Matrix

Def^n: An arrangement of "mn" elements in m-rows and n-columns is called Matrix of order $m \times n$.

Square matrix: A matrix with equal nos. of rows & columns is called Square matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ e & f & g \\ h & i & g \end{bmatrix}_{3\times 3}$$

Transpose of Matrix:

Transpose of matrix A is denoted by A' (A^t or A^T) and is defined as

$$A' = (a_{ji})_{n\times m} \text{ where } A = (a_{ij})_{m\times n}$$

Symmetric Matrix:

A square matrix is said to be symmetric if $A = A'$

eg $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{bmatrix}$, $A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ $\therefore A = A'$

Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric if $A = -A'$

eg $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$, $A' = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$

$\therefore A = -A'$

$\therefore A$ is skew-symmetric matrix.

we have $\boxed{a_{ii} = 0 \ \forall i}$

Thm:

"Every square matrix can be uniquely expressed as a sum of a symmetric matrix and a skew-symmetric matrix"

Proof: Let A be a square matrix with transpose A'.

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\therefore A = P + Q \quad \text{— (1)}$$

Where $P = \frac{1}{2}(A + A')$ & $Q = \frac{1}{2}(A - A')$

We have

$P' = \left[\frac{1}{2}(A + A')\right]'$

$= \frac{1}{2}\left[(A') + (A')'\right] \quad \because (A')' = A$

$= \frac{1}{2}(A + A') = P$

$\therefore P' = P$

$\therefore P$ is symmetric matrix.

Now consider

$Q' = \left[\frac{1}{2}(A - A')\right]'$

$= \frac{1}{2}\left[(A') - (A')'\right] \quad \because (A')' = A$

$= \frac{1}{2}\left[A' - A\right] = -\frac{1}{2}(A - A') = -Q$

$Q' = -Q$

$\therefore Q$ is Skew-symmetric Matrix.

Let R & S be the square matrices

Such that $A = R + S$ with $R' = R$, $S' = -S$

$\therefore A' = R' + S' = R - S$

we have $P = \frac{1}{2}(A + A') = \frac{1}{2}\left[(R+S) + (R-S)\right]$

$= \frac{1}{2}\left[R + S + R - S\right] = \frac{1}{2}(2R) = R$

$\boxed{\therefore P = R}$

Consider $Q = \frac{1}{2}(A - A') = \frac{1}{2}\left[(R+S) - (R-S)\right] = \frac{1}{2}\left[R + S - R + S\right]$

$= \frac{1}{2}(2S) = S \quad \boxed{\therefore Q = S}$

$\therefore P = \frac{1}{2}(A + A')$ & $Q = \frac{1}{2}(A - A')$ are unique.

Homework

- Revise all basic def^{ns} related to matrices
  - Minor
  - Cofactor
  - Adjoint
  - Singular/non-singular matrices
  - Reciprocal/Inverse matrix
  - Scalar matrix/Diagonal matrix
    Identity (Unity) matrix
  - Upper triangular matrix/Lower triangular matrix.