Appendix S2. Deriving Fourier Coefficients from a Periodic Function

Let us assume we want to model periodic dispersal rates in species who breed once per year, and estimate that signal in a dynamic occupancy model. Therefore, let us consider a periodic pulse (Fig. S1) that illustrates such a biological pattern and define it over a single period:

$$x_P(t) = \begin{cases} A, |t| \leq \frac{P_u}{2} \\ 0, |t| > \frac{P_u}{2} \end{cases}, \; -\frac{P}{2} < t < \frac{P}{2} \tag{S1}$$

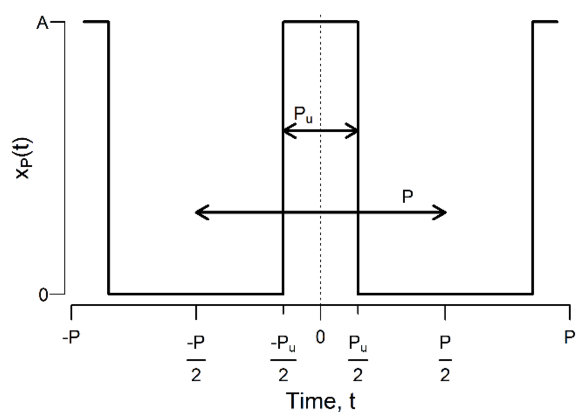

Figure S1. A graphical representation of the periodic pulse function in Eq. S1. Here, a pulse occurs with amplitude *A* once over period *P*.

Eq. S1 has a period of P, an amplitude of A, and a pulse width of P_u. Over a single period when the absolute value of t is less than or equal to P_u /2, Eq. S1 takes the value *A*. Otherwise, it is zero. This represents a short pulse that is separated by a longer trough over time (Fig. S1).

With our species and data, Eq. S1 could represent periodic pulses of increased colonization rates as juveniles disperse at a particular season over the period P = 4 (i.e., from sampling that occurs in the spring, summer, fall, and winter). As Eq. S1 is symmetric around its origin of zero, it is an even function (Kreyszig 2010). Therefore, *bn* = 0 for all *n* and this function can be approximated with a Fourier cosine series

$$x_P(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nw_0 t)) \tag{S2}$$

Which, following Kreyszig (2010), means that a_n can be written as

$$a_n = \frac{2}{P}\int_P x_P(t)\cos(nw_0 t)\,dt\,, \qquad n \neq 0 \tag{S3}$$

Given $x_P(t) = A$ at any time between $-P_u/2$ to $P_u/2$ and 0 at any other point (Eq. S1), a_n can be simplified and then solved. We illustrate this over the interval $-P_u/2$ to $P_u/2$

$$a_n = \frac{2}{P}\int_{-\frac{P_u}{2}}^{\frac{P_u}{2}} x_P(t)\cos(nw_0 t)\,dt \tag{S4}$$

$$= \frac{2}{P}\frac{A}{nw_0}\sin(nw_0 t)\Big|_{-\frac{P_u}{2}}^{+\frac{P_u}{2}} \tag{S5}$$

$$= \frac{2}{P}\frac{A}{nw_0}\left(\sin(nw_0\frac{P_u}{2}) - \sin(-nw_0\frac{P_u}{2})\right) \tag{S6}$$

Furthermore, as sin(a)-sin(-a) = 2sin(a) and $w_0 = 2\pi/P$, Eq. S6 can be reduced to

$$a_n = \frac{2A}{n\pi}\sin(\frac{n\pi P_u}{P}) \tag{S7}$$

As $n \to \infty$, Eq. S2 more closely approximates the graphical representation of the pulse in Fig. S1 over continuous time. However, when included within a statistical model it is impossible to include infinite terms and computational efficiency is reduced as more *n* are included.

Fortunately, when modeling events over discrete time (e.g., t = 1,2,…, *T*) fewer *n* are needed to reach a steady state. Our own trials suggest that *n* should at least range from 1,2,…,*P* for model fitting. In general, dynamic occupancy models necessitate modeling with discrete time and the estimated parameters associated to a species colonization or persistence rates are related to the length of time between primary sampling periods (MacKenzie et al. 2006). If data are collected on a more continuous scale (e.g., year-round monitoring), an investigator should discretize these data in such a way that is relevant to a species biology and the question at hand. For example, if a species breeds once per year and juvenile dispersal occurs during the fall, setting primary sampling periods in the spring, summer, fall, and winter would be an adequate approach so that a periodic component may be included within a model.

The pulse width, P_u, is only a parameter of interest if one expects the pulse to occur over multiple time steps, which may be the case if *P* is large and the time between steps is small. If the pulse should only happen at one specific time step, as in our case, then P_u can be reduced to a value such that $t + P_u/2 < t + 1$, so the pulse does not reach into the surrounding time steps. The natural choice is to set P_u to 1 so that it can then be removed from the equation. Further, Eq. S2 allows us to estimate the amplitude of a pulse, *A*, but it does not let us estimate when the pulse occurs over *P*. We add a phase shift term, δ, to the equation:

$$x_P(t) = a_0 + \sum_{n=1}^{P} \frac{2A}{\pi n} \sin\left(\frac{\pi n}{P}\right) \cos\left(\frac{2\pi n}{P}(t - \delta)\right) \tag{S8}$$

So that the model can then estimate the presence of a periodic pulse at any time step over period P with two parameters, *A* and δ (Fig. S2).

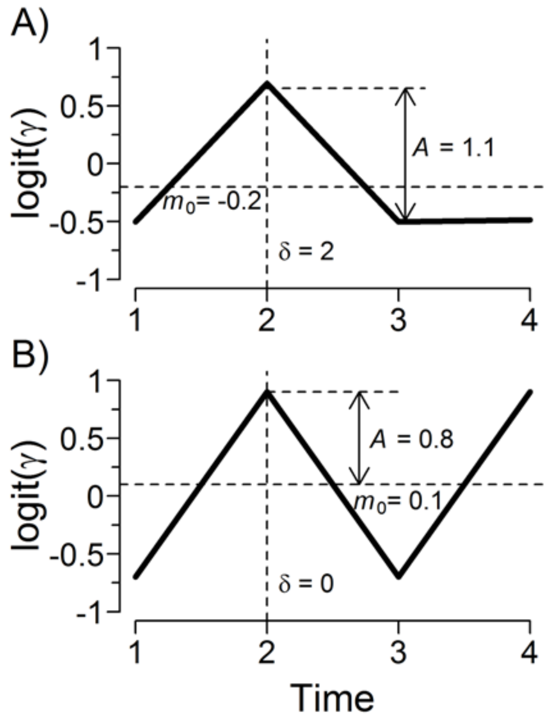

Figure S2. Examples of periodic curves when only discrete integers (i.e., t = 1,2,3,4) are supplied for A) a single season pulse with period P = 4 as specified in Eq. S2 and B) a boom-bust sinusoidal curve with period P = 2 as specified in Eq. 10 of the main text. Plots A and B are on the logit scale. m_0 controls the average colonization rate, A controls the amplitude, and δ controls when a pulse occurs.

Literature Cited

Kreyszig, E., 2010. *Advanced engineering mathematics*. John Wiley & Sons.