# On the existence and linear approximation of the power flow solution in power distribution networks – Numerical analysis

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August 4, 2014

**Abstract**

This technical note reports an extensive numerical analysis of the approximate power flow method proposed in the paper by S. Bolognani and S. Zampieri, *"On the existence and linear approximation of the power flow solution in power distribution networks"* [1]. Simulations are based on a modified version of the IEEE 123 test feeder. Different scenarios have been considered, in order to assess the performance of the approximate method for different practical cases. The technical details of the simulations, together with the simulation results and their interpretation, are included in this document. The GNU Octave [2] and Matpower [3] code used in the simulations is available online [4].

## 1 Introduction

The testbed used in these simulations has been obtained as a modification of the IEEE 123 Test Feeder [5] (available online [6, 4]). The following modifications have been introduced.

- Only the three-phase overhead lines and underground cables are considered. All loads are therefore located, as spot loads, at the point where their single phase feeder departs from the three phase feeder.
- Lines are assumed symmetric.
- Loads are assumed balanced and are modeled as PQ buses.
- Shunt capacitors are assumed balanced.
- Switches are in their normal position.
- Voltage regulators are modeled as ideal transformers with variable tap position.
- The tap changer at the substation is modeled as a slack node with variable voltage reference.
- A microgenerator is connected at bus 15, and can be operated as a PV bus.

The modified testbed is available online as a MatPower case file [4].

## 2 Grid topology and power line impedance

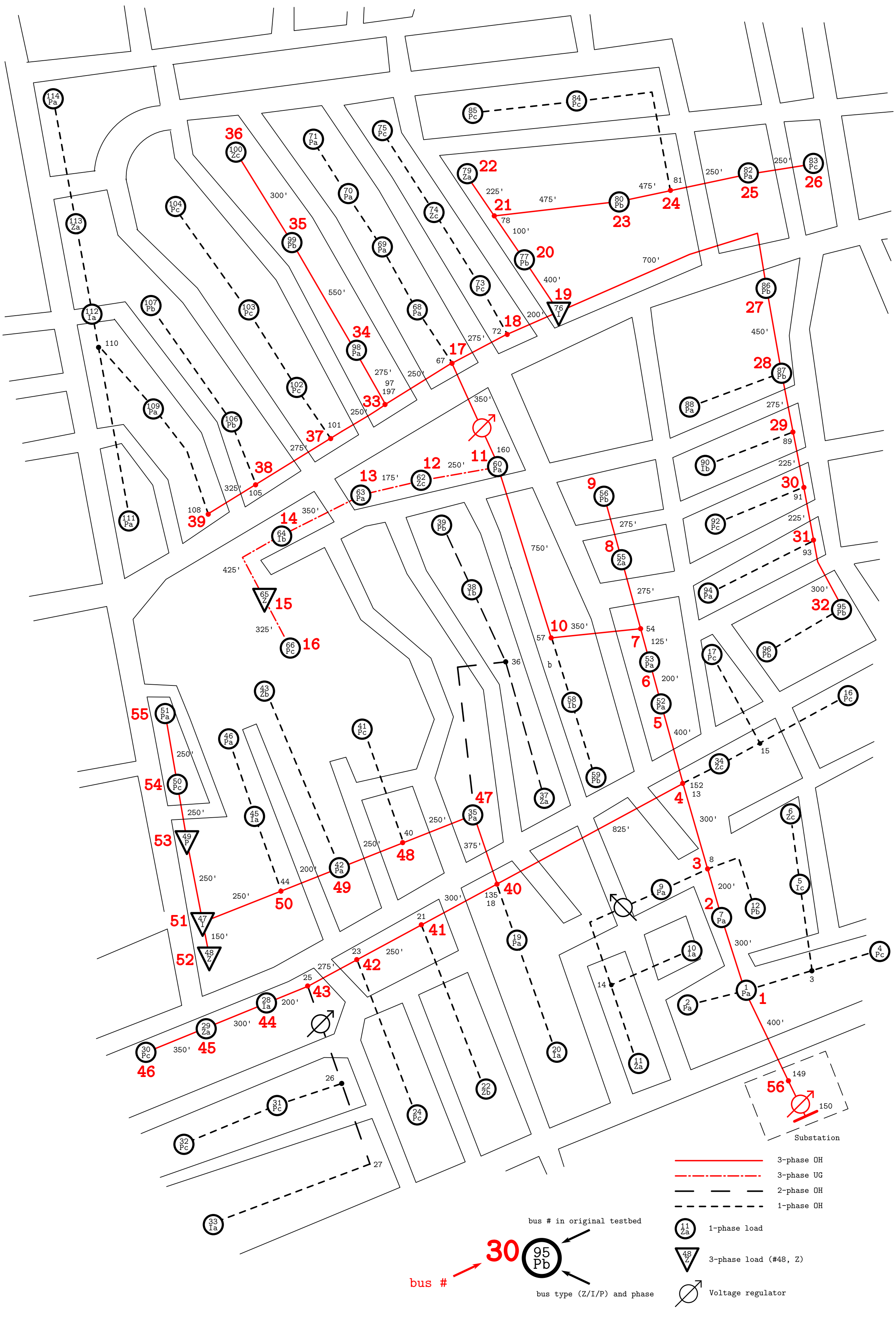
The topology of the modified testbed is represented in the attached figure, where the original bus numbers from the IEEE 123 test feeder are also reported. The network is radial, contains 56 nodes, and its segments are either overhead lines (OH) or underground cables (UG), with the following impedances. Distances (in yards) are reported in the attached figure, and corresponds to the distances in the original testbed.

Three-phase impedance $Z_{\mathrm{OH}}^{3\phi}$ in $\Omega$/mile:

$$\begin{bmatrix} 0.46190 + 1.0638j & 0.15583 + 0.43673j & 0.15583 + 0.43673j \\ & 0.46190 + 1.0638j & 0.15583 + 0.43673j \\ & & 0.46190 + 1.0638j \end{bmatrix}$$

Positive sequence impedance:

$$Z_{\mathrm{OH}} = 0.30607 + 0.62707j \ [\Omega/\text{mile}]$$


Substation
3-phase OH
3-phase UG
2-phase OH
1-phase OH
1-phase load
3-phase load (#48, Z)
Voltage regulator
bus # in original testbed
bus type (Z/I/P) and phase
bus #
30
95
Pb

Three-phase shunt admittance $Y_{\text{OH}}^{3\phi}$ in $\mu$ S/mile:

$$\begin{bmatrix} 5.6848j & -1.2315j & -1.2315j \\ & 5.6848j & -1.2315j \\ & & 5.6848j \end{bmatrix}$$

Positive sequence shunt admittance:

$$Y_{\text{OH}} = 6.9163j \; [\mu\text{S/mile}]$$

Three-phase impedance $Z_{\text{UG}}^{3\phi}$ in $\Omega$/mile:

$$\begin{bmatrix} 1.5249 + 0.74013j & 0.51067 + 0.25690j & 0.51067 + 0.25690j \\ & 1.5249 + 0.74013j & 0.51067 + 0.25690j \\ & & 1.5249 + 0.74013j \end{bmatrix}$$

Positive sequence impedance:

$$Z_{\text{UG}} = 1.01423 + 0.48323j \; [\Omega/\text{mile}]$$

Three-phase shunt admittance $Y_{\text{UG}}^{3\phi}$ in $\mu$ S/mile:

$$\begin{bmatrix} 67.2242j & & \\ & 67.2242j & \\ & & 67.2242j \end{bmatrix}$$

Positive sequence shunt admittance:

$$Y_{\text{UG}} = 67.2242j \; [\mu\text{S/mile}]$$

Simulations showed that, in this testbed, the shunt admittance of both the overhead lines and of the underground cables, is negligible. As discussed in [1], the model can be seamlessly extended to the case of non-zero shunt admittances (see 3.4 for a practical implementation).

# 3 Simulations

The following scenarios have been considered.

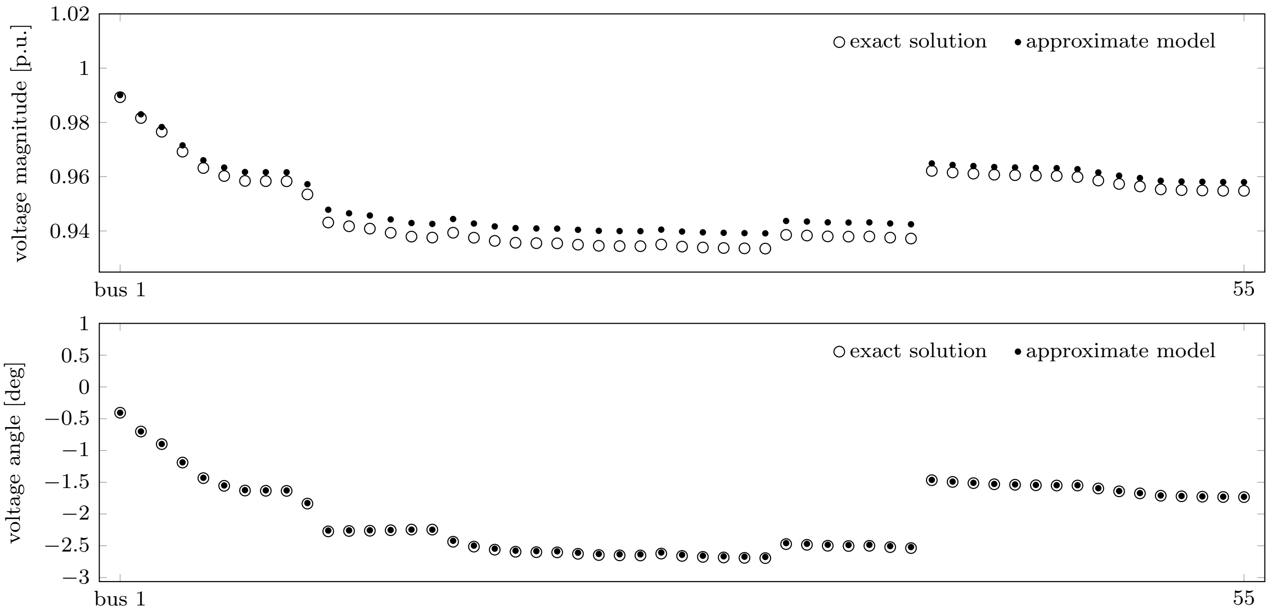
1. Nominal scenario – the nominal scenario with no shunt capacitor, nominal tap position at the voltage regulator, nominal tap position at the PCC. All the following scenarios depart from the nominal scenario as detailed hereafter.
2. Uniform overload – the load of all the PQ buses is increased proportionally.
3. Lumped overload – the load of one bus is increased.
4. Shunt capacitor – shunt capacitors are connected for static voltage regulation.
5. Voltage regulation – the tap position of the voltage regulator is changed.
6. Tap changer – the tap position at the PCC is changed.
7. PV bus – a voltage-regulated generator is connected to bus 15.

In the following, each one of these scenarios has been detailed, and the approximate power flow solution proposed in [1] has been plotted in red, together with the solution returned by the MatPower nonlinear power flow solver. The nominal scenario is always plotted, as a reference, in black.

## 3.1 Nominal scenario

The nominal scenario corresponds to the testbed described in Section 1, and corresponds to the numerical experiment reported in the original paper [1]. As illustrated in the following figure, the approximation error is uniformly small across the grid.

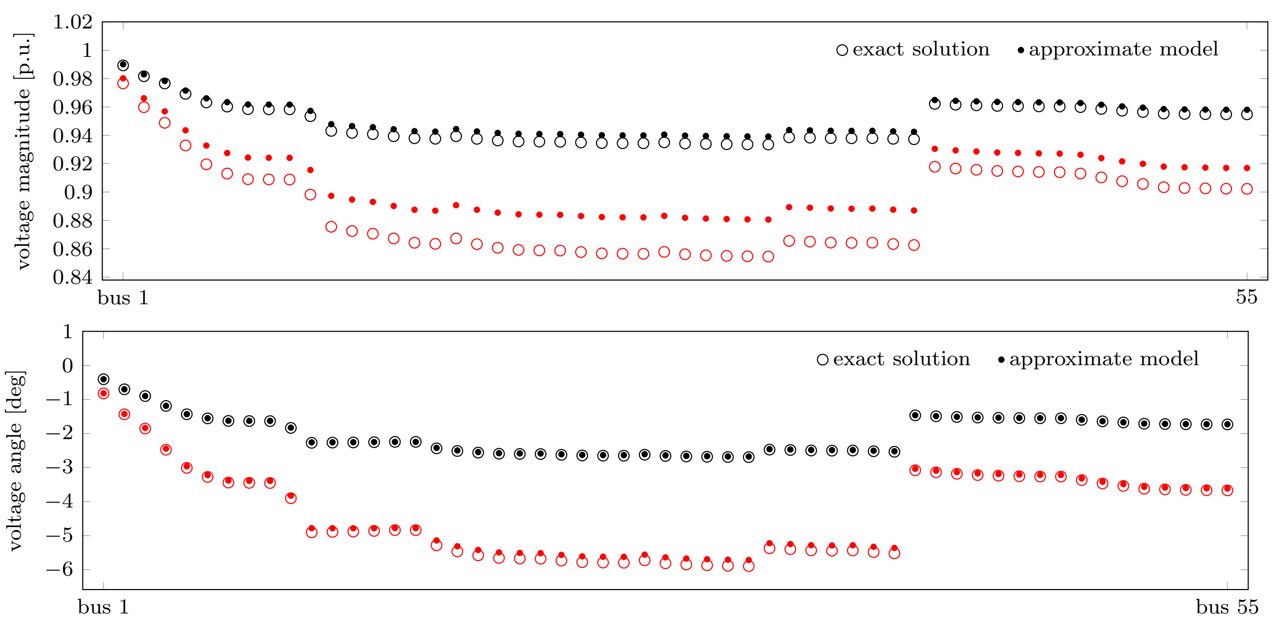
| | Nominal scenario |
|---|---|
| Maximum voltage drop | 6.55% |
| Maximum approximation error | 0.56% |




### 3.2 Uniform overload

In this scenario, the power demand of all buses has been doubled, in order to represent a case of relative overload of the power distribution grid. As illustrated in the following figure, for larger voltage drops (red), the approximation error increases compared to the nominal case (black), especially in the voltage magnitude.

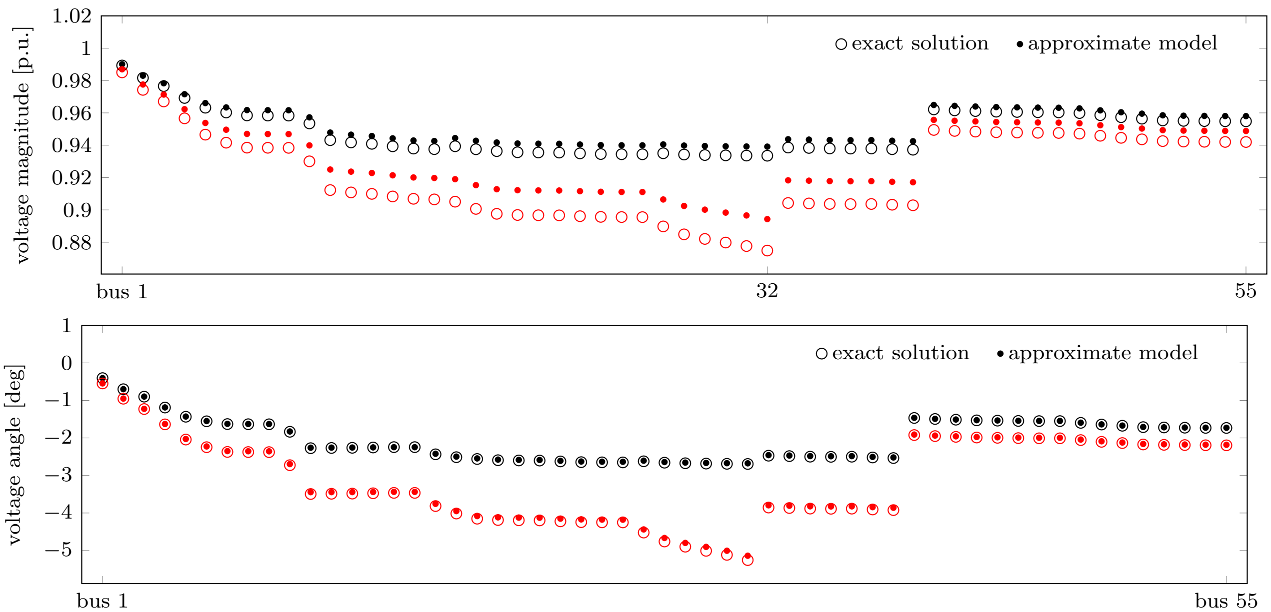
| | Uniform overload | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 14.55% | 6.55% |
| Maximum approximation error | 2.61% | 0.56% |




### 3.3 Lumped overload

In this scenario, the power demand of bus 32 has been increased to 1.2 MW and 0.6 MVAR. This lumped overload of the grid produces a localized drop in the voltage profile, which is also present in the approximate power flow solution. Also in this case, the overload condition slightly deteriorates the quality of the approximation.

| | Lumped overload | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 12.51% | 6.55% |
| Maximum approximation error | 1.95% | 0.56% |

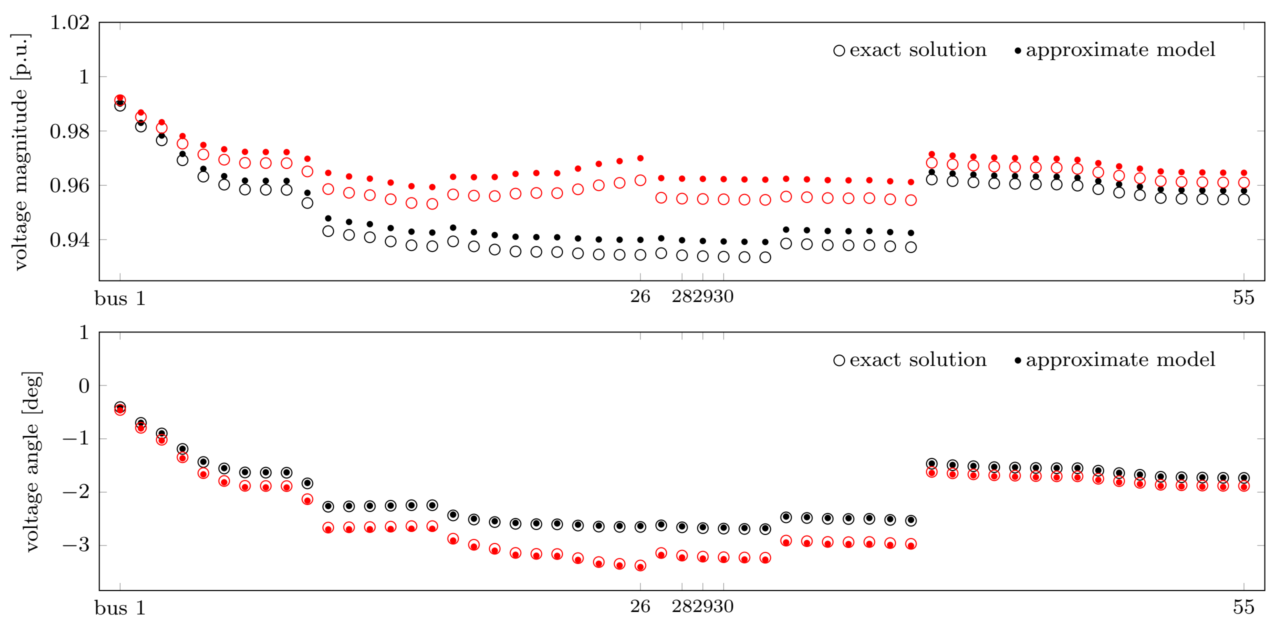



## 3.4 Shunt capacitor

In this scenario, shunt capacitors have been connected to buses 26 (600 KVAR), 28 (50 KVAR), 29 (50 KVAR), 30 (50 KVAR), according to what is suggested in the original testbed, in order to support the voltage.

As explained in the original paper [1], shunt admittances can be incorporated in the bus admittance matrix, and the resulting approximate power flow solution remains linear.

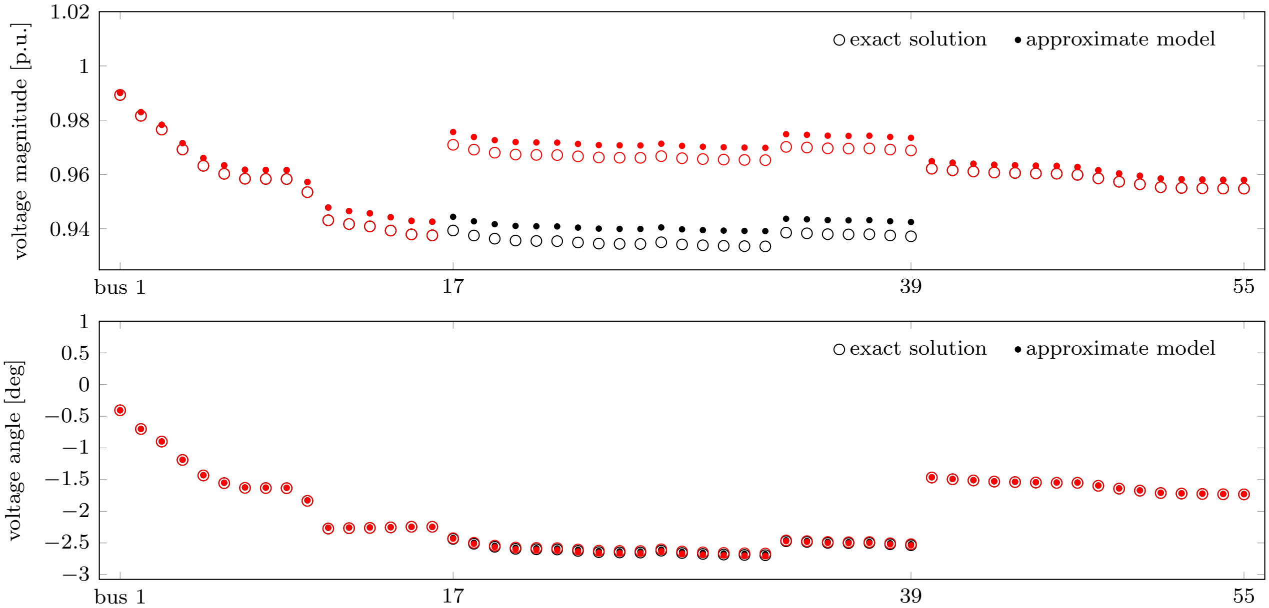
| | Shunt capacitor | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 4.69% | 6.55% |
| Maximum approximation error | 0.81% | 0.56% |



### 3.5 Voltage regulation

In this scenario, the voltage regulator that connects bus 11 with bus 17 is operated at a tap different from the nominal tap, obtaining a voltage ratio of 124/120. Because the voltage regulator is modeled as an ideal transformer, this device can be included in the model by proper scaling of the portion of the bus admittance matrix that involves the buses fed by the voltage regulator (buses 17 to 39).

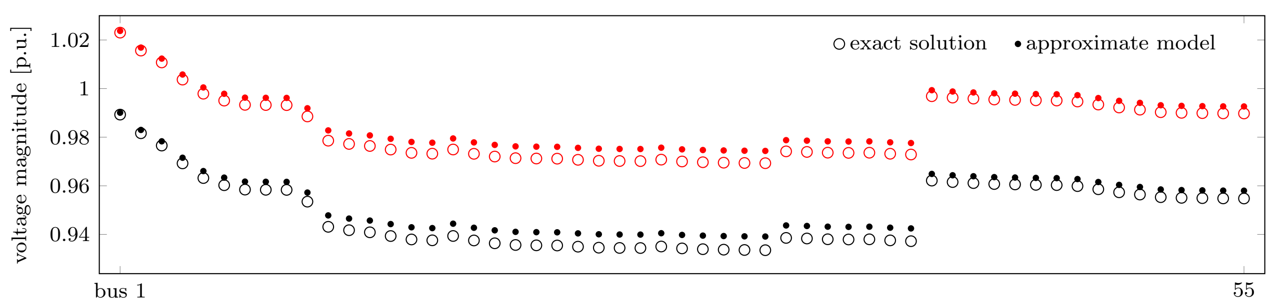
| | Voltage regulation | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 6.24% | 6.55% |
| Maximum approximation error | 0.5% | 0.56% |

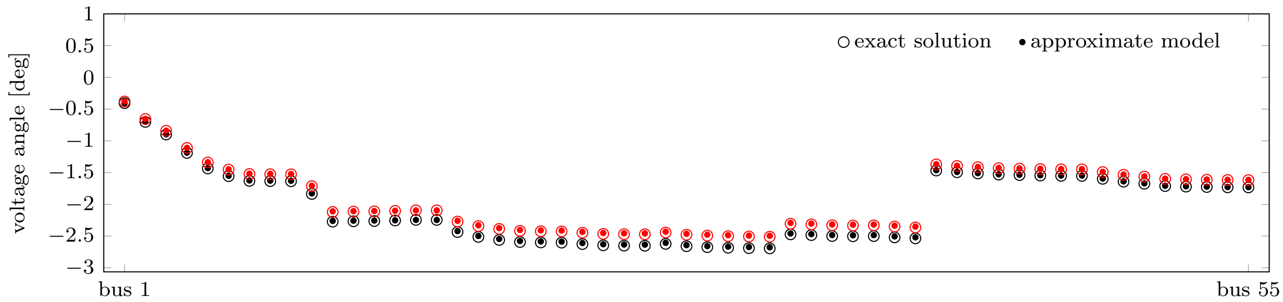



### 3.6 Tap changer

In this scenario, the tap changer at the PCC of the test feeder is operated at a tap different from the nominal tap, obtaining a secondary voltage of 124/120.

| | Tap changer | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 6.40% | 6.55% |
| Maximum approximation error | 0.51% | 0.56% |

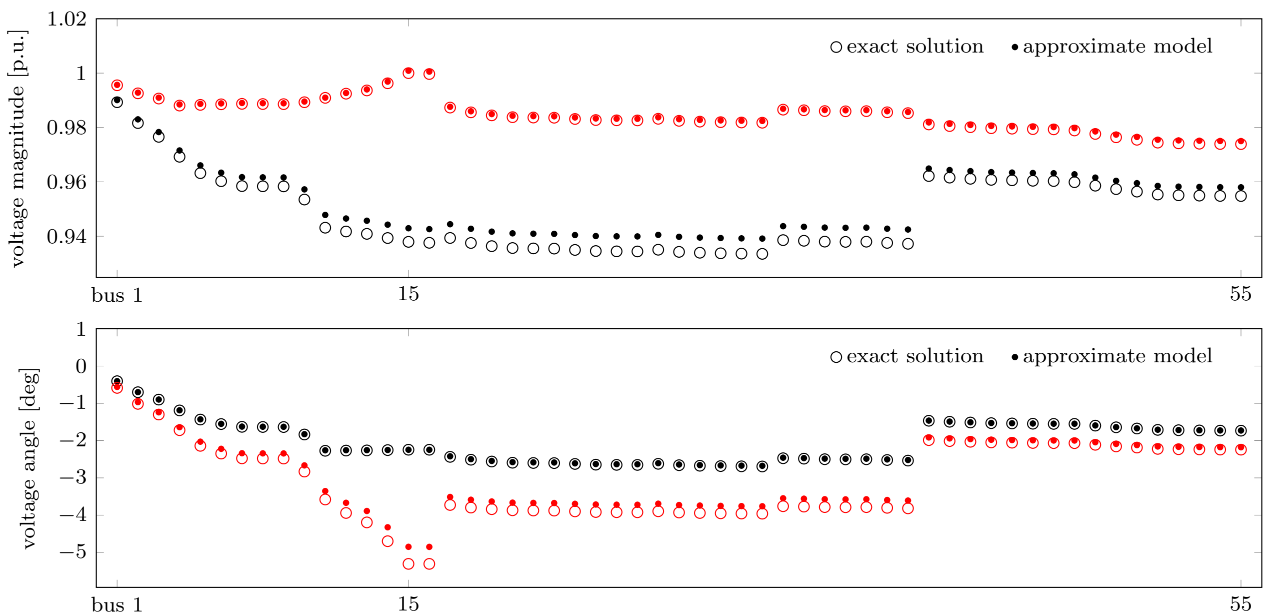





### 3.7 PV bus

In this scenario, the generator connected to bus 15 is voltage regulated, with a voltage reference of 1 p.u. As described in the original paper [1], the approximate model can be properly modified, so that the resulting model is linear in the power demands of the PQ buses, in the active power injection of the PV bus, and in the voltage reference of the PV bus.

| | Tap changer | Nominal scenario |
|---|---|---|
| Maximum voltage drop | 2.61% | 6.55% |
| Maximum approximation error | 0.11% | 0.56% |




# References


[1] S. Bolognani and S. Zampieri, "On the existence and linear approximation of the power flow solution in power distribution networks," *IEEE Transactions on Power Systems*, submitted.

[2] Octave community, "Gnu octave 3.8.1," 2014. [Online]. Available: www.gnu.org/software/octave/

[3] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "Matpower: Steady-state operations, planning and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, Feb. 2011.

[4] S. Bolognani, "Approximate linear solution of power flow equations in power distribution networks (source code)," GitHub, 2014. [Online]. Available: http://github.com/saveriob/approx-pf

[5] W. H. Kersting, "Radial distribution test feeders," in *IEEE Power Engineering Society Winter Meeting*, vol. 2, Jan. 2001, pp. 908–912.

[6] IEEE Power & Energy Society, “Distribution test feeders.” [Online]. Available: http://ewh.ieee.org/soc/pes/dsacom/testfeeders.html