Question Bank No.1 2413FEB1T1 : Engineering Mathematics-1
Topic: Matrices

1 Symmetric and Skew-Symmetric matrices

Q.Express the following matrices as sum of a symmetric and a Skew-Symmetric matrices.

1.
$$A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

2.
$$A = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix}$$

3.
$$A = \begin{bmatrix} 1 & 0 & 5 & 3 \\ -2 & 1 & 6 & 1 \\ 3 & 2 & 7 & 1 \\ 4 & -4 & 2 & 0 \end{bmatrix}$$

2 Orthogonal Matrices

Q. Verify that the following matrix is orthogonal and hence find its inverse.

1.
$$A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$$

2.
$$B = \begin{bmatrix} sin\theta & -cos\theta & 0 \\ cos\theta & sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Find a,b,c if the following matrix A is an orthogonal matrix.

$$A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$$

3 Hermitian and Skew-Hermitian matrices

Q.Express the following matrices as sum of a Hermitian and a Skew-Hermitian matrices.

1.
$$A = \begin{bmatrix} 1 & 1+i & 2+3i \\ 1-i & 2 & -i \\ 2-3i & i & 0 \end{bmatrix}$$

2.
$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

3. Prove that if A is Hermitian (skew Hermitian) then iA is skew Hermitian.

4 Unitary Matrices

1. Prove that A is an unitary matrix and hence find the inverse of A.
$$A = \frac{1}{2}\begin{bmatrix} i & \sqrt{3} \\ \sqrt{3} & i \end{bmatrix}$$

2.
$$A = \frac{1}{3}\begin{bmatrix} 2+i & 2i \\ 2i & 2-i \end{bmatrix}$$

3. Show that the matrix A is unitary, if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$.
$$A = \begin{bmatrix} \alpha + i\gamma & \beta + i\delta \\ \beta + i\delta & \alpha + i\gamma \end{bmatrix}$$

5 Rank of a matrix

Q.Find the Rank of the following matrix by reducing to echelon form.

1.
$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$$

2.

$$B = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & 7 \end{bmatrix}$$

3.

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

Q.Find the Rank of the following matrix by reducing to normal form.

1.

$$A = \begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

2.

$$B = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

3.

$$C = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 3 & 0 \\ 9 & 8 & 0 & 8 \end{bmatrix}$$

Q.Find the Non-Singular Matrices P and Q such that PAQ is in Normal Form and hence, find ρ(A) for the following Matrices.

1.

$$A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$$

2.

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

3.

$$C = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

6 Non-Homogeneous System of Equations

Q.Test the consistency of the system and solve if possible.

1. 2x-y+z=8, 3x-y+z=6, 4x-y+2z=7, -x+y-z=4
2. x+y+z=6, x-y+2z=5, 3x+y+z=8, 2x-2y+3z=7
3. 4x-2y+6z=8, x+2y-3z=-1, 15x-3y+9z=21
4. Investigate the values of α and β such that the equations $x+y+z=6, x+2y+4z=9, x+2y+\alpha z=\beta$, have
   (a) no solution,
   (b) a unique solution,
   (c) infinitely many solutions.
5. Discuss the values of α and β for which following system of equations $x+2y+3z=0, x+3y+5z=9, 2x+5y+\alpha z=\beta$, have
   (a) no solution,
   (b) a unique solution,
   (c) infinitely many solutions.

7 Homogeneous System of Equations

Q.Solve the following system of equations.

1. $3x_1+4x_2-x_3-9x_4=0, 2x_1+3x_2+2x_3-3x_4=0, 2x_1+x_2-14x_3-12x_4=0, x_1+3x_2+13x_3+3x_4=0.$
2. 2x-2y-5z=0, 4x-y+z=0, 3x-2y+3z=0, x-3y+7z=0
3. x+y+z=0, 2x+2y+3z=0, 3x+3y+3z=0
4. 2x+y+z=0, x-y+2z=0, 3x+2y+4z=0
5. Determine the value of k for which the following system of equations $2x-2y+z=kx, 2x-3y+2z=ky, -x+2y=kz$, has solutions and find all solutions.

8 Application to Coding Theory

1. Encode 'SECRET_CODE' with the key matrix

$$A=\begin{bmatrix}1 & 1\\ 2 & 6\end{bmatrix}$$

2. Encode 'WAR IS ON' with the key matrix

$$A=\begin{bmatrix}3 & 3\\ 4 & 5\end{bmatrix}$$

3. Encode 'GOD EXISTS' with the key matrix

$$A = \begin{bmatrix} 1 & 0 \\ 4 & 13 \end{bmatrix}$$

4. Encode 'ALL IS WELL' with the key matrix

$$A = \begin{bmatrix} 1 & 1 \\ O & 1 \end{bmatrix}$$

5. Encode 'TIT FOR TAT' with the key matrix

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$$

6. Decode the message 4, 15, 0, 15, 18, 0, 4, 9, 5 encoded with the key matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$