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Homework 5

COSC 3100
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Problem 1

Following the procedure of the book, end of Chapter 9, prove that the existence of a constant c > 0 such that T(n) ≤ cn .

Proof:

Assume that T(n) ≤ cn for a large c and all n > 500.

Therefore:

$$T(n) \leq c\left\lceil \frac{n}{7} \right\rceil + c\left(\frac{5\,n}{7} + 8\right) + an \tag{1}$$

$$T(n) \leq \frac{cn}{7} + c + \frac{5\,cn}{7} + 8\,c + an \tag{2}$$

$$= \frac{6\,cn}{7} + 9\,c + an \tag{3}$$

$$= cn + (-\frac{cn}{7} + 9c + an) \tag{4}$$

T(n) is bounded at upper limit by cn if:

$$-\frac{cn}{7} + 9c + an \leq 0 \tag{5}$$

$$-cn + 63c + 7an \leq 0 \tag{6}$$

$$7an \leq cn - 63c \tag{7}$$

$$c(n - 63) \geq 7an \tag{8}$$

$$c \geq \frac{7an}{n - 63} \tag{9}$$

$$c \geq 7a \cdot \frac{n}{n - 63} \tag{10}$$

$\frac{n}{n-63}$ exists and is positive when $n > 63$.

c > 7a would imply for a > 0, $\exists c : c >$ 0 when n > 63

$\therefore$ T(n) is $\Theta(n)$ or bounded by cn and is in the worst case linear.