

**Question 1** Use the tableau algorithm to test the satisfiability of  $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C$  (Hint: to test the satisfiability of a concept X, you have to show that it is possible for some element y to be member of X, in other words you have to use a tableau to construct a model for y:X).

**Answer 1** We try to construct a model:

1	$y : \exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C$	$\mathcal{A}$
2	$y : \exists S.C$	$(\sqcap : 1)$
3	$y : \forall S.(\neg C \sqcup \neg D)$	$(\sqcap : 1)$
4	$y : \exists R.C$	$(\sqcap : 1)$
5	$(y, p) : S$	$(\exists : 2)$
6	$p : C$	$(\exists : 2)$
7	$(y, q) : R$	$(\exists : 4)$
8	$q : C$	$(\exists : 4)$
9	$p : \neg C \sqcup \neg D$	$(\forall : 3, 5)$
$10^{\text{split}_1}$	$p : \neg C$	$(\sqcup : 9)$
$11^{\text{split}_1}$	clash	$(6, 10^1)$
$10^{\text{split}_2}$	$p : \neg D$	$(\sqcup : 9)$

A model exists that demonstrates the satisfiability.

**Question 2** Given the following interpretation of the concepts

- Person = {Daisy, Dan, Dirk, Pat, Paul}
- Doctor = {Daisy, Dan, Dirk}
- Patient = {Pat, Paul, Dan}
- Treats = {(Daisy, Pat), (Dan, Paul), (Dirk, Dan), (Dan, Dan)}
- Suffers\_from = {(Pat, pain), (Paul, pain), (Dan, pain), (Dan, sickness)}

Compute the interpretation of the following expressions:

- a) Doctor  $\sqcap$  Patient
- b) Person  $\sqcap \exists \text{treats}.\text{Doctor}$

**Answer 2**

- a) (Doctor  $\sqcap$  Patient) $^{\mathcal{I}} = \{\text{Dan}\}$
- b) (Person  $\sqcap \exists \text{treats}.\text{Doctor}$ ) $^{\mathcal{I}} = \{\text{Dirk}, \text{Dan}\}$

**Question 3** Show that from the knowledge base:  $P \sqsubseteq (E \sqcap U) \sqcup (E \sqcap \neg D)$  we can derive that  $P \sqsubseteq E$

**Answer 3** we have to show that  $a : P \sqcap \neg E$  is not satisfiable given that the TBox is not empty.

1.	$a : P \sqcap \neg E$	$\mathcal{A}$
2.	$\top \sqsubseteq \neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D)$	$\mathcal{T}$
3.	$a : P$	$(\sqcap : 1)$
4.	$a : \neg E$	$(\sqcap : 1)$
5.	$a : \neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D)$	$(\equiv : 2)$
6.	$a : \neg P$	$(\sqcup : 5)$
	$\times (3, 6)$	
7.	$a : (E \sqcap U)$	$(\sqcup : 5)$
8.	$a : E$	$(\sqcap : 7)$
	$\times (4, 8)$	
9.	$a : (E \sqcap \neg D)$	$(\sqcup : 5)$
10.	$a : E$	$(\sqcap : 9)$
	$\times (4, 10)$	

Yes, given the TBox, P is subsumed by E.