DTU

Industrial IoT for Digitization of Electronis Assets

Kalman Filter Estimation and Inflow Prediction

Agenda

- Estimate the Unmeasured Inflow using The Kalman Filter

State Equation

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \mathbf{G}_k\mathbf{w}_k \\
\mathbf{z}_k &= \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k
\end{aligned}
$$

- $\mathbf{x}_k$: State vector at time k.
- $\mathbf{A}_k$: State transition matrix.
- $\mathbf{B}_k$: Control-input matrix.
- $\mathbf{G}_k$: Noise transformation matrix.
- $\mathbf{z}_k$: Measurement at time k.
- $\mathbf{H}_k$: Observation matrix.
- $\mathbf{w}_k$: Process noise
- $\mathbf{v}_k$: Measurement noise

Noise Characteristics and Assumptions

- Process noise $\mathbf{w}_k$ and measurement noise $\mathbf{v}_k$ are typically independent.
- Both noises are modeled as white noise (constant spectral density, uncorrelated in time).
- $\mathbf{G}_k$ scales or transforms process noise to align with the state space.

$x_0 \sim \tilde{x}_0$ and covariance P_0

Covariance, of the process noise w_k and the measurement noise v_k

$$\mathbb{E}\left[\begin{pmatrix} w_k \\ v_k \end{pmatrix}\begin{pmatrix} w'_\ell & v'_\ell \end{pmatrix}\right] = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix}\delta_{kl}$$

- The resulting matrix has zeros off diagonal, uncorrelated error with each other.
- The δ_{kl} means that the autocorrelation is non-zero on for $l = k$, thus they are independent in time (independece of sample in white noise)

Prior knowledge of state $\rightarrow$ $\mathbf{P}_{k-1|k-1}$ $\hat{\mathbf{x}}_{k-1|k-1}$

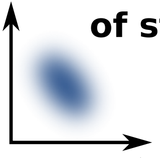


Figure:
Priori Knowledge from the previous measurements

Prediction Phase: Step 1.1

Let's start by considering the state update (no new measurement)

Predicted State Estimate (a priori)

$$\boxed{\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k}$$

- $\mathbf{x_{k|k-1}} \rightarrow$ estimate of x at time k given $k-1$.
- $\mathbf{x_{k-1|k-1}} \rightarrow$ all the estimation up to $k-1$ given $k-1$ observations.
- $\mathbf{Bu_{k-1}} \rightarrow$ deterministic input.

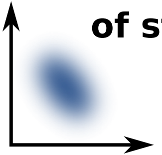
Prediction Phase: Step 1.2

Predicted Estimate Covariance (*a priori*)

$$\hat{P}_{k|k-1} = F_k P_{k|k-1} F_k^T + Q_k$$

- $\mathbf{P_{k|k-1}}$ $\rightarrow$ predicted state covariance matrix at time k based on P_{k-1}.
- $\mathbf{F_k}$ is the state transition matrix
- $\mathbf{Q_k}$ is the covariance matrix of the process noise

Prior knowledge of state



$$\mathbf{P}_{k-1|k-1}$$
$$\hat{\mathbf{x}}_{k-1|k-1}$$

Prediction step
Based on e.g. physical model

$$\mathbf{P}_{k|k-1}$$
$$\hat{\mathbf{x}}_{k|k-1}$$

Update Phase: Step 2.1

New measurement available

Measurement Update Equation

$$y_k = z_k - H_k \hat{x}_{k|k-1}$$

- $\mathbf{y_k} \rightarrow$ represents the measurement at time step k
- $\mathbf{z_k}$ is the predicted measurement at time step k.
- $\mathbf{H_k}$ is the observation matrix at time step k.

Update Phase: Step 2.2

New measurement available

Innovation of Covariance Matrix

$$S_{k|k-1} = H_k \hat{P}_{k|k-1} H_k^T + R_k$$

- $\mathbf{y_k} \rightarrow$ represents the measurement at time step k
- $\mathbf{z_k}$ is the predicted measurement at time step k.
- $\mathbf{H_k}$ is the observation matrix at time step k.

Update Phase: Step 2.3

New measurement available

Kalman Gain

$$S_{k|k-1} = H_k \hat{P}_{k|k-1} H_k^T + R_k$$

- $\mathbf{y_k} \rightarrow$ represents the measurement at time step k
- $\mathbf{z_k}$ is the predicted measurement at time step k.
- $\mathbf{H_k}$ is the observation matrix at time step k.

Update Phase: Step 2.4

Kalman Gain Calculation

$$K_k = \hat{P}_{k|k-1} H_k^T S_k^{-1}$$

$$K_k = \hat{P}_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

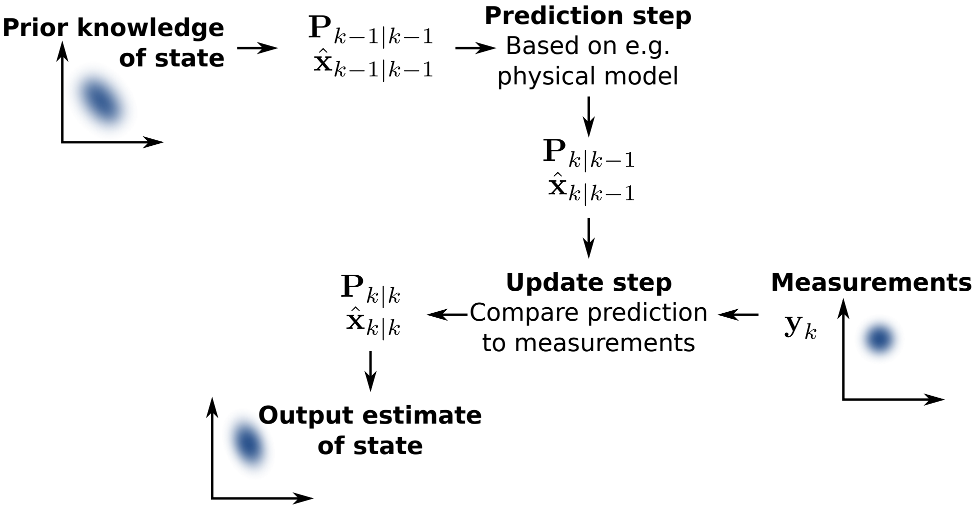
Update Phase: Step 2.5

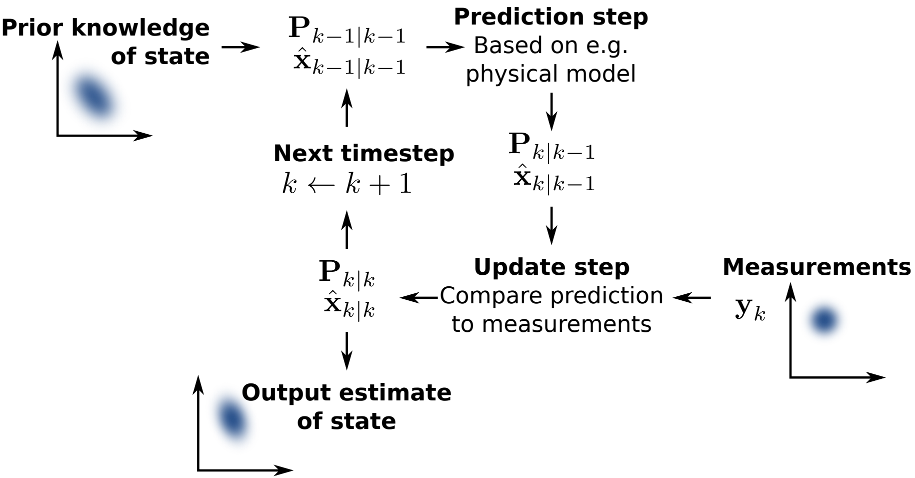
Updated State Estimate

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k y_k$$

Updated Estimate Covariance

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$


Prior knowledge of state
$\mathbf{P}_{k-1|k-1}$
$\hat{\mathbf{x}}_{k-1|k-1}$
Prediction step
Based on e.g. physical model
$\mathbf{P}_{k|k-1}$
$\hat{\mathbf{x}}_{k|k-1}$
Update step
Compare prediction to measurements
Measurements
$\mathbf{y}_k$
$\mathbf{P}_{k|k}$
$\hat{\mathbf{x}}_{k|k}$
Output estimate of state



Prior knowledge of state
$\mathbf{P}_{k-1|k-1}$
$\hat{\mathbf{x}}_{k-1|k-1}$
Prediction step
Based on e.g. physical model
Next timestep
$k \leftarrow k+1$
$\mathbf{P}_{k|k-1}$
$\hat{\mathbf{x}}_{k|k-1}$
$\mathbf{P}_{k|k}$
$\hat{\mathbf{x}}_{k|k}$
Update step
Compare prediction to measurements
Measurements
$\mathbf{y}_k$
Output estimate of state

Inflow Estimation of the Wastewater Tank

- The goal is to estimate the inflow rate (Q_{in}) in a reservoir system without direct measurements.
- Kalman Filter is utilized to estimate Q_{in} using the water height (h) and outflow rate (Q_{out}) measurements.
- The estimation is crucial for building the inflow forecast and the controller.
- State Variables: Inflow Rate (Q_{in}), Water Height (h), Outflow Rate (Q_{out}).
- Measurements: Water Height and Outflow Rate.

System Model

- The continuous-time model is given by the differential equation:

$$\frac{dh}{dt} = \frac{t_{sampling}}{A}(Q_{in}(t) - Q_{out}(t))$$

-

$$\frac{dh}{dt} = 0 \longrightarrow Q_{in}(t) = Q_{out}(t)$$

State Transition Matrix (A)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ \frac{t}{\text{area}} & 1 & -\frac{t}{\text{area}} \\ 0 & 0 & 1 \end{bmatrix}$$

- Describes the system's evolution over time.
- t is the time step, and area is a system parameter.

Observation Matrix (H) and Measurements

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Maps state variables to measurements.
- Observes water height (h) and outflow rate (Q_{out}).

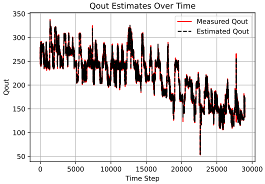
Process and Measurement Noise

- Process Noise: Error estimates for Q_{in}, h, Q_{out}.
- Measurement Noise: Error in measuring h and Q_{out}.

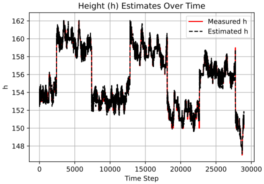
Initial State and Covariance

- Initial State: Estimates for Q_{in}, h, Q_{out}.
- Covariance: Represents initial uncertainty in state estimates.

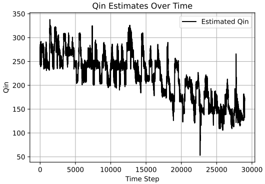
Python Tutorial: Build The Kalman Filter To Estimate the Inflow



(a) Outflow Estimation vs Measurements



(b) Height Estimation vs Measurements



(c) Indirect Inflow Estimation