

Problem 1

Following the procedure of the book, end of Chapter 9, prove that the existence of a constant $c > 0$ such that $T(n) \leq cn$.

Proof:

Assume that $T(n) \leq cn$ for a large c and all $n > 500$.

Therefore:

$$T(n) \leq c \left\lceil \frac{n}{7} \right\rceil + c \left(\frac{5n}{7} + 8 \right) + an \quad (1)$$

$$T(n) \leq \frac{cn}{7} + c + \frac{5cn}{7} + 8c + an \quad (2)$$

$$= \frac{6cn}{7} + 9c + an \quad (3)$$

$$= cn + \left(-\frac{cn}{7} + 9c + an \right) \quad (4)$$

Which is at most cn if

$$-\frac{cn}{7} + 9c + an \leq 0 \quad (5)$$

\therefore the run time of median of medians is $\Theta(n)$.