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Homework 2

COSC 3100
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Problem 1

Determine with proof the asymptotic relationship using Landau notation of the following pairs of functions:

1. $log_e(n)$ and $log_2(n)$
   Proof:

$$\text{Given: } c_1 log_e(n) \leq log_2(n) \leq c_2 log_e(n) \text{ for } n \geq n_0 \tag{1}$$

$$\text{Let } n_0 = 4 \text{ and } c_1 = 1 \text{ and } c_2 = 5 \tag{2}$$

$$log_e(n) \leq log_2(n) \leq 5 log_e(n) \text{ for } n \geq 4 \tag{3}$$

$$\therefore log_2(n) = \Theta(log_e(n)) \text{ for } n \to \infty \tag{4}$$

2. $log_2(log_2(n))n$ and $log_2(n)^2 n$
   Proof:

$$\text{Given: } 0 \leq log_2(log_2(n))n \leq c log_2(n)^2 n \text{ for } n \geq n_0 \tag{5}$$

$$\text{Let } n_0 = 5 \text{ and } c = 6 \tag{6}$$

$$\text{Then } 0 \leq log_2(log_2(n))n \leq 6 log_2(n)^2 n \text{ for } n \geq 5 \tag{7}$$

$$\therefore log_2(log_2(n))n = O(log_2(n)^2 n) \text{ for } n \to \infty \tag{8}$$

3. $\sqrt{n}$ and $log_2(n)$
   Proof:

$$\text{Given: } \lim_{x \to \infty} \frac{log_2(n)}{\sqrt{n}} \tag{9}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{2}\sqrt{n}}{n ln(2)} \tag{10}$$

$$= 0 \tag{11}$$

$$\therefore log_2(n) = \Theta(\sqrt{n}) \text{ for } n \to \infty \tag{12}$$

Problem 2

Given a NFA we must determine the states of the DFA by taking subsets of the NFA. We do this by starting at the starting state 'S' and form subsets of those states transitioned to upon an inuput of 0 or 1 (in this specific NFA). Once these subsets are formed as shown below we can determine that any subset containing either state 'B' or 'D' is accepting, as these are accepting states in the NFA.

Homework 2

	0	1
{}	{}	{}
{S}	{S,A}	{S,C}
{S,A}	{S,A,B}	{S,C}
{S,C}	{S,A}	{S,C,D}
{S,A,B}	{S,A,B}	{S,B}
{S,C,D}	{S,A,D}	{S,C,D}
{S,B}	{S,A,B}	{S,C,B}
{S,A,D}	{S,A,B,D}	{S,C,D}
{S,C,B}	{S,A,B}	{S,C,B,D}
{S,A,B,D}	{S,A,B,D}	{S,C,B,D}
{S,C,B,D}	{S,A,B,D}	{S,C,B,D}

Figure 1: Resulting DFA Table

Problem 3

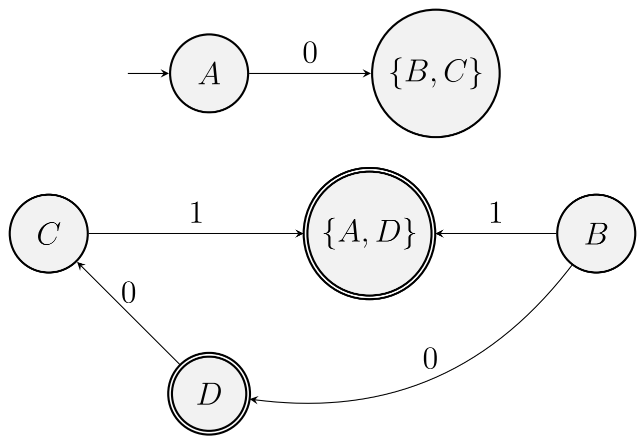

Figure 2: Resulting NFA without epsilon moves

	0	1
A	{B,C}	∅
B	{D}	{A,D}
C	∅	{A,D}
D	{C}	∅

Figure 3: Resulting NFA Table