Static Output Feedback Control Design for Takagi-Sugeno Descriptor Fuzzy Systems

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***Abstract*—An output feedback control design is an important problem for practical systems because some of the state variables may not measured and hence, a controller with state feedback is not feasible. Here, we are concerned with a new control design method via non-quadratic static output feedback control for fuzzy descriptor systems. Practical systems that are often modeled as a nonlinear system can be expressed by fuzzy descriptor systems. Such systems can describe nonlinear systems with equality constraints. For them, a control design method of admissible non-quadratic static output feedback controllers based on new design conditions is proposed. These conditions result from multiple Lyapunov matrix method and are relatively relaxed, and consequently they can be applied to a broader class of fuzzy descriptor systems. Admissibility conditions and admissible control design conditions are derived via Linear Matrix Inequalities(LMIs) with some given scalar and matrices. At the end, a simple simulation is provided to clarify our method.**

Main Contribution
A new Lyapunov function is introduced to reduce conservatism in control design conditions.

***Index Terms*—fuzzy descriptor models, static output feedback control design, non-PDC**

I. Introduction

In the literature, quite a few results on how to make a control design by state feedback controllers have been addressed. A controller with the state feedback is simple, but, in physical systems, not every state variable of the system is usually feasible for the feedback. Hence, the output feedback controller design is a crucial problem for physical systems. In modeling physical systems, a descriptor system plays an important role. A descriptor system is a general representation which is composed with differential equations and algebraic equations, and hence it can express a dynamical system with algebraic constraints, and can generalize an ordinary state-space system. In fact, a descriptor system is adopted in most engineering and scientific fields as mobile robots, mechanical systems and etc. The descriptor model is also called generalized state-space model, implicit model, differential-algebraic model, singular model, or semistate model. Due to its importance, many works on control synthesis and system analysis of descriptor systems have been made([2], [10], [7]). One of the crucial features of descriptor systems is the impulsive mode, which is undesirable in controlling descriptor systems. In [2] and [23], system behaviours of such descriptor systems are explained and definitions of regularity, impulsive modes, and admissibility are clarified. In [22], admissibility for discrete-time descriptor systems was studied. The continuous-time case was done in [9]. Furthermore, various problems on system analysis and synthesis for descriptor systems were investigated in [4], [21] and [23], and various design methods of admissible controllers were obtained there.

On the other hand, Takagi-Sugeno fuzzy system is now popular in describing nonlinear systems since it is capable of generally describing a class of nonlinear systems([14], [16], and [15]). In fact, Takagi-Sugeno fuzzy system plays a quite essential role in representing nonlinear systems. The stability and control design methods have been addressed in many previous works(for example, [5], [12], [16], [15], [20], and references therein.). Tremendous works for control design with output feedback for fuzzy systems have been published so far([1], [3], [8], [13], [25], [26], and [27]). The previous works in [25] and [27] proved the separatopn principle for fuzzy sytems with the well-known Parallel Distributed Compensator(PDC) and its dual observer, and then proposed the output feedback compensator. The results in [6] and [18] made use of a descriptor system approach that relaxes conditions for state feedback controllers' design. The result in [1] generalized the control design via descriptor system to non-quadratic static output feedback control design problem, while those in [3] and [8] considered the control design with dynamic output feedback control.

Here, the paper is interested in a static output feedback control design for fuzzy descriptor systems, based on new control design conditions. Conventionally, PDC is known to be popular for Takagi-Sugeno fuzzy systems, but it leads to a conservative controller that is available for only a limited class of systems. Due to its conservativeness, a static non-Parallel Distributed Compensator(non-PDC) with output feedback is employed and the resulting closed-loop system becomes admissible by such a non-PDC. Recently, a novel Lyapunov function possessing the integrals of the membership functions has been employed to synthesis the control design in [11] and [28]. Although a conventional multiple Lyapunov matrix approach relaxes the resulting stability conditions, the upper limits for the derivatives of the membership functions must be obtained in advance for conditions to be satisfied. It is not eventually practical and the general membership function is not always differentiable. In order to get rid of these difficulties, the integrals of the membership function was found useful. The integrals in Lyapunov function results in nonnecesity of any derivative information on the membership function. This kind of non-quadratic Lyapunov matrix approach further produces rather relaxed conditions for admissibility and control design conditions. Here, those resulting conditions are obtained based on Linear Matrix Inequalities(LMIs) that can numerically be solved by commonly used numerical softwares. The desired controllers are designed by control gains resulting from such LMI control design conditions. Finally, an simple illustrative example of output feedback control design is taken to emphasis that our design method is more powerful than the previous works.

II. Fuzzy Descriptor Models

At first, we give a Takagi-Sugeno fuzzy descriptor model under consideration. let us get started with the IF-THEN rules of the fuzzy model:

IF ξ_1 is M_{i1} and ξ_2 is M_{i2} $\cdots$ and ξ_p is M_{ip},
THEN $E\dot{x}(t) = A_i x(t) + B_i u(t)$,
$y(t) = C_i x(t) + D_i u(t),\ i = 1, 2, \cdots, r$

with $x(t) \in \Re^n,\ u(t) \in \Re^m$, and $y(t) \in \Re^q$ are the state variable, the control input, and the observation, respectively. The given constant matrices $E,\ A_i,\ B_i,\ C_i$, and D_i have compatible dimensions. The matrix E may not have a full rank; i.e., rank$E = s \leq n$. r is the number of IF-THEN rules. Each $\xi_i,\ i = 1, \cdots, p$ is a measurable premise variable, and M_{ij} is a fuzzy set. For simplicity, a vector $\xi = \begin{bmatrix} \xi_1 & \cdots & \xi_p \end{bmatrix}^T$ is defined. Then, the fuzzy descriptor system is described as

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r} \lambda_i(\xi)(A_i x(t) + B_i u(t)) \\ &\stackrel{def}{=} A_\lambda x(t) + B_\lambda u(t), \\ y(t) &= \sum_{i=1}^{r} \lambda_i(\xi)(C_i x(t) + D_i u(t)) \\ &\stackrel{def}{=} C_\lambda x(t) + D_\lambda u(t) \end{aligned} \tag{1}$$

where

$$\lambda_i(\xi) = \frac{\alpha_i(\xi)}{\sum_{i=1}^{r} \alpha_i(\xi)}, \quad \alpha_i(\xi) = \prod_{j=1}^{p} M_{ij}(\xi_j)$$

and $M_{ij}(\cdot)$ denotes the grade of the membership function of each M_{ij}. If the matrix E does not have a full rank, it is not unusual to assume that (1) can be transformed into the one with E being of the form:

$$E = \begin{bmatrix} I_s & 0 \\ 0 & 0 \end{bmatrix}.$$

It is also assumed that

$$\alpha_i(\xi) \geq 0,\ \forall\ i,\ \sum_{i=1}^{r} \alpha_i(\xi) > 0$$

for any premise variable ξ. Therefore, $\lambda_i(\xi)$ possesses the properties:

$$\lambda_i(\xi) \geq 0,\ \forall\ i,\ \sum_{i=1}^{r} \lambda_i(\xi) = 1$$

for any premise variable ξ.

Conventionally, the PDC is usually adopted so far. Control design conditions derived with PDC, however, are conservative. In order to overcome such weakness, we introduce a new class of controllers. Consider the following controller, which feeds back the observation output:

$$\begin{aligned} u(t) &= \sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} \lambda_j(\xi(t))\lambda_k(\xi(t-h))\mu_l(\xi(t))K_{jkl} \\ &\quad \times(\sum_{i=1}^{r} \mu_i(\xi)X_{5i})^{-1} y(t) \\ &\stackrel{def}{=} K_{\lambda\lambda^h\mu} X_{5\mu}^{-1} y(t) \end{aligned} \tag{2}$$

with

$$\mu_i(\lambda(t)) = \frac{1}{h}\int_{t-h}^{t} \lambda_i(\xi(\tau))d\tau, \tag{3}$$

h is a positive constant scalar to be determied, and $K_{jkl},\ X_{5l},\ \forall\ j, k, l$ are constant matrices to be calculated. The weighting function $\mu_i(\xi(t))$ in (3) owns the similar features to the membership function $\lambda_i(\xi(t))$. It eventually satisfies

$$\begin{aligned} \sum_{i=1}^{r} \mu_i(\xi) &= \frac{1}{h}\sum_{i=1}^{r}\int_{t-h}^{t} \lambda_i(\xi(\tau))d\tau \\ &= \frac{1}{h}\int_{t-h}^{t}\sum_{i=1}^{r} \lambda_i(\xi(\tau))d\tau \\ &= \frac{1}{h}\int_{t-h}^{t} 1 d\tau \\ &= 1, \end{aligned}$$

and $\mu_i(\xi(t))$ takes a nonnegative value for $i = 1, \cdots, r$.

Applying (2) to (1), we have the system below:

$$\tilde{E}\dot{\tilde{x}} = \tilde{A}_{\lambda\lambda\lambda^h\mu}\tilde{x} \tag{4}$$

with $\tilde{x}(t) = \begin{bmatrix} x^T & y^T & u^T \end{bmatrix}^T$, and

$$\tilde{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \tilde{A}_{\lambda\lambda\lambda^h\mu} = \begin{bmatrix} A_\lambda & 0 & B_\lambda \\ C_\lambda & -I & D_\lambda \\ 0 & K_{\lambda\lambda^h\mu}X_{5\mu}^{-1} & -I \end{bmatrix}.$$

When we discuss a descriptor system, the following notions are important.

Definition 2.1: ([2], [13]) (i) A system described by

$$E\dot{x} = Ax \tag{5}$$

is said to be impulse-free if deg(det$(sE - A)$) = rank$E$.
(ii) The system (5) is said to be stable if all roots of det$(sE -$

$A) = 0$ have negative real parts.
(iii) The system (5) is said to be regular if $\det(sE - A)$ is not identically zero.
(iv) The system (5) is said to be admissible if it is impulse-free, regular, and stable.
When we consider (1), it possibly possesses an impulsive solution. However, no impulsive mode and the regularity condition of $(E,\ A_{\lambda\lambda\lambda^h\mu})$ assure the uniqueness and existence of non-impulsive solution to the system (1) on $[0,\ \infty)$([21]).

Now, we state our problem: construct a static output feedback controller (2) that gets (1) to become admissible with more relaxed conditions than the previous works.

We emply a relaxation lemma below to make control design conditions less conservative.

Lemma 2.1: ([19])

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi)\lambda_j(\xi)\Phi_{ij} < 0$$

is satisfied as far as the followings hold:

$$\begin{aligned} \Phi_{ii} < 0, &\qquad \forall\ i, \\ \frac{2}{r-1}\Phi_{ii} + \Phi_{ij} + \Phi_{ji} < 0, &\quad \forall\ i,j,\ i \neq j. \end{aligned}$$

III. Design of Admissible Controllers

We attempt to obtain relaxed conditions for (4) to become admissible, and construct a non-quadratic output feedback control (2) for (1), based on such admissibility conditions. The following is a main result:

Theorem 3.1: The system (1) becomes admissible via a static output feedback control (2), provided that there exist a positive scalar h and matrices $X_{1l},\ X_{5l},\ X_{7l},\ X_{8l},\ X_{9l},\ l = 1,\cdots,r,\ K_{jkl}$ for all j,k,l such that

$$X_{1i}^TE^T = EX_{1i} \geq 0, \qquad \forall\ i, \tag{6}$$

$$\Psi_{ii}^{kl} < 0, \quad \forall\ i,k,l, \tag{7}$$

$$\frac{2}{r-1}\Psi_{ii}^{kl} + \Psi_{ij}^{kl} + \Psi_{ji}^{kl} < 0, \quad \forall\ i \neq j,k,l \tag{8}$$

with

$$\begin{aligned}
\Psi_{ij}^{kl} &= \begin{bmatrix} \Psi_{11i}^{kl} & \Psi_{12i}^{l} & \Psi_{13ij}^{kl} \\ * & \Psi_{22i}^{l} & \Psi_{23ij}^{kl} \\ * & * & \Psi_{33j}^{kl} \end{bmatrix}, \\
\Psi_{11i}^{kl} &= A_iX_{1l} + X_{1l}^TA_i^T + B_iX_{7l} + X_{7l}^TB_i^T \\
&\quad -\frac{1}{h}E(X_{1i} - X_{1k}), \\
\Psi_{12i}^{l} &= X_{1l}^TC_i^T - P_4^TX_{5l}^T + X_{7l}^TD_i^T + B_iX_{8l} \\
\Psi_{13ij}^{kl} &= P_4^TK_{jkl}^T - X_{7l}^T + B_iX_{9l}, \\
\Psi_{22i}^{l} &= D_iX_{8l} + X_{8l}^TD_i^T - X_{5l} - X_{5l}, \\
\Psi_{23ij}^{kl} &= K_{jkl}^T + D_iX_{9l} - X_{8l}^T - X_{5l}P_6, \\
\Psi_{33j}^{kl} &= K_{jkl}P_6 + P_6^TK_{jkl}^T - X_{9l} - X_{9l}^T
\end{aligned}$$

and we need to prescribe P_4 and P_6 as given matrices of compatible dimensions.

Proof: The important idea is what Lyapunov function is adopted to guarantee the admissibility of (4). Here, we adopt the Lyapunov function below:

$$V(\tilde{x}) = \tilde{x}^T\tilde{E}^T\tilde{X}_\mu^{-1}\tilde{x} \tag{9}$$

where $\tilde{X}_\mu$ is a matrix that depends on the function $\mu_i(t)$ to be determined, but must be an invertible matrix. Assume the structure of $\tilde{X}_\mu$ to be

$$\begin{aligned}
\tilde{X}_\mu = &\sum_{i=1}^{r}\mu_i(\xi)\begin{bmatrix} X_{1i} & X_{2i} & X_{3i} \\ X_{4i} & X_{5i} & X_{6i} \\ X_{7i} & X_{8i} & X_{9i} \end{bmatrix} \\
\overset{def}{=} &\begin{bmatrix} X_{1\mu} & X_{2\mu} & X_{3\mu} \\ X_{4\mu} & X_{5\mu} & X_{6\mu} \\ X_{7\mu} & X_{8\mu} & X_{9\mu} \end{bmatrix}.
\end{aligned}$$

Then, the matrix $\tilde{X}_\mu$ must satisfy

$$\tilde{E}^T\tilde{X}_\mu^{-1} = \tilde{X}_\mu^{-T}\tilde{E} \geq 0. \tag{10}$$

Pre-multiplying both sides of the equation (10) by $\tilde{X}_\mu^T$ and post-multiplying by $\tilde{X}_\mu$yield

$$\tilde{X}_\mu^T\tilde{E}^T = \tilde{E}\tilde{X}_\mu \geq 0. \tag{11}$$

It follows from (11) that $X_{2\mu} = 0,\ X_{3\mu} = 0$, and

$$\begin{aligned}
X_{1\mu}^TE^T &= EX_{1\mu} \geq 0, \\
\sum_{i=1}^{r}\mu_i(\xi)X_{1i}^TE^T &= \sum_{i=1}^{r}\mu_i(\xi)EX_{1i} \geq 0,
\end{aligned}$$

which results in (6) and implies that $X_{1\mu}$ is of the form

$$X_{1\mu} = \begin{bmatrix} X_{11\mu} & 0 \\ X_{13\mu} & X_{14\mu} \end{bmatrix}$$

with matrices are of compatible dimensions.

We are now ready to take the time derivative of $V(\tilde{x})$ along (4):

$$\begin{aligned}
\frac{dV(\tilde{x})}{dt} &= \dot{\tilde{x}}^T\tilde{E}\tilde{X}_\mu^{-1}\tilde{x} + \tilde{x}^T\tilde{E}\dot{\tilde{X}}_\mu^{-1}\tilde{x} \\
&\quad +\tilde{x}^T\tilde{E}\tilde{X}_\mu^{-1}\dot{\tilde{x}} \\
&= \tilde{x}^T\tilde{X}_\mu^{-T}[\tilde{X}_\mu^T\tilde{A}_{\lambda\lambda\lambda^h\mu}^T + \tilde{A}_{\lambda\lambda\lambda^h\mu}\tilde{X}_\xi \\
&\qquad -\tilde{E}\dot{\tilde{X}}_\mu]\tilde{X}_\mu^{-1}\tilde{x} \\
&= \dot{\tilde{x}}^T\tilde{X}_\mu^{-T}[\tilde{X}_\mu^T\tilde{A}_{\lambda\lambda\lambda^h\mu}^T + \tilde{A}_{\lambda\lambda\lambda^h\mu}\tilde{X}_\xi \\
&\qquad -\frac{1}{h}\tilde{E}(\tilde{X}_\lambda - \tilde{X}_{\lambda^h})]\tilde{X}_\mu^{-1}\tilde{x}
\end{aligned} \tag{12}$$

where we used the identity

$$\begin{aligned}
\tilde{E}\dot{\tilde{X}}_\mu^{-1} &= -\tilde{E}\tilde{X}_\mu^{-1}\dot{\tilde{X}}_\mu\tilde{X}_\mu^{-1} \\
&= -\tilde{X}_\mu^{-T}\left(\tilde{E}\dot{\tilde{X}}_\mu\right)\tilde{X}_\mu^{-1} \\
&= -\tilde{X}_\mu^{-T}\tilde{E}\left(\sum_{i=1}^{r}\dot{\mu}_i(\xi)\tilde{X}_i\right)\tilde{X}_\mu^{-1} \\
&= -\frac{1}{h}\tilde{X}_\mu^{-T}\tilde{E}\left(\sum_{i=1}^{r}\{\lambda_i(\xi(t))\right. \\
&\qquad \left. -\lambda_i(\xi(t-h))\}\tilde{X}_i\right)\tilde{X}_\mu^{-1} \\
&= -\frac{1}{h}\tilde{X}_\mu^{-T}\tilde{E}(\tilde{X}_\lambda - \tilde{X}_{\lambda^h})\tilde{X}_\mu^{-1}.
\end{aligned}$$

If we have

$$\Psi_{\lambda\lambda\lambda^h\mu} = \tilde{X}_\mu^T \tilde{A}_{\lambda\lambda\lambda^h\mu}^T + \tilde{A}_{\lambda\lambda\lambda^h\mu}\tilde{X}_\mu - \frac{1}{h}\tilde{E}(\tilde{X}_\lambda - \tilde{X}_{\lambda^h}) < 0 \quad (13)$$

it is not difficult to see from (12) that the derivative $\dot{V}(\tilde{x})$ is negative and the admissibility of (4) is established.

We calculate

$$\Psi_{\lambda\lambda\lambda^h\mu} = \begin{bmatrix} \Psi_{11\lambda\lambda^h\mu} & \Psi_{12\lambda\mu} & \Psi_{13\lambda\lambda\lambda^h\mu} \\ * & \Psi_{22\lambda\mu} & \Psi_{23\lambda\lambda\lambda^h\mu} \\ * & * & \Psi_{33\lambda\lambda\lambda^h\mu} \end{bmatrix} \quad (14)$$

where

$$\begin{aligned}
\Psi_{11\lambda\lambda^h\mu} &= A_\lambda X_{1\mu} + X_{1\mu}^T A_\lambda^T + B_\lambda X_{7\mu} + X_{7\mu}^T B_\lambda^T \\
&\quad -\frac{1}{h}E(X_{1\lambda} - X_{1\lambda^h}), \\
\Psi_{12\lambda\mu} &= X_{1\mu}^T C_\lambda^T - X_{4\mu}^T + X_{7\mu}^T D_\lambda^T + B_\lambda X_{8\mu}, \\
\Psi_{13\lambda\lambda\lambda^h\mu} &= X_{4\mu} X_{5\mu}^{-1} K_{\lambda\lambda^h\mu}^T - X_{7\mu}^T + B_\lambda X_{9\mu}, \\
\Psi_{22\lambda\mu} &= D_\lambda X_{8\mu} + X_{8\mu}^T D_\lambda^T - X_{5\mu} - X_{5\mu}^T, \\
\Psi_{23\lambda\lambda\lambda^h\mu} &= K_{\lambda\lambda^h\mu}^T + D_\lambda X_{9\mu} - X_{8\mu}^T - X_{6\mu}, \\
\Psi_{33\lambda\lambda\lambda^h\mu} &= K_{\lambda\lambda^h\mu} X_{5\mu}^{-1} X_{6\mu} + X_{6\mu}^T X_{5\mu}^{-T} K_\mu^T \\
&\quad -X_{9\mu} - X_{9\mu}^T.
\end{aligned}$$

According to the Lyapunov function based on the descriptor system, the matrices $X_{4\mu}$ and $X_{6\mu}$ can be free matrices to be freely chosen. Here, we take

$$X_{4\mu} = X_{5\mu}P_4,\ X_{6\mu} = X_{5\mu}P_6 \quad (15)$$

where both constant matrices P_4 and P_6 of compatible dimensions are given. Substituting these slack matrices into (14), we have

$$\Psi_{\lambda\lambda\lambda^h\mu} = \begin{bmatrix} \Psi_{11\lambda\lambda^h\mu} & \Psi_{12\lambda\mu} & \Psi_{13\lambda\lambda\lambda^h\mu} \\ * & \Psi_{22\lambda\mu} & \Psi_{23\lambda\lambda\lambda^h\mu} \\ * & * & \Psi_{33\lambda\lambda\lambda^h\mu} \end{bmatrix} \quad (16)$$

where

$$\begin{aligned}
\Psi_{11\lambda\lambda^h\mu} &= A_\lambda X_{1\mu} + X_{1\mu}^T A_\lambda^T + B_\lambda X_{7\mu} + X_{7\mu}^T B_\lambda^T \\
&\quad -\frac{1}{h}E(X_{1\lambda} - X_{1\lambda^h}), \\
\Psi_{12\lambda\mu} &= X_{1\mu}^T C_\lambda^T - P_4^T X_{5\mu}^T + X_{7\mu}^T D_\lambda^T + B_\lambda X_{8\mu}, \\
\Psi_{13\lambda\lambda\lambda^h\mu} &= P_4^T K_{\lambda\lambda^h\mu}^T - X_{7\mu}^T + B_\lambda X_{9\mu}, \\
\Psi_{22\lambda\mu} &= D_\lambda X_{8\mu} + X_{8\mu}^T D_\lambda^T - X_{5\mu} - X_{5\mu}^T, \\
\Psi_{23\lambda\lambda\lambda^h\mu} &= K_{\lambda\lambda^h\mu}^T + D_\lambda X_{9\mu} - X_{8\mu}^T - X_{5\mu}P_6, \\
\Psi_{33\lambda\lambda^h\mu} &= K_{\lambda\lambda^h\mu}P_6 + P_6^T K_{\lambda\lambda^h\mu}^T - X_{9\mu} - X_{9\mu}^T.
\end{aligned}$$

Finally, application of Lemma 2.1 to (16) leads to the conditions (7)-(8).

Remark 3.1: The importance of control design conditions (6)-(8) in the above theorem lies in the fact that they do not require whether or not the membership function $\lambda_i(\xi(t))$ for all i is differentiable. Although the control design conditions given in [1], [3], [13] and [17] are based on multiple Lyapunov matrix approach, the upper bounds of the derivative of $\lambda_i(\xi(t))$ for all i, which is possiblly unknown in advance and is not always differentiable. It is a highly restrictive condition.

The following result readily follows from the above theorem.

Corollary 3.1: The system (1) becomes admissible via a static output feedback control, provided that there exist a positive scalar h and matrices $X_{1l},\ X_{5l},\ X_{7l},\ X_{8l},\ X_{9l},\ \forall\ l,\ K_j,\ \forall\ j$ such that

$$X_{1i}^T E^T = EX_{1i} \geq 0, \qquad \forall\ i, \quad (17)$$

$$\Psi_{ii}^{kl} < 0, \quad \forall\ i,k,l, \quad (18)$$

$$\frac{2}{r-1}\Psi_{ii}^{kl} + \Psi_{ij}^{kl} + \Psi_{ji}^{kl} < 0, \quad \forall\ i \neq j,k,l \quad (19)$$

with

$$\begin{aligned}
\Psi_{ij}^{kl} &= \begin{bmatrix} \Psi_{11i}^{kl} & \Psi_{12i}^{l} & \Psi_{13ij}^{l} \\ * & \Psi_{22i}^{l} & \Psi_{23ij}^{l} \\ * & * & \Psi_{33j}^{l} \end{bmatrix}, \\
\Psi_{11i}^{kl} &= A_i X_{1l} + X_{1l}^T A_i^T + B_i X_{7l} + X_{7l}^T B_i^T \\
&\quad -\frac{1}{h}E(X_{1i} - X_{1k}), \\
\Psi_{12i}^{l} &= X_{1l}^T C_i^T - P_4^T X_{5l}^T + X_{7l}^T D_i^T + B_i X_{8l} \\
\Psi_{13ij}^{l} &= P_4^T K_j^T - X_{7l}^T + B_i X_{9l}, \\
\Psi_{22i}^{l} &= D_i X_{8l} + X_{8l}^T D_i^T - X_{5l} - X_{5l}, \\
\Psi_{23ij}^{l} &= K_j^T + D_i X_{9l} - X_{8l}^T - X_{5l}P_6, \\
\Psi_{33j}^{l} &= K_j P_6 + P_6^T K_j^T - X_{9l} - X_{9l}^T
\end{aligned}$$

and we need to prescribe P_4 and P_6 as given matrices of compatible dimensions. Then, our controller is designed as

$$\begin{aligned}
u(t) &= \sum_{j=1}^{r} \lambda_j(\xi) K_j (\sum_{i=1}^{r} \mu_i(\xi) X_{5i})^{-1} y(t) \\
&\overset{def}{=} K_\lambda X_{5\mu}^{-1} y(t)
\end{aligned} \quad (20)$$

Remark 3.2: A less number of matrix slack variables is required in Corollary 3.1 than in Theorem 3.1, but the corollary still gives similar effectiveness of Theorem 3.1.

Remark 3.3: The conditions (6)-(8) and (17)-(19) are not strictly LMIs. We need to prescribe scalar h and matrices P_4 and P_6, and we may give them a priori as in the above numerical example. However, some numerical methods how to solve this class of bilinear matrix inequalities have been found in the literature.

IV. Example

The fuzzy descriptor model below is taken to be considered.

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{2} \lambda_i(x_1)(A_i x(t) + B_i u(t)), \\
y(t) &= \sum_{i=1}^{2} \lambda_i(x_1) C_i x(t)
\end{aligned}$$

with $\lambda_1(x_1) = \sin^2(x_1),\ \lambda_2(x_1) = \cos^2(x_1)$, and

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} -2 & -3 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} -3 & 4 \\ 10 & -1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ 5 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 3 & -5 \end{bmatrix},\ C_2 = \begin{bmatrix} -2 & 10 \end{bmatrix}.$$

Matrices P_4 and P_6 are given as

$$P_4 = \begin{bmatrix} 0.2 & 0 \end{bmatrix} \text{ and } P_6 = 0.2.$$

Then, the conditions (6)-(8) are satisfied. Some control gains are obtained as

$$K_{111} = -4363.4,\ K_{222} = -824.3,$$

$$X_{51} = 1343.0,\ X_{52} = 1902.2.$$

By letting both P_4 and P_6 be zero matrices, which recover design methods proposed in [1], Theorem 3.1 does not give control gain matrices for this example. The result in [13] requires certain conditions and assumptions on the membership functions and their derivatives, which are strict conditions, while our results do not require any conditions on the membership functions. It clearly implies that our proposed method uses more relaxed conditions and can be applied for a broader class of nonlinear descriptor systems than those in the literature.

For the initial conditions $x = [2\ \ -3]^T$, we simulate the trajectories of the states in Fig.1. As in Fig.1, the states in the system are well controlled and well stabilized by our proposed controller.

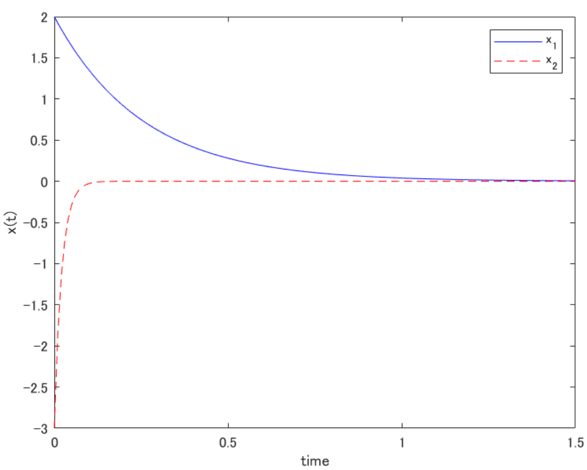

Fig. 1. The state trajectories

V. Conclusions

New methods of controller design by a static non-quadratic output feedback control for a fuzzy descriptor system were proposed by using new multiple Lyapunov matrix conditions. Unlike the traditional PDC controller for fuzzy systems, a different class of output feedback controllers with a new Lyapunov function made us obtain new control design methods since it helps produce more matrix variables which can freely be chosen. Freedom of slack variables and use of a new multiple Lyapunov matrix method have generalized control design conditions that are fairly more relaxed than those of PDC. At the end, a numerical simulation for a simple system was illustrated to show how our control design was made and was used to compare the previous works. It was illustrated in the numerical example that our method is more effective than those in the previous results.

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