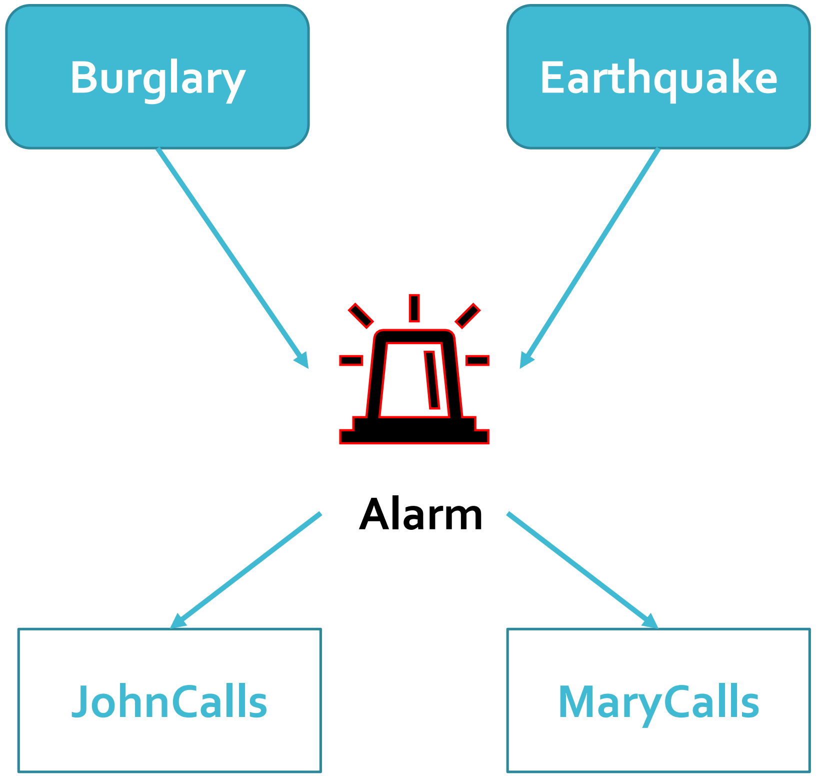
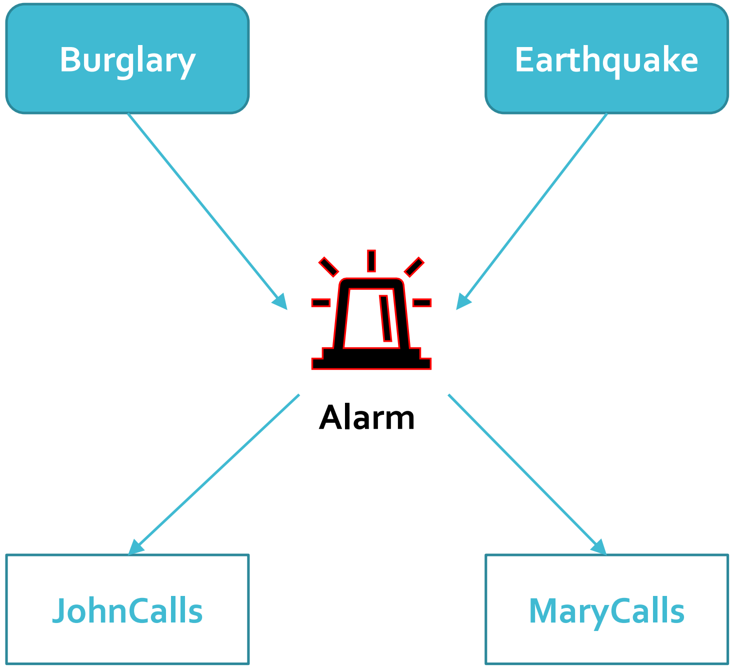
Lab4
Exact Inference

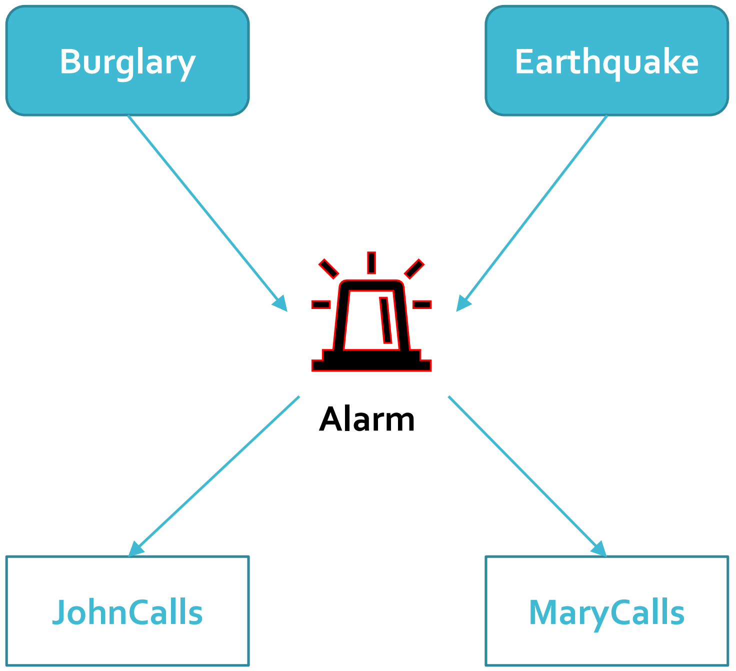
TA 王玥奕





Given a Bayes Net

Query P(Burglary| JohnCalls= True, MaryCalls= True)



P(Burglary | JohnCalls=True, MaryCalls=True)

$$P(B|j,m) = \alpha P(B,j,m)$$

$$= \alpha \sum_e \sum_a P(B,j,m,e,a)$$

Enumeration

$$= \alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$

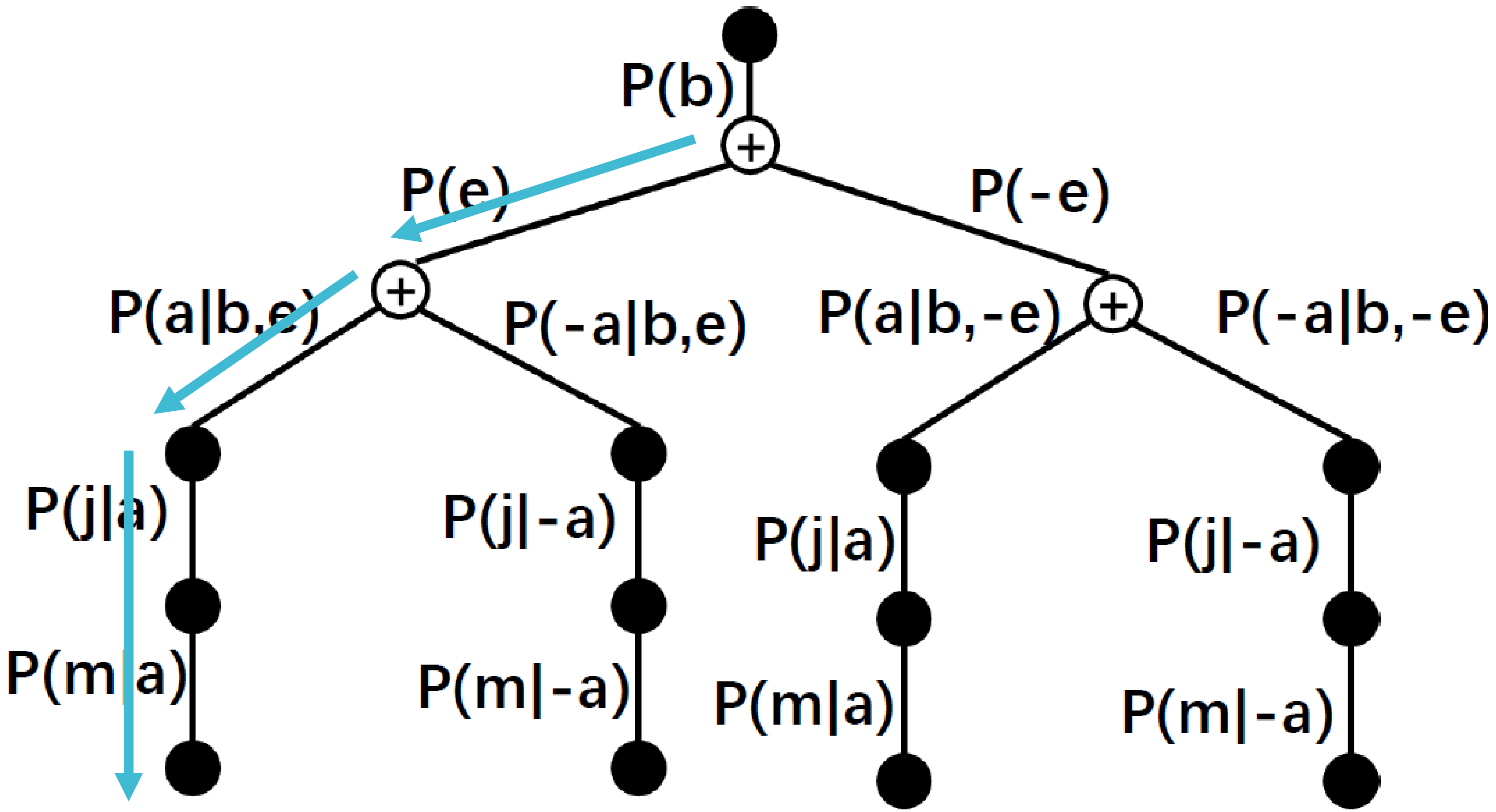
Elimination

$$= \alpha P(b) \boxed{\sum_e} P(e) \boxed{\sum_a} P(a|b,e)P(j|a)P(m|a)$$

Part 1 Enumeration

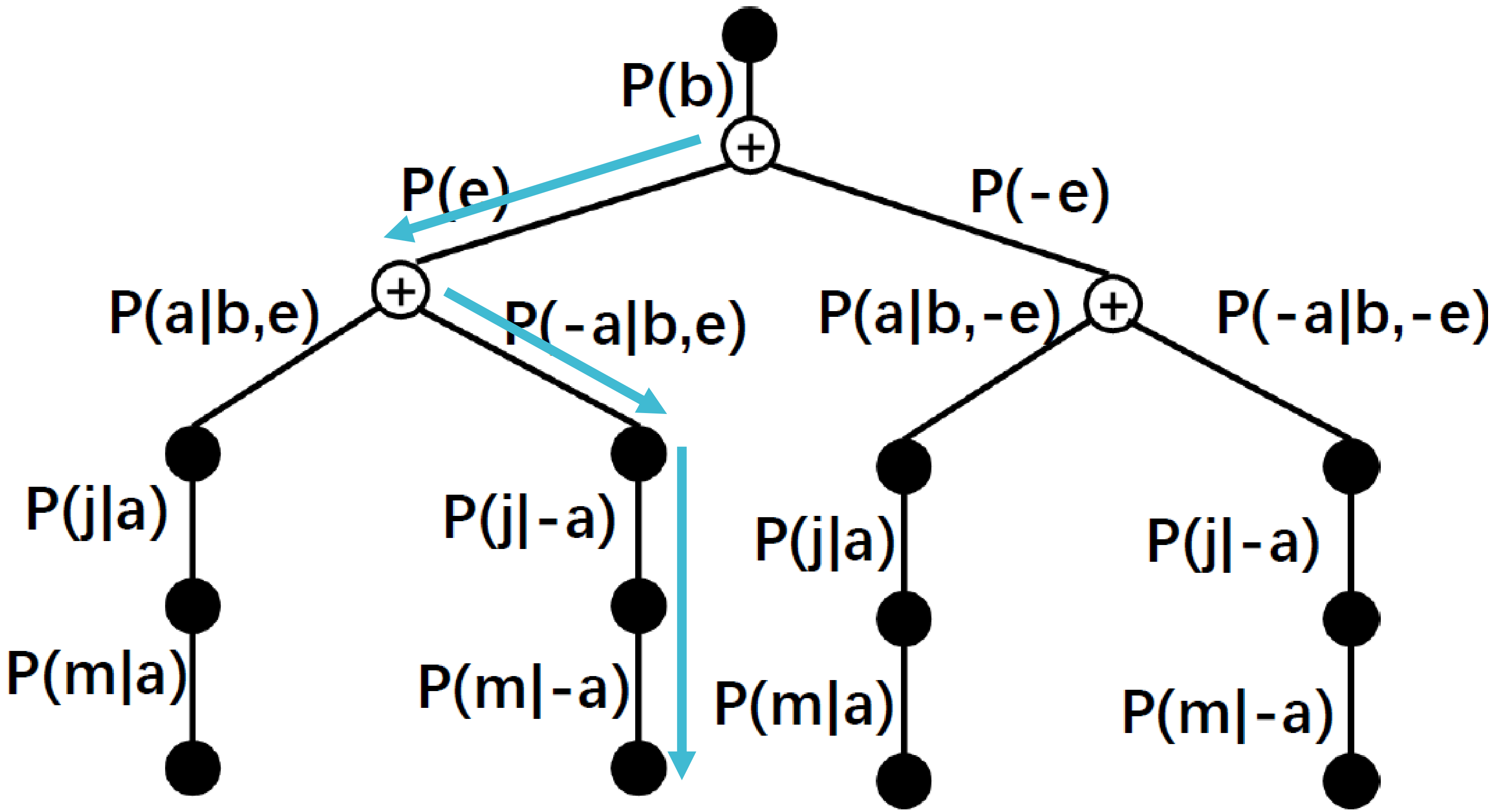
DFS

$$\alpha \sum_{e} \sum_{a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$



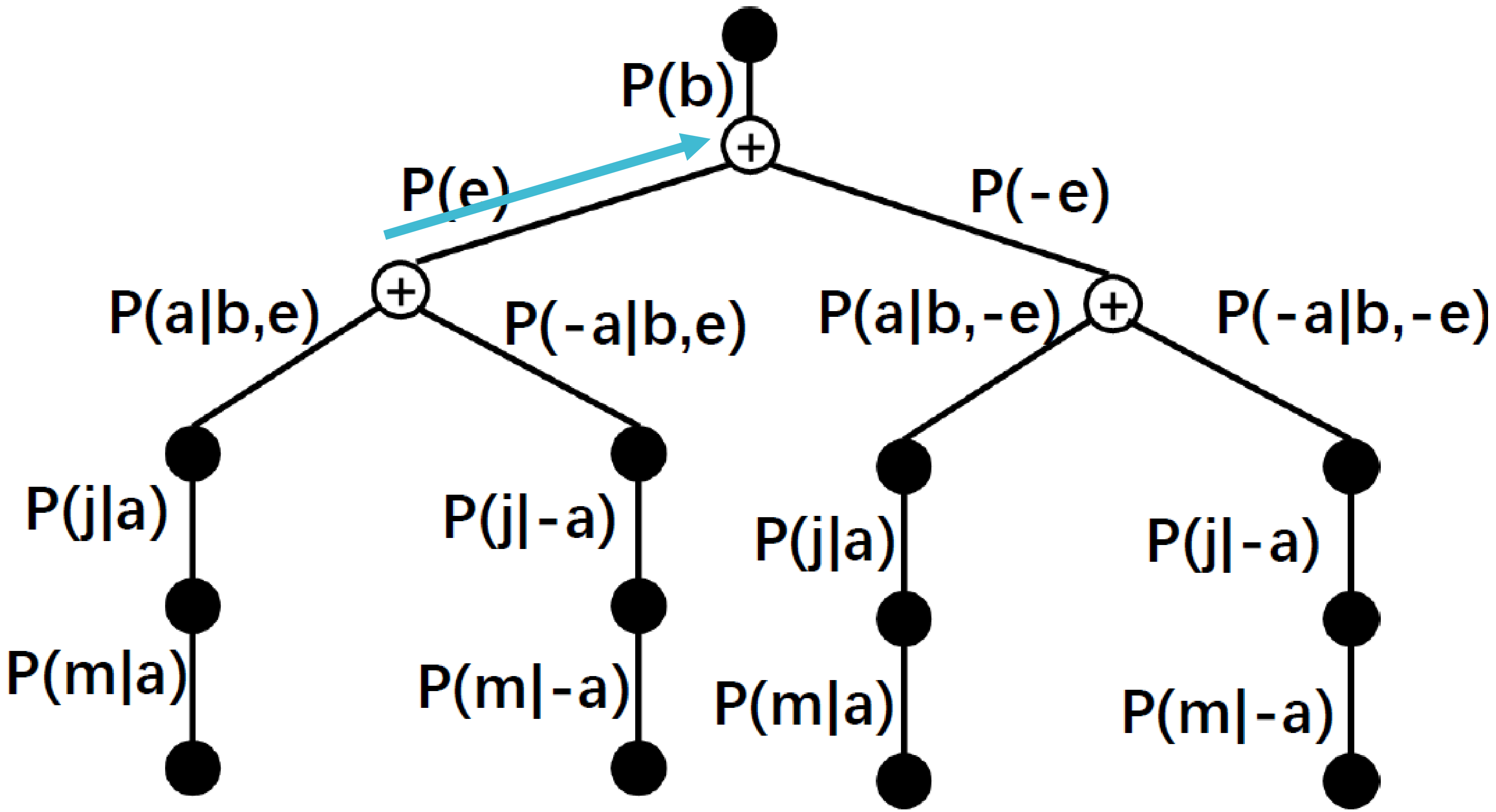
DFS

$$\alpha \sum_{e} \sum_{a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$



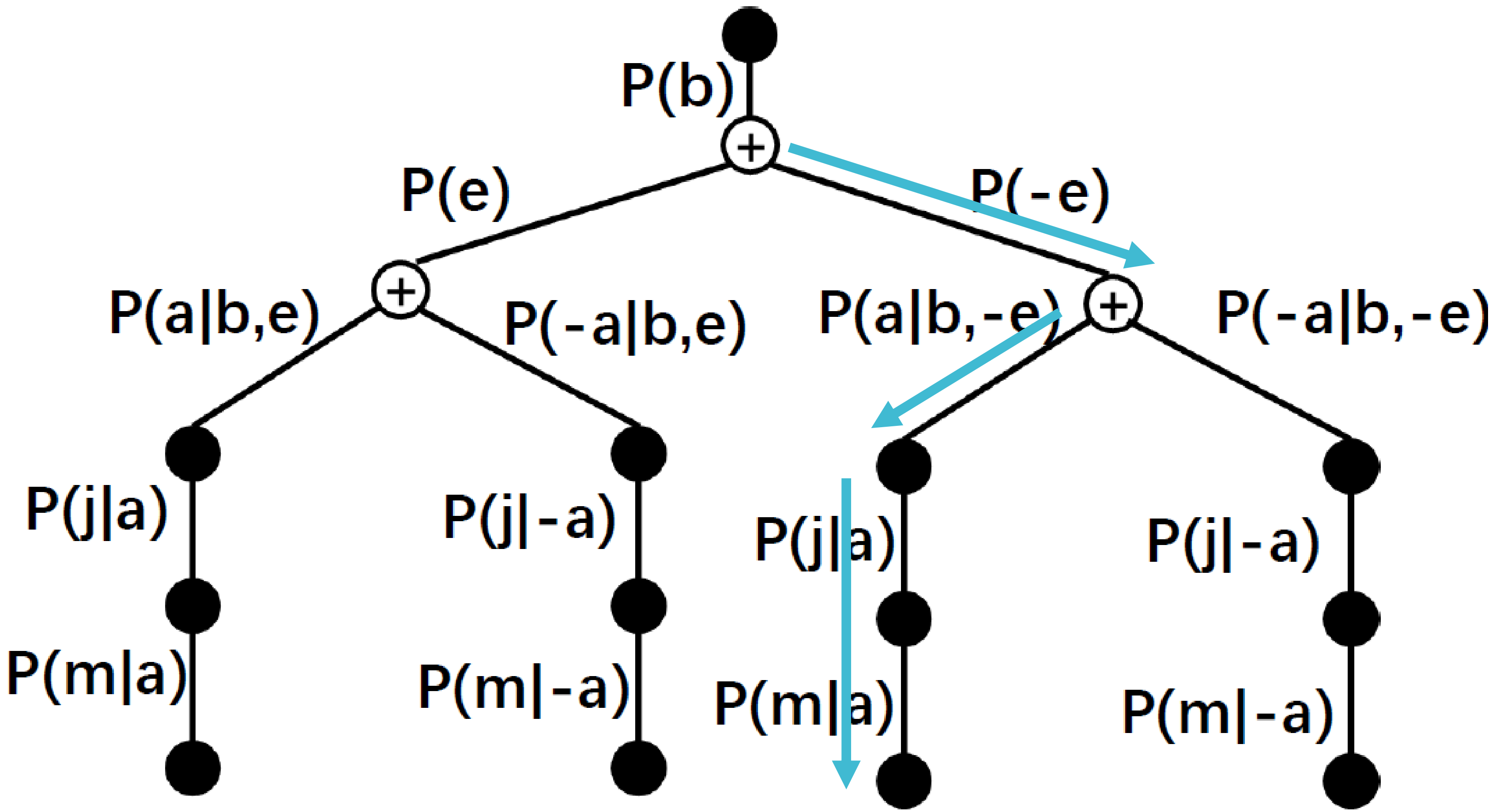
DFS

$$\alpha \sum_{e} \sum_{a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$



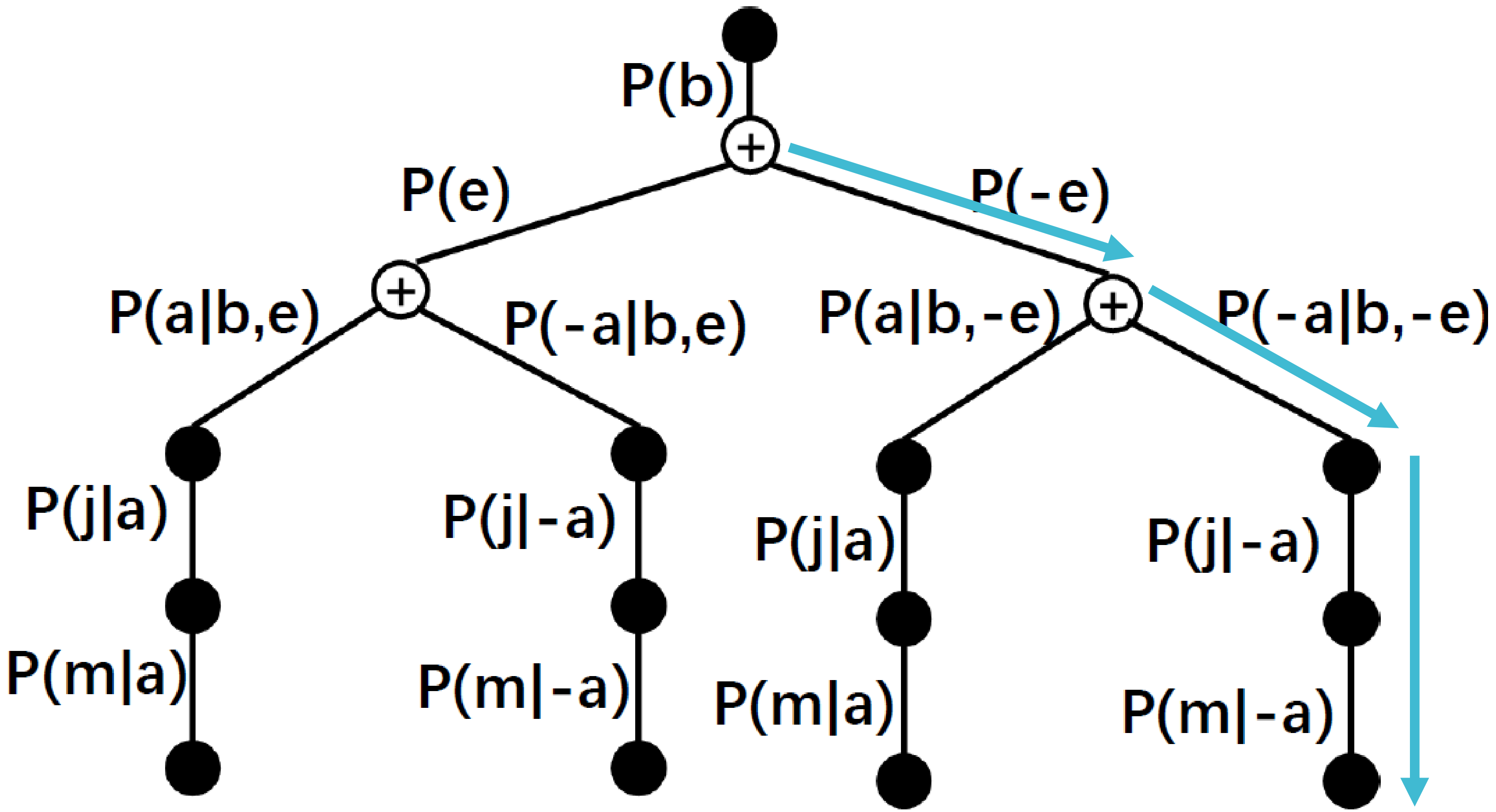
DFS

$$\alpha \sum_{e} \sum_{a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$



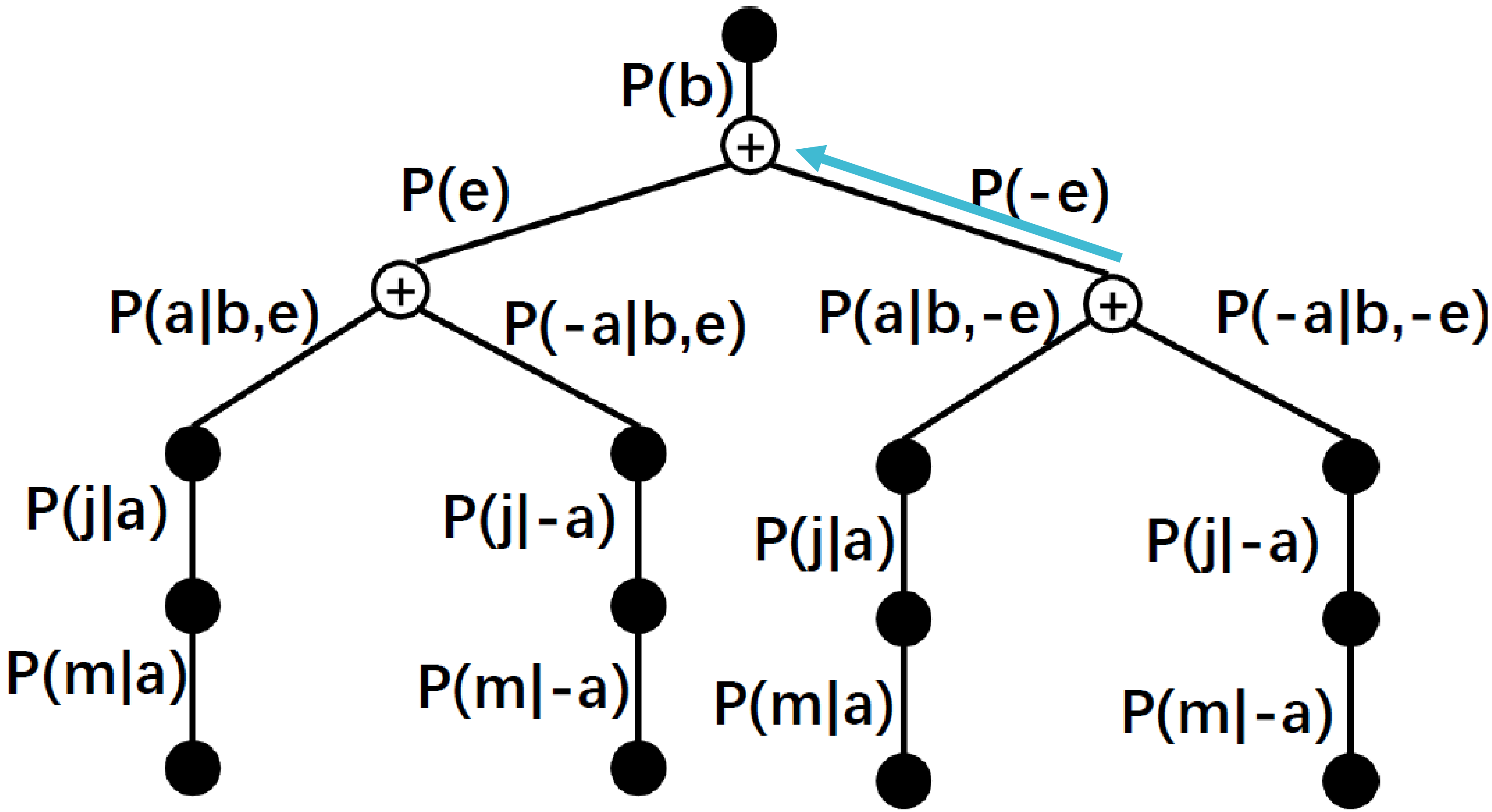
DFS

$$\alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$

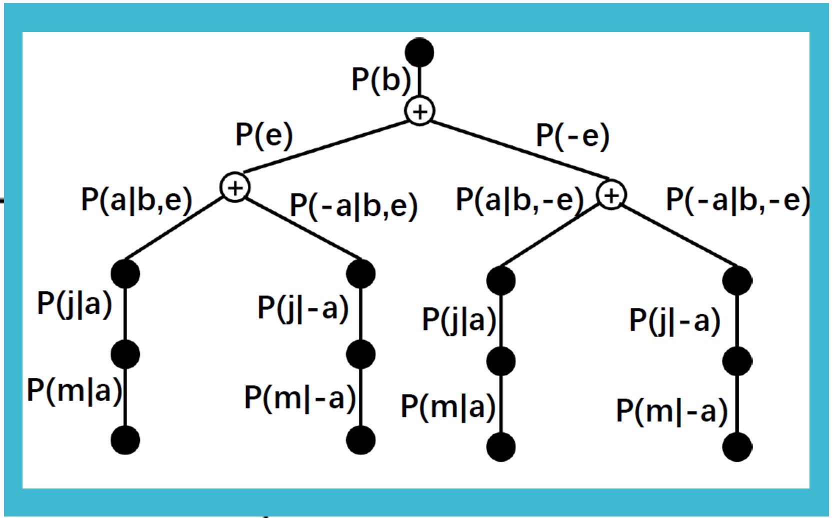


$$\alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$

DFS



Enumeration - Ask Algorithm



function ENUMERATION-ASK(X, **e**, bn) **returns** a distribution over X
  inputs: X, the query variable
    e, observed values for variables **E**
    bn, a Bayes net with variables $\{X\} \cup \mathbf{E} \cup \mathbf{Y}$ /* **Y** = *hidden variables* */

  $\mathbf{Q}(X) \leftarrow$ a distribution over X, initially empty
  for each value x_i of X **do**
    $\mathbf{Q}(x_i) \leftarrow$ ENUMERATE-ALL(bn.VARS, $\mathbf{e}_{x_i}$)
      where $\mathbf{e}_{x_i}$ is **e** extended with $X = x_i$
  return NORMALIZE($\mathbf{Q}(X)$)

---

function ENUMERATE-ALL($vars$, **e**) **returns** a real number

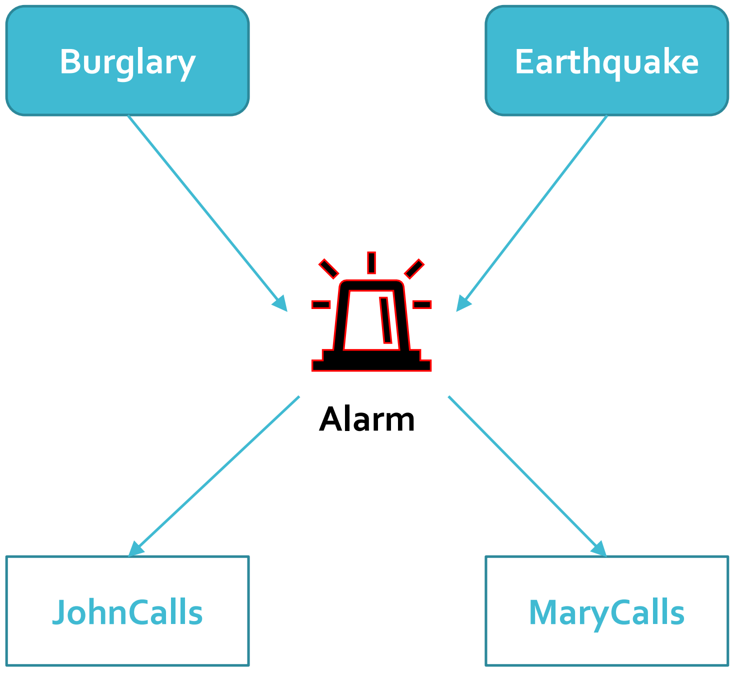
DFS

  if EMPTY?($vars$) **then return** 1.0
  $Y \leftarrow$ FIRST($vars$)
  if Y has value y in **e**
    then return $\boxed{P(y \mid parents(Y))}$ $\times$ ENUMERATE-ALL(REST($vars$), **e**)
    else return $\sum_y P(y \mid parents(Y)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}_y$)
      where $\mathbf{e}_y$ is **e** extended with $Y = y$

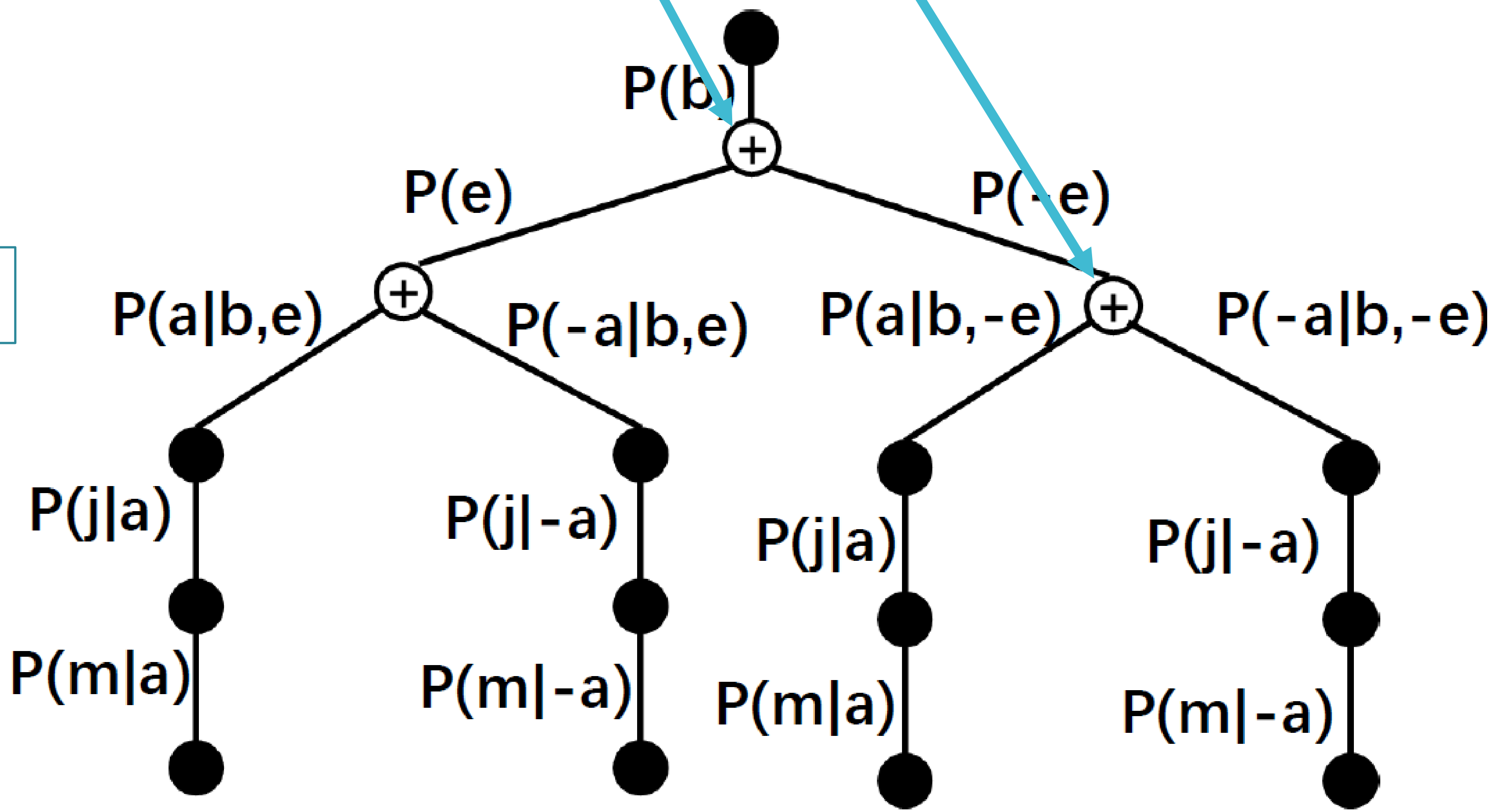
You can use Ynode.p()

Figure 14.9 The enumeration algorithm for answering queries on Bayesian networks.

Part 2 Elimination



$$P(B|j,m) = \alpha P(b)\boxed{\sum_e} P(e)\boxed{\sum_a} P(a|b,e)P(j|a)P(m|a)$$



Elimination – Introduce Factors

$$\mathbf{P}(B \mid j, m) = \alpha \underbrace{\mathbf{P}(B)}_{\mathbf{f}_1(B)} \sum_e \underbrace{P(e)}_{\mathbf{f}_2(E)} \boxed{\sum_a} \underbrace{\mathbf{P}(a \mid B, e)}_{\mathbf{f}_3(A,B,E)} \underbrace{P(j \mid a)}_{\mathbf{f}_4(A)} \underbrace{P(m \mid a)}_{\mathbf{f}_5(A)}$$

Step1: Make factors

Step2: Join factors &
eliminate hidden vars

$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \sum_a \mathbf{f}_3(A, B, E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A)$$

- First, we sum out A from the product of $\mathbf{f}_3$, $\mathbf{f}_4$, and $\mathbf{f}_5$. This gives us a new 2×2 factor $\mathbf{f}_6(B, E)$ whose indices range over just B and E:

$$\begin{aligned}\mathbf{f}_6(B, E) &= \sum_a \mathbf{f}_3(A, B, E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A) \\ &= (\mathbf{f}_3(a, B, E) \times \mathbf{f}_4(a) \times \mathbf{f}_5(a)) + (\mathbf{f}_3(\neg a, B, E) \times \mathbf{f}_4(\neg a) \times \mathbf{f}_5(\neg a)) .\end{aligned}$$

  Now we are left with the expression

$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \mathbf{f}_6(B, E) .$$

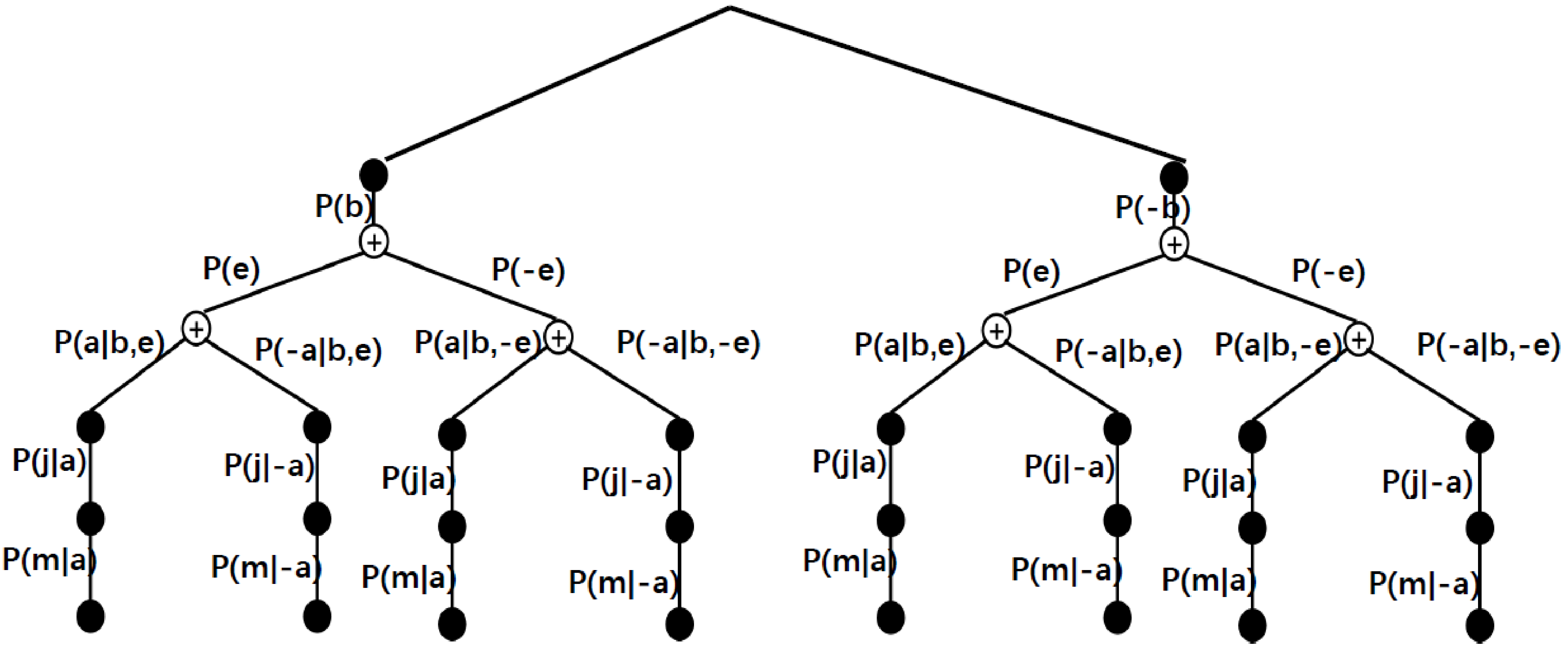
- Next, we sum out E from the product of $\mathbf{f}_2$ and $\mathbf{f}_6$:

$$\begin{aligned}\mathbf{f}_7(B) &= \sum_e \mathbf{f}_2(E) \times \mathbf{f}_6(B, E) \\ &= \mathbf{f}_2(e) \times \mathbf{f}_6(B, e) + \mathbf{f}_2(\neg e) \times \mathbf{f}_6(B, \neg e) .\end{aligned}$$

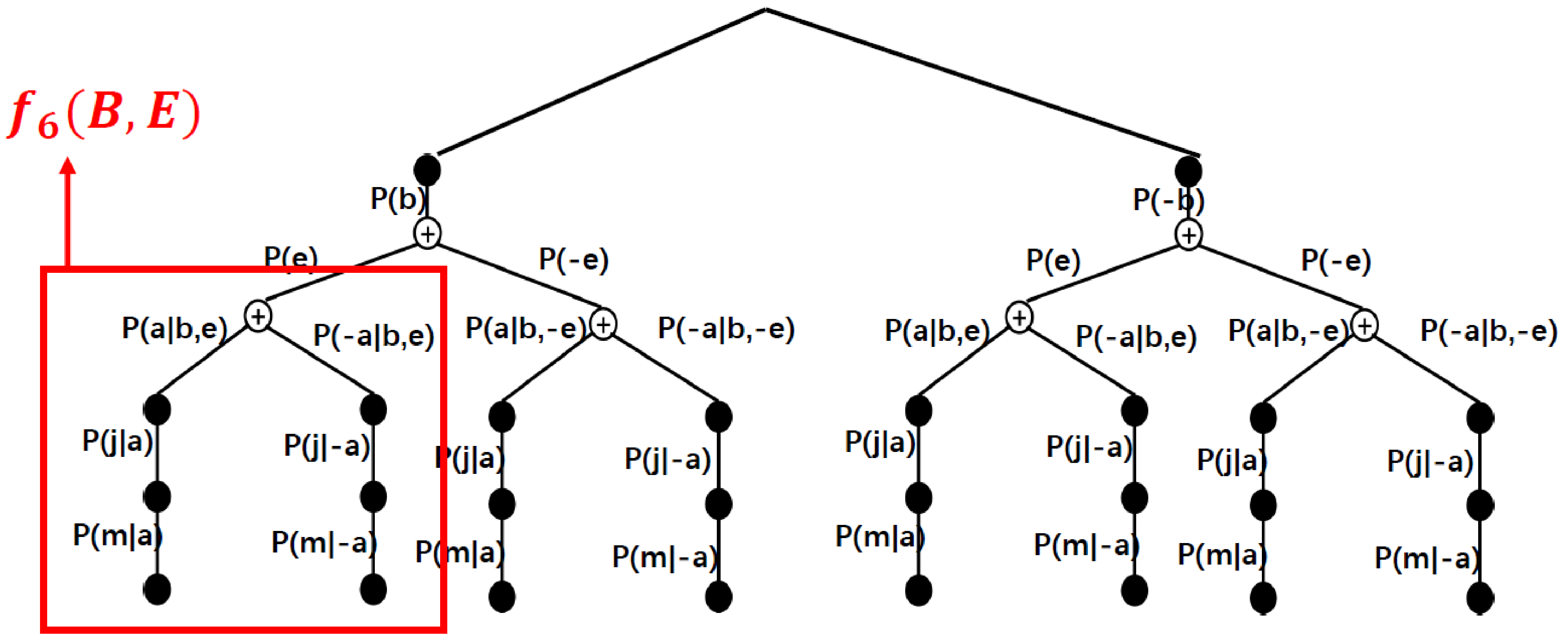
  This leaves the expression

$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \mathbf{f}_7(B)$$

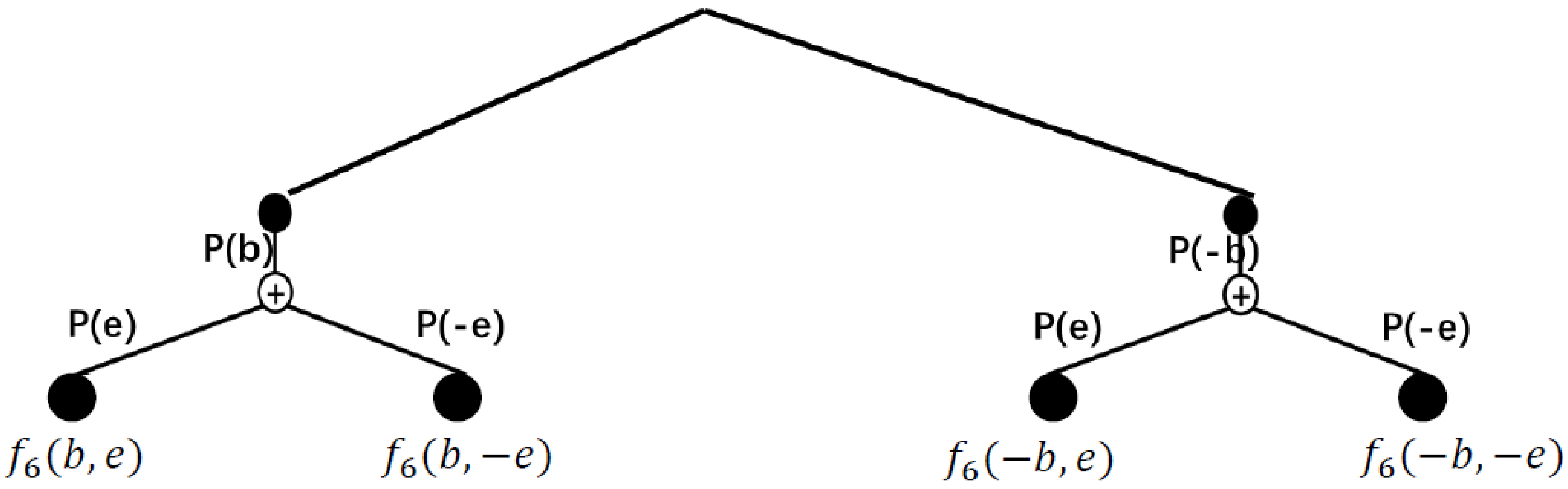
$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \sum_a \mathbf{f}_3(A, B, E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A)$$



$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \sum_a \mathbf{f}_3(A, B, E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A)$$



$$\mathbf{P}(B \mid j, m) = \alpha\, \mathbf{f}_1(B) \times \sum_e \mathbf{f}_2(E) \times \sum_a \mathbf{f}_3(A, B, E) \times \mathbf{f}_4(A) \times \mathbf{f}_5(A)$$



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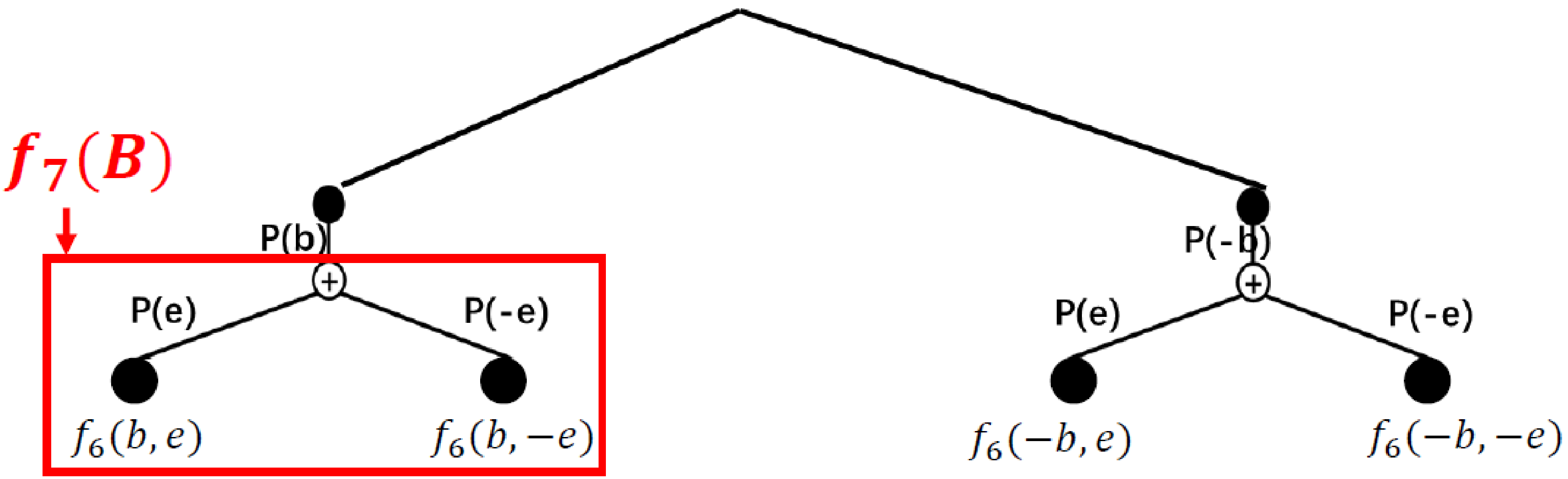

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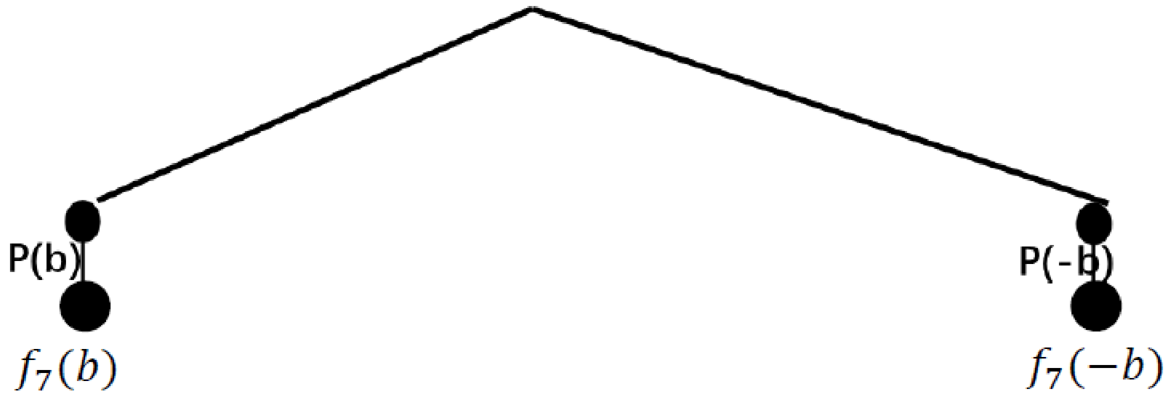


Elimination – Ask Algorithm

function ELIMINATION-ASK(X, **e**, bn) **returns** a distribution over X
  inputs: X, the query variable
    e, observed values for variables **E**
    bn, a Bayesian network specifying joint distribution $\mathbf{P}(X_1, \ldots, X_n)$

  $factors \leftarrow [\,]$
  for each var **in** ORDER(bn.VARS) **do**
    $factors \leftarrow$ [MAKE-FACTOR(var, **e**)$\boxed{|factors]}$
    if $\boxed{var \text{ is a hidden variable}}$ **then** $factors \leftarrow$ SUM-OUT(var, $factors$)
  return NORMALIZE(POINTWISE-PRODUCT($factors$))

Actually, this is an "append" operation

How to express this?

Review

- **Enumeration algorithm:**
- • Step 1: Select the entries consistent with the evidence
- • Step 2: Sum out hidden vars to get joint of Query and evidence
- • Step 3: Normalize

- **Elimination algorithm:**
- • Make factors
- • Join all factors and eliminate all hidden vars

Part 3 Lab

Functions to be implemented:

Enumeration algorithm:

- Def enumeration_ask(X, e, bn):
- Def enumeration_all(X, e, bn):

Elimination algorithm:

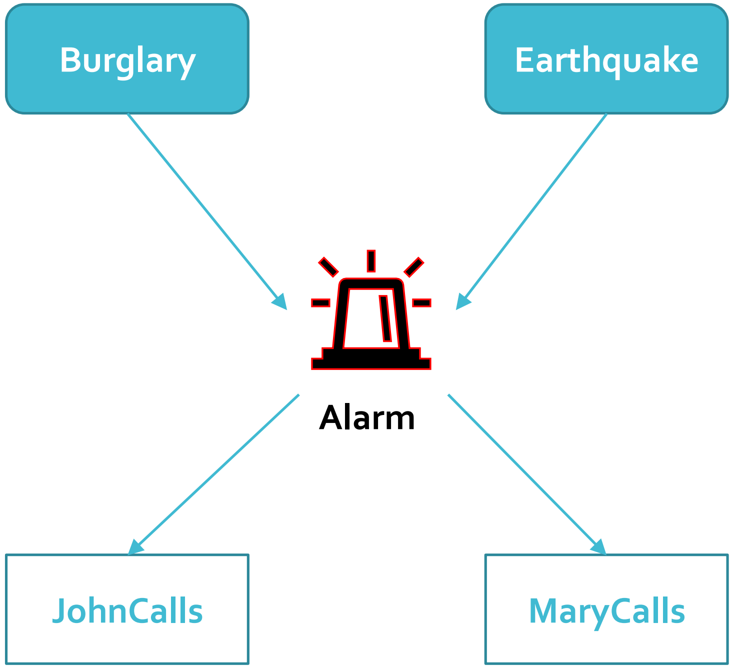
- Def elimination_ask(X, e, bn):

Problem Description:

Input

P(Earthquake = -)
P(Burglary = + | John = +, Mary = +)
******
Burglary
0.001
***
Earthquake
0.002
***
Alarm | Burglary Earthquake
0.95 + +
0.94 + -
0.29 - +
0.001 - -

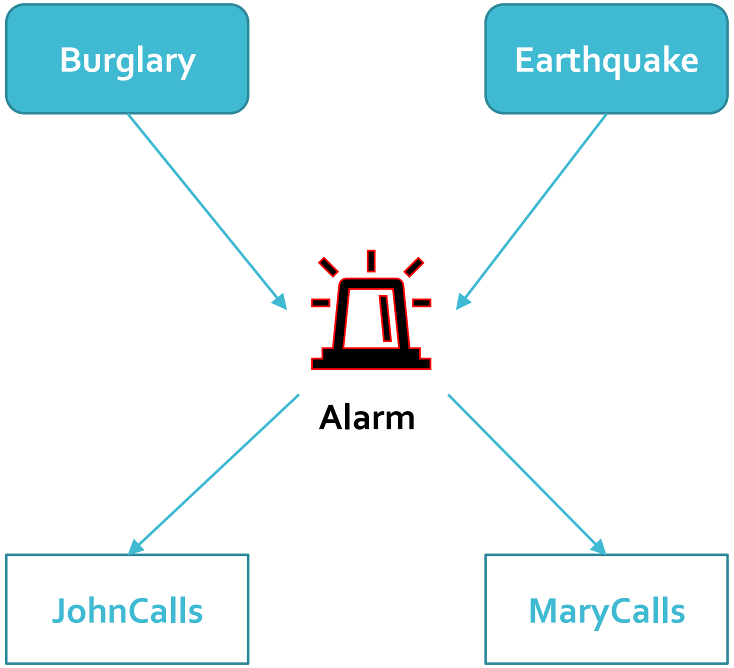
***
John | Alarm
0.9 +
0.05 -
***
Mary | Alarm
0.7 +
0.01 –



Problem Description:

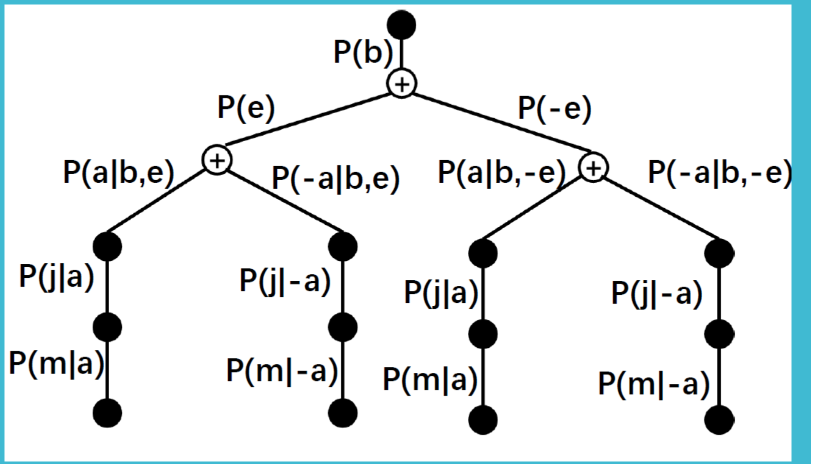
Output

probability by enumeration: 0.998
probability by elimination: 0.998
****
probability by enumeration: 0.284
probability by elimination: 0.284
****



P(Earthquake = -)
P(Burglary = + | John = +, Mary = +)

Enumeration - Ask Algorithm



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---

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DFS

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You can use Ynode.p()

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Figure 14.9 The enumeration algorithm for answering queries on Bayesian networks.

Elimination – Ask Algorithm

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    if var is a hidden variable **then** $factors \leftarrow$ SUM-OUT(var, $factors$)
  return NORMALIZE(POINTWISE-PRODUCT($factors$))

How to express this?

PJ4 Car

Due 2021.01.10

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