MATH50003
Numerical Analysis

I.2 Divided Differences

Office Hour: Thursdays, 16:00–17:00, Huxley 6M40

Thursday Demo Class (Huxley 340): setting up Git/Julia and labs
Problem Sheet 1 help (Huxley 341)
Lab 1 help (Huxley 342)

Dr Sheehan Olver

Part I

Calculus on a Computer

1. Rectangular rules for integration
2. Divided differences for differentiation
3. Dual numbers for differentiation

(Right-sided) divided differences

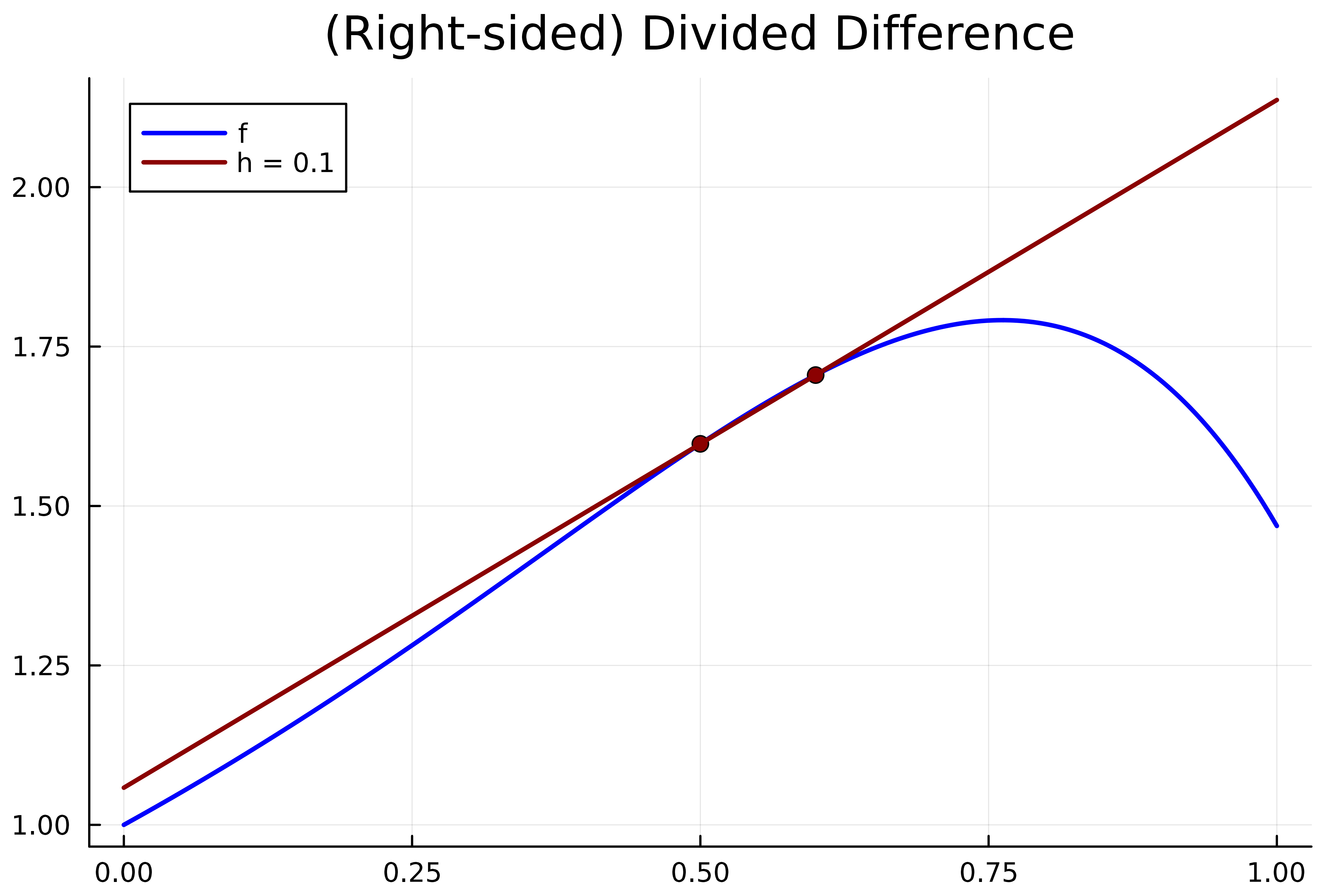
Approximating derivatives from function values

- Start with the definition of a derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Idea: make h small and use

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$



Divided differences is slope of line and approximates derivative

Proposition 1 (divided differences error)**.** *Suppose that f is twice differrentable on the interval $(x, x+h)$. The error in approximating the derivative using divided differences is*

$$f'(x) = \frac{f(x+h) - f(x)}{h} + \delta$$

where $|\delta| \leq Mh/2$ for $M = \sup_{x \leq t \leq x+h} |f''(t)|$.

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We bound

$$|\delta| = \frac{h|f''(t)|}{2} \le \frac{Mh}{2}$$

Other approximations to derivatives

Explored in Problem Sheets/Lab

Central differences

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second-order divided differences

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Since applying central differences twice w/ $h/2$:

$$\begin{aligned} f''(x) &\approx \frac{f'(x+h/2) - f'(x-h/2)}{h} \\ &\approx \frac{\frac{f(x+h)-f(x)}{h} - \frac{f(x)-f(x-h)}{h}}{h} \end{aligned}$$

Now for implementation

Can we get an idea of what goes wrong with h very small?