

ERRATA AND SOME NOTES FOR TOPICS IN ALGEBRAIC GEOMETRY BY LUC ILLUSIE

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Abstract

These notes correct a few typos, errors and some notes in *Topics in Algebraic Geometry* by Prof. Luc Illusie. The original book is [Illusie].

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1 Errata

- ◆ 1. (Page 10, line -6) Actually L, M are considered as two bicomplexes centered at 0-th column instead of mapping cones;

◆ 2. (Page 18, line 2) Replace
$$\begin{array}{ccc} & N & \\ +1 \nearrow & \uparrow v & \\ L & \xrightarrow{u} & M \end{array}$$
 by
$$\begin{array}{ccc} & N & \\ +1 \nwarrow & \uparrow v & \\ L & \xrightarrow{u} & M \end{array};$$

- ◆ 3. (Page 20, line 5) Replace $L \xrightarrow{u} M \rightarrow C(u) \xrightarrow{-pr} L[1]$ by $L \xrightarrow{u} M \xrightarrow{i} C(u) \xrightarrow{-pr} L[1]$;

- ◆ 4. (Page 21, line -5) Replace $uf = 0$ by $uf = f$;

- ◆ 5. (Page 24, line 12) Replace $\mathrm{Hom}_{\mathcal{C}(S^{-1})} = H(X, Y)/\sim$ by $\mathrm{Hom}_{\mathcal{C}(S^{-1})}(X, Y) = H(X, Y)/\sim$;

◆ 6. (Page 25, line -3) Replace
$$\begin{array}{ccc} M[-1] & \longrightarrow & Y \\ f' \uparrow & \nearrow f & \\ X & & \end{array}$$
 by
$$\begin{array}{ccc} M[-1] & \xrightarrow{t'} & Y \\ f' \uparrow & \nearrow f & \\ X & & \end{array};$$

- ◆ 7. (Page 27, the first paragraph) Replace I^Y by I_Y twice and replace $(I_X)^\circ$ by $(I^X)^\circ$;

- ◆ 8. (Page 27, line 9) Replace \mathcal{A} by \mathcal{C} ;

- ◆ 9. (Page 27, line 10) Replace $\varinjlim_{X' \xrightarrow{s} X \in (I_X)^\circ} \mathrm{Hom}_{\mathcal{C}}(X', Y)$ by $\varinjlim_{X' \xrightarrow{s} X \in (I^X)^\circ} \mathrm{Hom}_{\mathcal{C}}(X', Y)$;

- ◆ 10. (Page 27, line 12) Replace (X', t, f) by (X', s, f) ;

- ◆ 11. (Page 27, line -4) Replace $u \in (I_X)^\circ$ by $u \in (I^X)^\circ$;

- ◆ 12. (Page 28, line 6) Replace $(C(S^{-1}, Q)$ by $(C(S^{-1}), Q)$;
- ◆ 13. (Page 28, line -6) Replace $C(\mathcal{A})(S^{-1})$ by $C(\mathcal{A})(\text{qis}^{-1})$;
- ◆ 14. (Page 32, line 3) Replace $\tau_{\leq a}K \xrightarrow{f} K \xrightarrow{g} \tau_{\geq a+1}K \rightarrow$ by $\tau_{\leq a}K \xrightarrow{f} K \xrightarrow{g} \tau_{\geq a+1}K \rightarrow$ three times;
- ◆ 15. (Page 36, the second paragraph) Replace all $\tau_{[a,b]}L$ by $\tau_{[a+1,b]}L$ and replace $\tau_{[b-1,b]}L$ by $\tau_{[b,b]}L$;
- ◆ 16. (Page 40, line 18) Replace $K^+(\mathcal{J})(\text{qis}^{-1})$ by $K^+(\mathcal{I})(\text{qis}^{-1})$;
- ◆ 17. (Page 40, line -7) Replace (3.8) by (3.10);
- ◆ 18. (Page 41, line 1) Replace $\{M \rightarrow M'', \text{ where } M'' \in K^+(\mathcal{A})\}$ by $\{M \rightarrow M'', \text{ where } M'' \in K^+(\mathcal{A})\}$;
- ◆ 19. (Page 41, line 2) Replace (e.g. 4.13) by (4.18);
- ◆ 20. (Page 41, second paragraph) This proof probably has a mistake that pashout may not preserve monomorphism, see [Ka];
- ◆ 21. (Page 42, lemma 4.29) This proof probably has a mistake that pashout may not preserve monomorphism;
- ◆ 22. (Page 43, line -1) Replace $E' \in \mathcal{A}$ by $E' \in \mathcal{A}'$;
- ◆ 23. (Page 45, line 4) Replace (4.18) by (4.27);
- ◆ 24. (Page 45, line -4) Replace $\eta : FQ \rightarrow QG$ by $\eta : QF \rightarrow GQ$;
- ◆ 25. (Page 46, line 2,3) Replace $F(\varepsilon(L'))$ by $\varepsilon(L')$;
- ◆ 26. (Page 58, line 11) Replace Lemma 6.7 by Proposition 6.7;
- ◆ 27. (Page 60, line -3,-5) Replace zero by trivial;
- ◆ 28. (Page 64, line 6) Replace 6.8 by 6.7;
- ◆ 29. (Page 68, line -4) Replace $C^n(\mathcal{U} \cap V, F)$ by $\check{C}^n(\mathcal{U} \cap V, F)$;
- ◆ 30. (Page 71, line 4) The proof is same as Theorem 8.3 which reduce to the case of Lemma 8.4, so here we use the same homotopy operator k in 8.4;

2 Some Notes

♣(Page 72, Theorem 8.12) **THEOREM OF LERAY.** Let (X, \mathcal{O}_X) be a ringed space and F be an \mathcal{O}_X -module. Let $\mathfrak{U} = \{U_i\}_{i \in I}$ be an open covering of it. If for every nonempty finite subset $J \subset I$ and every $q > 0$ such that $H^q(U_J, F) = 0$ where $U_J = \bigcap_{j \in J} U_j$, then $\check{H}^n(\mathfrak{U}, F) \cong H^n(X, F)$.

The first proof. Consider $\mathcal{H}^q(X, F)$ be a presheaf with $U \mapsto H^q(U, F)$. By Grothendieck spectral sequence, there exists a spectral sequence such that $E_2^{p,q} = \check{H}^p(\mathfrak{U}, \mathcal{H}^q(X, F)) \Rightarrow H^{p+q}(X, F)$ and $\check{H}^p(\mathfrak{U}, \mathcal{H}^q(X, F)) = 0$ for $p > 0$ in this situation. Then the E_2 page is

$$\begin{array}{ccccccc}
 \dots & & 0 & & 0 & & 0 & & \dots \\
 & & \searrow & & \searrow & & & & \\
 \dots & & \check{H}^{p-1}(\mathfrak{U}, F) & & \check{H}^p(\mathfrak{U}, F) & & \check{H}^{p+1}(\mathfrak{U}, F) & & \dots
 \end{array}$$

Since it converge to $H^p(X, F)$ and for now $E_2 = E_\infty$, then we win. Here we use the fact that $\check{H}^p(\mathfrak{U}, -)$ as the right derived functor of $\check{H}^0(\mathfrak{U}, -)$, see St 01EN in [St]. \square

The second proof. See St 01EV in [St]. \square

3 Remarks

- ♠ 1. Here we assume that a single commutative diagram occupies one line;
- ♠ 2. I omitted the section (4.14) about Ext and extensions of groups;
- ♠ 3. If you find errors in my errata, please send to my email: 1225046792@qq.com.

References

- [Illusie] Luc Illusie, *Topics in Algebraic Geometry*, Université de Paris-Sud Département de Mathématiques, <http://staff.ustc.edu.cn/~yiouyang/Illusie.pdf>.
- [Ka] Masaki Kashiwara, Pierre Schapira, *Sheaves on Manifolds*, Springer, 1994.
- [St] Stacks project collaborators, *Stacks project*, <https://stacks.math.columbia.edu/>.