

Ques 4.  $T(n) = 2T(n-1) - 1 \quad \text{--- (1)}$

Let  $n = (n-1)$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 2 \times 2T(n-2) - 2 \quad \text{--- (2)}$

$T(n-2) = 2T(n-4) - 1$   
 put value in eq (2)

$T(n) = 8$

~~$T(n) = 2T(n-1)$~~

$T(n) = 4T(n-2) - 2 \quad \text{--- (3)}$

$n = n-2$

$T(n-2) = 2T(n-3) - 1$

$T(n) = 8T(n-3) - 3$

$T(n) = 2^k T(n-k) - k$

Let  $n-k = 1$

$k = n-1$

$T(n) = 2^{n-1} T(n-(n-1)) - (n-1)$

$= 2^{n-1} - (n-1)$

$= 2^{n-1}$

$= \frac{2^n - 1}{2} = O(2^n)$

Q. 5

$i = 1 \Rightarrow i++$ ,  $i = 2$

$S = 3$ ,  $i = 3$

$S = 6$ ,  $i = 4$

$S = 10$ ,  $i = 5$

$S = 15$ ,  ~~$i = 6$~~

$i = 2 \quad 3 \quad 4 \quad 5$

$S = S+1+2$ ,  $S+1+2+3$ ,  $S+1+2+3+4$ ,  $S+1+2+3+4+5$

$S = S+(1+2+3+4+5+\dots+K)$

$$S(K) = \frac{K(K+1)}{2} \leq n$$

$$= \frac{(K^2 + K)}{2} \leq n$$

$$K^2 \leq n$$

$$K \leq \sqrt{n}$$

$$T(n) = O(\sqrt{n})$$

Q. 6

$i = 1 \quad 2 \quad 3 \quad 4 \quad \dots \quad K$

$i^2 = 1 \quad 4 \quad 9 \quad 16 \quad \dots \quad K^2$

$$K^2 \leq n$$

$$K \leq \sqrt{n}$$

$$T(n) = O(\sqrt{n})$$

Q. 8

$$T(n) = n * n * [T(n-3)]$$

$$T(n) = n^2 + T(n-3)$$

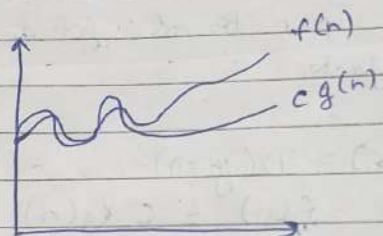
$$T(n) = O(n^2)$$

(iii) Big Omega

Gives <sup>lower</sup> bound for a  $f(n)$  within constant factor

$$f(n) = O(g(n))$$

it  $\exists c, n_0$  such that  $f(n) \geq cg(n)$  for  $c > 0$  and  $n \geq n_0$



Ques 2

$$i = 1 \quad 2 \quad 4 \quad 8 \quad \dots \quad n$$

$$= 2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad \dots \quad 2^k$$

$$G.P = 1, 2^{k-1}$$

$$n \Rightarrow \frac{2^k}{2}$$

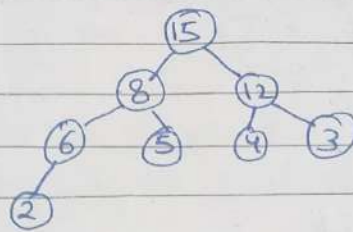
$$2n = 2^k$$

$$\log 2n = k \log 2$$

$$\log n = \frac{k}{2}$$

$$T(n) = O(\log(n))$$





### Assignment

Que 3.

$$T(n) = 3T(n-1) \quad \text{if } n > 0$$

When  $n=1$ ,

$$T(1) = 3T(0)$$

$$= 3$$

When  $n=2$

$$T(2) = 3T(1)$$

$$= 3 \times 3 = 9$$

When  $n=3$

$$T(3) = 3T(2)$$

$$= 9 \times 3 = 27$$

$$T(n) = 3, 9, 27, 81, \dots, 3^n$$

$$T(n) = O(3^n)$$

## Assignment.

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Ques 1. Asymptotic notations are used to represent the complexities of algorithms for asymptotic analysis.

Section - DS1

Name - Divyansh Deyan

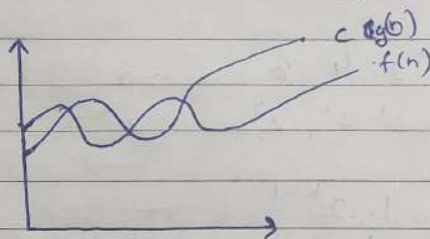
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(i) Big Oh notation:-

Gives an upper bound for a  $f(n)$  within a constant factor.

$$f(n) = O(g(n))$$

if  $f(n) \leq c \cdot g(n)$   
for  $c > 0$  &  $n \leq n_0$

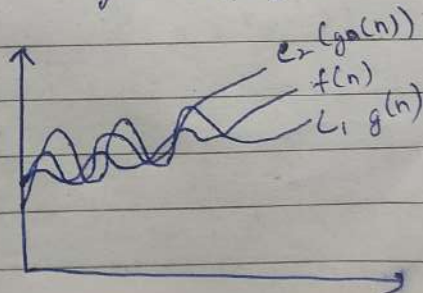


(ii) Big Theta:-

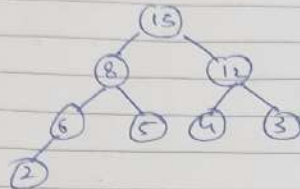
Gives ~~lower~~ bound for a  $f(n)$  within constant factor

$$f(n) = \Theta(g(n))$$

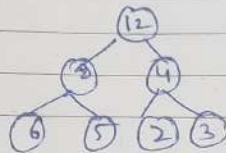
if  $c_1 g(n) \leq f(n) \leq c_2 g(n)$   
for  $c_1, c_2 > 0$  &  $n > n_0$



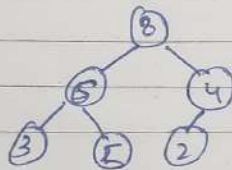
Ques 12.



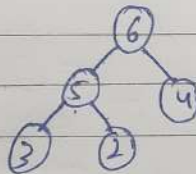
root 15 erased



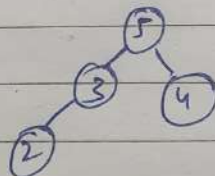
root 12 erased



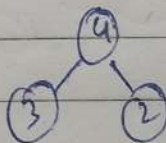
root 8 erased



root 6 erased



root 5 erased



root 4 erased



root 3 erased



root 2 erased

heap is empty.



Ques 9.

$i=1$	, $j=1$	2	3	4	...	$n/1$
$i=2$	, $j=1$	3	5	7	...	$n/2$
$i=3$	, $j=1$	4	7	11	...	$n/3$
$i=n-1$	, $j=1$	$n$				$n/n=1$

$$n + \frac{n}{2} + \frac{n}{3} + \dots + \frac{n}{n} = \log(n)$$

Harmonic Series.

$$T(n) = n * \log n = O(n \log n)$$

Ques 11.

```

int extractMin (vector<int> &arr)
{
    if (arr.empty())
        return -1;    // O(1)
    swap(arr[0], arr.back()); // O(1)
    int min = arr.back();
    arr.popback();    // O(1)
    heapify(arr, 0);  // O(log n)
    return min;
}

```

$$T(n) = O(\log(n))$$