A Comparative Study of EMD, EWT and VMD for Detecting the Oscillation in Control Loop

Awang N. I. Wardana
Department of Nuclear Engineering and Engineering Physics
Faculty of Engineering, Universitas Gadjah Mada
Yogyakarta, Indonesia
e-mail: awang.wardana@ugm.ac.id

***Abstract*—Adaptive time series decompositions can be used as a basis for oscillation detection in control loops at chemical plant. In general, the decomposition extracts a process time series into different intrinsic mode functions (IMFs). Later, IMFs are characterized to detect the oscillation. This paper presents a comparative study of three well-known adaptive time series decomposition, i.e. empirical mode decomposition (EMD), empirical wavelet transform (EWT) and variational mode decomposition (VMD). The study compares significancy, regularity and sparseness of each method with different process time series from chemical plant. The time series represents oscillation due to process disturbance, mistuned controller, sticking valve and faulty sensor. The comparison shows that EWT outperforms in consistent detection results over EMD and VMD.**

***Keywords*—empirical mode decomposition; empirical wavelet transorm; process control; performance evaluation; signal detection; variational mode decomposition**

I. Introduction

Nowadays, one of the important issues in process industry is utilization big data of process time series in respect to plant performances [1], [2]. One of the developed utilization is oscillation detection in control loops [3]. In oscillation detection, characterization of process time series in control loop should be developed. Many researchers have been developed these detection methods. In early development, the methods are developed using time or frequency based characterizing factors, i.e. integral absolute error, shape of autocorrelation function and power spectrum analysis.

In order to improve performance of the developed methods, time series decomposition also has been used by researchers for detecting the oscillation of process time series in control loops, i.e. wavelet packet transform [4], [5], [6], discrete cosine transform [7], [8], intrinsic time scale decomposition [9]. The newest developments been utilized adaptive time series decomposition that capable to analyze detail characters of nonlinear and non-stationary oscillation time series [10], [11], [12]. To investigate criteria of adaptive time series decomposition that influences performance of characterization methods, the comparative study of adaptive time series decomposition based methods should be done.

This paper presents a comparative study of three well-known adaptive time series decomposition, i.e. empirical mode decomposition (EMD), empirical wavelet transform (EWT) and variational mode decomposition (VMD) for detecting oscillation in control loops. As results of adaptive time series decomposition, intrinsic mode functions (IMFs) are analyzed for the comparison. Each IMFs has been calculated their cross-correlation coefficients with decomposed time series, regularity index and sparseness index. These values are represented significancy, regularity and sparsity of IMF and their spectrums. For comparing the performances, each decompositions is applied to extract the IMFs of oscillating process time series $e(t) = pv(t) - sp(t)$ (Fig. 1) from chemical plant. The industrial data presents the oscillation due to process disturbance, mistuned controller, sticking valve and faulty sensor.

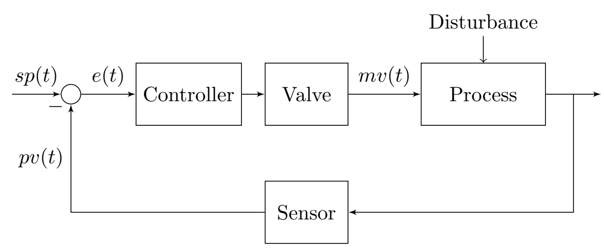


Fig. 1. Typical control loop in process industries with $sp(t)$ is set point, $pv(t)$ is process variable, $mv(t)$ is manipulated variable and $e(t) = pv(t) - sp(t)$.

To present this comparative study, the paper is structured as follows. Section 2 starts with basic theory regarding adaptive time series decomposition, i.e. EWT, EMD and VMD. The comparison methodology used in this work is described in Section 3, respectively, the explanation about characterizing factors, i.e. significancy, regularity and sparsity. Section 4 presents the results regarding the comparative study using different problems in the industrial time series. This paper ends in Section 5 with conclusion of this work.

II. Adaptive Time Series Decomposition

Adaptive time series decomposition is a method to decompose time series into principal modes directly based on signature of time series. It can be used to extract detail character of time series. The well-known adaptive time series decompositions are empirical mode decomposition (EMD), empirical wavelet transform (EWT) and variational mode decomposition (VMD). These decompositions have been implemented in many fields of application, e.g. medical, mechanical, geophysics [13], [14], [15].

A. Empirical Mode Decomposition

Empirical mode decomposition (EMD) [16] was developed to separate stationary & non-stationary or linear & non-linear components of a single time series. It is empirically decomposed a single time series into intrinsic mode functions (IMFs) as principal modes and a residual. In oscillation detection using EMD, a process time series $e(t)$ can be described as

$$e(t) = \sum_{i=1}^{N} u_i(t) + r(t), \tag{1}$$

where $u_i(t)$ is an IMF, $i = 1, 2, 3...N$, N is number of IMFs and $r(t)$ is a residual. $u_i(t)$ can extracting by shifting and iteration process. EMD procedure is detailed described in [16], [17]. The procedure begins with identification all extrema in a process time series $e(t)$ for calculating the average of upper envelope $e_{up}(t)$ and lower envelope $e_{lw}(t)$ as $m(t) = (e_{up}(t) + e_{lw}(t))/2$. The detail is calculated by $d(t) = e(t) - m(t)$. The iteration done until reaching stopping criteria of the residual $r(t)$. The detail $d(t)$ is refined by a shifting process iteratively. If the stopping criteria are reached then the detail $d(t)$ is extracted as IMF $u_1(t)$ while the residual $r(t)$ is calculated to define the next candidate IMF $u_2(t)$. The iteration process finished until the last IMF $u_N(t)$ is extracted.

B. Empirical Wavelet Transform

As described in [18], empirical wavelet transform (EWT) was developed to extract the principle modes based on segmentation of Fourier spectrum. The segmentation is done by detecting the local maxima in the spectrum. In oscillation detection, the IMF reconstructs a process time series $e(t)$ that is described by

$$e(t) = u_o(t) + \sum_{i=1}^{N} u_i(t), \tag{2}$$

where

$$u_0(t) = \left(\int e(\tau)\overline{\phi_1(\tau - t)} d\tau \right) * \phi_1(t) = \left(\hat{e}(\omega)\overline{\hat{\phi}_1(\omega)} \right)^{\vee}, \tag{3}$$

and

$$u_i(t) = \left(\int e(\tau)\overline{\psi_i(\tau - t)} d\tau \right) * \psi_i(t) = \left(\hat{e}(\omega)\overline{\hat{\psi}_i(\omega)} \right)^{\vee}. \tag{4}$$

$\hat{\phi}_i(\omega)$ is empirical scaling function (5) and $\hat{\psi}_i(\omega)$ is empirical wavelets (6) that are defined based on the spectrum segmentation $(\omega_1, \omega_2, \omega_3 .... \omega_k)$.

$$\hat{\phi}_n(\omega) = \begin{cases} 1 \quad \text{if } |\omega| \leq \omega_n - \tau_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\tau_n}(|\omega| - \omega_n + \tau_n)\right)\right] \\ \quad \text{if } \omega_n - \tau_n \leq |\omega| \leq \omega_n + \tau_n \\ 0 \quad\quad \text{otherwise} \end{cases}, \tag{5}$$

$$\hat{\psi}_n(\omega) = \begin{cases} 1 \quad \text{if } \omega_n + \tau_n \leq |\omega| \leq \omega_{n+1} - \tau_{n+1} \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\tau_{n+1}}(|\omega| - \omega_{n+1} + \tau_{n+1})\right)\right] \\ \quad \text{if } \omega_{n+1} - \tau_{n+1} \leq |\omega| \leq \omega_{n+1} + \tau_{n+1} \\ \sin\left[\frac{\pi}{2}\beta\left(\frac{1}{2\tau_n}(|\omega| - \omega_n + \tau_n)\right)\right] \\ \quad \text{if } \omega_n - \tau_n \leq |\omega| \leq \omega_n + \tau_n \\ 0 \quad \text{otherwise} \end{cases}, \tag{6}$$

where the $\beta(x) = x^4(35 - 84x + 70x^2 - 20x^3)$ [18].

C. Variational Mode Decomposition

Variational mode decomposition (VMD) [19] was developed to extract the specific sparsity properties of a time series $e(t)$. The sparsity properties are represented in the IMFs $u_i(t)$ and their centre of frequencies ω_i for $i = 1, 2, 3...N$, N is number of IMFs. The VMD algorithm is calculate the IMFs $u_i(t)$ and its centre of frequencies ω_i by solving the variational problem as described by

$$\min_{\{u_i(t)\},\{\omega_i\}} \sum_{i=1}^{N} \left\| \partial_t \left[\left\{ \delta(t) + \frac{j}{\pi t} \right\} * u_i(t) \right] e^{-i\omega_i t} \right\|_2^2, \tag{7}$$

subject to

$$\sum_{i=1}^{N} u_i(t) = e(t). \tag{8}$$

To solve the variational problem, the algorithm provides the complete optimization procedure in Fourier domain based on alternate direction method of multipliers (ADMM), described detail in [19] .

Algorithm 1 Complete optimization procedure of VMD [19]

Initialize $\{\hat{u}_i^1\}, \{\omega_i^1\}, \hat{\lambda}^1, n \leftarrow 0$
repeat
$n \leftarrow n + 1$
for $i = 1 : N$ **do**
update $\hat{u}_i$ for all $\omega \geq 0$:

$$\hat{u}_i^{n+1}(\omega) \leftarrow \frac{\hat{e}(\omega) - \sum_{k<i} \hat{u}_k^{n+1}(\omega) - \sum_{k>i} \hat{u}_k^n(\omega) + \frac{\hat{\lambda}^n(\omega)}{2}}{1 + 2\alpha(\omega - \omega_i^n)^2}$$

update ω_i:

$$\omega_i^{n+1} \leftarrow \frac{\int_o^\infty \omega \left|\hat{u}_i^{n+1}(\omega)\right|^2 d\omega}{\int_o^\infty \left|\hat{u}_i^{n+1}(\omega)\right|^2 d\omega}$$

end for
dual ascent for all $\omega \geq 0$:

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^n(\omega) + \tau \left(\hat{e}(\omega) - \textstyle\sum_k \hat{u}_k^{n+1}(\omega) \right)$$

until convergence: $\sum_i \left\| \hat{u}_i^{n+1} - \hat{u}_i^n \right\|_2^2 / \left\| \hat{u}_i^n \right\|_2 < \epsilon$

III. Comparison Method

This study compare the performance of EMD, EWT and VMD for characterizing oscillation in control loop. The comparison is done by investigating significancy, regularity and sparseness of IMFs $u_i(t)$ for $i = 1, 2, 3$ as the decomposition result of a normalized process time series $e(t)$.

A. Significancy

The objectives of process time series $e(t)$ decomposition in oscillation diagnosis is to extract oscillation characters. It means the IMFs $u_i(t)$ as the decomposition results should have significancy with process time series. As described in [20] to measure this significancy, it can be used cross correlation coefficient between a time series $e(t)$ and its IMFs $u_i(t)$ as described by

$$c(u_i, e) = \frac{\sum_{t=1}^{T}(u_i(t) - \bar{u}_i)(e(t) - \bar{e})}{\sqrt{\sum_{t=1}^{T}(u_i(t) - \bar{u}_i)^2 \sum_{t=1}^{n}(e(t) - \bar{e})^2}}, \quad (9)$$

where $\bar{u}_i$ is mean value of $u_i(t)$ while $\bar{e}$ is mean value of $e(t)$, $t = 1, 2, 3...T$ and T is number of sampling in a time series. The value of $c(u_i, e)$ is always between -1 and 1. $c(u_i, e) = 0$ means $u_i(t)$ has the lowest significancy with $e(t)$ while $|c(u_i, e)| = 1$ means $u_i(t)$ has the highest significancy with $e(t)$.

B. Regularity

The oscillation in time domain can be represented by periods of oscillation. The regular periods are indicated oscillation tendency of a time series [21]. To measure this regularity, it can be used the regularity index that can be represented as

$$r(t_p) = \left| \frac{\bar{t}_p - \sqrt{\frac{\sum_{k=1}^{K}(t_p(k) - \bar{t}_p)^2}{K}}}{\bar{t}_p} \right|, \quad (10)$$

where $\bar{t}_p$ is mean value of the periods $t_p(k)$, $k = 1, 2, 3...K$ and K is periods number of a time series $e(t)$. The value of $r(t_p)$ is always between 0 and 1. $r(t_p) = 0$ means $u_i(t)$ has the lowest regularity of periods while $r(t_p) = 1$ means $u_i(t)$ has the highest regularity of periods.

C. Sparseness

High significant similarity of IMF $u_i(t)$ with a time series $e(t)$ has consequence that it can be noisy when the time series is corrupted with noise. As described in [11] to differentiate between noisy and non-noisy IMF, it represents as sparseness index [22] that is expressed by

$$s(\hat{u}_i) = \frac{\sqrt{T} - \left(\sum_{i=1}^{T}|\hat{u}_i|\right) / \sqrt{\sum_{i=1}^{T}(\hat{u}_i)^2}}{\sqrt{T} - 1}, \quad (11)$$

where $\hat{u}_i$ is Fourier transform of $u_i(t)$ and T is number of sampling in a time series. The value of $s(\hat{u}_i)$ is always between 0 and 1. $s(\hat{u}_i) = 1$ means $u_i(t)$ is non-noisy IMF while $s(\hat{u}_i) = 0$ means $u_i(t)$ is noisy IMF.

EMD, EWT and VMD is applied in four oscillating process time series $e(t)$ of chemical plant obtained in [23] for comparing their performances. Each process time series represents oscillations due to process disturbance (CHEM38), mistuned controller (CHEM4), sticking valve (CHEM5) and faulty sensor (CHEM14). The decompositions are applied in each process time series for extracting three IMFs $u_i(t)$ for $i = 1, 2, 3$.

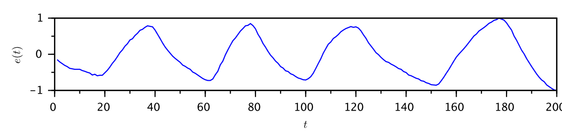
IV. Results

A. Oscillation due to Process Disturbance

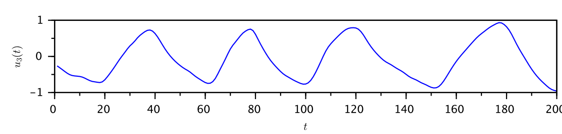
External disturbance directly influences to process dynamics (Fig. 1). The external harmonic disturbance can cause an oscillating process time series $e(t)$ (Fig. 2a). The oscillation has a consistent linear character with deterministic auto-correlation function [24].

TABLE I
THE COMPARATIVE RESULTS USING OSCILLATING $e(t)$ DUE TO PROCESS DISTURBANCE

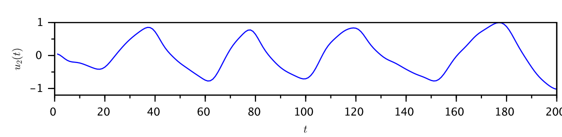
Decomposition	IMF	Comparative Values		
		$c(u_i, e)$	$r(t_p)$	$s(u_i)$
EMD	$u_1(t)$	-0.03	-	0.78
	$u_2(t)$	0.08	0.93	0.78
	$u_3(t)$	0.99	0.82	0.72
EWT	$u_1(t)$	0.11	-	0.66
	$u_2(t)$	0.99	0.85	0.70
	$u_3(t)$	0.01	0.67	0.31
VMD	$u_1(t)$	0.82	0.98	0.79
	$u_2(t)$	0.54	0.92	0.72
	$u_3(t)$	0.78	0.98	0.82



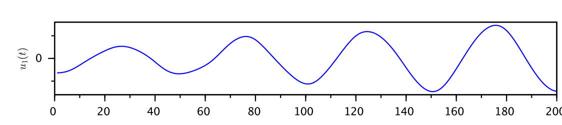
(a) $e(t)$ a normalized process time series.



(b) $u_3(t)$ the most significant IMF produced by EMD.



(c) $u_2(t)$ the most significant IMF produced by EWT.



(d) $u_1(t)$ the most significant IMF produced by VMD.

Fig. 2. An oscillating process time series due to process disturbance with its most significant IMF of each decomposition.

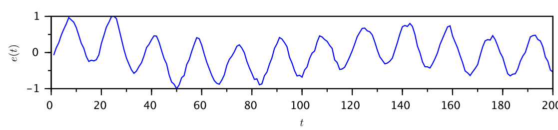
The comparative results (Table I) show that EMD and EWT produce one IMF with $c(u_i, e)$ are higher than VMD. It means that EMD and EWT can produce the IMFs that are higher significancy than VMD. Meanwhile, the most significant IMF for all decompositions, i.e. $u_3(t)$ for EMD, $u_2(t)$ for EWT and $u_3(t)$ for VMD, can characterize the periods of oscillation (Fig. 2). It is also shown that the regularity index $r(t_p)$ and sparseness index $s(u_i)$ of the most significant IMF of EMD, EWT and VMD show the relative equal values (Table I).

B. Oscillation due to Mistuned Controller

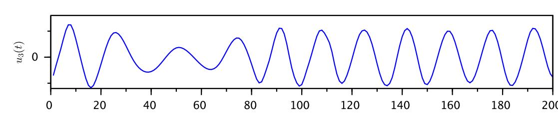
As part of control loop, parameters of a controller has effect on dynamics of a control loop. It should be well-tuned. The mistuned controller can produce the oscillating process time series $e(t)$ (Fig. 3a). As descibed in [24], the oscillation has consistent linear oscillation character with stochastic auto-correlation function.

TABLE II
THE COMPARATIVE RESULTS USING OSCILLATING $e(t)$ DUE TO MISTUNED CONTROLLER

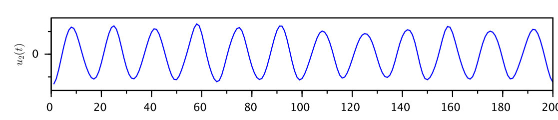
Decomposition	IMF	Comparative Values		
		$c(u_i, e)$	$r(t_p)$	$s(u_i)$
EMD	$u_1(t)$	0.46	-	0.66
	$u_2(t)$	0.05	0.76	0.63
	$u_3(t)$	0.71	0.84	0.75
EWT	$u_1(t)$	0.45	-	0.72
	$u_2(t)$	0.85	0.97	0.90
	$u_3(t)$	0.05	0.72	0.43
VMD	$u_1(t)$	0.49	-	0.72
	$u_2(t)$	0.86	0.97	0.92
	$u_3(t)$	0.84	0.96	0.84



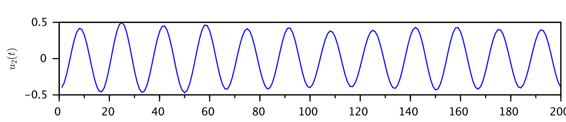
(a) $e(t)$ a normalized process time series.



(b) $u_3(t)$ the most significant IMF produced by EMD.



(c) $u_2(t)$ the most significant IMF produced by EWT.



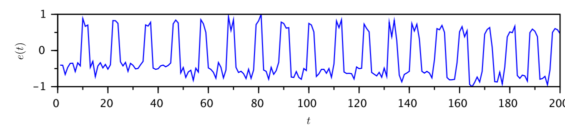
(d) $u_2(t)$ the most significant IMF produced by VMD.

Fig. 3. An oscillating process time series due to mistuned controller with its most significant IMF of each decomposition.

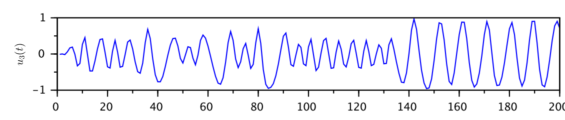
Comparison in Table II presents that the value of cross correlation cooeficient $c(u_i, e)$, regularity index $r(t_p)$ and sparseness index $s(u_i)$ of the most significant IMF of EWT and VMD $u_2(t)$ are higher than the most significant IMF of EMD $u_3(t)$. It is also presented visually in Fig. 3 that the most significant IMF of EWT and VMD can characterize the periods of oscillation with constant amplitudes better than the most significant IMF of EMD.

TABLE III
THE COMPARATIVE RESULTS USING OSCILLATING $e(t)$ DUE TO STICKING VALVE

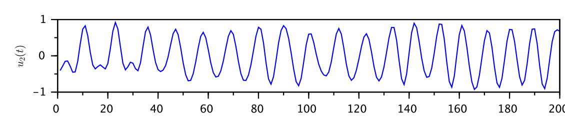
Decomposition	IMF	Comparative Values		
		$c(u_i, e)$	$r(t_p)$	$s(u_i)$
EMD	$u_1(t)$	0.05	0.53	0.74
	$u_2(t)$	0.56	0.84	0.64
	$u_3(t)$	0.72	0.67	0.46
EWT	$u_1(t)$	0.01	-	0.66
	$u_2(t)$	0.87	0.89	0.59
	$u_3(t)$	0.36	0.65	0.36
VMD	$u_1(t)$	0.20	-	0.85
	$u_2(t)$	0.63	0.95	0.81
	$u_3(t)$	0.66	0.91	0.75



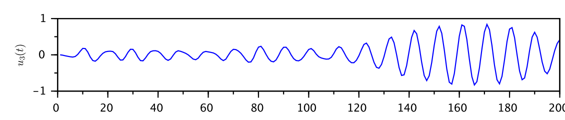
(a) $e(t)$ a normalized process time series.



(b) $u_3(t)$ the most significant IMF produced by EMD.



(c) $u_2(t)$ the most significant IMF produced by EWT.



(d) $u_3(t)$ the most significant IMF produced by VMD.

Fig. 4. An oscillating process time series due to sticking valve with its most significant IMF of each decomposition.

C. Oscillation due to Sticking Valve

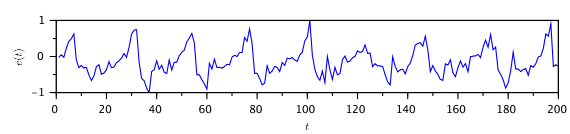
Stiction is a major problem of control valves that is affected the plant performance and product quality in chemical plant [25]. It can produce nonlinear oscillating process time series $e(t)$ (Fig. 4a). The produced oscillation cannot be solved by only retuning the controller. The affected valve should be maintained to eliminate this problem.

The comparative results (Table III) show the value of $c(u_i, e)$, $r(t_p)$, and $s(u_i)$ for each IMF extracted by decompositions. EMD decomposes the process time series $e(t)$ with

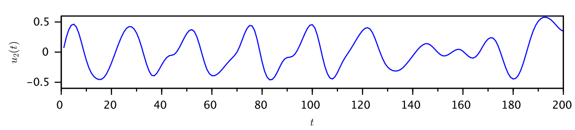
$u_3(t)$ as the most significant IMF indicated by $c(u_i, e) = 0.72$. It has regularity index $r(t_p) = 0.67$ and sparseness index $s(u_i) = 0.46$. These values indicate that it has irregular periods and amplitudes (Fig. 4b). EWT decomposes oscillation due to sticking valve into three IMFs with $u_2(t)$ as the most significant IMF ($c(u_i, e) = 0.87$). This IMF has regularity index $r(t_p) = 0.89$ and sparseness index $s(u_i = 0.59)$. It means EWT can extracted regular periods of the oscillation with constant amplitudes (Fig. 4c). For VMD, the comparative results describe that it difficult to determine the most significant IMF because $c(u_i, e)$ of $u_3(t)$ is slightly higher than $u_2(t)$. Nevertheless, $u_3(t)$ has regular periods with irregular amplitudes (Fig. 4d).

TABLE IV
THE COMPARATIVE RESULTS USING OSCILLATING $e(t)$ DUE TO FAULTY SENSOR

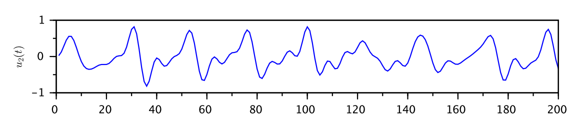
Decomposition	IMF	Comparative Values		
		$c(u_i, e)$	$r(t_p)$	$s(u_i)$
EMD	$u_1(t)$	0.13	-	0.79
	$u_2(t)$	0.69	0.75	0.63
	$u_3(t)$	0.57	0.66	0.44
EWT	$u_1(t)$	0.01	-	0.66
	$u_2(t)$	0.93	0.92	0.59
	$u_3(t)$	0.35	0.66	0.31
VMD	$u_1(t)$	0.25	-	0.93
	$u_2(t)$	0.71	0.95	0.81
	$u_3(t)$	0.55	0.91	0.77



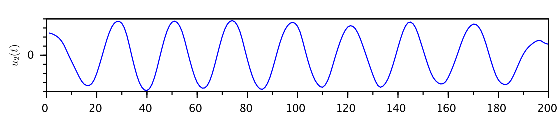
(a) $e(t)$ a normalized process time series.



(b) $u_2(t)$ the most significant IMF produced by EMD.



(c) $u_2(t)$ the most significant IMF produced by EWT.



(d) $u_2(t)$ the most significant IMF produced by VMD.

Fig. 5. An oscillating process time series due to faulty sensor with its most significant IMF of each decomposition.

D. Oscillation due to Faulty Sensor

Faulty sensor can effect to measurement deviation of process varibles. It has consequence to controller input in a control loop. It can result to the oscillating process time series $e(t)$ (Fig. 5). The oscillation characteristics can be related with type of faulty sensor. It can be caused by short spike reading, noisy reading or anomalous offset reading [26].

EMD decomposes a oscillating process time series $e(t)$ (Fig. 5a) with $u_2(t)$ as the most significant IMF indicated by $c(u_i, e) = 0.69$ (Table IV). It has regularity index $r(t_p) = 0.75$ and sparseness index $s(u_i) = 0.63$. These values indicate that $u_2(t)$ has irregular periods and amplitudes (Fig. 5b). For EWT, $u_2(t)$ also indicate as the most significant IMF with $c(u_i, e) = 0.93$, $r(t_p) = 0.92$ and $s(u_i) = 0.59$ (Table IV). It means this IMF $u_2(t)$ can extract regular periods and amplitudes of the oscillation but it is slightly corrupted by noise (Fig. 5c). VMD provides $u_2(t)$ as the most significant IMF with $c(u_i, e) = 0.71$, $r(t_p) = 0.95$ and $s(u_i) = 0.81$ (Table IV). For VMD, $u_2(t)$ has regular periods and amplitudes (Fig. 5d).

V. CONCLUSION

This paper presents a comparative study of EMD, EWT and VMD for detecting the oscillation in control loop. The study compares significancy, regularity and sparseness of each methods with different process time series from chemical plant. The process time series of plant presents four oscillation type based on the problems, i.e. process disturbance, mistuned controller, sticking valve and faulty sensor. In each type of oscillation, the comparative results show that EMD, EWT and VMD present different perfomances. For detecting the oscillation due to external process disturbance, the results show that EMD, EWT and VMD present equal performances. For detecting oscillation due to mistuned controller and faulty sensor, EWT and VMD perform better results than EMD. For detecting oscillation due to sticking valve, EWT can extract the most significant IMF that has better oscillation characters than EMD and VMD. Overall, it can be concluded that EWT outperforms in consistent results over EMD and VMD.

For next future work, EWT will be used as a characterization basis of oscillation in control loop, especially for differentiation between mistuned controller or external process disturbance problem and quantification of valve stiction problem.

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