$$
\begin{aligned}
\frac{\partial {}^{G}\widehat{\mathbf{p}}_j^{\kappa}}{\partial \delta\boldsymbol{\theta}} &= \frac{\partial\left(\widehat{\mathbf{R}}_{I_k}^{\kappa}\ {}^{I_k}\mathbf{p}_j\right)}{\partial \delta\boldsymbol{\theta}} && (1)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \mathrm{Exp}(\Delta(\delta\boldsymbol{\theta}))\ {}^{I_k}\mathbf{p}_j - \widehat{\mathbf{R}}_{I_k}^{\kappa}\ {}^{I_k}\mathbf{p}_j}{\Delta(\delta\boldsymbol{\theta})} && (2)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left(\boldsymbol{I} + [\Delta(\delta\boldsymbol{\theta})]_{\times}\right)\ {}^{I_k}\mathbf{p}_j - \widehat{\mathbf{R}}_{I_k}^{\kappa}\ {}^{I_k}\mathbf{p}_j}{\Delta(\delta\boldsymbol{\theta})} && (3)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{\widehat{\mathbf{R}}_{I_k}^{\kappa}\ [\Delta(\delta\boldsymbol{\theta})]_{\times}\ {}^{I_k}\mathbf{p}_j}{\Delta(\delta\boldsymbol{\theta})} && (4)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left(\Delta(\delta\boldsymbol{\theta}) \times {}^{I_k}\mathbf{p}_j\right)}{\Delta(\delta\boldsymbol{\theta})} && (5)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{-\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left({}^{I_k}\mathbf{p}_j \times \Delta(\delta\boldsymbol{\theta})\right)}{\Delta(\delta\boldsymbol{\theta})} && (6)\\
&= \lim_{\Delta(\delta\boldsymbol{\theta})\to 0} \frac{-\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left(\left[{}^{I_k}\mathbf{p}_j\right]_{\times} \Delta(\delta\boldsymbol{\theta})\right)}{\Delta(\delta\boldsymbol{\theta})} && (7)\\
&= -\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left[{}^{I_k}\mathbf{p}_j\right]_{\times} && (8)\\
&= -\widehat{\mathbf{R}}_{I_k}^{\kappa}\ \left[{}^{I}\mathbf{R}_L\ {}^{L_k}\mathbf{p}_j + {}^{I}\mathbf{p}_L\right]_{\times} && (9)
\end{aligned}
$$