FISHPACK Documentation

Documentation Notes

The most complete source of documentation for FISHPACK is this present document.

Also viewable is older document Efficient FORTRAN Subprograms for the Solution of Elliptic Partial Differential Equations by Paul Swarztrauber and Roland Sweet, NCAR Technical Note-TN/IA-109,July 1975. Readers will notice certain name changes e.g. PWSCRT instead of HWSCRT, and the fact that only 7 of the 19 solvers are discussed in the document. However, it contains technical discussion not available elsewhere.

Document Partial.pdf for solvers HWSCRT and HWSPLR is available when you download from the NCAR FISHPACK home page. It is also viewable from the tabs at the top of the NCAR FISHPACK homepage.

Obtaining the Software and Documentation

Programs, solvers and support files including some documentation are available at the download tab on the top of the NCAR FISHPACK home page. This distribution favors users running Linux, Mac, or Unix systems with a Fortran compiler, because it uses a Unix-based build system.

available on those systems.

Overview

The following table summarizes the contents of FISHPACK. Descriptions can be obtained by clicking on the solver name. Descriptions of the PDEs solved are also included after the table.

An Overview of FISHPACK Solvers		
computation	**subprogram**	**test program**

2D Helmholtz in Cartesian coordinates (centered grid)	hwscrt	thwscrt
2D Helmholtz in polar coordinates (centered grid)	hwsplr	thwsplr
2D Helmholtz in cylindrical coordinates (centered grid)	hwscyl	thwscyl
2D Helmholtz in spherical coordinates (centered grid)	hwsssp	thwsssp
2D Helmholtz in spherical coordinates (centered grid, axisymmetric)	hwscsp	thwscsp
2D Helmholtz in Cartesian coordinates (staggered grid)	hstcrt	thstcrt
2D Helmholtz in polar coordinates (staggered grid)	hstplr	thstplr
2D Helmholtz in cylindrical coordinates (staggered grid)	hstcyl	thstcyl
2D Helmholtz in spherical coordinates (staggered grid)	hstssp	thstssp
2D Helmholtz in spherical coordinates (staggered grid, axisymmetric)	hstcsp	thstcsp
3D Helmholtz in Cartesian coordinates (centered grid)	hw3crt	thw3crt
2D General Separable PDE (second or fourth order, centered grid)	sepeli	tsepeli
2D Separable PDE (second or fourth order, centered grid)	sepx4	tsepx4
real linear systems solver (centered grid, **sepx4**)	genbun	tgenbun
real block tridiagonal linear systems solver (centered grid, **sepeli**)	blktri	tblktri
real linear systems solver (staggered grid)	poistg	tpoistg
real linear systems solver (3D, centered grid)	pois3d	tpois3d
complex linear systems solver (centered grid)	cmgnbn	tcmgnbn

complex block tridiagonal linear systems solver (centered grid)	cblktri	tcblktri
real and complex fft package	fftpack	use with 3D solvers

Description

The form of the elliptic equations approximated are outlined below. The solvers allow periodic, specified, or derivative boundary conditions.

hwscrt

Subroutine for solving the standard five-point finite difference approximation to the Helmholtz equation in cartesian coordinates using a centered finite difference grid.

(d/dx)(du/dx) + (d/dy)(du/dy) + lambda*u = f(x,y)

Additional files required: **genbun, gnbnaux, comf**
Sample program file: thwscrt

---

hwsplr

Subroutine for solving a five-point finite difference approximation to the Helmholtz equation in polar coordinates using a centered finite difference grid.

(1/r)(d/dr)(r(du/dr)) + (1/r**2)(d/dtheta)(du/dtheta) + lambda*u = f(r,theta)

Additional files required: **genbun, gnbnaux, comf**
Sample program file: thwsplr

---

hwscyl

Subroutine for solving a five-point finite difference approximation to the modified Helmholtz equation in cylindrical coordinates using a centered finite difference grid.

(1/r)(d/dr)(r(du/dr)) + (d/dz)(du/dz) + (lambda/r**2)*u = f(r,z)

Additional files required: **genbun, gnbnaux, comf**
Sample program file: thwscyl

---

hwsssp

Subroutine for solving a five-point finite difference approximation to the Helmholtz equation in spherical coordinates and on the surface of the unit sphere using a centered finite difference grid

(1/sin(theta))(d/dtheta)(sin(theta)(du/dtheta)) +
(1/sin(theta)**2)(d/dphi)(du/dphi) + lambda*u = f(theta,phi)

Additional files required: **genbun, gnbnaux, comf**
Sample program file: thwsssp

---

hwscsp

Subroutine for solving a five-point finite difference approximation to the modified Helmholtz equation in spherical coordinates assuming axisymmetry (no dependence on longitude) using a centered finite difference grid.

(1/r**2)(d/dr)(r**2(du/dr)) +
1/(r**2*sin(theta))(d/dtheta)(sin(theta)(du/dtheta)) +
(lambda/(r*sin(theta)**2))*u = f(theta,r)

Additional files required: **blktri, comf**
Sample program file: thwscsp

---

hstcrt

Subroutine for solving the standard five-point finite difference approximation to the Helmholtz equation in cartesian coordinates using a staggered finite difference grid

(d/dx)(du/dx) + (d/dy)(du/dy) + lambda*u = f(x,y)

Additional files required: **genbun, poistg, gnbnaux, comf**
Sample program file: thstcrt

---

hstplr

Subroutine for solving a five-point finite difference approximation to the Helmholtz equation in polar coordinates using a staggered finite difference grid

(1/r)(d/dr)(r(du/dr)) + (1/r**2)(d/dtheta)(du/dtheta) + lambda*u = f(r,theta)

Additional files required: **genbun, poistg, gnbnaux, comf**
Sample program file: thstplr

---

hstcyl

Subroutine for solving a five-point finite difference approximation to the modified Helmholtz equation in cylindrical coordinates using a staggered finite difference grid.

(1/r)(d/dr)(r(du/dr)) + (d/dz)(du/dz) + (lambda/r**2)*u = f(r,z)

Additional files required: **genbun, poistg, gnbnaux, comf**
Sample program file: thstcyl

---

hwsssp

Subroutine for solving a five-point finite difference approximation to the Helmholtz equation in spherical coordinates and on the surface of the unit sphere using a staggered finite difference grid

(1/sin(theta))(d/dtheta)(sin(theta)(du/dtheta)) +
(1/sin(theta)**2)(d/dphi)(du/dphi) + lambda*u = f(theta,phi)

Additional files required: **genbun, poistg, gnbnaux, comf**
Sample program file: thstssp

---

hstcsp

Subroutine for solving a five-point finite difference approximation to the modified Helmholtz equation in spherical coordinates assuming axisymmetry (no dependence on longitude) using a staggered finite difference grid.

(1/r**2)(d/dr)(r**2(du/dr)) +
1/(r**2*sin(theta))(d/dtheta)(sin(theta)(du/dtheta)) +
(lambda/(r*sin(theta)**2))*u = f(theta,r)

Additional files required: **blktri, comf**
Sample program file: thwscsp

---

hw3crt

Subroutine for solving the standard seven-point finite difference approximation to the Helmholtz equation in cartesian coordinates using a centered finite difference grid.

(d/dx)(du/dx) + (d/dy)(du/dy) + (d/dz)(du/dz) + lambda*u = f(x,y,z)

Additional files required: **pois3d, comf, fftpack**
Sample program file: thw3crt

---

sepx4

Subroutine for automatically discretizing and solving second and (optionally) fourth order finite difference approximations on a uniform grid to certain separable elliptic partial differential equations with constant coefficients in one direction on a rectangle.

a(x)(d/dx)(du/dx) + b(x)du/dx + c(x)u + (d/dy)(du/dy) = g(x,y)

Additional files required: **genbun, gnbnaux, comf**
Sample program file: tsepx4

---

sepeli

Subroutine for automatically discretizing and solving second and (optionally) fourth order finite difference approximations on a uniform grid to the general separable elliptic partial differential equation on a rectangle.

a(x)(d/dx)(du/dx) + b(x)du/dx + c(x)u + d(y)(d/dy)(du/dy) + e(y)du/dy + f(y)u = g(x,y)

Additional files required: **blktri, comf**
Sample program file: tsepeli

---

genbun

Subroutine for solving the real linear system of equations that results from a finite difference approximation on a centered grid to certain two-dimensional elliptic partial differential equations (e.g., see **sepx4**) with constant coefficients in one direction.

Additional files required: **gnbnaux, comf**
Sample program file: tgenbun

---

blktri

Subroutine for solving block tridiagonal linear systems that arise from finite difference approximations to separable two-dimensional elliptic partial differential equations (see **sepeli**).

Additional files required: **comf**
Sample program file: tblktri

---

poistg

Subroutine for solving a block tridiagonal linear system of equations that arises from finite difference approximations on a staggered grid to two-dimensional elliptic partial differential equations with constant coefficients in one direction.

Additional files required: **gnbnaux, comf**
Sample program file: tpoistg

---

pois3d

Subroutine for solving a block tridiagonal linear system of equations that arises from finite difference approximations to three-dimensional elliptic partial differential equations in a box.

Additional files required: **comf, fftpack**
Sample program file: tpois3d

---

cmgnbn

Subroutine for solving a complex block tridiagonal linear system arising from finite difference approximations to separable complex two-dimensional elliptic partial differential equations in a box.

Additional files required: **comf**
Sample program file: tcmgnbn

---

cblktri

Subroutine for solving a complex block tridiagonal linear system of equations arising from finite difference approximation to separable complex two-dimensional elliptic partial differential equations.

Additional files required: **comf**
Sample program file: tcblktri

Return to beginning of this document

Text Below Contains Internal Files Referenced by Above Links

BLKTRI

```
      SUBROUTINE BLKTRI
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,CM,IDIMY,Y,

     1                    IERROR,W)

C

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C     *
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RESEARCH        *

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C     *                       FISHPACK version 4.1
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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                              BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *
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C     *                              OF
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C     *          THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH          *

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C     *                 BOULDER, COLORADO  (80307)
U.S.A.                 *

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C     *                THE NATIONAL SCIENCE FOUNDATION
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C

C

C DIMENSION OF
AN(N),BN(N),CN(N),AM(M),BM(M),CM(M),Y(IDIMY,N),
```

```
C ARGUMENTS                W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C USAGE                    CALL BLKTRI
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,

C
CM,IDIMY,Y,IERROR,W)

C

C PURPOSE                  BLKTRI SOLVES A SYSTEM OF
LINEAR EQUATIONS

C                          OF THE FORM

C

C                          AN(J)*X(I,J-1) + AM(I)*X(I-1,J)
+

C                          (BN(J)+BM(I))*X(I,J) +
CN(J)*X(I,J+1) +

C                          CM(I)*X(I+1,J) = Y(I,J)

C

C                          FOR I = 1,2,...,M  AND  J =
1,2,...,N.

C

C                          I+1 AND I-1 ARE EVALUATED
MODULO M AND

C                          J+1 AND J-1 MODULO N, I.E.,

C

C                          X(I,0) = X(I,N),  X(I,N+1) =
X(I,1),
```

```
C                         X(0,J) = X(M,J),  X(M+1,J) =
X(1,J).

C

C                         THESE EQUATIONS USUALLY RESULT
FROM THE

C                         DISCRETIZATION OF SEPARABLE
ELLIPTIC

C                         EQUATIONS.  BOUNDARY CONDITIONS
MAY BE

C                         DIRICHLET, NEUMANN, OR
PERIODIC.

C

C ARGUMENTS

C

C ON INPUT                IFLG

C

C                           = 0  INITIALIZATION ONLY.

C                                CERTAIN QUANTITIES THAT
DEPEND ON NP,

C                                N, AN, BN, AND CN ARE
COMPUTED AND

C                                STORED IN THE WORK ARRAY
W.

C

C                           = 1  THE QUANTITIES THAT WERE
COMPUTED

C                                IN THE INITIALIZATION
ARE USED
```

```
C                                TO OBTAIN THE SOLUTION
X(I,J).

C

C                                NOTE:

C                                A CALL WITH IFLG=0 TAKES

C                                APPROXIMATELY ONE HALF
THE TIME

C                                AS A CALL WITH IFLG = 1.

C                                HOWEVER, THE
INITIALIZATION DOES

C                                NOT HAVE TO BE REPEATED
UNLESS NP,

C                                N, AN, BN, OR CN CHANGE.

C

C                         NP

C                           = 0  IF AN(1) AND CN(N) ARE
NOT ZERO,

C                                WHICH CORRESPONDS TO
PERIODIC

C                                BOUNARY CONDITIONS.

C

C                           = 1  IF AN(1) AND CN(N) ARE
ZERO.

C

C                         N

C                           THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.

C                           N MUST BE GREATER THAN 4.
```

```
C                           THE OPERATION COUNT IS
PROPORTIONAL TO

C                           MNLOG2(N), HENCE N SHOULD BE
SELECTED

C                           LESS THAN OR EQUAL TO M.

C

C                         AN,BN,CN

C                           ONE-DIMENSIONAL ARRAYS OF
LENGTH N

C                           THAT SPECIFY THE COEFFICIENTS
IN THE

C                           LINEAR EQUATIONS GIVEN ABOVE.

C

C                         MP

C                           = 0  IF AM(1) AND CM(M) ARE
NOT ZERO,

C                                WHICH CORRESPONDS TO
PERIODIC

C                                BOUNDARY CONDITIONS.

C

C                           = 1  IF AM(1) = CM(M) = 0  .

C

C                         M

C                           THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.

C                            M MUST BE GREATER THAN 4.

C
```

```
C                         AM,BM,CM

C                           ONE-DIMENSIONAL ARRAYS OF
LENGTH M THAT

C                           SPECIFY THE COEFFICIENTS IN
THE LINEAR

C                           EQUATIONS GIVEN ABOVE.

C

C                         IDIMY

C                           THE ROW (OR FIRST) DIMENSION
OF THE

C                           TWO-DIMENSIONAL ARRAY Y AS IT
APPEARS

C                           IN THE PROGRAM CALLING
BLKTRI.

C                           THIS PARAMETER IS USED TO
SPECIFY THE

C                           VARIABLE DIMENSION OF Y.

C                           IDIMY MUST BE AT LEAST M.

C

C                         Y

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES

C                           THE VALUES OF THE RIGHT SIDE
OF THE LINEAR

C                           SYSTEM OF EQUATIONS GIVEN
ABOVE.

C                           Y MUST BE DIMENSIONED AT
LEAST M*N.

C
```

```
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C                           IF NP=1 DEFINE
K=INT(LOG2(N))+1 AND
C                           SET L=2**(K+1) THEN W MUST
HAVE DIMENSION
C                           (K-2)*L+K+5+MAX(2N,6M)
C
C                           IF NP=0 DEFINE K=INT(LOG2(N-
1))+1 AND
C                           SET L=2**(K+1) THEN W MUST
HAVE DIMENSION
C                           (K-2)*L+K+5+2N+MAX(2N,6M)
C
C                           **IMPORTANT**
C                           FOR PURPOSES OF CHECKING, THE
REQUIRED
C                           DIMENSION OF W IS COMPUTED BY
BLKTRI AND
C                           STORED IN W(1) IN FLOATING
POINT FORMAT.
C
C ARGUMENTS
C
C ON OUTPUT               Y
```

```
C                          CONTAINS THE SOLUTION X.

C

C                        IERROR

C                          AN ERROR FLAG THAT INDICATES
INVALID

C                          INPUT PARAMETERS.  EXCEPT FOR
NUMBER ZER0,

C                          A SOLUTION IS NOT ATTEMPTED.

C

C                        = 0  NO ERROR.

C                        = 1  M IS LESS THAN 5

C                        = 2  N IS LESS THAN 5

C                        = 3  IDIMY IS LESS THAN M.

C                        = 4  BLKTRI FAILED WHILE
COMPUTING RESULTS

C                             THAT DEPEND ON THE
COEFFICIENT ARRAYS

C                             AN, BN, CN.  CHECK THESE
ARRAYS.

C                        = 5  AN(J)*CN(J-1) IS LESS THAN
0 FOR SOME J.

C

C                             POSSIBLE REASONS FOR THIS
CONDITION ARE

C                             1. THE ARRAYS AN AND CN
ARE NOT CORRECT

C                             2. TOO LARGE A GRID
SPACING WAS USED
```

```
C                                IN THE DISCRETIZATION
OF THE ELLIPTIC
C                                EQUATION.
C                             3. THE LINEAR EQUATIONS
RESULTED FROM A
C                                PARTIAL DIFFERENTIAL
EQUATION WHICH
C                                WAS NOT ELLIPTIC.
C
C                        W
C                           CONTAINS INTERMEDIATE VALUES
THAT MUST
C                           NOT BE DESTROYED IF BLKTRI
WILL BE CALLED
C                           AGAIN WITH IFLG=1. W(1)
CONTAINS THE
C                           NUMBER OF LOCATIONS REQUIRED
BY W IN
C                           FLOATING POINT FORMAT.
C
C
C SPECIAL CONDITIONS     THE ALGORITHM MAY FAIL IF
ABS(BM(I)+BN(J))
C                        IS LESS THAN
ABS(AM(I))+ABS(AN(J))+
C                        ABS(CM(I))+ABS(CN(J))
C                        FOR SOME I AND J. THE ALGORITHM
WILL ALSO
```

```
C                        FAIL IF AN(J)*CN(J-1) IS LESS
THAN ZERO FOR

C                        SOME J.

C                        SEE THE DESCRIPTION OF THE
OUTPUT PARAMETER

C                        IERROR.

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       COMF FROM FISHPAK

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY PAUL SWARZTRAUBER AT
NCAR IN THE

C                        EARLY 1970'S.  REWRITTEN AND
RELEASED IN

C                        JANUARY, 1980.

C

C ALGORITHM              GENERALIZED CYCLIC REDUCTION

C

C PORTABILITY            FORTRAN 77.  APPROXIMATE
MACHINE ACCURACY
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C                               IS COMPUTED IN FUNCTION EPMACH.

C

C REFERENCES                     SWARZTRAUBER,P. AND R. SWEET,
'EFFICIENT

C                               FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF

C                               ELLIPTIC EQUATIONS'

C                               NCAR TN/IA-109, JULY, 1975, 138
PP.

C

C                               SWARZTRAUBER P. N.,A DIRECT
METHOD FOR

C                               THE DISCRETE SOLUTION OF
SEPARABLE

C                               ELLIPTIC EQUATIONS, S.I.A.M.

C                               J. NUMER. ANAL.,11(1974) PP.
1136-1150.

C*******************************************************
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CBLKTRI

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      SUBROUTINE CBLKTR
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,CM,IDIMY,Y,

     1                          IERROR,W)

C
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SOLUTION OF      *

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
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C     *                              BY
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C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

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C     *                           OF
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C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH         *

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C     *              THE NATIONAL SCIENCE FOUNDATION
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C

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C

C DIMENSION OF
AN(N),BN(N),CN(N),AM(M),BM(M),CM(M),Y(IDIMY,N),

C ARGUMENTS                W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C PURPOSE                  CBLKTR SOLVES A SYSTEM OF
LINEAR EQUATIONS

C                          OF THE FORM

C

C                          AN(J)*X(I,J-1) + AM(I)*X(I-1,J)
+

C                          (BN(J)+BM(I))*X(I,J) +
CN(J)*X(I,J+1) +

C                          CM(I)*X(I+1,J) = Y(I,J)

C

C                          FOR I = 1,2,...,M  AND  J =
1,2,...,N.

C

C                          I+1 AND I-1 ARE EVALUATED
MODULO M AND

C                          J+1 AND J-1 MODULO N, I.E.,

C

C                          X(I,0) = X(I,N),  X(I,N+1) =
X(I,1),

C                          X(0,J) = X(M,J),  X(M+1,J) =
X(1,J).
```

```
C

C                         THESE EQUATIONS USUALLY RESULT
FROM THE

C                         DISCRETIZATION OF SEPARABLE
ELLIPTIC

C                         EQUATIONS.  BOUNDARY CONDITIONS
MAY BE

C                         DIRICHLET, NEUMANN, OR
PERIODIC.

C

C                         CBLKTRI IS A COMPLEX VERSION OF
PACKAGE

C                         BLKTRI ON ULIB.

C

C USAGE                   CALL CBLKTR
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,

C
CM,IDIMY,Y,IERROR,W)

C

C ARGUMENTS

C

C ON INPUT                IFLG

C

C                           = 0  INITIALIZATION ONLY.

C                                CERTAIN QUANTITIES THAT
DEPEND ON NP,

C                                N, AN, BN, AND CN ARE
COMPUTED AND
```

```
C                                 STORED IN THE WORK ARRAY
W.

C

C                            = 1  THE QUANTITIES THAT WERE
COMPUTED

C                                 IN THE INITIALIZATION
ARE USED

C                                 TO OBTAIN THE SOLUTION
X(I,J).

C

C                                 NOTE:

C                                 A CALL WITH IFLG=0 TAKES

C                                 APPROXIMATELY ONE HALF
THE TIME

C                                 AS A CALL WITH IFLG = 1.

C                                 HOWEVER, THE
INITIALIZATION DOES

C                                 NOT HAVE TO BE REPEATED
UNLESS NP,

C                                 N, AN, BN, OR CN CHANGE.

C

C                         NP

C                            = 0  IF AN(1) AND CN(N) ARE
NOT ZERO,

C                                 WHICH CORRESPONDS TO
PERIODIC

C                                 BOUNARY CONDITIONS.

C
```

```
C                                 = 1  IF AN(1) AND CN(N) ARE
ZERO.

C

C                               N

C                                 THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.

C                                 N MUST BE GREATER THAN 4.

C                                 THE OPERATION COUNT IS
PROPORTIONAL TO

C                                 MNLOG2(N), HENCE N SHOULD BE
SELECTED

C                                 LESS THAN OR EQUAL TO M.

C

C                               AN,BN,CN

C                                 ONE-DIMENSIONAL ARRAYS OF
LENGTH N

C                                 THAT SPECIFY THE COEFFICIENTS
IN THE

C                                 LINEAR EQUATIONS GIVEN ABOVE.

C

C                               MP

C                                 = 0  IF AM(1) AND CM(M) ARE
NOT ZERO,

C                                      WHICH CORRESPONDS TO
PERIODIC

C                                      BOUNDARY CONDITIONS.

C

C                                 = 1  IF AM(1) = CM(M) = 0  .
```

```
C
C                        M
C                          THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.
C                           M MUST BE GREATER THAN 4.
C
C                        AM,BM,CM
C                          COMPLEX ONE-DIMENSIONAL
ARRAYS OF LENGTH M
C                          THAT SPECIFY THE COEFFICIENTS
IN THE LINEAR
C                          EQUATIONS GIVEN ABOVE.
C
C                        IDIMY
C                          THE ROW (OR FIRST) DIMENSION
OF THE
C                          TWO-DIMENSIONAL ARRAY Y AS IT
APPEARS
C                          IN THE PROGRAM CALLING
CBLKTR.
C                          THIS PARAMETER IS USED TO
SPECIFY THE
C                          VARIABLE DIMENSION OF Y.
C                          IDIMY MUST BE AT LEAST M.
C
C                        Y
C                          A COMPLEX TWO-DIMENSIONAL
ARRAY THAT
```

```
C                           SPECIFIES THE VALUES OF THE
RIGHT SIDE OF
C                           THE LINEAR SYSTEM OF
EQUATIONS GIVEN ABOVE.
C                           Y MUST BE DIMENSIONED
Y(IDIMY,N) WITH
C                           IDIMY .GE. M.
C
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C
C                           IF NP=1 DEFINE
K=INT(LOG2(N))+1 AND
C                           SET L=2**(K+1) THEN W MUST
HAVE DIMENSION
C                           (K-2)*L+K+5+MAX(2N,12M)
C
C                           IF NP=0 DEFINE K=INT(LOG2(N-
1))+1 AND
C                           SET L=2**(K+1) THEN W MUST
HAVE DIMENSION
C                           (K-2)*L+K+5+2N+MAX(2N,12M)
C
C                           **IMPORTANT**
C                           FOR PURPOSES OF CHECKING, THE
REQUIRED
```

```
C                           DIMENSION OF W IS COMPUTED BY
CBLKTR AND

C                           STORED IN W(1) IN FLOATING
POINT FORMAT.

C

C ARGUMENTS

C

C ON OUTPUT               Y

C                           CONTAINS THE SOLUTION X.

C

C                         IERROR

C                           AN ERROR FLAG THAT INDICATES
INVALID

C                           INPUT PARAMETERS.  EXCEPT FOR
NUMBER ZER0,

C                           A SOLUTION IS NOT ATTEMPTED.

C

C                         = 0  NO ERROR.

C                         = 1  M IS LESS THAN 5

C                         = 2  N IS LESS THAN 5

C                         = 3  IDIMY IS LESS THAN M.

C                         = 4  CBLKTR FAILED WHILE
COMPUTING RESULTS

C                              THAT DEPEND ON THE
COEFFICIENT ARRAYS

C                              AN, BN, CN.  CHECK THESE
ARRAYS.
```

```
C                        = 5  AN(J)*CN(J-1) IS LESS THAN
0 FOR SOME J.

C

C                             POSSIBLE REASONS FOR THIS
CONDITION ARE

C                             1. THE ARRAYS AN AND CN
ARE NOT CORRECT

C                             2. TOO LARGE A GRID
SPACING WAS USED

C                                IN THE DISCRETIZATION
OF THE ELLIPTIC

C                                EQUATION.

C                             3. THE LINEAR EQUATIONS
RESULTED FROM A

C                                PARTIAL DIFFERENTIAL
EQUATION WHICH

C                                WAS NOT ELLIPTIC.

C

C                        W

C                           CONTAINS INTERMEDIATE VALUES
THAT MUST

C                           NOT BE DESTROYED IF CBLKTR
WILL BE CALLED

C                           AGAIN WITH IFLG=1. W(1)
CONTAINS THE

C                           NUMBER OF LOCATIONS REQUIRED
BY W IN

C                           FLOATING POINT FORMAT.

C
```

```
C

C SPECIAL CONDITIONS     THE ALGORITHM MAY FAIL IF
ABS(BM(I)+BN(J))

C                        IS LESS THAN
ABS(AM(I))+ABS(AN(J))+

C                        ABS(CM(I))+ABS(CN(J))

C                        FOR SOME I AND J. THE ALGORITHM
WILL ALSO

C                        FAIL IF AN(J)*CN(J-1) IS LESS
THAN ZERO FOR

C                        SOME J.

C                        SEE THE DESCRIPTION OF THE
OUTPUT PARAMETER

C                        IERROR.

C

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       COMF FROM FISHPAK

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY PAUL SWARZTRAUBER AT
NCAR IN
```

```
C                         THE EARLY 1970'S.  REWRITTEN AN
RELEASED
C                         ON NCAR'S PUBLIC SOFTWARE
LIBRARIES IN
C                         JANUARY, 1980.
C
C ALGORITHM               GENERALIZED CYCLIC REDUCTION
C                         (SEE REFERENCE BELOW)
C
C PORTABILITY             FORTRAN 77
C                         THE APPROXIMATE MACHINE
ACCURACY IS COMPUTED
C                         IN FUNCTION EPMACH
C
C REFERENCES              SWARZTRAUBER,P. AND R. SWEET,
'EFFICIENT
C                         FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF
C                         ELLIPTIC EQUATIONS'
C                         NCAR TN/IA-109, JULY, 1975, 138
PP.
C
C                         SWARZTRAUBER P. N.,A DIRECT
METHOD FOR
C                         THE DISCRETE SOLUTION OF
SEPARABLE
C                         ELLIPTIC EQUATIONS, S.I.A.M.
```

```
C                                J. NUMER. ANAL.,11(1974) PP.
1136-1150.

C

C*****************************************************************
***************
```

CMGNBN

```
      SUBROUTINE CMGNBN
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,IERROR,W)

C

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*

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*

C     *
*

C     *                             FISHPACK version 4.1
*
```

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C     *
*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                            BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                            OF
*

C     *
*

C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH         *

C     *
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C     *                BOULDER, COLORADO  (80307)
U.S.A.             *

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* * * * * * * *

C

C

C

C DIMENSION OF           A(M),B(M),C(M),Y(IDIMY,N),

C                        W(SEE PARAMETER LIST)

C ARGUMENTS

C

C LATEST REVISION        NOVEMBER 1988

C

C PURPOSE                THE NAME OF THIS PACKAGE IS A
MNEMONIC FOR THE

C                        COMPLEX GENERALIZED BUNEMAN
ALGORITHM.

C                        IT SOLVES THE COMPLEX LINEAR
SYSTEM OF EQUATION

C

C                        A(I)*X(I-1,J) + B(I)*X(I,J) +
C(I)*X(I+1,J)
```

```
C                         + X(I,J-1) - 2.*X(I,J) +
X(I,J+1) = Y(I,J)

C

C                         FOR I = 1,2,...,M  AND  J =
1,2,...,N.

C

C                         INDICES I+1 AND I-1 ARE
EVALUATED MODULO M,

C                         I.E., X(0,J) = X(M,J) AND
X(M+1,J) = X(1,J),

C                         AND X(I,0) MAY EQUAL 0, X(I,2),
OR X(I,N),

C                         AND X(I,N+1) MAY EQUAL 0,
X(I,N-1), OR X(I,1)

C                         DEPENDING ON AN INPUT
PARAMETER.

C

C USAGE                   CALL CMGNBN
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,

C                                      IERROR,W)

C

C ARGUMENTS

C

C ON INPUT                NPEROD

C

C                           INDICATES THE VALUES THAT
X(I,0) AND

C                           X(I,N+1) ARE ASSUMED TO HAVE.
```

```
C
C                          = 0  IF X(I,0) = X(I,N) AND
X(I,N+1) =
C                               X(I,1).
C                          = 1  IF X(I,0) = X(I,N+1) = 0
.
C                          = 2  IF X(I,0) = 0 AND
X(I,N+1) = X(I,N-1).
C                          = 3  IF X(I,0) = X(I,2) AND
X(I,N+1) =
C                               X(I,N-1).
C                          = 4  IF X(I,0) = X(I,2) AND
X(I,N+1) = 0.
C
C                        N
C                          THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.
C                          N MUST BE GREATER THAN 2.
C
C                        MPEROD
C                          = 0 IF A(1) AND C(M) ARE NOT
ZERO
C                          = 1 IF A(1) = C(M) = 0
C
C                        M
C                          THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.
C                          N MUST BE GREATER THAN 2.
```

```
C
C                        A,B,C
C                          ONE-DIMENSIONAL COMPLEX
ARRAYS OF LENGTH M
C                          THAT SPECIFY THE COEFFICIENTS
IN THE LINEAR
C                          EQUATIONS GIVEN ABOVE.  IF
MPEROD = 0
C                          THE ARRAY ELEMENTS MUST NOT
DEPEND UPON
C                          THE INDEX I, BUT MUST BE
CONSTANT.
C                          SPECIFICALLY, THE SUBROUTINE
CHECKS THE
C                          FOLLOWING CONDITION .
C
C                            A(I) = C(1)
C                            C(I) = C(1)
C                            B(I) = B(1)
C
C                          FOR I=1,2,...,M.
C
C                        IDIMY
C                          THE ROW (OR FIRST) DIMENSION
OF THE
C                          TWO-DIMENSIONAL ARRAY Y AS IT
APPEARS
```

```
C                                    IN THE PROGRAM CALLING
CMGNBN.
C                                    THIS PARAMETER IS USED TO
SPECIFY THE
C                                    VARIABLE DIMENSION OF Y.
C                                    IDIMY MUST BE AT LEAST M.
C
C                                 Y
C                                    A TWO-DIMENSIONAL COMPLEX
ARRAY THAT
C                                    SPECIFIES THE VALUES OF THE
RIGHT SIDE
C                                    OF THE LINEAR SYSTEM OF
EQUATIONS GIVEN
C                                    ABOVE.
C                                    Y MUST BE DIMENSIONED AT
LEAST M*N.
C
C                                 W
C                                    A ONE-DIMENSIONAL COMPLEX
ARRAY THAT
C                                    MUST BE PROVIDED BY THE USER
FOR WORK
C                                    SPACE.  W MAY REQUIRE UP TO
4*N +
C                                    (10 + INT(LOG2(N)))*M
LOCATIONS.
C                                    THE ACTUAL NUMBER OF
LOCATIONS USED IS
```

```
C                          COMPUTED BY CMGNBN AND IS
RETURNED IN

C                          LOCATION W(1).

C

C

C  ON OUTPUT             Y

C

C                          CONTAINS THE SOLUTION X.

C

C                        IERROR

C                          AN ERROR FLAG WHICH INDICATES
INVALID

C                          INPUT PARAMETERS  EXCEPT FOR
NUMBER

C                          ZERO, A SOLUTION IS NOT
ATTEMPTED.

C

C                          = 0  NO ERROR.

C                          = 1  M .LE. 2  .

C                          = 2  N .LE. 2

C                          = 3  IDIMY .LT. M

C                          = 4  NPEROD .LT. 0 OR NPEROD
.GT. 4

C                          = 5  MPEROD .LT. 0 OR MPEROD
.GT. 1

C                          = 6  A(I) .NE. C(1) OR C(I)
.NE. C(1) OR
```

```
C                               B(I) .NE. B(1) FOR
C                               SOME I=1,2,...,M.
C                          = 7  A(1) .NE. 0 OR C(M) .NE.
0 AND
C                                 MPEROD = 1
C
C                        W
C                          W(1) CONTAINS THE REQUIRED
LENGTH OF W.
C
C SPECIAL CONDITONS      NONE
C
C I/O                    NONE
C
C PRECISION              SINGLE
C
C REQUIRED LIBRARY       COMF AND ULIBER, WHICH ARE
LOADED BY DEFAULT
C FILES                  ON NCAR'S CRAY MACHINES.
C
C LANGUAGE               FORTRAN
C
C HISTORY                WRITTEN IN 1979 BY ROLAND SWEET
OF NCAR'S
C                        SCIENTIFIC COMPUTING DIVISION.
MADE AVAILABLE
```

```
C                         ON NCAR'S PUBLIC LIBRARIES IN
JANUARY, 1980.

C

C ALGORITHM               THE LINEAR SYSTEM IS SOLVED BY
A CYCLIC

C                         REDUCTION ALGORITHM DESCRIBED
IN THE

C                         REFERENCE BELOW.

C

C PORTABILITY             FORTRAN 77.  ALL MACHINE
DEPENDENT CONSTANTS

C                         ARE DEFINED IN FUNCTION P1MACH.

C

C REFERENCES              SWEET, R., 'A CYCLIC REDUCTION
ALGORITHM FOR

C                         SOLVING BLOCK TRIDIAGONAL
SYSTEMS OF ARBITRARY

C                         DIMENSIONS,' SIAM J. ON NUMER.
ANAL.,

C                           14(SEPT., 1977), PP. 706-720.

C

C ACCURACY                THIS TEST WAS PERFORMED ON A
CDC 7600:

C

C                         A UNIFORM RANDOM NUMBER
GENERATOR WAS USED

C                         TO CREATE A SOLUTION ARRAY X
FOR THE SYSTEM
```

```
C                        GIVEN IN THE 'PURPOSE'
DESCRIPTION ABOVE

C                        WITH

C                          A(I) = C(I) = -0.5*B(I) = 1,
I=1,2,...,M

C

C                        AND, WHEN MPEROD = 1

C

C                          A(1) = C(M) = 0

C                          A(M) = C(1) = 2.

C

C                        THE SOLUTION X WAS SUBSTITUTED
INTO THE

C                        GIVEN SYSTEM  AND A RIGHT SIDE
Y WAS

C                        COMPUTED.  USING THIS ARRAY Y,
SUBROUTINE

C                        CMGNBN WAS CALLED TO PRODUCE
APPROXIMATE

C                        SOLUTION Z.  THEN RELATIVE
ERROR

C                          E = MAX(CABS(Z(I,J)-X(I,J)))/

C                              MAX(CABS(X(I,J)))

C                        WAS COMPUTED, WHERE THE TWO
MAXIMA ARE TAKEN

C                        OVER I=1,2,...,M AND J=1,...,N.

C
```

```
C                         THE VALUE OF E IS GIVEN IN THE
TABLE

C                         BELOW FOR SOME TYPICAL VALUES
OF M AND N.

C

C                    M (=N)    MPEROD    NPEROD
T(MSECS)    E

C                    ------    ------    ------    ------
--  ------

C

C                      31         0         0         77
1.E-12

C                      31         1         1         45
4.E-13

C                      31         1         3         91
2.E-12

C                      32         0         0         59
7.E-14

C                      32         1         1         65
5.E-13

C                      32         1         3         97
2.E-13

C                      33         0         0         80
6.E-13

C                      33         1         1         67
5.E-13

C                      33         1         3         76
3.E-12

C                      63         0         0        350
5.E-12
```

```
C                      63         1          1          215
6.E-13

C                      63         1          3          412
1.E-11

C                      64         0          0          264
1.E-13

C                      64         1          1          287
3.E-12

C                      64         1          3          421
3.E-13

C                      65         0          0          338
2.E-12

C                      65         1          1          292
5.E-13

C                      65         1          3          329
1.E-11

C

C*********************************************************
****************
```

FFTPACK

```
C

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* * * * * * * *

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*

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C     *                      FISHPACK version 4.1
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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *
C     *
*
C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
C     *
*
C     *                             BY
*
C     *
*
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET           *
C     *
*
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```
C     *                             OF
*

C     *
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C     *          THE NATIONAL CENTER FOR ATMOSPHERIC
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C

C

C LATEST REVISION

C ---------------

C     NOVEMBER 1988  (VERSION 4.1)

C

C PURPOSE
```

```
C -------

C      THIS PACKAGE CONSISTS OF PROGRAMS WHICH PERFORM
FAST FOURIER

C      TRANSFORMS FOR BOTH COMPLEX AND REAL PERIODIC
SEQUENCES AND

C      CERTAIN OTHER SYMMETRIC SEQUENCES THAT ARE LISTED
BELOW.

C

C USAGE

C -----

C      1.   RFFTI     INITIALIZE  RFFTF AND RFFTB

C      2.   RFFTF     FORWARD TRANSFORM OF A REAL
PERIODIC SEQUENCE

C      3.   RFFTB     BACKWARD TRANSFORM OF A REAL
COEFFICIENT ARRAY

C

C      4.   EZFFTI    INITIALIZE EZFFTF AND EZFFTB

C      5.   EZFFTF    A SIMPLIFIED REAL PERIODIC FORWARD
TRANSFORM

C      6.   EZFFTB    A SIMPLIFIED REAL PERIODIC BACKWARD
TRANSFORM

C

C      7.   SINTI     INITIALIZE SINT

C      8.   SINT      SINE TRANSFORM OF A REAL ODD
SEQUENCE

C

C      9.   COSTI     INITIALIZE COST
```

```
C     10.  COST      COSINE TRANSFORM OF A REAL EVEN
SEQUENCE

C

C     11.  SINQI     INITIALIZE SINQF AND SINQB

C     12.  SINQF     FORWARD SINE TRANSFORM WITH ODD
WAVE NUMBERS

C     13.  SINQB     UNNORMALIZED INVERSE OF SINQF

C

C     14.  COSQI     INITIALIZE COSQF AND COSQB

C     15.  COSQF     FORWARD COSINE TRANSFORM WITH ODD
WAVE NUMBERS

C     16.  COSQB     UNNORMALIZED INVERSE OF COSQF

C

C     17.  CFFTI     INITIALIZE CFFTF AND CFFTB

C     18.  CFFTF     FORWARD TRANSFORM OF A COMPLEX
PERIODIC SEQUENCE

C     19.  CFFTB     UNNORMALIZED INVERSE OF CFFTF

C

C SPECIAL CONDITIONS

C ------------------

C     BEFORE CALLING ROUTINES EZFFTB AND EZFFTF FOR THE
FIRST TIME,

C     OR BEFORE CALLING EZFFTB AND EZFFTF WITH A
DIFFERENT LENGTH,

C     USERS MUST INITIALIZE BY CALLING ROUTINE EZFFTI.

C

C I/O
```

```
C ---

C     NONE

C

C PRECISION

C ---------

C     NONE

C

C REQUIRED LIBRARY FILES

C ----------------------

C     NONE

C

C LANGUAGE

C --------

C     FORTRAN

C

C HISTORY

C -------

C     DEVELOPED AT NCAR IN BOULDER, COLORADO BY PAUL N.
SWARZTRAUBER

C     OF THE SCIENTIFIC COMPUTING DIVISION.  RELEASED ON
NCAR'S PUBLIC

C     SOFTWARE LIBRARIES IN JANUARY 1980.  MODIFIED MAY
29, 1985 TO

C     INCREASE EFFICIENCY.

C

C PORTABILITY
```

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C -----------

C      FORTRAN 77

C

C
*********************************************************
**************

C

C      SUBROUTINE RFFTI(N,WSAVE)

C

C      SUBROUTINE RFFTI INITIALIZES THE ARRAY WSAVE WHICH
IS USED IN

C      BOTH RFFTF AND RFFTB. THE PRIME FACTORIZATION OF N
TOGETHER WITH

C      A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C      STORED IN WSAVE.

C

C      INPUT PARAMETER

C

C      N        THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED.

C

C      OUTPUT PARAMETER

C

C      WSAVE    A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 2*N+15.

C               THE SAME WORK ARRAY CAN BE USED FOR BOTH
RFFTF AND RFFTB
```

```
C              AS LONG AS N REMAINS UNCHANGED. DIFFERENT
WSAVE ARRAYS

C              ARE REQUIRED FOR DIFFERENT VALUES OF N.
THE CONTENTS OF

C              WSAVE MUST NOT BE CHANGED BETWEEN CALLS OF
RFFTF OR RFFTB.

C

C
********************************************************
**************

C

C     SUBROUTINE RFFTF(N,R,WSAVE)

C

C     SUBROUTINE RFFTF COMPUTES THE FOURIER COEFFICIENTS
OF A REAL

C     PERODIC SEQUENCE (FOURIER ANALYSIS). THE TRANSFORM
IS DEFINED

C     BELOW AT OUTPUT PARAMETER R.

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE ARRAY R TO BE
TRANSFORMED.  THE METHOD

C              IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C              N MAY CHANGE SO LONG AS DIFFERENT WORK
ARRAYS ARE PROVIDED

C
```

```
C     R       A REAL ARRAY OF LENGTH N WHICH CONTAINS
THE SEQUENCE

C             TO BE TRANSFORMED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 2*N+15.

C             IN THE PROGRAM THAT CALLS RFFTF. THE WSAVE
ARRAY MUST BE

C             INITIALIZED BY CALLING SUBROUTINE
RFFTI(N,WSAVE) AND A

C             DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C             VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C             REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C             TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C             THE SAME WSAVE ARRAY CAN BE USED BY RFFTF
AND RFFTB.

C

C

C     OUTPUT PARAMETERS

C

C     R       R(1) = THE SUM FROM I=1 TO I=N OF R(I)

C

C             IF N IS EVEN SET L =N/2   , IF N IS ODD
SET L = (N+1)/2

C
```

```
C                THEN FOR K = 2,...,L

C

C                   R(2*K-2) = THE SUM FROM I = 1 TO I =
N OF

C

C                        R(I)*COS((K-1)*(I-1)*2*PI/N)

C

C                   R(2*K-1) = THE SUM FROM I = 1 TO I =
N OF

C

C                       -R(I)*SIN((K-1)*(I-1)*2*PI/N)

C

C              IF N IS EVEN

C

C                   R(N) = THE SUM FROM I = 1 TO I = N OF

C

C                        (-1)**(I-1)*R(I)

C

C       *****  NOTE

C                   THIS TRANSFORM IS UNNORMALIZED SINCE
A CALL OF RFFTF

C                   FOLLOWED BY A CALL OF RFFTB WILL
MULTIPLY THE INPUT

C                   SEQUENCE BY N.

C
```

```
C     WSAVE   CONTAINS RESULTS WHICH MUST NOT BE
DESTROYED BETWEEN

C             CALLS OF RFFTF OR RFFTB.

C

C

C
***********************************************************
**************

C

C     SUBROUTINE RFFTB(N,R,WSAVE)

C

C     SUBROUTINE RFFTB COMPUTES THE REAL PERODIC
SEQUENCE FROM ITS

C     FOURIER COEFFICIENTS (FOURIER SYNTHESIS). THE
TRANSFORM IS DEFINED

C     BELOW AT OUTPUT PARAMETER R.

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE ARRAY R TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C             N MAY CHANGE SO LONG AS DIFFERENT WORK
ARRAYS ARE PROVIDED

C

C     R       A REAL ARRAY OF LENGTH N WHICH CONTAINS
THE SEQUENCE
```

```
C               TO BE TRANSFORMED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 2*N+15.

C               IN THE PROGRAM THAT CALLS RFFTB. THE WSAVE
ARRAY MUST BE

C               INITIALIZED BY CALLING SUBROUTINE
RFFTI(N,WSAVE) AND A

C               DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C               VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C               REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C               TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C               THE SAME WSAVE ARRAY CAN BE USED BY RFFTF
AND RFFTB.

C

C

C     OUTPUT PARAMETERS

C

C     R       FOR N EVEN AND FOR I = 1,...,N

C

C                    R(I) = R(1)+(-1)**(I-1)*R(N)

C

C                         PLUS THE SUM FROM K=2 TO K=N/2
OF
```

```
C

C                         2.*R(2*K-2)*COS((K-1)*(I-
1)*2*PI/N)

C

C                        -2.*R(2*K-1)*SIN((K-1)*(I-
1)*2*PI/N)

C

C              FOR N ODD AND FOR I = 1,...,N

C

C                   R(I) = R(1) PLUS THE SUM FROM K=2 TO
K=(N+1)/2 OF

C

C                        2.*R(2*K-2)*COS((K-1)*(I-
1)*2*PI/N)

C

C                       -2.*R(2*K-1)*SIN((K-1)*(I-
1)*2*PI/N)

C

C       *****  NOTE

C                   THIS TRANSFORM IS UNNORMALIZED SINCE
A CALL OF RFFTF

C                   FOLLOWED BY A CALL OF RFFTB WILL
MULTIPLY THE INPUT

C                   SEQUENCE BY N.

C

C      WSAVE   CONTAINS RESULTS WHICH MUST NOT BE
DESTROYED BETWEEN

C              CALLS OF RFFTB OR RFFTF.
```

```
C

C

C
*********************************************************
**************

C

C     SUBROUTINE EZFFTI(N,WSAVE)

C

C     SUBROUTINE EZFFTI INITIALIZES THE ARRAY WSAVE
WHICH IS USED IN

C     BOTH EZFFTF AND EZFFTB. THE PRIME FACTORIZATION OF
N TOGETHER WITH

C     A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C     STORED IN WSAVE.

C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED.

C

C     OUTPUT PARAMETER

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15.

C             THE SAME WORK ARRAY CAN BE USED FOR BOTH
EZFFTF AND EZFFTB
```

```
C              AS LONG AS N REMAINS UNCHANGED. DIFFERENT
WSAVE ARRAYS

C              ARE REQUIRED FOR DIFFERENT VALUES OF N.

C

C

C
********************************************************
**************

C

C      SUBROUTINE EZFFTF(N,R,AZERO,A,B,WSAVE)

C

C      SUBROUTINE EZFFTF COMPUTES THE FOURIER
COEFFICIENTS OF A REAL

C      PERODIC SEQUENCE (FOURIER ANALYSIS). THE TRANSFORM
IS DEFINED

C      BELOW AT OUTPUT PARAMETERS AZERO,A AND B. EZFFTF
IS A SIMPLIFIED

C      BUT SLOWER VERSION OF RFFTF.

C

C      INPUT PARAMETERS

C

C      N       THE LENGTH OF THE ARRAY R TO BE
TRANSFORMED.  THE METHOD

C              IS MUST EFFICIENT WHEN N IS THE PRODUCT OF
SMALL PRIMES.

C

C      R       A REAL ARRAY OF LENGTH N WHICH CONTAINS
THE SEQUENCE
```

```
C              TO BE TRANSFORMED. R IS NOT DESTROYED.

C

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15.

C              IN THE PROGRAM THAT CALLS EZFFTF. THE
WSAVE ARRAY MUST BE

C              INITIALIZED BY CALLING SUBROUTINE
EZFFTI(N,WSAVE) AND A

C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C              THE SAME WSAVE ARRAY CAN BE USED BY EZFFTF
AND EZFFTB.

C

C     OUTPUT PARAMETERS

C

C     AZERO   THE SUM FROM I=1 TO I=N OF R(I)/N

C

C     A,B     FOR N EVEN B(N/2)=0. AND A(N/2) IS THE SUM
FROM I=1 TO

C              I=N OF (-1)**(I-1)*R(I)/N

C
```

```
C              FOR N EVEN DEFINE KMAX=N/2-1

C              FOR N ODD  DEFINE KMAX=(N-1)/2

C

C              THEN FOR  K=1,...,KMAX

C

C                   A(K) EQUALS THE SUM FROM I=1 TO I=N
OF

C

C                        2./N*R(I)*COS(K*(I-1)*2*PI/N)

C

C                   B(K) EQUALS THE SUM FROM I=1 TO I=N
OF

C

C                        2./N*R(I)*SIN(K*(I-1)*2*PI/N)

C

C

C
*********************************************************
**************

C

C      SUBROUTINE EZFFTB(N,R,AZERO,A,B,WSAVE)

C

C      SUBROUTINE EZFFTB COMPUTES A REAL PERODIC SEQUENCE
FROM ITS

C      FOURIER COEFFICIENTS (FOURIER SYNTHESIS). THE
TRANSFORM IS
```

```
C     DEFINED BELOW AT OUTPUT PARAMETER R. EZFFTB IS A
SIMPLIFIED

C     BUT SLOWER VERSION OF RFFTB.

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE OUTPUT ARRAY R.  THE
METHOD IS MOST

C             EFFICIENT WHEN N IS THE PRODUCT OF SMALL
PRIMES.

C

C     AZERO   THE CONSTANT FOURIER COEFFICIENT

C

C     A,B     ARRAYS WHICH CONTAIN THE REMAINING FOURIER
COEFFICIENTS

C             THESE ARRAYS ARE NOT DESTROYED.

C

C             THE LENGTH OF THESE ARRAYS DEPENDS ON
WHETHER N IS EVEN OR

C             ODD.

C

C             IF N IS EVEN N/2    LOCATIONS ARE REQUIRED

C             IF N IS ODD (N-1)/2 LOCATIONS ARE REQUIRED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15.
```

```
C              IN THE PROGRAM THAT CALLS EZFFTB. THE
WSAVE ARRAY MUST BE

C              INITIALIZED BY CALLING SUBROUTINE
EZFFTI(N,WSAVE) AND A

C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C              THE SAME WSAVE ARRAY CAN BE USED BY EZFFTF
AND EZFFTB.

C

C

C     OUTPUT PARAMETERS

C

C     R        IF N IS EVEN DEFINE KMAX=N/2

C              IF N IS ODD  DEFINE KMAX=(N-1)/2

C

C              THEN FOR I=1,...,N

C

C                   R(I)=AZERO PLUS THE SUM FROM K=1 TO
K=KMAX OF

C

C                   A(K)*COS(K*(I-
1)*2*PI/N)+B(K)*SIN(K*(I-1)*2*PI/N)
```

```
C

C     ********************* COMPLEX NOTATION
**************************

C

C              FOR J=1,...,N

C

C              R(J) EQUALS THE SUM FROM K=-KMAX TO K=KMAX
OF

C

C                   C(K)*EXP(I*K*(J-1)*2*PI/N)

C

C              WHERE

C

C                   C(K) = .5*CMPLX(A(K),-B(K))   FOR
K=1,...,KMAX

C

C                   C(-K) = CONJG(C(K))

C

C                   C(0) = AZERO

C

C                        AND I=SQRT(-1)

C

C     *************** AMPLITUDE - PHASE NOTATION
***********************

C

C              FOR I=1,...,N
```

```
C

C              R(I) EQUALS AZERO PLUS THE SUM FROM K=1 TO
K=KMAX OF

C

C                   ALPHA(K)*COS(K*(I-1)*2*PI/N+BETA(K))

C

C              WHERE

C

C                   ALPHA(K) = SQRT(A(K)*A(K)+B(K)*B(K))

C

C                   COS(BETA(K))=A(K)/ALPHA(K)

C

C                   SIN(BETA(K))=-B(K)/ALPHA(K)

C

C
********************************************************
**************

C

C      SUBROUTINE SINTI(N,WSAVE)

C

C      SUBROUTINE SINTI INITIALIZES THE ARRAY WSAVE WHICH
IS USED IN

C      SUBROUTINE SINT. THE PRIME FACTORIZATION OF N
TOGETHER WITH

C      A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C      STORED IN WSAVE.
```

```
C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N+1 IS A PRODUCT OF
SMALL PRIMES.

C

C     OUTPUT PARAMETER

C

C     WSAVE   A WORK ARRAY WITH AT LEAST INT(2.5*N+15)
LOCATIONS.

C             DIFFERENT WSAVE ARRAYS ARE REQUIRED FOR
DIFFERENT VALUES

C             OF N. THE CONTENTS OF WSAVE MUST NOT BE
CHANGED BETWEEN

C             CALLS OF SINT.

C

C
********************************************************
**************

C

C     SUBROUTINE SINT(N,X,WSAVE)

C

C     SUBROUTINE SINT COMPUTES THE DISCRETE FOURIER SINE
TRANSFORM

C     OF AN ODD SEQUENCE X(I). THE TRANSFORM IS DEFINED
BELOW AT
```

```
C     OUTPUT PARAMETER X.

C

C     SINT IS THE UNNORMALIZED INVERSE OF ITSELF SINCE A
CALL OF SINT

C     FOLLOWED BY ANOTHER CALL OF SINT WILL MULTIPLY THE
INPUT SEQUENCE

C     X BY 2*(N+1).

C

C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE SINT
MUST BE

C     INITIALIZED BY CALLING SUBROUTINE SINTI(N,WSAVE).

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N+1 IS THE PRODUCT
OF SMALL PRIMES.

C

C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE
TRANSFORMED

C

C

C     WSAVE   A WORK ARRAY WITH DIMENSION AT LEAST
INT(2.5*N+15)

C             IN THE PROGRAM THAT CALLS SINT. THE WSAVE
ARRAY MUST BE
```

```
C              INITIALIZED BY CALLING SUBROUTINE
SINTI(N,WSAVE) AND A

C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C

C      OUTPUT PARAMETERS

C

C      X       FOR I=1,...,N

C

C                   X(I)= THE SUM FROM K=1 TO K=N

C

C                        2*X(K)*SIN(K*I*PI/(N+1))

C

C                   A CALL OF SINT FOLLOWED BY ANOTHER
CALL OF

C                   SINT WILL MULTIPLY THE SEQUENCE X BY
2*(N+1).

C                   HENCE SINT IS THE UNNORMALIZED
INVERSE

C                   OF ITSELF.

C
```

```
C     WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT BE

C             DESTROYED BETWEEN CALLS OF SINT.

C

C
********************************************************
**************

C

C     SUBROUTINE COSTI(N,WSAVE)

C

C     SUBROUTINE COSTI INITIALIZES THE ARRAY WSAVE WHICH
IS USED IN

C     SUBROUTINE COST. THE PRIME FACTORIZATION OF N
TOGETHER WITH

C     A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C     STORED IN WSAVE.

C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N-1 IS A PRODUCT OF
SMALL PRIMES.

C

C     OUTPUT PARAMETER

C
```

```
C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT LEAST 3*N+15.

C             DIFFERENT WSAVE ARRAYS ARE REQUIRED FOR DIFFERENT VALUES

C             OF N. THE CONTENTS OF WSAVE MUST NOT BE CHANGED BETWEEN

C             CALLS OF COST.

C

C ****************************************************************

C

C     SUBROUTINE COST(N,X,WSAVE)

C

C     SUBROUTINE COST COMPUTES THE DISCRETE FOURIER COSINE TRANSFORM

C     OF AN EVEN SEQUENCE X(I). THE TRANSFORM IS DEFINED BELOW AT OUTPUT

C     PARAMETER X.

C

C     COST IS THE UNNORMALIZED INVERSE OF ITSELF SINCE A CALL OF COST

C     FOLLOWED BY ANOTHER CALL OF COST WILL MULTIPLY THE INPUT SEQUENCE

C     X BY 2*(N-1). THE TRANSFORM IS DEFINED BELOW AT OUTPUT PARAMETER X

C

C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE COST MUST BE
```

```
C     INITIALIZED BY CALLING SUBROUTINE COSTI(N,WSAVE).

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE SEQUENCE X. N MUST BE
GREATER THAN 1.

C             THE METHOD IS MOST EFFICIENT WHEN N-1 IS A
PRODUCT OF

C             SMALL PRIMES.

C

C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE
TRANSFORMED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15

C             IN THE PROGRAM THAT CALLS COST. THE WSAVE
ARRAY MUST BE

C             INITIALIZED BY CALLING SUBROUTINE
COSTI(N,WSAVE) AND A

C             DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C             VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C             REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C             TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C

C     OUTPUT PARAMETERS
```

```
C

C     X       FOR I=1,...,N

C

C                 X(I) = X(1)+(-1)**(I-1)*X(N)

C

C                  + THE SUM FROM K=2 TO K=N-1

C

C                      2*X(K)*COS((K-1)*(I-1)*PI/(N-1))

C

C                  A CALL OF COST FOLLOWED BY ANOTHER
CALL OF

C                  COST WILL MULTIPLY THE SEQUENCE X BY
2*(N-1)

C                  HENCE COST IS THE UNNORMALIZED
INVERSE

C                  OF ITSELF.

C

C     WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT BE

C             DESTROYED BETWEEN CALLS OF COST.

C

C
*********************************************************
**************

C

C     SUBROUTINE SINQI(N,WSAVE)

C
```

```
C     SUBROUTINE SINQI INITIALIZES THE ARRAY WSAVE WHICH IS USED IN

C     BOTH SINQF AND SINQB. THE PRIME FACTORIZATION OF N TOGETHER WITH

C     A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE COMPUTED AND

C     STORED IN WSAVE.

C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE SEQUENCE TO BE TRANSFORMED. THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF SMALL PRIMES.

C

C     OUTPUT PARAMETER

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT LEAST 3*N+15.

C             THE SAME WORK ARRAY CAN BE USED FOR BOTH SINQF AND SINQB

C             AS LONG AS N REMAINS UNCHANGED. DIFFERENT WSAVE ARRAYS

C             ARE REQUIRED FOR DIFFERENT VALUES OF N. THE CONTENTS OF

C             WSAVE MUST NOT BE CHANGED BETWEEN CALLS OF SINQF OR SINQB.

C
```

```
C
****************************************************************
*************

C

C     SUBROUTINE SINQF(N,X,WSAVE)

C

C     SUBROUTINE SINQF COMPUTES THE FAST FOURIER
TRANSFORM OF QUARTER

C     WAVE DATA. THAT IS , SINQF COMPUTES THE
COEFFICIENTS IN A SINE

C     SERIES REPRESENTATION WITH ONLY ODD WAVE NUMBERS.
THE TRANSFORM

C     IS DEFINED BELOW AT OUTPUT PARAMETER X.

C

C     SINQB IS THE UNNORMALIZED INVERSE OF SINQF SINCE A
CALL OF SINQF

C     FOLLOWED BY A CALL OF SINQB WILL MULTIPLY THE
INPUT SEQUENCE X

C     BY 4*N.

C

C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE SINQF
MUST BE

C     INITIALIZED BY CALLING SUBROUTINE SINQI(N,WSAVE).

C

C

C     INPUT PARAMETERS

C
```

```
C     N       THE LENGTH OF THE ARRAY X TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C

C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE
TRANSFORMED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15.

C             IN THE PROGRAM THAT CALLS SINQF. THE WSAVE
ARRAY MUST BE

C             INITIALIZED BY CALLING SUBROUTINE
SINQI(N,WSAVE) AND A

C             DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C             VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C             REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C             TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C

C     OUTPUT PARAMETERS

C

C     X       FOR I=1,...,N

C

C                  X(I) = (-1)**(I-1)*X(N)

C
```

```
C                      + THE SUM FROM K=1 TO K=N-1 OF
C
C                        2*X(K)*SIN((2*I-1)*K*PI/(2*N))
C
C                   A CALL OF SINQF FOLLOWED BY A CALL OF
C                   SINQB WILL MULTIPLY THE SEQUENCE X BY
4*N.
C                   THEREFORE SINQB IS THE UNNORMALIZED
INVERSE
C                   OF SINQF.
C
C     WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT
C              BE DESTROYED BETWEEN CALLS OF SINQF OR
SINQB.
C
C
********************************************************
**************
C
C     SUBROUTINE SINQB(N,X,WSAVE)
C
C     SUBROUTINE SINQB COMPUTES THE FAST FOURIER
TRANSFORM OF QUARTER
C     WAVE DATA. THAT IS , SINQB COMPUTES A SEQUENCE
FROM ITS
C     REPRESENTATION IN TERMS OF A SINE SERIES WITH ODD
WAVE NUMBERS.
```

```
C     THE TRANSFORM IS DEFINED BELOW AT OUTPUT PARAMETER X.
C
C     SINQF IS THE UNNORMALIZED INVERSE OF SINQB SINCE A CALL OF SINQB
C     FOLLOWED BY A CALL OF SINQF WILL MULTIPLY THE INPUT SEQUENCE X
C     BY 4*N.
C
C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE SINQB MUST BE
C     INITIALIZED BY CALLING SUBROUTINE SINQI(N,WSAVE).
C
C
C     INPUT PARAMETERS
C
C     N       THE LENGTH OF THE ARRAY X TO BE TRANSFORMED.  THE METHOD
C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF SMALL PRIMES.
C
C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE TRANSFORMED
C
C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT LEAST 3*N+15.
C             IN THE PROGRAM THAT CALLS SINQB. THE WSAVE ARRAY MUST BE
```

```
C              INITIALIZED BY CALLING SUBROUTINE
SINQI(N,WSAVE) AND A

C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C

C     OUTPUT PARAMETERS

C

C     X        FOR I=1,...,N

C

C                   X(I)= THE SUM FROM K=1 TO K=N OF

C

C                     4*X(K)*SIN((2K-1)*I*PI/(2*N))

C

C                   A CALL OF SINQB FOLLOWED BY A CALL OF

C                   SINQF WILL MULTIPLY THE SEQUENCE X BY
4*N.

C                   THEREFORE SINQF IS THE UNNORMALIZED
INVERSE

C                   OF SINQB.

C

C     WSAVE    CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT
```

```
C             BE DESTROYED BETWEEN CALLS OF SINQB OR
SINQF.

C

C
***********************************************************
**************

C

C     SUBROUTINE COSQI(N,WSAVE)

C

C     SUBROUTINE COSQI INITIALIZES THE ARRAY WSAVE WHICH
IS USED IN

C     BOTH COSQF AND COSQB. THE PRIME FACTORIZATION OF N
TOGETHER WITH

C     A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C     STORED IN WSAVE.

C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE ARRAY TO BE TRANSFORMED.
THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C

C     OUTPUT PARAMETER

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15.
```

```
C              THE SAME WORK ARRAY CAN BE USED FOR BOTH
COSQF AND COSQB

C              AS LONG AS N REMAINS UNCHANGED. DIFFERENT
WSAVE ARRAYS

C              ARE REQUIRED FOR DIFFERENT VALUES OF N.
THE CONTENTS OF

C              WSAVE MUST NOT BE CHANGED BETWEEN CALLS OF
COSQF OR COSQB.

C

C
*********************************************************
**************

C

C     SUBROUTINE COSQF(N,X,WSAVE)

C

C     SUBROUTINE COSQF COMPUTES THE FAST FOURIER
TRANSFORM OF QUARTER

C     WAVE DATA. THAT IS , COSQF COMPUTES THE
COEFFICIENTS IN A COSINE

C     SERIES REPRESENTATION WITH ONLY ODD WAVE NUMBERS.
THE TRANSFORM

C     IS DEFINED BELOW AT OUTPUT PARAMETER X

C

C     COSQF IS THE UNNORMALIZED INVERSE OF COSQB SINCE A
CALL OF COSQF

C     FOLLOWED BY A CALL OF COSQB WILL MULTIPLY THE
INPUT SEQUENCE X

C     BY 4*N.

C
```

```
C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE COSQF
MUST BE

C     INITIALIZED BY CALLING SUBROUTINE COSQI(N,WSAVE).

C

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE ARRAY X TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C

C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE
TRANSFORMED

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 3*N+15

C             IN THE PROGRAM THAT CALLS COSQF. THE WSAVE
ARRAY MUST BE

C             INITIALIZED BY CALLING SUBROUTINE
COSQI(N,WSAVE) AND A

C             DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C             VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C             REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C             TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.
```

```
C
C     OUTPUT PARAMETERS
C
C     X       FOR I=1,...,N
C
C                  X(I) = X(1) PLUS THE SUM FROM K=2 TO
K=N OF
C
C                     2*X(K)*COS((2*I-1)*(K-1)*PI/(2*N))
C
C                  A CALL OF COSQF FOLLOWED BY A CALL OF
C                  COSQB WILL MULTIPLY THE SEQUENCE X BY
4*N.
C                  THEREFORE COSQB IS THE UNNORMALIZED
INVERSE
C                  OF COSQF.
C
C     WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT
C             BE DESTROYED BETWEEN CALLS OF COSQF OR
COSQB.
C
C
*********************************************************
**************
C
C     SUBROUTINE COSQB(N,X,WSAVE)
```

```
C

C     SUBROUTINE COSQB COMPUTES THE FAST FOURIER
TRANSFORM OF QUARTER

C     WAVE DATA. THAT IS , COSQB COMPUTES A SEQUENCE
FROM ITS

C     REPRESENTATION IN TERMS OF A COSINE SERIES WITH
ODD WAVE NUMBERS.

C     THE TRANSFORM IS DEFINED BELOW AT OUTPUT PARAMETER
X.

C

C     COSQB IS THE UNNORMALIZED INVERSE OF COSQF SINCE A
CALL OF COSQB

C     FOLLOWED BY A CALL OF COSQF WILL MULTIPLY THE
INPUT SEQUENCE X

C     BY 4*N.

C

C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE COSQB
MUST BE

C     INITIALIZED BY CALLING SUBROUTINE COSQI(N,WSAVE).

C

C

C     INPUT PARAMETERS

C

C     N       THE LENGTH OF THE ARRAY X TO BE
TRANSFORMED.  THE METHOD

C             IS MOST EFFICIENT WHEN N IS A PRODUCT OF
SMALL PRIMES.

C
```

```
C     X       AN ARRAY WHICH CONTAINS THE SEQUENCE TO BE
TRANSFORMED

C

C     WSAVE   A WORK ARRAY THAT MUST BE DIMENSIONED AT
LEAST 3*N+15

C             IN THE PROGRAM THAT CALLS COSQB. THE WSAVE
ARRAY MUST BE

C             INITIALIZED BY CALLING SUBROUTINE
COSQI(N,WSAVE) AND A

C             DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C             VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C             REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C             TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C

C     OUTPUT PARAMETERS

C

C     X       FOR I=1,...,N

C

C                  X(I)= THE SUM FROM K=1 TO K=N OF

C

C                    4*X(K)*COS((2*K-1)*(I-1)*PI/(2*N))

C

C                  A CALL OF COSQB FOLLOWED BY A CALL OF
```

```
C                   COSQF WILL MULTIPLY THE SEQUENCE X BY
4*N.

C                   THEREFORE COSQF IS THE UNNORMALIZED
INVERSE

C                   OF COSQB.

C

C     WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT

C             BE DESTROYED BETWEEN CALLS OF COSQB OR
COSQF.

C

C
********************************************************
**************

C

C     SUBROUTINE CFFTI(N,WSAVE)

C

C     SUBROUTINE CFFTI INITIALIZES THE ARRAY WSAVE WHICH
IS USED IN

C     BOTH CFFTF AND CFFTB. THE PRIME FACTORIZATION OF N
TOGETHER WITH

C     A TABULATION OF THE TRIGONOMETRIC FUNCTIONS ARE
COMPUTED AND

C     STORED IN WSAVE.

C

C     INPUT PARAMETER

C

C     N       THE LENGTH OF THE SEQUENCE TO BE
TRANSFORMED
```

```
C

C     OUTPUT PARAMETER

C

C     WSAVE   A WORK ARRAY WHICH MUST BE DIMENSIONED AT
LEAST 4*N+15

C             THE SAME WORK ARRAY CAN BE USED FOR BOTH
CFFTF AND CFFTB

C             AS LONG AS N REMAINS UNCHANGED. DIFFERENT
WSAVE ARRAYS

C             ARE REQUIRED FOR DIFFERENT VALUES OF N.
THE CONTENTS OF

C             WSAVE MUST NOT BE CHANGED BETWEEN CALLS OF
CFFTF OR CFFTB.

C

C
*******************************************************
**************

C

C     SUBROUTINE CFFTF(N,C,WSAVE)

C

C     SUBROUTINE CFFTF COMPUTES THE FORWARD COMPLEX
DISCRETE FOURIER

C     TRANSFORM (THE FOURIER ANALYSIS). EQUIVALENTLY ,
CFFTF COMPUTES

C     THE FOURIER COEFFICIENTS OF A COMPLEX PERIODIC
SEQUENCE.

C     THE TRANSFORM IS DEFINED BELOW AT OUTPUT PARAMETER
C.

C
```

```
C      THE TRANSFORM IS NOT NORMALIZED. TO OBTAIN A
NORMALIZED TRANSFORM

C      THE OUTPUT MUST BE DIVIDED BY N. OTHERWISE A CALL
OF CFFTF

C      FOLLOWED BY A CALL OF CFFTB WILL MULTIPLY THE
SEQUENCE BY N.

C

C      THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE CFFTF
MUST BE

C      INITIALIZED BY CALLING SUBROUTINE CFFTI(N,WSAVE).

C

C      INPUT PARAMETERS

C

C

C      N       THE LENGTH OF THE COMPLEX SEQUENCE C. THE
METHOD IS

C              MORE EFFICIENT WHEN N IS THE PRODUCT OF
SMALL PRIMES. N

C

C      C       A COMPLEX ARRAY OF LENGTH N WHICH CONTAINS
THE SEQUENCE

C

C      WSAVE   A REAL WORK ARRAY WHICH MUST BE
DIMENSIONED AT LEAST 4N+15

C               IN THE PROGRAM THAT CALLS CFFTF. THE WSAVE
ARRAY MUST BE

C               INITIALIZED BY CALLING SUBROUTINE
CFFTI(N,WSAVE) AND A
```

```
C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C              THE SAME WSAVE ARRAY CAN BE USED BY CFFTF
AND CFFTB.

C

C      OUTPUT PARAMETERS

C

C      C       FOR J=1,...,N

C

C                 C(J)=THE SUM FROM K=1,...,N OF

C

C                       C(K)*EXP(-I*(J-1)*(K-1)*2*PI/N)

C

C                             WHERE I=SQRT(-1)

C

C      WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT BE

C              DESTROYED BETWEEN CALLS OF SUBROUTINE
CFFTF OR CFFTB

C
```

```
C
*****************************************************************
C
C     SUBROUTINE CFFTB(N,C,WSAVE)
C
C     SUBROUTINE CFFTB COMPUTES THE BACKWARD COMPLEX
DISCRETE FOURIER
C     TRANSFORM (THE FOURIER SYNTHESIS). EQUIVALENTLY ,
CFFTB COMPUTES
C     A COMPLEX PERIODIC SEQUENCE FROM ITS FOURIER
COEFFICIENTS.
C     THE TRANSFORM IS DEFINED BELOW AT OUTPUT PARAMETER
C.
C
C     A CALL OF CFFTF FOLLOWED BY A CALL OF CFFTB WILL
MULTIPLY THE
C     SEQUENCE BY N.
C
C     THE ARRAY WSAVE WHICH IS USED BY SUBROUTINE CFFTB
MUST BE
C     INITIALIZED BY CALLING SUBROUTINE CFFTI(N,WSAVE).
C
C     INPUT PARAMETERS
C
C
C     N      THE LENGTH OF THE COMPLEX SEQUENCE C. THE
METHOD IS
```

```
C             MORE EFFICIENT WHEN N IS THE PRODUCT OF
SMALL PRIMES.

C

C     C       A COMPLEX ARRAY OF LENGTH N WHICH CONTAINS
THE SEQUENCE

C

C     WSAVE   A REAL WORK ARRAY WHICH MUST BE
DIMENSIONED AT LEAST 4N+15

C              IN THE PROGRAM THAT CALLS CFFTB. THE WSAVE
ARRAY MUST BE

C              INITIALIZED BY CALLING SUBROUTINE
CFFTI(N,WSAVE) AND A

C              DIFFERENT WSAVE ARRAY MUST BE USED FOR
EACH DIFFERENT

C              VALUE OF N. THIS INITIALIZATION DOES NOT
HAVE TO BE

C              REPEATED SO LONG AS N REMAINS UNCHANGED
THUS SUBSEQUENT

C              TRANSFORMS CAN BE OBTAINED FASTER THAN THE
FIRST.

C              THE SAME WSAVE ARRAY CAN BE USED BY CFFTF
AND CFFTB.

C

C     OUTPUT PARAMETERS

C

C     C       FOR J=1,...,N

C

C                 C(J)=THE SUM FROM K=1,...,N OF

C
```

```
C                       C(K)*EXP(I*(J-1)*(K-1)*2*PI/N)

C

C                             WHERE I=SQRT(-1)

C

C      WSAVE   CONTAINS INITIALIZATION CALCULATIONS WHICH
MUST NOT BE

C              DESTROYED BETWEEN CALLS OF SUBROUTINE
CFFTF OR CFFTB

C
********************************************************
**************
```

GENBUN

```
      SUBROUTINE GENBUN
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,IERROR,W)

C

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                              BY
*

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C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

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C

C

C

C DIMENSION OF           A(M),B(M),C(M),Y(IDIMY,N),

C                        W(SEE PARAMETER LIST)

C ARGUMENTS

C

C LATEST REVISION        NOVEMBER 1988

C

C PURPOSE                THE NAME OF THIS PACKAGE IS A
MNEMONIC FOR THE
```

```
C                         GENERALIZED BUNEMAN ALGORITHM.

C

C                         IT SOLVES THE REAL LINEAR
SYSTEM OF EQUATIONS

C

C                         A(I)*X(I-1,J) + B(I)*X(I,J) +
C(I)*X(I+1,J)

C                         + X(I,J-1) - 2.*X(I,J) +
X(I,J+1) = Y(I,J)

C

C                         FOR I = 1,2,...,M  AND  J =
1,2,...,N.

C

C                         INDICES I+1 AND I-1 ARE
EVALUATED MODULO M,

C                         I.E., X(0,J) = X(M,J) AND
X(M+1,J) = X(1,J),

C                         AND X(I,0) MAY EQUAL 0, X(I,2),
OR X(I,N),

C                         AND X(I,N+1) MAY EQUAL 0,
X(I,N-1), OR X(I,1)

C                         DEPENDING ON AN INPUT
PARAMETER.

C

C USAGE                   CALL GENBUN
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,

C                                      IERROR,W)

C

C ARGUMENTS
```

```
C
C ON INPUT                NPEROD
C
C                           INDICATES THE VALUES THAT
X(I,0) AND
C                           X(I,N+1) ARE ASSUMED TO HAVE.
C
C                           = 0  IF X(I,0) = X(I,N) AND
X(I,N+1) =
C                                X(I,1).
C                           = 1  IF X(I,0) = X(I,N+1) = 0
.
C                           = 2  IF X(I,0) = 0 AND
X(I,N+1) = X(I,N-1).
C                           = 3  IF X(I,0) = X(I,2) AND
X(I,N+1) =
C                                X(I,N-1).
C                           = 4  IF X(I,0) = X(I,2) AND
X(I,N+1) = 0.
C
C                         N
C                           THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.
C                           N MUST BE GREATER THAN 2.
C
C                         MPEROD
C                           = 0 IF A(1) AND C(M) ARE NOT
ZERO
```

```
C                          = 1 IF A(1) = C(M) = 0
C
C                        M
C                          THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.
C                          N MUST BE GREATER THAN 2.
C
C                        A,B,C
C                          ONE-DIMENSIONAL ARRAYS OF
LENGTH M THAT
C                          SPECIFY THE COEFFICIENTS IN
THE LINEAR
C                          EQUATIONS GIVEN ABOVE.  IF
MPEROD = 0
C                          THE ARRAY ELEMENTS MUST NOT
DEPEND UPON
C                          THE INDEX I, BUT MUST BE
CONSTANT.
C                          SPECIFICALLY, THE SUBROUTINE
CHECKS THE
C                          FOLLOWING CONDITION .
C
C                            A(I) = C(1)
C                            C(I) = C(1)
C                            B(I) = B(1)
C
C                          FOR I=1,2,...,M.
```

```
C
C                        IDIMY
C                          THE ROW (OR FIRST) DIMENSION
OF THE
C                          TWO-DIMENSIONAL ARRAY Y AS IT
APPEARS
C                          IN THE PROGRAM CALLING
GENBUN.
C                          THIS PARAMETER IS USED TO
SPECIFY THE
C                          VARIABLE DIMENSION OF Y.
C                          IDIMY MUST BE AT LEAST M.
C
C                        Y
C                          A TWO-DIMENSIONAL COMPLEX
ARRAY THAT
C                          SPECIFIES THE VALUES OF THE
RIGHT SIDE
C                          OF THE LINEAR SYSTEM OF
EQUATIONS GIVEN
C                          ABOVE.
C                          Y MUST BE DIMENSIONED AT
LEAST M*N.
C
C                        W
C                          A ONE-DIMENSIONAL ARRAY THAT
MUST
C                          BE PROVIDED BY THE USER FOR
WORK
```

```
C                           SPACE.  W MAY REQUIRE UP TO
4*N +
C                           (10 + INT(LOG2(N)))*M
LOCATIONS.
C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS
C                           COMPUTED BY GENBUN AND IS
RETURNED IN
C                           LOCATION W(1).
C
C
C  ON OUTPUT             Y
C
C                           CONTAINS THE SOLUTION X.
C
C                         IERROR
C                           AN ERROR FLAG WHICH INDICATES
INVALID
C                           INPUT PARAMETERS  EXCEPT FOR
NUMBER
C                           ZERO, A SOLUTION IS NOT
ATTEMPTED.
C
C                           = 0  NO ERROR.
C                           = 1  M .LE. 2  .
C                           = 2  N .LE. 2
C                           = 3  IDIMY .LT. M
```

```
C                           = 4  NPEROD .LT. 0 OR NPEROD
.GT. 4

C                           = 5  MPEROD .LT. 0 OR MPEROD
.GT. 1

C                           = 6  A(I) .NE. C(1) OR C(I)
.NE. C(1) OR

C                                B(I) .NE. B(1) FOR

C                                SOME I=1,2,...,M.

C                           = 7  A(1) .NE. 0 OR C(M) .NE.
0 AND

C                                  MPEROD = 1

C

C                         W

C                           W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C SPECIAL CONDITONS       NONE

C

C I/O                     NONE

C

C PRECISION               SINGLE

C

C REQUIRED LIBRARY        COMF AND GNBNAUX FROM FISHPAK

C FILES

C

C LANGUAGE                FORTRAN
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C

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C                         SCIENTIFIC COMPUTING DIVISION.
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C                         ON NCAR'S PUBLIC LIBRARIES IN
JANUARY, 1980.

C

C ALGORITHM               THE LINEAR SYSTEM IS SOLVED BY
A CYCLIC

C                         REDUCTION ALGORITHM DESCRIBED
IN THE

C                         REFERENCE.

C

C PORTABILITY             FORTRAN 77 --

C                         THE MACHINE DEPENDENT CONSTANT
PI IS

C                         DEFINED IN FUNCTION PIMACH.

C

C REFERENCES              SWEET, R., "A CYCLIC REDUCTION
ALGORITHM FOR

C                         SOLVING BLOCK TRIDIAGONAL
SYSTEMS OF ARBITRARY

C                         DIMENSIONS," SIAM J. ON NUMER.
ANAL., 14 (1977)

C                         PP. 706-720.

C

C ACCURACY                THIS TEST WAS PERFORMED ON A
CDC 7600:
```

```
C

C                         A UNIFORM RANDOM NUMBER
GENERATOR WAS USED

C                         TO CREATE A SOLUTION ARRAY X
FOR THE SYSTEM

C                         GIVEN IN THE 'PURPOSE'
DESCRIPTION ABOVE

C                         WITH

C                           A(I) = C(I) = -0.5*B(I) = 1,
I=1,2,...,M

C

C                         AND, WHEN MPEROD = 1

C

C                           A(1) = C(M) = 0

C                           A(M) = C(1) = 2.

C

C                         THE SOLUTION X WAS SUBSTITUTED
INTO THE

C                         GIVEN SYSTEM  AND, USING DOUBLE
PRECISION

C                         A RIGHT SIDE Y WAS COMPUTED.

C                         USING THIS ARRAY Y, SUBROUTINE
GENBUN

C                         WAS CALLED TO PRODUCE
APPROXIMATE

C                         SOLUTION Z.  THEN RELATIVE
ERROR

C                           E = MAX(ABS(Z(I,J)-X(I,J)))/
```

```
C                              MAX(ABS(X(I,J)))
C                        WAS COMPUTED, WHERE THE TWO
MAXIMA ARE TAKEN
C                        OVER I=1,2,...,M AND J=1,...,N.
C
C                        THE VALUE OF E IS GIVEN IN THE
TABLE
C                        BELOW FOR SOME TYPICAL VALUES
OF M AND N.
C
C                   M (=N)    MPEROD    NPEROD
T(MSECS)    E
C                   ------    ------    ------    ------
--  ------
C
C                     31         0         0          36
6.E-14
C                     31         1         1          21
4.E-13
C                     31         1         3          41
3.E-13
C                     32         0         0          29
9.E-14
C                     32         1         1          32
3.E-13
C                     32         1         3          48
1.E-13
C                     33         0         0          36
9.E-14
```

```
C                      33         1         1          30
4.E-13

C                      33         1         3          34
1.E-13

C                      63         0         0         150
1.E-13

C                      63         1         1          91
1.E-12

C                      63         1         3         173
2.E-13

C                      64         0         0         122
1.E-13

C                      64         1         1         128
1.E-12

C                      64         1         3         199
6.E-13

C                      65         0         0         143
2.E-13

C                      65         1         1         120
1.E-12

C                      65         1         3         138
4.E-13

C * * * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * *
```

GNBNAUX

```
C
```

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EQUATIONS        *

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C     *                            BY
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```
C PACKAGE GNBNAUX
C
C LATEST REVISION        NOVEMBER 1988
C
C PURPOSE                TO PROVIDE AUXILIARY ROUTINES
FOR FISHPAK
C                        ENTRIES GENBUN AND POISTG.
C
C USAGE                  THERE ARE NO USER ENTRIES IN
THIS PACKAGE.
C                        THE ROUTINES IN THIS PACKAGE
ARE NOT INTENDED
C                        TO BE CALLED BY USERS, BUT
RATHER BY ROUTINES
C                        IN PACKAGES GENBUN AND POISTG.
C
C SPECIAL CONDITIONS     NONE
C
C I/O                    NONE
C
C PRECISION              SINGLE
C
C REQUIRED LIBRARY       COMF FROM FISHPAK
C FILES
C
C LANGUAGE               FORTRAN
```

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C

C HISTORY                  WRITTEN IN 1979 BY ROLAND SWEET
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C                          SCIENTIFIC COMPUTING DIVISION.
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C

C PORTABILITY              FORTRAN 77

C
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HSTCRT

```
      SUBROUTINE HSTCRT
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                     ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                             BY
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C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
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C

C

C

C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N),

C ARGUMENTS              W(SEE ARGUMENT LIST)

C

C LATEST REVISION        NOVEMBER 1988

C
```

```
C PURPOSE                    SOLVES THE STANDARD FIVE-POINT
FINITE

C                            DIFFERENCE APPROXIMATION TO
THE HELMHOLTZ

C                            EQUATION

C                              (D/DX)(DU/DX) +
(D/DY)(DU/DY) + LAMBDA*U

C                              = F(X,Y)

C                            ON A STAGGERED GRID IN
CARTESIAN COORDINATES.

C

C USAGE                      CALL HSTCRT
(A,B,M,MBDCND,BDA,BDB,C,D

C
N,NBDCND,BDC,BDD,ELMBDA,

C
F,IDIMF,PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT

C

C                           A,B

C                             THE RANGE OF X, I.E. A .LE. X
.LE. B.

C                             A MUST BE LESS THAN B.

C

C                           M
```

```
C                           THE NUMBER OF GRID POINTS IN
THE
C                           INTERVAL (A,B).  THE GRID
POINTS
C                           IN THE X-DIRECTION ARE GIVEN
BY
C                           X(I) = A + (I-0.5)DX FOR
I=1,2,...,M
C                           WHERE DX =(B-A)/M.  M MUST BE
GREATER
C                           THAN 2.
C
C                         MBDCND
C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS
C                           AT X = A AND X = B.
C
C                           = 0  IF THE SOLUTION IS
PERIODIC IN X,
C                                U(M+I,J) = U(I,J).
C
C                           = 1  IF THE SOLUTION IS
SPECIFIED AT
C                                X = A AND X = B.
C
C                           = 2  IF THE SOLUTION IS
SPECIFIED AT
C                                X = A AND THE DERIVATIVE
```

```
C                                OF THE SOLUTION WITH
RESPECT TO X

C                                IS SPECIFIED AT X = B.

C

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO X IS
SPECIFIED

C                                AT X = A  AND X = B.

C

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO X IS
SPECIFIED

C                                AT X = A  AND THE
SOLUTION IS

C                                SPECIFIED AT X = B.

C

C                         BDA

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N

C                           THAT SPECIFIES THE BOUNDARY
VALUES

C                           (IF ANY) OF THE SOLUTION AT X
= A.

C

C                           WHEN MBDCND = 1 OR 2,

C                             BDA(J) = U(A,Y(J)) ,
J=1,2,...,N.
```

```
C
C                                WHEN MBDCND = 3 OR 4,
C                                  BDA(J) = (D/DX)U(A,Y(J)) ,
J=1,2,...,N.
C
C                              BDB
C                                A ONE-DIMENSIONAL ARRAY OF
LENGTH N
C                                THAT SPECIFIES THE BOUNDARY
VALUES
C                                OF THE SOLUTION AT X = B.
C
C                                WHEN MBDCND = 1 OR 4
C                                  BDB(J) = U(B,Y(J)) ,
J=1,2,...,N.
C
C                                WHEN MBDCND = 2 OR 3
C                                  BDB(J) = (D/DX)U(B,Y(J)) ,
J=1,2,...,N.
C
C                              C,D
C                                THE RANGE OF Y, I.E. C .LE. Y
.LE. D.
C                                C MUST BE LESS THAN D.
C
C
C                              N
```

```
C                           THE NUMBER OF UNKNOWNS IN THE
INTERVAL

C                           (C,D).  THE UNKNOWNS IN THE
Y-DIRECTION

C                           ARE GIVEN BY Y(J) = C + (J-
0.5)DY,

C                           J=1,2,...,N, WHERE DY = (D-
C)/N.

C                           N MUST BE GREATER THAN 2.

C

C                         NBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT Y = C   AND Y = D.

C

C

C                           = 0  IF THE SOLUTION IS
PERIODIC IN Y, I.E.

C                                U(I,J) = U(I,N+J).

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT Y = C

C                                AND Y = D.

C

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT Y = C

C                                AND THE DERIVATIVE OF
THE SOLUTION
```

```
C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                Y = D.

C

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                Y = C AND Y = D.

C

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                Y = C AND THE SOLUTION
IS SPECIFIED

C                                AT Y = D.

C

C                         BDC

C                           A ONE DIMENSIONAL ARRAY OF
LENGTH M THAT

C                           SPECIFIES THE BOUNDARY VALUES
OF THE

C                           SOLUTION AT Y = C.

C

C                           WHEN NBDCND = 1 OR 2,

C                             BDC(I) = U(X(I),C) ,
I=1,2,...,M.
```

```
C
C                            WHEN NBDCND = 3 OR 4,
C                              BDC(I) = (D/DY)U(X(I),C),
I=1,2,...,M.
C
C                            WHEN NBDCND = 0, BDC IS A
DUMMY VARIABLE.
C
C                          BDD
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH M THAT
C                            SPECIFIES THE BOUNDARY VALUES
OF THE
C                            SOLUTION AT Y = D.
C
C                            WHEN NBDCND = 1 OR 4,
C                              BDD(I) = U(X(I),D) ,
I=1,2,...,M.
C
C                            WHEN NBDCND = 2 OR 3,
C                              BDD(I) = (D/DY)U(X(I),D) ,
I=1,2,...,M.
C
C                            WHEN NBDCND = 0, BDD IS A
DUMMY VARIABLE.
C
C                          ELMBDA
```

```
C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                           EQUATION.  IF LAMBDA IS
GREATER THAN 0,
C                           A SOLUTION MAY NOT EXIST.
HOWEVER,
C                           HSTCRT WILL  ATTEMPT TO FIND
A SOLUTION.
C
C                         F
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES
C                           THE VALUES OF THE RIGHT SIDE
OF THE
C                           HELMHOLTZ EQUATION.  FOR
I=1,2,...,M
C                           AND J=1,2,...,N
C
C                             F(I,J) = F(X(I),Y(J)) .
C
C                           F MUST BE DIMENSIONED AT
LEAST M X N.
C
C                         IDIMF
C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAY
C                           F AS IT APPEARS IN THE
PROGRAM CALLING
```

```
C                              HSTCRT.  THIS PARAMETER IS
USED TO SPECIFY
C                              THE VARIABLE DIMENSION OF F.
C                              IDIMF MUST BE AT LEAST M.
C
C                            W
C                              A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                              PROVIDED BY THE USER FOR WORK
SPACE.
C                              W MAY REQUIRE UP TO 13M + 4N
+
C                              M*INT(LOG2(N)) LOCATIONS. THE
ACTUAL NUMBER
C                              OF LOCATIONS USED IS COMPUTED
BY HSTCRT
C                              AND IS RETURNED IN THE
LOCATION W(1).
C
C ON OUTPUT                  F
C                              CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
C                              DIFFERENCE APPROXIMATION FOR
THE GRID POINT
C                              (X(I),Y(J)) FOR  I=1,2,...,M,
J=1,2,...,N.
C
C                            PERTRB
```

```
C                          IF A COMBINATION OF PERIODIC
OR DERIVATIVE

C                          BOUNDARY CONDITIONS IS
SPECIFIED FOR A

C                          POISSON EQUATION (LAMBDA =
0), A SOLUTION

C                          MAY NOT EXIST.  PERTRB IS A
CONSTANT,

C                          CALCULATED AND SUBTRACTED
FROM F, WHICH

C                          ENSURES THAT A SOLUTION
EXISTS.  HSTCRT

C                          THEN COMPUTES THIS SOLUTION,
WHICH IS A

C                          LEAST SQUARES SOLUTION TO THE
ORIGINAL

C                          APPROXIMATION.  THIS SOLUTION
PLUS ANY

C                          CONSTANT IS ALSO A SOLUTION;
HENCE, THE

C                          SOLUTION IS NOT UNIQUE.  THE
VALUE OF

C                          PERTRB SHOULD BE SMALL
COMPARED TO THE

C                          RIGHT SIDE F.  OTHERWISE, A
SOLUTION IS

C                          OBTAINED TO AN ESSENTIALLY
DIFFERENT PROBLEM.

C                          THIS COMPARISON SHOULD ALWAYS
BE MADE TO
```

```
C                          INSURE THAT A MEANINGFUL
SOLUTION HAS BEEN

C                          OBTAINED.

C

C                        IERROR

C                          AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                          PARAMETERS.  EXCEPT TO
NUMBERS 0 AND  6,

C                          A SOLUTION IS NOT ATTEMPTED.

C

C                          =  0  NO ERROR

C

C                          =  1  A .GE. B

C

C                          =  2  MBDCND .LT. 0 OR MBDCND
.GT. 4

C

C                          =  3  C .GE. D

C

C                          =  4  N .LE. 2

C

C                         =  5  NBDCND .LT. 0 OR NBDCND
.GT. 4

C

C                         =  6  LAMBDA .GT. 0
```

```
C
C                          =  7  IDIMF .LT. M
C
C                          =  8  M .LE. 2
C
C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING
C                          A POSSIBLY INCORRECT CALL TO
HSTCRT, THE
C                          USER SHOULD TEST IERROR AFTER
THE CALL.
C
C                         W
C                           W(1) CONTAINS THE REQUIRED
LENGTH OF W.
C
C I/O                     NONE
C
C PRECISION               SINGLE
C
C REQUIRED LIBRARY        COMF, GENBUN, GNBNAUX, AND
POISTG
C FILES                   FROM FISHPAK
C
C LANGUAGE                FORTRAN
C
```

```
C HISTORY                    WRITTEN BY ROLAND SWEET AT NCAR
IN 1977.

C                            RELEASED ON NCAR'S PUBLIC
SOFTWARE LIBRARIES

C                            IN JANUARY 1980.

C

C PORTABILITY                FORTRAN 77

C

C ALGORITHM                  THIS SUBROUTINE DEFINES THE
FINITE-DIFFERENCE

C                            EQUATIONS, INCORPORATES
BOUNDARY DATA, ADJUSTS

C                            THE RIGHT SIDE WHEN THE SYSTEM
IS SINGULAR

C                            AND CALLS EITHER POISTG OR
GENBUN WHICH SOLVES

C                            THE LINEAR SYSTEM OF EQUATIONS.

C

C TIMING                     FOR LARGE M AND N, THE
OPERATION COUNT

C                            IS ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).

C

C ACCURACY                   THE SOLUTION PROCESS EMPLOYED
RESULTS IN A

C                            LOSS OF NO MORE THAN FOUR
SIGNIFICANT DIGITS

C                            FOR N AND M AS LARGE AS 64.
MORE DETAILED
```

```
C                        INFORMATION ABOUT ACCURACY CAN
BE FOUND IN

C                        THE DOCUMENTATION FOR PACKAGE
POISTG WHICH

C                        SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES             U. SCHUMANN AND R. SWEET,"A
DIRECT METHOD

C                        FOR THE SOLUTION OF POISSON'S
EQUATION WITH

C                        BOUNDARY CONDITIONS ON A
STAGGERED GRID OF

C                        ARBITRARY SIZE," J. COMP. PHYS.
20(1976),

C                        PP. 171-182.

C*******************************************************
****************
```

HSTCSP

```
      SUBROUTINE HSTCSP
(INTL,A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,

     1
BDD,ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

```
C     *
*

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*

C     *
*

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RESEARCH         *

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*

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*

C     *
*

C     *                        FISHPACK version 4.1
*

C     *
*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                               BY
*

C     *
*
```

```
C     *         JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                              OF
*

C     *
*

C     *          THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH          *

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*

C     *
*

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C
```

```
C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N),

C ARGUMENTS                W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C PURPOSE                  SOLVES THE STANDARD FIVE-POINT
FINITE

C                          DIFFERENCE APPROXIMATION ON A
STAGGERED

C                          GRID TO THE MODIFIED HELMHOLTZ
EQUATION IN

C                          SPHERICAL COORDINATES ASSUMING
AXISYMMETRY

C                          (NO DEPENDENCE ON LONGITUDE).

C

C                          THE EQUATION IS

C

C                             (1/R**2)(D/DR)(R**2(DU/DR))
+

C
1/(R**2*SIN(THETA))(D/DTHETA)

C                             (SIN(THETA)(DU/DTHETA)) +

C                             (LAMBDA/(R*SIN(THETA))**2)U
=  F(THETA,R)

C

C                          WHERE THETA IS COLATITUDE AND R
IS THE
```

```
C                         RADIAL COORDINATE. THIS TWO-
DIMENSIONAL

C                         MODIFIED HELMHOLTZ EQUATION
RESULTS FROM

C                         THE FOURIER TRANSFORM OF THE
THREE-

C                         DIMENSIONAL POISSON EQUATION.

C

C

C USAGE                   CALL HSTCSP
(INTL,A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                      PERTRB,IERROR,W)

C

C ARGUMENTS

C  ON INPUT               INTL

C

C                           = 0  ON INITIAL ENTRY TO
HSTCSP OR IF ANY

C                                OF THE ARGUMENTS C, D,
N, OR NBDCND

C                                ARE CHANGED FROM A
PREVIOUS CALL

C

C                           = 1  IF C, D, N, AND NBDCND
ARE ALL

C                                UNCHANGED FROM PREVIOUS
CALL TO HSTCSP
```

```
C

C                          NOTE:

C                          A CALL WITH INTL = 0 TAKES
APPROXIMATELY

C                          1.5 TIMES AS MUCH TIME AS A
CALL WITH

C                          INTL = 1.  ONCE A CALL WITH
INTL = 0

C                          HAS BEEN MADE THEN SUBSEQUENT
SOLUTIONS

C                          CORRESPONDING TO DIFFERENT F,
BDA, BDB,

C                          BDC, AND BDD CAN BE OBTAINED
FASTER WITH

C                          INTL = 1 SINCE INITIALIZATION
IS NOT

C                          REPEATED.

C

C                        A,B

C                          THE RANGE OF THETA
(COLATITUDE),

C                          I.E. A .LE. THETA .LE. B.  A

C                          MUST BE LESS THAN B AND A
MUST BE

C                          NON-NEGATIVE.  A AND B ARE IN
RADIANS.

C                          A = 0 CORRESPONDS TO THE
NORTH POLE AND

C                          B = PI CORRESPONDS TO THE
SOUTH POLE.
```

```
C
C                          * * *  IMPORTANT  * * *
C
C                          IF B IS EQUAL TO PI, THEN B
MUST BE
C                          COMPUTED USING THE STATEMENT
C                              B = PIMACH(DUM)
C                          THIS INSURES THAT B IN THE
USER'S PROGRAM
C                          IS EQUAL TO PI IN THIS
PROGRAM, PERMITTING
C                          SEVERAL TESTS OF THE INPUT
PARAMETERS THAT
C                          OTHERWISE WOULD NOT BE
POSSIBLE.
C
C                          * * * * * * * * * * * *
C
C                        M
C                          THE NUMBER OF GRID POINTS IN
THE INTERVAL
C                          (A,B).  THE GRID POINTS IN
THE THETA-
C                          DIRECTION ARE GIVEN BY
C                            THETA(I) = A + (I-
0.5)DTHETA
C                          FOR I=1,2,...,M WHERE DTHETA
=(B-A)/M.
```

```
C                          M MUST BE GREATER THAN 4.

C

C                         MBDCND

C                          INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                          AT THETA = A AND THETA = B.

C

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = A AND THETA = B.

C                               (SEE NOTES 1, 2 BELOW)

C

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = A AND THE
DERIVATIVE OF THE

C                               SOLUTION WITH RESPECT TO
THETA IS

C                               SPECIFIED AT THETA = B

C                               (SEE NOTES 1, 2 BELOW).

C

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO THETA IS
SPECIFIED

C                               AT THETA = A (SEE NOTES
1, 2 BELOW)

C                               AND THETA = B.
```

```
C

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED AT

C                                THETA = A (SEE NOTES 1,
2 BELOW) AND

C                                THE SOLUTION IS
SPECIFIED AT THETA = B.

C

C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = A = 0 AND THE
SOLUTION IS

C                                SPECIFIED AT THETA = B.

C                                (SEE NOTE 2 BELOW)

C

C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = A = 0 AND THE
DERIVATIVE OF

C                                THE SOLUTION WITH
RESPECT TO THETA IS

C                                SPECIFIED AT THETA = B

C                                (SEE NOTE 2 BELOW).

C

C                           = 7  IF THE SOLUTION IS
SPECIFIED AT
```

```
C                                THETA = A AND THE
SOLUTION IS

C                                UNSPECIFIED AT THETA = B
= PI.

C

C                           = 8  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED AT

C                                THETA = A (SEE NOTE 1
BELOW)

C                                AND THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = B = PI.

C

C                           = 9  IF THE SOLUTION IS
UNSPECIFIED AT

C                                 THETA = A = 0 AND THETA
= B = PI.

C

C                           NOTE 1:

C                           IF A = 0, DO NOT USE MBDCND =
1,2,3,4,7

C                           OR 8, BUT INSTEAD USE MBDCND
= 5, 6, OR 9.

C

C                           NOTE 2:

C                           IF B = PI, DO NOT USE MBDCND
= 1,2,3,4,5,
```

```
C                            OR 6, BUT INSTEAD USE MBDCND
= 7, 8, OR 9.

C

C                            NOTE 3:

C                            WHEN A = 0  AND/OR B = PI THE
ONLY

C                            MEANINGFUL BOUNDARY CONDITION
IS

C                            DU/DTHETA = 0.   SEE D.
GREENSPAN,

C                            'NUMERICAL ANALYSIS OF
ELLIPTIC

C                             BOUNDARY VALUE PROBLEMS,'

C                            HARPER AND ROW, 1965, CHAPTER
5.)

C

C                          BDA

C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT

C                            SPECIFIES THE BOUNDARY VALUES
(IF ANY) OF

C                            THE SOLUTION AT THETA = A.

C

C                            WHEN  MBDCND = 1, 2, OR 7,

C                              BDA(J) = U(A,R(J)),
J=1,2,...,N.

C

C                            WHEN MBDCND = 3, 4, OR 8,
```

```
C                             BDA(J) =
(D/DTHETA)U(A,R(J)), J=1,2,...,N.

C

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDA IS A

C                           DUMMY VARIABLE.

C

C                         BDB

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT

C                           SPECIFIES THE BOUNDARY VALUES
OF THE

C                           SOLUTION AT THETA = B.

C

C                           WHEN MBDCND = 1, 4, OR 5,

C                             BDB(J) = U(B,R(J)),
J=1,2,...,N.

C

C                           WHEN MBDCND = 2,3, OR 6,

C                             BDB(J) =
(D/DTHETA)U(B,R(J)), J=1,2,...,N.

C

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDB IS

C                           A DUMMY VARIABLE.

C

C                         C,D
```

```
C                                 THE RANGE OF R , I.E. C .LE.
R .LE. D.

C                                 C MUST BE LESS THAN D AND
NON-NEGATIVE.

C

C                              N

C                                 THE NUMBER OF UNKNOWNS IN THE
INTERVAL

C                                 (C,D).  THE UNKNOWNS IN THE
R-DIRECTION

C                                 ARE GIVEN BY R(J) = C + (J-
0.5)DR,

C                                 J=1,2,...,N, WHERE DR = (D-
C)/N.

C                                 N MUST BE GREATER THAN 4.

C

C                              NBDCND

C                                 INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                                 AT R = C AND R = D.

C

C

C                                 = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                      R = C AND R = D.

C

C                                 = 2  IF THE SOLUTION IS
SPECIFIED AT
```

```
C                                R = C AND THE DERIVATIVE
OF THE

C                                SOLUTION WITH RESPECT TO
R IS

C                                SPECIFIED AT R = D. (SEE
NOTE 1 BELOW)

C

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO R IS
SPECIFIED AT

C                                R = C AND R = D.

C

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO R IS

C                                SPECIFIED AT R = C AND
THE SOLUTION

C                                IS SPECIFIED AT R = D.

C

C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                                R = C = 0 (SEE NOTE 2
BELOW) AND THE

C                                SOLUTION IS SPECIFIED AT
R = D.

C

C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT
```

```
C                               R = C = 0 (SEE NOTE 2
BELOW)

C                               AND THE DERIVATIVE OF
THE SOLUTION

C                               WITH RESPECT TO R IS
SPECIFIED AT

C                               R = D.

C

C                          NOTE 1:

C                          IF C = 0 AND MBDCND = 3,6,8
OR 9, THE

C                          SYSTEM OF EQUATIONS TO BE
SOLVED IS

C                          SINGULAR.  THE UNIQUE
SOLUTION IS

C                          DETERMINED BY EXTRAPOLATION
TO THE

C                          SPECIFICATION OF
U(THETA(1),C).

C                          BUT IN THESE CASES THE RIGHT
SIDE OF THE

C                          SYSTEM WILL BE PERTURBED BY
THE CONSTANT

C                          PERTRB.

C

C                          NOTE 2:

C                          NBDCND = 5 OR 6 CANNOT BE
USED WITH

C                          MBDCND =1, 2, 4, 5, OR 7
```

```
C                           (THE FORMER INDICATES THAT
THE SOLUTION IS

C                           UNSPECIFIED AT R = 0; THE
LATTER INDICATES

C                           SOLUTION IS SPECIFIED).

C                           USE INSTEAD NBDCND = 1 OR 2.

C

C                         BDC

C                           A ONE DIMENSIONAL ARRAY OF
LENGTH M THAT

C                           SPECIFIES THE BOUNDARY VALUES
OF THE

C                           SOLUTION AT R = C.  WHEN
NBDCND = 1 OR 2,

C                             BDC(I) = U(THETA(I),C),
I=1,2,...,M.

C

C                           WHEN NBDCND = 3 OR 4,

C                             BDC(I) =
(D/DR)U(THETA(I),C), I=1,2,...,M.

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDC IS

C                           A DUMMY VARIABLE.

C

C                         BDD

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M THAT
```

```
C                           SPECIFIES THE BOUNDARY VALUES
OF THE

C                           SOLUTION AT R = D.  WHEN
NBDCND = 1 OR 4,

C                             BDD(I) = U(THETA(I),D) ,
I=1,2,...,M.

C

C                           WHEN NBDCND = 2 OR 3,

C                             BDD(I) =
(D/DR)U(THETA(I),D), I=1,2,...,M.

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDD IS

C                           A DUMMY VARIABLE.

C

C                         ELMBDA

C                           THE CONSTANT LAMBDA IN THE
MODIFIED

C                           HELMHOLTZ EQUATION.  IF
LAMBDA IS GREATER

C                           THAN 0, A SOLUTION MAY NOT
EXIST.

C                           HOWEVER, HSTCSP WILL ATTEMPT
TO FIND A

C                           SOLUTION.

C

C                         F

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE
```

```
C                           VALUES OF THE RIGHT SIDE OF
THE MODIFIED
C                           HELMHOLTZ EQUATION.  FOR
I=1,2,...,M AND
C                           J=1,2,...,N
C
C                                 F(I,J) =
F(THETA(I),R(J)) .
C
C                           F MUST BE DIMENSIONED AT
LEAST M X N.
C
C                         IDIMF
C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAY
C                           F AS IT APPEARS IN THE
PROGRAM CALLING
C                           HSTCSP.  THIS PARAMETER IS
USED TO SPECIFY
C                           THE VARIABLE DIMENSION OF F.
C                           IDIMF MUST BE AT LEAST M.
C
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C                           WITH K = INT(LOG2(N))+1 AND L
= 2**(K+1),
```

```
C                              W MAY REQUIRE UP TO
C                              (K-2)*L+K+MAX(2N,6M)+4(N+M)+5
LOCATIONS.
C                              THE ACTUAL NUMBER OF
LOCATIONS USED IS
C                              COMPUTED BY HSTCSP AND IS
RETURNED IN THE
C                              LOCATION W(1).
C
C
C ON OUTPUT                  F
C                              CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
C                              DIFFERENCE APPROXIMATION FOR
THE GRID POINT
C                              (THETA(I),R(J)) FOR
I=1,2,..,M, J=1,2,...,N.
C
C                            PERTRB
C                              IF A COMBINATION OF PERIODIC,
DERIVATIVE,
C                              OR UNSPECIFIED BOUNDARY
CONDITIONS IS
C                              SPECIFIED FOR A POISSON
EQUATION
C                              (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.
C                              PERTRB IS A CONSTANT,
CALCULATED AND
```

```
C                           SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                           SOLUTION EXISTS.  HSTCSP THEN
COMPUTES THIS

C                           SOLUTION, WHICH IS A LEAST
SQUARES SOLUTION

C                           TO THE ORIGINAL
APPROXIMATION.

C                           THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                           A SOLUTION; HENCE, THE
SOLUTION IS NOT

C                           UNIQUE.  THE VALUE OF PERTRB
SHOULD BE

C                           SMALL COMPARED TO THE RIGHT
SIDE F.

C                           OTHERWISE, A SOLUTION IS
OBTAINED TO AN

C                           ESSENTIALLY DIFFERENT
PROBLEM.

C                           THIS COMPARISON SHOULD ALWAYS
BE MADE TO

C                           INSURE THAT A MEANINGFUL
SOLUTION HAS BEEN

C                           OBTAINED.

C

C                         IERROR

C                           AN ERROR FLAG THAT INDICATES
INVALID INPUT
```

```
C                           PARAMETERS. EXCEPT FOR
NUMBERS 0 AND 10,

C                           A SOLUTION IS NOT ATTEMPTED.

C

C                           =  0  NO ERROR

C

C                           =  1  A .LT. 0 OR B .GT. PI

C

C                           =  2  A .GE. B

C

C                           =  3  MBDCND .LT. 1 OR MBDCND
.GT. 9

C

C                           =  4  C .LT. 0

C

C                           =  5  C .GE. D

C

C                           =  6  NBDCND .LT. 1 OR NBDCND
.GT. 6

C

C                           =  7  N .LT. 5

C

C                           =  8  NBDCND = 5 OR 6 AND

C                                 MBDCND = 1, 2, 4, 5, OR
7

C
```

```
C                          =  9  C .GT. 0 AND NBDCND
.GE. 5

C

C                          = 10  ELMBDA .GT. 0

C

C                          = 11  IDIMF .LT. M

C

C                          = 12  M .LT. 5

C

C                          = 13  A = 0 AND MBDCND
=1,2,3,4,7 OR 8

C

C                          = 14  B = PI AND MBDCND .LE.
6

C

C                          = 15  A .GT. 0 AND MBDCND =
5, 6, OR 9

C

C                          = 16  B .LT. PI AND MBDCND
.GE. 7

C

C                          = 17  LAMBDA .NE. 0 AND
NBDCND .GE. 5

C

C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                          A POSSIBLY INCORRECT CALL TO
HSTCSP,
```

```
C                             THE USER SHOULD TEST IERROR
AFTER THE CALL.

C

C                           W

C                             W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C                             ALSO  W CONTAINS INTERMEDIATE
VALUES THAT

C                             MUST NOT BE DESTROYED IF
HSTCSP WILL BE

C                             CALLED AGAIN WITH INTL = 1.

C

C I/O                       NONE

C

C PRECISION                 SINGLE

C

C REQUIRED LIBRARY          BLKTRI AND COMF FROM FISHPAK

C FILES

C

C LANGUAGE                  FORTRAN

C

C HISTORY                   WRITTEN BY ROLAND SWEET AT NCAR
IN 1977.

C                           RELEASED ON NCAR'S PUBLIC
SOFTWARE LIBRARIES

C                           IN JANUARY 1980.

C
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C PORTABILITY            FORTRAN 77
C
C ALGORITHM              THIS SUBROUTINE DEFINES THE
FINITE-DIFFERENCE
C                        EQUATIONS, INCORPORATES
BOUNDARY DATA, ADJUSTS
C                        THE RIGHT SIDE WHEN THE SYSTEM
IS SINGULAR
C                        AND CALLS BLKTRI WHICH SOLVES
THE LINEAR
C                        SYSTEM OF EQUATIONS.
C
C
C TIMING                 FOR LARGE M AND N, THE
OPERATION COUNT IS
C                        ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).  THE
C                        TIMING ALSO DEPENDS ON INPUT
PARAMETER INTL.
C
C ACCURACY               THE SOLUTION PROCESS EMPLOYED
RESULTS IN
C                        A LOSS OF NO MORE THAN FOUR
SIGNIFICANT
C                        DIGITS FOR N AND M AS LARGE AS
64.
C                        MORE DETAILED INFORMATION ABOUT
ACCURACY
C                        CAN BE FOUND IN THE
DOCUMENTATION FOR
```

```
C                         SUBROUTINE BLKTRI WHICH IS THE
ROUTINE

C                         SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES              P.N. SWARZTRAUBER, "A DIRECT
METHOD FOR

C                         THE DISCRETE SOLUTION OF
SEPARABLE ELLIPTIC

C                         EQUATIONS",

C                         SIAM J. NUMER. ANAL. 11(1974),
PP. 1136-1150.

C

C                         U. SCHUMANN AND R. SWEET, "A
DIRECT METHOD FOR

C                         THE SOLUTION OF POISSON'S
EQUATION WITH NEUMANN

C                         BOUNDARY CONDITIONS ON A
STAGGERED GRID OF

C                         ARBITRARY SIZE," J. COMP. PHYS.
20(1976),

C                         PP. 171-182.

C********************************************************
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```

HSTCYL

```
      SUBROUTINE HSTCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                   ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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*

C      *
*

C      *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C      *
*

C      *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
```

```
C     *
*

C     *                              BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

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C     *                              OF
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C

C

C

C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N),

C ARGUMENTS                W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C PURPOSE                  SOLVES THE STANDARD FIVE-POINT
FINITE

C                          DIFFERENCE APPROXIMATION ON A
STAGGERED

C                          GRID TO THE MODIFIED HELMHOLTZ
EQUATION

C                          IN CYLINDRICAL COORDINATES.
THIS EQUATION

C

C                            (1/R)(D/DR)(R(DU/DR)) +
(D/DZ)(DU/DZ)

C

C                            + LAMBDA*(1/R**2)*U = F(R,Z)

C

C                          IS A TWO-DIMENSIONAL MODIFIED
HELMHOLTZ
```

```
C                        EQUATION RESULTING FROM THE
FOURIER TRANSFORM

C                        OF A THREE-DIMENSIONAL POISSON
EQUATION.

C

C USAGE                  CALL HSTCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                     PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT               A,B

C

C                          THE RANGE OF R, I.E. A .LE. R
.LE. B.

C                          A MUST BE LESS THAN B AND A
MUST BE

C                          BE NON-NEGATIVE.

C

C                        M

C                          THE NUMBER OF GRID POINTS IN
THE INTERVAL

C                          (A,B).  THE GRID POINTS IN
THE R-DIRECTION

C                          R-DIRECTION ARE GIVEN BY

C                          R(I) = A + (I-0.5)DR FOR
I=1,2,...,M
```

```
C                            WHERE DR =(B-A)/M.

C                            M MUST BE GREATER THAN 2.

C

C                          MBDCND

C                            INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                            AT R = A AND R = B.

C

C                            = 1  IF THE SOLUTION IS
SPECIFIED AT R = A

C                                 (SEE NOTE BELOW) AND R =
B.

C

C                            = 2  IF THE SOLUTION IS
SPECIFIED AT R = A

C                                 (SEE NOTE BELOW) AND THE
DERIVATIVE

C                                 OF THE SOLUTION WITH
RESPECT TO R IS

C                                 SPECIFIED AT R = B.

C

C                            = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                 WITH RESPECT TO R IS
SPECIFIED AT

C                                 R = A (SEE NOTE BELOW)
AND R = B.

C
```

```
C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION
C                                WITH RESPECT TO R IS
SPECIFIED AT
C                                R = A (SEE NOTE BELOW)
AND THE
C                                SOLUTION IS SPECIFIED AT
R = B.
C
C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT
C                                R = A = 0 AND THE
SOLUTION IS
C                                SPECIFIED AT R = B.
C
C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT
C                                R = A = 0 AND THE
DERIVATIVE OF THE
C                                SOLUTION WITH RESPECT TO
R IS SPECIFIED
C                                AT R = B.
C
C                           NOTE:
C                           IF A = 0, DO NOT USE MBDCND =
1,2,3, OR 4,
C                           BUT INSTEAD USE MBDCND = 5 OR
6.
C                           THE RESULTING APPROXIMATION
GIVES THE ONLY
```

```
C                                MEANINGFUL BOUNDARY
CONDITION,

C                                I.E. DU/DR = 0.

C                                (SEE D. GREENSPAN,
'INTRODUCTORY NUMERICAL

C                                ANALYSIS OF ELLIPTIC BOUNDARY
VALUE

C                                PROBLEMS,' HARPER AND ROW,
1965, CHAPTER 5.)

C

C                              BDA

C                                A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT

C                                SPECIFIES THE BOUNDARY VALUES
(IF ANY)

C                                OF THE SOLUTION AT R = A.

C

C                                WHEN MBDCND = 1 OR 2,

C                                  BDA(J) = U(A,Z(J)) ,
J=1,2,...,N.

C

C                                WHEN MBDCND = 3 OR 4,

C                                  BDA(J) = (D/DR)U(A,Z(J)) ,
J=1,2,...,N.

C

C                                WHEN MBDCND = 5 OR 6, BDA IS
A DUMMY

C                                VARIABLE.
```

```
C
C                            BDB
C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT
C                              SPECIFIES THE BOUNDARY VALUES
OF THE
C                              SOLUTION AT R = B.
C
C                              WHEN MBDCND = 1,4,OR 5,
C                                BDB(J) = U(B,Z(J)) ,
J=1,2,...,N.
C
C                              WHEN MBDCND = 2,3, OR 6,
C                                BDB(J) = (D/DR)U(B,Z(J)) ,
J=1,2,...,N.
C
C                            C,D
C                              THE RANGE OF Z, I.E. C .LE. Z
.LE. D.
C                              C MUST BE LESS THAN D.
C
C                            N
C                              THE NUMBER OF UNKNOWNS IN THE
INTERVAL
C                              (C,D).  THE UNKNOWNS IN THE
Z-DIRECTION
C                              ARE GIVEN BY Z(J) = C + (J-
0.5)DZ,
```

```
C                           J=1,2,...,N, WHERE DZ = (D-
C)/N.

C                           N MUST BE GREATER THAN 2.

C

C                         NBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT Z = C  AND Z = D.

C

C                           = 0  IF THE SOLUTION IS
PERIODIC IN Z, I.E.

C                                U(I,J) = U(I,N+J).

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT Z = C

C                                AND Z = D.

C

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT Z = C

C                                AND THE DERIVATIVE OF
THE SOLUTION WITH

C                                RESPECT TO Z IS
SPECIFIED AT Z = D.

C

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION WITH

C                                RESPECT TO Z IS
SPECIFIED AT Z = C
```

```
C                                AND Z = D.
C
C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION WITH
C                                RESPECT TO Z IS
SPECIFIED AT Z = C AND
C                                THE SOLUTION IS
SPECIFIED AT Z = D.
C
C                         BDC
C                           A ONE DIMENSIONAL ARRAY OF
LENGTH M THAT
C                           SPECIFIES THE BOUNDARY VALUES
OF THE
C                           SOLUTION AT Z = C.
C
C                           WHEN NBDCND = 1 OR 2,
C                             BDC(I) = U(R(I),C)  ,
I=1,2,...,M.
C
C                           WHEN NBDCND = 3 OR 4,
C                             BDC(I) = (D/DZ)U(R(I),C),
I=1,2,...,M.
C
C                           WHEN NBDCND = 0, BDC IS A
DUMMY VARIABLE.
C
C                         BDD
```

```
C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M THAT

C                           SPECIFIES THE BOUNDARY VALUES
OF THE

C                           SOLUTION AT Z = D.

C

C                           WHEN NBDCND = 1 OR 4,

C                             BDD(I) = U(R(I),D) ,
I=1,2,...,M.

C

C                           WHEN NBDCND = 2 OR 3,

C                             BDD(I) = (D/DZ)U(R(I),D) ,
I=1,2,...,M.

C

C                           WHEN NBDCND = 0, BDD IS A
DUMMY VARIABLE.

C

C                         ELMBDA

C                           THE CONSTANT LAMBDA IN THE
MODIFIED

C                           HELMHOLTZ EQUATION.  IF
LAMBDA IS GREATER

C                           THAN 0, A SOLUTION MAY NOT
EXIST.

C                           HOWEVER, HSTCYL WILL ATTEMPT
TO FIND A

C                           SOLUTION.  LAMBDA MUST BE
ZERO WHEN

C                           MBDCND = 5 OR 6.
```

```
C
C                         F
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES
C                           THE VALUES OF THE RIGHT SIDE
OF THE
C                           MODIFIED HELMHOLTZ EQUATION.
C                           FOR I=1,2,...,M   AND
J=1,2,...,N
C                             F(I,J) = F(R(I),Z(J)) .
C                           F MUST BE DIMENSIONED AT
LEAST M X N.
C
C                         IDIMF
C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAY
C                           F AS IT APPEARS IN THE
PROGRAM CALLING
C                           HSTCYL.  THIS PARAMETER IS
USED TO SPECIFY
C                           THE VARIABLE DIMENSION OF F.
IDIMF MUST
C                           BE AT LEAST M.
C
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE. W MAY
```

```
C                            REQUIRE UP TO 13M + 4N +
M*INT(LOG2(N))

C                            LOCATIONS.  THE ACTUAL NUMBER
OF LOCATIONS

C                            USED IS COMPUTED BY HSTCYL
AND IS RETURNED

C                            IN THE LOCATION W(1).

C

C ON OUTPUT

C

C                          F

C                            CONTAINS THE SOLUTION U(I,J)
OF THE FINITE

C                            DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                            (R(I),Z(J)) FOR  I=1,2,...,M,
J=1,2,...,N.

C

C                          PERTRB

C                            IF A COMBINATION OF PERIODIC,
DERIVATIVE,

C                            OR UNSPECIFIED BOUNDARY
CONDITIONS IS

C                            SPECIFIED FOR A POISSON
EQUATION

C                            (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.

C                            PERTRB IS A CONSTANT,
CALCULATED AND
```

```
C                             SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                             SOLUTION EXISTS.  HSTCYL THEN
COMPUTES

C                             THIS SOLUTION, WHICH IS A
LEAST SQUARES

C                             SOLUTION TO THE ORIGINAL
APPROXIMATION.

C                             THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                             A SOLUTION; HENCE, THE
SOLUTION IS NOT

C                             UNIQUE.  THE VALUE OF PERTRB
SHOULD BE

C                             SMALL COMPARED TO THE RIGHT
SIDE F.

C                             OTHERWISE, A SOLUTION IS
OBTAINED TO AN

C                             ESSENTIALLY DIFFERENT
PROBLEM.

C                             THIS COMPARISON SHOULD ALWAYS
BE MADE TO

C                             INSURE THAT A MEANINGFUL
SOLUTION HAS BEEN

C                             OBTAINED.

C

C                           IERROR

C                             AN ERROR FLAG THAT INDICATES
INVALID INPUT
```

```
C                            PARAMETERS. EXCEPT TO NUMBERS
0 AND 11,

C                            A SOLUTION IS NOT ATTEMPTED.

C

C                            =  0  NO ERROR

C

C                            =  1  A .LT. 0

C

C                            =  2  A .GE. B

C

C                            =  3  MBDCND .LT. 1 OR MBDCND
.GT. 6

C

C                            =  4  C .GE. D

C

C                            =  5  N .LE. 2

C

C                            =  6  NBDCND .LT. 0 OR NBDCND
.GT. 4

C

C                            =  7  A = 0 AND MBDCND =
1,2,3, OR 4

C

C                            =  8  A .GT. 0 AND MBDCND
.GE. 5

C
```

```
C                          =  9  M .LE. 2

C

C                          = 10  IDIMF .LT. M

C

C                          = 11  LAMBDA .GT. 0

C

C                          = 12  A=0, MBDCND .GE. 5,
ELMBDA .NE. 0

C

C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                          A POSSIBLY INCORRECT CALL TO
HSTCYL, THE

C                          USER SHOULD TEST IERROR AFTER
THE CALL.

C

C                        W

C                          W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       COMF, GENBUN, GNBNAUX, AND
POISTG

C FILES                  FROM FISHPAK
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C

C LANGUAGE                 FORTRAN

C

C HISTORY                  WRITTEN BY ROLAND SWEET AT NCAR
IN 1977.

C                          RELEASED ON NCAR'S PUBLIC
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C                          IN JANUARY 1980.

C

C PORTABILITY              FORTRAN 77

C

C ALGORITHM                THIS SUBROUTINE DEFINES THE
FINITE-DIFFERENCE

C                          EQUATIONS, INCORPORATES
BOUNDARY DATA, ADJUSTS

C                          THE RIGHT SIDE WHEN THE SYSTEM
IS SINGULAR AND

C                          CALLS EITHER POISTG OR GENBUN
WHICH SOLVES THE

C                          LINEAR SYSTEM OF EQUATIONS.

C

C TIMING                   FOR LARGE M AND N, THE
OPERATION COUNT

C                          IS ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).

C

C ACCURACY                 THE SOLUTION PROCESS RESULTS IN
A LOSS
```

```
C                          OF NO MORE THAN FOUR
SIGNIFICANT DIGITS

C                          FOR N AND M AS LARGE AS 64.

C                          MORE DETAILED INFORMATION ABOUT
ACCURACY

C                          CAN BE FOUND IN THE
DOCUMENTATION FOR

C                          SUBROUTINE POISTG WHICH IS THE
ROUTINE THAT

C                          ACTUALLY SOLVES THE FINITE
DIFFERENCE

C                          EQUATIONS.

C

C REFERENCES               U. SCHUMANN AND R. SWEET, "A
DIRECT METHOD FOR

C                          THE SOLUTION OF POISSON'S
EQUATION WITH NEUMANN

C                          BOUNDARY CONDITIONS ON A
STAGGERED GRID OF

C                          ARBITRARY SIZE," J. COMP. PHYS.
20(1976),

C                          PP. 171-182.

C*******************************************************
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```

HSTPLR

```
      SUBROUTINE HSTPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                   ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
```

```
C     *
*

C     *                              BY
*

C     *
*

C     *         JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

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C

C

C

C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N),

C ARGUMENTS              W(SEE ARGUMENT LIST)

C

C LATEST REVISION        NOVEMBER 1988

C

C PURPOSE                SOLVES THE STANDARD FIVE-POINT
FINITE

C                        DIFFERENCE APPROXIMATION ON A
STAGGERED

C                        GRID TO THE HELMHOLTZ EQUATION
IN POLAR

C                        COORDINATES.  THE EQUATION IS

C

C                           (1/R)(D/DR)(R(DU/DR)) +

C
(1/R**2)(D/DTHETA)(DU/DTHETA) +

C                           LAMBDA*U = F(R,THETA)

C

C USAGE                  CALL HSTPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,
```

```
C
IDIMF,PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                A,B

C

C                           THE RANGE OF R, I.E. A .LE. R
.LE. B.

C                           A MUST BE LESS THAN B AND A
MUST BE

C                           NON-NEGATIVE.

C

C                         M

C                           THE NUMBER OF GRID POINTS IN
THE INTERVAL

C                           (A,B).  THE GRID POINTS IN
THE R-DIRECTION

C                           ARE GIVEN BY R(I) = A + (I-
0.5)DR FOR

C                           I=1,2,...,M WHERE DR =(B-
A)/M.

C                           M MUST BE GREATER THAN 2.

C

C                         MBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT R = A AND R = B.

C
```

```
C                              = 1  IF THE SOLUTION IS
SPECIFIED AT R = A

C                                   AND R = B.

C

C                              = 2  IF THE SOLUTION IS
SPECIFIED AT R = A

C                                   AND THE DERIVATIVE OF
THE SOLUTION

C                                   WITH RESPECT TO R IS
SPECIFIED AT R = B.

C                                   (SEE NOTE 1 BELOW)

C

C                              = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                   WITH RESPECT TO R IS
SPECIFIED AT

C                                   R = A (SEE NOTE 2 BELOW)
AND R = B.

C

C                              = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                   WITH RESPECT TO R IS
SPECIFIED AT

C                                   SPECIFIED AT R = A (SEE
NOTE 2 BELOW)

C                                   AND THE SOLUTION IS
SPECIFIED AT R = B.

C

C
```

```
C                          = 5  IF THE SOLUTION IS
UNSPECIFIED AT
C                               R = A = 0 AND THE
SOLUTION IS
C                               SPECIFIED AT R = B.
C
C                          = 6  IF THE SOLUTION IS
UNSPECIFIED AT
C                               R = A = 0 AND THE
DERIVATIVE OF THE
C                               SOLUTION WITH RESPECT TO
R IS SPECIFIED
C                               AT R = B.
C
C                          NOTE 1:
C                          IF A = 0, MBDCND = 2, AND
NBDCND = 0 OR 3,
C                          THE SYSTEM OF EQUATIONS TO BE
SOLVED IS
C                          SINGULAR.  THE UNIQUE
SOLUTION IS
C                          IS DETERMINED BY
EXTRAPOLATION TO THE
C                          SPECIFICATION OF
U(0,THETA(1)).
C                          BUT IN THIS CASE THE RIGHT
SIDE OF THE
C                          SYSTEM WILL BE PERTURBED BY
THE CONSTANT
C                          PERTRB.
```

```
C
C                            NOTE 2:
C                            IF A = 0, DO NOT USE MBDCND =
3 OR 4,
C                            BUT INSTEAD USE MBDCND =
1,2,5, OR 6.
C
C                          BDA
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT
C                            SPECIFIES THE BOUNDARY VALUES
(IF ANY) OF
C                            THE SOLUTION AT R = A.
C
C                            WHEN MBDCND = 1 OR 2,
C                              BDA(J) = U(A,THETA(J)) ,
J=1,2,...,N.
C
C                            WHEN MBDCND = 3 OR 4,
C                              BDA(J) =
(D/DR)U(A,THETA(J)) ,
C                              J=1,2,...,N.
C
C                            WHEN MBDCND = 5 OR 6, BDA IS
A DUMMY
C                            VARIABLE.
C
```

```
C                          BDB
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT
C                            SPECIFIES THE BOUNDARY VALUES
OF THE
C                            SOLUTION AT R = B.
C
C                            WHEN MBDCND = 1,4, OR 5,
C                              BDB(J) = U(B,THETA(J)) ,
J=1,2,...,N.
C
C                            WHEN MBDCND = 2,3, OR 6,
C                              BDB(J) =
(D/DR)U(B,THETA(J)) ,
C                              J=1,2,...,N.
C
C                          C,D
C                            THE RANGE OF THETA, I.E. C
.LE. THETA .LE. D.
C                            C MUST BE LESS THAN D.
C
C                          N
C                            THE NUMBER OF UNKNOWNS IN THE
INTERVAL
C                            (C,D).  THE UNKNOWNS IN THE
THETA-
C                            DIRECTION ARE GIVEN BY
THETA(J) = C +
```

```
C                           (J-0.5)DT,   J=1,2,...,N,
WHERE

C                           DT = (D-C)/N.  N MUST BE
GREATER THAN 2.

C

C                         NBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT THETA = C  AND THETA = D.

C

C                           = 0  IF THE SOLUTION IS
PERIODIC IN THETA,

C                                I.E. U(I,J) = U(I,N+J).

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = C AND THETA = D

C                                (SEE NOTE BELOW).

C

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = C AND THE
DERIVATIVE OF THE

C                                SOLUTION WITH RESPECT TO
THETA IS

C                                SPECIFIED AT THETA = D

C                                (SEE NOTE BELOW).

C
```

```
C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO THETA IS
SPECIFIED

C                               AT THETA = C AND THETA =
D.

C

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO THETA IS
SPECIFIED

C                               AT THETA = C AND THE
SOLUTION IS

C                               SPECIFIED AT THETA = D

C                               (SEE NOTE BELOW).

C

C                          NOTE:

C                          WHEN NBDCND = 1, 2, OR 4, DO
NOT USE

C                          MBDCND = 5 OR 6 (THE FORMER
INDICATES THAT

C                          THE SOLUTION IS SPECIFIED AT
R =  0; THE

C                          LATTER INDICATES THE SOLUTION
IS UNSPECIFIED

C                          AT R = 0).  USE INSTEAD
MBDCND = 1 OR 2.

C

C                        BDC
```

```
C                            A ONE DIMENSIONAL ARRAY OF
LENGTH M THAT
C                            SPECIFIES THE BOUNDARY VALUES
OF THE
C                            SOLUTION AT THETA = C.
C
C                            WHEN NBDCND = 1 OR 2,
C                              BDC(I) = U(R(I),C) ,
I=1,2,...,M.
C
C                            WHEN NBDCND = 3 OR 4,
C                              BDC(I) =
(D/DTHETA)U(R(I),C),
C                              I=1,2,...,M.
C
C                            WHEN NBDCND = 0, BDC IS A
DUMMY VARIABLE.
C
C                          BDD
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH M THAT
C                            SPECIFIES THE BOUNDARY VALUES
OF THE
C                            SOLUTION AT THETA = D.
C
C                            WHEN NBDCND = 1 OR 4,
C                              BDD(I) = U(R(I),D) ,
I=1,2,...,M.
```

```
C
C                           WHEN NBDCND = 2 OR 3,
C                             BDD(I)
=(D/DTHETA)U(R(I),D), I=1,2,...,M.
C
C                           WHEN NBDCND = 0, BDD IS A
DUMMY VARIABLE.
C
C                         ELMBDA
C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                           EQUATION.  IF LAMBDA IS
GREATER THAN 0,
C                           A SOLUTION MAY NOT EXIST.
HOWEVER, HSTPLR
C                           WILL ATTEMPT TO FIND A
SOLUTION.
C
C                         F
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE
C                           VALUES OF THE RIGHT SIDE OF
THE HELMHOLTZ
C                           EQUATION.
C
C                           FOR I=1,2,...,M AND
J=1,2,...,N
C                             F(I,J) = F(R(I),THETA(J)) .
```

```
C

C                              F MUST BE DIMENSIONED AT
LEAST M X N.

C

C                            IDIMF

C                              THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                              F AS IT APPEARS IN THE
PROGRAM CALLING

C                              HSTPLR.  THIS PARAMETER IS
USED TO SPECIFY

C                              THE VARIABLE DIMENSION OF F.

C                              IDIMF MUST BE AT LEAST M.

C

C                            W

C                              A ONE-DIMENSIONAL ARRAY THAT
MUST BE

C                              PROVIDED BY THE USER FOR WORK
SPACE.

C                              W MAY REQUIRE UP TO 13M + 4N
+

C                              M*INT(LOG2(N)) LOCATIONS.

C                              THE ACTUAL NUMBER OF
LOCATIONS USED IS

C                              COMPUTED BY HSTPLR AND IS
RETURNED IN

C                              THE LOCATION W(1).

C
```

```
C
C ON OUTPUT
C
C                         F
C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT
C                           (R(I),THETA(J)) FOR
I=1,2,...,M,
C                           J=1,2,...,N.
C
C                         PERTRB
C                           IF A COMBINATION OF PERIODIC,
DERIVATIVE,
C                           OR UNSPECIFIED BOUNDARY
CONDITIONS IS
C                           SPECIFIED FOR A POISSON
EQUATION
C                           (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.
C                           PERTRB IS A CONSTANT
CALCULATED AND
C                           SUBTRACTED FROM F, WHICH
ENSURES THAT A
C                           SOLUTION EXISTS.  HSTPLR THEN
COMPUTES THIS
C                           SOLUTION, WHICH IS A LEAST
SQUARES SOLUTION
```

```
C                            TO THE ORIGINAL
APPROXIMATION.

C                            THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                            A SOLUTION; HENCE, THE
SOLUTION IS NOT

C                            UNIQUE.  THE VALUE OF PERTRB
SHOULD BE

C                            SMALL COMPARED TO THE RIGHT
SIDE F.

C                            OTHERWISE, A SOLUTION IS
OBTAINED TO AN

C                            ESSENTIALLY DIFFERENT
PROBLEM.

C                            THIS COMPARISON SHOULD ALWAYS
BE MADE TO

C                            INSURE THAT A MEANINGFUL
SOLUTION HAS BEEN

C                            OBTAINED.

C

C                          IERROR

C                            AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                            PARAMETERS. EXCEPT TO NUMBERS
0 AND 11,

C                            A SOLUTION IS NOT ATTEMPTED.

C

C                            =  0  NO ERROR

C
```

```
C                           =  1  A .LT. 0

C

C                           =  2  A .GE. B

C

C                           =  3  MBDCND .LT. 1 OR MBDCND
.GT. 6

C

C                           =  4  C .GE. D

C

C                           =  5  N .LE. 2

C

C                           =  6  NBDCND .LT. 0 OR NBDCND
.GT. 4

C

C                           =  7  A = 0 AND MBDCND = 3 OR
4

C

C                           =  8  A .GT. 0 AND MBDCND
.GE. 5

C

C                           =  9  MBDCND .GE. 5 AND
NBDCND .NE. 0 OR 3

C

C                           = 10  IDIMF .LT. M

C

C                           = 11  LAMBDA .GT. 0
```

```
C
C                                  = 12  M .LE. 2
C
C                                  SINCE THIS IS THE ONLY MEANS
OF INDICATING
C                                  A POSSIBLY INCORRECT CALL TO
HSTPLR, THE
C                                  USER SHOULD TEST IERROR AFTER
THE CALL.
C
C                                W
C                                  W(1) CONTAINS THE REQUIRED
LENGTH OF W.
C
C I/O                            NONE
C
C PRECISION                      SINGLE
C
C REQUIRED LIBRARY               COMF, GENBUN, GNBNAUX, AND
POISTG
C FILES                          FROM FISHPAK
C
C LANGUAGE                       FORTRAN
C
C HISTORY                        WRITTEN BY ROLAND SWEET AT NCAR
IN 1977.
```

```
C                        RELEASED ON NCAR'S PUBLIC
SOFTWARE LIBRARIES

C                        IN JANUARY 1980.

C

C PORTABILITY            FORTRAN 77.

C

C ALGORITHM              THIS SUBROUTINE DEFINES THE
FINITE-

C                        DIFFERENCE EQUATIONS,
INCORPORATES BOUNDARY

C                        DATA, ADJUSTS THE RIGHT SIDE
WHEN THE SYSTEM

C                        IS SINGULAR AND CALLS EITHER
POISTG OR GENBUN

C                        WHICH SOLVES THE LINEAR SYSTEM
OF EQUATIONS.

C

C TIMING                 FOR LARGE M AND N, THE
OPERATION COUNT

C                        IS ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).

C

C ACCURACY               THE SOLUTION PROCESS EMPLOYED
RESULTS IN

C                        A LOSS OF NO MORE THAN FOUR
SIGNIFICANT

C                        DIGITS FOR N AND M AS LARGE AS
64.

C                        MORE DETAILED INFORMATION ABOUT
ACCURACY
```

```
C                                CAN BE FOUND IN THE
DOCUMENTATION FOR

C                                ROUTINE POISTG WHICH IS THE
ROUTINE THAT

C                                ACTUALLY SOLVES THE FINITE
DIFFERENCE

C                                EQUATIONS.

C

C REFERENCES                     U. SCHUMANN AND R. SWEET, "A
DIRECT METHOD

C                                FOR THE SOLUTION OF POISSON'S
EQUATION WITH

C                                NEUMANN BOUNDARY CONDITIONS ON
A STAGGERED

C                                GRID OF ARBITRARY SIZE," J.
COMP. PHYS.

C                                20(1976), PP. 171-182.

C********************************************************
****************
```

HSTSSP

```
      SUBROUTINE HSTSSP
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                           ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

```
C     *
*
C     *                    copyright (c) 1999 by UCAR
*
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RESEARCH        *
C     *
*
C     *                        all rights reserved
*
C     *
*
C     *                        FISHPACK version 4.1
*
C     *
*
C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *
C     *
*
C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS       *
C     *
*
C     *                               BY
*
C     *
*
```

```
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                          OF
*

C     *
*

C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH          *

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C     *                BOULDER, COLORADO  (80307)
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C     *
*

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C
```

```
C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N),

C ARGUMENTS                W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C PURPOSE                  SOLVES THE STANDARD FIVE-POINT
FINITE

C                          DIFFERENCE APPROXIMATION ON A
STAGGERED GRID

C                          TO THE HELMHOLTZ EQUATION IN
SPHERICAL

C                          COORDINATES AND ON THE SURFACE
OF THE UNIT

C                          SPHERE (RADIUS OF 1).  THE
EQUATION IS

C

C
(1/SIN(THETA))(D/DTHETA)(SIN(THETA)

C                            (DU/DTHETA)) +
(1/SIN(THETA)**2)

C                            (D/DPHI)(DU/DPHI) + LAMBDA*U
= F(THETA,PHI)

C

C                          WHERE THETA IS COLATITUDE AND
PHI IS

C                          LONGITUDE.

C
```

```
C USAGE                  CALL HSTSSP
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                       PERTRB,IERROR,W)

C

C

C ARGUMENTS

C ON INPUT

C

C                        A,B

C                          THE RANGE OF THETA
(COLATITUDE),

C                          I.E. A .LE. THETA .LE. B.

C                          A MUST BE LESS THAN B AND A
MUST BE

C                          NON-NEGATIVE.  A AND B ARE IN
RADIANS.

C                          A = 0 CORRESPONDS TO THE
NORTH POLE AND

C                          B = PI CORRESPONDS TO THE
SOUTH POLE.

C

C

C                            * * *  IMPORTANT  * * *

C

C                          IF B IS EQUAL TO PI, THEN B
MUST BE
```

```
C                           COMPUTED USING THE STATEMENT
C                             B = PIMACH(DUM)
C
C                           THIS INSURES THAT B IN THE
USER"S PROGRAM
C                           IS EQUAL TO PI IN THIS
PROGRAM WHICH
C                           PERMITS SEVERAL TESTS OF THE
INPUT
C                           PARAMETERS THAT OTHERWISE
WOULD NOT BE
C                           POSSIBLE.
C
C                             * * * * * * * * * * * *
C                         M
C                           THE NUMBER OF GRID POINTS IN
THE INTERVAL
C                           (A,B).  THE GRID POINTS IN
THE THETA
C                           DIRECTION ARE GIVEN BY
C                           THETA(I) = A + (I-0.5)DTHETA
C                           FOR I=1,2,...,M WHERE DTHETA
=(B-A)/M.
C                           M MUST BE GREATER THAN 2.
C
C                         MBDCND
C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS
```

```
C                          AT THETA = A AND THETA = B.

C

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = A AND THETA = B.

C                               (SEE NOTE 3 BELOW)

C

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = A AND THE
DERIVATIVE OF THE

C                               SOLUTION WITH RESPECT TO
THETA IS

C                               SPECIFIED AT THETA = B

C                               (SEE NOTES 2 AND 3
BELOW).

C

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                             WITH RESPECT TO THETA IS
SPECIFIED

C                             AT THETA = A

C                             (SEE NOTES 1, 2 BELOW) AND
THETA = B.

C

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO THETA IS
SPECIFIED
```

```
C                                AT THETA = A

C                                (SEE NOTES 1 AND 2
BELOW) AND THE

C                                SOLUTION IS SPECIFIED AT
THETA = B.

C

C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = A = 0 AND THE
SOLUTION IS

C                                SPECIFIED AT THETA = B.

C                                (SEE NOTE 3 BELOW)

C

C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = A = 0 AND THE
DERIVATIVE

C                                OF THE SOLUTION WITH
RESPECT TO THETA

C                                IS SPECIFIED AT THETA =
B

C                                (SEE NOTE 2 BELOW).

C

C                           = 7  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = A AND THE
SOLUTION IS

C                                UNSPECIFIED AT THETA = B
= PI.
```

```
C                                (SEE NOTE 3 BELOW)

C

C                           = 8  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED AT

C                                THETA = A (SEE NOTE 1
BELOW)

C                                AND THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = B = PI.

C

C                           = 9  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = A = 0 AND THETA
= B = PI.

C

C                           NOTE 1:

C                           IF A = 0, DO NOT USE MBDCND =
3, 4, OR 8,

C                           BUT INSTEAD USE MBDCND = 5,
6, OR 9.

C

C                           NOTE 2:

C                           IF B = PI, DO NOT USE MBDCND
= 2, 3, OR 6,

C                           BUT INSTEAD USE MBDCND = 7,
8, OR 9.

C
```

```
C                           NOTE 3:
C                           WHEN THE SOLUTION IS
SPECIFIED AT
C                           THETA = 0 AND/OR THETA = PI
AND THE OTHER
C                           BOUNDARY CONDITIONS ARE
COMBINATIONS
C                           OF UNSPECIFIED, NORMAL
DERIVATIVE, OR
C                           PERIODICITY A SINGULAR SYSTEM
RESULTS.
C                           THE UNIQUE SOLUTION IS
DETERMINED BY
C                           EXTRAPOLATION TO THE
SPECIFICATION OF THE
C                           SOLUTION AT EITHER THETA = 0
OR THETA = PI.
C                           BUT IN THESE CASES THE RIGHT
SIDE OF THE
C                           SYSTEM  WILL BE PERTURBED BY
THE CONSTANT
C                           PERTRB.
C
C                         BDA
C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT
C                           SPECIFIES THE BOUNDARY VALUES
(IF ANY) OF
C                           THE SOLUTION AT THETA = A.
C
```

```
C                          WHEN MBDCND = 1, 2, OR 7,

C                            BDA(J) = U(A,PHI(J)) ,
J=1,2,...,N.

C

C                          WHEN MBDCND = 3, 4, OR 8,

C                            BDA(J) =
(D/DTHETA)U(A,PHI(J)) ,

C                            J=1,2,...,N.

C

C                          WHEN MBDCND HAS ANY OTHER
VALUE,

C                          BDA IS A DUMMY VARIABLE.

C

C                        BDB

C                          A ONE-DIMENSIONAL ARRAY OF
LENGTH N THAT

C                          SPECIFIES THE BOUNDARY VALUES
OF THE

C                          SOLUTION AT THETA = B.

C

C                          WHEN MBDCND = 1,4, OR 5,

C                            BDB(J) = U(B,PHI(J)) ,
J=1,2,...,N.

C

C                          WHEN MBDCND = 2,3, OR 6,

C                            BDB(J) =
(D/DTHETA)U(B,PHI(J)) ,
```

```
C                              J=1,2,...,N.
C
C                             WHEN MBDCND HAS ANY OTHER
VALUE, BDB IS
C                             A DUMMY VARIABLE.
C
C                           C,D
C                             THE RANGE OF PHI (LONGITUDE),
C                             I.E. C .LE. PHI .LE. D.
C                             C MUST BE LESS THAN D.  IF D-
C = 2*PI,
C                             PERIODIC BOUNDARY CONDITIONS
ARE USUALLY
C                             USUALLY PRESCRIBED.
C
C                           N
C                             THE NUMBER OF UNKNOWNS IN THE
INTERVAL
C                             (C,D).  THE UNKNOWNS IN THE
PHI-DIRECTION
C                             ARE GIVEN BY PHI(J) = C + (J-
0.5)DPHI,
C                             J=1,2,...,N, WHERE DPHI = (D-
C)/N.
C                             N MUST BE GREATER THAN 2.
C
C                           NBDCND
```

```
C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT PHI = C  AND PHI = D.

C

C                           = 0  IF THE SOLUTION IS
PERIODIC IN PHI,

C                                I.E.  U(I,J) = U(I,N+J).

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                PHI = C AND PHI = D

C                                (SEE NOTE BELOW).

C

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                PHI = C AND THE
DERIVATIVE OF THE

C                                SOLUTION WITH RESPECT TO
PHI IS

C                                SPECIFIED AT PHI = D

C                                (SEE NOTE BELOW).

C

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO PHI IS
SPECIFIED

C                                AT PHI = C AND PHI = D.

C
```

```
C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO PHI IS
SPECIFIED

C                               AT PHI = C AND THE
SOLUTION IS

C                               SPECIFIED AT PHI = D

C                               (SEE NOTE BELOW).

C

C                          NOTE:

C                          WHEN NBDCND = 1, 2, OR 4, DO
NOT USE

C                          MBDCND = 5, 6, 7, 8, OR 9

C                          (THE FORMER INDICATES THAT
THE SOLUTION

C                          IS SPECIFIED AT A POLE; THE
LATTER

C                          INDICATES THE SOLUTION IS
UNSPECIFIED).

C                          USE INSTEAD MBDCND = 1 OR 2.

C

C                        BDC

C                          A ONE DIMENSIONAL ARRAY OF
LENGTH M THAT

C                          SPECIFIES THE BOUNDARY VALUES
OF THE

C                          SOLUTION AT PHI = C.

C
```

```
C                          WHEN NBDCND = 1 OR 2,

C                            BDC(I) = U(THETA(I),C) ,
I=1,2,...,M.

C

C                          WHEN NBDCND = 3 OR 4,

C                            BDC(I) =
(D/DPHI)U(THETA(I),C),

C                            I=1,2,...,M.

C

C                          WHEN NBDCND = 0, BDC IS A
DUMMY VARIABLE.

C

C                        BDD

C                          A ONE-DIMENSIONAL ARRAY OF
LENGTH M THAT

C                          SPECIFIES THE BOUNDARY VALUES
OF THE

C                          SOLUTION AT PHI = D.

C

C                          WHEN NBDCND = 1 OR 4,

C                            BDD(I) = U(THETA(I),D) ,
I=1,2,...,M.

C

C                          WHEN NBDCND = 2 OR 3,

C                            BDD(I) =
(D/DPHI)U(THETA(I),D) ,

C                            I=1,2,...,M.
```

```
C
C                            WHEN NBDCND = 0, BDD IS A
DUMMY VARIABLE.
C
C                          ELMBDA
C                            THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                            EQUATION.  IF LAMBDA IS
GREATER THAN 0,
C                            A SOLUTION MAY NOT EXIST.
HOWEVER,
C                            HSTSSP WILL ATTEMPT TO FIND A
SOLUTION.
C
C                          F
C                            A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES
C                            THE VALUES OF THE RIGHT SIDE
OF THE
C                            HELMHOLTZ EQUATION.
C                            FOR I=1,2,...,M AND
J=1,2,...,N
C
C                              F(I,J) = F(THETA(I),PHI(J))
.
C
C                            F MUST BE DIMENSIONED AT
LEAST M X N.
C
```

```
C                         IDIMF
C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAY
C                           F AS IT APPEARS IN THE
PROGRAM CALLING
C                           HSTSSP.  THIS PARAMETER IS
USED TO SPECIFY
C                           THE VARIABLE DIMENSION OF F.
C                           IDIMF MUST BE AT LEAST M.
C
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C                           W MAY REQUIRE UP TO 13M + 4N
+
C                           M*INT(LOG2(N))  LOCATIONS.
C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS
C                           COMPUTED BY HSTSSP AND IS
RETURNED IN
C                           THE LOCATION W(1).
C
C
C ON OUTPUT               F
C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
```

```
C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                           (THETA(I),PHI(J)) FOR

C                           I=1,2,...,M, J=1,2,...,N.

C

C                         PERTRB

C                           IF A COMBINATION OF PERIODIC,
DERIVATIVE,

C                           OR UNSPECIFIED BOUNDARY
CONDITIONS IS

C                           SPECIFIED FOR A POISSON
EQUATION

C                           (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.

C                           PERTRB IS A CONSTANT,
CALCULATED AND

C                           SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                           SOLUTION EXISTS.  HSTSSP THEN
COMPUTES

C                           THIS SOLUTION, WHICH IS A
LEAST SQUARES

C                           SOLUTION TO THE ORIGINAL
APPROXIMATION.

C                           THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                           A SOLUTION; HENCE, THE
SOLUTION IS NOT

C                           UNIQUE.  THE VALUE OF PERTRB
SHOULD BE
```

```
C                           SMALL COMPARED TO THE RIGHT
SIDE F.

C                           OTHERWISE, A SOLUTION IS
OBTAINED TO AN

C                           ESSENTIALLY DIFFERENT
PROBLEM.

C                           THIS COMPARISON SHOULD ALWAYS
BE MADE TO

C                           INSURE THAT A MEANINGFUL
SOLUTION HAS BEEN

C                           OBTAINED.

C

C                         IERROR

C                           AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                           PARAMETERS. EXCEPT TO NUMBERS
0 AND 14,

C                           A SOLUTION IS NOT ATTEMPTED.

C

C                           =  0  NO ERROR

C

C                           =  1  A .LT. 0 OR B .GT. PI

C

C                           =  2  A .GE. B

C

C                           =  3  MBDCND .LT. 1 OR MBDCND
.GT. 9

C
```

```
C                          =  4  C .GE. D

C

C                          =  5  N .LE. 2

C

C                          =  6  NBDCND .LT. 0 OR NBDCND
.GT. 4

C

C                          =  7  A .GT. 0 AND MBDCND =
5, 6, OR 9

C

C                          =  8  A = 0 AND MBDCND = 3,
4, OR 8

C

C                          =  9  B .LT. PI AND MBDCND
.GE. 7

C

C                          = 10  B = PI AND MBDCND =
2,3, OR 6

C

C                          = 11  MBDCND .GE. 5 AND
NDBCND = 1, 2, OR 4

C

C                          = 12  IDIMF .LT. M

C

C                          = 13  M .LE. 2

C

C                          = 14  LAMBDA .GT. 0
```

```
C
C                           SINCE THIS IS THE ONLY MEANS
OF INDICATING
C                           A POSSIBLY INCORRECT CALL TO
HSTSSP, THE
C                           USER SHOULD TEST IERROR AFTER
THE CALL.
C
C                         W
C                           W(1) CONTAINS THE REQUIRED
LENGTH OF W.
C
C I/O                     NONE
C
C PRECISION               SINGLE
C
C REQUIRED LIBRARY        COMF, GENBUN, GNBNAUX, AND
POISTG
C FILES                   FROM FISHPAK
C
C LANGUAGE                FORTRAN
C
C HISTORY                 WRITTEN BY ROLAND SWEET AT NCAR
IN 1977.
C                         RELEASED ON NCAR'S PUBLIC
SOFTWARE LIBRARIES
C                         IN JANUARY 1980.
```

```
C

C PORTABILITY                FORTRAN 77.

C

C ALGORITHM                  THIS SUBROUTINE DEFINES THE
FINITE-

C                            DIFFERENCE EQUATIONS,
INCORPORATES BOUNDARY

C                            DATA, ADJUSTS THE RIGHT SIDE
WHEN THE SYSTEM

C                            IS SINGULAR AND CALLS EITHER
POISTG OR GENBUN

C                            WHICH SOLVES THE LINEAR SYSTEM
OF EQUATIONS.

C

C TIMING                     FOR LARGE M AND N, THE
OPERATION COUNT

C                            IS ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).

C

C ACCURACY                   THE SOLUTION PROCESS EMPLOYED
RESULTS IN

C                            A LOSS OF NO MORE THAN FOUR
SIGNIFICANT

C                            DIGITS FOR N AND M AS LARGE AS
64.

C                            MORE DETAILED INFORMATION ABOUT
ACCURACY

C                            CAN BE FOUND IN THE
DOCUMENTATION FOR
```

```
C                        ROUTINE POISTG WHICH IS THE
ROUTINE THAT

C                        ACTUALLY SOLVES THE FINITE
DIFFERENCE

C                        EQUATIONS.

C

C REFERENCES             U. SCHUMANN AND R. SWEET,"A
DIRECT METHOD

C                        FOR THE SOLUTION OF POISSON'S
EQUATION WITH

C                        NEUMANN BOUNDARY CONDITIONS ON
A STAGGERED

C                        GRID OF ARBITRARY SIZE," J.
COMP. PHYS.

C                        20(1976), PP. 171-182.

C*******************************************************
****************
```

HW3CRT

```
      SUBROUTINE HW3CRT
(XS,XF,L,LBDCND,BDXS,BDXF,YS,YF,M,MBDCND,BDYS,

     1
BDYF,ZS,ZF,N,NBDCND,BDZS,BDZF,ELMBDA,LDIMF,

     2                   MDIMF,F,PERTRB,IERROR,W)

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

```
C     *
*

C     *                    copyright (c) 1999 by UCAR
*

C     *
*

C     *       UNIVERSITY CORPORATION for ATMOSPHERIC
RESEARCH         *

C     *
*

C     *                        all rights reserved
*

C     *
*

C     *                        FISHPACK version 4.1
*

C     *
*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                               BY
*

C     *
*
```

```
C     *         JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                              OF
*

C     *
*

C     *          THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH         *

C     *
*

C     *                 BOULDER, COLORADO  (80307)
U.S.A.             *

C     *
*

C     *                    WHICH IS SPONSORED BY
*

C     *
*

C     *               THE NATIONAL SCIENCE FOUNDATION
*

C     *
*

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C
```

```
C DIMENSION OF            BDXS(MDIMF,N+1),
BDXF(MDIMF,N+1),

C ARGUMENTS               BDYS(LDIMF,N+1),
BDYF(LDIMF,N+1),

C                         BDZS(LDIMF,M+1),
BDZF(LDIMF,M+1),

C                         F(LDIMF,MDIMF,N+1), W(SEE
ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988

C

C PURPOSE                 SOLVES THE STANDARD FIVE-POINT
FINITE

C                         DIFFERENCE APPROXIMATION TO THE
HELMHOLTZ

C                         EQUATION IN CARTESIAN
COORDINATES.  THIS

C                         EQUATION IS

C

C                           (D/DX)(DU/DX) + (D/DY)(DU/DY)
+

C                           (D/DZ)(DU/DZ) + LAMBDA*U =
F(X,Y,Z) .

C

C USAGE                   CALL HW3CRT
(XS,XF,L,LBDCND,BDXS,BDXF,YS,YF,M,

C
MBDCND,BDYS,BDYF,ZS,ZF,N,NBDCND,

C
BDZS,BDZF,ELMBDA,LDIMF,MDIMF,F,
```

```
C                                                PERTRB,IERROR,W)

C

C ARGUMENTS

C

C ON INPUT                    XS,XF

C

C                               THE RANGE OF X, I.E. XS .LE.
X .LE. XF .

C                               XS MUST BE LESS THAN XF.

C

C                             L

C                               THE NUMBER OF PANELS INTO
WHICH THE

C                               INTERVAL (XS,XF) IS
SUBDIVIDED.

C                               HENCE, THERE WILL BE L+1 GRID
POINTS

C                               IN THE X-DIRECTION GIVEN BY

C                               X(I) = XS+(I-1)DX FOR
I=1,2,...,L+1,

C                               WHERE DX = (XF-XS)/L IS THE
PANEL WIDTH.

C                               L MUST BE AT LEAST 5.

C

C                             LBDCND

C                               INDICATES THE TYPE OF
BOUNDARY CONDITIONS
```

```
C                          AT X = XS AND X = XF.

C

C                          = 0  IF THE SOLUTION IS
PERIODIC IN X,

C                               I.E. U(L+I,J,K) =
U(I,J,K).

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               X = XS AND X = XF.

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               X = XS AND THE
DERIVATIVE OF THE

C                               SOLUTION WITH RESPECT TO
X IS

C                               SPECIFIED AT X = XF.

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO X IS
SPECIFIED AT

C                               X = XS AND X = XF.

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO X IS
SPECIFIED AT

C                               X = XS AND THE SOLUTION
IS SPECIFIED

C                               AT X=XF.

C
```

```
C                         BDXS
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE
C                           VALUES OF THE DERIVATIVE OF
THE SOLUTION
C                           WITH RESPECT TO X AT X = XS.
C
C                           WHEN LBDCND = 3 OR 4,
C
C                             BDXS(J,K) =
(D/DX)U(XS,Y(J),Z(K)),
C                             J=1,2,...,M+1,
K=1,2,...,N+1.
C
C                           WHEN LBDCND HAS ANY OTHER
VALUE, BDXS
C                           IS A DUMMY VARIABLE. BDXS
MUST BE
C                           DIMENSIONED AT LEAST
(M+1)*(N+1).
C
C                         BDXF
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE
C                           VALUES OF THE DERIVATIVE OF
THE SOLUTION
C                           WITH RESPECT TO X AT X = XF.
C
```

```
C                          WHEN LBDCND = 2 OR 3,

C

C                            BDXF(J,K) =
(D/DX)U(XF,Y(J),Z(K)),

C                            J=1,2,...,M+1,
K=1,2,...,N+1.

C

C                          WHEN LBDCND HAS ANY OTHER
VALUE, BDXF IS

C                          A DUMMY VARIABLE.  BDXF MUST
BE

C                          DIMENSIONED AT LEAST
(M+1)*(N+1).

C

C                        YS,YF

C                          THE RANGE OF Y, I.E. YS .LE.
Y .LE. YF.

C                          YS MUST BE LESS THAN YF.

C

C                        M

C                          THE NUMBER OF PANELS INTO
WHICH THE

C                          INTERVAL (YS,YF) IS
SUBDIVIDED.

C                          HENCE, THERE WILL BE M+1 GRID
POINTS IN

C                          THE Y-DIRECTION GIVEN BY Y(J)
= YS+(J-1)DY

C                          FOR J=1,2,...,M+1,
```

```
C                          WHERE DY = (YF-YS)/M IS THE
PANEL WIDTH.

C                          M MUST BE AT LEAST 5.

C

C                        MBDCND

C                          INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                          AT Y = YS AND Y = YF.

C

C                          = 0  IF THE SOLUTION IS
PERIODIC IN Y, I.E.

C                               U(I,M+J,K) = U(I,J,K).

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               Y = YS AND Y = YF.

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               Y = YS AND THE
DERIVATIVE OF THE

C                               SOLUTION WITH RESPECT TO
Y IS

C                               SPECIFIED AT Y = YF.

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO Y IS
SPECIFIED AT

C                               Y = YS AND Y = YF.

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION
```

```
C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                AT Y = YS AND THE
SOLUTION IS

C                                SPECIFIED AT Y=YF.

C

C                         BDYS

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES

C                           THE VALUES OF THE DERIVATIVE
OF THE

C                           SOLUTION WITH RESPECT TO Y AT
Y = YS.

C

C                           WHEN MBDCND = 3 OR 4,

C

C                             BDYS(I,K) =
(D/DY)U(X(I),YS,Z(K)),

C                             I=1,2,...,L+1,
K=1,2,...,N+1.

C

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDYS

C                           IS A DUMMY VARIABLE. BDYS
MUST BE

C                           DIMENSIONED AT LEAST
(L+1)*(N+1).

C

C                         BDYF
```

```
C                                  A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES
C                                  THE VALUES OF THE DERIVATIVE
OF THE
C                                  SOLUTION WITH RESPECT TO Y AT
Y = YF.
C
C                                  WHEN MBDCND = 2 OR 3,
C
C                                    BDYF(I,K) =
(D/DY)U(X(I),YF,Z(K)),
C                                    I=1,2,...,L+1,
K=1,2,...,N+1.
C
C                                  WHEN MBDCND HAS ANY OTHER
VALUE, BDYF
C                                  IS A DUMMY VARIABLE. BDYF
MUST BE
C                                  DIMENSIONED AT LEAST
(L+1)*(N+1).
C
C                                ZS,ZF
C                                  THE RANGE OF Z, I.E. ZS .LE.
Z .LE. ZF.
C                                  ZS MUST BE LESS THAN ZF.
C
C                                N
C                                  THE NUMBER OF PANELS INTO
WHICH THE
```

```
C                              INTERVAL (ZS,ZF) IS
SUBDIVIDED.

C                              HENCE, THERE WILL BE N+1 GRID
POINTS

C                              IN THE Z-DIRECTION GIVEN BY

C                              Z(K) = ZS+(K-1)DZ FOR
K=1,2,...,N+1,

C                              WHERE DZ = (ZF-ZS)/N IS THE
PANEL WIDTH.

C                              N MUST BE AT LEAST 5.

C

C                            NBDCND

C                              INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                              AT Z = ZS AND Z = ZF.

C

C                              = 0  IF THE SOLUTION IS
PERIODIC IN Z, I.E.

C                                   U(I,J,N+K) = U(I,J,K).

C                              = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                   Z = ZS AND Z = ZF.

C                              = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                   Z = ZS AND THE
DERIVATIVE OF THE

C                                   SOLUTION WITH RESPECT TO
Z IS

C                                   SPECIFIED AT Z = ZF.
```

```
C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Z IS
SPECIFIED AT

C                                Z = ZS AND Z = ZF.

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Z IS
SPECIFIED AT

C                                Z = ZS AND THE SOLUTION
IS SPECIFIED

C                                AT Z=ZF.

C

C                         BDZS

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES

C                           THE VALUES OF THE DERIVATIVE
OF THE

C                           SOLUTION WITH RESPECT TO Z AT
Z = ZS.

C

C                           WHEN NBDCND = 3 OR 4,

C

C                             BDZS(I,J) =
(D/DZ)U(X(I),Y(J),ZS),

C                             I=1,2,...,L+1,
J=1,2,...,M+1.

C
```

```
C                          WHEN NBDCND HAS ANY OTHER
VALUE, BDZS

C                          IS A DUMMY VARIABLE. BDZS
MUST BE

C                          DIMENSIONED AT LEAST
(L+1)*(M+1).

C

C                        BDZF

C                          A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES

C                          THE VALUES OF THE DERIVATIVE
OF THE

C                          SOLUTION WITH RESPECT TO Z AT
Z = ZF.

C

C                          WHEN NBDCND = 2 OR 3,

C

C                            BDZF(I,J) =
(D/DZ)U(X(I),Y(J),ZF),

C                            I=1,2,...,L+1,
J=1,2,...,M+1.

C

C                          WHEN NBDCND HAS ANY OTHER
VALUE, BDZF

C                          IS A DUMMY VARIABLE. BDZF
MUST BE

C                          DIMENSIONED AT LEAST
(L+1)*(M+1).

C
```

```
C                         ELMBDA
C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                           EQUATION. IF LAMBDA .GT. 0, A
SOLUTION
C                           MAY NOT EXIST.  HOWEVER,
HW3CRT WILL
C                           ATTEMPT TO FIND A SOLUTION.
C
C                         LDIMF
C                           THE ROW (OR FIRST) DIMENSION
OF THE
C                           ARRAYS F,BDYS,BDYF,BDZS,AND
BDZF AS IT
C                           APPEARS IN THE PROGRAM
CALLING HW3CRT.
C                           THIS PARAMETER IS USED TO
SPECIFY THE
C                           VARIABLE DIMENSION OF THESE
ARRAYS.
C                           LDIMF MUST BE AT LEAST L+1.
C
C                         MDIMF
C                           THE COLUMN (OR SECOND)
DIMENSION OF THE
C                           ARRAY F AND THE ROW (OR
FIRST) DIMENSION
C                           OF THE ARRAYS BDXS AND BDXF
AS IT APPEARS
```

```
C                           IN THE PROGRAM CALLING
HW3CRT.  THIS

C                           PARAMETER IS USED TO SPECIFY
THE VARIABLE

C                           DIMENSION OF THESE ARRAYS.

C                           MDIMF MUST BE AT LEAST M+1.

C

C                         F

C                           A THREE-DIMENSIONAL ARRAY OF
DIMENSION AT

C                           AT LEAST (L+1)*(M+1)*(N+1),
SPECIFYING THE

C                           VALUES OF THE RIGHT SIDE OF
THE HELMHOLZ

C                           EQUATION AND BOUNDARY VALUES
(IF ANY).

C

C                           ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:

C                           FOR I=2,3,...,L,
J=2,3,...,M,

C                           AND K=2,3,...,N

C                           F(I,J,K) = F(X(I),Y(J),Z(K)).

C

C                           ON THE BOUNDARIES, F IS
DEFINED AS FOLLOWS:

C                           FOR J=1,2,...,M+1,
K=1,2,...,N+1,

C                           AND I=1,2,...,L+1
```

```
C

C                           LBDCND      F(1,J,K)
F(L+1,J,K)

C                           ------   ---------------   --
-------------

C

C                             0      F(XS,Y(J),Z(K))
F(XS,Y(J),Z(K))

C                             1      U(XS,Y(J),Z(K))
U(XF,Y(J),Z(K))

C                             2      U(XS,Y(J),Z(K))
F(XF,Y(J),Z(K))

C                             3      F(XS,Y(J),Z(K))
F(XF,Y(J),Z(K))

C                             4      F(XS,Y(J),Z(K))
U(XF,Y(J),Z(K))

C

C                           MBDCND      F(I,1,K)
F(I,M+1,K)

C                           ------   ---------------   --
-------------

C

C                             0      F(X(I),YS,Z(K))
F(X(I),YS,Z(K))

C                             1      U(X(I),YS,Z(K))
U(X(I),YF,Z(K))

C                             2      U(X(I),YS,Z(K))
F(X(I),YF,Z(K))

C                             3      F(X(I),YS,Z(K))
F(X(I),YF,Z(K))
```

```
C                             4      F(X(I),YS,Z(K))
U(X(I),YF,Z(K))

C

C                           NBDCND      F(I,J,1)
F(I,J,N+1)

C                           ------   ---------------   --
-------------

C

C                             0      F(X(I),Y(J),ZS)
F(X(I),Y(J),ZS)

C                             1      U(X(I),Y(J),ZS)
U(X(I),Y(J),ZF)

C                             2      U(X(I),Y(J),ZS)
F(X(I),Y(J),ZF)

C                             3      F(X(I),Y(J),ZS)
F(X(I),Y(J),ZF)

C                             4      F(X(I),Y(J),ZS)
U(X(I),Y(J),ZF)

C

C                           NOTE:

C                           IF THE TABLE CALLS FOR BOTH
THE SOLUTION

C                           U AND THE RIGHT SIDE F ON A
BOUNDARY,

C                           THEN THE SOLUTION MUST BE
SPECIFIED.

C

C                         W

C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
```

```
C                           PROVIDED BY THE USER FOR WORK
SPACE.

C                           THE LENGTH OF W MUST BE AT
LEAST

C                           30 + L + M + 5*N + MAX(L,M,N)
+

C                           7*(INT((L+1)/2) +
INT((M+1)/2))

C

C

C

C

C ON OUTPUT              F

C                           CONTAINS THE SOLUTION
U(I,J,K) OF THE

C                           FINITE DIFFERENCE
APPROXIMATION FOR THE

C                           GRID POINT (X(I),Y(J),Z(K))
FOR

C                           I=1,2,...,L+1, J=1,2,...,M+1,

C                           AND K=1,2,...,N+1.

C

C                         PERTRB

C                           IF A COMBINATION OF PERIODIC
OR DERIVATIVE

C                           BOUNDARY CONDITIONS IS
SPECIFIED FOR A

C                           POISSON EQUATION (LAMBDA =
0), A SOLUTION
```

```
C                                MAY NOT EXIST.  PERTRB IS A
CONSTANT,

C                                CALCULATED AND SUBTRACTED
FROM F, WHICH

C                                ENSURES THAT A SOLUTION
EXISTS.  PWSCRT

C                                THEN COMPUTES THIS SOLUTION,
WHICH IS A

C                                LEAST SQUARES SOLUTION TO THE
ORIGINAL

C                                APPROXIMATION.  THIS SOLUTION
IS NOT

C                                UNIQUE AND IS UNNORMALIZED.
THE VALUE OF

C                                PERTRB SHOULD BE SMALL
COMPARED TO THE

C                                THE RIGHT SIDE F.  OTHERWISE,
A SOLUTION

C                                IS OBTAINED TO AN ESSENTIALLY
DIFFERENT

C                                PROBLEM.  THIS COMPARISON
SHOULD ALWAYS

C                                BE MADE TO INSURE THAT A
MEANINGFUL

C                                SOLUTION HAS BEEN OBTAINED.

C

C                              IERROR

C                                AN ERROR FLAG THAT INDICATES
INVALID INPUT
```

```
C                          PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 12,

C                          A SOLUTION IS NOT ATTEMPTED.

C

C                          =  0  NO ERROR

C                          =  1  XS .GE. XF

C                          =  2  L .LT. 5

C                          =  3  LBDCND .LT. 0 .OR.
LBDCND .GT. 4

C                          =  4  YS .GE. YF

C                          =  5  M .LT. 5

C                          =  6  MBDCND .LT. 0 .OR.
MBDCND .GT. 4

C                          =  7  ZS .GE. ZF

C                          =  8  N .LT. 5

C                          =  9  NBDCND .LT. 0 .OR.
NBDCND .GT. 4

C                          = 10  LDIMF .LT. L+1

C                          = 11  MDIMF .LT. M+1

C                          = 12  LAMBDA .GT. 0

C

C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                          A POSSIBLY INCORRECT CALL TO
HW3CRT, THE

C                          USER SHOULD TEST IERROR AFTER
THE CALL.
```

```
C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       POIS3D, FFTPACK, AND COMF FROM
FISHPAK

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                        1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                        LIBRARIES IN JANUARY 1980.

C

C PORTABILITY            FORTRAN 77

C

C ALGORITHM              THIS SUBROUTINE DEFINES THE
FINITE DIFFERENCE

C                        EQUATIONS, INCORPORATES
BOUNDARY DATA, AND

C                        ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS AND
```

```
C                         THEN CALLS POIS3D TO SOLVE THE
SYSTEM.

C

C TIMING                  FOR LARGE L, M AND N, THE
OPERATION COUNT

C                         IS ROUGHLY PROPORTIONAL TO

C                           L*M*N*(LOG2(L)+LOG2(M)+5),

C                         BUT ALSO DEPENDS ON INPUT
PARAMETERS LBDCND

C                         AND MBDCND.

C

C ACCURACY                THE SOLUTION PROCESS EMPLOYED
RESULTS IN

C                         A LOSS OF NO MORE THAN FOUR
SIGNIFICANT

C                         DIGITS FOR L, M AND N AS LARGE
AS 32.

C                         MORE DETAILED INFORMATION ABOUT
ACCURACY

C                         CAN BE FOUND IN THE
DOCUMENTATION FOR

C                         ROUTINE POIS3D WHICH IS THE
ROUTINE THAT

C                         ACTUALLY SOLVES THE FINITE
DIFFERENCE

C                         EQUATIONS.

C

C REFERENCES              NONE
```

```
C*****************************************************
****************
```

HWSCRT

```
      SUBROUTINE HWSCRT
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                   ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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C      *                      FISHPACK version 4.1
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```
C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                          BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
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C     *                          OF
*

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C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
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C     *              THE NATIONAL SCIENCE FOUNDATION
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C

C

C

C DIMENSION OF           BDA(N),      BDB(N),
BDC(M),BDD(M),

C ARGUMENTS              F(IDIMF,N),  W(SEE ARGUMENT
LIST)

C

C LATEST REVISION        NOVEMBER 1988

C

C PURPOSE                SOLVES THE STANDARD FIVE-POINT
FINITE

C                        DIFFERENCE APPROXIMATION TO THE
HELMHOLTZ

C                        EQUATION IN CARTESIAN
COORDINATES.  THIS

C                        EQUATION IS

C

C                          (D/DX)(DU/DX) + (D/DY)(DU/DY)

C                          + LAMBDA*U = F(X,Y).
```

```
C

C USAGE                     CALL HWSCRT
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                        PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                  A,B

C

C                             THE RANGE OF X, I.E., A .LE.
X .LE. B.

C                             A MUST BE LESS THAN B.

C

C                           M

C                             THE NUMBER OF PANELS INTO
WHICH THE

C                             INTERVAL (A,B) IS SUBDIVIDED.

C                             HENCE, THERE WILL BE M+1 GRID
POINTS

C                             IN THE X-DIRECTION GIVEN BY

C                             X(I) = A+(I-1)DX FOR I =
1,2,...,M+1,

C                             WHERE DX = (B-A)/M IS THE
PANEL WIDTH.

C                             M MUST BE GREATER THAN 3.

C
```

```
C                          MBDCND

C                            INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                            AT X = A AND X = B.

C

C                            = 0  IF THE SOLUTION IS
PERIODIC IN X,

C                                 I.E., U(I,J) = U(M+I,J).

C                            = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                 X = A AND X = B.

C                            = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                 X = A AND THE DERIVATIVE
OF THE

C                                 SOLUTION WITH RESPECT TO
X IS

C                                 SPECIFIED AT X = B.

C                            = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                 WITH RESPECT TO X IS
SPECIFIED AT

C                                 AT X = A AND X = B.

C                            = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                 WITH RESPECT TO X IS
SPECIFIED AT

C                                 X = A AND THE SOLUTION
IS SPECIFIED
```

```
C                                AT X = B.

C

C                         BDA

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO X AT X = A.

C

C                           WHEN MBDCND = 3 OR 4,

C

C                             BDA(J) = (D/DX)U(A,Y(J)), J
= 1,2,...,N+1.

C

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDA IS

C                           A DUMMY VARIABLE.

C

C                         BDB

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1

C                           THAT SPECIFIES THE VALUES OF
THE DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO X AT X = B.

C

C                           WHEN MBDCND = 2 OR 3,
```

```
C
C                                BDB(J) = (D/DX)U(B,Y(J)), J
= 1,2,...,N+1
C
C                              WHEN MBDCND HAS ANY OTHER
VALUE BDB IS A
C                              DUMMY VARIABLE.
C
C                            C,D
C                              THE RANGE OF Y, I.E., C .LE.
Y .LE. D.
C                              C MUST BE LESS THAN D.
C
C                            N
C                              THE NUMBER OF PANELS INTO
WHICH THE
C                              INTERVAL (C,D) IS SUBDIVIDED.
HENCE,
C                              THERE WILL BE N+1 GRID POINTS
IN THE
C                              Y-DIRECTION GIVEN BY Y(J) =
C+(J-1)DY
C                              FOR J = 1,2,...,N+1, WHERE
C                              DY = (D-C)/N IS THE PANEL
WIDTH.
C                              N MUST BE GREATER THAN 3.
C
C                            NBDCND
```

```
C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS AT

C                           Y = C AND Y = D.

C

C                           = 0  IF THE SOLUTION IS
PERIODIC IN Y,

C                                I.E., U(I,J) = U(I,N+J).

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                Y = C AND Y = D.

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                Y = C AND THE DERIVATIVE
OF THE

C                                SOLUTION WITH RESPECT TO
Y IS

C                                SPECIFIED AT Y = D.

C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                Y = C AND Y = D.

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO Y IS
SPECIFIED AT

C                                Y = C AND THE SOLUTION
IS SPECIFIED

C                                AT Y = D.
```

```
C
C                         BDC
C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT
C                           SPECIFIES THE VALUES OF THE
DERIVATIVE
C                           OF THE SOLUTION WITH RESPECT
TO Y AT Y = C.
C
C                           WHEN NBDCND = 3 OR 4,
C
C                             BDC(I) = (D/DY)U(X(I),C), I
= 1,2,...,M+1
C
C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDC IS
C                           A DUMMY VARIABLE.
C
C                         BDD
C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT
C                           SPECIFIES THE VALUES OF THE
DERIVATIVE
C                           OF THE SOLUTION WITH RESPECT
TO Y AT Y = D.
C
C                           WHEN NBDCND = 2 OR 3,
C
```

```
C                            BDD(I) = (D/DY)U(X(I),D), I
= 1,2,...,M+1

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDD IS

C                           A DUMMY VARIABLE.

C

C                         ELMBDA

C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ

C                           EQUATION.  IF LAMBDA .GT. 0,
A SOLUTION

C                           MAY NOT EXIST.  HOWEVER,
HWSCRT WILL

C                           ATTEMPT TO FIND A SOLUTION.

C

C                         F

C                           A TWO-DIMENSIONAL ARRAY, OF
DIMENSION AT

C                           LEAST (M+1)*(N+1), SPECIFYING
VALUES OF THE

C                           RIGHT SIDE OF THE HELMHOLTZ
EQUATION AND

C                           BOUNDARY VALUES (IF ANY).

C

C                           ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:

C                           FOR I = 2,3,...,M AND J =
2,3,...,N
```

```
C                          F(I,J) = F(X(I),Y(J)).

C

C                          ON THE BOUNDARIES, F IS
DEFINED AS FOLLOWS:

C                          FOR J=1,2,...,N+1,
I=1,2,...,M+1,

C

C                          MBDCND     F(1,J)
F(M+1,J)

C                          ------     ---------     ----
----

C

C                            0        F(A,Y(J))
F(A,Y(J))

C                            1        U(A,Y(J))
U(B,Y(J))

C                            2        U(A,Y(J))
F(B,Y(J))

C                            3        F(A,Y(J))
F(B,Y(J))

C                            4        F(A,Y(J))
U(B,Y(J))

C

C

C                          NBDCND     F(I,1)
F(I,N+1)

C                          ------     ---------     ----
----

C
```

```
C                              0        F(X(I),C)
F(X(I),C)

C                              1        U(X(I),C)
U(X(I),D)

C                              2        U(X(I),C)
F(X(I),D)

C                              3        F(X(I),C)
F(X(I),D)

C                              4        F(X(I),C)
U(X(I),D)

C

C                            NOTE:

C                            IF THE TABLE CALLS FOR BOTH
THE SOLUTION U

C                            AND THE RIGHT SIDE F AT A
CORNER THEN THE

C                            SOLUTION MUST BE SPECIFIED.

C

C                          IDIMF

C                            THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                            F AS IT APPEARS IN THE
PROGRAM CALLING

C                            HWSCRT.  THIS PARAMETER IS
USED TO SPECIFY

C                            THE VARIABLE DIMENSION OF F.
IDIMF MUST

C                            BE AT LEAST M+1   .

C
```

```
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C                           W MAY REQUIRE UP TO 4*(N+1) +
C                           (13 + INT(LOG2(N+1)))*(M+1)
LOCATIONS.
C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS
C                           COMPUTED BY HWSCRT AND IS
RETURNED IN
C                           LOCATION W(1).
C
C
C ON OUTPUT               F
C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT
C                           (X(I),Y(J)), I = 1,2,...,M+1,
C                           J = 1,2,...,N+1  .
C
C                         PERTRB
C                           IF A COMBINATION OF PERIODIC
OR DERIVATIVE
C                           BOUNDARY CONDITIONS IS
SPECIFIED FOR A
```

```
C                          POISSON EQUATION (LAMBDA =
0), A SOLUTION

C                          MAY NOT EXIST.  PERTRB IS A
CONSTANT,

C                          CALCULATED AND SUBTRACTED
FROM F, WHICH

C                          ENSURES THAT A SOLUTION
EXISTS.  HWSCRT

C                          THEN COMPUTES THIS SOLUTION,
WHICH IS A

C                          LEAST SQUARES SOLUTION TO THE
ORIGINAL

C                          APPROXIMATION.  THIS SOLUTION
PLUS ANY

C                          CONSTANT IS ALSO A SOLUTION.
HENCE, THE

C                          SOLUTION IS NOT UNIQUE.  THE
VALUE OF

C                          PERTRB SHOULD BE SMALL
COMPARED TO THE

C                          RIGHT SIDE F.  OTHERWISE, A
SOLUTION IS

C                          OBTAINED TO AN ESSENTIALLY
DIFFERENT

C                          PROBLEM. THIS COMPARISON
SHOULD ALWAYS

C                          BE MADE TO INSURE THAT A
MEANINGFUL

C                          SOLUTION HAS BEEN OBTAINED.

C
```

```
C                            IERROR

C                              AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                              PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 6,

C                              A SOLUTION IS NOT ATTEMPTED.

C

C                              = 0  NO ERROR

C                              = 1  A .GE. B

C                              = 2  MBDCND .LT. 0 OR MBDCND
.GT. 4

C                              = 3  C .GE. D

C                              = 4  N .LE. 3

C                              = 5  NBDCND .LT. 0 OR NBDCND
.GT. 4

C                              = 6  LAMBDA .GT. 0

C                              = 7  IDIMF .LT. M+1

C                              = 8  M .LE. 3

C

C                              SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                              A POSSIBLY INCORRECT CALL TO
HWSCRT, THE

C                              USER SHOULD TEST IERROR AFTER
THE CALL.

C

C                            W
```

```
C                          W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       GENBUN, GNBNAUX, AND COMF

C FILES                  FROM FISHPAK

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                        1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                        LIBRARIES IN JANUARY 1980.

C

C PORTABILITY            FORTRAN 77

C

C ALGORITHM              THE ROUTINE DEFINES THE FINITE
DIFFERENCE

C                        EQUATIONS, INCORPORATES
BOUNDARY DATA, AND
```

```
C                        ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS

C                        AND THEN CALLS GENBUN TO SOLVE
THE SYSTEM.

C

C TIMING                 FOR LARGE  M AND N, THE
OPERATION COUNT

C                        IS ROUGHLY PROPORTIONAL TO

C                          M*N*(LOG2(N)

C                        BUT ALSO DEPENDS ON INPUT
PARAMETERS NBDCND

C                        AND MBDCND.

C

C ACCURACY               THE SOLUTION PROCESS EMPLOYED
RESULTS IN A LOSS

C                        OF NO MORE THAN THREE
SIGNIFICANT DIGITS FOR N

C                        AND M AS LARGE AS 64.  MORE
DETAILS ABOUT

C                        ACCURACY CAN BE FOUND IN THE
DOCUMENTATION FOR

C                        SUBROUTINE GENBUN WHICH IS THE
ROUTINE THAT

C                        SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES             SWARZTRAUBER,P. AND R. SWEET,
"EFFICIENT

C                        FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF
```

```
C                              ELLIPTIC EQUATIONS"
C                                NCAR TN/IA-109, JULY, 1975,
138 PP.
C*******************************************************
****************
      DIMENSION       F(IDIMF,1)
      DIMENSION       BDA(*)     ,BDB(*)     ,BDC(*)
,BDD(*)     ,
     1                W(*)
C
C     CHECK FOR INVALID PARAMETERS.
C
      IERROR = 0
      IF (A .GE. B) IERROR = 1
      IF (MBDCND.LT.0 .OR. MBDCND.GT.4) IERROR = 2
      IF (C .GE. D) IERROR = 3
      IF (N .LE. 3) IERROR = 4
      IF (NBDCND.LT.0 .OR. NBDCND.GT.4) IERROR = 5
      IF (IDIMF .LT. M+1) IERROR = 7
      IF (M .LE. 3) IERROR = 8
      IF (IERROR .NE. 0) RETURN
      NPEROD = NBDCND
      MPEROD = 0
      IF (MBDCND .GT. 0) MPEROD = 1
      DELTAX = (B-A)/FLOAT(M)
      TWDELX = 2./DELTAX
```

```
      DELXSQ = 1./DELTAX**2
      DELTAY = (D-C)/FLOAT(N)
      TWDELY = 2./DELTAY
      DELYSQ = 1./DELTAY**2
      NP = NBDCND+1
      NP1 = N+1
      MP = MBDCND+1
      MP1 = M+1
      NSTART = 1
      NSTOP = N
      NSKIP = 1
      GO TO (104,101,102,103,104),NP
  101 NSTART = 2
      GO TO 104
  102 NSTART = 2
  103 NSTOP = NP1
      NSKIP = 2
  104 NUNK = NSTOP-NSTART+1
C
C     ENTER BOUNDARY DATA FOR X-BOUNDARIES.
C
      MSTART = 1
      MSTOP = M
      MSKIP = 1
```

```
    GO TO (117,105,106,109,110),MP
105 MSTART = 2
    GO TO 107
106 MSTART = 2
    MSTOP = MP1
    MSKIP = 2
107 DO 108 J=NSTART,NSTOP
       F(2,J) = F(2,J)-F(1,J)*DELXSQ
108 CONTINUE
    GO TO 112
109 MSTOP = MP1
    MSKIP = 2
110 DO 111 J=NSTART,NSTOP
       F(1,J) = F(1,J)+BDA(J)*TWDELX
111 CONTINUE
112 GO TO (113,115),MSKIP
113 DO 114 J=NSTART,NSTOP
       F(M,J) = F(M,J)-F(MP1,J)*DELXSQ
114 CONTINUE
    GO TO 117
115 DO 116 J=NSTART,NSTOP
       F(MP1,J) = F(MP1,J)-BDB(J)*TWDELX
116 CONTINUE
117 MUNK = MSTOP-MSTART+1
```

```
C
C     ENTER BOUNDARY DATA FOR Y-BOUNDARIES.
C
      GO TO (127,118,118,120,120),NP
  118 DO 119 I=MSTART,MSTOP
         F(I,2) = F(I,2)-F(I,1)*DELYSQ
  119 CONTINUE
      GO TO 122
  120 DO 121 I=MSTART,MSTOP
         F(I,1) = F(I,1)+BDC(I)*TWDELY
  121 CONTINUE
  122 GO TO (123,125),NSKIP
  123 DO 124 I=MSTART,MSTOP
         F(I,N) = F(I,N)-F(I,NP1)*DELYSQ
  124 CONTINUE
      GO TO 127
  125 DO 126 I=MSTART,MSTOP
         F(I,NP1) = F(I,NP1)-BDD(I)*TWDELY
  126 CONTINUE
C
C    MULTIPLY RIGHT SIDE BY DELTAY**2.
C
  127 DELYSQ = DELTAY*DELTAY
      DO 129 I=MSTART,MSTOP
```

```
         DO 128 J=NSTART,NSTOP
            F(I,J) = F(I,J)*DELYSQ
  128    CONTINUE
  129 CONTINUE
C
C     DEFINE THE A,B,C COEFFICIENTS IN W-ARRAY.
C
      ID2 = MUNK
      ID3 = ID2+MUNK
      ID4 = ID3+MUNK
      S = DELYSQ*DELXSQ
      ST2 = 2.*S
      DO 130 I=1,MUNK
         W(I) = S
         J = ID2+I
         W(J) = -ST2+ELMBDA*DELYSQ
         J = ID3+I
         W(J) = S
  130 CONTINUE
      IF (MP .EQ. 1) GO TO 131
      W(1) = 0.
      W(ID4) = 0.
  131 CONTINUE
      GO TO (135,135,132,133,134),MP
```

```
  132 W(ID2) = ST2
      GO TO 135
  133 W(ID2) = ST2
  134 W(ID3+1) = ST2
  135 CONTINUE
      PERTRB = 0.
      IF (ELMBDA) 144,137,136
  136 IERROR = 6
      GO TO 144
  137 IF ((NBDCND.EQ.0 .OR. NBDCND.EQ.3) .AND.
     1    (MBDCND.EQ.0 .OR. MBDCND.EQ.3)) GO TO 138
      GO TO 144
C
C     FOR SINGULAR PROBLEMS MUST ADJUST DATA TO INSURE
THAT A SOLUTION
C     WILL EXIST.
C
  138 A1 = 1.
      A2 = 1.
      IF (NBDCND .EQ. 3) A2 = 2.
      IF (MBDCND .EQ. 3) A1 = 2.
      S1 = 0.
      MSP1 = MSTART+1
      MSTM1 = MSTOP-1
      NSP1 = NSTART+1
```

```
      NSTM1 = NSTOP-1
      DO 140 J=NSP1,NSTM1
         S = 0.
         DO 139 I=MSP1,MSTM1
            S = S+F(I,J)
  139    CONTINUE
         S1 = S1+S*A1+F(MSTART,J)+F(MSTOP,J)
  140 CONTINUE
      S1 = A2*S1
      S = 0.
      DO 141 I=MSP1,MSTM1
         S = S+F(I,NSTART)+F(I,NSTOP)
  141 CONTINUE
      S1 =
S1+S*A1+F(MSTART,NSTART)+F(MSTART,NSTOP)+F(MSTOP,NSTART)
+
     1      F(MSTOP,NSTOP)
      S = (2.+FLOAT(NUNK-2)*A2)*(2.+FLOAT(MUNK-2)*A1)
      PERTRB = S1/S
      DO 143 J=NSTART,NSTOP
         DO 142 I=MSTART,MSTOP
            F(I,J) = F(I,J)-PERTRB
  142    CONTINUE
  143 CONTINUE
      PERTRB = PERTRB/DELYSQ
```

```
C
C     SOLVE THE EQUATION.
C
  144 CALL GENBUN
(NPEROD,NUNK,MPEROD,MUNK,W(1),W(ID2+1),W(ID3+1),
     1
IDIMF,F(MSTART,NSTART),IERR1,W(ID4+1))
      W(1) = W(ID4+1)+3.*FLOAT(MUNK)
C
C     FILL IN IDENTICAL VALUES WHEN HAVE PERIODIC
BOUNDARY CONDITIONS.
C
      IF (NBDCND .NE. 0) GO TO 146
      DO 145 I=MSTART,MSTOP
         F(I,NP1) = F(I,1)
  145 CONTINUE
  146 IF (MBDCND .NE. 0) GO TO 148
      DO 147 J=NSTART,NSTOP
         F(MP1,J) = F(1,J)
  147 CONTINUE
      IF (NBDCND .EQ. 0) F(MP1,NP1) = F(1,NP1)
  148 CONTINUE
      RETURN
C
C REVISION HISTORY---
C
```

```
C SEPTEMBER 1973    VERSION 1

C APRIL     1976    VERSION 2

C JANUARY   1978    VERSION 3

C DECEMBER  1979    VERSION 3.1

C FEBRUARY  1985    DOCUMENTATION UPGRADE

C NOVEMBER  1988    VERSION 3.2, FORTRAN 77 CHANGES

C-------------------------------------------------------
----------------

      END
```

HWSCSP

```
      SUBROUTINE HWSCSP
(INTL,TS,TF,M,MBDCND,BDTS,BDTF,RS,RF,N,NBDCND,

     1
BDRS,BDRF,ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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C     *                       FISHPACK version 4.1
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C     *
*
C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
C     *
*
C     *                              BY
*
C     *
*
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *
C     *
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C     *                              OF
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C     *
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C DIMENSION OF            BDTS(N+1),     BDTF(N+1),
BDRS(M+1), BDRF(M+1),

C ARGUMENTS               F(IDIMF,N+1),  W(SEE ARGUMENT
LIST)

C

C LATEST REVISION         NOVEMBER 1988

C

C PURPOSE                 SOLVES A FINITE DIFFERENCE
APPROXIMATION
```

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C                          TO THE MODIFIED HELMHOLTZ
EQUATION IN

C                          SPHERICAL COORDINATES ASSUMING
AXISYMMETRY

C                          (NO DEPENDENCE ON LONGITUDE).
THE EQUATION

C                          IS

C

C                            (1/R**2)(D/DR)((R**2)(D/DR)U)
+

C

C
(1/(R**2)SIN(THETA))(D/DTHETA)

C

C                            (SIN(THETA)(D/DTHETA)U) +

C

C                            (LAMBDA/(RSIN(THETA))**2)U =
F(THETA,R).

C

C                          THIS TWO DIMENSIONAL MODIFIED
HELMHOLTZ

C                          EQUATION RESULTS FROM THE
FOURIER TRANSFORM

C                          OF THE THREE DIMENSIONAL
POISSON EQUATION.

C

C USAGE                    CALL HWSCSP
(INTL,TS,TF,M,MBDCND,BDTS,BDTF,
```

```
C
RS,RF,N,NBDCND,BDRS,BDRF,ELMBDA,

C
F,IDIMF,PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                INTL

C                             = 0  ON INITIAL ENTRY TO
HWSCSP OR IF ANY

C                                  OF THE ARGUMENTS RS, RF,
N, NBDCND

C                                  ARE CHANGED FROM A
PREVIOUS CALL.

C                             = 1  IF RS, RF, N, NBDCND ARE
ALL UNCHANGED

C                                  FROM PREVIOUS CALL TO
HWSCSP.

C

C                             NOTE:

C                             A CALL WITH INTL=0 TAKES
APPROXIMATELY

C                             1.5 TIMES AS MUCH TIME AS A
CALL WITH

C                             INTL = 1  .  ONCE A CALL WITH
INTL = 0

C                             HAS BEEN MADE THEN SUBSEQUENT
SOLUTIONS

C                             CORRESPONDING TO DIFFERENT F,
BDTS, BDTF,
```

```
C                          BDRS, BDRF CAN BE OBTAINED
FASTER WITH

C                          INTL = 1 SINCE INITIALIZATION
IS NOT

C                          REPEATED.

C

C                        TS,TF

C                          THE RANGE OF THETA
(COLATITUDE), I.E.,

C                          TS .LE. THETA .LE. TF. TS
MUST BE LESS

C                          THAN TF.  TS AND TF ARE IN
RADIANS. A TS OF

C                          ZERO CORRESPONDS TO THE NORTH
POLE AND A

C                          TF OF PI CORRESPONDS TO THE
SOUTH POLE.

C

C                          **** IMPORTANT ****

C

C                          IF TF IS EQUAL TO PI THEN IT
MUST BE

C                          COMPUTED USING THE STATEMENT

C                          TF = PIMACH(DUM). THIS
INSURES THAT TF

C                          IN THE USER'S PROGRAM IS
EQUAL TO PI IN

C                          THIS PROGRAM WHICH PERMITS
SEVERAL TESTS
```

```
C                           OF THE  INPUT PARAMETERS THAT
OTHERWISE

C                           WOULD NOT BE POSSIBLE.

C

C                         M

C                           THE NUMBER OF PANELS INTO
WHICH THE

C                           INTERVAL (TS,TF) IS
SUBDIVIDED.

C                           HENCE, THERE WILL BE M+1 GRID
POINTS

C                           IN THE THETA-DIRECTION GIVEN
BY

C                           THETA(K) = (I-1)DTHETA+TS FOR

C                           I = 1,2,...,M+1, WHERE DTHETA
= (TF-TS)/M

C                           IS THE PANEL WIDTH.

C

C                         MBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITION

C                           AT THETA = TS AND  THETA =
TF.

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = TS AND THETA =
TF.
```

```
C                             = 2  IF THE SOLUTION IS
SPECIFIED AT

C                                  THETA = TS AND THE
DERIVATIVE OF THE

C                                  SOLUTION WITH RESPECT TO
THETA IS

C                                  SPECIFIED AT THETA = TF

C                                  (SEE NOTE 2 BELOW).

C                             = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                                  WITH RESPECT TO THETA IS
SPECIFIED

C                                  AT THETA = TS AND THETA
= TF

C                                  (SEE NOTES 1,2 BELOW).

C                             = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                  WITH RESPECT TO THETA IS
SPECIFIED

C                                  AT THETA = TS (SEE NOTE
1 BELOW) AND

C                                  SOLUTION IS SPECIFIED AT
THETA = TF.

C                             = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                                  THETA = TS = 0 AND THE
SOLUTION IS

C                                   SPECIFIED AT THETA =
TF.
```

```
C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = TS = 0 AND THE
DERIVATIVE

C                                OF THE SOLUTION WITH
RESPECT TO THETA

C                                IS SPECIFIED AT THETA =
TF

C                                (SEE NOTE 2 BELOW).

C                           = 7  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = TS AND THE
SOLUTION IS

C                                 UNSPECIFIED AT THETA =
TF = PI.

C                           = 8  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED

C                                AT THETA = TS (SEE NOTE
1 BELOW)

C                                AND THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = TF = PI.

C                           = 9  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = TS = 0 AND THETA
= TF = PI.

C

C                           NOTE 1:
```

```
C                              IF TS = 0, DO NOT USE MBDCND
= 3,4, OR 8,

C                              BUT INSTEAD USE MBDCND = 5,6,
OR 9  .

C

C                              NOTE 2:

C                              IF TF = PI, DO NOT USE MBDCND
= 2,3, OR 6,

C                              BUT INSTEAD USE MBDCND = 7,8,
OR 9  .

C

C                            BDTS

C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                              SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                              THE SOLUTION WITH RESPECT TO
THETA AT

C                              THETA = TS.  WHEN MBDCND =
3,4, OR 8,

C

C                                BDTS(J) =
(D/DTHETA)U(TS,R(J)),

C                                J = 1,2,...,N+1  .

C

C                              WHEN MBDCND HAS ANY OTHER
VALUE, BDTS IS

C                              A DUMMY VARIABLE.

C
```

```
C                        BDTF
C                          A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT
C                          SPECIFIES THE VALUES OF THE
DERIVATIVE OF
C                          THE SOLUTION WITH RESPECT TO
THETA AT
C                          THETA = TF.  WHEN MBDCND =
2,3, OR 6,
C
C                          BDTF(J) =
(D/DTHETA)U(TF,R(J)),
C                          J = 1,2,...,N+1  .
C
C                          WHEN MBDCND HAS ANY OTHER
VALUE, BDTF IS
C                          A DUMMY VARIABLE.
C
C                        RS,RF
C                          THE RANGE OF R, I.E., RS .LE.
R .LT. RF.
C                          RS MUST BE LESS THAN RF.  RS
MUST BE
C                          NON-NEGATIVE.
C
C                        N
C                          THE NUMBER OF PANELS INTO
WHICH THE
```

```
C                                 INTERVAL (RS,RF) IS
SUBDIVIDED.
C                                 HENCE, THERE WILL BE N+1 GRID
POINTS IN THE
C                                 R-DIRECTION GIVEN BY R(J) =
(J-1)DR+RS
C                                 FOR J = 1,2,...,N+1, WHERE DR
= (RF-RS)/N
C                                 IS THE PANEL WIDTH.
C                                 N MUST BE GREATER THAN 2
C
C                               NBDCND
C                                 INDICATES THE TYPE OF
BOUNDARY CONDITION
C                                 AT R = RS AND R = RF.
C
C                                 = 1  IF THE SOLUTION IS
SPECIFIED AT
C                                      R = RS AND R = RF.
C                                 = 2  IF THE SOLUTION IS
SPECIFIED AT
C                                      R = RS AND THE
DERIVATIVE
C                                      OF THE SOLUTION WITH
RESPECT TO R
C                                      IS SPECIFIED AT R = RF.
C                                 = 3  IF THE DERIVATIVE OF THE
SOLUTION
```

```
C                                WITH RESPECT TO R IS
SPECIFIED AT

C                                R = RS AND R = RF.

C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO R IS
SPECIFIED AT

C                                RS AND THE SOLUTION IS
SPECIFIED AT

C                                R = RF.

C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                                R = RS = 0 (SEE NOTE
BELOW)  AND THE

C                                SOLUTION IS SPECIFIED AT
R = RF.

C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT

C                                R = RS = 0 (SEE NOTE
BELOW) AND THE

C                                DERIVATIVE OF THE
SOLUTION WITH

C                                RESPECT TO R IS
SPECIFIED AT R = RF.

C

C                           NOTE:

C                           NBDCND = 5 OR 6 CANNOT BE
USED WITH

C                           MBDCND = 1,2,4,5, OR 7.  THE
FORMER
```

```
C                            INDICATES THAT THE SOLUTION
IS UNSPECIFIED

C                            AT R = 0, THE LATTER
INDICATES THAT THE

C                            SOLUTION IS SPECIFIED).

C                            USE INSTEAD   NBDCND = 1 OR 2
.

C

C                          BDRS

C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                            SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                            THE SOLUTION WITH RESPECT TO
R AT R = RS.

C

C                            WHEN NBDCND = 3 OR 4,

C                              BDRS(I) =
(D/DR)U(THETA(I),RS),

C                              I = 1,2,...,M+1  .

C

C                            WHEN NBDCND HAS ANY OTHER
VALUE, BDRS IS

C                            A DUMMY VARIABLE.

C

C                          BDRF

C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1
```

```
C                           THAT SPECIFIES THE VALUES OF
THE
C                           DERIVATIVE OF THE SOLUTION
WITH RESPECT
C                           TO R AT R = RF.
C
C                           WHEN NBDCND = 2,3, OR 6,
C                             BDRF(I) =
(D/DR)U(THETA(I),RF),
C                             I = 1,2,...,M+1  .
C
C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDRF IS
C                           A DUMMY VARIABLE.
C
C                         ELMBDA
C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                           EQUATION.  IF LAMBDA .GT. 0,
A SOLUTION
C                           MAY NOT EXIST.  HOWEVER,
HWSCSP WILL
C                           ATTEMPT TO FIND A SOLUTION.
IF NBDCND = 5
C                           OR 6 OR  MBDCND = 5,6,7,8, OR
9, ELMBDA
C                           MUST BE ZERO.
C
```

```
C                         F

C                           A TWO-DIMENSIONAL ARRAY, OF
DIMENSION AT

C                           LEAST (M+1)*(N+1), SPECIFYING
VALUES OF THE

C                           RIGHT SIDE OF THE HELMHOLTZ
EQUATION AND

C                           BOUNDARY VALUES (IF ANY).

C

C                           ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:

C                           FOR I = 2,3,...,M AND J =
2,3,...,N

C                           F(I,J) = F(THETA(I),R(J)).

C

C                           ON THE BOUNDARIES, F IS
DEFINED AS FOLLOWS:

C                           FOR J=1,2,...,N+1,
I=1,2,...,M+1,

C

C                           MBDCND   F(1,J)
F(M+1,J)

C                           ------   ----------        --
--------

C

C                             1      U(TS,R(J))
U(TF,R(J))

C                             2      U(TS,R(J))
F(TF,R(J))
```

```
C                             3      F(TS,R(J))
F(TF,R(J))

C                             4      F(TS,R(J))
U(TF,R(J))

C                             5      F(0,R(J))
U(TF,R(J))

C                             6      F(0,R(J))
F(TF,R(J))

C                             7      U(TS,R(J))
F(PI,R(J))

C                             8      F(TS,R(J))
F(PI,R(J))

C                             9      F(0,R(J))
F(PI,R(J))

C

C                             NBDCND   F(I,1)
F(I,N+1)

C                             ------   --------------
--------------

C

C                               1      U(THETA(I),RS)
U(THETA(I),RF)

C                               2      U(THETA(I),RS)
F(THETA(I),RF)

C                               3      F(THETA(I),RS)
F(THETA(I),RF)

C                               4      F(THETA(I),RS)
U(THETA(I),RF)

C                               5      F(TS,0)
U(THETA(I),RF)
```

```
C                              6      F(TS,0)
F(THETA(I),RF)

C

C                              NOTE:

C                              IF THE TABLE CALLS FOR BOTH
THE SOLUTION

C                              U AND THE RIGHT SIDE F AT A
CORNER THEN

C                              THE SOLUTION MUST BE
SPECIFIED.

C

C                            IDIMF

C                              THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                              F AS IT APPEARS IN THE
PROGRAM CALLING

C                              HWSCSP.  THIS PARAMETER IS
USED TO SPECIFY

C                              THE VARIABLE DIMENSION OF F.
IDIMF MUST

C                              BE AT LEAST M+1  .

C

C                            W

C                              A ONE-DIMENSIONAL ARRAY THAT
MUST BE

C                              PROVIDED BY THE USER FOR WORK
SPACE.

C                              ITS LENGTH CAN BE COMPUTED
FROM THE
```

```
C                              FORMULA BELOW WHICH DEPENDS
ON THE VALUE

C                              OF NBDCND

C

C                              IF NBDCND=2,4 OR 6 DEFINE
NUNK=N

C                              IF NBDCND=1 OR 5   DEFINE
NUNK=N-1

C                              IF NBDCND=3        DEFINE
NUNK=N+1

C

C                              NOW SET K=INT(LOG2(NUNK))+1
AND

C                              L=2**(K+1) THEN W MUST BE
DIMENSIONED

C                              AT LEAST (K-
2)*L+K+5*(M+N)+MAX(2*N,6*M)+23

C

C                              **IMPORTANT**

C                              FOR PURPOSES OF CHECKING, THE
REQUIRED

C                              LENGTH OF W IS COMPUTED BY
HWSCSP AND

C                              STORED IN W(1) IN FLOATING
POINT FORMAT.

C

C ON OUTPUT              F

C                              CONTAINS THE SOLUTION U(I,J)
OF THE FINITE
```

```
C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                           (THETA(I),R(J)),  I =
1,2,...,M+1,

C                                             J =
1,2,...,N+1  .

C

C                         PERTRB

C                           IF A COMBINATION OF PERIODIC
OR DERIVATIVE

C                           BOUNDARY CONDITIONS IS
SPECIFIED FOR A

C                           POISSON EQUATION (LAMBDA =
0), A SOLUTION

C                           MAY NOT EXIST.  PERTRB IS A
CONSTANT,

C                           CALCULATED AND SUBTRACTED
FROM F, WHICH

C                           ENSURES THAT A SOLUTION
EXISTS.  HWSCSP

C                           THEN COMPUTES THIS SOLUTION,
WHICH IS A

C                           LEAST SQUARES SOLUTION TO THE
ORIGINAL

C                           APPROXIMATION. THIS SOLUTION
IS NOT UNIQUE

C                           AND IS UNNORMALIZED. THE
VALUE OF PERTRB

C                           SHOULD BE SMALL COMPARED TO
THE RIGHT SIDE
```

```
C                          F. OTHERWISE , A SOLUTION IS
OBTAINED TO

C                          AN ESSENTIALLY DIFFERENT
PROBLEM. THIS

C                          COMPARISON SHOULD ALWAYS BE
MADE TO INSURE

C                          THAT A MEANINGFUL SOLUTION
HAS BEEN OBTAINED.

C

C                        IERROR

C                          AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                          PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 10,

C                          A SOLUTION IS NOT ATTEMPTED.

C

C                          = 1  TS.LT.0. OR TF.GT.PI

C                          = 2  TS.GE.TF

C                          = 3  M.LT.5

C                          = 4  MBDCND.LT.1 OR
MBDCND.GT.9

C                          = 5  RS.LT.0

C                          = 6  RS.GE.RF

C                          = 7  N.LT.5

C                          = 8  NBDCND.LT.1 OR
NBDCND.GT.6

C                          = 9  ELMBDA.GT.0

C                          = 10 IDIMF.LT.M+1
```

```
C                                  = 11 ELMBDA.NE.0 AND
MBDCND.GE.5

C                                  = 12 ELMBDA.NE.0 AND NBDCND
EQUALS 5 OR 6

C                                  = 13 MBDCND EQUALS 5,6 OR 9
AND TS.NE.0

C                                  = 14 MBDCND.GE.7 AND TF.NE.PI

C                                  = 15 TS.EQ.0 AND MBDCND
EQUALS 3,4 OR 8

C                                  = 16 TF.EQ.PI AND MBDCND
EQUALS 2,3 OR 6

C                                  = 17 NBDCND.GE.5 AND RS.NE.0

C                                  = 18 NBDCND.GE.5 AND MBDCND
EQUALS 1,2,4,5 OR

C

C                                  SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                                  A POSSLIBY INCORRECT CALL TO
HWSCSP, THE

C                                  USER SHOULD TEST IERROR AFTER
A CALL.

C

C                                W

C                                  CONTAINS INTERMEDIATE VALUES
THAT MUST NOT

C                                  BE DESTROYED IF HWSCSP WILL
BE CALLED AGAIN

C                                  WITH INTL = 1.  W(1) CONTAINS
THE NUMBER
```

```
C                              OF LOCATIONS WHICH W MUST
HAVE

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       BLKTRI, AND COMF FROM FISHPAK

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                        1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                        LIBRARIES IN JANUARY 1980.

C

C PORTABILITY            FORTRAN 77.

C

C ALGORITHM              THE ROUTINE DEFINES THE FINITE
DIFFERENCE

C                        EQUATIONS, INCORPORATES
BOUNDARY DATA, AND
```

```
C                              ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS

C                              AND THEN CALLS BLKTRI TO SOLVE
THE SYSTEM.

C

C REFERENCES                   SWARZTRAUBER,P. AND R. SWEET,
"EFFICIENT

C                              FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF

C                              ELLIPTIC EQUATIONS"

C                                NCAR TN/IA-109, JULY, 1975,
138 PP.

C********************************************************
****************
```

HWSCYL

```
      SUBROUTINE HWSCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                         ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C     *
*

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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                             BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

C     *
*
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C     *                              OF
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C

C

C

C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N+1),

C ARGUMENTS               W(SEE ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988
```

```
C

C PURPOSE                  SOLVES A FINITE DIFFERENCE
APPROXIMATION

C                          TO THE HELMHOLTZ EQUATION IN
CYLINDRICAL

C                          COORDINATES.  THIS MODIFIED
HELMHOLTZ EQUATION

C

C                            (1/R)(D/DR)(R(DU/DR)) +
(D/DZ)(DU/DZ)

C

C                            + (LAMBDA/R**2)U = F(R,Z)

C

C                          RESULTS FROM THE FOURIER
TRANSFORM OF THE

C                          THREE-DIMENSIONAL POISSON
EQUATION.

C

C USAGE                    CALL HWSCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                       PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                 A,B

C                            THE RANGE OF R, I.E., A .LE.
R .LE. B.
```

```
C                           A MUST BE LESS THAN B AND A
MUST BE

C                           NON-NEGATIVE.

C

C                         M

C                           THE NUMBER OF PANELS INTO
WHICH THE

C                           INTERVAL (A,B) IS SUBDIVIDED.
HENCE,

C                           THERE WILL BE M+1 GRID POINTS
IN THE

C                           R-DIRECTION GIVEN BY R(I) =
A+(I-1)DR,

C                           FOR I = 1,2,...,M+1, WHERE DR
= (B-A)/M

C                           IS THE PANEL WIDTH. M MUST BE
GREATER

C                           THAN 3.

C

C                         MBDCND

C                           INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                           AT R = A AND R = B.

C

C                           = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                R = A AND R = B.

C                           = 2  IF THE SOLUTION IS
SPECIFIED AT
```

```
C                              R = A AND THE DERIVATIVE
OF THE
C                              SOLUTION WITH RESPECT TO
R IS
C                              SPECIFIED AT R = B.
C                         = 3  IF THE DERIVATIVE OF THE
SOLUTION
C                              WITH RESPECT TO R IS
SPECIFIED AT
C                              R = A (SEE NOTE BELOW)
AND R = B.
C                         = 4  IF THE DERIVATIVE OF THE
SOLUTION
C                              WITH RESPECT TO R IS
SPECIFIED AT
C                              R = A (SEE NOTE BELOW)
AND THE
C                              SOLUTION IS SPECIFIED AT
R = B.
C                         = 5  IF THE SOLUTION IS
UNSPECIFIED AT
C                              R = A = 0 AND THE
SOLUTION IS
C                              SPECIFIED AT R = B.
C                         = 6  IF THE SOLUTION IS
UNSPECIFIED AT
C                              R = A = 0 AND THE
DERIVATIVE OF THE
C                              SOLUTION WITH RESPECT TO
R IS SPECIFIED
```

```
C                                      AT R = B.

C

C                                IF A = 0, DO NOT USE MBDCND =
3 OR 4,

C                                BUT INSTEAD USE MBDCND =
1,2,5, OR 6  .

C

C                              BDA

C                                A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                                SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                                THE SOLUTION WITH RESPECT TO
R AT R = A.

C

C                                WHEN MBDCND = 3 OR 4,

C                                  BDA(J) = (D/DR)U(A,Z(J)), J
= 1,2,...,N+1.

C

C                                WHEN MBDCND HAS ANY OTHER
VALUE, BDA IS

C                                A DUMMY VARIABLE.

C

C                              BDB

C                                A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                                SPECIFIES THE VALUES OF THE
DERIVATIVE
```

```
C                              OF THE SOLUTION WITH RESPECT
TO R AT R = B.

C

C                              WHEN MBDCND = 2,3, OR 6,

C                                BDB(J) = (D/DR)U(B,Z(J)), J
= 1,2,...,N+1.

C

C                              WHEN MBDCND HAS ANY OTHER
VALUE, BDB IS

C                              A DUMMY VARIABLE.

C

C                            C,D

C                              THE RANGE OF Z, I.E., C .LE.
Z .LE. D.

C                              C MUST BE LESS THAN D.

C

C                            N

C                              THE NUMBER OF PANELS INTO
WHICH THE

C                              INTERVAL (C,D) IS SUBDIVIDED.
HENCE,

C                              THERE WILL BE N+1 GRID POINTS
IN THE

C                              Z-DIRECTION GIVEN BY Z(J) =
C+(J-1)DZ,

C                              FOR J = 1,2,...,N+1,

C                              WHERE DZ = (D-C)/N IS THE
PANEL WIDTH.
```

```
C                          N MUST BE GREATER THAN 3.

C

C                        NBDCND

C                          INDICATES THE TYPE OF
BOUNDARY CONDITIONS

C                          AT Z = C AND Z = D.

C

C                          = 0  IF THE SOLUTION IS
PERIODIC IN Z,

C                               I.E., U(I,1) = U(I,N+1).

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               Z = C AND Z = D.

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               Z = C AND THE DERIVATIVE
OF

C                               THE SOLUTION WITH
RESPECT TO Z IS

C                               SPECIFIED AT Z = D.

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO Z IS

C                               SPECIFIED AT Z = C AND Z
= D.

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO Z IS
SPECIFIED AT
```

```
C                                Z = C AND THE SOLUTION
IS SPECIFIED

C                                AT Z = D.

C

C                         BDC

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO Z AT Z = C.

C

C                           WHEN NBDCND = 3 OR 4,

C                             BDC(I) = (D/DZ)U(R(I),C), I
= 1,2,...,M+1.

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDC IS

C                           A DUMMY VARIABLE.

C

C                         BDD

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                           THE SOLUTION WITH RESPECT TO
Z AT Z = D.

C
```

```
C                          WHEN NBDCND = 2 OR 3,

C                            BDD(I) = (D/DZ)U(R(I),D), I
= 1,2,...,M+1

C

C                          WHEN NBDCND HAS ANY OTHER
VALUE, BDD IS

C                          A DUMMY VARIABLE.

C

C                        ELMBDA

C                          THE CONSTANT LAMBDA IN THE
HELMHOLTZ

C                          EQUATION.  IF LAMBDA .GT. 0,
A SOLUTION

C                          MAY NOT EXIST.  HOWEVER,
HWSCYL WILL

C                          ATTEMPT TO FIND A SOLUTION.
LAMBDA MUST

C                          BE ZERO WHEN MBDCND = 5 OR 6
.

C

C                        F

C                          A TWO-DIMENSIONAL ARRAY, OF
DIMENSION AT

C                          LEAST (M+1)*(N+1), SPECIFYING
VALUES

C                          OF THE RIGHT SIDE OF THE
HELMHOLTZ

C                          EQUATION AND BOUNDARY DATA
(IF ANY).
```

```
C

C                           ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:

C                           FOR I = 2,3,...,M AND J =
2,3,...,N

C                           F(I,J) = F(R(I),Z(J)).

C

C                           ON THE BOUNDARIES F IS
DEFINED AS FOLLOWS:

C                           FOR J = 1,2,...,N+1 AND I =
1,2,...,M+1

C

C                           MBDCND   F(1,J)
F(M+1,J)

C                           ------   ---------         --
-------

C

C                             1      U(A,Z(J))
U(B,Z(J))

C                             2      U(A,Z(J))
F(B,Z(J))

C                             3      F(A,Z(J))
F(B,Z(J))

C                             4      F(A,Z(J))
U(B,Z(J))

C                             5      F(0,Z(J))
U(B,Z(J))

C                             6      F(0,Z(J))
F(B,Z(J))

C
```

```
C                           NBDCND   F(I,1)
F(I,N+1)

C                           ------   ---------        --
-------

C

C                             0      F(R(I),C)
F(R(I),C)

C                             1      U(R(I),C)
U(R(I),D)

C                             2      U(R(I),C)
F(R(I),D)

C                             3      F(R(I),C)
F(R(I),D)

C                             4      F(R(I),C)
U(R(I),D)

C

C                           NOTE:

C                           IF THE TABLE CALLS FOR BOTH
THE SOLUTION

C                           U AND THE RIGHT SIDE F AT A
CORNER THEN

C                           THE SOLUTION MUST BE
SPECIFIED.

C

C                         IDIMF

C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                           F AS IT APPEARS IN THE
PROGRAM CALLING
```

```
C                           HWSCYL.  THIS PARAMETER IS
USED TO SPECIFY

C                           THE VARIABLE DIMENSION OF F.
IDIMF MUST

C                           BE AT LEAST M+1  .

C

C                         W

C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE

C                           PROVIDED BY THE USER FOR WORK
SPACE.

C                           W MAY REQUIRE UP TO 4*(N+1) +

C                           (13 + INT(LOG2(N+1)))*(M+1)
LOCATIONS.

C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS

C                           COMPUTED BY HWSCYL AND IS
RETURNED IN

C                           LOCATION W(1).

C

C

C ON OUTPUT               F

C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE

C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                           (R(I),Z(J)), I =1,2,...,M+1,
J =1,2,...,N+1.

C
```

```
C                         PERTRB

C                           IF ONE SPECIFIES A
COMBINATION OF PERIODIC,

C                           DERIVATIVE, AND UNSPECIFIED
BOUNDARY

C                           CONDITIONS FOR A POISSON
EQUATION

C                           (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.

C                           PERTRB IS A CONSTANT,
CALCULATED AND

C                           SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                           SOLUTION EXISTS.  HWSCYL THEN
COMPUTES

C                           THIS SOLUTION, WHICH IS A
LEAST SQUARES

C                           SOLUTION TO THE ORIGINAL
APPROXIMATION.

C                           THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                           A SOLUTION.  HENCE, THE
SOLUTION IS NOT

C                           UNIQUE. THE VALUE OF PERTRB
SHOULD BE

C                           SMALL COMPARED TO THE RIGHT
SIDE F.

C                           OTHERWISE, A SOLUTION IS
OBTAINED TO AN

C                           ESSENTIALLY DIFFERENT
PROBLEM.  THIS
```

```
C                            COMPARISON SHOULD ALWAYS BE
MADE TO INSURE

C                            THAT A MEANINGFUL SOLUTION
HAS BEEN OBTAINED.

C

C                          IERROR

C                            AN ERROR FLAG WHICH INDICATES
INVALID INPUT

C                            PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 11,

C                            A SOLUTION IS NOT ATTEMPTED.

C

C                            =  0  NO ERROR.

C                            =  1  A .LT. 0  .

C                            =  2  A .GE. B.

C                            =  3  MBDCND .LT. 1 OR MBDCND
.GT. 6  .

C                            =  4  C .GE. D.

C                            =  5  N .LE. 3

C                            =  6  NBDCND .LT. 0 OR NBDCND
.GT. 4  .

C                            =  7  A = 0, MBDCND = 3 OR 4
.

C                            =  8  A .GT. 0, MBDCND .GE. 5
.

C                            =  9  A = 0, LAMBDA .NE. 0,
MBDCND .GE. 5  .

C                            = 10  IDIMF .LT. M+1  .
```

```
C                          = 11  LAMBDA .GT. 0  .

C                          = 12  M .LE. 3

C

C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING

C                          A POSSIBLY INCORRECT CALL TO
HWSCYL, THE

C                          USER SHOULD TEST IERROR AFTER
THE CALL.

C

C                        W

C                          W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       GENBUN, GNBNAUX, AND COMF

C FILES                  FROM FISHPAK

C

C LANGUAGE               FORTRAN

C
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```
C HISTORY                 WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                         1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                         LIBRARIES IN JANUARY 1980.

C

C PORTABILITY             FORTRAN 77

C

C ALGORITHM               THE ROUTINE DEFINES THE FINITE
DIFFERENCE

C                         EQUATIONS, INCORPORATES
BOUNDARY DATA, AND

C                         ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS

C                         AND THEN CALLS GENBUN TO SOLVE
THE SYSTEM.

C

C TIMING                  FOR LARGE  M AND N, THE
OPERATION COUNT

C                         IS ROUGHLY PROPORTIONAL TO

C                           M*N*(LOG2(N)

C                         BUT ALSO DEPENDS ON INPUT
PARAMETERS NBDCND

C                         AND MBDCND.

C

C ACCURACY                THE SOLUTION PROCESS EMPLOYED
RESULTS IN A LOSS

C                         OF NO MORE THAN THREE
SIGNIFICANT DIGITS FOR N
```

```
C                            AND M AS LARGE AS 64.  MORE
DETAILS ABOUT

C                            ACCURACY CAN BE FOUND IN THE
DOCUMENTATION FOR

C                            SUBROUTINE GENBUN WHICH IS THE
ROUTINE THAT

C                            SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES                 SWARZTRAUBER,P. AND R. SWEET,
"EFFICIENT

C                            FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF

C                            ELLIPTIC EQUATIONS"

C                              NCAR TN/IA-109, JULY, 1975,
138 PP.

C********************************************************
****************
```

HWSPLR

```
      SUBROUTINE HWSPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,

     1                       ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

```
C     *
*

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*

C     *
*

C     *                        FISHPACK version 4.1
*

C     *
*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                               BY
*

C     *
*
```

```
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                          OF
*

C     *
*

C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH          *

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C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C
```

```
C DIMENSION OF
BDA(N),BDB(N),BDC(M),BDD(M),F(IDIMF,N+1),

C ARGUMENTS                 W(SEE ARGUMENT LIST)

C

C LATEST REVISION          NOVEMBER 1988

C

C PURPOSE                   SOLVES A FINITE DIFFERENCE
APPROXIMATION TO

C                           THE HELMHOLTZ EQUATION IN POLAR
COORDINATES.

C                           THE EQUATION IS

C

C                               (1/R)(D/DR)(R(DU/DR)) +

C
(1/R**2)(D/DTHETA)(DU/DTHETA) +

C                               LAMBDA*U = F(R,THETA).

C

C USAGE                     CALL HWSPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,

C
NBDCND,BDC,BDD,ELMBDA,F,IDIMF,

C                                        PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                  A,B

C                             THE RANGE OF R, I.E., A .LE.
R .LE. B.
```

```
C                             A MUST BE LESS THAN B AND A
MUST BE

C                             NON-NEGATIVE.

C

C                           M

C                             THE NUMBER OF PANELS INTO
WHICH THE

C                             INTERVAL (A,B) IS SUBDIVIDED.
HENCE,

C                             THERE WILL BE M+1 GRID POINTS
IN THE

C                             R-DIRECTION GIVEN BY R(I) =
A+(I-1)DR,

C                             FOR I = 1,2,...,M+1,

C                             WHERE DR = (B-A)/M IS THE
PANEL WIDTH.

C                             M MUST BE GREATER THAN 3.

C

C                           MBDCND

C                             INDICATES THE TYPE OF
BOUNDARY CONDITION

C                             AT R = A AND R = B.

C

C                             = 1  IF THE SOLUTION IS
SPECIFIED AT

C                                  R = A AND R = B.

C                             = 2  IF THE SOLUTION IS
SPECIFIED AT
```

```
C                              R = A AND THE DERIVATIVE
OF

C                              THE SOLUTION WITH
RESPECT TO R IS

C                              SPECIFIED AT R = B.

C                         = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                              WITH RESPECT TO R IS
SPECIFIED AT

C                              R = A (SEE NOTE BELOW)
AND R = B.

C                         = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                              WITH RESPECT TO R IS
SPECIFIED AT

C                              R = A (SEE NOTE BELOW)
AND THE

C                              SOLUTION IS SPECIFIED AT
R = B.

C                         = 5  IF THE SOLUTION IS
UNSPECIFIED AT

C                              R = A = 0 AND THE
SOLUTION IS

C                              SPECIFIED AT R = B.

C                         = 6  IF THE SOLUTION IS
UNSPECIFIED AT

C                              R = A = 0 AND THE
DERIVATIVE OF THE

C                              SOLUTION WITH RESPECT TO
R IS SPECIFIED
```

```
C                                AT R = B.

C

C                              NOTE:

C                              IF A = 0, DO NOT USE MBDCND =
3 OR 4, BUT

C                              INSTEAD USE MBDCND = 1,2,5,
OR 6  .

C

C                            BDA

C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                              SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                              THE SOLUTION WITH RESPECT TO
R AT R = A.

C

C                              WHEN MBDCND = 3 OR 4,

C                                BDA(J) =
(D/DR)U(A,THETA(J)),

C                                J = 1,2,...,N+1  .

C

C                              WHEN MBDCND HAS ANY OTHER
VALUE, BDA IS

C                              A DUMMY VARIABLE.

C

C                            BDB

C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT
```

```
C                              SPECIFIES THE VALUES OF THE
DERIVATIVE OF

C                              THE SOLUTION WITH RESPECT TO
R AT R = B.

C

C                              WHEN MBDCND = 2,3, OR 6,

C                                BDB(J) =
(D/DR)U(B,THETA(J)),

C                                J = 1,2,...,N+1  .

C

C                              WHEN MBDCND HAS ANY OTHER
VALUE, BDB IS

C                              A DUMMY VARIABLE.

C

C                            C,D

C                              THE RANGE OF THETA, I.E., C
.LE.

C                              THETA .LE. D.  C MUST BE LESS
THAN D.

C

C                            N

C                              THE NUMBER OF PANELS INTO
WHICH THE

C                              INTERVAL (C,D) IS SUBDIVIDED.
HENCE,

C                              THERE WILL BE N+1 GRID POINTS
IN THE

C                              THETA-DIRECTION GIVEN BY
```

```
C                          THETA(J) = C+(J-1)DTHETA FOR
C                          J = 1,2,...,N+1, WHERE
C                          DTHETA = (D-C)/N IS THE PANEL
WIDTH.
C                          N MUST BE GREATER THAN 3.
C
C                        NBDCND
C                          INDICATES THE TYPE OF
BOUNDARY CONDITIONS
C                          AT THETA = C AND AT THETA =
D.
C
C                          = 0  IF THE SOLUTION IS
PERIODIC IN THETA,
C                               I.E., U(I,J) = U(I,N+J).
C                          = 1  IF THE SOLUTION IS
SPECIFIED AT
C                               THETA = C AND THETA = D
C                               (SEE NOTE BELOW).
C                          = 2  IF THE SOLUTION IS
SPECIFIED AT
C                               THETA = C AND THE
DERIVATIVE OF THE
C                               SOLUTION WITH RESPECT TO
THETA IS
C                               SPECIFIED AT THETA = D
C                               (SEE NOTE BELOW).
```

```
C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED

C                                AT THETA = C AND THE
SOLUTION IS

C                                SPECIFIED AT THETA = D

C                                (SEE NOTE BELOW).

C

C                           NOTE:

C                           WHEN NBDCND = 1,2, OR 4, DO
NOT USE

C                           MBDCND = 5 OR 6

C                           (THE FORMER INDICATES THAT
THE SOLUTION

C                           IS SPECIFIED AT R = 0, THE
LATTER INDICATES

C                           THE SOLUTION IS UNSPECIFIED
AT R = 0).

C                           USE INSTEAD MBDCND = 1 OR 2
.

C

C                         BDC

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO THETA AT
```

```
C                           THETA = C.  WHEN NBDCND = 3
OR 4,

C

C                             BDC(I) =
(D/DTHETA)U(R(I),C),

C                             I = 1,2,...,M+1  .

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDC IS

C                           A DUMMY VARIABLE.

C

C                         BDD

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO THETA AT

C                           THETA = D.  WHEN NBDCND = 2
OR 3,

C

C                             BDD(I) =
(D/DTHETA)U(R(I),D),

C                             I = 1,2,...,M+1  .

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDD IS

C                           A DUMMY VARIABLE.
```

```
C
C                        ELMBDA
C                          THE CONSTANT LAMBDA IN THE
HELMHOLTZ
C                          EQUATION.  IF LAMBDA .LT. 0,
A SOLUTION
C                          MAY NOT EXIST.  HOWEVER,
HWSPLR WILL
C                          ATTEMPT TO FIND A SOLUTION.
C
C                        F
C                          A TWO-DIMENSIONAL ARRAY, OF
DIMENSION AT
C                          LEAST (M+1)*(N+1), SPECIFYING
VALUES
C                          OF THE RIGHT SIDE OF THE
HELMHOLTZ
C                          EQUATION AND BOUNDARY DATA
(IF ANY).
C
C                          ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:
C                          FOR I = 2,3,...,M AND J =
2,3,...,N
C                          F(I,J) = F(R(I),THETA(J)).
C
C                          ON THE BOUNDARIES F IS
DEFINED AS FOLLOWS:
```

```
C                              FOR J = 1,2,...,N+1 AND I =
1,2,...,M+1

C

C                              MBDCND   F(1,J)
F(M+1,J)

C                              ------   -------------     --
-----------

C

C                                1      U(A,THETA(J))
U(B,THETA(J))

C                                2      U(A,THETA(J))
F(B,THETA(J))

C                                3      F(A,THETA(J))
F(B,THETA(J))

C                                4      F(A,THETA(J))
U(B,THETA(J))

C                                5      F(0,0)
U(B,THETA(J))

C                                6      F(0,0)
F(B,THETA(J))

C

C                              NBDCND   F(I,1)
F(I,N+1)

C                              ------   ---------         --
-------

C

C                                0      F(R(I),C)
F(R(I),C)

C                                1      U(R(I),C)
U(R(I),D)
```

```
C                              2      U(R(I),C)
F(R(I),D)

C                              3      F(R(I),C)
F(R(I),D)

C                              4      F(R(I),C)
U(R(I),D)

C

C                            NOTE:

C                            IF THE TABLE CALLS FOR BOTH
THE SOLUTION

C                            U AND THE RIGHT SIDE F AT A
CORNER THEN

C                            THEN THE SOLUTION MUST BE
SPECIFIED.

C

C                          IDIMF

C                            THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                            F AS IT APPEARS IN THE
PROGRAM CALLING

C                            HWSPLR.  THIS PARAMETER IS
USED TO SPECIFY

C                            THE VARIABLE DIMENSION OF F.
IDIMF MUST

C                            BE AT LEAST M+1.

C

C                          W

C                            A ONE-DIMENSIONAL ARRAY THAT
MUST BE
```

```
C                           PROVIDED BY THE USER FOR WORK
SPACE.

C                           W MAY REQUIRE UP TO 4*(N+1) +

C                           (13 + INT(LOG2(N+1)))*(M+1)
LOCATIONS.

C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS

C                           COMPUTED BY HWSPLR AND IS
RETURNED IN

C                           LOCATION W(I).

C

C

C ON OUTPUT               F

C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE

C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                           (R(I),THETA(J)),

C                           I = 1,2,...,M+1, J =
1,2,...,N+1  .

C

C                         PERTRB

C                           IF A COMBINATION OF PERIODIC,
DERIVATIVE,

C                           OR UNSPECIFIED BOUNDARY
CONDITIONS IS

C                           SPECIFIED FOR A POISSON
EQUATION
```

```
C                           (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.

C                           PERTRB IS A CONSTANT,
CALCULATED AND

C                           SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                           SOLUTION EXISTS.  HWSPLR THEN
COMPUTES

C                           THIS SOLUTION, WHICH IS A
LEAST SQUARES

C                           SOLUTION TO THE ORIGINAL
APPROXIMATION.

C                           THIS SOLUTION PLUS ANY
CONSTANT IS ALSO

C                           A SOLUTION.  HENCE, THE
SOLUTION IS NOT

C                           UNIQUE.  PERTRB SHOULD BE
SMALL COMPARED

C                           TO THE RIGHT SIDE. OTHERWISE,
A SOLUTION

C                           IS OBTAINED TO AN ESSENTIALLY
DIFFERENT

C                           PROBLEM.  THIS COMPARISON
SHOULD ALWAYS

C                           BE MADE TO INSURE THAT A
MEANINGFUL

C                           SOLUTION HAS BEEN OBTAINED.

C

C                         IERROR
```

```
C                           AN ERROR FLAG THAT INDICATES
INVALID INPUT
C                           PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 11,
C                           A SOLUTION IS NOT ATTEMPTED.
C
C                           =  0  NO ERROR.
C                           =  1  A .LT. 0  .
C                           =  2  A .GE. B.
C                           =  3  MBDCND .LT. 1 OR MBDCND
.GT. 6  .
C                           =  4  C .GE. D.
C                           =  5  N .LE. 3
C                           =  6  NBDCND .LT. 0 OR .GT. 4
.
C                           =  7  A = 0, MBDCND = 3 OR 4
.
C                           =  8  A .GT. 0, MBDCND .GE. 5
.
C                           =  9  MBDCND .GE. 5, NBDCND
.NE. 0
C                                 AND NBDCND .NE. 3  .
C                           = 10  IDIMF .LT. M+1  .
C                           = 11  LAMBDA .GT. 0  .
C                           = 12  M .LE. 3
C
C                           SINCE THIS IS THE ONLY MEANS
OF INDICATING
```

```
C                          A POSSIBLY INCORRECT CALL TO
HWSPLR, THE

C                          USER SHOULD TEST IERROR AFTER
THE CALL.

C

C                        W

C                          W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       GENBUN, GNBNAUX, AND COMF

C FILES                  FROM FISHPAK

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                        1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                        LIBRARIES IN JANUARY 1980.

C
```

```
C PORTABILITY            FORTRAN 77

C

C ALGORITHM              THE ROUTINE DEFINES THE FINITE
DIFFERENCE

C                        EQUATIONS, INCORPORATES
BOUNDARY DATA, AND

C                        ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS

C                        AND THEN CALLS GENBUN TO SOLVE
THE SYSTEM.

C

C TIMING                 FOR LARGE  M AND N, THE
OPERATION COUNT

C                        IS ROUGHLY PROPORTIONAL TO

C                          M*N*(LOG2(N)

C                        BUT ALSO DEPENDS ON INPUT
PARAMETERS NBDCND

C                        AND MBDCND.

C

C ACCURACY               THE SOLUTION PROCESS EMPLOYED
RESULTS IN A LOSS

C                        OF NO MORE THAN THREE
SIGNIFICANT DIGITS FOR N

C                        AND M AS LARGE AS 64.  MORE
DETAILS ABOUT

C                        ACCURACY CAN BE FOUND IN THE
DOCUMENTATION FOR

C                        SUBROUTINE GENBUN WHICH IS THE
ROUTINE THAT
```

```
C                         SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES               SWARZTRAUBER,P. AND R. SWEET,
"EFFICIENT

C                         FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF

C                         ELLIPTIC EQUATIONS"

C                           NCAR TN/IA-109, JULY, 1975,
138 PP.

C*******************************************************
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```

HWSSSP

```
      SUBROUTINE HWSSSP
(TS,TF,M,MBDCND,BDTS,BDTF,PS,PF,N,NBDCND,BDPS,

     1
BDPF,ELMBDA,F,IDIMF,PERTRB,IERROR,W)

C

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C     *                      FISHPACK version 4.1
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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                             BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

C     *
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C     *                             OF
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C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
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C     *                BOULDER, COLORADO  (80307)
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C

C

C

C DIMENSION OF           BDTS(N+1),    BDTF(N+1),
BDPS(M+1), BDPF(M+1),

C ARGUMENTS              F(IDIMF,N+1), W(SEE ARGUMENT
LIST)

C

C LATEST REVISION        NOVEMBER 1988

C
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```
C PURPOSE                    SOLVES A FINITE DIFFERENCE
APPROXIMATION TO

C                            THE HELMHOLTZ EQUATION IN
SPHERICAL

C                            COORDINATES AND ON THE SURFACE
OF THE UNIT

C                            SPHERE (RADIUS OF 1).  THE
EQUATION IS

C

C
(1/SIN(THETA))(D/DTHETA)(SIN(THETA)

C                              (DU/DTHETA)) +
(1/SIN(THETA)**2)(D/DPHI)

C                              (DU/DPHI)  + LAMBDA*U =
F(THETA,PHI)

C

C                            WHERE THETA IS COLATITUDE AND
PHI IS

C                            LONGITUDE.

C

C USAGE                      CALL HWSSSP
(TS,TF,M,MBDCND,BDTS,BDTF,PS,PF,

C
N,NBDCND,BDPS,BDPF,ELMBDA,F,

C
IDIMF,PERTRB,IERROR,W)

C

C ARGUMENTS

C ON INPUT                   TS,TF
```

```
C

C                           THE RANGE OF THETA
(COLATITUDE), I.E.,

C                           TS .LE. THETA .LE. TF. TS
MUST BE LESS

C                           THAN TF.  TS AND TF ARE IN
RADIANS.

C                           A TS OF ZERO CORRESPONDS TO
THE NORTH

C                           POLE AND A TF OF PI
CORRESPONDS TO

C                           THE SOUTH POLE.

C

C                           * * * IMPORTANT * * *

C

C                           IF TF IS EQUAL TO PI THEN IT
MUST BE

C                           COMPUTED USING THE STATEMENT

C                           TF = PIMACH(DUM). THIS
INSURES THAT TF

C                           IN THE USER'S PROGRAM IS
EQUAL TO PI IN

C                           THIS PROGRAM WHICH PERMITS
SEVERAL TESTS

C                           OF THE INPUT PARAMETERS THAT
OTHERWISE

C                           WOULD NOT BE POSSIBLE.

C

C
```

```
C                        M

C                          THE NUMBER OF PANELS INTO
WHICH THE

C                          INTERVAL (TS,TF) IS
SUBDIVIDED.

C                          HENCE, THERE WILL BE M+1 GRID
POINTS IN THE

C                          THETA-DIRECTION GIVEN BY

C                          THETA(I) = (I-1)DTHETA+TS FOR

C                          I = 1,2,...,M+1, WHERE

C                          DTHETA = (TF-TS)/M IS THE
PANEL WIDTH.

C                          M MUST BE GREATER THAN 5

C

C                        MBDCND

C                          INDICATES THE TYPE OF
BOUNDARY CONDITION

C                          AT THETA = TS AND THETA = TF.

C

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = TS AND THETA =
TF.

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               THETA = TS AND THE
DERIVATIVE OF

C                               THE SOLUTION WITH
RESPECT TO THETA IS
```

```
C                                SPECIFIED AT THETA = TF
C                                (SEE NOTE 2 BELOW).
C                           = 3  IF THE DERIVATIVE OF THE
SOLUTION
C                                WITH RESPECT TO THETA IS
SPECIFIED
C                                SPECIFIED AT THETA = TS
AND
C                                THETA = TF (SEE NOTES
1,2 BELOW).
C                           = 4  IF THE DERIVATIVE OF THE
SOLUTION
C                                WITH RESPECT TO THETA IS
SPECIFIED
C                                AT THETA = TS (SEE NOTE
1 BELOW)
C                                AND THE SOLUTION IS
SPECIFIED AT
C                                THETA = TF.
C                           = 5  IF THE SOLUTION IS
UNSPECIFIED AT
C                                THETA = TS = 0 AND THE
SOLUTION
C                                IS SPECIFIED AT THETA =
TF.
C                           = 6  IF THE SOLUTION IS
UNSPECIFIED AT
C                                THETA = TS = 0 AND THE
DERIVATIVE
```

```
C                                OF THE SOLUTION WITH
RESPECT TO THETA

C                                IS SPECIFIED AT THETA =
TF

C                                (SEE NOTE 2 BELOW).

C                           = 7  IF THE SOLUTION IS
SPECIFIED AT

C                                THETA = TS AND THE
SOLUTION IS

C                                IS UNSPECIFIED AT THETA
= TF = PI.

C                           = 8  IF THE DERIVATIVE OF THE
SOLUTION

C                                WITH RESPECT TO THETA IS
SPECIFIED

C                                AT THETA = TS (SEE NOTE
1 BELOW) AND

C                                THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = TF = PI.

C                           = 9  IF THE SOLUTION IS
UNSPECIFIED AT

C                                THETA = TS = 0 AND THETA
= TF = PI.

C

C                           NOTES:

C                           IF TS = 0, DO NOT USE MBDCND
= 3,4, OR 8,

C                           BUT INSTEAD USE MBDCND = 5,6,
OR 9  .
```

```
C
C                            IF TF = PI, DO NOT USE MBDCND
= 2,3, OR 6,
C                            BUT INSTEAD USE MBDCND = 7,8,
OR 9  .
C
C                          BDTS
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT
C                            SPECIFIES THE VALUES OF THE
DERIVATIVE OF
C                            THE SOLUTION WITH RESPECT TO
THETA AT
C                            THETA = TS.  WHEN MBDCND =
3,4, OR 8,
C
C                            BDTS(J) =
(D/DTHETA)U(TS,PHI(J)),
C                            J = 1,2,...,N+1  .
C
C                            WHEN MBDCND HAS ANY OTHER
VALUE, BDTS IS
C                            A DUMMY VARIABLE.
C
C                          BDTF
C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1
C                            THAT SPECIFIES THE VALUES OF
THE DERIVATIVE
```

```
C                           OF THE SOLUTION WITH RESPECT
TO THETA AT

C                           THETA = TF.  WHEN MBDCND =
2,3, OR 6,

C

C                           BDTF(J) =
(D/DTHETA)U(TF,PHI(J)),

C                           J = 1,2,...,N+1  .

C

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDTF IS

C                           A DUMMY VARIABLE.

C

C                         PS,PF

C                           THE RANGE OF PHI (LONGITUDE),
I.E.,

C                           PS .LE. PHI .LE. PF.  PS MUST
BE LESS

C                           THAN PF.  PS AND PF ARE IN
RADIANS.

C                           IF PS = 0 AND PF = 2*PI,
PERIODIC

C                           BOUNDARY CONDITIONS ARE
USUALLY PRESCRIBED.

C

C                           * * * IMPORTANT * * *

C

C                           IF PF IS EQUAL TO 2*PI THEN
IT MUST BE
```

```
C                           COMPUTED USING THE STATEMENT
C                           PF = 2.*PIMACH(DUM). THIS
INSURES THAT
C                           PF IN THE USERS PROGRAM IS
EQUAL TO
C                           2*PI IN THIS PROGRAM WHICH
PERMITS TESTS
C                           OF THE INPUT PARAMETERS THAT
OTHERWISE
C                           WOULD NOT BE POSSIBLE.
C
C                         N
C                           THE NUMBER OF PANELS INTO
WHICH THE
C                           INTERVAL (PS,PF) IS
SUBDIVIDED.
C                           HENCE, THERE WILL BE N+1 GRID
POINTS
C                           IN THE PHI-DIRECTION GIVEN BY
C                           PHI(J) = (J-1)DPHI+PS  FOR
C                           J = 1,2,...,N+1, WHERE
C                           DPHI = (PF-PS)/N IS THE PANEL
WIDTH.
C                           N MUST BE GREATER THAN 4
C
C                         NBDCND
C                           INDICATES THE TYPE OF
BOUNDARY CONDITION
```

```
C                          AT PHI = PS AND PHI = PF.

C

C                          = 0  IF THE SOLUTION IS
PERIODIC IN PHI,

C                               I.U., U(I,J) = U(I,N+J).

C                          = 1  IF THE SOLUTION IS
SPECIFIED AT

C                               PHI = PS AND PHI = PF

C                               (SEE NOTE BELOW).

C                          = 2  IF THE SOLUTION IS
SPECIFIED AT

C                               PHI = PS (SEE NOTE
BELOW)

C                               AND THE DERIVATIVE OF
THE SOLUTION

C                               WITH RESPECT TO PHI IS
SPECIFIED

C                               AT PHI = PF.

C                          = 3  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO PHI IS
SPECIFIED

C                               AT PHI = PS AND PHI =
PF.

C                          = 4  IF THE DERIVATIVE OF THE
SOLUTION

C                               WITH RESPECT TO PHI IS
SPECIFIED

C                               AT PS AND THE SOLUTION
IS SPECIFIED
```

```
C                                AT PHI = PF
C
C                           NOTE:
C                           NBDCND = 1,2, OR 4 CANNOT BE
USED WITH
C                           MBDCND = 5,6,7,8, OR 9.  THE
FORMER INDICATES
C                           THAT THE SOLUTION IS
SPECIFIED AT A POLE, THE
C                           LATTER INDICATES THAT THE
SOLUTION IS NOT
C                           SPECIFIED.  USE INSTEAD
MBDCND = 1 OR 2.
C
C                         BDPS
C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT
C                           SPECIFIES THE VALUES OF THE
DERIVATIVE
C                           OF THE SOLUTION WITH RESPECT
TO PHI AT
C                           PHI = PS.  WHEN NBDCND = 3 OR
4,
C
C                             BDPS(I) =
(D/DPHI)U(THETA(I),PS),
C                             I = 1,2,...,M+1  .
C
```

```
C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDPS IS

C                           A DUMMY VARIABLE.

C

C                         BDPF

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                           SPECIFIES THE VALUES OF THE
DERIVATIVE

C                           OF THE SOLUTION WITH RESPECT
TO PHI AT

C                           PHI = PF.  WHEN NBDCND = 2 OR
3,

C

C                             BDPF(I) =
(D/DPHI)U(THETA(I),PF),

C                             I = 1,2,...,M+1  .

C

C                           WHEN NBDCND HAS ANY OTHER
VALUE, BDPF IS

C                           A DUMMY VARIABLE.

C

C                         ELMBDA

C                           THE CONSTANT LAMBDA IN THE
HELMHOLTZ

C                           EQUATION.  IF LAMBDA .GT. 0,
A SOLUTION

C                           MAY NOT EXIST.  HOWEVER,
HWSSSP WILL
```

```
C                           ATTEMPT TO FIND A SOLUTION.

C

C                         F

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE

C                           VALUE OF THE RIGHT SIDE OF
THE HELMHOLTZ

C                           EQUATION AND BOUNDARY VALUES
(IF ANY).

C                           F MUST BE DIMENSIONED AT
LEAST (M+1)*(N+1).

C

C                           ON THE INTERIOR, F IS DEFINED
AS FOLLOWS:

C                           FOR I = 2,3,...,M AND J =
2,3,...,N

C                           F(I,J) = F(THETA(I),PHI(J)).

C

C                           ON THE BOUNDARIES F IS
DEFINED AS FOLLOWS:

C                           FOR J = 1,2,...,N+1 AND I =
1,2,...,M+1

C

C                           MBDCND   F(1,J)
F(M+1,J)

C                           ------   ------------      --
----------

C
```

```
C                               1       U(TS,PHI(J))
U(TF,PHI(J))

C                               2       U(TS,PHI(J))
F(TF,PHI(J))

C                               3       F(TS,PHI(J))
F(TF,PHI(J))

C                               4       F(TS,PHI(J))
U(TF,PHI(J))

C                               5       F(0,PS)
U(TF,PHI(J))

C                               6       F(0,PS)
F(TF,PHI(J))

C                               7       U(TS,PHI(J))
F(PI,PS)

C                               8       F(TS,PHI(J))
F(PI,PS)

C                               9       F(0,PS)
F(PI,PS)

C

C                             NBDCND   F(I,1)
F(I,N+1)

C                             ------   --------------    --
------------

C

C                               0       F(THETA(I),PS)
F(THETA(I),PS)

C                               1       U(THETA(I),PS)
U(THETA(I),PF)

C                               2       U(THETA(I),PS)
F(THETA(I),PF)
```

```
C                              3      F(THETA(I),PS)
F(THETA(I),PF)

C                              4      F(THETA(I),PS)
U(THETA(I),PF)

C

C                            NOTE:

C                            IF THE TABLE CALLS FOR BOTH
THE SOLUTION U

C                            AND THE RIGHT SIDE F AT A
CORNER THEN THE

C                            SOLUTION MUST BE SPECIFIED.

C

C                          IDIMF

C                            THE ROW (OR FIRST) DIMENSION
OF THE ARRAY

C                            F AS IT APPEARS IN THE
PROGRAM CALLING

C                            HWSSSP.  THIS PARAMETER IS
USED TO SPECIFY

C                            THE VARIABLE DIMENSION OF F.
IDIMF MUST BE

C                            AT LEAST M+1  .

C

C                          W

C                            A ONE-DIMENSIONAL ARRAY THAT
MUST BE

C                            PROVIDED BY THE USER FOR WORK
SPACE.

C                            W MAY REQUIRE UP TO
```

```
C
4*(N+1)+(16+INT(LOG2(N+1)))(M+1) LOCATIONS

C                           THE ACTUAL NUMBER OF
LOCATIONS USED IS

C                           COMPUTED BY HWSSSP AND IS
OUTPUT IN

C                           LOCATION W(1). INT( ) DENOTES
THE

C                           FORTRAN INTEGER FUNCTION.

C

C

C ON OUTPUT               F

C                           CONTAINS THE SOLUTION U(I,J)
OF THE FINITE

C                           DIFFERENCE APPROXIMATION FOR
THE GRID POINT

C                           (THETA(I),PHI(J)),  I =
1,2,...,M+1  AND

C                           J = 1,2,...,N+1  .

C

C                         PERTRB

C                           IF ONE SPECIFIES A
COMBINATION OF PERIODIC,

C                           DERIVATIVE OR UNSPECIFIED
BOUNDARY

C                           CONDITIONS FOR A POISSON
EQUATION

C                           (LAMBDA = 0), A SOLUTION MAY
NOT EXIST.
```

```
C                             PERTRB IS A CONSTANT,
CALCULATED AND

C                             SUBTRACTED FROM F, WHICH
ENSURES THAT A

C                             SOLUTION EXISTS.  HWSSSP THEN
COMPUTES

C                             THIS SOLUTION, WHICH IS A
LEAST SQUARES

C                             SOLUTION TO THE ORIGINAL
APPROXIMATION.

C                             THIS SOLUTION IS NOT UNIQUE
AND IS

C                             UNNORMALIZED. THE VALUE OF
PERTRB SHOULD

C                             BE SMALL COMPARED TO THE
RIGHT SIDE F.

C                             OTHERWISE , A SOLUTION IS
OBTAINED TO AN

C                             ESSENTIALLY DIFFERENT
PROBLEM. THIS

C                             COMPARISON SHOULD ALWAYS BE
MADE TO INSURE

C                             THAT A MEANINGFUL SOLUTION
HAS BEEN

C                             OBTAINED

C

C                           IERROR

C                             AN ERROR FLAG THAT INDICATES
INVALID INPUT
```

```
C                           PARAMETERS.  EXCEPT FOR
NUMBERS 0 AND 8,

C                           A SOLUTION IS NOT ATTEMPTED.

C

C                           = 0  NO ERROR

C                           = 1  TS.LT.0 OR TF.GT.PI

C                           = 2  TS.GE.TF

C                           = 3  MBDCND.LT.1 OR
MBDCND.GT.9

C                           = 4  PS.LT.0 OR PS.GT.PI+PI

C                           = 5  PS.GE.PF

C                           = 6  N.LT.5

C                           = 7  M.LT.5

C                           = 8  NBDCND.LT.0 OR
NBDCND.GT.4

C                           = 9  ELMBDA.GT.0

C                           = 10 IDIMF.LT.M+1

C                           = 11 NBDCND EQUALS 1,2 OR 4
AND MBDCND.GE.5

C                           = 12 TS.EQ.0 AND MBDCND
EQUALS 3,4 OR 8

C                           = 13 TF.EQ.PI AND MBDCND
EQUALS 2,3 OR 6

C                           = 14 MBDCND EQUALS 5,6 OR 9
AND TS.NE.0

C                           = 15 MBDCND.GE.7 AND TF.NE.PI

C
```

```
C                          SINCE THIS IS THE ONLY MEANS
OF INDICATING
C                          A POSSIBLY INCORRECT CALL TO
HWSSSP, THE
C                          USER SHOULD TEST IERROR AFTER
A CALL.
C
C                        W
C                          CONTAINS INTERMEDIATE VALUES
THAT MUST NOT
C                          BE DESTROYED IF HWSSSP WILL
BE CALLED AGAIN
C                          WITH INTL = 1. W(1) CONTAINS
THE REQUIRED
C                          LENGTH OF W .
C
C SPECIAL CONDITIONS     NONE
C
C I/O                    NONE
C
C PRECISION              SINGLE
C
C REQUIRED LIBRARY       GENBUN, GNBNAUX, AND COMF
C FILES                  FROM FISHPAK
C
C LANGUAGE               FORTRAN
C
```

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C HISTORY                 WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                         1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                         LIBRARIES IN JANUARY 1980.

C

C PORTABILITY             FORTRAN 77

C

C ALGORITHM               THE ROUTINE DEFINES THE FINITE
DIFFERENCE

C                         EQUATIONS, INCORPORATES
BOUNDARY DATA, AND

C                         ADJUSTS THE RIGHT SIDE OF
SINGULAR SYSTEMS

C                         AND THEN CALLS GENBUN TO SOLVE
THE SYSTEM.

C

C TIMING                  FOR LARGE  M AND N, THE
OPERATION COUNT

C                         IS ROUGHLY PROPORTIONAL TO

C                           M*N*(LOG2(N)

C                         BUT ALSO DEPENDS ON INPUT
PARAMETERS NBDCND

C                         AND MBDCND.

C

C ACCURACY                THE SOLUTION PROCESS EMPLOYED
RESULTS IN A LOSS

C                         OF NO MORE THAN THREE
SIGNIFICANT DIGITS FOR N
```

```
C                         AND M AS LARGE AS 64.  MORE
DETAILS ABOUT

C                         ACCURACY CAN BE FOUND IN THE
DOCUMENTATION FOR

C                         SUBROUTINE GENBUN WHICH IS THE
ROUTINE THAT

C                         SOLVES THE FINITE DIFFERENCE
EQUATIONS.

C

C REFERENCES              P. N. SWARZTRAUBER, "THE DIRECT
SOLUTION OF

C                         THE DISCRETE POISSON EQUATION
ON THE SURFACE OF

C                         A SPHERE", S.I.A.M. J. NUMER.
ANAL.,15(1974),

C                         PP 212-215.

C

C                         SWARZTRAUBER,P. AND R. SWEET,
"EFFICIENT

C                         FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF

C                         ELLIPTIC EQUATIONS", NCAR
TN/IA-109, JULY,

C                         1975, 138 PP.

C********************************************************
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POIS3D

```
      SUBROUTINE POIS3D
(LPEROD,L,C1,MPEROD,M,C2,NPEROD,N,A,B,C,LDIMF,

     1                  MDIMF,F,IERROR,W)

C

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C     *                      FISHPACK version 4.1
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*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
```

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C     *
*

C     *                              BY
*

C     *
*

C     *         JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

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C     *                              OF
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C     *          THE NATIONAL CENTER FOR ATMOSPHERIC
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C

C

C

C DIMENSION OF            A(N), B(N), C(N),
F(LDIMF,MDIMF,N),

C ARGUMENTS               W(SEE ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988

C

C PURPOSE                 SOLVES THE LINEAR SYSTEM OF
EQUATIONS

C                         FOR UNKNOWN X VALUES, WHERE
I=1,2,...,L,

C                         J=1,2,...,M, AND K=1,2,...,N

C

C                         C1*(X(I-1,J,K) -2.*X(I,J,K)
+X(I+1,J,K)) +

C                         C2*(X(I,J-1,K) -2.*X(I,J,K)
+X(I,J+1,K)) +

C                         A(K)*X(I,J,K-1) +B(K)*X(I,J,K)+
C(K)*X(I,J,K+1)

C                         = F(I,J,K)

C

C                         THE INDICES K-1 AND K+1 ARE
EVALUATED MODULO N,
```

```
C                         I.E. X(I,J,0)=X(I,J,N) AND
X(I,J,N+1)=X(I,J,1).

C                         THE UNKNOWNS

C                         X(0,J,K), X(L+1,J,K), X(I,0,K),
AND X(I,M+1,K)

C                         ARE ASSUMED TO TAKE ON CERTAIN
PRESCRIBED

C                         VALUES DESCRIBED BELOW.

C

C USAGE                   CALL POIS3D
(LPEROD,L,C1,MPEROD,M,C2,NPEROD,

C                         N,A,B,C,LDIMF,MDIMF,F,IERROR,W)

C

C ARGUMENTS

C

C ON INPUT

C                         LPEROD

C                           INDICATES THE VALUES THAT
X(0,J,K) AND

C                           X(L+1,J,K) ARE ASSUMED TO
HAVE.

C                           = 0  X(0,J,K)=X(L,J,K),
X(L+1,J,K)=X(1,J,K)

C                           = 1  X(0,J,K) = 0,
X(L+1,J,K) = 0

C                           = 2  X(0,J,K)=0,
X(L+1,J,K)=X(L-1,J,K)

C                           = 3  X(0,J,K)=X(2,J,K),
X(L+1,J,K)=X(L-1,J,K)
```

```
C                          = 4  X(0,J,K)=X(2,J,K),
X(L+1,J,K) = 0.

C

C                         L

C                          THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.

C                          L MUST BE AT LEAST 3.

C

C                         C1

C                          REAL CONSTANT IN THE ABOVE
LINEAR SYSTEM

C                          OF EQUATIONS TO BE SOLVED.

C

C                         MPEROD

C                          INDICATES THE VALUES THAT
X(I,0,K) AND

C                          X(I,M+1,K) ARE ASSUMED TO
HAVE.

C                          = 0  X(I,0,K)=X(I,M,K),
X(I,M+1,K)=X(I,1,K)

C                          = 1  X(I,0,K)=0,
X(I,M+1,K)=0

C                          = 2  X(I,0,K)=0,
X(I,M+1,K)=X(I,M-1,K)

C                          = 3  X(I,0,K)=X(I,2,K)
X(I,M+1,K)=X(I,M-1,K)

C                          = 4  X(I,0,K)=X(I,2,K)
X(I,M+1,K)=0

C
```

```
C                        M
C                          THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.
C                          M MUST BE AT LEAST 3.
C
C                        C2
C                          REAL CONSTANT IN THE ABOVE
LINEAR SYSTEM
C                          OF EQUATIONS TO BE SOLVED.
C
C                        NPEROD
C                          = 0  IF A(1) AND C(N) ARE NOT
ZERO.
C                          = 1  IF A(1) = C(N) = 0.
C
C                        N
C                          THE NUMBER OF UNKNOWNS IN THE
K-DIRECTION.
C                          N MUST BE AT LEAST 3.
C
C                        A, B, C
C                          ONE-DIMENSIONAL ARRAYS OF
LENGTH N THAT
C                          SPECIFY THE COEFFICIENTS IN
THE LINEAR
C                          EQUATIONS GIVEN ABOVE.
C
```

```
C                          IF NPEROD = 0 THE ARRAY
ELEMENTS MUST NOT

C                          DEPEND UPON INDEX K, BUT MUST
BE CONSTANT.

C                          SPECIFICALLY,THE SUBROUTINE
CHECKS THE

C                          FOLLOWING CONDITION

C                            A(K) = C(1)

C                            C(K) = C(1)

C                            B(K) = B(1)

C                          FOR K=1,2,...,N.

C

C                        LDIMF

C                          THE ROW (OR FIRST) DIMENSION
OF THE THREE-

C                          DIMENSIONAL ARRAY F AS IT
APPEARS IN THE

C                          PROGRAM CALLING POIS3D.  THIS
PARAMETER IS

C                          USED TO SPECIFY THE VARIABLE
DIMENSION

C                          OF F.  LDIMF MUST BE AT LEAST
L.

C

C                        MDIMF

C                          THE COLUMN (OR SECOND)
DIMENSION OF THE THREE

C                          DIMENSIONAL ARRAY F AS IT
APPEARS IN THE
```

```
C                          PROGRAM CALLING POIS3D.  THIS
PARAMETER IS

C                          USED TO SPECIFY THE VARIABLE
DIMENSION

C                          OF F.  MDIMF MUST BE AT LEAST
M.

C

C                        F

C                          A THREE-DIMENSIONAL ARRAY
THAT SPECIFIES THE

C                          VALUES OF THE RIGHT SIDE OF
THE LINEAR SYSTEM

C                          OF EQUATIONS GIVEN ABOVE.  F
MUST BE

C                          DIMENSIONED AT LEAST L X M X
N.

C

C                        W

C                          A ONE-DIMENSIONAL ARRAY THAT
MUST BE PROVIDED

C                          BY THE USER FOR WORK SPACE.
THE LENGTH OF W

C                          MUST BE AT LEAST

C                            30 + L + M + 2*N +
MAX(L,M,N) +

C                            7*(INT((L+1)/2) +
INT((M+1)/2)).

C

C ON OUTPUT
```

```
C
C                         F
C                           CONTAINS THE SOLUTION X.
C
C                         IERROR
C                           AN ERROR FLAG THAT INDICATES
INVALID INPUT
C                           PARAMETERS.  EXCEPT FOR
NUMBER ZERO, A
C                           SOLUTION IS NOT ATTEMPTED.
C                           = 0  NO ERROR
C                           = 1  IF LPEROD .LT. 0 OR .GT.
4
C                           = 2  IF L .LT. 3
C                           = 3  IF MPEROD .LT. 0 OR .GT.
4
C                           = 4  IF M .LT. 3
C                           = 5  IF NPEROD .LT. 0 OR .GT.
1
C                           = 6  IF N .LT. 3
C                           = 7  IF LDIMF .LT. L
C                           = 8  IF MDIMF .LT. M
C                           = 9  IF A(K) .NE. C(1) OR
C(K) .NE. C(1)
C                                OR B(I) .NE.B(1) FOR
SOME K=1,2,...,N.
C                           = 10 IF NPEROD = 1 AND A(1)
.NE. 0
```

```
C                                  OR C(N) .NE. 0

C

C                             SINCE THIS IS THE ONLY MEANS
OF INDICATING A

C                             POSSIBLY INCORRECT CALL TO
POIS3D, THE USER

C                             SHOULD TEST IERROR AFTER THE
CALL.

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C PRECISION              SINGLE

C

C REQUIRED LIBRARY       COMF AND FFTPACK FROM FISHPAK

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                        1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                        LIBRARIES IN JANUARY, 1980.

C
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```
C PORTABILITY            FORTRAN 77

C

C ALGORITHM              THIS SUBROUTINE SOLVES THREE-
DIMENSIONAL BLOCK

C                        TRIDIAGONAL LINEAR SYSTEMS
ARISING FROM FINITE

C                        DIFFERENCE APPROXIMATIONS TO
THREE-DIMENSIONAL

C                        POISSON EQUATIONS USING THE FFT
PACKAGE

C                        FFTPACK WRITTEN BY PAUL
SWARZTRAUBER.

C

C TIMING                 FOR LARGE L, M AND N, THE
OPERATION COUNT

C                        IS ROUGHLY PROPORTIONAL TO

C                          L*M*N*(LOG2(L)+LOG2(M)+5)

C                        BUT ALSO DEPENDS ON INPUT
PARAMETERS LPEROD

C                        AND MPEROD.

C

C ACCURACY               TO MEASURE THE ACCURACY OF THE
ALGORITHM A

C                        UNIFORM RANDOM NUMBER GENERATOR
WAS USED TO

C                        CREATE A SOLUTION ARRAY X FOR
THE SYSTEM GIVEN

C                        IN THE 'PURPOSE' SECTION WITH
```

```
C                             A(K) = C(K) = -0.5*B(K) = 1,
K=1,2,...,N

C                           AND, WHEN NPEROD = 1

C                             A(1) = C(N) = 0

C                             A(N) = C(1) = 2.

C

C                           THE SOLUTION X WAS SUBSTITUTED
INTO THE GIVEN

C                           SYSTEM AND, USING DOUBLE
PRECISION, A RIGHT

C                           SIDE Y WAS COMPUTED.  USING
THIS ARRAY Y

C                           SUBROUTINE POIS3D WAS CALLED TO
PRODUCE AN

C                           APPROXIMATE SOLUTION Z.
RELATIVE ERROR

C

C                           E = MAX(ABS(Z(I,J,K)-
X(I,J,K)))/MAX(ABS(X(I,J,K

C

C                           WAS COMPUTED, WHERE THE TWO
MAXIMA ARE TAKEN

C                           OVER I=1,2,...,L, J=1,2,...,M
AND K=1,2,...,N.

C                           VALUES OF E ARE GIVEN IN THE
TABLE BELOW FOR

C                           SOME TYPICAL VALUES OF L,M AND
N.

C
```

```
C                            L(=M=N)    LPEROD     MPEROD
E

C                            ------     ------     ------
------

C

C                              16          0          0
1.E-13

C                              15          1          1
4.E-13

C                              17          3          3
2.E-13

C                              32          0          0
2.E-13

C                              31          1          1
2.E-12

C                              33          3          3
7.E-13

C

C REFERENCES                    NONE

C
********************************************************
************
```

POISTG

```
      SUBROUTINE POISTG
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,IERROR,W)

C
```

```
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C     *                     FISHPACK version 4.1
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*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                          BY
*
```

```
C     *
*
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *
C     *
*
C     *                             OF
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* * * * * * * *
C
C
```

```
C

C DIMENSION OF            A(M),  B(M),  C(M),
Y(IDIMY,N),

C ARGUMENTS               W(SEE ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988

C

C PURPOSE                 SOLVES THE LINEAR SYSTEM OF
EQUATIONS

C                         FOR UNKNOWN X VALUES, WHERE
I=1,2,...,M

C                         AND J=1,2,...,N

C

C                         A(I)*X(I-1,J) + B(I)*X(I,J) +
C(I)*X(I+1,J)

C                         + X(I,J-1) - 2.*X(I,J) +
X(I,J+1)

C                         = Y(I,J)

C

C                         THE INDICES I+1 AND I-1 ARE
EVALUATED MODULO M,

C                         I.E. X(0,J) = X(M,J) AND
X(M+1,J) = X(1,J), AND

C                         X(I,0) MAY BE EQUAL TO X(I,1)
OR -X(I,1), AND

C                         X(I,N+1) MAY BE EQUAL TO X(I,N)
OR -X(I,N),

C                         DEPENDING ON AN INPUT
PARAMETER.
```

```
C

C USAGE                        CALL POISTG
(NPEROD,N,MPEROD,M,A,B,C,IDIMY,Y,

C                                                IERROR,W)

C

C ARGUMENTS

C

C ON INPUT

C

C                        NPEROD

C                          INDICATES VALUES WHICH X(I,0)
AND X(I,N+1)

C                          ARE ASSUMED TO HAVE.

C                          = 1 IF X(I,0) = -X(I,1) AND
X(I,N+1) = -X(I,N

C                          = 2 IF X(I,0) = -X(I,1) AND
X(I,N+1) =  X(I,N

C                          = 3 IF X(I,0) =  X(I,1) AND
X(I,N+1) =  X(I,N

C                          = 4 IF X(I,0) =  X(I,1) AND
X(I,N+1) = -X(I,N

C

C                        N

C                          THE NUMBER OF UNKNOWNS IN THE
J-DIRECTION.

C                          N MUST BE GREATER THAN 2.

C
```

```
C                         MPEROD

C                           = 0 IF A(1) AND C(M) ARE NOT
ZERO

C                           = 1 IF A(1) = C(M) = 0

C

C                         M

C                           THE NUMBER OF UNKNOWNS IN THE
I-DIRECTION.

C                           M MUST BE GREATER THAN 2.

C

C                         A,B,C

C                           ONE-DIMENSIONAL ARRAYS OF
LENGTH M THAT

C                           SPECIFY THE COEFFICIENTS IN
THE LINEAR

C                           EQUATIONS GIVEN ABOVE.  IF
MPEROD = 0 THE

C                           ARRAY ELEMENTS MUST NOT
DEPEND ON INDEX I,

C                           BUT MUST BE CONSTANT.
SPECIFICALLY, THE

C                           SUBROUTINE CHECKS THE
FOLLOWING CONDITION

C                             A(I) = C(1)

C                             B(I) = B(1)

C                             C(I) = C(1)

C                           FOR I = 1, 2, ..., M.

C
```

```
C                         IDIMY

C                           THE ROW (OR FIRST) DIMENSION
OF THE TWO-

C                           DIMENSIONAL ARRAY Y AS IT
APPEARS IN THE

C                           PROGRAM CALLING POISTG.  THIS
PARAMETER IS

C                           USED TO SPECIFY THE VARIABLE
DIMENSION OF Y.

C                           IDIMY MUST BE AT LEAST M.

C

C                         Y

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE

C                           VALUES OF THE RIGHT SIDE OF
THE LINEAR SYSTEM

C                           OF EQUATIONS GIVEN ABOVE.

C                           Y MUST BE DIMENSIONED AT
LEAST M X N.

C

C                         W

C                           A ONE-DIMENSIONAL WORK ARRAY
THAT MUST BE

C                           PROVIDED BY THE USER FOR WORK
SPACE.  W MAY

C                           REQUIRE UP TO 9M + 4N +
M(INT(LOG2(N)))

C                           LOCATIONS.  THE ACTUAL NUMBER
OF LOCATIONS
```

```
C                          USED IS COMPUTED BY POISTG
AND RETURNED IN

C                          LOCATION W(1).

C

C ON OUTPUT

C

C                        Y

C                          CONTAINS THE SOLUTION X.

C

C                        IERROR

C                          AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                          PARAMETERS.  EXCEPT FOR
NUMBER ZERO, A

C                          SOLUTION IS NOT ATTEMPTED.

C                          = 0  NO ERROR

C                          = 1  IF M .LE. 2

C                          = 2  IF N .LE. 2

C                          = 3  IDIMY .LT. M

C                          = 4  IF NPEROD .LT. 1 OR
NPEROD .GT. 4

C                          = 5  IF MPEROD .LT. 0 OR
MPEROD .GT. 1

C                          = 6  IF MPEROD = 0 AND A(I)
.NE. C(1)

C                               OR B(I) .NE. B(1) OR
C(I) .NE. C(1)
```

```
C                                FOR SOME I = 1, 2, ...,
M.

C                              = 7  IF MPEROD .EQ. 1 .AND.

C                                (A(1).NE.0 .OR.
C(M).NE.0)

C

C                         W

C                           W(1) CONTAINS THE REQUIRED
LENGTH OF W.

C

C

C I/O                     NONE

C

C PRECISION               SINGLE

C

C REQUIRED LIBRARY        GNBNAUX AND COMF FROM FISHPAK

C FILES

C

C LANGUAGE                FORTRAN

C

C HISTORY                 WRITTEN BY ROLAND SWEET AT NCAR
IN THE LATE

C                         1970'S.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                         LIBRARIES IN JANUARY, 1980.

C
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```
C PORTABILITY            FORTRAN 77
C
C ALGORITHM              THIS SUBROUTINE IS AN
IMPLEMENTATION OF THE
C                        ALGORITHM PRESENTED IN THE
REFERENCE BELOW.
C
C TIMING                 FOR LARGE M AND N, THE
EXECUTION TIME IS
C                        ROUGHLY PROPORTIONAL TO
M*N*LOG2(N).
C
C ACCURACY               TO MEASURE THE ACCURACY OF THE
ALGORITHM A
C                        UNIFORM RANDOM NUMBER GENERATOR
WAS USED TO
C                        CREATE A SOLUTION ARRAY X FOR
THE SYSTEM GIVEN
C                        IN THE 'PURPOSE' SECTION ABOVE,
WITH
C                          A(I) = C(I) = -0.5*B(I) = 1,
I=1,2,...,M
C                        AND, WHEN MPEROD = 1
C                          A(1) = C(M) = 0
C                          B(1) = B(M) =-1.
C
C                        THE SOLUTION X WAS SUBSTITUTED
INTO THE GIVEN
```

```
C                         SYSTEM AND, USING DOUBLE
PRECISION, A RIGHT SID
C                         Y WAS COMPUTED.  USING THIS
ARRAY Y SUBROUTINE
C                         POISTG WAS CALLED TO PRODUCE AN
APPROXIMATE
C                         SOLUTION Z.  THEN THE RELATIVE
ERROR, DEFINED A
C                           E = MAX(ABS(Z(I,J)-
X(I,J)))/MAX(ABS(X(I,J)))
C                         WHERE THE TWO MAXIMA ARE TAKEN
OVER I=1,2,...,M
C                         AND J=1,2,...,N, WAS COMPUTED.
VALUES OF E ARE
C                         GIVEN IN THE TABLE BELOW FOR
SOME TYPICAL VALUE
C                         OF M AND N.
C
C                         M (=N)    MPEROD    NPEROD
E
C                         ------    ------    ------    -
-----
C
C                           31        0-1       1-4
9.E-13
C                           31        1         1
4.E-13
C                           31        1         3
3.E-13
```

```
C                           32          0-1         1-4
3.E-12

C                           32          1           1
3.E-13

C                           32          1           3
1.E-13

C                           33          0-1         1-4
1.E-12

C                           33          1           1
4.E-13

C                           33          1           3
1.E-13

C                           63          0-1         1-4
3.E-12

C                           63          1           1
1.E-12

C                           63          1           3
2.E-13

C                           64          0-1         1-4
4.E-12

C                           64          1           1
1.E-12

C                           64          1           3
6.E-13

C                           65          0-1         1-4
2.E-13

C                           65          1           1
1.E-11

C                           65          1           3
4.E-13

C
```

```
C REFERENCES                SCHUMANN, U. AND R. SWEET,"A
DIRECT METHOD

C                           FOR THE SOLUTION OF POISSON"S
EQUATION WITH

C                           NEUMANN BOUNDARY CONDITIONS ON
A STAGGERED

C                           GRID OF ARBITRARY SIZE," J.
COMP. PHYS.

C                           20(1976), PP. 171-182.

C
********************************************************
*************
```

SEPELI

```
      SUBROUTINE SEPELI
(INTL,IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,

     1
D,N,NBDCND,BDC,GAMA,BDD,XNU,COFX,COFY,GRHS,

     2                      USOL,IDMN,W,PERTRB,IERROR)

C

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SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                             BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

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C     *                             OF
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C

C

C

C DIMENSION OF            BDA(N+1), BDB(N+1), BDC(M+1),
BDD(M+1),

C ARGUMENTS               USOL(IDMN,N+1),GRHS(IDMN,N+1),

C                         W (SEE ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988
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```
C

C PURPOSE                SEPELI SOLVES FOR EITHER THE
SECOND-ORDER

C                        FINITE DIFFERENCE APPROXIMATION
OR A

C                        FOURTH-ORDER APPROXIMATION TO A
SEPARABLE

C                        ELLIPTIC EQUATION

C

C                          2    2

C                          AF(X)*D U/DX + BF(X)*DU/DX  +
CF(X)*U +

C                          2    2

C                          DF(Y)*D U/DY  + EF(Y)*DU/DY +
FF(Y)*U

C

C                          = G(X,Y)

C

C                        ON A RECTANGLE (X GREATER THAN
OR EQUAL TO A

C                        AND LESS THAN OR EQUAL TO B; Y
GREATER THAN

C                        OR EQUAL TO C AND LESS THAN OR
EQUAL TO D).

C                        ANY COMBINATION OF PERIODIC OR
MIXED BOUNDARY

C                        CONDITIONS IS ALLOWED.

C
```

```
C                         THE POSSIBLE BOUNDARY
CONDITIONS ARE:

C                         IN THE X-DIRECTION:

C                         (0) PERIODIC, U(X+B-A,Y)=U(X,Y)
FOR ALL

C                             Y,X (1) U(A,Y), U(B,Y) ARE
SPECIFIED FOR

C                             ALL Y

C                         (2) U(A,Y),
DU(B,Y)/DX+BETA*U(B,Y) ARE

C                             SPECIFIED FOR ALL Y

C                         (3)
DU(A,Y)/DX+ALPHA*U(A,Y),DU(B,Y)/DX+

C                             BETA*U(B,Y) ARE SPECIFIED
FOR ALL Y

C                         (4)
DU(A,Y)/DX+ALPHA*U(A,Y),U(B,Y) ARE

C                             SPECIFIED FOR ALL Y

C

C                         IN THE Y-DIRECTION:

C                         (0) PERIODIC, U(X,Y+D-C)=U(X,Y)
FOR ALL X,Y

C                         (1) U(X,C),U(X,D) ARE SPECIFIED
FOR ALL X

C                         (2)
U(X,C),DU(X,D)/DY+XNU*U(X,D) ARE

C                             SPECIFIED FOR ALL X

C                         (3)
DU(X,C)/DY+GAMA*U(X,C),DU(X,D)/DY+
```

```
C                             XNU*U(X,D) ARE SPECIFIED
FOR ALL X

C                         (4)
DU(X,C)/DY+GAMA*U(X,C),U(X,D) ARE

C                             SPECIFIED FOR ALL X

C

C USAGE                    CALL SEPELI
(INTL,IORDER,A,B,M,MBDCND,BDA,

C
ALPHA,BDB,BETA,C,D,N,NBDCND,BDC,

C
GAMA,BDD,XNU,COFX,COFY,GRHS,USOL,

C
IDMN,W,PERTRB,IERROR)

C

C ARGUMENTS

C ON INPUT                 INTL

C                           = 0 ON INITIAL ENTRY TO
SEPELI OR IF ANY

C                               OF THE ARGUMENTS C,D, N,
NBDCND, COFY

C                               ARE CHANGED FROM A
PREVIOUS CALL

C                           = 1 IF C, D, N, NBDCND, COFY
ARE UNCHANGED

C                               FROM THE PREVIOUS CALL.

C

C                          IORDER
```

```
C                                 = 2 IF A SECOND-ORDER
APPROXIMATION

C                                     IS SOUGHT

C                                 = 4 IF A FOURTH-ORDER
APPROXIMATION

C                                     IS SOUGHT

C

C                              A,B

C                                 THE RANGE OF THE X-
INDEPENDENT VARIABLE,

C                                 I.E., X IS GREATER THAN OR
EQUAL TO A

C                                 AND LESS THAN OR EQUAL TO B.
A MUST BE

C                                 LESS THAN B.

C

C                              M

C                                 THE NUMBER OF PANELS INTO
WHICH THE

C                                 INTERVAL [A,B] IS SUBDIVIDED.
HENCE,

C                                 THERE WILL BE M+1 GRID POINTS
IN THE X-

C                                 DIRECTION GIVEN BY XI=A+(I-
1)*DLX

C                                 FOR I=1,2,...,M+1 WHERE
DLX=(B-A)/M IS

C                                 THE PANEL WIDTH.  M MUST BE
LESS THAN
```

```
C                              IDMN AND GREATER THAN 5.

C

C                            MBDCND

C                              INDICATES THE TYPE OF
BOUNDARY CONDITION

C                              AT X=A AND X=B

C

C                              = 0 IF THE SOLUTION IS
PERIODIC IN X, I.E.,

C                                  U(X+B-A,Y)=U(X,Y)  FOR
ALL Y,X

C                              = 1 IF THE SOLUTION IS
SPECIFIED AT X=A

C                                  AND X=B, I.E., U(A,Y) AND
U(B,Y) ARE

C                                  SPECIFIED FOR ALL Y

C                              = 2 IF THE SOLUTION IS
SPECIFIED AT X=A AND

C                                  THE BOUNDARY CONDITION IS
MIXED AT X=B,

C                                  I.E., U(A,Y) AND
DU(B,Y)/DX+BETA*U(B,Y)

C                                  ARE SPECIFIED FOR ALL Y

C                              = 3 IF THE BOUNDARY
CONDITIONS AT X=A AND

C                                  X=B ARE MIXED, I.E.,

C                                  DU(A,Y)/DX+ALPHA*U(A,Y)
AND
```

```
C                               DU(B,Y)/DX+BETA*U(B,Y)
ARE SPECIFIED

C                               FOR ALL Y

C                           = 4 IF THE BOUNDARY CONDITION
AT X=A IS

C                               MIXED AND THE SOLUTION IS
SPECIFIED

C                               AT X=B, I.E.,
DU(A,Y)/DX+ALPHA*U(A,Y)

C                               AND U(B,Y) ARE SPECIFIED
FOR ALL Y

C

C                         BDA

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1

C                           THAT SPECIFIES THE VALUES OF

C                           DU(A,Y)/DX+ ALPHA*U(A,Y) AT
X=A, WHEN

C                           MBDCND=3 OR 4.

C                           BDA(J) =
DU(A,YJ)/DX+ALPHA*U(A,YJ),

C                           J=1,2,...,N+1. WHEN MBDCND
HAS ANY OTHER

C                           OTHER VALUE, BDA IS A DUMMY
PARAMETER.

C

C                         ALPHA

C                           THE SCALAR MULTIPLYING THE
SOLUTION IN
```

```
C                           CASE OF A MIXED BOUNDARY
CONDITION AT X=A

C                           (SEE ARGUMENT BDA).  IF
MBDCND IS NOT

C                           EQUAL TO 3 OR 4 THEN ALPHA IS
A DUMMY

C                           PARAMETER.

C

C                         BDB

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1

C                           THAT SPECIFIES THE VALUES OF

C                           DU(B,Y)/DX+ BETA*U(B,Y) AT
X=B.

C                           WHEN MBDCND=2 OR 3

C                           BDB(J) =
DU(B,YJ)/DX+BETA*U(B,YJ),

C                           J=1,2,...,N+1. WHEN MBDCND
HAS ANY OTHER

C                           OTHER VALUE, BDB IS A DUMMY
PARAMETER.

C

C                         BETA

C                           THE SCALAR MULTIPLYING THE
SOLUTION IN

C                           CASE OF A MIXED BOUNDARY
CONDITION AT

C                           X=B (SEE ARGUMENT BDB).  IF
MBDCND IS
```

```
C                                NOT EQUAL TO 2 OR 3 THEN BETA
IS A DUMMY
C                                PARAMETER.
C
C                              C,D
C                                THE RANGE OF THE Y-
INDEPENDENT VARIABLE,
C                                I.E., Y IS GREATER THAN OR
EQUAL TO C
C                                AND LESS THAN OR EQUAL TO D.
C MUST BE
C                                LESS THAN D.
C
C                              N
C                                THE NUMBER OF PANELS INTO
WHICH THE
C                                INTERVAL [C,D] IS SUBDIVIDED.
C                                HENCE, THERE WILL BE N+1 GRID
POINTS
C                                IN THE Y-DIRECTION GIVEN BY
C                                YJ=C+(J-1)*DLY FOR
J=1,2,...,N+1 WHERE
C                                DLY=(D-C)/N IS THE PANEL
WIDTH.
C                                IN ADDITION, N MUST BE
GREATER THAN 4.
C
C                              NBDCND
```

```
C                           INDICATES THE TYPES OF
BOUNDARY CONDITIONS

C                           AT Y=C AND Y=D

C

C                           = 0 IF THE SOLUTION IS
PERIODIC IN Y,

C                               I.E., U(X,Y+D-C)=U(X,Y)
FOR ALL X,Y

C                           = 1 IF THE SOLUTION IS
SPECIFIED AT Y=C

C                               AND Y = D, I.E., U(X,C)
AND U(X,D)

C                               ARE SPECIFIED FOR ALL X

C                           = 2 IF THE SOLUTION IS
SPECIFIED AT Y=C

C                               AND THE BOUNDARY
CONDITION IS MIXED

C                               AT Y=D, I.E., U(X,C) AND

C                               DU(X,D)/DY+XNU*U(X,D) ARE
SPECIFIED

C                               FOR ALL X

C                           = 3 IF THE BOUNDARY
CONDITIONS ARE MIXED

C                               AT Y=C AND Y=D, I.E.,

C                               DU(X,D)/DY+GAMA*U(X,C)
AND

C                               DU(X,D)/DY+XNU*U(X,D) ARE
SPECIFIED

C                               FOR ALL X
```

```
C                              = 4 IF THE BOUNDARY CONDITION
IS MIXED

C                                  AT Y=C AND THE SOLUTION
IS SPECIFIED

C                                  AT Y=D, I.E.
DU(X,C)/DY+GAMA*U(X,C)

C                                  AND U(X,D) ARE SPECIFIED
FOR ALL X

C

C                            BDC

C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1

C                              THAT SPECIFIES THE VALUE OF

C                              DU(X,C)/DY+GAMA*U(X,C) AT
Y=C.

C                              WHEN NBDCND=3 OR 4 BDC(I) =
DU(XI,C)/DY +

C                              GAMA*U(XI,C), I=1,2,...,M+1.

C                              WHEN NBDCND HAS ANY OTHER
VALUE, BDC

C                              IS A DUMMY PARAMETER.

C

C                            GAMA

C                              THE SCALAR MULTIPLYING THE
SOLUTION IN

C                              CASE OF A MIXED BOUNDARY
CONDITION AT

C                              Y=C (SEE ARGUMENT BDC).  IF
NBDCND IS
```

```
C                            NOT EQUAL TO 3 OR 4 THEN GAMA
IS A DUMMY

C                            PARAMETER.

C

C                          BDD

C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1

C                            THAT SPECIFIES THE VALUE OF

C                            DU(X,D)/DY + XNU*U(X,D) AT
Y=C.

C                            WHEN NBDCND=2 OR 3 BDD(I) =
DU(XI,D)/DY +

C                            XNU*U(XI,D), I=1,2,...,M+1.

C                            WHEN NBDCND HAS ANY OTHER
VALUE, BDD

C                            IS A DUMMY PARAMETER.

C

C                          XNU

C                            THE SCALAR MULTIPLYING THE
SOLUTION IN

C                            CASE OF A MIXED BOUNDARY
CONDITION AT

C                            Y=D (SEE ARGUMENT BDD).  IF
NBDCND IS

C                            NOT EQUAL TO 2 OR 3 THEN XNU
IS A

C                            DUMMY PARAMETER.

C
```

```
C                         COFX

C                           A USER-SUPPLIED SUBPROGRAM
WITH

C                           PARAMETERS X, AFUN, BFUN,
CFUN WHICH

C                           RETURNS THE VALUES OF THE X-
DEPENDENT

C                           COEFFICIENTS AF(X), BF(X),
CF(X) IN THE

C                           ELLIPTIC EQUATION AT X.

C

C                         COFY

C                           A USER-SUPPLIED SUBPROGRAM
WITH PARAMETERS

C                           Y, DFUN, EFUN, FFUN WHICH
RETURNS THE

C                           VALUES OF THE Y-DEPENDENT
COEFFICIENTS

C                           DF(Y), EF(Y), FF(Y) IN THE
ELLIPTIC

C                           EQUATION AT Y.

C

C                           NOTE:  COFX AND COFY MUST BE
DECLARED

C                           EXTERNAL IN THE CALLING
ROUTINE.

C                           THE VALUES RETURNED IN AFUN
AND DFUN

C                           MUST SATISFY AFUN*DFUN
GREATER THAN 0
```

```
C                           FOR A LESS THAN X LESS THAN
B, C LESS

C                           THAN Y LESS THAN D (SEE
IERROR=10).

C                           THE COEFFICIENTS PROVIDED MAY
LEAD TO A

C                           MATRIX EQUATION WHICH IS NOT
DIAGONALLY

C                           DOMINANT IN WHICH CASE
SOLUTION MAY FAIL

C                           (SEE IERROR=4).

C

C                         GRHS

C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE

C                           VALUES OF THE RIGHT-HAND SIDE
OF THE

C                           ELLIPTIC EQUATION, I.E.,

C                           GRHS(I,J)=G(XI,YI), FOR
I=2,...,M,

C                           J=2,...,N.  AT THE
BOUNDARIES, GRHS IS

C                           DEFINED BY

C

C                           MBDCND   GRHS(1,J)
GRHS(M+1,J)

C                           ------   ---------   --------
---

C                             0      G(A,YJ)     G(B,YJ)
```

```
C                           1         *           *

C                           2         *        G(B,YJ)
J=1,2,...,N+1

C                           3      G(A,YJ)     G(B,YJ)

C                           4      G(A,YJ)        *

C

C                         NBDCND   GRHS(I,1)
GRHS(I,N+1)

C                         ------   ---------   --------
---

C                           0      G(XI,C)     G(XI,D)

C                           1         *           *

C                           2         *        G(XI,D)
I=1,2,...,M+1

C                           3      G(XI,C)     G(XI,D)

C                           4      G(XI,C)        *

C

C                         WHERE * MEANS THESE
QUANTITIES ARE NOT USED.

C                         GRHS SHOULD BE DIMENSIONED
IDMN BY AT LEAST

C                         N+1 IN THE CALLING ROUTINE.

C

C                       USOL

C                         A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE

C                         VALUES OF THE SOLUTION ALONG
THE BOUNDARIES.
```

```
C                           AT THE BOUNDARIES, USOL IS
DEFINED BY

C

C                           MBDCND   USOL(1,J)
USOL(M+1,J)

C                           ------   ---------   --------
---

C                             0          *           *

C                             1      U(A,YJ)     U(B,YJ)

C                             2      U(A,YJ)         *
J=1,2,...,N+1

C                             3          *           *

C                             4          *       U(B,YJ)

C

C                           NBDCND   USOL(I,1)
USOL(I,N+1)

C                           ------   ---------   --------
---

C                             0          *           *

C                             1      U(XI,C)     U(XI,D)

C                             2      U(XI,C)         *
I=1,2,...,M+1

C                             3          *           *

C                             4          *       U(XI,D)

C

C                           WHERE * MEANS THE QUANTITIES
ARE NOT USED

C                           IN THE SOLUTION.
```

```
C

C                           IF IORDER=2, THE USER MAY
EQUIVALENCE GRHS

C                           AND USOL TO SAVE SPACE.  NOTE
THAT IN THIS

C                           CASE THE TABLES SPECIFYING
THE BOUNDARIES

C                           OF THE GRHS AND USOL ARRAYS
DETERMINE THE

C                           BOUNDARIES UNIQUELY EXCEPT AT
THE CORNERS.

C                           IF THE TABLES CALL FOR BOTH
G(X,Y) AND

C                           U(X,Y) AT A CORNER THEN THE
SOLUTION MUST

C                           BE CHOSEN.  FOR EXAMPLE, IF
MBDCND=2 AND

C                           NBDCND=4, THEN U(A,C),
U(A,D), U(B,D) MUST

C                           BE CHOSEN AT THE CORNERS IN
ADDITION

C                           TO G(B,C).

C

C                           IF IORDER=4, THEN THE TWO
ARRAYS, USOL AND

C                           GRHS, MUST BE DISTINCT.

C

C                           USOL SHOULD BE DIMENSIONED
IDMN BY AT LEAST

C                           N+1 IN THE CALLING ROUTINE.
```

```
C
C                         IDMN
C                           THE ROW (OR FIRST) DIMENSION
OF THE ARRAYS
C                           GRHS AND USOL AS IT APPEARS
IN THE PROGRAM
C                           CALLING SEPELI.  THIS
PARAMETER IS USED
C                           TO SPECIFY THE VARIABLE
DIMENSION OF GRHS
C                           AND USOL.  IDMN MUST BE AT
LEAST 7 AND
C                           GREATER THAN OR EQUAL TO M+1.
C
C                         W
C                           A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                           PROVIDED BY THE USER FOR WORK
SPACE.
C                           LET K=INT(LOG2(N+1))+1 AND
SET L=2**(K+1).
C                           THEN (K-2)*L+K+10*N+12*M+27
WILL SUFFICE
C                           AS A LENGTH OF W.  THE ACTUAL
LENGTH OF W
C                           IN THE CALLING ROUTINE MUST
BE SET IN W(1)
C                           (SEE IERROR=11).
C
```

```
C ON OUTPUT              USOL

C                          CONTAINS THE APPROXIMATE
SOLUTION TO THE

C                          ELLIPTIC EQUATION.

C                          USOL(I,J) IS THE
APPROXIMATION TO U(XI,YJ)

C                          FOR I=1,2...,M+1 AND
J=1,2,...,N+1.

C                          THE APPROXIMATION HAS ERROR

C                          O(DLX**2+DLY**2) IF CALLED
WITH IORDER=2

C                          AND O(DLX**4+DLY**4) IF
CALLED WITH

C                          IORDER=4.

C

C                        W

C                          CONTAINS INTERMEDIATE VALUES
THAT MUST NOT

C                          BE DESTROYED IF SEPELI IS
CALLED AGAIN WITH

C                          INTL=1.  IN ADDITION W(1)
CONTAINS THE

C                          EXACT MINIMAL LENGTH (IN
FLOATING POINT)

C                          REQUIRED FOR THE WORK SPACE
(SEE IERROR=11).

C

C                        PERTRB
```

```
C                            IF A COMBINATION OF PERIODIC
OR DERIVATIVE

C                            BOUNDARY CONDITIONS

C                            (I.E., ALPHA=BETA=0 IF
MBDCND=3;

C                            GAMA=XNU=0 IF NBDCND=3) IS
SPECIFIED

C                            AND IF THE COEFFICIENTS OF
U(X,Y) IN THE

C                            SEPARABLE ELLIPTIC EQUATION
ARE ZERO

C                            (I.E., CF(X)=0 FOR X GREATER
THAN OR EQUAL

C                            TO A AND LESS THAN OR EQUAL
TO B;

C                            FF(Y)=0 FOR Y GREATER THAN OR
EQUAL TO C

C                            AND LESS THAN OR EQUAL TO D)
THEN A

C                            SOLUTION MAY NOT EXIST.
PERTRB IS A

C                            CONSTANT CALCULATED AND
SUBTRACTED FROM

C                            THE RIGHT-HAND SIDE OF THE
MATRIX EQUATIONS

C                            GENERATED BY SEPELI WHICH
INSURES THAT A

C                            SOLUTION EXISTS. SEPELI THEN
COMPUTES THIS

C                            SOLUTION WHICH IS A WEIGHTED
MINIMAL LEAST
```

```
C                           SQUARES SOLUTION TO THE
ORIGINAL PROBLEM.

C

C                         IERROR

C                           AN ERROR FLAG THAT INDICATES
INVALID INPUT

C                           PARAMETERS OR FAILURE TO FIND
A SOLUTION

C                           = 0 NO ERROR

C                           = 1 IF A GREATER THAN B OR C
GREATER THAN D

C                           = 2 IF MBDCND LESS THAN 0 OR
MBDCND GREATER

C                               THAN 4

C                           = 3 IF NBDCND LESS THAN 0 OR
NBDCND GREATER

C                               THAN 4

C                           = 4 IF ATTEMPT TO FIND A
SOLUTION FAILS.

C                               (THE LINEAR SYSTEM
GENERATED IS NOT

C                               DIAGONALLY DOMINANT.)

C                           = 5 IF IDMN IS TOO SMALL

C                               (SEE DISCUSSION OF IDMN)

C                           = 6 IF M IS TOO SMALL OR TOO
LARGE

C                               (SEE DISCUSSION OF M)

C                           = 7 IF N IS TOO SMALL (SEE
DISCUSSION OF N)
```

```
C                           = 8 IF IORDER IS NOT 2 OR 4

C                           = 9 IF INTL IS NOT 0 OR 1

C                           = 10 IF AFUN*DFUN LESS THAN
OR EQUAL TO 0

C                                FOR SOME INTERIOR MESH
POINT (XI,YJ)

C                           = 11 IF THE WORK SPACE LENGTH
INPUT IN W(1)

C                                IS LESS THAN THE EXACT
MINIMAL WORK

C                                SPACE LENGTH REQUIRED
OUTPUT IN W(1).

C

C                           NOTE (CONCERNING IERROR=4):
FOR THE

C                           COEFFICIENTS INPUT THROUGH
COFX, COFY,

C                           THE DISCRETIZATION MAY LEAD
TO A BLOCK

C                           TRIDIAGONAL LINEAR SYSTEM
WHICH IS NOT

C                           DIAGONALLY DOMINANT (FOR
EXAMPLE, THIS

C                           HAPPENS IF CFUN=0 AND
BFUN/(2.*DLX) GREATER

C                           THAN AFUN/DLX**2).  IN THIS
CASE SOLUTION

C                           MAY FAIL.  THIS CANNOT HAPPEN
IN THE LIMIT
```

```
C                           AS DLX, DLY APPROACH ZERO.
HENCE, THE

C                           CONDITION MAY BE REMEDIED BY
TAKING LARGER

C                           VALUES FOR M OR N.

C

C SPECIAL CONDITIONS      SEE COFX, COFY ARGUMENT
DESCRIPTIONS ABOVE.

C

C I/O                     NONE

C

C PRECISION               SINGLE

C

C REQUIRED LIBRARY        BLKTRI, COMF, AND SEPAUX

C FILES                   FROM FISHPAK

C

C LANGUAGE                FORTRAN

C

C HISTORY                 DEVELOPED AT NCAR DURING 1975-
76 BY

C                         JOHN C. ADAMS OF THE SCIENTIFIC
COMPUTING

C                         DIVISION.  RELEASED ON NCAR'S
PUBLIC SOFTWARE

C                         LIBRARIES IN JANUARY 1980.

C

C PORTABILITY             FORTRAN 77
```

```
C
C ALGORITHM                SEPELI AUTOMATICALLY
DISCRETIZES THE
C                          SEPARABLE ELLIPTIC EQUATION
WHICH IS THEN
C                          SOLVED BY A GENERALIZED CYCLIC
REDUCTION
C                          ALGORITHM IN THE SUBROUTINE,
BLKTRI.  THE
C                          FOURTH-ORDER SOLUTION IS
OBTAINED USING
C                          'DEFERRED CORRECTIONS' WHICH IS
DESCRIBED
C                          AND REFERENCED IN SECTIONS,
REFERENCES AND
C                          METHOD.
C
C TIMING                   THE OPERATIONAL COUNT IS
PROPORTIONAL TO
C                          M*N*LOG2(N).
C
C ACCURACY                 THE FOLLOWING ACCURACY RESULTS
WERE OBTAINED
C                          ON A CDC 7600.  NOTE THAT THE
FOURTH-ORDER
C                          ACCURACY IS NOT REALIZED UNTIL
THE MESH IS
C                          SUFFICIENTLY REFINED.
C
```

```
C                                      SECOND-ORDER
FOURTH-ORDER

C                                M    N     ERROR
ERROR

C

C                                 6    6    6.8E-1
1.2E0

C                                14   14    1.4E-1
1.8E-1

C                                30   30    3.2E-2
9.7E-3

C                                62   62    7.5E-3
3.0E-4

C                               126  126    1.8E-3
3.5E-6

C

C

C REFERENCES                 KELLER, H.B., NUMERICAL METHODS
FOR TWO-POINT

C                            BOUNDARY-VALUE PROBLEMS,
BLAISDEL (1968),

C                            WALTHAM, MASS.

C

C                            SWARZTRAUBER, P., AND R. SWEET
(1975):

C                            EFFICIENT FORTRAN SUBPROGRAMS
FOR THE

C                            SOLUTION OF ELLIPTIC PARTIAL
DIFFERENTIAL

C                            EQUATIONS.  NCAR TECHNICAL NOTE
```

```
C                            NCAR-TN/IA-109, PP. 135-137.

C********************************************************
****************
```

SEPX4

```
      SUBROUTINE SEPX4
(IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,D,N,

     1
NBDCND,BDC,BDD,COFX,GRHS,USOL,IDMN,W,PERTRB,

     2                   IERROR)

C

C      * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C      *
*

C      *                 copyright (c) 1999 by UCAR
*

C      *
*

C      *       UNIVERSITY CORPORATION for ATMOSPHERIC
RESEARCH         *

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*

C      *                     all rights reserved
*

C      *
*
```

```
C     *                      FISHPACK version 4.1
*

C     *
*

C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *

C     *
*

C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                             BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                             OF
*

C     *
*

C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH         *

C     *
*

C     *                BOULDER, COLORADO  (80307)
U.S.A.            *
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*

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C

C DIMENSION OF            BDA(N+1), BDB(N+1), BDC(M+1),
BDD(M+1),

C ARGUMENTS               USOL(IDMN,N+1),
GRHS(IDMN,N+1),

C                         W (SEE ARGUMENT LIST)

C

C LATEST REVISION         NOVEMBER 1988

C

C PURPOSE                 SEPX4 SOLVES FOR EITHER THE
SECOND-ORDER

C                         FINITE DIFFERENCE APPROXIMATION
OR A

C                         FOURTH-ORDER APPROXIMATION TO A
SEPARABLE
```

```
C                         ELLIPTIC EQUATION

C

C
AF(X)*UXX+BF(X)*UX+CF(X)*U+UYY = G(X,Y)

C

C                         ON A RECTANGLE (X GREATER THAN
OR EQUAL TO

C                         A AND LESS THAN OR EQUAL TO B,
Y GREATER THAN

C                         OR EQUAL TO C AND LESS THAN OR
EQUAL TO D).

C                         ANY COMBINATION OF PERIODIC OR
MIXED BOUNDARY

C                         CONDITIONS IS ALLOWED.  IF
BOUNDARY

C                         CONDITIONS IN THE X DIRECTION
ARE PERIODIC

C                         (SEE MBDCND=0 BELOW) THEN THE
COEFFICIENTS

C                         MUST SATISFY

C

C                           AF(X)=C1,BF(X)=0,CF(X)=C2 FOR
ALL X.

C

C                         HERE C1,C2 ARE CONSTANTS,
C1.GT.0.

C

C                         THE POSSIBLE BOUNDARY
CONDITIONS ARE:
```

```
C                         IN THE X-DIRECTION:

C                           (0) PERIODIC, U(X+B-
A,Y)=U(X,Y) FOR

C                               ALL Y,X

C                           (1) U(A,Y), U(B,Y) ARE
SPECIFIED FOR ALL Y

C                           (2) U(A,Y),
DU(B,Y)/DX+BETA*U(B,Y) ARE

C                               SPECIFIED FOR ALL Y

C                           (3)
DU(A,Y)/DX+ALPHA*U(A,Y),DU(B,Y)/DX+

C                               BETA*U(B,Y) ARE SPECIFIED
FOR ALL Y

C                           (4)
DU(A,Y)/DX+ALPHA*U(A,Y),U(B,Y) ARE

C                               SPECIFIED FOR ALL Y

C

C                         IN THE Y-DIRECTION:

C                           (0) PERIODIC, U(X,Y+D-
C)=U(X,Y) FOR ALL X,Y

C                           (1) U(X,C),U(X,D) ARE
SPECIFIED FOR ALL X

C                           (2) U(X,C),DU(X,D)/DY ARE
SPECIFIED FOR

C                               ALL X

C                           (3) DU(X,C)/DY,DU(X,D)/DY ARE
SPECIFIED FOR

C                               ALL X
```

```
C                           (4) DU(X,C)/DY,U(X,D) ARE
SPECIFIED FOR

C                               ALL X

C

C USAGE                      CALL
SEPX4(IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,

C
BETA,C,D,N,NBDCND,BDC,BDD,COFX,

C
GRHS,USOL,IDMN,W,PERTRB,IERROR)

C

C ARGUMENTS

C ON INPUT                   IORDER

C                             = 2 IF A SECOND-ORDER
APPROXIMATION IS

C                               SOUGHT

C                             = 4 IF A FOURTH-ORDER
APPROXIMATION IS

C                               SOUGHT

C

C                            A,B

C                             THE RANGE OF THE X-
INDEPENDENT VARIABLE,

C                             I.E., X IS GREATER THAN OR
EQUAL TO A

C                             AND LESS THAN OR EQUAL TO B.
A MUST BE

C                             LESS THAN B.
```

```
C
C                        M
C                          THE NUMBER OF PANELS INTO
WHICH THE
C                          INTERVAL (A,B) IS SUBDIVIDED.
HENCE,
C                          THERE WILL BE M+1 GRID POINTS
IN THE X-
C                          DIRECTION GIVEN BY XI=A+(I-
1)*DLX
C                          FOR I=1,2,...,M+1 WHERE
DLX=(B-A)/M IS
C                          THE PANEL WIDTH.  M MUST BE
LESS THAN
C                          IDMN AND GREATER THAN 5.
C
C                        MBDCND
C                          INDICATES THE TYPE OF
BOUNDARY CONDITION
C                          AT X=A AND X=B
C                          = 0 IF THE SOLUTION IS
PERIODIC IN X, I.E.,
C                              U(X+B-A,Y)=U(X,Y)  FOR
ALL Y,X
C                          = 1 IF THE SOLUTION IS
SPECIFIED AT X=A
C                              AND X=B, I.E., U(A,Y) AND
U(B,Y) ARE
C                              SPECIFIED FOR ALL Y
```

```
C                            = 2 IF THE SOLUTION IS
SPECIFIED AT X=A

C                                AND THE BOUNDARY
CONDITION IS MIXED AT

C                                X=B, I.E., U(A,Y) AND

C                                DU(B,Y)/DX+BETA*U(B,Y)
ARE SPECIFIED

C                                FOR ALL Y

C                            = 3 IF THE BOUNDARY
CONDITIONS AT X=A AND

C                                X=B ARE MIXED, I.E.,

C                                DU(A,Y)/DX+ALPHA*U(A,Y)
AND

C                                DU(B,Y)/DX+BETA*U(B,Y)
ARE SPECIFIED

C                                FOR ALL Y

C                            = 4 IF THE BOUNDARY CONDITION
AT X=A IS

C                                MIXED AND THE SOLUTION IS
SPECIFIED

C                                AT X=B, I.E.,
DU(A,Y)/DX+ALPHA*U(A,Y)

C                                AND U(B,Y) ARE SPECIFIED
FOR ALL Y

C

C                          BDA

C                            A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                            SPECIFIES THE VALUES OF
```

```
C                           DU(A,Y)/DX+ ALPHA*U(A,Y) AT
X=A, WHEN

C                           MBDCND=3 OR 4.

C                           BDA(J) =
DU(A,YJ)/DX+ALPHA*U(A,YJ),

C                           J=1,2,...,N+1

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDA IS

C                           A DUMMY PARAMETER.

C

C                         ALPHA

C                           THE SCALAR MULTIPLYING THE
SOLUTION IN CASE

C                           OF A MIXED BOUNDARY CONDITION
AT X=A

C                           (SEE ARGUMENT BDA).  IF
MBDCND IS NOT EQUAL

C                           TO EITHER 3 OR 4, THEN ALPHA
IS A DUMMY

C                           PARAMETER.

C

C                         BDB

C                           A ONE-DIMENSIONAL ARRAY OF
LENGTH N+1 THAT

C                           SPECIFIES THE VALUES OF

C                           DU(B,Y)/DX+ BETA*U(B,Y) AT
X=B.

C                           WHEN MBDCND=2 OR 3
```

```
C                           BDB(J) =
DU(B,YJ)/DX+BETA*U(B,YJ),

C                           J=1,2,...,N+1

C                           WHEN MBDCND HAS ANY OTHER
VALUE, BDB IS

C                           A DUMMY PARAMETER.

C

C                         BETA

C                           THE SCALAR MULTIPLYING THE
SOLUTION IN

C                           CASE OF A MIXED BOUNDARY
CONDITION AT X=B

C                           (SEE ARGUMENT BDB).  IF
MBDCND IS NOT EQUAL

C                           TO 2 OR 3, THEN BETA IS A
DUMMY PARAMETER.

C

C                         C,D

C                           THE RANGE OF THE Y-
INDEPENDENT VARIABLE,

C                           I.E., Y IS GREATER THAN OR
EQUAL TO C AND

C                           LESS THAN OR EQUAL TO D.  C
MUST BE LESS

C                           THAN D.

C

C                         N

C                           THE NUMBER OF PANELS INTO
WHICH THE
```

```
C                           INTERVAL (C,D) IS SUBDIVIDED.
HENCE,

C                           THERE WILL BE N+1 GRID POINTS
IN THE Y-

C                           DIRECTION GIVEN BY YJ=C+(J-
1)*DLY FOR

C                           J=1,2,...,N+1 WHERE DLY=(D-
C)/N IS THE

C                           PANEL WIDTH.  IN ADDITION, N
MUST BE

C                           GREATER THAN 4.

C

C                         NBDCND

C                           INDICATES THE TYPES OF
BOUNDARY CONDITIONS

C                           AT Y=C AND Y=D

C                           = 0 IF THE SOLUTION IS
PERIODIC IN Y,

C                               I.E., U(X,Y+D-C)=U(X,Y)
FOR ALL X,Y

C                           = 1 IF THE SOLUTION IS
SPECIFIED AT Y=C

C                               AND Y = D, I.E., U(X,C)
AND U(X,D)

C                               ARE SPECIFIED FOR ALL X

C                           = 2 IF THE SOLUTION IS
SPECIFIED AT Y=C

C                               AND THE BOUNDARY
CONDITION IS MIXED
```

```
C                              AT Y=D, I.E., DU(X,C)/DY
AND U(X,D)

C                              ARE SPECIFIED FOR ALL X

C                          = 3 IF THE BOUNDARY
CONDITIONS ARE MIXED

C                              AT Y=CAND Y=D I.E.,

C                              DU(X,D)/DY AND DU(X,D)/DY
ARE

C                              SPECIFIED FOR ALL X

C                          = 4 IF THE BOUNDARY CONDITION
IS MIXED

C                              AT Y=C AND THE SOLUTION
IS SPECIFIED

C                              AT Y=D, I.E.
DU(X,C)/DY+GAMA*U(X,C)

C                              AND U(X,D) ARE SPECIFIED
FOR ALL X

C

C                        BDC

C                          A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                          SPECIFIES THE VALUE
DU(X,C)/DY AT Y=C.

C

C                          WHEN NBDCND=3 OR 4

C                            BDC(I) = DU(XI,C)/DY
I=1,2,...,M+1.

C
```

```
C                              WHEN NBDCND HAS ANY OTHER
VALUE, BDC IS

C                              A DUMMY PARAMETER.

C

C                            BDD

C                              A ONE-DIMENSIONAL ARRAY OF
LENGTH M+1 THAT

C                              SPECIFIED THE VALUE OF
DU(X,D)/DY AT Y=D.

C

C                              WHEN NBDCND=2 OR 3

C                                BDD(I)=DU(XI,D)/DY
I=1,2,...,M+1.

C

C                              WHEN NBDCND HAS ANY OTHER
VALUE, BDD IS

C                              A DUMMY PARAMETER.

C

C                            COFX

C                              A USER-SUPPLIED SUBPROGRAM
WITH PARAMETERS

C                              X, AFUN, BFUN, CFUN WHICH
RETURNS THE

C                              VALUES OF THE X-DEPENDENT
COEFFICIENTS

C                              AF(X), BF(X), CF(X) IN THE
ELLIPTIC

C                              EQUATION AT X.  IF BOUNDARY
CONDITIONS IN
```

```
C                              THE X DIRECTION ARE PERIODIC
THEN THE

C                              COEFFICIENTS MUST SATISFY
AF(X)=C1,BF(X)=0,

C                              CF(X)=C2 FOR ALL X.  HERE
C1.GT.0

C                              AND C2 ARE CONSTANTS.

C

C                              NOTE THAT COFX MUST BE
DECLARED EXTERNAL

C                              IN THE CALLING ROUTINE.

C

C                            GRHS

C                              A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE

C                              VALUES OF THE RIGHT-HAND SIDE
OF THE

C                              ELLIPTIC EQUATION,
I.E.,GRHS(I,J)=G(XI,YI),

C                              FOR I=2,...,M, J=2,...,N.  AT
THE

C                              BOUNDARIES, GRHS IS DEFINED
BY

C

C                              MBDCND   GRHS(1,J)
GRHS(M+1,J)

C                              ------   ---------   --------
---

C                                0      G(A,YJ)     G(B,YJ)
```

```
C                             1         *           *
C                             2         *        G(B,YJ)
J=1,2,...,N+1
C                             3      G(A,YJ)     G(B,YJ)
C                             4      G(A,YJ)        *
C
C                           NBDCND   GRHS(I,1)
GRHS(I,N+1)
C                           ------   ---------   --------
---
C                             0      G(XI,C)     G(XI,D)
C                             1         *           *
C                             2         *        G(XI,D)
I=1,2,...,M+1
C                             3      G(XI,C)     G(XI,D)
C                             4      G(XI,C)        *
C
C                           WHERE * MEANS THESE QUANTITES
ARE NOT USED.
C                           GRHS SHOULD BE DIMENSIONED
IDMN BY AT LEAST
C                           N+1 IN THE CALLING ROUTINE.
C
C                         USOL
C                           A TWO-DIMENSIONAL ARRAY THAT
SPECIFIES THE
C                           VALUES OF THE SOLUTION ALONG
THE BOUNDARIES.
```

```
C                           AT THE BOUNDARIES, USOL IS
DEFINED BY
C
C                           MBDCND   USOL(1,J)
USOL(M+1,J)
C                           ------   ---------   --------
---
C                             0          *            *
C                             1      U(A,YJ)      U(B,YJ)
C                             2      U(A,YJ)          *
J=1,2,...,N+1
C                             3          *            *
C                             4          *        U(B,YJ)
C
C                           NBDCND   USOL(I,1)
USOL(I,N+1)
C                           ------   ---------   --------
---
C                             0          *            *
C                             1      U(XI,C)      U(XI,D)
C                             2      U(XI,C)          *
I=1,2,...,M+1
C                             3          *            *
C                             4          *        U(XI,D)
C
C                           WHERE * MEANS THE QUANTITES
ARE NOT USED
C                           IN THE SOLUTION.
```

```
C
C                            IF IORDER=2, THE USER MAY
EQUIVALENCE GRHS
C                            AND USOL TO SAVE SPACE.  NOTE
THAT IN THIS
C                            CASE THE TABLES SPECIFYING
THE BOUNDARIES
C                            OF THE GRHS AND USOL ARRAYS
DETERMINE THE
C                            BOUNDARIES UNIQUELY EXCEPT AT
THE CORNERS.
C                            IF THE TABLES CALL FOR BOTH
G(X,Y) AND
C                            U(X,Y) AT A CORNER THEN THE
SOLUTION MUST
C                            BE CHOSEN.
C                            FOR EXAMPLE, IF MBDCND=2 AND
NBDCND=4,
C                            THEN U(A,C), U(A,D),U(B,D)
MUST BE CHOSEN
C                            AT THE CORNERS IN ADDITION TO
G(B,C).
C
C                            IF IORDER=4, THEN THE TWO
ARRAYS, USOL AND
C                            GRHS, MUST BE DISTINCT.
C
C                            USOL SHOULD BE DIMENSIONED
IDMN BY AT LEAST
C                            N+1 IN THE CALLING ROUTINE.
```

```
C
C                          IDMN
C                            THE ROW (OR FIRST) DIMENSION
OF THE ARRAYS
C                            GRHS AND USOL AS IT APPEARS
IN THE PROGRAM
C                            CALLING SEPELI.  THIS
PARAMETER IS USED
C                            TO SPECIFY THE VARIABLE
DIMENSION OF GRHS
C                            AND USOL.  IDMN MUST BE AT
LEAST 7 AND
C                            GREATER THAN OR EQUAL TO M+1.
C
C                          W
C                            A ONE-DIMENSIONAL ARRAY THAT
MUST BE
C                            PROVIDED BY THE USER FOR WORK
SPACE.
C
10*N+(16+INT(LOG2(N+1)))*(M+1)+11 WILL
C                            SUFFICE AS A LENGTH FOR W.
THE ACTUAL
C                            LENGTH OF W IN THE CALLING
ROUTINE
C                            MUST BE SET IN W(1) (SEE
IERROR=11).
C
C
```

```
C ON OUTPUT                USOL

C                            CONTAINS THE APPROXIMATE
SOLUTION TO THE

C                            ELLIPTIC EQUATION. USOL(I,J)
IS THE

C                            APPROXIMATION TO U(XI,YJ) FOR
I=1,2...,M+1

C                            AND J=1,2,...,N+1.  THE
APPROXIMATION HAS

C                            ERROR O(DLX**2+DLY**2) IF
CALLED WITH

C                            IORDER=2 AND O(DLX**4+DLY**4)
IF CALLED

C                            WITH IORDER=4.

C

C                          W

C                            CONTAINS INTERMEDIATE VALUES
THAT MUST NOT

C                            BE DESTROYED IF SEPELI IS
CALLED AGAIN

C                            WITH INTL=1.  IN ADDITION
W(1) CONTAINS

C                            THE EXACT MINIMAL LENGTH (IN
FLOATING POINT)

C                            REQUIRED FOR THE WORK SPACE
(SEE IERROR=11).

C

C                          PERTRB

C                            IF A COMBINATION OF PERIODIC
OR DERIVATIVE
```

```
C                          BOUNDARY CONDITIONS (I.E.,
ALPHA=BETA=0 IF

C                          MBDCND=3) IS SPECIFIED AND IF
CF(X)=0 FOR

C                          ALL X THEN A SOLUTION TO THE
DISCRETIZED

C                          MATRIX EQUATION MAY NOT EXIST

C                          (REFLECTING THE NON-
UNIQUENESS OF SOLUTIONS

C                          TO THE PDE).

C                          PERTRB IS A CONSTANT
CALCULATED AND

C                          SUBTRACTED FROM THE RIGHT
HAND SIDE OF THE

C                          MATRIX EQUATION INSURING THE
EXISTENCE OF A

C                          SOLUTION.  SEPX4 COMPUTES
THIS SOLUTION

C                          WHICH IS A WEIGHTED MINIMAL
LEAST SQUARES

C                          SOLUTION TO THE ORIGINAL
PROBLEM.  IF

C                          SINGULARITY IS NOT DETECTED
PERTRB=0.0 IS

C                          RETURNED BY SEPX4.

C

C                        IERROR

C                          AN ERROR FLAG THAT INDICATES
INVALID INPUT
```

```
C                            PARAMETERS OR FAILURE TO FIND
A SOLUTION

C

C                            =  0 NO ERROR

C                            =  1 IF A GREATER THAN B OR C
GREATER

C                                 THAN D

C                            =  2 IF MBDCND LESS THAN 0 OR
MBDCND

C                                 GREATER THAN 4

C                            =  3 IF NBDCND LESS THAN 0 OR
NBDCND

C                                 GREATER THAN 4

C                            =  4 IF ATTEMPT TO FIND A
SOLUTION FAILS.

C                                 (THE LINEAR SYSTEM
GENERATED IS NOT

C                                 DIAGONALLY DOMINANT.)

C                            =  5 IF IDMN IS TOO SMALL
(SEE DISCUSSION

C                                 OF IDMN)

C                            =  6 IF M IS TOO SMALL OR TOO
LARGE

C                                 (SEE DISCUSSION OF M)

C                            =  7 IF N IS TOO SMALL (SEE
DISCUSSION OF N)

C                            =  8 IF IORDER IS NOT 2 OR 4

C                            =  9 IF INTL IS NOT 0 OR 1
```

```
C                          = 10 IF AFUN IS LESS THAN OR
EQUAL TO ZERO

C                               FOR SOME INTERIOR MESH
POINT XI SOME

C                               INTERIOR MESH POINT
(XI,YJ)

C                          = 11 IF THE WORK SPACE LENGTH
INPUT IN W(1)

C                               IS LESS THAN THE EXACT
MINIMAL WORK

C                               SPACE LENGTH REQUIRED
OUTPUT IN W(1).

C                          = 12 IF MBDCND=0 AND
AF(X)=CF(X)=CONSTANT

C                               OR BF(X)=0 FOR ALL X IS
NOT TRUE.

C

C SPECIAL CONDITIONS     NONE

C

C I/O                    NONE

C

C REQUIRED LIBRARY       COMF, GENBUN, GNBNAUX, AND
SEPAUX

C FILES                  FROM FISHPAK

C

C

C PRECISION              SINGLE

C
```

```
C REQUIRED LIBRARY       NONE

C FILES

C

C LANGUAGE               FORTRAN

C

C HISTORY                SEPX4 WAS DEVELOPED AT NCAR BY
JOHN C.

C                        ADAMS OF THE SCIENTIFIC
COMPUTING DIVISION

C                        IN OCTOBER 1978.  THE BASIS OF
THIS CODE IS

C                        NCAR ROUTINE SEPELI.  BOTH
PACKAGES WERE

C                        RELEASED ON NCAR'S PUBLIC
LIBRARIES IN

C                        JANUARY 1980.

C

C PORTABILITY            FORTRAN 77

C

C ALGORITHM              SEPX4 AUTOMATICALLY DISCRETIZES
THE SEPARABLE

C                        ELLIPTIC EQUATION WHICH IS THEN
SOLVED BY A

C                        GENERALIZED CYCLIC REDUCTION
ALGORITHM IN THE

C                        SUBROUTINE POIS.  THE FOURTH
ORDER SOLUTION

C                        IS OBTAINED USING THE TECHNIQUE
OF DEFFERRED
```

```
C                             CORRECTIONS REFERENCED BELOW.

C

C TIMING                      WHEN POSSIBLE, SEPX4 SHOULD BE
USED INSTEAD

C                             OF PACKAGE SEPELI.  THE
INCREASE IN SPEED

C                             IS AT LEAST A FACTOR OF THREE.

C

C REFERENCES                  KELLER, H.B., NUMERICAL METHODS
FOR TWO-POINT

C                             BOUNDARY-VALUE PROBLEMS,
BLAISDEL (1968),

C                             WALTHAM, MASS.

C

C                             SWARZTRAUBER, P., AND R. SWEET
(1975):

C                             EFFICIENT FORTRAN SUBPROGRAMS
FOR THE

C                             SOLUTION OF ELLIPTIC PARTIAL
DIFFERENTIAL

C                             EQUATIONS.  NCAR TECHNICAL NOTE

C                               NCAR-TN/IA-109, PP. 135-137.

C*******************************************************
****************
```

TBLKTRI

```
C
C     file tblktri.f
C
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
C     *
*
C     *                  copyright (c) 1999 by UCAR
*
C     *
*
C     *       UNIVERSITY CORPORATION for ATMOSPHERIC
RESEARCH         *
C     *
*
C     *                      all rights reserved
*
C     *
*
C     *                      FISHPACK version 4.1
*
C     *
*
C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE
SOLUTION OF      *
C     *
*
C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *
```

```
C     *
*

C     *                              BY
*

C     *
*

C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

C     *
*

C     *                              OF
*

C     *
*

C     *         THE NATIONAL CENTER FOR ATMOSPHERIC
RESEARCH         *

C     *
*

C     *                BOULDER, COLORADO  (80307)
U.S.A.             *

C     *
*

C     *                   WHICH IS SPONSORED BY
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*

C     *              THE NATIONAL SCIENCE FOUNDATION
*

C     *
*
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C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *

C

C

C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE BLKTRI
TO

C     SOLVE THE EQUATION

C

C     .5/S*(D/DS)(.5/S*DU/DS)+.5/T*(D/DT)(.5/T*DU/DT)

C
(1)

C                  = 15/4*S*T*(S**4+T**4)

C

C     ON THE RECTANGLE 0 .LT. S .LT. 1 AND 0 .LT. T .LT.
1

C     WITH THE BOUNDARY CONDITIONS

C

C     U(0,T) = 0

C                              0 .LE. T .LE. 1

C     U(1,T) = T**5

C

C     AND

C

C     U(S,0) = 0

C                              0 .LE. S .LE. 1
```

```
C     U(S,1) = S**5

C

C     THE EXACT SOLUTION OF THIS PROBLEM IS U(S,T) =
(S*T)**5

C

C     DEFINE THE INTEGERS M = 50 AND N = 63. THEN DEFINE
THE

C     GRID INCREMENTS DELTAS = 1/(M+1) AND DELTAT =
1/(N+1).

C

C     THE GRID IS THEN GIVEN BY S(I) = I*DELTAS FOR I =
1,...,M

C     AND T(J) = J*DELTAT FOR J = 1,...,N.

C

C     THE APPROXIMATE SOLUTION IS GIVEN AS THE SOLUTION
TO

C     THE FOLLOWING FINITE DIFFERENCE APPROXIMATION OF
EQUATION (1).

C

C     .5/(S(I)*DELTAS)*((U(I+1,J)-
U(I,J))/(2*S(I+.5)*DELTAS)

C                      -(U(I,J)-U(I-1,J))/(2*S(I-
.5)*DELTAS))

C     +.5/(T(I)*DELTAT)*((U(I,J+1)-
U(I,J))/(2*T(I+.5)*DELTAT) (2)

C                      -(U(I,J)-U(I,J-1))/(2*T(I-
.5)*DELTAT))

C                = 15/4*S(I)*T(J)*(S(I)**4+T(J)**4)

C
```

```
C              WHERE S(I+.5) = .5*(S(I+1)+S(I))
C                    S(I-.5) = .5*(S(I)+S(I-1))
C                    T(I+.5) = .5*(T(I+1)+T(I))
C                    T(I-.5) = .5*(T(I)+T(I-1))
C
C      THE APPROACH IS TO WRITE EQUATION (2) IN THE FORM
C
C      AM(I)*U(I-1,J)+BM(I,J)*U(I,J)+CM(I)*U(I+1,J)
C        +AN(J)*U(I,J-1)+BN(J)*U(I,J)+CN(J)*U(I,J+1)
(3)
C            = Y(I,J)
C
C      AND THEN CALL SUBROUTINE BLKTRI TO DETERMINE
U(I,J)
C
C
C
      DIMENSION       Y(75,105)  ,AM(75)     ,BM(75)
,CM(75)     ,
     1                AN(105)    ,BN(105)    ,CN(105)
,W(823)     ,
     2                S(75)      ,T(105)
C
      IFLG = 0
      NP = 1
      N = 63
```

```
      MP = 1

      M = 50

      IDIMY = 75

C

C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING THE

C     COEFFICIENTS AND THE ARRAY Y.

C

      DELTAS = 1./FLOAT(M+1)

      DO 101 I=1,M

         S(I) = FLOAT(I)*DELTAS

  101 CONTINUE

      DELTAT = 1./FLOAT(N+1)

      DO 102 J=1,N

         T(J) = FLOAT(J)*DELTAT

  102 CONTINUE

C

C     COMPUTE THE COEFFICIENTS AM,BM,CM CORRESPONDING TO
THE S DIRECTION

C

      HDS = DELTAS/2.

      TDS = DELTAS+DELTAS

      DO 103 I=1,M

         TEMP1 = 1./(S(I)*TDS)

         TEMP2 = 1./((S(I)-HDS)*TDS)
```

```
         TEMP3 = 1./((S(I)+HDS)*TDS)
         AM(I) = TEMP1*TEMP2
         CM(I) = TEMP1*TEMP3
         BM(I) = -(AM(I)+CM(I))
  103 CONTINUE
C
C     COMPUTE THE COEFFICIENTS AN,BN,CN CORRESPONDING TO
THE T DIRECTION
C
      HDT = DELTAT/2.
      TDT = DELTAT+DELTAT
      DO 104 J=1,N
         TEMP1 = 1./(T(J)*TDT)
         TEMP2 = 1./((T(J)-HDT)*TDT)
         TEMP3 = 1./((T(J)+HDT)*TDT)
         AN(J) = TEMP1*TEMP2
         CN(J) = TEMP1*TEMP3
         BN(J) = -(AN(J)+CN(J))
  104 CONTINUE
C
C     COMPUTE RIGHT SIDE OF EQUATION
C
      DO 106 J=1,N
         DO 105 I=1,M
            Y(I,J) = 3.75*S(I)*T(J)*(S(I)**4+T(J)**4)
```

```
  105    CONTINUE

  106 CONTINUE

C

C      THE NONZERO BOUNDARY CONDITIONS ENTER THE LINEAR
SYSTEM VIA

C      THE RIGHT SIDE Y(I,J). IF THE EQUATIONS (3) GIVEN
ABOVE

C      ARE EVALUATED AT I=M AND J=1,...,N THEN THE TERM
CM(M)*U(M+1,J)

C      IS KNOWN FROM THE BOUNDARY CONDITION TO BE
CM(M)*T(J)**5.

C      THEREFORE THIS TERM CAN BE INCLUDED IN THE RIGHT
SIDE Y(M,J).

C      THE SAME ANALYSIS APPLIES AT J=N AND I=1,..,M.
NOTE THAT THE

C      CORNER AT J=N,I=M INCLUDES CONTRIBUTIONS FROM BOTH
BOUNDARIES.

C

      DO 107 J=1,N

         Y(M,J) = Y(M,J)-CM(M)*T(J)**5

  107 CONTINUE

      DO 108 I=1,M

         Y(I,N) = Y(I,N)-CN(N)*S(I)**5

  108 CONTINUE

C

C      DETERMINE THE APPROXIMATE SOLUTION U(I,J)

C
```

```
  109 CALL BLKTRI
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,CM,IDIMY,Y,IERROR,W)

      IFLG = IFLG+1

      IF (IFLG-1) 109,109,110

C

C     COMPUTE DISCRETIZATION ERROR

C

  110 ERR = 0.

      DO 112 J=1,N

         DO 111 I=1,M

            Z = ABS(Y(I,J)-(S(I)*T(J))**5)

            IF (Z .GT. ERR) ERR = Z

  111    CONTINUE

  112 CONTINUE

      IW = INT(W(1))

      PRINT 1001 , IERROR,ERR,IW

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE BLKTRI EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/

     3        18X,34HDISCRETIZATION ERROR = 1.64478E-05/

     4        12X,32HREQUIRED LENGTH OF W ARRAY = 823//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
```

```
     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7        12X,28HREQUIRED LENGTH OF W ARRAY =,I4)

C

      END
```

TCBLKTRI

```
C

C     file tcblktri.f

C

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

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C

C     THIS PROGRAM ILLUSTRATES THE USE OF SUBROUTINE
CBLKTR WHICH IS

C     THE COMPLEX VERSION OF BLKTRI. THE PROGRAM SOLVES
THE EQUATION

C

C     .5/S*(D/DS)(.5/S*DU/DS)+.5/T*(D/DT)(.5/T*DU/DT)-
SQRT(-1)*U

C
(1)

C           = 15/4*S*T*(S**4+T**4)-SQRT(-1)*(S*T)**5

C

C     ON THE RECTANGLE 0 .LT. S .LT. 1 AND 0 .LT. T .LT.
1

C     WITH THE BOUNDARY CONDITIONS

C
```

```
C     U(0,T) = 0

C                              0 .LE. T .LE. 1

C     U(1,T) = T**5

C

C     AND

C

C     U(S,0) = 0

C                              0 .LE. S .LE. 1

C     U(S,1) = S**5

C

C     THE EXACT SOLUTION OF THIS PROBLEM IS U(S,T) =
(S*T)**5

C

C     DEFINE THE INTEGERS M = 50 AND N = 63. THEN DEFINE
THE

C     GRID INCREMENTS DELTAS = 1/(M+1) AND DELTAT =
1/(N+1).

C

C     THE GRID IS THEN GIVEN BY S(I) = I*DELTAS FOR I =
1,...,M

C     AND T(J) = J*DELTAT FOR J = 1,...,N.

C

C     THE APPROXIMATE SOLUTION IS GIVEN AS THE SOLUTION
TO

C     THE FOLLOWING FINITE DIFFERENCE APPROXIMATION OF
EQUATION (1).

C
```

```
C     .5/(S(I)*DELTAS)*((U(I+1,J)-
U(I,J))/(2*S(I+.5)*DELTAS)

C                     -(U(I,J)-U(I-1,J))/(2*S(I-
.5)*DELTAS))

C     +.5/(T(I)*DELTAT)*((U(I,J+1)-
U(I,J))/(2*T(I+.5)*DELTAT) (2)

C                     -(U(I,J)-U(I,J-1))/(2*T(I-
.5)*DELTAT))

C                        -SQRT(-1)*U(I,J)

C                  = 15/4*S(I)*T(J)*(S(I)**4+T(J)**4)

C                        -SQRT(-1)*(S(I)*T(J))**5

C

C             WHERE S(I+.5) = .5*(S(I+1)+S(I))

C                   S(I-.5) = .5*(S(I)+S(I-1))

C                   T(I+.5) = .5*(T(I+1)+T(I))

C                   T(I-.5) = .5*(T(I)+T(I-1))

C

C     THE APPROACH IS TO WRITE EQUATION (2) IN THE FORM

C

C     AM(I)*U(I-1,J)+BM(I,J)*U(I,J)+CM(I)*U(I+1,J)

C       +AN(J)*U(I,J-1)+BN(J)*U(I,J)+CN(J)*U(I,J+1)
(3)

C           = Y(I,J)

C

C     AND THEN CALL SUBROUTINE CBLKTR TO DETERMINE
U(I,J)

C
```

```
C

C

C

      DIMENSION       Y(75,105)  ,AM(75)     ,BM(75)
,CM(75)     ,

     1                AN(105)    ,BN(105)    ,CN(105)
,S(75)      ,

     2                T(105)     ,W(1123)

      COMPLEX         Y          ,AM         ,BM
,CM

C

      IFLG = 0

      NP = 1

      N = 63

      MP = 1

      M = 50

      IDIMY = 75

C

C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING THE

C     COEFFICIENTS AND THE ARRAY Y.

C

      DELTAS = 1./FLOAT(M+1)

      DO 101 I=1,M

         S(I) = FLOAT(I)*DELTAS

  101 CONTINUE
```

```
      DELTAT = 1./FLOAT(N+1)
      DO 102 J=1,N
         T(J) = FLOAT(J)*DELTAT
  102 CONTINUE
C
C     COMPUTE THE COEFFICIENTS AM,BM,CM CORRESPONDING TO
THE S DIRECTION
C
      HDS = DELTAS/2.
      TDS = DELTAS+DELTAS
      DO 103 I=1,M
         TEMP1 = 1./(S(I)*TDS)
         TEMP2 = 1./((S(I)-HDS)*TDS)
         TEMP3 = 1./((S(I)+HDS)*TDS)
         AM(I) = CMPLX(TEMP1*TEMP2,0.)
         CM(I) = CMPLX(TEMP1*TEMP3,0.)
         BM(I) = -(AM(I)+CM(I))-(0.,1.)
  103 CONTINUE
C
C     COMPUTE THE COEFFICIENTS AN,BN,CN CORRESPONDING TO
THE T DIRECTION
C
      HDT = DELTAT/2.
      TDT = DELTAT+DELTAT
      DO 104 J=1,N
```

```
          TEMP1 = 1./(T(J)*TDT)
          TEMP2 = 1./((T(J)-HDT)*TDT)
          TEMP3 = 1./((T(J)+HDT)*TDT)
          AN(J) = TEMP1*TEMP2
          CN(J) = TEMP1*TEMP3
          BN(J) = -(AN(J)+CN(J))
  104 CONTINUE
C
C     COMPUTE RIGHT SIDE OF EQUATION
C
      DO 106 J=1,N
         DO 105 I=1,M
            Y(I,J) = 3.75*S(I)*T(J)*(S(I)**4+T(J)**4)-
     1               (0.,1.)*(S(I)*T(J))**5
  105    CONTINUE
  106 CONTINUE
C
C     THE NONZERO BOUNDARY CONDITIONS ENTER THE LINEAR
SYSTEM VIA
C     THE RIGHT SIDE Y(I,J). IF THE EQUATIONS (3) GIVEN
ABOVE
C     ARE EVALUATED AT I=M AND J=1,...,N THEN THE TERM
CM(M)*U(M+1,J)
C     IS KNOWN FROM THE BOUNDARY CONDITION TO BE
CM(M)*T(J)**5.
```

```
C     THEREFORE THIS TERM CAN BE INCLUDED IN THE RIGHT
SIDE Y(M,J).

C     THE SAME ANALYSIS APPLIES AT J=N AND I=1,..,M.
NOTE THAT THE

C     CORNER AT J=N,I=M INCLUDES CONTRIBUTIONS FROM BOTH
BOUNDARIES.

C

      DO 107 J=1,N

         Y(M,J) = Y(M,J)-CM(M)*T(J)**5

  107 CONTINUE

      DO 108 I=1,M

         Y(I,N) = Y(I,N)-CN(N)*S(I)**5

  108 CONTINUE

C

  109 CALL CBLKTR
(IFLG,NP,N,AN,BN,CN,MP,M,AM,BM,CM,IDIMY,Y,IERROR,W)

      IFLG = IFLG+1

      IF (IFLG-1) 109,109,110

C

C     COMPUTE DISCRETIZATION ERROR

C

  110 ERR = 0.

      DO 112 J=1,N

         DO 111 I=1,M

            Z = CABS(Y(I,J)-(S(I)*T(J))**5)

            IF (Z .GT. ERR) ERR = Z
```

```
  111    CONTINUE

  112 CONTINUE

      IW = INT(W(1))

      PRINT 1001 , IERROR,ERR,IW

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE CBLKTR EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/

     3        18X,34HDISCRETIZATION ERROR = 1.64572E-05/

     4        12X,33HREQUIRED LENGTH OF W ARRAY = 1123//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7        12X,28HREQUIRED LENGTH OF W ARRAY =,I5)

C

      END
```

TCMGNBN

```
C

C file tcmgnbn.f

C
```

```
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C     *                              BY
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C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
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C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE CMGNBN
TO SOLVE
```

```
C     THE EQUATION

C

C     (1+X)**2*(D/DX)(DU/DX) - 2(1+X)(DU/DX) +
(D/DY)(DU/DY)

C

C              - SQRT(-1)*U = (3 - SQRT(-
1))*(1+X)**4*SIN(Y)          (1)

C

C     ON THE RECTANGLE 0 .LT. X .LT. 1 AND -PI .LT. Y
.LT. PI

C     WITH THE BOUNDARY CONDITIONS

C

C     (DU/DX)(0,Y) = 4SIN(Y)
(2)

C                                -PI .LE. Y .LE. PI

C     U(1,Y) = 16SIN(Y)
(3)

C

C     AND WITH U PERIODIC IN Y USING FINITE DIFFERENCES
ON A

C     GRID WITH DELTAX (= DX) = 1/20 AND DELTAY (= DY) =
PI/20.

C        TO SET UP THE FINITE DIFFERENCE EQUATIONS WE
DEFINE

C     THE GRID POINTS

C

C     X(I) = (I-1)DX            I=1,2,...,21

C
```

```
C     Y(J) = -PI + (J-1)DY      J=1,2,...,41
C
C     AND LET V(I,J) BE AN APPROXIMATION TO
U(X(I),Y(J)).
C     NUMBERING THE GRID POINTS IN THIS FASHION GIVES
THE SET
C     OF UNKNOWNS AS V(I,J) FOR I=1,2,...,20 AND
J=1,2,...,40.
C     HENCE, IN THE PROGRAM M = 20 AND N = 40.  AT THE
INTERIOR
C     GRID POINT (X(I),Y(J)), WE REPLACE ALL DERIVATIVES
IN
C     EQUATION (1) BY SECOND ORDER CENTRAL FINITE
DIFFERENCES,
C     MULTIPLY BY DY**2, AND COLLECT COEFFICIENTS OF
V(I,J) TO
C     GET THE FINITE DIFFERENCE EQUATION
C
C     A(I)V(I-1,J) + B(I)V(I,J) + C(I)V(I+1,J)
C
C     + V(I,J-1) - 2V(I,J) + V(I,J+1) = F(I,J)
(4)
C
C     WHERE S = (DY/DX)**2, AND FOR I=2,3,...,19
C
C     A(I) = (1+X(I))**2*S + (1+X(I))*S*DX
C
C     B(I) = -2(1+X(I))**2*S - SQRT(-1)*DY**2
```

```
C

C     C(I) = (1+X(I))**2*S - (1+X(I))*S*DX

C

C     F(I,J) = (3 - SQRT(-
1))*(1+X(I))**4*DY**2*SIN(Y(J))

C              FOR J=1,2,...,40.

C

C        TO OBTAIN EQUATIONS FOR I = 1, WE REPLACE THE

C     DERIVATIVE IN EQUATION (2) BY A SECOND ORDER
CENTRAL

C     FINITE DIFFERENCE APPROXIMATION, USE THIS EQUATION
TO

C     ELIMINATE THE VIRTUAL UNKNOWN V(0,J) IN EQUATION
(4)

C     AND ARRIVE AT THE EQUATION

C

C     B(1)V(1,J) + C(1)V(2,J) + V(1,J-1) - 2V(1,J) +
V(1,J+1)

C

C                         = F(1,J)

C

C     WHERE

C

C     B(1) = -2S - SQRT(-1)*DY**2 , C(1) = 2S

C

C     F(1,J) = (11-SQRT(-1)+8/DX)*DY**2*SIN(Y(J)),
J=1,2,...,40.
```

```
C
C     FOR COMPLETENESS, WE SET A(1) = 0.
C        TO OBTAIN EQUATIONS FOR I = 20, WE INCORPORATE
C     EQUATION (3) INTO EQUATION (4) BY SETTING
C
C     V(21,J) = 16SIN(Y(J))
C
C     AND ARRIVE AT THE EQUATION
C
C     A(20)V(19,J) + B(20)V(20,J)
C
C     + V(20,J-1) - 2V(20,J) + V(20,J+1) = F(20,J)
C
C     WHERE
C
C     A(20) = (1+X(20))**2*S + (1+X(20))*S*DX
C
C     B(20) = -2*(1+X(20))**2*S - SQRT(-1)*DY**2
C
C     F(20,J) = ((3-SQRT(-1))*(1+X(20))**4*DY**2 -
16(1+X(20))**2*S
C                + 16(1+X(20))*S*DX)*SIN(Y(J))
C
C                    FOR J=1,2,...,40.
C
```

```
C     FOR COMPLETENESS, WE SET C(20) = 0.  HENCE, IN THE

C     PROGRAM MPEROD = 1.

C        THE PERIODICITY CONDITION ON U GIVES THE
CONDITIONS

C

C     V(I,0) = V(I,40) AND V(I,41) = V(I,1) FOR
I=1,2,...,20.

C

C     HENCE, IN THE PROGRAM NPEROD = 0.

C

C          THE EXACT SOLUTION TO THIS PROBLEM IS

C

C                  U(X,Y) = (1+X)**4*SIN(Y) .

C

      COMPLEX        F          ,A          ,B
,C          ,W

      DIMENSION      F(22,40)   ,A(20)      ,B(20)
,C(20)      ,

     1               X(21)      ,Y(41)      ,W(380)

C

C     FROM THE DIMENSION STATEMENT WE GET THAT IDIMF =
22 AND THAT W

C     HAS BEEN DIMENSIONED

C

C     4N + (10+INT(LOG2(N)))M = 4*20 + (10+5)*20 = 380 .

C
```

```
      IDIMF = 22
      M = 20
      MP1 = M+1
      MPEROD = 1
      DX = 0.05
      N = 40
      NPEROD = 0
      PI = PIMACH(DUM)
      DY = PI/20.
C
C     GENERATE GRID POINTS FOR LATER USE.
C
      DO 101 I=1,MP1
         X(I) = FLOAT(I-1)*DX
  101 CONTINUE
      DO 102 J=1,N
         Y(J) = -PI+FLOAT(J-1)*DY
  102 CONTINUE
C
C     GENERATE COEFFICIENTS.
C
      S = (DY/DX)**2
      DO 103 I=2,19
         T = 1.+X(I)
```

```
         TSQ = T**2
         A(I) = CMPLX((TSQ+T*DX)*S,0.)
         B(I) = -2.*TSQ*S-(0.,1.)*DY**2
         C(I) = CMPLX((TSQ-T*DX)*S,0.)
  103 CONTINUE
      A(1) = (0.,0.)
      B(1) = -2.*S-(0.,1.)*DY**2
      C(1) = CMPLX(2.*S,0.)
      B(20) = -2.*S*(1.+X(20))**2-(0.,1.)*DY**2
      A(20) = CMPLX(S*(1.+X(20))**2+(1.+X(20))*DX*S,0.)
      C(20) = (0.,0.)
C
C     GENERATE RIGHT SIDE.
C
      DO 105 I=2,19
         DO 104 J=1,N
            F(I,J) = (3.,-
1.)*(1.+X(I))**4*DY**2*SIN(Y(J))
  104    CONTINUE
  105 CONTINUE
      T = 1.+X(20)
      TSQ = T**2
      T4 = TSQ**2
      DO 106 J=1,N
         F(1,J) = ((11.,-1.)+8./DX)*DY**2*SIN(Y(J))
```

```
         F(20,J) = ((3.,-1.)*T4*DY**2-
16.*TSQ*S+16.*T*S*DX)*SIN(Y(J))

  106 CONTINUE

      CALL CMGNBN
(NPEROD,N,MPEROD,M,A,B,C,IDIMF,F,IERROR,W)

C

C     COMPUTE DISCRETIAZATION ERROR.  THE EXACT SOLUTION
IS

C

C            U(X,Y) = (1+X)**4*SIN(Y) .

C

      ERR = 0.

      DO 108 I=1,M

         DO 107 J=1,N

            T = CABS(F(I,J)-(1.+X(I))**4*SIN(Y(J)))

            IF (T .GT. ERR) ERR = T

  107    CONTINUE

  108 CONTINUE

      T = REAL(W(1))

      PRINT 1001 , IERROR,ERR,T

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE CMGNBN EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/
```

```
     3         18X,34HDISCRETIZATION ERROR = 9.16200E-03/

     4         12X,32HREQUIRED LENGTH OF W ARRAY = 380//

     5         10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6         32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7         12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)

C

      END
```

TGENBUN

```
C

C file tgenbun.f

C

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C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE GENBUN
TO

C     SOLVE THE EQUATION

C

C     (1+X)**2*(D/DX)(DU/DX) - 2(1+X)(DU/DX) +
(D/DY)(DU/DY)

C

C                  = 3(1+X)**4*SIN(Y)
(1)

C

C     ON THE RECTANGLE 0 .LT. X .LT. 1 AND -PI .LT. Y
.LT. PI

C     WITH THE BOUNDARY CONDITIONS
```

```
C
C     (DU/DX)(0,Y) = 4SIN(Y)
(2)
C                               -PI .LE. Y .LE. PI
C     U(1,Y) = 16SIN(Y)
(3)
C
C     AND WITH U PERIODIC IN Y USING FINITE DIFFERENCES
ON A
C     GRID WITH DELTAX (= DX) = 1/20 AND DELTAY (= DY) =
PI/20.
C        TO SET UP THE FINITE DIFFERENCE EQUATIONS WE
DEFINE
C     THE GRID POINTS
C
C     X(I) = (I-1)DX            I=1,2,...,21
C
C     Y(J) = -PI + (J-1)DY      J=1,2,...,41
C
C     AND LET V(I,J) BE AN APPROXIMATION TO
U(X(I),Y(J)).
C     NUMBERING THE GRID POINTS IN THIS FASHION GIVES
THE SET
C     OF UNKNOWNS AS V(I,J) FOR I=1,2,...,20 AND
J=1,2,...,40.
C     HENCE, IN THE PROGRAM M = 20 AND N = 40.  AT THE
INTERIOR
C     GRID POINT (X(I),Y(J)), WE REPLACE ALL DERIVATIVES
IN
```

```
C     EQUATION (1) BY SECOND ORDER CENTRAL FINITE
DIFFERENCES,

C     MULTIPLY BY DY**2, AND COLLECT COEFFICIENTS OF
V(I,J) TO

C     GET THE FINITE DIFFERENCE EQUATION

C

C     A(I)V(I-1,J) + B(I)V(I,J) + C(I)V(I+1,J)

C

C     + V(I,J-1) - 2V(I,J) + V(I,J+1) = F(I,J)
(4)

C

C     WHERE S = (DY/DX)**2, AND FOR I=2,3,...,19

C

C     A(I) = (1+X(I))**2*S + (1+X(I))*S*DX

C

C     B(I) = -2(1+X(I))**2*S

C

C     C(I) = (1+X(I))**2*S - (1+X(I))*S*DX

C

C     F(I,J) = 3(1+X(I))**4*DY**2*SIN(Y(J))  FOR
J=1,2,...,40.

C

C        TO OBTAIN EQUATIONS FOR I = 1, WE REPLACE THE

C     DERIVATIVE IN EQUATION (2) BY A SECOND ORDER
CENTRAL

C     FINITE DIFFERENCE APPROXIMATION, USE THIS EQUATION
TO
```

```
C     ELIMINATE THE VIRTUAL UNKNOWN V(0,J) IN EQUATION
(4)

C     AND ARRIVE AT THE EQUATION

C

C     B(1)V(1,J) + C(1)V(2,J) + V(1,J-1) - 2V(1,J) +
V(1,J+1)

C

C                          = F(1,J)

C

C     WHERE

C

C     B(1) = -2S , C(1) = 2S

C

C     F(1,J) = (11+8/DX)*DY**2*SIN(Y(J)),  J=1,2,...,40.

C

C     FOR COMPLETENESS, WE SET A(1) = 0.

C        TO OBTAIN EQUATIONS FOR I = 20, WE INCORPORATE

C     EQUATION (3) INTO EQUATION (4) BY SETTING

C

C     V(21,J) = 16SIN(Y(J))

C

C     AND ARRIVE AT THE EQUATION

C

C     A(20)V(19,J) + B(20)V(20,J)

C
```

```
C     + V(20,J-1) - 2V(20,J) + V(20,J+1) = F(20,J)

C

C     WHERE

C

C     A(20) = (1+X(20))**2*S + (1+X(20))*S*DX

C

C     B(20) = -2*(1+X(20))**2*S

C

C     F(20,J) = (3(1+X(20))**4*DY**2 - 16(1+X(20))**2*S

C                + 16(1+X(20))*S*DX)*SIN(Y(J))

C

C                    FOR J=1,2,...,40.

C

C     FOR COMPLETENESS, WE SET C(20) = 0.  HENCE, IN THE

C     PROGRAM MPEROD = 1.

C        THE PERIODICITY CONDITION ON U GIVES THE
CONDITIONS

C

C     V(I,0) = V(I,40) AND V(I,41) = V(I,1) FOR
I=1,2,...,20.

C

C     HENCE, IN THE PROGRAM NPEROD = 0.

C

C          THE EXACT SOLUTION TO THIS PROBLEM IS

C
```

```
C                  U(X,Y) = (1+X)**4*SIN(Y) .
C
      DIMENSION       F(22,40)   ,A(20)      ,B(20)
,C(20)      ,
     1                X(21)      ,Y(41)      ,W(380)
C
C     FROM THE DIMENSION STATEMENT WE GET THAT IDIMF =
22 AND THAT W
C     HAS BEEN DIMENSIONED
C
C     4N + (10+INT(LOG2(N)))M = 4*20 + (10+5)*20 = 380 .
C
      IDIMF = 22
      M = 20
      MP1 = M+1
      MPEROD = 1
      DX = 0.05
      N = 40
      NPEROD = 0
      PI = PIMACH(DUM)
      DY = PI/20.
C
C     GENERATE GRID POINTS FOR LATER USE.
C
      DO 101 I=1,MP1
```

```
          X(I) = FLOAT(I-1)*DX
  101 CONTINUE
      DO 102 J=1,N
         Y(J) = -PI+FLOAT(J-1)*DY
  102 CONTINUE
C
C     GENERATE COEFFICIENTS.
C
      S = (DY/DX)**2
      DO 103 I=2,19
         T = 1.+X(I)
         TSQ = T**2
         A(I) = (TSQ+T*DX)*S
         B(I) = -2.*TSQ*S
         C(I) = (TSQ-T*DX)*S
  103 CONTINUE
      A(1) = 0.
      B(1) = -2.*S
      C(1) = -B(1)
      B(20) = -2.*S*(1.+X(20))**2
      A(20) = -B(20)/2.+(1.+X(20))*DX*S
      C(20) = 0.
C
C     GENERATE RIGHT SIDE.
```

```
C

      DO 105 I=2,19

         DO 104 J=1,N

            F(I,J) = 3.*(1.+X(I))**4*DY**2*SIN(Y(J))

  104    CONTINUE

  105 CONTINUE

      T = 1.+X(20)

      TSQ = T**2

      T4 = TSQ**2

      DO 106 J=1,N

         F(1,J) = (11.+8./DX)*DY**2*SIN(Y(J))

         F(20,J) = (3.*T4*DY**2-
16.*TSQ*S+16.*T*S*DX)*SIN(Y(J))

  106 CONTINUE

      CALL GENBUN
(NPEROD,N,MPEROD,M,A,B,C,IDIMF,F,IERROR,W)

C

C     COMPUTE DISCRETIAZATION ERROR.  THE EXACT SOLUTION
IS

C

C            U(X,Y) = (1+X)**4*SIN(Y) .

C

      ERR = 0.

      DO 108 I=1,M

         DO 107 J=1,N

            T = ABS(F(I,J)-(1.+X(I))**4*SIN(Y(J)))
```

```
            IF (T .GT. ERR) ERR = T
  107    CONTINUE
  108 CONTINUE
      PRINT 1001 , IERROR,ERR,W(1)
      STOP
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE GENBUN EXAMPLE///
     1         10X,46HTHE OUTPUT FROM THE NCAR CONTROL DATA 7600 WAS//
     2         32X,10HIERROR = 0/
     3         18X,34HDISCRETIZATION ERROR = 9.64063E-03/
     4         12X,32HREQUIRED LENGTH OF W ARRAY = 380//
     5         10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6         32X,8HIERROR =,I2/18X,22HDISCRETIZATION ERROR =,E12.5/
     7         12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)
C
      END
```

THSTCRT

```
C
C file thstcrt.f
C
```

```
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C

C
```

```
C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE HSTCRT
TO SOLVE

C     THE EQUATION

C

C    (D/DX)(DU/DX) + (D/DY)(DU/DY) - 2*U = -
2(PI**2+1)SIN(PI*X)COS(PI*Y)

C

C     WHERE  1 .LE. X .LE. 3 AND -1 .LE. Y .LE. 1 AND
THE BOUNDARY

C     CONDITIONS ARE

C

C     U = 0 ON X = 1,  DU/DX = -PI*COS(PI*Y) ON X = 3

C

C     AND U IS PERIODIC IN Y .

C

C     WE WANT TO HAVE 48 UNKNOWNS IN THE X-INTERVAL AND
53 UNKNOWNS

C     IN THE Y-INTERVAL.

C

      DIMENSION       F(50,53)   ,BDA(53)    ,BDB(53)
,W(1076)    ,

     1                X(48)      ,Y(53)

C

C     FROM THE DIMENSION STATEMENT WE GET IDIMF = 50.
ALSO NOTE THAT

C     W IS DIMENSIONED (13 + INT(LOG2(N))*M + 4*N.

C
```

```
      IDIMF = 50
      A = 1.
      B = 3.
      M = 48
      DX = (B-A)/FLOAT(M)
      MBDCND = 2
      C = -1.
      D = 1.
      N = 53
      DY = (D-C)/FLOAT(N)
      NBDCND = 0
      ELMBDA = -2.
C
C     AUXILIARY QUANTITIES
C
      PI = PIMACH(DUM)
      PISQ = PI*PI
C
C     GENERATE AND STORE GRID POINTS FOR COMPUTATION OF BOUNDARY DATA
C     AND THE RIGHT SIDE OF THE HELMHOLTZ EQUATION.
C
      DO 101 I=1,M
         X(I) = A+(FLOAT(I)-0.5)*DX
  101 CONTINUE
```

```
      DO 102 J=1,N
         Y(J) = C+(FLOAT(J)-0.5)*DY
  102 CONTINUE
C
C     GENERATE BOUNDARY DATA.
C
      DO 103 J=1,N
         BDA(J) = 0.
         BDB(J) = -PI*COS(PI*Y(J))
  103 CONTINUE
C
C     BDC AND BDD ARE DUMMY ARGUMENTS IN THIS EXAMPLE.
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      T = -2.*(PISQ+1.)
      DO 105 I=1,M
         DO 104 J=1,N
            F(I,J) = T*SIN(PI*X(I))*COS(PI*Y(J))
  104    CONTINUE
  105 CONTINUE
      CALL HSTCRT
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,
     1             IDIMF,PERTRB,IERROR,W)
C
```

```
C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS

C

C                 U(X,Y) = SIN(PI*X)*COS(PI*Y) .

C

      ERR = 0.

      DO 107 I=1,M

         DO 106 J=1,N

            T = ABS(F(I,J)-SIN(PI*X(I))*COS(PI*Y(J)))

            IF (T .GT. ERR) ERR = T

  106    CONTINUE

  107 CONTINUE

      PRINT 1001 , IERROR,ERR,W(1)

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE HSTCRT EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/

     3        18X,34HDISCRETIZATION ERROR = 1.26001E-03/

     4        12X,32HREQUIRED LENGTH OF W ARRAY = 884//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)

C
```

```
      END
```

THSTCYL

THSTCSP

```
C

C file thstcsp.f

C

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C

C

C     THIS PROGRAM ILLUSTRATES THE USE OF SUBROUTINE
HSTCSP TO SOLVE

C     THE EQUATION

C

C                (1/R**2)(D/DR)(R**2(DU/DR)) +

C

C
(1/R**2*SIN(THETA))(D/DTHETA)(SIN(THETA)(DU/DTHETA))

C

C                      = 12*(R*COS(THETA))**2

C

C     ON THE RECTANGLE 0 .LT. THETA .LT. PI , 0 .LT. R
.LT. 1

C     WITH THE BOUNDARY CONDITIONS

C

C     U(THETA,1) = COS(THETA)**4 , O .LE. THETA .LE. PI

C
```

```
C     AND THE SOLUTION UNSPECIFIED ON THE REMAINING
BOUNDARIES.

C     WE WILL USE 45 UNKNOWNS IN THE THETA-INTERVAL AND
15 UNKNOWNS

C     IN THE R-INTERVAL.

C

      DIMENSION       F(47,16)   ,BDD(45)    ,W(615)
,THETA(45)   ,

     1                R(15)      ,COST(45)

C

C     NOTE THAT FROM DIMENSION STATEMENT WE GET THAT
IDIMF = 47  AND

C     THAT W IS DIMENSIONED ACCORDING TO THE STATEMENT
IN THE

C     DESCRIPTION OF W.

C

      IDIMF = 47

      A = 0.

      B = PIMACH(DUM)

C

C     NOTE THAT B IS SET TO PI USING THE FUNCTION PIMACH
AS REQUIRED.

C

      M = 45

      MBDCND = 9

      DT = (B-A)/FLOAT(M)

C
```

```
C     DEFINE GRID POINTS THETA(I) AND COS(THETA(I))
C
      DO 101 I=1,M
         THETA(I) = A+(FLOAT(I)-0.5)*DT
         COST(I) = COS(THETA(I))
  101 CONTINUE
      C = 0.
      D = 1.
      N = 15
      NBDCND = 5
      DR = (D-C)/FLOAT(N)
C
C     DEFINE GRID POINTS R(J)
C
      DO 102 J=1,N
         R(J) = C+(FLOAT(J)-0.5)*DR
  102 CONTINUE
C
C     DEFINE BOUNDARY ARRAY BDD.  BDA, BDB, AND BDC ARE
DUMMY
C     VARIABLES IN THIS EXAMPLE.
C
      DO 103 I=1,M
         BDD(I) = COST(I)**4
  103 CONTINUE
```

```
      ELMBDA = 0.
C
C     DEFINE RIGHT SIDE F
C
      DO 105 I=1,M
         DO 104 J=1,N
            F(I,J) = 12.*(R(J)*COST(I))**2
  104    CONTINUE
  105 CONTINUE
      INTL = 0
      CALL HSTCSP
(INTL,A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,
     1             ELMBDA,F,IDIMF,PERTRB,IERROR,W)
C
C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS
C
C     U(THETA,R) = (R*COS(THETA))**4
C
      ERR = 0.
      DO 107 I=1,M
         DO 106 J=1,N
            Z = ABS(F(I,J)-(R(J)*COST(I))**4)
            IF (Z .GT. ERR) ERR = Z
  106    CONTINUE
```

```
  107 CONTINUE
      PRINT 1001 , IERROR,ERR,W(1)
      STOP
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HSTCSP EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/
     3        18X,34HDISCRETIZATION ERROR = 5.58432E-03/
     4        12X,32HREQUIRED LENGTH OF W ARRAY = 583//
     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/
     7        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)
C
      END
```

THSTCYL

```
C
C file thstcyl.f
C
C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

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C

C     PROGRAM TO ILLUSTRATE THE USE OF HSTCYL TO SOLVE
THE EQUATION

C
```

```
C    (1/R)(D/DR)(R*DU/DR) + (D/DZ)(DU/DZ) =
(2*R*Z)**2*(4*Z**2 + 3*R**2)

C

C     ON THE RECTANGLE 0 .LT. R .LT. 1 , 0 .LT. Z .LT. 1
WITH THE

C     BOUNDARY CONDITIONS

C

C      (DU/DR)(1,Z) = 4*Z**2  FOR  0 .LE. Z .LE. 1

C

C     AND

C

C      (DU/DZ)(R,0) = 0 AND (DU/DZ)(R,1) = 4*R**2  FOR  0
.LE. R .LE. 1 .

C

C     THE SOLUTION TO THIS PROBLEM IS NOT UNIQUE.  IT IS
A

C     ONE-PARAMETER FAMILY OF SOLUTIONS GIVEN BY

C

C            U(R,Z) = (R*Z)**4 + ARBITRARY CONSTANT .

C

C     THE R-INTERVAL WILL CONTAIN 50 UNKNOWNS AND THE Z-
INTERVAL WILL

C     CONTAIN 52 UNKNOWNS.

C

      DIMENSION       F(51,52)   ,BDB(52)    ,BDC(50)
,BDD(50)    ,

     1                W(1108)    ,R(50)      ,Z(52)
```

```
C
C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF.
ALSO NOTE THAT W
C     IS DIMENSIONED (13 + INT(LOG2(N)))*M + 4*N .
C
      IDIMF = 51
      A = 0.
      B = 1.
      M = 50
      MBDCND = 6
      C = 0.
      D = 1.
      N = 52
      NBDCND = 3
      ELMBDA = 0.
C
C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING
C     BOUNDARY DATA AND THE RIGHT SIDE OF THE POISSON
EQUATION.
C
      DO 101 I=1,M
         R(I) = (FLOAT(I)-0.5)/50.
  101 CONTINUE
      DO 102 J=1,N
         Z(J) = (FLOAT(J)-0.5)/52.
```

```
  102 CONTINUE
C
C     GENERATE BOUNDARY DATA.
C
      DO 103 J=1,N
         BDB(J) = 4.*Z(J)**4
  103 CONTINUE
      DO 104 I=1,M
         BDC(I) = 0.
         BDD(I) = 4.*R(I)**4
  104 CONTINUE
C
C     BDA IS A DUMMY VARIABLE.
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      DO 106 I=1,M
         DO 105 J=1,N
            F(I,J) =
4.*R(I)**2*Z(J)**2*(4.*Z(J)**2+3.*R(I)**2)
  105    CONTINUE
  106 CONTINUE
      CALL HSTCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,
     1             IDIMF,PERTRB,IERROR,W)
```

```
C
C     COMPUTE DISCRETIZATION ERROR BY MINIMIZING OVER
ALL A THE FUNCTION
C     NORM(F(I,J) - A*1 - U(R(I),Z(J))).  THE EXACT
SOLUTION IS
C                U(R,Z) = (R*Z)**4 + ARBITRARY CONSTANT.
C
      X = 0.
      DO 108 I=1,M
         DO 107 J=1,N
            X = X+F(I,J)-(R(I)*Z(J))**4
  107    CONTINUE
  108 CONTINUE
      X = X/FLOAT(M*N)
      DO 110 I=1,M
         DO 109 J=1,N
            F(I,J) = F(I,J)-X
  109    CONTINUE
  110 CONTINUE
      ERR = 0.
      DO 112 I=1,M
         DO 111 J=1,N
            X = ABS(F(I,J)-(R(I)*Z(J))**4)
            IF (X .GT. ERR) ERR = X
  111    CONTINUE
```

```
  112 CONTINUE
      PRINT 1001 , IERROR,PERTRB,ERR,W(1)
      STOP
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HSTCYL EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/32X,20HPERTRB =-
4.43114E-04/
     3        18X,34HDISCRETIZATION ERROR = 7.52796E-05/
     4        12X,32HREQUIRED LENGTH OF W ARRAY = 958//
     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6        32X,8HIERROR =,I2/32X,8HPERTRB =,E12.5/
     7        18X,22HDISCRETIZATION ERROR =,E12.5/
     8        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)
C
      END
```

THSTPLR

```
C
C file thstplr.f
C
C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
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C

C

C     PROGRAM TO ILLUSTRATE THE USE OF HSTPLR TO SOLVE
THE EQUATION
```

```
C

C     (1/R)(D/DR)(R*DU/DR) +
(1/R**2)(D/DTHETA)(DU/DTHETA) = 16*R**2

C

C     ON THE QUARTER-DISK 0 .LT. R .LT. 1 AND 0 .LT.
THETA .LT. PI/2

C     WITH THE BOUNDARY CONDITIONS

C

C     U(1,THETA) = 1 - COS(4*THETA)  FOR  0 .LE. THETA
.LE. PI/2

C

C     AND

C

C     (DU/DR)(R,0) = (DU/DR)(R,PI/2) = 0  FOR  O .LT. R
.LT. 1 .

C

C     NOTE THAT U AT THE ORIGIN IS UNSPECIFIED.  THE
EXACT SOLUTION TO

C     THIS PROBLEM IS

C

C               U(R,THETA) = (R**4)(1-COS(4*THETA)) .

C

C     WE WILL USE 50 UNKNOWNS IN THE R-INTERVAL AND 48
UNKNOWNS IN

C     THE THETA-INTERVAL.

C
```

```
      DIMENSION       F(51,50)   ,BDB(48)    ,BDC(50)
,BDD(50)    ,

     1                W(1092)    ,R(50)      ,THETA(48)

C

C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF.
ALSO NOTE THAT W

C     IS DIMENSIONED (13 + LOG2(N))*M + 4*N .

C

      IDIMF = 51

      A = 0.

      B = 1.

      M = 50

      MBDCND = 5

      C = 0.

      PI = PIMACH(DUM)

      D = PI/2.

      N = 48

      NBDCND = 3

      ELMBDA = 0.

C

C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING

C     BOUNDARY DATA AND THE RIGHT SIDE OF THE POISSON
EQUATION.

C

      DO 101 I=1,M
```

```
         R(I) = (FLOAT(I)-0.5)/50.
  101 CONTINUE
      DO 102 J=1,N
         THETA(J) = (FLOAT(J)-0.5)*PI/96.
  102 CONTINUE
C
C     GENERATE BOUNDARY DATA.
C
      DO 103 J=1,N
         BDB(J) = 1.-COS(4.*THETA(J))
  103 CONTINUE
      DO 104 I=1,M
         BDC(I) = 0.
         BDD(I) = 0.
  104 CONTINUE
C
C     BDA IS A DUMMY VARIABLE.
C
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      DO 106 I=1,M
         DO 105 J=1,N
            F(I,J) = 16.*R(I)**2
```

```
  105    CONTINUE

  106 CONTINUE

      CALL HSTPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,

     1             IDIMF,PERTRB,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS

C

C                U(R,THETA) = R**4*(1 - COS(4*THETA))

C

      ERR = 0.

      DO 108 I=1,M

         DO 107 J=1,N

            Z = ABS(F(I,J)-R(I)**4*(1.-
COS(4.*THETA(J))))

            IF (Z .GT. ERR) ERR = Z

  107    CONTINUE

  108 CONTINUE

      PRINT 1001 , IERROR,ERR,W(1)

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE HSTPLR EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/
```

```
     3         18X,34HDISCRETIZATION ERROR = 1.13038E-03/

     4         12X,33HREQUIRED LENGTH OF W ARRAY = 1042//

     5         10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6         32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7         12X,28HREQUIRED LENGTH OF W ARRAY =,F5.0)

C

      END
```

THSTSSP

```
C

C file thstssp.f

C

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C

C     PROGRAM TO ILLUSTRATE THE USE OF HSTSSP TO SOLVE
POISSON"S

C     EQUATION

C

C
(1/SIN(THETA))(D/DTHETA)(SIN(THETA)*DU/DTHETA) +

C

C    (1/SIN(THETA)**2)(D/DPHI)(DU/DPHI) = 2 -
6*(SIN(THETA)*SIN(PHI))**2

C     ON THE NORTHERN HEMISPHERE SUBJECT TO EQUATORIAL
SYMMETRY, I.E.

C     THE DERIVATIVE OF THE SOLUTION AT THETA = PI/2 IS
ZERO.  A
```

```
C     5-DEGREE GRID IS TO BE USED.

C

C     THE EXACT SOLUTION IS NOT UNIQUE.  ANY FUNCTION OF
THE FORM

C

C           U(THETA,PHI) = (SIN(THETA)*SIN(PHI))**2 +
CONSTANT

C

C     IS A SOLUTION.

C

      DIMENSION       F(18,72)   ,BDB(72)    ,SINT(18)
,SINP(72)   ,

     1                W(630)

C

C     THE VALUE OF IDIMF IS THE FIRST DIMENSION OF F. W
IS DIMENSIONED

C     (13 + INT(LOG2(N)))*M + 4*N

C

      PI = PIMACH(DUM)

      A = 0.

      B = PI/2.

      M = 18

      MBDCND = 6

      C = 0.

      D = 2.*PI

      N = 72
```

```
      NBDCND = 0
      ELMBDA = 0.
      IDIMF = 18
C
C     GENERATE SINES FOR USE IN SUBSEQUENT COMPUTATIONS
C
      DTHETA = B/FLOAT(M)
      DO 101 I=1,M
         SINT(I) = SIN((FLOAT(I)-0.5)*DTHETA)
  101 CONTINUE
      DPHI = D/FLOAT(N)
      DO 102 J=1,N
         SINP(J) = SIN((FLOAT(J)-0.5)*DPHI)
  102 CONTINUE
C
C     COMPUTE RIGHT SIDE OF EQUATION AND STORE IN F
C
      DO 104 J=1,N
         DO 103 I=1,M
            F(I,J) = 2.-6.*(SINT(I)*SINP(J))**2
  103    CONTINUE
  104 CONTINUE
C
C     STORE DERIVATIVE DATA AT THE EQUATOR
```

```
C
      DO 105 J=1,N
         BDB(J) = 0.
  105 CONTINUE
C
C     BDA, BDC, AND BDD ARE DUMMY VARIABLES.
C
      CALL HSTSSP
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,
     1             IDIMF,PERTRB,IERROR,W)
C
C     COMPUTE DISCRETIZATION ERROR. SINCE PROBLEM IS
SINGULAR, THE
C     SOLUTION MUST BE NORMALIZED.
C
      ERR = 0.
      DO 107 J=1,N
         DO 106 I=1,M
            Z = ABS(F(I,J)-(SINT(I)*SINP(J))**2-F(1,1))
            IF (Z .GT. ERR) ERR = Z
  106    CONTINUE
  107 CONTINUE
C
      PRINT 1001 , IERROR,PERTRB,ERR,W(1)
      STOP
```

```
C

 1001 FORMAT (1H1,20X,25HSUBROUTINE HSTSSP EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/32X,20HPERTRB =
6.35830E-04/

     3        18X,34HDISCRETIZATION ERROR = 3.37523E-03/

     4        12X,32HREQUIRED LENGTH OF W ARRAY = 540//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/32X,8HPERTRB =,E12.5/

     7        18X,22HDISCRETIZATION ERROR =,E12.5/

     8        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)

C

      END
```

THW3CRT

```
C

C file thw3crt.f

C

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C

C        PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE
HW3CRT TO

C     SOLVE THE HELMHOLTZ EQUATION

C

C     (D/DX)(DU/DX) + (D/DY)(DU/DY) + (D/DZ)(DU/DZ) - 3U
```

```
C
C        =  4X**2*(3-X**2):SIN(Y)*COS(Z)
C
C     ON THE PARALLELEPIPED 0 .LT. X .LT. 1, 0 .LT. Y
.LT. 2*PI,
C     0 .LT. Z .LT. PI/2 WITH THE BOUNDARY CONDITIONS
C
C     U(0,Y,Z) = 0
C                           0 .LE. Y .LE. 2*PI , 0 .LE.
Z .LE. PI/2
C     U(1,Y,Z) = SIN(Y)*COS(Z)
C
C     U PERIODIC IN Y,
C
C     U(X,Y,0) = X**4*SIN(Y)
C                           0 .LE. X .LE. 1 , 0 .LE. Y
.LE. 2*PI
C     (DU/DX)(X,Y,PI/2) = -X**4*SIN(Y)
C
C     USING A FINITE DIFFERENCE GRID WITH PANEL WIDTHS
C
C     DELTAX (=DX) = 1/10 , DELTAY (=DY) = 1/40 , DELTAZ
(=DZ) = 1/15.
C
C        THE EXACT SOLUTION OF THIS PROBLEM IS
C
```

```
C     U(X,Y,Z) = X**4*SIN(Y)*COS(Z) .

C

      DIMENSION       F(11,41,16),BDZF(11,41),W(370)
,X(11)      ,

     1                Y(41)      ,Z(16)

C

C        FROM THE DESCRIPTION OF THE PROBLEM GIVEN
ABOVE, WE DEFINE

C     THE FOLLOWING QUANTITIES

C

      ELMBDA = -3.

      XS = 0.

      XF = 1.

      LBDCND = 1

      YS = 0.

      PI = PIMACH(DUM)

      YF = 2.*PI

      MBDCND = 0

      ZS = 0.

      ZF = PI/2.

      NBDCND = 2

      L = 10

      M = 40

      N = 15

C
```

```
C     FROM THE DIMENSION STATEMENT ABOVE WE DEFINE
C
      LDIMF = 11
      MDIMF = 41
C
C     ALSO NOTE THAT W HAS BEEN DIMENSIONED
C
30+L+M+5*N+MAX(L,M,N)+7*(INT((L+1)/2)+INT((M+1)/2))
C      = 30+10+40+75+40+7*(5+20) = 370
C     WE DEFINE THE GRID POINTS FOR LATER USE.
C
      LP1 = L+1
      DX = (XF-XS)/FLOAT(L)
      DO 101 I=1,LP1
         X(I) = XS+FLOAT(I-1)*DX
  101 CONTINUE
      MP1 = M+1
      DY = (YF-YS)/FLOAT(M)
      DO 102 J=1,MP1
         Y(J) = YS+FLOAT(J-1)*DY
  102 CONTINUE
      NP1 = N+1
      DZ = (ZF-ZS)/FLOAT(N)
      DO 103 K=1,NP1
         Z(K) = ZS+FLOAT(K-1)*DZ
```

```
  103 CONTINUE
C
C     WE DEFINE THE ARRAY OF DERIVATIVE BOUNDARY VALUES.
C
      DO 105 I=1,LP1
         DO 104 J=1,MP1
            BDZF(I,J) = -X(I)**4*SIN(Y(J))
  104    CONTINUE
  105 CONTINUE
C
C     NOTE THAT FOR THIS EXAMPLE ALL OTHER BOUNDARY
ARRAYS ARE
C     DUMMY VARIABLES.
C     WE DEFINE THE FUNCTION BOUNDARY VALUES IN THE F
ARRAY.
C
      DO 107 J=1,MP1
         DO 106 K=1,NP1
            F(1,J,K) = 0.
            F(LP1,J,K) = SIN(Y(J))*COS(Z(K))
  106    CONTINUE
  107 CONTINUE
      DO 109 I=1,LP1
         DO 108 J=1,MP1
            F(I,J,1) = X(I)**4*SIN(Y(J))
```

```
  108    CONTINUE

  109 CONTINUE

C

C     WE NOW DEFINE THE VALUES OF THE RIGHT SIDE OF THE
HELMHOLTZ

C     EQUATION.

C

      DO 112 I=2,L

         DO 111 J=1,MP1

            DO 110 K=2,NP1

               F(I,J,K) = 4.*X(I)**2*(3.-
X(I)**2)*SIN(Y(J))*COS(Z(K))

  110       CONTINUE

  111    CONTINUE

  112 CONTINUE

C

C     CALL HW3CRT TO GENERATE AND SOLVE THE FINITE
DIFFERENCE EQUATION.

C

      CALL HW3CRT
(XS,XF,L,LBDCND,BDXS,BDXF,YS,YF,M,MBDCND,BDYS,BDYF,

     1
ZS,ZF,N,NBDCND,BDZS,BDZF,ELMBDA,LDIMF,MDIMF,F,

     2             PERTRB,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
TO THE
```

```
C     PROBLEM IS
C
C        U(X,Y,Z) = X**4*SIN(Y)*COS(Z)
C
      ERR = 0.
      DO 115 I=1,LP1
         DO 114 J=1,MP1
            DO 113 K=1,NP1
               T = ABS(F(I,J,K)-
X(I)**4*SIN(Y(J))*COS(Z(K)))
               IF (T .GT. ERR) ERR = T
  113       CONTINUE
  114    CONTINUE
  115 CONTINUE
      PRINT 1001 , IERROR,ERR
      STOP
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HW3CRT EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/
     3        18X,34HDISCRETIZATION ERROR = 9.64802E-
03//
     4        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     5        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5)
```

```
C

      END
```

THWSCRT

```
C

C file thwscrt.f

C

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C

C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE HWSCRT
TO SOLVE

C     THE EQUATION

C

C     (D/DX)(DU/DX) + (D/DY)(DU/DY) - 4*U

C

C     = (2 - (4 + PI**2/4)*X**2)*COS((Y+1)*PI/2)

C

C     WITH THE BOUNDARY CONDITIONS

C     ON THE RECTANGLE 0 .LT. X .LT. 2, -1 .LT. Y .LT. 3
WITH THE

C

C     U(0,Y) = 0

C                                           -1 .LE. Y
.LE. 3

C     (DU/DX)(2,Y) = 4*COS((Y+1)*PI/2)

C
```

```
C     AND WITH U PERIODIC IN Y.

C           THE X-INTERVAL WILL BE DIVIDED INTO 40 PANELS
AND THE

C     Y-INTERVAL WILL BE DIVIDED INTO 80 PANELS.

C

      DIMENSION       F(45,82)   ,BDB(81)    ,W(1103)
,X(41)      ,

     1                Y(81)

C

C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF.
ALSO NOTE THAT W

C     IS DIMENSIONED 4*(N+1) + (13 +
INT(LOG2(N+1)))*(M+1)  .

C

      IDIMF = 45

      A = 0.

      B = 2.

      M = 40

      MBDCND = 2

      C = -1.

      D = 3.

      N = 80

      NBDCND = 0

      ELMBDA = -4.

C

C     AUXILIARY QUANTITIES.
```

```
C
      PI = PIMACH(DUM)
      PIBY2 = PI/2.
      PISQ = PI**2
      MP1 = M+1
      NP1 = N+1
C
C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING
C     BOUNDARY DATA AND THE RIGHT SIDE OF THE HELMHOLTZ
EQUATION.
C
      DO 101 I=1,MP1
         X(I) = FLOAT(I-1)/20.
  101 CONTINUE
      DO 102 J=1,NP1
         Y(J) = -1.+FLOAT(J-1)/20.
  102 CONTINUE
C
C     GENERATE BOUNDARY DATA.
C
      DO 103 J=1,NP1
         BDB(J) = 4.*COS((Y(J)+1.)*PIBY2)
  103 CONTINUE
C
```

```
C     BDA, BDC, AND BDD ARE DUMMY VARIABLES.

C

      DO 104 J=1,NP1

         F(1,J) = 0.

  104 CONTINUE

C

C     GENERATE RIGHT SIDE OF EQUATION.

C

      DO 106 I=2,MP1

         DO 105 J=1,NP1

            F(I,J) = (2.-
(4.+PISQ/4.)*X(I)**2)*COS((Y(J)+1.)*PIBY2)

  105    CONTINUE

  106 CONTINUE

      CALL HWSCRT
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,

     1             IDIMF,PERTRB,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS

C                U(X,Y) = X**2*COS((Y+1)*PIBY2)

C

      ERR = 0.

      DO 108 I=1,MP1

         DO 107 J=1,NP1

            Z = ABS(F(I,J)-X(I)**2*COS((Y(J)+1.)*PIBY2))
```

```
            IF (Z .GT. ERR) ERR = Z
  107    CONTINUE
  108 CONTINUE
      PRINT 1001 , IERROR,ERR,W(1)
      STOP
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HWSCRT EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/
     3        18X,34HDISCRETIZATION ERROR = 5.36508E-04/
     4        12X,32HREQUIRED LENGTH OF W ARRAY = 880//
     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/
     7        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)
C
      END
```

THWSCSP

```
C
C file thwscsp.f
C
```

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```

```
C     PROGRAM TO ILLUSTRATE THE USE OF HWSCSP

C

      DIMENSION       F(48,33)   ,BDTF(33)   ,W(775)
,R(33)      ,

     1                THETA(48)

C

      PI = PIMACH(DUM)

      INTL = 0

      TS = 0.

      TF = PI/2.

      M = 36

      MBDCND = 6

      RS = 0.

      RF = 1.

      N = 32

      NBDCND = 5

      ELMBDA = 0.

      IDIMF = 48

C

C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING THE

C     BOUNDARY DATA AND THE RIGHT SIDE OF THE EQUATION.

C

      MP1 = M+1

      DTHETA = TF/FLOAT(M)
```

```
      DO 101 I=1,MP1
         THETA(I) = FLOAT(I-1)*DTHETA
  101 CONTINUE
      NP1 = N+1
      DR = 1./FLOAT(N)
      DO 102 J=1,NP1
         R(J) = FLOAT(J-1)*DR
  102 CONTINUE
C
C     GENERATE NORMAL DERIVATIVE DATA AT EQUATOR
C
      DO 103 J=1,NP1
         BDTF(J) = 0.
  103 CONTINUE
C
C     COMPUTE BOUNDARY DATA ON THE SURFACE OF THE SPHERE
C
      DO 104 I=1,MP1
         F(I,N+1) = COS(THETA(I))**4
  104 CONTINUE
C
C     COMPUTE RIGHT SIDE OF EQUATION
C
      DO 106 I=1,MP1
```

```
         CI4 = 12.*COS(THETA(I))**2
         DO 105 J=1,N
            F(I,J) = CI4*R(J)**2
  105    CONTINUE
  106 CONTINUE
C
      CALL HWSCSP
(INTL,TS,TF,M,MBDCND,BDTS,BDTF,RS,RF,N,NBDCND,BDRS,
     1             BDRF,ELMBDA,F,IDIMF,PERTRB,IERROR,W)
C
C     COMPUTE DISCRETIZATION ERROR
C
      ERR = 0.
      DO 108 I=1,MP1
         CI4 = COS(THETA(I))**4
         DO 107 J=1,N
            Z = ABS(F(I,J)-CI4*R(J)**4)
            IF (Z .GT. ERR) ERR = Z
  107    CONTINUE
  108 CONTINUE
      IW = INT(W(1))
      PRINT 1001 , IERROR,ERR,IW
C
C     THE FOLLOWING PROGRAM ILLUSTRATES THE USE OF
HWSCSP TO SOLVE
```

```
C     A THREE DIMENSIONAL PROBLEM WHICH HAS LONGITUDNAL
DEPENDENCE

C

      MBDCND = 2

      NBDCND = 1

      DPHI = PI/72.

      ELMBDA = -2.*(1.-COS(DPHI))/DPHI**2

C

C     COMPUTE BOUNDARY DATA ON THE SURFACE OF THE SPHERE

C

      DO 109 I=1,MP1

         F(I,N+1) = SIN(THETA(I))

  109 CONTINUE

C

C     COMPUTE RIGHT SIDE OF THE EQUATION

C

      DO 111 J=1,N

         DO 110 I=1,MP1

            F(I,J) = 0.

  110    CONTINUE

  111 CONTINUE

C

      CALL HWSCSP
(INTL,TS,TF,M,MBDCND,BDTS,BDTF,RS,RF,N,NBDCND,BDRS,

     1             BDRF,ELMBDA,F,IDIMF,PERTRB,IERROR,W)
```

```
C

C     COMPUTE DISCRETIZATION ERROR   (FOURIER
COEFFICIENTS)

C

      ERR = 0

      DO 113 I=1,MP1

         SI = SIN(THETA(I))

         DO 112 J=1,NP1

            Z = ABS(F(I,J)-R(J)*SI)

            IF (Z .GT. ERR) ERR = Z

  112    CONTINUE

  113 CONTINUE

C

      IW = INT(W(1))

      PRINT 1002 , IERROR,ERR,IW

      STOP

C

 1001 FORMAT (1H1,20X,27HSUBROUTINE HWSCSP EXAMPLE 1///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/

     3        18X,34HDISCRETIZATION ERROR = 7.99842E-04/

     4        12X,32HREQUIRED LENGTH OF W ARRAY = 775//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/
```

```
     7         12X,28HREQUIRED LENGTH OF W ARRAY =,I4)

 1002 FORMAT (1H1,20X,27HSUBROUTINE HWSCSP EXAMPLE 2///

     1         10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2         32X,10HIERROR = 0/

     3         18X,34HDISCRETIZATION ERROR = 5.86824E-05/

     4         12X,32HREQUIRED LENGTH OF W ARRAY = 775//

     5         10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6         32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7         12X,28HREQUIRED LENGTH OF W ARRAY =,I4)

C

      END
```

THWSCYL

```
C

C file thwscyl.f

C

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C

C

C          PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE
HWSCYL TO SOLVE

C     THE EQUATION

C

C     (1/R)(D/DR)(R*(DU/DR)) + (D/DZ)(DU/DZ)

C
```

```
C     = (2*R*Z)**2*(4*Z**2 + 3*R**2)

C

C     ON THE RECTANGLE 0 .LT. R .LT. 1, 0 .LT. Z .LT. 1
WITH THE

C     BOUNDARY CONDITIONS

C

C     U(0,Z) UNSPECIFIED

C                                            0 .LE. Z
.LE. 1

C     (DU/DR)(1,Z) = 4*Z**4

C

C     AND

C

C     (DU/DZ)(R,0) = 0

C                                            0 .LE. R
.LE. 1

C     (DU/DZ)(R,1) = 4*R**4 .

C

C          THE R-INTERVAL WILL BE DIVIDED INTO 50 PANELS
AND THE

C     Z-INTERVAL WILL BE DIVIDED INTO 100 PANELS.

C

      DIMENSION       F(75,105)  ,BDA(101)   ,BDB(101)
,BDC(51)    ,

     1                BDD(51)    ,W(1373)    ,R(51)
,Z(101)

C
```

```
C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF.
ALSO NOTE THAT W

C     IS DIMENSIONED 4*(N+1) + (13 +
INT(LOG2(N+1)))*(M+1) .

C

      IDIMF = 75

      A = 0.

      B = 1.

      M = 50

      MBDCND = 6

      C = 0.

      D = 1.

      N = 100

      NBDCND = 3

      ELMBDA = 0.

C

C     AUXILIARY QUANTITIES.

C

      MP1 = M+1

      NP1 = N+1

C

C     GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING

C     BOUNDARY DATA AND THE RIGHT SIDE OF THE POISSON
EQUATION.

C
```

```
      DO 101 I=1,MP1
         R(I) = FLOAT(I-1)/50.
  101 CONTINUE
      DO 102 J=1,NP1
         Z(J) = FLOAT(J-1)/100.
  102 CONTINUE
C
C     GENERATE BOUNDARY DATA.
C
      DO 103 J=1,NP1
         BDB(J) = 4.*Z(J)**4
  103 CONTINUE
      DO 104 I=1,MP1
         BDC(I) = 0.
         BDD(I) = 4.*R(I)**4
  104 CONTINUE
C
C     BDA IS A DUMMY VARIABLE.
C
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      DO 106 I=1,MP1
         DO 105 J=1,NP1
```

```
            F(I,J) =
4.*R(I)**2*Z(J)**2*(4.*Z(J)**2+3.*R(I)**2)

  105    CONTINUE

  106 CONTINUE

      CALL HWSCYL
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,

     1             IDIMF,PERTRB,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR BY MINIMIZING OVER
ALL A THE FUNCTION

C     NORM(F(I,J) - A*1 - U(R(I),Z(J))).  THE EXACT
SOLUTION IS

C                U(R,Z) = (R*Z)**4 + ARBITRARY CONSTANT.

C

      X = 0.

      DO 108 I=1,MP1

         DO 107 J=1,NP1

            X = X+F(I,J)-(R(I)*Z(J))**4

  107    CONTINUE

  108 CONTINUE

      X = X/FLOAT(NP1*MP1)

      DO 110 I=1,MP1

         DO 109 J=1,NP1

            F(I,J) = F(I,J)-X

  109    CONTINUE

  110 CONTINUE
```

```
      ERR = 0.

      DO 112 I=1,MP1

         DO 111 J=1,NP1

            X = ABS(F(I,J)-(R(I)*Z(J))**4)

            IF (X .GT. ERR) ERR = X

  111    CONTINUE

  112 CONTINUE

      PRINT 1001 , IERROR,PERTRB,ERR,W(1)

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE HWSCYL EXAMPLE///

     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/32X,20HPERTRB =
2.26734E-04/

     3        18X,34HDISCRETIZATION ERROR = 3.73672E-04/

     4        12X,33HREQUIRED LENGTH OF W ARRAY = 1118//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/32X,8HPERTRB =,E12.5/

     7        18X,22HDISCRETIZATION ERROR =,E12.5/

     8        12X,28HREQUIRED LENGTH OF W ARRAY =,F5.0)

C

      END
```

THWSPLR

```
C

C file thwsplr.f

C

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C

C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE HWSPLR
TO SOLVE

C     THE EQUATION

C

C     (1/R)(D/DR)(R*(DU/DR)) +
(1/R**2)(D/DTHETA)(DU/DTHETA) = 16*R**2

C

C     ON THE QUARTER-DISK 0 .LT. R .LT. 1, 0 .LT. THETA
.LT. PI/2 WITH

C     WITH THE BOUNDARY CONDITIONS

C

C     U(1,THETA) = 1 - COS(4*THETA), 0 .LE. THETA .LE. 1

C

C     AND

C

C     (DU/DTHETA)(R,0) = (DU/DTHETA)(R,PI/2) = 0,  0
.LE. R .LE. 1.

C

C     (NOTE THAT THE SOLUTION U IS UNSPECIFIED AT R =
0.)

C          THE R-INTERVAL WILL BE DIVIDED INTO 50 PANELS
AND THE

C     THETA-INTERVAL WILL BE DIVIDED INTO 48 PANELS.
```

```
C

      DIMENSION       F(100,50)  ,BDC(51)    ,BDD(51)
,W(1114)    ,

     1                R(51)      ,THETA(49)

C

C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF.
ALSO NOTE THAT W

C     IS DIMENSIONED 4*(N+1) + (13 +
INT(LOG2(N+1)))*(M+1) .

C

      IDIMF = 100

      A = 0.

      B = 1.

      M = 50

      MBDCND = 5

      C = 0.

      PI = PIMACH(DUM)

      D = PI/2.

      N = 48

      NBDCND = 3

      ELMBDA = 0.

C

C     AUXILIARY QUANTITIES.

C

      MP1 = M+1

      NP1 = N+1
```

```
C
C      GENERATE AND STORE GRID POINTS FOR THE PURPOSE OF
COMPUTING
C      BOUNDARY DATA AND THE RIGHT SIDE OF THE POISSON
EQUATION.
C
      DO 101 I=1,MP1
         R(I) = FLOAT(I-1)/50.
  101 CONTINUE
      DO 102 J=1,NP1
         THETA(J) = FLOAT(J-1)*PI/96.
  102 CONTINUE
C
C      GENERATE BOUNDARY DATA.
C
      DO 103 I=1,MP1
         BDC(I) = 0.
         BDD(I) = 0.
  103 CONTINUE
C
C      BDA AND BDB ARE DUMMY VARIABLES.
C
      DO 104 J=1,NP1
         F(MP1,J) = 1.-COS(4.*THETA(J))
  104 CONTINUE
```

```
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      DO 106 I=1,M
         DO 105 J=1,NP1
            F(I,J) = 16.*R(I)**2
  105    CONTINUE
  106 CONTINUE
      CALL HWSPLR
(A,B,M,MBDCND,BDA,BDB,C,D,N,NBDCND,BDC,BDD,ELMBDA,F,
     1             IDIMF,PERTRB,IERROR,W)
C
C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS
C                U(R,THETA) = R**4*(1 - COS(4*THETA))
C
      ERR = 0.
      DO 108 I=1,MP1
         DO 107 J=1,NP1
            Z = ABS(F(I,J)-R(I)**4*(1.-
COS(4.*THETA(J))))
            IF (Z .GT. ERR) ERR = Z
  107    CONTINUE
  108 CONTINUE
      PRINT 1001 , IERROR,ERR,W(1)
      STOP
```

```
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HWSPLR EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/
     3        18X,34HDISCRETIZATION ERROR = 6.19134E-04/
     4        12X,32HREQUIRED LENGTH OF W ARRAY = 882//
     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/
     7        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)
C
      END
```

THWSSSP

```
C
C file thwsssp.f
C
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C

C

C

C     PROGRAM TO ILLUSTRATE THE USE OF HWSSSP

C

      DIMENSION       F(19,73)   ,BDTF(73)   ,SINT(19)
,SINP(73)   ,

     1                W(600)
```

```
C
      PI = PIMACH(DUM)
      TS = 0
      TF = PI/2.
      M = 18
      MBDCND = 6
      PS = 0
      PF = PI+PI
      N = 72
      NBDCND = 0
      ELMBDA = 0.
      IDIMF = 19
C
C     GENERATE SINES FOR USE IN SUBSEQUENT COMPUTATIONS
C
      DTHETA = TF/FLOAT(M)
      MP1 = M+1
      DO 101 I=1,MP1
         SINT(I) = SIN(FLOAT(I-1)*DTHETA)
  101 CONTINUE
      DPHI = (PI+PI)/FLOAT(N)
      NP1 = N+1
      DO 102 J=1,NP1
         SINP(J) = SIN(FLOAT(J-1)*DPHI)
```

```
  102 CONTINUE
C
C     COMPUTE RIGHT SIDE OF EQUATION AND STORE IN F
C
      DO 104 J=1,NP1
         DO 103 I=1,MP1
            F(I,J) = 2.-6.*(SINT(I)*SINP(J))**2
  103    CONTINUE
  104 CONTINUE
C
C     STORE DERIVATIVE DATA AT THE EQUATOR
C
      DO 105 J=1,NP1
         BDTF(J) = 0.
  105 CONTINUE
C
      CALL HWSSSP
(TS,TF,M,MBDCND,BDTS,BDTF,PS,PF,N,NBDCND,BDPS,BDPF,
     1             ELMBDA,F,IDIMF,PERTRB,IERROR,W)
C
C     COMPUTE DISCRETIZATION ERROR. SINCE PROBLEM IS
SINGULAR, THE
C     SOLUTION MUST BE NORMALIZED.
C
      ERR = 0
```

```
      DO 107 J=1,NP1
         DO 106 I=1,MP1
            Z = ABS(F(I,J)-(SINT(I)*SINP(J))**2-F(1,1))
            IF (Z .GT. ERR) ERR = Z
  106    CONTINUE
  107 CONTINUE
C
      IW = INT(W(1))
      PRINT 1001 , IERROR,ERR,IW
      STOP
C
C
 1001 FORMAT (1H1,20X,25HSUBROUTINE HWSSSP EXAMPLE///
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//
     2        32X,10HIERROR = 0/
     3        18X,34HDISCRETIZATION ERROR = 3.38107E-03/
     4        12X,32HREQUIRED LENGTH OF W ARRAY = 600//
     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//
     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5 /
     7        12X,28HREQUIRED LENGTH OF W ARRAY =,I4)
C
      END
```

TPOIS3D

```
C
C file tpois3d.f
C
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C

C

C     THIS PROGRAM ILLUSTRATES THE USE OF THE SUBROUTINE
POIS3D TO SOLVE

C     THE EQUATION

C

C     (D/DX)(DU/DX) + (D/DY)(DU/DY) +
(1+Z)**2*(D/DZ)(DU/DZ)

C

C     - 2*(1+Z)*(DU/DZ) = 2*SIN(X)*SIN(Y)*(1+Z)**4
(1)

C

C     ON THE PARALLELEPIPED -PI .LT. X .LT. PI, -PI .LT.
Y .LT. PI,

C     0 .LT. Z .LT. 1 WITH BOUNDARY CONDITIONS

C

C     U PERIODIC IN X

C

C     U PERIODIC IN Y

C

C     (DU/DZ)(X,Y,0) = 4*SIN(X)*SIN(Y)    -PI .LT. X,Y
.LT. PI       (2)

C
```

```
C     U(X,Y,1) = 16*SIN(X)*SIN(Y)         -PI .LT. X,Y
.LT. PI       (3)

C

C     USING A FINITE DIFFERENCE GRID WITH DELTAX (= DX)
= 2*PI/30 ,

C     DELTAY (= DY) = 2*PI/30, AND DELTAZ (= DZ) = 1/10.

C         TO SET UP THE FINITE DIFFERENCE EQUATIONS WE
DEFINE THE GRID

C     POINTS

C

C         X(I) = -PI + (I-1)*DX         I=1,2,...,31

C

C         Y(J) = -PI + (J-1)*DY         J=1,2,...,31

C

C         Z(K) = (K-1)*DZ               K=1,2,...,11

C

C     AND LET V(I,J,K) BE AN APPROXIMATION TO
U(X(I),Y(J),Z(K)).

C     NUMBERING THE GRID POINTS IN THIS FASHION GIVES
THE SET OF

C     UNKNOWNS AS V(I,J,K) FOR I=1,2,...,30,
J=1,2,...,30, K=1,2,...,10.

C     HENCE, IN THE PROGRAM L=30, M = 30, AND N = 10.
AT THE INTERIOR

C     GRID POINT (X(I),Y(J),Z(K)), WE REPLACE ALL
DERIVATIVES IN

C     EQUATION (1) BY SECOND ORDER CENTRAL FINITE
DIFFERENCES AND
```

```
C     COLLECT COEFFICIENTS OF V(I,J,K) TO GET THE FINITE
DIFFERENCE

C     EQUATION

C

C        (V(I-1,J,K) - 2V(I,J,K) + V(I+1,J,K))/DX**2

C

C      + (V(I,J-1,K) - 2V(I,J,K) + V(I,J+1,K))/DY**2

C

C      + A(K)V(I,J,K-1) + B(K)V(I,J,K) + C(K)V(I,J,K+1)
= F(I,J,K)   (4)

C

C     WHERE FOR K=2,3,...,9

C

C     A(K) = (1+Z(K))**2/DZ**2 + (1+Z(K))/DZ

C

C     B(K) = -2(1+Z(K))**2/DZ**2

C

C     C(K) = (1+Z(K))**2/DZ**2 - (1+Z(K))/DZ

C

C     F(I,J,K) = 2SIN(X(I))*SIN(Y(J))*(1+Z(K))**4  FOR
I,J=1,2,...,30.

C

C         TO OBTAIN EQUATIONS FOR K=1, WE REPLACE THE
DERIVATIVE IN

C     EQUATION (2) BY A SECOND ORDER CENTRAL FINITE
DIFFERENCE APPROX-
```

```
C     IMATION, USE THIS EQUATION TO ELIMINATE THE
VIRTUAL UNKNOWN

C     V(I,J,0) IN EQUATION (4) AND ARRIVE AT THE
EQUATION

C

C              (V(I-1,J,1) -2V(I,J,1) +
V(I+1,J,1))/DX**2

C

C            + (V(I,J-1,1) -2V(I,J,1) +
V(I,J+1,1))/DY**2

C

C            + B(1)V(I,J,1) + C(1)V(I,J,2) = F(I,J,1)

C

C     WHERE

C            B(1) = -C(1) = -2(1+Z(1))**2/DZ**2 = -
2/DZ**2

C

C            F(I,J,1) = (10 + 8/DZ)SIN(X(I))*SIN(Y(J))

C

C            FOR I,J=1,2,...,30.  FOR COMPLETENESS WE
SET A(1) = 0.

C

C         TO OBTAIN EQUATIONS FOR K=10, WE INCORPORATE
EQUATION (3) INTO

C     EQUATION (4) BY SETTING

C

C            V(I,J,11) = U(X(I),Y(J),1) =
16SIN(X(I))*SIN(Y(J))
```

```
C
C     AND ARRIVE AT THE EQUATION
C
C              (V(I-1,J,10) - 2V(I,J,10) +
V(I+1,J,10))/DX**2
C
C            + (V(I,J-1,10) - 2V(I,J,10) +
V(I,J+1,10))/DY**2
C
C            + A(10)V(I,J,9) + B(10)V(I,J,10) =
F(I,J,10)
C
C     WHERE
C
C            A(10) = (1+Z(10))**2/DZ**2 + (1+Z(10))/DZ
C
C            B(10) = -2(1+Z(10))**2/DZ**2
C
C            F(I,J,10) =
2SIN(X(I))*SIN(Y(J))*((1+Z(10))**4
C                        -8*((1+Z(10))**2/DZ**2 -
(1+Z(10))/DZ))
C
C                        FOR I,J=1,2,...,30.
C
C     FOR COMPLETENESS, WE SET C(10) = 0.  HENCE, IN THE
PROGRAM,
```

```
C     NPEROD = 1.

C         THE PERIODICITY CONDITIONS ON U GIVE THE
CONDITIONS

C

C             V(0,J,K) = V(30,J,K) AND V(31,J,K) =
V(1,J,K)

C                        FOR J=1,2,...,30 AND
K=1,2,...,10,

C             AND

C             V(I,0,K) = V(I,30,K) AND V(I,31,K) =
V(I,1,K)

C                        FOR I=1,2,...,30 AND
K=1,2,...,10.

C

C     HENCE, IN THE PROGRAM LPEROD = MPEROD = 0.

C

      DIMENSION       F(32,33,10),A(10)      ,B(10)
,C(10)      ,

     1                W(350)     ,X(30)      ,Y(30)
,Z(10)

C

C     FROM THE DIMENSION STATEMENT WE GET THAT LDIMF =
32, MDIMF = 33,

C     AND NOTE THAT W HAS BEEN DIMENSIONED ACCORDING TO
ITS DESCRIPTION.

C

      LDIMF = 32

      MDIMF = 33
```

```
      PI = PIMACH(DUM)
      LPEROD = 0
      L = 30
      DX = 2.*PI/FLOAT(L)
      C1 = 1./DX**2
      MPEROD = 0
      M = 30
      DY = 2.*PI/FLOAT(M)
      C2 = 1./DY**2
      NPEROD = 1
      N = 10
      DZ = 1./FLOAT(N)
      DZSQ = 1./DZ**2
C
C     GENERATE GRID POINTS FOR LATER USE.
C
      DO 101 I=1,L
         X(I) = -PI+FLOAT(I-1)*DX
  101 CONTINUE
      DO 102 J=1,M
         Y(J) = -PI+FLOAT(J-1)*DY
  102 CONTINUE
C
C     GENERATE COEFFICIENTS
```

```
C
      A(1) = 0.
      B(1) = -2.*DZSQ
      C(1) = -B(1)
      Z(1) = 0.
      DO 103 K=2,N
         Z(K) = FLOAT(K-1)*DZ
         T = 1.+Z(K)
         A(K) = T**2*DZSQ+T/DZ
         B(K) = -2.*T**2*DZSQ
         C(K) = T**2*DZSQ-T/DZ
  103 CONTINUE
C
C     GENERATE RIGHT SIDE OF EQUATION
C
      DO 106 I=1,L
         DO 105 J=1,M
            DO 104 K=2,N
               F(I,J,K) =
2.*SIN(X(I))*SIN(Y(J))*(1.+Z(K))**4
  104       CONTINUE
  105    CONTINUE
  106 CONTINUE
      DO 108 I=1,L
         DO 107 J=1,L
```

```
            F(I,J,1) = (10.+8./DZ)*SIN(X(I))*SIN(Y(J))

            F(I,J,N) = F(I,J,N)-
C(N)*16.*SIN(X(I))*SIN(Y(J))

  107    CONTINUE

  108 CONTINUE

      C(N) = 0.

C

C     CALL POIS3D TO SOLVE EQUATIONS.

C

      CALL POIS3D
(LPEROD,L,C1,MPEROD,M,C2,NPEROD,N,A,B,C,LDIMF,MDIMF,

     1             F,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS

C

C              U(X,Y,Z) = SIN(X)*SIN(Y)*(1+Z)**4

C

      ERR = 0.

      DO 111 I=1,L

         DO 110 J=1,M

            DO 109 K=1,N

               T = ABS(F(I,J,K)-
SIN(X(I))*SIN(Y(J))*(1.+Z(K))**4)

               IF (T .GT. ERR) ERR = T

  109       CONTINUE
```

```
  110     CONTINUE

  111 CONTINUE

      PRINT 1001 ,  IERROR,ERR

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE POIS3D EXAMPLE///

     1         10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2         32X,10HIERROR = 0/

     3         18X,34HDISCRETIZATION ERROR = 2.93277E-
02//

     4         10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     5         32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5)

C

      END
```

TPOISTG

```
C

C file tpoistg.g

C

C     * * * * * * * * * * * * * * * * * * * * * * * * *
* * * * * * * *
```

```
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*

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS        *

C     *
*

C     *                               BY
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C     *
*
```

```
C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET         *

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C     *                          OF
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* * * * * * * *

C

C     PROGRAM TO ILLUSTRATE THE USE OF SUBROUTINE POISTG
TO

C     SOLVE THE EQUATION
```

```
C

C      (1/COS(X))(D/DX)(COS(X)(DU/DX)) + (D/DY)(DU/DY) =

C

C            2*Y**2*(6-Y**2)*SIN(X)

C

C      ON THE RECTANGLE -PI/2 .LT. X .LT. PI/2 AND

C      0 .LT. Y .LT. 1 WITH THE BOUNDARY CONDITIONS

C

C      (DU/DX) (-PI/2,Y) = (DU/DX)(PI/2,Y) = 0 , 0 .LE. Y
.LE. 1  (2)

C

C      U(X,0) = 0
(3)

C                                  -PI/2 .LE. X .LE. PI/2

C      (DU/DY)(X,1) = 4SIN(X)
(4)

C

C      USING FINITE DIFFERENCES ON A STAGGERED GRID WITH

C      DELTAX (= DX) = PI/40 AND DELTAY (= DY) = 1/20 .

C         TO SET UP THE FINITE DIFFERENCE EQUATIONS WE
DEFINE

C      THE GRID POINTS

C

C      X(I) = -PI/2 + (I-0.5)DX            I=1,2,...,40

C

C      Y(J) = (J-O.5)DY                    J=1,2,...,20
```

```
C

C     AND LET V(I,J) BE AN APPROXIMATION TO
U(X(I),Y(J)).

C     NUMBERING THE GRID POINTS IN THIS FASHION GIVES
THE SET

C     OF UNKNOWNS AS V(I,J) FOR I=1,2,...,40 AND
J=1,2,...,20.

C     HENCE, IN THE PROGRAM M = 40 AND N = 20.  AT THE
INTERIOR

C     GRID POINT (X(I),Y(J)), WE REPLACE ALL DERIVATIVES
IN

C     EQUATION (1) BY SECOND ORDER CENTRAL FINITE
DIFFERENCES,

C     MULTIPLY BY DY**2, AND COLLECT COEFFICIENTS OF
V(I,J) TO

C     GET THE FINITE DIFFERENCE EQUATION

C

C     A(I)V(I-1,J) + B(I)V(I,J) + C(I)V(I+1,J)

C

C     + V(I,J-1) - 2V(I,J) + V(I,J+1) = F(I,J)
(5)

C

C     WHERE S = (DY/DX)**2, AND FOR I=2,3,...,39

C

C     A(I) = S*COS(X(I)-DX/2)

C

C     B(I) = -S*(COS(X(I)-DX/2)+COS(X(I)+DX/2))

C
```

```
C     C(I) = S*COS(X(I)+DX/2)

C

C     F(I,J) = 2DY**2*Y(J)**2*(6-Y(J)**2)*SIN(X(I)) ,
J=1,2,...,19.

C

C        TO OBTAIN EQUATIONS FOR I = 1, WE REPLACE
EQUATION (2)

C     BY THE SECOND ORDER APPROXIMATION

C

C     (V(1,J)-V(0,J))/DX = 0

C

C     AND USE THIS EQUATION TO ELIMINATE V(0,J) IN
EQUATION (5)

C     TO ARRIVE AT THE EQUATION

C

C     B(1)V(1,J) + C(1)V(2,J) + V(1,J-1) - 2V(1,J) +
V(1,J+1)

C

C                        = F(1,J)

C

C     WHERE

C

C     B(1) = -S*(COS(X(1)-DX/2)+COS(X(1)+DX/2))

C

C     C(1) = -B(1)

C
```

```
C     FOR COMPLETENESS, WE SET A(1) = 0.

C        TO OBTAIN EQUATIONS FOR I = 40, WE REPLACE THE
DERIVATIVE

C     IN EQUATION (2) AT X=PI/2 IN A SIMILAR FASHION,
USE THIS

C     EQUATION TO ELIMINATE THE VIRTUAL UNKNOWN V(41,J)
IN EQUATION

C     (5) AND ARRIVE AT THE EQUATION

C

C     A(40)V(39,J) + B(40)V(40,J)

C

C     + V(40,J-1) - 2V(40,J) + V(40,J+1) = F(40,J)

C

C     WHERE

C

C     A(40) = -B(40) = -S*(COS(X(40)-
DX/2)+COS(X(40)+DX/2))

C

C     FOR COMPLETENESS, WE SET C(40) = 0.  HENCE, IN THE

C     PROGRAM MPEROD = 1.

C        FOR J = 1, WE REPLACE EQUATION (3) BY THE
SECOND ORDER

C     APPROXIMATION

C

C                (V(I,0) + V(I,1))/2 = 0

C
```

```
C     TO ARRIVE AT THE CONDITION

C

C                V(I,0) = -V(I,1) .

C

C     FOR J = 20, WE REPLACE EQUATION (4) BY THE SECOND
ORDER

C     APPROXIMATION

C

C                (V(I,21) - V(I,20))/DY = 4*SIN(X)

C

C     AND COMBINE THIS EQUATION WITH EQUATION (5) TO
ARRIVE AT

C     THE EQUATION

C

C     A(I)V(I-1,20) + B(I)V(I,20) + C(I)V(I+1,20)

C

C     + V(I,19) - 2V(I,20) + V(I,21) = F(I,20)

C

C     WHERE

C

C     V(I,21) = V(I,20)  AND

C

C     F(I,20) = 2*DY**2*Y(J)**2*(6-Y(J)**2)*SIN(X(I)) -
4*DY*SIN(X(I))

C

C     HENCE, IN THE PROGRAM NPEROD = 2 .
```

```
C        THE EXACT SOLUTION TO THIS PROBLEM IS
C
C        U(X,Y) = Y**4*COS(X) .
C
      DIMENSION       F(42,20)   ,A(40)      ,B(40)
,C(40)      ,
     1                W(600)     ,X(40)      ,Y(20)
C
C     FROM DIMENSION STATEMENT WE GET VALUE OF IDIMF =
42.  ALSO
C     NOTE THAT W HAS BEEN DIMENSIONED
C     9M + 4N + M(INT(LOG2(N))) = 360 + 80 + 160 = 600 .
C
      IDIMF = 42
      MPEROD = 1
      M = 40
      PI = PIMACH(DUM)
      DX = PI/FLOAT(M)
      NPEROD = 2
      N = 20
      DY = 1./FLOAT(N)
C
C     GENERATE AND STORE GRID POINTS FOR COMPUTATION.
C
      DO 101 I=1,M
```

```
          X(I) = -PI/2.+(FLOAT(I)-0.5)*DX
  101 CONTINUE
      DO 102 J=1,N
          Y(J) = (FLOAT(J)-0.5)*DY
  102 CONTINUE
C
C     GENERATE COEFFICIENTS .
C
      S = (DY/DX)**2
      A(1) = 0.
      B(1) = -S*COS(-PI/2.+DX)/COS(X(1))
      C(1) = -B(1)
      DO 103 I=2,M
          A(I) = S*COS(X(I)-DX/2.)/COS(X(I))
          C(I) = S*COS(X(I)+DX/2.)/COS(X(I))
          B(I) = -(A(I)+C(I))
  103 CONTINUE
      A(40) = -B(40)
      C(40) = 0.
C
C     GENERATE RIGHT SIDE OF EQUATION.
C
      DO 105 I=1,M
          DO 104 J=1,N
```

```
            F(I,J) = 2.*DY**2*Y(J)**2*(6.-
Y(J)**2)*SIN(X(I))

  104    CONTINUE

  105 CONTINUE

      DO 106 I=1,M

         F(I,N) = F(I,N)-4.*DY*SIN(X(I))

  106 CONTINUE

      CALL POISTG
(NPEROD,N,MPEROD,M,A,B,C,IDIMF,F,IERROR,W)

C

C     COMPUTE DISCRETIZATION ERROR.  THE EXACT SOLUTION
IS

C

C     U(X,Y) = Y**4*SIN(X)

C

      ERR = 0.

      DO 108 I=1,M

         DO 107 J=1,N

            T = ABS(F(I,J)-Y(J)**4*SIN(X(I)))

            IF (T .GT. ERR) ERR = T

  107    CONTINUE

  108 CONTINUE

      PRINT 1001 , IERROR,ERR,W(1)

      STOP

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE POISTG EXAMPLE///
```

```
     1        10X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2        32X,10HIERROR = 0/

     3        18X,34HDISCRETIZATION ERROR = 5.64171E-04/

     4        12X,32HREQUIRED LENGTH OF W ARRAY = 560//

     5        10X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     6        32X,8HIERROR =,I2/18X,22HDISCRETIZATION
ERROR =,E12.5/

     7        12X,28HREQUIRED LENGTH OF W ARRAY =,F4.0)

C

      END
```

TSEPELI

```
C

C file tsepeli.f

C

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C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL
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C     *                             BY
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C     *        JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
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C

C

C     AN EXAMPLE SHOWING THE USE OF SEPELI TO SOLVE THE
SEPARABLE

C     ELLIPTIC PARTIAL DIFFERENTIAL EQUATION . . .

C     (X+1)**2*UXX+2*(X+1)*UX+EXP(Y)*UYY-(X+Y)*U =
G(X,Y) ON

C     0.LE.X.LE.1, 0.LE.Y.LE.1  WITH SPECIFIED BOUNDARY
CONDITIONS

C     AT Y=0,1 AND MIXED BOUNDARY CONDITIONS OF THE FORM
```

```
C      UX(0,Y)+U(0,Y), UX(1,Y)+U(1,Y) AT X=0,1.

C      THE APPROXIMATION IS GENERATED ON A UNIFORM 33 BY
33 GRID.

C      THE EXACT SOLUTION U(X,Y)=(X*Y)**3+1 IS USED TO
SET THE

C      RIGHT HAND SIDE, BOUNDARY CONDITIONS, AND COMPUTE
SECOND AND

C      FOURTH ORDER DISCRETIZATION ERROR

C      THE EXACT WORK SPACE LENGTH REQUIRED IS 1118
WORDS.

C      THIS WAS DETERMINED BY A PREVIOUS CALL TO SEPELI
AND PRINT

C      OUT OF W(1).

C

       DIMENSION       USOL(33,33),GRHS(33,33),BDA(33)
,BDB(33)    ,

     1                 W(1118)

C

C      DECLARE COEFFICIENT SUBROUTINES EXTERNAL

C

       EXTERNAL        COFX       ,COFY

C

C      DEFINE ARITHMETIC FUNCTIONS GIVING EXACT SOLUTION

C

       UE(S,T) = (S*T)**3+1.0

       UXE(S,T) = 3.0*S**2*T**3

       UXXE(S,T) = 6.0*S*T**3
```

```
      UYE(S,T) = 3.0*S**3*T**2
      UYYE(S,T) = 6.0*S**3*T
C
C     SET LIMITS ON REGION
C
      A = 0.0
      B = 1.0
      C = 0.0
      D = 1.0
C
C     SET GRID SIZE
C
      M = 32
      N = 32
      DLX = (B-A)/FLOAT(M)
      DLY = (D-C)/FLOAT(N)
      NX = M+1
      NY = N+1
      DO 102 I=1,NX
         X = A+FLOAT(I-1)*DLX
C
C     SET SPECIFIED BOUNDARY CONDITIONS AT Y=C,D
C
         USOL(I,1) = UE(X,C)
```

```
          USOL(I,NY) = UE(X,D)

          CALL COFX (X,AF,BF,CF)

          DO 101 J=1,NY

             Y = C+FLOAT(J-1)*DLY

             CALL COFY (Y,DF,EF,FF)

C

C     SET RIGHT HAND SIDE

C

             GRHS(I,J) =
AF*UXXE(X,Y)+BF*UXE(X,Y)+CF*UE(X,Y)+

     1
DF*UYYE(X,Y)+EF*UYE(X,Y)+FF*UE(X,Y)

  101    CONTINUE

  102 CONTINUE

C

C     SET MIXED BOUNDARY CONDITIONS AT X=A,B

C

      ALPHA = 1.0

      BETA = 1.0

      DO 103 J=1,NY

         Y = C+FLOAT(J-1)*DLY

         BDA(J) = UXE(A,Y)+ALPHA*UE(A,Y)

         BDB(J) = UXE(B,Y)+BETA*UE(B,Y)

  103 CONTINUE

C
```

```
C     SET BOUNDARY SWITHCES

C

      MBDCND = 3

      NBDCND = 1

C

C     SET FIRST DIMENSION OF USOL,GRHS AND WORK SPACE
LENGTH

C

      IDMN = 33

      W(1) = 1118.

C

C     SET WORK SPACE LENGTH IN FIRST WORD

C     SET INITIAL CALL PARAMETER TO ZERO

C

      INTL = 0

C

C     OBTAIN SECOND ORDER APPROXIMATION

C

      IORDER = 2

      CALL SEPELI
(INTL,IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,D,N,

     1
NBDCND,DUM,DUM,DUM,DUM,COFX,COFY,GRHS,USOL,IDMN,W,

     2             PERTRB,IERROR)

      ERR = 0.0

      DO 105 I=1,NX
```

```
          X = A+FLOAT(I-1)*DLX

          DO 104 J=1,NY

             Y = C+FLOAT(J-1)*DLY

             ERR = AMAX1(ERR,ABS((USOL(I,J)-
UE(X,Y))/UE(X,Y)))

  104    CONTINUE

  105 CONTINUE

      ERR2 = ERR

C

C     OBTAIN FOURTH ORDER APPROXIMATION

C

      IORDER = 4

C

C     NON-INITIAL CALL

C

      INTL = 1

      CALL SEPELI
(INTL,IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,D,N,

     1
NBDCND,DUM,DUM,DUM,DUM,COFX,COFY,GRHS,USOL,IDMN,W,

     2             PERTRB,IERROR)

C

C     COMPUTE DISCRETIZATION ERROR

C

      ERR = 0.0

      DO 107 J=1,NY
```

```
         Y = C+FLOAT(J-1)*DLY

         DO 106 I=1,NX

            X = A+FLOAT(I-1)*DLX

            ERR = AMAX1(ERR,ABS((USOL(I,J)-
UE(X,Y))/UE(X,Y)))

  106    CONTINUE

  107 CONTINUE

      ERR4 = ERR

      IW = INT(W(1))

      PRINT 1001 , IERROR,ERR2,ERR4,IW

C

C

 1001 FORMAT (1H1,20X,25HSUBROUTINE SEPELI EXAMPLE///

     1         20X,46HTHE OUTPUT FROM THE NCAR CONTROL
DATA 7600 WAS//

     2         20X,10HIERROR = 0/

     3         20X,47HSECOND ORDER DISCRETIZATION ERROR =
9.78910E-05/

     4         20X,47HFOURTH ORDER DISCRETIZATION ERROR =
1.47351E-06/

     5         20X,33HREQUIRED LENGTH OF W ARRAY = 1118//

     6         20X,32HTHE OUTPUT FROM YOUR COMPUTER IS//

     7         20X,8HIERROR =,I2/

     8         20X,36HSECOND ORDER DISCRETIZATION ERROR =
, E12.5/

     9         20X,36HFOURTH ORDER DISCRETIZATION ERROR =
, E12.5/
```

```
     +        20X,29HREQUIRED LENGTH OF W ARRAY = , I4)
C
      END
      SUBROUTINE COFX (X,AF,BF,CF)
C
C     SET COEFFICIENTS IN THE X-DIRECTION.
C
      AF = (X+1.)**2
      BF = 2.0*(X+1.)
      CF = -X
      RETURN
      END
      SUBROUTINE COFY (Y,DF,EF,FF)
C
C     SET COEFFICIENTS IN Y DIRECTION
C
      DF = EXP(Y)
      EF = 0.0
      FF = -Y
      RETURN
      END
```

TSEPX4

```
C
C file tsepx4.f
C
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *                                                               *
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C     *     A PACKAGE OF FORTRAN SUBPROGRAMS FOR THE SOLUTION OF      *
C     *                                                               *
C     *      SEPARABLE ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS        *
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C     *                              BY
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C     *         JOHN ADAMS, PAUL SWARZTRAUBER AND ROLAND
SWEET          *

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C     EXAMPLE SHOWING THE USE OF SEPX4 TO SOLVE THE
ELLIPTIC PDE

C     (X+1)**2*UXX+2*(X+1)*UX+UYY-X*U=G(X,Y) ON THE
REGION

C     0.LE.X.LE.1, 0.LE.Y.LE.1  WITH SPECIFIED BOUNDARY
CONDITIONS

C     AT Y=0,1 AND MIXED BOUNDARY CONDITIONS OF THE FORM

C     UX(0,Y)+U(0,Y), UX(1,Y)+U(1,Y) AT X=0,1.

C     THE APPROXIMATION IS GENERATED ON A UNIFORM 33 BY
33 GRID.

C     THE EXACT SOLUTION U(X,Y)=(X*Y)**3+1 IS USED TO
SET THE

C     RIGHT HAND SIDE, BOUNDARY CONDITIONS, AND COMPUTE
SECOND AND

C     FOURTH ORDER DISCRETIZATION ERROR

C     THE EXACT WORK SPACE LENGTH REQUIRED IS 1005
WORDS.  THIS

C     WAS DETERMINED BY A PREVIOUS CALL TO SEPX4 AND
PRINT OUT OF

C     W(1).

C

      DIMENSION       USOL(33,33),GRHS(33,33),BDA(33)
,BDB(33)    ,

     1                W(1024)
```

```
      EXTERNAL COFX4
C
C     DEFINE ARITHMETIC FUNCTIONS GIVING EXACT SOLUTION
C
      UE(S,T)=(S*T)**3+1.0
      UXE(S,T)=3.0*S**2*T**3
      UXXE(S,T)=6.0*S*T**3
      UYE(S,T)=3.0*S**3*T**2
      UYYE(S,T)=6.0*S**3*T
C
C     SET LIMITS ON REGION
C
      A = 0.0
      B = 1.0
      C = 0.0
      D = 1.0
C
C     SET GRID SIZE
C
      M = 32
      N = 32
      DLX = (B-A)/FLOAT(M)
      DLY = (D-C)/FLOAT(N)
      NX = M+1
```

```
      NY = N+1
          DO 102 I=1,NX
          X = A+FLOAT(I-1)*DLX
C
C     SET SPECIFIED BOUNDARY CONDITIONS AT Y=C,D
C
          USOL(I,1) = UE(X,C)
          USOL(I,NY) = UE(X,D)
          CALL COFX4 (X,AF,BF,CF)
              DO 101 J=1,NY
              Y = C+FLOAT(J-1)*DLY
C
C     SET RIGHT HAND SIDE
C
              GRHS(I,J) =
AF*UXXE(X,Y)+BF*UXE(X,Y)+CF*UE(X,Y)+UYYE(X,Y)
  101         CONTINUE
  102     CONTINUE
C
C     SET MIXED BOUNDARY CONDITIONS AT X=A,B
C
      ALPHA = 1.0
      BETA = 1.0
          DO 103 J=1,NY
          Y = C+FLOAT(J-1)*DLY
```

```
          BDA(J) = UXE(A,Y)+ALPHA*UE(A,Y)

          BDB(J) = UXE(B,Y)+BETA*UE(B,Y)

  103     CONTINUE

C

C     SET BOUNDARY SWITHCES

C

      MBDCND = 3

      NBDCND = 1

C

C     SET FIRST DIMENSION OF USOL,GRHS AND WORK SPACE
LENGTH

C

      IDMN = 33

      W(1) = 1024.

C

C     OBTAIN SECOND ORDER APPROXIMATION

C

      IORDER = 2

      CALL SEPX4
(IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,D,N,NBDCND,

     1
DUM,DUM,COFX4,GRHS,USOL,IDMN,W,PERTRB,IERROR)

C

C     COMPUTE SECOND ORDER DISCRETIZATION ERROR
(RELATIVE)
```

```
C     ALSO RESET SPECIFIED BOUNDARIES AND RIGHT HAND
SIDE.

C

      ERR = 0.0

          DO 105 I=1,NX

          X = A+FLOAT(I-1)*DLX

          USOL(I,1) = UE(X,C)

          USOL(I,NY) = UE(X,D)

          CALL COFX4 (X,AF,BF,CF)

              DO 104 J=1,NY

              Y = C+FLOAT(J-1)*DLY

              ERR = AMAX1(ERR,ABS((USOL(I,J)-
UE(X,Y))/UE(X,Y)))

              GRHS(I,J) =
AF*UXXE(X,Y)+BF*UXE(X,Y)+CF*UE(X,Y)+UYYE(X,Y)

  104         CONTINUE

  105     CONTINUE

      ERR2=ERR

C

C     OBTAIN FOURTH ORDER APPROXIMATION

C

      IORDER = 4

      CALL SEPX4
(IORDER,A,B,M,MBDCND,BDA,ALPHA,BDB,BETA,C,D,N,NBDCND,

     1
DUM,DUM,COFX4,GRHS,USOL,IDMN,W,PERTRB,IERROR)
```

```
C

C     COMPUTE FOURTH ORDER DISCRETIZATION ERROR
(RELATIVE)

C

      ERR = 0.0

          DO 107 J=1,NY

          Y = C+FLOAT(J-1)*DLY

              DO 106 I=1,NX

              X = A+FLOAT(I-1)*DLX

              ERR = AMAX1(ERR,ABS((USOL(I,J)-
UE(X,Y))/UE(X,Y)))

  106         CONTINUE

  107     CONTINUE

      ERR4=ERR

      IW = INT(W(1)+0.5)

      PRINT 1001,IERROR,ERR2,ERR4,IW

 1001 FORMAT(1H1,20X,25HSUBROUTINE SEPX4  EXAMPLE ///

     120X,46HTHE OUTPUT FROM THE NCAR CONTROL DATA 7600
WAS //

     220X,10HIERROR = 0 /

     320X,48HSECOND ORDER DISCRETIZATION ERROR =
1.5985E-04  /

     420X,48HFOURTH ORDER DISCRETIZATION ERROR =
1.85749E-06  /

     520X,33HREQUIRED LENGTH OF W ARRAY = 1024 //

     620X, 32HTHE OUTPUT FROM YOUR COMPUTER IS //
```

```
     720X, 8HIERROR = I2 /
     820X,36HSECOND ORDER DISCRETIZATION ERROR = E12.5 /
     920X,36HFOURTH ORDER DISCRETIZATION ERROR = E12.5 /
     920X,29HREQUIRED LENGTH OF W ARRAY = I5)
C
      END
      SUBROUTINE COFX4(X,AF,BF,CF)
C
C     SET COEFFICIENTS IN THE X-DIRECTION.
C
      AF = (X+1.)**2
      BF = 2.0*(X+1.)
      CF = -X
      RETURN
      END
```