Module 4
Power Electronic Converters
Introductory aspects for modeling and control

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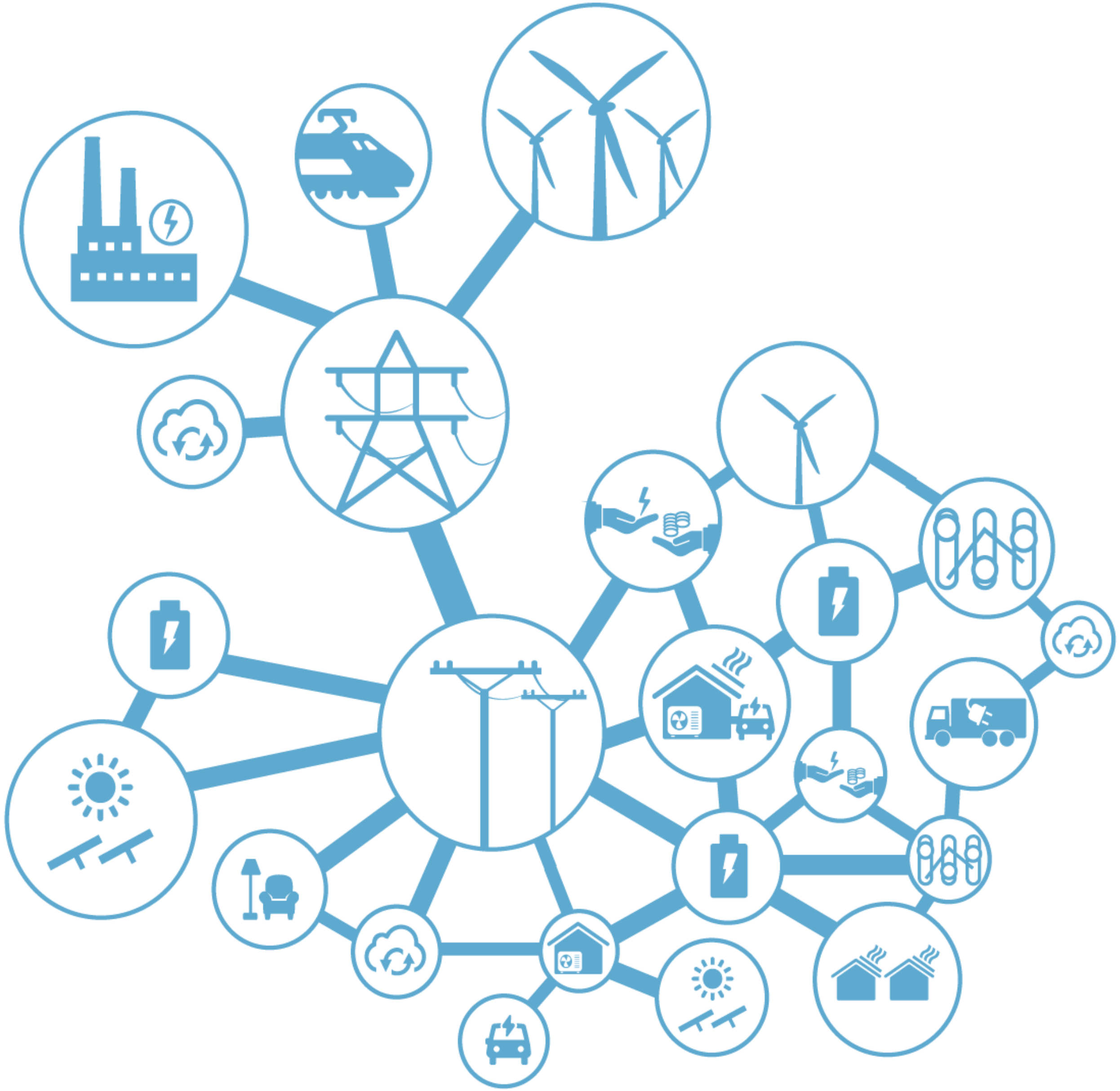
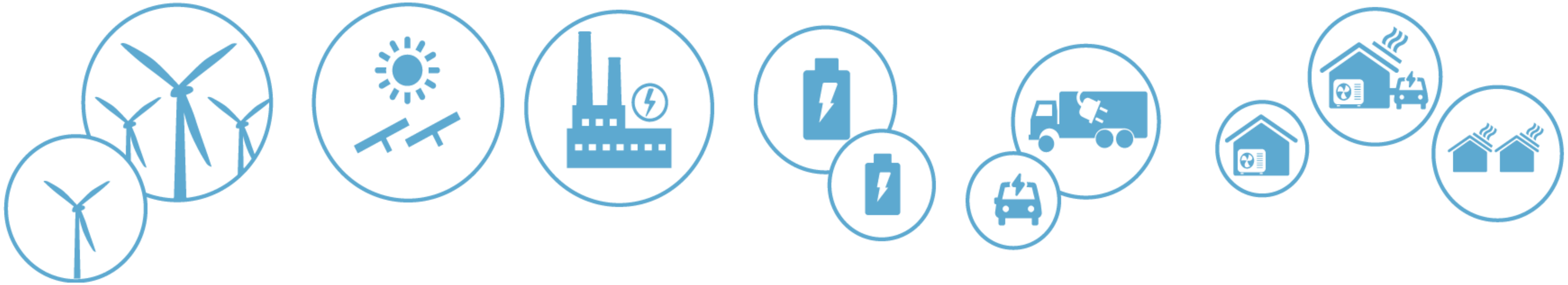
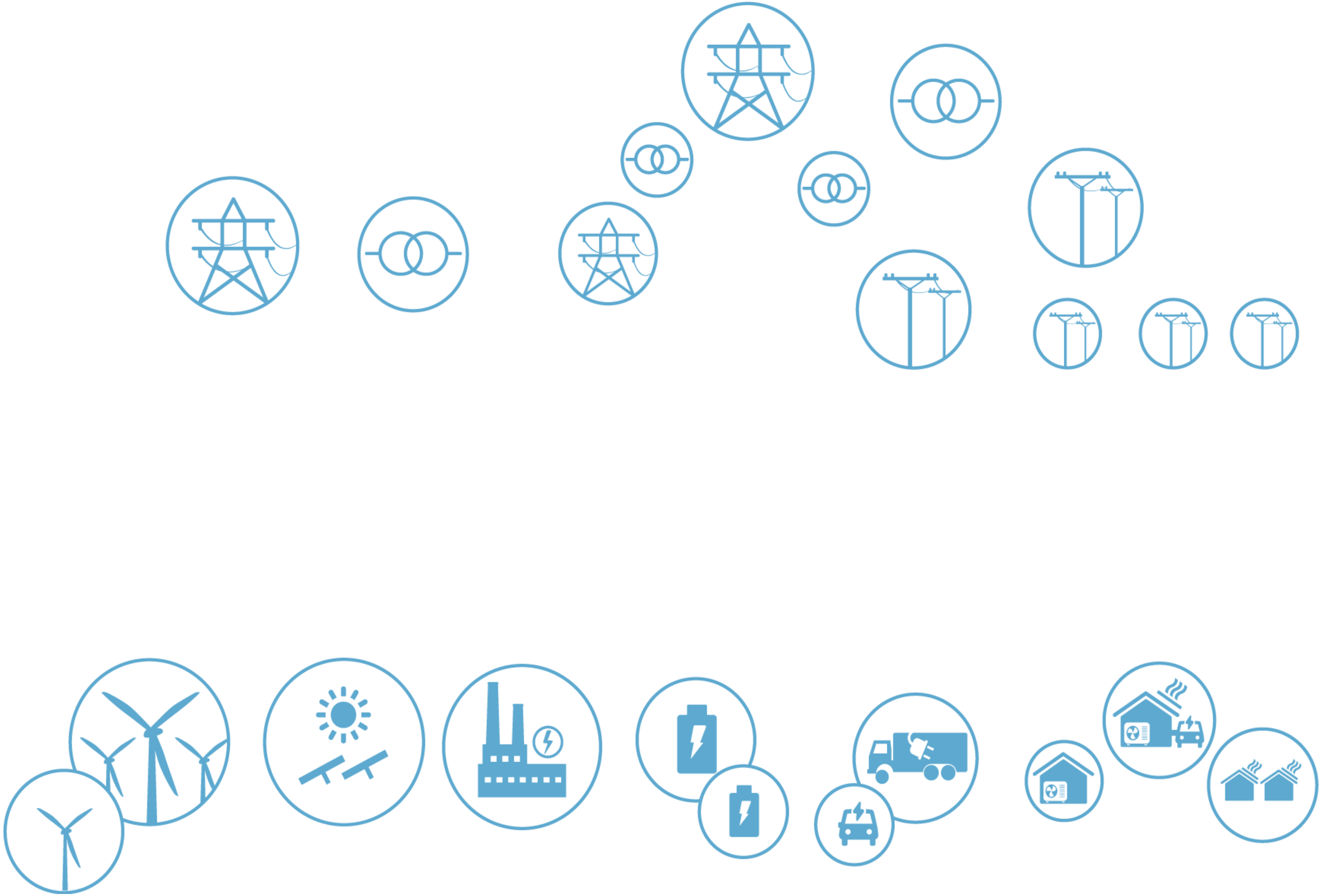
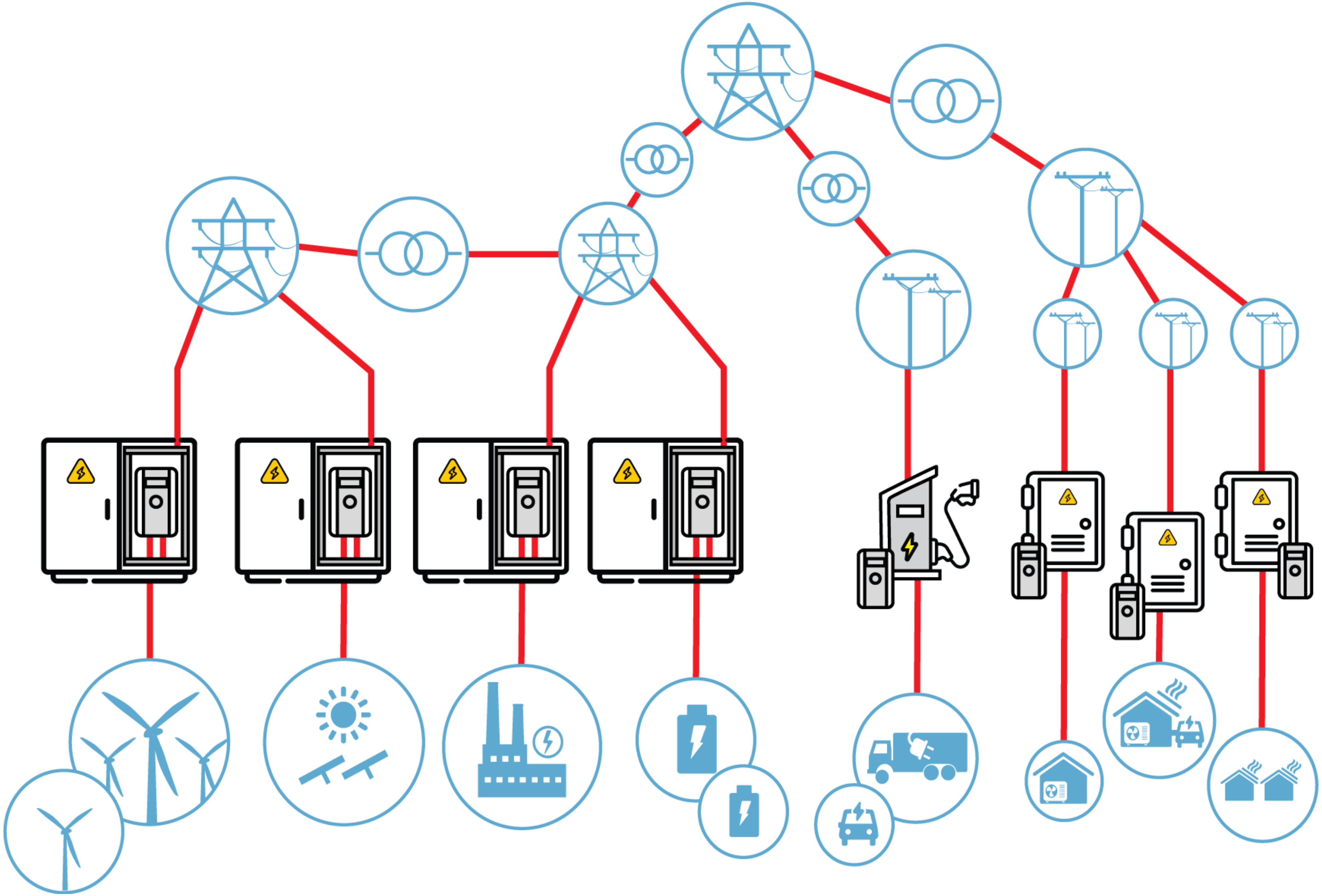
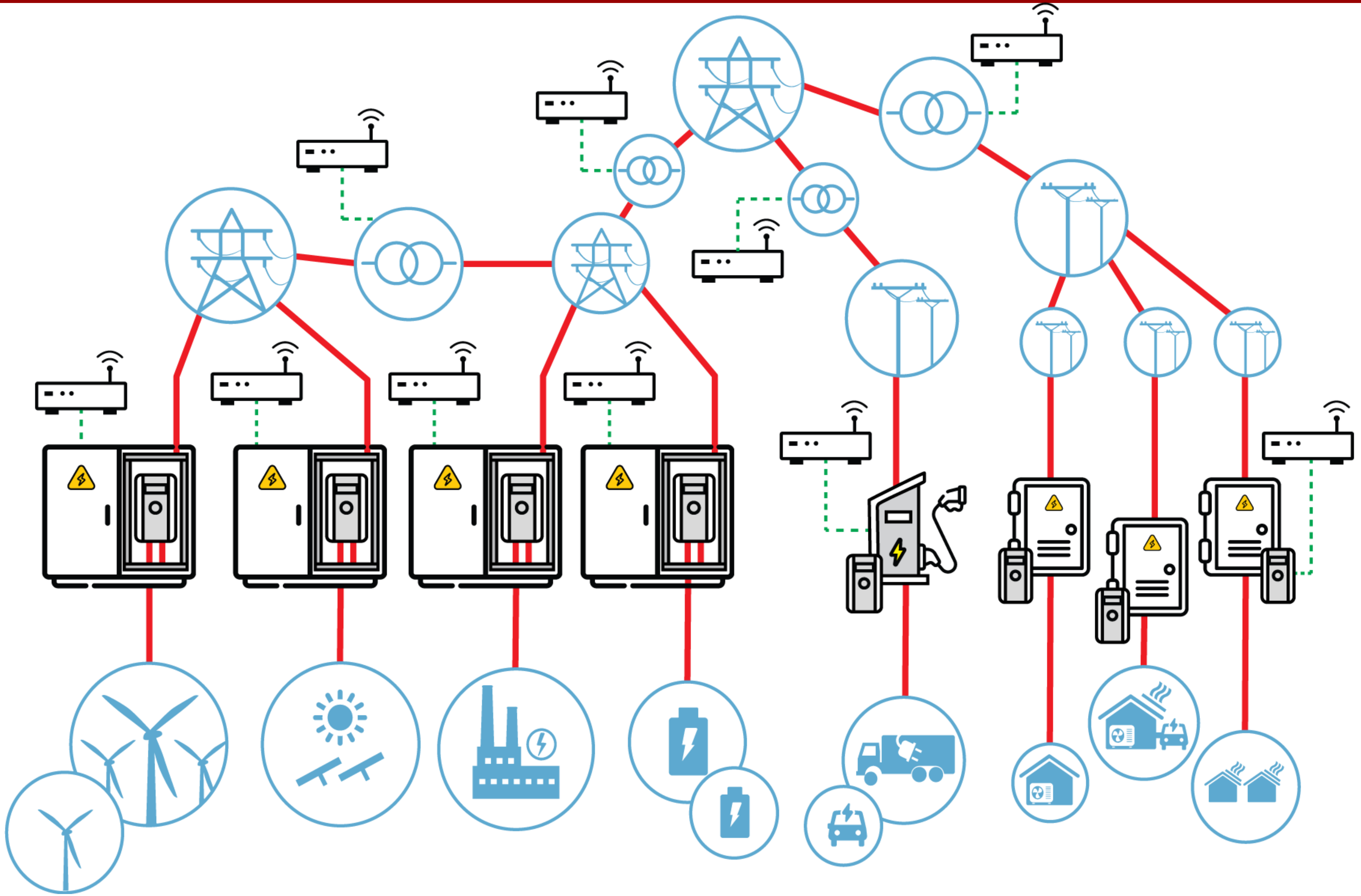
Motivation











Why is needed power conversion?

We need to **adapt** power for its use

Electrical equipment with different electrical requirements

Power converter

General overview and an example of a Smart Drive System

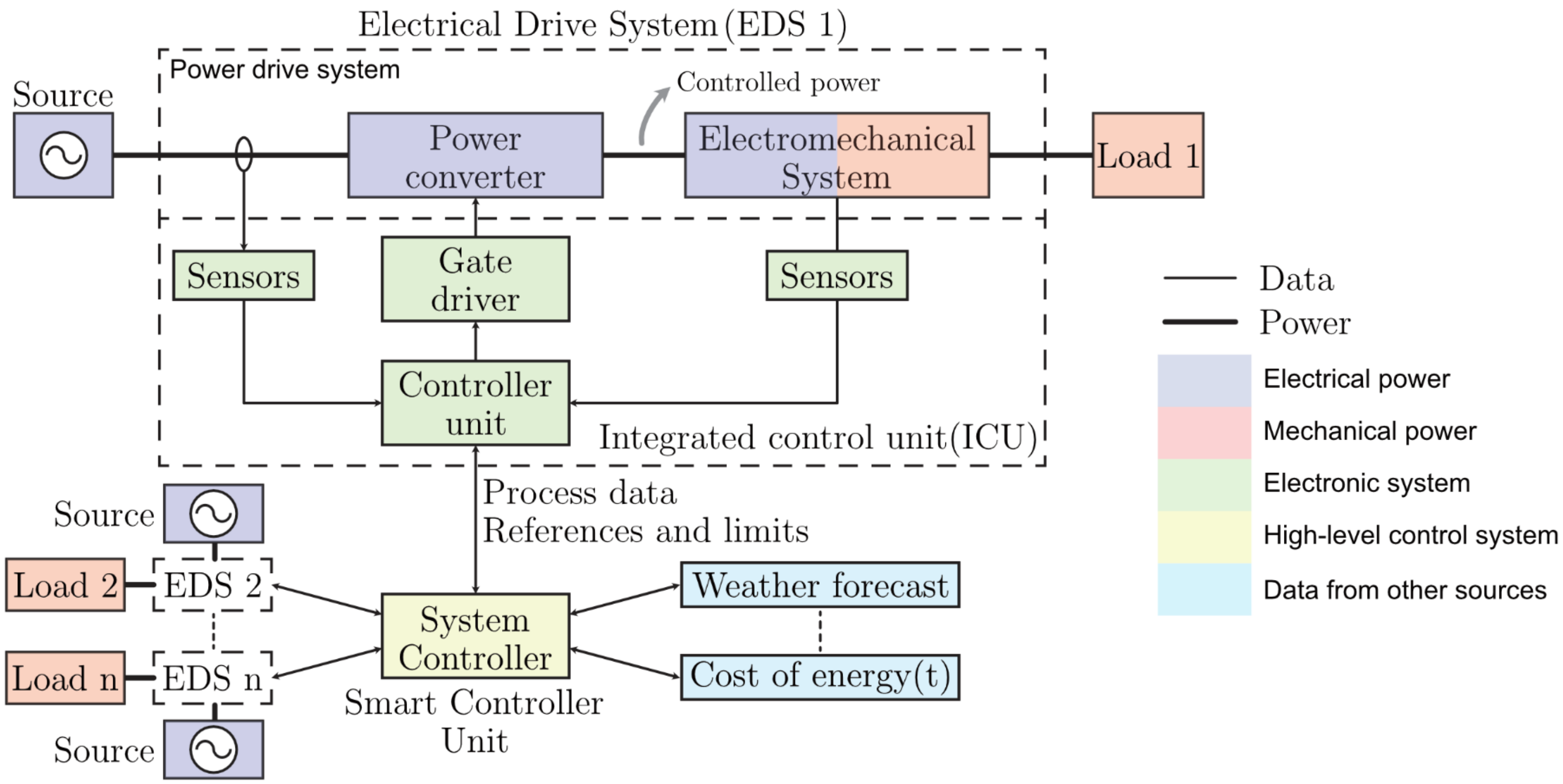
A *power converter* is an electronic circuit, conformed by:

- Semiconductors
- Passive components (e.g., capacitors, inductors, transformers, resistors)

which convert electrical power from one fixed state (amplitude and frequency) to another.

- The performance indicators for its selection can consider cost, efficiency, power density, size or weight.

- The converter to use strongly depends on the target application.



Example of a Smart Drive System

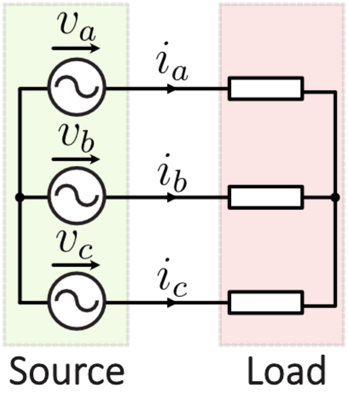
Module objectives

- Comprehend the idea of the **Space Vector Theory** used to control of Power Electronic Converters (PECs) connected to AC systems.
- Develop the **mathematical model** for PECs in both continuous and discrete time domains.
- Understand what is a **modulator** for PECs, why we use it, and how it works.
- Understand and be capable to **implement control algorithms** for PECs, including linear controllers and Model Predictive Control (MPC).
- **Utilize simulation software** to model, simulation, and control an Active Front End (AFE) converter effectively.

Mathematical foundation
Introduction to Space Vector Theory

Mathematical foundation
Definitions

Consider the following 3-phase balanced (or symmetrical) system:



$$v_a(t) = \hat{V} \cdot \cos(wt)$$
$$v_b(t) = \hat{V} \cdot \cos\left(wt - \frac{2\pi}{3}\right)$$
$$v_c(t) = \hat{V} \cdot \cos\left(wt - \frac{4\pi}{3}\right)$$

$$i_a(t) = \hat{I} \cdot \cos(wt - \phi)$$
$$i_b(t) = \hat{I} \cdot \cos\left(wt - \frac{2\pi}{3} - \phi\right)$$
$$i_c(t) = \hat{I} \cdot \cos\left(wt - \frac{4\pi}{3} - \phi\right)$$

Note:
A balanced system has:
- Equal magnitude
- Phase displacement (separated $2\pi/3$ radians)

where,
- The amplitude (peak values) of voltage and current: $\hat{V}$ and $\hat{I}$
- Grid frequency is $w = 2\pi \cdot f$ (e.g., f=50Hz or T=20ms)
- Time is *t*.

Mathematical foundation
Definitions

The Root Mean Square (RMS) value of a continuous function *x* is defined as:

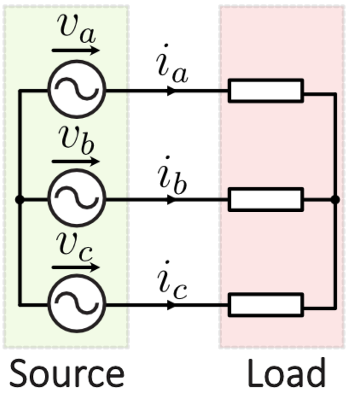
$$x_{RMS} = \sqrt{\frac{1}{T}\int_0^T x(t)^2\,dt}$$

Note:

If $v(t) = \hat{V} \cdot \cos(wt + \theta_v)$, then:

$$v_{rms} = \frac{\hat{V}}{\sqrt{2}} = \mathrm{V_{rms}}$$

Thus, we can also consider:



$$v_a(t) = \sqrt{2}V_{rms} \cdot \cos(wt)$$

$$v_b(t) = \sqrt{2}V_{rms} \cdot \cos\left(wt - \frac{2\pi}{3}\right)$$

$$v_c(t) = \sqrt{2}V_{rms} \cdot \cos\left(wt - \frac{4\pi}{3}\right)$$

$$i_a(t) = \sqrt{2}I_{rms} \cdot \cos(wt - \phi)$$

$$i_b(t) = \sqrt{2}I_{rms} \cdot \cos\left(wt - \frac{2\pi}{3} - \phi\right)$$

$$i_c(t) = \sqrt{2}I_{rms} \cdot \cos\left(wt - \frac{4\pi}{3} - \phi\right)$$

Mathematical foundation

Definitions

The active power P transferred from the source is equal to:

$$P(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) = V_{rms}I_{rms}\Bigg[\cos(\phi) + \cos(2wt - \phi) + \cos(\phi) + \cos\left(2wt - \phi + \frac{2\pi}{3}\right) + \cos(\phi) + \cos\left(2wt - \phi - \frac{2\pi}{3}\right)\Bigg] = 3V_{rms}I_{rms} \cdot \cos(\phi)\ \text{[W]}$$

The reactive power Q transferred from the source is equal to:

$$Q = 3V_{rms}I_{rms} \cdot \sin(\phi)\ \text{[VAr]}$$

The apparent power transferred from the source

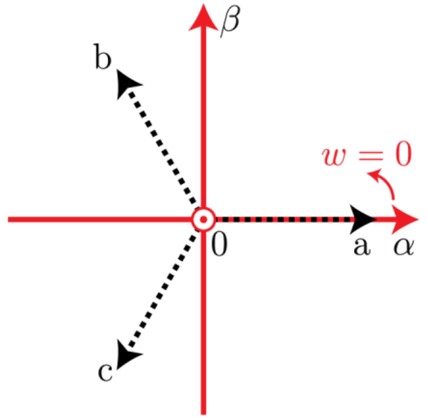
$$S = \sqrt{P^2 + Q^2}\ [VA]$$

Mathematical foundation

Reference-frame Theory / Space Vector Theory

For a 3-phase system, consider the following reference **frames** in the complex plane:

- abc frame
- Stationary reference frame $\alpha\beta 0$



We define the operator:

$$a = e^{j2\pi/3} = \cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)$$

For instance, in a 3-phase system:

$$v_a(t) = \hat{V}\cdot\cos(wt) \qquad i_a(t) = \hat{I}\cdot\cos(wt-\phi)$$

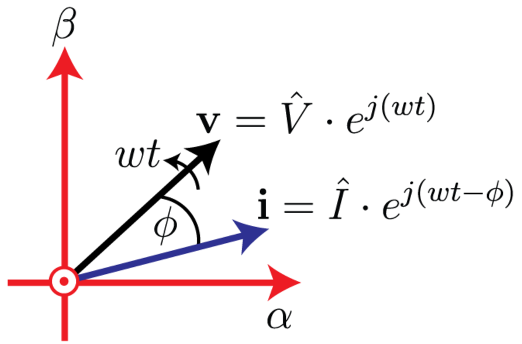
$$v_b(t) = \hat{V}\cdot\cos\left(wt-\frac{2\pi}{3}\right) \qquad i_b(t) = \hat{I}\cdot\cos\left(wt-\frac{2\pi}{3}-\phi\right)$$

$$v_c(t) = \hat{V}\cdot\cos\left(wt-\frac{4\pi}{3}\right) \qquad i_c(t) = \hat{I}\cdot\cos\left(wt-\frac{4\pi}{3}-\phi\right)$$

We can use the operator to define the projection from *abc* frame to $\alpha\beta 0$ as:

$$\vec{v} = \boldsymbol{v} = \frac{2}{3}(v_a + a\cdot v_b + a^2\cdot v_c) = \hat{V}\cdot e^{jwt}$$

$$\vec{i} = \boldsymbol{i} = \frac{2}{3}(i_a + a\cdot i_b + a^2\cdot i_c) = \hat{I}\cdot e^{j(wt-\phi)}$$



Mathematical foundation

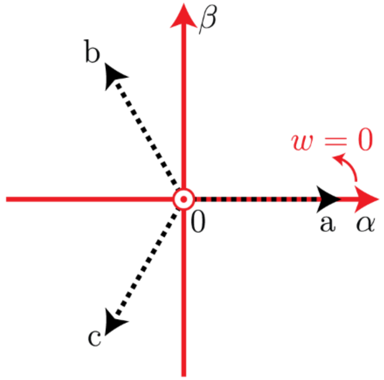
Definitions – Clarke transform

The previous idea introduces the Clarke Transform:

$$\vec{x} = \boldsymbol{x}^{\alpha\beta 0} = \frac{2}{3}(x_a + a \cdot x_b + a^2 \cdot x_c) = [T_{\alpha\beta 0}] \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

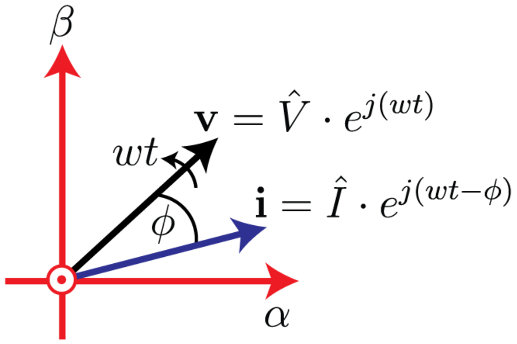
where,

$$[T_{\alpha\beta 0}] = \frac{2}{3} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$



Note:

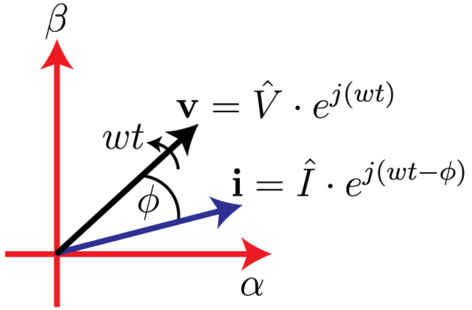
- The $\alpha\beta 0$ reference frame is stationary (angular speed of $\alpha\beta$ is 0).
- Analysis from 3-phases (abc) to two phases ($\alpha\beta$).
- Allows to model dynamic 3-phase systems.
- This modeling provides a vector representation of the system.



Show animation

Mathematical foundation – review

Instantaneous power



From previous example: $\boldsymbol{v}^{\alpha\beta0} = \hat{V} \cdot e^{jwt}$ $\quad$ $\boldsymbol{i}^{\alpha\beta0} = \hat{I} \cdot e^{j(wt-\phi)}$

We define active and reactive power using the $\boldsymbol{v}$ and $\boldsymbol{i}$ vectors:

$$P = \frac{3}{2}\Re e\left\{\boldsymbol{v}^{\alpha\beta0} \cdot \left(\boldsymbol{i}^{\alpha\beta0}\right)^*\right\} = 3\frac{\hat{V}}{\sqrt{2}}\frac{\hat{I}}{\sqrt{2}}\Re e\left\{e^{j\theta}\right\} = 3V_{rms}I_{rms}\cos(\phi)$$

$$Q = \frac{3}{2}\Im m\left\{\boldsymbol{v}^{\alpha\beta0} \cdot \left(\boldsymbol{i}^{\alpha\beta0}\right)^*\right\} = 3\frac{\hat{V}}{\sqrt{2}}\frac{\hat{I}}{\sqrt{2}}\Im m\left\{e^{j\theta}\right\} = 3V_{rms}I_{rms}\sin(\phi)$$

Note: * represents the conjugate operator, ensuring consistency with the sign for reactive power (positive inductive, negative capacitive).

Similarly, the active power, considering $i_a + i_b + i_c = 0$, can be obtained as follows:

$$P = \frac{3}{2}\Re e\{\boldsymbol{v}\boldsymbol{i}^*\} = \frac{3}{2}\Re e\left\{\left(\frac{2}{3}(v_a + a \cdot v_b + a^2 \cdot v_c)\right) \cdot \left(\frac{2}{3}(i_a + a^2 \cdot i_b + a \cdot i_c)\right)\right\} = v_a i_a + v_b i_b + v_c i_c = 3V_{rms}I_{rms} \cdot \cos(\phi)$$

P and Q are known as the instantaneous active and reactive power respectively.

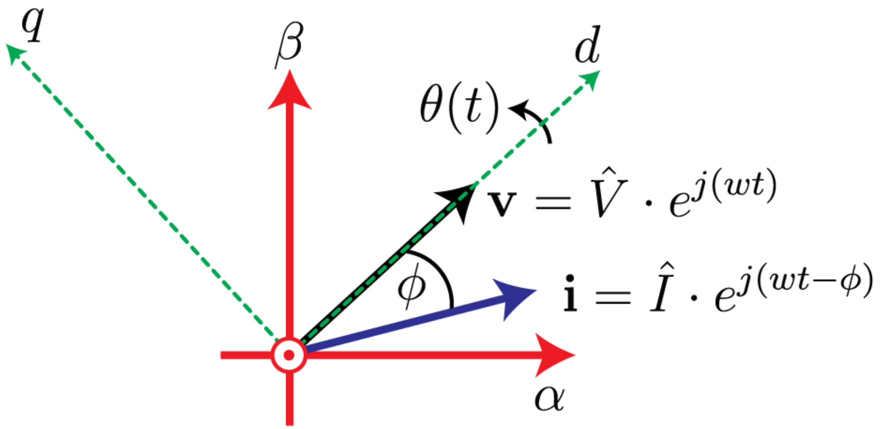
Note: constant 3/2 for power calculation provides the correct expression

Mathematical foundation – review

Definitions – Park transform

Idea: synchronize the variable of interest with respect to how is changing the phase θ, where

$$\theta(t) = \theta_0 + \int_0^t w\,dt$$



This leads to a **rotating reference frame**, represented as *dq0.*

The relation between variables in stationary to rotating frame can be modeled as:

$$\boldsymbol{X}^{\alpha\beta} = \boldsymbol{X}^{dq} \cdot \boldsymbol{e}^{jwt} \longrightarrow \boldsymbol{X}^{dq} = \boldsymbol{X}^{\alpha\beta} \cdot \boldsymbol{e}^{-jwt}$$

$$\longrightarrow X^d + jX^q = \left(X^\alpha + jX^\beta\right) \cdot (\cos(wt) - j \cdot sin(wt))$$

Which introduces the Park transform: $\boldsymbol{X}^{dq} = \left[T_{dq0}\right] \cdot \boldsymbol{X}^{\alpha\beta}$

where $\left[T_{dq0}\right] = \begin{bmatrix} \cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

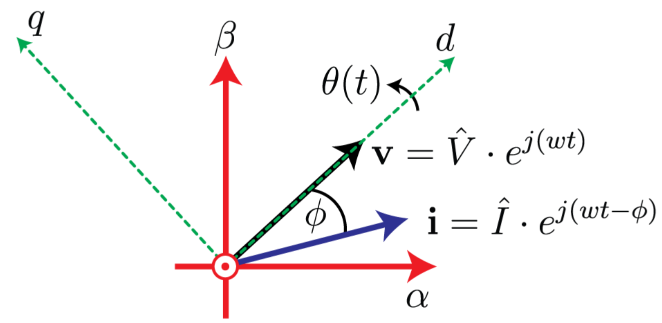
Using previous expression, active and reactive power calculation result:

$$P = \frac{3}{2}\Re e\{\boldsymbol{v} \cdot \boldsymbol{i}^*\} = \frac{3}{2}\Re e\left\{\boldsymbol{v}^{\boldsymbol{dq}} \cdot \left(\boldsymbol{i}^{\boldsymbol{dq}}\right)^*\right\} = \frac{3}{2}\left(v^d \cdot i^d + v^q \cdot i^q\right)$$

$$Q = \frac{3}{2}\Im m\{\boldsymbol{v} \cdot \boldsymbol{i}^*\} = \frac{3}{2}\Im m\left\{\boldsymbol{v}^{\boldsymbol{dq}} \cdot \left(\boldsymbol{i}^{\boldsymbol{dq}}\right)^*\right\} = \frac{3}{2}\left(v^d \cdot i^q - v^q \cdot i^d\right)$$

Mathematical foundation – review

Exercise



Consider a voltage source of $\hat{V} = V_g = 230\sqrt{2}[V]$, $f_g = 50[Hz]$:

$$v_a(t) = \hat{V} \cdot \sin(wt)$$

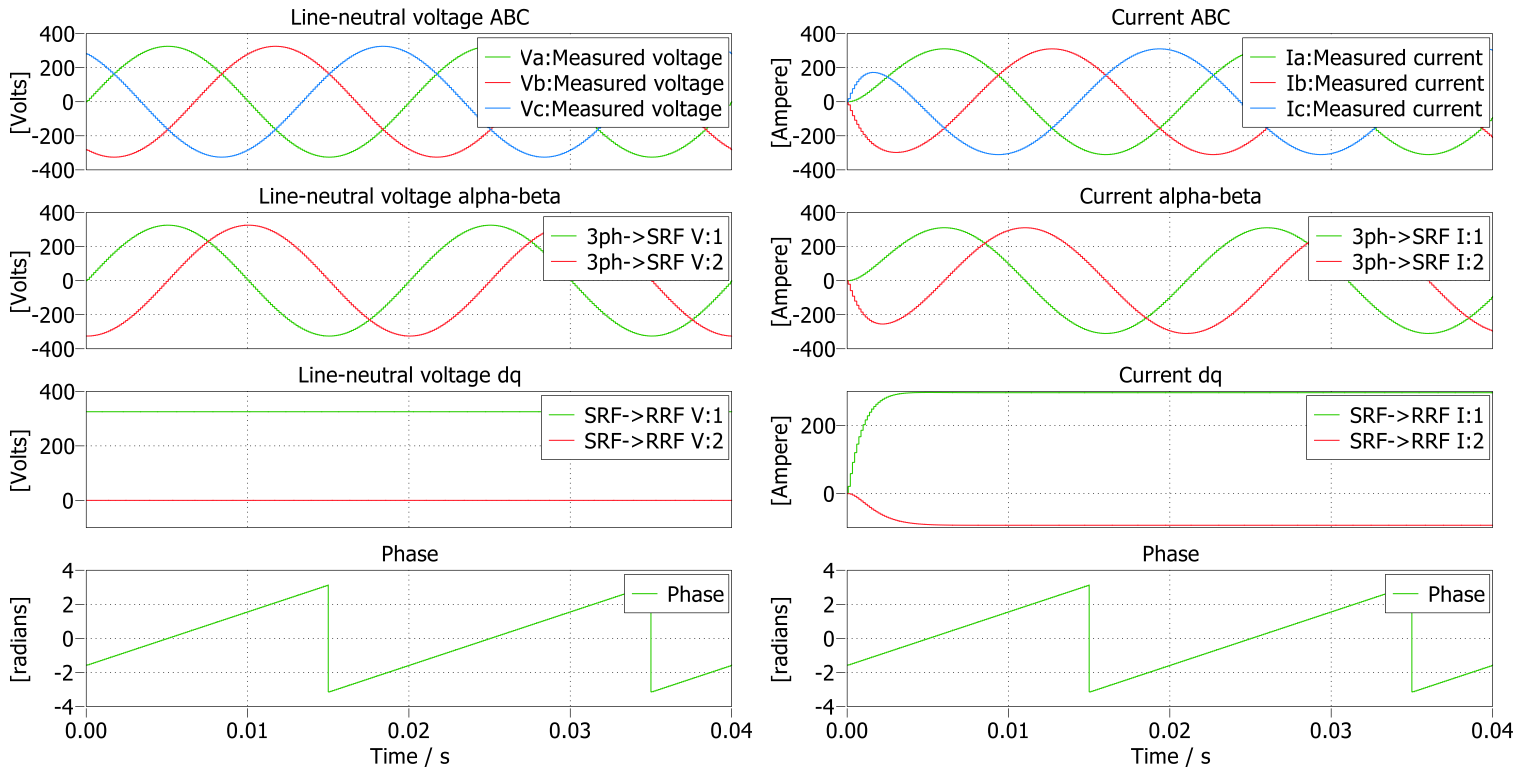
$$v_b(t) = \hat{V} \cdot \sin\left(wt - \frac{2\pi}{3}\right)$$

$$v_c(t) = \hat{V} \cdot \sin\left(wt - \frac{4\pi}{3}\right)$$

A load $L = 1[mH]$ and $\mathrm{R} = 1[\Omega]$.

Simulate the system in PLECS

1. Generate the voltage and current waveforms in abc frame, stationary frame (*αβ*) and rotating frame (*dq*).
2. Calculate power transfer (active, reactive and apparent) and contrast with power expression based on phasors.



Hint: To get phase, use “atan2” trigonometric function.

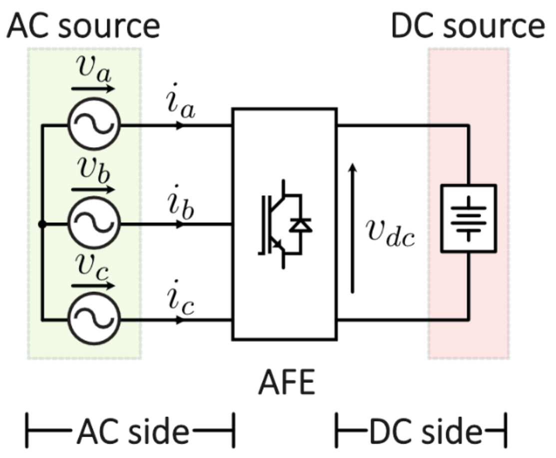
Modeling

Objectives

- Understand and apply the space vector theory **to model** a PECs connected to the AC grid.
- Understand the role of the **modulator** in PECs.
- Practice the learned knowledge by simulating an Active Front End converter in open loop.

Active Front End

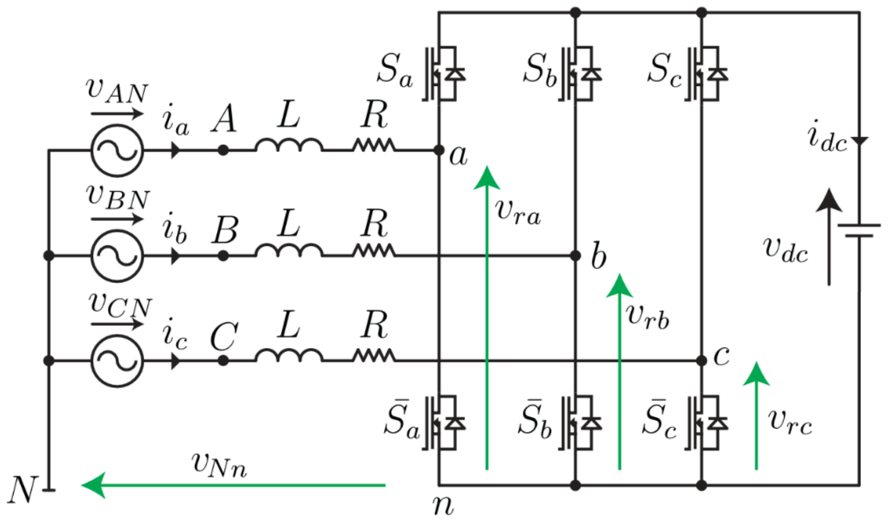
Introduction



System under study

An Active Front End (AFE) converter is a **bidirectional** power electronic converter used either for:

- Rectification: AC to DC
- or inversion: DC to AC



Two-level grid-connected converter as Energy Storage System

Applications:

- Renewable energy systems (e.g. wind and solar)
- Motor drives.
- Energy storage systems (e.g. batteries, supercapacitors)
- Grid-tied systems for power quality improvement.

Active Front End

Modeling

In the ac side:

$$v_{ra} = S_a \cdot v_{dc}$$

$$v_{rb} = S_b \cdot v_{dc}$$

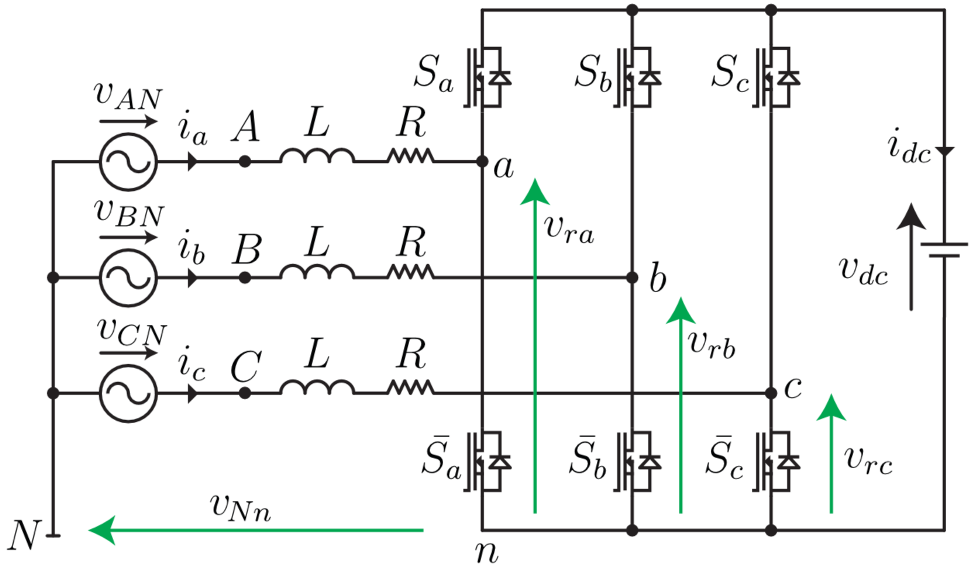
$$v_{rc} = S_c \cdot v_{dc}$$

$$v_{aN} = L \cdot \frac{di_a}{dt} + R \cdot i_a + v_{ra} + v_{nN}$$

$$v_{bN} = L \cdot \frac{di_b}{dt} + R \cdot i_b + v_{rb} + v_{nN}$$

$$v_{cN} = L \cdot \frac{di_c}{dt} + R \cdot i_c + v_{rc} + v_{nN}$$

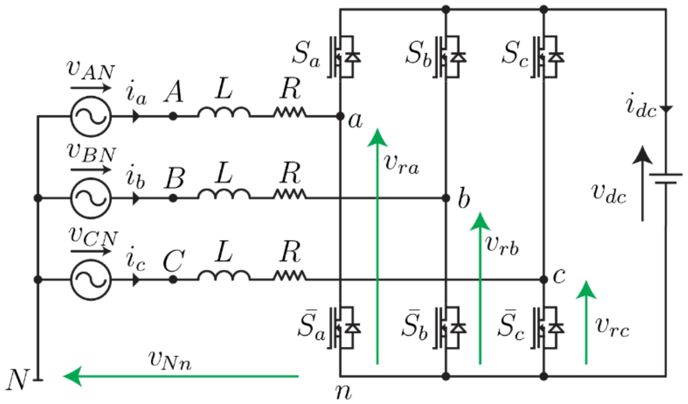
where S_a, S_b and S_c = {0,1}

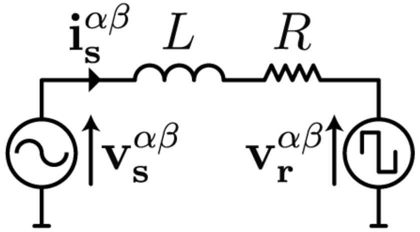


In the dc side: $i_{dc} = S_a \cdot i_a + S_b \cdot i_b + S_c \cdot i_c$

Active Front End

Modeling





In the ac side:

$$v_{ra} = S_a \cdot v_{dc}$$

$$v_{rb} = S_b \cdot v_{dc}$$

$$v_{rc} = S_c \cdot v_{dc}$$

$$v_{aN} = L \cdot \frac{di_a}{dt} + R \cdot i_a + v_{ra} + v_{nN}$$

$$v_{bN} = L \cdot \frac{di_b}{dt} + R \cdot i_b + v_{rb} + v_{nN}$$

$$v_{cN} = L \cdot \frac{di_c}{dt} + R \cdot i_c + v_{rc} + v_{nN}$$

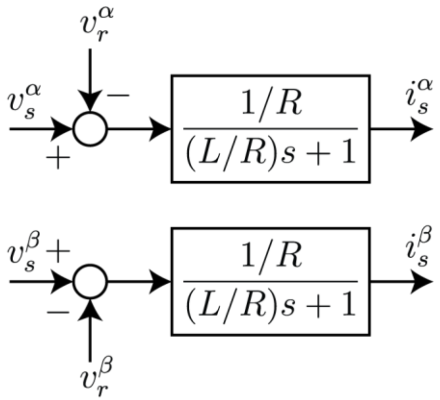
In $\alpha\beta$ stationary frame:

$$\boldsymbol{v_s^{\alpha\beta}} = L \cdot \frac{d\boldsymbol{i_s^{\alpha\beta}}}{dt} + R \cdot \boldsymbol{i_s^{\alpha\beta}} + \boldsymbol{v_r^{\alpha\beta}}$$

$$v_s^\alpha + jv_s^\beta = L \cdot \frac{d}{dt}\left(i_s^\alpha + ji_s^\beta\right) + R \cdot \left(i_s^\alpha + ji_s^\beta\right) + \left(v_r^\alpha + jv_r^\beta\right)$$

$$v_s^\alpha = L \cdot \frac{d}{dt} i_s^\alpha + R \cdot i_s^\alpha + v_r^\alpha$$

$$v_s^\beta = L \cdot \frac{d}{dt} i_s^\beta + R \cdot i_s^\beta + v_r^\beta$$



Reminder: Clark transform

$$\vec{x} = \boldsymbol{x^{\alpha\beta 0}} = \frac{2}{3}\left(x_a + a \cdot x_b + a^2 \cdot x_c\right) = \left[T_{\alpha\beta 0}\right] \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

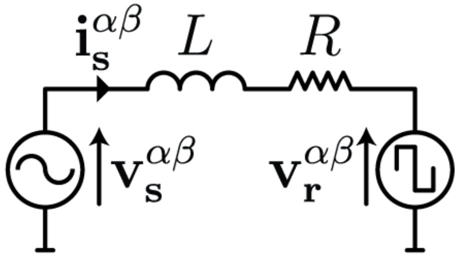
Note: αβ components are not coupled

Active Front End

Modeling

In $\alpha\beta$ stationary frame:

$$\boldsymbol{v}_s^{\alpha\beta} = L \cdot \frac{d\boldsymbol{i}_s^{\alpha\beta}}{dt} + R \cdot \boldsymbol{i}_s^{\alpha\beta} + \boldsymbol{v}_r^{\alpha\beta}$$



In the ac side:

$$v_{ra} = S_a \cdot v_{dc}$$
$$v_{rb} = S_b \cdot v_{dc}$$
$$v_{rc} = S_c \cdot v_{dc}$$

$$\left.\begin{aligned} v_{aN} &= L \cdot \frac{di_a}{dt} + R \cdot i_a + v_{ra} + v_{nN} \\ v_{bN} &= L \cdot \frac{di_b}{dt} + R \cdot i_b + v_{rb} + v_{nN} \\ v_{cN} &= L \cdot \frac{di_c}{dt} + R \cdot i_c + v_{rc} + v_{nN} \end{aligned}\right\}$$

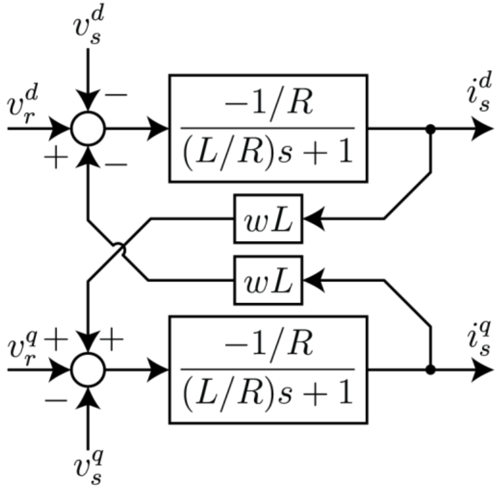
To transform between stationary and rotating (dq) rotating frame, a generalized variable X:

$$\boldsymbol{X}^{\alpha\beta} = \boldsymbol{X}^{dq} \cdot \boldsymbol{e}^{jwt}$$

$$\Rightarrow \quad \boldsymbol{v}_s^{dq} = L \cdot \frac{d\boldsymbol{i}_s^{dq}}{dt} + R \cdot \boldsymbol{i}_s^{dq} + jwL \cdot \boldsymbol{i}_s^{dq} + \boldsymbol{v}_r^{\alpha\beta}$$

$$v_s^d + jv_s^q = L \cdot \frac{d}{dt}(i_s^d + ji_s^q) + (R + jwL) \cdot \left(i_s^d + ji_s^q\right) + (v_r^d + jv_r^q)$$

$$\Rightarrow \quad \begin{aligned} v_s^d &= L \cdot \frac{d}{dt} i_s^d + R \cdot i_s^d - wL \cdot i_s^q + v_r^d \\ v_s^q &= L \cdot \frac{d}{dt} i_s^q + R \cdot i_s^q + wL \cdot i_s^d + v_r^q \end{aligned}$$



Note: dq components are coupled

Active Front End

Modeling: summary

In abc frame:

$$v_{ra} = S_a \cdot v_{dc}$$

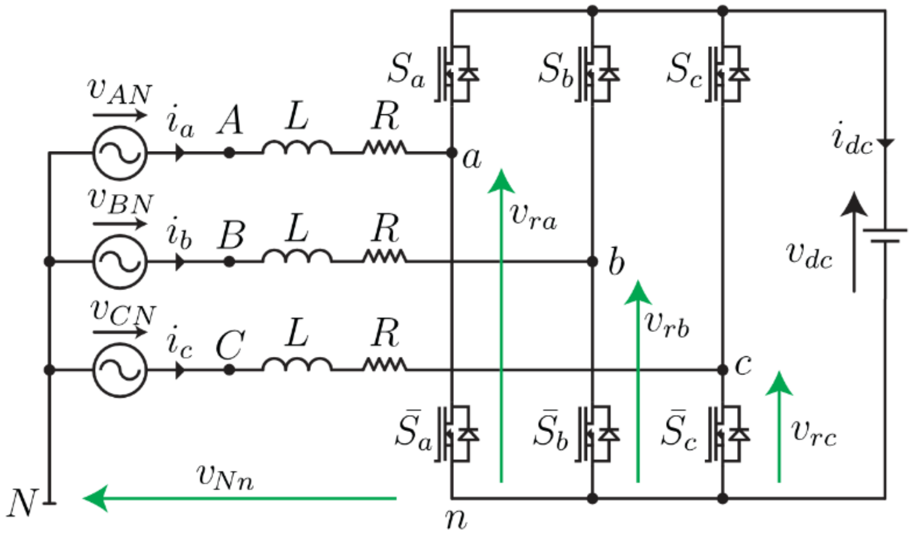
$$v_{rb} = S_b \cdot v_{dc}$$

$$v_{rc} = S_c \cdot v_{dc}$$

$$v_{aN} = L \cdot \frac{di_a}{dt} + R \cdot i_a + v_{ra} + v_{nN}$$

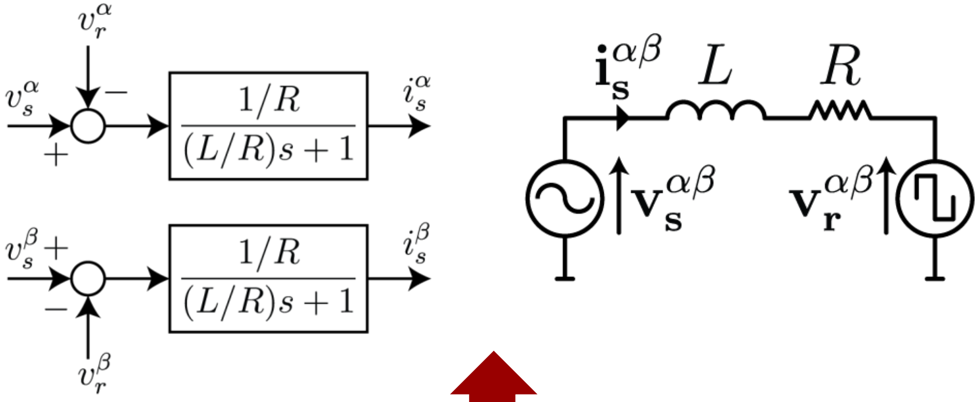
$$v_{bN} = L \cdot \frac{di_b}{dt} + R \cdot i_b + v_{rb} + v_{nN}$$

$$v_{cN} = L \cdot \frac{di_c}{dt} + R \cdot i_c + v_{rc} + v_{nN}$$



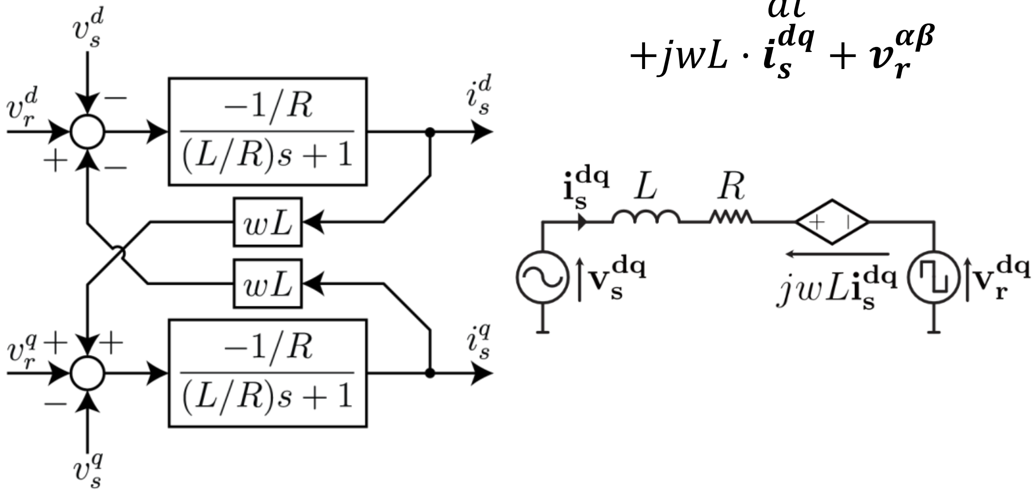
In $\alpha\beta$ stationary frame:

$$\boldsymbol{v_s^{\alpha\beta}} = L \cdot \frac{d\boldsymbol{i_s^{\alpha\beta}}}{dt} + R \cdot \boldsymbol{i_s^{\alpha\beta}} + \boldsymbol{v_r^{\alpha\beta}}$$



In dq rotating frame:

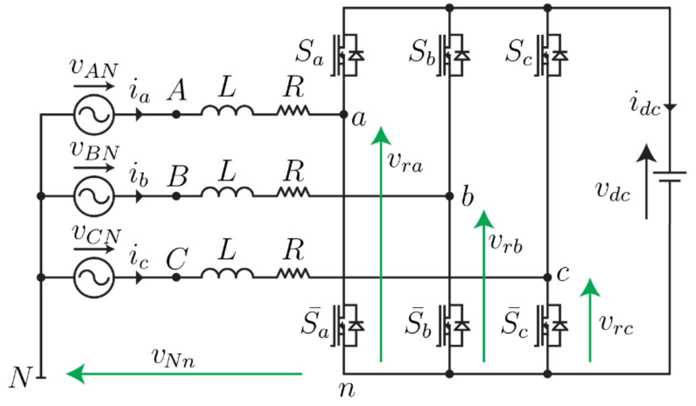
$$\boldsymbol{v_s^{dq}} = L \cdot \frac{d\boldsymbol{i_s^{dq}}}{dt} + R \cdot \boldsymbol{i_s^{dq}} + jwL \cdot \boldsymbol{i_s^{dq}} + \boldsymbol{v_r^{\alpha\beta}}$$

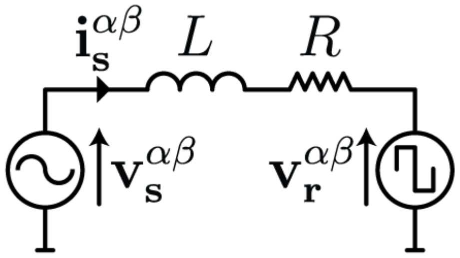


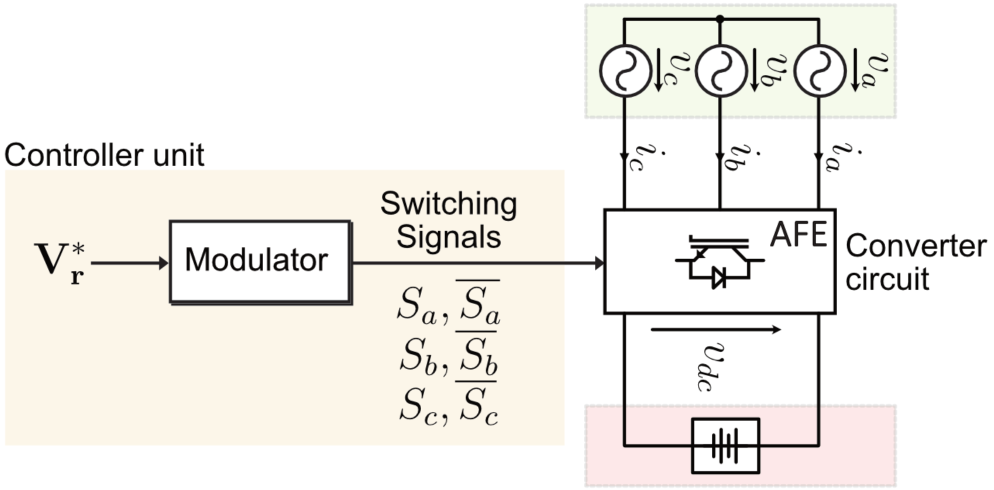
Active Front End

Modulator

The switching nature of the converter makes not possible to apply a pure sinusoidal AC voltage.







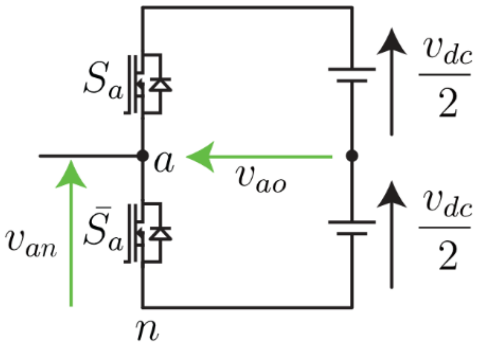
A **modulator** is a component of the controller unit, responsible of providing the gate signals to trigger the power semiconductors to act according with the desired actuation. It translates control commands into the desired actuation.

Different modulators have been proposed. We will study two: Carrier-based PWM (CB-PWM) and Space Vector Modulation (SVM).

Active Front End

Modulation: notion of average value and PWM

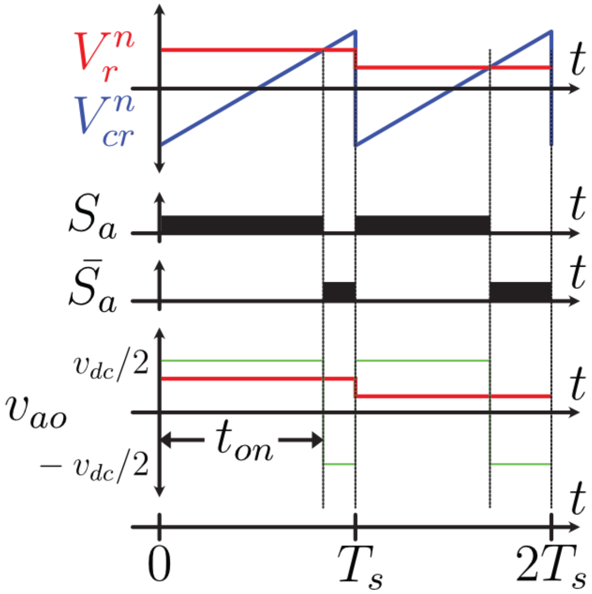
Consider only one inverter leg as shown in the circuit



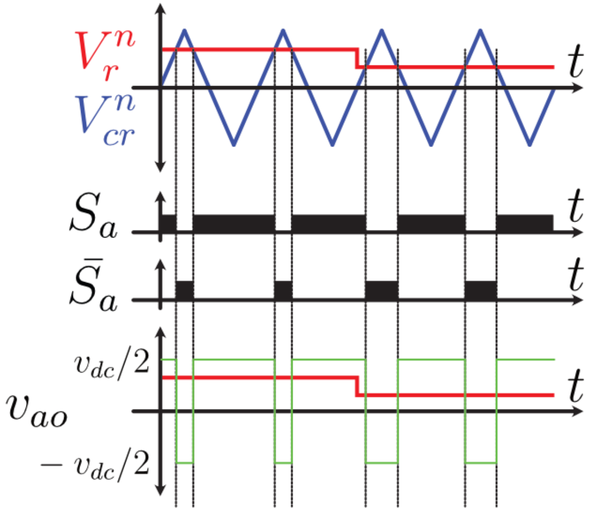
Its principle of operation is based on the “average value principle”, according to the duty cycle.

$$\overline{v_{ao}} = \frac{1}{T_s}\int_0^T v_{ao}(t)dt = \frac{1}{T_s}\left(\int_0^{t_{on}} \frac{v_{dc}}{2}dt + \int_{t_{on}}^{T} -\frac{v_{dc}}{2}dt\right) = v_{dc}\cdot\frac{t_{on}}{T} - \frac{v_{dc}}{2}$$

$$\overline{v_{an}} = \frac{1}{T_s}\int_0^T v_{an}(t)dt = \frac{1}{T_s}\left(\int_0^{t_{on}} v_{dc}dt + \int_{t_{on}}^{T} 0dt\right) = v_{dc}\cdot\frac{t_{on}}{T}$$



Example 1: Sawtooth carrier

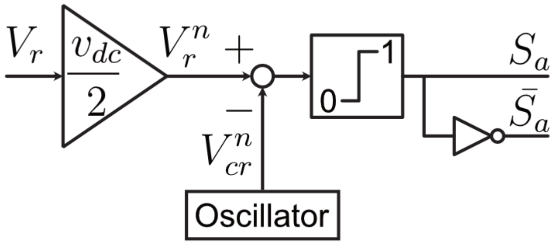


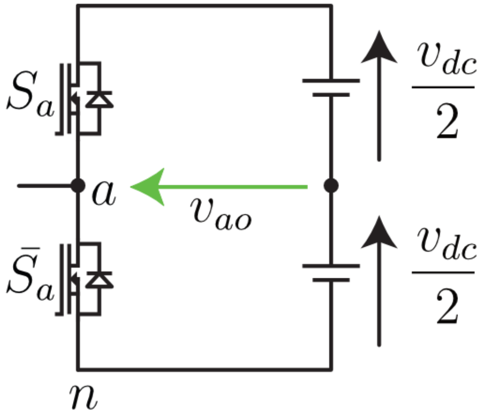
Example 2: Triangular carrier

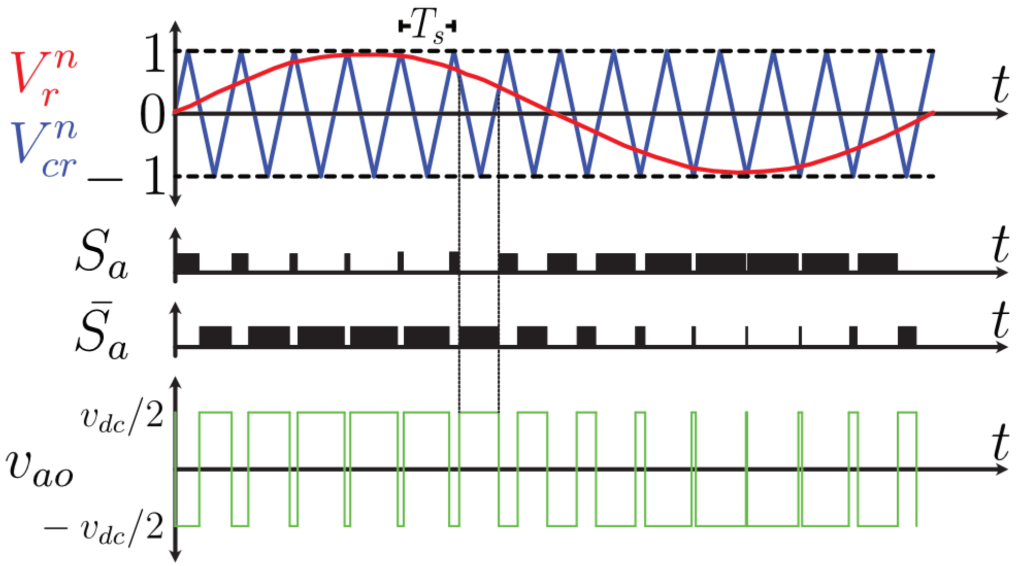
Active Front End

Carrier-based PWM: Sinusoidal PWM (SPWM)

SPWM modulator scheme changes the duty cycle every period T which, in average, generates the desired AC signal.







- V_{cr} is known as the carrier, with fixed period T_s.
- V_r is a voltage that we want to apply.
- Superindex *n* refers to normalize.

Note:

- Fixed switching frequency
- Controllable amplitude by changing duty cycle
- It can be extended for more phases.

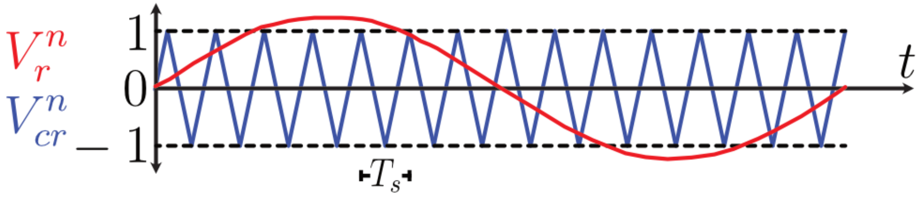
Question:

Using a triangular waveform of period T_s, what is the resulting fundamental period of AC voltage v_{ao}?

Active Front End

Carrier-based PWM: actuation limitation and 3er harmonic injection

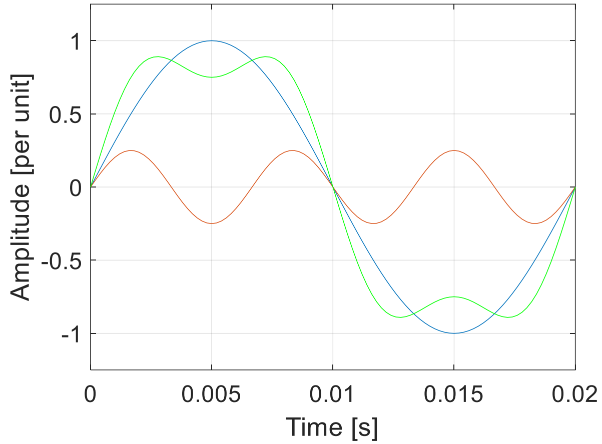
There is a limited range where actuation signal remains in the linear range without modifying the fundamental amplitude.



This issue is known as *overmodulation*.

One alternative to extend it is by injecting a common mode signal, e.g. a third harmonic.

$$v_r^* = \widehat{v_1} \cdot \sin(wt) + \widehat{v_3} \cdot \sin(3wt)$$



By doing this, considering for actuation reference:

$$v_{ra}^* = \widehat{v_1} \cdot \sin(wt) + \widehat{v_3} \cdot \sin(3wt)$$

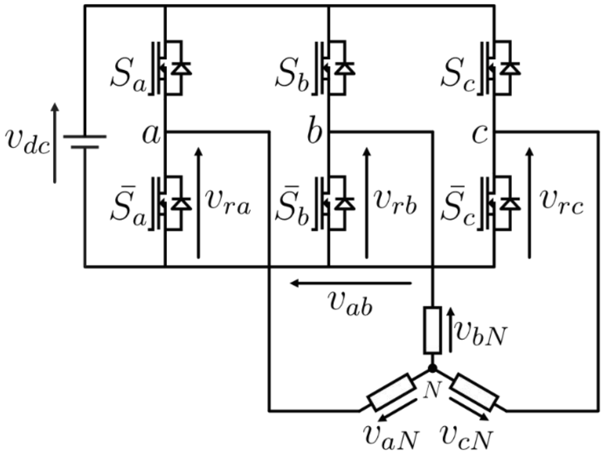
$$v_{rb}^* = \widehat{v_1} \cdot \sin(wt - 2\pi/3) + \widehat{v_3} \cdot \sin(3(wt - 2\pi/3))$$

$$v_{rc}^* = \widehat{v_1} \cdot \sin(wt - 4\pi/3) + \widehat{v_3} \cdot \sin(3(wt - 4\pi/3))$$

$$v_{ra} = \widehat{v_1} \cdot \sin(wt) + \widehat{v_3} \cdot \sin(3wt) + \mathrm{v_a^{dc+HF}}$$

$$v_{rb} = \widehat{v_1} \cdot \sin(wt - 2\pi/3) + \widehat{v_3} \cdot \sin(3(wt - 2\pi/3)) + \mathrm{v_b^{dc+HF}}$$

$$v_{rc} = \widehat{v_1} \cdot \sin(wt - 4\pi/3) + \widehat{v_3} \cdot \sin\big(3(wt - 4\pi/3)\big) + \mathrm{v_a^{dc+HF}}$$



We can calculate:

$$v_{ab} = v_{ra} - v_{rb}$$
$$= \sqrt{3}\widehat{v_1} \cdot \sin(wt) + \mathrm{v_{ab}^{HF}}$$

Note: We can achieve higher actuation magnitude and 3th harmonic cancelation

Active Front End

Carrier-based PWM: 3er harmonic injection and minmax function

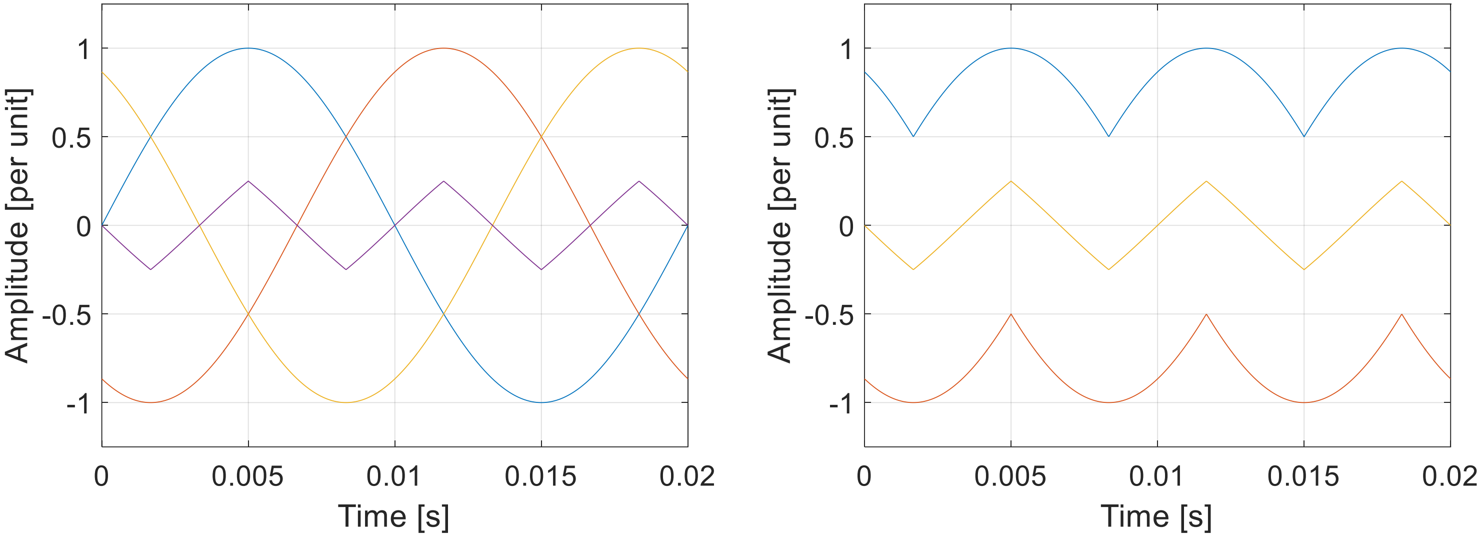
Issue: To generate a third harmonic you have to know the actuation frequency, phase and amplitude, in order to synchronize and inject the 3rd harmonic.

One solution is to inject a min-max

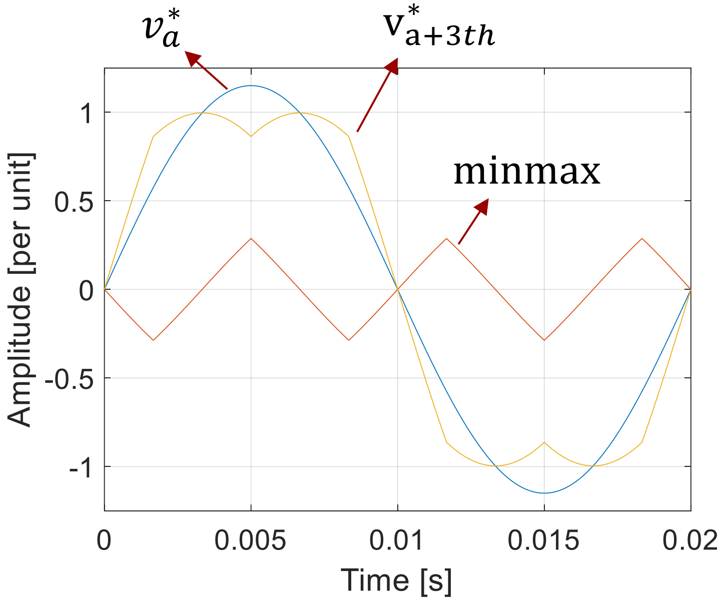
$$\text{minmax} = \frac{\min(v_a^*, v_b^*, v_c^*) + \max(v_a^*, v_b^*, v_c^*)}{2}$$

The actuation in phase a can be calculated as:

$$\mathrm{v}_{\mathrm{a}+3th}^* = \mathrm{v}_{\mathrm{a}}^* - \text{minmax}$$



Example of waveforms v_a^*, v_b^*, v_c^*, and minmax



Example of minmax injection

Active Front End

SVM: Space Vector Modulation

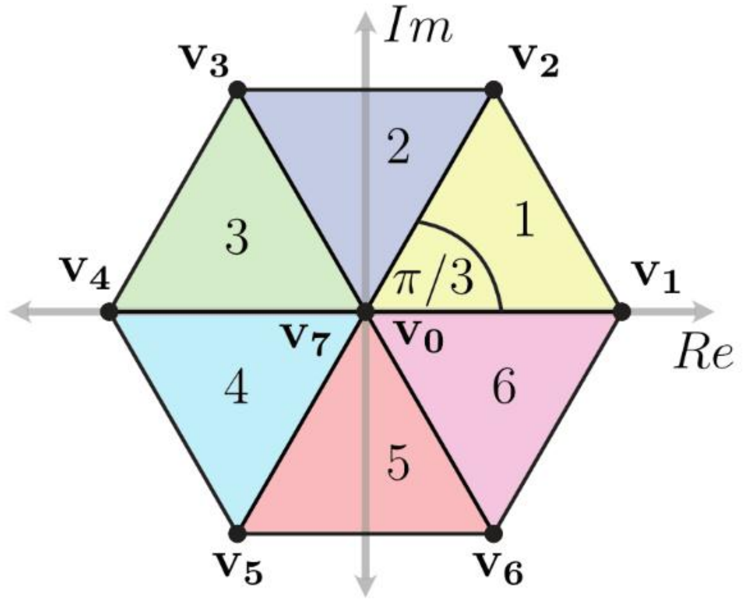
Space vector modulation (SVM) is an equivalent for the “average value principle” in a vectorial form.

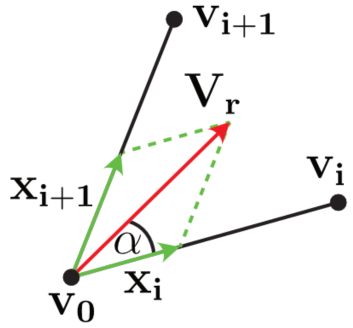
Idea: generate in average V_r^* by applying the **voltage vector** for **certain periods of time.**

In other words: an inverter can generate V_r^* in the complex plane by combining two adjacent voltage vectors and the null vector.

For this reason, we need to model:

1. Voltage apply to the converter as space vector.
2. Periods to apply voltage vector





Active Front End

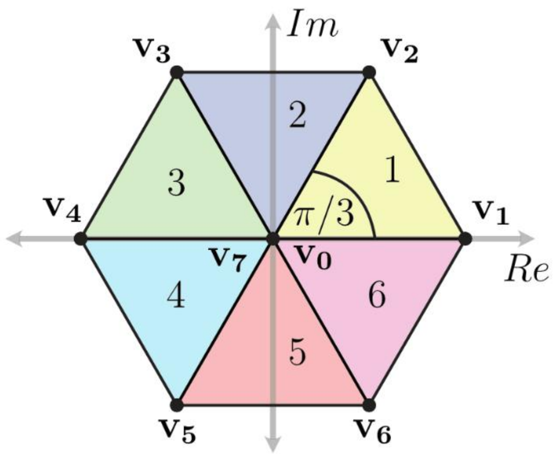
SVM: space vector voltage model

Considering that any switching signal $S = \{0{,}1\}$ (e.g. S_a, $\overline{S_b}$), 8 possible voltage vectors can be applied (see table).

For the space vector voltage model, consider the equation:

$$\boldsymbol{v}_{\boldsymbol{r}}^{\boldsymbol{\alpha\beta}} = \frac{2}{3} v_{dc}\left(S_a + \boldsymbol{a} S_b + \boldsymbol{a}^2 S_c\right)$$

The 8 voltage vectors are plotted in the complex plane in figure.



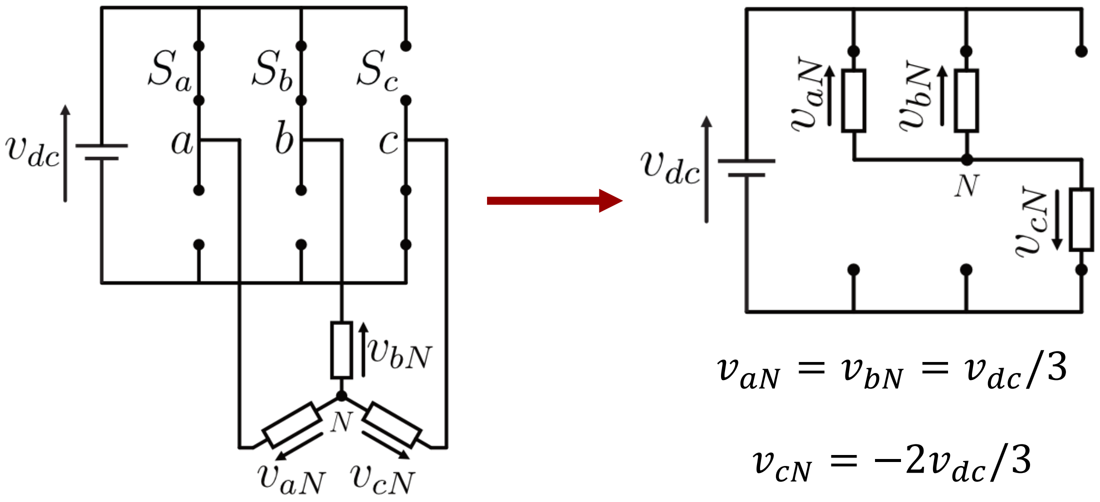
State (S_a, S_b, S_c)	Space vector voltage $v_r^{\alpha\beta}$
(0,0,0)	$\boldsymbol{v_0} = (0{,}0)$
(1,0,0)	$\boldsymbol{v_1} = \frac{2}{3} v_{dc}(1{,}0)$
(1,1,0)	$\boldsymbol{v_2} = \frac{2}{3} v_{dc}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
(0,1,0)	$\boldsymbol{v_3} = \frac{2}{3} v_{dc}\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
(0,1,1)	$\boldsymbol{v_4} = \frac{2}{3} v_{dc}(-1{,}0)$
(0,0,1)	$\boldsymbol{v_5} = \frac{2}{3} v_{dc}\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(1,0,1)	$\boldsymbol{v_6} = \frac{2}{3} v_{dc}\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(1,1,1)	$\boldsymbol{v_7} = (0{,}0)$

Active Front End

SVM: space vector voltage model

A second approach uses the state applied to the switches, assuming balanced loads.

For instance, if $S_a = S_b = 1$ and $S_c = 0$, a simplification of the resulting circuit is:



$$v_{aN} = v_{bN} = v_{dc}/3$$

$$v_{cN} = -2v_{dc}/3$$

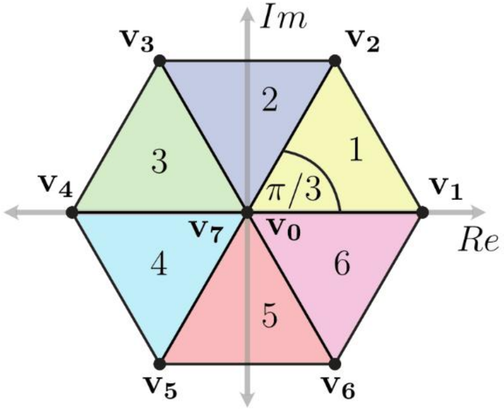
By doing the same procedure for the rest of the states, we can get the same voltage vectors.

State (S_a, S_b, S_c)	v_{aN}	v_{bN}	v_{cN}	Space vector voltage $v_r^{\alpha\beta} = \frac{2}{3}(v_{aN} + a v_{bN} + a^2 v_{cN})$
(0,0,0)	0	0	0	$\boldsymbol{v_0} = (0,0)$
(1,0,0)	$\frac{2}{3}v_{dc}$	$-\frac{1}{3}v_{dc}$	$-\frac{1}{3}v_{dc}$	$\boldsymbol{v_1} = \frac{2}{3}v_{dc}(1,0)$
(1,1,0)	$\frac{1}{3}v_{dc}$	$\frac{1}{3}v_{dc}$	$-\frac{2}{3}v_{dc}$	$\boldsymbol{v_2} = \frac{2}{3}v_{dc}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
(0,1,0)	$-\frac{1}{3}v_{dc}$	$\frac{2}{3}v_{dc}$	$-\frac{1}{3}v_{dc}$	$\boldsymbol{v_3} = \frac{2}{3}v_{dc}\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
(0,1,1)	$-\frac{2}{3}v_{dc}$	$\frac{1}{3}v_{dc}$	$\frac{1}{3}v_{dc}$	$\boldsymbol{v_4} = \frac{2}{3}v_{dc}(-1,0)$
(0,0,1)	$-\frac{1}{3}v_{dc}$	$-\frac{1}{3}v_{dc}$	$\frac{2}{3}v_{dc}$	$\boldsymbol{v_5} = \frac{2}{3}v_{dc}\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(1,0,1)	$\frac{1}{3}v_{dc}$	$-\frac{2}{3}v_{dc}$	$\frac{1}{3}v_{dc}$	$\boldsymbol{v_6} = \frac{2}{3}v_{dc}\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(1,1,1)	0	0	0	$\boldsymbol{v_7} = (0,0)$

Active Front End

Space Vector Modulation

The hexagon consider 6 sectors.



Consider a general case as shows the figure. It is possible to generate V_r **based on $\boldsymbol{v_i}$, $\boldsymbol{v_{i+1}}$, and $\boldsymbol{v_0}$.**

$$\Delta t_i = \frac{3}{2}\frac{|V_r|}{v_{dc}}\left(\cos(\alpha) - \frac{1}{\sqrt{3}}\sin(\alpha)\right)T_s$$

Based on the geometry, we get:

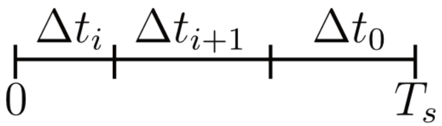
$$|\boldsymbol{x_i}| = |V_r|\cos(\alpha) - \frac{|V_r|\sin(\alpha)}{\tan\left(\frac{\pi}{3}\right)} \qquad |\boldsymbol{x_{i+1}}| = \frac{|V_r|\sin(\alpha)}{\sin\left(\frac{\pi}{3}\right)}$$

Besides, we can get the magnitude of vectors $\boldsymbol{x_i}$ and $\boldsymbol{x_{i+1}}$ based on the period where we will apply the corresponding voltage vector:

$$|\boldsymbol{x_i}| = |v_i| \cdot \frac{\Delta t_i}{T_s} \qquad |\boldsymbol{x_{i+1}}| = |v_{i+1}| \cdot \frac{\Delta t_{i+1}}{T_s} \qquad |v_i| = |v_{i+1}| = \frac{2v_{dc}}{3}$$

Using previous equations, the period to apply vectors $\boldsymbol{v_i}$ (Δt_i), $\boldsymbol{v_{i+1}}$ (Δt_{i+1}), and $\boldsymbol{v_0}$ (Δt_0) are:

$$\Delta t_i = \frac{3}{2}\frac{|V_r|}{v_{dc}}\left(\cos(\alpha) - \frac{1}{\sqrt{3}}\sin(\alpha)\right)T_s \qquad \Delta t_{i+1} = \sqrt{3}\frac{|V_r|}{v_{dc}}\sin(\alpha)T_s \qquad \Delta t_0 = T_s - (\Delta t_i + \Delta t_{i+1})$$



Active Front End

Exercise 2

Open the simulation “02_AFE_modulation”

For a CB-PWM modulator:

1. Study the simulation, in particular block ”Modulation”. What is the role of the its inputs? How are they generated?
2. Open subsystem “CB-PWM modulation”. Contrast the block diagram explained in class with the implemented one.

For a SVM:

1. Open block SVM and contrast the implementation with the explanation provided in class.

Once you undestand both modulator implementations:

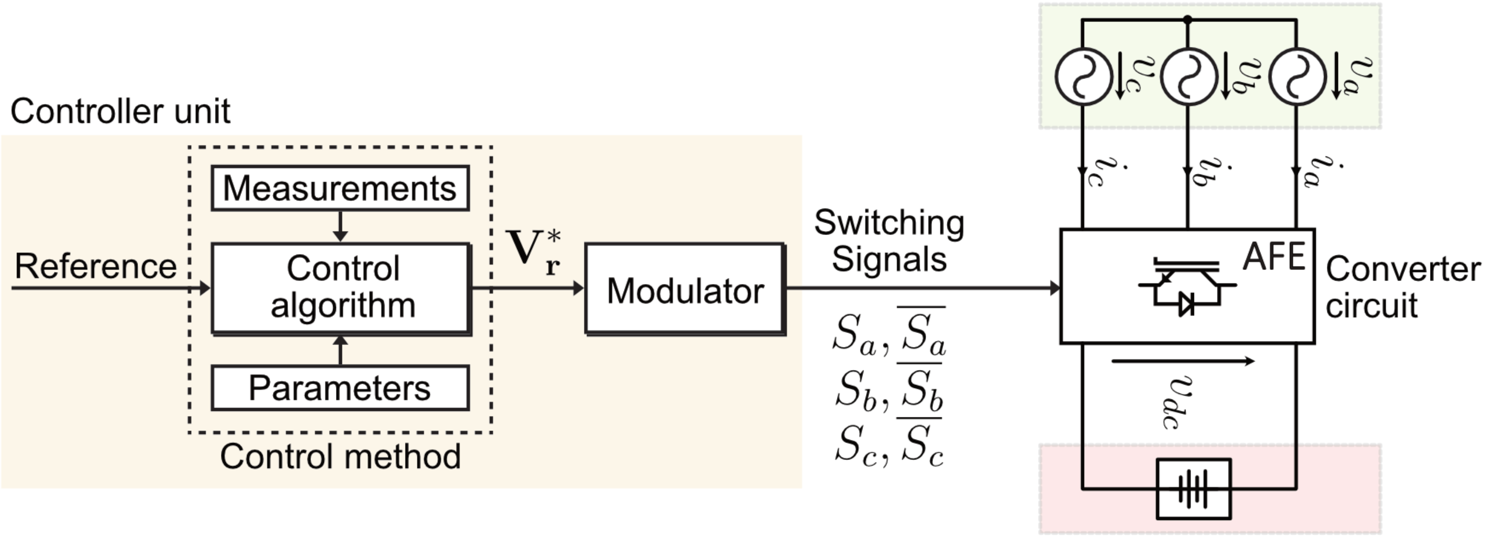
1. How can you select the modulation to apply?
2. Select a pair of values for “Vr_d_act” and “Vr_q_act” within [-200,200] volts. On steady-state, how can you determine the value of the current per phase?
3. Run the script provided (go to “Simulation -> Simulation script…”). What is doing this code?
   Suggestion: observe scope “Current_comp”.
4. Compare the performance of both modulator in time and frequency domain. Do you observe a difference? Why? Can you correct this?
5. Provide Vr_d_act= Vr_q_act=320V and compare performance of CB-PWM (without and with 3th harmonic injection) and SVM. Do you observe any difference? Which method do you prefer?

Control

Active Front End

Control architecture

One of the most common used control architectures for AFE converters is shown below



The controller unit under study is conformed by:

- A **control method** unit, which implements a **control algorithm** based on **measurements** and **parameters** to provide **an actuation** capable (based on the converter capability) to follow the **reference** (one or multiple).
- A **Modulator** unit.

We will study *two controller structures*: a *PI-based control* and *Model Predictive Control*.

Active Front End
PID Control in continuous time

This control algorithm generates an actuation as a linear function of the error between the reference and measurement

$$u(t) = k_p \cdot e(t) + \frac{k_p}{T_i}\int e(t)dt + k_p T_d \cdot \frac{d}{dt}(e(t))$$

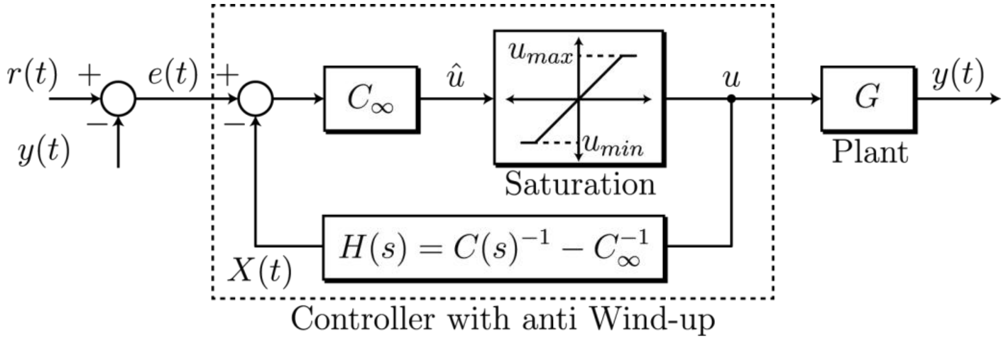
Note: Controller parameters are k_p, T_i, and T_d.

Using Laplace Transform, and neglecting the derivative part (not needed for the system to control) the transfer function that model actuation (u) and error (e) is:

$$\frac{u(s)}{e(s)} = C(s) = k_p\left(1 + \frac{1}{T_i s}\right)$$

Considering that the actuation have a limited capability (e.g., limited voltage to apply), the controller structure considers:

1. A saturation unit to constrain the maximum actuation.
2. An anti wind-up structure to avoid integration of constant error, where $C_\infty = k_p$.



This is known as a *PI-based controller with anti wind-up*.

Active Front End

PI Control in discrete time

Microcontroller units works in discrete time at specific sampling and switching frequencies.

For simplicity, switching and sampling time are equal.

One method to model a continuous time system is by approximating it in discrete time, for instance using the following methods:

Euler method	Tustin method
$s \sim \frac{1 - z^{-1}}{T_s}$	$s \sim \frac{2}{T_s} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$

These methods can be used to get the approximation in discrete states for fundamental linear operations from s-domain. For instance, using Euler method in derivatives or an integrator:

Examples	Continuous time	Laplace domain	Discrete time
Derivative	$u(t) = \frac{d}{dt}(e(t))$	$u(s) = s \cdot (\, e(s)\,)$	$u(k) \sim \frac{e(k) - e(k-1)}{T_s}$
Integration	$u(t) = \int e(t)dt$	$u(t) = \frac{e(t)}{s}$	$u(k) \sim u(k-1) + e(k) \cdot T_s$

For example, using Euler method in the transfer function C(s), we obtain:

$$\frac{u(s)}{e(s)} = C(s) = k_p\left(1 + \frac{1}{T_i s}\right) \rightarrow u(k) = \underbrace{k_p \cdot e(k)}_{\text{Proportional part}} + \underbrace{u(k-1) + \frac{k_p}{T_i} \cdot T_s \cdot e(k)}_{\text{Integral part}}$$

Note: Similar calculation is required for H(s) for the anti wind-up controller or any transfer function in s-domain.

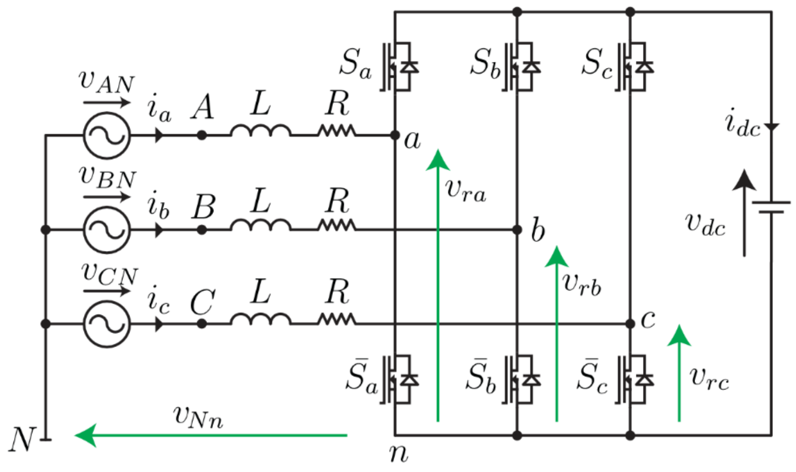
Active Front End

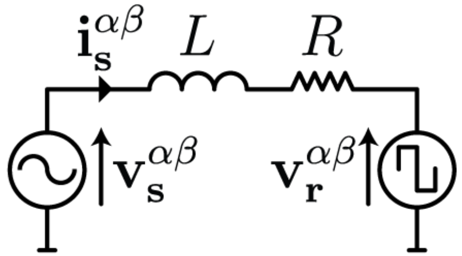
Control method: Identify system variables

The control structure is based on the modeling of the system of interest.

For the AFE converter, grid currents are a function of grid and converter voltages. In this sense:

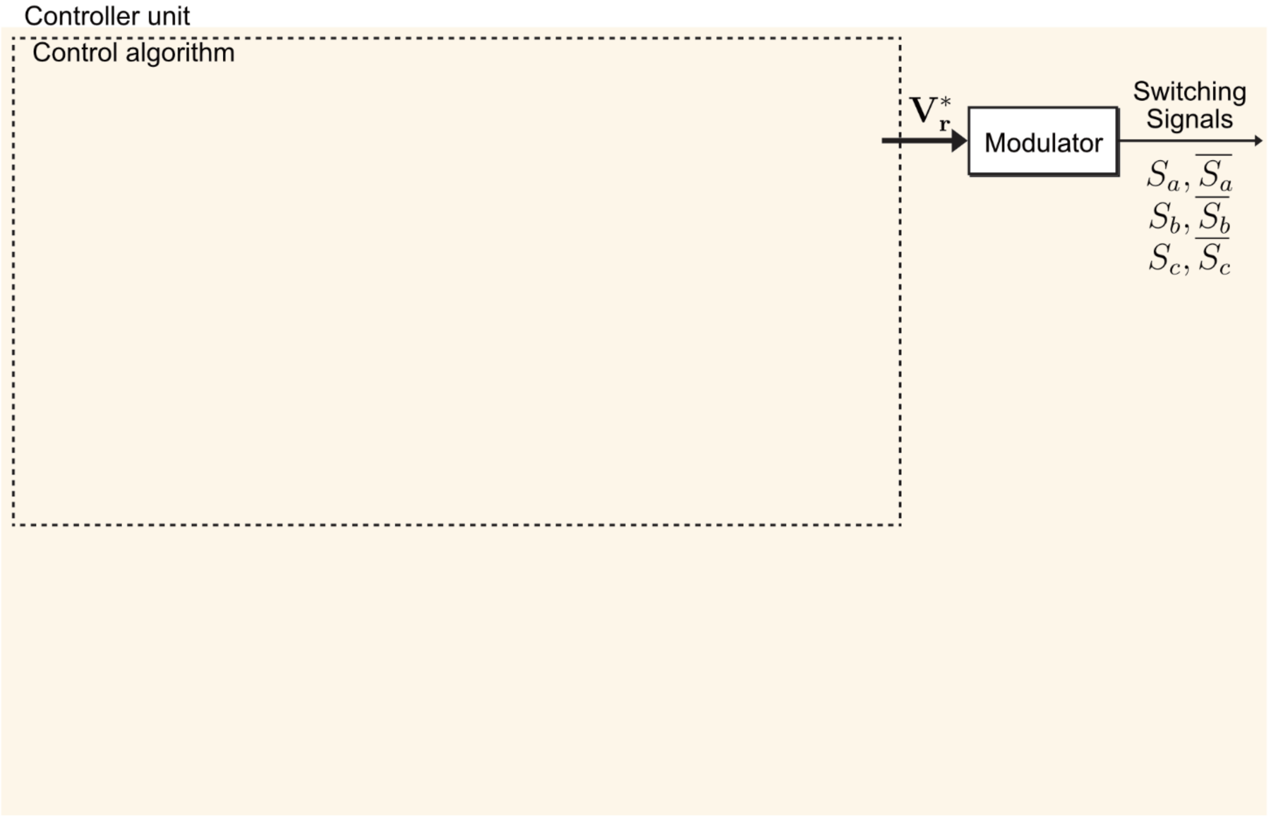
- Converter voltage $\boldsymbol{v_r}$ is the **actuation**
- Grid current $\boldsymbol{i_s}$ are the **output** and **reference** to control
- Grid voltage $\boldsymbol{v_s}$ is a **perturbation**.

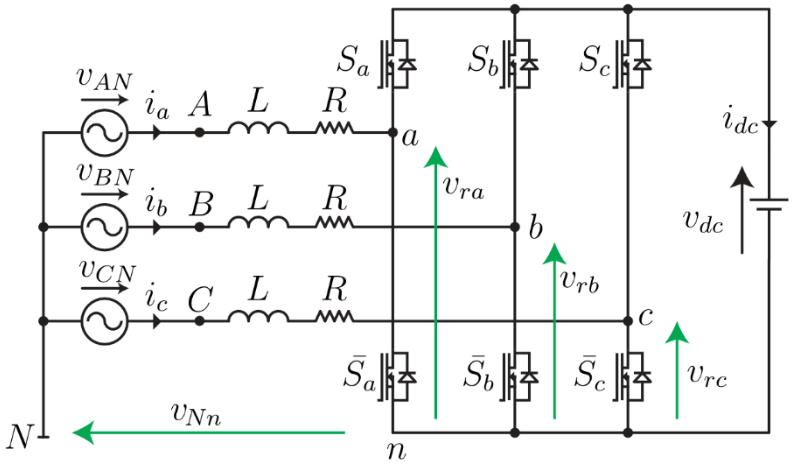


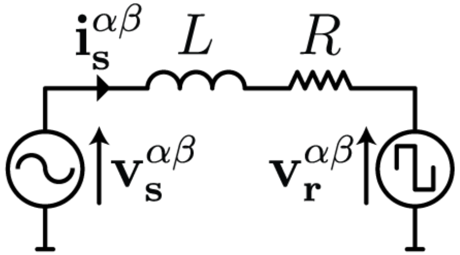


Active Front End

Control structure: Modulator and control algorithm





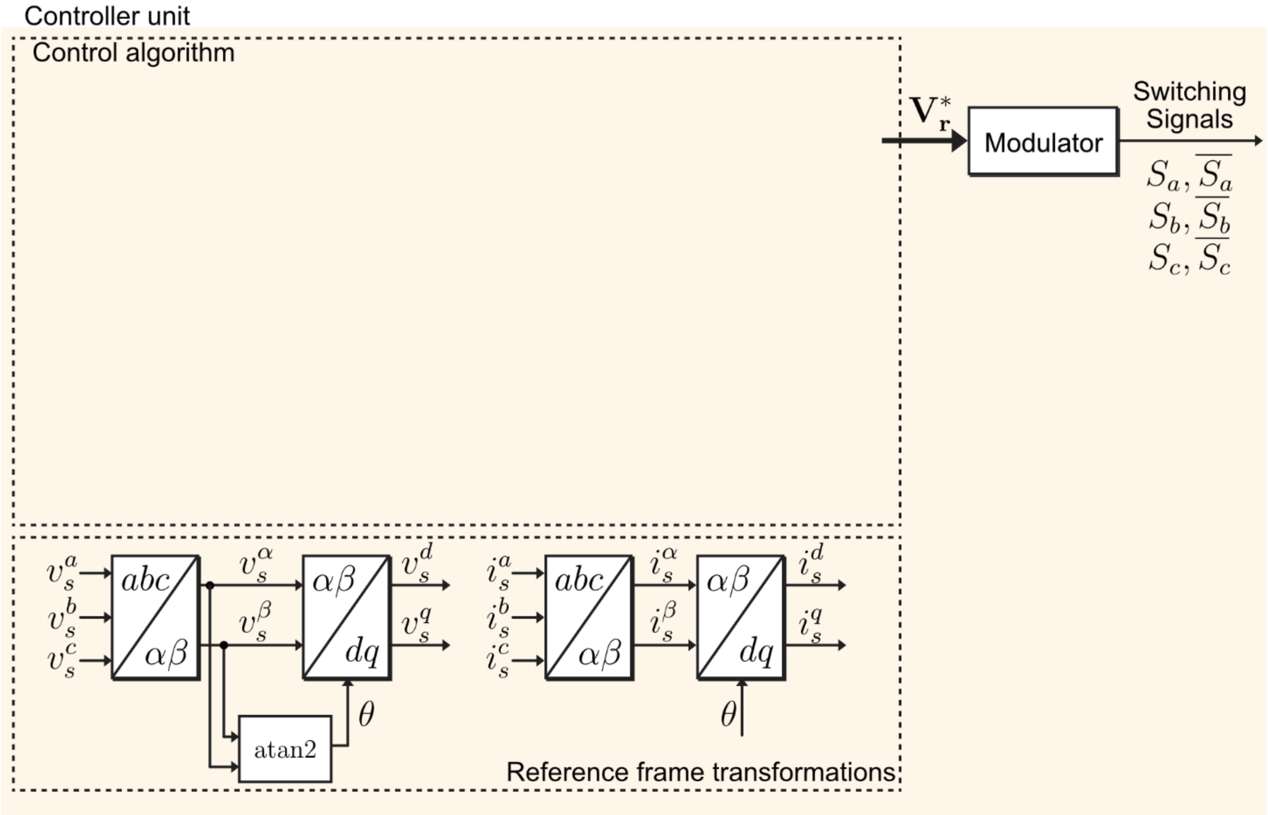


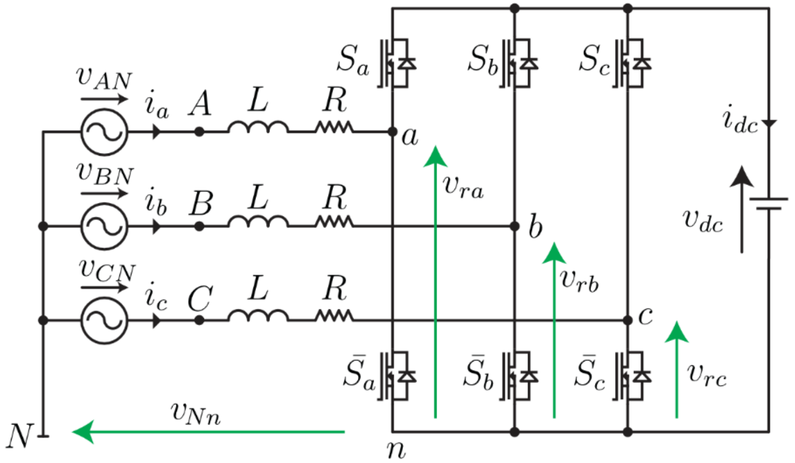
Note that:

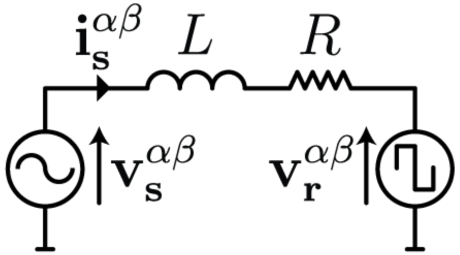
- The output of the control algorithm is the input of the modulator unit

Active Front End

Control structure: reference frame orientation





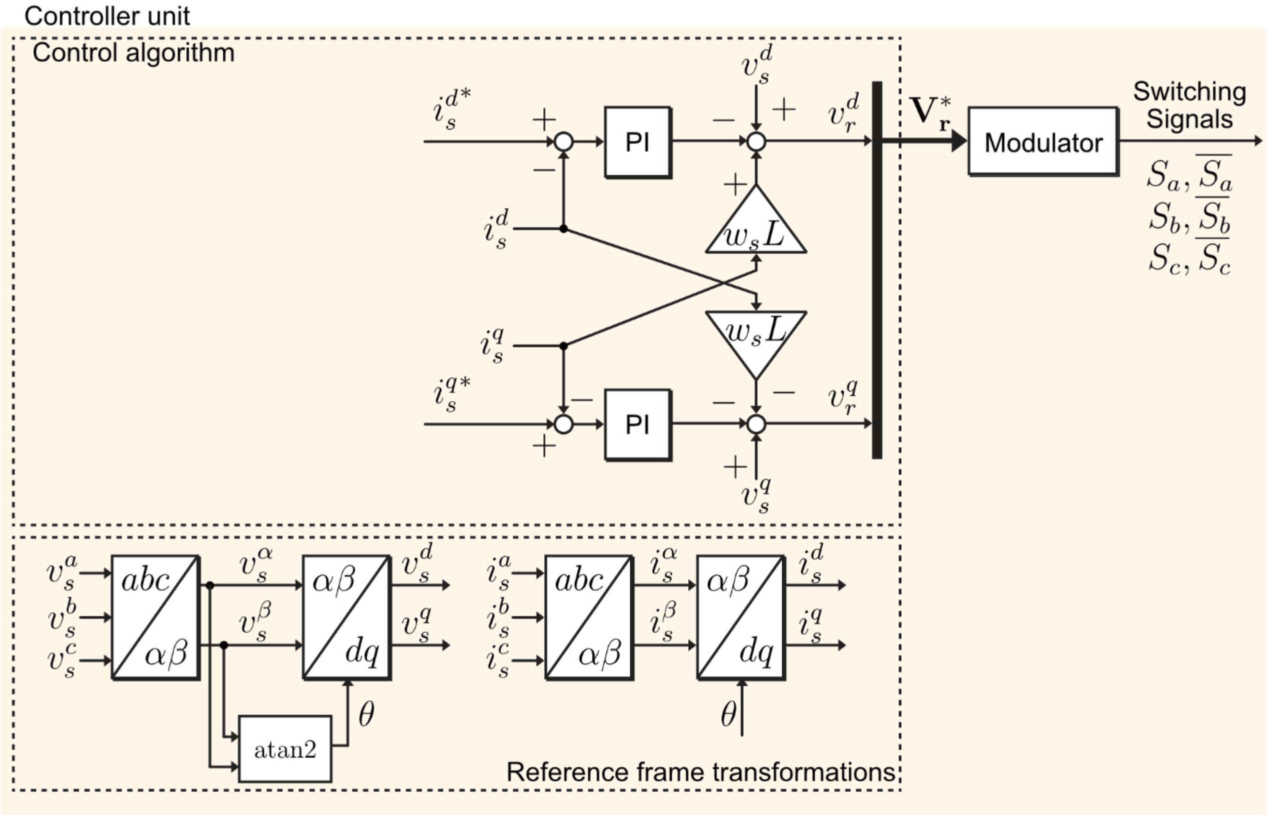


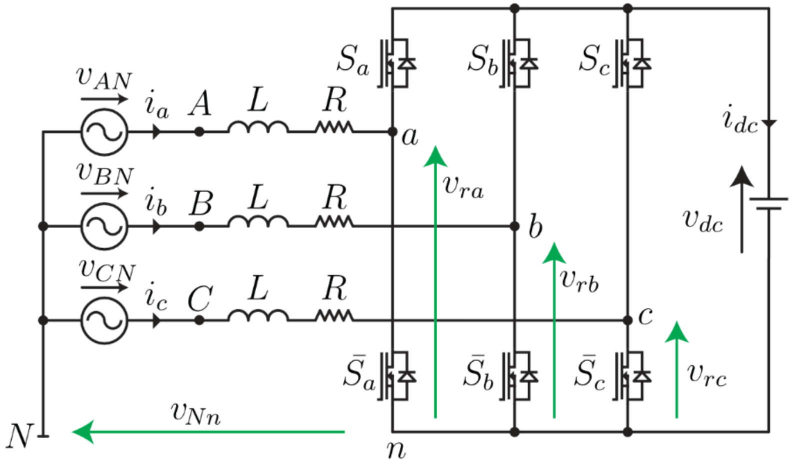
Note that:

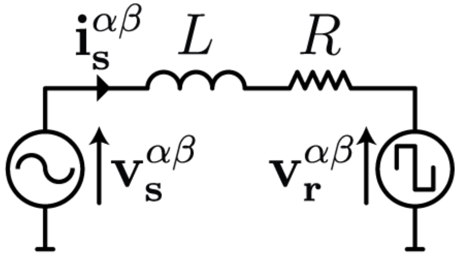
- Measurements on each phase (phase voltages and currents) are used to calculate its equivalent in stationary ($\alpha\beta$) and rotating (dq) frame, using estimation of voltage phase (θ).

Active Front End

Control structure: PI current controller







Using model equations

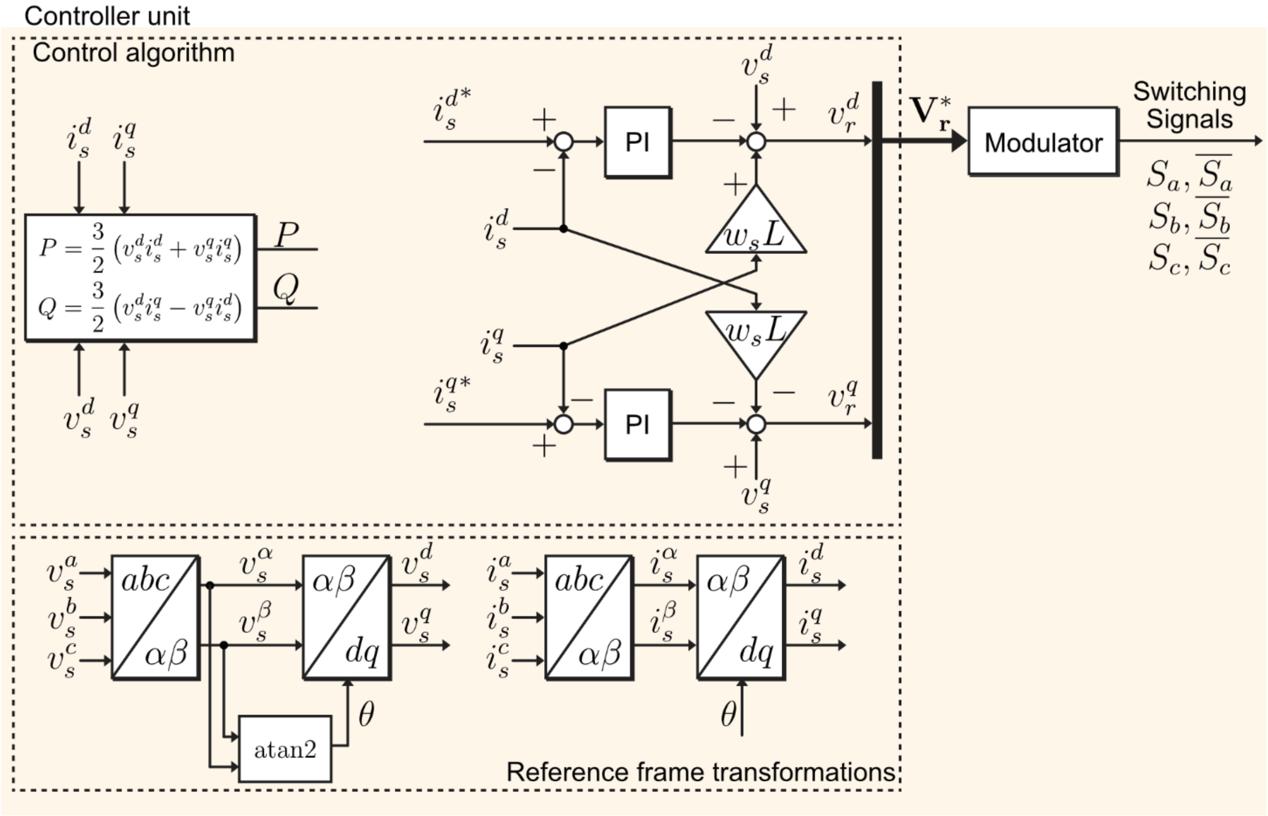
$$\begin{cases} v_s^d = \boxed{L \cdot \frac{d}{dt} i_s^d + R \cdot i_s^d} - wL \cdot i_s^q + v_r^d \\ v_s^q = \boxed{L \cdot \frac{d}{dt} i_s^q + R \cdot i_s^q} + wL \cdot i_s^d + v_r^q \end{cases}$$

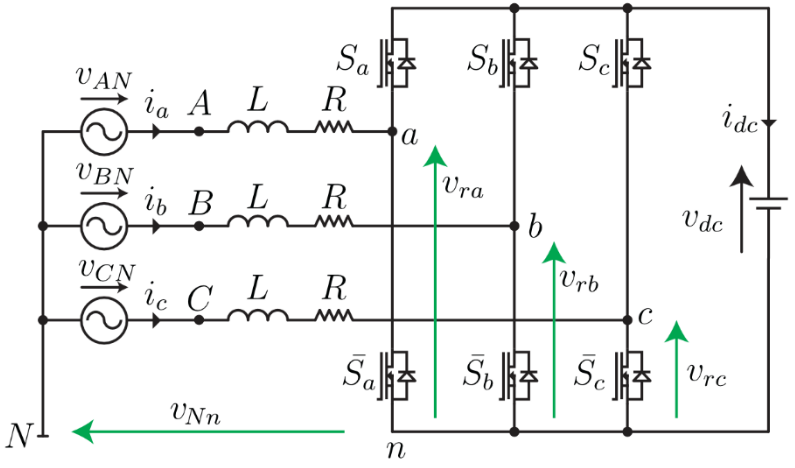
Note that:

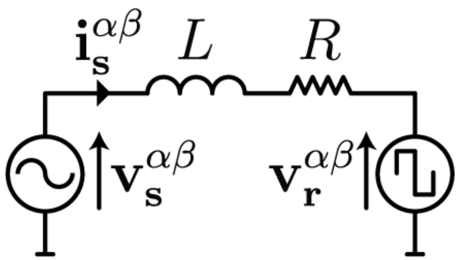
- Voltage drop that PI controls is inside red square.
- Inversely proportional relationship between $\boldsymbol{v_r}$ and $\boldsymbol{i_s}$

Active Front End

Control structure: Power estimation





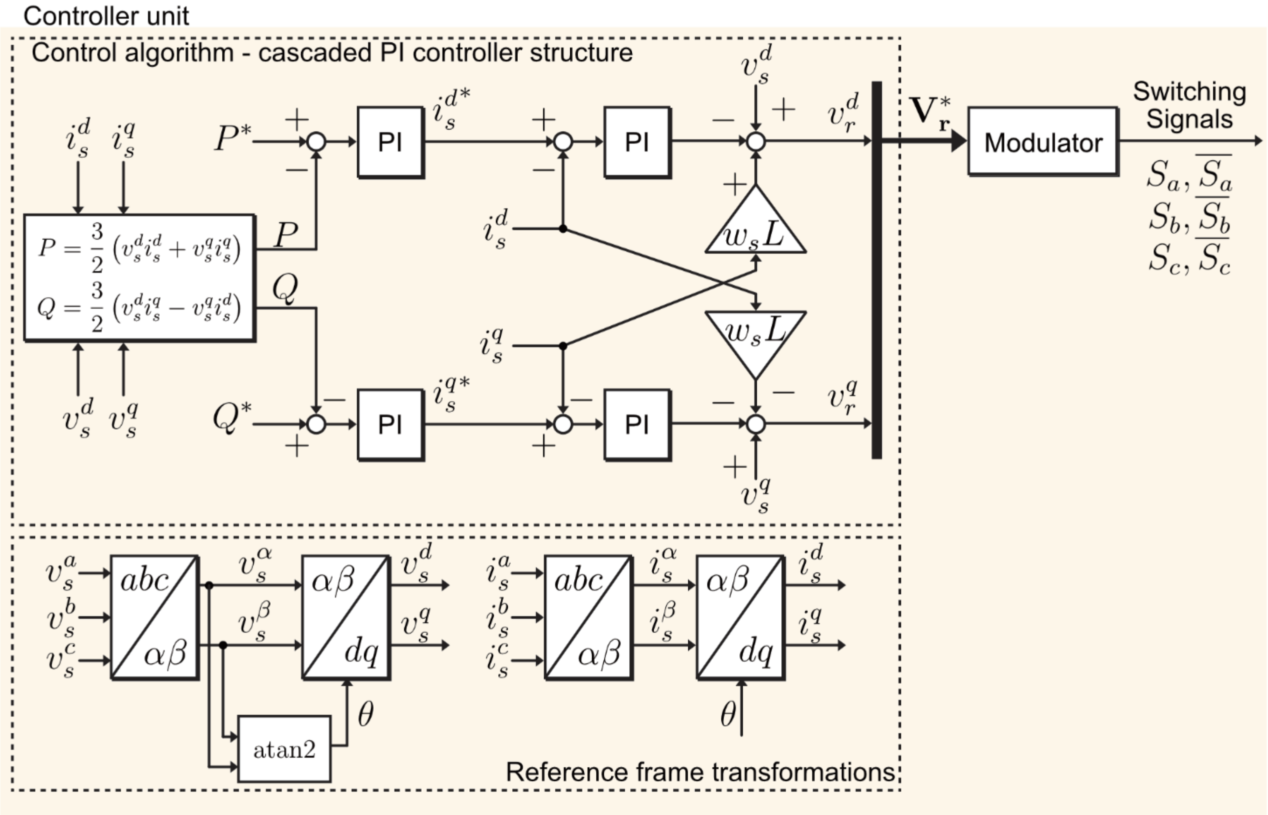


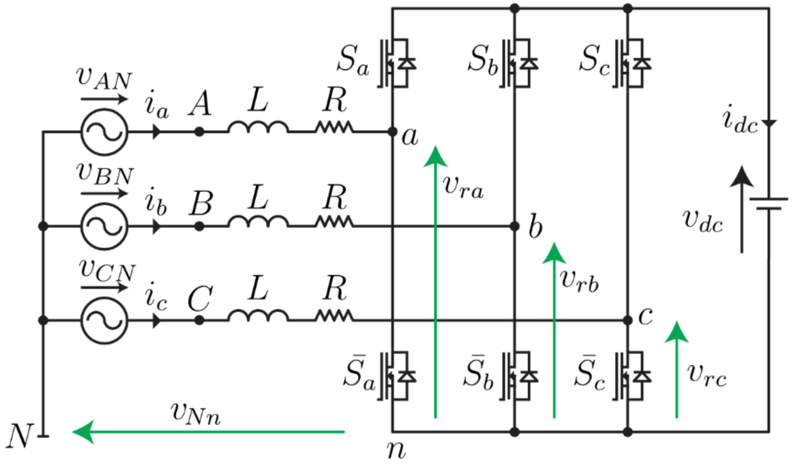
Note that:

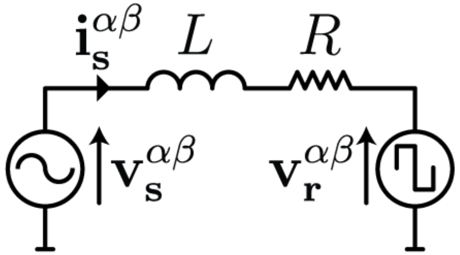
- Power is calculated in the rotating reference frame, but it can be also used stationary or abc frames.

Active Front End

Control structure: Cascaded PI power controller







Note that:

- Dynamic inner loop (current) should be faster than outer controller to avoid dynamic coupling.

Active Front End

Exercise 3.1 Linear controller

Open the simulation “03_A_AFE_linear_controller”

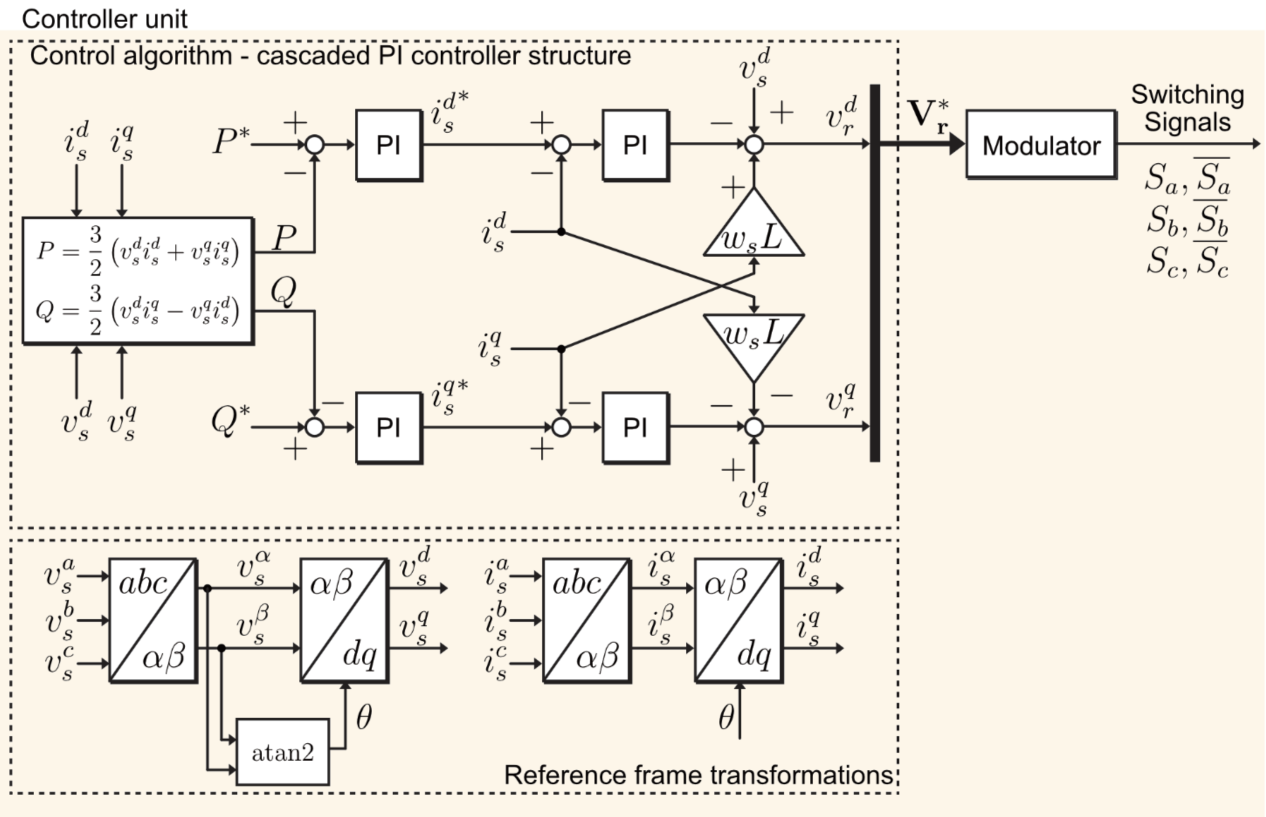
In this simulation, a PI-based current controller structure is provided. Open the block “Controller”:

- Contrast the implementation of each controller with the control algorithm explained in the class. What is the function of:
  - “Kp_i” and “Ti_i”?
  - “u_max_i”, “u_min_i”? Are these values correct?

TODO: Implement a power controller based on the controller structure in the Figure. Evaluate on time-domain the performance.

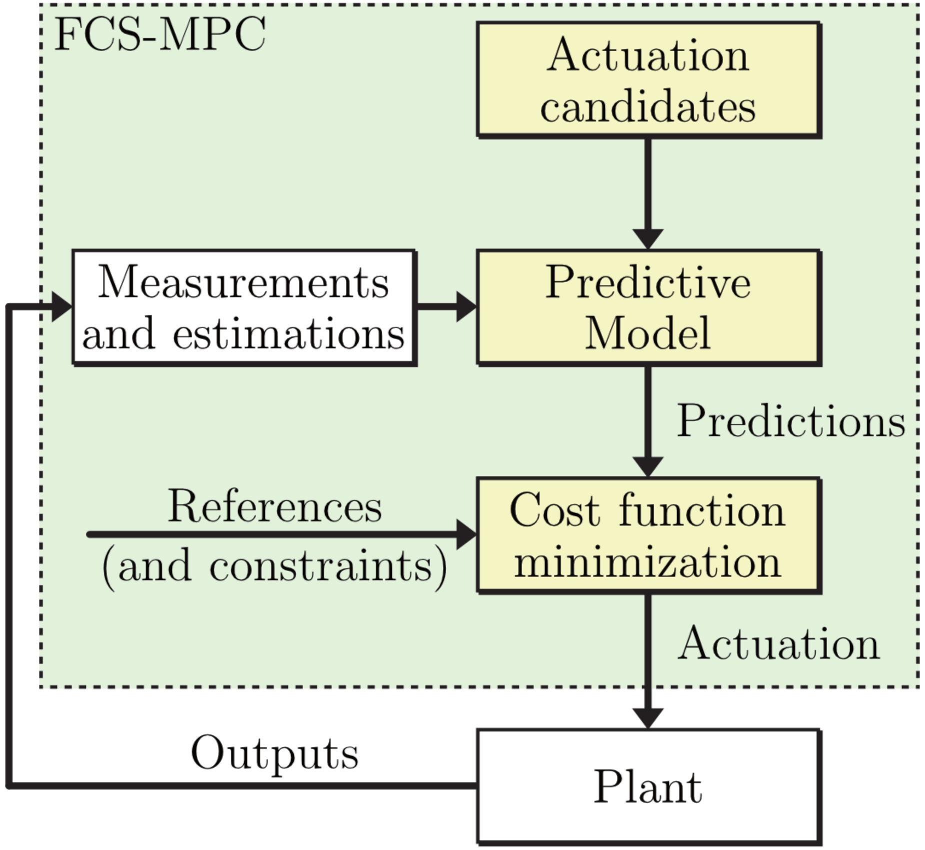
Use the global variable “power_control” to enable this mode (0: current control, 1: power control).

Check the performance using “Scope – power control”.



Active Front End

Predictive Control: Finite Control Set – Model Predictive Control (FCS-MPC)



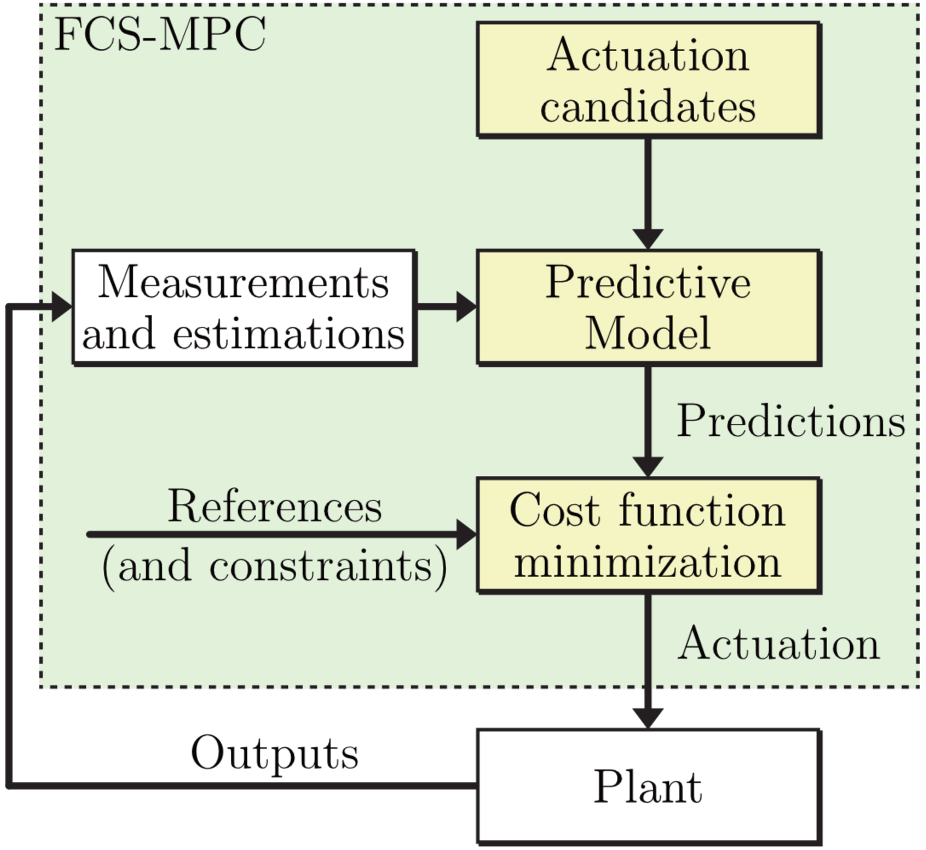
For a **Finite Set** of *n* possible control actuation candidates (**$\boldsymbol{u}$**)

Using a mathematical model of the system, a set of **prediction** of the controlled variables are computed for each actuation candidate.

A **cost function** evaluates these predictions to identify the **optimal control actuation** that minimizes it, which is then applied at the next sampling period.

Active Front End

Predictive Control: Finite Control Set – Model Predictive Control (FCS-MPC)



Advantages:

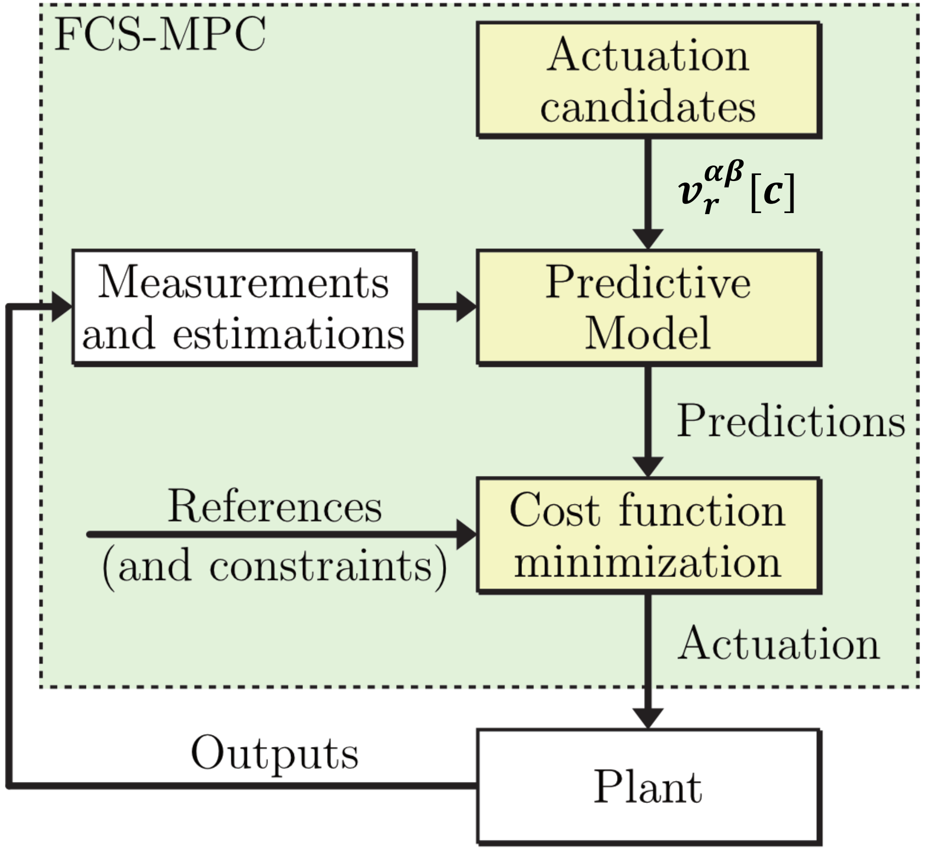
- Concept is straightforward and intuitive.
- Applicable to a wide range of systems.
- Multiple variables can be effectively controlled.
- Nonlinearities in the system model can be naturally incorporated.
- System constraints are easily integrated into the control design.
- Easy to implement (program) on microcontroller uC unit.

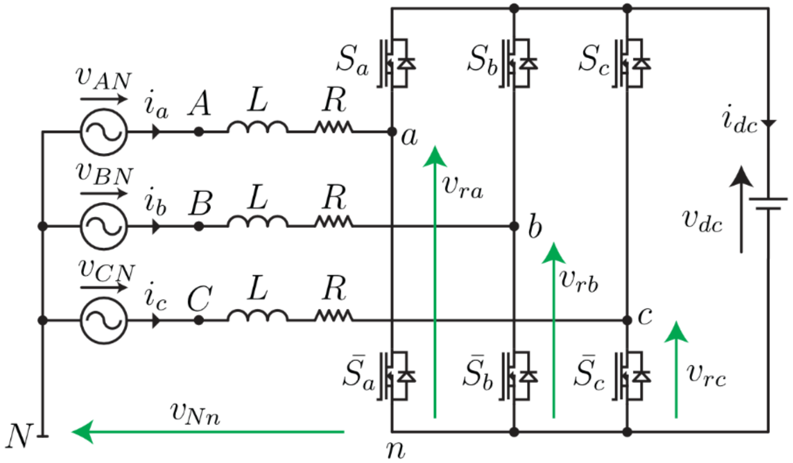
Disadvantages:

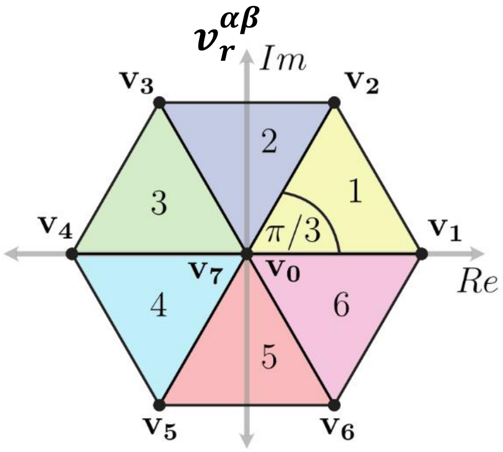
- The controller performance highly depends on the model and parameters.
- High computational burden compared with classic linear controllers

Active Front End

Example of FCS-MPC for AFE converter: Actuation candidates





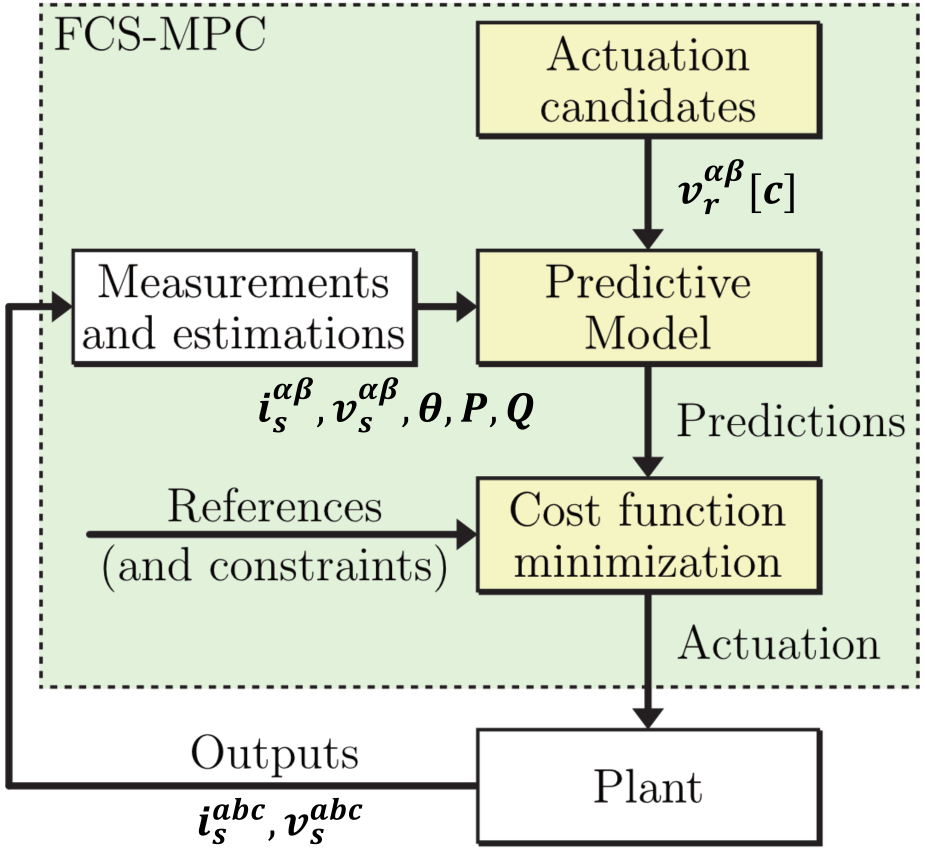


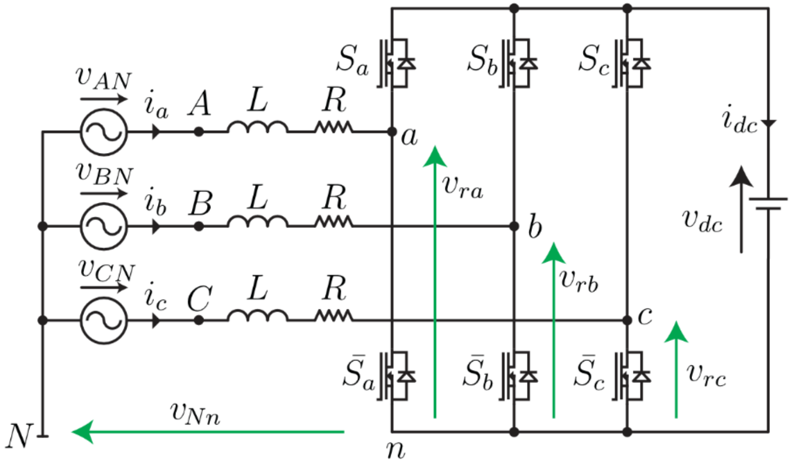
We can consider the 8 switching states for the voltage vector of actuation candidates ($\boldsymbol{v}_r^{\alpha\beta}[\boldsymbol{c}] = \{\boldsymbol{v}_0, \boldsymbol{v}_1, \dots, \boldsymbol{v}_7\}$).

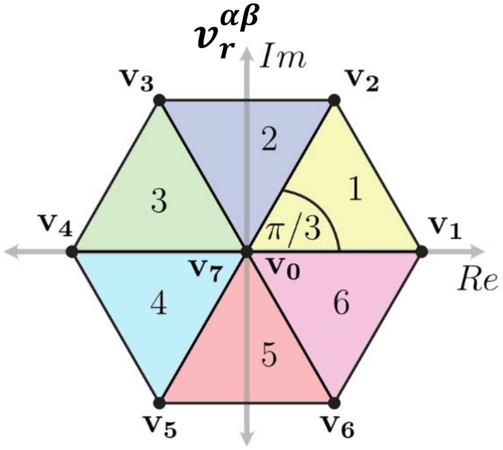
Note that, by doing this, we do not need to use a modulator.

Active Front End

Example of FCS-MPC for AFE converter: Measurements and estimations



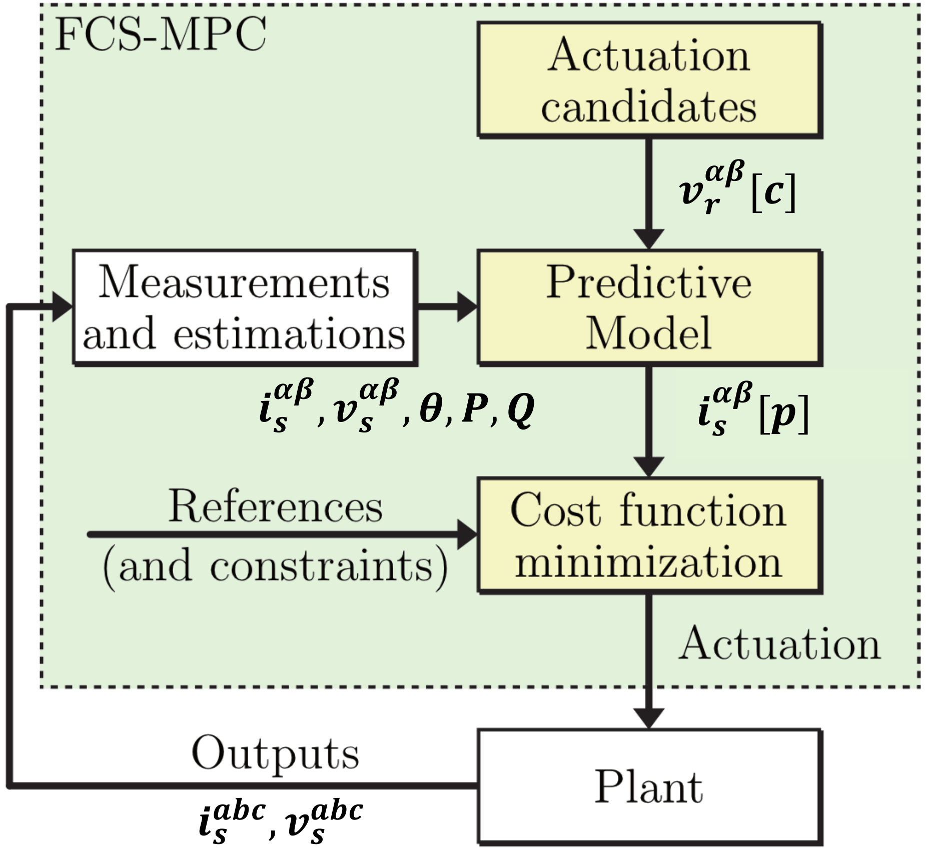


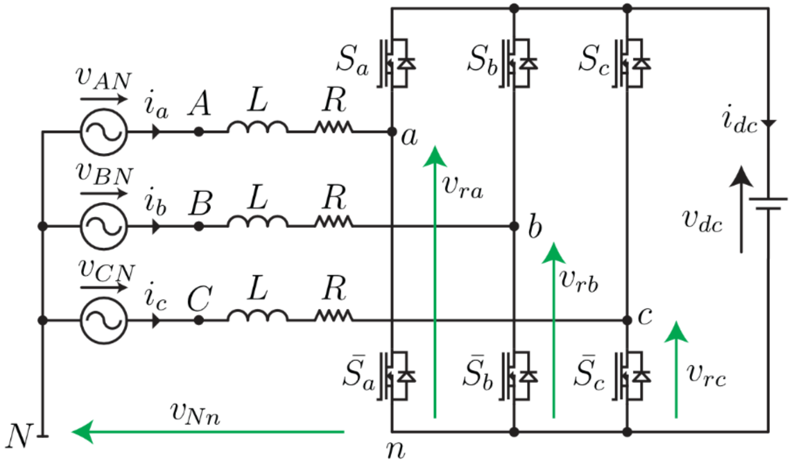


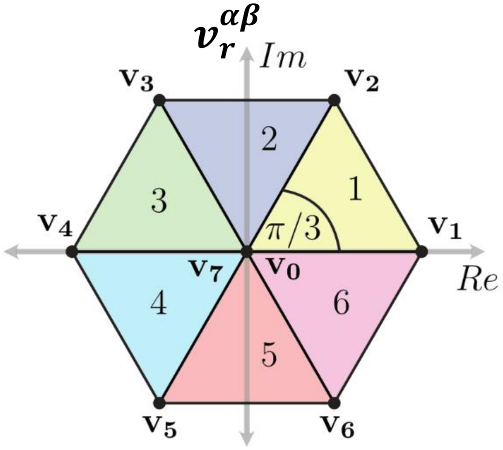
By measuring $\boldsymbol{i_s^{abc}}, \boldsymbol{v_s^{abc}}$, we can calculate $\boldsymbol{i_s^{\alpha\beta}}, \boldsymbol{v_s^{\alpha\beta}}$ and $\boldsymbol{\theta}$, together with active and reactive power $\boldsymbol{P}$ and $\boldsymbol{Q}$.

Active Front End

Example of FCS-MPC for AFE converter: Predictive Current Model







We want to control P and Q. To control them, we can use the current model. By applying Euler approximation:

$$\frac{d\boldsymbol{x}}{dt} \sim \frac{x(k+1) - x(k)}{T_s} \quad \text{in equation} \quad \boldsymbol{v}_s^{\alpha\beta} = L \cdot \frac{d\boldsymbol{i}_s^{\alpha\beta}}{dt} + R \cdot \boldsymbol{i}_s^{\alpha\beta} + \boldsymbol{v}_r^{\alpha\beta}$$

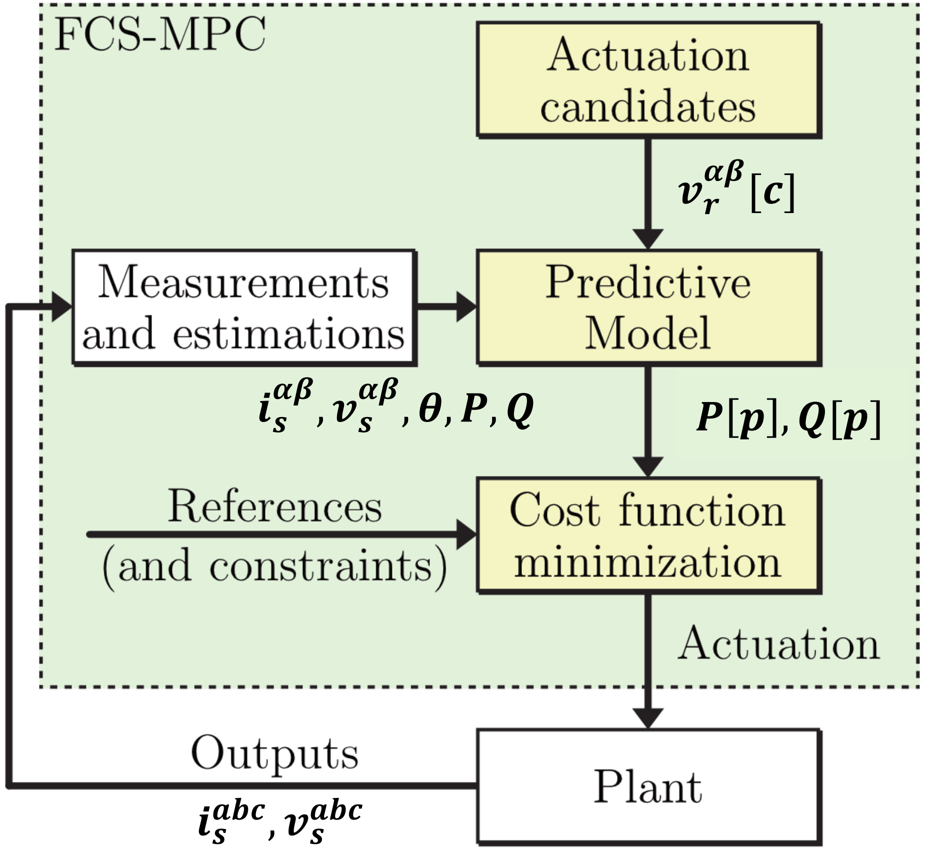
A predictive current model is derived below:

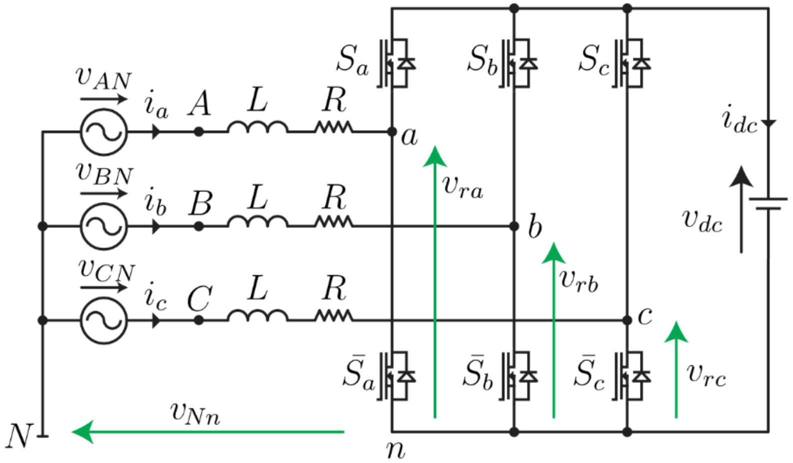
$$\boldsymbol{i}_s^{\alpha\beta}(k+1) = \boldsymbol{i}_s^{\alpha\beta}(k) \cdot \left(1 - \frac{RT_s}{L}\right) + \left(\boldsymbol{v}_s^{\alpha\beta} + \boldsymbol{v}_r^{\alpha\beta}\right) \cdot \frac{T_s}{L}$$

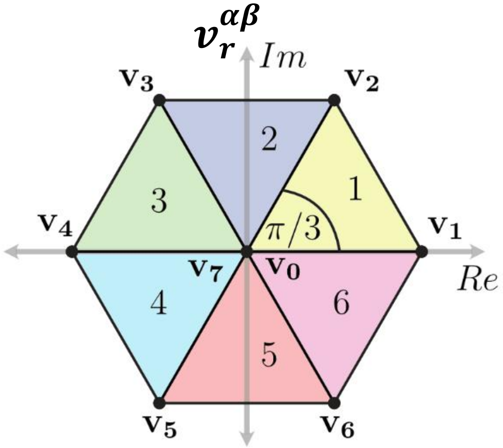
This block generates 8 predictions $\boldsymbol{i}_s^{\alpha\beta}[\boldsymbol{p}](k+1) = [\boldsymbol{i}_s^0, \dots, \boldsymbol{i}_s^7]$ for each $\boldsymbol{v}_r^{\alpha\beta}[\boldsymbol{c}]$

Active Front End

Example of FCS-MPC for AFE converter: Predictive Power Model







Assuming that the grid voltages will not change significantly between consecutive sampling periods:

$$\boldsymbol{v_s^{\alpha\beta}}(\mathrm{k}+1) \sim \boldsymbol{v_s^{\alpha\beta}}(\mathrm{k})$$

The active and reactive power predictions can be estimated:

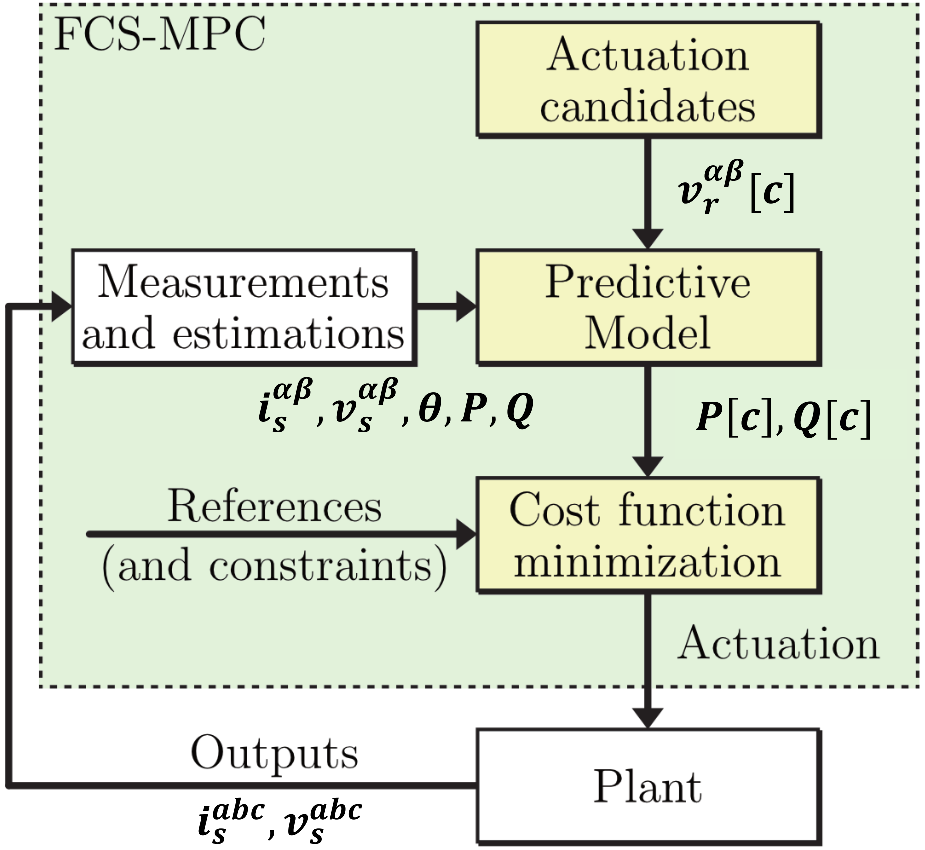
$$\mathrm{P}(\mathrm{k}+1) \sim \frac{3}{2}\left(v_s^{\alpha}(\mathrm{k}) \cdot i_s^{\alpha}(\mathrm{k}+1) + v_s^{\beta}(\mathrm{k}) \cdot i_s^{\beta}(\mathrm{k}+1)\right)$$

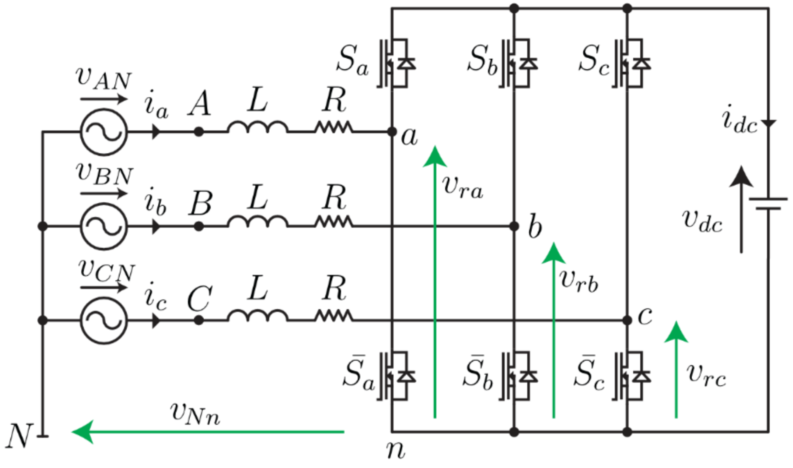
$$\mathrm{Q}(\mathrm{k}+1) \sim \frac{3}{2}\left(v_s^{\alpha}(\mathrm{k}) \cdot i_s^{\beta}(\mathrm{k}+1) - v_s^{\beta}(\mathrm{k}) \cdot i_s^{\alpha}(\mathrm{k}+1)\right)$$

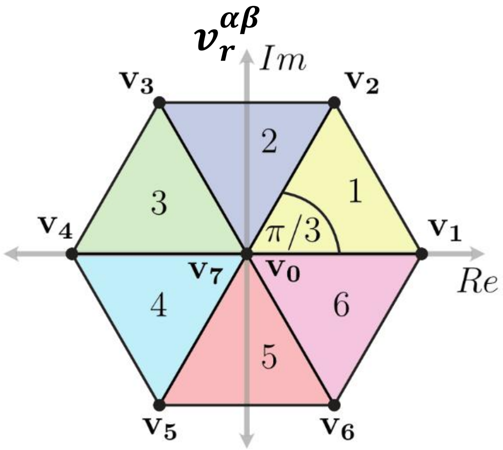
This block generates 8 predictions $\boldsymbol{P}[\boldsymbol{p}](k+1)$ and $\boldsymbol{Q}[\boldsymbol{p}](k+1)$ for each $\boldsymbol{v_r^{\alpha\beta}}[\boldsymbol{c}]$

Active Front End

Example of FCS-MPC for AFE converter: Cost function minimization







Note: Cost function is positive defined.

We define a cost function to control the active and reactive power as:

$$g = (P^* - P^c)^2 + (Q^* - Q^c)^2$$

We will select the actuation that minimizes the cost function:

$$v_r^{\alpha\beta}(k+1) = \underset{v_r^{\alpha\beta}[c]}{arg\,min}\ g$$

The selected actuation is applied at the next sampling period.

Active Front End

Exercise 3.2 FCS-MPC method

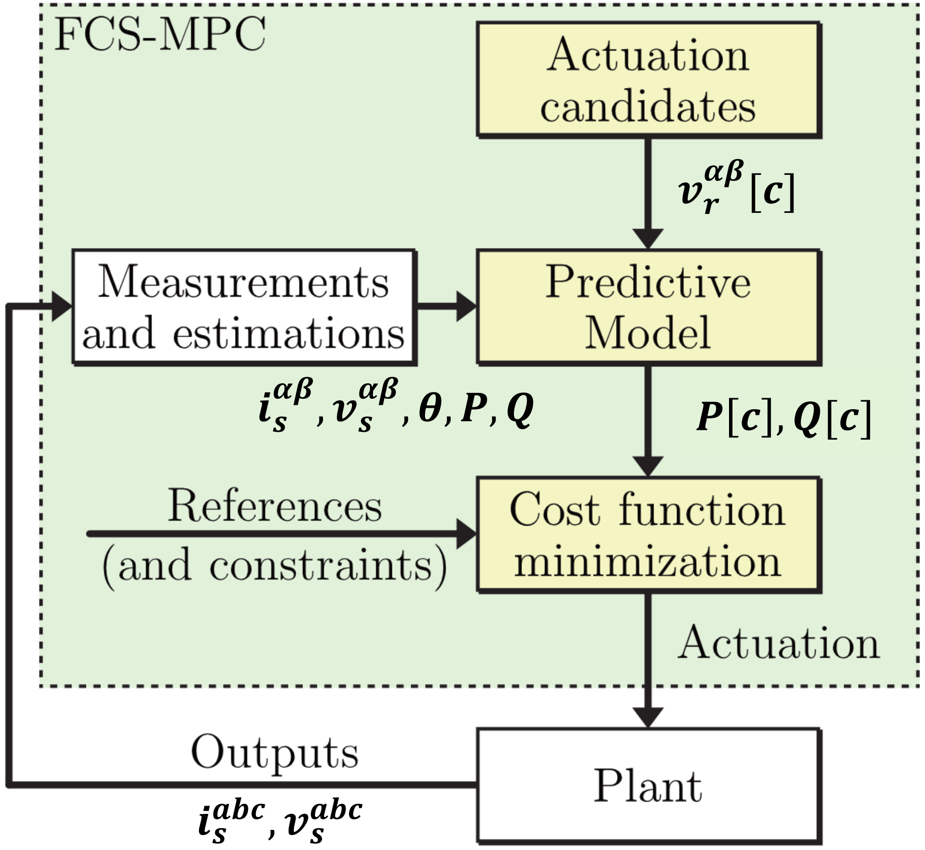
Open the simulation “03_AFE_Control”

TODO: Program an FCS-MPC structure to control of active and reactive power P and Q. Operate at $f_{sw} = 50kHz$.

Hints: Inside the unit “controller” implement the control algorithm:

- Check if you have all the required inputs for modeling.
- Consider at least two user inputs:
  - Variable “controller_mode” allows you to select the controller to use (PI or FCS-MPC).
  - Variable “mode” let you choose between current or power control.

TODO: Contrast the performance of FCS-MPC with PI-controller. What can you observe?



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Thank you for your attention