3400	0.406	1.795
3600	0.426	1.858
3800	0.446	1.918
4000	0.467	1.975
4200	0.490	2.031
4400	0.515	2.085
4600	0.541	2.138
4800	0.571	2.190
5000	0.606	2.242



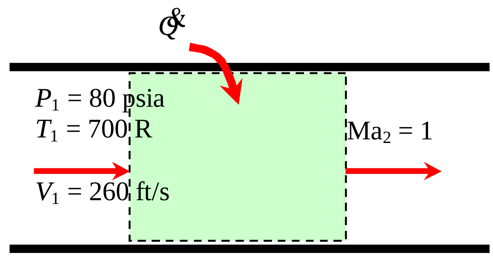
Discussion Note that the entropy of air increases during this heating process, as expected.

12-92E

Solution Air flowing with a subsonic velocity in a square duct is accelerated by heating until the flow is choked at the exit. The rate of heat transfer and the entropy change are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** The flow is choked at the duct exit. **3** Mass flow rate remains constant.

Properties We take the properties of air to be k = 1.4, c_p = 0.2400 Btu/lbm·R, and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R.



Analysis The inlet density and mass flow rate of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{80 \text{ psia}}{(0.3704 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(700 \text{ R})} = 0.3085 \text{ lbm/ft}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.3085 \text{ lbm/ft}^3)(6\times 6/144 \text{ ft}^2)(260 \text{ ft/s}) = 20.06 \text{ lbm/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 700 \text{ R} + \frac{(260 \text{ ft/s})^2}{2\times 0.2400 \text{ Btu/lbm}\cdot\text{R}}\left(\frac{1 \text{ Btu/lbm}}{25{,}037 \text{ ft}^2/\text{s}^2}\right) = 705.6 \text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855 \text{ Btu/lbm}\cdot\text{R})(700 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/lbm}}\right)} = 1297 \text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{260 \text{ ft/s}}{1297 \text{ ft/s}} = 0.2005$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\text{Ma}_1 = 0.2005$: $T_1/T^* = 0.2075$, $P_1/P^* = 2.272$, $T_{01}/T_0^* = 0.1743$
$\text{Ma}_2 = 1$: $T_2/T^* = 1$, $P_2/P^* = 1$, $T_{02}/T_0^* = 1$

Then the exit temperature, pressure, and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.2075} \quad \rightarrow \quad T_2 = T_1/0.2075 = (700 \text{ R})/0.2075 = 3374 \text{ R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1}{2.272} \quad \rightarrow \quad P_2 = P_1/2.272 = (80 \text{ psia})/2.272 = 35.2 \text{ psia}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.1743} \quad \rightarrow \quad T_{02} = T_{01}/0.1743 = (705.6 \text{ R})/0.1743 = 4048 \text{ R}$$

Then the rate of heat transfer and entropy change become

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (20.06 \text{ lbm/s})(0.2400 \text{ Btu/lbm}\cdot\text{R})(4048 - 705.6) \text{ R} = \mathbf{16{,}090 \ Btu/s}$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = (0.2400 \text{ Btu/lbm}\cdot\text{R})\ln\frac{3374 \text{ R}}{700 \text{ R}} - (0.06855 \text{ Btu/lbm}\cdot\text{R})\ln\frac{35.2 \text{ psia}}{80 \text{ psia}} = \mathbf{0.434 \ Btu/lbm\cdot R}$$

Discussion Note that the entropy of air increases during this heating process, as expected.

12-93

Solution Fuel is burned in a rectangular duct with compressed air. For specified heat transfer, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

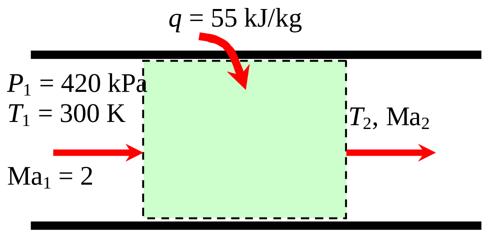
Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 347.2\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(347.2\text{ m/s}) = 694.4\text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300\text{ K} + \frac{(694.4\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 539.9\text{ K}$$



The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9\text{ K} + \frac{55\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 594.6\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-15 or from the appropriate relation. At Ma$_1$ = 2 we read T_{01}/T_0^* = 0.7934. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9\text{ K}}{0.7934} = 680.5\text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{02}}{T_0^*} = \frac{594.6\text{ K}}{680.5\text{ K}} = 0.8738 \quad\rightarrow\quad \text{Ma}_2 = 1.642 \cong \mathbf{1.64}$$

Also,

$$\text{Ma}_1 = 2 \quad\rightarrow\quad T_1/T^* = 0.5289$$
$$\text{Ma}_2 = 1.642 \quad\rightarrow\quad T_2/T^* = 0.6812$$

Then the exit temperature becomes

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.6812}{0.5289} = 1.288 \quad\rightarrow\quad T_2 = 1.288T_1 = 1.288(300\text{ K}) = \mathbf{386\text{ K}}$$

Discussion Note that the temperature increases during this supersonic Rayleigh flow with heating. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

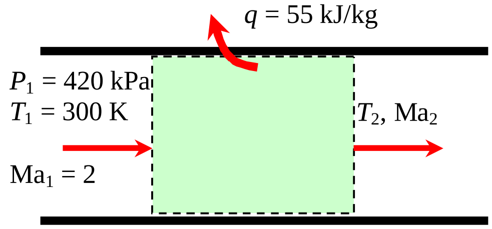
12-94

Solution Compressed air is cooled as it flows in a rectangular duct. For specified heat rejection, the exit temperature and Mach number are to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The stagnation temperature and Mach number at the inlet are



$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(300\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 347.2\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(347.2\text{ m/s}) = 694.4\text{ m/s}$$

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 300\text{ K} + \frac{(694.4\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 539.9\text{ K}$$

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 539.9\text{ K} + \frac{-55\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 485.2\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A-15 or from the appropriate relation. At $\text{Ma}_1 = 2$ we read $T_{01}/T_0^* = 0.7934$. Therefore,

$$T_0^* = \frac{T_{01}}{0.7934} = \frac{539.9\text{ K}}{0.7934} = 680.5\text{ K}$$

The stagnation temperature ratio at the exit and the Mach number corresponding to it are, from Table A-15,

$$\frac{T_{02}}{T_0^*} = \frac{485.2\text{ K}}{680.5\text{ K}} = 0.7130 \quad \rightarrow \quad \text{Ma}_2 = 2.479 \cong \mathbf{2.48}$$

Also,

$$\text{Ma}_1 = 2 \quad \rightarrow \quad T_1/T^* = 0.5289$$
$$\text{Ma}_2 = 2.479 \quad \rightarrow \quad T_2/T^* = 0.3838$$

Then the exit temperature becomes

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.3838}{0.5289} = 0.7257 \quad \rightarrow \quad T_2 = 0.7257T_1 = 0.7257(300\text{ K}) = \mathbf{218\text{ K}}$$

Discussion Note that the temperature decreases and Mach number increases during this supersonic Rayleigh flow with cooling. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

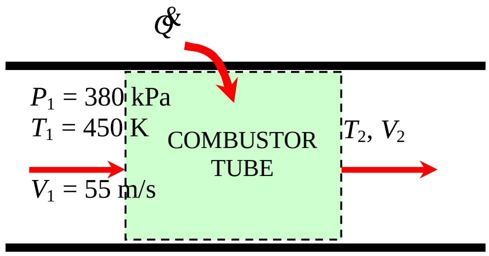
12-95

Solution Fuel is burned in a tubular combustion chamber with compressed air. For a specified exit Mach number, the exit temperature and the rate of fuel consumption are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Combustion is complete, and it is treated as a heat addition process, with no change in the chemical composition of flow. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The inlet density and mass flow rate of air are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{380 \text{ kPa}}{(0.287 \text{ kJ/kgK})(450 \text{ K})} = 2.942 \text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.942 \text{ kg/m}^3)[\pi(0.16 \text{ m})^2/4](55 \text{ m/s}) = 3.254 \text{ kg/s}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 450 \text{ K} + \frac{(55 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg}\cdot\text{K}}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 451.5 \text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(450 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 425.2 \text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{55 \text{ m/s}}{425.2 \text{ m/s}} = 0.1293$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15) (We used analytical functions):

Ma_1 = 0.1293:	T_1/T^* = 0.09201,	T_{01}/T^* = 0.07693,	V_1/V^* = 0.03923
Ma_2 = 0.8:	T_2/T^* = 1.0255,	T_{02}/T^* = 0.9639,	V_2/V^* = 0.8101

The exit temperature, stagnation temperature, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0255}{0.09201} = 11.146 \quad \rightarrow \quad T_2 = 11.146T_1 = 11.146(450 \text{ K}) = \mathbf{5016\ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.07693} = 12.530 \quad \rightarrow \quad T_{02} = 12.530T_{01} = 12.530(451.5 \text{ K}) = 5658 \text{ K}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.8101}{0.03923} = 20.650 \quad \rightarrow \quad V_2 = 20.650V_1 = 20.650(55 \text{ m/s}) = 1136 \text{ m/s}$$

Then the mass flow rate of the fuel is determined to be

$$q = c_p(T_{02} - T_{01}) = (1.005 \text{ kJ/kg}\cdot\text{K})(5658 - 451.5) \text{ K} = 5232 \text{ kJ/kg}$$

$$\dot{Q} = \dot{m}_{air} q = (3.254 \text{ kg/s})(5232 \text{ kJ/kg}) = 17{,}024 \text{ kW}$$

$$\dot{m}_{fuel} = \frac{\dot{Q}}{HV} = \frac{17{,}024 \text{ kJ/s}}{39{,}000 \text{ kJ/kg}} = \mathbf{0.4365\ kg/s}$$

Discussion Note that both the temperature and velocity increase during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

12-96

Solution Air flowing at a supersonic velocity in a duct is decelerated by heating. The highest temperature air can be heated by heat addition and the rate of heat transfer are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Mass flow rate remains constant.

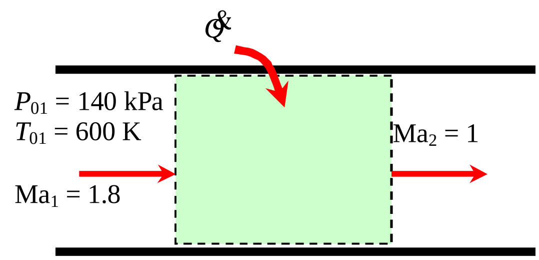
Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\mathrm{Ma}_2 = V_2/c_2 = 1$. Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (600\text{ K})\left(1+\frac{1.4\text{-}1}{2}1.8^2\right)^{-1} = 364.1\text{ K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (140\text{ K})\left(1+\frac{1.4\text{-}1}{2}1.8^2\right)^{-1.4/0.4}$$
$$= 24.37\text{ kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{24.37\text{ kPa}}{(0.287\text{ kJ/kgK})(364.1\text{ K})} = 0.2332\text{ kg/m}^3$$



Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(364.1\text{K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 382.5\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.8(382.5\text{ m/s}) = 688.5\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.2332\text{ kg/m}^3)[\pi(0.07\text{ m})^2/4](688.5\text{ m/s}) = 0.6179\text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 1.8$:	$T_1/T^* = 0.6089$,	$T_{01}/T_0^* = 0.8363$
$\mathrm{Ma}_2 = 1$:	$T_2/T^* = 1$,	$T_{02}/T_0^* = 1$

Then the exit temperature and stagnation temperature are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1}{0.6089} \quad\rightarrow\quad T_2 = T_1/0.6089 = (364.1\text{ K})/0.6089 = \mathbf{598\ K}$$

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.8363} \quad\rightarrow\quad T_{02} = T_{01}/0.8363 = (600\text{ K})/0.8363 = 717.4\text{ K} \cong \mathbf{717\ K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (0.6179\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(717.4 - 600)\text{ K} = \mathbf{72.9\ kW}$$

Discussion Note that this is the highest temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature will cause the mass flow rate to decrease. Also, once the sonic conditions are reached, the thermodynamic temperature can be increased further by cooling the fluid and reducing the velocity (see the *T-s* diagram for Rayleigh flow).

Adiabatic Duct Flow with Friction (Fanno Flow)

12-97C

Solution We are to discuss the effect of friction on velocity in Fanno flow.

Analysis Friction **increases the flow velocity in subsonic Fanno flow**, but **decreases the flow velocity in supersonic flow**.

Discussion These results may not be intuitive, but they come from following the Fanno line, which satisfies the conservation equations.

12-98C

Solution We are to discuss the T-s diagram for Fanno flow.

Analysis The points on the Fanno line on a *T-s* diagram represent the **states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations for a given inlet state**. Therefore, for a given initial state, the fluid cannot exist at any downstream state outside the Fanno line on a *T-s* diagram.

Discussion The *T-s* diagram is quite useful, since any downstream state must lie on the Fanno line.

12-99C

Solution We are to discuss the effect of friction on the entropy during Fanno flow.

Analysis In Fanno flow, the effect of friction is **always to increase the entropy of the fluid**. Therefore Fanno flow always proceeds in the direction of increasing entropy.

Discussion To do otherwise would violate the second law of thermodynamics.

12-100C

Solution We are to discuss what happens to supersonic Fanno flow, initially sonic at the exit, when the duct is extended.

Analysis **The flow at the duct exit remains sonic**. The **mass flow rate must remain constant** since upstream conditions are not affected by the added duct length.

Discussion The mass flow rate is fixed by the upstream stagnation conditions and the size of the throat – therefore, the mass flow rate does not change by extending the duct. However, a *shock wave* appears in the duct when it is extended.

12-101C

Solution We are to examine what happens when the Mach number of air decreases in supersonic Fanno flow.

Analysis During supersonic Fanno flow, the **stagnation temperature T_0 remains constant**, **stagnation pressure P_0 decreases**, and **entropy s increases**.

Discussion Friction leads to irreversible losses, which are felt as a loss of stagnation pressure and an increase of entropy. However, since the flow is adiabatic, the stagnation temperature does not change downstream.

12-102C

Solution We are to discuss the characteristic aspect of Fanno flow and its main assumptions.

Analysis The characteristic aspect of Fanno flow is its **consideration of friction**. The main assumptions associated with Fanno flow are: the flow is **steady**, **one-dimensional**, and **adiabatic** through a **constant-area duct**, and the fluid is an **ideal gas** with **constant specific heats**.

Discussion Compared to Rayleigh flow, Fanno flow accounts for friction but neglects heat transfer effects, whereas Rayleigh flow accounts for heat transfer but neglects frictional effects.

12-103C

Solution We are to discuss what happens to choked subsonic Fanno flow when the duct is extended.

Analysis The flow is choked, and thus **the flow at the duct exit must remain sonic**. The **mass flow rate has to decrease** as a result of extending the duct length in order to compensate.

Discussion Since there is no way for the flow to become supersonic (e.g., there is no throat), the upstream flow must adjust itself such that the flow at the exit plan remains sonic.

12-104C

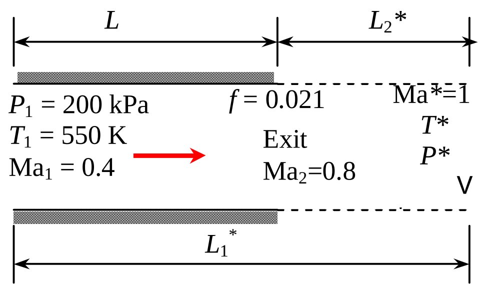
Solution We are to examine what happens when the Mach number of air increases in subsonic Fanno flow.

Analysis During subsonic Fanno flow, the **stagnation temperature T_0 remains constant**, **stagnation pressure P_0 decreases**, and **entropy s increases**.

Discussion Friction leads to irreversible losses, which are felt as a loss of stagnation pressure and an increase of entropy. However, since the flow is adiabatic, the stagnation temperature does not change downstream.

12-105

Solution Subsonic airflow in a constant cross-sectional area adiabatic duct is considered. For a specified exit Mach number, the duct length, temperature, pressure, and velocity at the duct exit are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.021.

Analysis The inlet velocity is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(550\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 470.1\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.4(470.1\text{ m/s}) = 188.0\text{ m/s}$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$Ma_1 = 0.4$: $(fL^*/D_h)_1 = 2.3085$ $T_1/T^* = 1.1628$, $P_1/P^* = 2.6958$, $V_1/V^* = 0.4313$

$Ma_2 = 0.8$: $(fL^*/D_h)_2 = 0.0723$ $T_2/T^* = 1.0638$, $P_2/P^* = 1.2893$, $V_2/V^* = 0.8251$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0638}{1.1628} = 0.9149 \quad \rightarrow \quad T_2 = 0.9149T_1 = 0.9149(550\text{ K}) = \mathbf{503.2\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.2893}{2.6958} = 0.4783 \quad \rightarrow \quad P_2 = 0.4783P_1 = 0.4783(200\text{ kPa}) = \mathbf{95.65\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.8251}{0.4313} = 1.9131 \quad \rightarrow \quad V_2 = 1.9131V_1 = 1.9131(188.0\text{ m/s}) = \mathbf{359.7\ m/s}$$

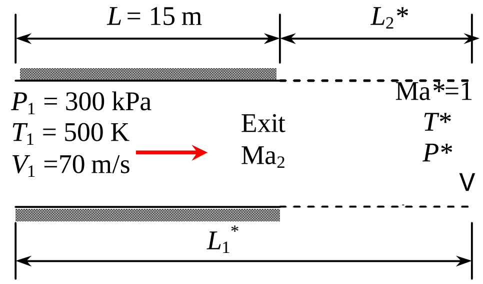
Finally, the actual duct length is determined to be

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (2.3085 - 0.0723)\frac{0.12\text{ m}}{0.021} = \mathbf{12.8\ m}$$

Discussion Note that it takes a duct length of 12.8 m for the Mach number to increase from 0.4 to 0.8. The Mach number rises at a much higher rate as sonic conditions are approached. The maximum (or sonic) duct lengths at the inlet and exit states in this case are L_1^* = 13.2 m and L_2^* = 0.413 m. Therefore, the flow would reach sonic conditions if a 0.413-m long section were added to the existing duct.

12-106

Solution Air enters a constant-area adiabatic duct of given length at a specified state. The exit Mach number, exit velocity, and the mass flow rate are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.023.

Analysis The first thing we need to know is whether the flow is choked at the exit or not. Therefore, we first determine the inlet Mach number and the corresponding value of the function fL^*/D_h ,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(500\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 448.2\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{70\text{ m/s}}{448.2\text{ m/s}} = 0.1562$$

Corresponding to this Mach number we calculate (or read) from Table A-16), $(fL^*/D_h)_1$ = 25.540. Also, using the actual duct length L, we have

$$\frac{fL}{D_h} = \frac{(0.023)(15\text{ m})}{0.04\text{ m}} = 8.625 \quad < \quad 25.540$$

Therefore, flow is *not* choked and exit Mach number is less than 1. Noting that $L = L_1^* - L_2^*$, the function fL^*/D_h at the exit state is calculated from

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL}{D_h} = 25.540 - 8.625 = 16.915$$

The Mach number corresponding to this value of fL^*/D is obtained from Table A-16 to be

Ma_2 = **0.187**

which is the Mach number at the duct exit. The mass flow rate of air is determined from the inlet conditions to be

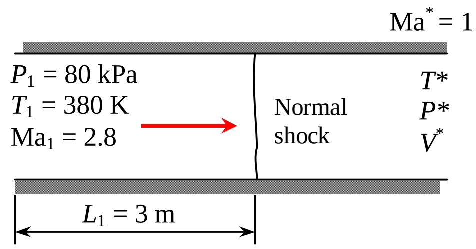
$$\rho_1 = \frac{P_1}{RT_1} = \frac{300\text{ kPa}}{(0.287\text{ kJ/kgK})(500\text{ K})}\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = 2.091\text{ kg/m}^3$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.091\text{ kg/m}^3)[\pi(0.04\text{ m})^2/4](70\text{ m/s}) = \mathbf{0.184\ kg/s}$$

Discussion It can be shown that L_2^* = 29.4 m, indicating that it takes a duct length of 15 m for the Mach number to increase from 0.156 to 0.187, but only 29.4 m to increase from 0.187 to 1. Therefore, the Mach number rises at a much higher rate as sonic conditions are approached.

12-107

Solution Air enters a constant-area adiabatic duct at a specified state, and undergoes a normal shock at a specified location. The exit velocity, temperature, and pressure are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.007.

Analysis The Fanno flow functions corresponding to the inlet Mach number of 2.8 are, from Table A-16,

$\mathrm{Ma}_1 = 2.8$: $(fL^*/D_h)_1 = 0.4898$ $\quad T_1/T^* = 0.4673$, $\quad P_1/P^* = 0.2441$

First we check to make sure that the flow everywhere upstream the shock is supersonic. The required duct length from the inlet L_1^* for the flow to reach sonic conditions is

$$L_1^* = 0.4898\frac{D}{f} = 0.4898\frac{0.05\text{ m}}{0.007} = 3.50\text{ m}$$

which is greater than the actual length 3 m. Therefore, the flow is indeed supersonic when the normal shock occurs at the indicated location. Also, using the actual duct length L_1, we have $\dfrac{fL_1}{D_h} = \dfrac{(0.007)(3\text{ m})}{0.05\text{ m}} = 0.4200$. Noting that $L_1 = L_1^* - L_2^*$, the function fL^*/D_h at the exit state and the corresponding Mach number are

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL_1}{D_h} = 0.4898 - 0.4200 = 0.0698 \quad \to \quad \mathrm{Ma}_2 = 1.315$$

From Table A-16, at Ma_2 =1.315: T_2/T^* = 0.8918 and P_2/P^* = 0.7183. Then the temperature, pressure, and velocity before the shock are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.8918}{0.4673} = 1.9084 \qquad \to \quad T_2 = 1.9084T_1 = 1.9084(380\text{ K}) = 725.2\text{ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.7183}{0.2441} = 2.9426 \qquad \to \quad P_2 = 2.9426P_1 = 2.9426(80\text{ kPa}) = 235.4\text{ kPa}$$

The normal shock functions corresponding to a Mach number of 1.315 are, from Table A-14,

Ma_2 =1.315: $\mathrm{Ma}_3 = 0.7786$, $\quad T_3/T_2 = 1.2001$, $\quad P_3/P_2 = 1.8495$

Then the temperature and pressure after the shock become

$$T_3 = 1.2001T_2 = 1.2001(725.2\text{ K}) = 870.3\text{ K} \quad \text{and} \quad P_3 = 1.8495P_2 = 1.8495(235.4\text{ kPa}) = 435.4\text{ kPa}$$

Sonic conditions exist at the duct exit, and the flow downstream the shock is still Fanno flow. From Table A-16,

$\mathrm{Ma}_3 = 0.7786$: $\quad T_3/T^* = 1.0702$, $\quad P_3/P^* = 1.3286$
$\mathrm{Ma}_4 = 1$: $\quad T_4/T^* = 1$, $\quad P_4/P^* = 1$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_4}{T_3} = \frac{T_4/T^*}{T_3/T^*} = \frac{1}{1.0702} \qquad \to \quad T_4 = T_3/1.0702 = (870.3\text{ K})/1.0702 = \mathbf{813\text{ K}}$$

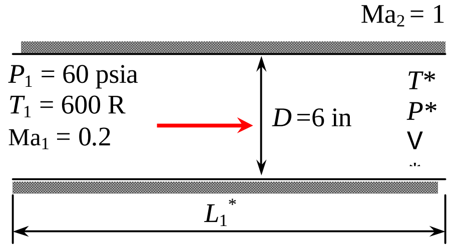
$$\frac{P_4}{P_3} = \frac{P_4/P^*}{P_3/P^*} = \frac{1}{1.3286} \qquad \to \quad P_4 = P_3/1.3286 = (435.4\text{ kPa})/1.3286 = \mathbf{328\text{ kPa}}$$

$$V_4 = \mathrm{Ma}_4 c_4 = (1)\sqrt{kRT_4} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(813\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{572\text{ m/s}}$$

Discussion It can be shown that L_3^* = 0.67 m, and thus the total length of this duct is 3.67 m. If the duct is extended, the normal shock will move further upstream, and eventually to the inlet of the duct.

12-108E

Solution Helium enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of helium to be k = 1.667, c_p = 1.2403 Btu/lbm·R, and R = 0.4961 Btu/lbm·R. The friction factor is given to be f = 0.025.

Analysis The Fanno flow function fL^*/D corresponding to the inlet Mach number of 0.2 is (Table A-16)

$$\frac{fL_1^*}{D} = 14.5333$$

Noting that * denotes sonic conditions, which exist at the exit state, the duct length is determined to be

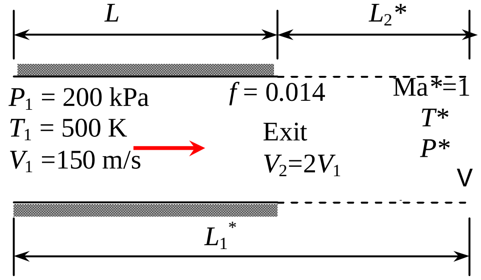
$$L_1^* = 14.5333D/f = 14.5333(6/12\ \text{ft})/0.025 = \mathbf{291\ ft}$$

Thus, for the given friction factor, the duct length must be 291 ft for the Mach number to reach Ma = 1 at the duct exit.

Discussion This problem can also be solved using equations instead of tabulated values for the Fanno functions.

12-109

Solution Subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The duct length from the inlet where the inlet velocity doubles and the pressure drop in that section are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.014.

Analysis The inlet Mach number is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(500\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 448.2\ \text{m/s} \quad \rightarrow \quad \text{Ma}_1 = \frac{V_1}{c_1} = \frac{150\ \text{m/s}}{448.2\ \text{m/s}} = 0.3347$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$\text{Ma}_1 = 0.3347$: $(fL^*/D_h)_1 = 3.924$ $P_1/P^* = 3.2373$, $V_1/V^* = 0.3626$

Therefore, $V_1 = 0.3626V^*$. Then the Fanno function V_2/V^* becomes $\dfrac{V_2}{V^*} = \dfrac{2V_1}{V^*} = \dfrac{2\times 0.3626V^*}{V^*} = 0.7252$.

The corresponding Mach number and Fanno flow functions are, from Table A-16,

$\text{Ma}_2 = 0.693$, $(fL^*/D_h)_1 = 0.2220$, and $P_2/P^* = 1.5099$.

Then the duct length where the velocity doubles, the exit pressure, and the pressure drop become

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (3.924 - 0.2220)\frac{0.15\ \text{m}}{0.014} = \mathbf{39.7\ m}$$

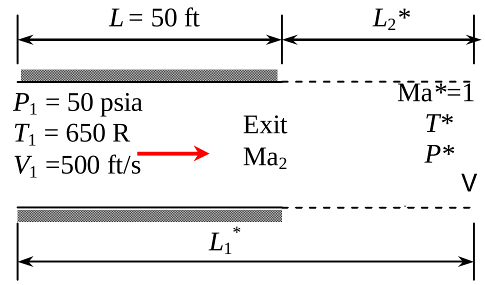
$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.5099}{3.2373} = 0.4664 \qquad \rightarrow \quad P_2 = 0.4664P_1 = 0.4664(200\ \text{kPa}) = 93.3\ \text{kPa}$$

$$\Delta P = P_1 - P_2 = 200 - 93.3 = 106.7\ \text{kPa} \cong \mathbf{107\ kPa}$$

Discussion Note that it takes a duct length of 39.7 m for the velocity to double, and the Mach number to increase from 0.3347 to 0.693. The maximum (or sonic) duct lengths at the inlet and exit states in this case are L_1^* = 42.1 m and L_2^* = 2.38 m. Therefore, the flow would reach sonic conditions if there is an additional 2.38 m of duct length.

12-110E

Solution Air enters a constant-area adiabatic duct of given length at a specified state. The velocity, temperature, and pressure at the duct exit are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of helium to be k = 1.4, c_p = 0.2400 Btu/lbm·R, and R = 0.06855 Btu/lbm·R = 0.3704 psia·ft^3/lbm·R. The friction factor is given to be f = 0.025.

Analysis The first thing we need to know is whether the flow is choked at the exit or not. Therefore, we first determine the inlet Mach number and the corresponding value of the function fL^*/D_h ,

$$T_1 = T_{01} - \frac{V_1^2}{2c_p} = 650\text{ R} - \frac{(500\text{ ft/s})^2}{2\times 0.2400\text{ Btu/lbm}\cdot\text{R}}\left(\frac{1\text{ Btu/lbm}}{25{,}037\text{ ft}^2/\text{s}^2}\right) = 629.2\text{ R}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.06855\text{ Btu/lbm}\cdot\text{R})(629.2\text{ R})\left(\frac{25{,}037\text{ ft}^2/\text{s}^2}{1\text{ Btu/1bm}}\right)} = 1230\text{ ft/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{500\text{ m/s}}{1230\text{ ft/s}} = 0.4066$$

Corresponding to this Mach number we calculate (or read) from Table A-16), $(fL^*/D_h)_1$ = 2.1911. Also, using the actual duct length L, we have

$$\frac{fL}{D_h} = \frac{(0.02)(50\text{ ft})}{6/12\text{ ft}} = 2 \;<\; 2.1911$$

Therefore, the flow is *not* choked and exit Mach number is less than 1. Noting that $L = L_1^* - L_2^*$, the function fL^*/D_h at the exit state is calculated from

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL}{D_h} = 2.1911 - 2 = 0.1911$$

The Mach number corresponding to this value of fL^*/D is obtained from Table A-16 to be Ma$_2$ = 0.7091. The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

Ma$_1$ = 0.4066: T_1/T^* = 1.1616, P_1/P^* = 2.6504, V_1/V^* = 0.4383
Ma$_2$ = 0.7091: T_2/T^* = 1.0903, P_2/P^* = 1.4726, V_2/V^* = 0.7404

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.0903}{1.1616} = 0.9386 \quad\rightarrow\quad T_2 = 0.9386T_1 = 0.9386(629.2\text{ R}) = \mathbf{591\,R}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{1.4726}{2.6504} = 0.5556 \quad\rightarrow\quad P_2 = 0.5556P_1 = 0.5556(50\text{ psia}) = \mathbf{27.8\,psia}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.7404}{0.4383} = 1.6893 \quad\rightarrow\quad V_2 = 1.6893V_1 = 1.6893(500\text{ ft/s}) = \mathbf{845\,ft/s}$$

Discussion It can be shown that L_2^* = 4.8 ft, indicating that it takes a duct length of 50 ft for the Mach number to increase from 0.4066 to 0.7091, but only 4.8 ft to increase from 0.7091 to 1. Therefore, the Mach number rises at a much higher rate as sonic conditions are approached.

12-111

Solution Choked subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The variation of duct length with Mach number is to be investigated, and the results are to be plotted.

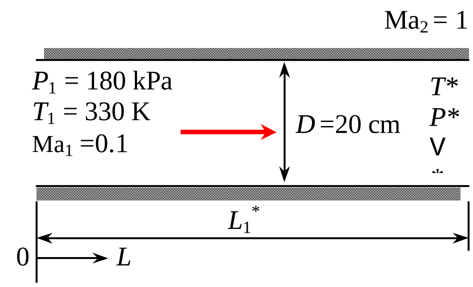
Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.02.

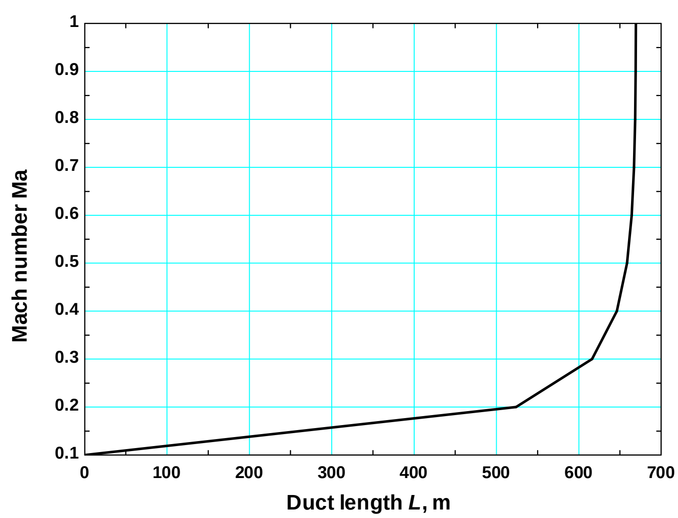
Analysis The flow is choked, and thus Ma_2 =1. Corresponding to the inlet Mach number of Ma_1 = 0.1 we have, from Table A-16, fL^*/D_h = 66.922, Therefore, the original duct length is

$$L_1^* = 66.922\frac{D}{f} = 66.922\frac{0.20\text{ m}}{0.02} = 669\text{ m}$$

Repeating the calculations for different Ma_2 as it varies from 0.1 to 1 results in the following table for the location on the duct from the inlet. The EES *Equations* window is printed below, along with the plotted results.



Mach number, Ma	Duct length L, m
0.10	0
0.20	524
0.30	616
0.40	646
0.50	659
0.60	664
0.70	667
0.80	668
0.90	669
1.00	669



EES program:

```
k=1.4
cp=1.005
R=0.287

P1=180
T1=330
Ma1=0.1
"Ma2=1"
f=0.02
D=0.2

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))
```

```
rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that the Mach number increases very mildly at the beginning, and then rapidly near the duct outlet. It takes 262 m of duct length for Mach number to increase from 0.1 to 0.2, but only 1 m to increase from 0.7 to 1.

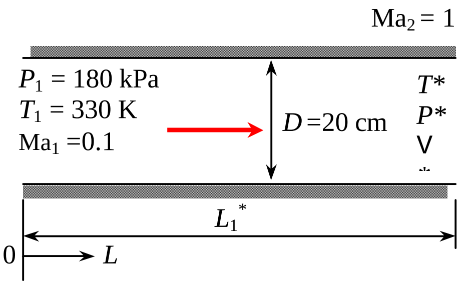
12-112

Solution Choked subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The variation of duct length with Mach number is to be investigated, and the results are to be plotted.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of helium to be k = 1.667, c_p = 5.193 kJ/kg·K, and R = 2.077 kJ/kg·K. The average friction factor is given to be f = 0.02.

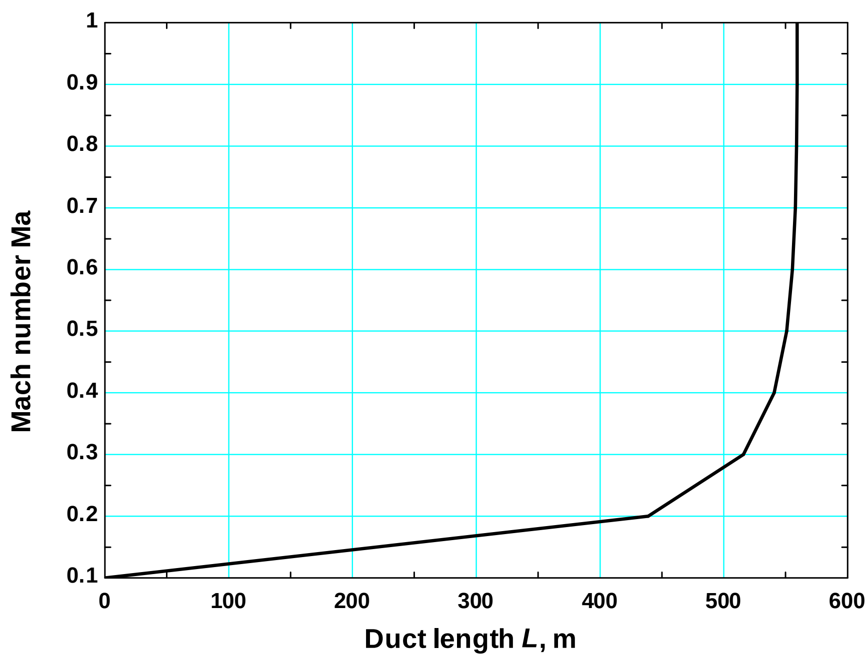
Analysis The flow is choked, and thus Ma_2 =1. Corresponding to the inlet Mach number of Ma_1 = 0.1 we have, from Table A-16, fL^*/D_h = 66.922, Therefore, the original duct length is



$$L_1^* = 66.922\frac{D}{f} = 66.922\frac{0.20\text{ m}}{0.02} = 669\text{ m}$$

Repeating the calculations for different Ma_2 as it varies from 0.1 to 1 results in the following table for the location on the duct from the inlet. The EES *Equations* window is printed below, along with the plotted results.

Mach number, Ma	Duct length L, m
0.10	0
0.20	439
0.30	516
0.40	541
0.50	551
0.60	555
0.70	558
0.80	559
0.90	559
1.00	559



EES program:

```
k=1.667
cp=5.193
R=2.077

P1=180
T1=330
Ma1=0.1
"Ma2=1"
f=0.02
D=0.2
```

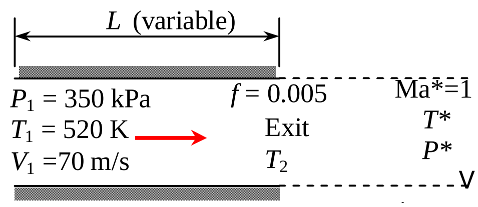
```
C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that the Mach number increases very mildly at the beginning, and then rapidly near the duct outlet. It takes 262 m of duct length for Mach number to increase from 0.1 to 0.2, but only 1 m to increase from 0.7 to 1.

12-113

Solution The flow of argon gas in a constant cross-sectional area adiabatic duct is considered. The variation of entropy change with exit temperature is to be investigated, and the calculated results are to be plotted on a *T-s* diagram.

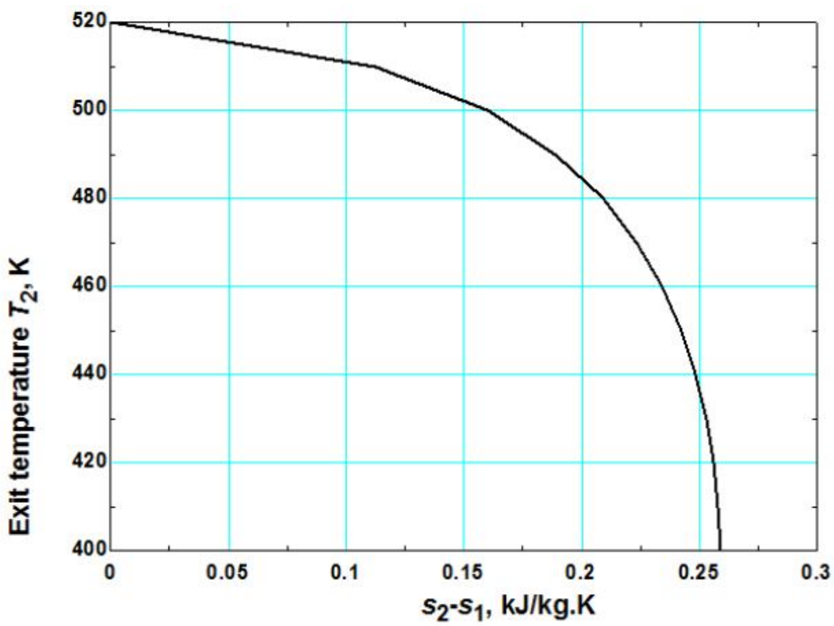
Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.



Properties The properties of argon are given to be k = 1.667, c_p = 0.5203 kJ/kg·K, and R = 0.2081 kJ/kg·K. The average friction factor is given to be f = 0.005.

Analysis Using EES, we determine the entropy change and tabulate and plot the results as follows:

Exit temp. T_2, K	Mach umber Ma_2	Entropy change Δs, kg/kg·K
520	0.165	0.000
510	0.294	0.112
500	0.385	0.160
490	0.461	0.189
480	0.528	0.209
470	0.591	0.224
460	0.649	0.234
450	0.706	0.242
440	0.760	0.248
430	0.813	0.253
420	0.865	0.256
410	0.916	0.258
400	0.967	0.259



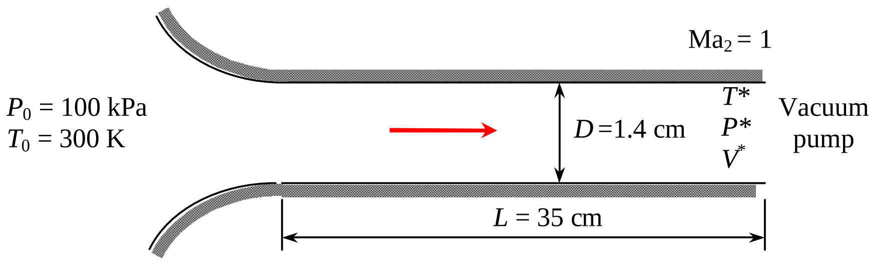
EES Program:

```
k=1.667
cp=0.5203
R=0.2081
P1=350
T1=520
V1=70
"T2=400"
f=0.005
D=0.08
C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))
rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1
P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
T2=T2Ts/T1Ts*T1
V2=V2Vs/V1Vs*V1
Del_s=cp*ln(T2/T1)-R*ln(P2/P1)
```

Discussion Note that entropy increases with increasing duct length and Mach number (and thus decreasing temperature). It reached a maximum value of 0.259 kJ/kg.K when the Mach number reaches Ma_2 =1 and thus the flow is choked.

12-114

Solution Air enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.018.

Analysis The mass flow rate will be maximum when the flow is choked, and thus the exit Mach number is $\mathrm{Ma}_2 = 1$. In that case we have

$$\frac{fL_1^*}{D} = \frac{fL_1}{D} = \frac{(0.018)(0.35\ \mathrm{m})}{0.014\ \mathrm{m}} = 0.45$$

The Mach number corresponding to this value of fL^*/D at the tube inlet is obtained from Table A-16 to be $\mathrm{Ma}_1 = 0.6107 \approx$ **0.611**. This value is obtained using the analytical relation. An interpolation on Table 16 gives 0.614. Noting that the flow in the nozzle section is isentropic, the thermodynamic temperature, pressure, and density at the tube inlet become

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (300\ \mathrm{K})\left(1+\frac{1.4-1}{2}(0.6107)^2\right)^{-1} = 279.2\ \mathrm{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (100\ \mathrm{kPa})\left(1+\frac{1.4-1}{2}(0.6107)^2\right)^{-1.4/0.4} = 77.74\ \mathrm{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{77.74\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kgK})(279.2\ \mathrm{K})} = 0.9702\ \mathrm{kg/m^3}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(279.2\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 334.9\ \mathrm{m/s}$$

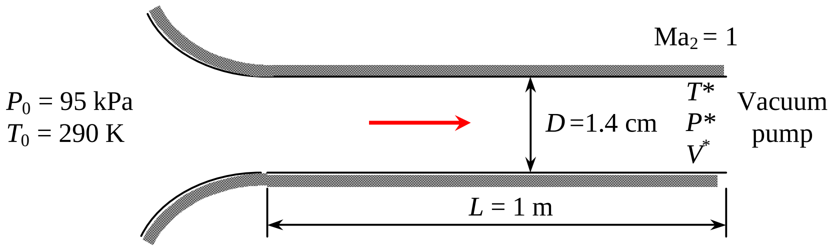
$$V_1 = \mathrm{Ma}_1 c_1 = 0.6107(334.9\ \mathrm{m/s}) = 204.5\ \mathrm{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.9702\ \mathrm{kg/m^3})[\pi(0.014\ \mathrm{m})^2/4](204.5\ \mathrm{m/s}) = \mathbf{0.0305\ kg/s}$$

Discussion This is the maximum mass flow rate through the tube for the specified stagnation conditions at the inlet. The flow rate will remain at this level even if the vacuum pump drops the pressure even further.

12-115

Solution Air enters a constant-area adiabatic duct at a specified state, and leaves at sonic state. The maximum duct length that will not cause the mass flow rate to be reduced is to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.025.

Analysis The mass flow rate will be maximum when the flow is choked, and thus the exit Mach number is Ma_2 = 1. In that case we have

$$\frac{fL_1^*}{D} = \frac{fL_1}{D} = \frac{(0.025)(1\,\text{m})}{0.014\,\text{m}} = 1.786$$

The Mach number corresponding to this value of fL^*/D at the tube inlet is obtained from Table A-16 to be Ma_1 = **0.4422**. Noting that the flow in the nozzle section is isentropic, the thermodynamic temperature, pressure, and density at the tube inlet become

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\text{Ma}_1^2\right)^{-1} = (290\,\text{K})\left(1+\frac{1.4\text{-}1}{2}(0.4422)^2\right)^{-1} = 279.1\,\text{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\text{Ma}_1^2\right)^{-k/(k-1)} = (95\,\text{kPa})\left(1+\frac{1.4\text{-}1}{2}(0.4422)^2\right)^{-1.4/0.4} = 83.06\,\text{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{83.06\,\text{kPa}}{(0.287\,\text{kJ/kgK})(279.1\,\text{K})} = 1.037\,\text{kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\,\text{kJ/kg}\cdot\text{K})(279.1\,\text{K})\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right)} = 334.9\,\text{m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.4422(334.9\,\text{m/s}) = 148.1\,\text{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.037\,\text{kg/m}^3)[\pi(0.014\,\text{m})^2/4](148.1\,\text{m/s}) = \mathbf{0.0236\ kg/s}$$

Discussion This is the maximum mass flow rate through the tube for the specified stagnation conditions at the inlet. The flow rate will remain at this level even if the vacuum pump drops the pressure even further.

Review Problems

12-116

Solution The thrust developed by the engine of a Boeing 777 is about 380 kN. The mass flow rate of gases through the nozzle is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow of combustion gases through the nozzle is isentropic. **3** Choked flow conditions exist at the nozzle exit. **4** The velocity of gases at the nozzle inlet is negligible.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K, and it can also be used for combustion gases. The specific heat ratio of combustion gases is k = 1.33.

Analysis The velocity at the nozzle exit is the sonic speed, which is determined to be

$$V = c = \sqrt{kRT} = \sqrt{(1.33)(0.287\ \text{kJ/kg}\cdot\text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)(220\ \text{K})} = 289.8\ \text{m/s}$$

Noting that thrust F is related to velocity by $F = \dot{m}V$, the mass flow rate of combustion gases is determined to be

$$\dot{m} = \frac{F}{V} = \frac{380{,}000\ \text{N}}{289.8\ \text{m/s}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 1311\ \text{kg/s} \cong \mathbf{1310\ kg/s}$$

Discussion The combustion gases are mostly nitrogen (due to the 78% of N_2 in air), and thus they can be treated as air with a good degree of approximation.

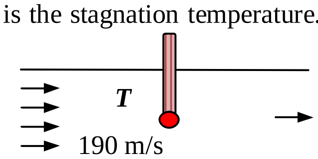
12-117

Solution A stationary temperature probe is inserted into an air duct reads 85°C. The actual temperature of air is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg·K.

Analysis The air that strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature. The actual air temperature is determined from



$$T = T_0 - \frac{V^2}{2c_p} = 85°\text{C} - \frac{(190\ \text{m/s})^2}{2\times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{67.0°C}$$

Discussion Temperature rise due to stagnation is very significant in high-speed flows, and should always be considered when compressibility effects are not negligible.

12-118

Solution Nitrogen flows through a heat exchanger. The stagnation pressure and temperature of the nitrogen at the inlet and the exit states are to be determined.

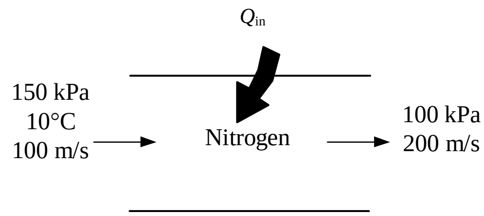
Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow of nitrogen through the heat exchanger is isentropic.

Properties The properties of nitrogen are c_p = 1.039 kJ/kg·K and k = 1.4.

Analysis The stagnation temperature and pressure of nitrogen at the inlet and the exit states are determined from

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 10°\text{C} + \frac{(100\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.8°C}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (150\text{ kPa})\left(\frac{288.0\text{ K}}{283.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{159\ kPa}$$



From the energy balance relation $E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$ with w = 0

$$q_{\text{in}} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + \Delta pe^{\nearrow 0}$$

$$150\text{ kJ/kg} = (1.039\text{ kJ/kg}\cdot°\text{C})(T_2 - 10°\text{C}) + \frac{(200\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

$$T_2 = 139.9°\text{C}$$

and

$$T_{02} = T_2 + \frac{V_2^2}{2c_p} = 139.9°\text{C} + \frac{(200\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{159°C}$$

$$P_{02} = P_2\left(\frac{T_{02}}{T_2}\right)^{k/(k-1)} = (100\text{ kPa})\left(\frac{432.3\text{ K}}{413.1\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{117\ kPa}$$

Discussion Note that the stagnation temperature and pressure can be very different than their thermodynamic counterparts when dealing with compressible flow.

12-119

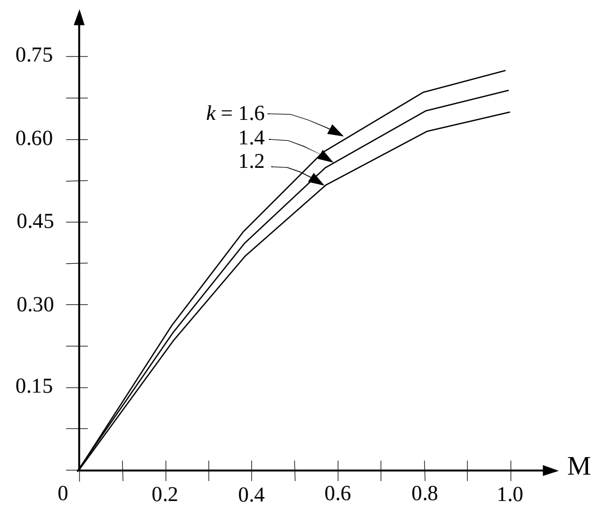
Solution The mass flow parameter $\dot{m}\sqrt{RT_0}/(AP_0)$ versus the Mach number for k = 1.2, 1.4, and 1.6 in the range of $0 \le \text{Ma} \le 1$ is to be plotted.

Analysis The mass flow rate parameter $(\dot{m}\sqrt{RT_0})/P_0 A$ can be expressed as

$$\frac{\dot{m}\sqrt{RT_0}}{P_0 A} = \text{Ma}\sqrt{k}\left(\frac{2}{2+(k-1)M^2}\right)^{(k+1)/2(k-1)}$$

Thus,

Ma	***k* = 1.2**	***k* = 1.4**	***k* = 1.6**
0.0	0	0	0
0.1	0.1089	0.1176	0.1257
0.2	0.2143	0.2311	0.2465
0.3	0.3128	0.3365	0.3582
0.4	0.4015	0.4306	0.4571
0.5	0.4782	0.5111	0.5407
0.6	0.5411	0.5763	0.6077
0.7	0.5894	0.6257	0.6578
0.8	0.6230	0.6595	0.6916
0.9	0.6424	0.6787	0.7106
1.0	0.6485	0.6847	0.7164



Discussion Note that the mass flow rate increases with increasing Mach number and specific heat ratio. It levels off at Ma = 1, and remains constant (choked flow).

12-120

Solution The equivalent relation for the speed of sound is to be verified using thermodynamic relations.

Analysis The two relations are $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$ and $c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$

From $r = 1/v \longrightarrow dr = -dv/v^2$. Thus, $c^2 = \left(\frac{\partial P}{\partial r}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\frac{\partial T}{\partial v}\right)_s = -v^2\left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial v}\right)_s$

From the cyclic rule,

$$(P,T,s):\left(\frac{\partial P}{\partial T}\right)_s\left(\frac{\partial T}{\partial s}\right)_P\left(\frac{\partial s}{\partial P}\right)_T = -1 \longrightarrow \left(\frac{\partial P}{\partial T}\right)_s = -\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T$$

$$(T,v,s):\left(\frac{\partial T}{\partial v}\right)_s\left(\frac{\partial v}{\partial s}\right)_T\left(\frac{\partial s}{\partial T}\right)_v = -1 \longrightarrow \left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v$$

Substituting,

$$c^2 = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial P}{\partial s}\right)_T\left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial T}{\partial s}\right)_v = -v^2\left(\frac{\partial s}{\partial T}\right)_P\left(\frac{\partial T}{\partial s}\right)_v\left(\frac{\partial P}{\partial v}\right)_T$$

Recall that $\frac{c_p}{T} = \left(\frac{\partial s}{\partial T}\right)_P$ and $\frac{c_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v$. Substituting,

$$c^2 = -v^2\left(\frac{c_p}{T}\right)\left(\frac{T}{c_v}\right)\left(\frac{\partial P}{\partial v}\right)_T = -v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

Replacing $-dv/v^2$ by dρ, we get $\boxed{c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T}$, which is the desired expression

Discussion Note that the differential thermodynamic property relations are very useful in the derivation of other property relations in differential form.

12-121

Solution For ideal gases undergoing isentropic flows, expressions for P/P^*, T/T^*, and ρ/ρ^* as functions of k and Ma are to be obtained.

Analysis Equations 12-18 and 12-21 are given to be $\frac{T_0}{T} = \frac{2+(k-1)\mathrm{Ma}^2}{2}$ and $\frac{T^*}{T_0} = \frac{2}{k+1}$

Multiplying the two, $\left(\frac{T_0}{T}\frac{T^*}{T_0}\right) = \left(\frac{2+(k-1)\mathrm{Ma}^2}{2}\right)\left(\frac{2}{k+1}\right)$

Simplifying and inverting, $\boxed{\frac{T}{T^*} = \frac{k+1}{2+(k-1)\mathrm{Ma}^2}}$ (1)

From $\frac{P}{P^*} = \left(\frac{T}{T^*}\right)^{k/(k-1)} \longrightarrow \boxed{\frac{P}{P^*} = \left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)}}$ (2)

From $\frac{\rho}{\rho^*} = \left(\frac{\rho}{\rho^*}\right)^{k/(k-1)} \longrightarrow \boxed{\frac{\rho}{\rho^*} = \left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{k/(k-1)}}$ (3)

Discussion Note that some very useful relations can be obtained by very simple manipulations.

12-122

Solution It is to be verified that for the steady flow of ideal gases $dT_0/T = dA/A + (1\text{-Ma}^2)\, dV/V$. The effect of heating and area changes on the velocity of an ideal gas in steady flow for subsonic flow and supersonic flow are to be explained.

Analysis We start with the relation $\dfrac{V^2}{2} = c_p(T_0 - T)$ (1)

Differentiating, $V\,dV = c_p(dT_0 - dT)$ (2)

We also have $\dfrac{d\rho}{\rho} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$ (3)

and $\dfrac{dP}{\rho} + V\,dV = 0$ (4)

Differentiating the ideal gas relation $P = \rho RT$, $\dfrac{dP}{P} = \dfrac{d\rho}{\rho} + \dfrac{dT}{T} = 0$ (5)

From the speed of sound relation, $c^2 = kRT = (k-1)c_pT = kP/\rho$ (6)

Combining Eqs. (3) and (5), $\dfrac{dP}{P} - \dfrac{dT}{T} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$ (7)

Combining Eqs. (4) and (6), $\dfrac{dP}{\rho} = \dfrac{dP}{kP/c^2} = -V dV$

or, $\dfrac{dP}{P} = -\dfrac{k}{C^2}V\,dV = -k\dfrac{V^2}{C^2}\dfrac{dV}{V} = -k\text{Ma}^2\dfrac{dV}{V}$ (8)

Combining Eqs. (2) and (6), $dT = dT_0 - V\dfrac{dV}{c_p}$

or, $\dfrac{dT}{T} = \dfrac{dT_0}{T} - \dfrac{V^2}{C_pT}\dfrac{dV}{V} = \dfrac{dT}{T} = \dfrac{dT_0}{T} - \dfrac{V^2}{C^2/(k-1)}\dfrac{dV}{V} = \dfrac{dT_0}{T} - (k-1)\text{Ma}^2\dfrac{dV}{V}$ (9)

Combining Eqs. (7), (8), and (9), $-(k-1)\text{Ma}^2\dfrac{dV}{V} - \dfrac{dT_0}{T} + (k-1)\text{Ma}^2\dfrac{dV}{V} + \dfrac{dA}{A} + \dfrac{dV}{V} = 0$

or, $\dfrac{dT_0}{T} = \dfrac{dA}{A} + \left[-k\text{Ma}^2 + (k-1)\text{Ma}^2 + 1\right]\dfrac{dV}{V}$

Thus, $\boxed{\dfrac{dT_0}{T} = \dfrac{dA}{A} + (1-\text{Ma}^2)\dfrac{dV}{V}}$ (10)

Differentiating the steady-flow energy equation $q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$

$\delta q = c_p dT_0$ (11)

Eq. (11) relates the stagnation temperature change dT_0 to the net heat transferred to the fluid. Eq. (10) relates the velocity changes to area changes dA, and the stagnation temperature change dT_0 or the heat transferred.

(*a*) When Ma < 1 (subsonic flow), **the fluid accelerates if the duct converges** ($dA < 0$) **or the fluid is heated** ($dT_0 > 0$ or $\delta q > 0$). **The fluid decelerates if the duct converges** ($dA < 0$) **or the fluid is cooled** ($dT_0 < 0$ or $\delta q < 0$).

(*b*) When Ma > 1 (supersonic flow), **the fluid accelerates if the duct diverges** ($dA > 0$) **or the fluid is cooled** ($dT_0 < 0$ or $\delta q < 0$). **The fluid decelerates if the duct converges** ($dA < 0$) **or the fluid is heated** ($dT_0 > 0$ or $\delta q > 0$).

Discussion Some of these results are not intuitively obvious, but come about by satisfying the conservation equations.

12-123

Solution A Pitot-static probe measures the difference between the static and stagnation pressures for a subsonic airplane. The speed of the airplane and the flight Mach number are to be determined.

Assumptions **1** Air is an ideal gas with a constant specific heat ratio. **2** The stagnation process is isentropic.

Properties The properties of air are R = 0.287 kJ/kg·K and k = 1.4.

Analysis The stagnation pressure of air at the specified conditions is

$$P_0 = P + \Delta P = 54 + 16 = 70 \text{ kPa}$$

Then,

$$\frac{P_0}{P} = \left(1 + \frac{(k-1)\text{Ma}^2}{2}\right)^{k/k-1} \longrightarrow \frac{70}{54} = \left(1 + \frac{(1.4-1)\text{Ma}^2}{2}\right)^{1.4/0.4}$$

It yields **Ma = 0.620**

The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(256 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 320.7 \text{ m/s}$$

Thus,

$$V = \text{Ma} \times c = (0.620)(320.7 \text{ m/s}) = \mathbf{199 \text{ m/s}}$$

Discussion Note that the flow velocity can be measured in a simple and accurate way by simply measuring pressure.

12-124

Solution An expression for the speed of sound based on van der Waals equation of state is to be derived. Using this relation, the speed of sound in carbon dioxide is to be determined and compared to that obtained by ideal gas behavior.

Properties The properties of CO_2 are R = 0.1889 kJ/kg·K and k = 1.279 at T = 50°C = 323.2 K.

Analysis Van der Waals equation of state can be expressed as $P = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$.

Differentiating, $\left(\dfrac{\partial P}{\partial v}\right)_T = \dfrac{RT}{(v-b)^2} + \dfrac{2a}{v^3}$

Noting that $\rho = 1/v \longrightarrow d\rho = -dv/v^2$, the speed of sound relation becomes

Substituting,

$$c^2 = k\left(\frac{\partial P}{\partial r}\right)_T = v^2 k\left(\frac{\partial P}{\partial v}\right)_T$$

$$c^2 = \frac{v^2 kRT}{(v-b)^2} - \frac{2ak}{v}$$

Using the molar mass of CO_2 (M = 44 kg/kmol), the constant a and b can be expressed per unit mass as

$$a = 0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^2 \quad \text{and} \qquad b = 9.70\times10^{-4}\ \text{m}^3/\text{kg}$$

The specific volume of CO_2 is determined to be

$$200\ \text{kPa} = \frac{(0.1889\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323.2\ \text{K})}{\text{v} - 0.000970\ \text{m}^3/\text{kg}} - \frac{2\times 0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^2}{v^2} \to v = 0.300\ \text{m}^3/\text{kg}$$

Substituting,

$$\text{c} = \left(\left(\frac{(0.300\ \text{m}^3/\text{kg})^2(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})}{(0.300 - 0.000970\ \text{m}^3/\text{kg})^2} - \frac{2(0.1882\ \text{kPa}\cdot\text{m}^6/\text{kg}^3)(1.279)}{(0.300\ \text{m}^3/\text{kg})^2}\right)\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kPa}\cdot\text{m}^3/\text{kg}}\right)\right)^{1/2}$$

$$= \mathbf{271\ m/s}$$

If we treat CO_2 as an ideal gas, the speed of sound becomes

$$c = \sqrt{kRT} = \sqrt{(1.279)(0.1889\ \text{kJ/kg}\cdot\text{K})(323.2\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{279\ m/s}$$

Discussion Note that the ideal gas relation is the simplest equation of state, and it is very accurate for most gases encountered in practice. At high pressures and/or low temperatures, however, the gases deviate from ideal gas behavior, and it becomes necessary to use more complicated equations of state.

12-125

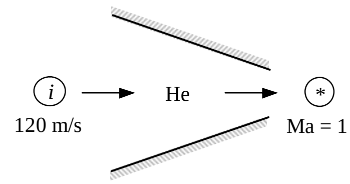
Solution Helium gas is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667.

Analysis The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *. We first determine the stagnation temperature and pressure, which remain constant throughout the nozzle since the flow is isentropic.

$$T_0 = T_i + \frac{V_i^2}{2c_p} = 560\text{ K} + \frac{(120\text{ m/s})^2}{2\times 5.1926\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 561.4\text{ K}$$



and

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{561.4\text{ K}}{560\text{ K}}\right)^{1.667/(1.667-1)} = 0.6037\text{ MPa}$$

The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (561.4\text{ K})\left(\frac{2}{1.667+1}\right) = 421.0\text{ K} = \mathbf{421\text{ K}}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.6037\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 0.2941\text{ MPa} \cong \mathbf{0.294\text{ MPa}}$$

The speed of sound and the Mach number at the nozzle inlet are

$$c_i = \sqrt{kRT}_i = \sqrt{(1.667)(2.0769\text{ kJ/kg}\cdot\text{K})(560\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1392\text{ m/s}$$

$$\text{Ma}_i = \frac{V_i}{c_i} = \frac{120\text{ m/s}}{1392\text{ m/s}} = 0.08618$$

The ratio of the entrance-to-throat area is

$$\frac{A_i}{A^*} = \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]}$$

$$= \frac{1}{0.08618}\left[\left(\frac{2}{1.667+1}\right)\left(1+\frac{1.667-1}{2}(0.08618)^2\right)\right]^{2.667/(2\times 0.667)}$$

$$= 8.745$$

Then the ratio of the throat area to the entrance area becomes

$$\frac{A^*}{A_i} = \frac{1}{8.745} = 0.1144 \cong \mathbf{0.114}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

12-126

Solution Helium gas enters a nozzle with negligible velocity, and is accelerated in a nozzle. The pressure and temperature of helium at the location where Ma = 1 and the ratio of the flow area at this location to the inlet flow area are to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The entrance velocity is negligible.

Properties The properties of helium are R = 2.0769 kJ/kg·K, c_p = 5.1926 kJ/kg·K, and k = 1.667.

Analysis We treat helium as an ideal gas with k = 1.667. The properties of the fluid at the location where Ma = 1 are the critical properties, denoted by superscript *.

The stagnation temperature and pressure in this case are identical to the inlet temperature and pressure since the inlet velocity is negligible. They remain constant throughout the nozzle since the flow is isentropic.

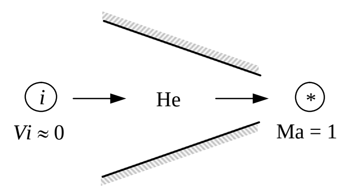
$$T_0 = T_i = 560 \text{ K}$$

$$P_0 = P_i = 0.6 \text{ MPa}$$

The Mach number at the nozzle exit is given to be Ma = 1. Therefore, the properties at the nozzle exit are the *critical properties* determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (560\text{ K})\left(\frac{2}{1.667+1}\right) = \mathbf{420\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (0.6\text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = \mathbf{0.292\ MPa}$$



The ratio of the nozzle inlet area to the throat area is determined from

$$\frac{A_i}{A^*} = \frac{1}{\text{Ma}_i}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}_i^2\right)\right]^{(k+1)/[2(k-1)]}$$

But the Mach number at the nozzle inlet is Ma = 0 since $V_i \cong 0$. Thus the ratio of the throat area to the nozzle inlet area is

$$\frac{A^*}{A_i} = \frac{1}{\infty} = \mathbf{0}$$

Discussion The compressible flow functions are essential tools when determining the proper shape of the compressible flow duct.

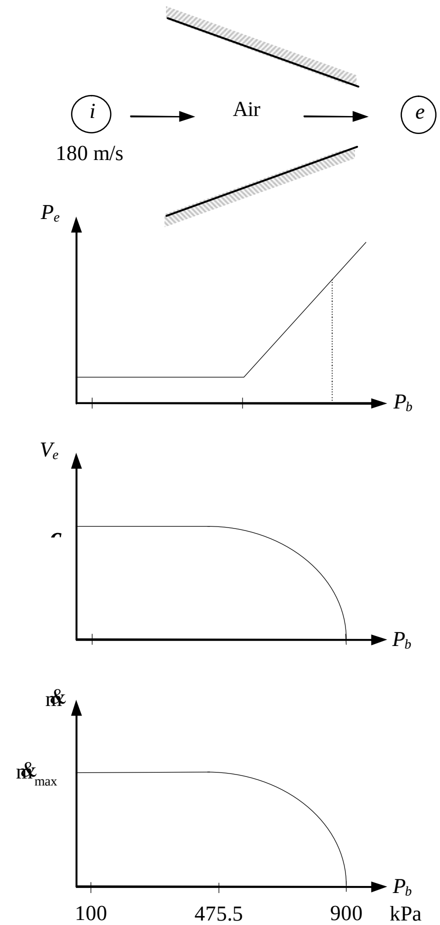
12-127

Solution Air enters a converging nozzle. The mass flow rate, the exit velocity, the exit Mach number, and the exit pressure-stagnation pressure ratio versus the back pressure-stagnation pressure ratio for a specified back pressure range are to be calculated and plotted.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The properties of air at room temperature are R = 0.287 kJ/kg·K, c_p = 1.005 kJ/kg·K, and k = 1.4.

Analysis We use EES to tabulate and plot the results. The stagnation properties remain constant throughout the nozzle since the flow is isentropic. They are determined from



$$T_0 = T_i + \frac{V_i^2}{2c_p} = 400\text{ K} + \frac{(180\text{ m/s})^2}{2\times 1.005\text{ kJ/kg}\cdot\text{K}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 416.1\text{ K}$$

$$P_0 = P_i\left(\frac{T_0}{T_i}\right)^{k/(k-1)} = (900\text{ kPa})\left(\frac{416.1\text{ K}}{400\text{ K}}\right)^{1.4/(1.4-1)} = 1033.3\text{ kPa}$$

The critical pressure is determined to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1033.3\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/0.4} = 545.9\text{ kPa}$$

Then the pressure at the exit plane (throat) is

$P_e = P_b$ for $P_b \geq 545.9$ kPa

$P_e = P^* = 545.9$ kPa for $P_b < 545.9$ kPa (choked flow)

Thus the back pressure does not affect the flow when $100 < P_b < 545.9$ kPa. For a specified exit pressure P_e, the temperature, velocity, and mass flow rate are

Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (416.1\text{K})\left(\frac{\text{P}_\text{e}}{1033.3}\right)^{0.4/1.4}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.005\text{ kJ/kg}\cdot\text{K})(416.1 - T_e)\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Speed of sound $$c_e = \sqrt{kRT_e} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Mach number $$\text{Ma}_e = V_e / c_e$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.001\text{ m}^2)$$

P_b, kPa	P_b, P_0	P_e, kPa	P_b, P_0	T_e, K	V_e, m/s	Ma	ρ_e, kg/m^3	$\dot{m}$, kg / s
900	0.871	900	0.871	400.0	180.0	0.45	7.840	0
800	0.774	800	0.774	386.8	162.9	0.41	7.206	1.174
700	0.677	700	0.677	372.3	236.0	0.61	6.551	1.546
600	0.581	600	0.581	356.2	296.7	0.78	5.869	1.741
545.9	0.528	545.9	0.528	333.3	366.2	1.00	4.971	1.820
500	0.484	545.9	0.528	333.2	366.2	1.00	4.971	1.820
400	0.387	545.9	0.528	333.3	366.2	1.00	4.971	1.820
300	0.290	545.9	0.528	333.3	366.2	1.00	4.971	1.820
200	0.194	545.9	0.528	333.3	366.2	1.00	4.971	1.820
100	0.097	545.9	0.528	333.3	366.2	1.00	4.971	1.820

Discussion Once the back pressure drops below 545.0 kPa, the flow is choked, and $\dot{m}$ remains constant from then on.

12-128

Solution Nitrogen gas enters a converging nozzle. The properties at the nozzle exit are to be determined.

Assumptions **1** Nitrogen is an ideal gas with $k = 1.4$. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The schematic of the duct is shown in Fig. 12–25. For isentropic flow through a duct, the area ratio A/A^* (the flow area over the area of the throat where Ma = 1) is also listed in Table A–13. At the initial Mach number of Ma = 0.3, we read

$$\frac{A_1}{A^*} = 2.0351, \qquad \frac{T_1}{T_0} = 0.9823, \quad \text{and} \qquad \frac{P_1}{P_0} = 0.9395$$

With a 20 percent reduction in flow area, $A_2 = 0.8A_1$, and

$$\frac{A_2}{A^*} = \frac{A_2}{A_1}\frac{A_1}{A^*} = (0.8)(2.0351) = 1.6281$$

For this value of A_2/A^* from Table A–13, we read

$$\frac{T_2}{T_0} = 0.9791, \qquad \frac{P_2}{P_0} = 0.8993, \quad \text{and} \quad \text{Ma}_2 =$$

Here we chose the subsonic Mach number for the calculated A_2/A^* instead of the supersonic one because the duct is converging in the flow direction and the initial flow is subsonic. Since the stagnation properties are constant for isentropic flow, we can write

$$\frac{T_2}{T_1} = \frac{T_2/T_0}{T_1/T_0} \quad \rightarrow \quad T_2 = T_1\left(\frac{T_2/T_0}{T_1/T_0}\right) = (400\text{ K})\left(\frac{0.9791}{0.9823}\right) = \mathbf{399\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P_0}{P_1/P_0} \quad \rightarrow \quad P_2 = P_1\left(\frac{P_2/P_0}{P_1/P_0}\right) = (100\text{ kPa})\left(\frac{0.8993}{0.9395}\right) = \mathbf{95.7\ K}$$

which are the temperature and pressure at the desired location.

Discussion Note that the temperature and pressure drop as the fluid accelerates in a converging nozzle.

12-129

Solution Nitrogen gas enters a converging nozzle. The properties at the nozzle exit are to be determined.

Assumptions **1** Nitrogen is an ideal gas with $k = 1.4$. **2** Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The schematic of the duct is shown in Fig. 12–25. For isentropic flow through a duct, the area ratio A/A^* (the flow area over the area of the throat where Ma = 1) is also listed in Table A–13. At the initial Mach number of Ma = 0.5, we read

$$\frac{A_1}{A^*} = 1.3398, \qquad \frac{T_1}{T_0} = 0.9524, \quad \text{and} \qquad \frac{P_1}{P_0} = 0.8430$$

With a 20 percent reduction in flow area, $A_2 = 0.8A_1$, and

$$\frac{A_2}{A^*} = \frac{A_2}{A_1}\frac{A_1}{A^*} = (0.8)(1.3398) = 1.0718$$

For this value of A_2/A^* from Table A–13, we read

$$\frac{T_2}{T_0} = 0.9010, \qquad \frac{P_2}{P_0} = 0.6948, \quad \text{and } \mathrm{Ma}_2 = \mathbf{0.740}$$

Here we chose the subsonic Mach number for the calculated A_2/A^* instead of the supersonic one because the duct is converging in the flow direction and the initial flow is subsonic. Since the stagnation properties are constant for isentropic flow, we can write

$$\frac{T_2}{T_1} = \frac{T_2/T_0}{T_1/T_0} \quad \rightarrow \quad T_2 = T_1\left(\frac{T_2/T_0}{T_1/T_0}\right) = (400\ \mathrm{K})\left(\frac{0.9010}{0.9524}\right) = \mathbf{378\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P_0}{P_1/P_0} \quad \rightarrow \quad P_2 = P_1\left(\frac{P_2/P_0}{P_1/P_0}\right) = (100\ \mathrm{kPa})\left(\frac{0.6948}{0.8430}\right) = \mathbf{82.4\ K}$$

which are the temperature and pressure at the desired location.

Discussion Note that the temperature and pressure drop as the fluid accelerates in a converging nozzle.

12-130

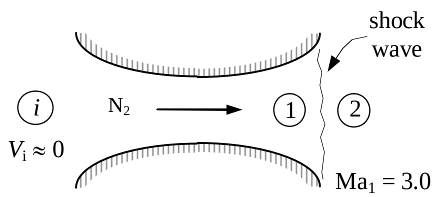
Solution Nitrogen entering a converging-diverging nozzle experiences a normal shock. The pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock are to be determined. The results are to be compared to those of air under the same conditions.

Assumptions **1** Nitrogen is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of nitrogen are R = 0.297 kJ/kg·K and k = 1.4.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Assuming the flow before the shock to be isentropic,

$$P_{01} = P_i = 620\ \text{kPa}$$
$$T_{01} = T_i = 310\ \text{K}$$



Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (310\ \text{K})\left(\frac{2}{2+(1.4\text{-}1)3^2}\right) = 110.7\ \text{K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{k/(k-1)} = (620\ \text{kPa})\left(\frac{110.7}{310}\right)^{1.4/0.4} = 16.88\ \text{kPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For $\text{Ma}_1 = 3.0$ we read

$$\text{Ma}_2 = 0.4752 \cong \mathbf{0.475}, \quad \frac{P_{02}}{P_{01}} = 0.32834, \quad \frac{P_2}{P_1} = 10.333, \quad \text{and} \quad \frac{T_2}{T_1} = 2.679$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.32834 P_{01} = (0.32834)(620\ \text{kPa}) = 203.6\ \text{kPa} \cong \mathbf{204\ kPa}$$
$$P_2 = 10.333 P_1 = (10.333)(16.88\ \text{kPa}) = 174.4\ \text{kPa} \cong \mathbf{174\ kPa}$$
$$T_2 = 2.679 T_1 = (2.679)(110.7\ \text{K}) = 296.6\ \text{K} \cong \mathbf{297\ K}$$

The velocity after the shock can be determined from $V_2 = \text{Ma}_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (0.4752)\sqrt{(1.4)(0.297\ \text{kJ/kg}\cdot\text{K})(296.6\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 166.9\ \text{m/s} \cong \mathbf{167\ m/s}$$

Discussion For *air* at specified conditions k = 1.4 (same as nitrogen) and R = 0.287 kJ/kg·K. Thus the only quantity which will be different in the case of air is the velocity after the normal shock, which happens to be 164.0 m/s.

12-131

Solution The diffuser of an aircraft is considered. The static pressure rise across the diffuser and the exit area are to be determined.

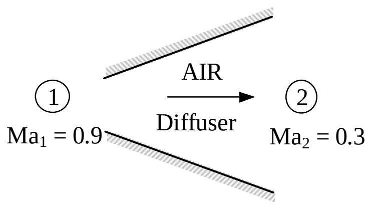
Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the diffuser is steady, one-dimensional, and isentropic. **3** The diffuser is adiabatic.

Properties Air properties at room temperature are R = 0.287 kJ/kg·K, c_p = 1.005 kJ/kg·K, and k = 1.4.

Analysis The inlet velocity is

$$V_1 = \mathrm{Ma}_1 c_1 = M_1\sqrt{kRT_1} = (0.9)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(242.7\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 281.0\ \mathrm{m/s}$$

Then the stagnation temperature and pressure at the diffuser inlet become



$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 242.7 + \frac{(281.0\ \mathrm{m/s})^2}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 282.0\ \mathrm{K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{282.0\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 69.50\ \mathrm{kPa}$$

For an adiabatic diffuser, the energy equation reduces to $h_{01} = h_{02}$. Noting that $h = c_pT$ and the specific heats are assumed to be constant, we have

$$T_{01} = T_{02} = T_0 = 282.0\ \mathrm{K}$$

The isentropic relation between states 1 and 02 gives

$$P_{02} = P_{02} = P_1\left(\frac{T_{02}}{T_1}\right)^{k/(k-1)} = (41.1\ \mathrm{kPa})\left(\frac{282.0\ \mathrm{K}}{242.7\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 69.50\ \mathrm{kPa}$$

The exit velocity can be expressed as

$$V_2 = \mathrm{Ma}_2 c_2 = \mathrm{Ma}_2\sqrt{kRT_2} = (0.3)\sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})\,T_2\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 6.01\sqrt{T_2}$$

Thus $$T_2 = T_{02} - \frac{V_2^2}{2c_p} = (282.0) - \frac{6.01^2\,\mathrm{T_2\ m^2/s^2}}{2(1.005\ \mathrm{kJ/kg\cdot K})}\left(\frac{1\ \mathrm{kJ/kg}}{1000\ \mathrm{m^2/s^2}}\right) = 277.0\ \mathrm{K}$$

Then the static exit pressure becomes

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{k/(k-1)} = (69.50\ \mathrm{kPa})\left(\frac{277.0\ \mathrm{K}}{282.0\ \mathrm{K}}\right)^{1.4/(1.4-1)} = 65.28\ \mathrm{kPa}$$

Thus the static pressure rise across the diffuser is

$$\Delta P = P_2 - P_1 = 65.28 - 41.1 = \mathbf{24.2\ kPa}$$

Also, $$\rho_2 = \frac{P_2}{RT_2} = \frac{65.28\ \mathrm{kPa}}{(0.287\ \mathrm{kPa\cdot m^3/kg\cdot K})(277.0\ \mathrm{K})} = 0.8211\ \mathrm{kg/m^3}$$

$$V_2 = 6.01\sqrt{T_2} = 6.01\sqrt{277.0} = 100.0\ \mathrm{m/s}$$

Thus $$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{38\ \mathrm{kg/s}}{(0.8211\ \mathrm{kg/m^3})(100.0\ \mathrm{m/s})} = \mathbf{0.463\ m^2}$$

Discussion The pressure rise in actual diffusers will be lower because of the irreversibilities. However, flow through well-designed diffusers is very nearly isentropic.

12-132

Solution The critical temperature, pressure, and density of an equimolar mixture of oxygen and nitrogen for specified stagnation properties are to be determined.

Assumptions Both oxygen and nitrogen are ideal gases with constant specific heats at room temperature.

Properties The specific heat ratio and molar mass are k = 1.395 and M = 32 kg/kmol for oxygen, and k = 1.4 and M = 28 kg/kmol for nitrogen.

Analysis The gas constant of the mixture is

$$M_m = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5\times 32 + 0.5\times 28 = 30 \text{ kg/kmol}$$

$$R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{30 \text{ kg/kmol}} = 0.2771 \text{ kJ/kg}\cdot\text{K}$$

The specific heat ratio is 1.4 for nitrogen, and nearly 1.4 for oxygen. Therefore, the specific heat ratio of the mixture is also 1.4. Then the critical temperature, pressure, and density of the mixture become

$$T^* = T_0\left(\frac{2}{k+1}\right) = (550\text{ K})\left(\frac{2}{1.4+1}\right) = 458.3\text{ K} \cong \mathbf{458\ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (350\text{ kPa})\left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)} = 184.9\text{ kPa} \cong \mathbf{185\ kPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{184.9\text{ kPa}}{(0.2771\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(458.3\text{ K})} = \mathbf{1.46\ kg/m^3}$$

Discussion If the specific heat ratios k of the two gases were different, then we would need to determine the k of the mixture from $k = c_{p,m}/c_{v,m}$ where the specific heats of the mixture are determined from

$$C_{p,m} = \text{mf}_{O_2} c_{p,O_2} + \text{mf}_{N_2} c_{p,N_2} = (y_{O_2} M_{O_2}/M_m)c_{p,O_2} + (y_{N_2} M_{N_2}/M_m)c_{p,N_2}$$

$$C_{v,m} = \text{mf}_{O_2} c_{v,O_2} + \text{mf}_{N_2} c_{v,N_2} = (y_{O_2} M_{O_2}/M_m)c_{v,O_2} + (y_{N_2} M_{N_2}/M_m)c_{v,N_2}$$

where mf is the mass fraction and y is the mole fraction. In this case it would give

$$c_{p,m} = (0.5\times 32/30)\times 0.918 + (0.5\times 28/30)\times 1.039 = 0.974 \text{ kJ/kg.K}$$

$$c_{p,m} = (0.5\times 32/30)\times 0.658 + (0.5\times 28/30)\times 0.743 = 0.698 \text{ kJ/kg.K}$$

and

k = 0.974/0.698 = 1.40

12-133E

Solution Helium gas is accelerated in a nozzle. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined for the case of isentropic nozzle.

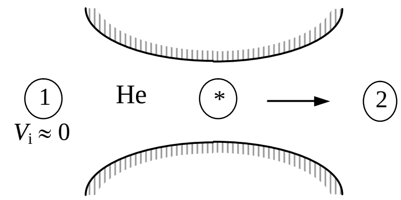
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 0.4961 Btu/lbm·R = 2.6809 psia·ft^3/lbm·R, c_p = 1.25 Btu/lbm·R, and k = 1.667.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 740 \text{ R}$$

$$P_{01} = P_1 = 220 \text{ psia}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 740 \text{ R}$$

$$P_{02} = P_{01} = 220 \text{ psia}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (740 \text{ R})\left(\frac{2}{1.667+1}\right) = 554.9 \text{ R}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (220 \text{ psia})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 107.2 \text{ psia}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{107.2 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(554.9 \text{ R})} = 0.072031 \text{bm/ft}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(554.9 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = 3390 \text{ ft/s}$$

and $$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.2 \text{ 1bm/s}}{(0.072031 \text{bm/ft}^3)(3390 \text{ ft/s})} = \mathbf{8.19 \times 10^{-4} \ ft^2}$$

At the nozzle exit the pressure is P_2 = 15 psia. Then the other properties at the nozzle exit are determined to be

$$\frac{p_0}{p_2} = \left(1 + \frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{220 \text{ psia}}{15 \text{ psia}} = \left(1 + \frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma$_2$ = 2.405, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (740 \text{ R})\left(\frac{2}{2+(1.667-1)\times 2.405^2}\right) = 252.6 \text{ R}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{15 \text{ psia}}{(2.6809 \text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(252.6 \text{ R})} = 0.02215 \text{ 1bm/ft}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.405)\sqrt{(1.667)(0.4961 \text{ Btu/lbm}\cdot\text{R})(252.6 \text{ R})\left(\frac{25{,}037 \text{ ft}^2/\text{s}^2}{1 \text{ Btu/1bm}}\right)} = 5500 \text{ ft/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.2 \text{ lbm/s}}{(0.02215 \text{ lbm/ft}^3)(5500 \text{ ft/s})} = \mathbf{0.00164 \ ft^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

12-134

Solution Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for an ideal gas with k = 1.667.

Properties The specific heat ratio of the ideal gas is given to be k = 1.667.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\text{Ma}^* = \text{Ma}\sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\text{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\text{Ma}^2\right)^{-1}$$

```
k=1.667
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0.0	0	∞	1.0000	1.0000	1.0000
0.1	0.1153	5.6624	0.9917	0.9950	0.9967
0.2	0.2294	2.8879	0.9674	0.9803	0.9868
0.3	0.3413	1.9891	0.9288	0.9566	0.9709
0.4	0.4501	1.5602	0.8782	0.9250	0.9493
0.5	0.5547	1.3203	0.8186	0.8869	0.9230
0.6	0.6547	1.1760	0.7532	0.8437	0.8928
0.7	0.7494	1.0875	0.6850	0.7970	0.8595
0.8	0.8386	1.0351	0.6166	0.7482	0.8241
0.9	0.9222	1.0081	0.5501	0.6987	0.7873
1.0	1.0000	1.0000	0.4871	0.6495	0.7499
1.2	1.1390	1.0267	0.3752	0.5554	0.6756
1.4	1.2572	1.0983	0.2845	0.4704	0.6047
1.6	1.3570	1.2075	0.2138	0.3964	0.5394
1.8	1.4411	1.3519	0.1603	0.3334	0.4806
2.0	1.5117	1.5311	0.1202	0.2806	0.4284
2.2	1.5713	1.7459	0.0906	0.2368	0.3825
2.4	1.6216	1.9980	0.0686	0.2005	0.3424
2.6	1.6643	2.2893	0.0524	0.1705	0.3073
2.8	1.7007	2.6222	0.0403	0.1457	0.2767
3.0	1.7318	2.9990	0.0313	0.1251	0.2499
5.0	1.8895	9.7920	0.0038	0.0351	0.1071
∝	1.9996	∞	0	0	0

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case k = 1.667.

12-135

Solution Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for an ideal gas with k = 1.667.

Properties The specific heat ratio of the ideal gas is given to be k = 1.667.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2=\sqrt{\frac{(k-1)\mathrm{Ma}_1^2+2}{2k\mathrm{Ma}_1^2-k+1}} \qquad \frac{P_2}{P_1}=\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}=\frac{2k\mathrm{Ma}_1^2-k+1}{k+1}$$

$$\frac{T_2}{T_1}=\frac{2+\mathrm{Ma}_1^2(k-1)}{2+\mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1}=\frac{P_2/P_1}{T_2/T_1}=\frac{(k+1)\mathrm{Ma}_1^2}{2+(k-1)\mathrm{Ma}_1^2}=\frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}}=\frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1+\mathrm{Ma}_2^2(k-1)/2}{1+\mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1}=\frac{(1+k\mathrm{Ma}_1^2)[1+\mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1+k\mathrm{Ma}_2^2}$$

```
k=1.667
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	2.0530
1.1	0.9131	1.2625	1.1496	1.0982	0.999	2.3308
1.2	0.8462	1.5500	1.2972	1.1949	0.9933	2.6473
1.3	0.7934	1.8626	1.4413	1.2923	0.9813	2.9990
1.4	0.7508	2.2001	1.5805	1.3920	0.9626	3.3838
1.5	0.7157	2.5626	1.7141	1.4950	0.938	3.8007
1.6	0.6864	2.9501	1.8415	1.6020	0.9085	4.2488
1.7	0.6618	3.3627	1.9624	1.7135	0.8752	4.7278
1.8	0.6407	3.8002	2.0766	1.8300	0.8392	5.2371
1.9	0.6227	4.2627	2.1842	1.9516	0.8016	5.7767
2.0	0.6070	4.7503	2.2853	2.0786	0.763	6.3462
2.1	0.5933	5.2628	2.3802	2.2111	0.7243	6.9457
2.2	0.5814	5.8004	2.4689	2.3493	0.6861	7.5749
2.3	0.5708	6.3629	2.5520	2.4933	0.6486	8.2339
2.4	0.5614	6.9504	2.6296	2.6432	0.6124	8.9225
2.5	0.5530	7.5630	2.7021	2.7989	0.5775	9.6407
2.6	0.5455	8.2005	2.7699	2.9606	0.5442	10.3885
2.7	0.5388	8.8631	2.8332	3.1283	0.5125	11.1659
2.8	0.5327	9.5506	2.8923	3.3021	0.4824	11.9728
2.9	0.5273	10.2632	2.9476	3.4819	0.4541	12.8091
3.0	0.5223	11.0007	2.9993	3.6678	0.4274	13.6750
4.0	0.4905	19.7514	3.3674	5.8654	0.2374	23.9530
5.0	0.4753	31.0022	3.5703	8.6834	0.1398	37.1723
∞	0.4473	∞	3.9985	∞	0	∞

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case k = 1.667.

12-136

Solution Helium gas is accelerated in a nozzle isentropically. For a specified mass flow rate, the throat and exit areas of the nozzle are to be determined.

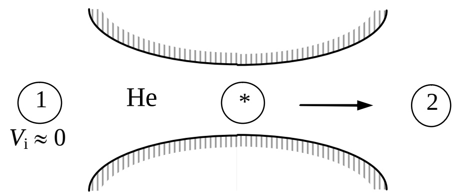
Assumptions **1** Helium is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The properties of helium are R = 2.0769 kJ/kg.K, c_p = 5.1926 kJ/kg.K, and k = 1.667.

Analysis The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible,

$$T_{01} = T_1 = 500 \text{ K}$$

$$P_{01} = P_1 = 1.0 \text{ MPa}$$



The flow is assumed to be isentropic, thus the stagnation temperature and pressure remain constant throughout the nozzle,

$$T_{02} = T_{01} = 500 \text{ K}$$

$$P_{02} = P_{01} = 1.0 \text{ MPa}$$

The critical pressure and temperature are determined from

$$T^* = T_0\left(\frac{2}{k+1}\right) = (500 \text{ K})\left(\frac{2}{1.667+1}\right) = 375.0 \text{ K}$$

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (1.0 \text{ MPa})\left(\frac{2}{1.667+1}\right)^{1.667/(1.667-1)} = 0.487 \text{ MPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{487 \text{ kPa}}{(2.0769 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(375 \text{ K})} = 0.625 \text{ kg/m}^3$$

$$V^* = c^* = \sqrt{kRT^*} = \sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(375 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 1139.4 \text{ m/s}$$

Thus the throat area is

$$A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{0.46 \text{ kg/s}}{(0.625 \text{ kg/m}^3)(1139.4 \text{ m/s})} = 6.460\times10^{-4} \text{ m}^2 = \mathbf{6.46 \text{ cm}^2}$$

At the nozzle exit the pressure is P_2 = 0.1 MPa. Then the other properties at the nozzle exit are determined to be

$$\frac{P_0}{P_2} = \left(1+\frac{k-1}{2}\text{Ma}_2^2\right)^{k/(k-1)} \longrightarrow \frac{1.0 \text{ MPa}}{0.1 \text{ MPa}} = \left(1+\frac{1.667-1}{2}\text{Ma}_2^2\right)^{1.667/0.667}$$

It yields Ma_2 = 2.130, which is greater than 1. Therefore, the nozzle must be converging-diverging.

$$T_2 = T_0\left(\frac{2}{2+(k-1)\text{Ma}_2^2}\right) = (500 \text{ K})\left(\frac{2}{2+(1.667-1)\times 2.13^2}\right) = 199.0 \text{ K}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{100 \text{ kPa}}{(2.0769 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(199 \text{ K})} = 0.242 \text{ kg/m}^3$$

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2\sqrt{kRT_2} = (2.13)\sqrt{(1.667)(2.0769 \text{ kJ/kg}\cdot\text{K})(199 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 1768.0 \text{ m/s}$$

Thus the exit area is

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{0.46 \text{ kg/s}}{(0.242 \text{ kg/m}^3)(1768 \text{ m/s})} = 0.1075\times10^{-3} \text{ m}^2 \cong \mathbf{10.8 \text{ cm}^2}$$

Discussion Flow areas in actual nozzles would be somewhat larger to accommodate the irreversibilities.

12-137

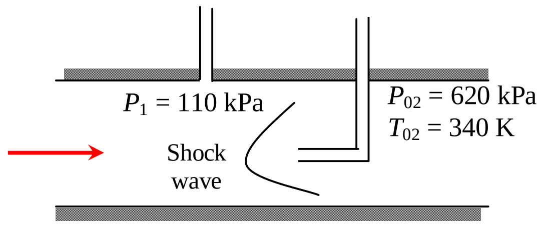
Solution The flow velocity of air in a channel is to be measured using a Pitot-static probe, which causes a shock wave to occur. For measured values of static pressure before the shock and stagnation pressure and temperature after the shock, the flow velocity before the shock is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady and one-dimensional.

Properties The specific heat ratio of air at room temperature is k= 1.4.

Analysis The nose of the probe is rounded (instead of being pointed), and thus it will cause a bow shock wave to form. Bow shocks are difficult to analyze. But they are normal to the body at the nose, and thus we can approximate them as normal shocks in the vicinity of the probe. It is given that the static pressure before the shock is P_1 = 110 kPa, and the stagnation pressure and temperature after the shock are P_{02} = 620 kPa, and T_{02} = 340 K. Noting that the stagnation temperature remains constant, we have

$$T_{01} = T_{02} = 340\text{ K}$$



Also, $$\frac{P_{02}}{P_1} = \frac{620\text{ kPa}}{110\text{ kPa}} = 5.6364 \approx 5.64$$

The fluid properties after the shock are related to those before the shock through the functions listed in Table A-14.
For $P_{02} / P_1 = 5.64$ we read

$$\text{Ma}_1 = 2.0, \quad \text{Ma}_2 = 0.5774, \quad \frac{P_{02}}{P_{01}} = 0.7209, \quad \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = 2.6667,$$

Then the stagnation pressure and temperature before the shock become

$$P_{01} = P_{02}/0.7209 = (620\text{ kPa})/0.7209 = 860\text{ kPa}$$

$$T_1 = T_{01}\left(\frac{P_1}{P_{01}}\right)^{(k-1)/k} = (340\text{ K})\left(\frac{110\text{ kPa}}{860\text{ kPa}}\right)^{(1.4-1)/1.4} = 188.9\text{ K}$$

The flow velocity before the shock can be determined from $V_1 = \text{Ma}_1 c_1$, where c_1 is the speed of sound before the shock,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(188.9\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 275.5\text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(275.5\text{ m/s}) = \mathbf{551\ m/s}$$

Discussion The flow velocity after the shock is $V_2 = V_1/2.6667$ = 551/2.6667 = 207 m/s. Therefore, the velocity measured by a Pitot-static probe would be very different that the flow velocity.

12-138

Solution Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for air.

Properties The specific heat ratio is given to be k = 1.4 for air.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k+1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

Air:

```
k=1.4
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8929
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.5	0.5130	7.1250	3.3333	2.1375	0.499	8.5261
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
3.5	0.4512	14.1250	4.2609	3.3151	0.2129	16.2420
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
4.5	0.4236	23.4583	4.8119	4.8751	0.0917	26.5387
5.0	0.4152	29.0000	5.0000	5.8000	0.06172	32.6535
5.5	0.4090	35.1250	5.1489	6.8218	0.04236	39.4124
6.0	0.4042	41.8333	5.2683	7.9406	0.02965	46.8152
6.5	0.4004	49.1250	5.3651	9.1564	0.02115	54.8620
7.0	0.3974	57.0000	5.4444	10.4694	0.01535	63.5526
7.5	0.3949	65.4583	5.5102	11.8795	0.01133	72.8871
8.0	0.3929	74.5000	5.5652	13.3867	0.008488	82.8655
8.5	0.3912	84.1250	5.6117	14.9911	0.006449	93.4876
9.0	0.3898	94.3333	5.6512	16.6927	0.004964	104.7536
9.5	0.3886	105.1250	5.6850	18.4915	0.003866	116.6634
10.0	0.3876	116.5000	5.7143	20.3875	0.003045	129.2170

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case k = 1.4.

12-139

Solution Using the normal shock relations, the normal shock functions are to be evaluated and tabulated as in Table A-14 for methane.

Properties The specific heat ratio is given to be $k = 1.3$ for methane.

Analysis The normal shock relations listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - k + 1}} \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} = \frac{2k\mathrm{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)} \qquad \frac{\rho_2}{\rho_1} = \frac{P_2 / P_1}{T_2 / T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2},$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left[\frac{1 + \mathrm{Ma}_2^2(k-1)/2}{1 + \mathrm{Ma}_1^2(k-1)/2}\right]^{\frac{k+1}{2(k-1)}} \qquad \frac{P_{02}}{P_1} = \frac{(1 + k\mathrm{Ma}_1^2)[1 + \mathrm{Ma}_2^2(k-1)/2]^{k/(k-1)}}{1 + k\mathrm{Ma}_2^2}$$

Methane:

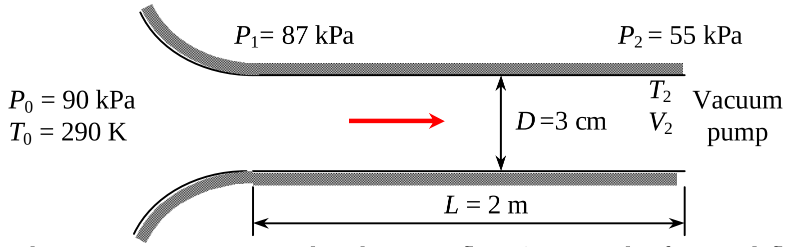
```
k=1.3
My=SQRT((Mx^2+2/(k-1))/(2*Mx^2*k/(k-1)-1))
PyPx=(1+k*Mx^2)/(1+k*My^2)
TyTx=(1+Mx^2*(k-1)/2)/(1+My^2*(k-1)/2)
RyRx=PyPx/TyTx
P0yP0x=(Mx/My)*((1+My^2*(k-1)/2)/(1+Mx^2*(k-1)/2))^(0.5*(k+1)/(k-1))
P0yPx=(1+k*Mx^2)*(1+My^2*(k-1)/2)^(k/(k-1))/(1+k*My^2)
```

Ma_1	Ma_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_{02}/P_1
1.0	1.0000	1.0000	1.0000	1.0000	1	1.8324
1.5	0.6942	2.4130	1.9346	1.2473	0.9261	3.2654
2.0	0.5629	4.3913	2.8750	1.5274	0.7006	5.3700
2.5	0.4929	6.9348	3.7097	1.8694	0.461	8.0983
3.0	0.4511	10.0435	4.4043	2.2804	0.2822	11.4409
3.5	0.4241	13.7174	4.9648	2.7630	0.1677	15.3948
4.0	0.4058	17.9565	5.4118	3.3181	0.09933	19.9589
4.5	0.3927	22.7609	5.7678	3.9462	0.05939	25.1325
5.0	0.3832	28.1304	6.0526	4.6476	0.03613	30.9155
5.5	0.3760	34.0652	6.2822	5.4225	0.02243	37.3076
6.0	0.3704	40.5652	6.4688	6.2710	0.01422	44.3087
6.5	0.3660	47.6304	6.6218	7.1930	0.009218	51.9188
7.0	0.3625	55.2609	6.7485	8.1886	0.006098	60.1379
7.5	0.3596	63.4565	6.8543	9.2579	0.004114	68.9658
8.0	0.3573	72.2174	6.9434	10.4009	0.002827	78.4027
8.5	0.3553	81.5435	7.0190	11.6175	0.001977	88.4485
9.0	0.3536	91.4348	7.0837	12.9079	0.001404	99.1032
9.5	0.3522	101.8913	7.1393	14.2719	0.001012	110.367
10.0	0.3510	112.9130	7.1875	15.7096	0.000740	122.239

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case $k = 1.3$.

12-140

Solution Air enters a constant-area adiabatic duct at a specified state, and leaves at a specified pressure. The mass flow rate of air, the exit velocity, and the average friction factor are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The friction factor is given to be f = 0.025.

Analysis Noting that the flow in the nozzle section is isentropic, the Mach number, thermodynamic temperature, and density at the tube inlet become

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} \quad \rightarrow \quad 87\text{ kPa} = (90\text{ kPa})\left(1+\frac{1.4\text{-}1}{2}\mathrm{Ma}_1^2\right)^{-1.4/0.4} \quad \rightarrow \quad \mathrm{Ma}_1 = 0.2206$$

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (290\text{ K})\left(1+\frac{1.4\text{-}1}{2}(0.2206)^2\right)^{-1} = 287.2\text{ K}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{87\text{ kPa}}{(0.287\text{ kJ/kgK})(287.2\text{ K})} = 1.055\text{ kg/m}^3$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(287.2\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 339.7\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 0.2206(339.7\text{ m/s}) = 74.94\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.055\text{ kg/m}^3)[\pi(0.03\text{ m})^2/4](74.94\text{ m/s}) = \mathbf{0.0559\ kg/s}$$

The Fanno flow functions corresponding to the inlet Mach number are, from Table A-16 (we used analytical relations),

Ma_1 = 0.2206: $(fL^*/D_h)_1$ =11.520 T_1/T^* = 1.1884, P_1/P^* = 4.9417, V_1/V^* = 0.2405

Therefore, $P_1 = 4.9417P^*$. Then the Fanno function P_2/P^* becomes

$$\frac{P_2}{P^*} = \frac{P_2}{P_1/5.2173} = \frac{4.9417(55\text{ kPa})}{87\text{ kPa}} = 3.124$$

The corresponding Mach number and Fanno flow functions are, from Table A-16,

Ma_2 = 0.3465, $(fL^*/D_h)_2$ = 3.5536, and V_2/V^* =0.3751.

Then the air velocity at the duct exit and the average friction factor become

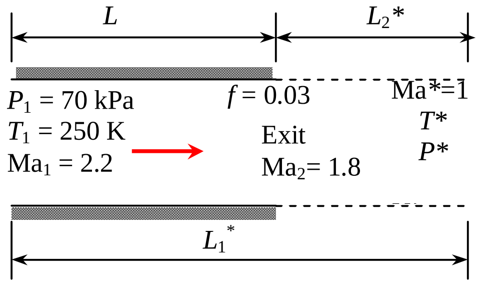
$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.3751}{0.2405} = 1.5597 \quad \rightarrow \quad V_2 = 1.5597V_1 = 1.5597(74.94\text{ m/s}) = \mathbf{117\ m/s}$$

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} \quad \rightarrow \quad 2\text{ m} = (11.520 - 3.5536)\frac{0.03\text{ m}}{f} \quad \rightarrow \quad f = \mathbf{0.120}$$

Discussion Note that the mass flow rate and the average friction factor can be determined by measuring static pressure, as in incompressible flow.

12-141

Solution Supersonic airflow in a constant cross-sectional area adiabatic duct is considered. For a specified exit Mach number, the temperature, pressure, and velocity at the duct exit are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K. The average friction factor is given to be $f = 0.03$.

Analysis The inlet velocity is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(250\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 316.9\ \text{m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2.2(316.9\ \text{m/s}) = 697.3\ \text{m/s}$$

The Fanno flow functions corresponding to the inlet and exit Mach numbers are, from Table A-16,

$\text{Ma}_1 = 2.2$: $(fL^*/D_h)_1 = 0.3609$ $T_1/T^* = 0.6098$, $P_1/P^* = 0.3549$, $V_1/V^* = 1.7179$

$\text{Ma}_2 = 1.8$: $(fL^*/D_h)_2 = 0.2419$ $T_2/T^* = 0.7282$, $P_2/P^* = 0.4741$, $V_2/V^* = 1.5360$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.7282}{0.6098} = 1.1942 \quad \rightarrow \quad T_2 = 1.1942T_1 = 1.1942(250\ \text{K}) = \mathbf{299\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.4741}{0.3549} = 1.3359 \quad \rightarrow \quad P_2 = 1.3359P_1 = 1.3359(70\ \text{kPa}) = \mathbf{93.5\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{1.5360}{1.7179} = 0.8941 \quad \rightarrow \quad V_2 = 0.8941V_1 = 0.8941(697.3\ \text{m/s}) = \mathbf{623\ m/s}$$

Discussion The duct length is determined to be

$$L = L_1^* - L_2^* = \left(\frac{fL_1^*}{D_h} - \frac{fL_2^*}{D_h}\right)\frac{D_h}{f} = (0.3609 - 0.2419)\frac{0.055\ \text{m}}{0.03} = \mathbf{0.218\ m}$$

Note that it takes a duct length of only 0.218 m for the Mach number to decrease from 2.2 to 1.8. The maximum (or sonic) duct lengths at the inlet and exit states in this case are $L_1^* = 0.662$ m and $L_2^* = 0.443$ m. Therefore, the flow would reach sonic conditions if a 0.443-m long section were added to the existing duct.

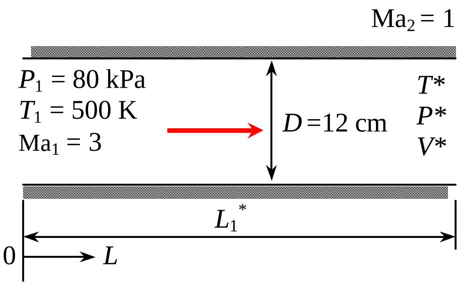
12-142

Solution Choked supersonic airflow in a constant cross-sectional area adiabatic duct is considered. The variation of duct length with Mach number is to be investigated, and the results are to be plotted.

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K. The average friction factor is given to be f = 0.03.

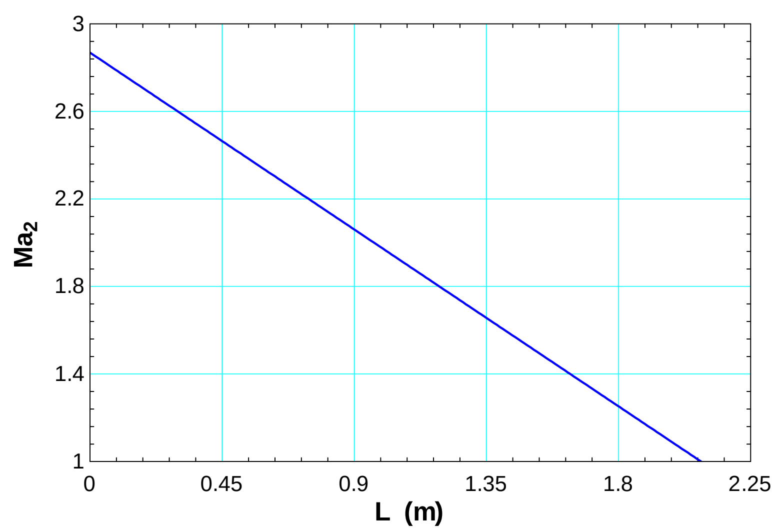
Analysis We use EES to solve the problem. The flow is choked, and thus Ma_2 =1. Corresponding to the inlet Mach number of Ma_1 = 3 we have, from Table A-16, fL^*/D_h = 0.5222, Therefore, the original duct length is



$$L_1^* = 0.5222\frac{D}{f} = 0.5222\frac{0.18\,\mathrm{m}}{0.03} = 3.13\,\mathrm{m}$$

Repeating the calculations for different Ma_2 as it varies from 3 to 1 results in the following table for the location on the duct from the inlet:

Mach number, Ma	Duct length L, m
1.00	2.09
1.25	1.89
1.50	1.54
1.75	1.19
2.00	0.87
2.25	0.59
2.50	0.36
2.75	0.16
3.00	0.00



EES program:

```
k=1.4
cp=1.005
R=0.287

P1=80
T1=500
Ma1=3
"Ma2=1"
f=0.03
D=0.12

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
```

```
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that the Mach number decreases nearly linearly along the duct.

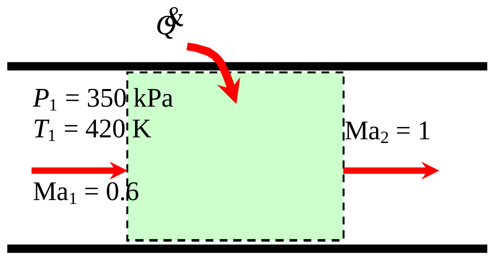
12-143

Solution Air flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\mathrm{Ma}_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{350\text{ kPa}}{(0.287\text{ kJ/kgK})(420\text{ K})} = 2.904\text{ kg/m}^3$$

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (420\text{ K})\left(1+\frac{1.4\text{-}1}{2}0.6^2\right) = 450.2\text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(420\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 410.8\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 0.6(410.8\text{ m/s}) = 246.5\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (2.904\text{ kg/m}^3)(0.1\times 0.1\text{ m}^2)(246.5\text{ m/s}) = 7.157\text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are $T_{02}/T_0^* = 1$ (since $\mathrm{Ma}_2 = 1$) .

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\mathrm{Ma}_1^2[2+(k-1)\mathrm{Ma}_1^2]}{(1+k\mathrm{Ma}_1^2)^2} = \frac{(1.4+1)0.6^2[2+(1.4-1)0.6^2]}{(1+1.4\times 0.6^2)^2} = 0.8189$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.8189} \quad \rightarrow \quad T_{02} = T_{01}/0.8189 = (450.2\text{ K})/0.8189 = 549.8\text{ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (7.157\text{ kg/s})(1.005\text{ kJ/kg}\cdot\text{K})(549.8 - 450.2)\text{ K} = \mathbf{716\ kW}$$

Discussion It can also be shown that T_2 = 458 K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. We can also solve this problem using the Rayleigh function values listed in Table A-15.

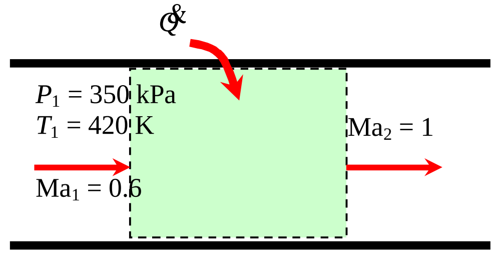
12-144

Solution Helium flowing at a subsonic velocity in a duct is accelerated by heating. The highest rate of heat transfer without affecting the inlet conditions is to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of helium to be k = 1.667, c_p = 5.193 kJ/kg·K, and R = 2.077 kJ/kg·K.

Analysis Heat transfer will stop when the flow is choked, and thus $\mathrm{Ma}_2 = V_2/c_2 = 1$. The inlet density and stagnation temperature are



$$\rho_1 = \frac{P_1}{RT_1} = \frac{350\text{ kPa}}{(2.077\text{ kJ/kgK})(420\text{ K})} = 0.4012\text{ kg/m}^3$$

$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (420\text{ K})\left(1+\frac{1.667-1}{2}0.6^2\right) = 470.4\text{ K}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.667)(2.077\text{ kJ/kg}\cdot\text{K})(420\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 1206\text{ m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 0.6(1206\text{ m/s}) = 723.5\text{ m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.4012\text{ kg/m}^3)(0.1\times 0.1\text{ m}^2)(723.5\text{ m/s}) = 2.903\text{ kg/s}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are $T_{02}/T_0^* = 1$ (since $\mathrm{Ma}_2 = 1$)

$$\frac{T_{01}}{T_0^*} = \frac{(k+1)\mathrm{Ma}_1^2[2+(k-1)\mathrm{Ma}_1^2]}{(1+k\mathrm{Ma}_1^2)^2} = \frac{(1.667+1)0.6^2[2+(1.667-1)0.6^2]}{(1+1.667\times 0.6^2)^2} = 0.8400$$

Therefore,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{1}{0.8400} \quad \rightarrow \quad T_{02} = T_{01}/0.8400 = (470.4\text{ K})/0.8400 = 560.0\text{ K}$$

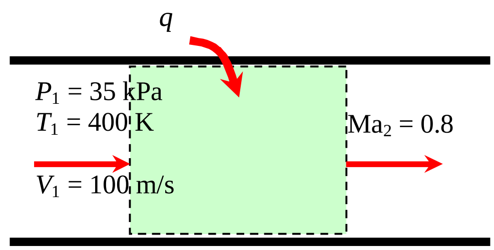
Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}_{\text{air}} c_p (T_{02} - T_{01}) = (2.903\text{ kg/s})(5.193\text{ kJ/kg}\cdot\text{K})(560.0 - 470.4)\text{ K} = \mathbf{1350\ kW}$$

Discussion It can also be shown that T_2 = 420 K, which is the highest thermodynamic temperature that can be attained under stated conditions. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease. Also, in the solution of this problem, we cannot use the values of Table A-15 since they are based on k = 1.4.

12-145

Solution Air flowing at a subsonic velocity in a duct is accelerated by heating. For a specified exit Mach number, the heat transfer for a specified exit Mach number as well as the maximum heat transfer are to be determined.



Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Inlet conditions (and thus the mass flow rate) remain constant.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis The inlet Mach number and stagnation temperature are

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg}\cdot\text{K})(400\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 400.9\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{100\text{ m/s}}{400.9\text{ m/s}} = 0.2494$$

$$T_{01} = T_1\left(1+\frac{k-1}{2}\text{Ma}_1^2\right) = (400\text{ K})\left(1+\frac{1.4\text{-}1}{2}0.2494^2\right) = 405.0\text{ K}$$

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A-15):

$Ma_1 = 0.2494$: $T_{01}/T^* = 0.2559$
$Ma_2 = 0.8$: $T_{02}/T^* = 0.9639$

Then the exit stagnation temperature and the heat transfer are determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{0.9639}{0.2559} = 3.7667 \quad \rightarrow \quad T_{02} = 3.7667T_{01} = 3.7667(405.0\text{ K}) = 1526\text{ K}$$

$$q = c_p\left(T_{02}-T_{01}\right) = \left(1.005\text{ kJ/kg}\cdot\text{K}\right)\left(1526-405\right)\text{ K} = 1126\text{ kJ/kg} \cong \mathbf{1130\ kJ/kg}$$

Maximum heat transfer will occur when the flow is choked, and thus $Ma_2 = 1$ and thus $T_{02}/T^* = 1$. Then,

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T^*}{T_{01}/T^*} = \frac{1}{0.2559} \quad \rightarrow \quad T_{02} = T_{01}/0.2559 = (405\text{ K})/0.2559 = 1583\text{ K}$$

$$q_{max} = c_p(T_{02}-T_{01}) = (1.005\text{ kJ/kg}\cdot\text{K})(1583-405)\text{ K} = 1184\text{ kJ/kg} \cong \mathbf{1180\ kJ/kg}$$

Discussion This is the maximum heat that can be transferred to the gas without affecting the mass flow rate. If more heat is transferred, the additional temperature rise will cause the mass flow rate to decrease.

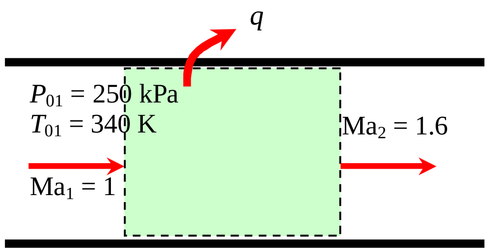
12-146

Solution Air flowing at sonic conditions in a duct is accelerated by cooling. For a specified exit Mach number, the amount of heat transfer per unit mass is to be determined.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis Noting that $Ma_1 = 1$, the inlet stagnation temperature is



$$T_{01} = T_1\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right) = (340\text{ K})\left(1+\frac{1.4-1}{2}1^2\right) = 408\text{ K}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-15):

$Ma_1 = 1$: $T_{01}/T_0^* = 1$

$Ma_2 = 1.6$: $T_{02}/T_0^* = 0.8842$

Then the exit stagnation temperature and heat transfer are determined to be

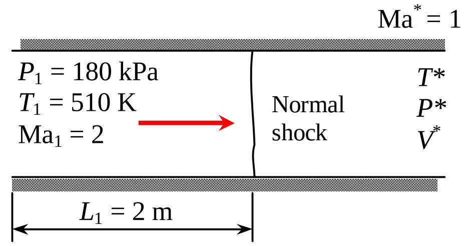
$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.8842}{1} = 0.8842 \quad\rightarrow\quad T_{02} = 0.8842T_{01} = 0.8842(408\text{ K}) = 360.75\text{ K}$$

$$q = c_p(T_{02}-T_{01}) = (1.005\text{ kJ/kg}\cdot\text{K})(360.75-408)\text{ K} = -47.49\text{ kJ/kg} \cong \mathbf{-47.5\text{ kJ/kg}}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated.

12-147

Solution Combustion gases enter a constant-area adiabatic duct at a specified state, and undergo a normal shock at a specified location. The exit velocity, temperature, and pressure are to be determined.



Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties The specific heat ratio and gas constant of combustion gases are given to be k = 1.33 and R = 0.280 kJ/kg·K. The friction factor is given to be f = 0.010.

Analysis The Fanno flow functions corresponding to the inlet Mach number of 2 are calculated from the relations in Table A-16 for k = 1.33 to be

$\mathrm{Ma}_1 = 2$: $(fL^*/D_h)_1 = 0.3402$ $T_1/T^* = 0.7018$, $P_1/P^* = 0.4189$

First we check to make sure that the flow everywhere upstream the shock is supersonic. The required duct length from the inlet L_1^* for the flow to reach sonic conditions is $L_1^* = 0.3402\dfrac{D}{f} = 0.3402\dfrac{0.10\text{ m}}{0.010} = 3.40\text{ m}$, which is greater than the actual length of 2 m. Therefore, the flow is indeed supersonic when the normal shock occurs at the indicated location. Also, using the actual duct length L_1, we have $\dfrac{fL_1}{D_h} = \dfrac{(0.010)(2\text{ m})}{0.10\text{ m}} = 0.2000$. Noting that $L_1 = L_1^* - L_2^*$, the function fL^*/D_h at the exit state and the corresponding Mach number are $\left(\dfrac{fL^*}{D_h}\right)_2 = \left(\dfrac{fL^*}{D_h}\right)_1 - \dfrac{fL_1}{D_h} = 0.3402 - 0.2000 = 0.1402 \quad \to \quad \mathrm{Ma}_2 = 1.476.$

From the relations in Table A-16, at Ma_2 =1.476: $T_2/T^* = 0.8568$, $P_2/P^* = 0.6270$. Then the temperature, pressure, and velocity before the shock are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.8568}{0.7018} = 1.2209 \quad \to \quad T_2 = 1.2209T_1 = 1.2209(510\text{ K}) = 622.7\text{ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{0.6270}{0.4189} = 1.4968 \quad \to \quad P_2 = 1.4968P_1 = 1.4968(180\text{ kPa}) = 269.4\text{ kPa}$$

The normal shock functions corresponding to a Mach number of 1.476 are, from the relations in Table A-14,

Ma_2 =1.476: $\mathrm{Ma}_3 = 0.7052$, $T_3/T_2 = 1.2565$, $P_3/P_2 = 2.3466$

Then the temperature and pressure after the shock become

$$T_3 = 1.2565T_2 = 1.2565(622.7\text{ K}) = 782.4\text{ K}$$

$$P_3 = 2.3466P_2 = 2.3466(269.4\text{ kPa}) = 632.3\text{ kPa}$$

Sonic conditions exist at the duct exit, and the flow downstream of the shock is still Fanno flow. From the relations in Table A-16,

$\mathrm{Ma}_3 = 0.7052$: $T_3/T^* = 1.0767$, $P_3/P^* = 1.4713$

$\mathrm{Ma}_4 = 1$: $T_4/T^* = 1$, $P_4/P^* = 1$

Then the temperature, pressure, and velocity at the duct exit are determined to be

$$\frac{T_4}{T_3} = \frac{T_4/T^*}{T_3/T^*} = \frac{1}{1.0767} \quad \to \quad T_4 = T_3/1.0767 = (782.4\text{ K})/1.0767 = \mathbf{727\ K}$$

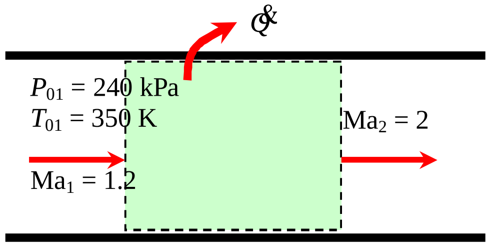
$$\frac{P_4}{P_3} = \frac{P_4/P^*}{P_3/P^*} = \frac{1}{1.4713} \quad \to \quad P_4 = P_3/1.4713 = (632.3\text{ kPa})/1.4713 = \mathbf{430\ kPa}$$

$$V_4 = \mathrm{Ma}_4 c_4 = (1)\sqrt{kRT_4} = \sqrt{(1.33)(0.280\text{ kJ/kg}\cdot\text{K})(727\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{520\ m/s}$$

Discussion It can be shown that L_3^* = 2.13 m, and thus the total length of this duct is 4.13 m. If the duct is extended, the normal shock will move farther upstream, and eventually to the inlet of the duct.

12-148

Solution Air flowing at a supersonic velocity in a duct is accelerated by cooling. For a specified exit Mach number, the rate of heat transfer is to be determined.



Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Properties We take the properties of air to be k = 1.4, c_p = 1.005 kJ/kg·K, and R = 0.287 kJ/kg·K.

Analysis Knowing stagnation properties, the static properties are determined to be

$$T_1 = T_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} = (350\ \mathrm{K})\left(1+\frac{1.4\text{-}1}{2}1.2^2\right)^{-1} = 271.7\ \mathrm{K}$$

$$P_1 = P_{01}\left(1+\frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-k/(k-1)} = (240\ \mathrm{kPa})\left(1+\frac{1.4\text{-}1}{2}1.2^2\right)^{-1.4/0.4} = 98.97\ \mathrm{kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{98.97\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kgK})(271.7\ \mathrm{K})} = 1.269\ \mathrm{kg/m^3}$$

Then the inlet velocity and the mass flow rate become

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(271.7\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 330.4\ \mathrm{m/s}$$

$$V_1 = \mathrm{Ma}_1 c_1 = 1.2(330.4\ \mathrm{m/s}) = 396.5\ \mathrm{m/s}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (1.269\ \mathrm{kg/m^3})[\pi(0.20\ \mathrm{m})^2/4](396.5\ \mathrm{m/s}) = 15.81\ \mathrm{kg/s}$$

The Rayleigh flow functions T_0/T_0^* corresponding to the inlet and exit Mach numbers are (Table A-15):

$\mathrm{Ma}_1 = 1.8$: $T_{01}/T_0^* = 0.9787$
$\mathrm{Ma}_2 = 2$: $T_{02}/T_0^* = 0.7934$

Then the exit stagnation temperature is determined to be

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}/T_0^*}{T_{01}/T_0^*} = \frac{0.7934}{0.9787} = 0.8107 \quad \rightarrow \quad T_{02} = 0.8107T_{01} = 0.8107(350\ \mathrm{K}) = 283.7\ \mathrm{K}$$

Finally, the rate of heat transfer is

$$\dot{Q} = \dot{m}_{\mathrm{air}} c_p\left(T_{02}-T_{01}\right) = (15.81\ \mathrm{kg/s})(1.005\ \mathrm{kJ/kg\cdot K})(283.7-350)\ \mathrm{K} = -1053\ \mathrm{kW} \cong \mathbf{-1050\ kW}$$

Discussion The negative sign confirms that the gas needs to be cooled in order to be accelerated. Also, it can be shown that the thermodynamic temperature drops to 158 K at the exit, which is extremely low. Therefore, the duct may need to be heavily insulated to maintain indicated flow conditions.

12-149

Solution Choked subsonic airflow in a constant cross-sectional area adiabatic duct is considered. The effect of duct length on the mass flow rate and the inlet conditions is to be investigated as the duct length is doubled.

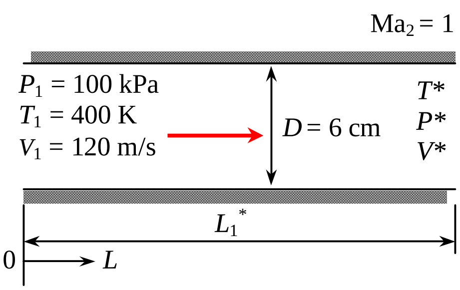
Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor remains constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K. The average friction factor is given to be $f = 0.02$.

Analysis We use EES to solve the problem. The flow is choked, and thus $\mathrm{Ma}_2 = 1$. The inlet Mach number is

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \mathrm{kJ/kg\cdot K})(400\ \mathrm{K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)} = 400.9\ \mathrm{m/s}$$

$$\mathrm{Ma}_1 = \frac{V_1}{c_1} = \frac{120\ \mathrm{m/s}}{400.9\ \mathrm{m/s}} = 0.2993$$



Corresponding to this Mach number we have, from Table A-16, $fL^*/D_h = 5.3312$, Therefore, the original duct length is

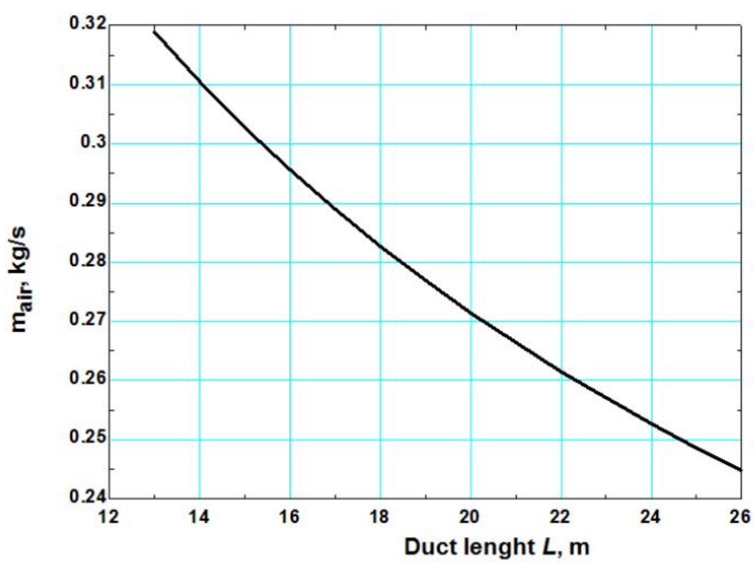
$$L = L_1^* = 5.3312\frac{D}{f} = 5.3312\frac{0.06\ \mathrm{m}}{0.02} = 16.0\ \mathrm{m}$$

Then the initial mass flow rate becomes

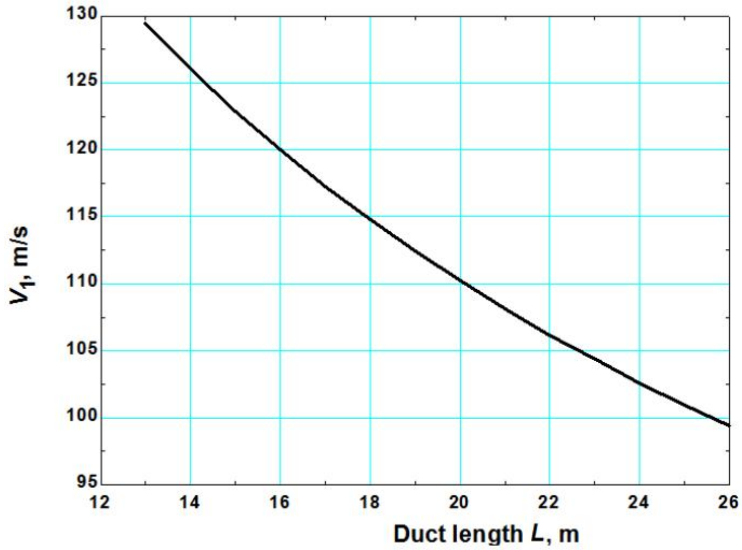
$$\rho_1 = \frac{P_1}{RT_1} = \frac{100\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kgK})(400\ \mathrm{K})} = 0.8711\ \mathrm{kg/m^3}$$

$$\dot{m}_{air} = \rho_1 A_{c1} V_1 = (0.8711\ \mathrm{kg/m^3})[\pi(0.06\ \mathrm{m})^2/4](120\ \mathrm{m/s}) = 0.296\ \mathrm{kg/s}$$

Duct length L, m	Inlet velocity V_1, m/s	Mass flow rate $\dot{m}_{air}$, kg/s
13	129	0.319
14	126	0.310
15	123	0.303
16	120	0.296
17	117	0.289
18	115	0.283
19	112	0.277
20	110	0.271
21	108	0.266
22	106	0.262
23	104	0.257
24	103	0.253
25	101	0.249
26	99	0.245



The EES program is listed below, along with a plot of inlet velocity vs. duct length:




```
k=1.4
cp=1.005
R=0.287

P1=100
T1=400
"L=26"
Ma2=1
f=0.02
D=0.06

C1=sqrt(k*R*T1*1000)
Ma1=V1/C1
T01=T02
T01=T1*(1+0.5*(k-1)*Ma1^2)
T02=T2*(1+0.5*(k-1)*Ma2^2)
P01=P1*(1+0.5*(k-1)*Ma1^2)^(k/(k-1))

rho1=P1/(R*T1)
Ac=pi*D^2/4
mair=rho1*Ac*V1

P01Ps=((2+(k-1)*Ma1^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma1
P1Ps=((k+1)/(2+(k-1)*Ma1^2))^0.5/Ma1
T1Ts=(k+1)/(2+(k-1)*Ma1^2)
R1Rs=((2+(k-1)*Ma1^2)/(k+1))^0.5/Ma1
V1Vs=1/R1Rs
fLs1=(1-Ma1^2)/(k*Ma1^2)+(k+1)/(2*k)*ln((k+1)*Ma1^2/(2+(k-1)*Ma1^2))
Ls1=fLs1*D/f

P02Ps=((2+(k-1)*Ma2^2)/(k+1))^(0.5*(k+1)/(k-1))/Ma2
P2Ps=((k+1)/(2+(k-1)*Ma2^2))^0.5/Ma2
T2Ts=(k+1)/(2+(k-1)*Ma2^2)
R2Rs=((2+(k-1)*Ma2^2)/(k+1))^0.5/Ma2
V2Vs=1/R2Rs
fLs2=(1-Ma2^2)/(k*Ma2^2)+(k+1)/(2*k)*ln((k+1)*Ma2^2/(2+(k-1)*Ma2^2))
Ls2=fLs2*D/f
L=Ls1-Ls2

P02=P02Ps/P01Ps*P01
P2=P2Ps/P1Ps*P1
V2=V2Vs/V1Vs*V1
```

Discussion Note that once the flow is choked, any increase in duct length results in a decrease in the mass flow rate and the inlet velocity.

12-150

Solution Using EES (or other) software, the shape of a converging-diverging nozzle is to be determined for specified flow rate and stagnation conditions. The nozzle and the Mach number are to be plotted.

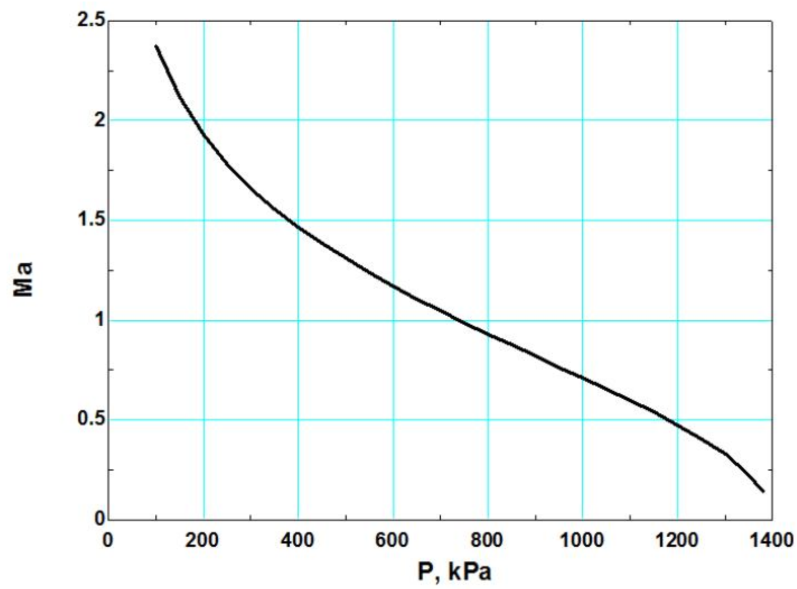
Assumptions **1** Air is an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

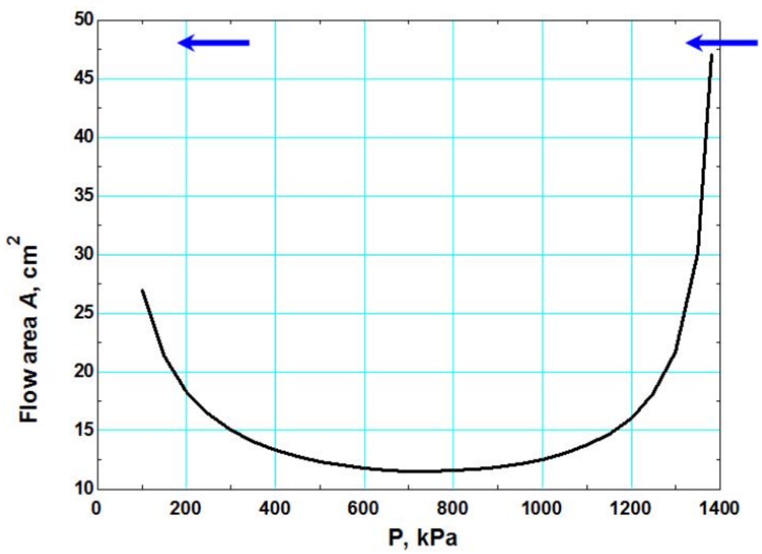
Properties The specific heat ratio of air at room temperature is 1.4.

Analysis The problem is solved using EES, and the results are tabulated and plotted below.

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
P0=1400 "kPa"
T0=200+273 "K"
m=3 "kg/s"
rho_0=P0/(R*T0)
rho=P/(R*T)
T=T0*(P/P0)^((k-1)/k)
V=SQRT(2*Cp*(T0-T)*1000)
A=m/(rho*V)*10000 "cm2"
C=SQRT(k*R*T*1000)
Ma=V/C
```

Pressure *P*, kPa	Flow area *A*, cm^2	Mach number Ma
1400	∞	0
1350	30.1	0.229
1300	21.7	0.327
1250	18.1	0.406
1200	16.0	0.475
1150	14.7	0.538
1100	13.7	0.597
1050	13.0	0.655
1000	12.5	0.710
950	12.2	0.766
900	11.9	0.820
850	11.7	0.876
800	11.6	0.931
750	11.5	0.988
700	11.5	1.047
650	11.6	1.107
600	11.8	1.171
550	12.0	1.237
500	12.3	1.308
450	12.8	1.384
400	13.3	1.467
350	14.0	1.559
300	15.0	1.663
250	16.4	1.784
200	18.3	1.929
150	21.4	2.114
100	27.0	2.373





Discussion The shape is not actually to scale since the horizontal axis is pressure rather than distance. If the pressure decreases linearly with distance, then the shape *would* be to scale.

12-151

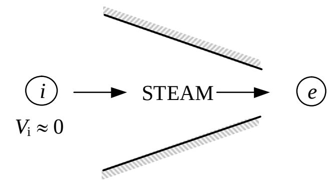
Solution Steam enters a converging nozzle. The exit pressure, the exit velocity, and the mass flow rate versus the back pressure for a specified back pressure range are to be plotted.

Assumptions **1** Steam is to be treated as an ideal gas with constant specific heats. **2** Flow through the nozzle is steady, one-dimensional, and isentropic. **3** The nozzle is adiabatic.

Properties The ideal gas properties of steam are R = 0.462 kJ/kg.K, c_p = 1.872 kJ/kg.K, and k = 1.3.

Analysis We use EES to solve the problem. The stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Since the flow is isentropic, they remain constant throughout the nozzle,

$P_0 = P_i$ = 6 MPa and $T_0 = T_i$ = 700 K



The critical pressure is determined to be

$$P^* = P_0\left(\frac{2}{k+1}\right)^{k/(k-1)} = (6\text{ MPa})\left(\frac{2}{1.3+1}\right)^{1.3/0.3} = 3.274\text{ MPa}$$

Then the pressure at the exit plane (throat) is

$P_e = P_b$ for $P_b \geq 3.274$ MPa

$P_e = P^* = 3.274$ MPa for $P_b < 3.274$ MPa (choked flow)

Thus the back pressure does not affect the flow when $3 < P_b < 3.274$ MPa. For a specified exit pressure P_e, the temperature, velocity, and mass flow rate are

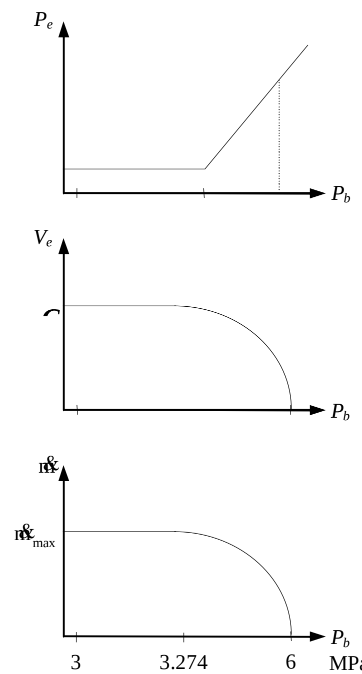
Temperature $$T_e = T_0\left(\frac{P_e}{P_0}\right)^{(k-1)/k} = (700\text{ K})\left(\frac{P_e}{6}\right)^{0.3/1.3}$$

Velocity $$V = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(1.872\text{ kJ/kg}\cdot\text{K})(700 - T_e)\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

Density $$\rho_e = \frac{P_e}{RT_e} = \frac{P_e}{(0.462\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})T_e}$$

Mass flow rate $$\dot{m} = \rho_e V_e A_e = \rho_e V_e (0.0008\text{ m}^2)$$

The results of the calculations are tabulated as follows:



P_b, MPa	P_e, MPa	T_e, K	V_e, m/s	ρ_e, kg/m^3	$\dot{m}$, kg/s
6.0	6.0	700	0	18.55	0
5.5	5.5	686.1	228.1	17.35	3.166
5.0	5.0	671.2	328.4	16.12	4.235
4.5	4.5	655.0	410.5	14.87	4.883
4.0	4.0	637.5	483.7	13.58	5.255
3.5	3.5	618.1	553.7	12.26	5.431
3.274	3.274	608.7	584.7	11.64	5.445
3.0	3.274	608.7	584.7	11.64	5.445

Discussion Once the back pressure drops below 3.274 MPa, the flow is choked, and $\dot{m}$ remains constant from then on.

12-152

Solution An expression for the ratio of the stagnation pressure after a shock wave to the static pressure before the shock wave as a function of k and the Mach number upstream of the shock wave is to be found.

Analysis The relation between P_1 and P_2 is

$$\frac{P_2}{P_1}=\frac{1+k\mathrm{Ma}_2^2}{1+k\mathrm{Ma}_1^2}\longrightarrow P_2=P_1\left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)$$

We substitute this into the isentropic relation

$$\frac{P_{02}}{P_2}=\left(1+\left(k-1\right)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

which yields

$$\frac{P_{02}}{P_1}=\left(\frac{1+k\mathrm{Ma}_1^2}{1+k\mathrm{Ma}_2^2}\right)\left(1+(k-1)\mathrm{Ma}_2^2/2\right)^{k/(k-1)}$$

where

$$\mathrm{Ma}_2^2=\frac{\mathrm{Ma}_1^2+2/(k-1)}{2k\mathrm{Ma}_2^2/(k-1)-1}$$

Substituting,

$$\boxed{\frac{P_{02}}{P_1}=\left(\frac{(1+k\mathrm{Ma}_1^2)(2k\mathrm{Ma}_1^2-k+1)}{k\mathrm{Ma}_1^2(k+1)-k+3}\right)\left(1+\frac{(k-1)\mathrm{Ma}_1^2/2+1}{2k\mathrm{Ma}_1^2/(k-1)-1}\right)^{k/(k-1)}}$$

Discussion Similar manipulations of the equations can be performed to get the ratio of other parameters across a shock.

12-153

Solution Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for air.

Properties The specific heat ratio is given to be k = 1.4 for air.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}} \qquad \frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-k/(k-1)} \qquad \frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1}$$

Air:

```
k=1.4
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.5	1.3646	1.1762	0.2724	0.3950	0.6897
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.5	1.8257	2.6367	0.0585	0.1317	0.4444
3.0	1.9640	4.2346	0.0272	0.0762	0.3571
3.5	2.0642	6.7896	0.0131	0.0452	0.2899
4.0	2.1381	10.7188	0.0066	0.0277	0.2381
4.5	2.1936	16.5622	0.0035	0.0174	0.1980
5.0	2.2361	25.0000	0.0019	0.0113	0.1667
5.5	2.2691	36.8690	0.0011	0.0076	0.1418
6.0	2.2953	53.1798	0.0006	0.0052	0.1220
6.5	2.3163	75.1343	0.0004	0.0036	0.1058
7.0	2.3333	104.1429	0.0002	0.0026	0.0926
7.5	2.3474	141.8415	0.0002	0.0019	0.0816
8.0	2.3591	190.1094	0.0001	0.0014	0.0725
8.5	2.3689	251.0862	0.0001	0.0011	0.0647
9.0	2.3772	327.1893	0.0000	0.0008	0.0581
9.5	2.3843	421.1314	0.0000	0.0006	0.0525
10.0	2.3905	535.9375	0.0000	0.0005	0.0476

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case k = 1.4.

12-154

Solution Using the compressible flow relations, the one-dimensional compressible flow functions are to be evaluated and tabulated as in Table A-13 for methane.

Properties The specific heat ratio is given to be k = 1.3 for methane.

Analysis The compressible flow functions listed below are expressed in EES and the results are tabulated.

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}} \qquad \frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-k/(k-1)} \qquad \frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1}$$

Methane:

```
k=1.3
PP0=(1+(k-1)*M^2/2)^(-k/(k-1))
TT0=1/(1+(k-1)*M^2/2)
DD0=(1+(k-1)*M^2/2)^(-1/(k-1))
Mcr=M*SQRT((k+1)/(2+(k-1)*M^2))
AAcr=((2/(k+1))*(1+0.5*(k-1)*M^2))^(0.5*(k+1)/(k-1))/M
```

Ma	Ma^*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
1.0	1.0000	1.0000	0.5457	0.6276	0.8696
1.5	1.3909	1.1895	0.2836	0.3793	0.7477
2.0	1.6956	1.7732	0.1305	0.2087	0.6250
2.5	1.9261	2.9545	0.0569	0.1103	0.5161
3.0	2.0986	5.1598	0.0247	0.0580	0.4255
3.5	2.2282	9.1098	0.0109	0.0309	0.3524
4.0	2.3263	15.9441	0.0050	0.0169	0.2941
4.5	2.4016	27.3870	0.0024	0.0095	0.2477
5.0	2.4602	45.9565	0.0012	0.0056	0.2105
5.5	2.5064	75.2197	0.0006	0.0033	0.1806
6.0	2.5434	120.0965	0.0003	0.0021	0.1563
6.5	2.5733	187.2173	0.0002	0.0013	0.1363
7.0	2.5978	285.3372	0.0001	0.0008	0.1198
7.5	2.6181	425.8095	0.0001	0.0006	0.1060
8.0	2.6350	623.1235	0.0000	0.0004	0.0943
8.5	2.6493	895.5077	0.0000	0.0003	0.0845
9.0	2.6615	1265.6040	0.0000	0.0002	0.0760
9.5	2.6719	1761.2133	0.0000	0.0001	0.0688
10.0	2.6810	2416.1184	0.0000	0.0001	0.0625

Discussion The tabulated values are useful for quick calculations, but be careful – they apply only to one specific value of k, in this case k = 1.3.

Fundamentals of Engineering (FE) Exam Problems

12-155

An aircraft is cruising in still air at 5°C at a velocity of 400 m/s. The air temperature at the nose of the aircraft where stagnation occurs is

(a) 5°C (b) 25°C (c) 55°C (d) 80°C (e) 85°C

Answer (e) 85°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=5 "C"
Vel1= 400 "m/s"
T1_stag=T1+Vel1^2/(2*Cp*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Tstag=T1 "Assuming temperature rise"
W2_Tstag=Vel1^2/(2*Cp*1000) "Using just the dynamic temperature"
W3_Tstag=T1+Vel1^2/(Cp*1000) "Not using the factor 2"
```

12-156

Air is flowing in a wind tunnel at 25°C, 80 kPa, and 250 m/s. The stagnation pressure at a probe inserted into the flow stream is

(a) 87 kPa (b) 93 kPa (c) 113 kPa (d) 119 kPa (e) 125 kPa

Answer (c) 113 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
T1=25 "K"
P1=80 "kPa"
Vel1= 250 "m/s"
T1_stag=(T1+273)+Vel1^2/(2*Cp*1000) "C"
T1_stag/(T1+273)=(P1_stag/P1)^((k-1)/k)

"Some Wrong Solutions with Common Mistakes:"
T11_stag/T1=(W1_P1stag/P1)^((k-1)/k);   T11_stag=T1+Vel1^2/(2*Cp*1000)  "Using deg. C for temperatures"
T12_stag/(T1+273)=(W2_P1stag/P1)^((k-1)/k);   T12_stag=(T1+273)+Vel1^2/(Cp*1000)  "Not using the factor 2"
T13_stag/(T1+273)=(W3_P1stag/P1)^(k-1);   T13_stag=(T1+273)+Vel1^2/(2*Cp*1000)  "Using wrong isentropic relation"
```

12-157

An aircraft is reported to be cruising in still air at -20°C and 40 kPa at a Mach number of 0.86. The velocity of the aircraft is

(a) 91 m/s (b) 220 m/s (c) 186 m/s (d) 280 m/s (e) 378 m/s

Answer (d) 280 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=-20+273 "K"
P1=40 "kPa"
Mach=0.86
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1

"Some Wrong Solutions with Common Mistakes:"
W1_vel=Mach*VS2; VS2=SQRT(k*R*T1) "Not using the factor 1000"
W2_vel=VS1/Mach "Using Mach number relation backwards"
W3_vel=Mach*VS3; VS3=k*R*T1 "Using wrong relation"
```

12-158

Air is flowing in a wind tunnel at 12°C and 66 kPa at a velocity of 230 m/s. The Mach number of the flow is

(a) 0.54 (b) 0.87 (c) 3.3 (d) 0.36 (e) 0.68

Answer (e) 0.68

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
T1=12+273 "K"
P1=66 "kPa"
Vel1=230 "m/s"
VS1=SQRT(k*R*T1*1000)
Mach=Vel1/VS1

"Some Wrong Solutions with Common Mistakes:"
W1_Mach=Vel1/VS2; VS2=SQRT(k*R*(T1-273)*1000) "Using C for temperature"
W2_Mach=VS1/Vel1 "Using Mach number relation backwards"
W3_Mach=Vel1/VS3; VS3=k*R*T1 "Using wrong relation"
```

12-159

Consider a converging nozzle with a low velocity at the inlet and sonic velocity at the exit plane. Now the nozzle exit diameter is reduced by half while the nozzle inlet temperature and pressure are maintained the same. The nozzle exit velocity will

(a) remain the same (b) double (c) quadruple (d) go down by half (e) go down to one-fourth

Answer (a) remain the same

12-160

Air is approaching a converging-diverging nozzle with a low velocity at 12°C and 200 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of air at the throat of the nozzle is

(a) 338 m/s (b) 309 m/s (c) 280 m/s (d) 256 m/s (e) 95 m/s

Answer (b) 309 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.4
Cp=1.005 "kJ/kg.K"
R=0.287 "kJ/kg.K"
"Properties at the inlet"
T1=12+273 "K"
P1=200 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(To-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"
```

12-161

Argon gas is approaching a converging-diverging nozzle with a low velocity at 20°C and 120 kPa, and it leaves the nozzle at a supersonic velocity. If the cross-sectional area of the throat is 0.015 m^2, the mass flow rate of argon through the nozzle is

(a) 0.41 kg/s (b) 3.4 kg/s (c) 5.3 kg/s (d) 17 kg/s (e) 22 kg/s

Answer (c) 5.3 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.667
Cp=0.5203 "kJ/kg.K"
R=0.2081 "kJ/kg.K"
A=0.015 "m^2"
"Properties at the inlet"
T1=20+273 "K"
P1=120 "kPa"
Vel1=0 "m/s"
To=T1 "since velocity is zero"
Po=P1
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
rho_throat=P_throat/(R*T_throat)
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)
m=rho_throat*A*V_throat

"Some Wrong Solutions with Common Mistakes:"
W1_mass=rho_throat*A*V1_throat;  V1_throat=SQRT(k*R*T1_throat*1000); T1_throat=2*(To-273)/(k+1) "Using C for temp"
W2_mass=rho2_throat*A*V_throat; rho2_throat=P1/(R*T1)  "Using density at inlet"
```

12-162

Carbon dioxide enters a converging-diverging nozzle at 60 m/s, 310°C, and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of carbon dioxide at the throat of the nozzle is

(a) 125 m/s (b) 225 m/s (c) 312 m/s (d) 353 m/s (e) 377 m/s

Answer (d) 353 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.289
Cp=0.846 "kJ/kg.K"
R=0.1889 "kJ/kg.K"
"Properties at the inlet"
T1=310+273 "K"
P1=300 "kPa"
Vel1=60 "m/s"
To=T1+Vel1^2/(2*Cp*1000)
To/T1=(Po/P1)^((k-1)/k)
"Throat properties"
T_throat=2*To/(k+1)
P_throat=Po*(2/(k+1))^(k/(k-1))
"The velocity at the throat is the velocity of sound,"
V_throat=SQRT(k*R*T_throat*1000)

"Some Wrong Solutions with Common Mistakes:"
W1_Vthroat=SQRT(k*R*T1*1000) "Using T1 for temperature"
W2_Vthroat=SQRT(k*R*T2_throat*1000); T2_throat=2*(T_throat-273)/(k+1) "Using C for temperature"
W3_Vthroat=k*R*T_throat "Using wrong relation"
```

12-163

Consider gas flow through a converging-diverging nozzle. Of the five statements below, select the one that is incorrect:

(a) The fluid velocity at the throat can never exceed the speed of sound.

(b) If the fluid velocity at the throat is below the speed of sound, the diversion section will act like a diffuser.

(c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

(d) There will be no flow through the nozzle if the back pressure equals the stagnation pressure.

(e) The fluid velocity decreases, the entropy increases, and stagnation enthalpy remains constant during flow through a normal shock.

Answer (c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

12-164

Combustion gases with k = 1.33 enter a converging nozzle at stagnation temperature and pressure of 350°C and 400 kPa, and are discharged into the atmospheric air at 20°C and 100 kPa. The lowest pressure that will occur within the nozzle is

(a) 13 kPa (b) 100 kPa (c) 216 kPa (d) 290 kPa (e) 315 kPa

Answer (c) 216 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

```
k=1.33
Po=400 "kPa"
"The critical pressure is"
P_throat=Po*(2/(k+1))^(k/(k-1))
"The lowest pressure that will occur in the nozzle is the higher of the critical or atmospheric pressure."

"Some Wrong Solutions with Common Mistakes:"
W2_Pthroat=Po*(1/(k+1))^(k/(k-1)) "Using wrong relation"
W3_Pthroat=100 "Assuming atmospheric pressure"
```

Design and Essay Problems

12-165 to 12-167

Solution Students' essays and designs should be unique and will differ from each other.