Homework 1

Analyzing the Fibonacci Sequence

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Develop a recurrence relation for the number of recursive calls $\mathbf{r}_n$.

$$\mathbf{r}_n = \begin{cases} 0 & \textbf{for } n = 0 \\ 0 & \textbf{for } n = 1 \\ r_{n-1} + r_{n-2} + 2 & \textbf{for } n \geq 2 \end{cases}$$

Prove by induction that $\mathbf{r}_n = -2 + f_{n-1} + f_n + f_{n+1}$ for $n > 1$.

Proof:

Base Case (n = 2, n = 3)

$$r_2 = -2 + f_1 + f_2 + f_3 \tag{1}$$

$$r_2 = -2 + 1 + 1 + 2 \tag{2}$$

$$r_2 = 2 \tag{3}$$

This works. Now n = 3

$$r_3 = -2 + f_2 + f_3 + f_4 \tag{4}$$

$$r_3 = -2 + 1 + 2 + 3 \tag{5}$$

$$r_3 = 4 \tag{6}$$

This works. Now to the inductive step.

If we can assume that P(n) : $r_n = -2 + f_{n-1} + f_n + f_{n+1}$ **for all n $\leq$ k,**
then P(k+1): $r_{k+1} = -2 + f_k + f_{k+1} + f_{k+2}$

$$P(k) : r_k = -2 + f_{k-1} + f_k + f_{k+1} \tag{7}$$

$$P(k+1) : r_{k+1} = -2 + f_{k-2} + f_{k-1} + f_k \tag{8}$$

$$r_{k+1} = 2 + r_k + r_{k-1} = 2 + -2 + f_{k-1} + f_k + f_{k+1} + -2 + f_{k-2} + f_{k-1} + f_k \tag{9}$$

$$r_{k+1} = -2 + (f_{k-1} + f_{k-2}) + (f_k + f_{k-1}) + (f_{k+1} + f_k) \tag{10}$$

$$P(k+1) : r_{k+1} = -2 + f_k + f_{k+1} + f_{k+2} \tag{11}$$