Direct Current Flowgate Transmission Rights

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This document serves as an electronic companion for the letter "Direct Current Flowgate Transmission Rights" submitted to IEEE Power Engineering Letters. It provides the detailed procedure to obtain the budget balance condition for a power system comprising both AC and DC networks.

For completeness, we first provide the optimal power flow problem formulation following the same assumptions and notation as in [1].

$$
\begin{aligned}
&\min_{p,\theta,u} && \mathcal{F}(p) && \text{(1a)}\\
&\text{subject to} && && \\
&\lambda_i && p_i = \sum_{j\in\mathcal{N}_i^{\mathrm{AC}}} b_{ij}(\theta_i-\theta_j) + \sum_{j\in\mathcal{N}_i^{\mathrm{DC}}} u_{ij}, i \in \mathcal{N}, && \text{(1b)}\\
&\xi_i^l, \xi_i^u \geq 0 \perp && \underline{p}_i \leq p_i \leq \overline{p}_i, \quad i\in\mathcal{N}, && \text{(1c)}\\
&\mu_{ij}^{\mathrm{AC}} \geq 0 \perp && b_{ij}(\theta_i-\theta_j) \leq \overline{s}_{ij}, \quad (i,j)\in\mathcal{L}^{AC}, && \text{(1d)}\\
&\mu_{ij}^{\mathrm{DC}} \geq 0 \perp && u_{ij} \leq \overline{s}_{ij}, \quad (i,j)\in\mathcal{L}^{DC}, && \text{(1e)}\\
&\nu_{ij} && u_{ij}+u_{ji}=0, \quad (i,j)\in\mathcal{L}^{DC}. && \text{(1f)}
\end{aligned}
$$

The budget balance condition for model (1) writes as

$$
\sum_{i\in\mathcal{N}} \lambda_i p_i + \sum_{ij\in\mathcal{L}^{\mathrm{AC}}} \mu_{ij}^{\mathrm{AC}} \overline{s}_{ij} + \sum_{ij\in\mathcal{L}^{\mathrm{DC}}} \mu_{ij}^{\mathrm{DC}} \overline{s}_{ij} = 0. \tag{2}
$$

The stationarity conditions of model (1) are:

$$
\begin{aligned}
&\frac{d\mathcal{F}(p)}{dp_i} - \lambda_i + \xi_i^u - \xi_i^l = 0, \quad i\in\mathcal{N}, && \text{(3a)}\\
&\sum_{j\in\mathcal{N}_i^{\mathrm{AC}}} b_{ij}(\lambda_i - \lambda_j + \mu_{ij}^{\mathrm{AC}} - \mu_{ji}^{\mathrm{AC}}) = 0, \quad i\in\mathcal{N}, && \text{(3b)}\\
&\lambda_i + \mu_{ij}^{\mathrm{DC}} + \nu_{ij} + \nu_{ji} = 0, \quad (i,j)\in\mathcal{L}^{DC}. && \text{(3c)}
\end{aligned}
$$

Using the above definitions, the budget balance equation (2) is derived based on the following reformulations.

Equation (1b) multiplied by λ_i becomes

$$
\lambda_i p_i = \sum_{j\in\mathcal{N}_i^{\mathrm{AC}}} b_{ij}\lambda_i(\theta_i-\theta_j) + \sum_{j\in\mathcal{N}_i^{\mathrm{DC}}} \lambda_i u_{ij}, \ \forall i. \tag{4}
$$

Multiply equation (3b) by $(\theta_i - \theta_j)$ and rearrange its terms as

$$
\begin{aligned}
&\sum_{j\in\mathcal{N}_i^{\mathrm{AC}}} b_{ij}\left[\lambda_i(\theta_i-\theta_j)+\lambda_j(\theta_j-\theta_i)\right] = \\
&-\sum_{j\in\mathcal{N}_i^{\mathrm{AC}}} b_{ij}\left[\mu_{ij}^{\mathrm{AC}}(\theta_i-\theta_j)+\mu_{ji}^{\mathrm{AC}}(\theta_j-\theta_i)\right], \ \forall i. && \text{(5)}
\end{aligned}
$$

Summing over all i, equation (5) writes as

$$\sum_{i \in \mathcal{N}_j^{\mathrm{AC}}} \sum_{j \in \mathcal{N}_i^{\mathrm{AC}}} b_{ij} \lambda_i (\theta_i - \theta_j) = -\sum_{i \in \mathcal{N}_j^{\mathrm{AC}}} \sum_{j \in \mathcal{N}_i^{\mathrm{AC}}} b_{ij} \mu_{ij}^{\mathrm{AC}} (\theta_i - \theta_j). \tag{6}$$

The complementarity condition (1d) can be written as

$$\mu_{ij}^{\mathrm{AC}} b_{ij} (\theta_i - \theta_j) = \mu_{ij}^{\mathrm{AC}} \overline{s}_{ij}, \quad (i,j) \in \mathcal{L}^{AC} \tag{7}$$

and thus equation (6) can be reformulated as

$$\sum_{i \in \mathcal{N}_j^{\mathrm{AC}}} \sum_{j \in \mathcal{N}_i^{\mathrm{AC}}} b_{ij} \lambda_i (\theta_i - \theta_j) = \sum_{i \in \mathcal{N}_j^{\mathrm{AC}}} \sum_{j \in \mathcal{N}_i^{\mathrm{AC}}} \mu_{ij}^{\mathrm{AC}} \overline{s}_{ij}, \quad (i,j) \in \mathcal{L}^{AC}. \tag{8}$$

Similarly, multiply equation (3c) with u_{ij} to obtain

$$\lambda_i u_{ij} + \mu_{ij}^{\mathrm{DC}} u_{ij} + (\nu_{ij} + \nu_{ji}) u_{ij} = 0, \quad (i,j) \in \mathcal{L}^{DC}. \tag{9}$$

The term $\mu_{ij}^{\mathrm{DC}} u_{ij}$ in (9) can be replaced by $\mu_{ij}^{\mathrm{DC}} \overline{s}_{ij}$, based on the complementarity constraint (1d) that writes as

$$\mu_{ij}^{\mathrm{DC}} u_{ij} = \mu_{ij}^{\mathrm{DC}} \overline{s}_{ij}, \quad (i,j) \in \mathcal{L}^{\mathrm{DC}}, \tag{10}$$

and thus (9) writes as

$$\lambda_i u_{ij} + \mu_{ij}^{\mathrm{DC}} \overline{s}_{ij} + (\nu_{ij} + \nu_{ji}) u_{ij} = 0, \quad (i,j) \in \mathcal{L}^{DC}. \tag{11}$$

Summing (11) for the sending and receiving ends of DC line (i,j) and considering that $u_{ij} = -u_{ji}$, according to constraint (1e), it follows that

$$\lambda_i u_{ij} + \lambda_j u_{ji} = -\mu_{ij}^{\mathrm{DC}} \overline{s}_{ij} - \mu_{ji}^{\mathrm{DC}} \overline{s}_{ji}, \quad (i,j) \in \mathcal{L}^{\mathrm{DC}}. \tag{12}$$

Finally, summing over all i, the right-hand side of equation (4) is replaced with equivalent expressions from (8) and (12) to obtain the budget balance equation

$$\sum_i \lambda_i p_i = -\sum_{ij \in \mathcal{L}^{\mathrm{AC}}} \mu_{ij}^{\mathrm{AC}} \overline{s}_{ij} - \sum_{ij \in \mathcal{L}^{\mathrm{DC}}} \mu_{ij}^{\mathrm{DC}} \overline{s}_{ij}. \tag{13}$$

References

[1] S. Delikaraoglou and J.A. Taylor and P. Pinson "Direct Current Flowgate Transmission Rights," *submitted to* IEEE Power Engineering Letters, 2017.