1 Motivation

Electric quadruped robots are now performing useful work in real applications including exploration of abandoned mines [1], inspection of power plants [2, 3], transportation of payloads on rough terrain [4], and more. The remarkable recent translation from research platforms to useful robots was enabled by a *fundamental innovation on the actuation technology* using powerful electric actuators that are capable of rapid and precise torque control [5]. Moreover, the significant decrease in the cost of these motors due to the prosperous drone industry enabled the commercialization of quadruped robots as personal robots at an staggering price of $2,700 [6], a minuscule fraction of the cost from five years ago. But even capable quadrupedal robots are still not able to use tools created for humans and work in environments designed for the anthropomorphic form, such as narrow corridors or climbing ladders. On the other hand, capable humanoid robots could help firefighters in emergency response; could help nurses transport weakened patients; or help workers move heavy boxes within warehouses. These physically demanding occupations are often the cause of injury and disability for numerous human workers in the US [7], with lower back injuries affecting more than 600,000 American workers per year and with an overall cost of $50 billion each year [8]. The fundamental challenge preventing the advent of widely adopted humanoid robots is that their design and control is significantly more complex when compared to quadrupedal robots. The limbs of *bipedal* humanoid robots require additional joints, more powerful actuators, and sophisticated mechanical transmissions often arranged in parallel topologies for load sharing. *Humanoid robots have thus remained as research platforms since their design is still mostly done empirically due to a lack of formal tools supporting the design process.* However, we envision that the electric motors that enabled the remarkable success of quadruped robots can be translated to the creation of physically capable humanoid robots via a formal design process of their complex actuation topology, and its integration into design, simulation, and control.

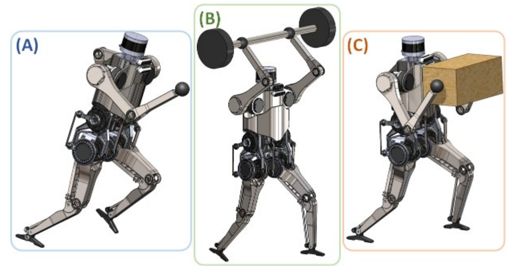

Figure 1: The humanoid Dash will realize multiple whole-body dynamic tasks. (A) Running. (B) Powerlifting. (C) Carrying.

Toward this goal, this project will create an *integration* framework for the co-design of the hardware and control of humanoid robots with electric motors to maximize their physical capabilities. Dynamic whole-body movements enable robots to amplify the forces that they can apply to the environment. For instance, the fast movement performed by weightlifters allows them to raise a heavy barbell over their heads, a feat that would be impossible to realize semi-statically. Designing robots that realize tasks dynamically enables minimizing the robot size and mass, making them safer to interact with humans and more cost effective, as demonstrated by nimble quadrupeds [6, 9]. The main **Intellectual Merit** of this proposal is a **holistic framework for robot integration** of dynamic humanoids and at the core of this framework is a novel co-design optimization formulation that is scalable to multiple tasks (Objective 3). The transformative nature of this formulation is amplified by **three fundamental scientific contributions** of this work: (i) Derivation of performance metrics and design principles tailored to dynamic humanoid robots (Objective 1); (ii) creation of efficient computational tools for simulation and identification of the complex mechanisms employed by these robots (Objective 2); and (iii) extension of whole-body control techniques for dynamic motions to prevent issues with self-collision typical in humanoid robots (Task 3.3). The proposed framework will be employed for the integration of the humanoid robot *Dash* to realize three demanding tasks that impose conflicting requirements on the robot's design (Fig. 1). Since these tasks require operation at the boundary of the robot's performance, the harmonic integration of software and hardware is imperative. We assume that if our co-design framework can create a robot that can perform multiple athletic behaviors, it can also be used to create machines that address most ordinary physical tasks performed by human workers. This effort will leverage the combined expertise of our team in design, fabrication, control, and simulation of legged robots to achieve three objectives:

Objective 1 (years 1-2): Formalize design guidelines for dynamic humanoid robots with electric motors. New *performance metrics* will be created, analyzed, and used as the cost function of the co-design formulation, reducing the dimensionality of the search space in comparison to traditional co-design methods. For practical impact of these fundamental developments, *design principles* will then be distilled and leveraged to guide the ideation process of candidate designs that seed parametric design optimization.

Objective 2 (years 1-2): Reduce the sim2real gap of humanoid robots with complex actuation topologies through fundamental computational advances for *efficient and accurate simulation* of dynamic systems with closed kinematic chains and for *precise parameter identification* of parallel actuation mechanisms. These complex mechanisms are used in the limbs of humanoid robots to facilitate load sharing between motors.

Objective 3 (years 1-4): Formulation of a novel co-design framework and integration of the dynamic humanoid robot *Dash* with electric motors. The core of the framework is a numerical optimization formulation that incorporates results from Objectives 1 and 2 to reduce the dimensionality of the design search space. We will employ this framework to create the humanoid robot Dash to realize three whole-body dynamic tasks with performance around 90% of that of human athletes when normalized by size and mass [10]: (i) *Sprint running* at 3.5m/s ($7.8mph$), a locomotion behavior that requires whole-body coordination. (ii) *Powerlifting* a 15kg ($33lb$) barbell, a manipulation behavior that requires whole-body activation. And (iii) *loaded carrying* a 9kg ($20lb$) payload at 1.3m/s ($2.9mph$), which combines dynamic locomotion and forceful manipulation. *While there is currently no humanoid robot with electric motors that can perform these tasks, the successful integration of high-torque brushless DC motors into humanoids would bring the necessary capabilities to make these feats a reality.*

Broader Impact: Humanoid robots with physical capabilities comparable to those of the average human could protect workers that regularly realize demanding physical labor such as firefighters, policemen, or nurses. The anthropomorphic shape of the robot would allow it to use tools designed for humans and to be directly deployed to existing work environments without major structural modifications. Towards that vision, this work will leverage the novel co-design framework to create dynamic humanoid robots with the widely available electric actuators popularly used in quadrupeds. Even if the framework is unfruitful, we envision that the design guidelines and tools will represent a common language to evaluate and compare dynamic robots beyond humanoids, such as manipulators and prosthetic devices. To nurture the adoption of this common language, Broader Impact activities include a workshop featuring discussions with experts in humanoid design and control from academia and industry. The lessons learned from this research will be reflected in education through the creation of a course on the design and control of dynamic robots co-taught between the PI's universities. Moreover, in line with the collaborative essence of this research, the robot will be open-sourced to lower the barrier to entry for research, accelerate the progress of existing groups, and establish a common platform for benchmarking control approaches. To promote wide adoption of this platform, we target a maximum total cost of $60k, which is a realistic equipment budget for typical research proposals. For context, the NSF-funded Open Leg project with two motors has an estimated cost of $25k [11]. We propose a robot with total mass $\leq 40\text{kg}$ ($88lb$) and height $\leq 1.2\text{m}$ ($47in$). From our past experience with robots up to human scale, we believe more research groups will be willing to adopt a smaller and safer robots. We are confident that facilitating access to a capable platform will accelerate the research progress on humanoid robots and expedite their transition to useful tools in the US.

Fit within NSF's National Robotics Initiative 3.0: This proposal supports the vision of the NRI 3.0 by creating a novel framework for the *integration* of dynamic humanoid robots with electric motors. To enable this framework, this work makes fundamental contributions to the science of humanoid design, simulation, identification, and control. *While these fundamental science aspects are new, it is their integration that represents the ultimate focus and transformative potential of this work.* The ***Intellectual Merit regarding the integration of robotic systems*** is the co-design framework that pushes robot potential by considering how hardware and control software co-depend on one another. At the core of the framework is a formulation that employs condensed design metrics to enable the co-design process to encompass multiple behaviors. Powerful electric motors revolutionized the field of quadruped robots in the past decade. The integration barrier when translating this actuation technology to a humanoid system with more complex limbs represents the singular focus of this proposed work.

2 Previous relevant work by members of the proposal team

This four-year effort will leverage the combined expertise of the PI's in design, control, and simulation of legged robots (Fig. 2). **PI Ramos** is an expert in the integration of complex robotic systems, such as the

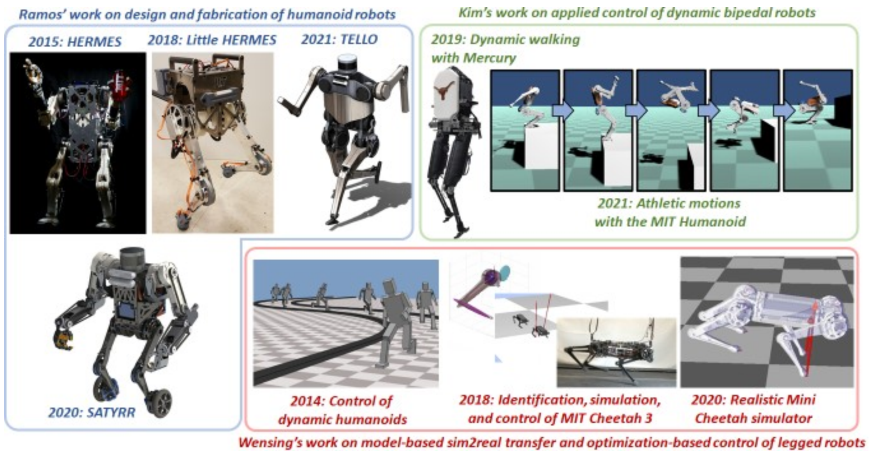

Figure 2: The combined experience of the PI's will be fundamental for the integration of a capable humanoid robot. PI Ramos has integrated several dynamic robots in the past, such as HERMES, Little HERMES, TELLO, and SATYRR. Co-PI Wensing has experience in the dynamics and control of legged robots, including fundamental advances to system identification and dynamic simulation that enabled sim2real transfer with the MIT Cheetah 3 and Mini Cheetah. Co-PI Kim has extensive experience on whole-body planning and control of athletic motions for humanoids.

full scale humanoid *HERMES* [12–15], the small biped *Little HERMES* [16–18], an exoskeleton for human augmentation [19, 20], assistive mobility devices [21], robotics kits for education [22], and more. The PI is currently developing the humanoid robot TELLO [23] and the wheeled-biped SATYRR [24]. The existing TELLO hardware will be used to experimentally validate the contributions from all objectives. Although the PI studied particular principles for the integration of legged robots, this proposal will formalize a holistic hardware and control co-design procedure specifically for dynamic humanoid robots with electric motors. Ramos will oversee the entire project and lead the work in Objective 1 to formalize fundamental performance metrics and design principles for dynamic humanoid robots. His expertise in hardware integration will complement the expertise of the Co-PI's for creating Dash in Objective 3.

Co-PI Wensing has extensive expertise in the dynamics [25–27], identification [28–30], and control [31–37] of legged robots, while also bringing dynamic systems perspectives to the design of these platforms [38–40]. Wensing will lead the efforts of Objective 2 where his expertise on dynamics [25–27] and efficient simulation for the MIT Cheetah robots [41, 42] will be primarily leveraged. The identification advances proposed will build naturally from his recent work on geometric aspects of identification [29, 30]. The proposed research as part of this project diverts from his past research in its study of mechanisms with closed kinematic chains, which present fundamental challenges compared to his past work on open-chain systems. Wensing will provide a supporting role in the work carried out in Objectives 1 and 3.

Co-PI Kim has extensive expertise in planning and control of athletic motions for legged robots. His previous work showed that a whole-body dynamics-based locomotion controller (WBLC) can balance point-foot biped robots that are not statically stable [43, 44]. Later, the WBLC was extended by the integration with model predictive control and the performance of the integrated control framework was validated by high-speed running of a quadruped robot, Mini-Cheetah [45]. Recently, he developed an actuator-aware trajectory optimization and control pipeline for acrobatic motions of a humanoid robot [46] and dynamic landing of a quadruped robot [47]. This work builds on his recent work to address the critical issue of self-collision avoidance during highly dynamic movements.

Other related work. The legged robots introduced by Raibert in the 80's where capable of impressive athletic feats even for today's standards [48–50]. The success of these machines was largely enabled by their clever hardware design, which leveraged low-inertia legs and high-force-density hydraulic actuation [51]. This proper design permitted the successful implementation of simple controllers based on intuitive heuristics. In contrast, although the research on humanoid robots experienced remarkable progress that culminated at the 2015 DARPA Robotics Challenge [52], none of the competing robots displayed the athletic capabilities of Raibert's robots. This performance gap can be attributed to most humanoids inheriting their hardware from position-controlled industrial manipulators due to the lack of design guidelines specific to humanoids [53, 54]. To this date, many of the studies that describe the hardware of humanoids showcase outstanding engineering work [55–66], but lack a holistic and principled framework to design them *from task to embodiment* and specifically for dynamic and demanding behaviors. **This research will fill this gap with a definite departure from the old design paradigm of industrial manipulators and will develop the design tools and metrics specifically tailored for dynamic humanoid robots.**

3 Research description

The proposed research will pursue three objectives along four years. The first is to study *performance metrics* that are tailored for dynamic humanoids and devise *design principles* that are based on key properties of the actuation topology. This proposal aims to formalize these concepts that have been empirically used by the robotics community to create legged robots. The design principles will seed the ideation process for candidate actuation topologies and the performance metrics will be employed as costs and constraints in the co-design optimization formulation developed in Objective 3. The second objective will develop a *novel computational tool* to accurately and efficiently simulate closed-chain (parallel) actuation mechanisms that are typically employed in humanoid robots. Moreover, a *novel algorithm for system identification* of this family of mechanisms will be devised. These simulation and model identification tools will be experimentally employed in Objective 3 for the co-design of the robot Dash. Finally, the third objective is to leverage the principles, metrics, and tools developed in the first two objectives to formulate a *novel co-design optimization framework* to create dynamic humanoid robots with electric motors for multiple tasks. This formulation with be used in Objective 3 to create the humanoid Dash by targeting three demanding dynamic tasks: (i) sprint running, (ii) powerlifting, and (iii) loaded carry. **A detailed timeline is provided in the Collaboration Plan** and the leading roles in Figure 3.

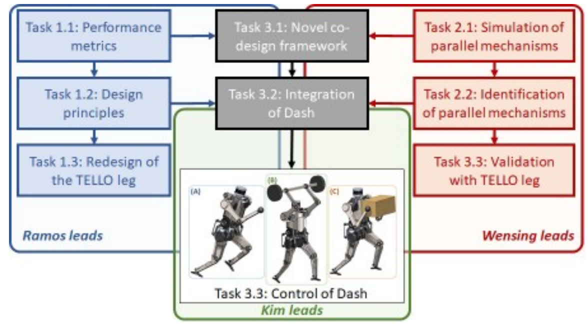

Figure 3: Objectives and leading roles for this proposal. Ramos will lead Objective 1, Wensing will lead Objective 2, and Kim will focus on the substantial software and control integration effort to achieve the targeted tasks. All PI's will support each other across all tasks, in particular for the co-design and integration of Dash in Task 3.2. Three copies of Dash will be integrated.

3.1 Objective 1: Formalize design guidelines for dynamic humanoid robots with electric motors.

Multiple challenges complicate the process of designing the actuation of capable humanoid robots, but effectively no formal guidelines exists. The robot must perform several activities, such as running and carrying objects, that impose demanding and conflicting requirements on the actuators (e.g. high speed *and* high torque). For instance, during running the leg must generate two to three times the force required to support the robot's entire body weight statically, and the same limb must swing within a fraction of a second to reach the next foothold. To address this challenge, existing co-design approaches augment the formulation of trajectory optimization (TO) to simultaneously select key parameters of the motors, transmissions, and limb for a particular task [40, 67–70]. However, this approach leads to two key limitations that hinder their use for designing multipurpose humanoid robots: (i) They *are not scalable to multiple tasks* due to the massive number of decision variables and the nonlinearity, hybrid nature, and constraints of the equations of motion of humanoids. And (ii) they *require a near-optimal seed topology* because the discontinuous and immense design space of humanoids cannot be easily parametrized. But this seed is difficult to conceive because there are infinite combinations of mechanisms that can potentially address the task requirements. Task 1.1 will address the first limitation by deriving condensed *performance metrics* to reduce the number of decision variables for co-design of humanoids and enable the simultaneous optimization of multiple tasks. Instead of searching for a complete trajectory for each of the robot's behaviors like is typically done in TO, the solver only optimizes for key instances that require peak performance scores. To mitigate the second limitation of existing co-design approaches, Task 1.2 will compile *design principles* to guide the ideation process for candidate actuation topologies. Designers will leverage these "design rules" to propose feasible mechanisms that will be fine tuned via optimization. Finally, Task 1.3 will leverage the metrics from Task 1.1 and the principles from Task 1.2 to revise the design of the existing TELLO hardware [71]. At the end of Objective 1, we will select an improved candidate actuation topology that will seed the co-design framework formulated in Task 3.1 as part of Objective 3.

Task 1.1. Create dynamic performance metrics for humanoid robots with electric motors: Conventional co-design approaches augment TO to include hardware parameters of the robot as decision variables. For instance, direct collocation methods discretize the equations of motion and select the position q_k, velocity

$\dot{q}_k$, acceleration $\ddot{q}_k$, and torque τ_k for each of the robot's joints at each discretization node k [72]. However, fine discretization of the whole-body trajectory of high-dimensional humanoids for multiple task results in a large and nonlinear formulation with multiple local minima and that is very challenging to solve robustly. To vastly facilitate this search, in this Task we will formulate performance metrics that succinctly describe the hardware and control requirements for dynamic humanoid robots with electric motors. Similar metrics, such as accuracy and repeatability, have been extensively used to quantify the performance of in industrial manipulators that realize position-control tasks like welding. However, for legged robots, the ability to perform fast motions and precise force control is more relevant than achieving sub-millimeter precision (c.f., [73, 74]), and hence, novel metrics are needed. We will use these special metrics to design dynamic humanoids by maximizing the performance scores at a few *key instances in time*, instead of optimizing for *all* time steps of the *entire* motion of *all* the joints for *each* task, as it is typically done in TO.

Proposed Research: First we compile performance metrics that summarize the hardware and control requirements for dynamic humanoids. These metrics include those adapted from other platforms and those we propose that are specific to dynamic humanoids. The metrics *adapted* from the literature include:

- **Generalized Inertia Ellipsoid (GIE) [75, 76]:** It describes the configuration dependent inertial properties of the limbs perceived by external forces (*task-space inertia*). It is affected by the inertia of the rotors, transmission, and links, and is augmented by the bidirectional energy efficiency [23].
- **Impact Mitigation Factor (IMF) [38]:** quantifies energetic losses and the shock loads during impact events. Given by the ratio between the inertia of a limb and the total inertia of the robot.
- **Kinematic Manipulability (KM) [77, 78]:** is a widely adopted measure of the dexterity of robot manipulators and an indicator for singular poses (also known as *manipulability index*).
- **Dynamic Manipulability (DM) [77, 79]:** given by the ratio between the achievable task-space force and the limb inertia. Describes the robot's ability to accelerate its limbs, for instance during leg swing.

However, these metrics do not address all requirements for dynamic humanoid robots, and additional metrics are needed. The *novel metrics* for dynamic humanoid robots that will be explored include:

- **Centroidal Inertia Isotropy (CII):** describes how the centroidal inertia of the robot's body varies according to the pose of the limbs [80]. *It quantifies the error between the full robot model and the centroidal dynamics* [81]. Dash will be designed for a maximum CII score so its model can be accurately approximated by a single rigid body.
- **Dynamic Force Density (DFD):** describes the robot's ability to actively generate contact forces during high-speed movements and normalized by the total robot mass. Similarly, the **Static Force Density (SFD)** describes the robot's ability to resist external forces (e.g. its own weight) semi-statically.
- **Contact Force Bandwidth (CFB):** describes the ability to rapidly produce contact forces to propel itself or manipulate objects. Proportional to the ratio between transmission stiffness and limb inertia.
- **Reactive Limb Acceleration (RLA):** quantifies reactive acceleration produced on the body due to the acceleration of the limbs. Swinging lightweight limbs minimally perturb the body. [48, 51].

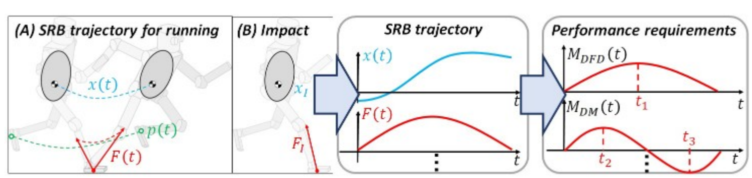

Figure 4: Computation of required metric scores and critical instances for running. Trajectory optimization computes high-level trajectories for the single rigid body model for running and impacting. These trajectories are use to compute critical instances t_i when peak performance is required.

These metrics are utilized to identify the instances t_i of peak performance requirements for targeted tasks. First, we use a general reduced model of the robot that *permits the abstraction of the actuation topology of the limbs*. Reduced-order models, such as the classic Linear Inverted Pendulum and others [25, 81–86], have been extensively used for plannning and control of legged robots, and here we extend this idea to co-design. We employ the single rigid body (SRB) model to compute "best case scenario" scores that will be targeted by candidate mechanisms during the co-design process. Figure 4 illustrates the proposed procedure to find peak scores and critical instances for the running task, although additional tasks could be considered simultaneously.

Running is described at a high-level by the trajectories of the robot's center of mass $x(t)$, the swing foot $p(t)$, and the ground reaction force $F(t)$. These high-level trajectories are obtained via conventional trajectory optimization of the SRB model *independently* from the leg actuation topology [87]. The ground reaction force $F(t)$ propels the robot forward, and hence, running fast means that the robot must generate, within a short stance time, large contact forces when compared to the total mass of the machine. This performance requirement is captured by the *Dynamic Force Density* score $M_{DFD}(t)$ which reaches a peak at instant t_1 in Figure 4. In addition, the foot must be rapidly accelerated during swing, leading to peak *Dynamic Manipulability* scores $M_{DM}(t)$ that are required at instants t_2 and t_3. Finally, during impact at time t_4, the *Impact Mitigation Factor* must be maximized to reduce the shock loads that could damage the robot. The other performance metrics are also affected by the trajectory of the SRB model for running. And hence, to create a humanoid robot that is capable of running, the hardware design must simultaneously meet the scores for each of the metrics for all critical instances t_i. If multiple tasks are targeted, additional SRB trajectories are computed and additional critical instances t_i are identified. *The goal of Task 1.1 is to use the SRB model to understand and quantify how each of the targeted behaviors (running, lifting, carrying) correlate to each of the performance metrics*. This information will allow us to formulate the numerical optimization in Task 3.1 that selects physical parameters of the robot by approximating the performance scores of the candidate actuation topology to those of the SRB model. Overall, the novelty of this Task 1.1 is to find concise and representative co-design criteria. Optimizing for concise performance metrics instead of the full body trajectories will enable scaling of the co-design formulation to multiple behaviors by reducing of the search space and avoiding local minima.

Task 1.2. Derive design principles for dynamic humanoid robots with electric motors: Existing numerical co-design approaches require a favorable seed candidate topology because the design space of humanoid robots is overly large and too complex to be parameterized. To address this challenge, in this Task we will compile *design principles* for dynamic humanoid robots to guide the ideation process for candidate actuation topologies. Towards this goal we study the correlation between *fundamental properties of the actuation* with the performance scores on the metrics from Task 1.1. This data will allow us to compile novel design principles and validate established empirical rules such as *"Employing gearboxes with low reduction ratio and high energy-efficiency improves the limb's backdrivability"*. Although this empirical principle has been extensively employed in the design of capable quadrupeds and co-bots [9, 38, 39, 88–90], it has only been theoretically validated in the context of legged robots in the PI's recent work [23]. It is worth transparently acknowledging that the distillation of design principles does not necessarily follow a scientific process. However, the PIs view this research task as critical to the **broad impact** of the research so that the fundamental merit of the previous task can be communicated in a way that is most valuable to designers.

The proposed research will study how the mechanical transmission and the limb topology modifies the interactions between the motor and the environment. To illustrate this challenge, Fig. 5 shows the mechanical transmission of the knee joint adopted by four established quadruped robots ordered by mass. Although all share almost identical kinematic topologies and similar speed reduction ratios, their knee transmissions employ vastly different designs. The Open Dynamics' Solo (2.2kg) [91] and MIT Mini-Cheetah (9kg) [9] employ highly efficient timing belts that are sufficiently strong components at their small scale. However, their load capacity and stiffness is insufficient at the scale of the MIT Cheetah robot (45kg) [39], which employs stronger but less energy-efficient metal chain. In contrast, Boston Dynamics' Spot (25kg) [92] uses a screw mechanism that is both stiffer and more efficient than metal chains, but substantially more expensive and complex. The selection of these components can be done empirically for quadrupeds due to the simplicity of their 3 DoF limbs. However, the limbs of humanoid robots typically have 6 DoF, additional distal joints, and often use

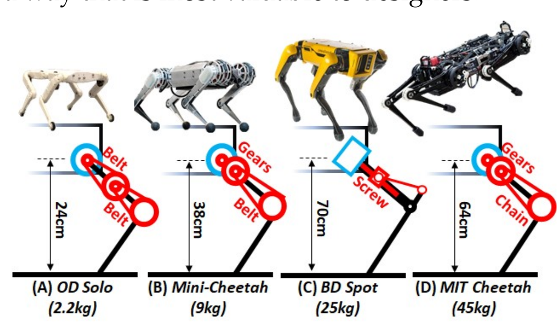

Figure 5: Comparison of knee transmission designs for (A) the Open Dynamics' Solo, (B) the MIT Mini Cheetah, (C) Boston Dynamics' Spot, and (D) the MIT Cheetah. The electric motor is shown in blue and the transmission in red.

complex hybrid serial and parallel actuation schemes [54, 93]. And hence, to understand the engineering trade-offs of transmission design for humanoid robots, we model how their fundamental properties affect the equations of motion of the *full robot*.

Proposed Research: We define the *most relevant* fundamental properties of transmissions for dynamic humanoid robots: **(i) Bidirectional efficiency:** Mechanical transmissions, such as gearboxes, have different energy-efficiency values if the input power flows from the motor to the load (forward driving efficiency η_F) or in the opposite direction (backward driving efficiency η_B) [94]. It is always true that $\eta_B \leq \eta_F \leq 1$ [95]. **(ii) Speed reduction ratio:** Given by the ratio between the speed of the rotor of the motor and the joint. **(iii) Passive stiffness:** Given by the *inherent* mechanical compliance of the transmission components. For instance, metal gears are considerably stiffer than cable-driven transmissions. **(iv) Load capacity:** Given by the maximum force/torque that the transmission can sustain before failure. **(v) Joint coupling:** Describes how multiple motors drive multiple joints simultaneously. Parallel actuation can reduce the torque requirements without increasing the limb's reflected inertia. This principle is used in TELLO's design.

Next, we model how each fundamental property affects the equations of motion of the robot and the performance metrics. For instance, the PI's work demonstrated that the *bidirectional efficiency* of the transmission modifies the limb's *Generalized Inertia Ellipsoid* [23]. When the backwards efficiency η_B is low, the end-effector appears to have larger inertia due to friction amplification (Fig. 6). We employ a new approach for coordinate reduction of the conventional Lagrangian formulation to obtain the augmented equations of motion of the full robot that capture this phenomena. The end result is that the inertia matrix $M(\eta_B^{-1}, q)$ is directly affected by the inverse of the backwards efficiency η_B. This means that decreasing the backwards efficiency increases the apparent inertia of the robot. The extreme case of "infinite" inertia occurs when $\eta_B \approx 0$ and external forces cannot backdrive the robot. This model captures how industrial manipulators are not backdrivable because they employ strain wave gearboxes ($\eta_F < 60\%$ and $\eta_B \approx 0\%$). The correlation between the actuation's *bidirectional efficiency* and the *Generalized Inertia Ellipsoid* allows us to devise the design rule: *"If backdrivability is imperative to the task, the designer should employ transmissions with high energy-efficiency, such as spur gears"*. This principle is commonly used in transparent haptic devices and other robots that physically interact with humans [90, 96]. All other fundamental properties will similarly modify the equations of motion of the robot to some extent. *The goal of Task 1.2 is to understand and quantify how each of the fundamental parameters of the limb's actuation correlate to each of the performance metrics.* A design principle is formulated when there is a high correlation between a property of the transmission and a metric.

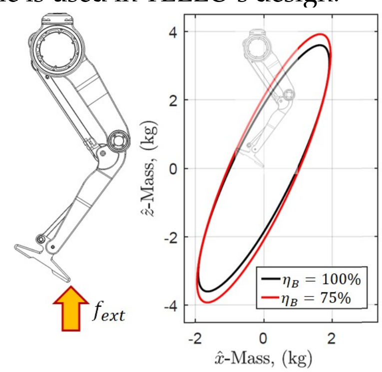

Figure 6: The Generalized Inertia Ellipsoids (GIE) perceived by an external force f_{ext}. Low backdriving efficiency (red) of gearboxes inflates the nominal apparent inertia (black) of the leg [23]. The GIE is infinite when $\eta_B \approx 0\%$.

Task 1.3. Redesign of the TELLO leg using performance metrics and design principles for dynamic humanoid robots with electric motors: Although TELLO was not intended to perform the demanding tasks targeted in this proposal, the design of its leg embodies empirical design principles for dynamic humanoid robots. The principles include: (i) motors with large gap radius to increase torque density [5, 38], (ii) energy-efficient ($> 90\%$) and low reduction ratio ($6:1$) planetary gearboxes with spur gears, (iii) actuation coupling to increase force density without inflating the reflected inertia, and (iv) motors placed close to the body to reduce the limb's moving inertia. However, the design and topology of TELLO's leg can be further optimized to perform dynamic motions. *The goal of Task 1.3 is to reformulate the design of the TELLO leg using the principles derived in Task 1.2. to address the tasks of running, powerlifting, and loaded carry.* The improved leg topology design will seed the co-design optimization formulation in Objective 3.

Proposed Research: First we will experimentally validate the performance of the TELLO leg compared to the required scores from the SRB model. We aim to understand how the limitations that were experimentally observed are captured by our performance metrics. For instance, because the motors that drive the ankle are located near the hip, the transmission is inherently more compliant due to the long belts, which leads to low-frequency undamped oscillations. However, previous work showed that the effective stiffness of the joint can be increased by placing the gearbox closer to the joint and after the belt transmission instead of near the motor [97]. And hence, we expect to compute a higher *Contact Force Bandwidth* for this improved design in comparison to TELLO's original design. Next we will employ the novel design principles devised in Task 1.2 to propose an improved actuation scheme and leg topology. For instance, in addition to the improved transmission stiffness, we expect that the hip and knee flexion joints will require motors with greater torque density for the running task.

3.2 Objective 2: Reduce the sim2real gap of robots with complex actuation topologies

In parallel, Objective 2 will advance the ability to accurately simulate and identify systems with the closed-chain parallel actuation mechanisms optimized in Objective 1. These advances are critical for reducing the sim-to-real gap of such machines. A common practice is to treat the actuators as torque sources, however, electric actuators with large radius themselves have non-trivial physical dynamics due to effects of quickly rotating high-inertia motor rotors. Addressing these effects has been key in improving the simulation accuracy and efficiency of the Cheetah robots [28], and likewise for the sim2real transfer of policies into experiments. While existing methods are applicable to the open-chain leg design of quadrupeds [28, 98], this objective will explore new strategies that are tailored to closed-chain designs.

Task 2.1. Efficient simulation of systems with parallel actuation: The first task of this objective will develop methods for rapidly simulating systems that include parallel actuation without sacrificing physical accuracy. This fundamental advance will leverage the Gauss principle of least constraint [99, 100], which is equivalent to more classical mechanics principles (e.g., D'Alembert) but is comparatively underemployed.

To illustrate the proposed advances, consider the standard equations of motion for a robot $\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{\tau}_g(\boldsymbol{q}) = \boldsymbol{\tau}$ where $\boldsymbol{q} \in \mathbb{R}^N$ gives the joint angles of the robot, $\mathbf{M}$ is the mass matrix, $\mathbf{C}\dot{\boldsymbol{q}}$ Coriolis and centripetal terms, and $\boldsymbol{\tau}_g$ gravity torques [101]. To address the reflected inertia of motors, a common strategy is simply to add a term such as $n_i^2 I_i^R$ to each diagonal element of $\mathbf{M}$, where n_i gives the gear ratio for the i-th gearbox and I_i^R is the rotational inertia the associated motor rotor. This approximation works well when the motor inertia I_i^R is small, since it neglects Coriolis effects or coupling effects onto other joints. With these assumptions, the most efficient algorithm for computing the forward dynamics of the system is the Articulated Body Algorithm (ABA) [101], whose computational cost scales as $O(N)$.

This approximation breaks down with actuation where the gear ratio is small and the actuator inertia is large. To reduce the sim-to-real gap in these systems, the dynamic effects of actuator rotors can be addressed using constraint handling techniques. Consider a set of maximal coordinates $\boldsymbol{q} = [\boldsymbol{q}_J^T, \boldsymbol{q}_R^T]^T$ where $\boldsymbol{q}_J$ represents the angles for the joints and $\boldsymbol{q}_R$ the angles between each rotor and its preceding link. Theses angles are related by the constraint $q_{J_i} = n_i q_{R_i}$. These constraints can be handled by either 1) modelling them as soft constraints or 2) treating them as hard constraints and solving for necessary constraint forces. The first approach, taken in MuJoCo [102], has the advantage that the ABA can be used for efficient computation, but its physical accuracy is low when large time-steps are used. The second approach, taken in Bullet [103], has an advantage of accuracy with longer integration time steps, but has $O(N^3)$ cost per step.

Proposed Research: This task will develop a method for simulating actuator dynamics in systems with parallel actuation topologies while maintaining accuracy and minimizing computational cost. To make this fundamental step forward, this work will explore the recursive application of the Gauss principle of least constraint to address actuator dynamics for parallel actuation schemes. Roughly, the Gauss principle of least constraint states that the accelerations of bodies in a constrained system deviate as little as possible from the accelerations they would experience in the absence of constraints. Returning to our maximal coordinates $\boldsymbol{q} = [\boldsymbol{q}_J^T, \boldsymbol{q}_R^T]^T$, this constraint problem would take the form:

$$\min_{\ddot{\boldsymbol{q}}} \; (\ddot{\boldsymbol{q}} - \ddot{\boldsymbol{q}}_{unc})^T \mathbf{M} (\ddot{\boldsymbol{q}} - \ddot{\boldsymbol{q}}_{unc}) \quad \text{subject to} \quad \ddot{q}_{J_i} = n_i \ddot{q}_{R_i} \tag{1}$$

where $\ddot{\boldsymbol{q}}_{unc}$ represents accelerations that would be experienced if joints and rotors could move independently. In a recent undergraduate thesis in Co-PI Wensing's group [42], we explored how to reformulate this problem at a body-by-body level using dynamic programming principles, providing an $O(N)$ algorithm to solve (1) that is implemented in the open-source Cheetah Software [41]. The resulting algorithm resembles the form of one in [98], however, the use of the Gauss principle provides a general tool for

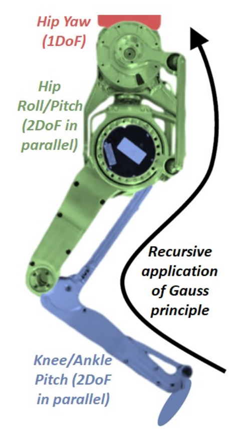

Figure 7: Conceptual approach for simulation strategy.

extension to a wider class of systems.

The main strategy to extend this approach to parallel actuation mechanisms is illustrated in Fig. 7. Rather than treating the bodies one-by-one, the approach will lump together sets of bodies connected via parallel actuation modules. This will result in a more involved invocation of the Gauss principle when treating the coupled constraints across these joints, which will cost $O(N_p^3)$ at each step where N_p is the number of bodies actuated in parallel. Since the approach will only pay a cubic penalty for coupled actuation modules, as opposed to for all bodies, it should be much faster than the second constraint-handling approach described above. Algorithms will be developed theoretically, benchmarked in C/C++, and distributed open source. We consider this work integrative in nature, since the fundamental theory has been demonstrated in our previous work, but the theory needs to be extended to the new actuator strategies pursued in this work. Given that the most commonly used simulators have downsides compared to this approach, and the path forward is clear, we view this task as low-risk and high-reward direction.

Task 2.2. Identifiability analysis of closed-chain mechanisms: The second subtask will provide computational tools that address the identifiability of dynamic properties of actuation components. In the classical open-chain (non parallel) actuation case, the majority of the inertial properties (e.g., mass, center of mass location, rotational inertias) of each actuator can be lumped into the mass of the preceding link [29]. For the robots designed in Objective 1, it is of fundamental importance to understand if and how the arrangement of parallel actuators (e.g., in line with one another, orthogonal, etc.) influence the ability to identify the actuators, and to deduce which experimental conditions are necessary to carry out the identification.

Proposed Research: This work will consider a new approach to identifiability analysis of robot mechanisms using tools from algebraic geometry. The main idea is as follows. While parallel actuators generally lend to complex kinematics without closed-form solutions, we can consider a set of maximal coordinates as before. In those maximal coordinates, the unconstrained equations of motion are polynomial in the set of variables $\{s_i, c_i, \dot{q}_i, \ddot{q}_i\}_{i=1}^{M}$ (see, e.g., [104]) where $c_i = \cos(q_i)$ and $s_i = \sin(q_i)$. Since closed-chain constraints (e.g., from a four bar mechanism) are polynomial in the same variables [104], it then follows that all such constraints can be written down in the form $\mathbf{g}(\mathbf{s}_{\boldsymbol{q}}, \mathbf{c}_{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = 0$. Collecting constraints as $\mathbf{h} = [\mathbf{g}\ ;\ \mathbf{s}_{\boldsymbol{q}}^2 + \mathbf{c}_{\boldsymbol{q}}^2 - 1] = 0$, these observations imply the system evolves on the following algebraic set:

$$\mathcal{P} = \{(\mathbf{s}_{\boldsymbol{q}}, \mathbf{c}_{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \mid \mathbf{h}(\mathbf{s}_{\boldsymbol{q}}, \mathbf{c}_{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = 0\}\,. \tag{2}$$

The novelty of this subtask comes from exploring the use of Gröbner basis tools from algebraic geometry to address model identifiability with closed-chain actuation mechanisms. Toward this aim, we return to our original equations of motion in an alternative form $\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{\tau}_g(\boldsymbol{q}) = \mathbf{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\pi}$ where $\mathbf{Y}$ is a regressor matrix [105] and $\boldsymbol{\pi}$ denotes inertial parameters of all of the bodies and actuators. For open-chain systems, one can understand which parameters are identifiable by looking at the set $\{\boldsymbol{\pi} \mid \mathbf{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\pi} = 0,\ \forall \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}\}$ [29]. For closed-chain systems, however, there are no theoretically established methods due to the added complexity from constraints (2). The fundamental insight we explore is that the regressor $\mathbf{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ itself is polynomial in $\mathbf{s}_{\boldsymbol{q}}, \mathbf{c}_{\boldsymbol{q}}, \dot{\boldsymbol{q}}$,, and $\ddot{\boldsymbol{q}}$. Using this insight, identifiability analysis proceeds by considering the remainder of the regressor entires following polynomial long division by a Gröbner basis for the constraint polynomials $\mathbf{h}$ [106]. This process will "factor out" the constraints, reducing the identifiability problem down to an unconstrained one.

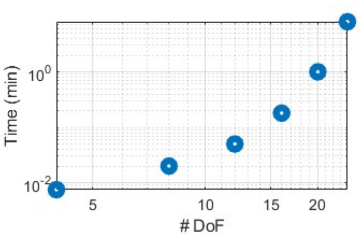

Figure 8: Scaling of Gröbner basis calculations for a humanoid without parallel actuation.

Since Gröbner basis strategies are known to inherently suffer from poor scaling (i.e., exponential complexity or worse), we considered applying this method for the open-chain case to benchmark expected compute times. Figure 8 shows the computation required to carry out the proposed strategy as we add joints to the limbs of a humanoid (the target being six joints per limb for a total of 24 joints). Despite showing exponential scaling up to this point, the absolute time required is still under 10 minutes. While we expect this time to increase when loop closures are added, multiple orders of magnitude increase would still not prevent success of this task. Figure 8 was generated using MATALB, and there are more efficient tools available if needed (e.g., [107, 108]). While these methods should be interesting from a theoretical

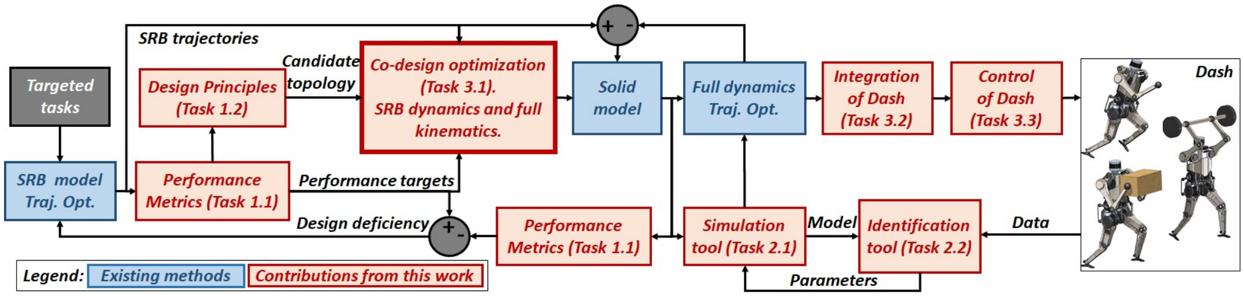

Figure 9: Proposed framework for the co-design of robot hardware and control. At its core is a scalable formulation based on condensed metrics.

perspective, practically, they will be valuable to assess which dynamic parameters of the actuators can be identified in table-top experiments, and which dynamic parameters will affect the robot's movement "in the wild". These theoretical insights will be used to design a set of experimental conditions that can be used to fully identify the novel actuation systems considered herein.

Task 2.3. Simulation and identification and validation with the TELLO leg: The final subtask will quantify the effects of the actuator modelling advances with the TELLO Leg available in the lab of PI Ramos. To validate the advances in Task 2.1, a preliminary model of the TELLO leg will be implemented in the open-source simulator developed by the proposing team. The Gauss principle simulation solution will be compared with competing strategies that rely upon soft and hard constraints. The methods will be compared by looking at the simulation time required while maintaining a target level of constraint tolerance. To validate the advances in Task 2.2, identifiability analysis will first be used to design identification experiments for TELLO. For example, these experiments might involve attaching the system to a tabletop in different configurations (e.g., so that gravity effects are felt differently) or attaching it to a force plate. The designed experiments will then be run while collecting measurements of actuator torques, joint angles, and any external load cells. Identifiability results from the algebraic geometry analysis will be validated against a numerical analysis from these tests. To assess the benefits of actuator modelling, identification will be run with and without actuator models, with estimation residuals compared.

3.3 Objective 3: Formulation of a novel co-design framework for the integration of dynamic humanoid robots with electric motors

The third objective of this work is to formulate a novel co-design framework and employ it for the integration of the dynamic humanoid robot Dash. *This framework, and its synthesis with the previous Objectives, represents the main contribution to the science of robot integration from this research.* The joint consideration of the design of the robot's hardware and control strategy often leads to issues with methodological scalability due to their individual complexity. The nonlinear, hybrid, and constrained nature of the equations of motion of the robot is commonly the bottleneck for scalability, especially when involving parallel mechanisms due to the added constraints. The key idea to enable scalable co-design in Objective 3 will be to make use of the new and existing design metrics from Objective 1 to optimize over a concise description of robot performance without having to consider the complexity of the robot's full dynamics when evaluating candidate designs. The core of our framework (Fig. 9) is an optimization formulation developed in Task 3.1 that combines the simple dynamics of the SRB model with the full kinematics of the robot's limbs [87]. Employing reduced order models is a common approach to robot control [87] and here we translate it to co-design by combining the results from Objectives 1 and 2 with the formulation in Task 3.1 to formalize the co-design framework. This framework will then be used to create the humanoid robot *Dash* in Task 3.2. Finally, Task 3.3 focuses on the controller design, addressing self-collisions that often represent a safety barrier to executing dynamic motions on robot hardware.

Task 3.1. Scalable Co-Design Methodology Based on Performance Metrics: The goal of this task is to formulate and solve a numerical optimization problem that employs design metrics to facilitate the search for optimal robot design parameters. The goal of the optimization is to maximize the summed score of a candidate design across multiple required tasks. For instance, as mentioned in Objective 1, sprint running requires peak performance scores for metrics that include *Dynamic Force Density*, *Dynamic Manipulability*,

Impact Mitigation Factor, and others. These metrics reach a peak requirements at multiple keyframes in time. And hence, for each task i, metric j, and keyframe k we consider the metric score $M_j(\boldsymbol{\lambda}, \boldsymbol{q}_{ijk}, \dot{\boldsymbol{q}}_{ijk})$. To place all metrics on even footing, we consider each to be normalized by the performance score from the SRB model from Objective 1, with the normalized metric denoted $\overline{M}_\ell$, where $\ell = \{i, j, k\}$. Then, our goal is to select parameters $\boldsymbol{\lambda}$ for a design that outperforms the SRB model's targets as much as possible. This formulation combines the simple dynamics of the SRB model with the full kinematics of the candidate design. The design parameters $\boldsymbol{\lambda}$ include the robot's geometry (e.g., length of limb segments), the reduction ration of each transmission, and the characteristics of the electric motors (mass and radius). Previous work from the co-PI showed that the large radius electric motors employed in quadrupeds follow a scaling factor that is proportional to the motor's total mass and radius [38]. This means that other key motor parameters such as torque constant, rotor inertia, and peak torque can be derived from the motor's mass and radius.
Proposed Research: We will implement the following preliminary version of the co-design formulation:

$$
\begin{aligned}
\underset{\substack{\text{Design Params } \boldsymbol{\lambda} \\ \text{Keyframe Configs } \{\boldsymbol{q}_\ell\}}}{\text{maximize}} \quad & \sum_{\substack{\text{Tasks } i \\ \text{Metrics } j \\ \text{Keyframes } k}} \overline{M}_\ell(\boldsymbol{\lambda}, \boldsymbol{q}_\ell, \dot{\boldsymbol{q}}_\ell) && \text{(Normalized Summed Score)} \\
\text{subject to} \quad & \underline{\boldsymbol{\tau}}(\boldsymbol{\lambda}, \dot{\boldsymbol{q}}_\ell) \le \mathbf{J}_c(\boldsymbol{q}_\ell)^T \mathbf{f}_\ell \le \overline{\boldsymbol{\tau}}(\boldsymbol{\lambda}, \dot{\boldsymbol{q}}_\ell) && \text{(Torque Feasibility)} \\
& f_{FK}(\boldsymbol{\lambda}, \boldsymbol{q}_\ell) = \mathbf{x}_\ell && \text{(Matching Simple-Model Task Plan)} \\
& \mathbf{J}(\boldsymbol{q}_\ell)\dot{\boldsymbol{q}}_\ell = \dot{\mathbf{x}}_\ell && \\
& \underline{\boldsymbol{\lambda}} \le \boldsymbol{\lambda} \le \overline{\boldsymbol{\lambda}} && \text{(Parameter Limits)} \\
& \overline{M}_\ell(\boldsymbol{\lambda}, \boldsymbol{q}_\ell, \dot{\boldsymbol{q}}_\ell) \ge 1 && \text{(Target Performance Minimums)}
\end{aligned}
$$

where $\underline{\boldsymbol{\tau}}$ and $\overline{\boldsymbol{\tau}}$ represent the velocity-dependent torque boundaries of a design, f_{FK} represents the forward task kinematics to match the SRB model trajectory, and $\mathbf{f}_\ell$ represents the contact forces from the SRB model. Overall, this representation of the co-design problem removes the need to consider the full dynamics of the robot, which is the core bottleneck for scalability of conventional methods. The full robot dynamics is approximately accounted for via the chosen metrics, which are dynamics aware. Despite these benefits, we envision that the above formulation may converge to candidate designs that reach the boundaries of the parameters (i.e., that the problem may be "overfitting" to the specific metrics used). In this regard, we will make use of full trajectory optimization using the models from Objective 2 to perform a cross-validation-like step for early termination of the above problem. Overall, *the goal of Task 3.1 is to formulate an optimization that leverages the SRB model dynamics to select the best physical parameters for a candidate design topology.* The optimal design is used as the backbone for the creation of the solid model of the physical robot.

Task 3.2. Integration of the dynamic humanoid robot Dash with electric motors: We follow the novel co-design framework (Fig. 9) for the integration of the dynamic humanoid robot *Dash*.

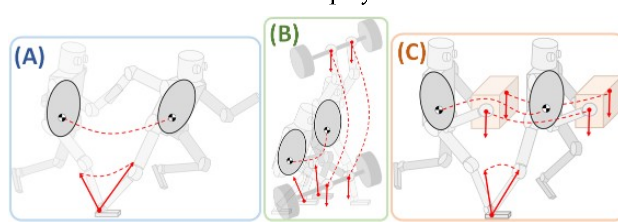

Figure 10: SRB trajectories of Dash: **(A)** running, **(B)** powerlifting, and **(C)** loaded carry.

Proposed Research: First, we employ conventional methods for trajectory optimization to compute the motion of the SRB model for each of the targeted behaviors (Fig. 10). We evaluate these trajectories using the performance metrics (Task 1.1) to identify keyframes of the motion that require peak performance. Once the most important performance metrics have been identified and quantified we use the design principles (Task 1.2) to select a candidate mechanism topology that employs appropriate power transmission elements. We will use the mechanism topology of the TELLO leg as one of the candidate designs. Next, the co-design optimization (Task 3.1) will use this seed design to select optimal parameters for the robot by targeting the performance scores from the trajectories from the SRB model. Once an optimal design has been selected, we create the solid model of the robot using off-the-shelf components and existing Computer Aided Design (CAD) software. The parameters from the solid model are used for the simulation and trajectory optimization of the full robot dynamics, including the closed chain mechanisms (Task. 2.1). The full dynamics motion is compared with that of the SRB model to inform the design iteration of the solid model. Once the robot is fabricated from the

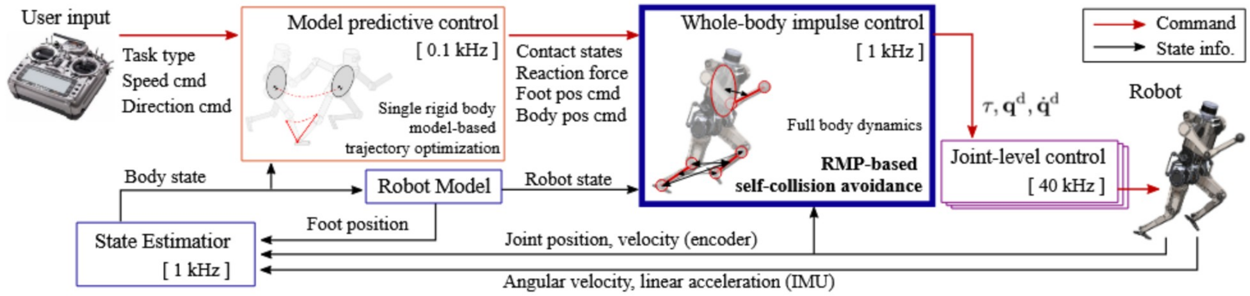

Figure 11: Control framework for a dynamic humanoid robot, Dash.

solid model, the real parameters are estimated using the system identification tool (Task 2.2) and the robot simulation is updated. Once an accurate parameters of the real robot are obtained, the accurate model is utilized to derive model-based control approaches (Task 3.3) that are tailored for dynamic behaviors. *The goal of Task 3.2 is to fabricate, identify, and model a humanoid that is designed to realize running, lifting, and carrying tasks.* Finally, three copies of the robot Dash will be assembled (at UIUC, ND, and UMass).

Task 3.3. Control and evaluation of the dynamic humanoid robot Dash: This task addresses the implementation of the controller tailored to dynamic movements, which comprises a *vital and substantial* part of the integration of Dash. The controller will combine a Model Predictive Controller (MPC) that computes the high-level trajectory of the robot using the SRB model, and a Whole-Body Impulse Controller (WBIC), that drives the real robot to track the SRB reference (Fig. 11). This strategy combines the proven advantages of reduced-model based MPC for the control of legged robots [31] with a WBIC that considers the full dynamics, constraints, and limitations of the machine [45]. A similar controller has been implemented by the co-PI to achieve high speed running of the Mini Cheetah quadruped [45] and to realize acrobatic behaviors of a humanoid robot in simulation [46].

A fundamental challenge that arises when using reduced-order models for the control of humanoid robots is the difficulty of addressing self-collisions, since the model neglects the motion of the limbs. This *critical* challenge, is particularly prominent in humanoids due to the length and range of motion of their limbs. Although the nominal motion from the co-design process should be free from self-collisions, when the robot deviates from the trajectory because of uncertainty or disturbances, self-collision becomes a key issue that must be addressed for successful integration. Existing approaches for self-collision avoidance are computationally expensive, sensitive to noise, and often do not consider full body dynamics [109–113]. On the other hand, we propose the incorporation of a novel algorithm for collision avoidance based on a Riemannian Motion Policy (RMP) [114] into the WBIC formulation. In essence, this approach will enable smooth and safe collision avoidance between multiple limbs during extremely dynamic motions without adding notable computation load. RMP-based algorithms have been used in manipulators [115] or wheeled robots control [116] but have not yet been demonstrated with legged robots.

Proposed Research: To implement the control of Dash, we first design and tune the controller shown in Fig. 11 using the simulation tool developed in Task 2.1. The MPC receives high-level commands from an operator (e.g., the desired CoM velocity) and solves a convex optimization for the motion of the SRB. Preliminary work shows this planning can be accomplished at 100Hz when planning for the next 0.7 seconds, which is about the time for two walking steps. Next, the WBIC considers the full-body dynamics to find joint torques and states at 1kHz that allow the robot to track the SRB reference from the MPC. At this stage, the RMP-based algorithm simultaneously modifies the motion to prevent the robot's limbs from colliding with each other. This is achieved by summing a motion command and repulsive collision avoidance signal in a way that considers the whole-body dynamics. Unlike potential field-based algorithms [113, 117] that overlap repulsive forces, the RMP-based algorithm constructs the repulsive actions at the acceleration level, sums them up with a motion command, and translates them to full-body motion using a Riemannian metric that warps configuration-space distances in a way that ensures collision avoidance. This approach will enable naturally accounting for multiple collision pairs via the metric, making the resulting behavior smooth and safe. Moreover, by designing the RMP offline, the process promises reduced computation costs compared to other attractive methods based on control barrier functions [118, 119].

Since RMP was originally designed for manipulators, its incorporation into humanoid robot control will require mathematical improvements and analysis to address features of a floating base. Our strategy will leverage Co-PI Kim's past work on whole-body control while adding collision avoidance RMPs. More specifically, in our formulation, we will first find the full-body acceleration that coincides with the floating base motion, $\ddot{\boldsymbol{q}}^{\mathrm{cmd}}_{\mathrm{pre}}$, and then will add limb motion such as from swing feet using the equation,

$$\ddot{\boldsymbol{q}}^{\mathrm{cmd}} = \ddot{\boldsymbol{q}}^{\mathrm{cmd}}_{\mathrm{pre}} + (\boldsymbol{J}^{*\top}\boldsymbol{\Lambda}^{*}\boldsymbol{J}^{*})^{-1}\boldsymbol{J}^{*\top}\boldsymbol{\Lambda}^{*}\ddot{\mathbf{x}}^{*}, \tag{3}$$

where $\boldsymbol{J}^{*}$ is a limb Jacobian, $\boldsymbol{\Lambda}^{*}$ is the Riemannian metric defined by the sum of an operational space inertia and collision-related Riemannian metrics, and $\ddot{\mathbf{x}}^{*}$ is the acceleration for the commanded limb motion and collision-avoidance. In effect, the RMP approach will enable to robot to proactively avoid configurations that will lead to self-collisions, while having online reactivity to respond to disturbances.

4 Evaluation

Evaluation Plan for Objective 1: The goal of Objective 1 is to formalize performance metrics and design principles for dynamic humanoid robots with electric motors. We assume that we can use these special metrics to design dynamic humanoids by maximizing the performance scores at a few *key instances in time*, instead of optimizing for *all* time steps of the *entire* motion of *all* the joints for *each* task, as it is typically done in co-design formulations. We will use the existing TELLO leg to evaluate if the proposed performance metrics can concisely describe the hardware and control requirements of humanoid robots. For that, we will quantitatively compare the performance scores of the SRB model with those for the full TELLO model for the targeted tasks. Although the optimal trajectories for these metrics are likely not identical, we assume that the peak performance scores will be similar.

Evaluation Plan for Objective 2: To evaluate the new algorithms and theory from Objective 2, we will carry out a series of simulation and experimental tests centered on the existing TELLO leg. The main metric for evaluating the simulation advance will be the computation time required versus a target level of constraint accuracy. This will be compared to the approaches taken in MuJoCo and Bullet. The validation of the identification advances with the TELLO hardware will compare estimation residuals with and without actuator models. The algebraic geometry methods for identifiability analysis will be verified by considering the singular values from the experimental regressor data to validate the geometric analysis.

Evaluation Plan for Objective 3: Our goal at the end of Objective 3 is to demonstrate that the real robot can consistently execute all targeted tasks at the designed level of performance. To estimate the robot's motion performance we will leverage the Co-PI's previous work about uncertainty analysis of biped locomotion [44]. On a high-level, the modeling, sensing, and actuation uncertainties at joint level are projected into task space to estimate the motion error from the SRB model. The motion error is obtained from experimental data by comparing the joint states and torques of the real robot with the trajectories obtained from simulation with the SRB and full rigid-body models. Some deviation from the designed motion is expected, but the fundamental behavior should be mostly consistent. The data obtained from the execution of the targeted tasks will allow us to compute the performance scores of the real robot in the metrics from Task 1.1 that were employed in the co-design process. For instance, the ground contact forces generated for sprint running allows the computation of the peak force density of the legs [38]. Similarly, we compute the force density of the arms from the trajectory of the payload during powerlifting. The targeted tasks are normalized to the projected scale of the robot Dash (mass $\leq 40\mathrm{kg}$ and height $\leq 1.2\mathrm{m}$). However, the results from this work are general to any robot scale. Each task will be specifically evaluated:

1- Sprint running: We target a speed of $3.5\mathrm{m/s}$ ($7.8mph$), which is 90% of the average sprinting speed for human athletes in body-lengths per second. The robot will sprint across a prescribed distance and over a flat and rigid surface. From standing still, the robot will accelerate to maximum speed, maintain the speed for a prescribed time, and safely decelerate to static standing. We will record the maximum acceleration and velocity that the machine can achieve and compared with the value expected from simulation analysis.

2- Powerlifting: We target a payload of $15\mathrm{kg}$ ($33lb$), which is 90% of the average weight human athletes can snatch normalized by mass. The robot will stand on a flat and rigid surface and lift a payload similar to an Olympic barbell from the ground and over its head in a single fast motion. The maximum payload the robot can lift will be compared to the value expected from the theoretical design analysis. To isolate the gripper requirements from this experiment, the end of the arms will be rigidly connected to the bar through a passive 3 DoF joint such that the robot cannot let go from the payload. We will record the torque and speed trajectories of the joints, specially knees and shoulders. In addition, a force plate will monitor the forces applied to the ground and the forces applied to the payload will be estimated using the torque profile of the joints and the kinematics of the limbs.

3- Loaded carry: We target a payload of 9kg ($20lb$) at 1.3m/s ($2.9mph$), which is 90% of the average payload humans can carry divided by body mass at a speed that is 90% of the average jogging speed in body-lengths per second. The robot will dynamically walk over a flat and rigid surface while carrying a heavy box with both arms. The hands will be rigidly connected to the box using a passive joint such that the robot cannot let go of the object. Our goal is to evaluate the relationship between the maximum achievable walking speed for a given payload compared to the simulation analysis.

5 Risk & Mitigation

Risk management for Objective 1: Design metrics such as those suggested in Task 1.1 have been extensively employed to create position controlled industrial manipulators [77]. And hence, we are confident that pursuing similar metrics in the context of dynamic humanoid robots represents a low-risk and high-reward goal *even if the ultimate goal of this proposal is not achieved*. We believe the design principles will also represent valuable guidelines for designers of dynamic or collaborative robots. One potential risk of using the SRB dynamics to guide the co-design optimization may be that the optimal trajectories for the full robot model may be too far from the that of the reduced model. To ensure that the dynamics of the robot can be approximated by the SRB model, the robot's *Centroidal Inertia Isotropy* score is embedded in the formulation as a core performance metric [120]. The CII measures the ratio between the largest and smallest full body inertia of the robot. In the extreme case when these are equal, the robot's dynamics can be precisely approximated by the SRB model.

Risk management for Objective 2: Preliminary theoretical and computational results [42] for simulation of single-DoF actuator dynamics (i.e., not parallel topologies) mitigate risk for Task 2.1. This task is low risk but could have broad impact on the community (e.g., to accelerate machine learning research as well). Task 2.2 could present some computational challenges with scalability using Gröbner basis techniques. However, preliminary results shows that reasonable computation times should be expected for robots of the complexity considered here. For Task 2.3, the use of the pre-existing TELLO leg decouples the success of this Objective from that of Objective 1.

Risk management for Objective 3: While the successes of Objectives 1 and 2 will enrich Objective 3, their uniform success is not strictly necessary for accomplishing Objective 3 overall. The humanoid robot TELLO developed by the PI is already a promising alternative for a dynamic humanoid design, and the analysis proposed in this project will only improve its capabilities. Given the challenges of working with robot hardware, we acknowledge possible delays in the final integration of the hardware and down times for maintenance. To mitigate this issue, the existing TELLO hardware will be extensively used for preliminary experiments and the simulator developed in Objective 2 will be as truthful to the robot dynamic as possible. These two tools will allow us to test candidate controllers beforehand and during down times of the Dash hardware. Moreover, because three copies of the robot will be assembled, it is unlikely that all robots will be unavailable for experiments at a given time. We target a scaled robot with total mass $\leq 40kg$ and height $\leq 1.2m$. From our team's past experience with robots up to human scale, we believe more research groups will be willing to adopt the design if it is smaller and safer to work with.

6 Intelectual Merit:

The main **Intellectual Merit** of this proposal is a **holistic framework for robot integration** of dynamic humanoids with electric motors. This framework is amplified by **fundamental scientific contributions** to design, simulation, and control of these robots. We envision that our contributions to integration can be naturally adapted to the hardware and control co-design of exoskeletons, prosthetic devices, and other platforms that perform physical tasks *dynamically*. In addition, achieving fast and accurate dynamic simulation of robots reduces the sim2real gap and supports approaches, such as reinforcement learning, which require massive amounts of training data but are infeasible to be developed in hardware alone.

7 Broader Impacts:

Projected impact on human welfare: Anthropomorphic robots with physical capabilities comparable to those of the average human could support firefighters, nurses, policemen, miners, loggers, and a number of other workers that perform physically demanding work daily in their occupations. This work focuses on creating a design framework that maximizes the dynamic physical capabilities of humanoid robots with electric actuators. We envision that if this framework can create machines capable of performing athletic behaviors, it can also be employed to design machines that address most of the physically demanding activities performed by human workers. For instance, the powerlifting and loaded carry tasks are fundamental for improving logistics in the US, with immediate relevance to the last mile problem of automating delivery. In addition, the combination of backdrivable electric motors and low inertia limbs enable machines that are inherently safe for human interaction [90,121]. Beyond helping workers, these technologies could have impact within the home, helping people move furniture or other heavy objects, which frequently lead to back injuries. Due to their human-like shape, these robots could be directly inserted in homes or working environments without significant modifications to tools or infrastructure.

Projected impact on the robotics community: This work will provide an accessible and yet capable platform for accelerating research toward practical humanoid robots in the US. At the end of the final year, we will organize a workshop on *Design and Control of Dynamic Humanoid Robots* to demonstrate the physical capabilities of Dash and release the source files of the platform to the robotics community. This open-sourcing will include CAD drawings, circuit designs, the dynamics simulator, control software produced in this project, and a list of best practices and when dealing with dynamic robots. We will service a website for these materials including a wiki and Q&A pages. Our Github repositories will be used to distribute simulation and control software packages. These materials will be maintained by co-PI Kim, who has experience distributing quadruped robots (Mini-Cheetah) to nine universities, a research institute, and companies. Our final goal is to build an ecosystem that allows collaboration and benefits from the shared experiences and ideas of multiple research groups.

Projected impact on education: The Co-PIs will use this project to offer a collaborative course on the *Design and Control of Dynamic Robots*, jointly offered at the University of Illinois Urbana-Champaign (UIUC) and the University of Notre Dame (ND). This course will be offered at the advanced undergraduate / early graduate level and will be co-taught by Ramos and Wensing. The course will meet in person at UIUC and ND, with one PI leading the class each time and the other section attending over Zoom. This strategy will enable students on the project (and many of those that are not) to benefit from the complementary expertise in design (Ramos) and control (Wensing) by the instructional team, and further follows the spirit of shared hardware from this proposal. The PIs will have a friendly competition between schools at the end of the semester, where teams will be given access to a boom-mounted open-source hopping leg robot [22] and can make their own design upgrades. The semester will conclude with a robot race, where the synergy of design and control will be evaluated based on an ability to run the fastest lap, and travelling the furthest on a limited battery charge. Since UIUC and ND are in close proximity, this final competition may take place on-site at UIUC (depending on the class size at ND).

8 Results from Prior NSF Support:

Dr. Ramos is the PI of a NSF M3X grant (NSF CMMI-2043339): *CAREER: Remote Control of Humanoid Robot Locomotion using Human Whole-body Movement and Mutual Adaptation*, $736,877, 04/21 to 03/26. **IM:** Achieve bilateral teleoperation of dynamic walking on uneven terrain with a humanoid robot. **BI:** Enhance engineering education, community outreach, academic training and mentoring. **Pubs:** [22,23].

Dr. Wensing's most relevant prior support was a NSF DCSD Grant, (NSF IIS-1835186): *EAGER/Collaborative Research: Unlocking Legged Mobility Through Structured Prediction,"* 08/18-07/20, $75,000. **IM:** Established a new approach to robot control via a rigorous optimal control over hierarchical abstractions. **BI:** Results enable legged machines to make rapid decisions necessary to keep their balance and avoid falls, improving robustness for deployment as first responders or explorers. **Pubs:** [27,33,34,37,122–125].

Dr. Kim does not yet have NSF support to report.

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