

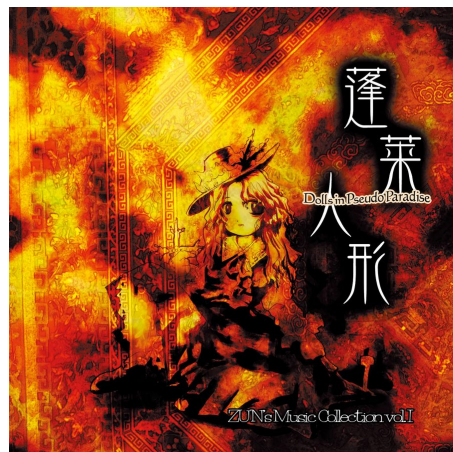
Our village of honest men originally consisted of only eight people.  
We all picked up and moved to a mountain in the east. Two years of honest and boring daily life passed us by.  
One day, one of us found a little hole by a peach tree.  
Yes, after that we wandered into this paradise.  
And right away, I quit being human.

---

— *Dolls in Pseudo Paradise*

## 蓬莱人形算法模板库

REFERENCE DOCUMENT for *Dolls in Pseudo Paradise*



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Harbin Institute of Technology

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```
1 #include "../header.cpp"
2 int F[MAXN];
3 int main(){
4     int n, m; cin >> n >> m;
5     for(int i = 1; i <= n; ++ i){
```

```

6   int w, v, c; cin >> w >> v >> c;
7   // w: value, v: volume, c: count
8   for(int j = 0; j < v; ++j){
9       deque<tuple<int, int>> > Q;
10      for(int k = 0; j + k * v ≤ m; ++k)
11          {
12              int x = j + k * v;
13              int f = F[x] - (x / v) * w;
14              while(!Q.empty() && get<0>(Q.back()) ≤ f)
15                  Q.pop_back();
16              Q.push_back({f, x});
17              while(!Q.empty() && get<1>(Q.front()) < x - c * v)
18                  Q.pop_front();
19              F[x] = get<0>(Q.front()) + (x / v) * w;
20          }
21      }
22      cout << F[m] << endl;
23      return 0;
24  }

```

## 1.2 树形背包

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  typedef long long i64;
4  const int MAXN = 2e3 + 3;
5  vector<int> E[MAXN];
6  int W[MAXN];
7  int F[MAXN][MAXN], S[MAXN];
8  void dfs(int u, int f){
9      F[u][1] = W[u], S[u] = 1;
10     for(auto &v : E[u]) if(v ≠ f){
11         dfs(v, u);
12         for(int i = S[u]; i ≥ 1; --i)
13             for(int j = S[v]; j ≥ 1; --j)
14                 F[u][i + j] = max(F[u][i + j], F[u][i]
15                                     + F[v][j]);
16         S[u] += S[v];
17     }
18 }
19 int main(){
20     int n, m;
21     cin >> n >> m;
22     for(int i = 1; i ≤ n; ++i){
23         int f;
24         cin >> f >> W[i];
25         E[f].push_back(i);
26     }

```

```

26     dfs(0, 0);
27     cout << F[0][m + 1] << endl;
28     return 0;
29 }

```

## 1.3 动态动态规划 1

### 1.3.1 例题

给定一棵  $n$  个点的树，点有点权，求最大独立集。 $m$  次修改，每次把  $x$  的权值修改成  $y$ 。

```

1  #include "../header.cpp"
2  int W[MAXN];
3  struct Mat{ int M[2][2]; };
4  struct Vec{ int V[2]; };
5  Mat operator *(const Mat &a, const Mat &b){
6      Mat c;
7      c.M[0][0] = max(a.M[0][0] + b.M[0][0], a.M[0][1]
8                      + b.M[1][0]);
9      c.M[0][1] = max(a.M[0][0] + b.M[0][1], a.M[0][1]
10                     + b.M[1][1]);
11     c.M[1][0] = max(a.M[1][0] + b.M[0][0], a.M[1][1]
12                     + b.M[1][0]);
13     c.M[1][1] = max(a.M[1][0] + b.M[0][1], a.M[1][1]
14                     + b.M[1][1]);
15     return c;
16 }
17 Vec operator *(const Mat &a, const Vec &b){
18     Vec r;
19     r.V[0] = max(a.M[0][0] + b.V[0], a.M[0][1] + b.V[1]);
20     r.V[1] = max(a.M[1][0] + b.V[0], a.M[1][1] + b.V[1]);
21     return r;
22 }
23 namespace Gra{
24     vector<int> E[MAXN];
25     int G[MAXN], S[MAXN], D[MAXN], T[MAXN], F[MAXN];
26     int X[MAXN], Y[MAXN];
27     int H[MAXN][2];
28     int K[MAXN][2];
29     struct Mat M[MAXN];
30     void dfs1(int u, int f){
31         S[u] = 1;
32         F[u] = f;
33         for(auto &v : E[u]) if(v ≠ f){
34             dfs1(v, u);
35             S[u] += S[v];
36             if(S[v] > S[G[u]]) G[u] = v;
37         }
38     }

```

```

34 }
35 int o;
36 void dfs2(int u, int f){
37     if(u = G[f])
38         X[u] = X[f];
39     else
40         X[u] = u;
41     H[u][0] = H[u][1] = 0;
42     K[u][0] = K[u][1] = 0;
43     const int &g = G[u];
44     D[u] = ++o;
45     T[o] = u;
46     if(g){
47         dfs2(g, u);
48         Y[u] = Y[g];
49         K[u][0] += max(K[g][0], K[g][1]);
50         K[u][1] += K[g][0];
51     } else {
52         Y[u] = u;
53     }
54     for(auto &v : E[u]) if(v ≠ f && v ≠ g){
55         dfs2(v, u);
56         H[u][0] += max(K[v][0], K[v][1]);
57         H[u][1] += K[v][0];
58     }
59     M[u].M[0][0] = H[u][0];
60     M[u].M[0][1] = H[u][0];
61     M[u].M[1][0] = H[u][1] + W[u];
62     M[u].M[1][1] = -INF;
63     K[u][0] += H[u][0];
64     K[u][1] += H[u][1] + W[u];
65 }
66 }
67 namespace Seg{
68     const int SZ = 4e5 + 3;
69     struct Mat M[SZ];
70     #define lc(t) (t << 1)
71     #define rc(t) (t << 1 | 1)
72     void pushup(int t, int a, int b){
73         M[t] = M[lc(t)] * M[rc(t)];
74     }
75     void build(int t, int a, int b){
76         if(a = b){
77             M[t] = Gra :: M[Gra :: T[a]];
78         } else {
79             int c = a + b >> 1;
80             build(lc(t), a, c);
81             build(rc(t), c + 1, b);
82             pushup(t, a, b);
83         }
84     }
85     void modify(int t, int a, int b, int p,

```

```

86  const Mat &w){
87  if(a == b){
88      M[t] = w;
89  } else {
90      int c = a + b >> 1;
91      if(p ≤ c) modify(lc(t), a, c, p, w);
92      else modify(rc(t), c + 1, b, p, w);
93      pushup(t, a, b);
94  }
95  Mat query(int t, int a, int b, int l, int r)
96  {
97      if(l ≤ a && b ≤ r){
98          return M[t];
99      } else {
100          int c = a + b >> 1;
101          if(r ≤ c) return query(lc(t), a, c, l, r); else
102          if(l > c) return query(rc(t), c + 1, b, l, r); else
103          return query(lc(t), a, c, l, r) *
104          query(rc(t), c + 1, b, l, r);
105      }
106  }
107  int qread();
108  int main(){
109      int n = qread(), m = qread();
110      up(1, n, i)
111      W[i] = qread();
112      up(2, n, i){
113          int u = qread(), v = qread();
114          Gra :: E[u].push_back(v);
115          Gra :: E[v].push_back(u);
116      }
117      Gra :: dfs1(1, 0);
118      Gra :: dfs2(1, 0);
119      Seg :: build(1, 1, n);
120      Vec v0;
121      v0.V[0] = v0.V[1] = 0;
122      up(1, m, i){
123          using namespace Gra;
124          int x = qread(), y = qread();
125          W[x] = y;
126          int u = x;
127          while(u ≠ 0){
128              const int &v = X[u];
129              const int &f = F[v];
130              M[u].M[0][0] = H[u][0];
131              M[u].M[0][1] = H[u][0];
132              M[u].M[1][0] = H[u][1] + W[u];
133              M[u].M[1][1] = -INF;

```

```

134  const Vec p = Seg :: query(1, 1, n, D[v
135  ], D[Y[u]]) * v0;
136  Seg :: modify(1, 1, n, D[u], M[u]);
137  const Vec q = Seg :: query(1, 1, n, D[v
138  ], D[Y[u]]) * v0;
139  if(f ≠ 0){
140      H[f][0] = H[f][0] - max(p.V[0], p.V
141      [1]) + max(q.V[0], q.V[1]);
142      H[f][1] = H[f][1] - p.V[0] + q.V[0];
143      u = f;
144  }
145  Vec v1 = Seg :: query(1, 1, n, D[1], D[Y
146  [1]]) * v0;
147  printf("%d\n", max(v1.V[0], v1.V[1]));
148  }
149  return 0;
150  }

```

## 1.4 插头 dp

### 1.4.1 例题

给出  $n \times m$  的方格，有些格子不能铺线，其它格子必须铺，形成一个闭合回路。问有多少种铺法？

```

1  #include "../header.cpp"
2  namespace HashT{
3      const int SIZ = 19999997;
4      int H[SIZ], V[SIZ], N[SIZ], t;
5      bool F[SIZ];
6      i64 W[SIZ];
7      void add(int u, int v, bool f, i64 w){
8          V[++ t] = v, N[t] = H[u], F[t] = f, W[t] =
9          w, H[u] = t;
10     }
11     i64& find(int u, bool f){
12         for(int p = H[u % SIZ]; p; p = N[p])
13             if(V[p] == u && F[p] == f)
14                 return W[p];
15         add(u % SIZ, u, f, 0);
16         return W[t];
17     }
18     char S[MAXN][MAXN];
19     int qread();
20     int n, m;
21     vector <pair<pair<int, bool>, i64>> M[2];
22     int getp(int s, int p){
23         return (s >> (2 * p - 2)) & 3;
24     }
25     int setw(int s, int p, int w){

```

```

26     return (s & ~(3 << (2 * p - 2))) | (w << (2
27     * p - 2));
28 }
29 int findr(int s, int p){
30     int c = 0;
31     for(int q = p; q ≤ m + 1; ++ q){
32         if(((s >> (2 * q - 2)) & 3) == 1) ++ c;
33         if(((s >> (2 * q - 2)) & 3) == 2) -- c;
34         if(c == 0)
35             return q;
36     }
37     return -1;
38 }
39 int findl(int s, int p){
40     int c = 0;
41     for(int q = p; q ≥ 1; -- q){
42         if(((s >> (2 * q - 2)) & 3) == 2) ++ c;
43         if(((s >> (2 * q - 2)) & 3) == 1) -- c;
44         if(c == 0)
45             return q;
46     }
47     return -1;
48 }
49 void state(int s){
50     return ;
51     up(1, m + 1, i){
52         switch(getp(s, i)){
53             case 0 : putchar('#'); break;
54             case 1 : putchar('('); break;
55             case 2 : putchar(')'); break;
56             case 3 : putchar('E');
57         }
58     }
59     puts("");
60 }
61 int main(){
62     n = qread(), m = qread();
63     up(1, n, i)
64     scanf("%s", S[i] + 1);
65     int o = 0;
66     #define X M[ o]
67     #define Y M[!o]
68     vector <pair<int, bool>> T;
69     X.push_back({0, 0}, 1});
70     up(1, n, i){
71         Y.clear();
72         for(auto &u : X){
73             auto [s0, c] = u;
74             auto [s, f] = s0;
75             if(getp(s, m + 1) == 0)
76                 Y.push_back({s << 2, f}, c);
77         }

```

```

77 o ^= 1;
78 up(1, m, j){
79     int x = j, y = j + 1;
80     for(auto &u : X){
81         auto [s0, c] = u;
82         auto [s, f] = s0;
83         int a = getp(s, x);
84         int b = getp(s, y);
85         int t = setw(setw(s, x, 0), y, 0);
86         #define update(t, c) HashT :: find(t,
            f) += c, T.push_back({t, f})
87         if(S[i][j] == '.'){ // 经过该格
88             if(a == 1 && b == 1){
89                 t = setw(t, findr(s, y), 1),
90                 update(t, c);
91             } else
92             if(a == 2 && b == 2){
93                 t = setw(t, findl(s, x), 2),
94                 update(t, c);
95             } else
96             if(a == 1 && b == 2){
97                 if(f == false) // 还没有闭合回路
98                     f = true, update(t, c);
99             } else
100             if(a == 2 && b == 1){
101                 update(t, c);
102             } else
103             if(a == 0 && b == 0){
104                 t = setw(t, x, 1);
105                 t = setw(t, y, 2);
106                 update(t, c);
107             } else { // a == 0 || b == 0
108                 int t1 = setw(t, x, a | b);
109                 int t2 = setw(t, y, a | b);
110                 update(t1, c);
111                 update(t2, c);
112             }
113         }
114         if(S[i][j] == '*'){ // 不经过该格
115             if(a == 0 && b == 0)
116                 update(t, c);
117         }
118     }
119     Y.clear();
120     for(auto &u : T){
121         auto [s, f] = u;
122         if(HashT :: find(s, f) != 0){
123             Y.push_back({{s, f}, HashT :: find(s,
                f)});
124             HashT :: find(s, f) = 0;
125         }
126     }

```

```

127     T.clear(), o ^= 1;
128 }
129 }
130 i64 ans = 0;
131 for(auto &u : X){
132     auto [s0, c] = u;
133     auto [s, f] = s0;
134     bool g = true;
135     up(1, m + 1, i)
136     g &= getp(s, i) == 0;
137     f &= g;
138     if(f)
139         ans = c;
140 }
141 printf("%lld\n", ans);
142 return 0;
143 }

```

## 1.5 斜率优化

### 1.5.1 形式

考虑一个经典的 dp 转移方程如下：

$$f_i = \max_{j < i} \{f(j) + w(j, i)\}$$

我们将式子拆成三个部分：只跟  $i$  有关或者与  $i, j$  均不相关的部分  $a(i)$ ，只跟  $j$  有关的部分  $b(j)$ ，跟  $i, j$  均有关的部分  $c(i, j)$ ：

$$f_i = a(i) + \max_{j < i} \{b(j) + c(i, j)\}$$

斜率优化可被用来解决这样一个情形： $c(i, j) = ic_j$ 。此时  $b(j) + c(i, j)$  可视为关于  $j$  的一次函数。如果  $c_j$  随着  $j$  的增大而单调，那么可用单调栈维护；否则可以考虑 CDQ 分治或者在凸包上二分。在凸包上可以使用二分查询最高/最低点。

### 1.5.2 例题

玩具装箱。原始转移方程为：

$$f_i = \max_{j < i} \{f_j + (s_i - s_j - L')^2\}$$

其中  $s_i = i + \sum_{j \leq i} c_i, L' = L + 1$ 。将其分类得到：

$$f_i = \max_{j < i} \{f_j + s_i^2 + s_j^2 + L'^2 - 2s_i s_j + 2s_j L' - 2s_i L'\}$$

$$= (s_i^2 - 2s_i L' + L'^2) + \max_{j < i} \{(f_j + s_j^2 + 2s_j L') - 2s_i s_j\}$$

在原始的玩具装箱中， $s_j$  单调增加，也就是斜率单调增加。因此可以直接使用单调栈维护凸包。同时  $s_i$  也单调增加，因此可以用指针维护。

```

1 #include "../header.cpp"
2 int n, L, p, e, C[MAXN], Q[MAXN];
3 f80 S[MAXN], F[MAXN];
4 f80 gtx(int x){ return S[x]; }
5 f80 gty(int x){ return F[x] + S[x] * S[x]; }
6 f80 gtw(int x){ return -2.0 * (L - S[x]); }
7 f80 gtk(int x, int y){ return (gty(y) - gty(x))
    / (gtx(y) - gtx(x)); }
8 int main(){
9     cin >> n >> L;
10    for(int i = 1; i <= n; ++ i){
11        cin >> C[i];
12        S[i] = S[i - 1] + C[i];
13    }
14    for(int i = 1; i <= n; ++ i){
15        S[i] += i;
16    }
17    e = p = 1, L ++, Q[p] = 0;
18    for(int i = 1; i <= n; ++ i){
19        while(e < p && gtk(Q[e], Q[e + 1]) < gtw(i
            ))
20            ++ e;
21        int j = Q[e];
22        F[i] = F[j] + pow(S[i] - S[j] - L, 2);
23        while(1 < p && gtk(Q[p - 1], Q[p]) > gtk(Q
            [p], i))
24            e -= (e == p), -- p;
25        Q[++ p] = i;
26    }
27    printf("%.0Lf\n", F[n]);
28    return 0;
29 }

```

## 2 数据结构

### 2.1 平衡树

#### 2.1.1 无旋 Treap



```

1 #include "../header.cpp"
2 mt19937_64 MT(114514);
3 namespace Treap{
4     const int SZ = 1e6 + 1e5 + 3;
5     int F[SZ], C[SZ], S[SZ], W[SZ], X[SZ][2], sz;
6     u64 H[SZ];
7     int newnode(int w){
8         W[++sz] = w, C[sz] = S[sz] = 1; H[sz] = MT();
9         return sz;
10    }
11    void pushup(int x){
12        S[x] = C[x] + S[X[x][0]] + S[X[x][1]];
13    }
14    pair<int, int> split(int u, int x){
15        if(u == 0)
16            return make_pair(0, 0);
17        if(W[u] > x){
18            auto [a, b] = split(X[u][0], x);
19            X[u][0] = b, pushup(u);
20            return make_pair(a, u);
21        } else {
22            auto [a, b] = split(X[u][1], x);
23            X[u][1] = a, pushup(u);
24            return make_pair(u, b);
25        }
26    }
27    int merge(int a, int b){
28        if(a == 0 || b == 0)
29            return a | b;
30        if(H[a] < H[b]){
31            X[a][1] = merge(X[a][1], b), pushup(a);
32            return a;
33        } else {
34            X[b][0] = merge(a, X[b][0]), pushup(b);
35            return b;
36        }
37    }
38    void insert(int &root, int w){
39        auto [p, q] = split(root, w);
40        auto [a, b] = split(p, w - 1);
41        if(b != 0){
42            ++S[b], ++C[b];
43        } else b = newnode(w);
44        p = merge(a, b);
45        root = merge(p, q);
46    }
47    void erase(int &root, int w){
48        auto [p, q] = split(root, w);
49        auto [a, b] = split(p, w - 1);

```

```

50        -- C[b], -- S[b];
51        p = C[b] == 0 ? a : merge(a, b);
52        root = merge(p, q);
53    }
54    int find_rank(int &root, int w){
55        int x = root, o = x, a = 0;
56        for(;;x){
57            if(w < W[x])
58                o = x, x = X[x][0];
59            else {
60                a += S[X[x][0]];
61                if(w == W[x]){
62                    o = x; break;
63                }
64                a += C[x];
65                o = x, x = X[x][1];
66            }
67        }
68        return a + 1;
69    }
70    int find_kth(int &root, int w){
71        int x = root, o = x, a = 0;
72        for(;;x){
73            if(w <= S[X[x][0]])
74                o = x, x = X[x][0];
75            else {
76                w -= S[X[x][0]];
77                if(w <= C[x]){
78                    o = x; break;
79                }
80                w -= C[x];
81                o = x, x = X[x][1];
82            }
83        }
84        return W[x];
85    }
86    int find_pre(int &root, int w){
87        return find_kth(root, find_rank(root, w) - 1);
88    }
89    int find_suc(int &root, int w){
90        return find_kth(root, find_rank(root, w) + 1);
91    }
92 }

```

### 2.1.2 Splay

```

1 #include "../header.cpp"
2 namespace Splay{
3     const int SZ = 1e6 + 1e5 + 3;
4     int F[SZ], C[SZ], S[SZ], X[SZ][2], size;

```

```

5     bool T[SZ];
6     bool is_root(int x){ return F[x] == 0; }
7     bool is_rson(int x){ return X[F[x]][1] == x; }
8     void push_down(int x){
9         if(!T[x]) return;
10        int lc = X[x][0], rc = X[x][1];
11        if(lc) T[lc] ^= 1, swap(X[lc][0], X[lc][1]);
12        if(rc) T[rc] ^= 1, swap(X[rc][0], X[rc][1]);
13        T[x] = 0;
14    }
15    void pushup(int x){
16        S[x] = C[x] + S[X[x][0]] + S[X[x][1]];
17    }
18    void rotate(int x){
19        int y = F[x], z = F[y];
20        bool f = is_rson(x);
21        bool g = is_rson(y);
22        int &t = X[x][!f];
23        if(z){ X[z][g] = t; }
24        if(t){ F[t] = y; }
25        X[y][f] = t, t = y;
26        F[y] = x, pushup(y);
27        F[x] = z, pushup(x);
28    }
29    void splay(int &r, int x, int g = 0){
30        for(int f; f = F[x], f != g; rotate(x))
31            if(F[f] != g) rotate(is_rson(x) == is_rson(f) ? f : x);
32            if(is_root(x)) r = x;
33    }
34    int get_kth(int &r, int w){
35        int x = r, o = x;
36        for(;;x){
37            push_down(x);
38            if(w <= S[X[x][0]]) o = x, x = X[x][0];
39            else {
40                w -= S[X[x][0]];
41                if(C[x] && w <= C[x]){ o = x; break; }
42                w -= C[x], o = x, x = X[x][1];
43            }
44        }
45        splay(r, o); return o;
46    }
47    int build(int l, int r){
48        if(l == r){
49            C[l] = S[l] = 1; return l;
50        }
51        int c = l + r >> 1, a = 0, b = 0;
52        if(l <= c - 1) a = build(l, c - 1), F[a] =

```

```

    c, X[c][0] = a;
    if(c + 1 ≤ r) b = build(c + 1, r), F[b] =
    c, X[c][1] = b;
    C[c] = 1, pushup(c); return c;
}
void output(int n, int &r){
    push_down(r);
    if(X[r][0]) output(n, X[r][0]);
    if(r ≠ 1 && r ≠ n + 2) printf("%d ", r -
    1);
    if(X[r][1]) output(n, X[r][1]);
}
}

```

### 2.1.3 Treap

```

1 #include "../header.cpp"
2 mt19937_64 MT(114514);
3 namespace Treap{
4     const int SIZ = 1e6 + 1e5 + 3;
5     int F[SIZ], C[SIZ], S[SIZ], W[SIZ], X[SIZ
6         ][2], sz;
7     u64 H[SIZ];
8     bool is_root(int x){ return F[x] == 0; }
9     bool is_rson(int x){ return X[F[x]][1] == x
10         ; }
11     int newnode(int w){
12         W[++sz] = w, C[sz] = S[sz] = 1; H[sz] =
13         MT();
14         return sz;
15     }
16     void pushup(int x){
17         S[x] = C[x] + S[X[x][0]] + S[X[x][1]];
18     }
19     void rotate(int &root, int x){
20         int y = F[x], z = F[y];
21         bool f = is_rson(x);
22         bool g = is_rson(y);
23         int &t = X[x][!f];
24         if(z){ X[z][g] = x; } else root = x;
25         if(t){ F[t] = y; }
26         X[y][f] = t, t = y;
27         F[y] = x, pushup(y);
28         F[x] = z, pushup(x);
29     }
30     void insert(int &root, int w){
31         if(root == 0) {root = newnode(w); return;}
32         int x = root, o = x;
33         for(;x;o = x, x = X[x][w > W[x]]){
34             ++S[x]; if(w == W[x]){ ++C[x], o = x;
35             break;}
36         }
37     }
38 }

```

```

33 if(W[o] ≠ w){
34     if(w < W[o]) X[o][0] = newnode(w), F[sz]
35     = o, o = sz;
36     else X[o][1] = newnode(w), F[sz] = o
37     , o = sz;
38 }
39 while(!is_root(o) && H[o] < H[F[o]])
40     rotate(root, o);
41 void erase(int &root, int w){
42     int x = root, o = x;
43     for(;x;o = x, x = X[x][w > W[x]]){
44         --S[x]; if(w == W[x]){ --C[x], o = x;
45         break;}
46     }
47     if(C[o] == 0){
48         while(X[o][0] || X[o][1]){
49             u64 wl = X[o][0] ? H[X[o][0]] :
50             ULLONG_MAX;
51             u64 wr = X[o][1] ? H[X[o][1]] :
52             ULLONG_MAX;
53             if(wl < wr){
54                 int p = X[o][0]; rotate(root, p);
55             } else {
56                 int p = X[o][1]; rotate(root, p);
57             }
58         }
59         if(is_root(o)){
60             root = o;
61         } else {
62             X[F[o]][is_rson(o)] = o;
63         }
64     }
65     int find_rank(int &root, int w){
66         int x = root, o = x, a = 0;
67         for(;x;o = x, x = X[x][w > W[x]]){
68             if(w < W[x])
69                 o = x, x = X[x][0];
70             else {
71                 a += S[X[x][0]];
72                 if(w == W[x]){
73                     o = x; break;
74                 }
75                 a += C[x];
76                 o = x, x = X[x][1];
77             }
78         }
79         return a + 1;
80     }
81     int find_kth(int &root, int w){
82         int x = root, o = x, a = 0;
83     }
84 }

```

```

80 for(;x;){
81     if(w ≤ S[X[x][0]])
82         o = x, x = X[x][0];
83     else {
84         w -= S[X[x][0]];
85         if(w ≤ C[x]){
86             o = x; break;
87         }
88         w -= C[x];
89         o = x, x = X[x][1];
90     }
91 }
92 return W[x];
93 }
94 int find_pre(int &root, int w){
95     return find_kth(root, find_rank(root, w) -
96     1);
97 }
98 int find_suc(int &root, int w){
99     return find_kth(root, find_rank(root, w +
100     1));
101 }

```

## 2.2 珂朵莉树

```

1 #include "../header.cpp"
2 namespace ODT {
3     // <pos_type, value_type>
4     map<int, long long> M;
5     // 分裂为 [1, p) 和 [p, +inf), 返回后者迭代
6     器
7     auto split(int p) {
8         auto it = prev(M.upper_bound(p));
9         return M.insert(
10             it,
11             make_pair(p, it → second)
12         );
13     }
14     // 区间赋值
15     void assign(int l, int r, int v) {
16         auto it = split(l);
17         split(r + 1);
18         while (it → first ≠ r + 1) {
19             it = M.erase(it);
20         }
21         M[l] = v;
22     }
23     // // 执行操作
24     // void perform(int l, int r) {
25     //     auto it = split(l);
26     //     split(r + 1);
27 }

```

```

26 // while (it → first ≠ r + 1) {
27 //     // Do something...
28 //     it = next(it);
29 // }
30 // }
31 };

```

### 2.3 可并堆

```

1 #include "../header.cpp"
2 namespace LeftHeap{
3     const int SIZ = 1e5 + 3;
4     int W[SIZ], D[SIZ], L[SIZ], R[SIZ], F[SIZ],
5     s;
6     bool E[SIZ];
7     int merge(int u, int v){
8         if(u == 0 || v == 0)
9             return u | v;
10        if(W[u] > W[v] || (W[u] == W[v] && u > v))
11            swap(u, v);
12        int &lc = L[u];
13        int &rc = R[u];
14        rc = merge(rc, v);
15        if(D[lc] < D[rc])
16            swap(lc, rc);
17        D[u] = min(D[lc], D[rc]) + 1;
18        if(lc ≠ 0) F[lc] = u;
19        if(rc ≠ 0) F[rc] = u;
20        return u;
21    }
22    void pop(int &root){
23        int root0 = merge(L[root], R[root]);
24        F[root0] = root0;
25        F[root] = root0;
26        E[root] = true;
27        root = root0;
28    }
29    int top(int &root){
30        return W[root];
31    }
32    int getfa(int u){
33        return u = F[u] ? u : F[u] = getfa(F[u]);
34    }
35    int newnode(int w){
36        ++ s;
37        W[s] = w;
38        F[s] = s;
39        D[s] = 1;
40        return s;
41    }

```

### 2.4 线性基

```

1 #include "../header.cpp"
2 namespace LB{
3     const int SIZ = 60 + 3;
4     i64 W[SIZ], h = 60;
5     void insert(i64 w){
6         for(int i = h; i ≥ 0; -- i){
7             if(w & (1ll << i)){
8                 if(!W[i]){
9                     W[i] = w;
10                    break;
11                } else {
12                    w ^= W[i];
13                }
14            }
15        }
16    }
17    i64 query(i64 x){
18        for(int i = h; i ≥ 0; -- i){
19            if(W[i]){
20                x = max(x, x ^ W[i]);
21            }
22        }
23        return x;
24    }
25 }
26 namespace realLB{
27     const int SIZ = 500 + 3;
28     long double W[SIZ][SIZ];
29     int n = 0;
30     void init(int n0){
31         n = n0;
32     }
33     bool zero(long double w){
34         return fabs(w) < 1e-9;
35     }
36     bool insert(long double X[]){
37         for(int i = 1; i ≤ n; ++ i){
38             if(!zero(X[i])){
39                 if(zero(W[i][i])){
40                     for(int j = 1; j ≤ n; ++ j)
41                         W[i][j] = X[j];
42                     return true;
43                 } else {
44                     long double t = X[i] / W[i][i];
45                     for(int j = 1; j ≤ n; ++ j)
46                         X[j] -= t * W[i][j];
47                 }
48             }
49        }
50        return false;
51    }

```

```

52 }
53 // === TEST ===
54 int qread();
55 const int MAXN = 500 + 3;
56 long double X[MAXN][MAXN], C[MAXN];
57 int I[MAXN];
58 bool cmp(int a, int b){
59     return C[a] < C[b];
60 }
61 int main(){
62     int n, m;
63     cin >> n >> m;
64     realLB :: init(m);
65     for(int i = 1; i ≤ n; ++ i){
66         for(int j = 1; j ≤ m; ++ j){
67             cin >> X[i][j];
68         }
69     }
70     for(int i = 1; i ≤ n; ++ i){
71         cin >> C[i];
72         I[i] = i;
73     }
74     sort(I + 1, I + 1 + n, cmp);
75     int ans = 0, cnt = 0;
76     for(int i = 1; i ≤ n; ++ i){
77         int x = I[i];
78         if(realLB :: insert(X[x]))
79             ans += C[x],
80             cnt += 1;
81     }
82     cout << cnt << " " << ans << endl;
83     return 0;
84 }

```

### 2.5 Link Cut 树

```

1 #include "../header.cpp"
2 namespace LinkCutTree{
3     const int SIZ = 1e5 + 3;
4     int F[SIZ], C[SIZ], S[SIZ], W[SIZ], A[SIZ],
5     X[SIZ][2], size;
6     bool T[SIZ];
7     bool is_root(int x){ return X[F[x]][0] ≠ x
8     && X[F[x]][1] ≠ x; }
9     bool is_rson(int x){ return X[F[x]][1] = x; }
10    int new_node(int w){
11        ++ size;
12        W[size] = w, C[size] = S[size] = 1;
13        A[size] = w, F[size] = 0;
14        X[size][0] = X[size][1] = 0;
15        return size;

```



```

14 }
15 void push_up(int x){
16     S[x] = C[x] + S[X[x][0]] + S[X[x][1]];
17     A[x] = W[x] ^ A[X[x][0]] ^ A[X[x][1]];
18 }
19 void push_down(int x){
20     if(!T[x]) return;
21     int lc = X[x][0], rc = X[x][1];
22     if(lc) T[lc] ^= 1, swap(X[lc][0], X[lc][1]);
23     if(rc) T[rc] ^= 1, swap(X[rc][0], X[rc][1]);
24     T[x] = false;
25 }
26 void update(int x){
27     if(!is_root(x)) update(F[x]); push_down(x);
28 }
29 void rotate(int x){
30     int y = F[x], z = F[y];
31     bool f = is_rson(x);
32     bool g = is_rson(y);
33     if(is_root(y)){
34         F[x] = z, F[y] = x;
35         X[y][f] = X[x][!f], F[X[x][!f]] = y;
36         X[x][!f] = y;
37     } else {
38         F[x] = z, F[y] = x;
39         X[z][g] = x;
40         X[y][f] = X[x][!f], F[X[x][!f]] = y;
41         X[x][!f] = y;
42     }
43     push_up(y), push_up(x);
44 }
45 void splay(int x){
46     update(x);
47     for(int f = F[x]; f = F[x], !is_root(x); rotate(x))
48         if(!is_root(f)) rotate(is_rson(x) == is_rson(f) ? f : x);
49 }
50 int access(int x){
51     int p;
52     for(p = 0; x; p = x, x = F[x]){
53         splay(x), X[x][1] = p, push_up(x);
54     }
55     return p;
56 }
57 void make_root(int x){
58     x = access(x);
59     T[x] ^= 1, swap(X[x][0], X[x][1]);
60 }

```

```

61 int find_root(int x){
62     access(x), splay(x), push_down(x);
63     while(X[x][0]) x = X[x][0], push_down(x);
64     splay(x);
65     return x;
66 }
67 void link(int x, int y){
68     make_root(x), splay(x), F[x] = y;
69 }
70 void cut(int x, int p){
71     make_root(x), access(p), splay(p), X[p][0] = F[x] = 0;
72 }
73 void modify(int x, int w){
74     splay(x), W[x] = w, push_up(x);
75 }
76 }
77 const int MAXN = 1e5 + 3;
78 map<pair<int, int>, bool> M;
79 int n, m;
80 int main(){
81     cin >> n >> m;
82     for(int i = 1; i ≤ n; ++ i){
83         int a; cin >> a;
84         LinkCutTree :: new_node(a);
85     }
86     for(int i = 1; i ≤ m; ++ i){
87         int o; cin >> o;
88         if(o == 0){
89             int u, v; cin >> u >> v;
90             LinkCutTree :: make_root(u);
91             int p = LinkCutTree :: access(v);
92             printf("%d\n", LinkCutTree :: A[p]);
93         } else if(o == 1){
94             int u, v; cin >> u >> v;
95             int a = LinkCutTree :: find_root(u);
96             int b = LinkCutTree :: find_root(v);
97             if(a ≠ b){
98                 LinkCutTree :: link(u, v);
99                 M[make_pair(min(u, v), max(u, v))] = true;
100             }
101         } else if(o == 2){
102             int u, v; cin >> u >> v;
103             if(M.count(make_pair(min(u, v), max(u, v)))){
104                 M.erase(make_pair(min(u, v), max(u, v)));
105                 LinkCutTree :: cut(u, v);
106             }
107         } else {
108             int u, w; cin >> u >> w;

```

```

109     LinkCutTree :: modify(u, w);
110 }
111 }
112 return 0;
113 }

```

## 2.6 线段树

### 2.6.1 李超树

```

1 #include "../..header.cpp"
2 struct Line{ int id; double k, b; Line() = default; };
3 namespace LCSeg{
4     const int SIZ = 2e5 + 3;
5     struct Line T[SIZ];
6     #define lc(t) (t << 1)
7     #define rc(t) (t << 1 | 1)
8     bool cmp(int p, Line x, Line y){
9         double w1 = x.k * p + x.b;
10        double w2 = y.k * p + y.b;
11        double d = w1 - w2;
12        if(fabs(d) < 1e-8) return x.id > y.id;
13        return d < 0;
14    }
15    void merge(int t, int a, int b, Line x, Line y){
16        int c = a + b >> 1;
17        if(cmp(c, x, y)) swap(x, y);
18        if(cmp(a, y, x)){
19            T[t] = x; if(a ≠ b) merge(rc(t), c + 1, b, T[rc(t)], y);
20        } else {
21            T[t] = x; if(a ≠ b) merge(lc(t), a, c, T[lc(t)], y);
22        }
23    }
24    // 插入线段 (l, f(l)) -- (r, f(r))
25    void modify(int t, int a, int b, int l, int r, Line x){
26        if(l ≤ a && b ≤ r) merge(t, a, b, T[t], x);
27        else {
28            int c = a + b >> 1;
29            if(l ≤ c) modify(lc(t), a, c, l, r, x);
30            if(r > c) modify(rc(t), c + 1, b, l, r, x);
31        }
32    }
33    // 查询 x = p 位置最高的线段 (有多条取编号最小)

```

```

34 void query(int t, int a, int b, int p, Line
    &x){
35     if(cmp(p, x, T[t])) x = T[t];
36     if(a != b){
37         int c = a + b >> 1;
38         if(p ≤ c) query(lc(t), a, c, p, x);
39         if(p > c) query(rc(t), c + 1, b, p, x);
40     }
41 }
42 }

```

### 2.6.2 线段树 3

```

1 #include "../header.cpp"
2 int A[MAXN];
3 struct Node{
4     i64 sum; int len, max1, max2, max_cnt,
        his_mx;
5     Node():
6         sum(0), max1(-INF), max2(-INF), max_cnt(0)
        , his_mx(-INF), len(0) {}
7     Node(int w):
8         sum(w), max1(w), max2(-INF), max_cnt(1)
        , his_mx(w), len(1) {}
9     bool update(int w1, int w2, int h1, int h2){
10         his_mx = max({his_mx, max1 + h1});
11         max1 += w1, max2 += w2;
12         sum += 1ll * w1 * max_cnt + 1ll * w2 * (
            len - max_cnt);
13         return max1 > max2;
14     }
15 };
16 struct Tag{
17     int max_add, max_his_add, umx_add,
        umx_his_add; bool have;
18     void update(int w1, int w2, int h1, int h2){
19         max_his_add = max(max_his_add, max_add +
            h1);
20         umx_his_add = max(umx_his_add, umx_add +
            h2);
21         max_add += w1, umx_add += w2, have = true;
22     }
23     void clear(){
24         max_add = max_his_add = umx_add =
            umx_his_add = have = 0;
25     }
26 };
27 struct Node operator +(Node a, Node b){
28     Node t;
29     t.max1 = max(a.max1, b.max1);
30     if(t.max1 != a.max1){
31         if(a.max1 > t.max2) t.max2 = a.max1;

```

```

32     } else{
33         if(a.max2 > t.max2) t.max2 = a.max2;
34         t.max_cnt += a.max_cnt;
35     }
36     if(t.max1 != b.max1){
37         if(b.max1 > t.max2) t.max2 = b.max1;
38     } else{
39         if(b.max2 > t.max2) t.max2 = b.max2;
40         t.max_cnt += b.max_cnt;
41     }
42     t.sum = a.sum + b.sum, t.len = a.len + b.len
        ;
43     t.his_mx = max(a.his_mx, b.his_mx);
44     return t;
45 }
46 namespace Seg{
47     const int SIZ = 2e6 + 3;
48     struct Node W[SIZ]; struct Tag T[SIZ];
49     #define lc(t) (t << 1)
50     #define rc(t) (t << 1 | 1)
51     void push_up(int t, int a, int b){
52         W[t] = W[lc(t)] + W[rc(t)];
53     }
54     void push_down(int t, int a, int b){
55         if(a == b) T[t].clear();
56         if(T[t].have){
57             int c = a + b >> 1, x = lc(t), y = rc(t)
                ;
58             int w = max(W[x].max1, W[y].max1);
59             int w1 = T[t].max_add, w2 = T[t].umx_add
                , w3 = T[t].max_his_add, w4 = T[t].
                umx_his_add;
60             if(w == W[x].max1)
61                 W[x].update(w1, w2, w3, w4),
62                 T[x].update(w1, w2, w3, w4);
63             else
64                 W[x].update(w2, w2, w4, w4),
65                 T[x].update(w2, w2, w4, w4);
66             if(w == W[y].max1)
67                 W[y].update(w1, w2, w3, w4),
68                 T[y].update(w1, w2, w3, w4);
69             else
70                 W[y].update(w2, w2, w4, w4),
71                 T[y].update(w2, w2, w4, w4);
72             T[t].clear();
73         }
74     }
75     void build(int t, int a, int b){
76         if(a == b){W[t] = Node(A[a]), T[t].clear()
            ;} else {
77             int c = a + b >> 1; T[t].clear();
78             build(lc(t), a, c);

```

```

79         build(rc(t), c + 1, b);
80         push_up(t, a, b);
81     }
82 }
83 void modiadd(int t, int a, int b, int l, int
    r, int w){
84     if(l ≤ a && b ≤ r){
85         T[t].update(w, w, w, w);
86         W[t].update(w, w, w, w);
87     } else {
88         int c = a + b >> 1; push_down(t, a, b);
89         if(l ≤ c) modiadd(lc(t), a, c, l, r,
            w);
90         if(r > c) modiadd(rc(t), c + 1, b, l, r
            , w);
91         push_up(t, a, b);
92     }
93 }
94 void modimin(int t, int a, int b, int l, int
    r, int w){
95     if(l ≤ a && b ≤ r){
96         if(w ≥ W[t].max1) return; else
97         if(w > W[t].max2){
98             int k = w - W[t].max1;
99             T[t].update(k, 0, k, 0);
100             W[t].update(k, 0, k, 0);
101         } else {
102             int c = a + b >> 1;
103             push_down(t, a, b);
104             modimin(lc(t), a, c, l, r, w);
105             modimin(rc(t), c + 1, b, l, r, w);
106             push_up(t, a, b);
107         }
108     } else {
109         int c = a + b >> 1; push_down(t, a, b);
110         if(l ≤ c) modimin(lc(t), a, c, l, r,
            w);
111         if(r > c) modimin(rc(t), c + 1, b, l, r
            , w);
112         push_up(t, a, b);
113     }
114 }
115 Node query(int t, int a, int b, int l, int r
    ){
116     if(l ≤ a && b ≤ r) return W[t];
117     int c = a + b >> 1; Node ret; push_down(t,
        a, b);
118     if(l ≤ c) ret = ret + query(lc(t), a, c
        , l, r);
119     if(r > c) ret = ret + query(rc(t), c + 1,
        b, l, r);
120     return ret;

```

```

121 }
122 }
123 int qread();
124 int main(){
125     int n = qread(), m = qread();
126     for(int i = 1; i ≤ n; ++ i)
127         A[i] = qread();
128     Seg :: build(1, 1, n);
129     for(int i = 1; i ≤ m; ++ i){
130         int op = qread();
131         if(op == 1){
132             int l = qread(), r = qread(), w = qread
                ();
133             Seg :: modiadd(1, 1, n, l, r, w);
134         } else if(op == 2){
135             int l = qread(), r = qread(), w = qread
                ();
136             Seg :: modimin(1, 1, n, l, r, w);
137         } else if(op == 3){
138             int l = qread(), r = qread();
139             auto p = Seg :: query(1, 1, n, l, r);
140             printf("%lld\n", p.sum);
141         } else if(op == 4){
142             int l = qread(), r = qread();
143             auto p = Seg :: query(1, 1, n, l, r);
144             printf("%d\n", p.max1);
145         } else if(op == 5){
146             int l = qread(), r = qread();
147             auto p = Seg :: query(1, 1, n, l, r);
148             printf("%d\n", p.his_mx);
149         }
150     }
151     return 0;
152 }

```

### 2.6.3 扫描线

```

1 #include "../header.cpp"
2 const int MAXN = 1e5 + 3;
3 int X1[MAXN], Y1[MAXN];
4 int X2[MAXN], Y2[MAXN];
5 int n, h, H[MAXN * 2];
6 namespace Seg{
7     #define lc(t) (t << 1)
8     #define rc(t) (t << 1 | 1)
9     const int SZ = 8e5 + 3;
10    int T[SZ], S[SZ], L[SZ];
11    void pushup(int t, int a, int b){
12        S[t] = 0;
13        if(a ≠ b){
14            S[t] = S[lc(t)] + S[rc(t)];
15            L[t] = L[lc(t)] + L[rc(t)];

```

```

16 }
17 if(T[t]) S[t] = L[t];
18 }
19 void modify(int t, int a, int b, int l, int
    r, int w){
20     if(l ≤ a && b ≤ r){
21         T[t] += w, pushup(t, a, b);
22     } else {
23         int c = a + b >> 1;
24         if(l ≤ c) modify(lc(t), a, c, l, r, w);
25         if(r > c) modify(rc(t), c + 1, b, l, r,
            w);
26         pushup(t, a, b);
27     }
28 }
29 void build(int t, int a, int b){
30     if(a == b){
31         L[t] = H[a] - H[a - 1];
32     } else {
33         int c = a + b >> 1;
34         build(lc(t), a, c);
35         build(rc(t), c + 1, b);
36         pushup(t, a, b);
37     }
38 }
39 int query(int t){
40     return S[t];
41 }
42 }
43 tuple <int, int, int> P[MAXN], Q[MAXN];
44 int main(){
45     n = qread();
46     for(int i = 1; i ≤ n; ++ i){
47         X1[i] = qread(), Y1[i] = qread();
48         X2[i] = qread(), Y2[i] = qread();
49         if(X1[i] > X2[i]) swap(X1[i], X2[i]);
50         if(Y1[i] > Y2[i]) swap(Y1[i], Y2[i]);
51         H[++ h] = Y1[i];
52         H[++ h] = Y2[i];
53         P[i] = make_tuple(X1[i], Y1[i], Y2[i]);
54         Q[i] = make_tuple(X2[i], Y1[i], Y2[i]);
55     }
56     sort(H + 1, H + 1 + h);
57     sort(P + 1, P + 1 + n);
58     sort(Q + 1, Q + 1 + n);
59     int o = unique(H + 1, H + 1 + h) - H - 1;
60     Seg :: build(1, 1, o);
61     i64 ans = 0, last = -1;
62     int p = 1, q = 1;
63     while(p ≤ n || q ≤ n){
64         int x = INF;
65         if(p ≤ n) x = min(x, get<0>(P[p]));

```

```

66     if(q ≤ n) x = min(x, get<0>(Q[q]));
67     if(last ≠ -1){
68         ans += 1ll * Seg :: query(1) * (x - last
            );
69     }
70     last = x;
71     while(q ≤ n && get<0>(Q[q]) == x){
72         auto [x, l, r] = Q[q]; ++ q;
73         l = lower_bound(H + 1, H + 1 + o, l) - H
            + 1;
74         r = lower_bound(H + 1, H + 1 + o, r) - H
            ;
75         Seg :: modify(1, 1, o, l, r, 1);
76     }
77     while(p ≤ n && get<0>(P[p]) == x){
78         auto [x, l, r] = P[p]; ++ p;
79         l = lower_bound(H + 1, H + 1 + o, l) - H
            + 1;
80         r = lower_bound(H + 1, H + 1 + o, r) - H
            ;
81         Seg :: modify(1, 1, o, l, r, -1);
82     }
83 }
84 printf("%lld\n", ans);
85 return 0;
86 }

```

## 2.7 根号数据结构

### 2.7.1 块状链表

```

1 #include "../header.cpp"
2 namespace BLOCK{
3     const int SZ = 1e6 + 1e5 + 3;
4     const int BSZ = 2000;
5     list <vector<int>> block;
6     void build(int n, const int A[]){
7         for(int l = 0, r = 0; r ≠ n; ){
8             l = r;
9             r = min(l + BSZ / 2, n);
10            vector <int> V0(A + l, A + r);
11            block.emplace_back(V0);
12        }
13    }
14    int get_kth(int k){
15        for(auto it = block.begin(); it ≠ block.
            end(); ++ it){
16            if(it → size() < k)
17                k -= it → size();
18            else return it → at(k - 1);
19        }
20        return -1;

```

```

21 }
22 int get_rank(int w){
23     int ans = 0;
24     for(auto it = block.begin(); it != block.
25         end(); ++ it){
26         if(it → back() < w)
27             ans += it → size();
28     }
29     ans += lower_bound(it → begin(), it
30         → end(), w) - it → begin();
31     break;
32 }
33 return ans + 1;
34 }
35 // 插入到第 k 个位置
36 void insert(int k, int w){
37     for(auto it = block.begin(); it != block.
38         end(); ++ it){
39         if(it → size() < k)
40             k -= it → size();
41     }
42     it → insert(it → begin() + k - 1, w)
43     ;
44     if(it → size() > BSZ){
45         vector<int> V1(it → begin(), it →
46             begin() + BSZ / 2);
47         vector<int> V2(it → begin() + BSZ
48             / 2, it → end());
49         *it = V2;
50         block.insert(it, V1);
51     }
52     return;
53 }
54 }
55 // 删除第 k 个数
56 void erase(int k){
57     for(auto it = block.begin(); it != block.
58         end(); ++ it){
59         if(it → size() < k)
60             k -= it → size();
61     }
62     it → erase(it → begin() + k - 1);
63     if(it → empty())
64         block.erase(it);
65     return;
66 }
67 }
68 int A[MAXN];

```

```

66 // == TEST ==
67 int main(){
68     ios :: sync_with_stdio(false);
69     cin.tie(nullptr);
70     int n, m;
71     cin >> n >> m;
72     for(int i = 1; i ≤ n; ++ i)
73         cin >> A[i];
74     sort(A + 1, A + 1 + n);
75     A[n + 1] = INT_MAX;
76     BLOCK :: build(n + 1, A + 1);
77     int last = 0;
78     int ans = 0;
79     // Do some op...
80     cout << ans << endl;
81     return 0;
82 }

```

### 2.7.2 莫队二次离线

```

1 #include " ../header.cpp"
2 int n, m, k, maxt = 16383, X[MAXM], C[MAXM], t
3 ;
4 int A[MAXN], bsize; i64 B[MAXN], R[MAXN];
5 struct Qry1{ int l, r, id; }O[MAXN];
6 struct Qry2{ int id, l, r; };
7 struct Qry3{ int id, l, r; };
8 bool cmp(Qry1 a, Qry1 b){
9     return a.l / bsize == b.l / bsize ? a.r < b.
10         r : a.l < b.l;
11 }
12 vector<Qry2> P[MAXN];
13 vector<Qry3> Q[MAXN];
14 int main(){
15     n = qread(), m = qread(), k = qread(), bsize
16         = sqrt(m + 1);
17     up(1, n, i) A[i] = qread();
18     up(1, m, i){
19         int l = qread(), r = qread(); O[i] = {l, r
20             , i};
21     }
22     sort(O + 1, O + 1 + m, cmp);
23     int l = 1, r = 0;
24     up(1, m, i){
25         int p = O[i].l, q = O[i].r;
26         if(r < q){
27             P[r].push_back({ i, r + 1, q});
28             Q[l - 1].push_back({-i, r + 1, q});
29         }
30         if(r > q){
31             P[q].push_back({-i, q + 1, r});
32             Q[l - 1].push_back({ i, q + 1, r});
33         }
34     }
35 }

```

```

29 }
30 r = q;
31 if(l > p){
32     P[p].push_back({-i, p, l - 1});
33     Q[r].push_back({ i, p, l - 1});
34 }
35 if(l < p){
36     P[l].push_back({ i, l, p - 1});
37     Q[r].push_back({-i, l, p - 1});
38 }
39 l = p;
40 }
41 up(0, maxt, i) if(__builtin_popcount(i) == k
42 ) X[++ t] = i;
43 up(0, n, i){
44     up(1, t, j) += C[A[i] ^ X[j]];
45     for(auto &o : P[i]){
46         if(o.id > 0) R[ o.id] += C[A[o.l]];
47         else R[-o.id] -= C[A[o.l]];
48         if(o.l < o.r)
49             P[i + 1].push_back({o.id, o.l + 1, o.r
50                 });
51     }
52     for(auto &o : Q[i]){
53         up(o.l, o.r, j){
54             if(o.id > 0) R[ o.id] += C[A[j]];
55             else R[-o.id] -= C[A[j]];
56         }
57     }
58     P[i].clear(), Q[i].clear();
59     P[i].shrink_to_fit();
60     Q[i].shrink_to_fit();
61 }
62 i64 ans = 0;
63 up(1, m, i){ ans += R[i], B[O[i].id] = ans;
64 }
65 up(1, m, i) printf("%lld\n", B[i]);
66 return 0;
67 }

```

## 3 树论

### 3.1 点分树

#### 3.1.1 例题

给定  $n$  个点组成的树，点有点权  $v_i$ 。  $m$  个操作，分为两种：

- $0 \times k$  查询距离  $x$  不超过  $k$  的所有点的点权之和；

- $0 \times y$  将点  $x$  的点权修改为  $y$ 。

```

1 #include "../header.cpp"
2 vector<int> E[MAXN];
3 namespace LCA{
4     const int SIZ = 1e5 + 3;
5     int D[SIZ], F[SIZ];
6     int P[SIZ], Q[SIZ], o;
7     void dfs(int u, int f){
8         P[u] = ++ o;
9         Q[o] = u;
10        F[u] = f;
11        D[u] = D[f] + 1;
12        for(auto &v : E[u]) if(v != f){
13            dfs(v, u);
14        }
15    }
16    const int MAXH = 18 + 3;
17    int h = 18;
18    int ST[SIZ][MAXH];
19    int cmp(int a, int b){
20        return D[a] < D[b] ? a : b;
21    }
22    int T[SIZ], n;
23    void init(int _n){
24        n = _n;
25        dfs(1, 0);
26        for(int i = 1; i <= n; ++ i)
27            ST[i][0] = Q[i];
28        for(int i = 2; i <= n; ++ i)
29            T[i] = T[i >> 1] + 1;
30        for(int i = 1; i <= h; ++ i){
31            for(int j = 1; j <= n; ++ j) if(j + (1 <<
32                i - 1) <= n){
33                ST[j][i] = cmp(ST[j][i - 1], ST[j + (1
34                    << i - 1)][i - 1]);
35            }
36        }
37        int lca(int a, int b){
38            if(a == b)
39                return a;
40            int l = P[a];
41            int r = P[b];
42            if(l > r)
43                swap(l, r);
44            ++ l;
45            int d = T[r - l + 1];
46            return F[cmp(ST[l][d], ST[r - (1 << d) +
47                1][d])];

```

```

48        return D[a] + D[b] - 2 * D[lca(a, b)];
49    }
50 }
51 namespace BIT{
52     void modify(int D[], int n, int p, int w){
53         ++ p;
54         while(p <= n)
55             D[p] += w, p += p & -p;
56     }
57     int query(int D[], int n, int p){
58         if(p < 0) return 0;
59         p = min(n, p + 1);
60         int r = 0;
61         while(p > 0)
62             r += D[p], p -= p & -p;
63         return r;
64     }
65 }
66 namespace PTree{
67     const int SIZ = 1e5 + 3;
68     bool V[SIZ];
69     int S[SIZ], L[SIZ];
70     vector<int> EE[MAXN];
71     int *D1[MAXN];
72     int *D2[MAXN];
73     void dfs1(int s, int &g, int u, int f){
74         S[u] = 1;
75         int maxsize = 0;
76         for(auto &v : E[u]) if(v != f && !V[v]){
77             dfs1(s, g, v, u);
78             if(S[v] > maxsize)
79                 maxsize = S[v];
80             S[u] += S[v];
81         }
82         maxsize = max(maxsize, s - S[u]);
83         if(maxsize <= s / 2)
84             g = u;
85     }
86     int n;
87     void build(int s, int &g, int u, int f){
88         dfs1(s, g, u, f);
89         V[g] = true, L[g] = s;
90         for(auto &u : E[g]) if(!V[u]){
91             int h = 0;
92             if(S[u] < S[g]) build(S[u], h, u, 0);
93             else build(s - S[g], h, u, 0);
94             EE[g].push_back(h);
95             EE[h].push_back(g);
96         }
97     }
98     int F[SIZ];
99     void dfs2(int u, int f){

```

```

100        F[u] = f;
101        for(auto &v : EE[u]) if(v != f){
102            dfs2(v, u);
103        }
104    }
105    void build(int _n){
106        n = _n;
107        int s = n, g = 0;
108        dfs1(s, g, 1, 0);
109        V[g] = true, L[g] = s;
110        for(auto &u : E[g]){
111            int h = 0;
112            if(S[u] < S[g]) build(S[u], h, u, 0);
113            else build(s - S[g], h, u, 0);
114            EE[g].push_back(h);
115            EE[h].push_back(g);
116        }
117        dfs2(g, 0);
118        for(int i = 1; i <= n; ++ i){
119            L[i] += 2;
120            D1[i] = new int[L[i] + 3];
121            D2[i] = new int[L[i] + 3];
122            for(int j = 0; j < L[i] + 3; ++ j)
123                D1[i][j] = D2[i][j] = 0;
124        }
125    }
126    void modify(int x, int w){
127        int u = x;
128        while(1){
129            BIT :: modify(D1[x], L[x], LCA :: dis(u,
130                x), w);
131            int y = F[x];
132            if(y != 0){
133                int e = LCA :: dis(x, y);
134                BIT :: modify(D2[x], L[x], LCA :: dis(
135                    u, y), w);
136                x = y;
137            } else break;
138        }
139    }
140    int query(int x, int d){
141        int ans = 0, u = x;
142        while(1){
143            ans += BIT :: query(D1[x], L[x], d - LCA
144                :: dis(u, x));
145            int y = F[x];
146            if(y != 0){
147                int e = LCA :: dis(x, y);
148                ans -= BIT :: query(D2[x], L[x], d -
149                    LCA :: dis(u, y));
150                x = y;
151            } else break;

```



```

148 }
149 return ans;
150 }
151 }
152 int W[MAXN];
153 int main(){
154     ios :: sync_with_stdio(false);
155     int n, m;
156     cin >> n >> m;
157     for(int i = 1; i ≤ n; ++ i){
158         cin >> W[i];
159     }
160     for(int i = 2; i ≤ n; ++ i){
161         int u, v;
162         cin >> u >> v;
163         E[u].push_back(v);
164         E[v].push_back(u);
165     }
166     LCA :: init(n);
167     PTree :: build(n);
168     for(int i = 1; i ≤ n; ++ i)
169         PTree :: modify(i, W[i]);
170     int lastans = 0;
171     for(int i = 1; i ≤ m; ++ i){
172         int op; cin >> op;
173         if(op == 0){
174             int x, d;
175             cin >> x >> d;
176             x ^= lastans;
177             d ^= lastans;
178             cout << (lastans = PTree :: query(x, d))
179                 << endl;
180         } else {
181             int x, w;
182             cin >> x >> w;
183             x ^= lastans;
184             w ^= lastans;
185             PTree :: modify(x, -W[x]);
186             PTree :: modify(x, W[x] = w);
187         }
188     }
189     return 0;
190 }

```

### 3.2 长链剖分

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 using i64 = long long;
4 const int INF = 1e9;
5 const i64 INFL = 1e18;
6 const int MAXN= 5e5 + 3;

```

```

7 const int MAXM= 19 + 3;
8 vector <int> P[MAXN];
9 vector <int> Q[MAXN];
10 vector <int> E[MAXN];
11 int h = 19;
12 int L[MAXN], F[MAXN], G[MAXN], D[MAXN], S[MAXM
    ][MAXN];
13 void dfs1(int u, int f){
14     L[u] = 1, S[0][u] = f;
15     F[u] = f, D[u] = D[f] + 1;
16     for(int i = 1; i ≤ h; ++ i)
17         S[i][u] = S[i - 1][S[i - 1][u]];
18     for(auto &v : E[u]) if(v ≠ f){
19         dfs1(v, u);
20         if(L[v] > L[G[u]])
21             G[u] = v;
22         L[u] = max(L[u], L[v] + 1);
23     }
24 }
25 int T[MAXN];
26 void dfs2(int u, int f){
27     if(u == G[f]){
28         T[u] = T[f];
29         P[T[u]].push_back(u);
30         Q[T[u]].push_back(F[Q[T[u]].back()]);
31     } else {
32         T[u] = u;
33         P[u].push_back(u);
34         Q[u].push_back(u);
35     }
36     if(G[u]) dfs2(G[u], u);
37     for(auto &v : E[u]) if(v ≠ f && v ≠ G[u
        ])
38         dfs2(v, u);
39 }
40 typedef unsigned int u32;
41 typedef unsigned long long u64;
42 int n, q; u32 s;
43 u32 get(u32 x) {
44     x ^= x << 13;
45     x ^= x >> 17;
46     x ^= x << 5;
47     return s = x;
48 }
49 int qread();
50 int H[MAXN];
51 int main(){
52     scanf("%d%d%u", &n, &q, &s);
53     int root = 0; H[0] = -1;
54     for(int i = 1; i ≤ n; ++ i){
55         int f = qread();
56         if(f == 0)

```

```

57         root = i;
58     } else {
59         E[f].push_back(i);
60         E[i].push_back(f);
61     }
62     H[i] = H[i >> 1] + 1;
63 }
64 dfs1(root, 0);
65 dfs2(root, 0);
66 int lastans = 0;
67 i64 realans = 0;
68 for(int i = 1; i ≤ q; ++ i){
69     int x = (get(s) ^ lastans) % n + 1;
70     int k = (get(s) ^ lastans) % D[x];
71     if(k == 0){
72         lastans = x;
73     } else {
74         int h = H[k];
75         k -= 1 << h;
76         x = S[h][x];
77         int t = T[x];
78         k -= D[x] - D[t];
79         if(k > 0){
80             x = Q[t][k];
81         } else {
82             x = P[t][-k];
83         }
84         lastans = x;
85     }
86     realans ^= 1ll * i * lastans;
87 }
88 printf("%lld\n", realans);
89 return 0;
90 }

```

### 3.3 重链剖分

```

1 #include "../header.cpp"
2 int n, m, root, MOD, A[MAXN];
3 int qread();
4 vector <int> E[MAXN];
5 int S[MAXN], G[MAXN], D[MAXN], F[MAXN];
6 void dfs1(int u, int f){
7     S[u] = 1, G[u] = 0, D[u] = D[f] + 1, F[u] =
8     f;
9     for(auto &v : E[u]) if(v ≠ f){
10         dfs1(v, u);
11         S[u] += S[v];
12         if(S[v] > S[G[u]])
13             G[u] = v;
14     }

```

```

15 int B[MAXN];
16 int P[MAXN], Q[MAXN], T[MAXN], L[MAXN], R[MAXN]
    ], cnt;
17 void dfs2(int u, int f){
18     P[++ cnt] = u, B[cnt] = A[u], Q[u] = cnt;
19     L[u] = cnt;
20     if(u ≠ G[f]) T[u] = u;
21     else T[u] = T[f];
22     if(G[u]) dfs2(G[u], u);
23     for(auto &v : E[u]) if(v ≠ f && v ≠ G[u]){
24         dfs2(v, u);
25     }
26     R[u] = cnt;
27 }
28 namespace Seg{
29     const int SIZ = 4e5 + 3;
30     i64 S[SIZ], T[SIZ];
31     void pushup(int t, int a, int b);
32     void pushdown(int t, int a, int b);
33     void modify(int t, int a, int b, int l, int
        r, int w);
34     i64 query(int t, int a, int b, int l, int r)
        ;
35     void build(int t, int a, int b);
36 }
37 int main(){
38     n = qread(), m = qread(), root = qread(),
        MOD = qread();
39     for(int i = 1; i ≤ n; ++ i)
40         A[i] = qread();
41     for(int i = 2; i ≤ n; ++ i){
42         int u = qread(), v = qread();
43         E[u].push_back(v);
44         E[v].push_back(u);
45     }
46     dfs1(root, 0);
47     dfs2(root, 0);
48     Seg :: build(1, 1, n);
49     for(int i = 1; i ≤ m; ++ i){
50         int op = qread();
51         if(op == 1){
52             int u = qread(), v = qread(), k = qread
                ();
53             while(T[u] ≠ T[v]){
54                 if(D[T[u]] < D[T[v]])
55                     swap(u, v);
56                 Seg :: modify(1, 1, n, Q[T[u]], Q[u],
                    k);
57                 u = F[T[u]];
58             }
59             if(D[u] < D[v]) swap(u, v);
60             Seg :: modify(1, 1, n, Q[v], Q[u], k);

```

```

61     } else if(op == 2){
62         int u = qread(), v = qread();
63         i64 ans = 0;
64         while(T[u] ≠ T[v]){
65             if(D[T[u]] < D[T[v]])
66                 swap(u, v);
67             ans = (ans + Seg :: query(1, 1, n, Q[T
                [u]], Q[u])) % MOD;
68             u = F[T[u]];
69         }
70         if(D[u] < D[v]) swap(u, v);
71         ans = (ans + Seg :: query(1, 1, n, Q[v],
            Q[u])) % MOD;
72         printf("%lld\n", ans);
73     } else if(op == 3){
74         int x = qread(), w = qread();
75         Seg :: modify(1, 1, n, L[x], R[x], w);
76     } else {
77         int x = qread();
78         printf("%lld\n", Seg :: query(1, 1, n, L
            [x], R[x]));
79     }
80 }
81 return 0;
82 }

```

### 3.4 树哈希

#### 3.4.1 用法

给定大小为  $n$  的以 1 为根的树，计算  $h_i$  表示子树  $i$  的哈希值，计算有多少个本质不同的值。

```

1 #include "../header.cpp"
2 u64 xor_shift(u64 x);
3 u64 H[MAXN];
4 vector<int> E[MAXN];
5 void dfs(int u, int f){
6     H[u] = 1;
7     for(auto &v : E[u]) if(v ≠ f){
8         dfs(v, u);
9         H[u] += H[v];
10    }
11    H[u] = xor_shift(H[u]); // !important
12 }
13 int main(){
14     int n;
15     cin >> n;
16     for(int i = 2; i ≤ n; ++ i){
17         int u, v;
18         cin >> u >> v;
19         E[u].push_back(v);

```

```

20     E[v].push_back(u);
21 }
22 dfs(1, 0);
23 sort(H + 1, H + 1 + n);
24 cout << (unique(H + 1, H + 1 + n) - H - 1)
    << endl;
25 return 0;
26 }

```

### 3.5 Prufer 序列

```

1 #include "../header.cpp"
2 int D[MAXN], F[MAXN], P[MAXN];
3 vector<int> tree2prufer(int n){
4     vector<int> P(n);
5     for(int i = 1, j = 1; i ≤ n - 2; ++ i, ++ j){
6         while(D[j]) ++ j;
7         P[i] = F[j];
8         while(i ≤ n - 2 && !--D[P[i]] && P[i] < j
            )
9             P[i + 1] = F[P[i]], i ++;
10    }
11    return P;
12 }
13 vector<int> prufer2tree(int n){
14     vector<int> F(n);
15     for(int i = 1, j = 1; i ≤ n - 1; ++ i, ++ j){
16         while(D[j]) ++ j;
17         F[j] = P[i];
18         while(i ≤ n - 1 && !--D[P[i]] && P[i] < j
            )
19             F[P[i]] = P[i + 1], i ++;
20    }
21    return F;
22 }

```

### 3.6 虚树

```

1 #include "../header.cpp"
2 vector<pair<int, int> > E[MAXN];
3 namespace LCA{
4     const int SIZ = 5e5 + 3;
5     int D[SIZ], H[SIZ], F[SIZ], P[SIZ], Q[SIZ],
        o;
6     void dfs(int u, int f){
7         P[u] = ++ o, Q[o] = u, F[u] = f, D[u] = D[
            f] + 1;
8         for(auto &[v, w] : E[u]) if(v ≠ f){
9             H[v] = H[u] + w, dfs(v, u);
10        }
11    }

```

## 4 图论

## 4.1 仙人掌

## 4.1.1 例题

给定一个仙人掌，多组询问  $u, v$  之间最短路长度。

```

1 #include "../header.cpp"
2 const int MAXD= 18 + 3;
3 struct edge{int u, v, w;};
4 vector <edge> V1[MAXN];
5 vector <edge> V2[MAXN];
6 vector <int> H[MAXN];
7 int n, D[MAXN], W[MAXN], F[MAXD][MAXN];
8 int o, X[MAXN], L[MAXN];
9 bool E[MAXN];
10 void dfs1(int u, int f){
11     D[u] = D[f] + 1, F[0][u] = f;
12     for(auto &e : V1[u]) if(e.v != f){
13         if(D[e.v] && D[e.v] < D[u]){
14             int a = e.u;
15             int b = e.v;
16             int c = ++ o, t = c + n;
17             H[c].push_back(a);
18             L[c] = W[a] - W[b] + e.w;
19             while(a != b)
20                 E[a] = true, a = F[0][a], H[c].
21                     push_back(a);
22             for(auto &x : H[c]){
23                 int w = min(W[x] - W[b], L[c] - W[x] +
24                     W[b]);
25                 V2[x].push_back(edge{x, t, w});
26                 V2[t].push_back(edge{t, x, w});
27             } else if(!D[e.v]){
28                 W[e.v] = W[u] + e.w, dfs1(e.v, u);
29             }
30         }
31     }
32     for(auto &e : V1[u]) if(D[e.v] > D[u]){
33         if(!E[e.v]){
34             V2[e.u].push_back({e.u, e.v, e.w});
35             V2[e.v].push_back({e.v, e.u, e.w});
36         }
37     }
38 }
39 int d = 18;
40 void dfs2(int u, int f){
41     D[u] = D[f] + 1, F[0][u] = f;
42     up(1, d, i) F[i][u] = F[i - 1][F[i - 1][u]];
43     for(auto &e : V2[u]) if(e.v != f){
44         X[e.v] = X[e.u] + e.w;
45         dfs2(e.v, u);
46     }
47 }

```

```

44 }
45 }
46 int lca(int u, int v){
47     if(D[u] < D[v]) swap(u, v);
48     dn(d, 0, i) if(D[F[i][u]] ≥ D[v]) u = F[i][
49         u];
50     if(u == v) return u;
51     dn(d, 0, i) if(F[i][u] ≠ F[i][v]) u = F[i][
52         u], v = F[i][v];
53     return F[0][u];
54 }
55 int jump(int u, int v){
56     dn(d, 0, i) if(D[F[i][v]] > D[u]) v = F[i][
57         v];
58     return v;
59 }
60 int dis(int x, int y){
61     int t = lca(x, y);
62     if(t > n){
63         int u = jump(t, x);
64         int v = jump(t, y);
65         int w = abs(W[u] - W[v]);
66         int l = min(w, L[t - n] - w);
67         return X[x] - X[u] + X[y] - X[v] + l;
68     } else {
69         return X[x] + X[y] - 2 * X[t];
70     }
71 }
72 int m, q;
73 int qread();
74 int main(){
75     n = qread(), m = qread(), q = qread();
76     up(1, m, i){
77         int u = qread(), v = qread(), w = qread();
78         V1[u].push_back(edge{u, v, w});
79         V1[v].push_back(edge{v, u, w});
80     }
81     dfs1(1, 0);
82     dfs2(1, 0);
83     up(1, q, i){
84         int u = qread(), v = qread();
85         printf("%d\n", dis(u, v));
86     }
87     return 0;
88 }

```

## 4.2 三元环计数

### 4.2.1 三元环计数

无向图：考虑将所有点按度数从小往大排序，然后将每条边定向，由排在前面的指向排在后面的，得到一个有向图。然后考虑枚举一个点，再枚举一个点，暴力数，具体见代码。结论是，这样定向后，每个点的出度是  $O(\sqrt{m})$  的。复杂度  $O(m\sqrt{m})$ 。有向图：不难发现，上述方法枚举了三个点，计算有向图三元环也就只需要处理下方向的事，这个由于算法够暴力，随便改改就能做了。

```

34         ] : G[j]) {
35             if (w1 == w2) ans += w1 ? in[k] : out[
36                 k];
37         }
38     for (auto [j, w] : G[i]) in[j] = out[j] =
        0;
    }
    cout << ans << '\n';

```

## 4.3 四元环计数

### 4.3.1 四元环计数

From zpk

- 无向图：类似，由于定向后出度结论过于强大，可以暴力。讨论了三种情况。
- 有向图：缺少题目，但应当类似三元环计数有向形式记录定向边和原边的正反关系。因此此法最强的结论是定向后出度  $O(\sqrt{m})$ ，实际上方法很暴力，应当不难数有向形式的。

```

1 ll n, m; cin >> n >> m;
2 vector<pair<ll, ll>> Edges(m);
3 vector<vector<ll>> G(n + 2);
4 vector<ll> deg(n + 2);
5 for (auto &[i, j] : Edges) cin >> i >> j, ++
    deg[i], ++deg[j];
6 for (auto [i, j] : Edges) {
7     if (deg[i] > deg[j] || (deg[i] == deg[j]
8         && i > j)) swap(i, j);
9     G[i].emplace_back(j), iG[j].emplace_back(i
10 );
11 }
12 ll ans = 0;
13 vector<ll> v1(n + 2), v2(n + 2);
14 for (ll i = 1; i <= n; ++i) {
15     for (auto j : G[i]) for (auto k : G[j]) ++
        v1[k];
16     for (auto j : iG[i]) for (auto k : G[j])
17         ans += v1[k], ++v2[k];
18     for (auto j : G[i]) for (auto k : G[j])
        ans += v1[k] * (v1[k] - 1) / 2, v1[k] =
        0;
19     for (auto j : iG[i]) for (auto k : G[j]) {
20         if (deg[k] > deg[i] || (deg[k] == deg[
21             i] && k > i)) ans += v2[k] * (v2[k]
22                 - 1) / 2;
23         v2[k] = 0;
24     }
25 }

```

```

19     }
20 }
21 cout << ans << '\n';

```

## 4.4 基环树

```

1 #include "../header.cpp"
2 using edge = tuple<int, int, int>;
3 vector<edge> E[MAXN];
4 vector<edge> W;
5 vector<int> C;
6 edge F[MAXN];
7 bool V[MAXN];
8 int I[MAXN], o;
9 void dfs0(int u, int e){
10     V[u] = true;
11     I[u] = ++o;
12     for(auto &[i, v, w] : E[u]) if(i != e){
13         if(V[v]){
14             if(I[v] < I[u]){
15                 for(int p = u; p != v;){
16                     auto &[j, f, x] = F[p];
17                     C.push_back(p);
18                     W.push_back({j, p, x});
19                     p = f;
20                 }
21                 C.push_back(v);
22                 W.push_back({i, v, w});
23             } else {
24                 F[v] = {i, u, w};
25                 dfs0(v, i);
26             }
27         }
28     }
29 }
30 namespace Problem2{
31 // === 删除环上第 i 条边，求直径 ===
32 i64 H[MAXN], A1[MAXN], B1[MAXN], A2[MAXN],
    B2[MAXN], A3[MAXN], B3[MAXN];
33 i64 L[MAXN];
34 i64 dis = 0;
35 void dfs1(int u, int e){
36     for(auto &[i, v, w] : E[u]) if(i != e){
37         if(!V[v]){
38             dfs1(v, i);
39             dis = max(dis, L[u] + w + L[v]);
40             L[u] = max(L[u], L[v] + w);
41         }
42     }
43 }
44 int main(){
45     int n;

```

```

1 // 无向图
2 ll n, m; cin >> n >> m;
3 vector<pair<ll, ll>> Edges(m);
4 vector<vector<ll>> G(n + 2);
5 vector<ll> deg(n + 2);
6 for (auto &[i, j] : Edges) cin >> i >> j, ++
    deg[i], ++deg[j];
7 for (auto [i, j] : Edges) {
8     if (deg[i] > deg[j] || (deg[i] == deg[j]
9         && i > j)) swap(i, j);
10     G[i].emplace_back(j);
11 }
12 vector<ll> val(n + 2);
13 ll ans = 0;
14 for (ll i = 1; i <= n; ++i) {
15     for (auto j : G[i]) ++val[j];
16     for (auto j : G[i]) for (auto k : G[j])
        ans += val[k];
17     for (auto j : G[i]) val[j] = 0;
18 }
19 // 有向图
20 ll n, m; cin >> n >> m;
21 vector<pair<ll, ll>> Edges(m);
22 vector<vector<pll>> G(n + 2);
23 vector<ll> deg(n + 2);
24 for (auto &[i, j] : Edges) cin >> i >> j, ++
    deg[i], ++deg[j];
25 for (auto [i, j] : Edges) {
26     ll flg = 0;
27     if (deg[i] > deg[j] || (deg[i] == deg[j]
28         && i > j)) swap(i, j), flg = 1;
29     G[i].emplace_back(j, flg);
30 }
31 vector<ll> in(n + 2), out(n + 2);
32 ll ans = 0;
33 for (ll i = 1; i <= n; ++i) {
34     for (auto [j, w] : G[i]) w ? (++in[j]) : (
        ++out[j]);
35     for (auto [j, w1] : G[i]) for (auto [k, w2]

```

```

46 cin >> n;
47 for(int i = 1; i ≤ n; ++ i){
48     int u, v, w;
49     cin >> u >> v >> w;
50     E[u].push_back({i, v, w});
51     E[v].push_back({i, u, w});
52 }
53 dfs0(1, 0);
54 memset(V, 0, sizeof(V));
55 for(auto &u : C)
56     V[u] = true;
57 for(auto &u : C){
58     dfs1(u, 0);
59 }
60 int l = 0, r = C.size() - 1;
61 for(int i = l; i ≤ r; ++ i){
62     int x = C[i];
63     if(i > 0)
64         H[i] = H[i - 1] + get<2>(W[i - 1]);
65     A1[i] = L[x] + H[i];
66     B1[i] = L[x] - H[i];
67     A2[i] = L[x] - H[i];
68     B2[i] = L[x] + H[i];
69 }
70 i64 h = H[r] + get<2>(W.back());
71 for(int i = l; i ≤ r; ++ i)
72     A1[i] = max(i = l ? -INFL : A1[i - 1],
73                 L[C[i]] + H[i]),
74     A2[i] = max(i = l ? -INFL : A2[i - 1],
75                 L[C[i]] - H[i]);
76 for(int i = r; i ≥ l; -- i)
77     B1[i] = max(i = r ? -INFL : B1[i + 1],
78                 L[C[i]] - H[i]),
79     B2[i] = max(i = r ? -INFL : B2[i + 1],
80                 L[C[i]] + H[i]);
81 A3[l] = -INFL, B3[r] = -INFL;
82 for(int i = l + 1; i ≤ r; ++ i){
83     int x = C[i];
84     i64 w = A2[i - 1] + L[x] + H[i];
85     A3[i] = max(A3[i - 1], w);
86 }
87 for(int i = r - 1; i ≥ l; -- i){
88     int x = C[i];
89     i64 w = B2[i + 1] + L[x] - H[i];
90     B3[i] = max(B3[i + 1], w);
91 }
92 i64 t = INFL;
93 for(int i = l; i < r; ++ i){
94     i64 d = A1[i] + B1[i + 1] + h;
95     i64 g = A2[i] + B2[i + 1] + 0;
96     d = max({d, dis, A3[i], B3[i + 1]});
97     t = min(t, d);

```

```

94 }
95 t = min(t, max(A3[r], dis));
96 if(t % 2 == 0)
97     cout << t / 2 << ".0" << endl;
98 if(t % 2 == 1)
99     cout << t / 2 << ".5" << endl;
100 return 0;
101 }
102 }
103 namespace Problem3{
104 // === 求最大点权独立集 ===
105 int A[MAXN];
106 i64 X[MAXN], Y[MAXN];
107 i64 P[MAXN][2], Q[MAXN][2];
108 void dfs1(int u, int e){
109     for(auto &[i, v, w] : E[u]) if(i ≠ e){
110         if(!V[v]){
111             dfs1(v, i);
112             Y[u] += max(X[v], Y[v]);
113             X[u] += Y[v];
114         }
115     }
116     X[u] += A[u];
117 }
118 int main(){
119     int n;
120     cin >> n;
121     for(int i = 1; i ≤ n; ++ i){
122         cin >> A[i];
123     }
124     for(int i = 1; i ≤ n; ++ i){
125         int u, v;
126         cin >> u >> v;
127         ++ u, ++ v;
128         E[u].push_back({i, v, 0});
129         E[v].push_back({i, u, 0});
130     }
131     double p;
132     cin >> p;
133     dfs0(1, 0);
134     memset(V, 0, sizeof(V));
135     for(auto &u : C)
136         V[u] = true;
137     for(auto &u : C){
138         dfs1(u, 0);
139     }
140     int l = 0, r = C.size() - 1;
141     P[0][1] = X[C[0]];
142     P[0][0] = -INFL;
143     Q[0][0] = Y[C[0]];
144     Q[0][1] = -INFL;
145     for(int i = l + 1; i ≤ r; ++ i){

```

```

146     int x = C[i];
147     P[i][1] = X[x] + P[i - 1][0];
148     P[i][0] = Y[x] + max(P[i - 1][0], P[i -
149     1][1]);
150     Q[i][1] = X[x] + Q[i - 1][0];
151     Q[i][0] = Y[x] + max(Q[i - 1][0], Q[i -
152     1][1]);
153 }
154 i64 ans = max({P[r][0], Q[r][0], Q[r][1]});
155 ;
156 cout << fixed << setprecision(1) << ans *
157 p << endl;
158 return 0;
159 }
160 }
161 int main(){
162     return Problem3 :: main();
163 }

```

## 4.5 2-SAT

### 4.5.1 例题

$n$  个变量  $m$  个条件, 形如若  $x_i = a$  则  $y_j = b$ , 找到任意一组可行解或者报告无解。

```

1 #include "../header.cpp"
2 namespace SCC{
3     const int MAXN = 2e6 + 3;
4     vector<int> V[MAXN];
5     stack<int> S;
6     int D[MAXN], L[MAXN], C[MAXN], o, s;
7     bool F[MAXN], I[MAXN];
8     void add(int u, int v){ V[u].push_back(v); }
9     void dfs(int u){
10         L[u] = D[u] = ++ o, S.push(u), I[u] = F[u]
11         = true;
12         for(auto &v : V[u]){
13             if(F[v]){
14                 if(I[v]) L[u] = min(L[u], D[v]);
15             } else {
16                 dfs(v), L[u] = min(L[u], L[v]);
17             }
18         }
19         if(L[u] == D[u]){
20             int c = ++ s;
21             while(S.top() ≠ u){
22                 int v = S.top(); S.pop();
23                 I[v] = false;
24                 C[v] = c;
25             }
26             S.pop(), I[u] = false, C[u] = c;

```



```

26 }
27 }
28 }
29 const int MAXN = 1e6 + 3;
30 int X[MAXN][2], o;
31 int main(){
32     ios :: sync_with_stdio(false);
33     int n, m;
34     cin >> n >> m;
35     for(int i = 1; i ≤ n; ++ i)
36         X[i][0] = ++ o;
37     for(int i = 1; i ≤ n; ++ i)
38         X[i][1] = ++ o;
39     for(int i = 1; i ≤ m; ++ i){
40         int a, x, b, y;
41         cin >> a >> x >> b >> y;
42         SCC :: add(X[a][!x], X[b][y]);
43         SCC :: add(X[b][!y], X[a][x]);
44     }
45     for(int i = 1; i ≤ o; ++ i)
46         if(!SCC :: F[i])
47             SCC :: dfs(i);
48     bool ok = true;
49     for(int i = 1; i ≤ n; ++ i){
50         if(SCC :: C[X[i][0]] == SCC :: C[X[i][1]])
51             ok = false;
52     }
53     if(ok){
54         cout << "POSSIBLE" << endl;
55         for(int i = 1; i ≤ n; ++ i){
56             int a = SCC :: C[X[i][0]];
57             int b = SCC :: C[X[i][1]];
58             if(a < b)
59                 cout << 0 << " ";
60             else
61                 cout << 1 << " ";
62         }
63         cout << endl;
64     } else {
65         cout << "IMPOSSIBLE" << endl;
66     }
67     return 0;
68 }

```

#### 4.6 割点

```

1 #include "../header.cpp"
2 vector<int> V[MAXN];
3 int n, m, o, D[MAXN], L[MAXN];
4 bool F[MAXN], C[MAXN];
5 void dfs(int u, int g){
6     L[u] = D[u] = ++ o, F[u] = true; int s = 0;

```

```

7     for(auto &v : V[u]){
8         if(!F[v]){
9             dfs(v, g), ++ s;
10            L[u] = min(L[u], L[v]);
11            if(u ≠ g && L[v] ≥ D[u]) C[u] = true;
12        } else {
13            L[u] = min(L[u], D[v]);
14        }
15    }
16    if(u == g && s > 1) C[u] = true;
17 }
18 int main(){
19     cin >> n >> m;
20     for(int i = 1; i ≤ m; ++ i){
21         int u, v;
22         cin >> u >> v;
23         V[u].push_back(v);
24         V[v].push_back(u);
25     }
26     for(int i = 1; i ≤ n; ++ i)
27         if(!F[i]) dfs(i, i);
28     vector<int> ANS;
29     for(int i = 1; i ≤ n; ++ i)
30         if(C[i]) ANS.push_back(i);
31     cout << ANS.size() << endl;
32     for(auto &u : ANS)
33         cout << u << " ";
34     return 0;
35 }

```

#### 4.7 边双连通分量

```

1 #include "../header.cpp"
2 vector<vector<int>> A;
3 vector<pair<int, int>> V[MAXN];
4 stack<int> S;
5 int D[MAXN], L[MAXN], o;
6 bool I[MAXN];
7 void dfs(int u, int l){
8     D[u] = L[u] = ++ o; I[u] = true, S.push(u);
9     int s = 0;
10    for(auto &p : V[u]) {
11        int v = p.first, id = p.second;
12        if(id ≠ l){
13            if(D[v]){
14                if(I[v]) L[u] = min(L[u], D[v]);
15            } else {
16                dfs(v, id), L[u] = min(L[u], L[v]), ++ s;
17            }
18        }
19    }
20 }

```

```

19     if(D[u] == L[u]){
20         vector<int> T;
21         while(S.top() ≠ u){
22             int v = S.top(); S.pop();
23             T.push_back(v), I[v] = false;
24         }
25         T.push_back(u), S.pop(), I[u] = false;
26         A.push_back(T);
27     }
28 }

```

#### 4.8 点双连通分量

```

1 #include "../header.cpp"
2 vector<vector<int>> A;
3 vector<int> V[MAXN];
4 stack<int> S;
5 int D[MAXN], L[MAXN], o; bool I[MAXN];
6 void dfs(int u, int f){
7     D[u] = L[u] = ++ o; I[u] = true, S.push(u);
8     int s = 0;
9     for(auto &v : V[u]) if(v ≠ f){
10         if(D[v]) L[u] = min(L[u], D[v]);
11     } else {
12         dfs(v, u), L[u] = min(L[u], L[v]), ++ s;
13         if(L[v] ≥ D[u]){
14             vector<int> T;
15             while(S.top() ≠ v){
16                 int t = S.top(); S.pop();
17                 T.push_back(t), I[t] = false;
18             }
19             T.push_back(v), S.pop(), I[v] = false;
20             T.push_back(u);
21             A.push_back(T);
22         }
23     }
24 }
25 if(f == 0 && s == 0){
26     A.push_back({u});
27 }
28 }

```

#### 4.9 强连通分量

```

1 #include "../header.cpp"
2 vector<int> V[MAXN];
3 stack<int> S;
4 int D[MAXN], L[MAXN], C[MAXN], o, s;
5 bool F[MAXN], I[MAXN];
6 void add(int u, int v){ V[u].push_back(v); }

```

```

7 void dfs(int u){
8     L[u] = D[u] = ++ o, S.push(u), I[u] = F[u] =
      true;
9     for(auto &v : V[u]){
10         if(F[v]){
11             if(I[v]) L[u] = min(L[u], D[v]);
12         } else {
13             dfs(v), L[u] = min(L[u], L[v]);
14         }
15     }
16     if(L[u] == D[u]){
17         int c = ++ s;
18         while(S.top() != u){
19             int v = S.top(); S.pop();
20             I[v] = false;
21             C[v] = c;
22         }
23         S.pop(), I[u] = false, C[u] = c;
24     }
25 }
26 vector<int> ANS[MAXN];
27 int main(){
28     int n, m;
29     cin >> n >> m;
30     for(int i = 1; i ≤ m; ++ i){
31         int u, v;
32         cin >> u >> v;
33         V[u].push_back(v);
34     }
35     for(int i = 1; i ≤ n; ++ i)
36         if(!F[i])
37             dfs(i);
38     for(int i = 1; i ≤ n; ++ i){
39         ANS[C[i]].push_back(i);
40     }
41     cout << s << endl;
42     for(int i = 1; i ≤ n; ++ i) if(F[i]){
43         int c = C[i];
44         sort(ANS[c].begin(), ANS[c].end());
45         for(auto &u : ANS[c])
46             cout << u << " ", F[u] = false;
47         cout << endl;
48     }
49     return 0;
50 }

```

## 5 网络流

### 5.1 费用流

```

1 #include " ../header.cpp"

```

```

2 namespace MCMF{
3     int H[MAXN], V[MAXM], N[MAXM], W[MAXM], F[
      MAXM], o = 1, n;
4     void add(int u, int v, int f, int c){
5         V[++ o] = v, N[o] = H[u], H[u] = o, F[o] =
          f, W[o] = c;
6         V[++ o] = u, N[o] = H[v], H[v] = o, F[o] =
          0, W[o] = -c;
7         n = max(n, u);
8         n = max(n, v);
9     }
10    void clear(){
11        for(int i = 1; i ≤ n; ++ i)
12            H[i] = 0;
13        n = 0, o = 1;
14    }
15    bool I[MAXN];
16    i64 D[MAXN];
17    bool spfa(int s, int t){
18        queue<int> Q;
19        Q.push(s), I[s] = true;
20        for(int i = 1; i ≤ n; ++ i)
21            D[i] = INFL;
22        D[s] = 0;
23        while(!Q.empty()){
24            int u = Q.front(); Q.pop(), I[u] = false;
25            ;
26            for(int i = H[u]; i; i = N[i]){
27                const int &v = V[i];
28                const int &f = F[i];
29                const int &w = W[i];
30                if(f && D[u] + w < D[v]){
31                    D[v] = D[u] + w;
32                    if(!I[v]) Q.push(v), I[v] = true;
33                }
34            }
35            return D[t] != INFL;
36        }
37    }
38    int C[MAXN]; bool T[MAXN];
39    pair<i64, i64> dfs(int s, int t, int u, i64
      maxf){
40        if(u == t)
41            return make_pair(maxf, 0);
42        i64 totf = 0;
43        i64 totc = 0;
44        T[u] = true;
45        for(int &i = C[u]; i; i = N[i]){
46            const int &v = V[i];
47            const int &f = F[i];
48            const int &w = W[i];
49            if(f && D[v] == D[u] + w && !T[v]){

```

```

49            auto p = dfs(s, t, v, min(1ll * F[i],
              maxf));
50            i64 f = p.first;
51            i64 c = p.second;
52            F[i] -= f;
53            F[i ^ 1] += f;
54            totf += f;
55            totc += 1ll * f * W[i] + c;
56            maxf -= f;
57            if(maxf == 0){
58                T[u] = false;
59                return make_pair(totf, totc);
60            }
61        }
62        T[u] = false;
63        return make_pair(totf, totc);
64    }
65    pair<i64, i64> mcmf(int s, int t){
66        i64 ans1 = 0;
67        i64 ans2 = 0;
68        pair<i64, i64> r;
69        while(spfa(s, t)){
70            memcpy(C, H, sizeof(int) * (n + 3));
71            r = dfs(s, t, s, INFL);
72            ans1 += r.first;
73            ans2 += r.second;
74        }
75        return make_pair(ans1, ans2);
76    }
77 }
78 int qread();
79 int main(){
80     int n = qread(), m = qread(), s = qread(), t
      = qread();
81     for(int i = 1; i ≤ m; ++ i){
82         int u = qread(), v = qread(), f = qread(),
          c = qread();
83         MCMF :: add(u, v, f, c);
84     }
85     pair<long long, long long> ans = MCMF ::
      mcmf(s, t);
86     printf("%lld %lld\n", ans.first, ans.second);
87     ;
88     return 0;
89 }

```

## 5.2 最小割树

### 5.2.1 用法

给定无向图求出最小割树, 点  $u$  和  $v$  作为起点终点的  
最小割为树上  $u$  到  $v$  路径上边权的最小值。

```
1 #include "../header.cpp"
2 namespace Dinic{
3     const long long INF = 1e18;
4     const int SIZ = 1e5 + 3;
5     int n, m;
6     int H[SIZ], V[SIZ], N[SIZ], F[SIZ], t = 1;
7     int add(int u, int v, int f){
8         V[++ t] = v, N[t] = H[u], F[t] = f, H[u] =
9             t;
10        V[++ t] = u, N[t] = H[v], F[t] = 0, H[v] =
11            t;
12        n = max(n, u);
13        n = max(n, v);
14        return t - 1;
15    }
16    void clear(){
17        for(int i = 1; i ≤ n; ++ i)
18            H[i] = 0;
19        n = m = 0, t = 1;
20    }
21    int D[SIZ];
22    bool bfs(int s, int t){
23        queue<int> Q;
24        for(int i = 1; i ≤ n; ++ i)
25            D[i] = 0;
26        Q.push(s), D[s] = 1;
27        while(!Q.empty()){
28            int u = Q.front(); Q.pop();
29            for(int i = H[u]; i = N[i]){
30                const int &v = V[i];
31                const int &f = F[i];
32                if(f ≠ 0 && !D[v]){
33                    D[v] = D[u] + 1;
34                    Q.push(v);
35                }
36            }
37        }
38        return D[t] ≠ 0;
39    }
40    int C[SIZ];
41    long long dfs(int s, int t, int u, long long
42        maxf){
43        if(u = t)
44            return maxf;
45        long long totf = 0;
46        for(int &i = C[u]; i = N[i]){
47            const int &v = V[i];
48            const int &f = F[i];
49            if(D[v] = D[u] + 1){
50                long long resf = dfs(s, t, v, min(maxf
51                    , 1ll * f));
52                totf += resf;
53                maxf -= resf;
54                F[i] -= resf;
55                F[i ^ 1] += resf;
56                if(maxf = 0)
57                    return totf;
58            }
59        }
60        return totf;
61    }
62    long long dinic(int s, int t){
63        long long ans = 0;
64        while(bfs(s, t)){
65            memcpy(C, H, sizeof(int) * (n + 3));
66            ans += dfs(s, t, s, INF);
67        }
68        return ans;
69    }
70 }
71 namespace GHTree{
72     const int MAXN = 500 + 5;
73     const int MAXM = 1500 + 5;
74     const int INF = 1e9;
75     int n, m, U[MAXN], V[MAXN], W[MAXN], A[MAXN],
76         B[MAXN];
77     void add(int u, int v, int w){
78         ++ m;
79         U[m] = u;
80         V[m] = v;
81         W[m] = w;
82         A[m] = Dinic :: add(u, v, w);
83         B[m] = Dinic :: add(v, u, w);
84         n = max(n, u);
85         n = max(n, v);
86     }
87     vector<pair<int, int>> E[MAXN];
88     void build(vector<int> N){
89         int s = N.front();
90         int t = N.back();
91         if(s = t) return;
92         for(int i = 1; i ≤ m; ++ i){
93             int a = A[i]; Dinic :: F[a] = W[i],
94                 Dinic :: F[a ^ 1] = 0;
95             int b = B[i]; Dinic :: F[b] = W[i],
96                 Dinic :: F[b ^ 1] = 0;
97         }
98         int w = Dinic :: dinic(s, t);
99     }
100 }
```

```
44     const int &v = V[i];
45     const int &f = F[i];
46     if(D[v] = D[u] + 1){
47         long long resf = dfs(s, t, v, min(maxf
48             , 1ll * f));
49         totf += resf;
50         maxf -= resf;
51         F[i] -= resf;
52         F[i ^ 1] += resf;
53         if(maxf = 0)
54             return totf;
55     }
56 }
57 return totf;
58 }
59 long long dinic(int s, int t){
60     long long ans = 0;
61     while(bfs(s, t)){
62         memcpy(C, H, sizeof(int) * (n + 3));
63         ans += dfs(s, t, s, INF);
64     }
65     return ans;
66 }
67 namespace GHTree{
68     const int MAXN = 500 + 5;
69     const int MAXM = 1500 + 5;
70     const int INF = 1e9;
71     int n, m, U[MAXN], V[MAXN], W[MAXN], A[MAXN],
72         B[MAXN];
73     void add(int u, int v, int w){
74         ++ m;
75         U[m] = u;
76         V[m] = v;
77         W[m] = w;
78         A[m] = Dinic :: add(u, v, w);
79         B[m] = Dinic :: add(v, u, w);
80         n = max(n, u);
81         n = max(n, v);
82     }
83     vector<pair<int, int>> E[MAXN];
84     void build(vector<int> N){
85         int s = N.front();
86         int t = N.back();
87         if(s = t) return;
88         for(int i = 1; i ≤ m; ++ i){
89             int a = A[i]; Dinic :: F[a] = W[i],
90                 Dinic :: F[a ^ 1] = 0;
91             int b = B[i]; Dinic :: F[b] = W[i],
92                 Dinic :: F[b ^ 1] = 0;
93         }
94         int w = Dinic :: dinic(s, t);
95     }
96 }
```

```
92     E[s].push_back(make_pair(t, w));
93     E[t].push_back(make_pair(s, w));
94     vector<int> P;
95     vector<int> Q;
96     for(auto &u : N){
97         if(Dinic :: D[u] ≠ 0)
98             P.push_back(u);
99         else
100             Q.push_back(u);
101     }
102     build(P), build(Q);
103 }
104 int D[MAXN];
105 int cut(int s, int t){
106     queue<int> Q; Q.push(s);
107     for(int i = 1; i ≤ n; ++ i)
108         D[i] = -1;
109     D[s] = INF;
110     while(!Q.empty()){
111         int u = Q.front(); Q.pop();
112         for(auto &e : E[u]){
113             int v = e.first;
114             int w = e.second;
115             if(D[v] = -1){
116                 D[v] = min(D[u], w);
117                 Q.push(v);
118             }
119         }
120     }
121     return D[t];
122 }
123 }
```

### 5.3 最大流

```
1 #include "../header.cpp"
2 namespace Dinic{
3     const i64 INF = 1e18;
4     const int SIZ = 5e5 + 3;
5     int n;
6     int H[MAXN], V[MAXN], N[MAXN], F[MAXN], t =
7         1;
8     void add(int u, int v, int f){
9         V[++ t] = v, N[t] = H[u], F[t] = f, H[u] =
10             t;
11         V[++ t] = u, N[t] = H[v], F[t] = 0, H[v] =
12             t;
13         n = max(n, u);
14         n = max(n, v);
15     }
16    void clear(){
17        for(int i = 1; i ≤ n; ++ i)
```

```

15     H[i] = 0;
16     n = 0, t = 1;
17 }
18 i64 D[MAXN];
19 bool bfs(int s, int t){
20     queue<int> Q;
21     for(int i = 1; i ≤ n; ++ i)
22         D[i] = 0;
23     Q.push(s), D[s] = 1;
24     while(!Q.empty()){
25         int u = Q.front(); Q.pop();
26         for(int i = H[u]; i ≤ N[i]){
27             const int &v = V[i];
28             const int &f = F[i];
29             if(f ≠ 0 && !D[v]){
30                 D[v] = D[u] + 1;
31                 Q.push(v);
32             }
33         }
34     }
35     return D[t] ≠ 0;
36 }
37 int C[MAXN];
38 i64 dfs(int s, int t, int u, i64 maxf){
39     if(u == t)
40         return maxf;
41     i64 totf = 0;
42     for(int &i = C[u]; i ≤ N[i]){
43         const int &v = V[i];
44         const int &f = F[i];
45         if(f && D[v] == D[u] + 1){
46             i64 f = dfs(s, t, v, min(1ll * f, maxf));
47             F[i] -= f, F[i ^ 1] += f, totf += f,
48             maxf -= f;
49             if(maxf == 0)
50                 return totf;
51         }
52     }
53     return totf;
54 }
55 i64 dinic(int s, int t){
56     i64 ans = 0;
57     while(bfs(s, t)){
58         memcpy(C, H, sizeof(int) * (n + 3));
59         ans += dfs(s, t, s, INFL);
60     }
61     return ans;
62 }

```

## 5.4 上下界费用流

### 5.4.1 用法

- add(u, v, l, r, c): 连一条容量在  $[l, r]$  的从  $u$  到  $v$  的费用为  $c$  的边;
- solve(): 计算无源汇最小费用可行流;
- solve(s, t): 计算有源汇最小费用最大流。

```

1 #define add add0
2 #include "flow-cost.cpp"
3 #undef add
4 namespace MCMF{
5     i64 cost0;
6     int G[MAXN];
7     void add(int u, int v, int l, int r, int c){
8         G[v] += l;
9         G[u] -= l;
10        cost0 += 1ll * l * c;
11        add0(u, v, r - l, c);
12    }
13    i64 solve(){
14        int s = ++ n;
15        int t = ++ n;
16        i64 sum = 0;
17        for(int i = 1; i ≤ n - 2; ++ i){
18            if(G[i] < 0)
19                add0(i, t, -G[i], 0);
20            else
21                add0(s, i, G[i], 0), sum += G[i];
22        }
23        auto res = mcmf(s, t);
24        if(res.first ≠ sum)
25            return -1;
26        return res.second + cost0;
27    }
28    i64 solve(int s0, int t0){
29        add0(t0, s0, INF, 0);
30        int s = ++ n;
31        int t = ++ n;
32        i64 sum = 0;
33        for(int i = 1; i ≤ n - 2; ++ i){
34            if(G[i] < 0)
35                add0(i, t, -G[i], 0);
36            else
37                add0(s, i, G[i], 0), sum += G[i];
38        }
39        auto res = mcmf(s, t);
40        if(res.first ≠ sum)
41            return -1;
42        return res.second + cost0;
43    }

```

44 }

## 5.5 上下界最大流

### 5.5.1 用法

- add(u, v, l, r, c): 连一条容量在  $[l, r]$  的从  $u$  到  $v$  的边;
- solve(): 检查是否存在无源汇可行流;
- solve(s, t): 计算有源汇最大流。

```

1 #define add add0
2 #include "flow-max.cpp"
3 #undef add
4 namespace Dinic{
5     int G[MAXN];
6     void add(int u, int v, int l, int r){
7         G[v] += l;
8         G[u] -= l;
9         add0(u, v, r - l);
10    }
11    void clear(){
12        for(int i = 1; i ≤ t; ++ i){
13            N[i] = F[i] = V[i] = 0;
14        }
15        for(int i = 1; i ≤ n; ++ i){
16            H[i] = G[i] = C[i] = 0;
17        }
18        t = 1, n = 0;
19    }
20    bool solve(){
21        int s = ++ n;
22        int t = ++ n;
23        i64 sum = 0;
24        for(int i = 1; i ≤ n - 2; ++ i){
25            if(G[i] < 0)
26                add0(i, t, -G[i]);
27            else
28                add0(s, i, G[i]), sum += G[i];
29        }
30        auto res = dinic(s, t);
31        if(res ≠ sum)
32            return true;
33        return false;
34    }
35    i64 solve(int s0, int t0){
36        add0(t0, s0, INF);
37        int s = ++ n;
38        int t = ++ n;
39        i64 sum = 0;
40        for(int i = 1; i ≤ n - 2; ++ i){

```

```

41     if(G[i] < 0)
42         add0(i, t, -G[i]);
43     else
44         add0(s, i, G[i]), sum += G[i];
45 }
46 auto res = dinic(s, t);
47 if(res != sum)
48     return -1;
49 return dinic(s0, t0);
50 }
51 }

```

## 6 数学

### 6.1 线性代数

#### 6.1.1 行列式

```

1 #include "../header.cpp"
2 struct Mat{
3     int n, m, W[MAXN][MAXN];
4     Mat(int _n = 0, int _m = 0){
5         n = _n, m = _m;
6         for(int i = 1; i ≤ n; ++ i)
7             for(int j = 1; j ≤ m; ++ j)
8                 W[i][j] = 0;
9     }
10 };
11 int mat_det(Mat a){
12     int ans = 1;
13     const int &n = a.n;
14     for(int i = 1; i ≤ n; ++ i){
15         int f = -1;
16         for(int j = i; j ≤ n; ++ j) if(a.W[j][i] != 0){
17             f = j; break;
18         }
19         if(f == -1) return 0;
20         if(f != i){
21             for(int j = 1; j ≤ n; ++ j)
22                 swap(a.W[i][j], a.W[f][j]);
23             ans = MOD - ans;
24         }
25         for(int j = i + 1; j ≤ n; ++ j) if(a.W[j][i])
26             while(a.W[j][i]){
27                 int u = a.W[i][i], v = a.W[j][i];
28                 if(u > v){
29                     for(int k = 1; k ≤ n; ++ k)
30                         swap(a.W[i][k], a.W[j][k]);
31                     ans = MOD - ans, swap(u, v);

```

```

32     }
33     int rate = v / u;
34     for(int k = 1; k ≤ n; ++ k){
35         a.W[j][k] = (a.W[j][k] - 1ll * rate
36                     * a.W[i][k] % MOD + MOD) % MOD;
37     }
38 }
39 }
40 for(int i = 1; i ≤ n; ++ i)
41     ans = 1ll * ans * a.W[i][i] % MOD;
42 return ans;
43 }
44 int main(){
45     int n; cin >> n;
46     Mat A(n, n);
47     for(int i = 1; i ≤ n; ++ i)
48         for(int j = 1; j ≤ n; ++ j)
49             cin >> A.W[i][j], A.W[i][j] %= MOD;
50     cout << mat_det(A) << endl;
51     return 0;
52 }

```

#### 6.1.2 高斯消元与求秩 (实数)

```

1 #include "../header.cpp"
2 const double EPS = 1e-9;
3 struct Mat{
4     int n, m;
5     double W[MAXN][MAXN];
6     Mat(int _n = 0, int _m = 0){
7         n = _n;
8         m = _m;
9         for(int i = 1; i ≤ n; ++ i)
10             for(int j = 1; j ≤ m; ++ j)
11                 W[i][j] = 0;
12 }
13 };
14 bool zero(double f){
15     return fabs(f) < EPS;
16 }
17 int mat_rank(Mat &a){
18     const int &n = a.n;
19     const int &m = a.m;
20     int cnt = 0;
21     for(int i = 1; i ≤ m; ++ i){
22         int p = cnt + 1;
23         int f = -1;
24         for(int j = p; j ≤ n; ++ j){
25             if(!zero(a.W[j][i])){
26                 f = j;
27                 break;

```

```

28     }
29 }
30 if(f == -1)
31     continue;
32 if(f != p){
33     for(int j = 1; j ≤ m; ++ j)
34         swap(a.W[p][j], a.W[f][j]);
35 }
36 ++ cnt;
37 for(int j = p + 1; j ≤ n; ++ j){
38     double rate = a.W[j][i] / a.W[p][i];
39     for(int k = 1; k ≤ m; ++ k){
40         a.W[j][k] -= rate * a.W[p][k];
41     }
42 }
43 }
44 return cnt;
45 }
46 double X[MAXN];
47 int main(){
48     int n;
49     cin >> n;
50     Mat A(n, n);
51     Mat T(n, n + 1);
52     for(int i = 1; i ≤ n; ++ i){
53         for(int j = 1; j ≤ n; ++ j)
54             cin >> A.W[i][j];
55         for(int j = 1; j ≤ n; ++ j)
56             T.W[i][j] = A.W[i][j];
57         cin >> T.W[i][n + 1];
58     }
59     int res1 = mat_rank(A);
60     int res2 = mat_rank(T);
61     if(res1 != res2)
62         cout << -1 << endl;
63     else
64         if(res2 < n)
65             cout << 0 << endl;
66     else {
67         for(int i = n; i ≥ 1; -- i){
68             X[i] = T.W[i][n + 1] / T.W[i][i];
69             for(int j = i - 1; j ≥ 1; -- j){
70                 double rate = T.W[j][i] / T.W[i][i];
71                 T.W[j][i] -= rate * T.W[i][i];
72                 T.W[j][n + 1] -= rate * T.W[i][n + 1];
73             }
74         }
75         for(int i = 1; i ≤ n; ++ i)
76             cout << "x" << i << "=" << fixed <<
77                 setprecision(2) << X[i] << endl;
78     }
79     return 0;

```



```
79 }
```

### 6.1.3 高斯消元与求秩 (整数)

```
1 #include "../header.cpp"
2 struct Mat{
3     int n, m;
4     int W[MAXN][MAXN];
5     Mat(int _n = 0, int _m = 0){
6         n = _n;
7         m = _m;
8         for(int i = 1; i ≤ n; ++ i)
9             for(int j = 1; j ≤ m; ++ j)
10                 W[i][j] = 0;
11     }
12 };
13 int power(int a, int b){
14     int r = 1;
15     while(b){
16         if(b & 1) r = 1ll * r * a % MOD;
17         b >>= 1, a = 1ll * a * a % MOD;
18     }
19     return r;
20 }
21 int inv(int x){
22     return power(x, MOD - 2);
23 }
24 int mat_rank(Mat &a){
25     const int &n = a.n;
26     const int &m = a.m;
27     int cnt = 0;
28     for(int i = 1; i ≤ m; ++ i){
29         int p = cnt + 1;
30         int f = -1;
31         for(int j = p; j ≤ n; ++ j){
32             if(a.W[j][i] ≠ 0){
33                 f = j;
34                 break;
35             }
36         }
37         if(f == -1) continue;
38         if(f ≠ p){
39             for(int j = 1; j ≤ m; ++ j)
40                 swap(a.W[p][j], a.W[f][j]);
41         }
42         ++ cnt;
43         int invp = inv(a.W[p][i]);
44         for(int j = p + 1; j ≤ n; ++ j){
45             int rate = 1ll * a.W[j][i] * invp % MOD;
46             for(int k = 1; k ≤ m; ++ k){
47
```

```
48         a.W[j][k] = (a.W[j][k] - 1ll * rate *
49             a.W[p][k] % MOD + MOD) % MOD;
50     }
51 }
52 return cnt;
53 }
54 int X[MAXN];
55 int main(){
56     int n;
57     cin >> n;
58     Mat A(n, n);
59     Mat T(n, n + 1);
60     for(int i = 1; i ≤ n; ++ i){
61         for(int j = 1; j ≤ n; ++ j)
62             cin >> A.W[i][j];
63         for(int j = 1; j ≤ n; ++ j)
64             T.W[i][j] = A.W[i][j];
65         cin >> T.W[i][n + 1];
66     }
67     int res1 = mat_rank(A);
68     int res2 = mat_rank(T);
69     if(res1 ≠ res2)
70         cout << -1 << endl;
71     else
72         if(res2 < n)
73             cout << 0 << endl;
74     else {
75         for(int i = n; i ≥ 1; -- i){
76             int invp = inv(T.W[i][i]);
77             X[i] = 1ll * T.W[i][n + 1] * invp % MOD;
78             for(int j = i - 1; j ≥ 1; -- j){
79                 int rate = 1ll * T.W[j][i] * invp %
80                     MOD;
81                 T.W[j][i] = (T.W[j][i] - 1ll *
82                     rate * T.W[i][i] % MOD + MOD) %
83                     MOD;
84                 T.W[j][n + 1] = (T.W[j][n + 1] - 1ll *
85                     rate * T.W[i][n + 1] % MOD + MOD) %
86                     MOD;
87             }
88             for(int i = 1; i ≤ n; ++ i)
89                 cout << "x" << i << "=" << X[i] << endl;
90         }
91     }
92     return 0;
93 }
```

### 6.1.4 矩阵求逆

```
1 #include<bits/stdc++.h>
2 using namespace std;
```

```
3 using i64 = long long;
4 const int INF = 1e9;
5 const i64 INFL = 1e18;
6 const int MAXN = 400 + 3;
7 const int MOD = 1e9 + 7;
8 struct Mat{
9     int n, m;
10     int W[MAXN][MAXN];
11     Mat(int _n = 0, int _m = 0){
12         n = _n, m = _m;
13         for(int i = 1; i ≤ n; ++ i)
14             for(int j = 1; j ≤ m; ++ j)
15                 W[i][j] = 0;
16     }
17 };
18 int power(int a, int b){
19     int r = 1;
20     while(b){
21         if(b & 1) r = 1ll * r * a % MOD;
22         b >>= 1, a = 1ll * a * a % MOD;
23     }
24     return r;
25 }
26 int inv(int x){
27     return power(x, MOD - 2);
28 }
29 bool mat_inv(Mat &a){
30     const int &n = a.n;
31     Mat b(n, n);
32     for(int i = 1; i ≤ n; ++ i)
33         b.W[i][i] = 1;
34     for(int i = 1; i ≤ n; ++ i){
35         int f = -1;
36         for(int j = i; j ≤ n; ++ j) if(a.W[j][i]
37             ≠ 0){
38             f = j;
39             break;
40         }
41         if(f == -1){
42             return false;
43         }
44         if(f ≠ i){
45             for(int j = 1; j ≤ n; ++ j)
46                 swap(a.W[i][j], a.W[f][j]),
47                 swap(b.W[i][j], b.W[f][j]);
48         }
49         int invp = inv(a.W[i][i]);
50         for(int j = i + 1; j ≤ n; ++ j){
51             int rate = 1ll * a.W[j][i] * invp
52                 % MOD;
53             for(int k = 1; k ≤ n; ++ k){
54                 a.W[j][k] = (a.W[j][k] - 1ll *
```

```

        rate * a.W[i][k] % MOD +
        MOD) % MOD;
        b.W[j][k] = (b.W[j][k] - 1ll *
        rate * b.W[i][k] % MOD +
        MOD) % MOD;
    }
}
for(int i = n; i ≥ 1; -- i){
    int invp = inv(a.W[i][i]);
    for(int j = 1; j ≤ n; ++ j){
        a.W[i][j] = 1ll * a.W[i][j] * invp
        % MOD;
        b.W[i][j] = 1ll * b.W[i][j] * invp
        % MOD;
    }
    for(int j = i - 1; j ≥ 1; -- j){
        int rate = 1ll * a.W[j][i] % MOD;
        for(int k = 1; k ≤ n; ++ k){
            a.W[j][k] = (a.W[j][k] - 1ll *
            rate * a.W[i][k] % MOD +
            MOD) % MOD;
            b.W[j][k] = (b.W[j][k] - 1ll *
            rate * b.W[i][k] % MOD +
            MOD) % MOD;
        }
    }
}
for(int i = 1; i ≤ n; ++ i)
    for(int j = 1; j ≤ n; ++ j)
        a.W[i][j] = b.W[i][j];
return true;
}
int X[MAXN];
int main(){
    int n;
    cin >> n;
    Mat A(n, n);
    for(int i = 1; i ≤ n; ++ i)
        for(int j = 1; j ≤ n; ++ j)
            cin >> A.W[i][j];
    bool res = mat_inv(A);
    if(res == false){
        cout << "No Solution" << endl;
    } else {
        for(int i = 1; i ≤ n; ++ i)
            for(int j = 1; j ≤ n; ++ j)
                cout << A.W[i][j] << " \n"[j
                == n];
    }
    return 0;
}

```

### 6.1.5 矩阵树

**LGV 定理叙述** 设  $G$  是一张有向无环图，边带权，每个点的度数有限。给定起点集合  $A = \{a_1, a_2, \dots, a_n\}$ ，终点集合  $B = \{b_1, b_2, \dots, b_n\}$ 。

- 一段路径  $p: v_0 \xrightarrow{w_1} v_1 \xrightarrow{w_2} v_2 \rightarrow \dots \rightarrow^{w_k} v_k$  的边权被定义为  $\omega(p) = \prod w_i$ 。
- 一对顶点  $(a, b)$  的权值定义为  $e(a, b) = \sum_{p: a \rightarrow b} \omega(p)$ 。

设矩阵  $M$  如下：

$$M = \begin{pmatrix} e(a_1, b_1) & e(a_1, b_2) & \cdots & e(a_1, b_n) \\ e(a_2, b_1) & e(a_2, b_2) & \cdots & e(a_2, b_n) \\ \vdots & \vdots & \ddots & \vdots \\ e(a_n, b_1) & e(a_n, b_2) & \cdots & e(a_n, b_n) \end{pmatrix}$$

从  $A$  到  $B$  得到一个不相交的路径组  $p = (p_1, p_2, \dots, p_n)$ ，其中从  $a_i$  到达  $b_{\pi_i}$ ， $\pi$  是一个排列。定义  $\sigma(\pi)$  是  $\pi$  逆序对的数量。

给出 LGV 的叙述如下：

$$\det(M) = \sum_{p: A \rightarrow B} (-1)^{\sigma(\pi)} \prod_{i=1}^n \omega(p_i)$$

可以将边权视作边的重数，那么  $e(a, b)$  就可以视为从  $a$  到  $b$  的不同路径方案数。

**矩阵树定理** 对于无向图，

- 定义度数矩阵  $D_{i,j} = [i = j] \deg(i)$ ；
- 定义邻接矩阵  $E_{i,j} = E_{j,i}$  是从  $i$  到  $j$  的边数个数；
- 定义拉普拉斯矩阵  $L = D - E$ 。

对于无向图的矩阵树定理叙述如下：

$$t(G) = \det(L_i) = \frac{1}{n} \lambda_1 \lambda_2 \cdots \lambda_{n-1}$$

其中  $L_i$  是将  $L$  删去第  $i$  行和第  $i$  列得到的子式。

对于有向图，类似于无向图定义入度矩阵、出度矩阵、邻接矩阵  $D^{\text{in}}, D^{\text{out}}, E$ ，同时定义拉普拉斯矩阵  $L^{\text{in}} = D^{\text{in}} - E, L^{\text{out}} = E$ 。

$$t^{\text{leaf}}(G, k) = \det(L_k^{\text{in}})$$

$$t^{\text{root}}(G, k) = \det(L_k^{\text{out}})$$

其中  $t^{\text{leaf}}(G, k)$  表示以  $k$  为根的叶向树， $t^{\text{root}}(G, k)$  表示以  $k$  为根的根向树。

**BEST 定理** 对于一个有向欧拉图  $G$ ，记点  $i$  的出度为  $\text{out}_i$ ，同时  $G$  的根向生成树个数为  $T$ 。 $T$  可以任意选取根。则  $G$  的本质不同的欧拉回路个数为：

$$T \prod_i (\text{out}_i - 1)!$$

```

1 #include "../header.cpp"
2 struct Mat{
3     int n, m;
4     int W[MAXN][MAXN];
5     Mat(int _n = 0, int _m = 0){
6         n = _n;
7         m = _m;
8         for(int i = 1; i ≤ n; ++ i)
9             for(int j = 1; j ≤ m; ++ j)
10                 W[i][j] = 0;
11     }
12 };
13 int mat_det(Mat a){
14     int ans = 1;
15     const int &n = a.n;
16     for(int i = 1; i ≤ n; ++ i){
17         int f = -1;
18         for(int j = i; j ≤ n; ++ j) if(a.W[j][i] ≠
19             0){
20             f = j;
21             break;
22         }
23         if(f == -1){
24             return 0;
25         }
26         if(f ≠ i){
27             for(int j = 1; j ≤ n; ++ j)
28                 swap(a.W[i][j], a.W[f][j]);
29             ans = MOD - ans;
30         }
31     }
32 }

```

```

30 for(int j = i + 1; j ≤ n; ++ j) if(a.W[j][i]
31   ){
32   while(a.W[j][i]){
33     int u = a.W[i][i];
34     int v = a.W[j][i];
35     if(u > v){
36       for(int k = 1; k ≤ n; ++ k)
37         swap(a.W[i][k], a.W[j][k]);
38       ans = MOD - ans;
39       swap(u, v);
40     }
41     int rate = v / u;
42     for(int k = 1; k ≤ n; ++ k){
43       a.W[j][k] = (a.W[j][k] - 1ll * rate
44         * a.W[i][k] % MOD + MOD) % MOD;
45     }
46   }
47   for(int i = 1; i ≤ n; ++ i)
48     ans = 1ll * ans * a.W[i][i] % MOD;
49   return ans;
50 }
51 int D[MAXN];
52 int W[MAXN][MAXN];
53 int main(){
54   int n, m, t;
55   cin >> n >> m >> t;
56   for(int i = 1; i ≤ m; ++ i){
57     int u, v, w;
58     cin >> u >> v >> w;
59     if(u ≠ v){
60       if(t == 0){ // 无向图
61         D[u] = (D[u] + w) % MOD;
62         D[v] = (D[v] + w) % MOD;
63         W[u][v] = (W[u][v] + w) % MOD;
64         W[v][u] = (W[v][u] + w) % MOD;
65       } else { // 叶向树
66         D[v] = (D[v] + w) % MOD;
67         W[u][v] = (W[u][v] + w) % MOD;
68       }
69     }
70   }
71   Mat A(n - 1, n - 1);
72   for(int i = 2; i ≤ n; ++ i)
73     for(int j = 2; j ≤ n; ++ j) // 以 1 为根的
74       // 叶向树
75       A.W[i - 1][j - 1] = MOD - W[i][j];
76   for(int i = 2; i ≤ n; ++ i)
77     A.W[i - 1][i - 1] = (D[i] + A.W[i - 1][i - 1] % MOD);
78   cout << mat_det(A) << endl;

```

```

78 return 0;
79 }

```

## 6.2 大步小步

### 6.2.1 用法

给定  $a, p$  求出  $x$  使得  $a^x = y \pmod{p}$ , 其中  $p$  为质数。

```

1 #include "../header.cpp"
2 namespace BSGS {
3   unordered_map <int, int> M;
4   int solve(int a, int y, int p){ // a ^ x =
5     // y (mod p)
6     M.clear();
7     int B = sqrt(p);
8     int w1 = y, u1 = power(a, p - 2, p);
9     int w2 = 1, u2 = power(a, B, p);
10    for(int i = 0; i < B; ++ i){
11      M[w1] = i;
12      w1 = 1ll * w1 * u1 % p;
13    }
14    for(int i = 0; i < p / B; ++ i){
15      if(M.count(w2)){
16        return i * B + M[w2];
17      }
18      w2 = 1ll * w2 * u2 % p;
19    }
20    return -1;
21  }

```

## 6.3 中国剩余定理

### 6.3.1 定理

对于线性方程:

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \dots \\ x \equiv a_n \pmod{m_n} \end{cases}$$

如果  $a_i$  两两互质, 可以得到  $x$  的解  $x \equiv L \pmod{M}$ ,

其中  $M = \prod m_i$ , 而  $L$  由下式给出:

$$L = \left( \sum a_i m_i \times ((M/m_i)^{-1} \pmod{m_i}) \right) \pmod{M}$$

```

1 #include "../header.cpp"
2 i64 A[MAXN], B[MAXN], M = 1;
3 i64 exgcd(i64 a, i64 b, i64 &x, i64 &y);
4 int main(){
5   int n; cin >> n;
6   for(int i = 1; i ≤ n; ++ i){
7     cin >> B[i] >> A[i];
8     M = M * B[i];
9   }
10  i64 L = 0;
11  for(int i = 1; i ≤ n; ++ i){
12    i64 m = M / B[i], b, k;
13    exgcd(m, B[i], b, k);
14    L = (L + (__int128)A[i] * m * b) % M;
15  }
16  L = (L % M + M) % M;
17  cout << L << endl;
18  return 0;
19 }

```

## 6.4 狄利克雷前缀和

### 6.4.1 用法

计算:

$$s(i) = \sum_{d|i} f_d$$

```

1 #include "../header.cpp"
2 unsigned A[MAXN];
3 int p, P[MAXN]; bool V[MAXN];
4 void solve(int n){
5   for(int i = 2; i ≤ n; ++ i){
6     if(!V[i]){
7       P[++ p] = i;
8       for(int j = 1; j ≤ n / i; ++ j){ // 前缀
9         // 和
10        A[j * i] += A[j];
11      }
12      for(int j = 1; j ≤ p && P[j] ≤ n / i; ++ j){
13        V[i * P[j]] = true;
14        if(i % P[j] == 0) break;
15      }
16    }
17  }

```

## 6.5 万能欧几里得

### 6.5.1 类欧几里得 (万能欧几里得)

From *zpk*

一种神奇递归, 对  $y = \left\lfloor \frac{Ax+B}{C} \right\rfloor$  向右和向上走的每一步进行压缩, 做到  $O(\log V)$  复杂度。其中  $A \geq C$  就是直接压缩, 向右之后必有至少  $\lfloor A/C \rfloor$  步向上。  $A < C$  实际上切换  $x, y$  轴后, 相当于压缩了一个上取整折线, 而上取整下取整可以互化, 便又可以递归。

代码中从  $(0, 0)$  走到  $(n, \lfloor (An+B)/C \rfloor)$ , 假设了  $A, B, C \geq 0, C \neq 0$  (类欧基本都作此假设),  $U, R$  矩阵是从右往左乘的, 对列向量进行优化, 和实际操作顺序恰好相反。快速幂的  $\log$  据说可以被递归过程均摊掉, 实际上并不会导致变成两个  $\log$ 。

```
1 Matrix solve(ll n, ll A, ll B, ll C, Matrix R,
2   Matrix U) { // (0, 0) 走到 (n, (An+B)/C)
3   if (A ≥ C) return solve(n, A % C, B, C, U
4     .qpow(A / C) * R, U);
5   ll l = B / C, r = (A * n + B) / C;
6   if (l == r) return R.qpow(n) * U.qpow(l);
7   // l = r → l = r or A = 0 or n = 0.
8   ll p = (C * r - B - 1) / A + 1;
9   return R.qpow(n - p) * U * solve(r - l -
10     1, C, C - B % C + A - 1, A, U, R) * U.
11     qpow(l);
12 }
```

## 6.6 扩展欧几里得

### 6.6.1 内容

给定  $a, b$ , 求出  $ax + by = \gcd(a, b)$  的一组  $x, y$ 。

```
1 int exgcd(int a, int b, int &x, int &y){
2   if(a == 0){
3     x = 0, y = 1; return b;
4   } else {
5     int x0 = 0, y0 = 0;
6     int d = exgcd(b % a, a, x0, y0);
7     x = y0 - (b / a) * x0;
8     y = x0;
9     return d;
10  }
11 }
```

## 6.7 快速离散对数

### 6.7.1 用法

给定原根  $g$  以及模数  $\text{mod}$ ,  $T$  次询问  $x$  的离散对数。  
复杂度  $O(\text{mod}^{2/3} + T \log \text{mod})$ 。

```
1 #include "../header.cpp"
2 namespace BSGS {
3   unordered_map<int, int> M;
4   int B, U, P, g;
5   void init(int g, int P0, int B0);
6   int solve(int y);
7 }
8 const int MAXN = 1e5 + 3;
9 int H[MAXN], P[MAXN], H0, p, h, g, mod;
10 bool V[MAXN];
11 int solve(int x){
12   if(x ≤ h) return H[x];
13   int v = mod / x, r = mod % x;
14   if(r < x - r) return ((H0 + solve(r)) % (mod
15     - 1) - H[v] + mod - 1) % (mod - 1);
16   else return (solve(x - r) - H[v +
17     1] + mod - 1) % (mod - 1);
18 }
19 int main(){
20   ios :: sync_with_stdio(false);
21   cin.tie(nullptr);
22   cin >> g >> mod;
23   h = sqrt(mod) + 1;
24   BSGS :: init(g, mod, sqrt(1ll * mod * sqrt(
25     mod) / log10(mod)));
26   H0 = BSGS :: solve(mod - 1);
27   H[1] = 0;
28   for(int i = 2; i ≤ h; ++ i){
29     if(!V[i]){
30       P[++ p] = i;
31       H[i] = BSGS :: solve(i);
32     }
33   }
34   for(int j = 1; j ≤ p && P[j] ≤ h / i; ++ j)
35     {
36       int &p = P[j];
37       H[i * p] = (H[i] + H[p]) % (mod - 1);
38       V[i * p] = true;
39       if(i % p == 0) break;
40     }
41   int T; cin >> T;
42   while(T --){
43     int x; cin >> x;
44     cout << solve(x) << "\n";
45   }
46   return 0;
47 }
```

43

}

## 6.8 快速最大公约数

### 6.8.1 用法

已知小值域  $m$  以及  $n$  次询问,  $O(m)$  预处理,  $O(1)$  单次查询  $x, y$  的最大公约数。

```
1 #include "../header.cpp"
2 const int MAXT = 1e6 + 3;
3 int G[MAXM][MAXM], T[MAXT][3];
4 int A[MAXN], B[MAXN], o = 1e6, h = 1e3, V[MAXT
5   ];
6 int tgcd(int a, int b){
7   if(a ≤ h && b ≤ h) return G[a][b];
8   return a == b ? a : 1;
9 }
10 int qgcd(int a, int b){
11   int ans = 1;
12   up(0, 2, i){
13     if(T[b][i] > h){
14       if(a % T[b][i] == 0) a /= T[b][i], ans
15         *= T[b][i];
16     } else {
17       int d = G[a % T[b][i]][T[b][i]];
18       a /= d, ans *= d;
19     }
20   }
21   return ans;
22 }
23 int main(){
24   ios :: sync_with_stdio(false);
25   cin.tie(nullptr);
26   up(1, h, i) G[0][i] = G[i][0] = i;
27   up(1, h, i) up(1, h, j){
28     if(i ≥ j) G[i][j] = G[i - j][j];
29     else G[i][j] = G[i][j - i];
30   }
31   up(2, o, i) if(!V[i]){
32     V[i] = i;
33     for(int j = 2; i * j ≤ o; ++ j)
34       if(!V[i * j]) V[i * j] = i;
35   }
36   T[1][0] = T[1][1] = T[1][2] = 1;
37   up(2, o, i){
38     int p = V[i];
39     int a = T[i / p][0];
40     int b = T[i / p][1];
41     int c = T[i / p][2];
42     int x, y, z;
43     if(p ≥ h){
44       int
```

```

42     x = 1, y = i / p, z = p;
43 } else {
44     if(c * p ≤ h){
45         x = a, y = b, z = c * p;
46     }
47     else if(b * p ≤ h){
48         x = a, y = b * p, z = c;
49         if(y > z) swap(y, z);
50     }
51     else if(a * p ≤ h){
52         x = a * p, y = b, z = c;
53         if(x > y) swap(x, y);
54         if(y > z) swap(y, z);
55     } else {
56         x = a * b, y = c, z = p;
57         if(x > y) swap(x, y);
58         if(y > z) swap(y, z);
59         if(x > z) swap(x, z);
60     }
61 }
62 T[i][0] = x;
63 T[i][1] = y;
64 T[i][2] = z;
65 }
66 int n;
67 cin >> n;
68 up(1, n, i) cin >> A[i];
69 up(1, n, i) cin >> B[i];
70 up(1, n, i){
71     int s = 0, u = 1;
72     up(1, n, j){
73         int d = gcd(A[i], B[j]);
74         u = 1ll * u * i % MOD;
75         s = (s + 1ll * d * u) % MOD;
76     }
77     printf("%d\n", s);
78 }
79 return 0;
80 }

```

## 6.9 原根

### 6.9.1 用法

计算  $P$  的最小原根。

原根表, 其中  $P = r \times 2^k$ , 对应原根为  $g$ 。

Prime	$g$	Prime	$g$
104857601	3	7881299347898369	6
167772161	3	31525197391593473	3
469762049	3	180143985094819841	6
998244353	3	1945555039024054273	5
1004535809	3	4179340454199820289	3

```

1 #include "../header.cpp"
2 int getphi(int x){
3     int t = x, r = x;
4     for(int i = 2; i ≤ x / i; ++ i){
5         if(t % i == 0){
6             r = r / i * (i - 1);
7             while(t % i == 0)
8                 t /= i;
9         }
10    }
11    if(t ≠ 1){
12        r = r / t * (t - 1);
13    }
14    return r;
15 }
16 vector<int> getprime(int x){
17     vector<int> p;
18     int t = x;
19     for(int i = 2; i ≤ x / i; ++ i){
20         if(t % i == 0){
21             p.push_back(i);
22             while(t % i == 0)
23                 t /= i;
24         }
25    }
26    if(t ≠ 1)
27        p.push_back(x);
28    return p;
29 }
30 bool test(int g, int m, int mm, vector<int> &p)
31 ){
32     for(auto &p: P){
33         if(power(g, mm / p, m) == 1)
34             return false;
35     }
36     return true;
37 }
38 int get_genshin(int m){
39     int mm = getphi(m);
40     vector<int> P = getprime(mm);

```

```

40 for(int i = 1; ++ i){
41     if(test(i, m, mm, P))
42         return i;
43 }
44 }

```

## 6.10 快速乘法逆元 (离线)

### 6.10.1 用法

离线计算  $x = [x_1, x_2, \dots, x_n]$  在模  $p$  意义下的逆元。

```

1 #include "../header.cpp"
2 int A[MAXN], B[MAXN];
3 int P[MAXN], Q[MAXN];
4 int main(){
5     ios :: sync_with_stdio(false);
6     cin.tie(nullptr);
7     int n, p, K, S = 1;
8     cin >> n >> p >> K;
9     P[0] = 1;
10    for(int i = 1; i ≤ n; ++ i){
11        cin >> A[i];
12        P[i] = 1ll * P[i - 1] * A[i] % p;
13    }
14    Q[n] = power(P[n], p - 2, p);
15    for(int i = n; i ≥ 1; -- i){
16        Q[i - 1] = 1ll * Q[i] * A[i] % p;
17        B[i] = 1ll * Q[i] * P[i - 1] % p;
18    }
19    int ans = 0;
20    for(int i = 1; i ≤ n; ++ i){
21        S = 1ll * S * K % p;
22        ans = (ans + 1ll * S * B[i]) % p;
23    }
24    cout << ans << "\n";
25    return 0;
26 }

```

## 6.11 快速乘法逆元 (在线)

### 6.11.1 用法

在线计算  $x = [x_1, x_2, \dots, x_n]$  在模  $p$  意义下的逆元。

```

1 #include "../header.cpp"
2 pair<int, int> F[MAXN], G[MAXN];
3 int I[MAXN];
4 using u32 = uint32_t;
5 u32 read(u32 &seed);
6 int main(){
7     ios :: sync_with_stdio(false);
8     cin.tie(nullptr);

```



```

9  u32 seed;
10 int n, p;
11 cin >> n >> p >> seed;
12 int m = pow(p, 1.0 / 3.0);
13 I[1] = 1;
14 for(int i = 2; i ≤ p / m; ++ i){
15     I[i] = 1ll * (p / i) * (p - I[p % i]) % p;
16 }
17 for(int i = 1; i < m; ++ i){
18     for(int j = i + 1; j ≤ m; ++ j){
19         if(!F[i * m * m / j].second){
20             F[i * m * m / j] = { i, j };
21             G[i * m * m / j] = { i, j };
22         }
23     }
24 }
25 F[0] = G[0] = { 0, 1 };
26 F[m * m] = G[m * m] = { 1, 1 };
27 for(int i = 1; i < m * m; ++ i) if(!F[i].second)
28     F[i] = F[i - 1];
29 for(int i = m * m - 1; i ≥ 1; -- i) if(!G[i].second)
30     G[i] = G[i + 1];
31 int lastans = 0;
32 for(int i = 1; i ≤ n; ++ i){
33     int a, inv;
34     a = (read(seed) ^ lastans) % (p - 1) + 1;
35     int w = 1ll * a * m * m / p;
36     auto &yy1 = F[w].second; // *avoid y1 in <cmath>
37     if(1ll * a * yy1 % p ≤ p / m){
38         inv = 1ll * I[1ll * a * yy1 % p] * yy1 % p;
39     } else {
40         auto &yy2 = G[w].second;
41         inv = 1ll * I[1ll * a * (p - yy2) % p] * (p - yy2) % p;
42     }
43     lastans = inv;
44 }
45 cout << lastans << "\n";
46 return 0;
47 }

```

## 6.12 拉格朗日插值

### 6.12.1 定理

给定  $n$  个横坐标不同的点  $(x_i, y_i)$ , 可以唯一确定一个  $n-1$  阶多项式如下:

$$f(x) = \sum_{i=1}^n \frac{\prod_{j \neq i} (x - x_j)}{\prod_{j \neq i} (x_i - x_j)} \cdot y_i$$

## 6.13 min-max 容斥

### 6.13.1 定理

$$\max_{i \in S} \{x_i\} = \sum_{T \subseteq S} (-1)^{|T|-1} \min_{j \in T} \{x_j\}$$

$$\min_{i \in S} \{x_i\} = \sum_{T \subseteq S} (-1)^{|T|-1} \max_{j \in T} \{x_j\}$$

期望意义下上式依然成立。

另外设  $\max^k$  表示第  $k$  大的元素, 可以推广为如下式

子:

$$\max_{i \in S}^k \{x_i\} = \sum_{T \subseteq S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \min_{j \in T} \{x_j\}$$

此外在数论上可以得到:

$$\text{lcm}\{x_i\} = \prod_{T \subseteq S} \left( \gcd\{x_j\} \right)^{(-1)^{|T|-1}}$$

### 6.13.2 应用

对于计算 “ $n$  个属性都出现的期望时间” 问题, 设第  $i$  个属性第一次出现的时间是  $t_i$ , 所求即为  $\max(t_i)$ , 使用 min-max 容斥转为计算  $\min(t_i)$ 。

比如  $n$  个独立物品, 每次抽中物品  $i$  的概率是  $p_i$ , 问期望抽多少次抽中所有物品。那么就可以计算  $\min_S$  表示第一次抽中物品集合  $S$  内物品的时间, 可以得到:

$$\max_U = \sum_{S \subseteq U} (-1)^{|S|-1} \min_S = \sum_{S \subseteq U} (-1)^{|S|-1} \cdot \frac{1}{\sum_{x \in S} p_x}$$

## 6.14 Barrett 取模

### 6.14.1 用法

调用 init 计算出  $S$  和  $X$ , 得到计算  $\lfloor x/P \rfloor = (x \times X)/2^{60+S}$ 。从而计算  $x \bmod P = x - P \times \lfloor x/P \rfloor$ 。

```

1 #include "../header.cpp"
2 i64 S = 0, X = 0;
3 void init(int MOD){
4     while((1 << (S + 1)) < MOD) S++;
5     X = ((__int128)1 << 60 + S) / MOD + !(((__int128)1 << 60 + S) % MOD);
6     cerr << S << " " << X << endl;
7 }
8 int power(i64 x, int y, int MOD){
9     i64 r = 1;
10    while(y){
11        if(y & 1){
12            r = r * x;
13            r = r - MOD * ((__int128)r * X >> 60 + S);
14        }
15        x = x * x;
16        x = x - MOD * ((__int128)x * X >> 60 + S);
17        y >>= 1;
18    }
19    return r;
20 }

```

## 6.15 Pollard's Rho

### 6.15.1 用法

- 调用 test(n) 判断  $n$  是否是质数;
- 调用 rho(n) 计算  $n$  分解质因数后的结果, 不保证结果有序。

```

1 #include "../header.cpp"
2 i64 step(i64 a, i64 c, i64 m){
3     return ((__int128)a * a + c) % m;
4 }
5 i64 multi(i64 a, i64 b, i64 m){
6     return ((__int128)a * b % m;
7 }
8 i64 power(i64 a, i64 b, i64 m){
9     i64 r = 1;
10    while(b){
11        if(b & 1) r = multi(r, a, m);
12        b >>= 1, a = multi(a, a, m);
13    }

```

```

14 return r;
15 }
16 mt19937_64 MT;
17 bool test(i64 n){
18     if(n < 3 || n % 2 == 0) return n == 2;
19     i64 u = n - 1, t = 0;
20     while(u % 2 == 0) u /= 2, t += 1;
21     int test_time = 20;
22     for(int i = 1; i ≤ test_time; ++ i){
23         i64 a = MT() % (n - 2) + 2;
24         i64 v = power(a, u, n);
25         if(v == 1) continue;
26         int s;
27         for(s = 0; s < t; ++ s){
28             if(v == n - 1) break;
29             v = multi(v, v, n);
30         }
31         if(s == t) return false;
32     }
33     return true;
34 }
35 basic_string<i64> rho(i64 n){
36     if(n == 1) return { };
37     if(test(n)) return {n};
38     i64 a = MT() % (n - 1) + 1;
39     i64 x1 = MT() % (n - 1), x2 = x1;
40     for(int i = 1; i <= 1){
41         i64 tot = 1;
42         for(int j = 1; j ≤ i; ++ j){
43             x2 = step(x2, a, n);
44             tot = multi(tot, llabs(x1 - x2), n);
45             if(j % 127 == 0){
46                 i64 d = __gcd(tot, n);
47                 if(d > 1)
48                     return rho(d) + rho(n / d);
49             }
50         }
51         i64 d = __gcd(tot, n);
52         if(d > 1)
53             return rho(d) + rho(n / d);
54         x1 = x2;
55     }
56 }

```

## 6.16 polya 定理

### 6.16.1 Burnside 引理

记所有染色方案的集合为  $X$ , 其中单个染色方案为  $x_0$ . 一种对称操作  $g \in X$  作用于染色方案  $x \in X$  上可以得到另外一种染色  $x'$ .

将所有对称操作作为集合  $G$ , 那么  $Gx = \{gx \mid g \in G\}$  是与  $x$  本质相同的染色方案的集合, 形式化地称为  $x$  的轨道. 统计本质不同染色方案数, 就是统计不同轨道个数.

Burnside 引理说明如下:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

其中  $X^g$  表示在  $g \in G$  的作用下, 不动点的集合. 不动点被定义为  $x = gx$  的  $x$ .

### 6.16.2 Polya 定理

对于通常的染色问题,  $X$  可以看作一个长度为  $n$  的序列, 每个元素是 1 到  $m$  的整数. 可以将  $n$  看作面数、 $m$  看作颜色数. Polya 定理叙述如下:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} \sum_{m \in G} m^{c(g)}$$

其中  $c(g)$  表示对一个序列做轮换操作  $g$  可以分解成多少个置换环。

然而, 增加了限制 (比如要求某种颜色必须要多少个), 就无法直接应用 Polya 定理, 需要利用 Burnside 引理进行具体问题具体分析。

### 6.16.3 应用

给定  $n$  个点  $n$  条边的环, 现在有  $n$  种颜色, 给每个顶点染色, 询问有多少种本质不同的染色方案。

显然  $X$  是全体元素在 1 到  $n$  之间长度为  $n$  的序列,  $G$  是所有可能的单次旋转方案, 共有  $n$  种, 第  $i$  种方案会把 1 置换到  $i$ . 于是:

$$\begin{aligned}
 \text{ans} &= \frac{1}{|G|} \sum_{i=1}^n m^{c(g_i)} \\
 &= \frac{1}{n} \sum_{i=1}^n n^{\text{gcd}(i, n)} \\
 &= \frac{1}{n} \sum_{d|n} n^d \sum_{i=1}^n [\text{gcd}(i, n) = d] \\
 &= \frac{1}{n} \sum_{d|n} n^d \varphi(n/d)
 \end{aligned}$$

```

1 #include "../header.cpp"
2 vector <tuple<int, int> > P;
3 void solve(int step, int n, int d, int f, int
    &ans){
4     if(step == P.size()){
5         ans = (ans + 1ll * power(n, n / d) * f) %
            MOD;
6     } else {
7         auto [w, c] = P[step];
8         int dd = 1, ff = 1;
9         for(int i = 0; i ≤ c; ++ i){
10             solve(step + 1, n, d * dd, f * ff, ans);
11             ff = ff * (w - (i == 0));
12             dd = dd * w;
13         }
14     }
15 }
16 int main(){
17     int T; cin >> T;
18     while(T --){
19         int n, t;
20         cin >> n;
21         t = n;
22         for(int i = 2; i * i ≤ n; ++ i) if(n % i ==
            0){
23             int w = i, c = 0;
24             while(t % i == 0){
25                 t /= i, c ++;
26             }
27             P.push_back({ w, c });
28         }
29         if(t != 1){
30             P.push_back({ t, 1 });
31         }
32         int ans = 0;
33         solve(0, n, 1, 1, ans);
34         ans = 1ll * ans * power(n, MOD - 2) % MOD;
35         cout << ans << endl;
36         P.clear();
37     }
38     return 0;
39 }

```

## 6.17 min25 筛

设有一个积性函数  $f(n)$ , 满足  $f(p^k)$  可以快速求, 考虑搞一个在质数位置和  $f(n)$  相等的  $g(n)$ , 满足它有完全积性, 并且单点和前缀和都可以快速求, 然后通过第一部分筛出  $g$  在质数位置的前缀和, 从而相当于得到  $f$  在质数

位置的前缀和, 然后利用它, 做第二部分, 求出  $f$  的前缀和。

第一部分:  $G_k(n) = \sum_{i=1}^n [\text{mindiv}(i) > p_k \text{ or isprime}(i)]g(i)$  ( $p_0 = 1$ ), 则有  $G_k(n) = G_{k-1}(n) - g(p_k)(G_{k-1}(n/p_k) - G_{k-1}(p_{k-1}))$ , 复杂度  $O(n^{3/4}/\log n)$ 。

第二部分:  $F_k(n) = \sum_{i=1}^n [\text{mindiv}(i) \geq p_k]f(i)$ ,  $F_k(n) = \sum_{\substack{h \geq k \\ p_h^2 \leq n}} \sum_{\substack{c \geq 1 \\ p_h^{c+1} \leq n}} (f(p_h^c)F_{h+1}(n/p_h^c) + f(p_h^{c+1})) + F_{\text{prime}}(n) - F_{\text{prime}}(p_{k-1})$ , 在  $n \leq 10^{13}$  可以证明复杂度  $O(n^{3/4}/\log n)$ 。

常见细节问题:

- 由于  $n$  通常是  $10^{10}$  到  $10^{11}$  的数, 导致  $n$  会爆 int,  $n^2$  会爆 long long, 而且往往会用自然数幂和, 更容易爆, 所以要小心。
- 记  $s = \lfloor \sqrt{n} \rfloor$ , 由于  $F$  递归时会去找  $F_{h+1}$ , 会访问到  $s$  以内最大的质数往后的一个质数, 而已经证明对于所有  $n \in \mathbb{N}^+$ ,  $[n+1, 2n]$  中有至少一个质数, 所以只需要筛到  $2s$  即可。
- 注意补回  $f(1)$ 。

```

22         v = n / l, r = n / v;
23         if (v ≤ sqrt_n) id1[v] = ++cnt;
24         else id2[init_n / v] = ++cnt;
25         val.emplace_back(v);
26     }
27 }
28 ll id(ll n) {
29     if (n ≤ sqrt_n) return id1[n];
30     else return id2[init_n / n];
31 }
32 }
33 using namespace init;
34 // 计算 $G_k$, 两个参数分别是 $g$ 从 $2$ 开始
   的前缀和和 $g$
35 auto calcG = [&] (auto&& sum, auto&& g) →
   vector<ll> {
36     vector<ll> G(cnt + 1);
37     for (int i = 1; i ≤ cnt; ++i) G[i] = sum(
   val[i]);
38     ll pre = 0;
39     for (int i = 1; p[i] * p[i] ≤ n; ++i) {
40         for (int j = 1; j ≤ cnt; ++j) {
41             if (p[i] * p[j] > val[j]) break;
42             ll tmp = id(val[j] / p[i]);
43             G[j] = (G[j] - g(p[i]) * (G[tmp] -
   pre)) % MD;
44         }
45         pre = (pre + g(p[i])) % MD;
46     }
47     for (int i = 1; i ≤ cnt; ++i) G[i] = (G[i]
   % MD + MD) % MD;
48     return G;
49 };
50 // 计算 $F_k$, 直接搜, 不用记忆化。`fp` 是 $F_{\text{prime}}$, `pc` 是 $p^c$, 其中 `f(p[h]^c)` 要替换掉。
51 function<ll(ll, int)> calcF = [&] (ll m, int k) {
52     if (p[k] > m) return 0;
53     ll ans = (fp[id(m)] - fp[id(p[k - 1])]) %
   MD;
54     for (int h = k; p[h] * p[h] ≤ m; ++h) {
55         ll pc = p[h], c = 1;
56         while (pc * p[h] ≤ m) {
57             ans = (ans + calcF(m / pc, h + 1)
   * f(p[h] ^ c)) % MD;
58             ++c, pc = pc * p[h], ans = (ans +
   f(p[h] ^ c)) % MD;
59         }
60     }
61     return ans;
62 };

```

## 6.18 杜教筛

### 6.18.1 用法

对于积性函数  $f$ , 找到易求前缀和的积性函数  $g, h$  使得  $h = f * g$ , 根据递推式计算  $S(n) = \sum_{i=1}^n f(i)$ :

$$S(n) = H(n) - \sum_{d=1}^n g(d) \times S\left(\left\lfloor \frac{n}{d} \right\rfloor\right)$$

### 6.18.2 例题

- 对于  $f = \varphi$ , 寻找  $g = 1, h = \text{id}$ ;
- 对于  $f = \mu$ , 寻找  $g = 1, h = \varepsilon$ 。

```

1 #include "../header.cpp"
2 const int H = 1e7;
3 int P[MAXN], p; bool V[MAXN];
4 i64 ph[MAXN], sph[MAXN];
5 i64 mu[MAXN], smu[MAXN];
6 i64 tp[MAXN];
7 i64 solve_ph(i64 N){
8     for(int d = N / H; d ≥ 1; -- d){
9         i64 n = N / d;
10        i64 wh = 1ll * n * (n + 1) / 2;
11        tp[d] = wh;
12        for(i64 l = 2, r; l ≤ n; l = r + 1){
13            r = n / (n / l);
14            i64 wg = r - l + 1;
15            i64 ws = n / l ≤ H ? sph[n / l] : tp[N
   / (n / l)];
16            tp[d] -= wg * ws;
17        }
18    }
19    return N ≤ H ? sph[N] : tp[1];
20 }
21 i64 solve_mu(i64 N){
22     for(int d = N / H; d ≥ 1; -- d){
23         i64 n = N / d;
24         i64 wh = 1;
25         tp[d] = wh;
26         for(i64 l = 2, r; l ≤ n; l = r + 1){
27             r = n / (n / l);
28             i64 wg = r - l + 1;
29             i64 ws = n / l ≤ H ? smu[n / l] : tp[N
   / (n / l)];
30             tp[d] -= wg * ws;
31         }
32     }
33     return N ≤ H ? smu[N] : tp[1];
34 }
35 int main(){

```

```

1 // 预处理, $1$ 所在的块也算进去了
2 namespace init {
3     ll init_n, sqrt_n;
4     vector<ll> np, p, id1, id2, val;
5     ll cnt;
6     void main(ll n) {
7         init_n = n, sqrt_n = sqrt(n);
8         ll M = sqrt_n * 2; // 筛出一个 > floor
   (sqrt(n)) 的质数, 避免后续讨论边界
9         np.resize(M + 1), p.resize(M + 1);
10        for (ll i = 2; i ≤ M; ++i) {
11            if (!np[i]) p[++p[0]] = i;
12            for (ll j = 1; j ≤ p[0]; ++j) {
13                if (i * p[j] > M) break;
14                np[i * p[j]] = 1;
15                if (i % p[j] == 0) break;
16            }
17        }
18        p[0] = 1;
19        id1.resize(sqrt_n + 1), id2.resize(
   sqrt_n + 1);
20        val.resize(1);
21        for (ll l = 1, r, v; l ≤ n; l = r +
   1) {

```

```

36 ios :: sync_with_stdio(false);
37 cin.tie(nullptr);
38 ph[1] = 1;
39 mu[1] = 1;
40 for(int i = 2; i ≤ H; ++ i){
41     if(!V[i]){
42         P[++ p] = i;
43         ph[i] = i - 1;
44         mu[i] = -1;
45     }
46     for(int j = 1; j ≤ p && P[j] ≤ H / i; ++ j)
47     {
48         int &p = P[j];
49         V[i * p] = true;
50         if(i % p == 0){
51             ph[i * p] = ph[i] * p;
52             mu[i * p] = 0;
53             break;
54         } else {
55             ph[i * p] = ph[i] * (p - 1);
56             mu[i * p] = -mu[i];
57         }
58     }
59     for(int i = 1; i ≤ H; ++ i){
60         sph[i] = sph[i - 1] + ph[i];
61         smu[i] = smu[i - 1] + mu[i];
62     }
63     int T; cin >> T;
64     while(T --> 0){
65         int n; cin >> n;
66         cout << solve_ph(n) << " " << solve_mu(n)
67         << "\n";
68     }
69     return 0;

```

## 6.19 PN 筛

### 6.19.1 用法

对于积性函数  $f(x)$ , 寻找积性函数  $g(x)$  使得  $g(p) = f(p)$ , 且  $g$  易求前缀和  $G$ .

令  $h = f * g^{-1}$ , 可以证明只有 PN 处  $h$  的函数值非 0, PN 指每个素因子幂次都不小于 2 的数。同时可以证明  $n$  以内的 PN 只有  $\mathcal{O}(\sqrt{n})$  个, 且可以暴力枚举质因子幂次得到所有 PN。

可利用下面公式计算  $h(p^c)$ :

$$h(p^c) = f(p^c) - \sum_{i=1}^c g(p^i) \times h(p^{c-i})$$

### 6.19.2 例题

定义积性函数  $f(x)$  满足  $f(p^k) = p^k(p^k - 1)$ , 计算  $\sum f(i)$ 。

取  $g(p) = \text{id}(p)\varphi(p) = f(p)$ , 根据  $g * \text{id} = \text{id}_2$  利用杜教筛求解。 $h(p^c)$  的值利用递推式进行计算。

```

1 #include "../header.cpp"
2 const int H = 1e7;
3 const int MOD = 1e9 + 7;
4 const int DIV2 = 500000004;
5 const int DIV6 = 166666666;
6 int P[MAXN], p; bool V[MAXN];
7 int g[MAXN], le[MAXN], ge[MAXN];
8 int s1(i64 n){ // 1^1 + 2^1 + ... + n^1
9     n %= MOD;
10    return 1ll * n * (n + 1) % MOD * DIV2 % MOD;
11 }
12 int s2(i64 n){ // 1^2 + 2^2 + ... + n^2
13     n %= MOD;
14    return 1ll * n * (n + 1) % MOD * (2 * n + 1)
15    % MOD * DIV6 % MOD;
16 }
17 int sg(i64 n, i64 N){
18     return n ≤ H ? le[n] : ge[N / n];
19 }
20 int sieve_du(i64 N){
21     for(int d = N / H; d ≥ 1; -- d){
22         i64 n = N / d;
23         int wh = s2(n);
24         for(i64 l = 2, r; l ≤ n; l = r + 1){
25             r = n / (n / l);
26             int wg = (s1(r) - s1(l - 1) + MOD) % MOD;
27             int ws = sg(n / l, N);
28             ge[d] = (ge[d] + 1ll * wg * ws) % MOD;
29         }
30         ge[d] = (wh - ge[d] + MOD) % MOD;
31     }
32     return N ≤ H ? le[N] : ge[1];
33 }
34 vector<int> hc[MAXM], gc[MAXM];
35 int ANS;
36 void sieve_pn(int last, i64 x, int h, i64 N){
37     ANS = (ANS + 1ll * h * sg(N / x, N)) % MOD;

```

```

37 for(i64 i = last + 1; x ≤ N / P[i] / P[i]; ++ i){
38     int c = 2;
39     for(i64 t = x * P[i] * P[i]; t ≤ N; t *= P[i], c++){
40         int hh = 1ll * h * hc[i][c] % MOD;
41         sieve_pn(i, t, hh, N);
42     }
43 }
44 }
45 int main(){
46     ios :: sync_with_stdio(false);
47     cin.tie(nullptr);
48     g[1] = 1;
49     for(int i = 2; i ≤ H; ++ i){
50         if(!V[i]){
51             P[++ p] = i, g[i] = 1ll * i * (i - 1) % MOD;
52         }
53         for(int j = 1; j ≤ p && P[j] ≤ H / i; ++ j)
54         {
55             int &p = P[j];
56             V[i * p] = true;
57             if(i % p == 0){
58                 g[i * p] = 1ll * g[i] * p % MOD * p % MOD;
59                 break;
60             } else {
61                 g[i * p] = 1ll * g[i] * p % MOD * (p - 1) % MOD;
62             }
63         }
64     }
65     for(int i = 1; i ≤ H; ++ i){
66         le[i] = (le[i - 1] + g[i]) % MOD;
67     }
68     i64 N;
69     cin >> N;
70     for(int i = 1; i ≤ p && 1ll * P[i] * P[i] ≤ N; i++){
71         int &p = P[i];
72         hc[i].push_back(1);
73         gc[i].push_back(1);
74         for(i64 c = 1, t = p; t ≤ N; t = t * p, ++ c){
75             if(c == 1){
76                 gc[i].push_back(1ll * p * (p - 1) % MOD);
77             } else {
78                 gc[i].push_back(1ll * gc[i].back() * p % MOD * p % MOD);

```

```

1 #include "../header.cpp"
2 int inv(int x);
3 const int MAX_ = (1 << 19) + 3;
4 using cplx = complex<double>;
5 const long double pi = acos(-1);
6 namespace Poly{
7     void FFT(int n, cplx Z[]){
8         static int W[MAX_];
9         int l = 1; W[0] = 0;
10        while (n >= 1)
11            up(0, l - 1, i)
12            W[l++] = W[i] << 1 | 1, W[i] <= 1;
13        up(0, l - 1, i)
14        if(W[i] > i) swap(Z[i], Z[W[i]]);
15        for (n = l >> 1, l = 1; n >= 1, l <= 1)
16        {
17            cplx* S = Z, o(cos(pi / l), sin(pi / l))
18            ;
19            up(0, n - 1, i){
20                cplx s(1, 0);
21                up(0, l - 1, j){
22                    cplx x = S[j] + s * S[j + l];

```



```

21     cplx y = S[j] - s * S[j + l];
22     S[j] = x, S[j + l] = y, s = s * o;
23 }
24 S += l << 1;
25 }
26 }
27 }
28 void IFFT(int n, cplx Z[]){
29     FFT(n, Z); reverse(Z + 1, Z + n);
30     up(0, n - 1, i) Z[i] /= n;
31 }
32 void NTT(int n, int Z[]){
33     static int W[MAX_];
34     int g = 3, l = 1; W[0] = 0;
35     while (n >= 1)
36         up(0, l - 1, i)
37         W[l++] = W[i] << 1 | 1, W[i] <= 1;
38     up(0, l - 1, i)
39     if (W[i] > i) swap(Z[i], Z[W[i]]);
40     for (n = l >> 1, l = 1; n >= 1, l <= 1)
41     {
42         int* S = Z, o = power(g, (MOD - 1) / l /
43             2);
44         up(0, n - 1, i){
45             int s = 1;
46             up(0, l - 1, j){
47                 int x = (S[j] + 1ll * s * S[j + l] %
48                     MOD) % MOD;
49                 int y = (S[j] - 1ll * s * S[j + l] %
50                     MOD + MOD) % MOD;
51                 S[j] = x, S[j + l] = y;
52                 s = 1ll * s * o % MOD;
53             }
54             S += l << 1;
55         }
56     }
57 }
58 void INTT(int n, int Z[]){
59     NTT(n, Z); reverse(Z + 1, Z + n);
60     int o = inv(n);
61     up(0, n - 1, i)
62     Z[i] = 1ll * Z[i] * o % MOD;
63 }
64 void MUL(int n, int A[], int B[]){ //
65     乘法
66     NTT(n, A), NTT(n, B);
67     up(0, n - 1, i)
68     A[i] = 1ll * A[i] * B[i] % MOD;
69     INTT(n, A);
70 }
71 void INV(int n, int Z[], int T[]){ //
72     乘法逆

```

```

67     static int A[MAX_];
68     up(0, n - 1, i)
69     T[i] = 0;
70     T[0] = power(Z[0], MOD - 2);
71     for (int l = 1; l < n; l <= 1){
72         up(0, 2 * l - 1, i) A[i] = Z[i];
73         up(2 * l, 4 * l - 1, i) A[i] = 0;
74         NTT(4 * l, A), NTT(4 * l, T);
75         up(0, 4 * l - 1, i)
76         T[i] = (2ll * T[i] - 1ll * A[i] * T[i]
77             % MOD * T[i] % MOD + MOD) % MOD;
78         INTT(4 * l, T);
79         up(2 * l, 4 * l - 1, i)
80         T[i] = 0;
81     }
82 }
83 void DIF(int n, int Z[], int T[]){ //
84     微分
85     up(0, n - 2, i)
86     T[i] = 1ll * Z[i + 1] * (i + 1) % MOD;
87     T[n - 1] = 0;
88 }
89 void INT(int n, int c, int Z[], int T[]){
90     // 积分
91     up(1, n - 1, i)
92     T[i] = 1ll * Z[i - 1] * inv(i) % MOD;
93     T[0] = c;
94 }
95 void LN(int n, int* Z, int* T){ // 求
96     对数
97     static int A[MAX_], B[MAX_];
98     up(0, 2 * n - 1, i)
99     A[i] = B[i] = 0;
100     DIF(n, Z, A), INV(n, Z, B), MUL(2 * n, A,
101     B), INT(n, 0, A, T);
102 }
103 void EXP(int n, int* Z, int* T){ // 求
104     指数
105     static int A[MAX_], B[MAX_];
106     up(1, 2 * n - 1, i) T[i] = 0;
107     T[0] = 1;
108     for (int l = 1; l < n; l <= 1){
109         LN(2 * l, T, A);
110         up(0, 2 * l - 1, i)
111         B[i] = (-A[i] + Z[i] + MOD) % MOD;
112         B[0] = (B[0] + 1) % MOD;
113         up(2 * l, 4 * l - 1, i)
114         T[i] = B[i] = 0;
115         MUL(4 * l, T, B);
116     }
117 }
118 void SQT(int n, int* Z, int* T){ // 开

```

```

113     根
114     static int A[MAX_], B[MAX_];
115     up(1, 2 * n - 1, i) T[i] = 0;
116     T[0] = 1;
117     int o = inv(2);
118     for (int l = 1; l < n; l <= 1){
119         INV(2 * l, T, A);
120         up(0, 2 * l - 1, i)
121         B[i] = Z[i];
122         up(2 * l, 4 * l - 1, i)
123         A[i] = B[i] = 0;
124         MUL(4 * l, A, B);
125         up(0, 2 * l - 1, i)
126         T[i] = 1ll * (T[i] + A[i]) * o % MOD;
127     }
128 }
129 void SHF(int n, int c, int* Z, int* T){ //
130     平移
131     static int A[MAX_], B[MAX_], F[MAX_], G[
132     MAX_];
133     int o = 1;
134     up(1, n - 1, i)
135     F[i] = 1ll * F[i - 1] * i % MOD,
136     G[i] = 1ll * G[i - 1] * inv(i) % MOD;
137     up(0, n - 1, i)
138     A[i] = 1ll * Z[n - 1 - i] * F[n - 1 - i]
139     % MOD;
140     up(0, n - 1, i){
141         B[i] = 1ll * G[i] * o % MOD;
142         o = 1ll * o * c % MOD;
143     }
144     int l = 1; while (l < 2 * n - 1) l <= 1;
145     up(n, l - 1, i)
146     A[i] = B[i] = 0;
147     MUL(l, A, B);
148     up(0, n - 1, i)
149     T[n - 1 - i] = 1ll * G[n - 1 - i] * A[i]
150     % MOD;
151 }
152 }

```

## 7.2 FWT 全家桶

### 7.2.1 用法

沃尔什全家桶。

包含与卷积、或卷积、异或卷积，定义分别为二进制与、或、异或带下式：

$$b_k = \sum_{i \otimes j = k} a_i \times b_j$$

```

1 #include "../header.cpp"
2 namespace Solve1{ // or 卷积
3     void FWT(int n, int *A){
4         for(int l = 1 << n, u = 2, v = 1; u ≤ l; u
5             <=<= 1, v <=<= 1)
6             for(int j = 0; j < l; j += u)
7                 for(int k = 0; k < v; ++ k)
8                     A[j + v + k] = (A[j + v + k] + A[j +
9                         k]) % MOD;
10    }
11    void IFWT(int n, int *A){
12        for(int l = 1 << n, u = l, v = l / 2; u >
13            1; u >>= 1, v >>= 1)
14            for(int j = 0; j < l; j += u)
15                for(int k = 0; k < v; ++ k)
16                    A[j + v + k] = (A[j + v + k] - A[j +
17                        k] + MOD) % MOD;
18    }
19 }
20 namespace Solve2{ // and 卷积
21     void FWT(int n, int *A){
22         for(int l = 1 << n, u = 2, v = 1; u ≤ l; u
23             <=<= 1, v <=<= 1)
24             for(int j = 0; j < l; j += u)
25                 for(int k = 0; k < v; ++ k)
26                     A[j + k] = (A[j + k] + A[j + v + k])
27                         % MOD;
28    }
29    void IFWT(int n, int *A){
30        for(int l = 1 << n, u = l, v = l / 2; u >
31            1; u >>= 1, v >>= 1)
32            for(int j = 0; j < l; j += u)
33                for(int k = 0; k < v; ++ k){
34                    int a = A[j + k];
35                    int b = A[j + v + k];
36                    A[j + k] = (a + b + MOD) % MOD;
37                    A[j + v + k] = (a - b + MOD) % MOD;
38                }
39    }
40 }
41 namespace Solve3{ // xor 卷积
42     void FWT(int n, int *A){
43         for(int l = 1 << n, u = 2, v = 1; u ≤ l; u
44             <=<= 1, v <=<= 1)
45             for(int j = 0; j < l; j += u)
46                 for(int k = 0; k < v; ++ k){
47                     int a = A[j + k];
48                     int b = A[j + v + k];
49                     A[j + k] = (a + b + MOD) % MOD;
50                     A[j + v + k] = (a - b + MOD) % MOD;
51                 }
52    }
53    void IFWT(int n, int *A){
54        int div2 = (MOD + 1) / 2;
55        for(int l = 1 << n, u = l, v = l / 2; u >
56            1; u >>= 1, v >>= 1)
57            for(int j = 0; j < l; j += u)
58                for(int k = 0; k < v; ++ k)
59                    A[j + v + k] = (A[j + v + k] + A[j +
60                        k] * div2 % MOD + MOD) % MOD;
61    }
62 }

```

```

43     for(int l = 1 << n, u = l, v = l / 2; u >
44         1; u >>= 1, v >>= 1)
45         for(int j = 0; j < l; j += u)
46             for(int k = 0; k < v; ++ k){
47                 int a = A[j + k];
48                 int b = A[j + v + k];
49                 A[j + k] = 1ll * (a + b + MOD) *
50                     div2 % MOD;
51                 A[j + v + k] = 1ll * (a - b + MOD) *
52                     div2 % MOD;
53             }
54    }
55 }

```

### 7.3 任意模数 NTT

```

1 #include "poly-family.cpp"
2 const int BLOCK = 32768;
3 using cplx = complex<double>;
4 cplx A1[MAXN], A2[MAXN], B1[MAXN], B2[MAXN];
5 int n, m, L, mod;
6 cplx P[MAXN], Q[MAXN];
7 void FFTFFT(int L, cplx X[], cplx Y[]){
8     for(int i = 0; i < L; ++ i){
9         P[i] = { X[i].real(), Y[i].imag() };
10    }
11    Poly :: FFT(L, P);
12    for(int i = 0; i < L; ++ i){
13        Q[i] = (i == 0 ? P[0] : P[L - i]);
14        Q[i].imag(-Q[i].imag());
15    }
16    for(int i = 0; i < L; ++ i){
17        X[i] = (P[i] + Q[i]);
18        Y[i] = (Q[i] - P[i]) * cplx(0, 1);
19        X[i] /= 2, Y[i] /= 2;
20    }
21 }
22 int main(){
23     ios :: sync_with_stdio(false);
24     cin.tie(nullptr);
25     cin >> n >> m >> mod;
26     for(int i = 0; i ≤ n; ++ i){
27         int a; cin >> a; a %= mod;
28         A1[i].real(a / BLOCK);
29         A2[i].imag(a % BLOCK);
30     }
31     for(int i = 0; i ≤ m; ++ i){
32         int a; cin >> a; a %= mod;
33         B1[i].real(a / BLOCK);
34         B2[i].imag(a % BLOCK);
35     }
36     for(L = 1; L ≤ n + m; L <=<= 1);

```

```

37     FFTFFT(L, A1, A2), FFTFFT(L, B1, B2);
38     for(int i = 0; i < L; ++ i){
39         P[i] = A1[i] * B1[i] + cplx(0, 1) * A2[i]
40             * B1[i];
41         Q[i] = A1[i] * B2[i] + cplx(0, 1) * A2[i]
42             * B2[i];
43     }
44     Poly :: IFFT(L, P);
45     Poly :: IFFT(L, Q);
46     for(int i = 0; i < L; ++ i){
47         long long a1b1 = P[i].real() + 0.5;
48         long long a2b1 = P[i].imag() + 0.5;
49         long long a1b2 = Q[i].real() + 0.5;
50         long long a2b2 = Q[i].imag() + 0.5;
51         long long w = ((a1b1 % mod * (BLOCK *
52             BLOCK % mod)) + ((a2b1 + a1b2) % mod) *
53             BLOCK + a2b2) % mod;
54         if(i ≤ n + m) cout << w << " ";
55     }
56     return 0;
57 }

```

## 8 字符串

### 8.1 AC 自动机

```

1 #include "../header.cpp"
2 namespace ACAM{
3     int C[MAXN][MAXM], F[MAXN], o;
4     void insert(char *S){
5         int p = 0, len = 0;
6         for(int i = 0; S[i]; ++ i){
7             int e = S[i] - 'a';
8             if(C[p][e]) p = C[p][e];
9             else p = C[p][e] = ++ o;
10            ++ len;
11        }
12    }
13    void build(){
14        queue<int> Q; Q.push(0);
15        while(!Q.empty()){
16            int u = Q.front(); Q.pop();
17            for(int i = 0; i < 26; ++ i){
18                int v = C[u][i];
19                if(v == 0) continue;
20                int p = F[u];
21                while(!C[p][i] && p ≠ 0) p = F[p];
22                if(C[p][i] && C[p][i] ≠ v)
23                    F[v] = C[p][i];
24                Q.push(v);
25            }
26        }
27    }

```

```

26 }
27 }
28 }

```

## 8.2 扩展 KMP

### 8.2.1 定义

$$z_i^{(1)} = |\text{lcp}(b, \text{suffix}(b, i))|$$

$$z_i^{(2)} = |\text{lcp}(b, \text{suffix}(a, i))|$$

```

1 #include "../header.cpp"
2 char A[MAXN], B[MAXN * 2];
3 int n, m, l, r, Z[MAXN * 2];
4 i64 ans1, ans2;
5 int main(){
6     scanf("%s%s", A + 1, B + 1);
7     n = strlen(A + 1);
8     m = strlen(B + 1);
9     l = 0, r = 0; Z[1] = 0, ans1 = m + 1;
10    for(int i = 2; i ≤ m; ++ i){
11        if(i ≤ r) Z[i] = min(r - i + 1, Z[i - l + 1]);
12        else Z[i] = 0;
13        while(B[Z[i] + 1] == B[i + Z[i]])
14            ++ Z[i];
15        if(i + Z[i] - 1 > r)
16            r = i + Z[i] - 1, l = i;
17        ans1 ^= 1ll * i * (Z[i] + 1);
18    }
19    l = 0, r = 0;
20    Z[1] = 0, B[m + 1] = '#', strcat(B + 1, A + 1);
21    for(int i = 2; i ≤ n + m + 1; ++ i){
22        if(i ≤ r) Z[i] = min(r - i + 1, Z[i - l + 1]);
23        else Z[i] = 0;
24        while(B[Z[i] + 1] == B[i + Z[i]])
25            ++ Z[i];
26        if(i + Z[i] - 1 > r)
27            r = i + Z[i] - 1, l = i;
28    }
29    for(int i = m + 2; i ≤ n + m + 1; ++ i){
30        ans2 ^= 1ll * (i - m - 1) * (Z[i] + 1);
31    }
32    printf("%lld\n%lld\n", ans1, ans2);
33    return 0;
34 }

```

## 8.3 Manacher

```

1 #include "../header.cpp"
2 const int MAXN = 2.2e7 + 11;
3 char S[MAXN], T[MAXN]; int n, R[MAXN];
4 int main(){
5     scanf("%s", S + 1);
6     n = strlen(S + 1);
7     for(int i = 1; i ≤ n; ++ i){
8         T[2 * i - 1] = S[i], T[2 * i] = '#';
9     }
10    T[0] = '#', n = 2 * n;
11    int p = 0, x = 0, ans = 0;
12    for(int i = 1; i ≤ n; ++ i){
13        if(i ≤ p) R[i] = min(R[2 * x - i], p - i);
14        while(i - R[i] - 1 ≥ 0 && T[i + R[i] + 1] == T[i - R[i] - 1])
15            ++ R[i];
16        if(i + R[i] > p){
17            p = i + R[i];
18            x = i;
19        }
20        ans = max(ans, R[i]);
21    }
22    printf("%d\n", ans);
23    return 0;
24 }

```

## 8.4 回文自动机

```

1 #include "../header.cpp"
2 namespace PAM{
3     const int SIZ = 5e5 + 3;
4     int n, s, F[SIZ], L[SIZ], D[SIZ];
5     int M[SIZ][MAXM];
6     char S[SIZ];
7     void init(){
8         S[0] = '$', n = 1;
9         F[s = 0] = -1, L[0] = -1, D[0] = 0;
10        F[s = 1] = 0, L[1] = 0, D[1] = 0;
11    }
12    void extend(int &last, char c){
13        S[++ n] = c;
14        int e = c - 'a', a = last;
15        while(c ≠ S[n - 1 - L[a]]) a = F[a];
16        if(M[a][e]){
17            last = M[a][e];
18        } else {
19            int cur = M[a][e] = ++ s;
20            L[cur] = L[a] + 2;
21            if(a == 0){

```

```

22        F[cur] = 1;
23        } else {
24            int b = F[a];
25            while(c ≠ S[n - 1 - L[b]])
26                b = F[b];
27            F[cur] = M[b][e];
28        }
29        D[cur] = D[F[cur]] + 1;
30        last = cur;
31    }
32 }
33 }

```

## 8.5 后缀平衡树

### 8.5.1 本代码尚未完成

## 8.6 后缀数组 (倍增)

```

1 #include "../header.cpp"
2 int n, m, A[MAXN], B[MAXN];
3 int C[MAXN], R[MAXN], P[MAXN], Q[MAXN];
4 char S[MAXN];
5 int main(){
6     scanf("%s", S), n = strlen(S), m = 256;
7     for(int i = 0; i < n; ++ i) R[i] = S[i];
8     for (int k = 1; k ≤ n; k <= 1){
9         for(int i = 0; i < n; ++ i){
10            Q[i] = ((i + k > n - 1) ? 0 : R[i + k]);
11            P[i] = R[i];
12            m = max(m, R[i]);
13        }
14        #define fun(a, b, c) \
15            memset(C, 0, sizeof(int) * (m + 1));
16        for(int i = 0; i < n; ++ i) C[a] += 1;
17        for(int i = 1; i ≤ m; ++ i) C[i] += C[i - 1];
18        for(int i = n - 1; i ≥ 0; -- i) c[-- C[a]] = b;
19        fun(Q[i], i, B)
20        fun(P[B[i]], B[i], A)
21        #undef fun
22        int p = 1; R[A[0]] = 1;
23        for(int i = 1; i ≤ n - 1; ++ i){
24            bool f1 = P[A[i]] == P[A[i - 1]];
25            bool f2 = Q[A[i]] == Q[A[i - 1]];
26            R[A[i]] = f1 && f2 ? R[A[i - 1]] : ++ p;
27        }
28        if (m == n) break;
29    }

```

```

30 for(int i = 0; i < n; ++ i)
31     printf("%u ", A[i] + 1);
32 return 0;
33 }

```

## 8.7 后缀数组 (SAIS)

```

1 #include "../header.cpp"
2 #define LTYPE 0
3 #define STYPE 1
4 void induce_sort(int n, int S[], int T[], int
5 m, int LM[], int SA[], int C[]){
6     vector<int> BL(n), BS(n), BM(n);
7     fill(SA, SA + n, -1);
8     for(int i = 0; i < n; ++ i){ // 预处理
9         BM[i] = BS[i] = C[i] - 1;
10        BL[i] = i == 0 ? 0 : C[i - 1];
11    }
12    for(int i = m - 1; i ≥ 0; -- i) // 放置
13        LMS 后缀
14        SA[BM[S[LM[i]]] --] = LM[i];
15    for(int i = 0, p; i < n; ++ i) // 计算 L
16        类型后缀的位置
17        if(SA[i] > 0 && T[p = SA[i] - 1] == LTYPE)
18            SA[BL[S[p]] ++] = p;
19    for(int i = n - 1, p; i ≥ 0; -- i) // 计算 S
20        类型后缀的位置
21        if(SA[i] > 0 && T[p = SA[i] - 1] == STYPE)
22            SA[BS[S[p]] --] = p;
23    // 长度 n, 字符集 [0, n), 要求最后一个元素为 0
24    // 例如输入 ababa 传入 n = 6, S = [1 2 1 2 1
25    // 0]
26    void sais(int n, int S[], int SA[]){
27        vector<int> T(n), C(n), I(n, -1);
28        T[n - 1] = STYPE;
29        for(int i = n - 2; i ≥ 0; -- i){ // 递推类
30            型
31            T[i] = S[i] == S[i + 1] ? T[i + 1] : (S[i]
32            < S[i + 1] ? STYPE : LTYPE);
33        }
34        for(int i = 0; i < n; ++ i) // 统计个数
35            C[S[i]] ++;
36        for(int i = 1; i < n; ++ i) // 前缀累加
37            C[i] += C[i - 1];
38        vector<int> P;
39        for(int i = 0; i < n; ++ i){ // 统计 LMS 后
40            缀
41            if(T[i] == STYPE && (i == 0 || T[i - 1] ==
42            LTYPE)){
43                I[i] = P.size(), P.push_back(i);

```

```

36 }
37 }
38 int m = P.size(), tot = 0, cnt = 0;
39 induce_sort(n, S, T.data(), m, P.data(), SA,
40 C.data());
41 vector<int> S0(m), SA0(m);
42 for(int i = 0, x, y = -1; i < n; ++ i){
43     if((x = I[SA[i]]) ≠ -1){
44         if(tot == 0 || P[x + 1] - P[x] ≠ P[y +
45         1] - P[y])
46             tot ++;
47         else for(int p1 = P[x], p2 = P[y]; p2 ≤
48             P[y + 1]; ++ p1, ++ p2){
49             if((S[p1] << 1 | T[p1]) ≠ (S[p2] << 1
50             | T[p2])){
51                 tot ++; break;
52             }
53         }
54         S0[y = x] = tot - 1;
55     }
56 }
57 if(tot == m){
58     for(int i = 0; i < m; ++ i)
59         SA0[S0[i]] = i;
60 } else {
61     sais(m, S0.data(), SA0.data());
62 }
63 for(int i = 0; i < m; ++ i)
64     S0[i] = P[SA0[i]];
65 induce_sort(n, S, T.data(), m, S0.data(), SA
66 , C.data());
67 }
68 int S[MAXN], SA[MAXN], H[MAXM], G[MAXM];
69 int main(){
70     int n = 0, t = 0, m = 256;
71     for(char c = cin.get(); isgraph(c); c = cin.
72     get()){
73         S[n ++] = c;
74         H[c] ++;
75     }
76     for(int i = 0; i < m; ++ i){
77         t += !!H[i], G[i] = t;
78     }
79     for(int i = 0; i < n; ++ i){
80         S[i] = G[S[i]];
81     }
82     sais(n + 1, S, SA);
83     for(int i = 1; i ≤ n; ++ i){
84         cout << SA[i] + 1 << " ";
85     }
86     return 0;
87 }

```

## 8.8 广义后缀自动机 (离线)

```

1 #include "../header.cpp"
2 namespace SAM{
3     const int SIZ = 2e6 + 3;
4     int M[SIZ][MAXM];
5     int L[SIZ], F[SIZ], S[SIZ];
6     int s = 0, h = 25;
7     void init(){
8         F[0] = -1, s = 0;
9     }
10    void extend(int &last, char c){
11        int e = c - 'a';
12        int cur = ++ s;
13        L[cur] = L[last] + 1;
14        int p = last;
15        while(p ≠ -1 && !M[p][e])
16            M[p][e] = cur, p = F[p];
17        if(p == -1){
18            F[cur] = 0;
19        } else {
20            int q = M[p][e];
21            if(L[p] + 1 == L[q]){
22                F[cur] = q;
23            } else {
24                int clone = ++ s;
25                L[clone] = L[p] + 1;
26                F[clone] = F[q];
27                for(int i = 0; i ≤ h; ++ i)
28                    M[clone][i] = M[q][i];
29                while(p ≠ -1 && M[p][e] == q)
30                    M[p][e] = clone, p = F[p];
31                F[cur] = F[q] = clone;
32            }
33        }
34        last = cur;
35    }
36    void solve(){
37        i64 ans = 0;
38        for(int i = 1; i ≤ s; ++ i)
39            ans += L[i] - L[F[i]];
40        cout << ans << endl;
41    }
42 }
43 namespace Trie{
44     const int SIZ = 1e6 + 3;
45     int M[SIZ][MAXM], s, h = 25;
46     void insert(char *S){
47         int p = 0;
48         for(int i = 0; S[i]; ++ i){
49             int e = S[i] - 'a';
50             if(M[p][e]){
51                 p = M[p][e];

```

```

52     } else
53     p = M[p][e] = ++ s;
54 }
55 }
56 int O[SIZ];
57 void build_sam(){
58     queue <int> Q;
59     Q.push(0);
60     while(!Q.empty()){
61         int u = Q.front(); Q.pop();
62         for(int i = 0; i ≤ h; ++ i){
63             char c = i + 'a';
64             if(M[u][i]){
65                 int v = M[u][i];
66                 O[v] = O[u];
67                 SAM :: extend(O[v], c);
68                 Q.push(v);
69             }
70         }
71     }
72 }
73 }

```

## 8.9 广义后缀自动机 (在线)

```

1 #include "../header.cpp"
2 namespace SAM{
3     const int SIZ = 2e6 + 3;
4     int M[SIZ][MAXM];
5     int L[SIZ], F[SIZ], S[SIZ];
6     int s = 0, h = 25;
7     void init(){
8         F[0] = -1, s = 0;
9     }
10    void extend(int &last, char c){
11        int e = c - 'a';
12        if(M[last][e]){
13            int p = last;
14            int q = M[last][e];
15            if(L[q] == L[last] + 1){
16                last = q;
17            } else {
18                int clone = ++ s;
19                L[clone] = L[p] + 1;
20                F[clone] = F[q];
21                for(int i = 0; i ≤ h; ++ i)
22                    M[clone][i] = M[q][i];
23                while(p ≠ -1 && M[p][e] == q)
24                    M[p][e] = clone, p = F[p];
25                F[q] = clone;
26                last = clone;
27            }

```

```

28     } else {
29         int cur = ++ s;
30         L[cur] = L[last] + 1;
31         int p = last;
32         while(p ≠ -1 && !M[p][e])
33             M[p][e] = cur, p = F[p];
34         if(p == -1){
35             F[cur] = 0;
36         } else {
37             int q = M[p][e];
38             if(L[p] + 1 == L[q]){
39                 F[cur] = q;
40             } else {
41                 int clone = ++ s;
42                 L[clone] = L[p] + 1;
43                 F[clone] = F[q];
44                 for(int i = 0; i ≤ h; ++ i)
45                     M[clone][i] = M[q][i];
46                 while(p ≠ -1 && M[p][e] == q)
47                     M[p][e] = clone, p = F[p];
48                 F[cur] = F[q] = clone;
49             }
50         }
51         last = cur;
52     }
53 }
54 void solve(){
55     i64 ans = 0;
56     for(int i = 1; i ≤ s; ++ i)
57         ans += L[i] - L[F[i]];
58     cout << ans << endl;
59 }
60 }
61 // 每次插入新字符串前将 last 清零

```

## 8.10 后缀自动机

```

1 #include "../header.cpp"
2 namespace SAM{
3     const int SIZ = 2e6 + 3;
4     int M[SIZ][MAXM];
5     int L[SIZ], F[SIZ], S[SIZ];
6     int last = 0, s = 0, h = 25;
7     void init(){
8         F[0] = -1, last = s = 0;
9     }
10    void extend(char c){
11        int cur = ++ s, e = c - 'a';
12        L[cur] = L[last] + 1;
13        S[cur] = 1;
14        int p = last;
15        while(p ≠ -1 && !M[p][e])

```

```

16        M[p][e] = cur, p = F[p];
17        if(p == -1){
18            F[cur] = 0;
19        } else {
20            int q = M[p][e];
21            if(L[p] + 1 == L[q]){
22                F[cur] = q;
23            } else {
24                int clone = ++ s;
25                L[clone] = L[p] + 1;
26                F[clone] = F[q];
27                S[clone] = 0;
28                for(int i = 0; i ≤ h; ++ i)
29                    M[clone][i] = M[q][i];
30                while(p ≠ -1 && M[p][e] == q)
31                    M[p][e] = clone, p = F[p];
32                F[cur] = F[q] = clone;
33            }
34        }
35        last = cur;
36    }
37    vector <int> E[SIZ];
38    void build(){
39        for(int i = 1; i ≤ s; ++ i){
40            E[F[i]].push_back(i);
41        }
42    }
43    i64 ans = 0;
44    void dfs(int u){
45        for(auto &v : E[u]){
46            dfs(v), S[u] += S[v];
47        }
48        if(S[u] > 1)
49            ans = max(ans, 1ll * S[u] * L[u]);
50    }
51 }

```

# 9 计算几何

## 9.1 二维凸包

### 9.1.1 例题

给定  $n$  个点, 保证每三点不共线。要求找到一个简单多边形满足它不是凸包, 使得该多边形面积最大。

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using i64 = long long;
4 const int MAXN = 2e5 + 3;
5 int X[MAXN], Y[MAXN];

```



```

6 struct Frac {
7     int a, b;
8     Frac (int _a, int _b){
9         if(_b < 0){
10             a = -_a, b = -_b;
11         } else {
12             a = _a, b = _b;
13         }
14     }
15 };
16 struct Node {
17     int x, y;
18 }P[MAXN];
19 bool operator < (const Frac A, const Frac B){
20     return 1ll * A.a * B.b - 1ll * A.b * B.a < 0;
21 }
22 bool operator < (const Node A, const Node B){
23     return A.x == B.x ? A.y > B.y : A.x < B.x;
24 }
25 const Frac intersect(Node A, Node B){
26     int a = B.y - A.y;
27     int b = A.x - B.x;
28     assert(b != 0);
29     if(b < 0){
30         a = -a, b = -b;
31     }
32     return Frac(a, b);
33 }
34 bool F[MAXN];
35 int main(){
36     int TT;
37     cin >> TT;
38     while(TT -- ){
39         int n;
40         cin >> n;
41         int maxx = -1e9, minx = 1e9;
42         for(int i = 1; i ≤ n; ++ i){
43             auto &[x, y] = P[i];
44             cin >> x >> y;
45             F[i] = false;
46         }
47         sort(P + 1, P + 1 + n);
48         vector<int> Q1, Q2, Q;
49         // Q1 计算上凸壳, Q2 计算下凸壳
50         for(int i = 1; i ≤ n; ++ i){
51             auto &[x, y] = P[i];
52             if(Q1.size() ≤ 1){
53                 Q1.push_back(i);
54             } else {
55                 while(Q1.size() ≥ 2){
56                     auto &[x1, y1] = P[Q1[Q1.

```

```

57                     size() - 2]];
58                     long long cmp = 1ll * (y -
59                     y1) * (x1 - x2) - 1ll *
60                     (x - x1) * (y1 - y2);
61                     if(cmp > 0){
62                         Q1.pop_back();
63                     } else break;
64                 }
65                 Q1.push_back(i);
66             }
67             if(Q2.size() ≤ 1){
68                 Q2.push_back(i);
69             } else {
70                 while(Q2.size() ≥ 2){
71                     auto &[x1, y1] = P[Q2[Q2.
72                     size() - 1]];
73                     auto &[x2, y2] = P[Q2[Q2.
74                     size() - 2]];
75                     long long cmp = 1ll * (y -
76                     y1) * (x1 - x2) - 1ll *
77                     (x - x1) * (y1 - y2);
78                     if(cmp < 0){
79                         Q2.pop_back();
80                     } else break;
81                 }
82                 Q2.push_back(i);
83             }
84         }
85         Q = Q1;
86         for(int i = Q2.size(); i ≠ 0; i --){
87             if(i ≠ Q2.size())
88                 Q.push_back(Q2[i - 1]);
89         }
90         long long area = 0;
91         int x0 = P[Q[0]].x;
92         int y0 = P[Q[0]].y;
93         for(int i = 1; i + 1 < Q.size(); ++ i){
94             auto &[x1, y1] = P[Q[i]];
95             auto &[x2, y2] = P[Q[i + 1]];
96             area += 1ll * (x1 - x0) * (y2 - y0)
97             - 1ll * (x2 - x0) * (y1 - y0);
98         }
99         area = -area;
100         for(auto &i: Q1) F[i] = true;
101         for(auto &i: Q2) F[i] = true;
102         bool ok = false;
103         for(int i = 1; i ≤ n; ++ i) if(!F[i]){
104             ok = true;
105             maxx = max(maxx, P[i].x);
106             minx = min(minx, P[i].x);
107         }

```

```

101         auto &[x2, y2] = P[Q1[Q1.
102         size() - 2]];
103         long long cmp = 1ll * (y -
104         y1) * (x1 - x2) - 1ll *
105         (x - x1) * (y1 - y2);
106         if(cmp > 0){
107             Q1.pop_back();
108         } else break;
109     }
110     Q1.push_back(i);
111 }
112 if(Q2.size() ≤ 1){
113     Q2.push_back(i);
114 } else {
115     while(Q2.size() ≥ 2){
116         auto &[x1, y1] = P[Q2[Q2.
117         size() - 1]];
118         auto &[x2, y2] = P[Q2[Q2.
119         size() - 2]];
120         long long cmp = 1ll * (y -
121         y1) * (x1 - x2) - 1ll *
122         (x - x1) * (y1 - y2);
123         if(cmp < 0){
124             Q2.pop_back();
125         } else break;
126     }
127     Q2.push_back(i);
128 }
129 }
130 Q = Q1;
131 for(int i = Q2.size(); i ≠ 0; i --){
132     if(i ≠ Q2.size())
133         Q.push_back(Q2[i - 1]);
134 }
135 long long area = 0;
136 int x0 = P[Q[0]].x;
137 int y0 = P[Q[0]].y;
138 for(int i = 1; i + 1 < Q.size(); ++ i){
139     auto &[x1, y1] = P[Q[i]];
140     auto &[x2, y2] = P[Q[i + 1]];
141     area += 1ll * (x1 - x0) * (y2 - y0)
142     - 1ll * (x2 - x0) * (y1 - y0);
143 }
144 area = -area;
145 for(auto &i: Q1) F[i] = true;
146 for(auto &i: Q2) F[i] = true;
147 bool ok = false;
148 for(int i = 1; i ≤ n; ++ i) if(!F[i]){
149     ok = true;
150     maxx = max(maxx, P[i].x);
151     minx = min(minx, P[i].x);
152 }

```

```

if(!ok){
    cout << -1 << "\n";
    continue;
}
vector<int> L1;
vector<int> L2;
// L1 插入 kx + b 维护下凸壳
for(int i = 1; i ≤ n; ++ i) if(!F[i]){
    auto &[k, b] = P[i];
    if(!L1.empty() && k == P[L1.back()
    ].x)
        continue;
    while(L1.size() ≥ 2){
        auto &P1 = P[L1[L1.size() -
        1]];
        auto &P2 = P[L1[L1.size() -
        2]];
        Frac i1 = intersect(P1, P[i]);
        Frac i2 = intersect(P2, P[i]);
        if(i1 < i2){
            L1.pop_back();
        } else break;
    }
    L1.push_back(i);
}
// L2 插入 kx + b 维护上凸壳
for(int i = n; i ≥ 1; -- i) if(!F[i]){
    auto &[k, b] = P[i];
    if(!L2.empty() && k == P[L2.back()
    ].x)
        continue;
    while(L2.size() ≥ 2){
        auto &P1 = P[L2[L2.size() -
        1]];
        auto &P2 = P[L2[L2.size() -
        2]];
        Frac i1 = intersect(P1, P[i]);
        Frac i2 = intersect(P2, P[i]);
        if(i1 < i2){
            L2.pop_back();
        } else break;
    }
    L2.push_back(i);
}
vector<Frac> E1;
E1.push_back(Frac(-2e9, 1));
for(int i = 0; i + 1 < L1.size(); ++ i){
    auto &P1 = P[L1[i]];
    auto &P2 = P[L1[i + 1]];
    E1.push_back(intersect(P1, P2));
}
vector<Frac> E2;

```

```

147 E2.push_back(Frac( -2e9, 1 ));
148 for(int i = 0; i + 1 < L2.size(); ++ i){
149     auto &P1 = P[L2[i]];
150     auto &P2 = P[L2[i + 1]];
151     E2.push_back(intersect(P1, P2));
152 }
153 long long ans = 0;
154 for(int i = 0; i + 1 < Q.size(); ++ i){
155     auto &x1, y1 = P[Q[i]];
156     auto &x2, y2 = P[Q[i + 1]];
157     long long w = 1ll * x2 * y1 - 1ll
158         * x1 * y2;
159     int A = y2 - y1;
160     int B = x1 - x2;
161     int x = 0, y = 0;
162     if(B == 0){
163         if(A > 0){
164             x = minx, y = 0;
165         } else {
166             x = maxx, y = 0;
167         }
168     } else
169     if(B < 0){
170         Frac K = Frac(-A, -B);
171         int p = 0;
172         for(int k = 20; k ≥ 0; -- k){
173             int pp = p | 1 << k;
174             if(pp < E1.size() && E1[pp]
175                 < K){
176                 p = pp;
177             }
178         }
179         x = P[L1[p]].x;
180         y = P[L1[p]].y;
181     } else {
182         Frac K = Frac( A, B);
183         int p = 0;
184         for(int k = 20; k ≥ 0; -- k){
185             int pp = p | 1 << k;
186             if(pp < E2.size() && E2[pp]
187                 < K){
188                 p = pp;
189             }
190         }
191         x = P[L2[p]].x;
192         y = P[L2[p]].y;
193     }
194     ans = max(ans, area - (w + 1ll * A
195         * x + 1ll * B * y));
196 }
197 // cerr << "ans = " << ans << endl;
198 cout << ans << "\n";
199 }

```

```

196 return 0;
197 }

9.2 最小圆覆盖

1 #include "2d.cpp"
2 point geto(point a, point b, point c) {
3     double a1, a2, b1, b2, c1, c2;
4     point ans(0, 0);
5     a1 = 2 * (b.x - a.x), b1 = 2 * (b.y - a.y)
6     ,
7     c1 = sqr(b.x) - sqr(a.x) + sqr(b.y) - sqr(
8         a.y);
9     a2 = 2 * (c.x - a.x), b2 = 2 * (c.y - a.y)
10    ,
11    c2 = sqr(c.x) - sqr(a.x) + sqr(c.y) - sqr(
12        a.y);
13    if (equal(a1, 0)) {
14        ans.y = c1 / b1;
15        ans.x = (c2 - ans.y * b2) / a2;
16    } else if (equal(b1, 0)) {
17        ans.x = c1 / a1;
18        ans.y = (c2 - ans.x * a2) / b2;
19    } else {
20        ans.x = (c2 * b1 - c1 * b2) / (a2 * b1
21            - a1 * b2);
22        ans.y = (c2 * a1 - c1 * a2) / (b2 * a1
23            - b1 * a2);
24    }
25    return ans;
26 }
27 mt19937 MT;
28 circ minimal(vector <point> V){
29     shuffle(V.begin(), V.end(), MT);
30     point o = V[0];
31     double r = 0;
32     for(int i = 0; i < V.size(); ++ i) {
33         if (sign(dis(o, V[i]) - r) ≠ 1)
34             continue;
35         o.x = (V[i].x + V[0].x) / 2;
36         o.y = (V[i].y + V[0].y) / 2;
37         r = dis(V[i], V[0]) / 2;
38         for(int j = 0; j < i; ++ j) {
39             if (sign(dis(o, V[j]) - r) ≠ 1)
40                 continue;
41             o.x = (V[i].x + V[j].x) / 2;
42             o.y = (V[i].y + V[j].y) / 2;
43             r = dis(V[i], V[j]) / 2;
44             for(int k = 0; k < j; ++ k) {
45                 if (sign(dis(o, V[k]) - r) ≠
46                     1) continue;
47                 o = geto(V[i], V[j], V[k]);

```

```

39         r = dis(o, V[i]);
40     }
41 }
42 }
43 circ res;
44 res.o = o;
45 res.r = r;
46 return res;
47 }

```

### 9.3 最左转线

```

1 #include "2d.cpp"
2 namespace DSU{
3     const int MAXN = 1e5 + 3;
4     int F[MAXN];
5     int getfa(int u){
6         return u == F[u] ? u : F[u] = getfa(F[
7             u]);
8     }
9 }
10 namespace Dual{
11     const int MAXN = 1e5 + 3;
12     const int MAXM = 1e5 + 3;
13     int A[MAXN], B[MAXN], W[MAXN], I[MAXN], n,
14         m;
15     int outer;
16     bool cmp(int a, int b){
17         return W[a] < W[b];
18     }
19     vector <pair<int, int>> E[MAXN];
20     const int MAXT = 20 + 3;
21     int F[MAXN][MAXT], G[MAXN][MAXT], D[MAXN],
22         h = 20;
23     void dfs(int u, int f){
24         D[u] = D[f] + 1;
25         for(int i = 1; i ≤ h; ++ i)
26             F[u][i] = F[F[u][i - 1]][i - 1],
27             G[u][i] = max(G[u][i - 1], G[F[u][
28                 i - 1]][i - 1]);
29         for(auto &[v, w] : E[u]) if(v ≠ f){
30             G[v][0] = w;
31             F[v][0] = u;
32             dfs(v, u);
33         }
34     }
35     void build(){
36         for(int i = 1; i ≤ n; ++ i)
37             DSU :: F[i] = i;
38         for(int i = 1; i ≤ m; ++ i)
39             I[i] = i;
40         sort(I + 1, I + 1 + m, cmp);

```

```

37     for(int i = 1; i ≤ m; ++ i){
38         int a = A[I[i]];
39         int b = B[I[i]];
40         int w = W[I[i]];
41         int fa = DSU :: getfa(a);
42         int fb = DSU :: getfa(b);
43         if(fa ≠ fb){
44             DSU :: F[fa] = fb;
45             E[a].push_back({b, w});
46             E[b].push_back({a, w});
47         }
48     }
49     dfs(1, 0);
50 }
51 int solve(int u, int v){
52     if(u = outer || v = outer)
53         return -1;
54     int ans = 0;
55     if(D[u] < D[v]) swap(u, v);
56     for(int i = h; i ≥ 0; -- i)
57         if(D[F[u][i]] ≥ D[v]){
58             ans = max(ans, G[u][i]);
59             u = F[u][i];
60         }
61     if(u = v) return ans;
62     for(int i = h; i ≥ 0; -- i)
63         if(F[u][i] ≠ F[v][i]){
64             ans = max(ans, G[u][i]);
65             ans = max(ans, G[v][i]);
66             u = F[u][i];
67             v = F[v][i];
68         }
69     ans = max(ans, G[u][0]);
70     ans = max(ans, G[v][0]);
71     return ans;
72 }
73 }
74 namespace Planer{
75     const int MAXN = 1e5 + 3 + 3;
76     const int MAXE = 2e5 + 3;
77     const int MAXG = 1e5 + 3;
78     const int MAXQ = 2e5 + 3;
79     point P[MAXN];
80     using edge = tuple<int, int>;
81     double gety(int a, int b, double x){
82         return P[a].y + (x - P[a].x) / (P[b].x
83             - P[a].x) * (P[b].y - P[a].y);
84     }
85     double scanx;
86     struct Cmp1{
87         bool operator()(const pair<edge, int>
88             l1, const pair<edge, int> l2) const

```

```

87     {
88         const edge &e1 = l1.first;
89         const edge &e2 = l2.first;
90         double h1 = gety(get<0>(e1), get
91             <1>(e1), scanx);
92         double h2 = gety(get<0>(e2), get
93             <1>(e2), scanx);
94         return h1 < h2;
95     };
96     struct Cmp2{
97         bool operator()(const pair<edge, int>
98             l1, const pair<edge, int> l2) const
99     {
100         if(l1.second = l2.second)
101             return false;
102         const edge &e1 = l1.first;
103         const edge &e2 = l2.first;
104         vec v1 = P[get<1>(e1)] - P[get<0>(
105             e1)];
106         vec v2 = P[get<1>(e2)] - P[get<0>(
107             e2)];
108         if(sign(v1.y) ≠ sign(v2.y)){
109             return v1.y > 0;
110         } else {
111             return sign(mulx(v1, v2)) =
112                 1;
113         }
114     };
115     vector <pair<edge, int> > E[MAXN];
116     vector <int> G[MAXG];
117     int L[MAXE], R[MAXE], W[MAXE], n, m, q, o;
118     double theta;
119     int outer;
120     void rotate(){
121         srand(time(0));
122         theta = PI * rand() / RAND_MAX;
123     }
124     int add(double x, double y){
125         srand(time(0));
126         P[++ n] = rotate(vec(x, y), theta);
127         return n;
128     }
129     int link(int u, int v, int w){
130         ++ m;
131         E[u].push_back({{u, v}, ++ o});
132         L[o] = u, R[o] = v, W[o] = w;
133         E[v].push_back({{v, u}, ++ o});
134         L[o] = v, R[o] = u, W[o] = w;
135         return m;
136     }
137     int I[MAXE];

```

```

132 int polys;
133 pair<edge, int> findleft(int l, int r){
134     auto it = lower_bound(E[r].begin(), E[
135         r].end(), make_pair(edge(r, l), 0),
136         Cmp2());
137     if(it = E[r].begin())
138         return E[r].back();
139     else
140         return *(it - 1);
141 }
142 void leftmost(){
143     for(int i = 1; i ≤ n; ++ i){
144         sort(E[i].begin(), E[i].end(),
145             Cmp2());
146     }
147     for(int p = 1; p ≤ n; ++ p){
148         for(auto &[e1, id1] : E[p]){
149             auto &[x, y] = e1;
150             if(!I[id1]){
151                 int l = x;
152                 int r = y;
153                 I[id1] = ++ polys;
154                 G[polys].push_back(id1);
155                 while(r ≠ p){
156                     auto [e2, id2] =
157                         findleft(l, r);
158                     auto [a, b] = e2;
159                     I[id2] = polys;
160                     G[polys].push_back(id2
161                         );
162                     l = r;
163                     r = b;
164                 }
165             }
166         }
167     }
168     for(int i = 1; i ≤ polys; ++ i){
169         double area = 0;
170         for(int j = 0; j < G[i].size(); ++ j)
171             area += mulx(P[L[G[i][j]]], P[
172                 R[G[i][j]]]);
173         if(area < 0)
174             outer = i;
175     }
176 }
177 void dual(){
178     Dual :: n = polys;
179     Dual :: m = 0;
180     for(int i = 1; i ≤ m; ++ i){
181         int u = I[2 * i - 1], v = I[2 * i

```

```

    ], w = W[2 * i];
    if(u == outer || v == outer)
        w = 1e9L + 1;
    ++ Dual :: m;
    Dual :: A[ Dual :: m ] = u;
    Dual :: B[ Dual :: m ] = v;
    Dual :: W[ Dual :: m ] = w;
}
Dual :: build();
Dual :: outer = outer;
}
set <pair<edge, int>, Cmp1> S;
vector <pair<double, int>> T;
vector <pair<double, int>> Q;
double X[MAXQ], Y[MAXQ];
int Z[MAXQ];
int ask(double x, double y){
    ++ q;
    point p = rotate(vec(x, y), theta);
    X[q] = p.x;
    Y[q] = p.y;
    return q;
}
void locate(){
    T.clear(), Q.clear(), S.clear();
    for(int i = 1; i ≤ q; ++ i){
        Q.push_back(make_pair(X[i], i));
    }
    for(int i = 1; i ≤ polys; ++ i){
        for(auto &e : G[i]){
            int u = L[e];
            int v = R[e];
            if(P[u].x > P[v].x){
                T.push_back(make_pair(P[v].x + 1e-5, e));
                T.push_back(make_pair(P[u].x - 1e-5, -e));
            }
        }
    }
    sort(T.begin(), T.end());
    sort(Q.begin(), Q.end());
    int p1 = 0, p2 = 0;
    scanx = -1e9;
    Cmp1 CMP;
    while(p1 < Q.size() || p2 < T.size()){
        // for(auto it1 = S.begin(), it2 =
        // next(S.begin()); it2 ≠ S.end()
        // ++ it1, ++ it2)
        // assert(CMP(*it1, *it2));
        double x1 = p1 < Q.size() ? Q[p1].
        first : 1e9;

```

```

223 double x2 = p2 < T.size() ? T[p2].
224 first : 1e9;
225 scanx = min(x1, x2);
226 if(equal(scanx, x1)){
227     auto &x = X[Q[p1].second];
228     auto &y = Y[Q[p1].second];
229     auto &z = Z[Q[p1].second];
230     P[n + 1] = point(-1e9, y);
231     P[n + 2] = point(1e9, y);
232     auto it = S.lower_bound({{n +
233         1, n + 2}, 0});
234     if(it == S.end())
235         z = outer;
236     else
237         z = it → second;
238     ++ p1;
239 }
240 if(equal(scanx, x2)){
241     int g = T[p2].second;
242     if(g > 0){
243         assert(!S.count({{L[g], R[
244             g]}, I[g]}));
245         S.insert({{L[g], R[g]}, I[
246             g]});
247     } else {
248         g = -g;
249         assert(S.count({{L[g], R[
250             g]}, I[g]}));
251         S.erase({{L[g], R[g]}, I[
252             g]});
253     }
254     ++ p2;
255 }
256 }
257 }
258 const int MAXN = 1e5 + 3;
259 int A[MAXN], B[MAXN];
260 int main(){
261     #ifndef ONLINE_JUDGE
262         freopen("test.in", "r", stdin);
263         freopen("test.out", "w", stdout);
264     #endif
265     int n, m, q;
266     Planer :: rotate();
267     cin >> n >> m;
268     for(int i = 1; i ≤ n; ++ i){
        double x, y;
        cin >> x >> y;
        Planer :: add(x, y);
    }
    for(int i = 1; i ≤ m; ++ i){

```

```

269 int u, v, w;
270 cin >> u >> v >> w;
271 Planer :: link(u, v, w);
272 }
273 Planer :: leftmost();
274 Planer :: dual();
275 cin >> q;
276 for(int i = 1; i ≤ q; ++ i){
277     double a1, b1, a2, b2;
278     cin >> a1 >> b1;
279     A[i] = Planer :: ask(a1, b1);
280     cin >> a2 >> b2;
281     B[i] = Planer :: ask(a2, b2);
282 }
283 Planer :: locate();
284 for(int i = 1; i ≤ q; ++ i)
285     A[i] = Planer :: Z[A[i]],
286     B[i] = Planer :: Z[B[i]];
287 for(int i = 1; i ≤ q; ++ i){
288     int ans = Dual :: solve(A[i], B[i]);
289     cout << ans << endl;
290 }
291 return 0;
292 }

```

## 9.4 二维基础

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using i64 = long long;
4 const int INF = 1e9;
5 const i64 INFL = 1e18;
6 int qread();
7 const double EPS = 1e-9;
8 const double PI = acos(-1);
9 bool equal(double a, double b){
10     return fabs(a - b) < EPS;
11 }
12 int sign(double a){
13     if(equal(a, 0))
14         return 0;
15     return a > 0 ? 1 : -1;
16 }
17 double sqr(double x){
18     return x * x;
19 }
20 struct vec{ // 二维向量
21     double x;
22     double y;
23     vec(){}
24     vec(double _x, double _y) : x(_x), y(_y){}
25 };

```

```

26 vec operator +(const vec &a, const vec &b){
27     return vec(a.x + b.x, a.y + b.y);
28 }
29 vec operator -(const vec &a, const vec &b){
30     return vec(a.x - b.x, a.y - b.y);
31 }
32 double mulp(const vec &a, const vec &b){
33     return a.x * b.x + a.y * b.y;
34 }
35 double mulx(const vec &a, const vec &b){
36     return a.x * b.y - a.y * b.x;
37 }
38 vec mul(const double &r, const vec &a){
39     return vec(r * a.x, r * a.y);
40 }
41 bool equal(vec a, vec b){
42     return equal(a.x, b.x) && equal(a.y, b.y);
43 }
44 using point = vec;
45 point rotate(point a, double t){
46     double c = cos(t);
47     double s = sin(t);
48     return point(a.x * c - a.y * s, a.y * c +
49                 a.x * s);
50 }
51 bool cmpx(point a, point b){
52     return sign(a.x - b.x) == -1;
53 }
54 bool cmpy(point a, point b){
55     return sign(a.y - b.y) == -1;
56 }
57 struct line{    // 有向直线
58     point o;
59     vec p;
60     line(point _o, vec _p) : o(_o), p(_p){}
61 };
62 struct segm{    // 有向线段
63     point a, b;
64     segm(point _a, point _b) : a(_a), b(_b){}
65 };
66 int side(line l, point p){
67     return sign(mulx(l.p, p - l.o));
68 }
69 int side(segm s, point p){
70     return sign(mulx(s.b - s.a, p - s.a));
71 }
72 bool parallel(line a, line b){
73     return equal(0, mulx(a.p, b.p));
74 }
75 double abs(vec a){
76     return sqrt(a.x * a.x + a.y * a.y);

```

```

77 double dis(point a, point b){
78     return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y));
79 }
80 double abs(segm s){
81     return dis(s.a, s.b);
82 }
83 double dis(line a, point p){
84     return abs(mulx(p - a.o, a.p)) / abs(a.p);
85 }
86 point intersection(line a, line b){
87     return b.o + mul(mulx(b.o - a.o, a.p) /
88                     mulx(a.p, b.p), b.p);
89 }
90 bool intersect(double l1, double r1, double l2
91               , double r2){
92     if(l1 > r1) swap(l1, r1);
93     if(l2 > r2) swap(l2, r2);
94     if(equal(r1, l2) || equal(r2, l1))
95         return true;
96     return !equal(max(r1, r2) - min(l1, l2),
97                 r1 - l1 + r2 - l2);
98 }
99 bool intersect(segm s1, segm s2){
100     bool fx = intersect(s1.a.x, s1.b.x, s2.a.x
101                       , s2.b.x);
102     if(!fx) return false;
103     bool fy = intersect(s1.a.y, s1.b.y, s2.a.y
104                       , s2.b.y);
105     if(!fy) return false;
106     bool g1 = side(s1, s2.a) * side(s1, s2.b)
107             = 1;
108     if(g1) return false;
109     bool g2 = side(s2, s1.a) * side(s2, s1.b)
110             = 1;
111     if(g2) return false;
112     return true;
113 }
114 struct circ{    // 二维圆形
115     point o;
116     double r;
117 };
118 struct poly{    // 二维多边形
119     vector<point> P;
120 };
121 double area(point a, point b, point c){
122     return abs(mulx(b - a, c - a)) / 2;
123 }
124 double area(const poly &P){
125     double ans = 0;
126     for(int i = 0; i < P.P.size(); ++ i){
127         const point &l = P.P[i];

```

```

121     const point &r = P.P[i + 1 == P.P.size
122         () ? 0 : i + 1];
123     ans += mulx(l, r);
124 }
125 return ans / 2;

```

## 10 其他

### 10.1 笛卡尔树

```

1 #include "../header.cpp"
2 // Li: 左儿子; Ri: 右儿子
3 int n, L[MAXN], R[MAXN], A[MAXN];
4 void build(){
5     stack<int> S;
6     A[n + 1] = -1e9;
7     for(int i = 1; i ≤ n + 1; ++ i){
8         int v = 0;
9         while(!S.empty() && A[S.top()] > A[i]){
10             auto u = S.top();
11             R[u] = v, v = u, S.pop();
12         }
13         L[i] = v, S.push(i);
14     }
15 }

```

### 10.2 CDQ 分治

#### 10.2.1 例题

给定三元组序列  $(a_i, b_i, c_i)$ , 求解  $f(i) = \sum_j [a_j \leq a_i \wedge b_j \leq b_i \wedge c_j \leq c_i]$ .

```

1 #include "../header.cpp"
2 struct Node{
3     int id, a, b, c;
4 }A[MAXN], B[MAXN];
5 bool cmp(Node a, Node b){
6     if(a.a ≠ b.a) return a.a < b.a;
7     if(a.b ≠ b.b) return a.b < b.b;
8     if(a.c ≠ b.c) return a.c < b.c;
9     return a.id < b.id;
10 }
11 int K[MAXN], H[MAXN];
12 int qread();
13 int n, m, D[MAXM];
14 namespace BIT{
15     void increase(int x, int w){
16         while(x ≤ m) D[x] += w, x += x & -x;

```



```

17 }
18 void decrease(int x, int w){
19     while(x ≤ m) D[x] -= w, x += x & -x;
20 }
21 void query(int x, int &r){
22     while(x) r += D[x], x -= x & -x;
23 }
24 }
25 void cdq(int l, int r){
26     if(l ≠ r){
27         int t = l + r >> 1; cdq(l, t), cdq(t + 1,
28             r);
29         int p = l, q = t + 1, u = l;
30         while(p ≤ t && q ≤ r){
31             if(A[p].b ≤ A[q].b)
32                 BIT :: increase(A[p].c, 1), B[u ++] =
33                     A[p ++];
34             else
35                 BIT :: query(A[q].c, K[A[q].id]), B[u
36                     ++] = A[q ++];
37             up(l, t, i) BIT :: decrease(A[i].c, 1);
38             up(l, r, i) A[i] = B[i];
39         }
40     }
41     int main(){
42         n = qread(), m = qread();
43         up(1, n, i) A[i].id = i, A[i].a = qread(), A
44             [i].b = qread(), A[i].c = qread();
45         sort(A + 1, A + 1 + n, cmp), cdq(1, n);
46         sort(A + 1, A + 1 + n, cmp);
47         dn(n, 1, i){
48             if(A[i].a = A[i + 1].a && A[i].b = A[i +
49                 1].b && A[i].c = A[i + 1].c)
50                 K[A[i].id] = K[A[i + 1].id];
51                 H[K[A[i].id]] ++;
52         }
53         up(0, n - 1, i) printf("%d\n", H[i]);
54         return 0;
55     }

```

## 10.3 自适应辛普森

### 10.3.1 例题

计算

$$\int_0^{+\infty} x^{(a/x)-x}$$

```

1 #include "../header.cpp"
2 double simpson(double (*f)(double), double l,
3     double r){
4     double mid = (l + r) / 2;
5     return (r - l) * (f(l) + 4 * f(mid) + f(r))
6         / 6.0;
7 }
8 double adapt_simpson(double (*f)(double),
9     double l, double r, double EPS, int step){
10     double mid = (l + r) / 2;
11     double w0 = simpson(f, l, r);
12     double w1 = simpson(f, l, mid);
13     double w2 = simpson(f, mid, r);
14     if(fabs(w0 - w1 - w2) < EPS && step < 0)
15         return w1 + w2;
16     else
17         return adapt_simpson(f, l, mid, EPS, step
18             - 1) +
19             adapt_simpson(f, mid, r, EPS, step
20                 - 1);
21 }
22 double a, l, r;
23 double fun(double x){
24     return pow(x, a / x - x);
25 }
26 int main(){
27     cin >> a;
28     if(a < 0)
29         cout << "orz" << endl;
30     else {
31         l = 1e-9, r = 150;
32         cout << fixed << setprecision(5) <<
33             adapt_simpson(fun, l, r, 1e-9, 15);
34     }
35 }

```

## 10.4 模拟退火

### 10.4.1 例题

给定  $n$  个物品挂在洞下，第  $i$  个物品坐标  $(x_i, y_i)$  重量为  $w_i$ 。询问平衡点。

```

1 #include "../header.cpp"
2 const double T0 = 2e3, Tk = 1e-14, delta =
3     0.993, R = 1e-3;
4 mt19937 MT(114514);
5 double distance(double x, double y, double a,
6     double b){
7     return sqrt(pow(a - x, 2) + pow(b - y, 2));
8 }

```

```

7 const int MAXN = 1e3 + 3;
8 double X[MAXN], Y[MAXN], W[MAXN]; int n;
9 double calculate(double x, double y){
10     double gx, gy, a;
11     for(int i = 0; i < n; ++i){
12         a = atan2(y - Y[i], x - X[i]);
13         gx += cos(a) * W[i];
14         gy += sin(a) * W[i];
15     }
16     return pow(gx, 2) + pow(gy, 2);
17 }
18 double ex, ey, eans = 1e18;
19 void SA(){
20     double T = T0, x = 0, y = 0, ans = calculate
21         (x, y);
22     double ansx, ansy;
23     uniform_real_distribution<double> U;
24     while(T > Tk){
25         double nx, ny, nans;
26         nx = x + 2 * (U(MT) - .5) * T;
27         ny = y + 2 * (U(MT) - .5) * T;
28         if((nans = calculate(nx, ny)) < ans){
29             ans = nans;
30             ansx = x = nx;
31             ansy = y = ny;
32         } else if(exp(-distance(nx, ny, x, y) / T
33             / R) > U(MT)){
34             x = nx, y = ny;
35         }
36         T *= delta;
37     }
38     if(ans < eans) eans = ans, ex = ansx, ey =
39         ansy;
40 }

```

## 10.5 伪随机生成

```

1 #include "../header.cpp"
2 u32 xorshift32(u32 &x){
3     x ^= x << 13, x ^= x >> 17, x ^= x << 5;
4     return x;
5 }
6 u64 xorshift64(u64 &x){
7     x ^= x << 13, x ^= x >> 17, x ^= x << 17;
8     return x;
9 }

```

## 11 header

```

1 #include <bits/stdc++.h>

```

<pre>2 using namespace std; 3 #define up(l, r, i) for(int i = l, END##i = r;   i ≤ END##i; ++ i) 4 #define dn(r, l, i) for(int i = r, END##i = l;   i ≥ END##i; -- i) 5 using i64 = long long;</pre>	<pre>6 using f80 = long double; 7 using u32 = unsigned; 8 using u64 = unsigned long long; 9 const int INF = 1e9; 10 const i64 INFL = 1e18; 11 int qread();</pre>	<pre>12 int power(int a, int b); 13 int power(int a, int b, int p); 14 const int MAXN = 10 + 3, MAXM = 10 + 3; 15 const int MOD = 998244353;</pre>
--	--	--