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# Graph and Dijkstra Algorithm

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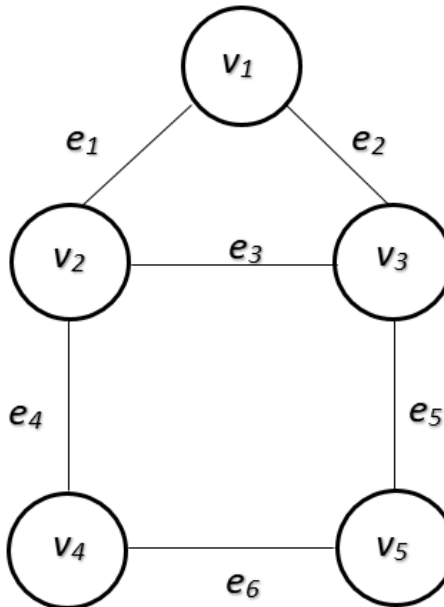
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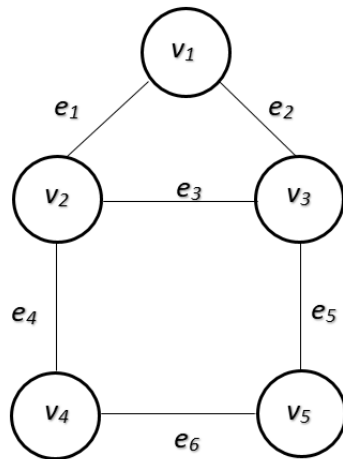
# Graph

- A graph  $G$  is a data structure that consists of a set of vertices  $V$  and edges  $E$ 
  - An edge  $e$  is a pair of vertices  $(v_i, v_j)$ , where  $v_i, v_j \in V$
  - For example, the following picture shows a graph with 5 vertices and 6 edges



# Adjacency Matrix

- If two vertices in a graph are connected by an edge, we say the vertices are adjacent
  - In our graph example, vertex  $v_1$  has two adjacent vertices,  $v_2$  and  $v_3$
  - Based on this property, we can use an adjacency matrix or adjacency list to represent a graph
  - We can also use an adjacency list to represent a graph



|       | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|-------|-------|-------|-------|-------|-------|
| $v_1$ | 0     | 1     | 1     | 0     | 0     |
| $v_2$ | 1     | 0     | 1     | 1     | 0     |
| $v_3$ | 1     | 1     | 0     | 0     | 1     |
| $v_4$ | 0     | 1     | 0     | 0     | 1     |
| $v_5$ | 0     | 0     | 1     | 1     | 0     |

Adjacency Matrix

| $v_1$ | $v_2, v_3$      |
|-------|-----------------|
| $v_2$ | $v_1, v_3, v_4$ |
| $v_3$ | $v_1, v_2, v_5$ |
| $v_4$ | $v_2, v_5$      |
| $v_5$ | $v_3, v_4$      |

Adjacency List

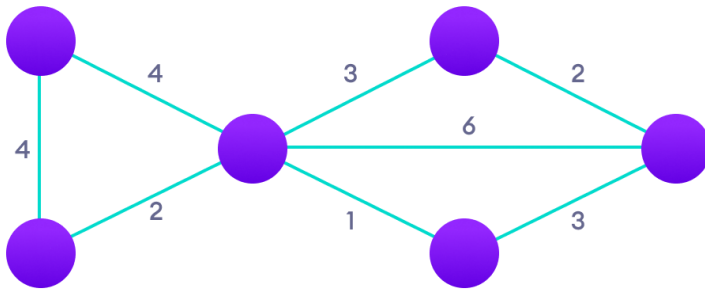
# Dijkstra Algorithm for Shortest Path

- The algorithm exists in many variants
  - Dijkstra's original algorithm finds the **shortest path between two given nodes**
  - But a more common variant fixes a single node as the "source" node and finds shortest paths from the **source to all other nodes** in the graph, producing a shortest-path tree

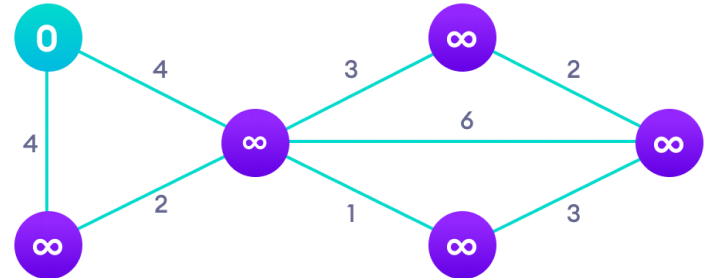
# Dijkstra Algorithm (Source to All)

- ❑ 1. Mark all nodes **unvisited**
- ❑ 2. Assign to every node a tentative distance value
  - **Zero for the initial node** and **infinity for all other nodes**
- ❑ 3. Select an **unvisited node** that is marked with the **smallest tentative distance**
- ❑ 4. **Update the tentative distances** of all the unvisited neighbors through the selected node and mark the selected node as visited
- ❑ 5. If all the nodes have been marked visited, then stop
  - Otherwise, go back to step 3

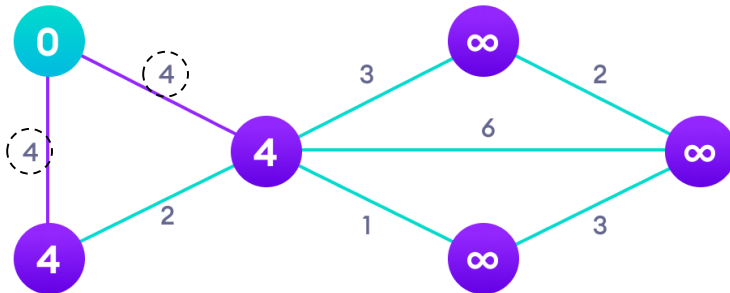
# Example (1/2)



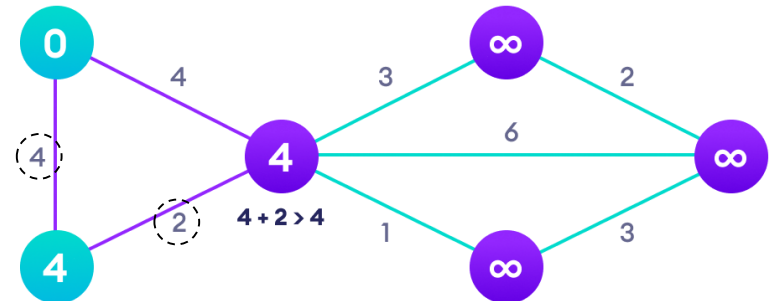
Step: 1



Step: 2

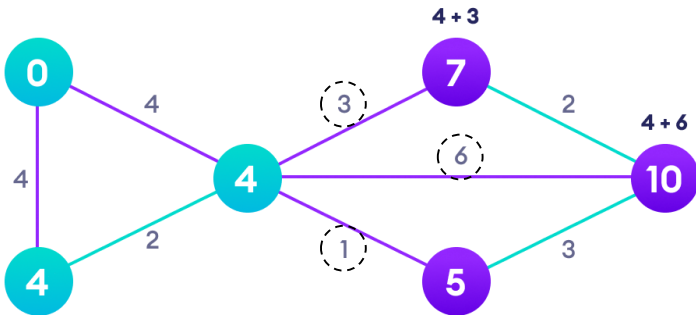


Step: 3

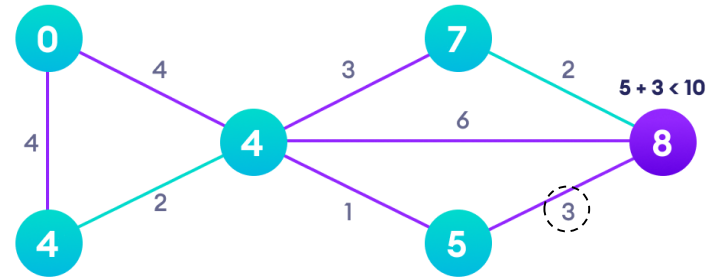


Step: 4

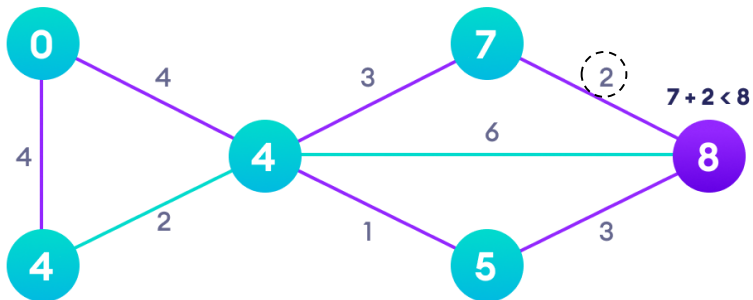
# Example (2/2)



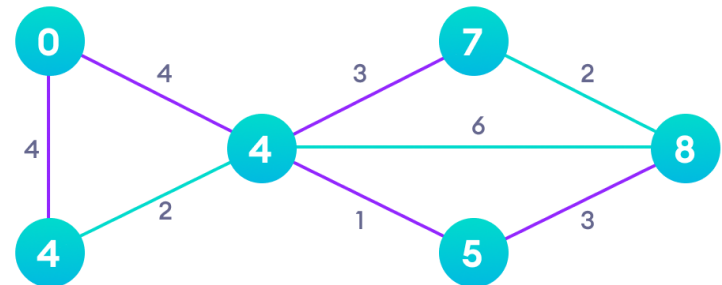
Step: 5



Step: 6



Step: 7



Step: 8



thank you!