Name: Daniel Throop
MUID: 006165173

Homework 2

COSC 3100
February 15, 2021

Problem 1

Determine with proof the asymptotic relationship using Landau notation of the following pairs of functions:

1. $\log_e(n)$ and $\log_2(n)$
   Proof:

$$\lim_{n\to\infty} \frac{log_e(n)}{log_2(n)} \tag{1}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{nln(2)}} \tag{2}$$

$$\lim_{n\to\infty} \frac{nln(2)}{n} = ln(2) \tag{3}$$

$$\therefore log_e(n) = \Theta(log_2(n)) \text{ for } n \to \infty \tag{4}$$

2. $\log_2(\log_2(n))n$ and $\log_2(n)^2 n$
   Proof:

$$\lim_{n\to\infty} \frac{log_2(log_2(n))}{log_2(n)^2} \tag{5}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n\ln(n)\ln(2)}}{\frac{2\ln(n)}{(\ln(2))^2 n}} \tag{6}$$

$$\lim_{n\to\infty} \frac{\ln(2)}{2\left(\ln(n)\right)^2} = 0 \tag{7}$$

$$\therefore log_2(log_2(n))n = o(log_2(n)^2 n) \text{ for } n \to \infty \tag{8}$$

3. $\sqrt{n}$ and $\log_2(n)$
   Proof:

$$\text{Given: } \lim_{x\to\infty} \frac{log_2(n)}{\sqrt{n}} \tag{9}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{2}\sqrt{n}}{nln(2)} \tag{10}$$

$$= 0 \tag{11}$$

$$\therefore log_2(n) = \Theta(\sqrt{n}) \text{ for } n \to \infty \tag{12}$$

Problem 2

Given a NFA we must determine the states of the DFA by taking subsets of the NFA. We do this by starting at the starting state 'S' and form subsets of those states transitioned to upon an inuput of 0 or 1 (in this specific NFA). Once these subsets are formed as shown below we can determine that any subset containing either state 'B' or 'D' is accepting, as these are accepting states in the NFA.

	0	1
{}	{}	{}
{S}	{S,A}	{S,C}
{S,A}	{S,A,B}	{S,C}
{S,C}	{S,A}	{S,C,D}
{S,A,B}	{S,A,B}	{S,B}
{S,C,D}	{S,A,D}	{S,C,D}
{S,B}	{S,A,B}	{S,C,B}
{S,A,D}	{S,A,B,D}	{S,C,D}
{S,C,B}	{S,A,B}	{S,C,B,D}
{S,A,B,D}	{S,A,B,D}	{S,C,B,D}
{S,C,B,D}	{S,A,B,D}	{S,C,B,D}

Figure 1: Resulting DFA Table

Problem 3

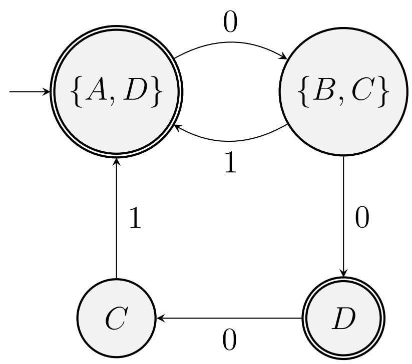

Figure 2: Resulting NFA without epsilon moves

	0	1
{A,D}	{B,C}	∅
{B,C}	{D}	{A,D}
C	∅	{A,D}
D	{C}	∅

Figure 3: Resulting NFA Table

Epsilon Closures
E(A) ={A,D}
E(B) = {B,C}
E(C) = {C}
E(D) = {D}