Basic Electrical Engineering

THIRD EDITION

About the Author

Ravish R Singh is presently Director at Thakur Ramnarayan College of Arts and Commerce, Mumbai. He obtained a BE degree from University of Mumbai in 1991, an MTech degree from IIT Bombay in 2001, and a PhD degree from Faculty of Technology, University of Mumbai, in 2013. He has published several books with McGraw Hill Education (India) on varied subjects like *Engineering Mathematics, Applied Mathematics, Electrical Networks, Network Analysis and Synthesis, Electrical Engineering, Basic Electrical and Electronics Engineering*, etc., for all-India curricula as well as regional curricula of some universities like Gujarat Technological University, Mumbai University, Pune University, Jawaharlal Nehru Technological University, Anna University, Uttarakhand Technical University, and Dr A P J Abdul Kalam Technical University. Dr Singh is a member of IEEE, ISTE, and IETE, and has published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems, and Engineering Mathematics.

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Third Edition

Ravish R Singh

Director
Thakur Ramnarayan College of Arts and Commerce
Mumbai, Maharashtra

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Dedicated
to
My son, *Aman*
and
daughter, *Aditri*

Contents

Preface

Basic Electrical Engineering is a core subject which is studied by first year engineering students of all branches. The syllabus of this course is designed to inculcate fundamental understanding of the subject amongst the students.

AIM

The second edition of *Basic Electrical Engineering* is designed for first year engineering students of University of Mumbai. Often, first year engineering students face difficulties while trying to grasp the nuances of the subject through various reference books. Hence, this text serve as a one stop easy solution to address all the pain points of the students. It comprises of a judicious mix of theory and a plethora of solved and unsolved problems.

Salient Features

- Solutions of MU examination question papers from 2012 to 2018 placed appropriately within the book
- Steps for drawing phasor diagrams have been covered in detail
- Each section concludes with exercises, review questions and multiple choice questions to test understanding of topics
- Rich exam-oriented pedagogy:
    - Solved MU problems within chapters: 106
    - Solved examples within chapters: 340
    - Unsolved exercise problems: 251
    - Chapter end review questions: 56
    - Multiple choice questions: 126

Chapter Organisation

This book is divided into **seven** chapters.

Chapters 1 and 2 comprehensively cover **dc circuits** and relevant topics like Kirchhoff's laws, ideal and practical voltage and current source, mesh and nodal analysis, supernode and supermesh analysis, source transformation, star-delta transformation, superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem. **Chapter 3** deals with **ac fundamentals** having detailed coverage on root mean square and phasor representations of alternating quantities.

Chapter 4 deals with analysis of **single-phase ac circuits**. Topics like behavior of *R*, *L*, *C*, series *R-L* circuits, series *R-C* circuits, series *R-L-C* circuits, series and parallel ac circuits and resonance are discussed in this chapter. **Chapter 5** discuses **three-phase ac circuits**

with lucid coverage on three-phase circuits, star or wye connection, delta or mesh connection and their relation. It also talks about measurement of three-phase power in detail.

Chapter 6 talks about **single-phase transformers** covering their construction, principle of working, emf equation, losses, ideal and practical transformers, and open-circuit and short-circuit tests.

Chapter 7 introduces principle of operation of DC motors and DC generators along with construction and classification of **DC machines** and emf equation.

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My acknowledgements would be incomplete without a mention of the contribution of my family members. I am indebted to my mother, Premsheela, for her lifelong inspiration. A heartfelt thanks is due to my wife, Nitu; son, Aman; and daughter, Aditri, for always motivating and supporting me in the whole project.

Any suggestions for improving the book will be gratefully acknowledged.

RAVISH R SINGH

Publisher's Note

Constructive suggestions and criticism always go a long way in enhancing any endeavour. We request all readers to email us their valuable comments/views/feedback for the betterment of the book at **info.india@mheducation.com**, mentioning the title and author name in the subject line.

Also, please feel free to report any piracy of the book spotted by you.

Roadmap to the Syllabus

Basic Electrical Engineering (FEC105)
(As per the latest syllabus of Mumbai University 2017-18 Regulation)

The text is useful for engineering students of all disciplines.

Unit 1: DC Circuits

Kirchhoff's laws, ideal and practical voltage and current source, mesh and nodal analysis, supernode and supermesh analysis, source transformation, star-delta transformation, superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem.

GO TO

Chapter 1 – Basic Concepts
Chapter 2 – DC Circuits

Unit 2: AC Circuits

Generation of alternating voltage and currents, rms and average value, form factor, crest factor, AC through resistance, inductance and capacitance, *R-L*, *R-C* and *R-L-C* series and parallel circuits, phasor diagrams, power and power factor, series and parallel resonance, *Q*-factor and bandwidth.

GO TO

Chapter 3 – AC Fundamentals
Chapter 4 – Single-Phase AC Circuits

Unit 3: Three-Phase Circuits

Three-phase voltage and current generation, star and delta connections (balanced load only), relationship between phase and line currents and voltages, phasor diagrams, basic principle of wattmeter, measurement of power by one and two-wattmeter methods.

GO TO

Chapter 5 – Three-Phase Circuits

Unit 4: Single-Phase Transformer

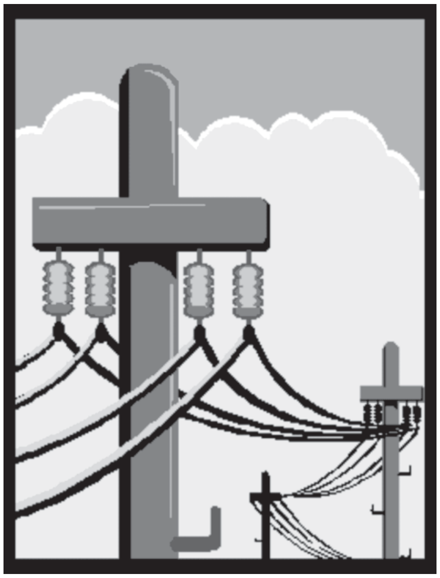
Construction, working principle, emf equation, ideal and practical transformer, transformer on no-load and on load, phasor diagrams, equivalent circuit, OC and SC test, regulation and efficiency.

GO TO **Chapter 6 – Single-Phase Transformers**

Unit 5: DC Machines

Principle of operation of DC motors and DC generators, construction and classification of DC machines, emf equation.

GO TO **Chapter 7 – DC Machines**



Chapter 1

Basic Concepts

Chapter Outline

1.1 VOLTAGE

We know that like charges repel each other whereas unlike charges attract each other. To overcome this force of attraction or repulsion, a certain amount of work or energy is required. When the charges are moved, it is said that a potential difference exists and the work or energy per unit charge utilized in this process is known as voltage or potential difference.

$$V = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$$

1.2 CURRENT

There are free electrons available in all conductors. These free electrons move at random in all directions within the structure in the absence of external voltage. If voltage is applied across the conductor, all the free electrons move in one direction depending on the polarity of the applied voltage. This movement of electrons constitutes an electric current. The conventional direction of current flow is opposite to that of electrons.

Current is defined as the rate of flow of electrons in a conductor. It is measured by the number of electrons that flow in unit time.

$$I = \frac{\text{charge}}{\text{time}} = \frac{Q}{t}$$

1.3 SOURCES

A source is a basic network element which supplies energy to the networks. There are two classes of sources, namely,

(i) Independent source (ii) Dependent source

1.3.1 Independent Sources

Output characteristics of independent sources are not dependent on any network variable such as a current or voltage. Its characteristics, however, may be time varying. There are two types of independent sources:

(i) Independent voltage source (ii) Independent current source

Independent Voltage Source An independent voltage source is a two-terminal network element that establishes a specified voltage across its terminals. The value of this voltage at any instant is independent of the value or direction of the current that flows through it. The symbols for such voltage sources are shown in Fig. 1.1.

The terminal voltage may be a constant, or it may be some specified function of time.

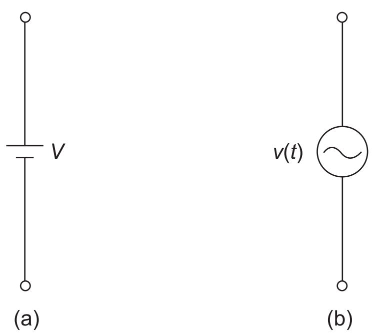


Fig. 1.1 *Independent voltage source*

Independent Current Source An independent current source is a two-terminal network element which produces a specified current. The value and direction of this current at any instant of time is independent of the value or direction of the voltage that appears across the terminals of the source. The symbols for such current sources are shown in Fig. 1.2.

The output current may be a constant or it may be a function of time.

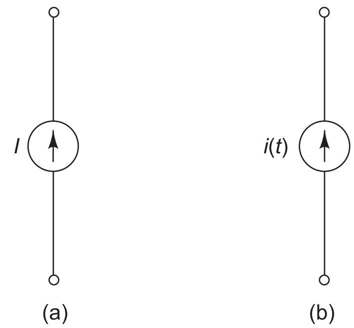


Fig. 1.2 *Independent current source*

1.3.2 Dependent Sources

If the voltage or current of a source depends in turn upon some other voltage or current, it is called as dependent or controlled source. The dependent sources are of four kinds depending on whether the control variable is a voltage or current and the source controlled is a voltage source or current source.

Voltage Controlled Voltage Source (VCVS) A voltage controlled voltage source is a four-terminal network component that establishes a voltage v_{cd} between two points c and d in the circuit that is proportional to a voltage v_{ab} between two points a and b.

The symbol for such a source is shown in Fig. 1.3.

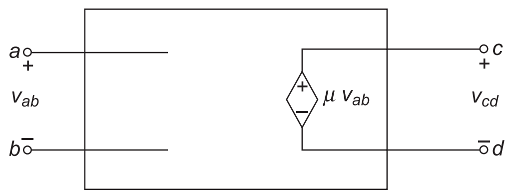


Fig. 1.3 *Voltage controlled voltage sources (VCVS)*

The (+) and (–) sign inside the diamond of the component symbol identify the component as a voltage source.

$$v_{cd} = \mu\, v_{ab}$$

The voltage v_{cd} depends upon the control voltage v_{ab} and the constant μ, a dimensionless constant called voltage gain.

Voltage Controlled Current Source (VCCS) A voltage controlled current source is a four-terminal network component that establishes a current i_{cd} in a branch of the circuit that is proportional to the voltage v_{ab} between two points a and b.

The symbol for such a source is shown in Fig. 1.4.

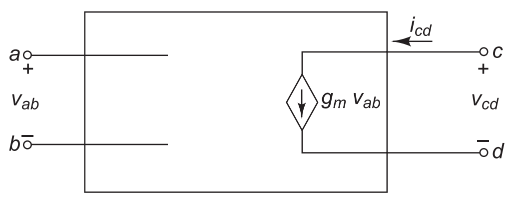


Fig. 1.4 *Voltage controlled current source (VCCS)*

The arrow inside the diamond of the component symbol identifies the component as a current source.

$$i_{cd} = g_m v_{ab}$$

The current i_{cd} depends upon the control voltage v_{ab} and the constant g_m, called the transconductance or mutual conductance. Constant g_m has dimension of ampere per volt or siemens (S).

Current Controlled Voltage Source (CCVS) A current controlled voltage source is a four terminal network component that establishes a voltage v_{cd} between two points c and d in the circuit that is proportional to current i_{ab} in some branch of the circuit.

The symbol for such a source is shown in Fig. 1.5.

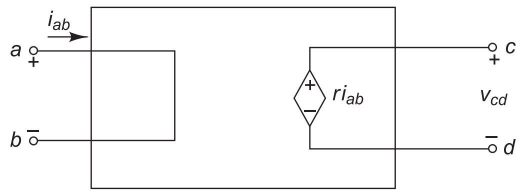


Fig. 1.5 *Current controlled voltage source (CCVS)*

$$v_{cd} = r\, i_{ab}$$

The voltage v_{cd} depends upon the control current i_{ab} and the constant r called the transresistance or mutual resistance. Constant r has dimension of volt per ampere or ohm (Ω).

Current Controlled Current Source (CCCS) A current controlled current source is a four-terminal network component that establishes a current i_{cd} in one branch of a circuit that is proportional to current i_{ab} in some branch of the network.

The symbol for such a source is shown in Fig. 1.6.

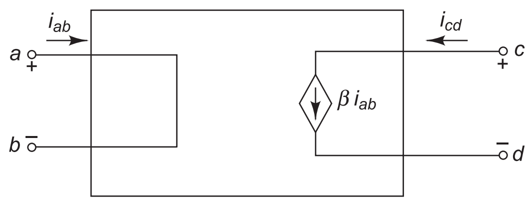


Fig. 1.6 *Current controlled current source (CCCS)*

$$i_{cd} = \beta\, i_{ab}$$

The current i_{cd} depends upon the control current i_{ab} and the dimensionless constant β, called the current gain.

1.4 OHM'S LAW

According to Ohm's law, the potential difference across any two points on a conductor is directly proportional to the current flowing through it, provided the physical conditions, viz., material length, cross-sectional area and temperature of the conductor remain constant.

$$V \propto I$$

$$V = R\,I$$

where R is the resistance between two points of the conductor.

Limitations

1. Ohm's law does not apply to nonmetallic conductors. For example, for silicon carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity.
2. Ohm's law also does not apply to nonlinear devices such as zener diodes, voltage regulator tubes, etc.
3. Ohm's law is true for metal conductors at constant temperature. If the temperature changes, the law is not applicable.

1.5 RESISTANCE

Resistance is the property of a material due to which it opposes the flow of electric current through it.

Certain materials offer very little opposition to the flow of electric current and are called *conductors*, e.g. metals, acids and salt solutions. Certain materials offer very high resistance to the flow of electric current and are called *insulators*, e.g. mica, glass, rubber, Bakelite, etc.

The SI unit of resistance is ohm and is represented by the symbol Ω. A conductor is said to have a resistance of one ohm if a potential difference of one volt across its terminals causes a current of one ampere to flow through it.

The resistance of a conductor depends on the following factors:

(i) It is directly proportional to its length.
(ii) It is inversely proportional to the area of cross section of the conductor.
(iii) It depends on the nature of the material.
(iv) It also depends on the temperature of the conductor.

Hence,

$$R \propto \frac{l}{A}$$

$$R = \rho\frac{l}{A}$$

where l is length of the conductor, A is the cross-sectional area and ρ is a constant known as the **specific resistance**, or **resistivity of the material**.

Specific Resistance The specific resistance, or the resistivity of a material, is the resistance offered by unit length of the material of unit cross-section. If the length is in metres and the area of cross-section in square metres, then the resistivity is expressed in ohm metres (Ω-m).

Table 1.1 shows the resistivities and temperature coefficients of various materials at 20 °C.

Table 1.1

Material	*Resistivity (Ω-m)*	*Temperature coefficient/ °C*
Silver	1.58×10^{-8}	0.0038
Copper	1.72×10^{-8}	0.0039
Gold	2.44×10^{-8}	0.0034
Aluminium	2.82×10^{-8}	0.0039
Calcium	3.36×10^{-8}	0.0041
Tungsten	5.60×10^{-8}	0.0045
Zinc	5.90×10^{-8}	0.0037
Nickel	6.99×10^{-8}	0.006
Iron	1.0×10^{-7}	0.005
Platinum	1.06×10^{-7}	0.00392
Tin	1.09×10^{-7}	0.0045
Lead	2.2×10^{-7}	0.0039
Manganin	4.82×10^{-7}	0.000002
Constantan	4.9×10^{-7}	0.000008
Mercury	9.8×10^{-7}	0.0009
Nichrome	1.10×10^{-6}	0.0004
Carbon	$5–8 \times 10^{-4}$	–0.0005
Germanium	4.6×10^{-1}	–0.048
Silicon	6.40×10^{2}	–0.075
Glass	$10^{10} – 10^{14}$	

Example 1

Calculate the resistance of a copper conductor having a length of 2 km and a cross-section of 22 mm². Assume the resistivity of copper is 1.72×10^{-8} Ω-m.

Solution

$$l = 2 \text{ km} = 2 \times 10^3 \text{ m}$$

$$A = 22 \text{ mm}^2 = 22 \times 10^{-6} \text{ m}^2$$

$$\rho = 1.72 \times 10^{-8} \ \Omega\text{-m}$$

Resistance of copper conductor

$$R = \rho \frac{l}{A}$$

$$= 1.72\times10^{-8}\times\frac{2\times10^{3}}{22\times10^{-6}}$$

$$= 1.56\ \Omega$$

Example 2

Calculate the resistance of a copper tube with the external diameter of 10 cm, internal diameter of 9 cm, length of 2 m and resistivity of copper as 1.72×10^{-8} Ω-m.

Solution

$$d_1 = 10\text{ cm} = 0.1\text{ m}$$

$$d_2 = 9\text{ cm} = 0.09\text{ m}$$

$$l = 2\text{ m}$$

$$\rho = 1.72\times10^{-8}\ \Omega\text{-m}$$

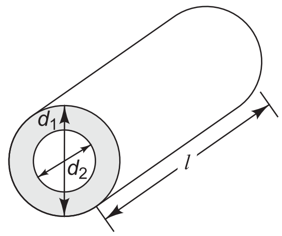


Fig. 1.7

Area of cross section of copper tube

$$A = \frac{\pi}{4}d_1^2 - \frac{\pi}{4}d_2^2$$

$$= \frac{\pi}{4}\times(0.1)^2 - \frac{\pi}{4}\times(0.09)^2$$

$$= 1.49\times10^{-3}\text{ m}^2$$

Resistance of copper tube

$$R = \rho\frac{l}{A}$$

$$= 1.72\times10^{-8}\times\frac{2}{1.49\times10^{-3}}$$

$$= 23.09\ \mu\Omega$$

Example 3

Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm^2, if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m. If the wire is drawn out to three times its original length, by how many times will the resistance increase?

Solution

$$l_1 = 100\text{ m}$$

$$A_1 = 0.1\text{ mm}^2 = 0.1\times10^{-6}\text{ m}^2$$

$$\rho_1 = 50\times10^{-8}\ \Omega\text{-m}$$

Resistance of the wire

$$R_1 = \rho_1\frac{l_1}{A_1}$$

$$= 50\times10^{-8}\times\frac{100}{0.1\times10^{-6}}$$

$$= 500\ \Omega$$

If the wire is drawn out to three times its original length, its volume remains constant.

Volume $= A_2 l_2 = A_1 l_1$

$$\frac{A_1}{A_2} = \frac{l_2}{l_1}$$

$$\rho_2 = \rho_1$$

$$l_2 = 3l_1$$

Now, $$R_2 = \rho_2 \frac{l_2}{l_1}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{l_2}{l_1} = 1 \cdot 3 \cdot 3 = 9$$

$$R_2 = 9R_1$$

Hence, resistance will increase by nine times its original value.

Example 4

A piece of silver wire has a resistance of 3 Ω. What will be the resistance of a manganin wire one-third the length and one-third the diameter, if the resistivity of manganin is 30 times that of silver?

Solution Let R_1, ρ_1, l_1, A_1 and R_2, ρ_2, l_2, A_2 be the resistance, resistivity, length and area of cross-section of silver and manganin wire respectively.

For silver wire, $R_1 = 3\ \Omega$

For manganin wire, $l_2 = \frac{1}{3} l_1, d_2 = \frac{1}{3} d_1, \rho_2 = 30\rho_1$

$$R_1 = \rho_1 \frac{l_1}{A_1}$$

$$R_2 = \rho_2 \frac{l_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{A_1}{A_2}$$

But $$A_1 = \frac{\pi}{4} d_1^2 \text{ and } A_2 = \frac{\pi}{4} d_2^2$$

where d_1 and d_2 are diameters of the silver and manganin wires respectively.

$$\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \left(\frac{d_1}{d_2}\right)^2$$

$$= (30)\left(\frac{1}{3}\right)(3)^2$$

$$= 90$$

$$R_2 = 90R_1 = 90(3) = 270\ \Omega$$

Resistance of manganin wire = 270 Ω

Example 5

A 10 m long aluminium wire of 2 mm diameter is connected in parallel to a 6 m long copper wire. A total current of 2 A is passed through the combination and found that current through the aluminium wire is 1.25 A. Calculate the diameter of the copper wire. Specific resistance of copper is 1.6×10^{-6} Ω-cm and that of aluminium is 2.6×10^{-6} Ω-cm.

Solution For aluminium wire, $l_1 = 10$ m, $d_1 = 2$ mm $= 2 \times 10^{-3}$ m, $I_1 = 1.25$ A

$$\rho_1 = 2.6 \times 10^{-6}\ \Omega\text{-cm} = 2.6 \times 10^{-8}\ \Omega\text{-m}$$

For copper wire, $l_2 = 6$ m

$$\rho_2 = 1.6 \times 10^{-6}\ \Omega\text{-cm} = 1.6 \times 10^{-8}\ \Omega\text{-m}$$

$$I_1 = 2\text{ A}$$

An aluminium wire and copper wire are connected in parallel.

For aluminium wire

$$R_1 = \frac{V}{I_1} = \frac{V}{1.25}$$

$$A_1 = \frac{\pi}{4}d_1^2$$

For copper wire

$$I_2 = 2 - 1.25 = 0.75\text{ A}$$

$$R_2 = \frac{V}{I_2} = \frac{V}{0.75}$$

$$A_2 = \frac{\pi}{4}d_2^2$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1}\cdot\frac{l_2}{l_1}\cdot\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1}\cdot\frac{l_2}{l_1}\cdot\frac{d_1^2}{d_2^2}$$

$$\frac{1.25}{0.75} = \frac{1.6\times10^{-8}}{2.6\times10^{-8}}\cdot\frac{6}{10}\cdot\frac{(2\times10^{-3})^2}{d_2^2}$$

$$d_2 = 0.94\text{ mm}$$

Diameter of copper wire = 0.94 mm

1.6 SERIES CIRCUIT

Resistors R_1 and R_2 are said to be connected in series when the same current flows through each resistor.

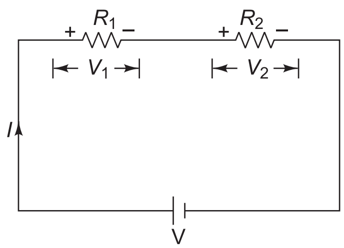


Fig. 1.8 *Series circuit*

Voltage across $R_1 = V_1 = R_1 I$

Voltage across $R_2 = V_2 = R_2 I$

The total voltage applied should be balanced by the sum of voltage drops around the circuit.

$$
\begin{aligned}
V &= V_1 + V_2 \\
&= R_1 I + R_2 I \\
&= (R_1 + R_2) I \\
&= R_T I \quad \text{where } R_T = R_1 + R_2
\end{aligned}
$$

Hence, when a number of resistors are connected in series, the equivalent resistance is the sum of all the individual resistance.

Note

1. Same current flows through each resistor.
2. Voltage drops are additive.
3. Resistances are additive.
4. Power is additive.
5. The applied voltage equals the sum of different voltage drops.

Voltage Division in a Series Circuit

$$I = \frac{V}{R_1 + R_2}$$

Hence, voltage across $R_1 = V_1 = R_1 I = R_1 \dfrac{V}{R_1 + R_2} = \dfrac{R_1}{R_1 + R_2} V$

Similarly, voltage across $R_2 = V_2 = R_2 I = R_2 \dfrac{V}{R_1 + R_2} = \dfrac{R_2}{R_1 + R_2} V$

1.7 PARALLEL CIRCUIT

Resistors R_1 and R_2 are said to be connected in parallel when the potential difference across each resistor is same.

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

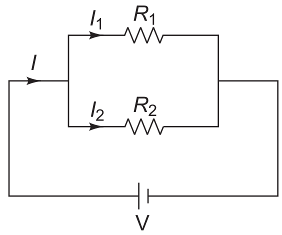


Fig. 1.9 *Parallel circuit*

Since, $$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I = \frac{V}{R_T} \quad \text{where} \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Hence, when a number of resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of reciprocals of individual resistances.

Note

1. Same voltage appears across all resistors.
2. Branch currents are additive.
3. Conductances are additive.
4. Power is additive.

Current Division in a Parallel Circuit

Case (i) When two resistances are connected in parallel,

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Also, $$V = R_T I = R_1 I_1 = R_2 I_2$$

Hence, current through $R_1 = I_1 = \dfrac{V}{R_1} = \dfrac{R_T I}{R_1} = \dfrac{R_2}{R_1 + R_2} I$

Similarly, current flowing through $R_2 = I_2 = \dfrac{V}{R_2} = \dfrac{R_T I}{R_2} = \dfrac{R_1}{R_1 + R_2} I$

Case (ii) When three resistances are connected in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{R_2 R_3 + R_3 R_1 + R_1 R_2}{R_1 R_2 R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2}$$

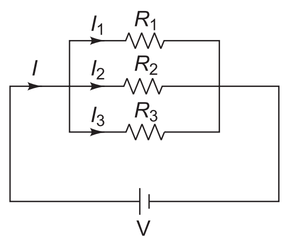


Fig. 1.10 *Parallel circuit*

Also, $$V = R_T I = R_1 I_1 = R_2 I_2 = R_3 I_3$$

Current through $R_1 = I_1 = \dfrac{V}{R_1} = \dfrac{R_T I}{R_1} = \dfrac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$

Current through $R_2 = I_2 = \dfrac{V}{R_2} = \dfrac{R_T I}{R_2} = \dfrac{R_3 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$

$$\text{Current through } R_3 = I_3 = \frac{V}{R_3} = \frac{R_T I}{R_3} = \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$$

1.8 SHORT AND OPEN CIRCUITS

When two terminals of a circuit are connected by a wire, they are said to be short circuited. A short circuit has following features:

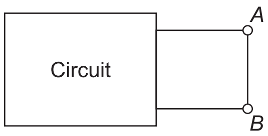


Fig. 1.11 *Short circuit*

(i) It has zero resistance.
(ii) Current through it is very large.
(iii) There is no voltage across it.

When two terminals of a circuit have no direct connection between them, they are said to be open circuited. An open circuit has the following features:

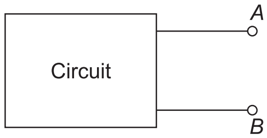


Fig. 1.12 *Open circuit*

(i) It has infinite resistance.
(ii) Current through it is zero.
(iii) The entire voltage appears across it.

1.8.1 Open Circuits and Short Circuits in a Series Circuit

When an open circuit appears in a series circuit, the equivalent resistance becomes infinite and no current flows through the circuit.

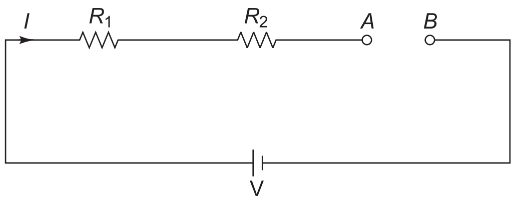


Fig. 1.13 *Open in series circuit*

$$I = \frac{V}{R_1 + R_2 + \infty} = \frac{V}{\infty} = 0$$

$$V_{AB} = V - R_1 I - R_2 I = V$$

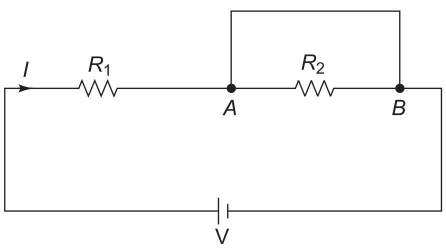


Fig. 1.14 *Short in series circuit*

When a short circuit appears in a series circuit, as shown in Fig. 1.14, the resistance R_2 becomes zero.

$$I = \frac{V}{R_1 + 0} = \frac{V}{R_1}$$

$$V_{AB} = 0$$

1.8.2 Open Circuits and Short Circuits in a Parallel Circuit

When an open circuit appears in a parallel circuit, no current flows through that branch. The other branch currents are not affected by the open circuit.

$$I_1 = 0$$

$$I_2 = \frac{V}{R_2}$$

$$V_{AB} = V$$

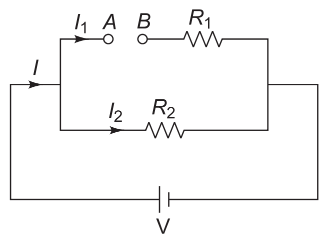


Fig. 1.15 *Open in parallel circuit*

When a short circuit appears in a parallel circuit, the equivalent resistance becomes zero.

$$I_1 = 0$$

$$I_2 = 0$$

$$V_{AB} = 0$$

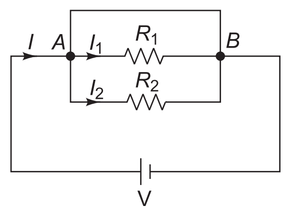


Fig. 1.16 *Short in parallel circuit*

Example 1

Find an equivalent resistance between terminals A and B.

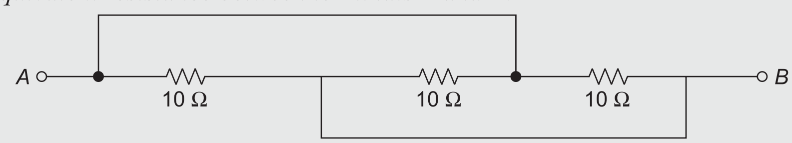


Fig. 1.17

Solution Marking all the junctions and redrawing the network,

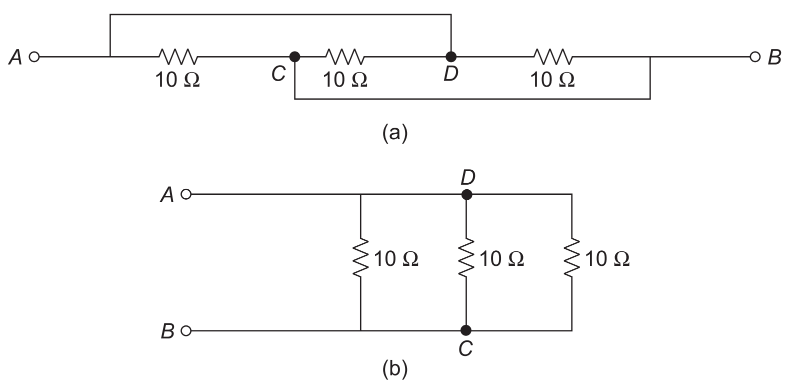


Fig. 1.18

$$R_{AB} = 10 \,\|10\|\, 10 = 3.33\ \Omega$$

Example 2

Find an equivalent resistance between terminals A and B.

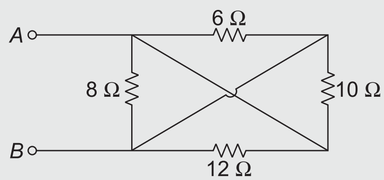


Fig. 1.21

Solution Marking all the junctions and redrawing the network,

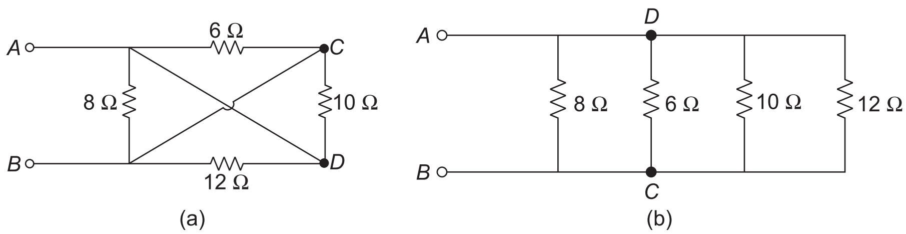


Fig. 1.22

$$R_{AB} = 8 \,\|6\|10\|\, 12 = 2.11\ \Omega$$

Example 3

Find the equivalent resistance between terminals A and B.

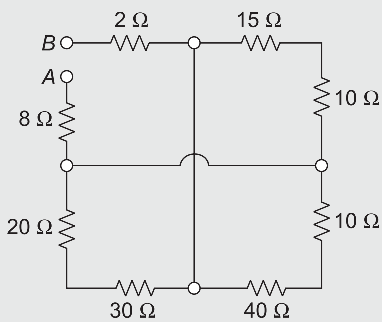


Fig. 1.23

Solution Marking all the junctions and redrawing the network,

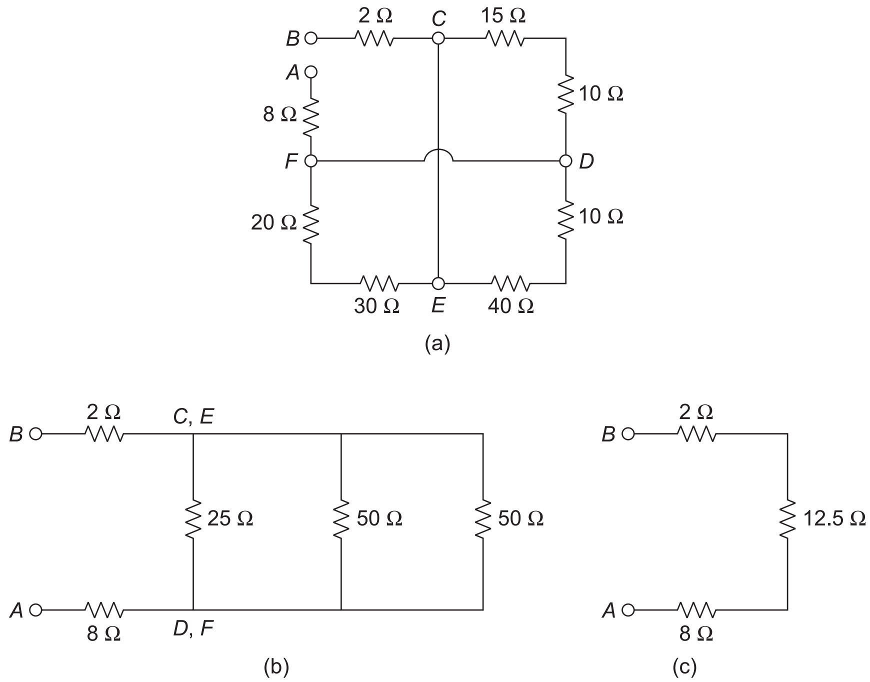


Fig. 1.24

$$R_{AB} = 22.5\ \Omega$$

Example 4

A resistor of 5 Ω is connected in series with a parallel combination of a number of resistors each of 5 Ω. If the total resistance of the combination is 6 Ω find the number of resistors connected in parallel. **[May 2016]**

Solution

$$R_S = 5\ \Omega$$

$$R_T = 6\ \Omega$$

If n resistors each of 5 Ω are connected in parallel, $R_p = \dfrac{5}{n}$

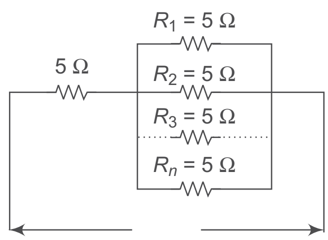


Fig. 1.25

$$R_T = R_s + R_p = 5 + \frac{5}{n}$$

$$6 = 5 + \frac{5}{n}$$

$$n = 5$$

Example 5

Find equivalent resistance across terminals A and B.

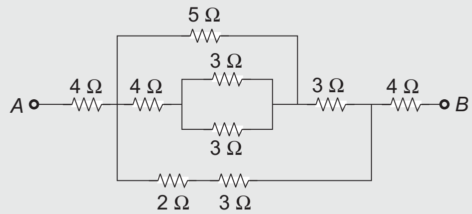


Fig. 1.19 **[Dec 2015]**

Solution Simplifying the network by series-parallel reduction technique,

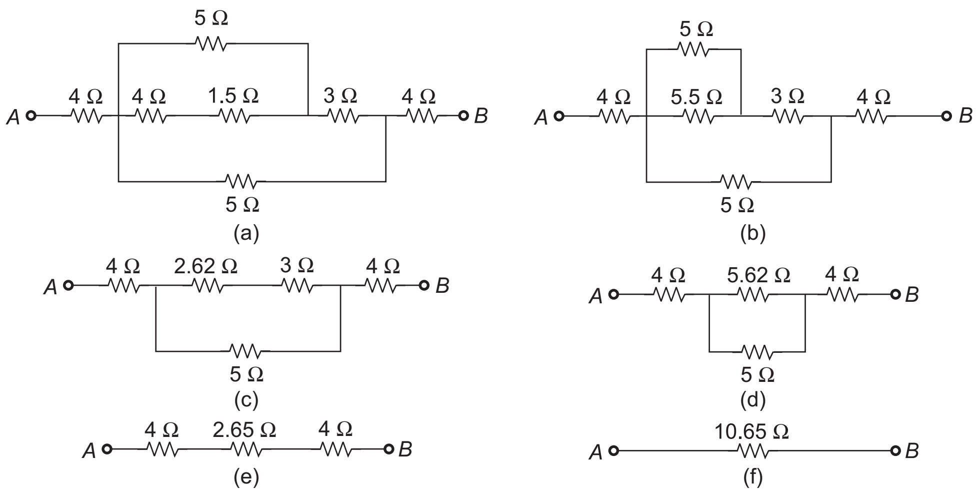


Fig. 1.20

$$R_{AB} = 10.65\ \Omega$$

Example 6

Determine the current delivered by the source.

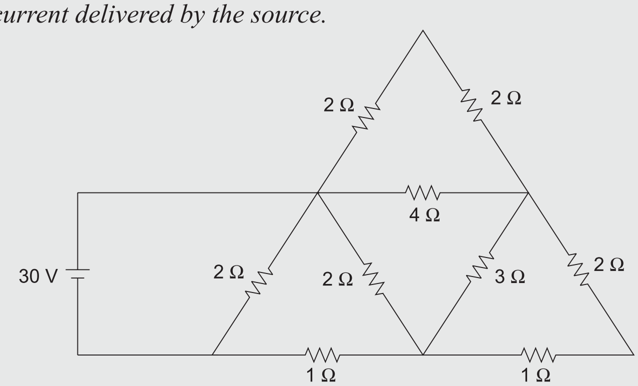


Fig. 1.26

Solution The network can be simplified by series–parallel reduction technique.

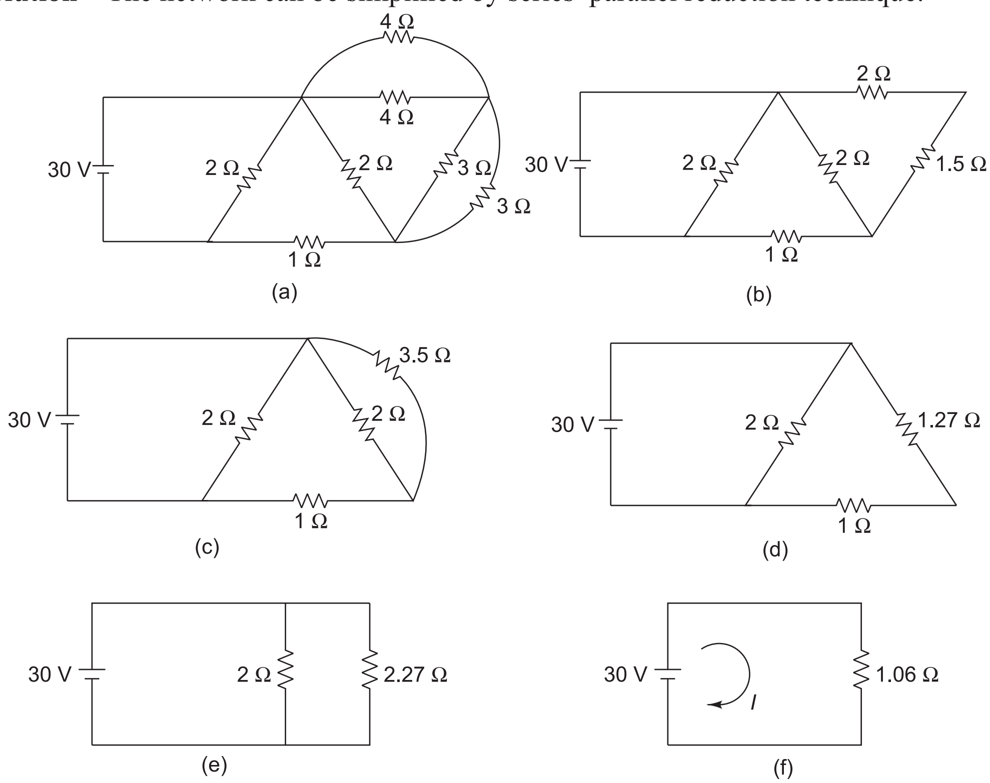


Fig. 1.27

$$I = \frac{30}{1.06} = 28.3 \text{ A}$$

Example 7

Three equal resistors of 30 Ω each are connected in parallel across a 120 V dc supply. What is the current through each of them (i) if one of the resistors burns out, or (ii) if one of the resistors gets shorted?

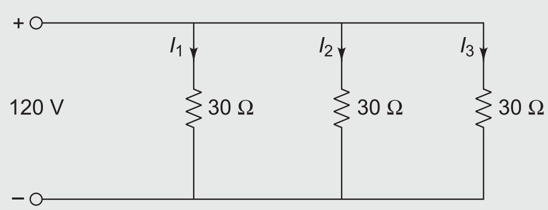


Fig. 1.28

Solution (i) If one of the resistors burns out, it will act as an open circuit.

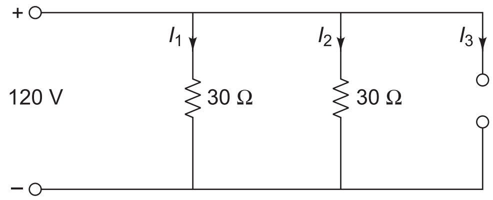


Fig. 1.29

$$I_3 = 0$$

$$I_1 = I_2 = \frac{120}{30} = 4 \text{ A}$$

(ii) If one of the resistors gets shorted, the effective resistance becomes zero.

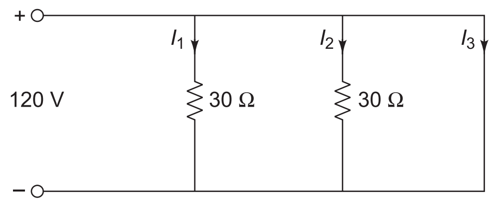


Fig. 1.30

$$I_1 = I_2 = 0$$

Example 8

A lamp rated at 100 V, 75 W is to be connected across a 230 V supply. Find the value of resistance to be connected in series with the lamp. Also find the power loss occurring in the resistor.

Solution

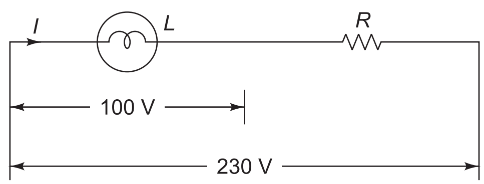


Fig. 1.31

$$V_1 = 100 \text{ V}$$
$$P_1 = 75 \text{ W}$$
$$V = 230 \text{ V}$$

(i) Value of resistance

Rated current of the lamp

$$I = \frac{P_1}{V_1} = \frac{75}{100} = 0.75 \text{ A}$$

Lamp will operate normally on 230 V supply if the current flowing through the lamp remains the rated current, i.e., 0.75 A.

Voltage across resistor R

$$V_2 = 230 - 100 = 130 \text{ V}$$

$$\text{Resistance } R = \frac{130}{0.75} = 173.33\,\Omega$$

(ii) Power loss occurring in the resistor

$$P_2 = \frac{V_2^2}{R} = \frac{(130)^2}{173.33} = 97.5 \text{ W}$$

Example 9

A 100 V, 60 W lamp is connected in series with a 100 V, 100 W lamp and the combination is connected across 200 V mains. Find the value of the resistance that should be connected in parallel with the first lamp so that each lamp may get the rated current at rated voltage.

Solution

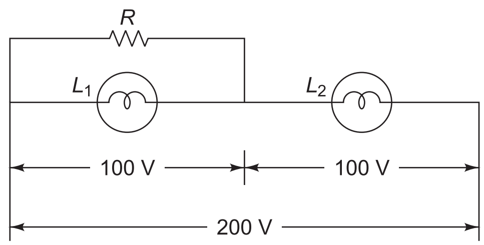


Fig. 1.32

$$V_1 = 100 \text{ V}$$
$$P_1 = 60 \text{ W}$$
$$V_2 = 100 \text{ V}$$
$$P_2 = 100 \text{ W}$$
$$V = 200 \text{ V}$$

Rated current of lamp L_1

$$I_1 = \frac{P_1}{V_1} = \frac{60}{100} = 0.6 \text{ A}$$

Rated current of lamp L_2

$$I_2 = \frac{P_2}{V_2} = \frac{100}{100} = 1 \text{ A}$$

Let R be the value of resistance that should be connected in parallel with the lamp L_1 so that rated current flows through lamp L_1.

Current through resistor R

$$I = 1 - 0.6 = 0.4 \text{ A}$$

$$\text{Resistance} \quad R = \frac{V_1}{I} = \frac{100}{0.4} = 250\,\Omega$$

Example 10

A 100 V, 60 W lamp is connected in series with a 100 V, 100 W lamp across 200 V supply. What will be the current drawn by the lamps? What will be the power consumed by each lamp and will such a combination work?

Solution $V_1 = 100$ V, $P_1 = 60$ W
$V_2 = 100$ V, $P_2 = 100$ W
$V = 200$ V

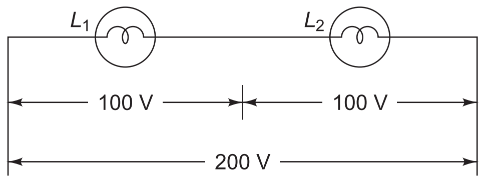


Fig. 1.33

(i) Current drawn by the lamps

Resistance of lamp L_1

$$R_1 = \frac{V_1^2}{P_1} = \frac{(100)^2}{60} = 166.67\ \Omega$$

Resistance of lamp L_2

$$R_2 = \frac{V_2^2}{P_2} = \frac{(100)^2}{100} = 100\ \Omega$$

Current drawn by the lamps

$$I = \frac{V}{R_1 + R_2} = \frac{200}{166.67 + 100} = 0.75\ \text{A}$$

(ii) Power consumed by the lamps

Power consumed by lamp $L_1 = I^2R_1 = (0.75)^2 \times 166.67 = 93.75$ W

Power consumed by lamp $L_2 = I^2R_2 = (0.75)^2 \times 100 = 56.25$ W

If a 100 V, 60 W lamp draws a power of 93.75 W, its filament will be overheated and will burn out. Hence, such a combination will not work.

Useful Formulae

1. $V = RI$

2. $R = \rho \dfrac{l}{A}$

3. Resistors R_1 and R_2 in series

(i) $R_T = R_1 + R_2$

(ii) $V = V_1 + V_2$

(iii) $P = P_1 + P_2$

(iv) $V_1 = \dfrac{R_1}{R_1 + R_2} V$

(v) $V_2 = \dfrac{R_2}{R_1 + R_2} V$

4. Resistors R_1 and R_2 in parallel

(i) $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

(ii) $I = I_1 + I_2$

(iii) $P = P_1 + P_2$

(iv) $I_1 = \dfrac{R_2}{R_1 + R_2} I$

(v) $I_2 = \dfrac{R_1}{R_1 + R_2} I$

Exercise 1.1

1.1 Calculate the resistance of a 100 m length of wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of material having a resistivity of 80 × 10^{-8} Ω-m. *[800 Ω]*

1.2 Find the resistance of a 2000 km cable at 20 °C, having a diameter of 0.7 cm. Assume specific resistance of copper at 20 °C as $\frac{1}{58}$ per °C for 1 m length and 1 mm^2 cross-section. *[896.01 Ω]*

1.3 A silver wire has a resistance of 2.5 Ω. What will be the resistance of a manganin wire having a diameter half of the silver wire and one-third length? The specific resistance of manganin is 30 times that of silver. *[100 Ω]*

1.4 Calculate the resistance of a 100 m length of wire having a cross-sectional area of 0.02 mm^2 and a resistivity of 40 μΩ-cm. If the wire is drawn out to four times its original length, calculate its new resistance. *[2000 Ω, 32000 Ω]*

1.5 A copper wire of 1 cm diameter had a resistance of 0.15 Ω. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire? *[2.4 Ω]*

1.6 A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in ratio 49 : 24. The former carries 80% more current than the latter and the latter is 47% longer than the former. Determine the ratio of their cross-sectional areas. *[0.4]*

1.7 Two wires, one of aluminium and one of copper, offer the same resistance. The diameter of cross-section of the aluminium wire is double that of the copper wire. If the resistivities of aluminium and copper are 0.028 μΩ-m and 0.0168 μΩ-m respectively, find the ratio of length of the wires. *[0.3]*

1.8 Find the resistance between terminals *A* and *B*.

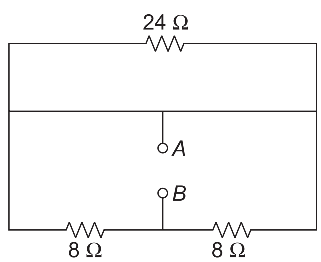


Fig. 1.34

[4 Ω]

1.9 Find the resistance between terminals A and B.

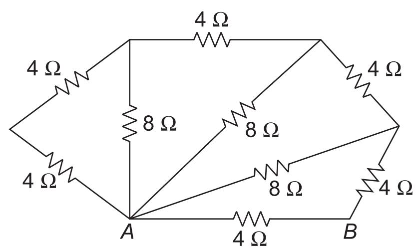


Fig. 1.35

[2.67 Ω]

1.10 Find the equivalent resistance between terminals A and B.

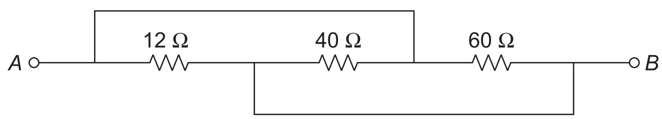


Fig. 1.36

[8 Ω]

1.11 What is the equivalent resistance between terminals A and B of the networks as shown in Fig. 1.37?

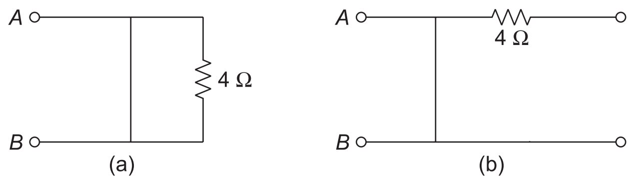


Fig. 1.37

[(a) 0 (b) 0]

1.12 Find the equivalent resistance between terminals A and B.

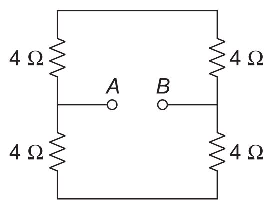


Fig. 1.38

[4 Ω]

1.13 Find the equivalent resistance between terminals A and B.

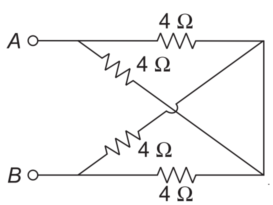


Fig. 1.39

[4 Ω]

1.14 Find the equivalent resistance between terminals A and B.

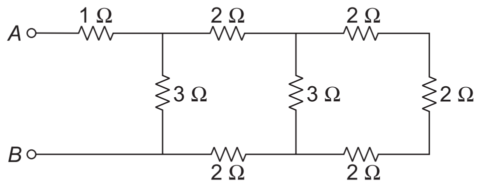


Fig. 1.40

[3 Ω]

1.15 A circuit consists of two parallel resistors, having resistances of 20 Ω and 30 Ω respectively, and is connected in series with a 15 Ω resistor. If the current through 15 Ω resistor is 3 A, find (i) current through 20 Ω and 30 Ω resistors, (ii) the voltage across the whole circuit, and (iii) total power.

[(i) 1.8 A, 1.2 A (ii) 81 V (iii) 243 W]

1.16 A resistor of 10 Ω is connected in series with two resistors each of 15 Ω arranged in parallel. What resistance must be connected across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied? *[6 Ω]*

1.17 A resistor of R Ω is connected in series with parallel circuit consisting of two resistors of 8 Ω and 12 Ω respectively. The total power dissipated in the circuit is 70 W when applied voltage is 20 V. Calculate the value of R. *[0.914 Ω]*

1.18 A lamp rated 110 V, 60 W is connected with another lamp rated 110 V, 100 W across 220 V mains. Calculate the resistance that should be joined in parallel with the first lamp, so that both the lamps may take their rated power. *[302.5 Ω]*

Review Questions

1.1 What are the factors governing the value of resistance? Explain the term resistivity.

Multiple Choice Questions

Choose the correct alternative in the following questions:

1.1 If the length of a wire of resistance R is uniformly stretched to n times its original value, its new resistance is

(a) nR (b) $\frac{R}{n}$

(c) n^2R (d) $\frac{R}{n^2}$

1.2 Two wires *A* and *B* of the same material and length *L* and 2*L* have radius *r* and 2*r* respectively. The ratio of their specific resistance will be

(a) 1 : 1 (b) 1 : 2
(c) 1 : 4 (d) 1 : 8

1.3 A length of wire having a resistance of 1Ω is cut into four equal parts and these four parts are bundled together side-by-side to form a wire. The new resistance will be

(a) $\frac{1}{4}$ Ω (b) $\frac{1}{16}$ Ω
(b) 4 Ω (c) 16 Ω

1.4 The hot resistance of the filament of a bulb is higher than the cold resistance because the temperature coefficient of the filament is

(a) negative (b) infinite
(c) zero (d) positive

1.5 A network contains linear resistors and ideal voltage sources. If values of all the resistors are doubled then the voltage across each resistor is

(a) halved (b) doubled
(c) increased by four times (d) not changed

1.6 A 10 V battery with an internal resistance of 1 Ω is connected across a nonlinear load whose *V*-*I* characteristic is given by $7I = V^2 + 2V$. The current delivered by the battery is

(a) 0 (b) 10 A (c) 5 A (d) 8 A

1.7 All the resistors in Fig. 1.41 are 1 Ω each. The value of *I* will be

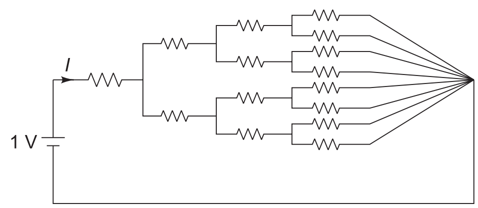


Fig. 1.41

(a) $\frac{1}{15}$ A (b) $\frac{2}{15}$ A (c) $\frac{4}{15}$ A (d) $\frac{8}{15}$ A

1.8 For the circuit shown in Fig. 1.42, the equivalent resistance will be

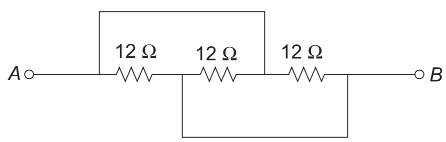


Fig. 1.42

(a) 36 Ω (b) 12 Ω (c) 6 Ω (d) 4 Ω

1.9 Two incandescent light bulbs of 40 W and 60 W rating are connected in series across the mains. Then

(a) the bulbs together consume 100 W
(b) the bulbs together consume 50 W
(c) the 60 W bulb glows brighter
(d) the 40 W bulb glows brighter

1.10 Twelve 1 Ω resistors are used as edges to form a cube. The resistance between the two diagonally opposite corners of the cube is

(a) $\frac{5}{6}\Omega$ (b) 1 Ω (c) $\frac{6}{5}\Omega$ (d) $\frac{3}{2}\Omega$

1.11 All resistors in the circuit shown in Fig. 1.43 are of R Ω each. The switch is initially open. When the switch is closed the lamp's intensity

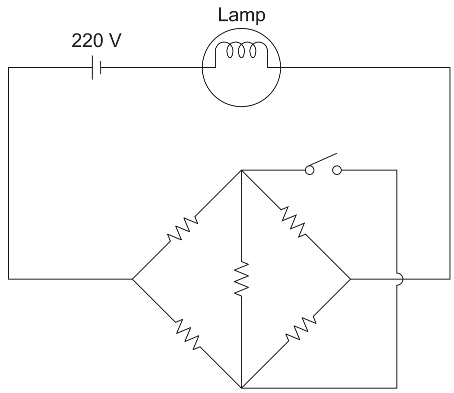


Fig. 1.43

(a) increases
(b) decreases
(c) remains the same
(d) depends on the value of R

1.12 Two 2 kΩ, 2 W resistors are connected in parallel. Then combined resistance and wattage ratings will be

(a) 4 kΩ, 4 W
(b) 1 kΩ, 4 W
(c) 1 kΩ, 2 W
(d) 1 kΩ, 1 W

1.13 Two electrical sub-networks N_1 and N_2 are connected through three resistors as shown in Fig. 1.44. The voltages across the 5 Ω resistor and 1 Ω resistor are given to be 10 V and 5 V respectively. Then the voltage across the 15 Ω resistor is

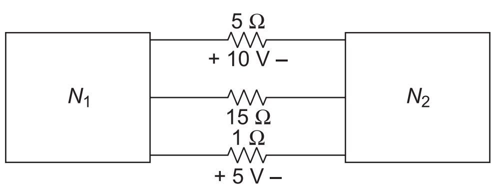
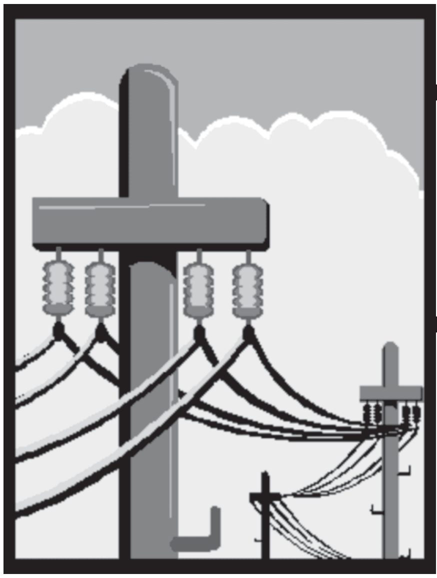


Fig. 1.44

(a) –105 V (b) 105 V (c) –15 V (d) 15 V

Answers to Multiple Choice Questions

1.1 (c)	**1.2** (b)	**1.3** (b)	**1.4** (d)	**1.5** (d)	**1.6** (c)
1.7 (d)	**1.8** (d)	**1.9** (d)	**1.10** (a)	**1.11** (c)	**1.12** (b)
1.13 (a)					



Chapter 2

DC Circuits

Chapter Outline

2.1 KIRCHHOFF'S LAWS

The entire study of electric circuit analysis is based mainly on Kirchhoff's laws. But before discussing this, it is essential to familiarise ourselves with the following terms:

Node A node is a junction where two or more circuit elements are connected together.

Branch An element or number of elements connected between two nodes constitute a branch.

Loop A loop is any closed part of the circuit.

Mesh A mesh is the most elementary form of a loop and cannot be further divided into other loops. All meshes are loops but all loops are not meshes.

Kirchhoff's Current Law (KCL) The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

Consider five conductors, carrying currents I_1, I_2, I_3, I_4 and I_5 meeting at a point O as shown in Fig. 2.1. Assuming the incoming currents to be positive and outgoing currents negative, we have

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

Thus, the above law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

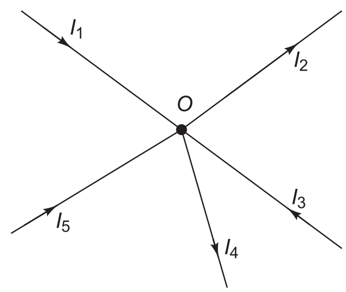


Fig. 2.1 *Kirchhoff's current law*

Kirchhoff's Voltage Law (KVL) The algebraic sum of all the voltages in any closed circuit or mesh or loop is zero.

If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of emfs and the sum of voltage drops or rises meeting on the way is zero.

Determination of Sign A rise in potential can be assumed to be positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(i) If we go from the positive terminal of the battery or source to the negative terminal, there is a fall in potential and the emf should be assigned a negative sign. If we go from the negative terminal of the battery or source to the positive terminal, there is a rise in potential and the emf should be given a positive sign.

Fig. 2.2 *Sign convention*

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in the potential and the sign of this voltage drop is negative. If we go opposite to the direction of the current flow, there is a rise in potential and hence, this voltage drop should be given a positive sign.

Fig. 2.3 *Sign convention*

Example 1

The voltage drop across the 15 Ω resistor is 30 V, having the polarity indicated. Find the value of R.

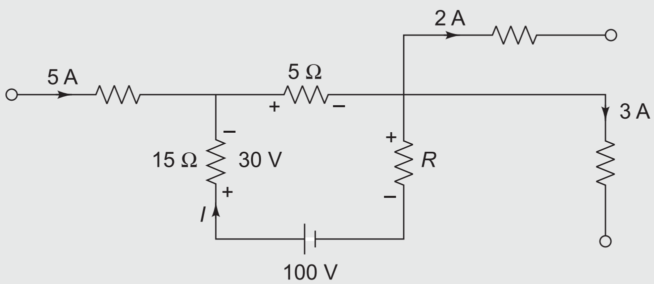


Fig. 2.4

Solution Current through the 15 Ω resistor

$$I = \frac{30}{15} = 2 \text{ A}$$

Current through the 5 Ω resistor = 5 + 2 = 7 A

Applying KVL to the closed path,

$$-5\,(7) - R(2) + 100 - 30 = 0$$

$$-35 - 2R + 100 - 30 = 0$$

$$R = 17.5\ \Omega$$

Example 2

Determine the currents I_1, I_2 and I_3.

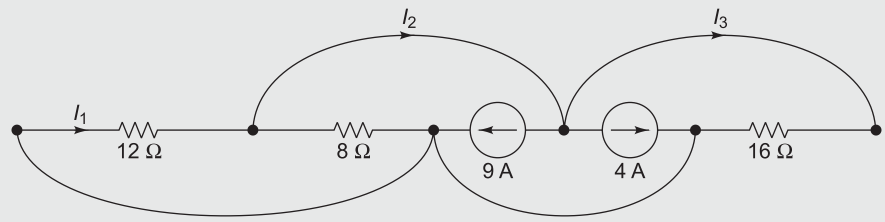


Fig. 2.5

Solution Assigning currents to all the branches,

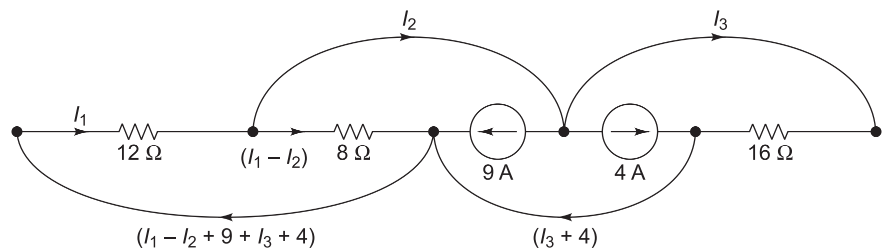


Fig. 2.6

From Fig. 2.6,

$$I_1 = I_1 - I_2 + 9 + I_3 + 4$$

$$I_2 - I_3 = 13 \tag{1}$$

Also, $$-12I_1 - 8(I_1 - I_2) = 0$$

$$-20I_1 + 8I_2 = 0 \tag{2}$$

and, $$-12I_1 - 16I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4 \text{ A}$$

$$I_2 = 10 \text{ A}$$

$$I_3 = -3 \text{ A}$$

Example 3

Find currents in all the branches of the network.

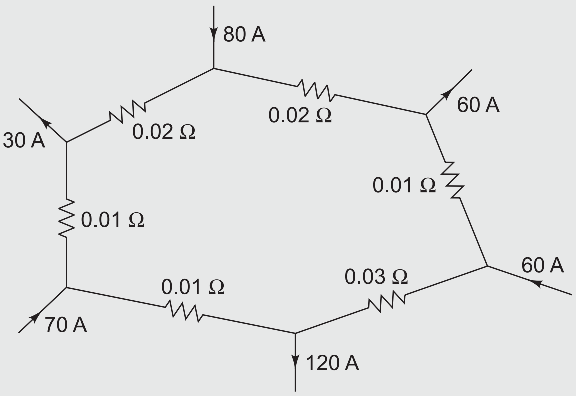


Fig. 2.7

Solution Let $I_{AF} = x$

Then $I_{FE} = x - 30$

$I_{ED} = x + 40$

$I_{DC} = x - 80$

$I_{CB} = x - 20$

$I_{BA} = x - 80$

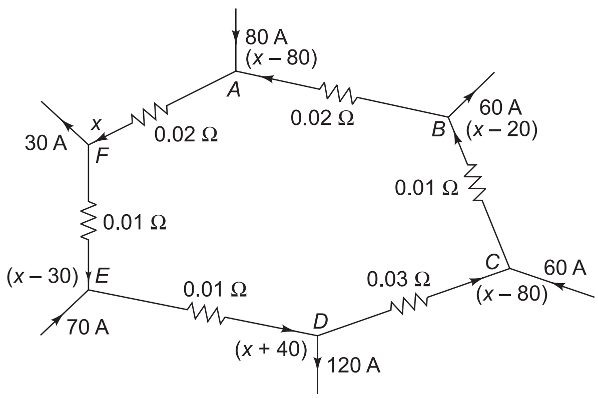


Fig. 2.8

Applying KVL to the closed path *AFEDCBA*,

$$-0.02\,x - 0.01\,(x - 30) - 0.01\,(x + 40) - 0.03\,(x - 80) - 0.01\,(x - 20) - 0.02\,(x - 80) = 0$$

$$x = 41\text{A}$$
$$I_{AF} = 41 \text{ A}$$
$$I_{FE} = 41 - 30 = 11 \text{ A}$$
$$I_{ED} = 41 + 40 = 81 \text{ A}$$
$$I_{DC} = 41 - 80 = -39 \text{ A}$$
$$I_{CD} = 39 \text{ A}$$
$$I_{CB} = 41 - 20 = 21 \text{ A}$$
$$I_{BA} = 41 - 80 = -39 \text{ A}$$
$$I_{AB} = 39 \text{ A}$$

Example 4

Find currents in all the branches of the network.

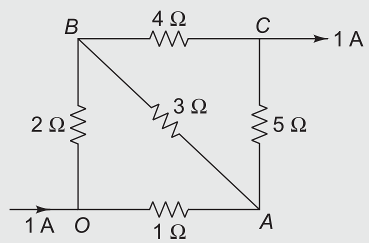


Fig. 2.9

Solution Assigning currents to all the branches,

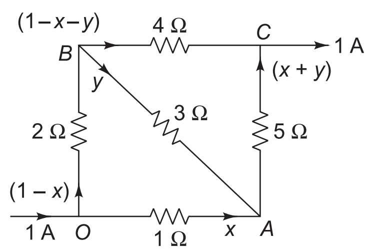


Fig. 2.10

Applying KVL to the closed path *OBAO*,

$$-2(1 - x) - 3y + 1(x) = 0$$
$$3x - 3y = 2 \tag{1}$$

Applying KVL to the closed path *ABCA*,

$$3y - 4(1 - x - y) + 5(x + y) = 0$$

$$9x + 12y = 4 \qquad (2)$$

Solving Eqs (1) and (2),

$$x = 0.57 \text{ A}$$

$$y = -0.095 \text{ A}$$

$$I_{OA} = 0.57 \text{ A}$$

$$I_{OB} = 1 - 0.57 = 0.43 \text{ A}$$

$$I_{AB} = 0.095 \text{ A}$$

$$I_{AC} = 0.57 - 0.095 = 0.475 \text{ A}$$

$$I_{BC} = 1 - 0.57 + 0.095 = 0.525 \text{ A}$$

Example 5

What is the potential difference between points x and y in the network?

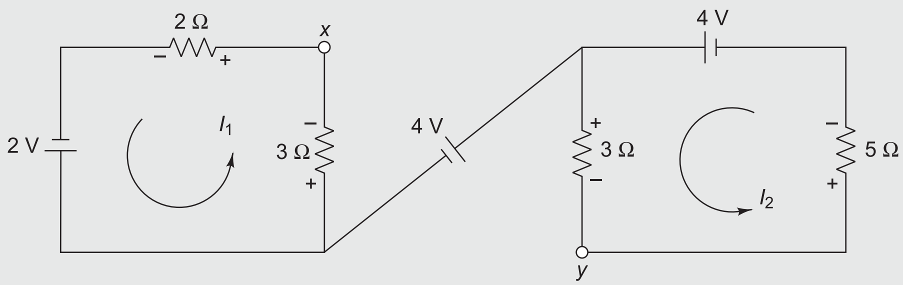


Fig. 2.11

Solution

$$I_1 = \frac{2}{2+3} = 0.4 \text{ A}$$

$$I_2 = \frac{4}{3+5} = 0.5 \text{ A}$$

Potential difference between points x and $y = V_{xy} = V_x - V_y$

Writing KVL equation for the path x to y,

$$V_x + 3I_1 + 4 - 3I_2 - V_y = 0$$

$$V_x + 3(0.4) + 4 - 3(0.5) - V_y = 0$$

$$V_x - V_y = -3.7$$

$$V_{xy} = -3.7 \text{ V}$$

Example 6

Find the voltage between points A and B.

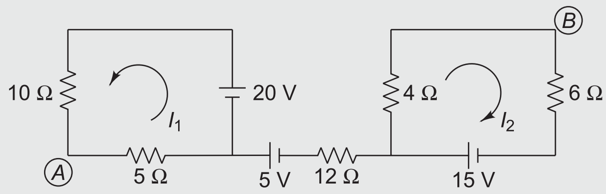


Fig. 2.12

Solution

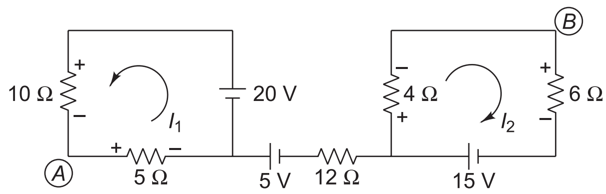


Fig. 2.13

$$I_1 = \frac{20}{10+5} = 1.33\text{ A}$$

$$I_2 = \frac{15}{4+6} = 1.5\text{ A}$$

Voltage between points A and $B = V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 5I_1 - 5 - 15 + 6I_2 - V_B = 0$$

$$V_A - 5(1.33) - 5 - 15 + 6(1.5) - V_B = 0$$

$$V_A - V_B = 17.65$$

$$V_{AB} = 17.65\text{ V}$$

Example 7

Determine the potential difference V_{AB} for the given network.

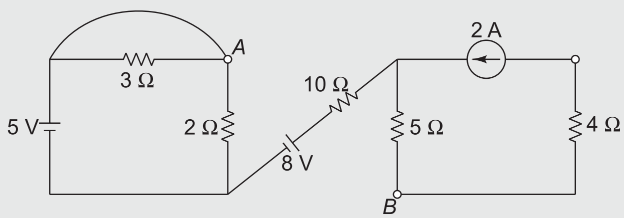


Fig. 2.14 **[May 2014]**

Solution The resistor of 3 Ω is connected across a short circuit. Hence, it gets shorted.

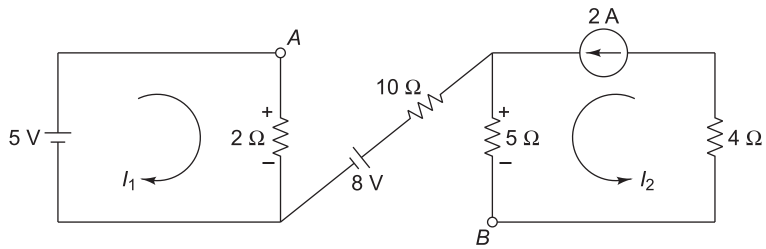


Fig. 2.15

$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Potential difference $\quad V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 8 - 5I_2 - V_B = 0$$

$$V_A - 2(2.5) + 8 - 5(2) - V_B = 0$$

$$V_A - V_B = 7$$

$$V_{AB} = 7 \text{ V}$$

Example 8

Find the voltage of the point A w.r.t. B.

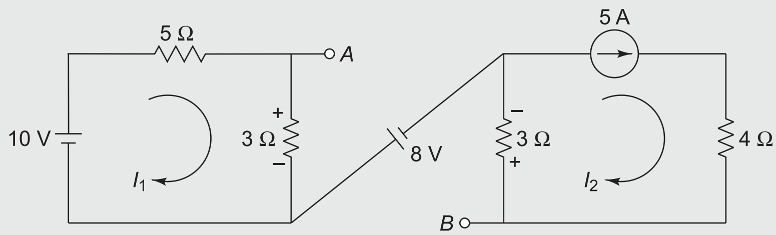


Fig. 2.16

Solution

$$I_1 = \frac{10}{5+3} = 1.25 \text{ A}$$

$$I_2 = 5 \text{ A}$$

Applying KVL to the path from A and B,

$$V_A - 3I_1 - 8 + 3I_2 - V_B = 0$$

$$V_A - 3(1.25) - 8 + 3(5) - V_B = 0$$

$$V_A - V_B = -3.25$$
$$V_{AB} = -3.25 \text{ V}$$

Example 9

Determine the value of resistance R as shown in Fig. 2.17.

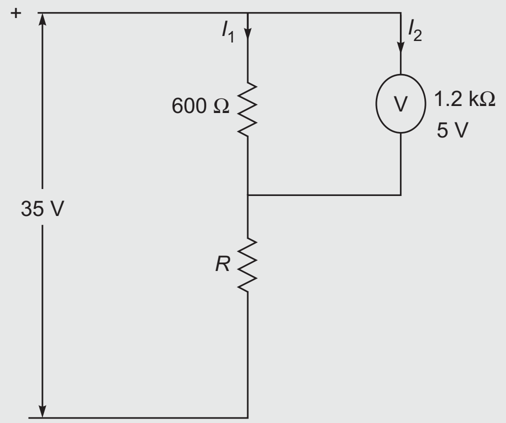


Fig. 2.17 **[Dec 2012]**

Solution

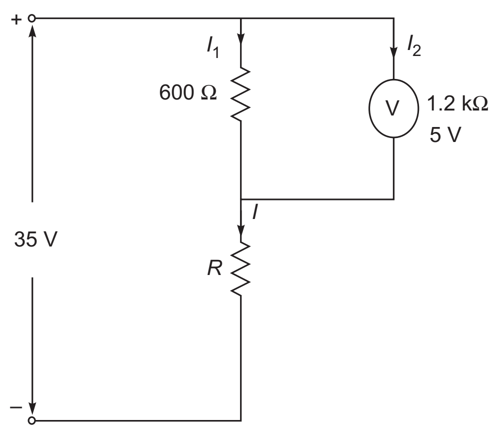


Fig. 2.18

$$I_2 = \frac{5}{1.2 \times 10^{-3}} = 4.17 \text{ mA}$$

$$I_1 = \frac{5}{600} = 8.33 \text{ mA}$$

$$I = I_1 + I_2 = 4.17 + 8.33 = 12.5 \text{ mA}$$

Applying KVL to the circuit,

$$35 - 600 I_1 - RI = 0$$

$$35 - 600(8.33 \times 10^{-3}) - R(12.5 \times 10^{-3}) = 0$$

$$R = 2.4 \text{ k}\Omega$$

Example 10

What values must R_1 and R_2 have

(a) when $I_1 = 4$ A and $I_2 = 6$ A both charging?

(b) when $I_1 = 2$ A discharging and $I_2 = 20$ A charging?

(c) when $I_1 = 0$?

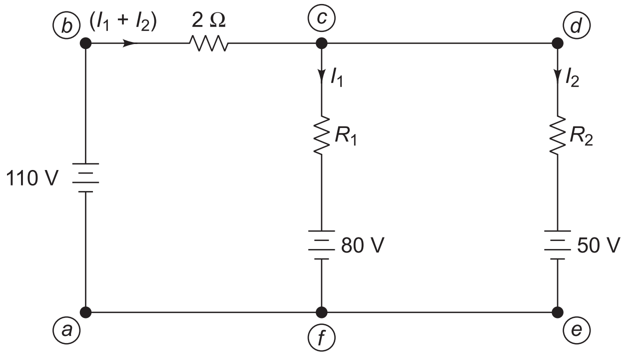


Fig. 2.19

Solution Applying KVL to closed path *abcfa*,

$$110 - 2(I_1 + I_2) - R_1 I_1 - 80 = 0$$

$$110 - 2I_1 - 2I_2 - R_1 I_1 - 80 = 0$$

$$(2 + R_1) I_1 + 2I_2 = 30 \tag{1}$$

Applying KVL to closed path *fcdef*,

$$80 + R_1 I_1 - R_2 I_2 - 50 = 0$$

$$R_1 I_1 - R_2 I_2 = -30 \tag{2}$$

Case (a) $I_1 = 4$ A and $I_2 = 6$ A both charging

i.e. $I_1 = 4$ A and $I_2 = 6$ A

Substituting I_1 and I_2 in Eq. (1),

$$(2 + R_1) 4 + 2(6) = 30$$

$$R_1 = 2.5\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (2),

$$2.5(4) - R_2(6) = -30$$

$$R_2 = 6.67\ \Omega$$

Case (b) $I_1 = 2$ A discharging and $I_2 = 20$ A charging

i.e. $$I_1 = -2 \text{ A and } I_2 = 20 \text{ A}$$

Substituting I_1 and I_2 in Eq. (1),

$$(2 + R_1)(-2) + 2(20) = 30$$
$$R_1 = 3\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (2),

$$3(-2) - R_2(20) = -30$$
$$R_2 = 1.2\ \Omega$$

Case (c) $I_1 = 0$

Substituting in Eq. (1),

$$(2 + R_1)(0) + 2I_2 = 30$$
$$I_2 = 15 \text{ A}$$

Substituting I_1 and I_2 in Eq. (2),

$$0 - 15R_2 = -30$$
$$R_2 = 2\ \Omega$$

Example 11

Find I_1 and I_2 when (a) $R = 2.3\ \Omega$, (b) $R = 0.5\ \Omega$, and (c) for what values of R is $I_1 = 0$?

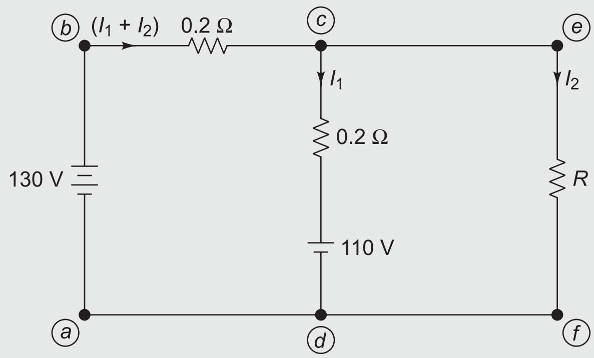


Fig. 2.20

Solution Applying KVL to closed path *abcda*,

$$130 - 0.2(I_1 + I_2) - 0.2I_1 - 110 = 0$$
$$0.4I_1 + 0.2I_2 = 20 \tag{1}$$

Applying KVL to closed path *dcefd*,

$$110 + 0.2I_1 - RI_2 = 0$$
$$0.2I_1 - RI_2 = -110 \tag{2}$$

Case (a) $R = 2.3\ \Omega$

Substituting R in Eq. (2),

$$0.2I_1 - 2.3I_2 = -110 \tag{3}$$

Solving Eqs (1) and (3),

$$I_1 = 25 \text{ A}$$

$$I_2 = 50 \text{ A}$$

Case (b) $R = 0.5\ \Omega$

Substituting R in Eq. (2),

$$0.2I_1 - 0.5I_2 = -110 \tag{4}$$

Solving Eqs (1) and (4),

$$I_1 = -50 \text{ A}$$

$$I_2 = 200 \text{ A}$$

Case (c) $I_1 = 0$

Substituting I_1 in Eq. (1),

$$0.2I_2 = 20$$

$$I_2 = 100 \text{ A}$$

Substituting I_1 and I_2 in Eq. (2),

$$0.2(0) - R\,(100) = -110$$

$$R = 1.1\ \Omega$$

Example 12

Find the value of R.

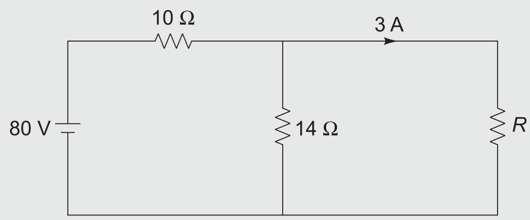


Fig. 2.21

Solution Assigning currents to all the branches,

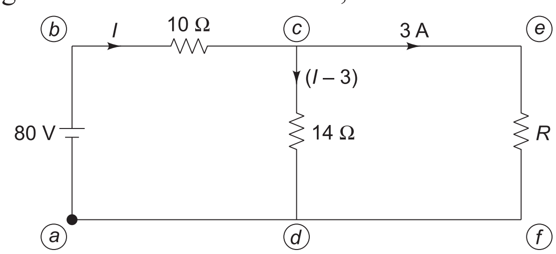


Fig. 2.22

Applying KVL to closed path *abcda*,

$$80 - 10I - 14\,(I - 3) = 0$$

$$I = 5.08 \text{ A}$$

Applying KVL to closed path *dcefd*,

$$14\,(I - 3) - 3R = 0$$

$$14\,(5.08 - 3) - 3R = 0$$

$$R = 9.71\ \Omega$$

Example 13

Determine current drawn by the ammeter shown in Fig. 2.23.

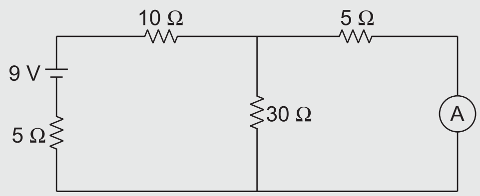


Fig. 2.23

Solution Assigning currents to all the branches,

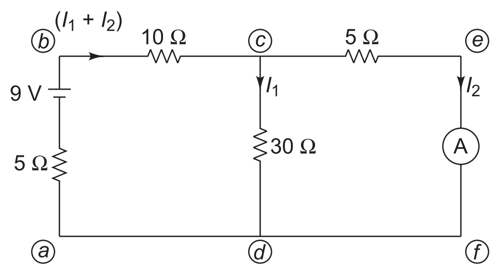


Fig. 2.24

Applying KVL to closed path *abcda*,

$$-5\,(I_1 + I_2) + 9 - 10\,(I_1 + I_2) - 30I_1 = 0$$

$$45I_1 + 15I_2 = 9 \qquad (1)$$

Applying KVL to closed path *dcefd*,

$$30I_1 - 5I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = 0.4 \text{ A}$$

Current drawn by ammeter = 0.4 A

Example 14

Find branch currents in various branches of Fig. 2.25.

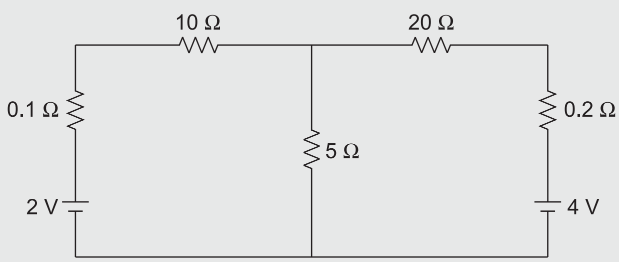


Fig. 2.25

Solution Assigning currents to various branches,

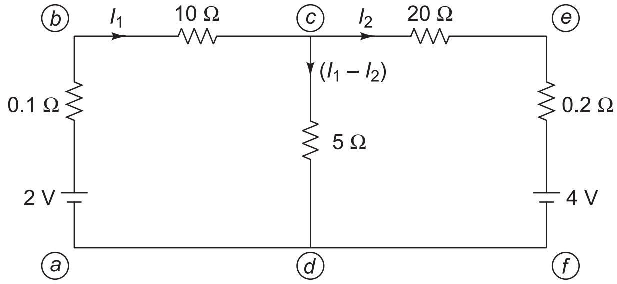


Fig. 2.26

Applying KVL to closed path *abcda*,

$$2 - 0.1I_1 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15.1I_1 - 5I_2 = 2 \tag{1}$$

Applying KVL to closed path *dcefd*,

$$5(I_1 - I_2) - 20I_2 - 0.2I_2 - 4 = 0$$

$$5I_1 - 25.2I_2 = 4 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 0.086 \text{ A}$$

$$I_2 = -0.142 \text{ A}$$

Example 15

Find the value of R and current flowing through it when the current is zero in the branch OA.

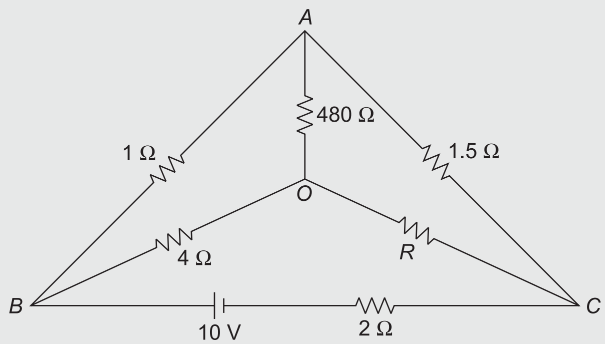


Fig. 2.27

Solution Assigning currents to all branches,

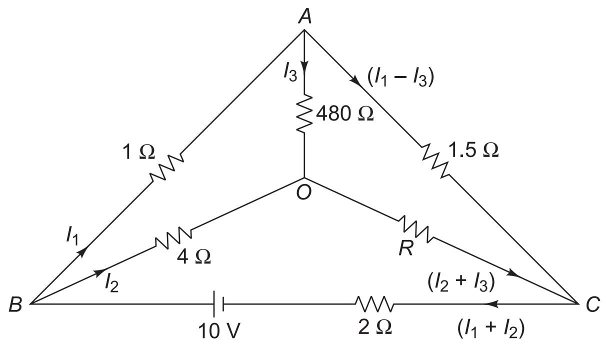


Fig. 2.28

Applying KVL to the closed path *OACO*,

$$480I_3 - 1.5\,(I_1 - I_3) + R\,(I_2 + I_3) = 0$$

But current in the branch *OA* is zero,

i.e. $$I_3 = 0$$

$$-1.5I_1 + RI_2 = 0 \tag{1}$$

Applying KVL to the closed path *BOCB*,

$$-4I_2 - R\,(I_2 + I_3) - 2\,(I_1 + I_2) + 10 = 0$$

But $$I_3 = 0$$

$$-2I_1 - (6 + R)\,I_2 = -10 \tag{2}$$

Applying KVL to the closed path *BOAB*,

$$-4I_2 + 480I_3 + I_1 = 0$$

But $$I_3 = 0$$

$$-4I_2 + I_1 = 0 \tag{3}$$

$$I_1 = 4I_2$$

Substituting I_1 in Eq. (1) and (2),

$$-6I_2 + RI_2 = 0 \tag{4}$$

and $$-14I_2 - RI_2 = -10 \tag{5}$$

From Eq. (4) and (5)

$$I_2 = 0.5 \text{ A}$$

Substituting I_2 in Eq. (4),

$$-6\,(0.5) + R\,(0.5) = 0$$

$$R = 6\ \Omega$$

Current in branch $OC = I_2 + I_3 = 0.5$ A

Example 16

Find the current supplied by the battery.

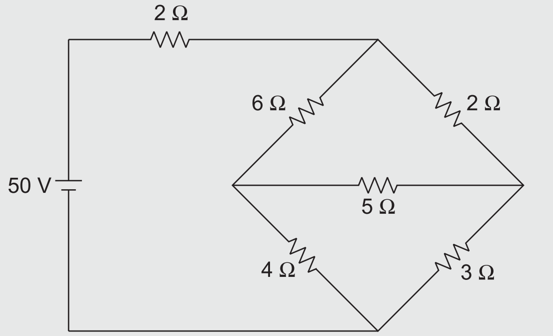


Fig. 2.29

Solution Assigning currents to all the branches,

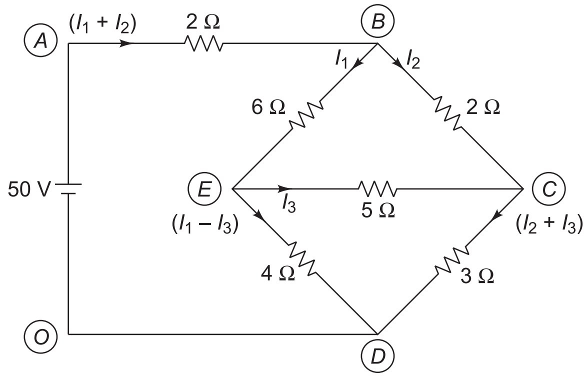


Fig. 2.30

Applying KVL to the closed path *OABEDO*,

$$50 - 2(I_1 + I_2) - 6I_1 - 4(I_1 - I_3) = 0$$

$$12I_1 + 2I_2 - 4I_3 = 50 \quad (1)$$

Applying KVL to the closed path *BCEB*,

$$-2I_2 + 5I_3 + 6I_1 = 0$$

$$6I_1 - 2I_2 + 5I_3 = 0 \quad (2)$$

Applying KVL to the closed path *ECDE*,

$$-5I_3 - 3(I_2 + I_3) + 4(I_1 - I_3) = 0$$

$$4I_1 - 3I_2 - 12I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2.817 \text{ A}$$

$$I_2 = 6.647 \text{ A}$$

$$I_3 = -0.723 \text{ A}$$

Current supplied by the battery $= I_1 + I_2$

$$= 2.817 + 6.647$$
$$= 9.464 \text{ A}$$

Example 17

Find the value of current flowing through the 2 Ω resistor.

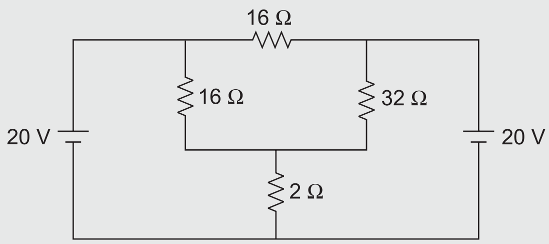


Fig. 2.31

Solution Assigning currents to all the branches,

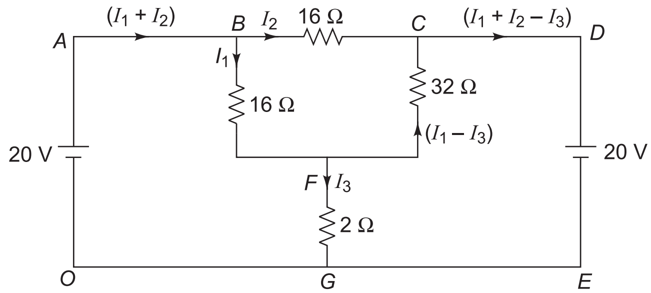


Fig. 2.32

Applying KVL to the closed path *OABFGO,*

$$20 - 16I_1 - 2I_3 = 0$$
$$16I_1 + 2I_3 = 20 \quad (1)$$

Applying KVL to the closed path *BCFB,*

$$-16I_2 + 32(I_1 - I_3) + 16I_1 = 0$$
$$48I_1 - 16I_2 + 32I_3 = 0 \quad (2)$$

Applying KVL to the closed path *GFCDEG,*

$$2I_3 - 32(I_1 - I_3) - 20 = 0$$
$$-32I_1 + 34I_3 = 20 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.05 \text{ A}$$
$$I_2 = 6.32 \text{ A}$$
$$I_3 = 1.58 \text{ A}$$

Current through the 2 Ω resistor $= I_3 = 1.58$ A

Example 18

Find the value of current flowing through the 4 Ω resistor.

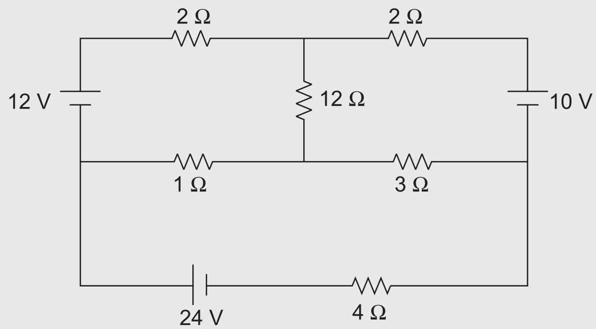


Fig. 2.33

Solution Assigning currents to all the branches,

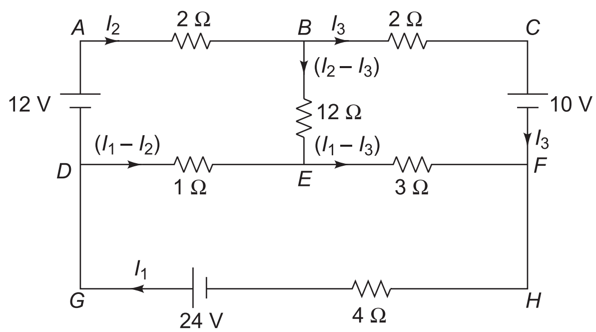


Fig. 2.34

Applying KVL to the closed path *ABEDA*,

$$-2I_2 - 12(I_2 - I_3) + 1(I_1 - I_2) + 12 = 0$$

$$I_1 - 15I_2 + 12I_3 = -12 \quad (1)$$

Applying KVL to the closed path *BCFEB*,

$$-2I_3 - 10 + 3(I_1 - I_3) + 12(I_2 - I_3) = 0$$

$$3I_1 + 12I_2 - 17I_3 = 10 \quad (2)$$

Applying KVL to the closed path *DEFHGD*,

$$-1(I_1 - I_2) - 3(I_1 - I_3) - 4I_1 + 24 = 0$$

$$-8I_1 + I_2 + 3I_3 = -24 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.11 \text{ A}$$

$$I_1 = 2.72 \text{ A}$$

$$I_3 = 2.06$$

Current through the 4 Ω resistor = I_1 = 4.11 A

Example 19

Find the value of current flowing through the 10 Ω resistor.

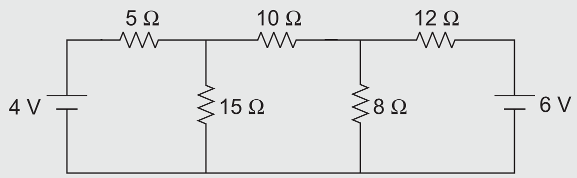


Fig. 2.35

Solution Assigning currents to all the branches,

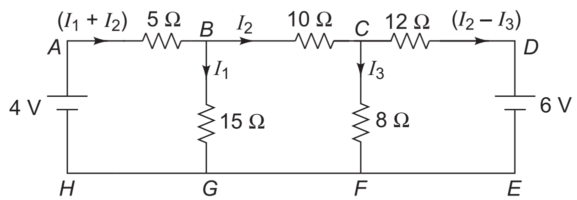


Fig. 2.36

Applying KVL to the closed path *ABGHA*,

$$-5(I_1 + I_2) - 15I_1 + 4 = 0$$

$$-20I_1 - 5I_2 = -4 \quad (1)$$

Applying KVL to the closed path *BCFGB*,

$$-10I_2 - 8I_3 + 15I_1 = 0$$

$$15I_1 - 10I_2 - 8I_3 = 0 \quad (2)$$

Applying KVL to the closed path *CDEFC,*

$$-12(I_2 - I_3) - 6 + 8I_3 = 0$$

$$-12I_2 + 20I_3 = 6 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.19 \text{ A}$$

$$I_2 = 0.032 \text{ A}$$

$$I_3 = 0.32 \text{ A}$$

Current through the 10 Ω resistor = I_2 = 0.032 A

Example 20

Determine the current supplied by each battery.

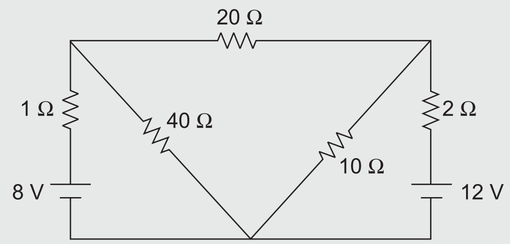


Fig. 2.37

Solution Assigning currents to all the branches,

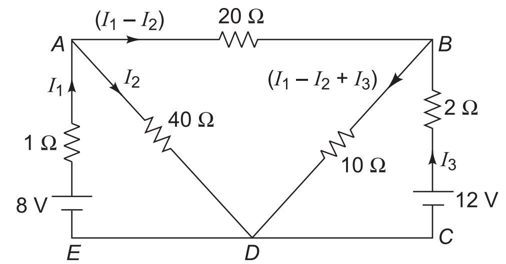


Fig. 2.38

Applying KVL to the closed path *ADEA*,

$$-40I_2 + 8 - 1I_1 = 0$$

$$I_1 + 40I_2 = 8 \tag{1}$$

Applying KVL to the closed path *ABDA*,

$$-20(I_1 - I_2) - 10(I_1 - I_2 + I_3) + 40I_2 = 0$$

$$-30I_1 + 70I_2 - 10I_3 = 0 \tag{2}$$

Applying KVL to the closed path *BCDB*,

$$2I_3 - 12 + 10\,(I_1 - I_2 + I_3) = 0$$

$$10I_1 - 10I_2 + 12I_3 = 12 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.1005 \text{ A}$$

$$I_2 = 0.197 \text{ A}$$

$$I_3 = 1.081 \text{ A}$$

Current supplied by the 8 V battery = I_1 = 0.1005 A

Current supplied by the 12 V battery = I_3 = 1.081 A

Example 21

Find the value of the unknown resistance R such that 2 A current flows through it.

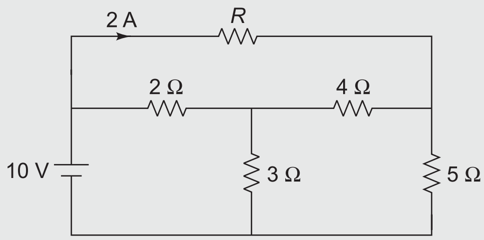


Fig. 2.39

Solution Assigning currents to all the branches,

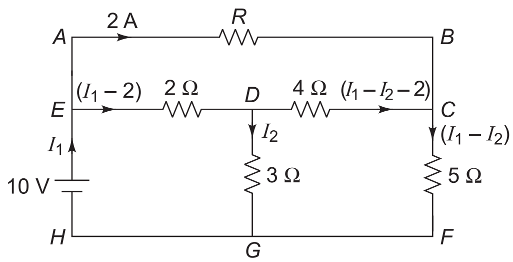


Fig. 2.40

Applying KVL to the closed path *ABCDEA*,

$$-2R + 4(I_1 - I_2 - 2) + 2(I_1 - 2) = 0$$

$$6I_1 - 4I_2 - 2R = 12 \qquad (1)$$

Applying KVL to the closed path *HEDGH*,

$$10 - 2(I_1 - 2) - 3I_2 = 0$$

$$2I_1 + 3I_2 = 14 \qquad (2)$$

Applying KVL to the closed path *GDCFG*,

$$3I_2 - 4(I_1 - I_2 - 2) - 5(I_1 - I_2) = 0$$

$$9I_1 - 12I_2 = 8 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 3.76 \text{ A}$$

$$I_2 = 2.16 \text{ A}$$

$$R = 0.98\ \Omega$$

Unknown resistance $R = 0.98\ \Omega$

Example 22

Find the current delivered by the 12V battery.

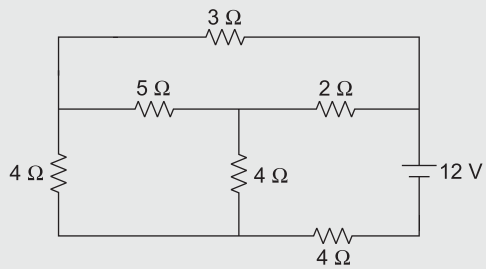


Fig. 2.41

Solution Assigning currents to all the branches,

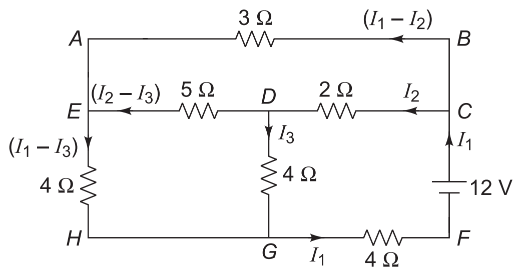


Fig. 2.42

Applying KVL to the closed path *ABCDEA*,

$$3(I_1 - I_2) - 2I_2 - 5\,(I_2 - I_3) = 0$$

$$3I_1 - 10I_2 + 5I_3 = 0 \quad (1)$$

Applying KVL to the closed path *HEDGH*,

$$4(I_1 - I_3) + 5\,(I_2 - I_3) - 4I_3 = 0$$

$$4I_1 + 5I_2 - 13I_3 = 0 \quad (2)$$

Applying KVL to the closed path *GDCFG*,

$$4I_3 + 2I_2 - 12 + 4I_1 = 0$$

$$4I_1 + 2I_2 + 4I_3 = 12 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.66\text{ A}$$

$$I_2 = 0.93\text{ A}$$

$$I_2 = 0.87\text{ A}$$

Current delivered by the 12 V battery = I_1 = 1.66 A

Exercise 2.1

2.1 Replace the network of sources shown below with
(i) $V_{aa'}$ (ii) $I_{bb'}$
(i)

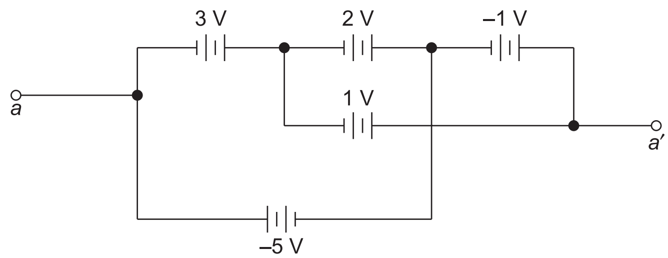


Fig. 2.43

[– 4 V]

(ii)

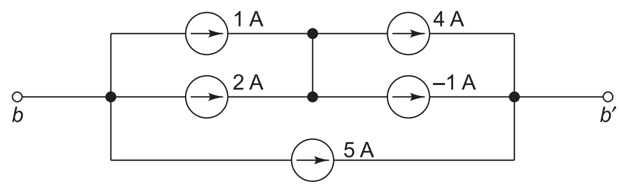


Fig. 2.44

[8 A]

2.2 Find I_x and V_x in the network shown in Fig. 2.45.

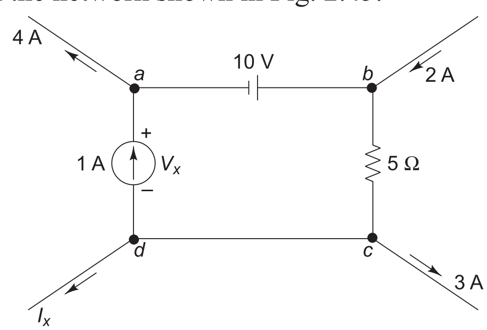


Fig. 2.45

[– 5 A, – 15 V]

2.3 Find V_1 and V_2 in the network shown in Fig. 2.46.

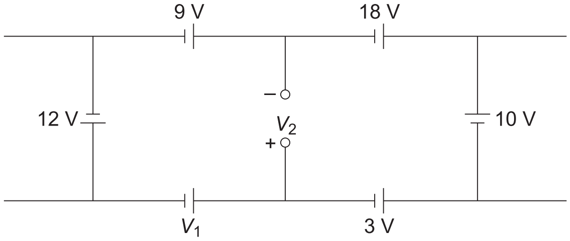


Fig. 2.46

[2 V, 5 V]

2.4 Find the values of unknown currents as shown in Fig. 2.47.

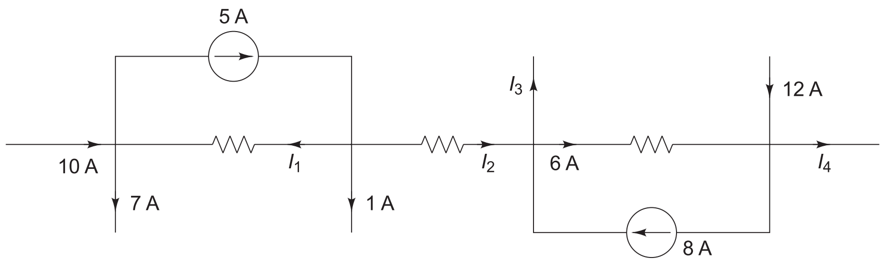


Fig. 2.47

[$I_1 = 2$ A, $I_2 = 2$ A, $I_3 = 4$ A, $I_4 = 10$ A]

2.5 Find the current in the branch *XY* as shown in Fig. 2.48.

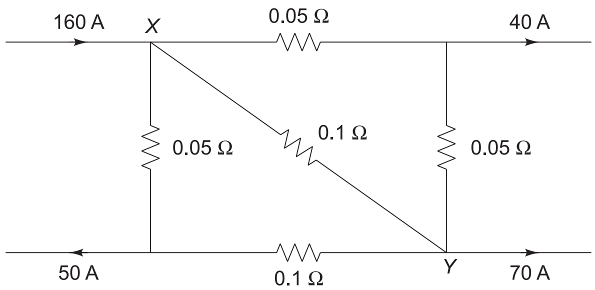


Fig. 2.48

[40 A]

2.6 Find I and V_{AB} for the network as shown in Fig. 2.49.

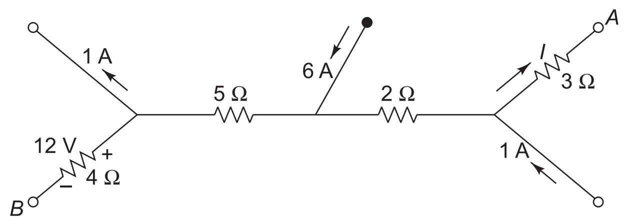


Fig. 2.49

[3 A, 19 V]

2.7 For the network as shown in Fig. 2.50, determine

(i) I_1, I_2 and I_3

(ii) R

(iii) E

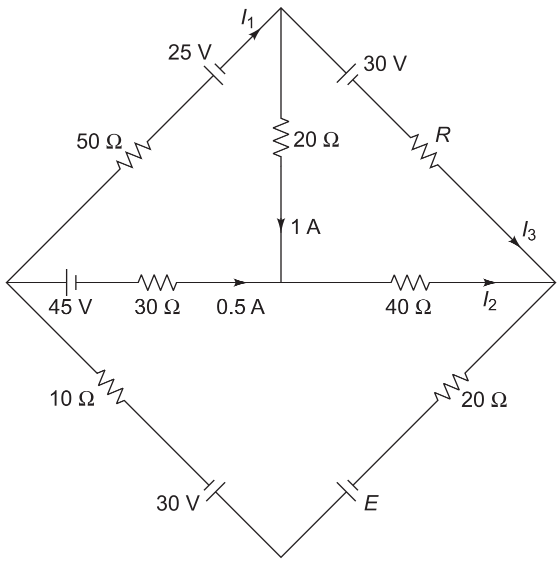


Fig. 2.50

[(i) 0.3 A, 1.5 A, – 0.7 A (ii) 71.43 Ω (iii) 174 V]

2.8 Find the value of current flowing through the 5 Ω resistor.

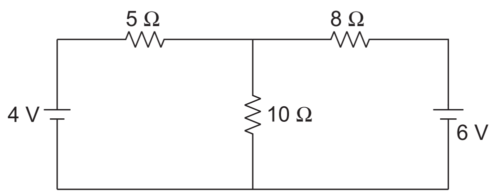


Fig. 2.51

[0.071 A]

2.9 Find the value of current flowing in the 4 Ω resistor.

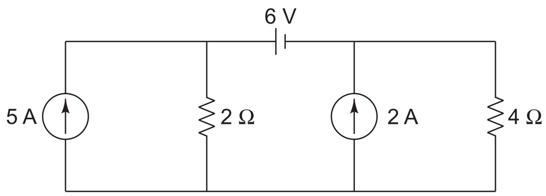


Fig. 2.52

[1.34 A]

2.10 Find the current I_1.

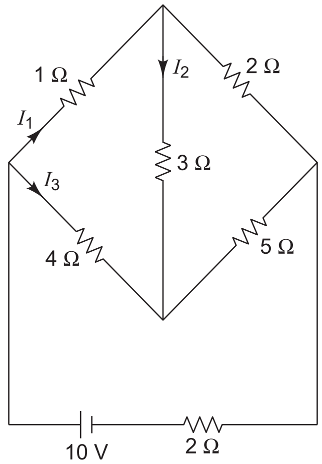


Fig. 2.53

[1.83 A]

2.11 In the network shown in Fig. 2.54, determine the value of E_2 which will reduce the galvanometer current to zero.

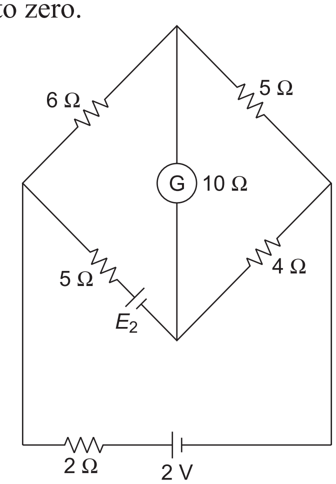


Fig. 2.54

[– 0.0322 V]

2.12 Determine the current supplied by the battery.

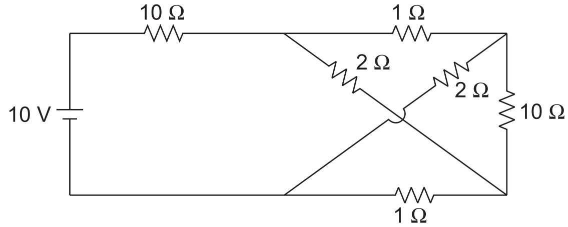


Fig. 2.55

[0.87 A]

2.13 In the network shown in Fig. 2.56, find the voltage between points A and B.

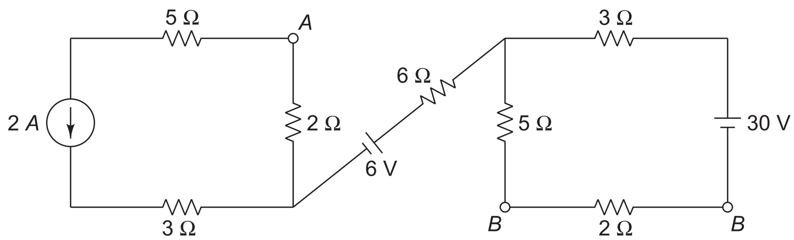


Fig. 2.56

[5 V]

2.14 In the network shown in Fig. 2.57, find the voltage between points A and B.

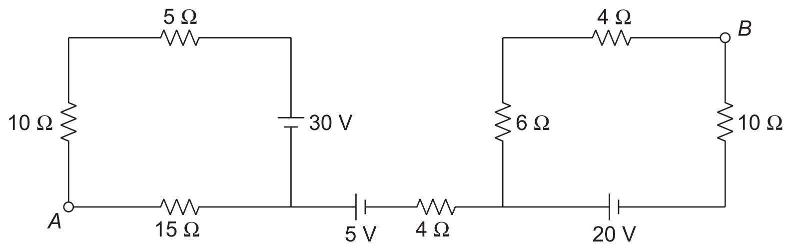


Fig. 2.57

[30 V]

2.15 Using KVL and KCL, find the values of V and I in Fig. 2.58.

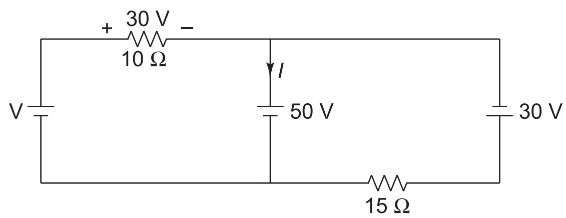


Fig. 2.58

[80 V, – 2.33 A]

2.16 Using KCL, find the values of V_{AB}, I_1, I_2 and I_3 in Fig. 2.59.

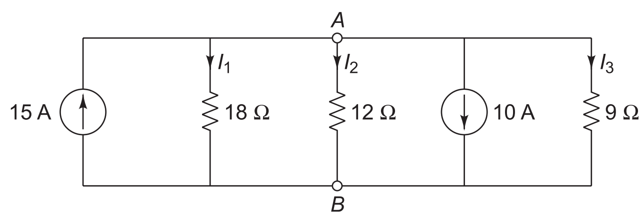


Fig. 2.59

[20 V, 1.11 A, 1.67 A, 2.22 A]

2.2 MESH ANALYSIS

A mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applicable only for planar networks. A network is said to be planar if it can be drawn on a plane surface without crossovers. In this method, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. If a network has a large number of voltage sources, it is useful to use mesh analysis. Basically, this analysis consists of writing mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents.

2.2.1 Steps to be Followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

Consider the network shown in Fig. 2.60 which has three meshes. Let the mesh currents for the three meshes be I_1, I_2 and I_3 and all the three mesh currents may be assumed to flow in the clockwise direction. The choice of direction for any mesh current is arbitrary.

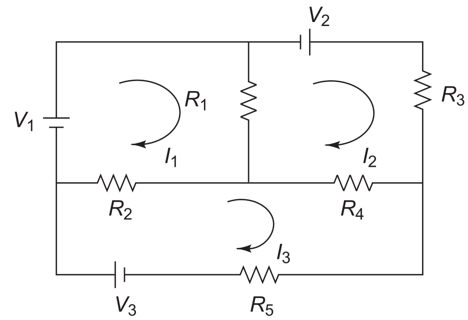


Fig. 2.60 *Mesh analysis*

Applying KVL to Mesh 1,

$$V_1 - R_1 (I_1 - I_2) - R_2 (I_1 - I_3) = 0$$

$$(R_1 + R_2) I_1 - R_1 I_2 - R_2 I_3 = V_1 \tag{2.1}$$

Applying KVL to Mesh 2,

$$V_2 - R_3 I_2 - R_4 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$-R_1 I_1 + (R_1 + R_3 + R_4) I_2 - R_4 I_3 = V_2 \tag{2.2}$$

Applying KVL to Mesh 3,

$$-R_2(I_3 - I_1) - R_4 (I_3 - I_2) - R_5 I_3 + V_3 = 0$$

$$-R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_5) I_3 = V_3 \qquad (2.3)$$

Writing Eqs (2.1), (2.2) and (2.3) in matrix form,

$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 \\ -R_1 & R_1 + R_3 + R_4 & -R_4 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

In general,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where,

R_{11} = Self-resistance or sum of all the resistances of Mesh 1

$R_{12} = R_{21}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 2

$R_{13} = R_{31}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 3

R_{22} = Self-resistance or sum of all the resistances of Mesh 2

$R_{23} = R_{32}$ = Mutual resistance or sum of all the resistances common to meshes 2 and 3

R_{33} = Self-resistance or sum of all the resistances of Mesh 3

If the directions of the currents passing through the common resistor are the same, the mutual resistance will have a positive sign, and if the direction of the currents passing through common resistor are opposite, then the mutual resistance will have a negative sign. If each mesh currents are assumed to flow in the clockwise direction, then all self-resistances will be always positive and all mutual resistances will always be negative.

The voltages V_1, V_2 and V_3 represent the algebraic sum of all the voltages in meshes 1, 2 and 3 respectively. While going along the current, if we go from negative terminal of the battery to the positive terminal, then its emf is taken as positive. Otherwise, it is taken as negative.

Example 1

Find the value of current flowing through 1 Ω resistor.

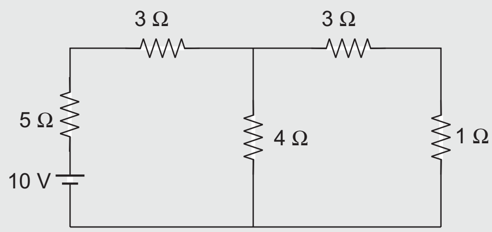


Fig. 2.61 **[May 2015]**

Solution Assigning clockwise currents in two meshes,

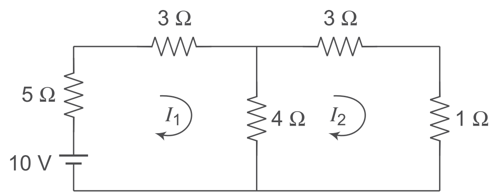


Fig. 2.62

Applying KVL to Mesh 1,

$$10 - 5I_1 - 3I_1 - 4(I_1 - I_2) = 0$$

$$12I_1 - 4I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4(I_2 - I_1) - 3I_2 - 1I_2 = 0$$

$$-4I_1 + 8I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1\text{ A}$$

$$I_2 = 0.5\text{ A}$$

$$I_{1\,\Omega} = I_2 = 1.5\text{ A}$$

Example 2

Find the value of current flowing through 5 Ω resistor.

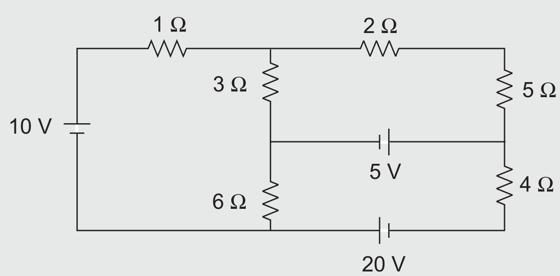


Fig. 2.63

Solution Assigning clockwise currents in three meshes,

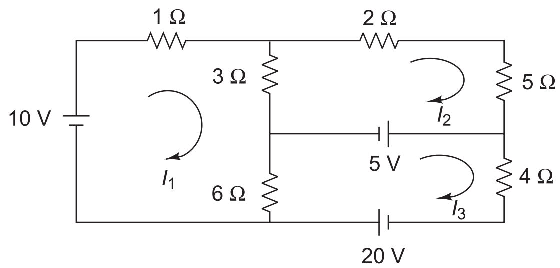


Fig. 2.64

Applying KVL to Mesh 1,

$$10 - 1(I_1) - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$

$$10I_1 - 3I_2 - 6I_3 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-3\,(I_2 - I_1) - 2I_2 - 5I_2 - 5 = 0$$

$$-3I_1 + 10I_2 = -5 \qquad (2)$$

Applying KVL to Mesh 3,

$$-6\,(I_3 - I_1) + 5 - 4I_3 + 20 = 0$$

$$-6I_1 + 10I_3 = 25 \qquad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix}$$

We can write matrix equation directly from Fig. 2.64.

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where R_{11} = Self-resistance of Mesh 1 = 1 + 3 + 6 = 10 Ω

R_{12} = Mutual resistance common to meshes 1 and 2 = –3 Ω

Here, negative sign indicates that the currents through common resistance are in opposite direction.

R_{13} = Mutual resistance common to meshes 1 and 3 = –6 Ω

Similarly,

$$R_{21} = -3\ \Omega$$

$$R_{22} = 3 + 2 + 5 = 10\ \Omega$$

$$R_{23} = 0$$

$$R_{31} = -6\ \Omega$$
$$R_{32} = 0$$
$$R_{33} = 6 + 4 = 10\ \Omega$$

For voltage matrix,

$$V_1 = 10\text{ V}$$
$$V_2 = -5\text{ V}$$
$$V_3 = \text{algebraic sum of all the voltages in Mesh 3} = 5 + 20 = 25\text{ V}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.27\text{ A}$$
$$I_2 = 0.78\text{ A}$$
$$I_3 = 5.06\text{ A}$$
$$I_{5\Omega} = I_2 = 0.78\text{ A}$$

Example 3

Find the value of current flowing through the 2 Ω resistor.

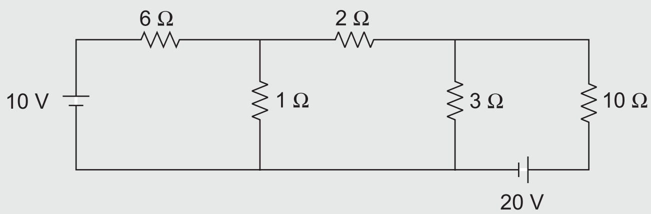


Fig. 2.65

Solution Assigning clockwise currents in the three meshes,

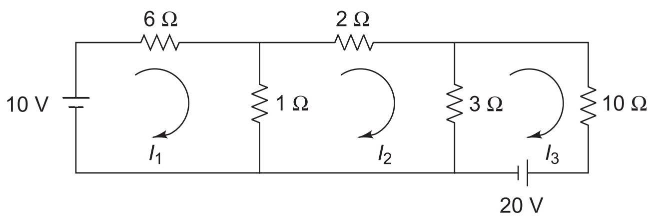


Fig. 2.66

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$
$$7I_1 - I_2 = 10 \tag{1}$$

Applying KVL to Mesh 2,

$$-(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$
$$-I_1 + 6I_2 - 3I_3 = 0 \tag{2}$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 10I_3 - 20 = 0$$

$$-3I_2 + 13I_3 = -20 \tag{3}$$

Writing equations in matrix form,

$$\begin{bmatrix} 7 & -1 & 0 \\ -1 & 6 & -3 \\ 0 & -3 & 13 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.34 \text{ A}$$
$$I_2 = -0.62 \text{ A}$$
$$I_3 = -1.68 \text{ A}$$
$$I_{2\Omega} = I_2 = -0.62 \text{ A}$$

Example 4

Determine the value of current flowing through the 5 Ω resistor.

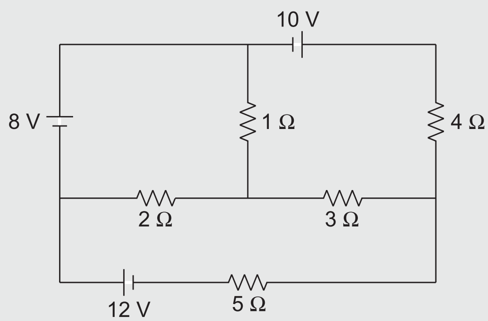


Fig. 2.67

Solution Assigning clockwise currents in the three meshes,

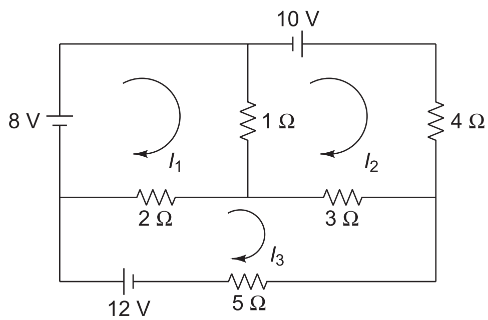


Fig. 2.68

Applying KVL to Mesh 1,

$$8 - 1(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$3I_1 - I_2 - 2I_3 = 8 \tag{1}$$

Applying KVL to Mesh 2,

$$10 - 4I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 3I_3 = 10 \tag{2}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 3(I_3 - I_2) - 5I_3 + 12 = 0$$

$$-2I_1 - 3I_2 + 10I_3 = 12 \tag{3}$$

Writing equations in matrix form,

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 8 & -3 \\ -2 & -3 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 6.01 \text{ A}$$

$$I_2 = 3.27 \text{ A}$$

$$I_3 = 3.38 \text{ A}$$

$$I_{5\Omega} = I_3 = 3.38 \text{ A}$$

Example 5

Find the value of current flowing through 4 Ω resistor.

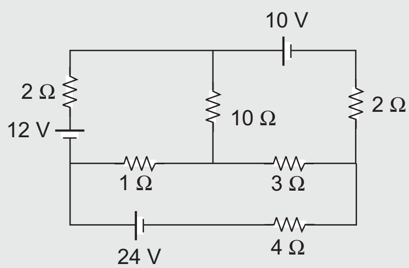


Fig. 2.69 **[Dec 2015]**

Solution Assigning clockwise currents in three meshes,

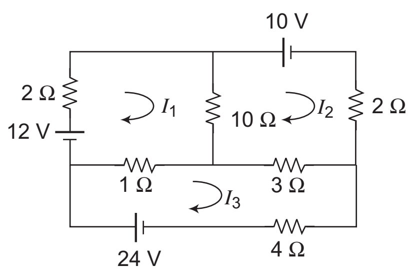


Fig. 2.70

Applying KVL to Mesh 1,

$$12 - 12I_1 - 10(I_1 - I_2) - 1(I_1 - I_3) = 0$$

$$13I_1 - 10I_2 - I_3 = 12 \qquad (1)$$

Applying KVL to Mesh 2,

$$-10(I_2 - I_1) - 10 - 2I_2 - 3(I_2 - I_3) = 0$$

$$-10I_1 + 15I_2 - 3I_3 = -10 \qquad (2)$$

Applying KVL to Mesh 3,

$$-1(I_3 - I_1) - 3(I_3 - I_2) - 4I_3 + 24 = 0$$

$$-I_1 - 3I_2 + 8I_3 = 24 \qquad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 13 & -10 & -1 \\ -10 & 15 & -3 \\ -1 & -3 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 24 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2.79 \text{ A}$$

$$I_2 = 2.01 \text{ A}$$

$$I_3 = 4.1 \text{ A}$$

$$I_{4\Omega} = I_3 = 4.1 \text{ A}$$

Example 6

Find the value of current supplied by the battery.

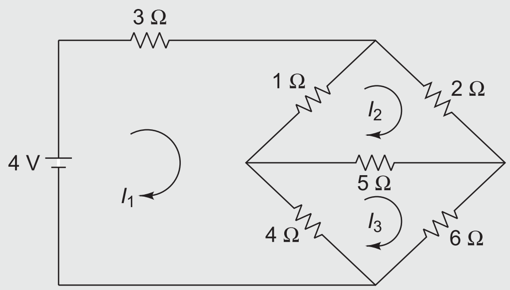


Fig. 2.71

Solution

Applying KVL to Mesh 1,

$$4 - 3I_1 - 1(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$8I_1 - I_2 - 4I_3 = 4 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2I_2 - 5(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 5I_3 = 0 \qquad (2)$$

Applying KVL to Mesh 3,

$$-6I_3 - 4(I_3 - I_1) - 5(I_3 - I_2) = 0$$

$$-4I_1 - 5I_2 + 15I_3 = 0 \qquad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 8 & -1 & -4 \\ -1 & 8 & -5 \\ -4 & -5 & 15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.66 \text{ A}$$

$$I_2 = 0.24 \text{ A}$$

$$I_3 = 0.26 \text{ A}$$

Current supplied by the battery = I_1 = 0.66 A.

Example 7

Determine the voltage V which cause the current I_1 to be zero.

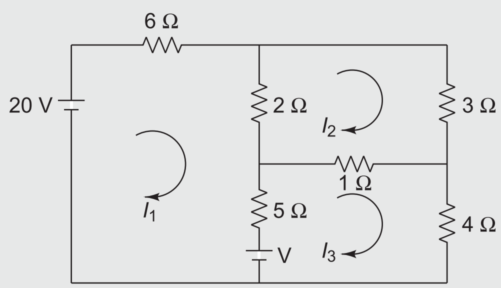


Fig. 2.72

Solution

Applying KVL to Mesh 1,

$$20 - 6I_1 - 2(I_1 - I_2) - 5(I_1 - I_3) - V = 0$$

$$V + 13I_1 - 2I_2 - 5I_3 = 20 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 3I_2 - 1(I_2 - I_3) = 0$$

$$2I_1 - 6I_2 + I_3 = 0 \qquad (2)$$

Applying KVL to Mesh 3,

$$-1(I_3 - I_2) - 4I_3 + V - 5(I_3 - I_1) = 0$$

$$V + 5I_1 + I_2 - 10I_3 = 0 \qquad (3)$$

Putting $I_1 = 0$ in Eqs (1), (2) and (3),

$$V - 2I_2 - 5I_3 = 20$$

$$-6I_2 + I_3 = 0$$

$$V + I_2 - 10I_3 = 0$$

Writing equations in matrix form,

$$\begin{bmatrix} 1 & -2 & -5 \\ 0 & -6 & 1 \\ 1 & 1 & -10 \end{bmatrix} \begin{bmatrix} V \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

Solving Eqs (1), (2), and (3),

$$V = 43.7 \text{ V}$$

Example 8

Find the value of current flowing through 10 Ω resistor in the circuit shown in Fig. 2.73.

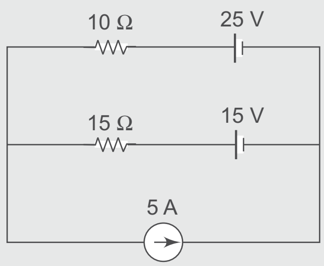


Fig. 2.73 **[May 2016]**

Solution Assigning clockwise currents in two meshes,

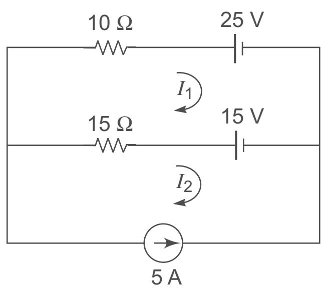


Fig. 2.74

Applying KVL to Mesh 1,

$$-10I_1 - 25 + 15 - 15(I_1 - I_2) = 0$$

$$25I_1 - 15I_2 = -10 \qquad (1)$$

Mesh 2 contains a current source of 5 A. Hence, we can write current equation for Mesh 2. Since direction of the current source and the mesh current I_2 are opposite,

$$I_2 = -5 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -3.4 \text{ A}$$

$$I_{10\Omega} = I_1 = -3.4 \text{ A}$$

Example 9

Find the value of current flowing through the 2 Ω resistor.

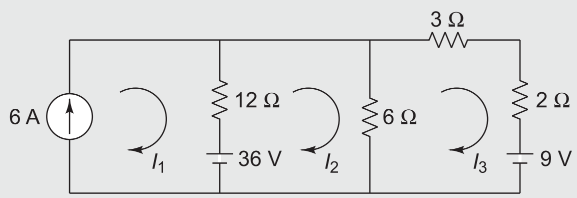


Fig. 2.75

Solution Mesh 1 contains a current source of 6 A. Hence, we can write current equation for Mesh 1. Since direction of the current source and the mesh current I_1 are same,

$$I_1 = 6 \qquad (1)$$

Applying KVL to Mesh 2,

$$36 - 12(I_2 - I_1) - 6(I_2 - I_3) = 0$$

$$-12I_1 + 18I_2 - 6I_3 = 36 \qquad (2)$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 - 2I_3 - 9 = 0$$

$$-6I_2 + 11I_3 = -9 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_2 = 7 \text{ A}$$

$$I_3 = 3 \text{ A}$$

$$I_{2\Omega} = I_3 = 3 \text{ A}$$

Example 10

Determine the mesh currents I_1, I_2 and I_3 in the network of Fig. 2.76.

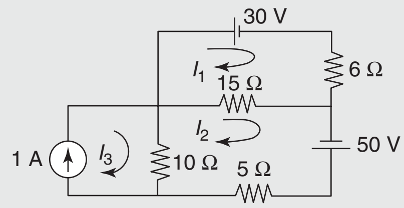


Fig. 2.76

Solution Applying KVL to Mesh 1,

$$-30-6I_1-15(I_1-I_2)=0$$

$$21I_1-15I_2=-30 \tag{1}$$

Applying KVL to Mesh 2,

$$-10(I_2-I_3)-15(I_2-I_1)+50-5I_2=0$$

$$-15I_1+30I_2-10I_3=50 \tag{2}$$

For Mesh 3,

$$I_3=1 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1=0$$

$$I_2=2\text{ A}$$

$$I_3=1\text{ A}$$

Example 11

Find the current through the 5 Ω resistor in the network of Fig. 2.77.

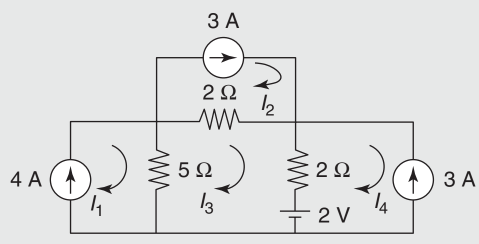


Fig. 2.77

Solution Writing current equations for Meshes 1, 2 and 4,

$$I_1=4 \tag{1}$$

$$I_2=3 \tag{2}$$

$$I_4=-3 \tag{3}$$

Applying KVL to Mesh 3,

$$-5(I_3 - I_1) - 2(I_3 - I_2) - 2(I_3 - I_4) - 2 = 0 \qquad (4)$$

Substituting Eqs (1), (2) and (3) in Eq. (4),

$$-5(I_3 - 4) - 2(I_3 - 3) - 2(I_3 + 3) - 2 = 0$$

$$I_3 = 2 \text{ A}$$

$$I_{5\Omega} = I_1 - I_3 = 4 - 2 = 2 \text{ A}$$

2.3 SUPERMESH ANALYSIS

Meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a supermesh. A path around a supermesh doesn't pass through a current source. A path around each mesh contained within a supermesh passes through a current source. The total number of equations required for a supermesh is equal to the number of meshes contained in the supermesh. A supermesh requires one mesh current equation, that is, a KVL equation. The remaining mesh current equations are KCL equations.

Example 1

Find the current through the 10 Ω resistor of the network shown in Fig. 2.78.

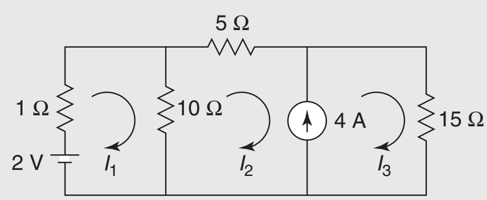


Fig. 2.78

Solution Applying KVL to Mesh 1,

$$2 - 1I_1 - 10(I_1 - I_2) = 0$$

$$11I_1 - 10I_2 = 2 \qquad (1)$$

Since meshes 2 and 3 contain a current source of 4 A, these two meshes will form a supermesh. A supermesh is formed by two adjacent meshes that have a common current source. The direction of the current source of 4 A and current $(I_3 - I_2)$ are same, i.e., in the upward direction.

Writing current equation to the supermesh,1

$$I_3 - I_2 = 4 \tag{2}$$

Applying KVL to the outer path of the supermesh,

$$-10(I_2 - I_1) - 5I_2 - 15I_3 = 0$$

$$10I_1 - 15I_2 - 15I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = -2.35 \text{ A}$$

$$I_2 = -2.78 \text{ A}$$

$$I_3 = 1.22 \text{ A} \tag{4}$$

Current through the 10 Ω resistor = $I_1 - I_2 = -(2.35) - (-2.78) = 0.43$ A

Example 2

Find the current in the 3 Ω resistor of the network shown in Fig. 2.79.

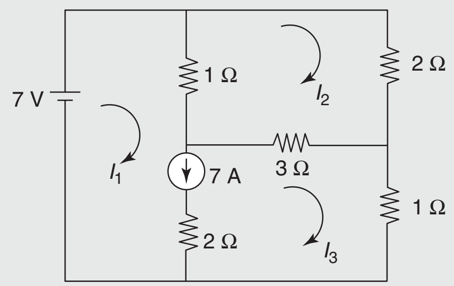


Fig. 2.79

Solution Meshes 1 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_3 = 7 \tag{1}$$

Applying KVL to the outer path of the supermesh,

$$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - 1I_3 = 0$$

$$-I_1 + 4I_2 - 4I_3 = -7 \tag{2}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$I_1 - 6I_2 + 3I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 9 \text{ A}$$
$$I_2 = 2.5 \text{ A}$$
$$I_3 = 2 \text{ A}$$

Current through the 3 Ω resistor = $I_2 - I_3$ = 2.5 − 2 = 0.5 A

Example 3

Find the current in the 5 Ω resistor of the network shown in Fig. 2.80.

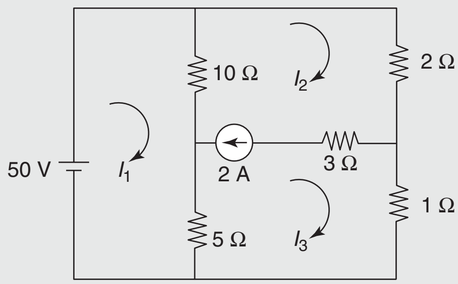


Fig. 2.80

Solution Applying KVL to Mesh 1,

$$50 - 10(I_1 - I_2) - 5(I_1 - I_3) = 0$$
$$15I_1 - 10I_2 - 5I_3 = 50 \tag{1}$$

Meshes 2 and 3 will form a supermesh as these two meshes share a common current source of 2 A.

Writing current equation for the supermesh,

$$I_2 - I_3 = 2 \tag{4}$$

Applying KVL to the outer path of the supermesh,

$$-10(I_2 - I_1) - 2I_2 - 1I_3 - 5(I_3 - I_1) = 0$$
$$-15I_1 + 12I_2 + 6I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 20 \text{ A}$$
$$I_2 = 17.33 \text{ A}$$
$$I_3 = 15.33 \text{ A}$$

Current through the 5 Ω resistor = $I_1 - I_3$ = 20 − 15.33 = 4.67 A

Example 4

Determine the power delivered by the voltage source and the current in the 10 Ω resistor of the network shown in Fig. 2.81.

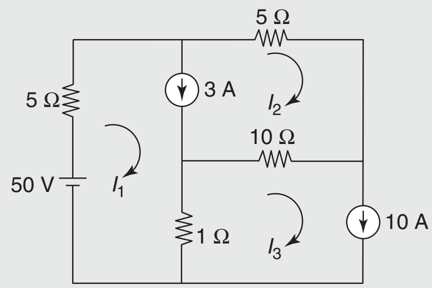


Fig. 2.81

Solution Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_1 - I_2 = 3 \tag{1}$$

Applying KVL to the outer path of the supermesh,

$$50 - 5I_1 - 5I_2 - 10(I_2 - I_3) - 1(I_1 - I_3) = 0$$

$$-6I_1 - 15I_2 + 11I_3 = -50 \tag{2}$$

For Mesh 3,

$$I_3 = 10 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 9.76 \text{ A}$$

$$I_2 = 6.76 \text{ A}$$

$$I_3 = 10 \text{ A}$$

Power delivered by the voltage source = 50 I_1 = 50 × 9.76 = 488 W

$$I_{10\,\Omega} = I_3 - I_2 = 10 - 6.76 = 3.24 \text{ A}$$

Example 5

For the network shown in Fig. 2.82, find current through the 8 Ω resistor.

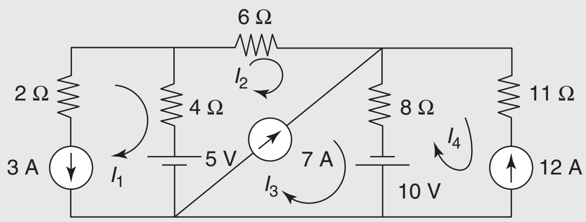


Fig. 2.82

Solution Writing current equations for meshes 1 and 4,

$$I_1 = -3 \tag{1}$$

$$I_4 = -12 \tag{2}$$

Meshes 2 and 3 will form a supermesh.
Writing current equation for the supermesh,

$$I_3 - I_2 = 7 \tag{3}$$

Applying KVL to the outer path of the supermesh,

$$5 - 4(I_2 - I_1) - 6I_2 - 8(I_3 - I_4) + 10 = 0$$

$$5 - 4(I_2 + 3) - 6I_2 - 8(I_3 + 12) + 10 = 0$$

$$-10I_2 - 8I_3 = 93 \tag{4}$$

Solving Eqs (3) and (4),

$$I_2 = -8.28 \text{ A}$$

$$I_3 = -1.28 \text{ A}$$

$$I_{8\,\Omega} = I_3 - I_4 = -1.28 + 12 = 10.72 \text{ A}$$

Exercise 2.2

2.1 Find the value of current flowing through the 10 Ω resistor.

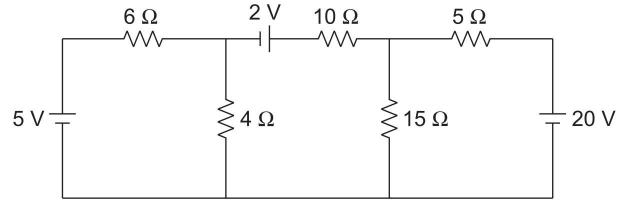


Fig. 2.83

[0.68 A]

2.2 Find the value of current flowing through the 20 Ω resistor.

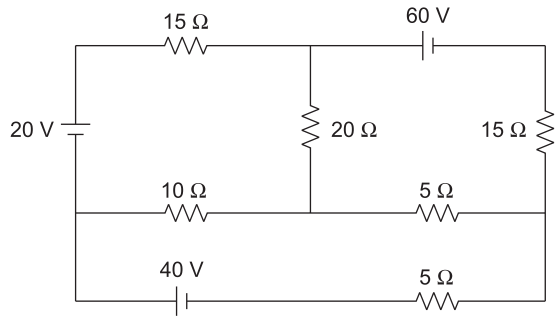


Fig. 2.84

[1.46 A]

2.3 Find the value of mesh currents.

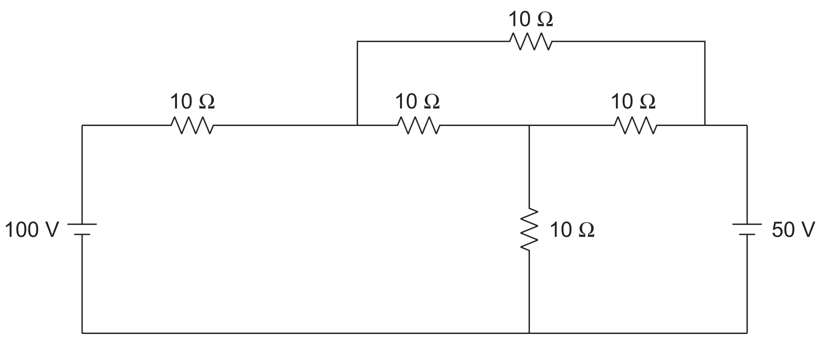


Fig. 2.85

[3.75 A, 0, 1.25 A]

2.4 Calculate the current through the 10 Ω resistor.

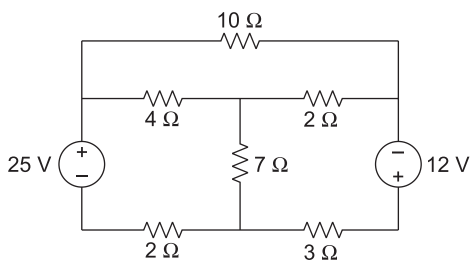


Fig. 2.86

[1.62 A]

2.5 Find the value of current flowing through 10 Ω resistor.

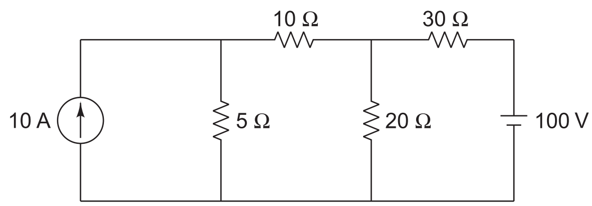


Fig. 2.87

[0.37 A]

2.6 Find the value of current flowing through the 2 Ω resistor.

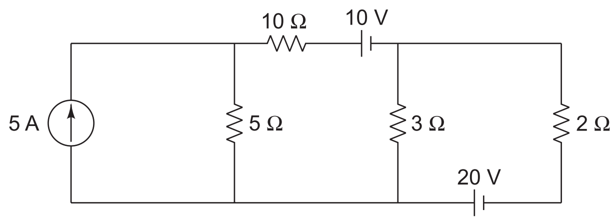


Fig. 2.88

[5 A]

2.7 Find the value of current flowing through the 20 Ω resistor.

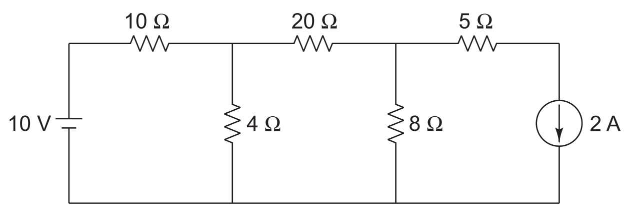


Fig. 2.89

[0.61 A]

2.8 Find the value of current flowing through the 8 Ω resistor.

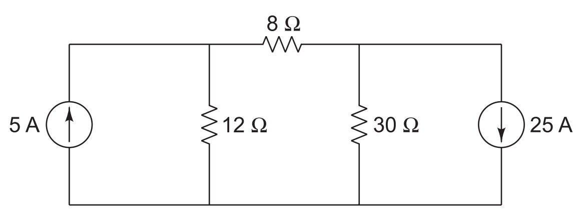


Fig. 2.90

[16.2 A]

2.9 Find the current through the 1 Ω resistor in the network shown in Fig. 2.91.

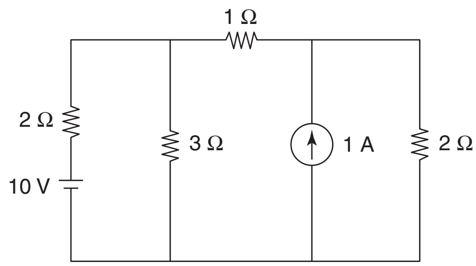


Fig. 2.91

[0.95 A]

2.10 Find the current through the 4 Ω resistor in the network shown in Fig. 2.92.

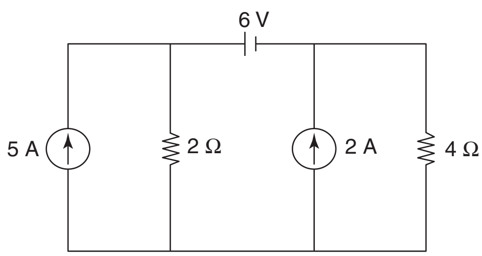


Fig. 2.92

[1.34 A]

2.4 NODAL ANALYSIS

Nodal analysis is based on Kirchhoff's current law which states that the algebraic sum of currents meeting at a point is zero. Every junction where two or more branches meet is regarded as a node. One of the nodes in the network is taken as *reference node* or *datum node*. If there are n nodes in any network, the number of simultaneous equations to be solved will be $(n - 1)$.

2.4.1 Steps to be followed in Nodal Analysis

1. Assuming that a network has n nodes, assign a reference node and the reference directions, and assign a current and a voltage name for each branch and node respectively.
2. Apply KCL at each node except for the reference node and apply Ohm's law to the branch currents.
3. Solve the simultaneous equations for the unknown node voltages.
4. Using these voltages, find any branch currents required.

Example 1

Calculate the current through 2 Ω resistor for the network shown in Fig. 2.93.

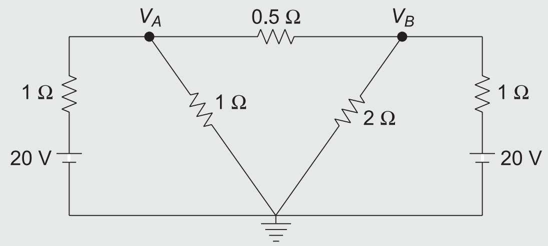


Fig. 2.93

Solution Assume that the currents are moving away from the nodes.

Applying KCL at node *A*,

$$\frac{V_A - 20}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{0.5}\right)V_A - \frac{1}{0.5}V_B = \frac{20}{1}$$

$$4V_A - 2V_B = 20 \tag{1}$$

Applying KCL at node *B*,

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$-\frac{1}{0.5}V_A + \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{1}\right)V_B = \frac{20}{1}$$

$$-2V_A + 3.5V_B = 20 \tag{2}$$

Solving Eqs (1) and (2),

$$V_A = 11\text{ V}$$

$$V_B = 12\text{ V}$$

$$\text{Current through 2 Ω resistor} = \frac{V_B}{2} = \frac{12}{2} = 6\text{ A}$$

Example 2

Find the voltage at nodes 1 and 2.

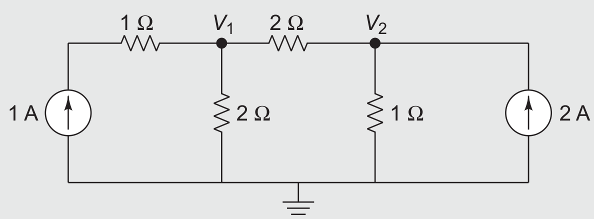


Fig. 2.94

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{2} + \frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 1$$

$$V_1 - 0.5\ V_2 = 1 \tag{1}$$

Applying KCL at Node 2,

$$2 = \frac{V_2}{1} + \frac{V_2 - V_1}{2}$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{1} + \frac{1}{2}\right)V_2 = 2$$

$$-0.5\ V_1 + 1.5\ V_2 = 4 \tag{2}$$

Solving Eqs (1) and (2),

$$V_1 = 2\text{ V}$$
$$V_2 = 2\text{ V}$$

Example 3

Find the value of current flowing through 4 Ω resister.

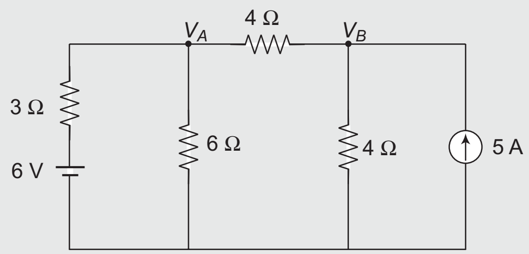


Fig. 2.95 **[Dec 2014]**

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node *A*,

$$\frac{V_A - 6}{3} + \frac{V_A}{6} + \frac{V_A - V_B}{4} = 0$$

$$\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right)V_A - \frac{1}{4}V_B = 2$$

$$0.75\ V_A - 0.25\ V_B = 2 \tag{1}$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{4} + \frac{V_B}{2} = 5$$

$$-\frac{1}{4}V_A + \left(\frac{1}{4} + \frac{1}{2}\right)V_B = 5$$

$$-0.25\ V_A + 0.75\ V_B = 5 \tag{2}$$

Solving Eqs (1) and (2),

$$V_A = 5.5 \text{ V}$$

$$V_B = 8.5 \text{ V}$$

$$I_{4\Omega} = \frac{V_B - V_A}{4} = \frac{8.5 - 5.5}{4} = 0.75 \text{ A}$$

Example 4

For the network given below, find value of current flowing through the 3 Ω resistor.

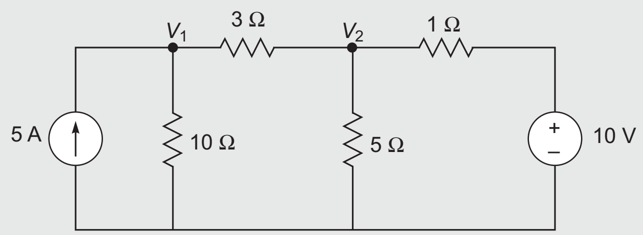


Fig. 2.96 **[May 2013]**

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$\left(\frac{1}{10} + \frac{1}{3}\right) V_1 - \frac{1}{3}\ V_2 = 5$$

$$0.433V_1 - 0.333V_2 = 5 \tag{1}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$-\frac{1}{3}\ V_1 + \left(\frac{1}{3} + \frac{1}{5} + 1\right) V_2 = 10$$

$$-0.333V_1 + 1.533V_2 = 10 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 19.89 \text{ V}$$

$$V_2 = 10.84 \text{ V}$$

$$I_{3\Omega} = \frac{V_1 - V_2}{3} = \frac{10.89 - 10.84}{3} = 3.02 \text{ A}$$

Example 5

Find the value of current flowing in the 100 Ω resistor.

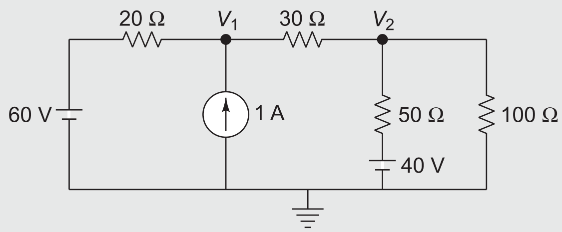


Fig. 2.97

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$\frac{V_1 - 60}{20} + \frac{V_1 - V_2}{30} = 1$$

$$\left(\frac{1}{20} + \frac{1}{30}\right)V_1 - \frac{1}{30}V_2 = \frac{60}{20} + 1$$

$$0.083\ V_1 - 0.033\ V_2 = 4 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{30} + \frac{V_2 - 40}{50} + \frac{V_2}{100} = 0$$

$$-\frac{1}{30}V_1 + \left(\frac{1}{30} + \frac{1}{50} + \frac{1}{100}\right)V_2 = \frac{40}{50}$$

$$-0.033\ V_1 + 0.063\ V_2 = 0.8 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 67.25 \text{ V}$$
$$V_2 = 48 \text{ V}$$

$$\text{Current through the } 100\ \Omega \text{ resistor} = \frac{V_2}{100} = \frac{48}{100} = 0.48 \text{ A}$$

Example 6

Find V_A and V_B.

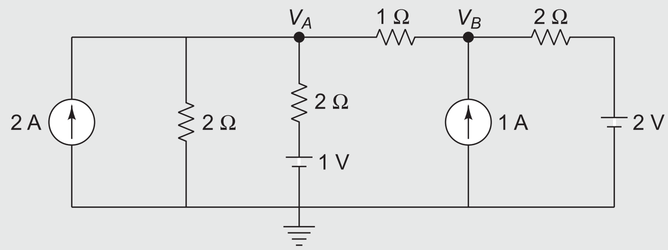


Fig. 2.98

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node A,

$$2 = \frac{V_A}{2} + \frac{V_A - 1}{2} + \frac{V_A - V_B}{1}$$

$$\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{1}\right)V_A - \left(\frac{1}{1}\right)V_B = 2 + \frac{1}{2}$$

$$2V_A - V_B = 2.5 \tag{1}$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{1} + \frac{V_B - 2}{2} = 1$$

$$-\left(\frac{1}{1}\right)V_A + \left(\frac{1}{1} + \frac{1}{2}\right)V_B = 1 + \frac{2}{2}$$

$$-V_A + 1.5\, V_B = 2 \tag{2}$$

Solving Eqs (1) and (2),

$$V_A = 2.875 \text{ V}$$
$$V_B = 3.25 \text{ V}$$

Example 7

Find the values of currents I_1, I_2 and I_3.

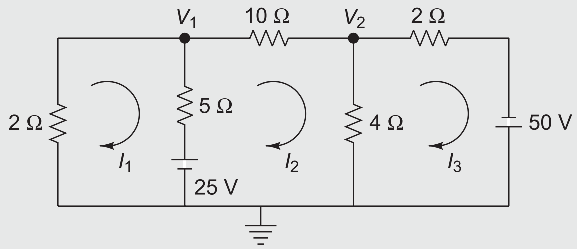


Fig. 2.99

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{2} + \frac{V_1 - 25}{5} + \frac{V_1 - V_2}{10} = 0$$

$$\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right) V_1 - \frac{1}{10} V_2 = \frac{25}{5}$$

$$0.8\ V_1 - 0.1\ V_2 = 5 \tag{1}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{4} + \frac{V_2 - (-50)}{2} = 0$$

$$-\frac{1}{10} V_1 + \left(\frac{1}{10} + \frac{1}{4} + \frac{1}{2}\right) V_2 = -\frac{50}{2}$$

$$-0.1\ V_1 + 0.85\ V_2 = -25 \tag{2}$$

Solving Eqs (1) and (2),

$$V_1 = 2.61\text{ V}$$

$$V_2 = -29.1\text{ V}$$

$$I_1 = -\frac{V_1}{2} = \frac{-2.61}{2} = -1.31\text{ A}$$

$$I_2 = \frac{V_1 - V_2}{10} = \frac{2.61 - (-29.1)}{10} = 3.17\text{ A}$$

$$I_3 = \frac{V_2 + 50}{2} = \frac{-29.1 + 50}{2} = 10.45\text{ A}$$

Example 8

Find the values of currents I_1, I_2 and I_3 and voltages V_a and V_b.

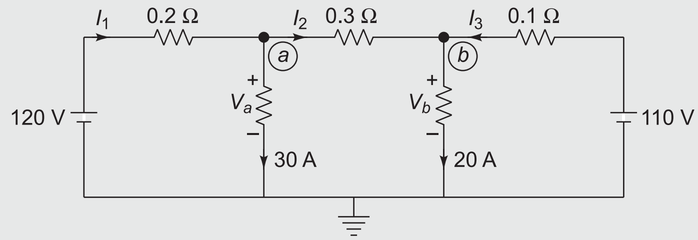


Fig. 2.100

Solution Applying KCL at Node a,

$$I_1 = 30 + I_2$$

$$\frac{120 - V_a}{0.2} = 30 + \frac{V_a - V_b}{0.3}$$

$$36 - 0.3V_a = 1.8 + 0.2V_a - 0.2V_b$$

$$0.5V_a - 0.2V_b = 34.2 \tag{1}$$

Applying KCL at Node b,

$$I_2 + I_3 = 20$$

$$\frac{V_a - V_b}{0.3} + \frac{110 - V_b}{0.1} = 20$$

$$\frac{0.1V_a - 0.1V_b + 33 - 0.3V_b}{0.03} = 20$$

$$0.1V_a - 0.4V_b = -32.4 \tag{2}$$

Solving Eqs (1) and (2),

$$V_a = 112 \text{ V}$$

$$V_b = 109 \text{ V}$$

$$I_1 = \frac{120 - V_a}{0.2} = \frac{120 - 112}{0.2} = 40 \text{ A}$$

$$I_2 = \frac{V_a - V_b}{0.3} = \frac{112 - 109}{0.3} = 10 \text{ A}$$

$$I_3 = \frac{110 - V_b}{0.1} = \frac{110 - 109}{0.1} = 10 \text{ A}$$

Example 9

Calculate the current through the 5 Ω resistor.

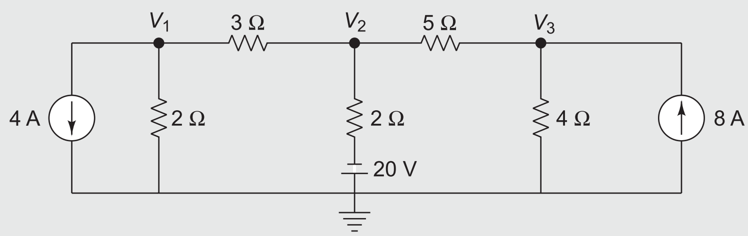


Fig. 2.101

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$4+\frac{V_1}{2}+\frac{V_1-V_2}{3}=0$$

$$\left(\frac{1}{2}+\frac{1}{3}\right)V_1-\frac{1}{3}V_2=4$$

$$0.83\ V_1-0.33\ V_2=-4 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{3}+\frac{V_2-(-20)}{2}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{3}V_1+\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=-\frac{20}{2}$$

$$-0.33\ V_1+1.03\ V_2-0.2\ V_3=-10 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{4}=8$$

$$-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{4}\right)V_3=8$$

$$-0.2\ V_2+0.45\ V_3=8 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1=-8.76\text{ V}$$

$$V_2=-9.92\text{ V}$$

$$V_3=13.37\text{ V}$$

Current through the 5 Ω resistor $= \dfrac{V_3 - V_2}{5} = \dfrac{13.37-(-9.92)}{5} = 4.66$ A

Example 10

Find the voltage across the 5 Ω resistor.

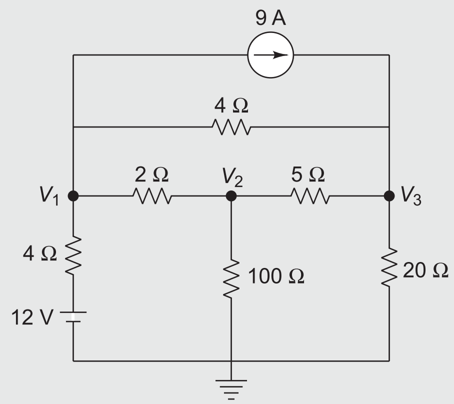


Fig. 2.102

Solution Assume that the currents are moving away from the node.

Applying KCL at Node 1,

$$\frac{V_1-12}{4}+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{4}+9 = 0$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=-9+\frac{12}{4}$$

$$V_1-0.5\ V_2-0.25\ V_3 = -6 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5} = 0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5\ V_1+0.71\ V_2-0.2\ V_3 = 0 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-V_1}{4} = 9$$

$$-\frac{1}{4}V_1 - \frac{1}{5}V_2 + \left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3 = 9$$

$$-0.25\ V_1 - 0.2\ V_2 + 0.5\ V_3 = 9 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 6.35\ \text{V}$$

$$V_2 = 11.76\ \text{V}$$

$$V_3 = 25.88\ \text{V}$$

Voltage across the 5 Ω resistor = $V_3 - V_2 = 25.88 - 11.76 = 14.12$ V

Example 11

Determine the value of current flowing through the 5 Ω resistor.

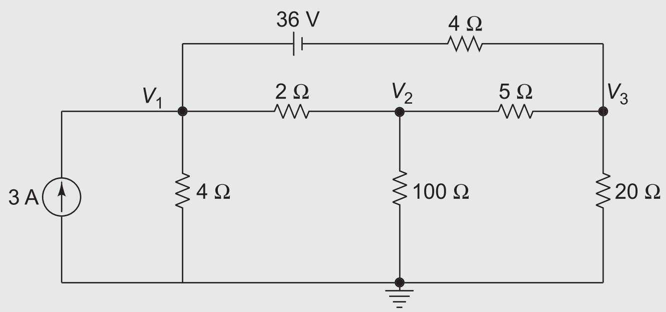


Fig. 2.103

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{4} + \frac{V_1 - V_2}{2} + \frac{V_1 - 36 - V_3}{4} = 3$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1 - \frac{1}{2}V_2 - \frac{1}{4}V_3 = 3 + \frac{36}{4}$$

$$V_1 - 0.5\ V_2 - 0.25\ V_3 = 12 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{100} + \frac{V_2 - V_3}{5} = 0$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2 - \frac{1}{5}V_3 = 0$$

$$-0.5V_1 + 0.71\ V_2 - 0.2\ V_3 = 0 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3 - V_2}{5} + \frac{V_3}{20} + \frac{V_3 - (-36) - V_1}{4} = 0$$

$$-\frac{1}{4}V_1 - \frac{1}{5}V_2 + \left(\frac{1}{5} + \frac{1}{20} + \frac{1}{4}\right)V_3 = -9$$

$$-0.25\ V_1 - 0.2\ V_2 + 0.5\ V_3 = -9 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 13.41\text{ V}$$

$$V_2 = 7.06\text{ V}$$

$$V_3 = -8.47\text{ V}$$

Current through the 5 Ω resistor $= \dfrac{V_2 - V_3}{5} = \dfrac{7.06 - (-8.47)}{5} = 3.11\text{ A}$

Example 12

Find the voltage drop across the 5 Ω resistor in the network shown in Fig. 2.104.

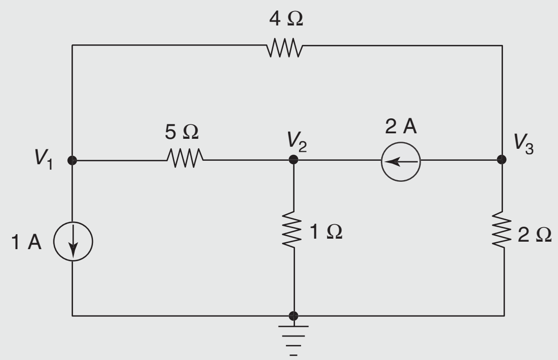


Fig 2.104

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$1 + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{4} = 0$$

$$\left(\frac{1}{5} + \frac{1}{4}\right)V_1 - \frac{1}{5}V_2 - \frac{1}{4}V_3 = -1$$

$$0.45\,V_1 - 0.2\,V_2 - 0.25\,V_3 = -1 \quad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{5} + \frac{V_2}{1} = 2$$

$$-\frac{1}{5}V_1 + \left(\frac{1}{5} + 1\right)V_2 = 2$$

$$-0.2\,V_1 + 1.2\,V_2 = 2 \tag{2}$$

Applying KCL at Node 3,

$$\frac{V_3}{2} + \frac{V_3 - V_1}{4} + 2 = 0$$

$$-\frac{1}{4}V_1 + \left(\frac{1}{2} + \frac{1}{4}\right)V_3 = -2$$

$$-0.25\,V_1 + 0.75\,V_3 = -2 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V_1 = -4\text{ V}$$

$$V_2 = 1\text{ V}$$

$$V_3 = -4\text{ V}$$

$$V_{5\,\Omega} = V_2 - V_1 = 1 - (-4) = 5\text{ V}$$

Example 13

Find the power dissipated in the 6 Ω resistor for the network shown in Fig. 2.105.

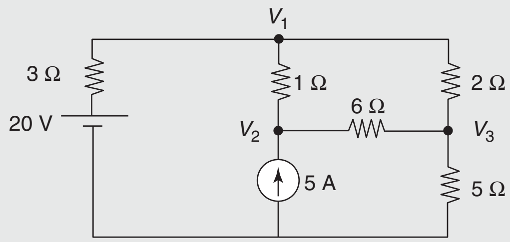


Fig. 2.105

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1 - 20}{3} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0$$

$$\left(\frac{1}{3} + 1 + \frac{1}{2}\right)V_1 - V_2 - \frac{1}{2}V_3 = \frac{20}{3}$$

$$1.83\,V_1 - V_2 - 0.5\,V_3 = 6.67 \tag{1}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{6} = 5$$

$$-V_1 + \left(1 + \frac{1}{6}\right)V_2 - \frac{1}{6}V_3 = 5$$

$$-V_1 + 1.17\,V_2 - 0.17\,V_3 = 5 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{2} + \frac{V_3}{5} + \frac{V_3 - V_2}{6} = 0$$

$$-\frac{1}{2}V_1 - \frac{1}{6}V_2 + \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{6}\right)V_3 = 0$$

$$-0.5\,V_1 - 0.17\,V_2 + 0.87\,V_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 23.82 \text{ V}$$

$$V_2 = 27.4 \text{ V}$$

$$V_3 = 19.04 \text{ V}$$

$$I_{6\,\Omega} = \frac{V_2 - V_3}{6} = \frac{27.4 - 19.04}{6} = 1.39 \text{ A}$$

Power dissipated in the 6 Ω resistor $= (1.39)^2 \times 6 = 11.59$ W

Example 14

Find the voltage V in the network shown in Fig. 2.106 which makes the current in the 10 Ω resistor zero.

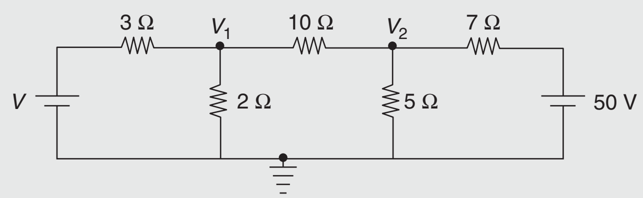


Fig. 2.106

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1 - V}{3} + \frac{V_1}{2} + \frac{V_1 - V_2}{10} = 0$$

$$\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{10}\right)V_1 - \frac{1}{10}V_2 - \frac{1}{3}V = 0$$

$$0.93\,V_1 - 0.1\,V_2 - 0.33\,V = 0 \tag{1}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + \frac{V_2 - 50}{7} = 0$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{7}\right)V_2 = \frac{50}{7}$$

$$-0.1\,V_1 + 0.44\,V_2 = 7.14 \tag{2}$$

$$I_{10\,\Omega} = \frac{V_1 - V_2}{10} = 0$$

$$V_1 - V_2 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V = 52.82\text{ V}$$

Example 15

Find node voltages.

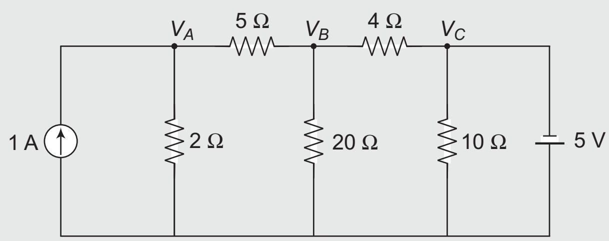


Fig. 2.107

[May 2016]

Solution Assume that the currents are moving away from the nodes,

Applying KCL at Node *A*,

$$\frac{V_A}{2} + \frac{V_A - V_B}{5} = 1$$

$$\left(\frac{1}{2}+\frac{1}{5}\right)V_A-\frac{1}{5}V_B=1$$

$$0.7\ V_A-0.2\ V_B=1 \tag{1}$$

Applying KCL at Node B,

$$\frac{V_B-V_A}{5}+\frac{V_B}{20}+\frac{V_B-V_C}{4}=0$$

$$-\frac{1}{5}V_A+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_B-\frac{1}{4}V_C=0$$

$$-0.2\ V_A+0.5\ V_B-0.25\ V_C=0 \tag{2}$$

Node C is directly connected to a voltage source of 5 V. Hence, we can write voltage equation at Node C.

$$V_C=-5 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V_A=0.81\text{ V}$$
$$V_B=-2.18\text{ V}$$
$$V_C=-5\text{ V}$$

Example 16

Find V_1 and V_2.

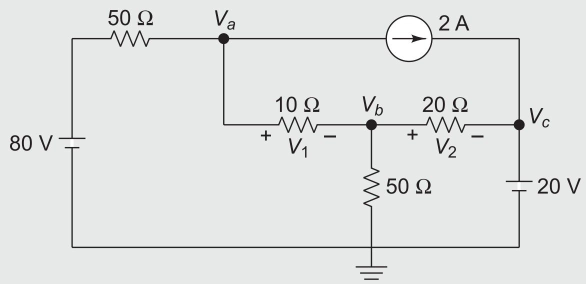


Fig. 2.108

Solution Assume that the currents are moving away from the nodes,

Applying KCL at Node a,

$$\frac{V_a-80}{50}+\frac{V_a-V_b}{10}+2=0$$

$$\left(\frac{1}{50}+\frac{1}{10}\right)V_a-\frac{1}{10}V_b=\frac{80}{50}-2$$

$$0.12\ V_a-0.1\ V_b=-0.4 \qquad (1)$$

Applying KCL at Node b,

$$\frac{V_b-V_a}{10}+\frac{V_b}{50}+\frac{V_b-V_c}{20}=0$$

$$-\frac{1}{10}V_a+\left(\frac{1}{10}+\frac{1}{50}+\frac{1}{20}\right)V_b-\frac{1}{20}V_c=0$$

$$-0.1\ V_a+0.17\ V_b-0.05\ V_c=0 \qquad (2)$$

Node c is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node c.

$$V_c=20 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_a=3.08\text{ V}$$
$$V_b=7.69\text{ V}$$
$$V_1=V_a-V_b=3.08-7.69=-4.61\text{ V}$$
$$V_2=V_b-V_c=7.69-20=-12.31\text{ V}$$

Example 17

Find the voltage across the 100 Ω resistor.

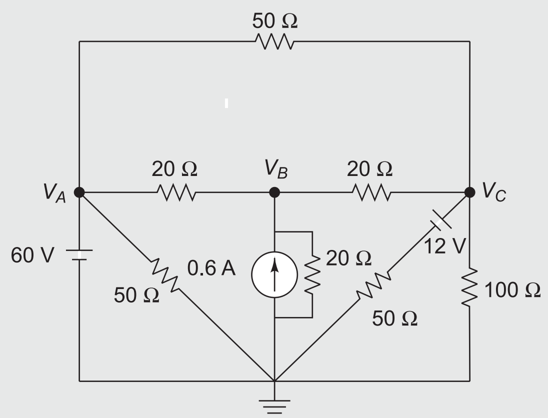


Fig. 2.109

Solution Node A is directly connected to a voltage source of 60 V. Hence, we can write voltage equation at Node A.

$$V_A = 60 \tag{1}$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node B,

$$\frac{V_B - V_A}{20} + \frac{V_B - V_C}{20} + \frac{V_B}{20} = 0.6$$

$$-\frac{1}{20}V_A + \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{20}\right)V_B - \frac{1}{20}V_C = 0.6$$

$$-0.05\ V_A + 0.15\ V_B - 0.05\ V_C = 0.6 \tag{2}$$

Applying KCL at Node C,

$$\frac{V_C - V_A}{50} + \frac{V_C - V_B}{20} + \frac{V_C - 12}{50} + \frac{V_C}{100} = 0$$

$$-\frac{1}{50}V_A - \frac{1}{20}V_B + \left(\frac{1}{50} + \frac{1}{20} + \frac{1}{50} + \frac{1}{100}\right)V_C = \frac{12}{50}$$

$$-0.02\ V_A - 0.05\ V_B + 0.1\ V_C = 0.24 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V_C = 31.68\text{ V}$$

$$\text{Voltage across the } 100\ \Omega \text{ resistor} = 31.68\text{ V}$$

2.5 SUPERNODE ANALYSIS

Nodes that are connected to each other by voltage sources, but not to the reference node by a path of voltage sources, form a *supernode*. A supernode requires one node voltage equation, that is, a KCL equation. The remaining node voltage equations are KVL equations.

Example 1

Determine the current in the 5 Ω resistor for the network shown in Fig. 2.110.

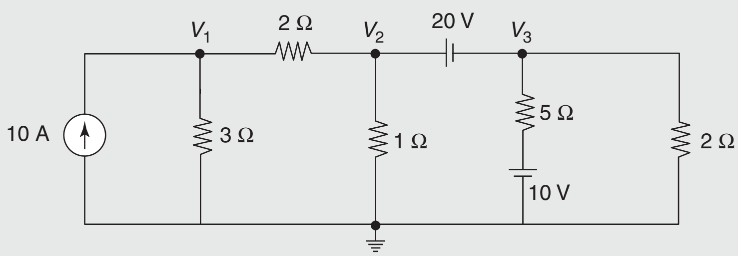


Fig. 2.110

Solution Assume that the currents are moving away from the nodes.
Applying KCL at Node 1,

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{3} + \frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 10$$

$$0.83\,V_1 - 0.5\,V_2 = 10 \qquad (1)$$

Nodes 2 and 3 will form a supernode.
Writing voltage equation for the supernode,

$$V_2 - V_3 = 20 \qquad (2)$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2} + 1\right)V_2 + \left(\frac{1}{5} + \frac{1}{2}\right)V_3 = 2$$

$$-0.5\,V_1 + 1.5\,V_2 + 0.7\,V_3 = 2 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 19.04 \text{ V}$$

$$V_2 = 11.6 \text{ V}$$

$$V_3 = -8.4 \text{ V}$$

$$I_{5\,\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4 - 10}{5} = -3.68 \text{ A}$$

Example 2

Find the power delivered by the 5 A current source in the network shown in Fig. 2.111.

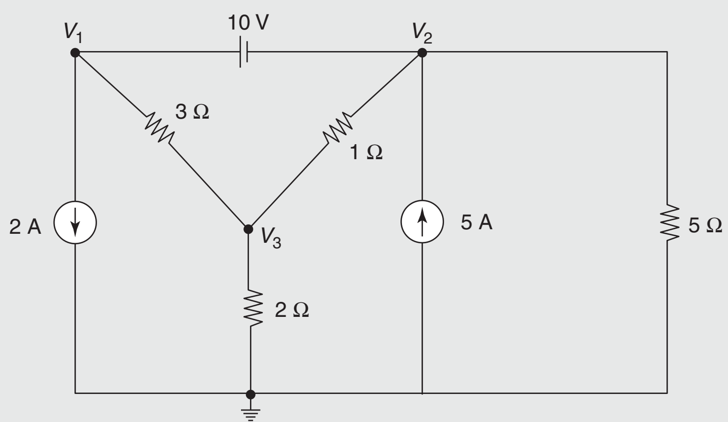


Fig. 2.111

Solution Assume that the currents are moving away from the nodes.

Nodes 1 and 2 will form a supernode.

Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \tag{1}$$

Applying KCL at the supernode,

$$2 + \frac{V_1 - V_3}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 5$$

$$\frac{1}{3}V_1 + \left(\frac{1}{5}+1\right)V_2 - \left(\frac{1}{3}+1\right)V_3 = 3$$

$$0.33\,V_1 + 1.2\,V_2 - 1.33\,V_3 = 3 \tag{2}$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$-\frac{1}{3}V_1 - V_2 + \left(\frac{1}{3}+1+\frac{1}{2}\right)V_3 = 0$$

$$-0.33\,V_1 - V_2 + 1.83\,V_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V_1 = 13.72\text{ V}$$

$$V_2 = 3.72\text{ V}$$

$$V_3 = 4.51\text{ V}$$

Power delivered by the 5 A source = 5 V_2 = 5 × 3.72 = 18.6 W

Example 3

In the network of Fig. 2.112, find the node voltages V_1, V_2 and V_3.

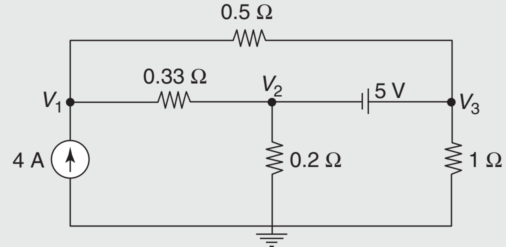


Fig. 2.112

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$4 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.5}$$

$$\left(\frac{1}{0.33} + \frac{1}{0.5}\right)V_1 - \frac{1}{0.33}V_2 - \frac{1}{0.5}V_3 = 4$$

$$5.03\,V_1 - 3.03\,V_2 - 2\,V_3 = 4 \qquad (1)$$

Nodes 2 and 3 will form a supernode.

Writing voltage equation for the supernode,

$$V_3 - V_2 = 5 \qquad (2)$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{0.33} + \frac{V_2}{0.2} + \frac{V_3}{1} + \frac{V_3 - V_1}{0.5} = 0$$

$$\left(-\frac{1}{0.33} - \frac{1}{0.5}\right)V_1 + \left(\frac{1}{0.33} + \frac{1}{0.2}\right)V_2 + \left(1 + \frac{1}{0.5}\right)V_3 = 0$$

$$-5.03\,V_1 + 8.03\,V_2 + 3\,V_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 2.62 \text{ V}$$
$$V_2 = -0.17 \text{ V}$$
$$V_3 = 4.83 \text{ V}$$

Exercise 2.3

2.1 Find the current I_x.

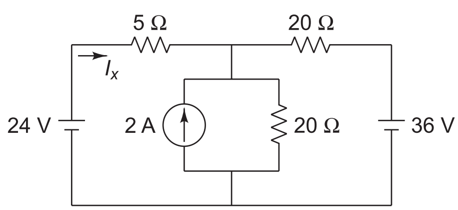


Fig. 2.113

[–0.93 A]

2.2 Find V_A and V_B.

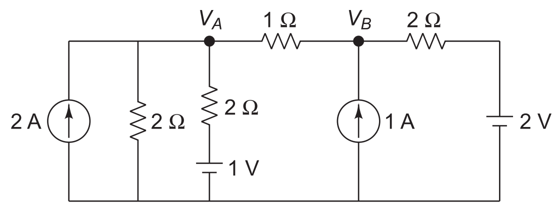


Fig. 2.114

[2.88 V, 3.25 V]

2.3 Find the current through the 6 Ω resistor.

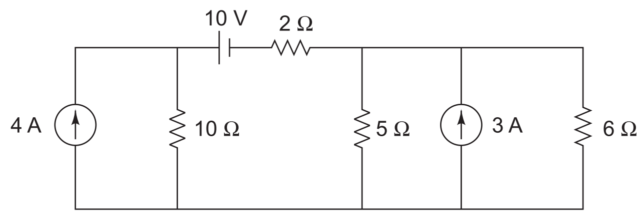


Fig. 2.115

[2.04 A]

2.4 Calculate the value of current flowing through the 10 Ω resistor.

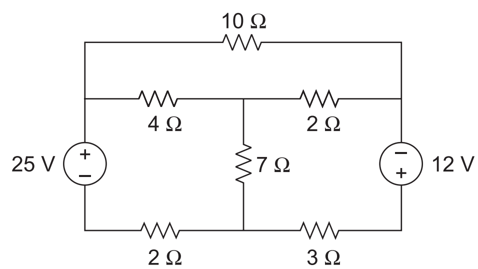


Fig. 2.116

[1.62 A]

2.5 Find the value of current flowing through the branch *ab*.

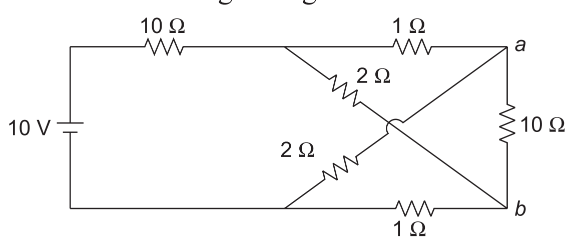


Fig. 2.117

[0.038 A]

2.6 Find the value of current flowing through the 4 Ω resistor.

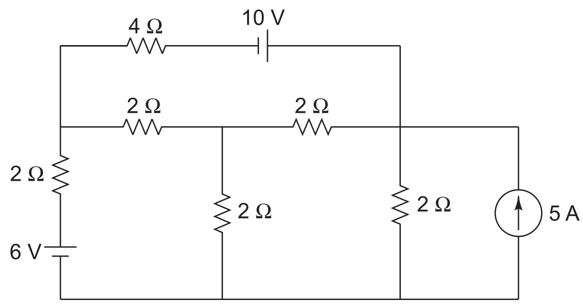


Fig. 2.118

[1.34 A]

2.7 Find the value of current flowing through the 40 Ω resistor.

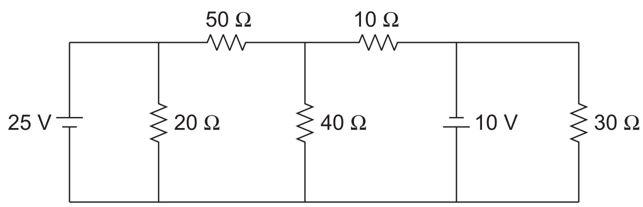


Fig. 2.119

[0.0862 A]

2.8 Find the value of current flowing through the 10 Ω resistor.

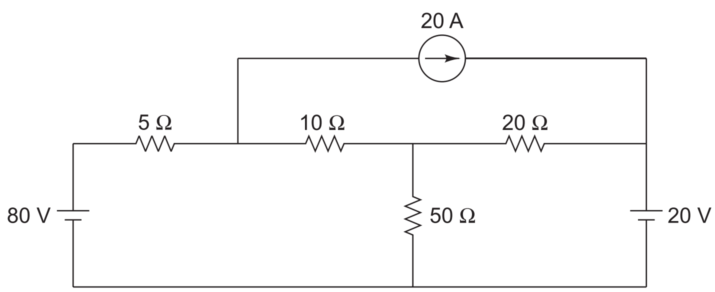


Fig. 2.120

[1.17 A]

2.9 Find the current through the 4 Ω resistor in the network shown in Fig. 2.121.

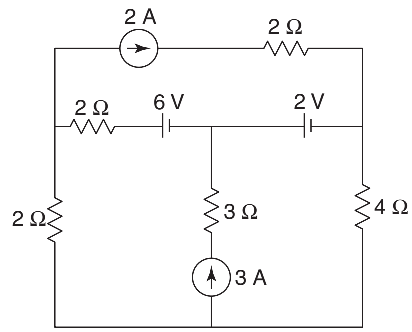


Fig. 2.121

[1 A]

2.6 SOURCE TRANSFORMATION

A voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor. Conversely, a current source with a parallel resistor can be converted into a voltage source with a series resistor.

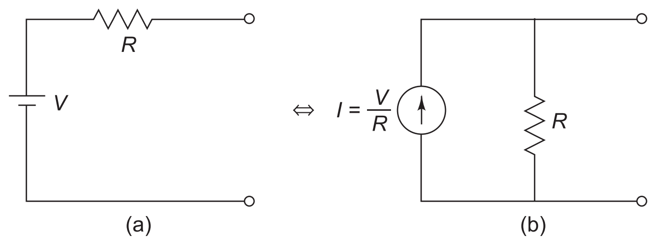


Fig. 2.122 *Source transformation*

Example 1

Replace the given network with a single current source and a resistor.

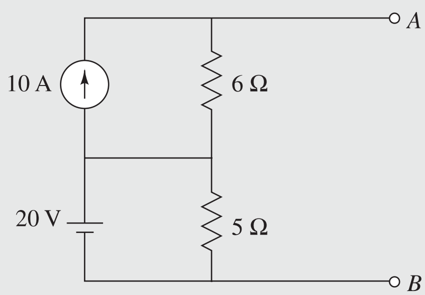


Fig. 2.123

Solution Since the resistor of 5 Ω is connected in parallel with the voltage source of 20 V it becomes redundant. Converting parallel combination of current source and resistor into equivalent voltage source and resistor,

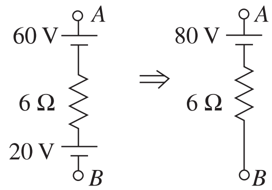


Fig. 2.124

By source transformation,

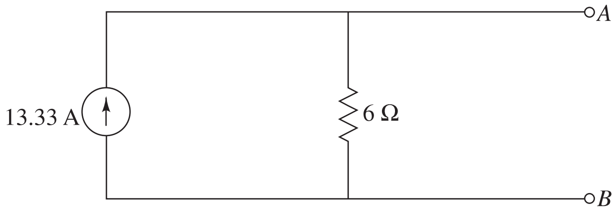


Fig. 2.125

Example 2

Convert the given circuit into a single current source in parallel with a single resistance between points A and B.

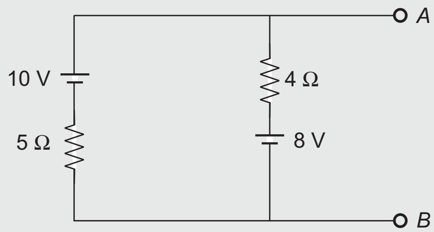


Fig. 2.126 **[Dec 2014]**

Solution Converting the series combination of the voltage source of 10 V and the resistor of 5 Ω and series combination of voltage source of 8 V and the resistor of 4 Ω into an equivalent current sources and resistors,

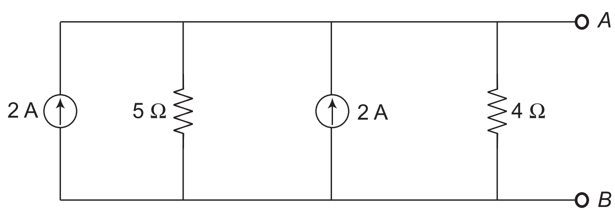


Fig. 2.127

Adding two current sources,

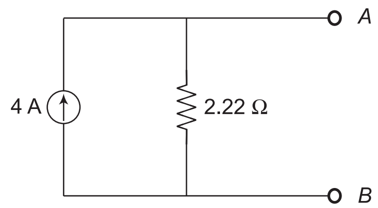


Fig. 2.128

Example 3

Convert the circuit given in Fig. 2.129 to a single voltage source in series with a resistor.

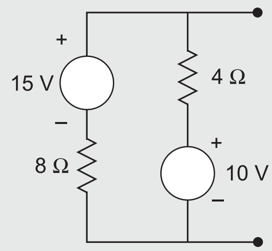


Fig. 2.129 **[May 2013]**

Solution By source transformation,

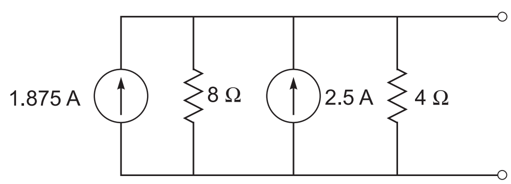


Fig. 2.130

Adding two current sources and simplifying,

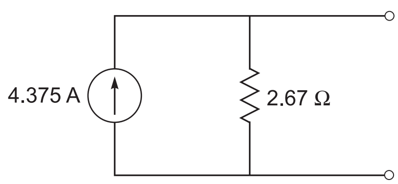


Fig. 2.131

Again by source transformation,

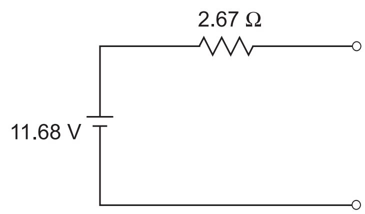


Fig. 2.132

Example 4

Reduce network shown into a single source and a single resistor between terminals A and B.

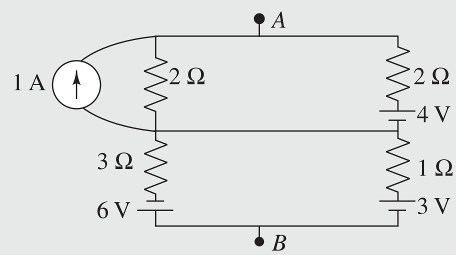


Fig. 2.133

Solution Converting all voltage sources into equivalent current sources,

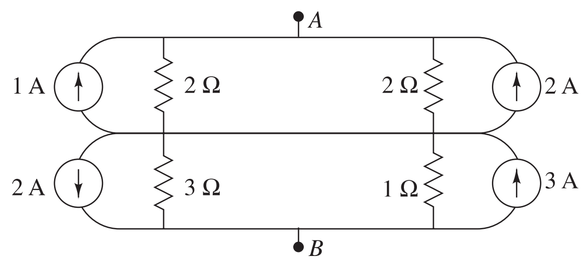


Fig. 2.134

Adding the current sources and simplifying the network,

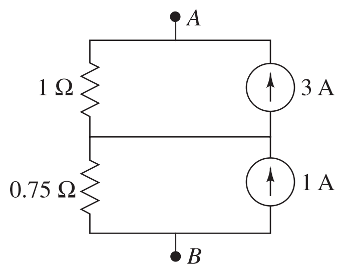


Fig. 2.135

Converting the current sources into equivalent voltage sources,

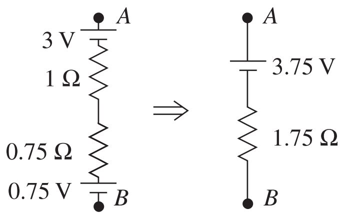


Fig. 2.136

Example 5

Replace the circuit between A and B with a voltage source in series with a single resistor.

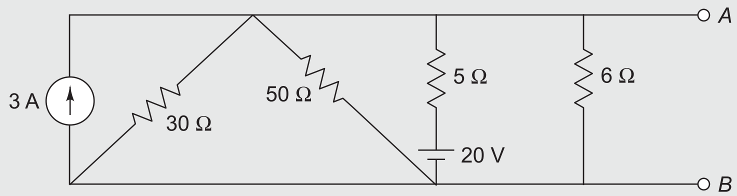


Fig. 2.137

Solution Converting the series combination of voltage source of 20 V and a resistor of 5 Ω into equivalent parallel combination of current source and resistor,

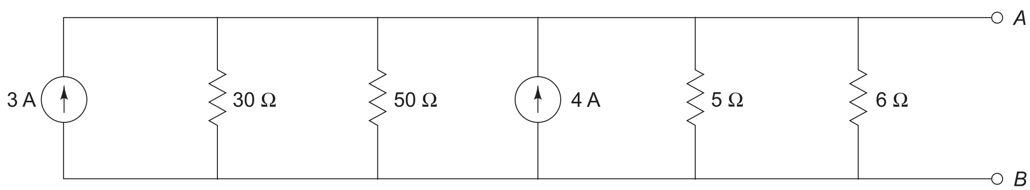


Fig. 2.138

Adding the two current sources and simplifying the circuit,

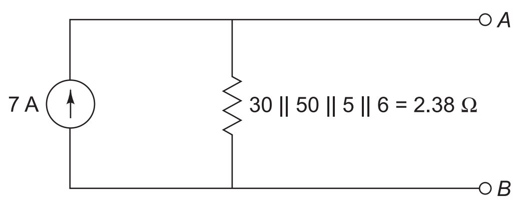


Fig. 2.139

By source transformation,

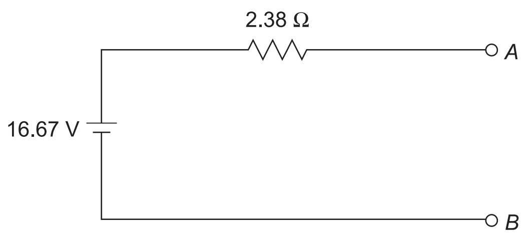


Fig. 2.140

Example 6

Reduce the circuit shown in Fig. 2.141 into a single current source in parallel with single resistance.

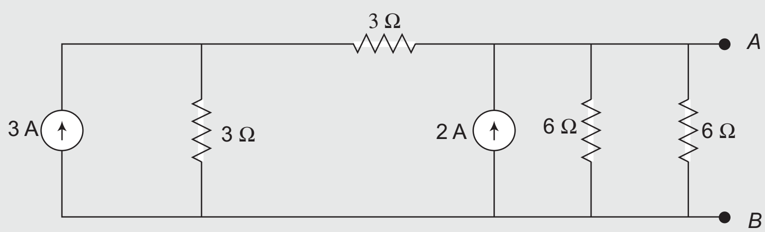


Fig. 2.141 **[May 2014]**

Solution Converting the parallel combination of the current source of 3 A and the resistor of 3 Ω into an equivalent series combination of voltage source and resistor,

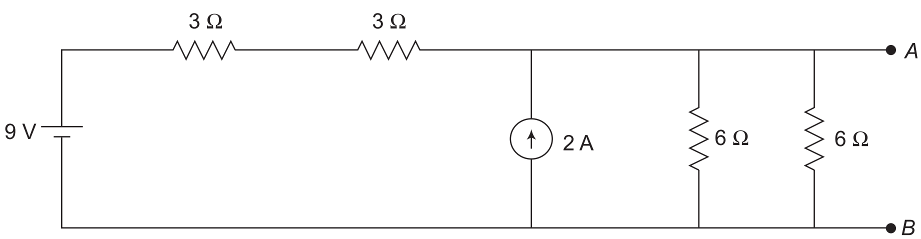


Fig. 2.142

Again by source transformation,

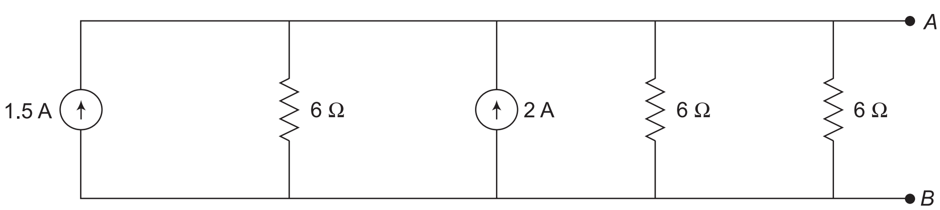


Fig. 2.143

Adding the two current sources and by series-parallel reduction technique,

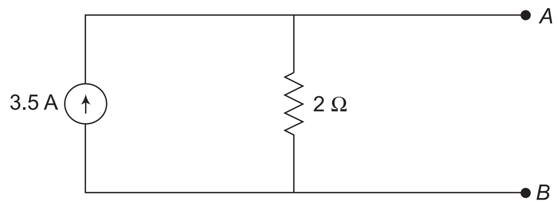


Fig. 2.144

Example 7

Find the power delivered by the 50 V source in the circuit.

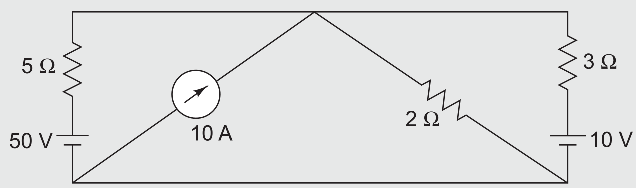


Fig. 2.145 [Dec 2012]

Solution Converting the series combination of voltage source of 10 V and resistor of 3 Ω into equivalent current source and resistor,

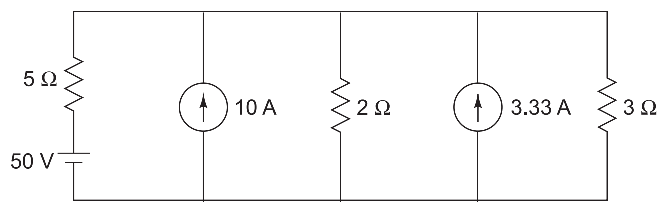


Fig. 2.146

Adding the two current sources and simplifying the circuit,

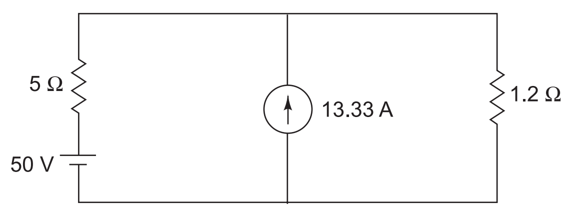


Fig. 2.147

By source transformation,

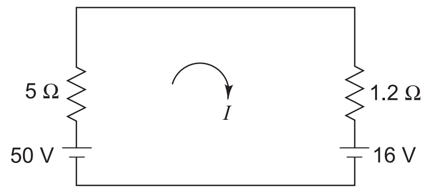


Fig. 2.148

Applying KVL to the circuit,

$$50 - 5I - 1.2I - 16 = 0$$

$$I = 5.48 \text{ A}$$

Power delivered by the 50 V source = $50 \times 5.48 = 274$ W

Example 8

Find the value of I.

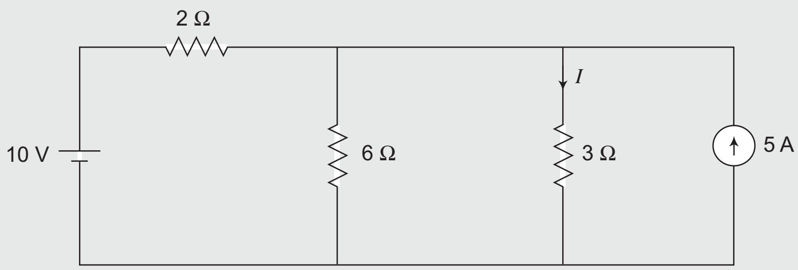


Fig. 2.149 **[Dec 2013]**

Solution Converting the series combination of the voltage source of 10 V and the resistor of 2 Ω into equivalent current source and resistor,

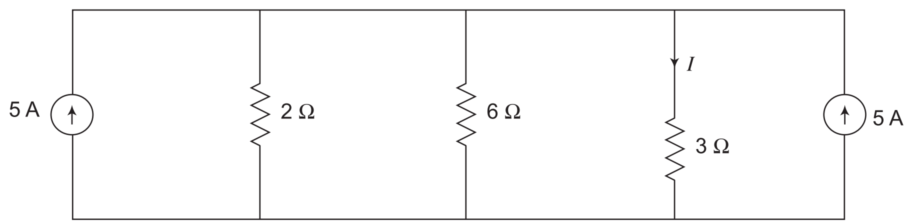


Fig. 2.150

Adding the two current sources,

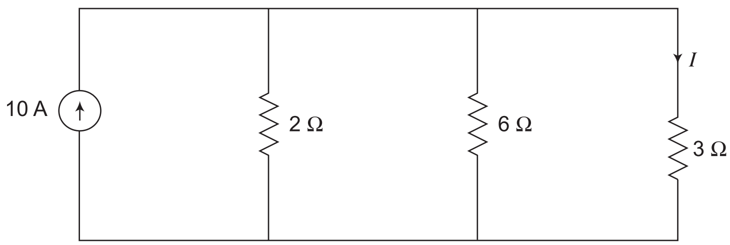


Fig. 2.151

Simplifying the circuit,

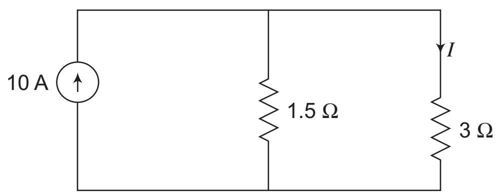


Fig. 2.152

By current-division rule,

$$I = 10 \times \frac{1.5}{1.5+3} = 3.33 \text{ A}$$

Example 9

Find the value of current flowing in the 4 Ω resistor.

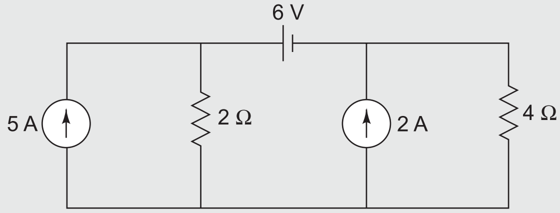


Fig. 2.153

Solution Converting the parallel combination of the current source of 5 A and the resistor of 2 Ω into an equivalent series combination of voltage source and resistor,

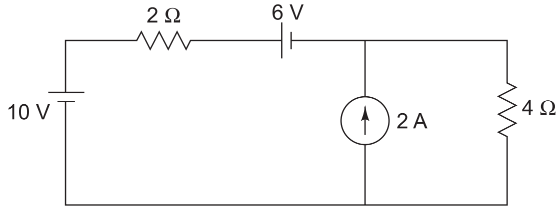


Fig. 2.154

Adding two voltage sources,

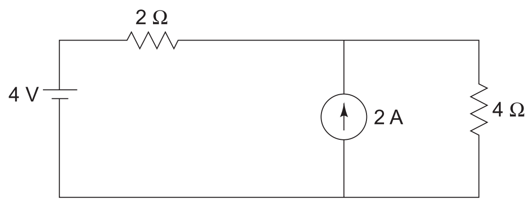


Fig. 2.155

Again by source transformation,

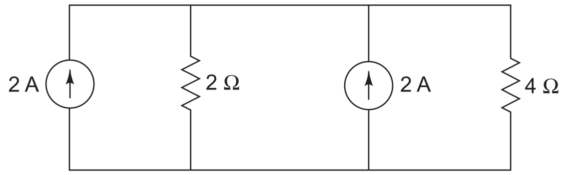


Fig. 2.156

Adding two current sources,

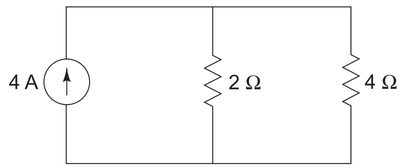


Fig. 2.157

By current-division rule,

$$I_{4\Omega} = 4 \times \frac{2}{2+4} = 1.33 \text{ A}$$

Example 10

Find the value of current flowing through the 10 Ω resistor.

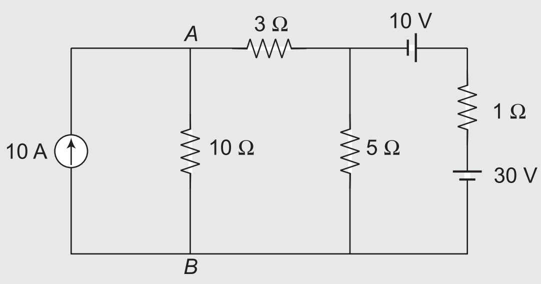


Fig. 2.158 **[Dec 2014]**

Solution Adding two voltage sources,

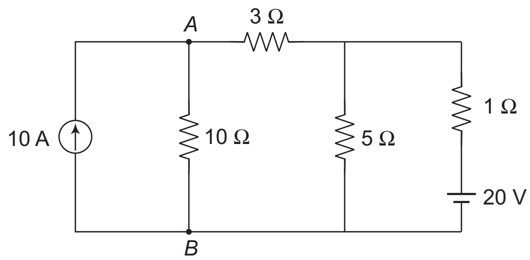


Fig. 2.159

Converting the series combination of the voltage source of 20 V and the resistor of 1 Ω into an equivalent current source and resistor,

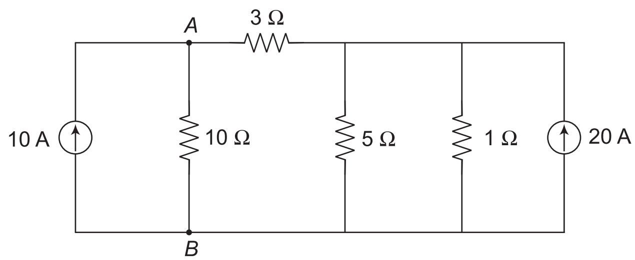


Fig. 2.160

Converting the parallel combination of the current source of 20 A and the resistors of 5 Ω and 1 Ω into an equivalent series combination of current source and resistor,

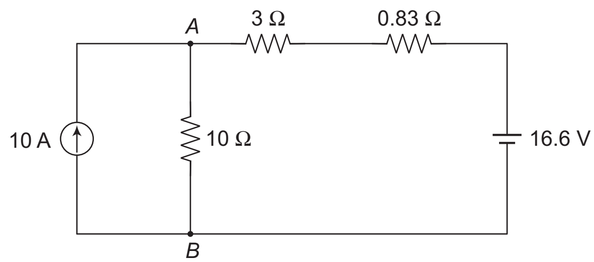


Fig. 2.161

Converting the series combination of the voltage source of 16.6 V and the resistors of 3 Ω and 0.83 Ω into an equivalent current source and resistor,

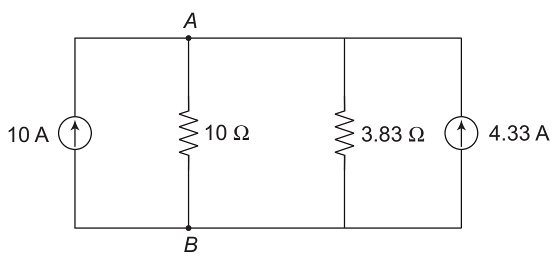


Fig. 2.162

Adding two current sources,

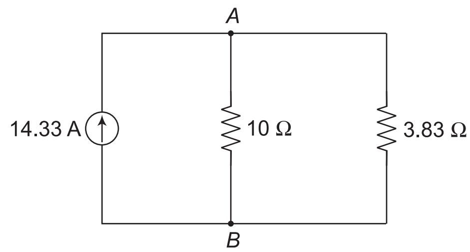


Fig. 2.163

By current-division rule,

$$I_{10\,\Omega} = \frac{14.33 \times 3.83}{3.83 + 10} = 3.97 \text{ A}$$

Example 11

Find the value of current flowing through the 8 Ω resistor.

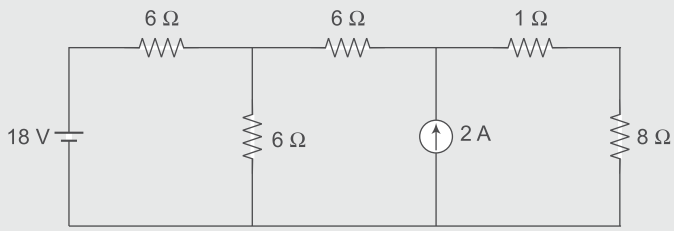
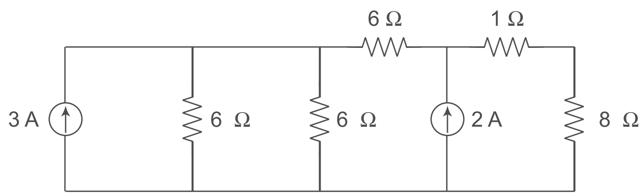
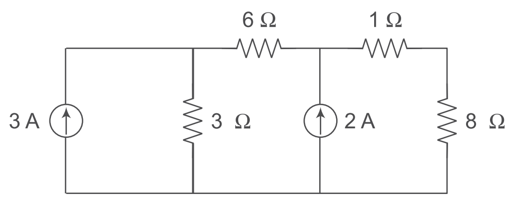


Fig. 2.164 **[May 2015]**

Solution Converting the series combination of the voltage source of 18 V and the resistor of 6 Ω in to equivalent current source and resistance.



(a)



(b)

Fig. 2.165

By source transformation,

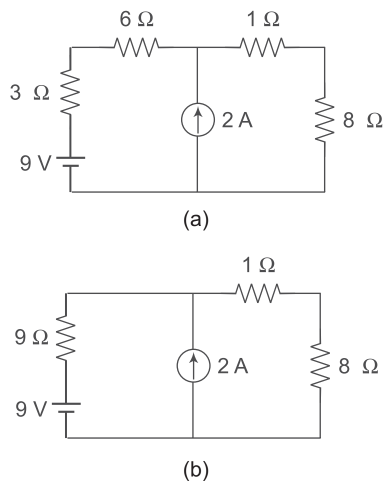


Fig. 2.166

By source transformation,

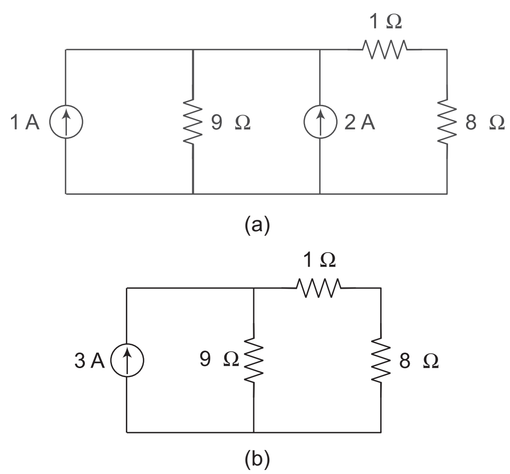


Fig. 2.167

By source transformation

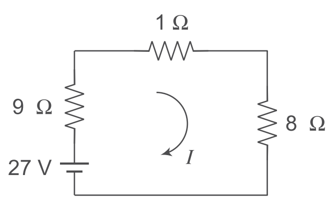


Fig. 2.168

$$I_{8\Omega} = \frac{27}{9+1+8} = 1.5 \text{ A}$$

Example 12

Find the voltage across the 4 Ω resistor.

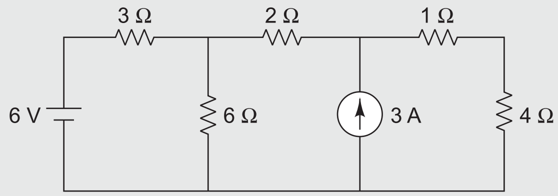


Fig. 2.169 [May 2016]

Solution Converting the series combination of the voltage source of 6 V and the resistor of 3 Ω into equivalent current source and resistor,

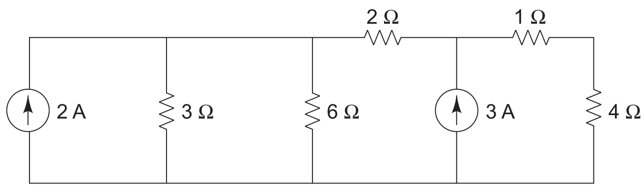


Fig. 2.170

By series–parallel reduction technique,

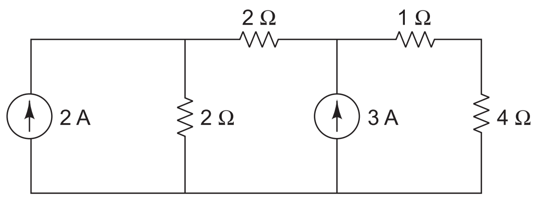
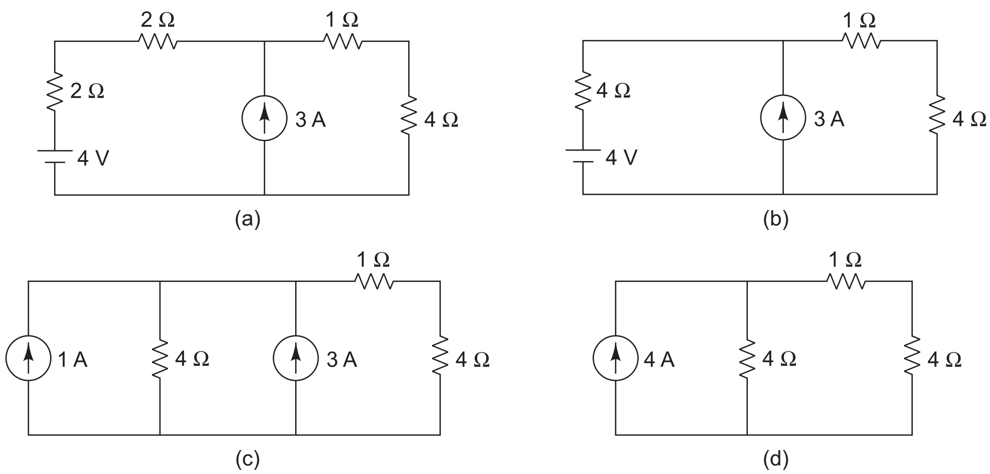
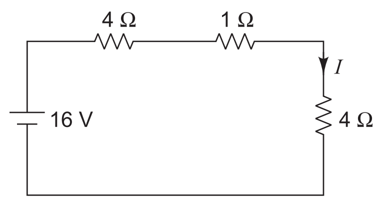


Fig. 2.171

By source transformation,





(e)

Fig. 2.172

$$I = \frac{16}{4+1+4} = 1.78 \text{ A}$$

$$\begin{aligned}\text{Voltage across the 4 Ω resistor} &= 4I \\ &= 4 \times 1.78 \\ &= 7.12 \text{ V}\end{aligned}$$

Exercise 2.4

1.1 Replace the given network with a single voltage source and a resistor.

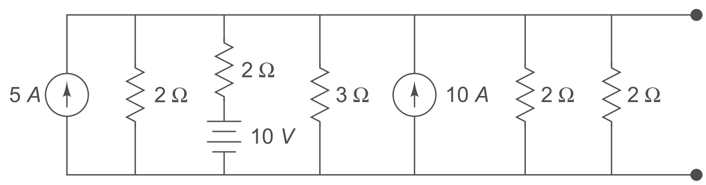


Fig. 2.173

[8.6 V, 0.43 Ω]

1.2 Use source transformation to simplify the network until two elements remain to the left of terminals *a* and *b*.

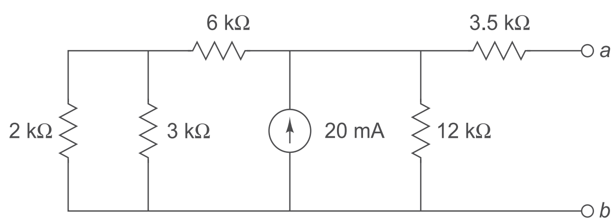


Fig. 2.174

[88.42 V, 7.92 kΩ]

2.7 STAR-DELTA TRANSFORMATION

When a circuit cannot be simplified by normal series–parallel reduction technique, the star-delta transformation can be used.

Figure 2.175(a) shows three resistors R_A, R_B and R_C connected in delta.

Figure 2.175(b) shows three resistors R_1, R_2 and R_3 connected in star.

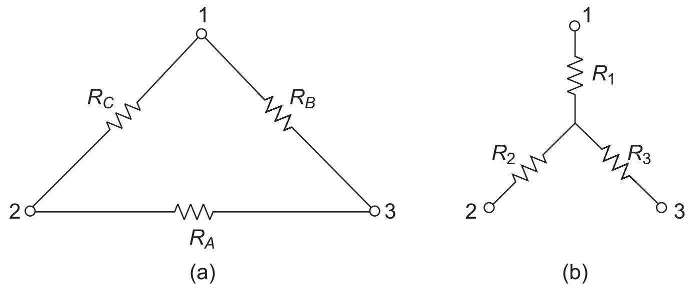


Fig. 2.175 *Delta and star networks*

These two networks will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements.

2.7.1 Delta to Star Transformation

Referring to delta network shown in Fig. 2.175(a),

the resistance between terminals 1 and 2 = $R_C \parallel (R_A + R_B)$

$$= \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \tag{2.4}$$

Referring to the star network shown in Fig. 2.175(b),

the resistance between terminals 1 and 2 = $R_1 + R_2$ (2.5)

Since the two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \tag{2.6}$$

Similarly, $$R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} \tag{2.7}$$

and $$R_3 + R_1 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \tag{2.8}$$

Subtracting Eq. (2.7) from Eq. (2.6),

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \tag{2.9}$$

Adding Eq. (2.9) and Eq. (2.8),

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{2.10}$$

Similarly,

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{2.11}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{2.12}$$

Thus, star resistor connected to a terminal is equal to the product of the two delta resistors connected to the same terminal divided by the sum of the delta resistors.

2.7.2 Star to Delta Transformation

Multiplying the above equations,

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A + R_B + R_C)^2} \tag{2.13}$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \tag{2.14}$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A + R_B + R_C)^2} \tag{2.15}$$

Adding Eqs (2.13), (2.14) and (2.15),

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$= \frac{R_A R_B R_C}{R_A + R_B + R_C}$$

$$= R_A R_1$$

$$= R_B R_2$$

$$= R_C R_3$$

Hence,

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$= R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$= R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Thus, delta resistor between the two terminals is the sum of two star resistors connected to the same terminals plus the product of the two resistors divided by the remaining third star resistor.

Note: When three equal resistors are connected in delta, the equivalent star resistance is given by

$$R_Y = \frac{R_\Delta R_\Delta}{R_\Delta + R_\Delta + R_\Delta} = \frac{R_\Delta}{3}$$

or $$R_\Delta = 3R_Y$$

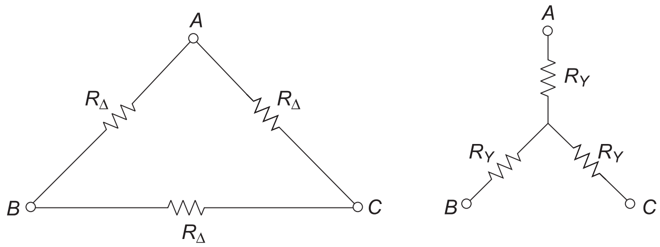


Fig. 2.176

Example 1

Convert the star circuit into its equivalent delta circuit.

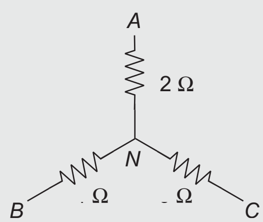


Fig. 2.177 **[May 2015]**

Solution Converting the given star network to delta,

$$R_A = 2 + 6 + \frac{2 \times 6}{4} = 11\,\Omega$$

$$R_B = 6 + 4 + \frac{6 \times 4}{2} = 22\,\Omega$$

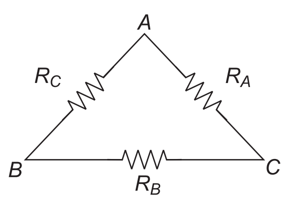


Fig. 2.178

$$R_C = 4 + 2 + \frac{4 \times 2}{6} = 7.33\ \Omega$$

Example 2

Find an equivalent resistance between terminals A and B.

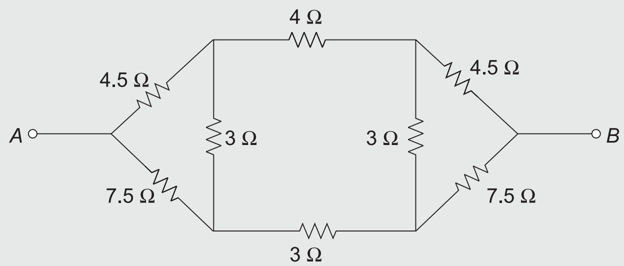


Fig. 2.179

Solution Converting the two delta networks formed by resistors of 4.5 Ω, 3 Ω and 7.5 Ω into equivalent star networks,

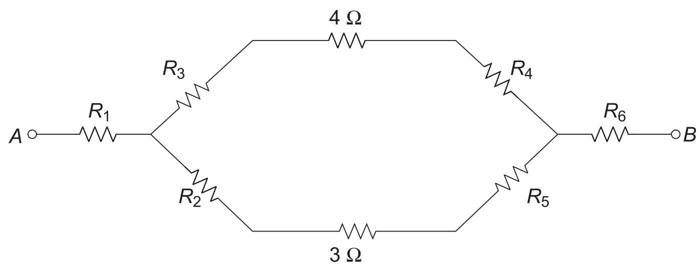


Fig. 2.180

$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\ \Omega$$

$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\ \Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\ \Omega$$

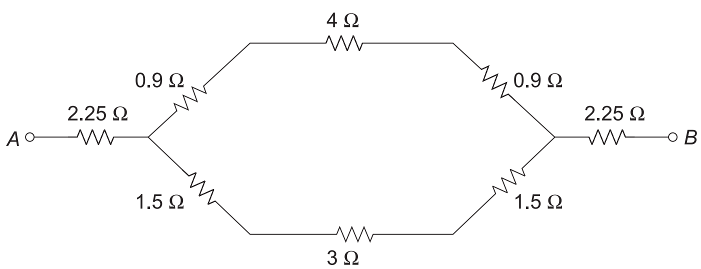


Fig. 2.181

Simplifying the network,

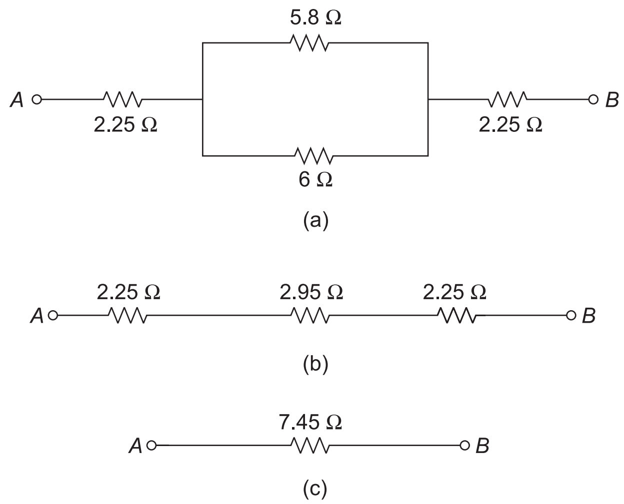


Fig. 2.182

$$R_{AB} = 7.45\ \Omega$$

Example 3

Find an equivalent resistance between terminals A and B.

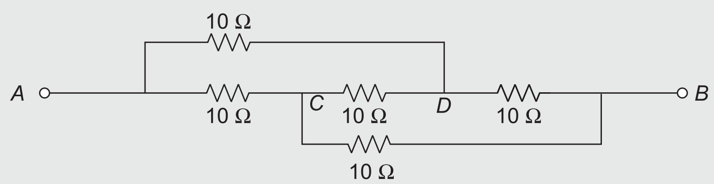


Fig. 2.183

Solution Redrawing the network,

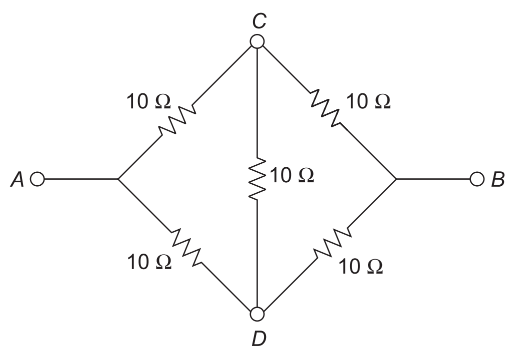


Fig. 2.184

Converting the delta network formed by three resistors of 10 Ω into an equivalent star network,

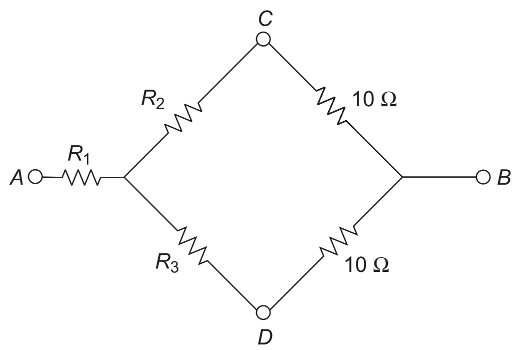


Fig. 2.185

$$R_1 = R_2 = R_3 = \frac{10 \times 10}{10+10+10} = \frac{10}{3}\ \Omega$$

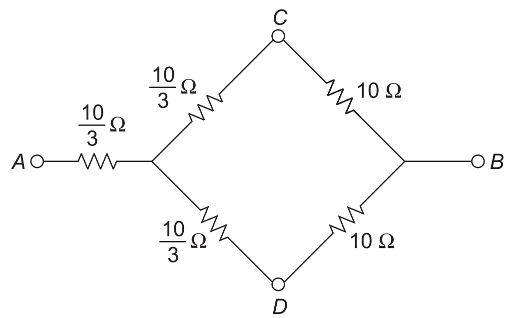


Fig. 2.186

Simplifying the network,

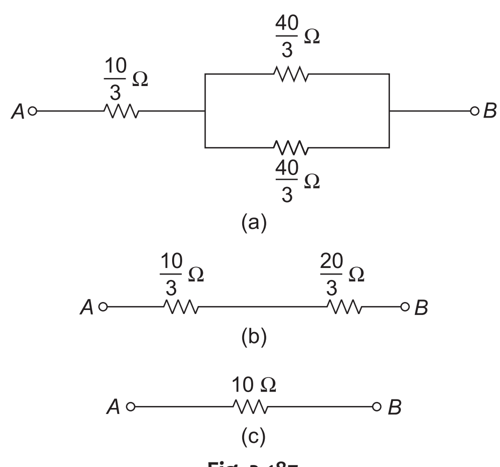


Fig. 2.187

$$R_{AB} = 10\ \Omega$$

Example 4

Calculate R_{xy} for the circuit shown in Fig. 2.188.

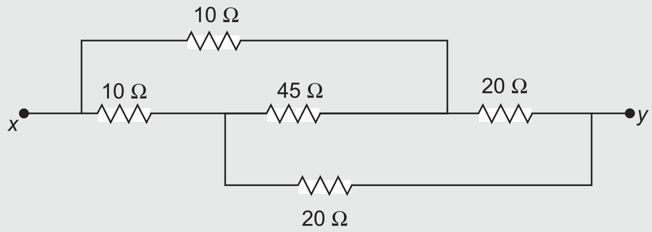


Fig. 2.188

[Dec 2012]

Solution Redrawing the network.

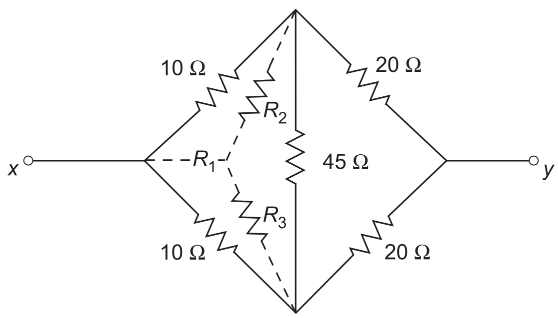


Fig. 2.189

Converting the delta network formed by resistors of 10 Ω, 10 Ω and 45 Ω into an equivalent star network,

$$R_1 = \frac{10 \times 10}{10 + 10 + 45} = 1.54\ \Omega$$

$$R_2 = R_3 = \frac{10 \times 45}{10 + 10 + 45} = 6.92\ \Omega$$

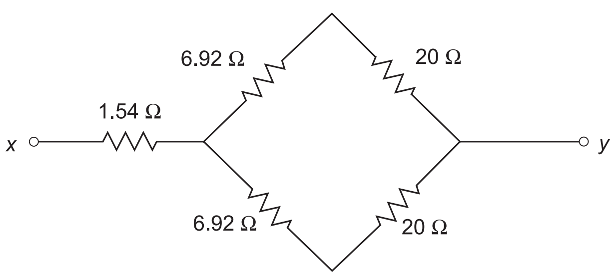


Fig. 2.190

Simplifying the network,

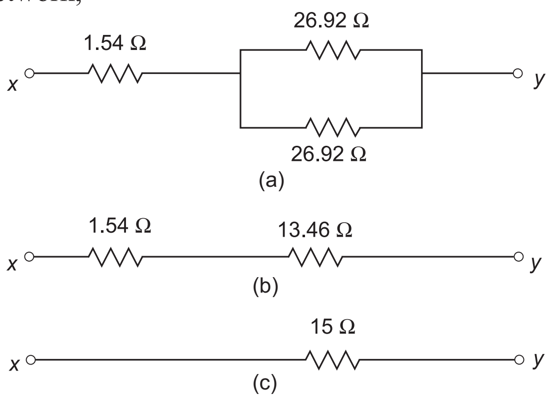


Fig. 2.191

$$R_{xy} = 15\ \Omega$$

Example 5

Find an equivalent resistance between terminals A and B.

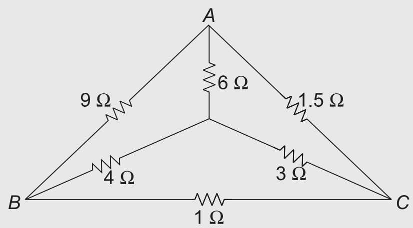


Fig. 2.192

Solution Converting the star network formed by resistors of 3 Ω, 4 Ω and 6 Ω into an equivalent delta network,

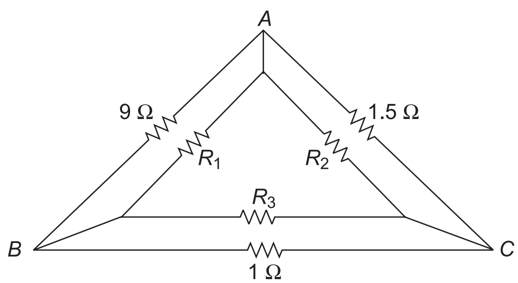


Fig. 2.193

$$R_1 = 6 + 4 + \frac{6 \times 4}{3} = 18\ \Omega$$

$$R_2 = 6 + 3 + \frac{6 \times 3}{4} = 13.5\ \Omega$$

$$R_3 = 4 + 3 + \frac{4 \times 3}{6} = 9\,\Omega$$

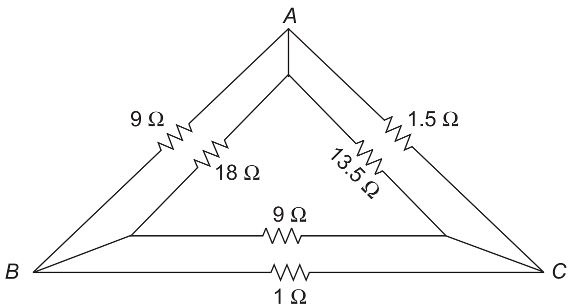


Fig. 2.194

Simplifying the network,

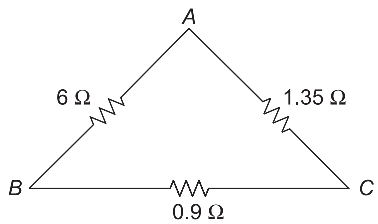


Fig. 2.195

$$R_{AB} = 6 \parallel (1.35 + 0.9)$$
$$= 6 \parallel 2.25$$
$$= 1.64\,\Omega$$

Example 6

Find an equivalent resistance between terminals A and N by solving outer delta ABC.

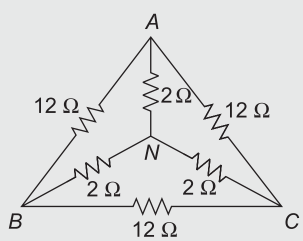


Fig. 2.196

Solution Converting outer delta ABC into a star network,

$$R_Y = \frac{12 \times 12}{12 + 12 + 12} = 4\,\Omega$$

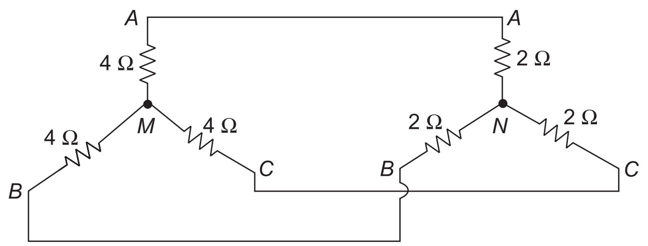


Fig. 2.197

Simplifying the network,

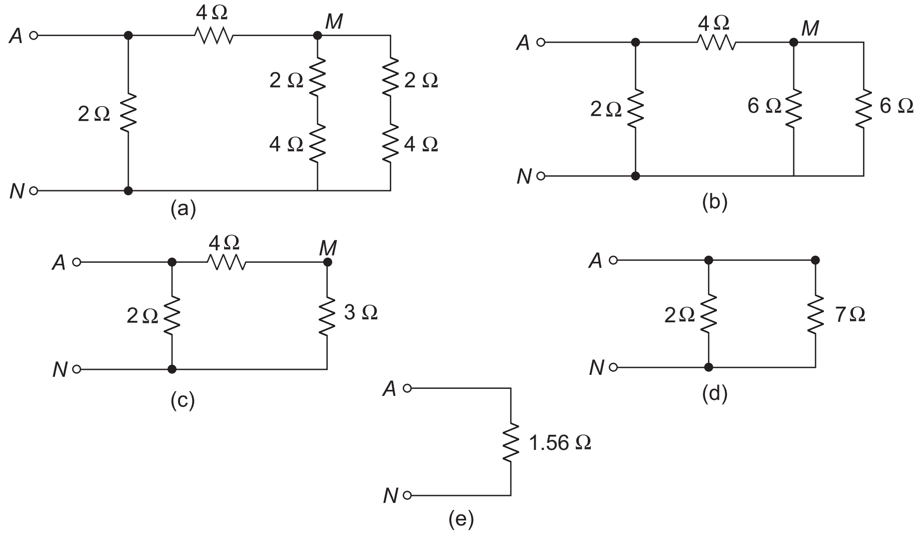


Fig. 2.198

$$R_{AN} = 1.56\ \Omega$$

Example 7

Find an equivalent resistance terminals between A and B.

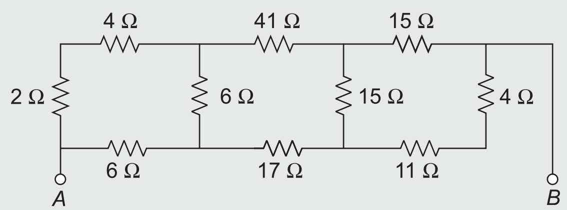


Fig. 2.199

Solution The resistors of 2 Ω and 4 Ω and the resistors of 4 Ω and 11 Ω are connected in series.

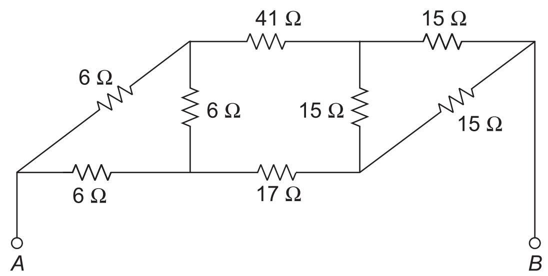


Fig. 2.200

Converting the two outer delta networks into equivalent star networks,

$$R_{Y_1} = \frac{6\times 6}{6+6+6} = 2\ \Omega$$

$$R_{Y_2} = \frac{15\times 15}{15+15+15} = 5\ \Omega$$

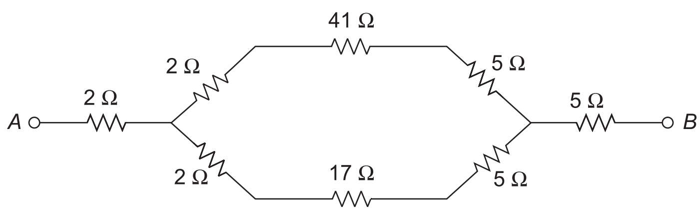


Fig. 2.201

Simplifying the network,

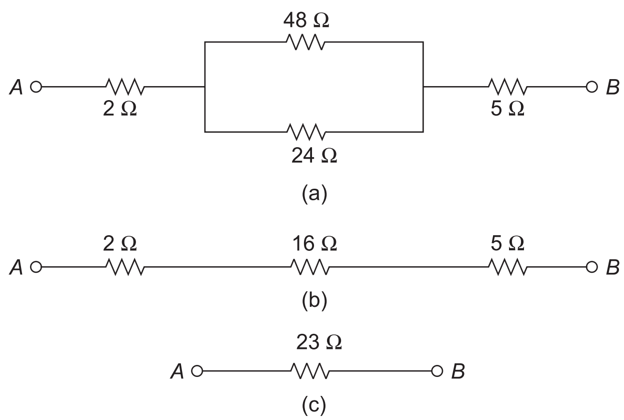


Fig. 2.202

$$R_{AB} = 23\ \Omega$$

Example 8

Find an equivalent resistance between terminals A and B.

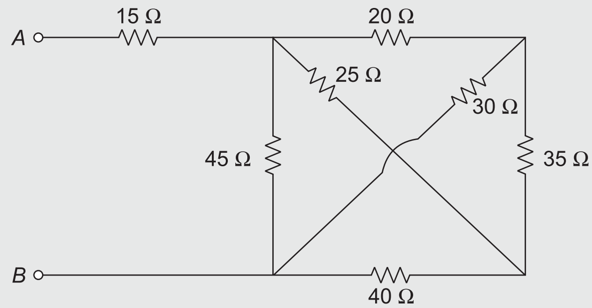


Fig. 2.203 **[May 2014]**

Solution Drawing the resistor of 30 Ω from outside,

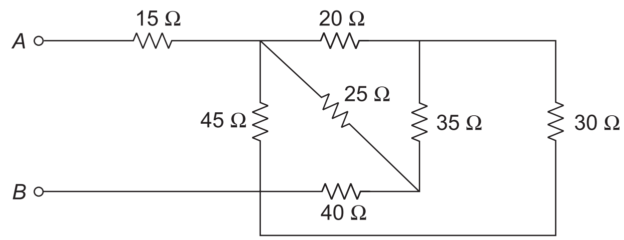


Fig. 2.204

Converting the delta network formed by resistors of 20 Ω, 25 Ω and 35 Ω into an equivalent star network,

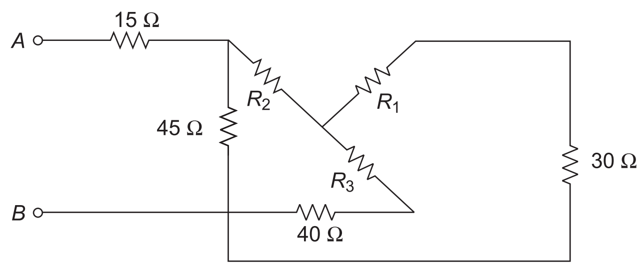


Fig. 2.205

$$R_1 = \frac{20 \times 35}{20 + 35 + 25} = 8.75\ \Omega$$

$$R_2 = \frac{20 \times 25}{20 + 35 + 25} = 6.25\ \Omega$$

$$R_3 = \frac{35 \times 25}{20+35+25} = 10.94\ \Omega$$

Redrawing the network,

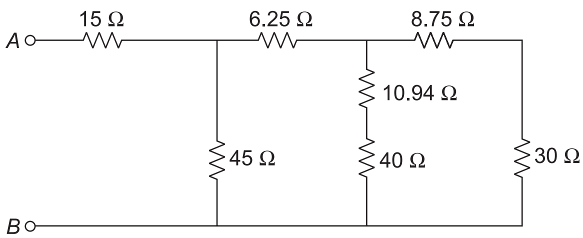


Fig. 2.206

Simplifying the network,

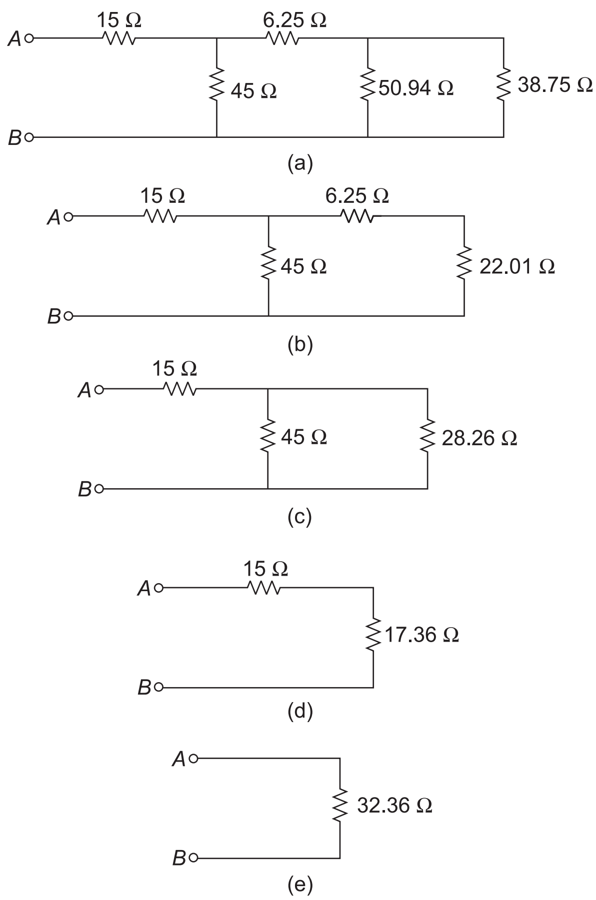


Fig. 2.207

$$R_{AB} = 32.36\ \Omega$$

Example 9

Find an equivalent resistance between terminals A and B.

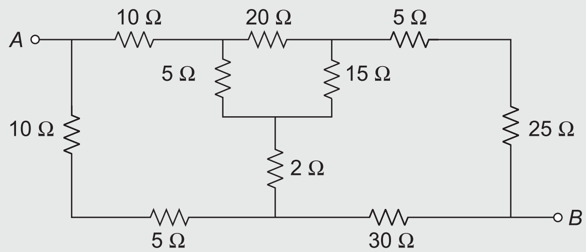


Fig. 2.208

Solution The resistors of 5 Ω and 25 Ω and the resistors of 10 Ω and 5 Ω are connected in series.

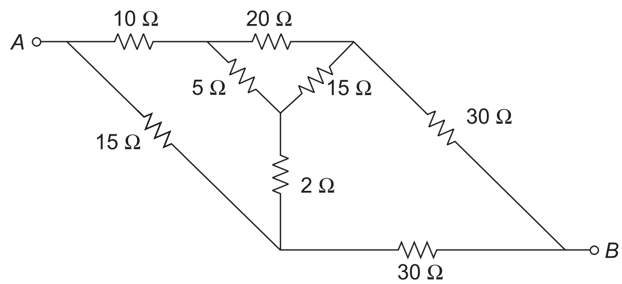


Fig. 2.209

Converting the delta network formed by the resistors of 20 Ω, 5 Ω and 15 Ω into an equivalent star network,

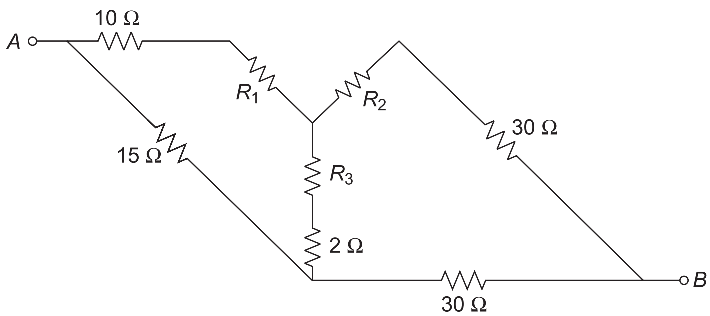


Fig. 2.210

$$R_1 = \frac{20 \times 5}{20+5+15} = 2.5\ \Omega$$

$$R_2 = \frac{20 \times 15}{20+5+15} = 7.5\ \Omega$$

$$R_3 = \frac{5 \times 15}{20+5+15} = 1.875\ \Omega$$

Redrawing the network,

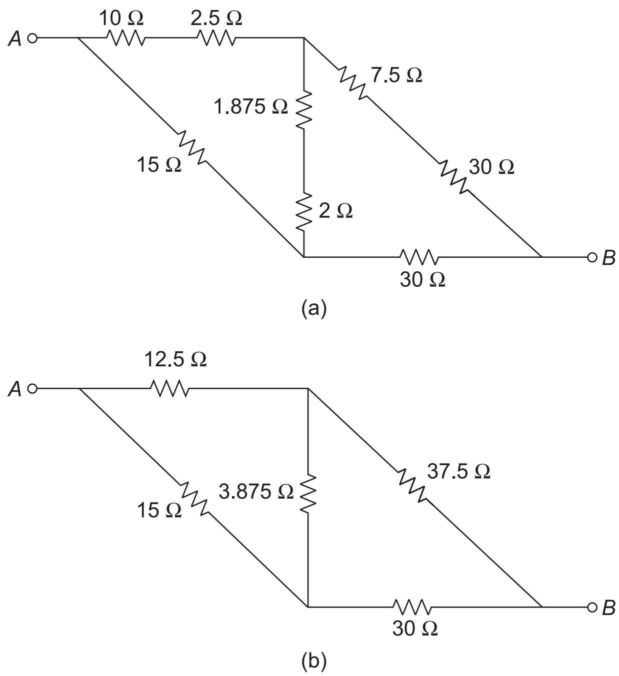


Fig. 2.211

Converting the delta network formed by the resistors of 3.875 Ω, 37.5 Ω and 30 Ω into an equivalent star network,

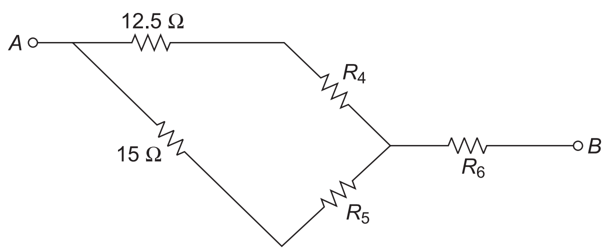


Fig. 2.212

$$R_4 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = 2.04\ \Omega$$

$$R_5 = \frac{3.875 \times 30}{3.875 + 37.5 + 30} = 1.63\ \Omega$$

$$R_6 = \frac{37.5 \times 30}{3.875 + 37.5 + 30} = 15.76\ \Omega$$

Simplifying the network,

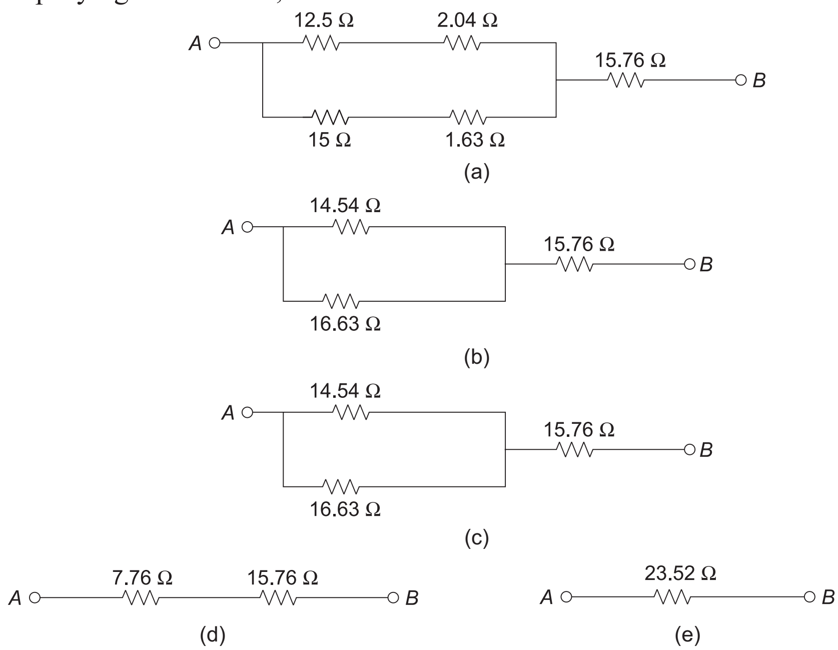


Fig. 2.213

$$R_{AB} = 23.52\ \Omega$$

Example 10

Find an equivalent resistance between terminals A and B.

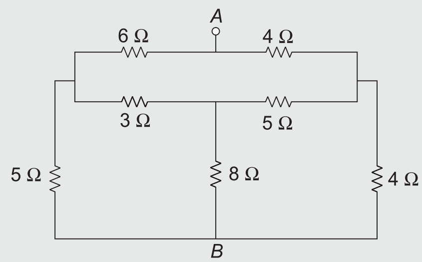


Fig. 2.214

Solution

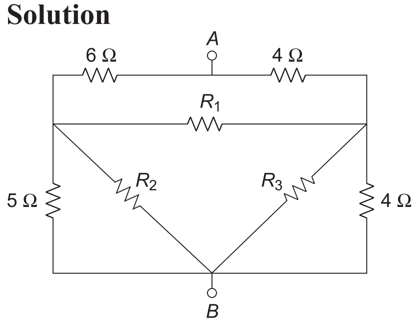


Fig. 2.215

Converting the star network formed by the resistors of 3 Ω, 5 Ω and 8 Ω into an equivalent delta network,

$$R_1 = 3 + 5 + \frac{3\times 5}{8} = 9.875\ \Omega$$

$$R_2 = 3 + 8 + \frac{3\times 8}{5} = 15.8\ \Omega$$

$$R_3 = 5 + 8 + \frac{5\times 8}{3} = 26.33\ \Omega$$

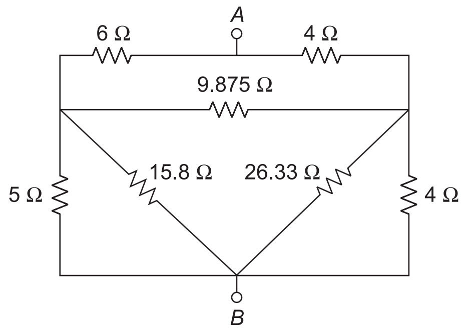


Fig. 2.216

The resistors of 15.8 Ω and 5 Ω and the resistors of 26.33 Ω and 4 Ω are connected in parallel.

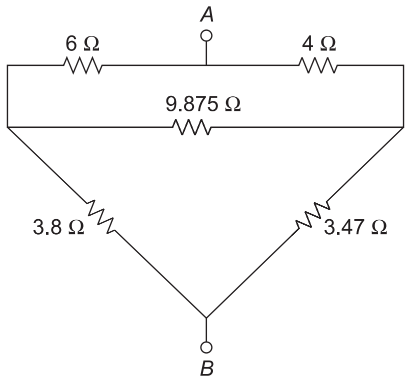


Fig. 2.217

Converting the delta network into a star network,

$$R_4 = \frac{3.8 \times 9.875}{3.8 + 9.875 + 3.47} = 2.19\ \Omega$$

$$R_5 = \frac{3.8 \times 3.47}{3.8 + 9.875 + 3.47} = 0.77\ \Omega$$

$$R_6 = \frac{3.47 \times 9.875}{3.8 + 9.875 + 3.47} = 2\ \Omega$$

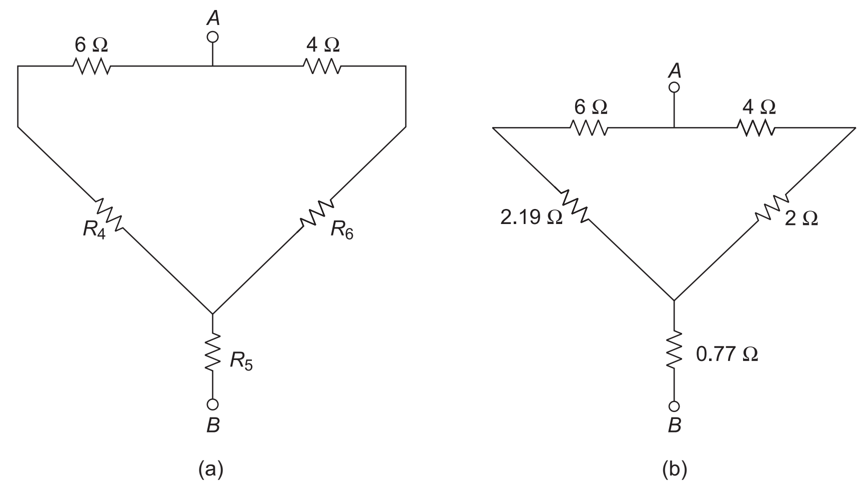


Fig. 2.218

Simplifying the network,

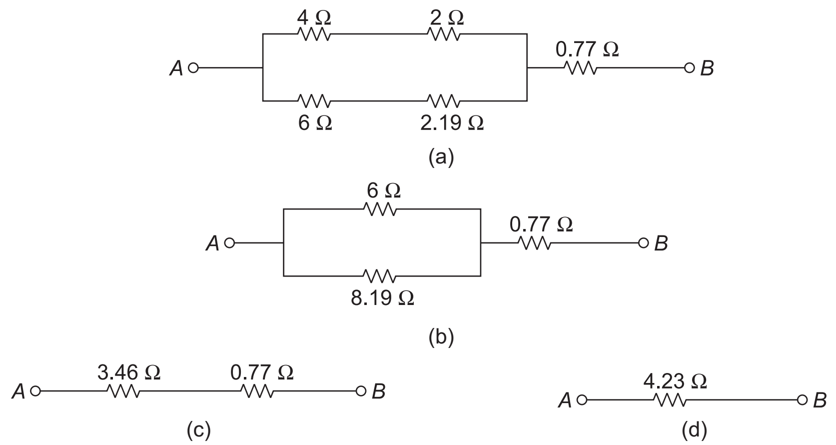


Fig. 2.219

$$R_{AB} = 4.23\ \Omega$$

Example 11

Find an equivalent resistance between terminals A and B.

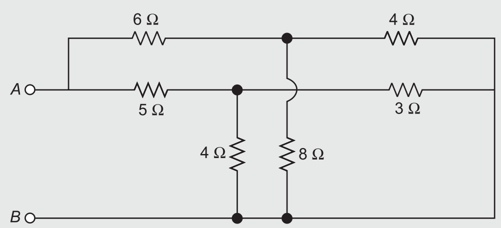


Fig. 2.220

Solution Converting the star network formed by the resistors of 3 Ω, 4 Ω and 5 Ω into an equivalent delta network,

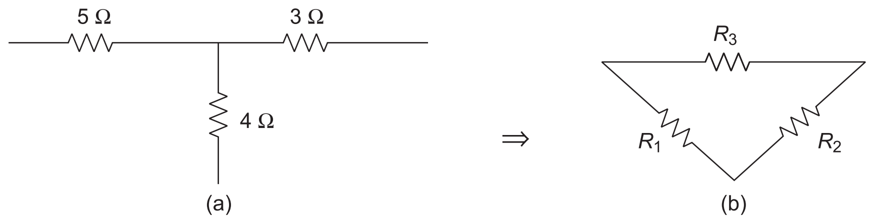


Fig. 2.221

$$R_1 = 5+4+\frac{5\times 4}{3} = 15.67\ \Omega$$

$$R_2 = 3+4+\frac{3\times 4}{5} = 9.4\ \Omega$$

$$R_3 = 5+3+\frac{5\times 3}{4} = 11.75\ \Omega$$

Similarly, converting the star network formed by the resistors of 4 Ω, 6 Ω and 8 Ω into an equivalent delta network,

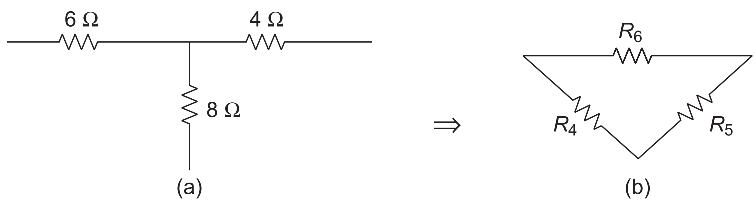


Fig. 2.222

$$R_4 = 6+8+\frac{6\times 8}{4} = 26\ \Omega$$

$$R_5 = 4+8+\frac{4\times 8}{6} = 17.33\ \Omega$$

$$R_6 = 6+4+\frac{6\times 4}{8} = 13\ \Omega$$

These two delta networks are connected in parallel between points A and B.

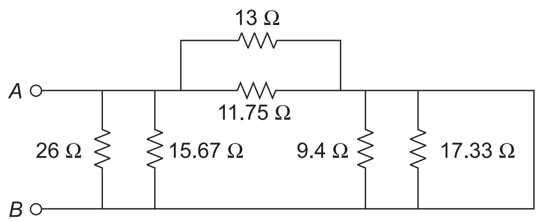


Fig. 2.223

The resistors of 9.4 Ω and 17.33 Ω are in parallel with a short. Hence, the equivalent resistance of this combination becomes zero.
Simplifying the parallel networks,

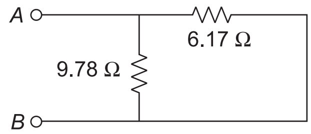


Fig. 2.224

$$R_{AB} = 6.17 \parallel 9.78 = 3.78\ \Omega$$

Example 12

Find the value of current flowing through 6 Ω resistor.

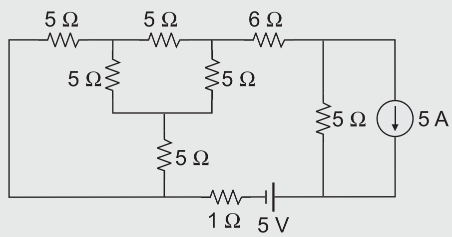


Fig. 2.225 **[Dec 2015]**

Solution Converting the parallel combination of the current source of 5 A and the resistor of 5 Ω into an equivalent series combination of voltage source and series resistor,

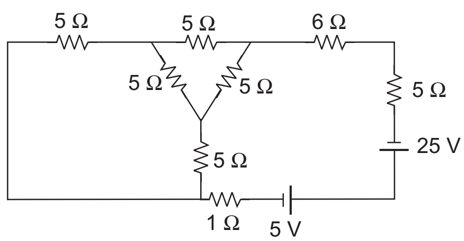


Fig. 2.226

Converting the delta network formed by three 5 Ω resistors into an equivalent star network,

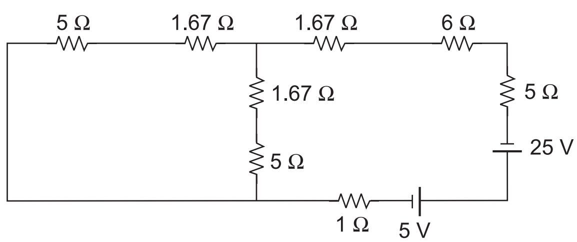


Fig. 2.227

By series-parallel reduction technique and adding two voltage sources,

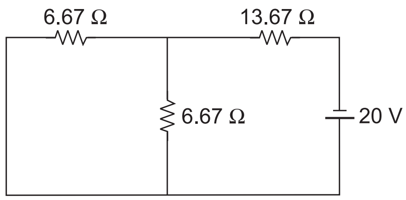


Fig. 2.228

Again by series-parallel reduction technique,

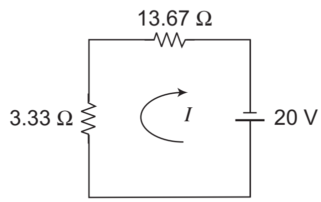


Fig. 2.229

$$I = \frac{20}{13.67+3.33} = 1.18\text{ A}$$

$$I_{6\Omega} = I_{13.67\Omega} = I = 1.18\text{ A}$$

Example 13

Determine the current supplied by the battery.

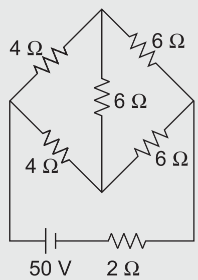


Fig. 2.230

Solution Converting the delta network formed by resistors of 6 Ω, 6 Ω and 6 Ω into an equivalent star network,

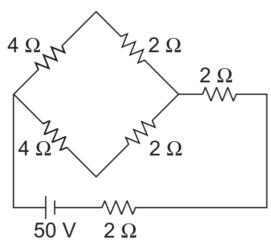


Fig. 2.231

Simplifying the network,

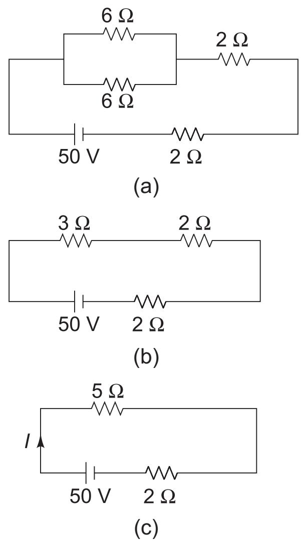


Fig. 2.232

$$I = \frac{50}{5+2} = 7.14\text{ A}$$

Example 14

Calculate the value of current flowing through the 10 Ω resistor.

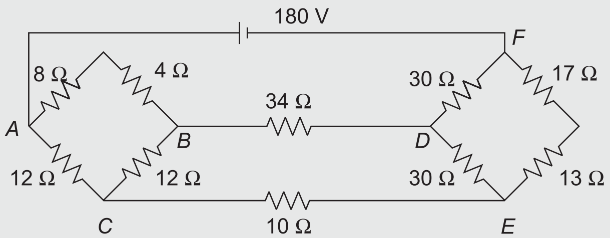


Fig. 2.233

Solution Between terminals A and B resistors of 8 Ω and 4 Ω are connected in series. Similarly, between terminals F and E, resistors of 17 Ω and 13 Ω are connected in series.

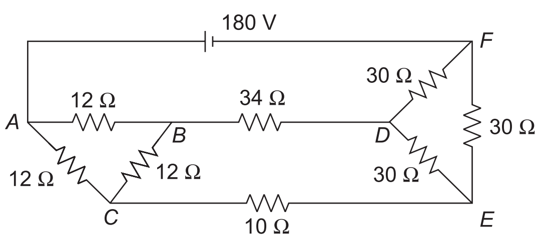


Fig. 2.234

Converting delta ABC and DEF into an equivalent star network,

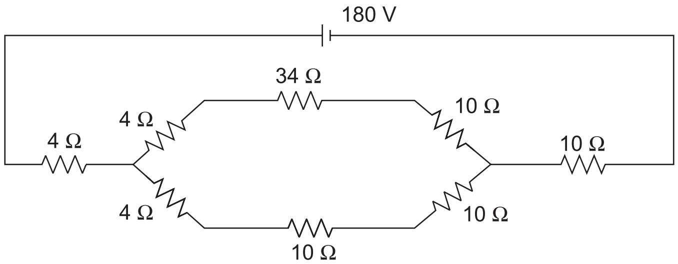
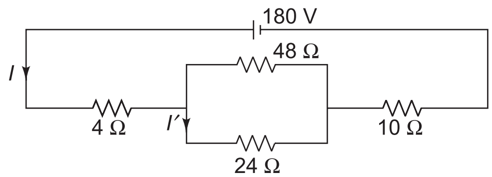
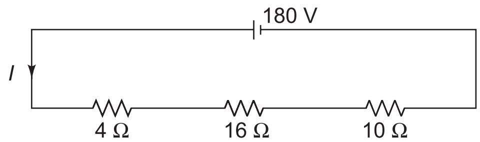


Fig. 2.235

Simplifying the network,



(a)



(b)

Fig. 2.236

$$I = \frac{180}{4+16+10} = 6\text{ A}$$

By current-division rule,

$$I' = I_{24\,\Omega} = I_{10\,\Omega} = 6 \times \frac{48}{24+48} = 4\text{ A}$$

Example 15

Determine current flow through the 20 Ω resistor in the following circuit in Fig. 2.237.

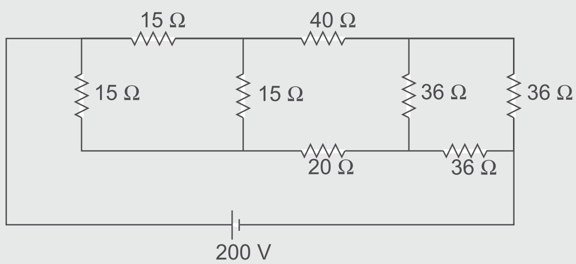


Fig. 2.237 **[Dec 2013]**

Solution Converting the two outer delta networks into equivalent star networks,

$$R_{Y1} = \frac{15 \times 15}{15+15+15} = 5\ \Omega$$

$$R_{Y2} = \frac{36 \times 36}{36+36+36} = 12\ \Omega$$

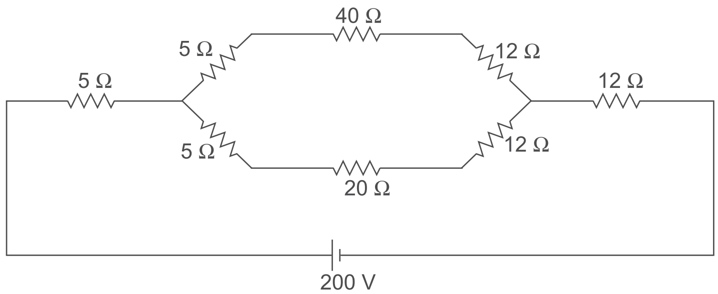


Fig. 2.238

Simplifying the network,

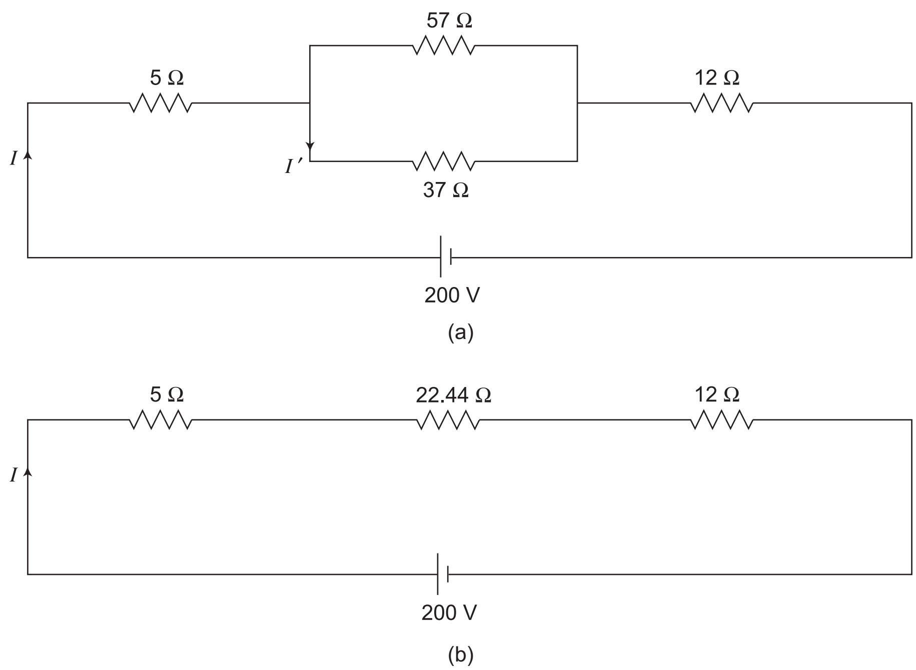


Fig. 2.239

$$I = \frac{200}{5 + 22.44 + 12} = 5.07\ \text{A}$$

By current-division rule,

$$I_{20\,\Omega} = I_{37\,\Omega} = 5.07 \times \frac{57}{57 + 37} = 3.07\ \text{A}$$

Example 16

Find the current supplied by the battery.

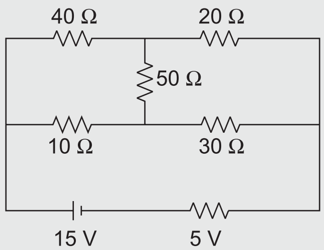


Fig. 2.240

Solution Converting the star network formed by resistors of 40 Ω, 20 Ω and 50 Ω into an equivalent delta network,

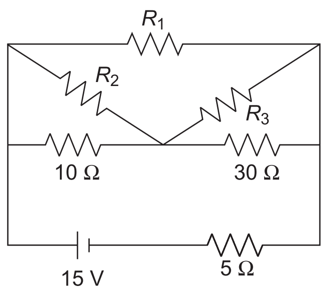


Fig. 2.241

$$R_1 = 40 + 20 + \frac{40 \times 20}{50} = 76\ \Omega$$

$$R_2 = 40 + 50 + \frac{40 \times 50}{20} = 190\ \Omega$$

$$R_3 = 20 + 50 + \frac{20 \times 50}{40} = 95\ \Omega$$

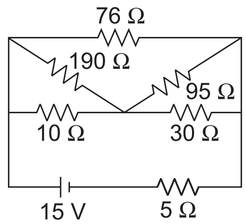


Fig. 2.242

The resistors of 190 Ω and 10 Ω and the resistors of 95 Ω and 30 Ω are connected in parallel.

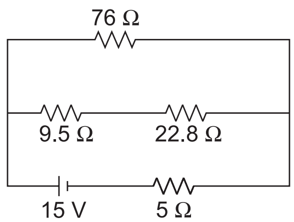


Fig. 2.243

Simplifying the network,

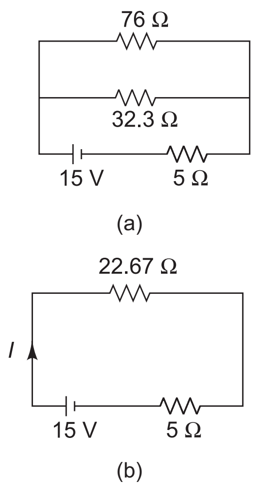
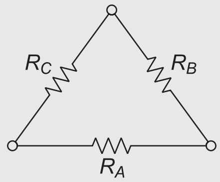


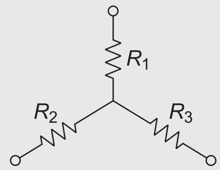
Fig. 2.244

$$I = \frac{15}{22.67 + 5} = 0.542 \text{ A}$$

Useful Formulae



Delta



Star

Delta to star transformation

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Star to delta transformation

$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Exercise 2.5

2.1 Find the equivalent resistance between terminals *A* and *B*.

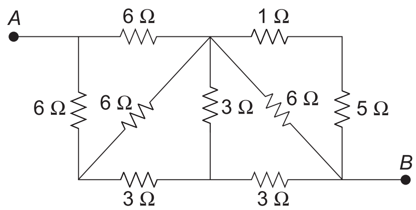


Fig. 2.245

[5 Ω]

2.2 Find the equivalent resistance between terminals *A* and *B*.

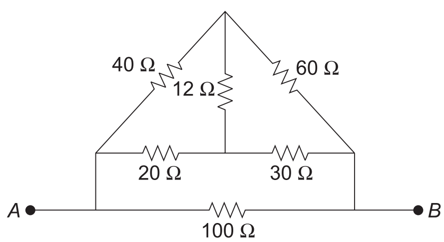


Fig. 2.246

[25 Ω]

2.3 Find the equivalent resistance between terminals A and B.

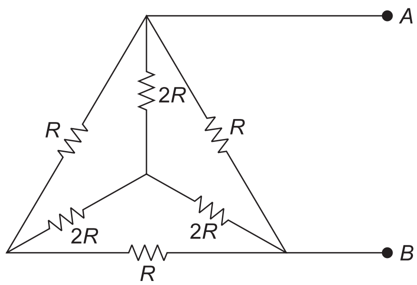


Fig. 2.247

$$\left[\frac{4}{7}R\right]$$

2.4 Find the equivalent resistance between terminals A and B.

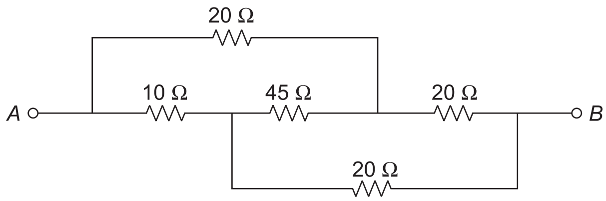


Fig. 2.248

[17 Ω]

2.5 Find R_{AB} by solving the outer delta (X–B–Y) only.

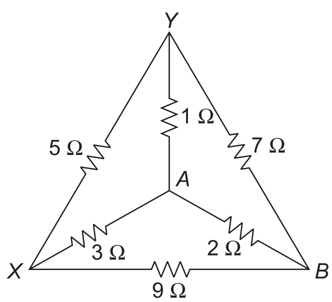


Fig. 2.249

[1.41 Ω]

2.6 Find the equivalent resistance between terminals A and B.

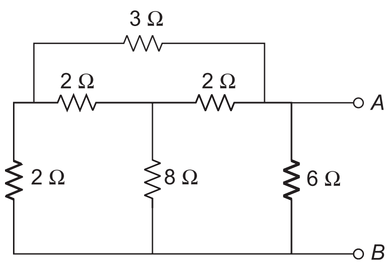


Fig. 2.250

[2.625 Ω]

2.7 Find the equivalent resistance between terminals *A* and *B*.

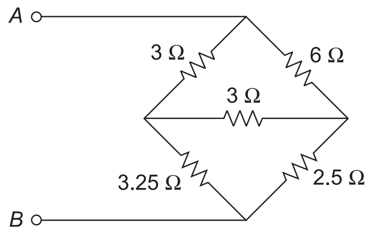


Fig. 2.251

[3.5 Ω]

2.8 Find the equivalent resistance between terminals *A* and *B*.

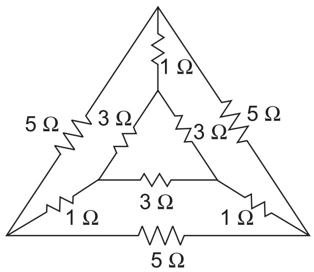


Fig. 2.252

[1.82 Ω]

2.9 Find the equivalent resistance between terminals *A* and *B*.

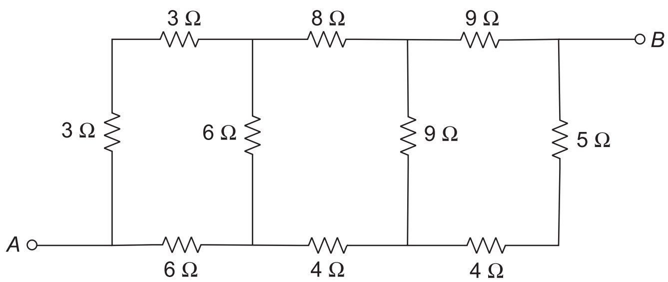


Fig. 2.253

[10.32 Ω]

2.10 Find the equivalent resistance between terminals *A* and *B*.

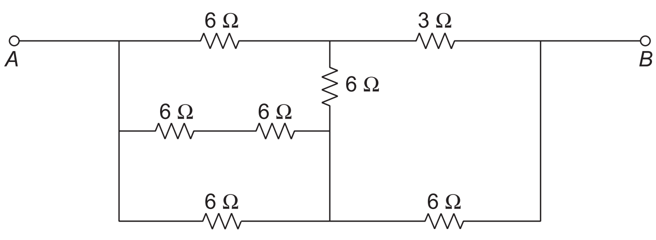


Fig. 2.254

[4.59 Ω]

2.11 Find the equivalent resistance between terminals A and B.

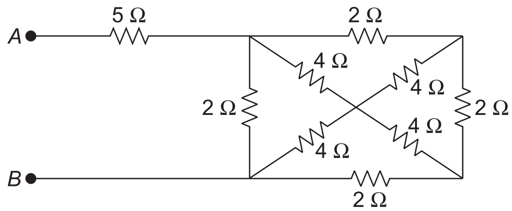


Fig. 2.255

[6.24 Ω]

2.12 Determine the value of current I.

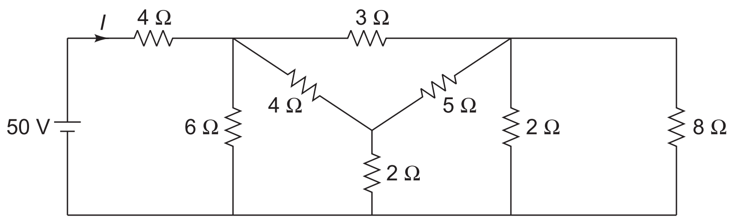


Fig. 2.256

[8.59 A]

2.13 Find the voltage between terminals A and B.

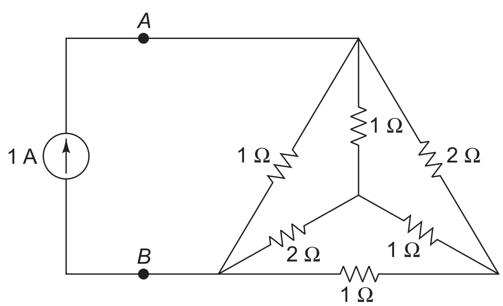


Fig. 2.257

[0.56 V]

2.14 Determine the power supplied to the network.

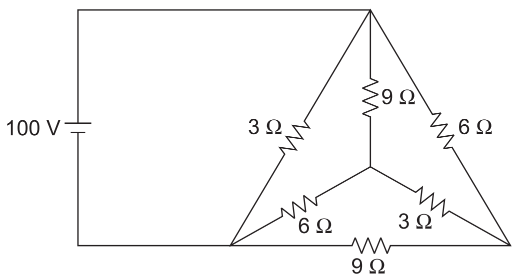


Fig. 2.258

[4705.88 Ω]

2.15 Find the value of current *I*.

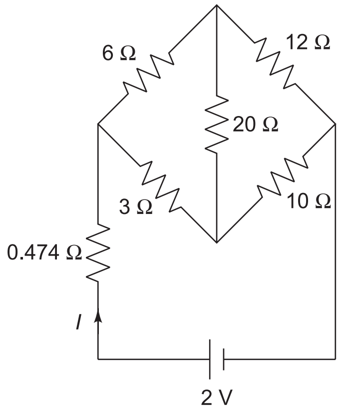


Fig. 2.259

[0.25 A]

2.16 Determine the value of current flowing through the 10 Ω resistor.

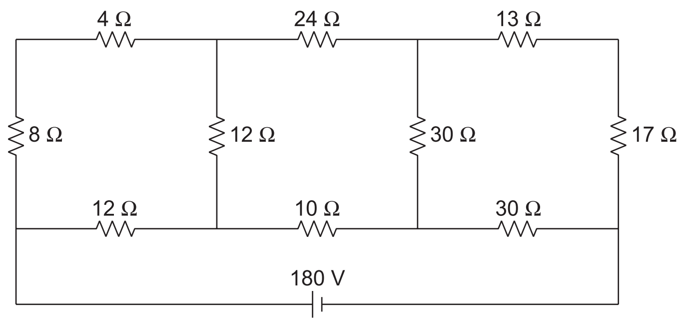


Fig. 2.260

[3.84 A]

2.8 SUPERPOSITION THEOREM

It states that '*In a linear network containing more than one independent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned.

The independent current sources are represented by infinite resistances, i.e., open circuits.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the circuit shown in Fig. 2.261. Suppose we have to find current I_4 flowing through R_4.

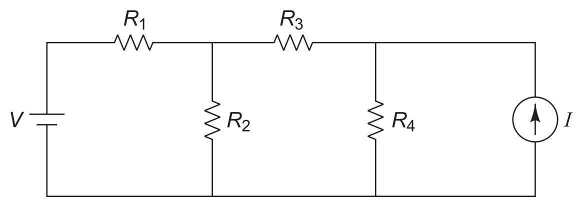


Fig. 2.261 *Superposition theorem*

2.8.1 Steps to be followed in Superposition Theorem

1. Find the current I_4' flowing through R_4 due to independent voltage source 'V', representing independent current source with infinite resistance, i.e., open circuit.

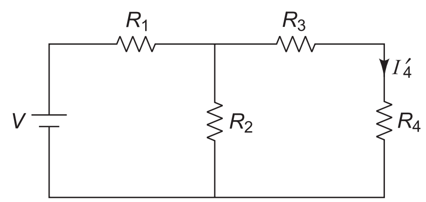


Fig. 2.262 *Step 1*

2. Find the current I_4'' flowing through R_4 due to independent current source 'I', representing the independent voltage source with zero resistance or short circuit.

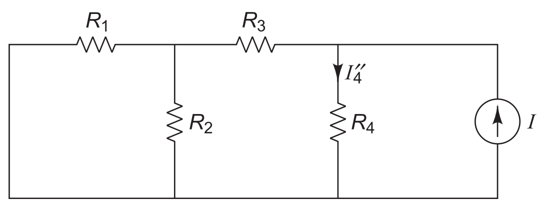


Fig. 2.263 *Step 2*

3. Find the resultant current I_4 through R_4 by the superposition theorem.

$$I_4 = I_4' + I_4''$$

Example 1

Find the value of current flowing through the 2 Ω resistor.

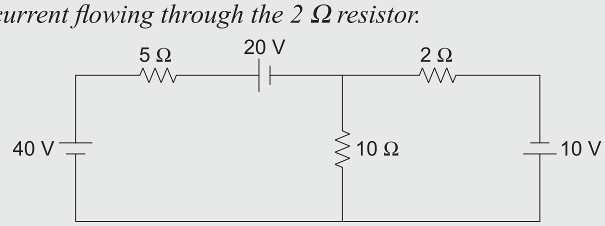


Fig. 2.264

Solution *Step I: When the 40 V source is acting alone*

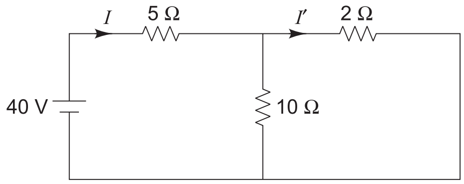


Fig. 2.265

By series–parallel reduction technique,

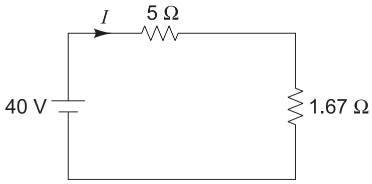


Fig. 2.266

$$I = \frac{40}{5+1.67} = 6 \text{ A}$$

From Fig. 2.265, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5 \text{ A} (\rightarrow)$$

Step II: When the 20 V source is acting alone

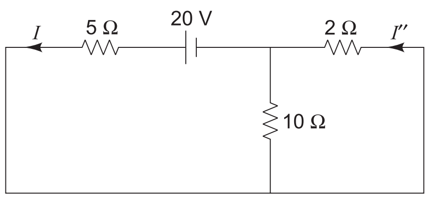


Fig. 2.267

By series–parallel reduction technique,

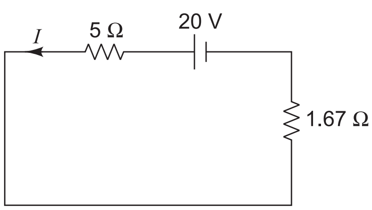


Fig. 2.268

$$I = \frac{20}{5+1.67} = 3 \text{ A}$$

From Fig. 2.267, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5 \text{ A } (\leftarrow) = -2.5\text{A } (\rightarrow)$$

Step III: *When the 10 V source is acting alone*

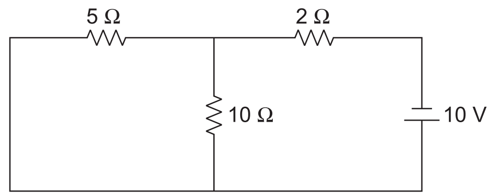


Fig. 2.269

By series–parallel reduction technique,

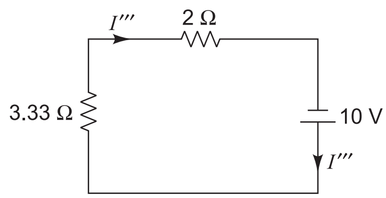


Fig. 2.270

$$I''' = \frac{10}{3.33+2} = 1.88 \text{ A } (\rightarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 5 - 2.5 + 1.88 \\ &= 4.38 \text{ A } (\rightarrow) \end{aligned}$$

Example 2

Find the value of current flowing through the 1 Ω resistor.

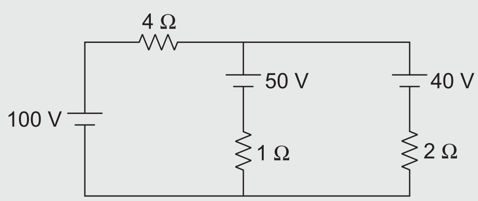


Fig. 2.271

Solution *Step I: When the 100 V source is acting alone*

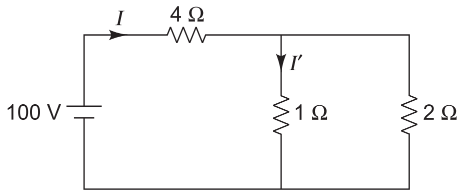


Fig. 2.272

By series–parallel reduction technique

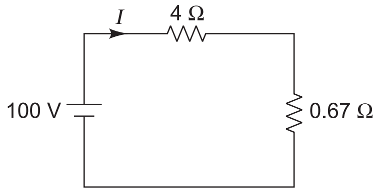


Fig. 2.273

$$I = \frac{100}{4+0.67} = 21.41 \text{ A}$$

From Fig. 2.272, by current-division rule,

$$I' = 21.41 \times \frac{2}{1+2} = 14.27 \text{ A} (\downarrow)$$

Step II: When the 50 V source is acting alone

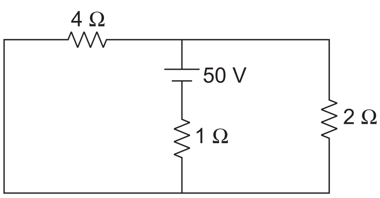


Fig. 2.274

By series–parallel reduction technique,

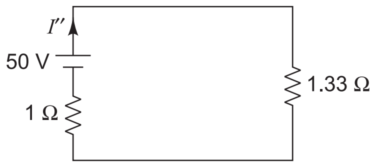


Fig. 2.275

$$I'' = \frac{50}{1+1.33} = 21.46 \text{ A} (\uparrow) = -21.46 \text{ A} (\downarrow)$$

Step III: When the 40 V source is acting alone

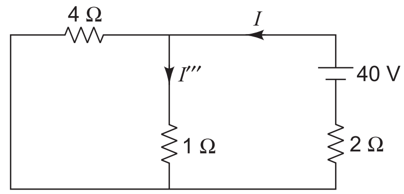


Fig. 2.276

By series–parallel reduction technique,

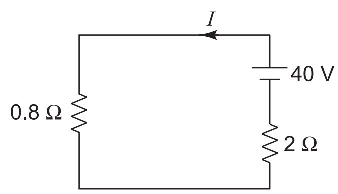


Fig. 2.277

$$I = \frac{40}{0.8+2} = 14.29 \text{ A}$$

From Fig. 2.276, by current-division rule,

$$I''' = 14.29 \times \frac{4}{4+1} = 11.43 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 14.27 - 21.46 + 11.43$$
$$= 4.24 \text{ A } (\downarrow)$$

Example 3

Find the value of current flowing through the 8 Ω resistor.

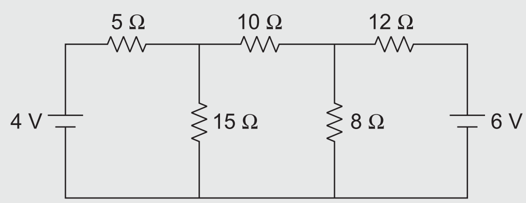


Fig. 2.278

Solution *Step I: When the 4 V source is acting alone*

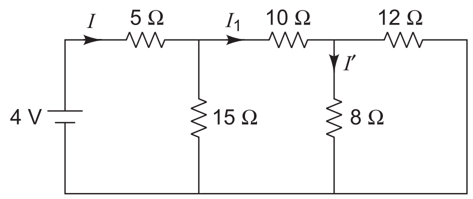


Fig. 2.279

By series–parallel reduction technique,

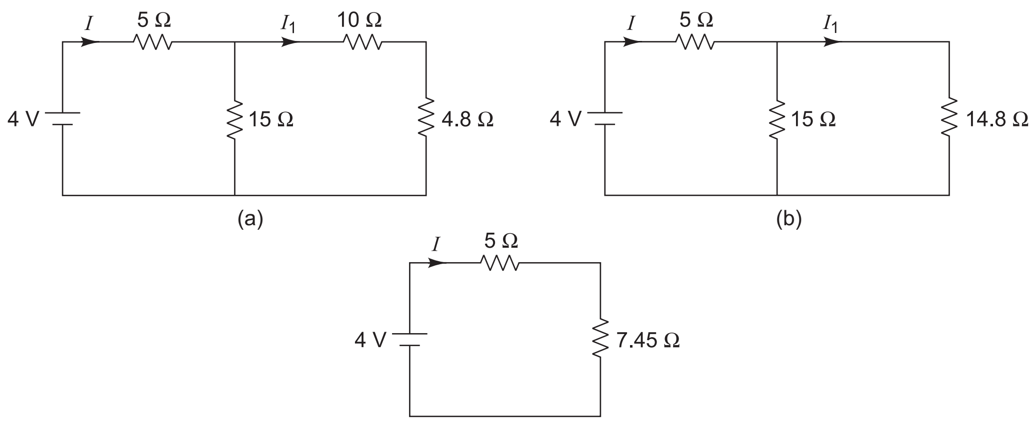


Fig. 2.280

$$I = \frac{4}{5+7.45} = 0.32 \text{ A}$$

From Fig. 2.280(b), by current-division rule,

$$I_1 = 0.32 \times \frac{15}{15+14.8} = 0.16 \text{ A}$$

From Fig. 2.279, by current-division rule,

$$I' = 0.16 \times \frac{12}{12+8} = 0.096 \text{ A} (\downarrow)$$

Step II: When the 6 V source is acting alone

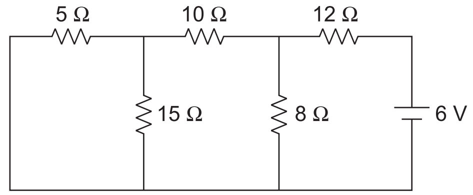


Fig. 2.281

By series–parallel reduction technique,

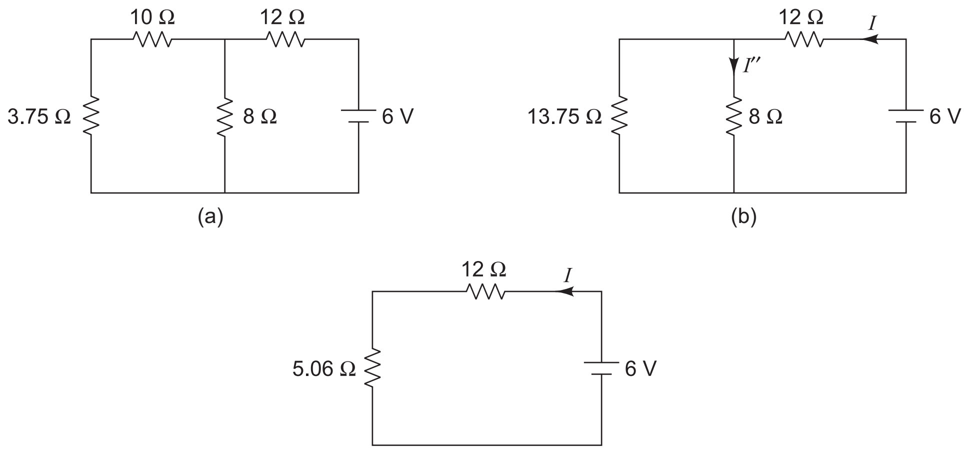


Fig. 2.282

$$I = \frac{6}{12 + 5.06} = 0.35 \text{ A}$$

From Fig. 2.282(b), by current division rule,

$$I'' = 0.35 \times \frac{13.75}{13.75 + 8} = 0.22 \text{ A} (\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 0.096 + 0.22 \\ &= 0.316 \text{ A} (\downarrow) \end{aligned}$$

Example 4

Find the value of current flowing through the 4 Ω resistor.

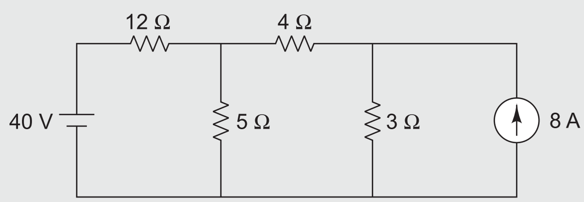


Fig. 2.283

Solution *Step I: When the 40 V source is acting alone*

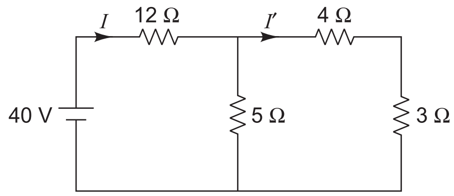


Fig. 2.284

By series–parallel reduction technique,

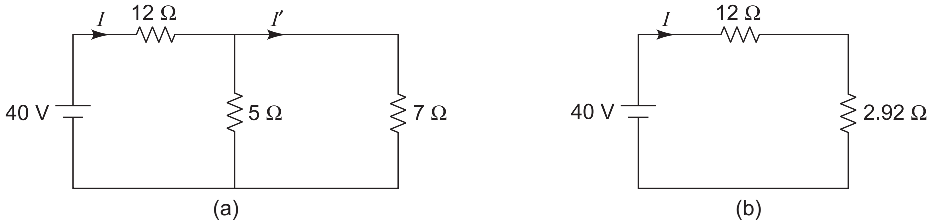


Fig. 2.285

$$I = \frac{40}{12 + 2.92} = 2.68 \text{ A}$$

From Fig. 2.285(a), by current-division rule,

$$I' = 2.68 \times \frac{5}{5+7} = 1.12 \text{ A} (\rightarrow) = -1.12 \text{ A} (\leftarrow)$$

Step II: When the 8 A source is acting alone

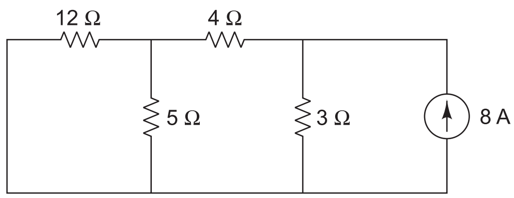


Fig. 2.286

By series–parallel reduction technique,

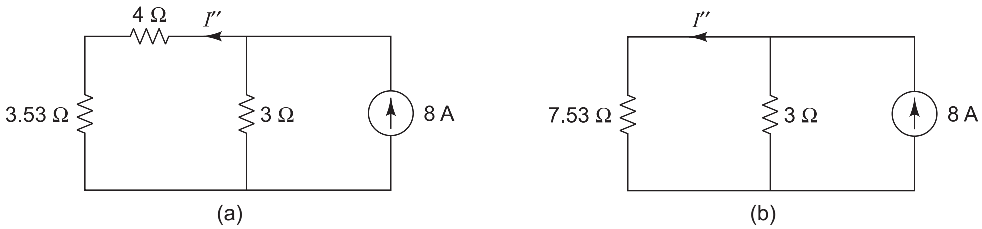


Fig. 2.287

From Fig. 2.287(b), by current-division rule,

$$I'' = 8 \times \frac{3}{7.53 + 3} = 2.28 \text{ A} (\leftarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= -1.12 + 2.28 \\ &= 1.16 \text{ A} (\leftarrow) \end{aligned}$$

Example 5

Find the value of current flowing in the 10 Ω resistor.

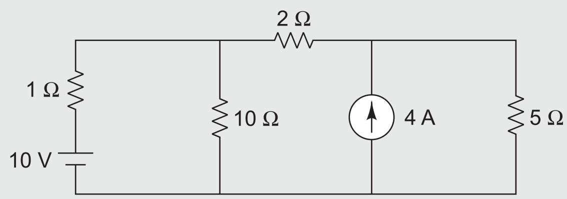


Fig. 2.288

Solution *Step I: When the 10 V source is acting alone*

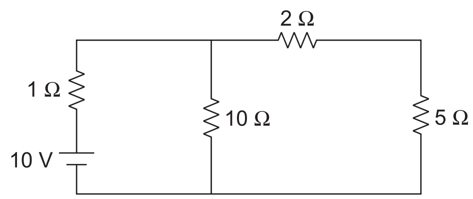


Fig. 2.289

By series–parallel reduction technique,

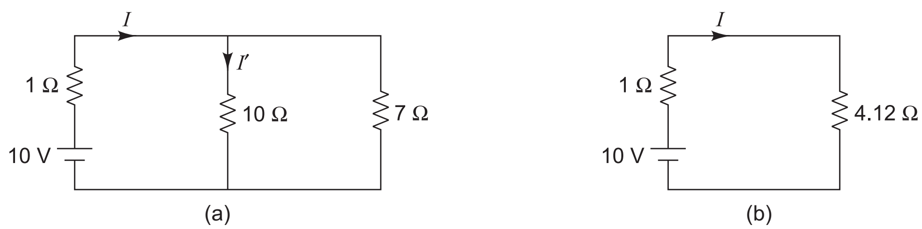


Fig. 2.290

$$I = \frac{10}{1 + 4.12} = 1.95 \text{ A}$$

From Fig. 2.290(a), by current-division rule,

$$I' = 1.95 \times \frac{7}{7+10} = 0.8 \text{ A } (\downarrow)$$

Step II: When the 4 A source is acting alone

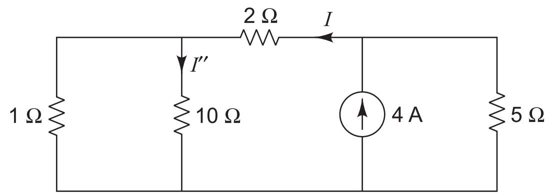
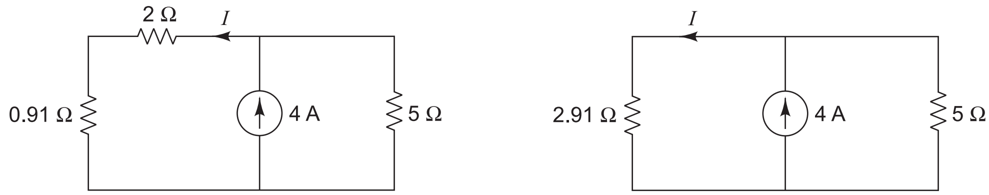


Fig. 2.291

By series–parallel reduction technique,



(a) (b)

Fig. 2.292

$$I = 4 \times \frac{5}{2.91+5} = 2.53 \text{ A}$$

From Fig. 2.291, by current-division rule,

$$I'' = 2.53 \times \frac{1}{1+10} = 0.23 \text{ A } (\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 0.8 + 0.23 \\ &= 1.03 \text{ A } (\downarrow) \end{aligned}$$

Example 6

Find the value of current flowing through the 8 Ω resistor.

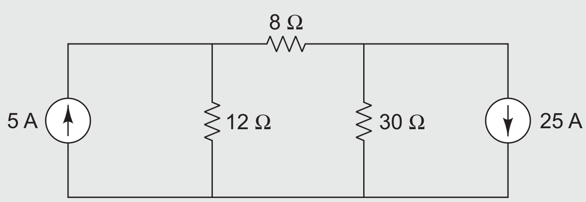


Fig. 2.293

Solution *Step I: When the 5A source is acting alone*

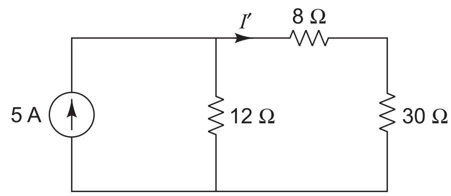


Fig. 2.294

By current-division rule,

$$I' = 5 \times \frac{12}{12 + 8 + 30} = 1.2\text{ A } (\rightarrow)$$

Step II: When the 25 A source is acting alone

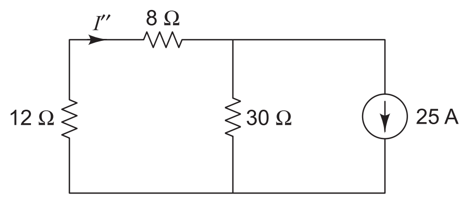


Fig. 2.295

By current-division rule,

$$I'' = 25 \times \frac{30}{30 + 12 + 8} = 15\text{ A } (\rightarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 1.2 + 15 \\ &= 16.2\text{ A } (\rightarrow) \end{aligned}$$

Example 7

Find the value of current flowing through the 4 Ω resistor.

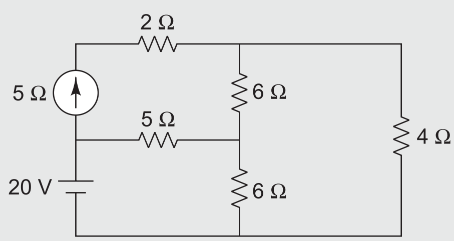


Fig. 2.296

Solution *Step I: When the 5 A source is acting alone*

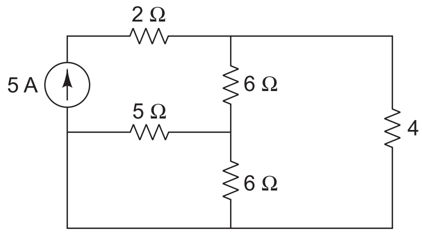
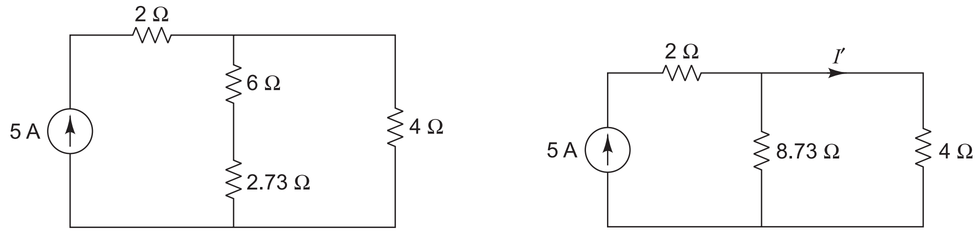


Fig. 2.297

By series–parallel reduction technique,



(a) (b)

Fig. 2.298

From Fig. 2.298(b), by current-division rule,

$$I' = 5 \times \frac{8.73}{8.73 + 4} = 3.43 \text{ A } (\downarrow)$$

Step II: When the 20 V source is acting alone

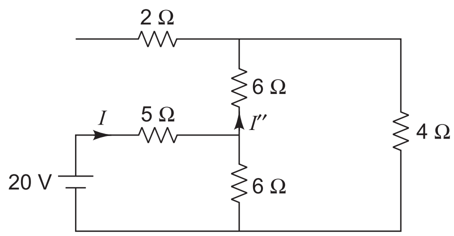
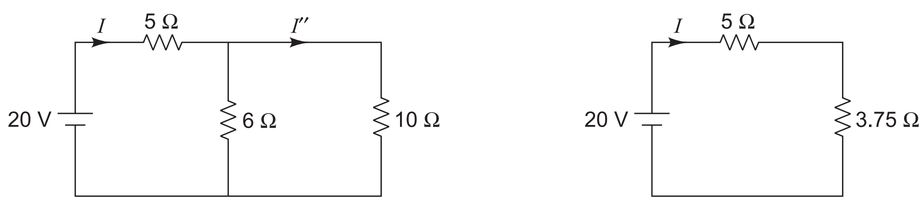


Fig. 2.299

By series–parallel reduction technique.



(a) (b)

Fig. 2.300

$$I = \frac{20}{5+3.75} = 2.29 \text{ A}$$

From Fig. 2.300(a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6+10} = 0.86 \text{ A } (\downarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 3.43 + 0.86 \\ &= 4.29 \text{ A } (\downarrow) \end{aligned}$$

Example 8

Find the value of current flowing through the 3 Ω resistor.

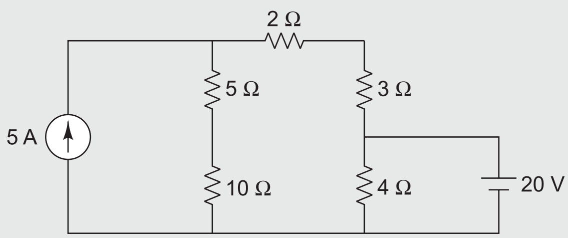


Fig. 2.301

Solution *Step I: When the 5 A source is acting alone*

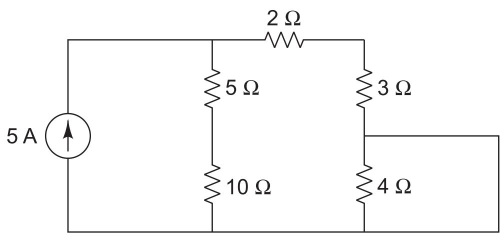


Fig. 2.302

By series–parallel reduction technique,

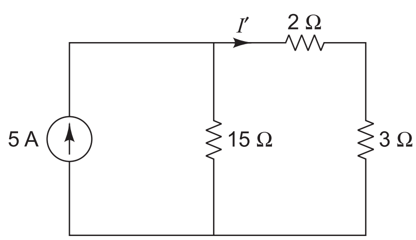


Fig. 2.303

By current-division rule,

$$I' = 5 \times \frac{15}{15+2+3} = 3.75\text{ A }(\downarrow)$$

Step II: When the 20 V source is acting alone

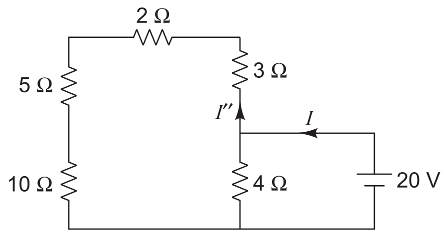
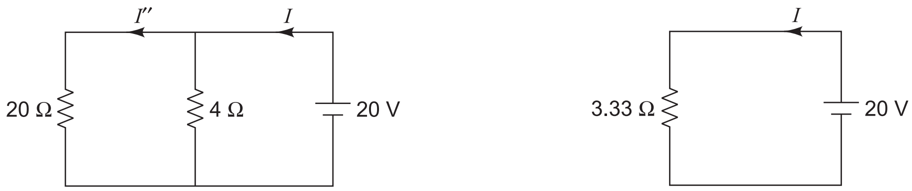


Fig. 2.304

By series–parallel reduction technique,



(a) (b)

Fig. 2.305

$$I = \frac{20}{3.33} = 6\text{ A}$$

From Fig. 2.305(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1\text{ A }(\uparrow) = -1\text{ A }(\downarrow)$$

Step III: By superposition theorem,

$$\begin{aligned} I &= I' + I'' \\ &= 3.75 - 1 \\ &= 2.75\text{ A }(\downarrow) \end{aligned}$$

Example 9

Find the value of current flowing in the 1 Ω resistor.

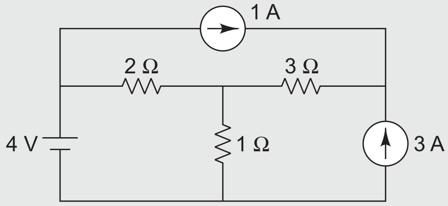


Fig. 2.306

Solution *Step I: When the 4 V source is acting alone*

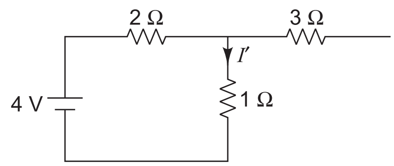


Fig. 2.307

By current-division rule,

$$I' = \frac{4}{2+1} = 1.33 \text{ A } (\downarrow)$$

Step II: When the 3 A source is acting alone

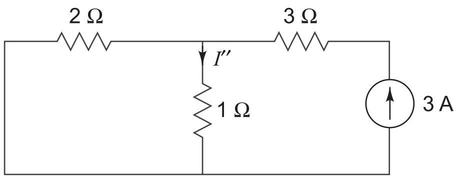


Fig. 2.308

By current-division rule,

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A } (\downarrow)$$

Step III: When the 1 A source is acting alone

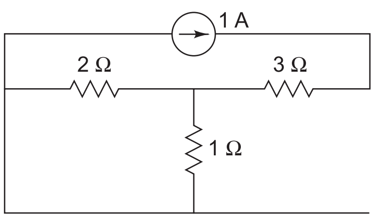


Fig. 2.309

Redrawing the circuit,

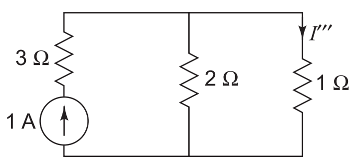


Fig. 2.310

By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 1.33 + 2 + 0.66$$
$$= 4\ \text{A}\ (\downarrow)$$

Example 10

Find the voltage V_{AB}.

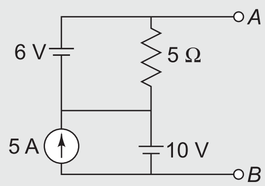


Fig. 2.311 [Dec 2014]

Solution *Step I: When the 6 V source is acting alone*

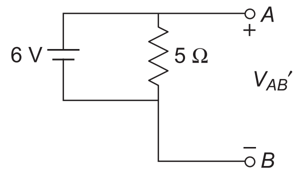


Fig. 2.312

$$V_{AB}' = 6\ \text{V}$$

Step II: When the 10 V source is acting alone

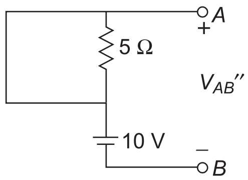


Fig. 2.313

Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V_{AB}'' = 10\ \text{V}$$

Step III: When the 5 A source is acting alone

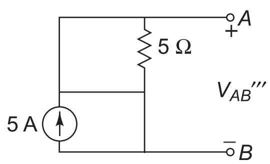


Fig. 2.314

Due to short circuit in both the parts,

$$V_{AB}''' = 0 \text{ V}$$

Step IV: By superposition theorem,

$$\begin{aligned} V_{AB} &= V_{AB}' + V_{AB}'' + V_{AB}''' \\ &= 6 + 10 + 0 \\ &= 16 \text{ V} \end{aligned}$$

Example 11

Find the voltage across 4 kΩ.

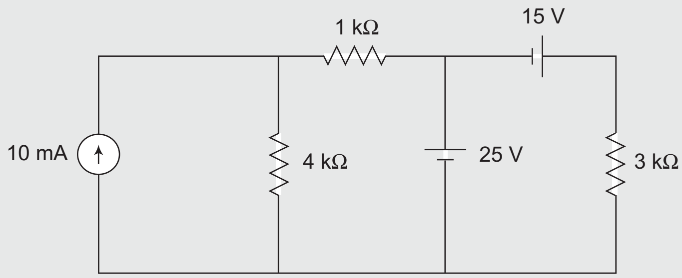


Fig. 2.315 **[May 2016]**

Solution *Step I: When the 10 mA source is acting alone*

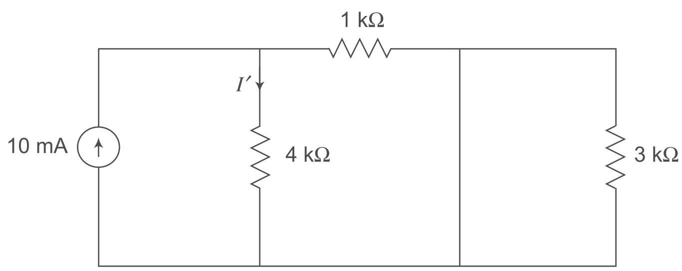


Fig. 2.316

Since 3 kΩ resistor is connected in parallel with short circuit, it gets shorted.

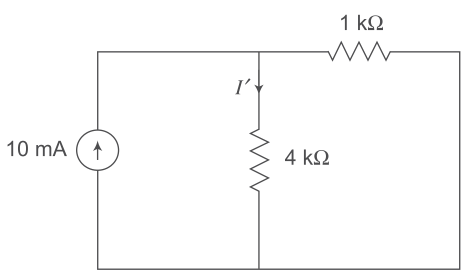


Fig. 2.317

By current division rule,

$$I' = 10\,m \times \frac{1k}{1k + 4k} = 2\text{ mA}(\downarrow)$$

Step II: When the 25 V source is acting alone

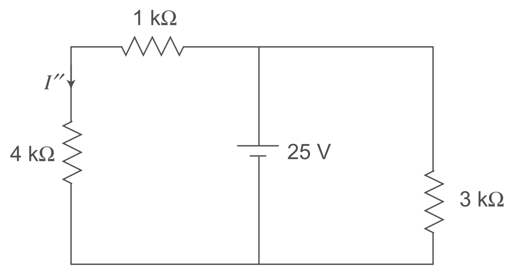


Fig. 2.318

Since 3 kΩ resistor is connected in parallel with 25 V source, it becomes redundant.

$$I'' = \frac{25}{4k + 1k} = 5\,mA(\downarrow)$$

Step III: When the 15 V source is acting alone

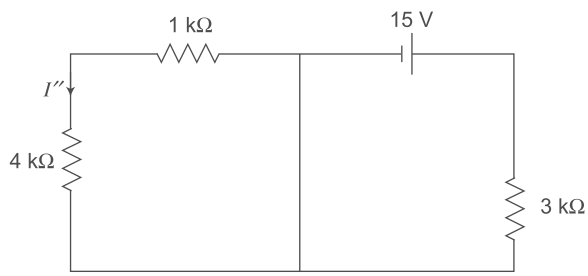


Fig. 2.319

Since series combination of 4 kΩ and 1 kΩ resistor is connected across a short circuit, it gets shorted.

$$I''' = 0$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 2\,mA + 5\,mA + 0 \\ &= 7\,mA(\downarrow) \end{aligned}$$

Example 12

Find the current through the 5 Ω resistor.

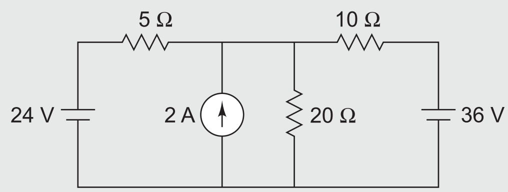


Fig. 2.320

Solution *Step I: When the 24 V source is acting alone*

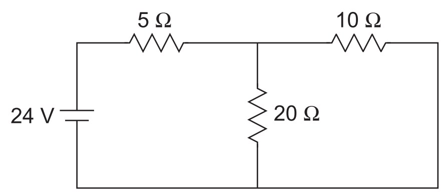


Fig. 2.321

By series–parallel reduction technique,

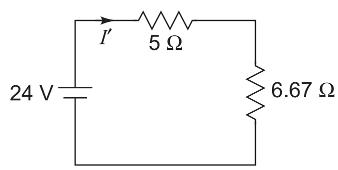


Fig. 2.322

$$I' = \frac{24}{5 + 6.67} = 2.06 \text{ A } (\rightarrow) = -\,2.06 \text{ A } (\leftarrow)$$

Step II: When the 2 A source is acting alone

By series-parallel reduction technique,

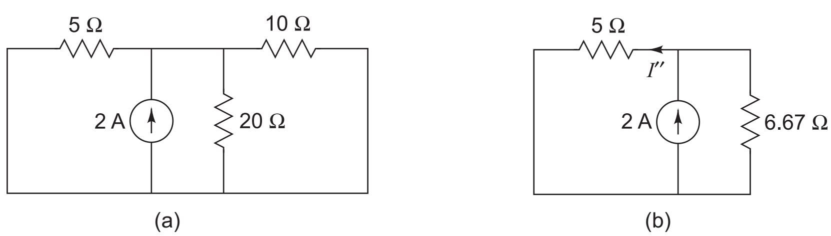


Fig. 2.323

From Fig. 2.323(b), by current-division rule,

$$I'' = 2 \times \frac{6.67}{5+6.67} = 1.14 \text{ A } (\leftarrow)$$

Step III: When the 36 V source is acting alone

By series-parallel reduction technique,

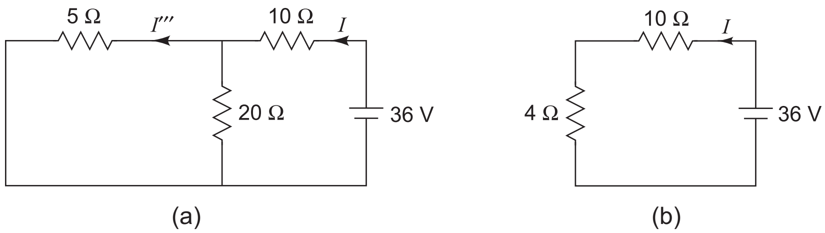


Fig. 2.324

$$I = \frac{36}{10+4} = 2.57 \text{ A}$$

From Fig. 2.324(a), by current-division rule,

$$I''' = 2.57 \times \frac{20}{20+5} = 2.06 \text{ A } (\leftarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= -2.06 + 1.14 + 2.06 \\ &= 1.14 \text{ A } (\leftarrow) \end{aligned}$$

Example 13

Find the value of current flowing through 30 Ω resistor.

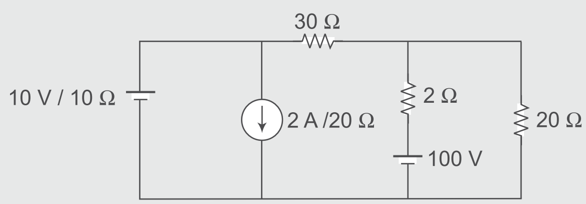


Fig. 2.325 **[Dec 2015]**

Solution *Step I: When the 10 V source is acting alone*

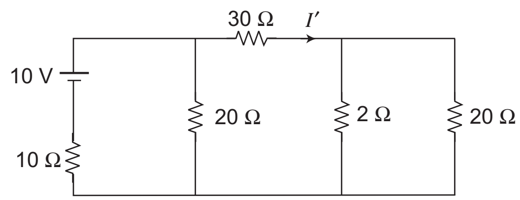


Fig. 2.326

By series-parallel reduction technique,

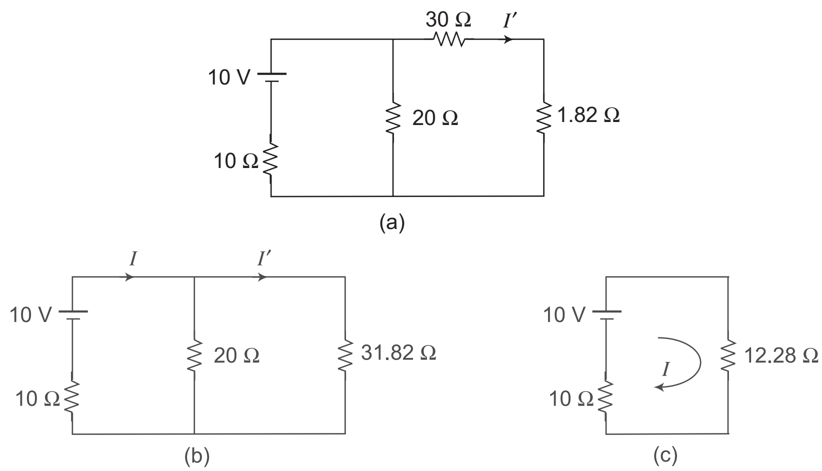


Fig. 2.327

$$I = \frac{10}{10+12.28} = 0.45\text{ A}$$

From Fig. 2.327(b), by current-division rule,

$$I' = 0.45 \times \frac{20}{20+31.82} = 0.17\text{ A }(\rightarrow) = -0.17\text{ A }(\leftarrow)$$

Step II: When the 2A source is acting alone

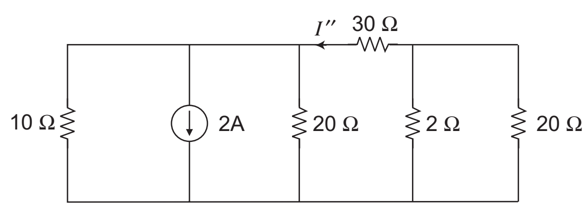


Fig. 2.328

By series-parallel reduction technique,

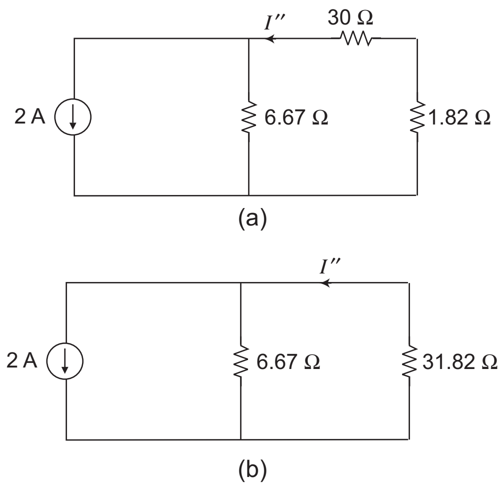


Fig. 2.329

By current-division rule,

$$I'' = 2 \times \frac{6.67}{6.67 + 31.82} = 0.35\,\text{A} \quad (\leftarrow)$$

Step-III: When the 100 V source is acting alone

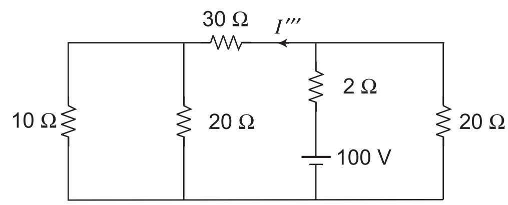
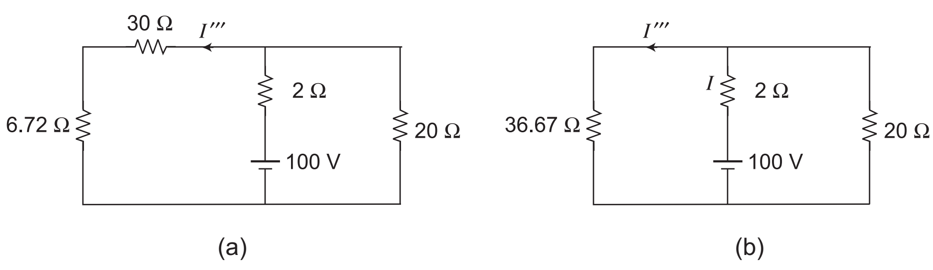
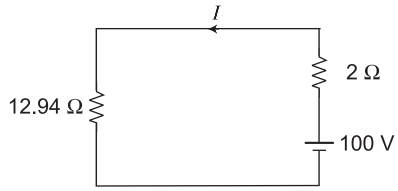


Fig. 2.330

By series-parallel reduction technique,





(c)

Fig. 2.331

$$I = \frac{100}{12.94+2} = 6.69 \text{ A}$$

By current-division rule,

$$I''' = 6.69 \times \frac{20}{20+36.67} = 2.36 \text{ A } (\leftarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= -0.17 + 0.35 + 2.36 \\ &= 2.54 \text{ A } (\leftarrow) \end{aligned}$$

Example 14

Find the value of current flowing through the 5 Ω resistor.

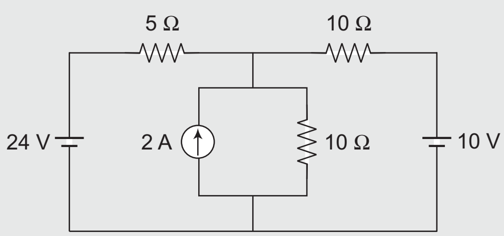
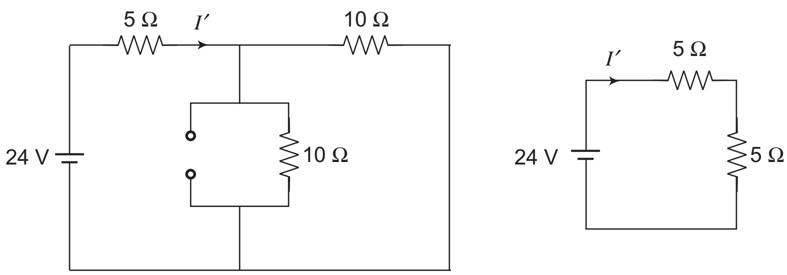


Fig. 2.332 **[May 2015]**

Solution *Step I: When the 24 V source is acting alone*



(a) (b)

Fig. 2.333

$$I' = \frac{24}{5+5} = 2.4 \text{ A } (\rightarrow) = -2.4 \text{ A } (\leftarrow)$$

Step II *When the 2 A source is acting alone*

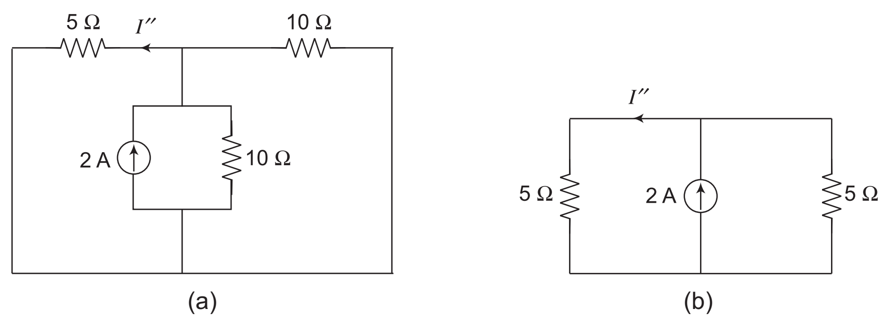


Fig. 2.334

$$I'' = 2 \times \frac{5}{5+5} = 1 \text{ A}(\leftarrow)$$

Step III When the 10 V source is acting alone

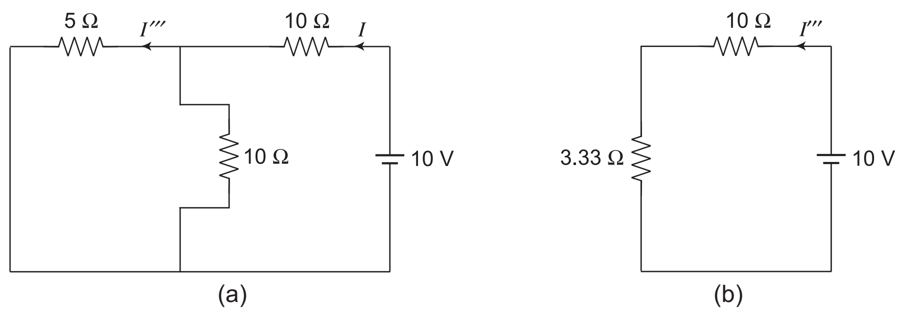


Fig. 2.335

$$I = \frac{10}{10+3.33} = 0.75 \text{ A}$$

By current-division rule,

$$I''' = 0.75 \times \frac{10}{10+5} = 0.5 \text{ A}(\leftarrow)$$

Step IV *By superposition theorem,*

$$I = I' + I'' + I'''$$
$$= -2.4 + 1 + 0.5$$
$$= -0.9 \text{ A } (\leftarrow)$$
$$I = 0.9 \text{ A } (\rightarrow)$$

Example 15

Find the value of current flowing through the 4 Ω resistor.

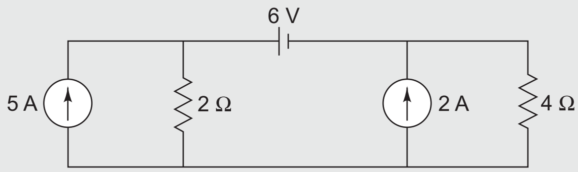


Fig. 2.336

Solution *Step I: When the 5 A source is acting alone*

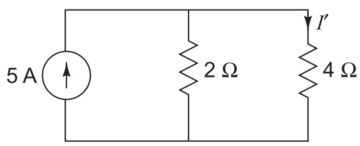


Fig. 2.337

By current-division rule,

$$I' = 5 \times \frac{2}{2+4} = 1.67 \text{ A } (\downarrow)$$

Step II: When the 2 A source is acting alone

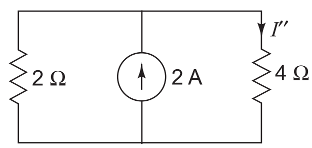


Fig. 2.338

By current-division rule,

$$I'' = 2 \times \frac{2}{2+4} = 0.67 \text{ A } (\downarrow)$$

Step III: When the 6 V source is acting alone

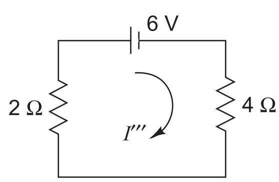


Fig. 2.339

Applying KVL to the mesh,

$$-2I''' - 6 - 4I''' = 0$$

$$I''' = -1 \text{ A} (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 1.67 + 0.67 - 1$$
$$= 1.34 \text{ A} (\downarrow)$$

Example 16

Find the value of current flowing through the 5 Ω resistor.

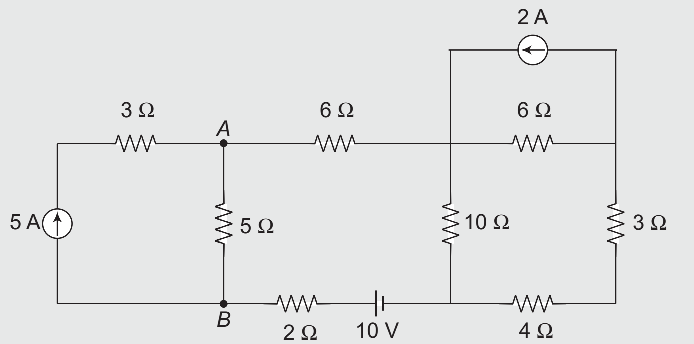


Fig. 2.340 [Dec 2014]

Solution *Step I: When the 5 A source is acting alone*

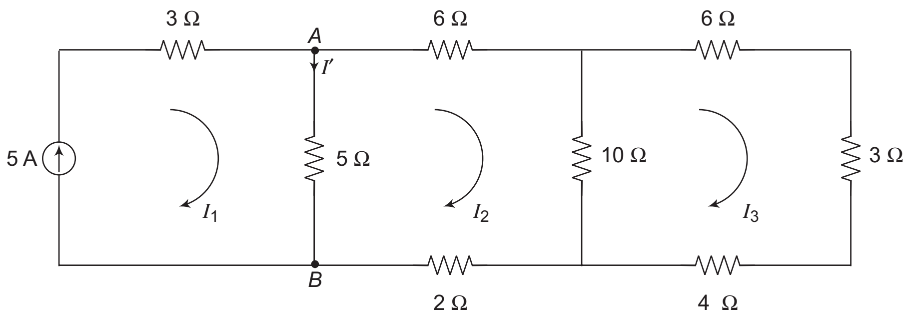


Fig. 2.341

Writing equations in matrix form,

$$\begin{bmatrix} 1 & 0 & 0 \\ -5 & 23 & -10 \\ 0 & -10 & 23 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = 5$$
$$I_2 = 1.34 \text{ A}$$

$$I_3 = 0.58 \text{ A}$$
$$I' = I_1 - I_2 = 5 - 1.34 = 3.66 \text{ A } (\downarrow)$$

Step II: When the 10 V source is acting alone

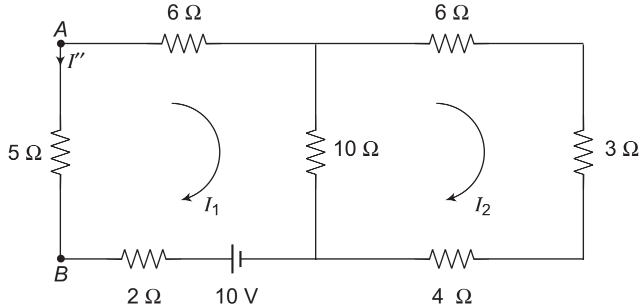


Fig. 2.342

Writing KVL equations in matrix form,

$$\begin{bmatrix} 23 & -10 \\ -10 & 23 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$I_1 = 0.54 \text{ A}$$
$$I_2 = 0.23 \text{ A}$$
$$I'' = -I_1 = -0.54 \text{ A } (\downarrow)$$

Step III: When the 2 A source is acting alone

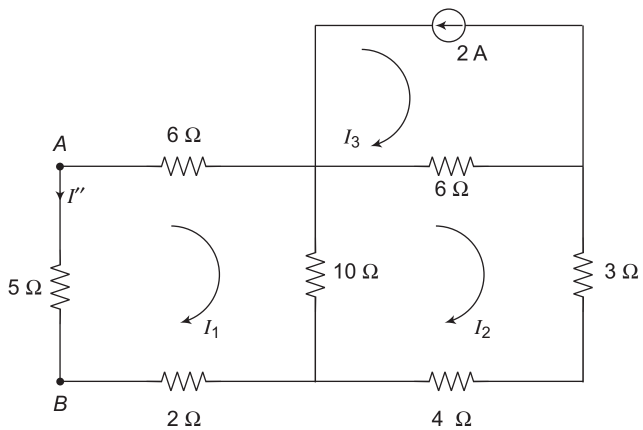


Fig. 2.343

Writing equations in matrix form,

$$\begin{bmatrix} 23 & -10 & 0 \\ -10 & 23 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

$$I_1 = -0.28 \text{ A}$$
$$I_2 = -0.64 \text{ A}$$
$$I_3 = -2 \text{ A}$$
$$I''' = -I_1 = -0.28 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 3.66 - 0.54 + 0.28 \\ &= 3.4 \text{ A} \end{aligned}$$

Example 16

Find the value of current flowing through the 3 Ω resistor.

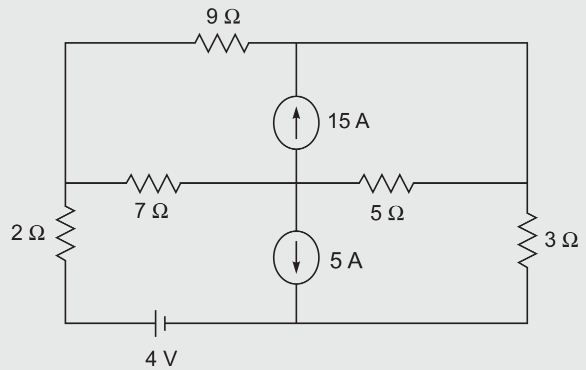


Fig. 2.344 **[Dec 2012]**

Solution *Step I:* When the 4 V source is acting alone

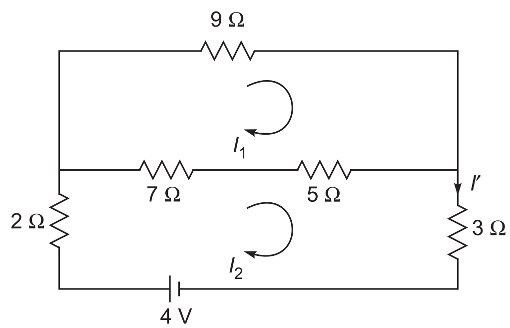


Fig. 2.345

Writing KVL equation in matrix form,

$$\begin{bmatrix} 21 & -12 \\ -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$I' = I_2 = 0.39 \text{ A} \quad (\downarrow)$$

Step II When the 15 A source is acting alone

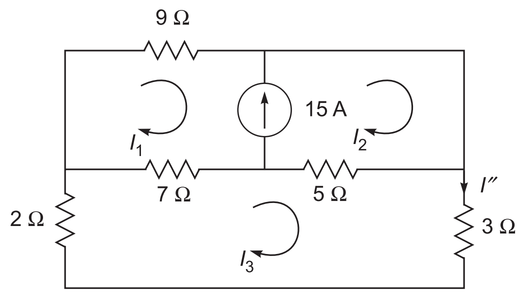


Fig. 2.346

Writing the current equation for the supermesh,

$$I_2 - I_1 = 15 \tag{1}$$

Writing the voltage equation for the supermesh,

$$-9I_1 - 5(I_2 - I_3) - 7(I_1 - I_3) = 0$$
$$-16I_1 - 5I_2 + 12I_3 = 0 \tag{2}$$

Applying KVL to Mesh 3,

$$-2I_3 - 7(I_3 - I_1) - 5(I_3 - I_2) - 3I_3 = 0$$
$$-7I_1 - 5I_2 + 17I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I'' = I_3 = 3.17 \text{ A} \quad (\downarrow)$$

Sep III When the 5 A source is acting alone

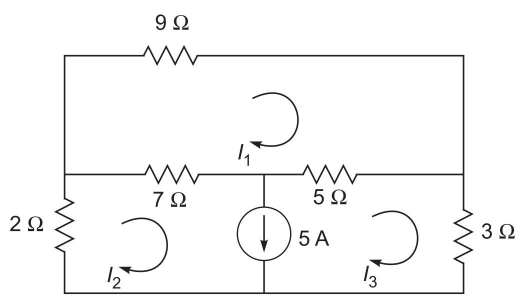


Fig. 2.347

Applying KVL to Mesh 1,

$$-9I_1 - 5(I_1 - I_3) - 7(I_1 - I_2) = 0$$
$$21I_1 - 7I_2 - 5I_3 = 0 \tag{1}$$

Writing the current equation for the supermesh,

$$I_2 - I_3 = 5 \tag{2}$$

Writing the voltage equation for the supermesh,

$$-2I_2 - 7(I_2 - I_1) - 5(I_3 - I_1) - 3I_3 = 0$$

$$12I_1 - 9I_2 - 8I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I''' = I_3 = -2.46 \text{ A} \quad (\downarrow)$$

Step IV *By superposition theorem*,

$$I = I' + I'' + I''' = 0.39 + 3.17 - 2.46 = 1.1 \text{ A}$$

$$V_{3\,\Omega} = 3I = 3(1.1) = 3.3 \text{ V}$$

Example 17

Determine the value of current flowing through R_L = 2 Ω in the circuit shown in Fig. 2.348.

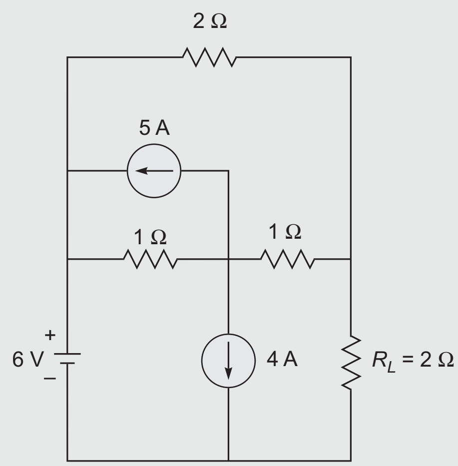


Fig. 2.348 **[May 2013]**

Solution *Step I:* When the 6 V source is acting alone

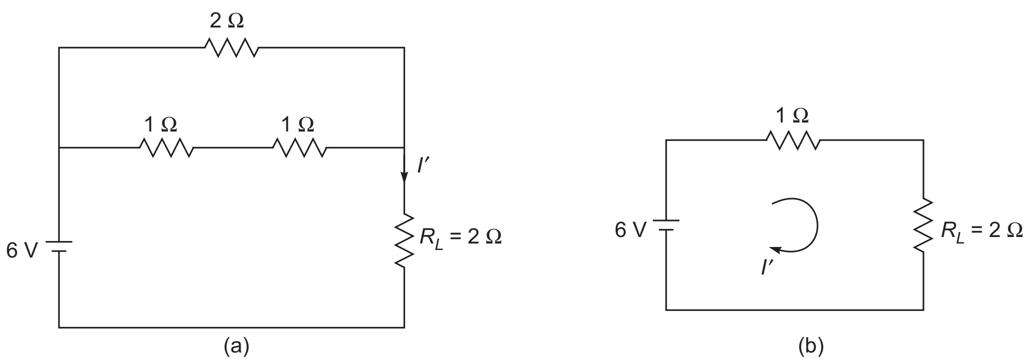


Fig. 2.349

$$I' = \frac{6}{1+2} = 2\text{ A} \quad (\downarrow)$$

Step II When the 4 A source is acting alone

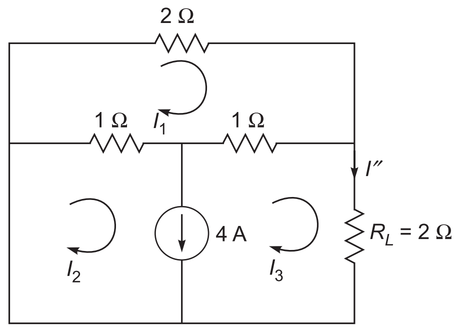


Fig. 2.350

Applying KVL to Mesh 1,

$$4I_1 - I_2 - I_3 = 0 \tag{1}$$

Writing the current equation for the supermesh,

$$I_2 - I_3 = 4 \tag{2}$$

Writing the voltage equation for the supermesh,

$$-1(I_2 - I_1) - 1(I_3 - I_1) - 2I_3 = 0$$

$$2I_1 - I_2 - 3I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I'' = I_3 = -0.67\text{ A} \quad (\downarrow)$$

Step III When the 5 A source is acting alone

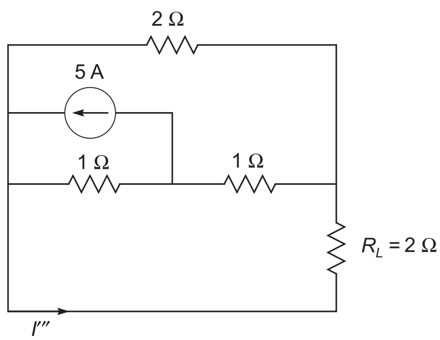


Fig. 2.351

Simplifying the circuit,

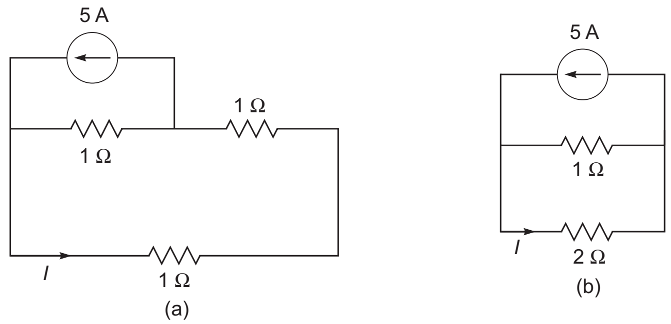


Fig. 2.352

$$I = 5 \times \frac{1}{1+2} = 1.67 \text{ A} \quad (\uparrow)$$

$$I''' = -1.67 \times \frac{1}{2} = -0.84 \text{ A} \quad (\downarrow)$$

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 2 - 0.67 - 0.84 = -0.49 \text{ A} \quad (\downarrow)$$

Example 18

Determine the value of current flowing in the 1 Ω resistor.

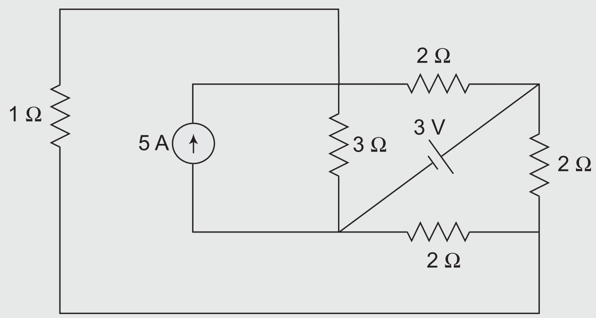


Fig. 2.353 [Dec 2013]

Solution *Step I: When the 5 A source is acting alone*

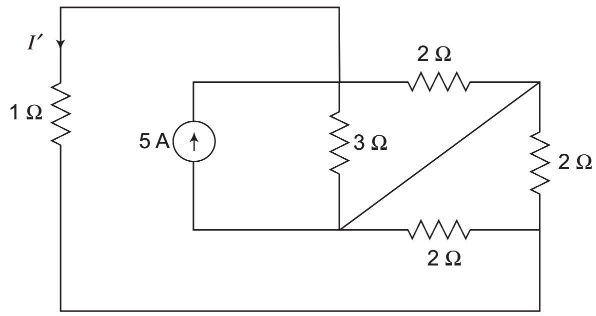


Fig. 2.354

Simplifying the network,

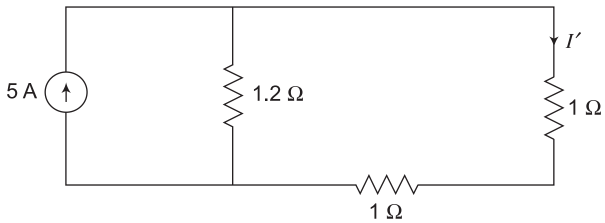


Fig. 2.355

By current-division rule,

$$I' = 5 \times \frac{1.2}{1.2+1+1} = 1.875 \text{ A } (\downarrow)$$

Step II When the 3 V source is acting alone

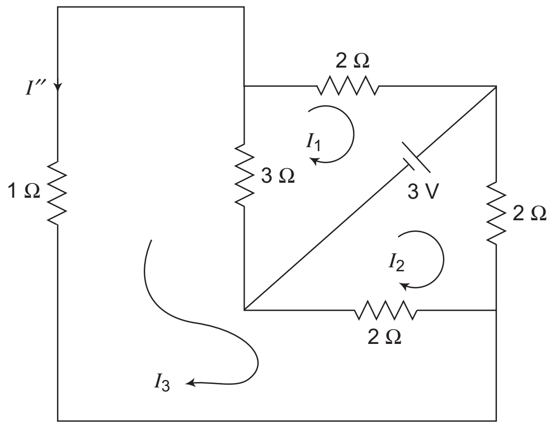


Fig. 2.356

Applying KVL to Mesh 1,

$$-2I_1 - 3 - 3\,(I_1 - I_3) = 0$$

$$5I_1 - 3I_3 = -3 \quad (1)$$

Applying KVL to Mesh 2,

$$3 - 2I_2 - 2(I_2 - I_3) = 0$$

$$4I_2 - 2\,I_3 = 3 \quad (2)$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_1) - 2\,(I_3 - I_2) - I_3 = 0$$

$$-3I_1 - 2I_2 + 6I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = -0.66 \text{ A}$$

$$I_2 = 0.7 \text{ A}$$

$$I_3 = -0.09 \text{ A}$$

$$I'' = -I_3 = 0.09 \text{ A}$$

Step III　By superposition theorem,

$$I = I' + I'' = 1.875 + 0.09 = 1.965 \text{ A } (\downarrow)$$

Example 19

Find the value of current flowing through the 6 Ω resistor.

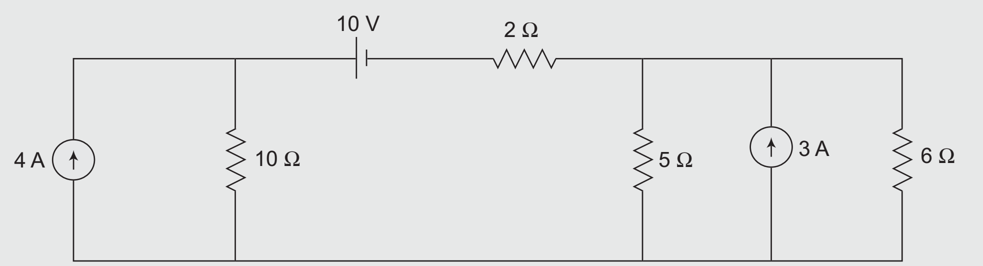


Fig. 2.357　**[May 2014]**

Solution　*Step I: When the 4 A source is acting alone*

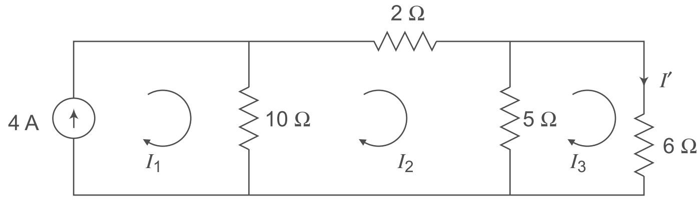


Fig. 2.358

Writing equations in matrix form,

$$\begin{bmatrix} 1 & 0 & 0 \\ -10 & 17 & -5 \\ 0 & -5 & 11 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$I' = I_3 = 1.23 \text{ A} \quad (\downarrow)$$

Step II When the 10 V source is acting alone

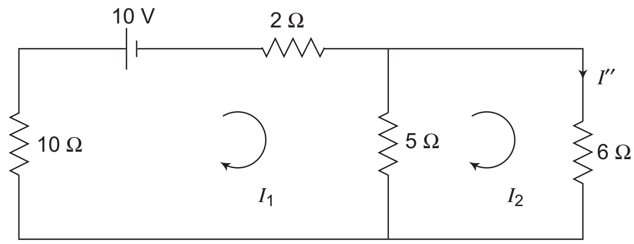


Fig. 2.359

Writing KVL equation in matrix form,

$$\begin{bmatrix} 17 & -5 \\ -5 & 11 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$$I'' = I_2 = -0.31 \text{ A} (\downarrow)$$

Step III When the 3 A source is acting alone

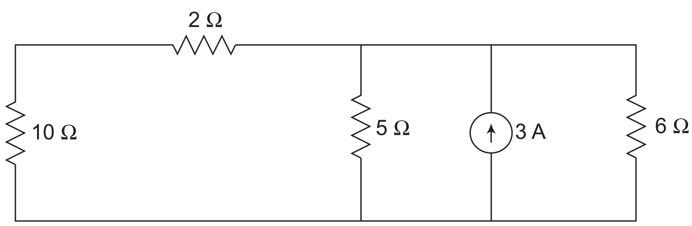


Fig. 2.360

By series-parallel reduction technique,

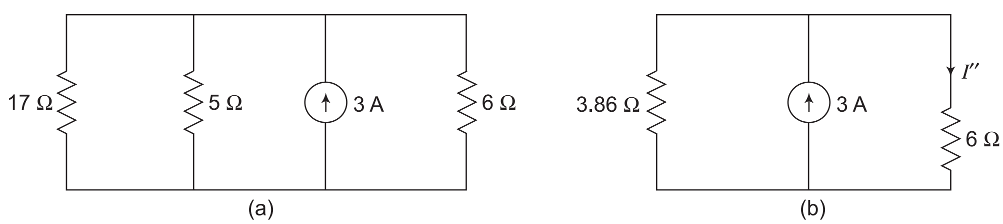


Fig. 2.361

By current-division rule,

$$I''' = 3 \times \frac{3.86}{3.86 + 6} = 1.17 \text{ A } (\downarrow)$$

Step IV *By superposition theorem,*

$$I = I' + I'' + I''' = 1.23 - 0.31 + 1.17 = 2.09 \text{ A } (\downarrow)$$

Exercise 2.6

2.1 Find the value of current flowing through the 1 Ω resistor.

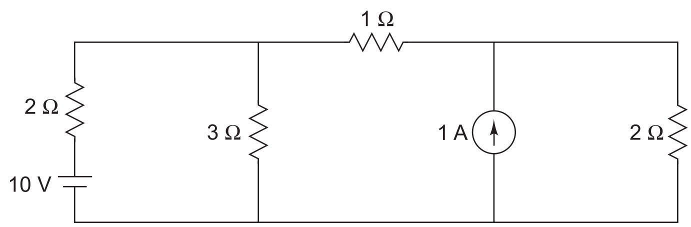


Fig. 2.362

[0.95 A]

2.2 Find the value of current flowing through the 10 Ω resistor.

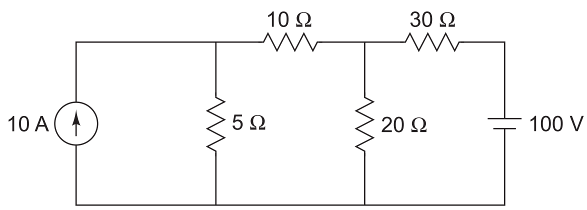


Fig. 2.363

[0.37 A]

2.3 Calculate the value of current flowing through the 10 Ω resistor.

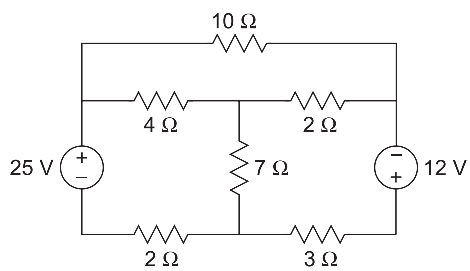


Fig. 2.364

[1.62 A]

2.4 Find the value of current flowing in the 2 Ω resistor. Also, find voltage across the current source.

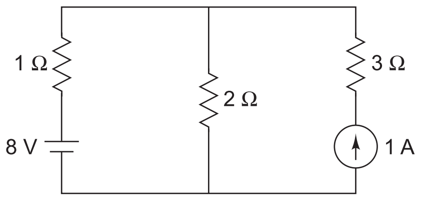


Fig. 2.365

[3 A, 9 V]

2.5 Find the current I_x.

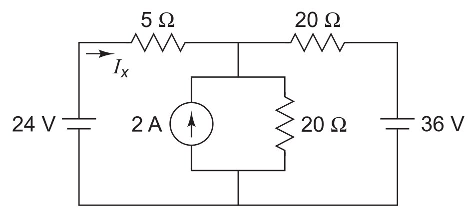


Fig. 2.366

[–0.93 A]

2.9 THEVENIN'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.'*

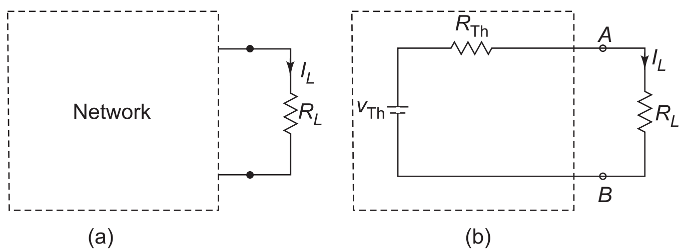


Fig. 2.367 *Thevenin's theorem*

Explanation The above method of determining the load current through a given load resistance can be explained with the help of the following circuit.

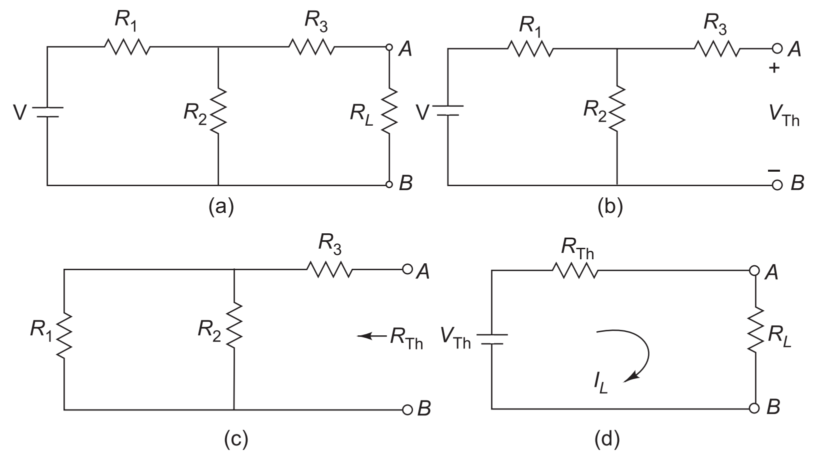


Fig. 2.368 *Steps in Thevenin's theorem*

2.9.1 Steps to be followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with the voltage sources and current sources replaced by internal resistances.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

Example 1

Find the value of current flowing through the 2 Ω resistor.

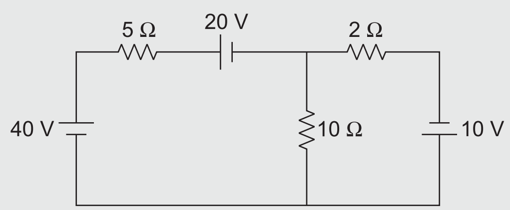


Fig. 2.369

Solution *Step I : Calculation of V_{Th}*

Removing the 2 Ω resistor from the network,

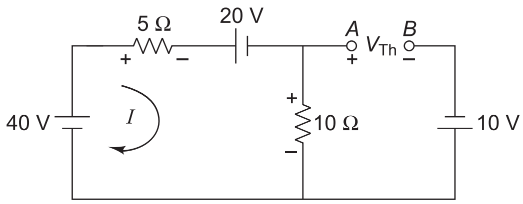


Fig. 2.370

Applying KVL to the mesh,

$$40 - 5I - 20 - 10I = 0$$
$$15I = 20$$
$$I = 1.33 \text{ A}$$

Writing V_{Th} equation,

$$10I - V_{Th} + 10 = 0$$
$$V_{Th} = 10I + 10$$
$$= 10\,(1.33) + 10$$
$$= 23.33 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

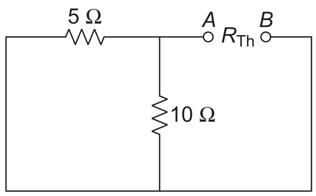


Fig. 2.371

$$R_{Th} = 5 \parallel 10 = 3.33\ \Omega$$

Step III: Calculation of I_L

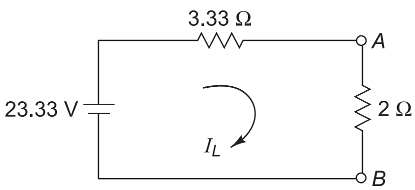


Fig. 2.372

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

Example 2

Find the value of current flowing through the 8 Ω resistor.

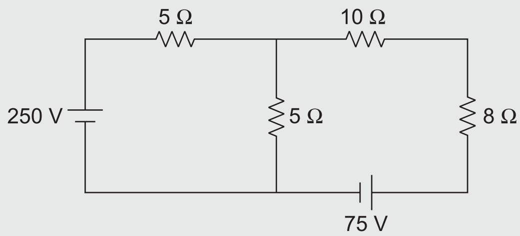


Fig. 2.373

Solution *Step I: Calculation of V_{Th}*

Removing the 8 Ω resistor from the network,

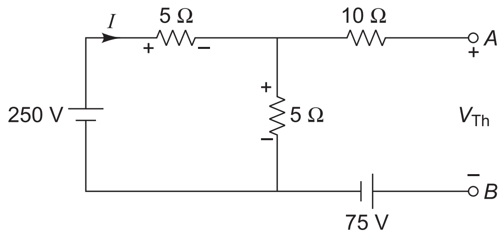


Fig. 2.374

$$I = \frac{250}{5+5} = 25 \text{ A}$$

Writing V_{Th} equation,

$$250 - 5I - V_{\text{Th}} - 75 = 0$$

$$V_{\text{Th}} = 175 - 5I$$

$$= 175 - 5\,(25)$$

$$= 50 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

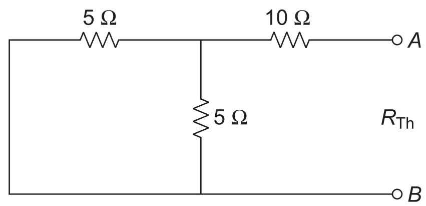


Fig. 2.375

$$R_{\text{Th}} = (5 \,||\, 5) + 10 = 12.5 \ \Omega$$

Step III: Calculation of I_L

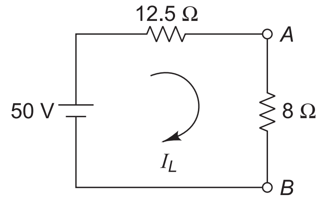


Fig. 2.376

$$I_L = \frac{50}{12.5+8} = 2.44 \text{ A}$$

Example 3

Find the value of current flowing through the 2 Ω resistor connected between terminals A and B.

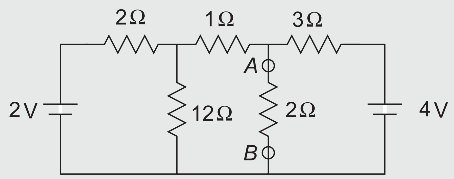


Fig. 2.377

Solution

Step I: Calculation of V_{Th}

Removing the 2 Ω resistor connected between terminals *A* and *B*,

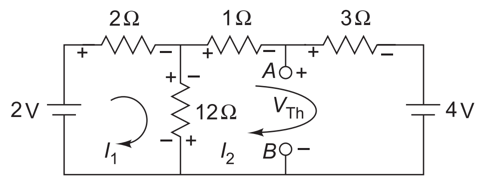


Fig. 2.378

Applying KVL to Mesh 1,

$$2 - 2I_1 - 12\,(I_1 - I_2) = 0$$

$$14I_1 - 12I_2 = 2 \tag{1}$$

Applying KVL to Mesh 2,

$$-\,12\,(I_2 - I_1) - 1I_2 - 3I_2 - 4 = 0$$

$$-\,12I_1 + 16I_2 = -4 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = -\,0.4 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 3I_2 - 4 = 0$$
$$V_{\text{Th}} = 4 + 3I_2$$
$$= 4 + 3\,(-0.4)$$
$$= 2.8 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing all voltage sources by short circuits,

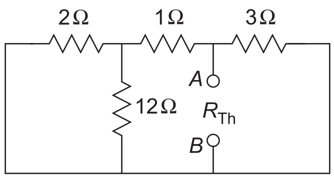


Fig. 2.379

$$R_{\text{Th}} = [|(2||\ 12) + 1]\ ||\ 3 = 1.43\ \Omega$$

Step III: Calculation of I_L

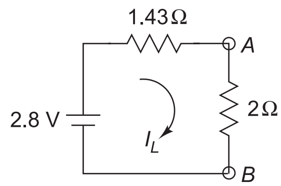


Fig. 2.380

$$I_L = \frac{40}{5+1.67} = 0.82 \text{ A}$$

Example 4

Find the value of current flowing through the 8 Ω resistor.

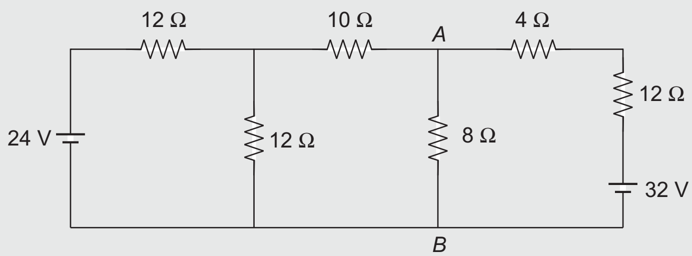


Fig. 2.381 **[May 2015]**

Solution *Step I: Calculation of* V_{Th}

Removing 8 Ω resistor connected between A and B,

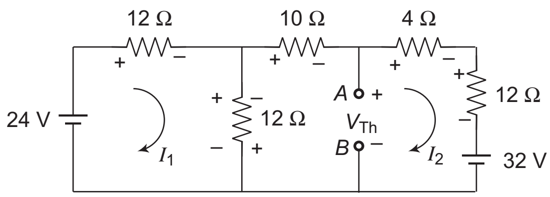


Fig. 2.382

Applying KVL to Mesh 1,

$$24 - 12I_1 - 12(I_1 - I_2) = 0$$

$$24I_1 - 12I_2 = 24 \tag{1}$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 10I_2 - 4I_2 - 12I_2 - 32 = 0$$

$$-12I_1 + 38I_2 = -32 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 0.69 \text{ A}$$

$$I_2 = -0.63 \text{ A}$$

Writing V_{Th} equation,

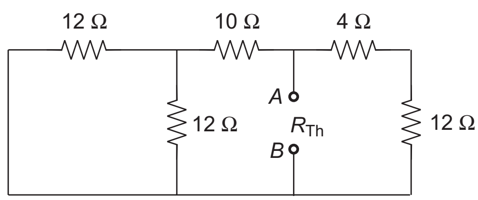
$$V_{Th} - 4I_2 - 12I_2 - 32 = 0$$

$$V_{Th} = 32 + 4(-0.63) + 12(-0.63)$$

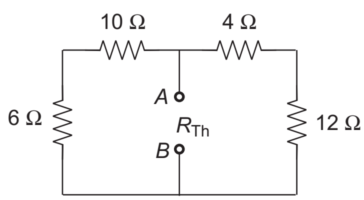
$$= 21.92 \text{ V}$$

Step II: Calculation of R_{TH}

Replacing all voltage sources by short circuits,



(a)



(b)

Fig. 2.383

$$R_{TH} = 8\ \Omega$$

Step III: Calculation of I_L

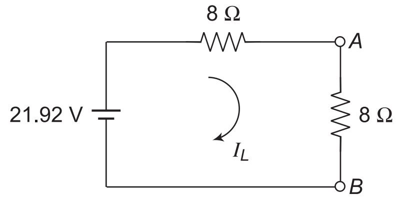


Fig. 2.384

$$I_L = \frac{21.92}{8+8} = 1.37 \text{ A}$$

Example 5

Find the value of current flowing through the 10 Ω resistor.

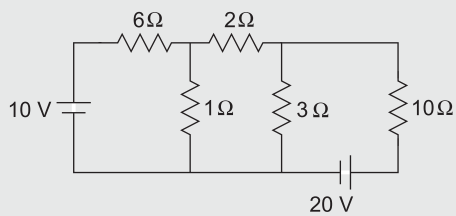


Fig. 2.385

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

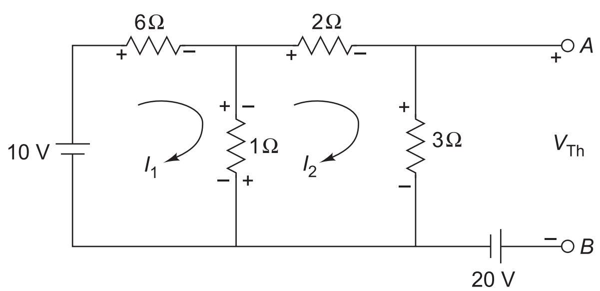


Fig. 2.386

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \tag{1}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$I_1 - 6I_2 = 0 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 0.24 \text{ A}$$

Writing V_{Th} equation,

$$3I_2 - V_{\text{Th}} - 20 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 3I_2 - 20 \\ &= 3\,(0.24) - 20 \\ &= -\,19.28 \text{ V} \\ &= 19.28 \text{ V (terminal } B \text{ is positive w.r.t } A) \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

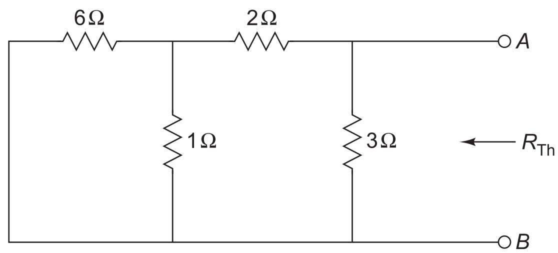


Fig. 2.387

$$R_{\text{Th}} = [(6 \| 1) + 2] \| 3 = 1.47\ \Omega$$

Step III: Calculation of I_L

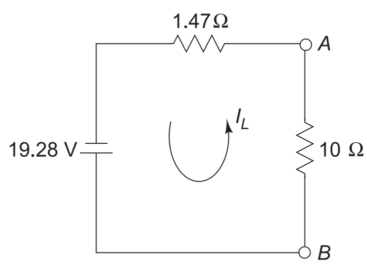


Fig. 2.388

$$I_L = 6 \times \frac{10}{10+2} = 1.68 \text{ A } (\uparrow)$$

Example 6

Find the value of current flowing through the 10 Ω resistor.

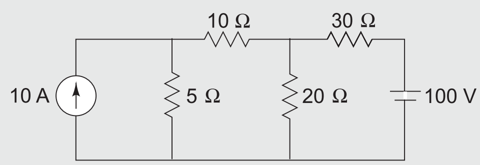


Fig. 2.389

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

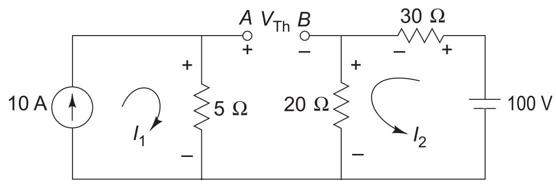


Fig. 2.390

For Mesh 1,

$$I_1 = 10$$

Applying KVL to Mesh 2,

$$100 - 30I_2 - 20I_2 = 0$$
$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$5I_1 - V_{\text{Th}} - 20I_2 = 0$$
$$V_{\text{Th}} = 5I_1 - 20I_2$$
$$= 5(10) - 20(2)$$
$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit and the voltage source by a short circuit,

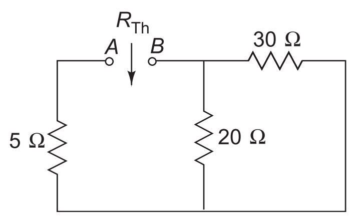


Fig. 2.391

$$R_{\text{Th}} = 5 + (20\,||\,30) = 17\ \Omega$$

Step III: Calculation of I_L

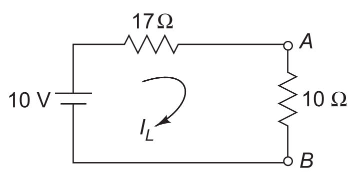


Fig. 2.392

$$I_L = \frac{20}{5 + 1.67} = 0.37 \text{ A}$$

Example 7

Find the value of current flowing through the 40 Ω resistor.

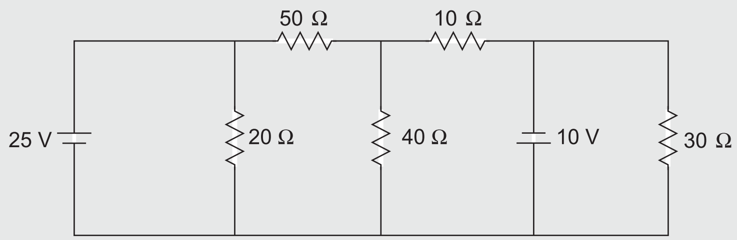


Fig. 2.393

Solution

Step I: Calculation of V_{Th}

Removing the 40 Ω resistor from the network,

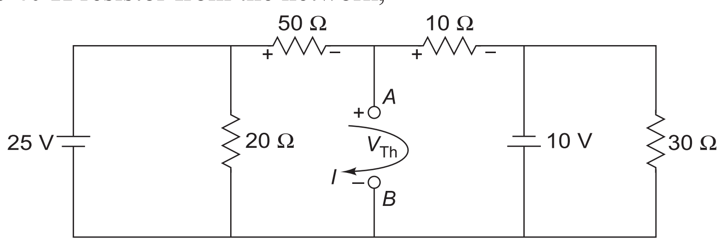


Fig. 2.394

Since the 20 Ω resistor is connected across the 25 V source, the resistor becomes redundant.

$$V_{20\,\Omega} = 25\text{ V}$$

Applying KVL to the mesh,

$$25 - 50I - 10I + 10 = 0$$
$$I = 0.58\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 10\text{I} + 10 = 0$$
$$V_{\text{Th}} = 10\,(I) - 10$$
$$= 10\,(0.58) - 10$$
$$= -4.2\text{ V}$$
$$= 4.2\text{ V (terminal } B \text{ is positive w.r.t. A)}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

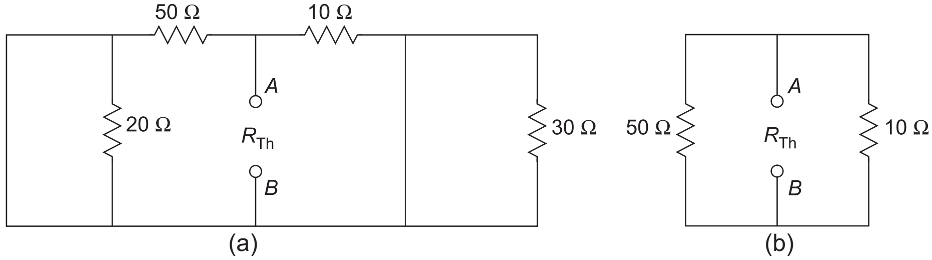


Fig. 2.395

$$R_{Th} = 50 \parallel 10 = 8.33\ \Omega$$

Step III: Calculation of I_L

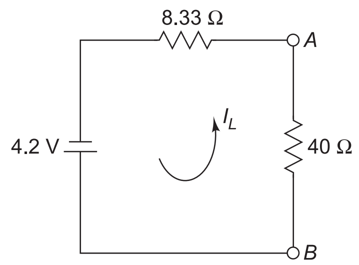


Fig. 2.396

$$I_L = \frac{10}{3.33 + 2} = 0.09\ \text{A}\ (\uparrow)$$

Example 8

Find the values of current flowing through the 10 Ω resistor.

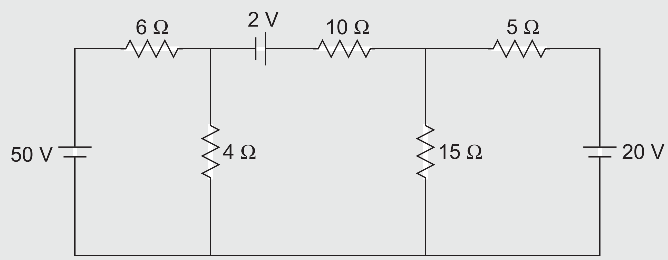


Fig. 2.397

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

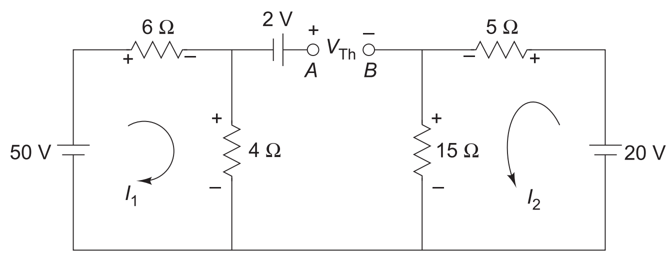


Fig. 2.398

$$I_1 = \frac{50}{10} = 5\ \text{A}$$

$$I_2 = \frac{20}{20} = 1\ \text{A}$$

Writing V_{Th} equation,

$$4I_1 + 2 - V_{Th} - 15I_2 = 0$$

$$V_{Th} = 4I_1 + 2 - 15I_2$$

$$= 4(5) + 2 - 15(1)$$

$$= 7\ V$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

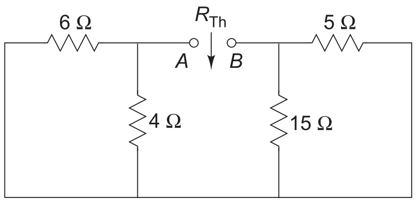


Fig. 2.399

$$R_{Th} = (6 \parallel 4) + (5 \parallel 15) = 6.15\ \Omega$$

Step III: Calculation of I_L

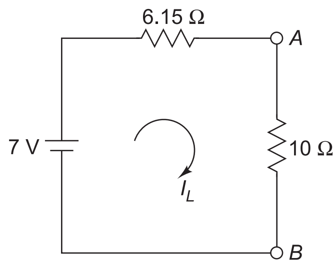


Fig. 2.400

$$I_L = \frac{50}{1+1.33} = 0.43\ \text{A}$$

Example 9

Determine the value of current flowing through the 24 Ω resistor.

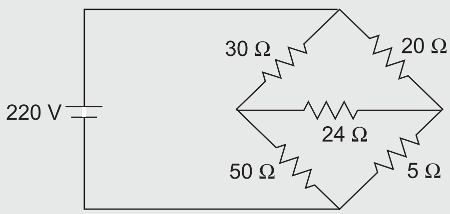


Fig. 2.401

Solution

Step I: Calculation of V_{Th}

Removing the 24 Ω resistor from the network,

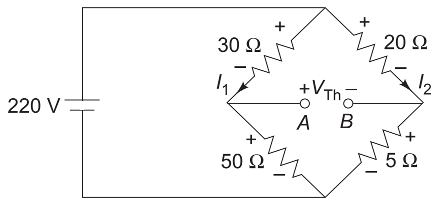


Fig. 2.402

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} + 30I_1 - 20I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 20I_2 - 30I_1 \\ &= 20\,(8.8) - 30\,(2.75) \\ &= 93.5 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

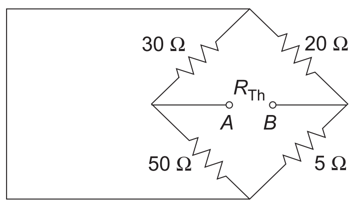


Fig. 2.403

Redrawing the circuit,

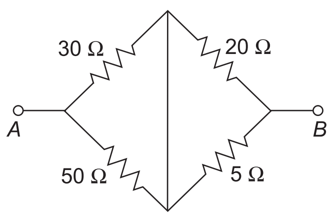


Fig. 2.404

$$R_{\text{Th}} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

Step III: Calculation of I_L

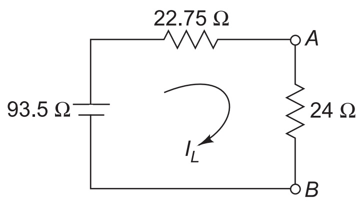


Fig. 2.405

$$I_L = \frac{93.5}{22.75 + 24} = 2 \text{ A}$$

Example 10

Find the value of current flowing through the 3 Ω resistor.

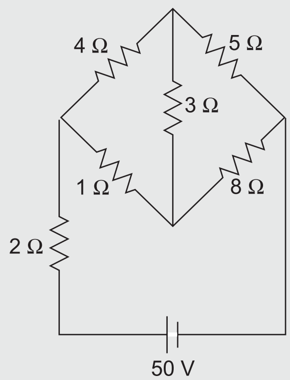


Fig. 2.406

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

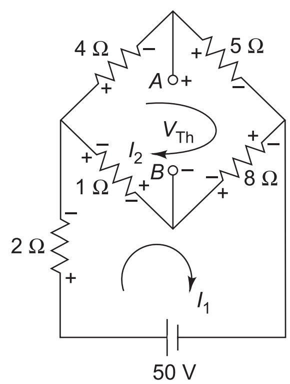


Fig. 2.407

Applying KVL to Mesh 1,

$$50 - 2I_1 - 1(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 50 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4I_2 - 5I_2 - 8(I_2 - I_1) - 1(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 7.69 \text{ A}$$

$$I_2 = 3.85 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 5I_2 - 8(I_2 - I_1) = 0$$

$$\begin{aligned} V_{Th} &= 5I_2 + 8\,(I_2 - I_1) \\ &= 5(3.85) + 8(3.85 - 7.69) \\ &= -11.47 \text{ V} \\ &= 11.47 \text{ V (the terminal } B \text{ is positive w.r.t. } A) \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

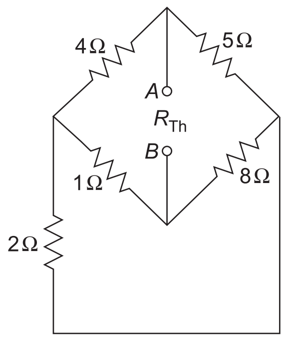


Fig. 2.408

Redrawing the network,

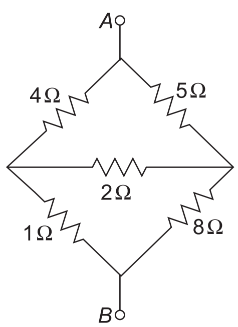


Fig. 2.409

Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\ \Omega$$

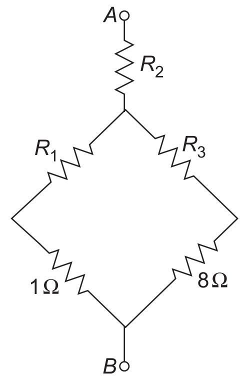


Fig. 2.410

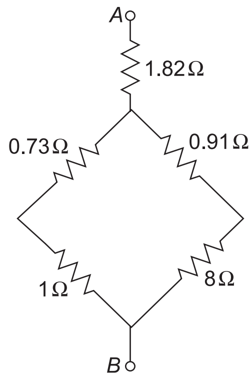


Fig. 2.411

Simplifying the network,

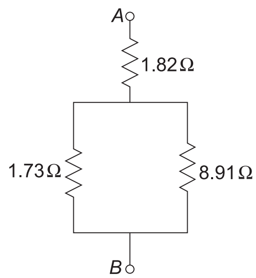


Fig. 2.412

$$R_{Th} = 1.82 + (1.73 \,||\, 8.91) = 3.27\ \Omega$$

Step III: Calculation of I_L

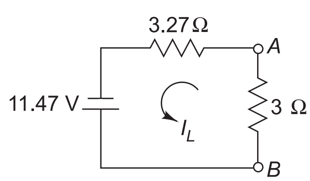


Fig. 2.413

$$I_L = \frac{40}{12+2.92} = 1.83\text{ A }(\uparrow)$$

Example 11

Find the value of current flowing through the 20 Ω resistor.

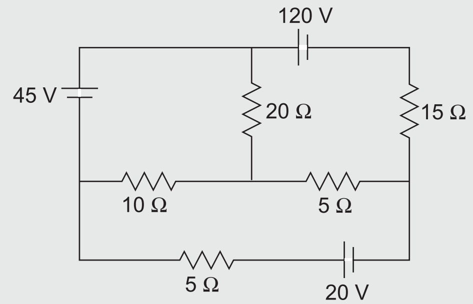


Fig. 2.414

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

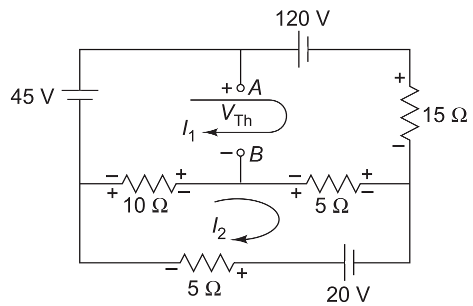


Fig. 2.415

Applying KVL to Mesh 1,

$$45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$$

$$30I_1 - 15I_2 = -75 \quad (1)$$

Applying KVL to Mesh 2,

$$20 - 5I_2 - 10(I_2 - I_1) - 5(I_2 - I_1) = 0$$

$$-15I_1 + 20I_2 = 20 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -3.2\text{ A}$$
$$I_2 = -1.4\text{ A}$$

Writing V_{Th} equation,

$$45 - V_{\text{Th}} - 10\,(I_1 - I_2) = 0$$

$$\begin{aligned} V_{Th} &= 45 - 10\,(I_1 - I_2) \\ &= 45 - 10\,[-3.2 - (-1.4)] \\ &= 63 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

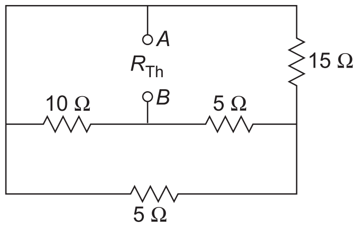


Fig. 2.416

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network,

$$R_1 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_3 = \frac{5 \times 5}{20} = 1.25\ \Omega$$

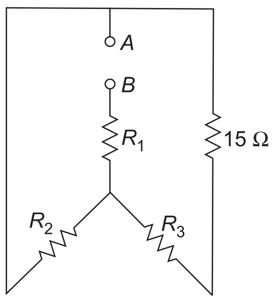


Fig. 2.417

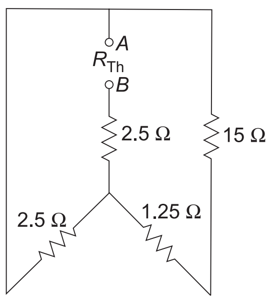


Fig. 2.418

Simplifying the network,

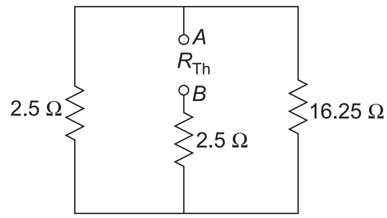


Fig. 2.419

$$R_{Th} = (16.25 \,||\, 2.5) + 2.5 = 4.67\ \Omega$$

Step III: Calculation of I_L

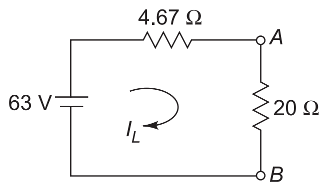


Fig. 2.420

$$I_L = \frac{63}{4.67+20} = 2.55\ \text{A}$$

Example 12

Find the value of current flowing through the 3 Ω resistor.

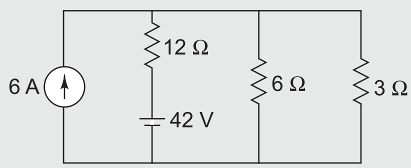


Fig. 2.421

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

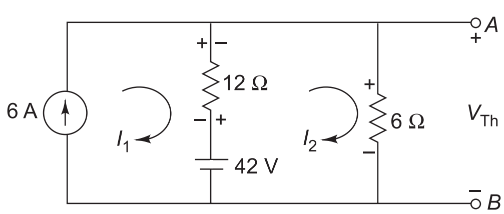


Fig. 2.422

Writing equation for Mesh 1,

$$I_1 = 6 \tag{1}$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6\,I_2 = 0$$

$$-12\,I_1 + 18\,I_2 = 42 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 6.33\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 6\,I_2 = 38\text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by short circuit and current source by open circuit,

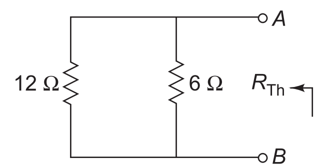


Fig. 2.423

$$R_{\text{Th}} = 6 \parallel 12 = 4\ \Omega$$

Step III: Calculation of I_L

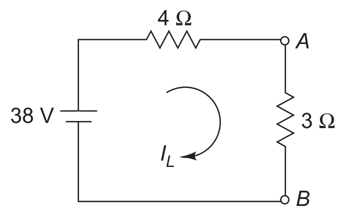


Fig. 2.424

$$I_L = \frac{38}{4+3} = 5.43\text{ A}$$

Example 13

Find the value of current flowing through the 30 Ω resistor.

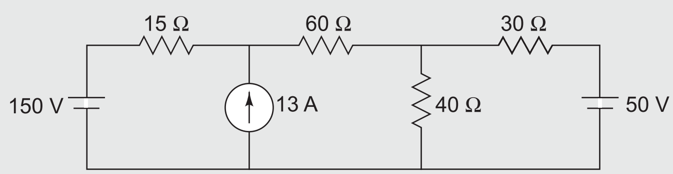


Fig. 2.425 **[May 2016]**

Solution

Step I: Calculation of V_{Th}

Removing the 30 Ω resistor from the network,

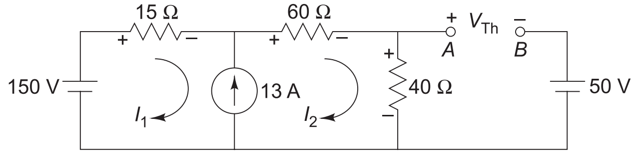


Fig. 2.426

Meshes 1 and 2 form a supermesh.

Writing current equation for supermesh,

$$I_2 - I_1 = 13 \tag{1}$$

Writing voltage equation for supermesh,

$$150 - 15I_1 - 60I_2 - 40I_2 = 0$$

$$15I_1 + 100I_2 = 150 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -10 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$40I_2 - V_{\text{Th}} - 50 = 0$$

$$V_{\text{Th}} = 40I_2 - 50$$

$$= 40(3) - 50$$

$$= 70 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

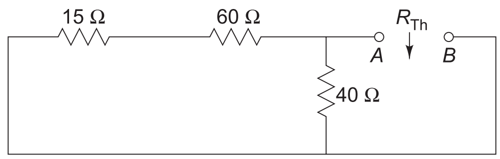


Fig. 2.427

$$R_{\text{Th}} = 75 \parallel 40 = 26.09 \ \Omega$$

Step III: Calculation of I_L

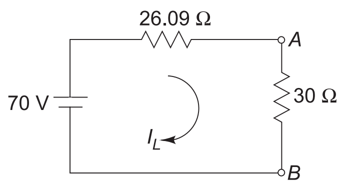


Fig. 2.428

$$I_L = \frac{70}{26.09+30} = 1.25 \text{ A}$$

Example 14

Find the value of current flowing through the 20 Ω resistor.

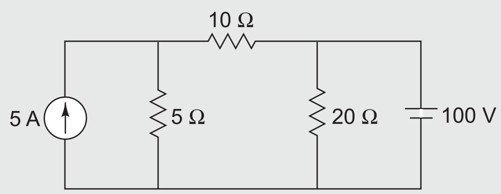


Fig. 2.429

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

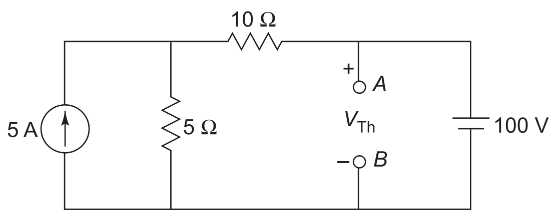


Fig. 2.430

From Fig. 2.430,

$$V_{\text{Th}} = 100 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

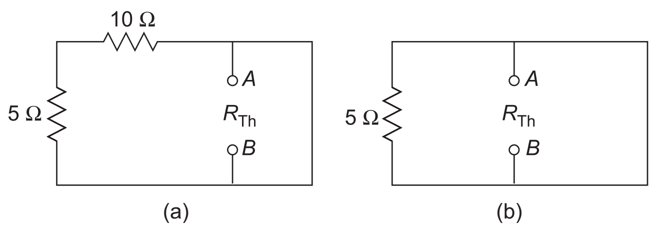


Fig. 2.431

$$R_{\text{Th}} = 0$$

Step III: Calculation of I_L

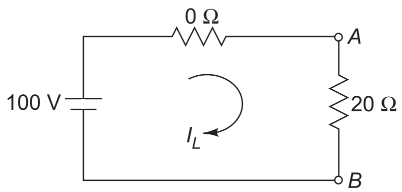


Fig. 2.432

$$I_L = \frac{100}{20} = 5 \text{ A}$$

Example 15

Find the value of current flowing through the 20 Ω resistor.

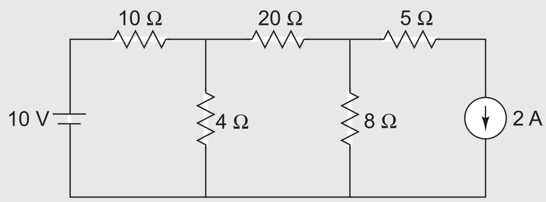


Fig. 2.433

Solution

Step 1: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

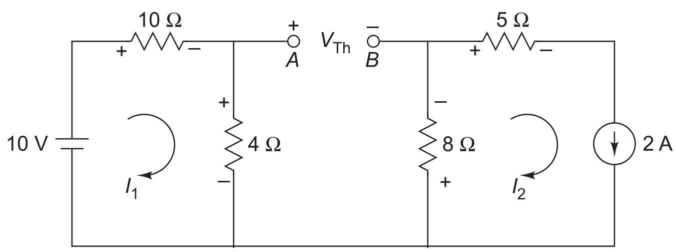


Fig. 2.434

$$I_1 = \frac{10}{10+4} = 0.71 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Writing the V_{Th} equation,

$$4\,I_1 - V_{Th} + 8\,I_2 = 0$$

$$V_{Th} = 4\,(0.71) + 8\,(2)$$

$$= 18.84 \text{ V}$$

Step II : Calculation of R_{Th}

Replacing the voltage source by short circuit and current source by an open circuit,

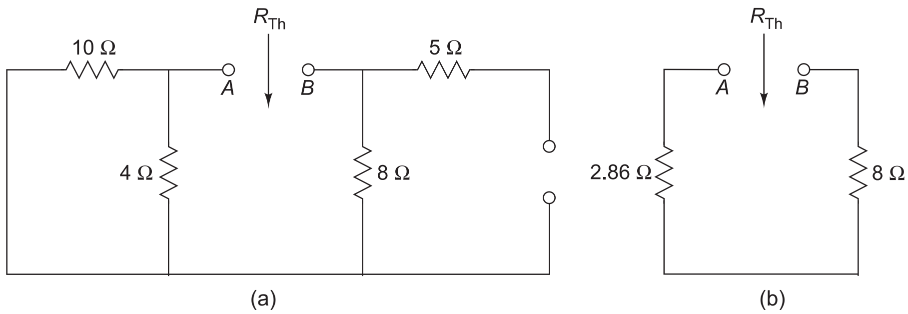


Fig. 2.435

$$R_{Th} = 10.86\ \Omega$$

Step III : Calculation of I_L

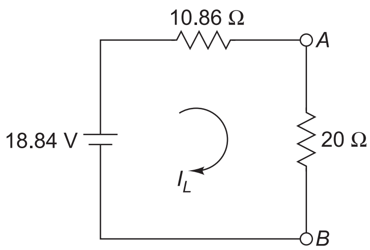


Fig. 2.436

$$I_L = \frac{18.84}{10.86 + 20} = 0.61\text{ A}$$

Example 16

Find the value of current flowing through the 5 Ω resistor.

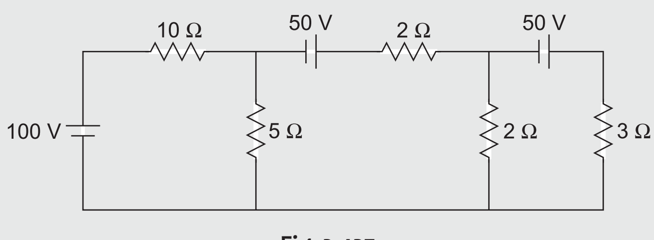


Fig. 2.437

Solution

Step I: Calculation of V_{Th}

Removing the 5 Ω resistor from the network,

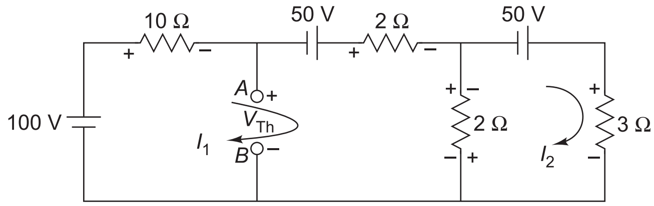


Fig. 2.438

Applying KVL to Mesh 1,

$$100 - 10\,I_1 + 50 - 2\,I_1 - 2\,(I_1 - I_2) = 0$$

$$14\,I_2 - 2\,I_2 = 150 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2\,(I_2 - I_1) + 50 - 3\,I_2 = 0$$

$$-2\,I_1 + 5\,I_2 = 50 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 12.88 \text{ A}$$

$$I_2 = 15.15 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 10\,I_1 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 100 - 10\,(12.88)$$

$$= -28.8 \text{ V}$$

$$= 28.8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

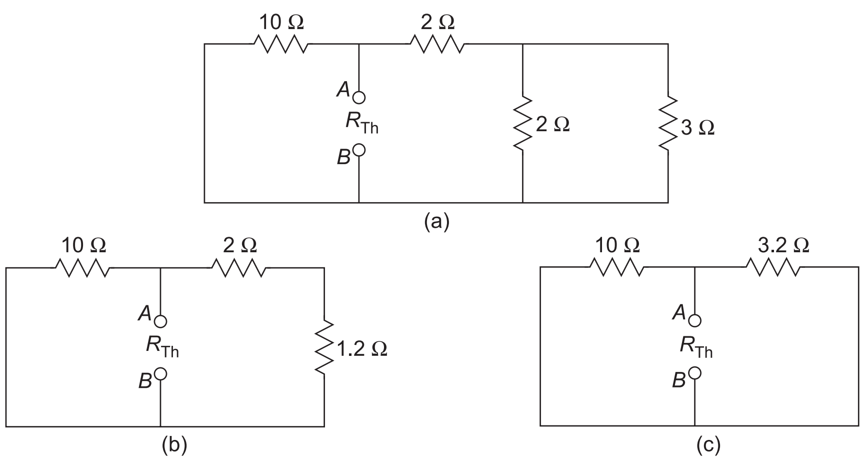


Fig. 2.439

$$R_{\text{Th}} = 10 \parallel 3.2 = 2.42\ \Omega$$

Step III: Calculation of I_L

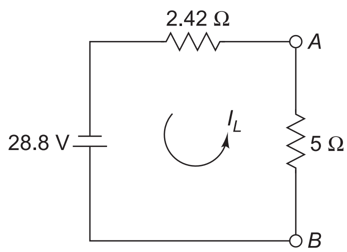


Fig. 2.440

$$I_L = \frac{28.8}{2.42 + 5} = 3.88 \text{ A} (\uparrow)$$

Example 17

Find the value of current flowing through the 10 Ω resistor.

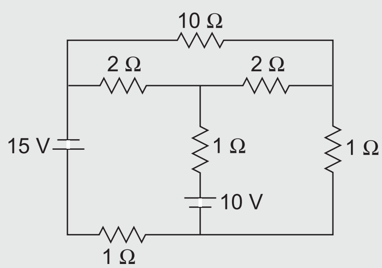


Fig. 2.441

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

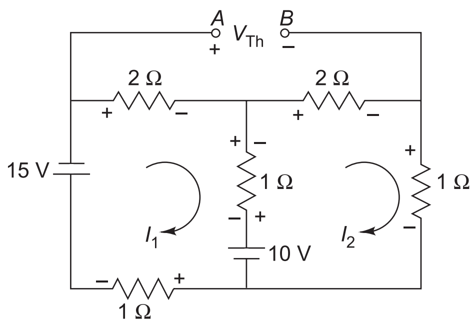


Fig. 2.442

Applying KVL to Mesh 1,

$$-15 - 2I_1 - 1\,(I_1 - I_2) - 10 - 1I_1 = 0$$

$$4I_1 - I_2 = -25 \qquad (1)$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - I_2 = 0$$

$$-I_1 + 4I_2 = 10 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -6 \text{ A}$$

$$I_2 = 1 \text{ A}$$

Writing V_{Th} equation,

$$-V_{Th} + 2I_2 + 2I_1 = 0$$

$$V_{Th} = 2I_1 + 2I_2$$

$$= 2(-6) + 2(1)$$

$$= -10 \text{ V}$$

$$= 10 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

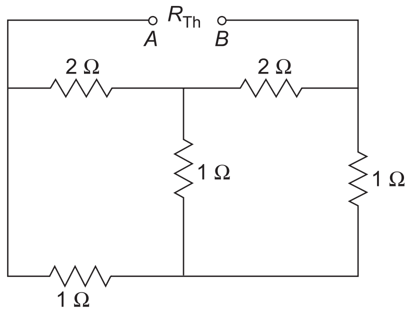


Fig. 2.443

Converting the star network formed by resistors of 2 Ω, 2 Ω and 1 Ω into an equivalent delta network.

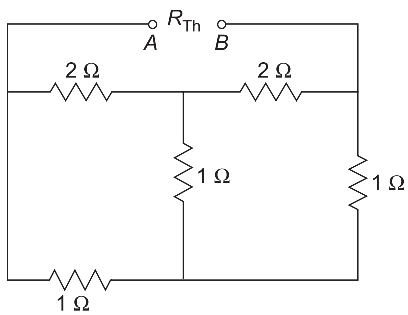


Fig. 2.444

$$R_1 = 2 + 2 + \frac{2 \times 2}{1} = 8\ \Omega$$

$$R_2 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

$$R_3 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

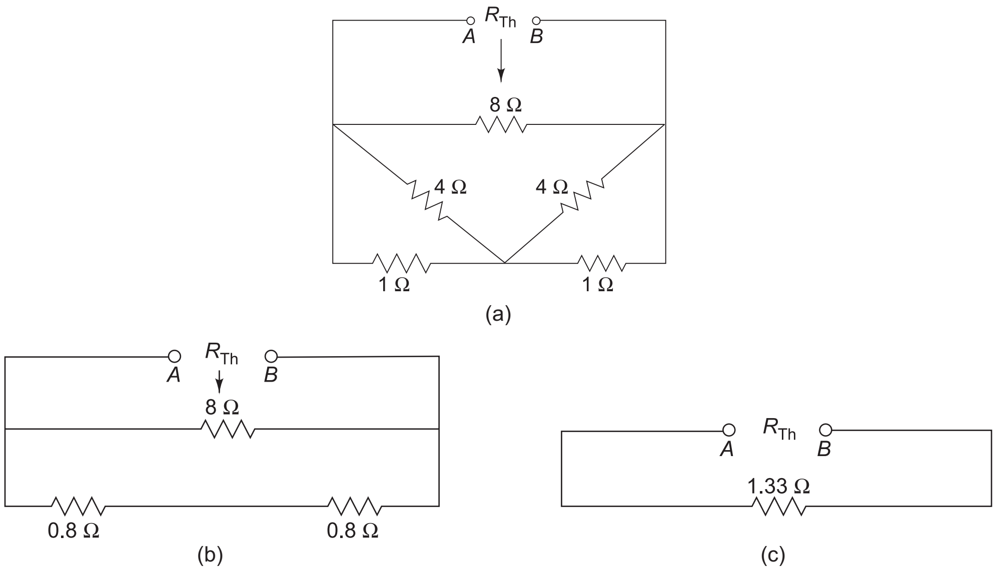


Fig. 2.445

$$R_{Th} = 1.33\ \Omega$$

Step III: Calculation of I_L

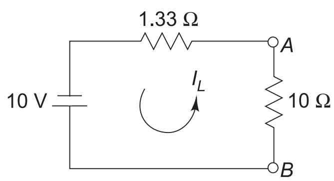


Fig. 2.446

$$I_L = \frac{10}{1.33 + 10} = 0.88\ \text{A}\ (\uparrow)$$

Example 18

Find the value of current flowing through the 1 Ω resistor.

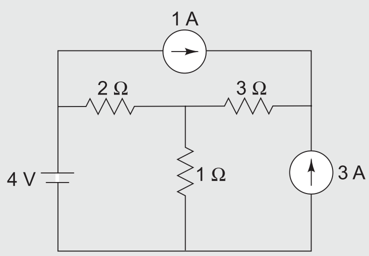


Fig. 2.447

Solution

Step I: Calculation of V_{Th}

Removing the 1 Ω resistor from the network,

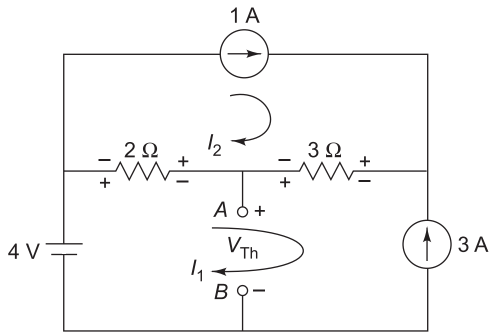


Fig. 2.448

Writing the current equation for meshes 1 and 2,

$$I_1 = -3$$

$$I_2 = 1$$

Writing V_{Th} equation,

$$4 - 2\,(I_1 - I_2) - V_{Th} = 0$$

$$V_{Th} = 4 - 2(-3 - 1)$$

$$= 4 - 2(-4)$$

$$= 12 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

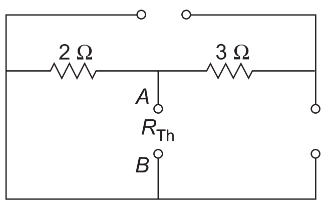


Fig. 2.449

$$R_{Th} = 2\ \Omega$$

Step III: Calculation of I_L

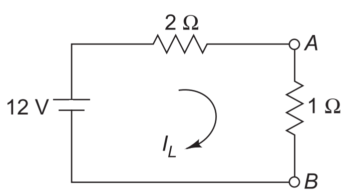


Fig. 2.450

$$I_L = \frac{12}{2+1} = 4\text{ A}$$

Example 19

Find the value of current flowing through the 3 Ω resistor.

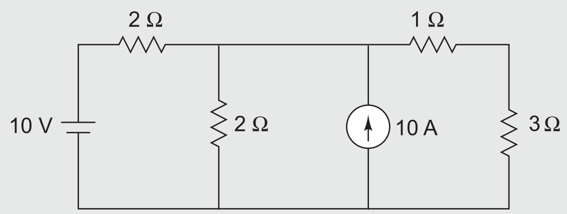


Fig. 2.451

Solution *Step I: Calculation of V_{Th}*

Removing the 3 Ω resistor from the network,

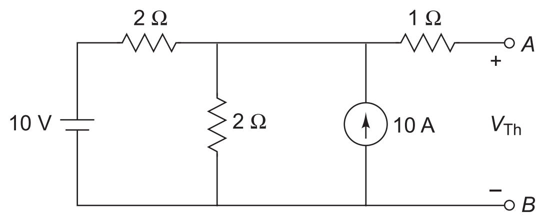


Fig. 2.452

By source transformation,

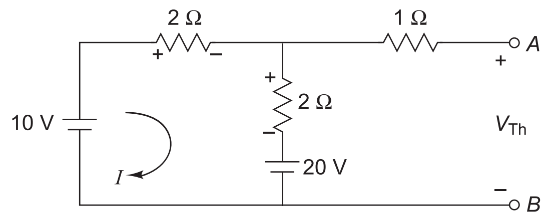


Fig. 2.453

Applying KVL to the mesh,

$$10 - 2I - 2I - 20 = 0$$
$$4I = -10$$
$$I = -2.5\text{ A}$$

Writing V_{Th} equation,

$$10 - 2I - V_{Th} = 0$$
$$V_{Th} = 10 - 2I$$

$$= 10 - 2(-2.5)$$
$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

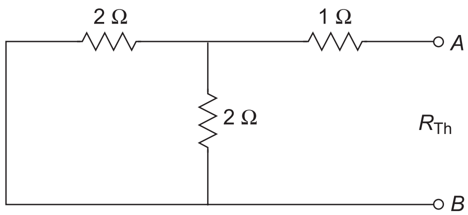


Fig. 2.454

$$R_{Th} = (2 \parallel 2) + 1 = 1 + 1 = 2\ \Omega$$

Step III: Calculation of I_L

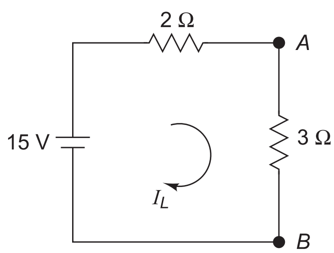


Fig. 2.455

$$I_L = \frac{15}{2+3} = 3 \text{ A}$$

Example 20

Find the value of current flowing through the 60 Ω resistor.

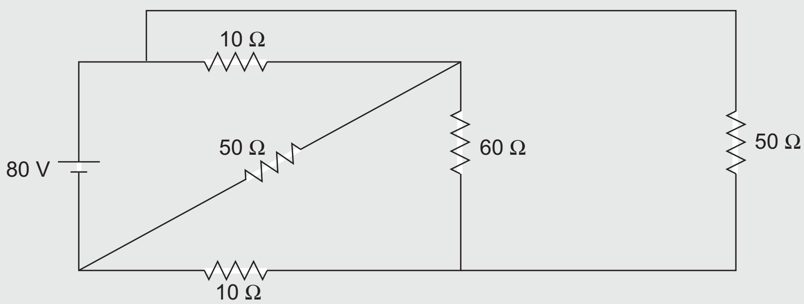


Fig. 2.456 **[May 2014]**

Solution *Step I: Calculation of V_{Th}*

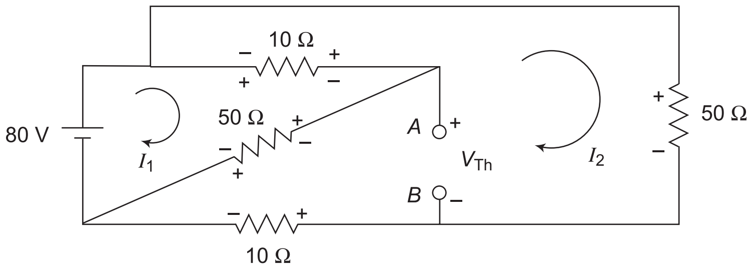


Fig. 2.457

Writing KVL equation in matrix form,

$$\begin{bmatrix} 60 & 0 \\ 0 & 120 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \end{bmatrix}$$

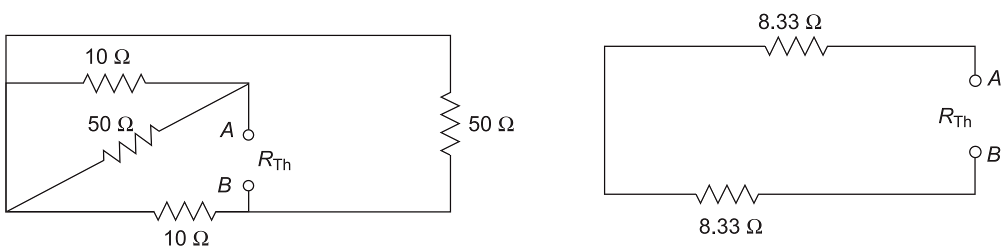
$$I_1 = 2.67 \text{ A}$$
$$I_2 = 1.33 \text{ A}$$

Writing V_{Th} equation,

$$80 - 10\,(I_1 - I_2) - V_{\text{Th}} - 10I_2 = 0$$
$$V_{\text{Th}} = 80 - 10(I_1 - I_2) - 10I_2$$
$$= 80 - 10(2.67 - 1.33) - 10\,(1.33)$$
$$= 53.3 \text{ V}$$

Step II Calculation of R_{Th}

Replacing voltage source by short circuit,



(a) (b)

Fig. 2.458

$$R_{\text{Th}} = 16.66\ \Omega$$

Step III Calculation of I_L

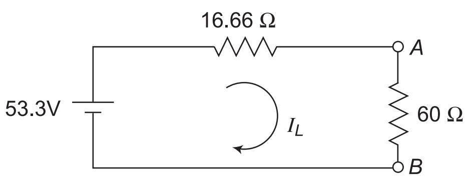


Fig. 2.459

$$I_L = \frac{53.3}{16.66 + 60} = 0.7\text{ A}$$

Exercise 2.7

2.1 Find the value of current flowing through the 6 Ω resistor.

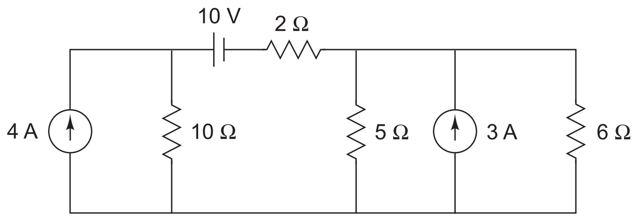


Fig. 2.460

[2.04 A]

2.2 Find the value of current flowing through the 2 Ω resistor connected between terminals *A* and *B*.

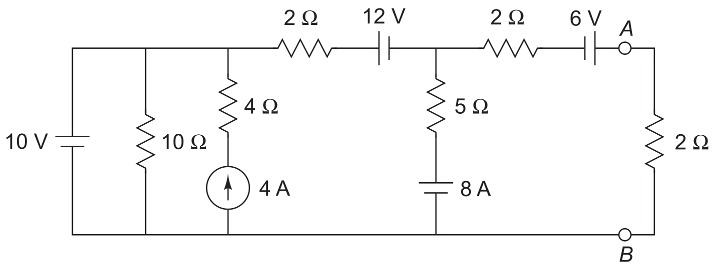


Fig. 2.461

[1.26 A]

2.3 Find the value of current flowing through the 5 Ω resistor.

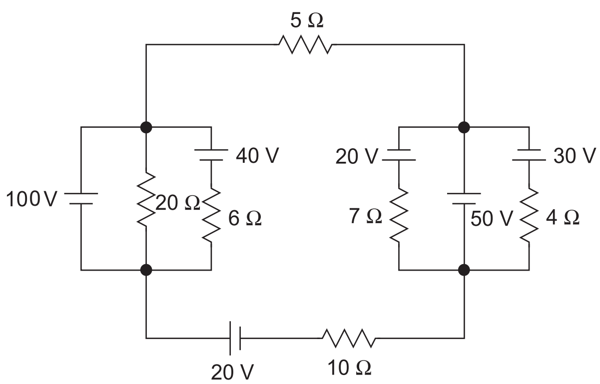


Fig. 2.462

[4.67 A]

2.4 Find the value of current flowing through the 20 Ω resistor.

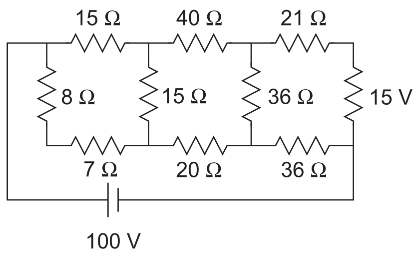


Fig. 2.463

[1.54 A]

2.5 Calculate the value of current flowing through the 10 Ω resistor.

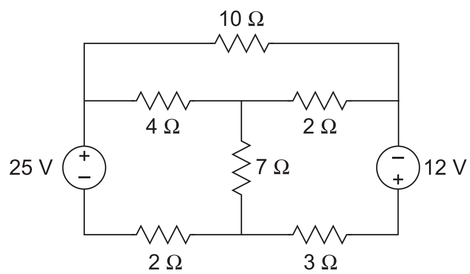


Fig. 2.464

[1.62 A]

2.6 Find the value of current flowing through the 2 Ω resistor.

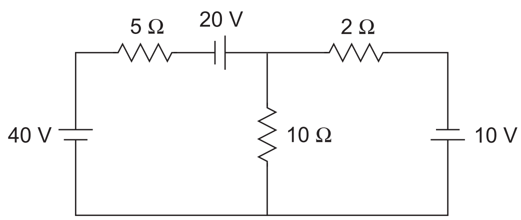


Fig. 2.465

[9.375 A]

2.7 Find the value of current flowing through the 5 Ω resistor.

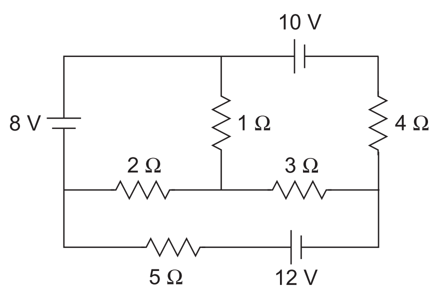


Fig. 2.466

[2 A]

2.10 NORTON'S THEOREM

[Dec 2013]

It states that *'Any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances*.

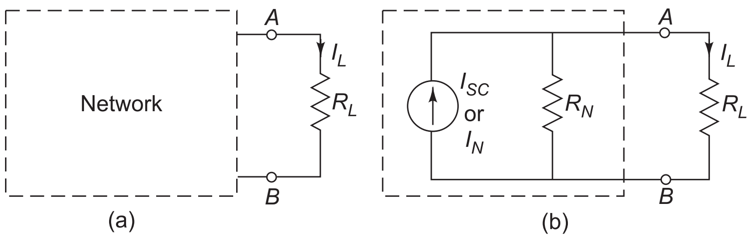


Fig. 2.467 *Norton's theorem*

Explanation The method of determining the load current through a given load resistance can be explained with the help of the following circuit.

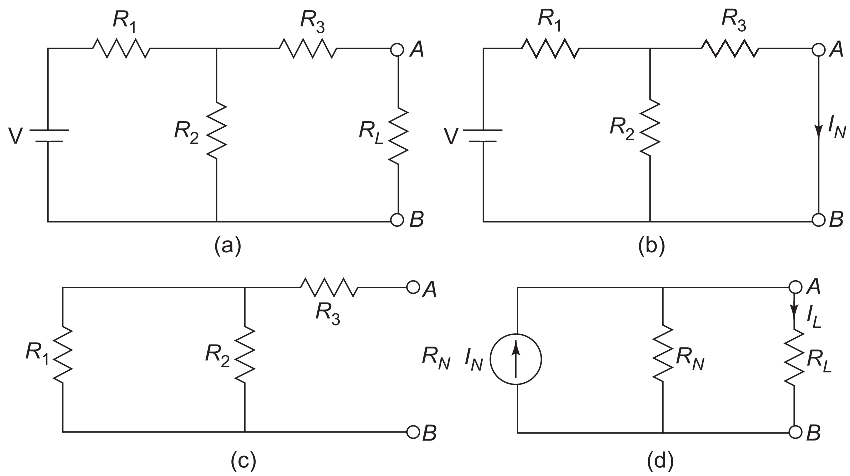


Fig. 2.468 *Steps in Norton's theorem*

2.10.1 Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{sc} or I_N.
3. Find the resistance R_N as seen from points A and B by replacing the voltage sources and current sources by internal resistances.

4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_N by current–division rule,

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 1

For the given circuit in Fig. 2.539, find the Norton equivalent between points A and B.

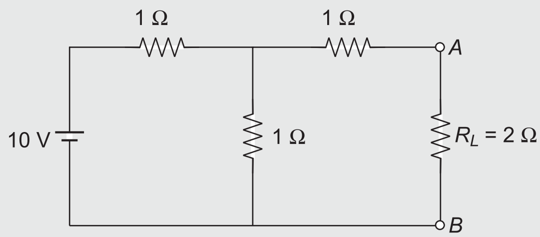


Fig. 2.469 **[May 2015]**

Solution

Step I: Calculation of I_N

Replacing 2 Ω resistor by short circuit,

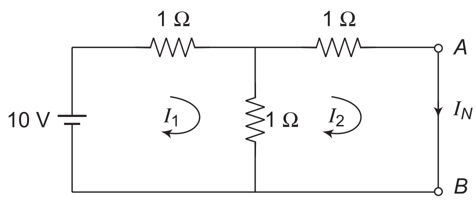


Fig. 2.470

Applying KVL to Mesh 1,

$$10 - 1I_1 - 1(I_1 - I_2) = 0$$

$$2I_1 = I_2 = 10 \qquad \text{...(1)}$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 1I_2 = 0$$

$$-I_1 + 2I_2 = 0 \qquad \text{...(2)}$$

Solving Eqs (1) and (2),

$$I_1 = 6.67 \text{ A}$$

$$I_2 = I_N = 3.33 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage source by short circuit,

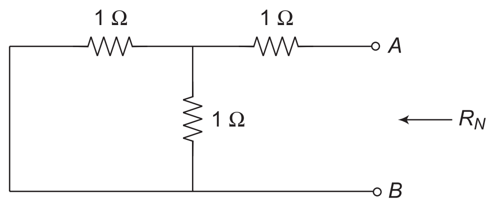


Fig. 2.471

$$R_N = 1.5\ \Omega$$

Step III: Norton's equivalent network

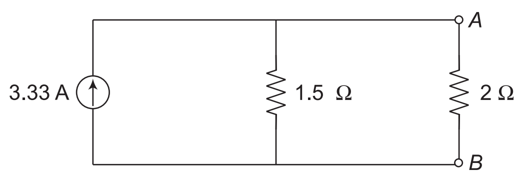


Fig. 2.172

Example 2

Find the value of current through the 10 Ω resistor.

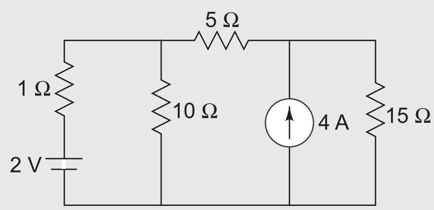


Fig. 2.473

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

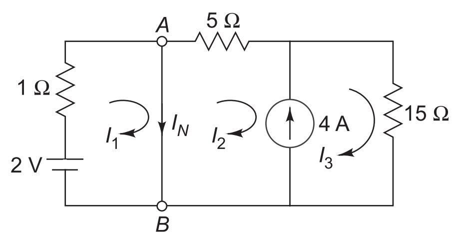


Fig. 2.474

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0 \quad (1)$$

$$I_1 = 2$$

Meshes 2 and 3 will form a supermesh.

Writing current equation for the supermesh,

$$I_3 - I_2 = 4 \quad (2)$$

Applying KVL to the supermesh,

$$-5I_2 - 15I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2 \text{ A}$$

$$I_2 = -3 \text{ A}$$

$$I_3 = 1 \text{ A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and current source by an open circuit,

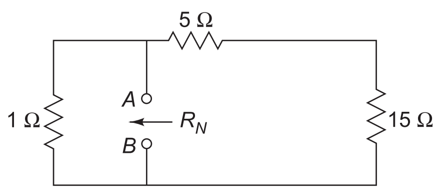


Fig. 2.475

$$R_N = 1 \parallel (5 + 15) = 0.95\ \Omega$$

Step III: Calculation of I_L

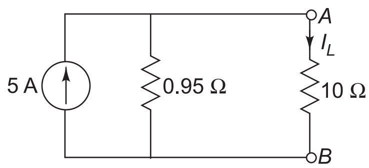


Fig. 2.476

$$I_L = 5 \times \frac{0.95}{10 + 0.95} = 0.43 \text{ A}$$

Example 3

Calculate the value of current flowing through the 15 Ω load resistor in the given circuit.

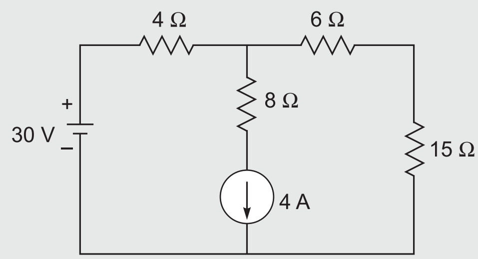


Fig. 2.477 **[May 2013]**

Solution

Step I: Calculation of I_N

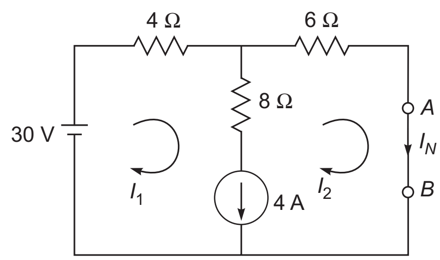


Fig. 2.478

Writing the current equation for the supermesh,

$$I_1 - I_2 = 4 \quad (1)$$

Writing the voltage equation for the supermesh,

$$30 - 4I_1 - 6I_2 = 0$$

$$4I_1 + 6I_2 = 30 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 5.4 \text{ A}$$

$$I_2 = 1.4 \text{ A}$$

$$I_N = I_2 = 1.4 \text{ A}$$

Step II: Calculation of R_N

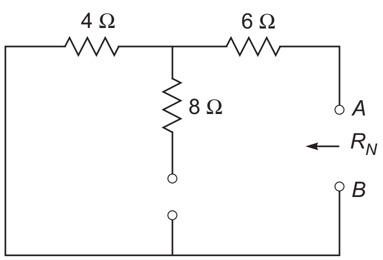


Fig. 2.479

Step III: Calculation of I_L

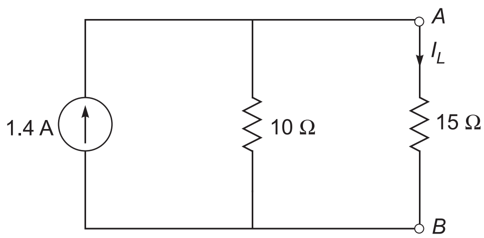


Fig. 2.480

$$I_L = 1.4 \times \frac{10}{10+15} = 0.56 \text{ A}$$

Example 4

Find the value of current flowing through the 10 Ω resistor.

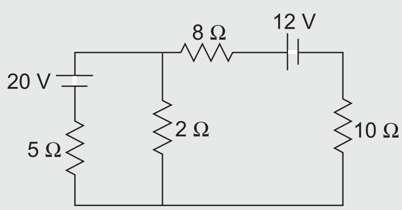


Fig. 2.481

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

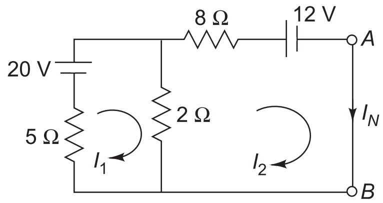


Fig. 2.482

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 20 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$

$$-2I_1 + 10I_2 = -12 \qquad (2)$$

Solving Eqs. (1) and (2),

$$I_2 = -0.67 \text{ A}$$

$$I_N = I_2 = -0.67 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

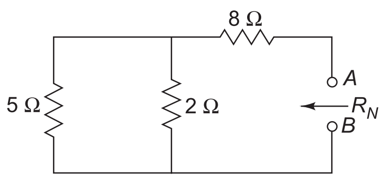


Fig. 2.483

$$R_N = (5 \,||\, 2) + 8 = 9.43 \ \Omega$$

Step III: Calculation of I_L

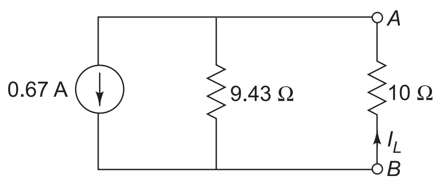


Fig. 2.484

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10} = 0.33 \text{ A} (\uparrow)$$

Example 5

Find the value of current flowing in the 10 Ω resistor.

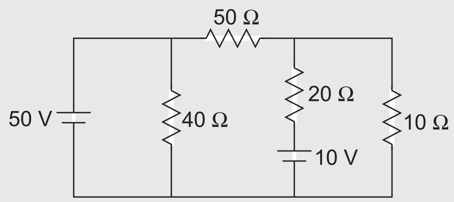


Fig. 2.485

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

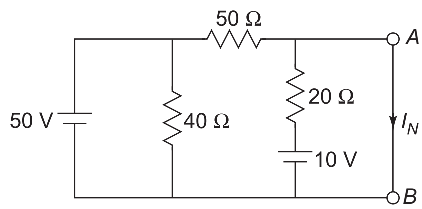


Fig. 2.486

The resistance of 40 Ω becomes redundant as it is connected across the 50 V source.

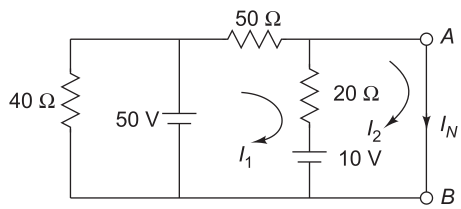


Fig. 2.487

Applying KVL to Mesh 1,

$$50 - 50\,I_1 - 20\,(I_1 - I_2) - 10 = 0$$

$$70\,I_1 - 20\,I_2 = 40 \qquad (1)$$

Applying KVL to Mesh 2,

$$10 - 20\,(I_2 - I_1) = 0$$

$$-20\,I_1 + 20\,I_2 = 10 \qquad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = 1\text{ A}$$

$$I_2 = 1.5\text{ A}$$

$$I_N = I_2 = 1.5\text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits, resistor of 40 Ω gets shorted.

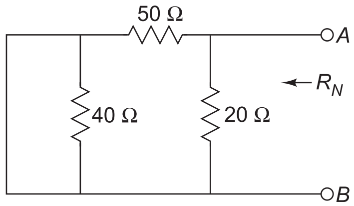


Fig. 2.488

$$R_N = 50||20 = 14.28\ \Omega$$

Step III: Calculation of I_L

$$I_L = 1.5 \times \frac{14.28}{14.28+10} = 0.88\text{ A}$$

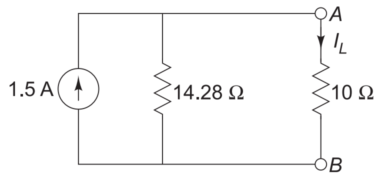


Fig. 2.489

Example 6

Find the value of current flowing through the 10 Ω resistor in Fig. 2.490.

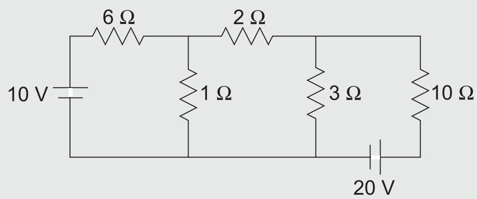


Fig. 2.490

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

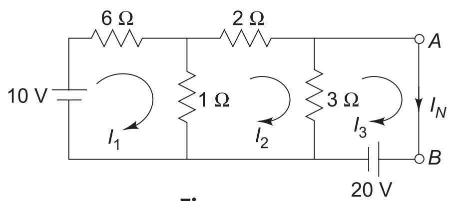


Fig. 2.491

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1\,(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1\,(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \qquad (2)$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 20 = 0$$

$$3I_2 - 3I_3 = 20 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_3 = -13.17 \text{ A}$$

$$I_N = I_3 = -13.17 \text{ A}$$

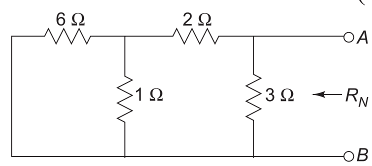


Fig. 2.492

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

$$R_N = [(6 \parallel 1) + 2] \parallel 3 = 1.46\ \Omega$$

Step III: Calculation of I_L

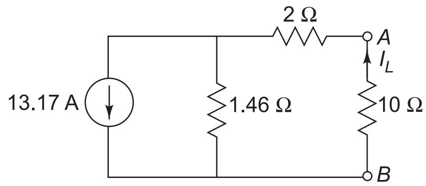


Fig. 2.493

$$I_L = 13.17 \times \frac{1.46}{1.46 + 10} = 1.68 \text{ A } (\uparrow)$$

Example 7

Find the value of current flowing through the 10 Ω resistor.

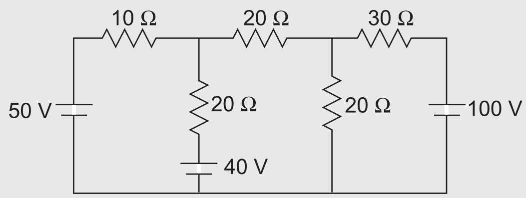


Fig. 2.494

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

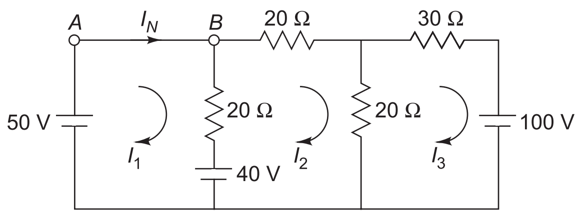


Fig. 2.495

Applying KVL to Mesh 1,

$$50 - 20(I_1 - I_2) - 40 = 0$$

$$20I_1 - 20I_2 = 10 \quad (1)$$

Applying KVL to Mesh 2,

$$40 - 20(I_2 - I_1) - 20I_2 - 20(I_2 - I_3) = 0$$

$$-20I_1 + 60I_2 - 20I_3 = 40 \quad (2)$$

Applying KVL to Mesh 3,

$$-20(I_3 - I_2) - 30I_3 - 100 = 0$$

$$-20I_2 + 50I_3 = -100 \quad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = 0.81 \text{ A}$$

$$I_N = I_1 = 0.81 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

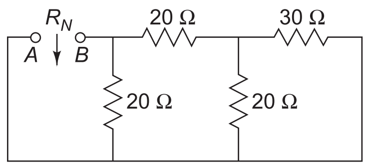


Fig. 2.496

$$R_N = [(20 \parallel 30) + 20] \parallel 20 = 12.3\ \Omega$$

Step III: Calculation of I_L

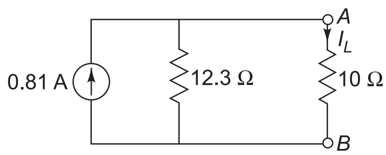


Fig. 2.497

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45 \text{ A}$$

Example 8

Obtain Norton's equivalent network as seen by R_L.

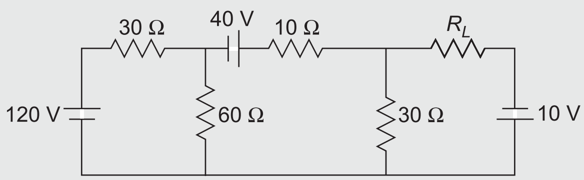


Fig. 2.498

Solution

Step I: Calculation of I_N

Replacing the resistor R_L by a short circuit,

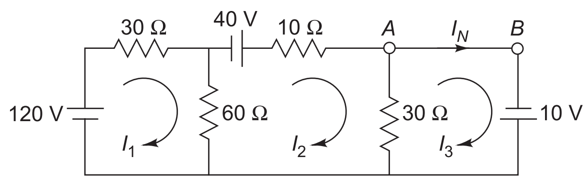


Fig. 2.499

Applying KVL to Mesh 1,

$$120 - 30I_1 - 60\,(I_1 - I_2) = 0$$

$$90I_1 - 60I_2 = 120 \tag{1}$$

Applying KVL to Mesh 2,

$$-60\,(I_2 - I_1) + 40 - 10I_2 - 30\,(I_2 - I_3) = 0$$

$$-60I_1 + 100I_2 - 30I_3 = 40 \tag{2}$$

Applying KVL to Mesh 3,

$$-30\,(I_3 - I_2) + 10 = 0$$

$$30I_2 - 30I_3 = -10 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_3 = 4.67\text{ A}$$

$$I_N = I_3 = 4.67\text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

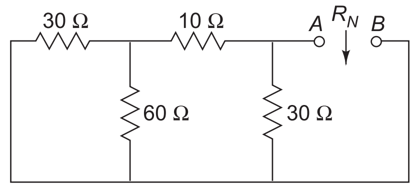


Fig. 2.500

$$R_N = [(30 \parallel 60) + 10] \parallel 30 = 15\ \Omega$$

Step III: Norton's equivalent network

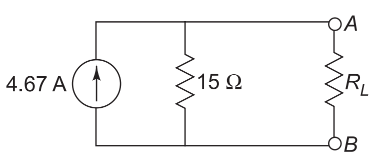


Fig. 2.501

Example 9

Find the value of current flowing through the 8 Ω resistor.

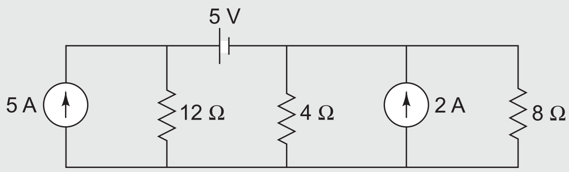


Fig. 2.502

Solution

Step I: Calculation of I_N

Replacing the 8 Ω resistor by a short circuit,

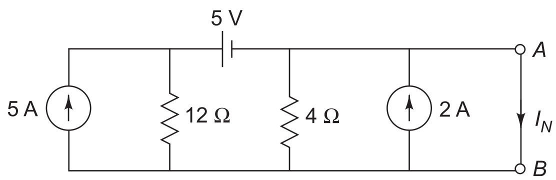


Fig. 2.503

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation,

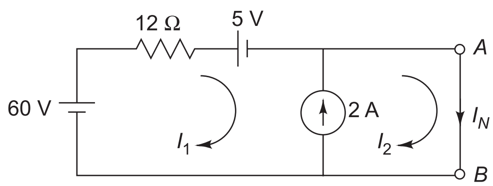


Fig. 2.504

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 2 \tag{1}$$

Applying KVL to the supermesh,

$$60 - 12I_1 - 5 = 0$$

$$12I_1 = 55 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 4.58 \text{ A}$$

$$I_2 = 6.58 \text{ A}$$

$$I_N = I_2 = 6.58 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

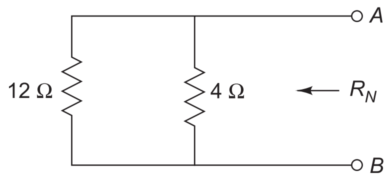


Fig. 2.505

$$R_N = 12 \parallel 4 = 3\ \Omega$$

Step III: Calculation of I_L

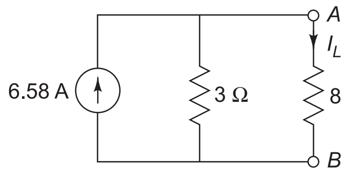


Fig. 2.506

$$I_L = 6.58 \times \frac{15}{2+3} = 1.79\ \text{A}$$

Example 10

Find value of current flowing through the 1 Ω resistor.

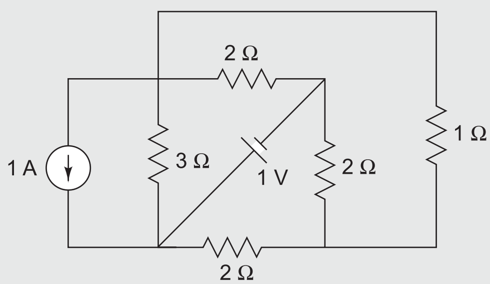


Fig. 2.507

Solution

Step I: Calculation of I_N

Replacing the 1 Ω resistor by a short circuit,

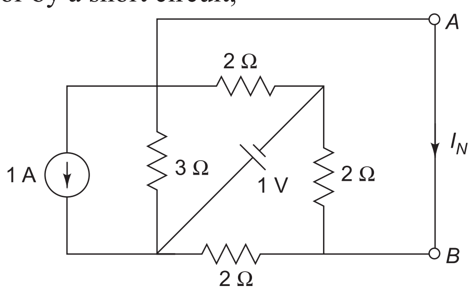


Fig. 2.508

By source transformation,

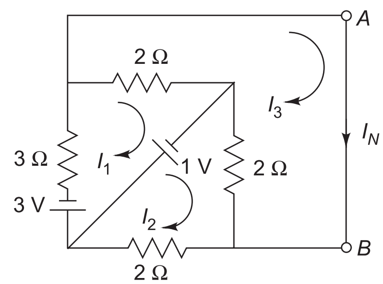


Fig. 2.509

Applying KVL to Mesh 1,

$$-3 - 3I_1 - 2(I_1 - I_3) + 1 = 0$$

$$5I_1 - 2I_3 = -2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1 - 2\,(I_2 - I_3) - 2\,I_2 = 0$$

$$4I_2 - 2I_3 = -1 \qquad (2)$$

Applying KVL to Mesh 3,

$$-2\,(I_3 - I_1) - 2\,(I_3 - I_2) = 0$$

$$-2I_1 - 2I_2 + 4I_3 = 0 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = -0.64 \text{ A}$$

$$I_2 = -0.55 \text{ A}$$

$$I_3 = -0.59 \text{ A}$$

$$I_N = I_3 = -0.59 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

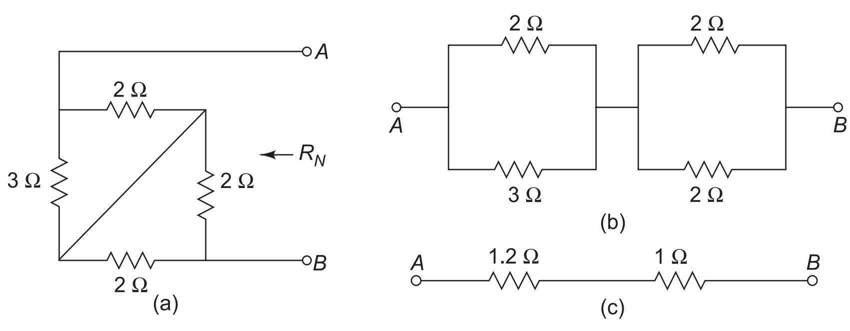


Fig. 2.510

$$R_N = 2.2\ \Omega$$

Step III: Calculation of I_L

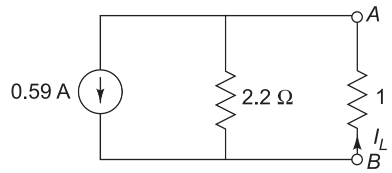


Fig. 2.511

$$I_L = 0.59 \times \frac{2.2}{2.2+1} = 0.41 \text{ A}$$

Exercise 2.8

2.1 Find the value of current flowing through the 10 Ω resistor.

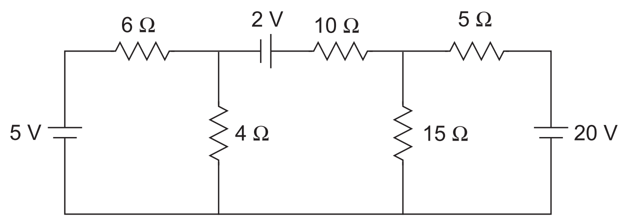


Fig. 2.512

[0.68 A]

2.2 Find the value of current flowing through the 20 Ω resistor.

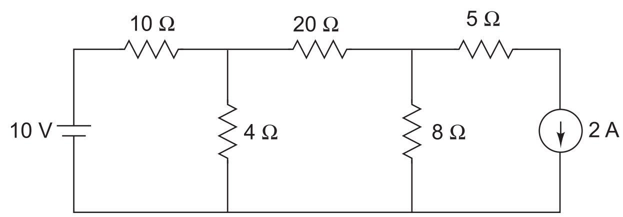


Fig. 2.513

[0.61 A]

2.3 Find the value of current flowing through the 2 Ω resistor.

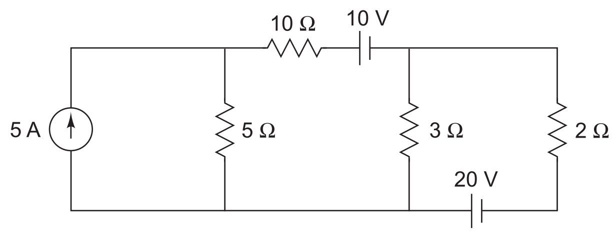


Fig. 2.514

[5 A]

2.4 Find the value of current flowing through the 5 Ω resistor.

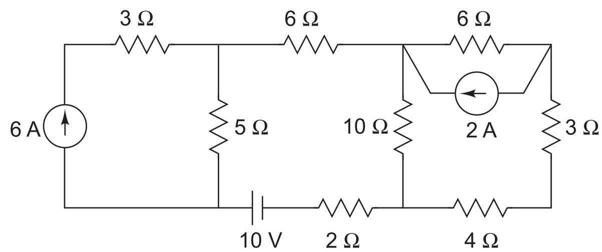


Fig. 2.515

[4.13 A]

2.5 Find the value of current flowing through the 15 Ω resistor.

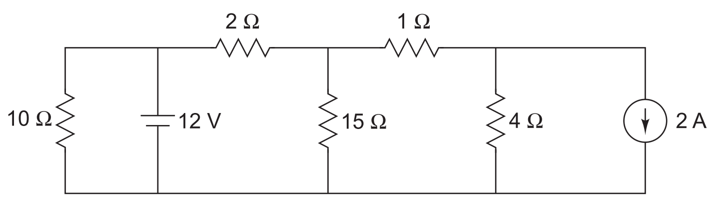


Fig. 2.516

[0.382 A]

2.6 Find Norton's equivalent network.

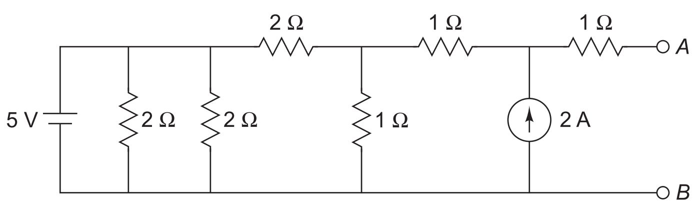


Fig. 2.517

[1.8 A, 1.67 Ω]

2.7 Find Norton's equivalent circuit for the portion of network shown in Fig. 2.518 to the left of *ab*. Hence obtain the current in the 10 Ω resistor.

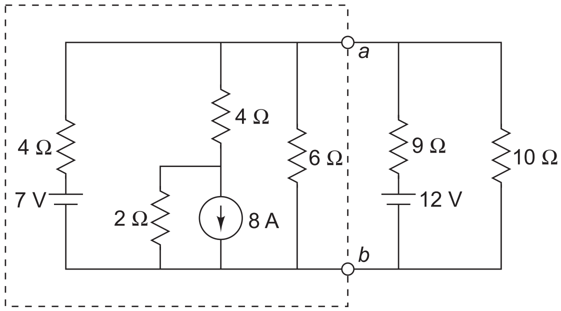


Fig. 2.518

[0.053 A]

2.11 MAXIMUM POWER TRANSFER THEOREM

[Dec 2012, 2015, May 2013, 2014]

It states that *'the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.'*

$$I = \frac{V}{R_S + R_L}$$

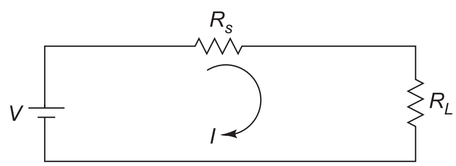


Fig. 2.519 *Maximum power transfer theorem*

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2[(R_S+R_L)^2-(2R_L)(R_S+R_L)]}{(R_S+R_L)^4}$$

$$(R_S+R_L)^2 - 2\,R_L\,(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_S R_L - 2R_L R_S - 2R_L^2 = 0$$

$$R_L = R_S$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

2.11.1 Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage sources and current sources replaced by internal resistances.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{\text{Th}}$$

5. Find the maximum power.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}}+R_L} = \frac{V_{\text{Th}}}{2R_{\text{Th}}}$$

$$P_{\max} = I_L^2\,R_L = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}^2} \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

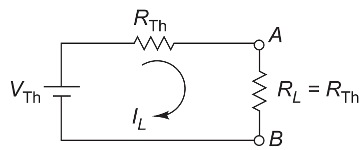


Fig. 2.520 *Equivalent circuit*

Example 1

Find the value of resistance R_L for maximum power transfer calculate maximum power.

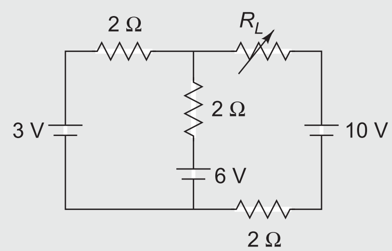


Fig. 2.521

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

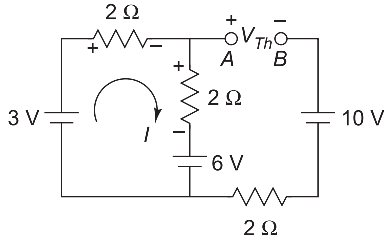


Fig. 2.522

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$
$$I = -0.75 \text{ A}$$

Writing V_{Th} equation,

$$6 + 2I - V_{\text{Th}} - 10 = 0$$
$$\begin{aligned} V_{\text{Th}} &= 6 + 2I - 10 \\ &= 6 + 2(-0.75) - 10 \\ &= -5.5 \text{ V} \\ &= 5.5 \text{ V (terminal } B \text{ is positive w.r.t } A) \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

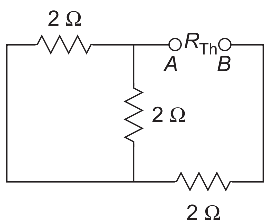


Fig. 2.523

$$R_{\text{Th}} = (2 \parallel 2) + 2 = 3\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 3\ \Omega$$

Step IV: Calculation of P_{max}

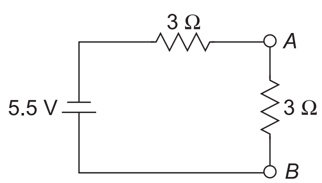


Fig. 2.524

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(5.5)^2}{4\times 3} = 2.52 \text{ W}$$

Example 2

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

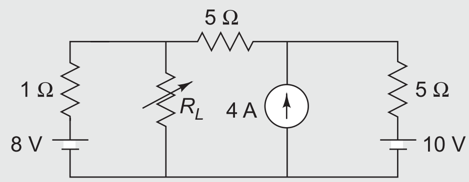


Fig. 2.525

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

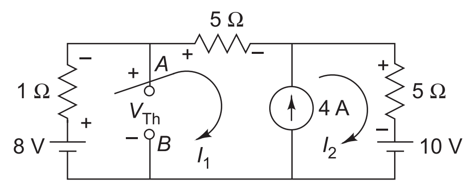


Fig. 2.526

Meshes 1 and 2 will form a supermesh.

Writing current equation for the supermesh,

$$I_2 - I_1 = 4 \tag{1}$$

Applying KVL to the supermesh,

$$8 - 1I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \tag{2}$$

Solving Eqs. (1) and (2),

$$I_1 = -2 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$8 - 1I_1 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8 - I_1$$

$$= 8 - (-2)$$

$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current source by an open circuit,

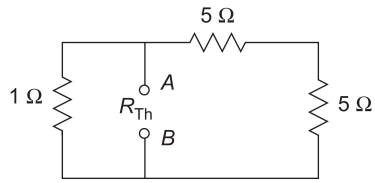


Fig. 2.527

$$R_{\text{Th}} = 10 \parallel 1 = 0.91\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 0.91\ \Omega$$

Step IV: Calculation of P_{max}

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(10)^2}{4\times 0.91} = 27.47\ \text{W}$$

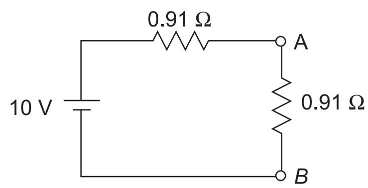


Fig. 2.528

Example 3

Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

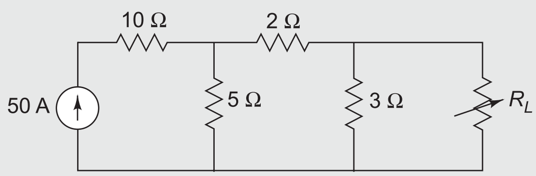


Fig. 2.529

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

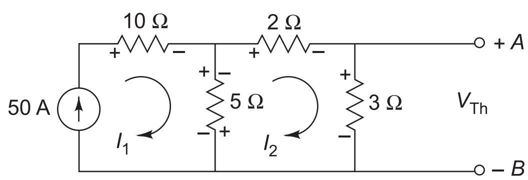


Fig. 2.530

For Mesh 1,

$$I_1 = 50$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25\text{ A}$$
$$V_{Th} = 3I_2 = 3(25) = 75\text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit,

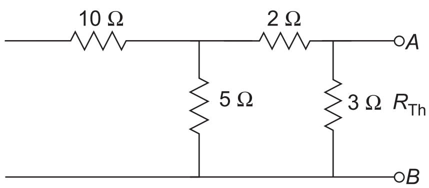


Fig. 2.531

$$R_{Th} = 7 \,||\, 3 = 2.1\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 2.1\ \Omega$$

Step IV: Calculation of P_{max}

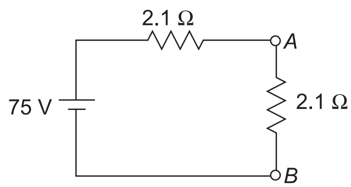


Fig. 2.532

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(75)^2}{4 \times 2.1} = 669.64\text{ W}$$

Example 4

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

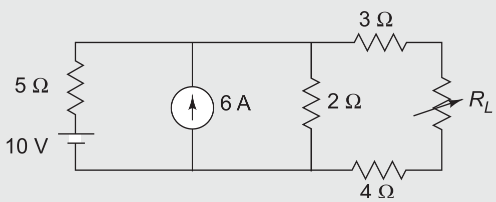


Fig. 2.533

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

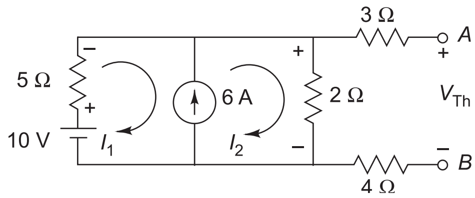


Fig. 2.534

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 6 \tag{1}$$

Applying KVL to the supermesh,

$$10 - 5I_1 - 2I_2 = 0$$

$$5I_1 + 2I_2 = 10 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -0.29 \text{ A}$$

$$I_2 = 5.71 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} = 2I_2 = 11.42 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

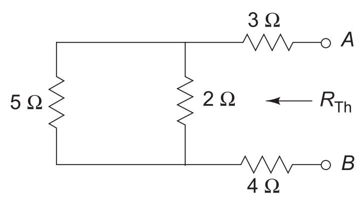


Fig. 2.535

$$R_{Th} = (5 \,||\, 2) + 3 + 4 = 8.43\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 8.43\ \Omega$$

Step IV: Calculation of P_{max}

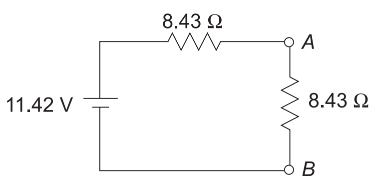


Fig. 2.536

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87 \text{ W}$$

Example 5

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

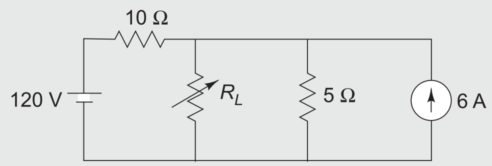


Fig. 2.537

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

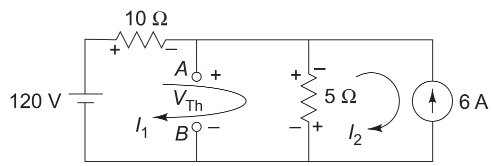


Fig. 2.538

Applying KVL to Mesh 1,

$$120 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15I_1 - 5I_2 = 120 \quad (1)$$

Writing current equation for Mesh 2,

$$I_2 = -6 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 6 \text{ A}$$

Writing V_{Th} equation,

$$120 - 10I_1 - V_{Th} = 0$$

$$V_{Th} = 120 - 10\,(6)$$

$$= 60 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

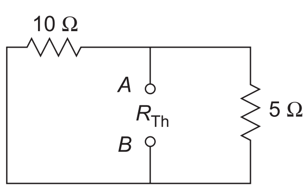


Fig. 2.539

$$R_{Th} = 10 \,||\, 5 = 3.33\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.33\ \Omega$$

Step IV: Calculation of P_{max}

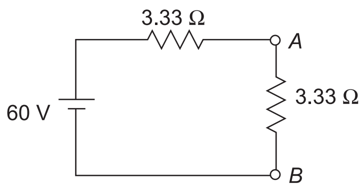


Fig. 2.540

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 3.33} = 270.27\ \text{W}$$

Example 6

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

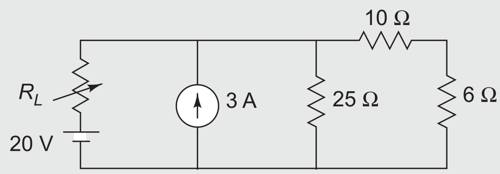


Fig. 2.541

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

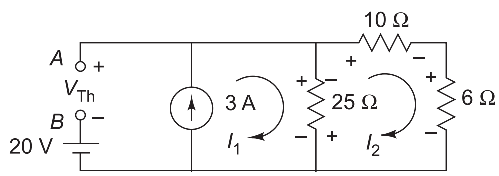


Fig. 2.542

For Mesh 1,

$$I_1 = 3 \tag{1}$$

Applying KVL to Mesh 2,

$$-25(I_2 - I_1) - 10I_2 - 6I_2 = 0$$

$$-25I_1 + 41I_2 = 0 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 1.83 \text{ A}$$

Writing V_{Th} equation,

$$20 + V_{\text{Th}} - 10I_2 - 6I_2 = 0$$

$$V_{\text{Th}} = -20 + 10\,(1.83) + 6\,(1.83)$$

$$= 9.28 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

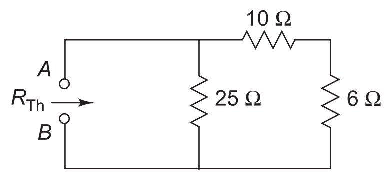


Fig. 2.543

$$R_{\text{Th}} = 25 \,||\, 16 = 9.76\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 9.76\ \Omega$$

Step IV: Calculation of P_{max}

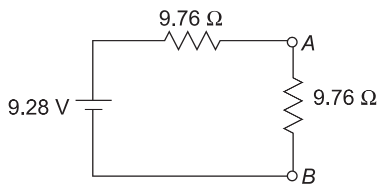


Fig. 2.544

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(9.28)^2}{4 \times 9.76} = 2.21 \text{ W}$$

Example 7

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

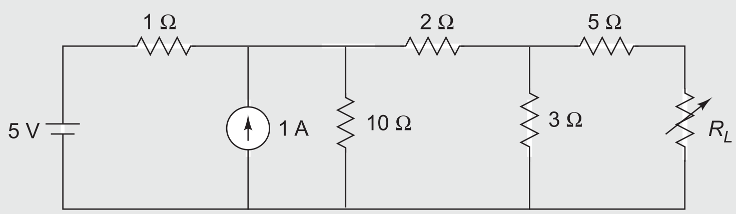


Fig. 2.545

Solution

Step I : Calculation of V_{Th}

Removing the variable resistor R_L from the network,

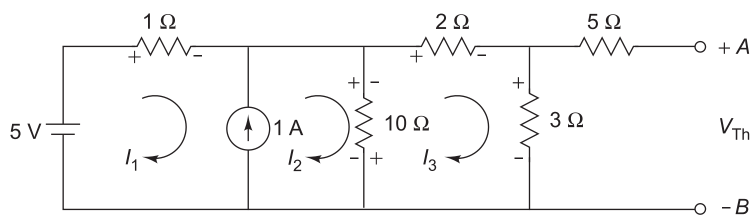


Fig. 2.546

Meshes 1 and 2 will form a supermesh.

Writing the current equation for the supermesh,

$$I_2 - I_1 = 1 \tag{1}$$

Writing the voltage equation for the supermesh,

$$5 - 1I_1 - 10\,(I_2 - I_3) = 0$$

$$I_1 + 10\,I_2 - 10\,I_3 = 5 \tag{2}$$

Applying KVL to Mesh 3,

$$-\,10\,(I_3 - I_2) - 2\,I_3 - 3\,I_3 = 0$$

$$-\,10\,I_2 + 15\,I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.38\text{ A}$$

$$I_2 = 1.38\text{ A}$$

$$I_3 = 0.92\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 3\,I_3 = 2.76\text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

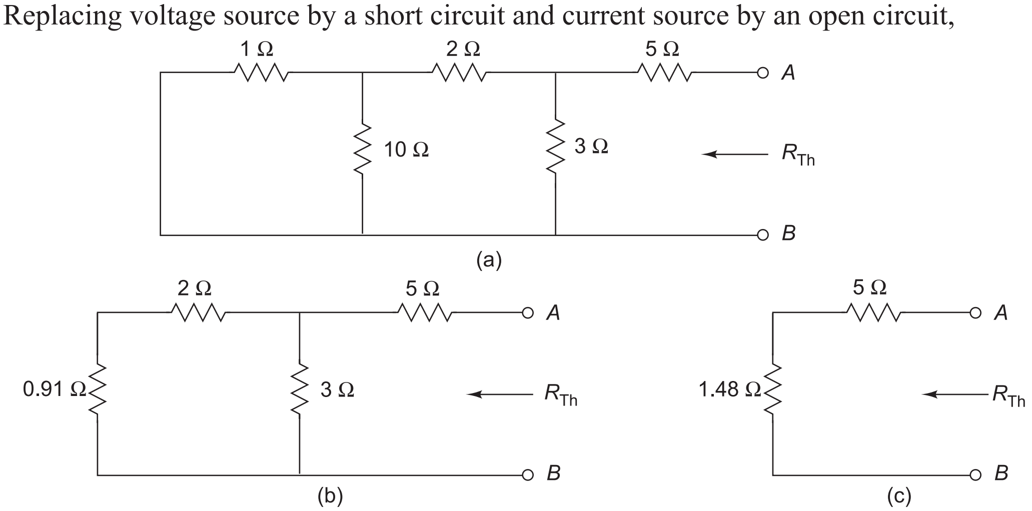


Fig. 2.547

$$R_{Th} = 6.48\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 6.48\ \Omega$$

Step IV: Calculation of P_{max}

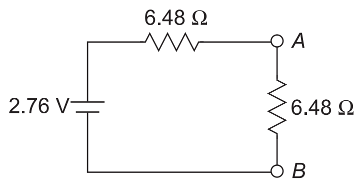


Fig. 2.548

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(2.76)^2}{4 \times 6.48} = 0.29\ \text{W}$$

Example 8

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

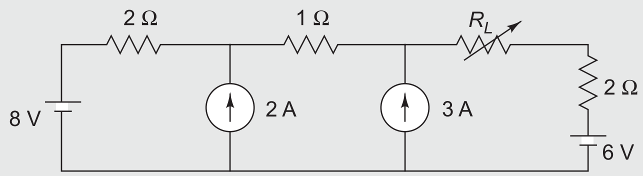


Fig. 2.549

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

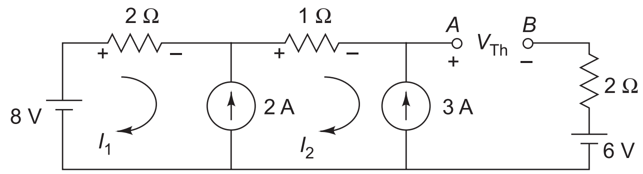


Fig. 2.550

From Fig. 2.550,

$$I_2 - I_1 = 2 \tag{1}$$

$$I_2 = -3\ \text{A} \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -5\ \text{A}$$

Writing V_{Th} equation,

$$8 - 2I_1 - 1I_2 - V_{\text{Th}} - 6 = 0$$

$$V_{\text{Th}} = 8 - 2(-5) - (-3) - 6$$

$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

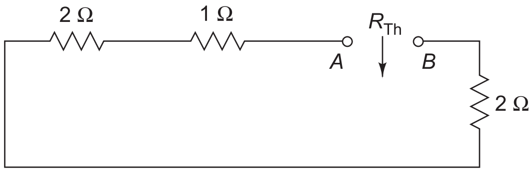


Fig. 2.551

$$R_{\text{Th}} = 5\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 5\ \Omega$$

Step IV: Calculation of P_{max}

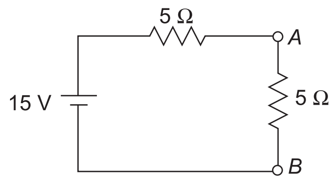


Fig. 2.552

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(15)^2}{4 \times 5} = 11.25 \text{ W}$$

Example 9

Find the value of resistance the R_L for maximum power transfer and calculate the maximum power.

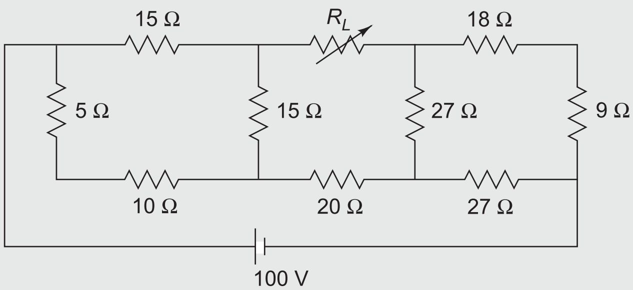


Fig. 2.553

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

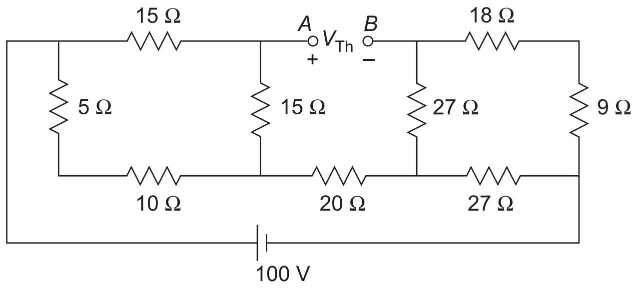


Fig. 2.554

By star-delta transformation,

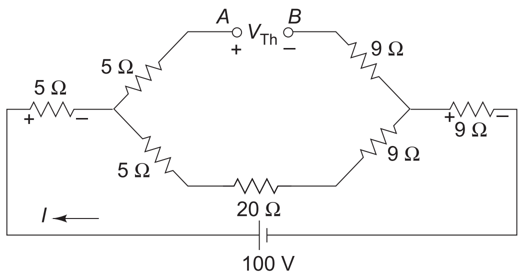


Fig. 2.555

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

Writing V_{Th} equation,

$$100 - 5I - V_{Th} - 9I = 0$$

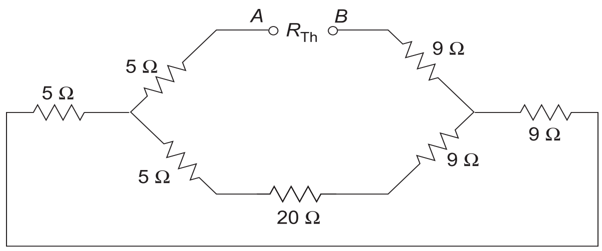
$$V_{Th} = 100 - 14I$$

$$= 100 - 14(2.08)$$

$$= 70.88 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,



(a)

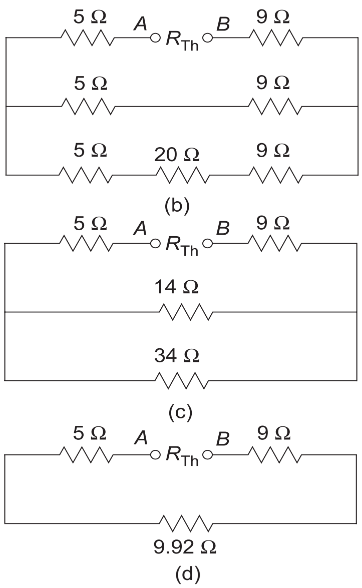


Fig. 2.556

$$R_{\text{Th}} = 23.92\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 23.92\ \Omega$$

Step IV: Calculation of P_{max}

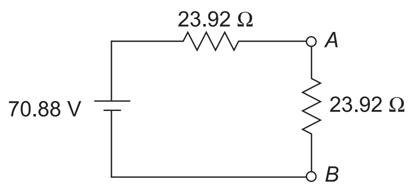


Fig. 2.557

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(70.88)^2}{4 \times 23.92} = 52.51\ \text{W}$$

Example 10

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

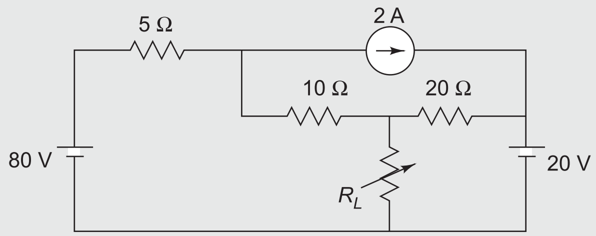


Fig. 2.558

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

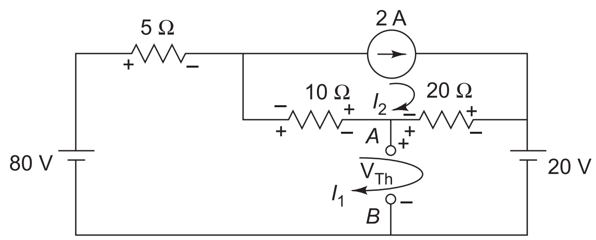


Fig. 2.559

Applying KVL to Mesh 1,

$$80 - 5I_1 - 10(I_1 - I_2) - 20(I_1 - I_2) - 20 = 0$$

$$35I_1 - 30I_2 = 60 \quad (1)$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 3.43 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 20\,(I_1 - I_2) - 20 = 0$$

$$V_{\text{Th}} = 20(3.43 - 2) + 20$$

$$= 48.6 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

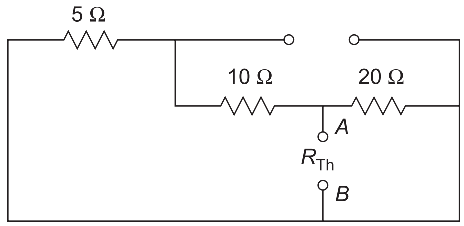


Fig. 2.560

$$R_{Th} = 15 \,||\, 20 = 8.57\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 8.57\ \Omega$$

Step IV: Calculation of P_{max}

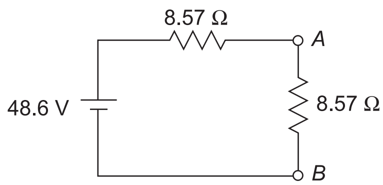


Fig. 2.561

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(48.6)^2}{4 \times 8.57} = 68.9\ \text{W}$$

Example 11

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

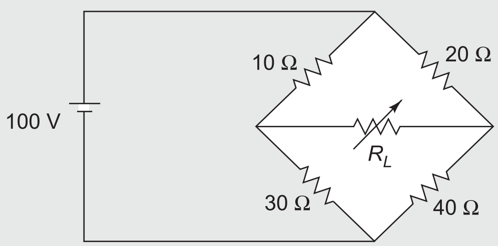


Fig. 2.562

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

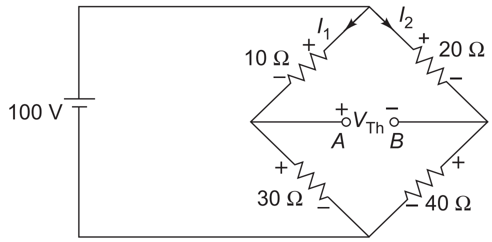


Fig. 2.563

$$I_1 = \frac{100}{10+30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20+40} = 1.66 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} + 10I_1 - 20I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 20I_2 - 10I_1 \\ &= 20(1.66) - 10(2.5) \\ &= 8.2 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

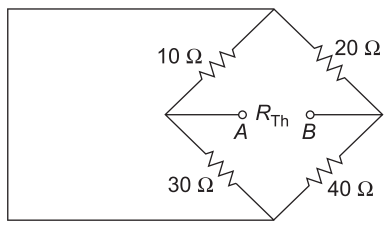


Fig. 2.564

Redrawing the network,

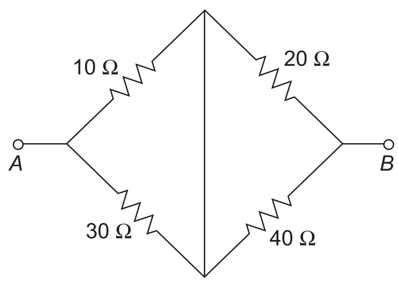


Fig. 2.565

$$R_{\text{Th}} = (10 \,||\, 30) + (20 \,||\, 40) = 20.83 \ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 20.83\ \Omega$$

Step IV: Calculation of P_{max}

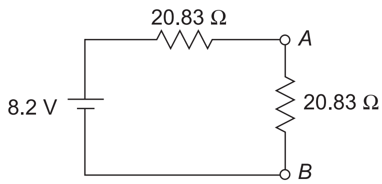


Fig. 2.566

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81\ \text{W}$$

Example 12

For the given circuit find the value of R_L for maximum power transfer and calculate the maximum power absorbed by R_L.

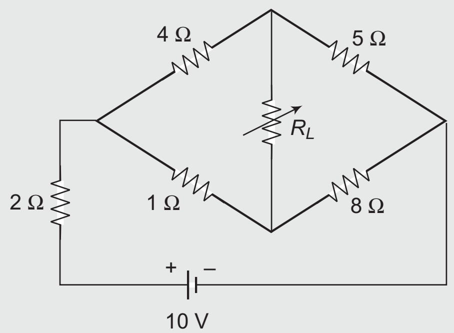


Fig. 2.567 **[Dec 2014]**

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

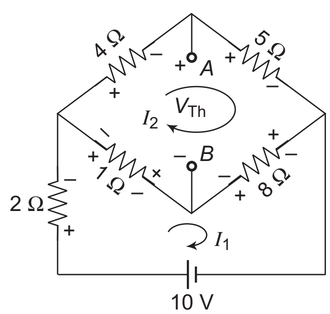


Fig. 2.568

Applying KVL to Mesh 1,

$$10 - 2I_1 - 1(I_1 - I_2) - 8\,(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2

$$-4I_2 - 5I_2 - 8\,(I_2 - I_1) - 1\,(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1.54 \text{ A}$$

$$I_2 = 0.77 \text{ A}$$

Writing V_{Th} equation,

$$-1\,(I_2 - I_1) - 4I_2 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = -1(I_2 - I_1) - 4I_2$$

$$= -1(0.77 - 1.54) - 4(0.77)$$

$$= -2.31 \text{ V}$$

$$= 2.31 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

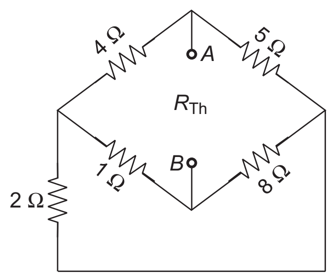


Fig. 2.569

Redrawing the network,

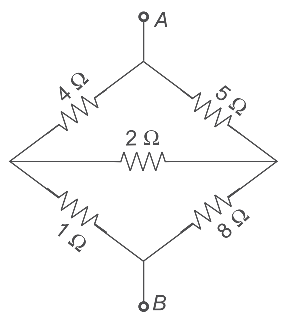


Fig. 2.570

Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\,\Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\,\Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\,\Omega$$

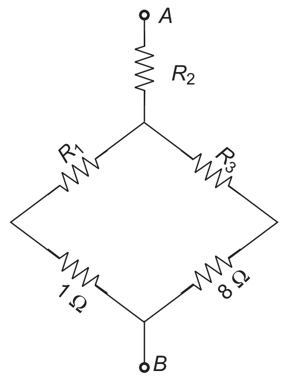


Fig. 2.571

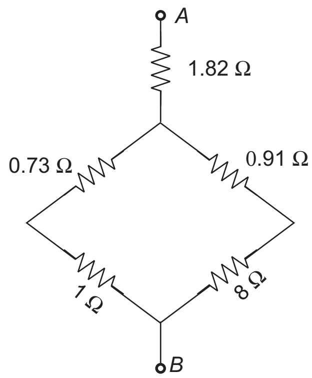


Fig. 2.572

Simplifying the network,

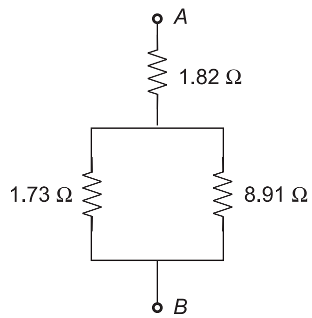


Fig. 2.573

$$R_{\mathrm{Th}} = 1.82 + (1.73 \parallel 8.91) = 3.27\,\Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\mathrm{Th}} = 3.27\,\Omega$$

Step IV: Calculation of P_{max}

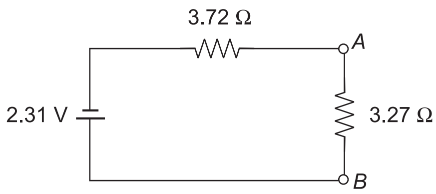


Fig. 2.574

$$P_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}} = \frac{(2.31)^2}{4 \times 3.27} = 0.41\ \mathrm{W}$$

Example 13

Determine the value of R for maximum power transfer. Also find the magnitude of maximum power transferred.

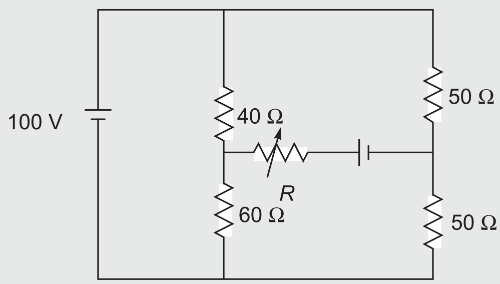


Fig. 2.575 **[Dec 2012]**

Solution

Step I: Calculation of V_{Th}

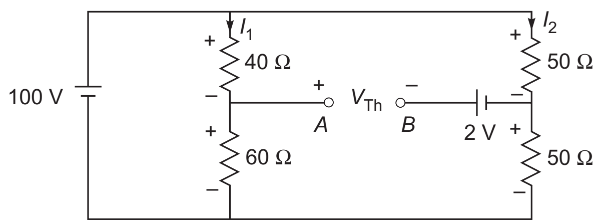


Fig. 2.576

$$I_1 = \frac{100}{40+60} = 1\ \mathrm{A}$$

$$I_2 = \frac{100}{50+50} = 1\ \mathrm{A}$$

Writing V_{Th} equation,

$$-40I_1 + V_{\text{Th}} - 2 + 50I_2 = 0$$
$$-40(1) + V_{\text{Th}} - 2 + 50(1) = 0$$
$$V_{\text{Th}} = -8 \text{ V}$$
$$= 8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

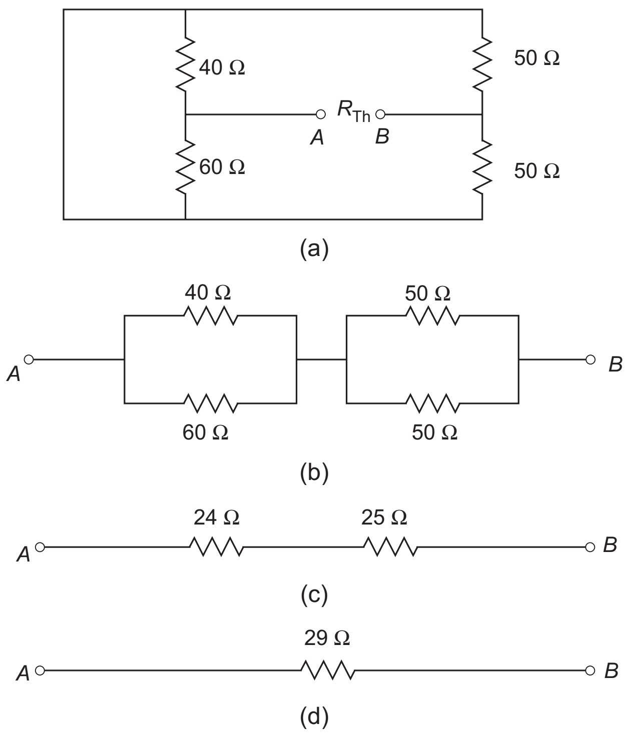


Fig. 2.577

$$R_{\text{Th}} = 49\ \Omega$$

Step III: Value of R

For maximum power transfer

$$R = R_{\text{Th}} = 49\ \Omega$$

Step IV: Calculation of P_{max}

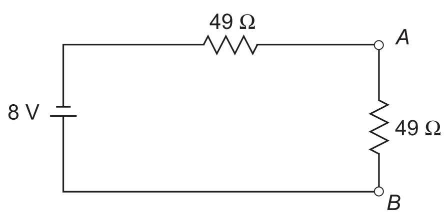


Fig. 2.578

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8)^2}{4\times 49} = 0.33 \text{ W}$$

Example 14

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

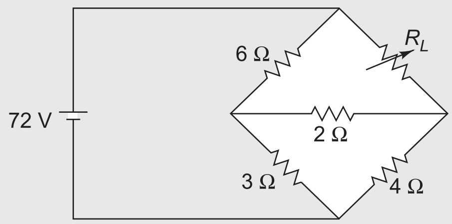


Fig. 2.579

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

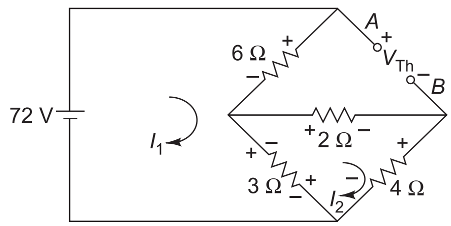


Fig. 2.580

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3\,(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 72 \quad (1)$$

Applying KVL to Mesh 2,

$$-3\,(I_2 - I_1) - 2I_2 - 4I_2 = 0$$

$$-3I_1 + 9I_2 = 0 \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 9 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 6I_1 - 2I_2 = 0$$

$$V_{\text{Th}} = 6I_1 + 2I_2 = 6\,(9) + 2\,(3) = 60 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit,

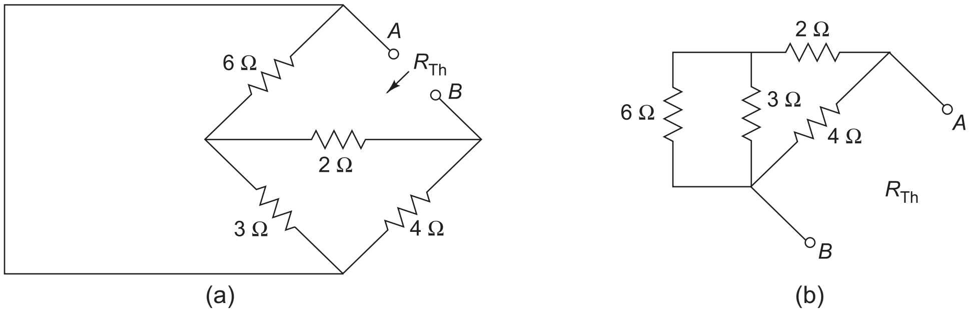


Fig. 2.581

$$R_{Th} = [(6 \,||\, 3) + 2] \,||\, 4 = 2\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 2\ \Omega$$

Step IV: Calculation of P_{max}

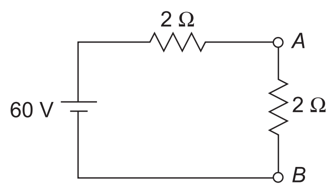


Fig. 2.582

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 2} = 450\ \text{W}$$

Example 15

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

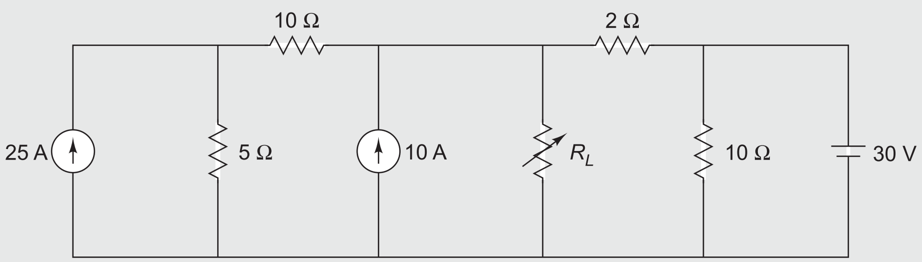


Fig. 2.583

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

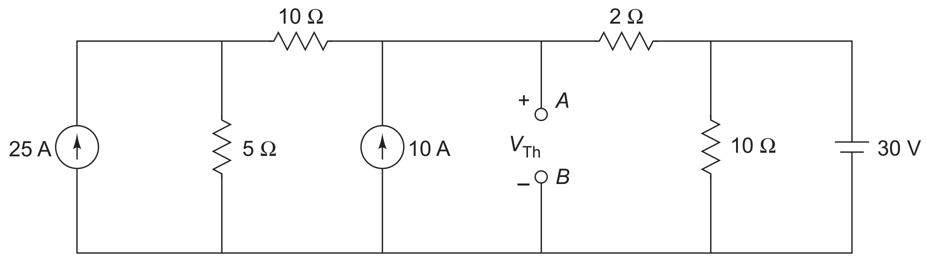


Fig. 2.584

By source transformation, the current source of 25 A and the 5 Ω resistor is converted into an equivalent voltage source of 125 V and a series resistor of 5 Ω. Also the voltage source of 30 V is connected across the 10 Ω resistor. Hence, the 10 Ω resistor becomes redundant.

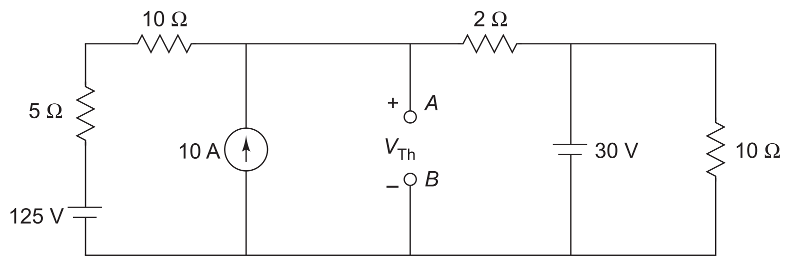


Fig. 2.585

Applying KCL at node,

$$\frac{V_{\text{Th}} - 125}{15} - 10 + \frac{V_{\text{Th}} - 30}{2} = 0$$

$$V_{\text{Th}} = 58.81 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current sources by open circuits,

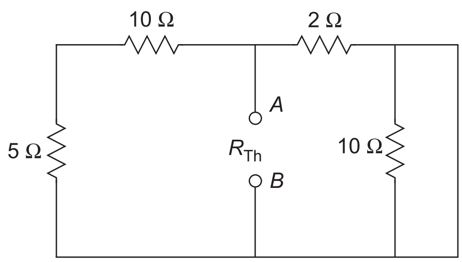


Fig. 2.586

Simplifying the network,

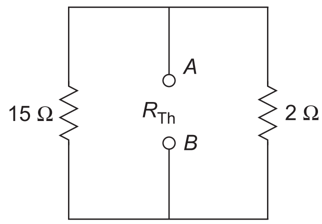


Fig. 2.587

$$R_{Th} = 15 \parallel 2 = 1.76\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 1.76\ \Omega$$

Step IV: Calculation of P_{max}

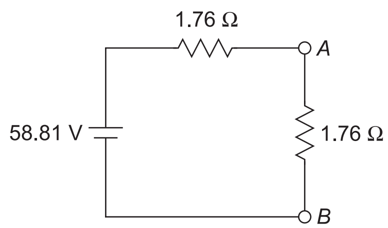


Fig. 2.588

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(58.81)^2}{4 \times 1.76} = 491.28\ \text{W}$$

Example 16

Find the value of R_L for maximum power transfer and calculate maximum power.

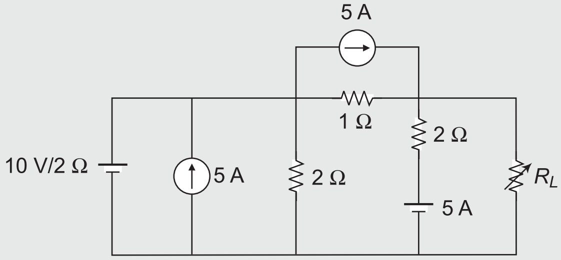


Fig. 2.589 **[Dec 2015]**

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

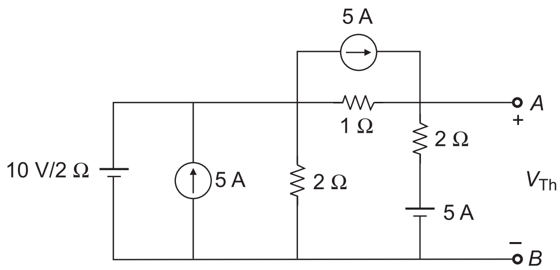


Fig. 2.590

By source transformation, the current source of 5 A and parallel resistor of 2 Ω is converted into an equivalent voltage source of 10 V and series resistor of 2 Ω. Similarly, the other current source of 5 A and parallel resistor of 1 Ω is converted into an equivalent voltage source of 5 V and series resistor of 1 Ω.

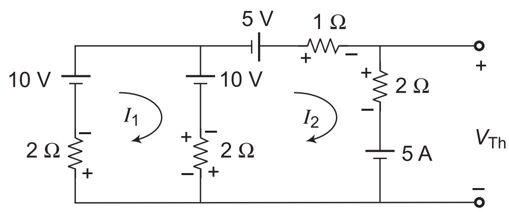


Fig. 2.591

Applying KVL to Mesh 1,

$$-2I_1 + 10 - 10 - 2(I_1 - I_2) = 0$$

$$4I_1 - 2I_2 = 0 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) + 10 + 5 - 1I_2 - 2I_2 - 5 = 0$$

$$-2I_1 + 5I_2 = 10 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1.25 \text{ A}$$

$$I_2 = 2.5 \text{ A}$$

Writing V_{Th} equation,

$$5 + 2I_2 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 5 + 2I_2$$

$$= 5 + 2\,(2.5)$$

$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current sources by open circuits,

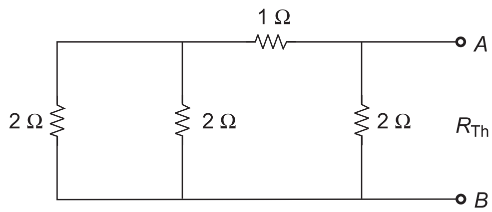


Fig. 2.592

By series-parallel reduction technique,

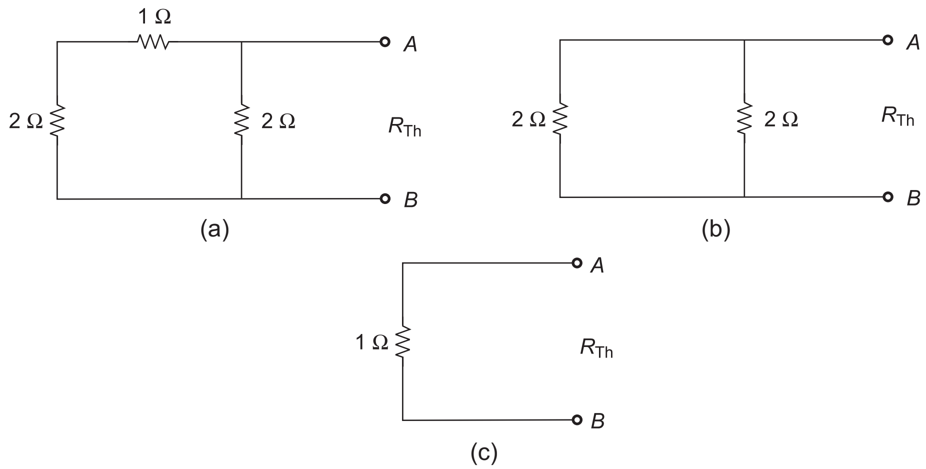


Fig. 2.593

$$R_{Th} = 1\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 1\ \Omega$$

Step IV: Calculation of P_{max}

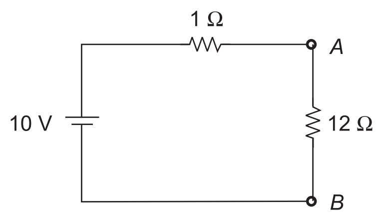


Fig. 2.594

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(10)^2}{4\times 1} = 25 \text{ W}$$

Example 17

For the given circuit, find the value of 'R_L' so that maximum power is dissipated in it. Also, find P_{max}.

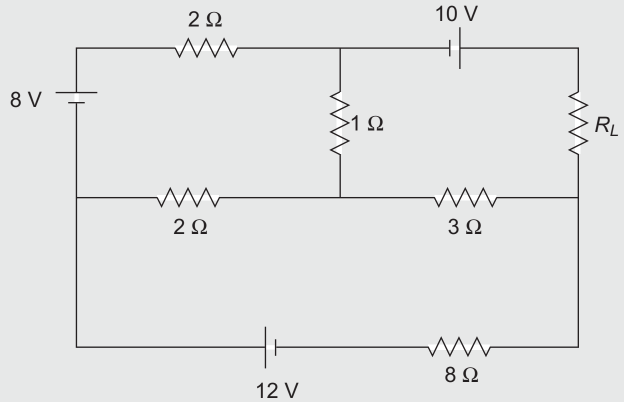


Fig. 2.595 **[Dec 2013]**

Solution

Step I: Calculation of V_{Th}

Removing the resistor R_L from the network,

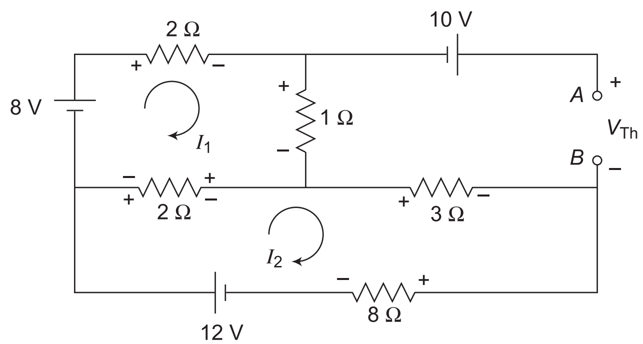


Fig. 2.596

Applying KVL to Mesh 1,

$$8 - 2I_1 - 1I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 8 \quad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 3I_2 - 8I_2 + 12 = 0$$

$$-2I_1 + 13I_2 = 12 \quad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = 2.1 \text{ A}$$
$$I_2 = 1.25 \text{ A}$$

Writing V_{Th} equation,

$$1I_1 + 10 - V_{\text{Th}} + 3I_2 = 0$$
$$V_{\text{Th}} = 1I_1 + 10 + 3I_2$$
$$= 1(2.1) + 10 + 3(1.25)$$
$$= 15.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

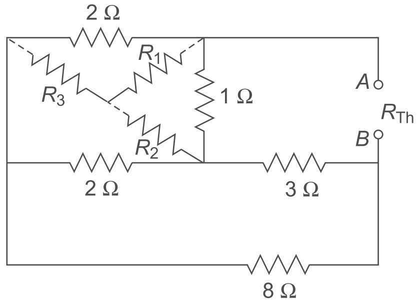


Fig. 2.597

Converting the delta network formed by resistors of 2 Ω, 1 Ω and 2 Ω into equivalent star network,

$$R_1 = \frac{2\times 1}{2+1+2} = 0.4\,\Omega$$
$$R_2 = \frac{2\times 1}{2+1+2} = 0.4\,\Omega$$
$$R_3 = \frac{2\times 2}{2+1+2} = 0.8\,\Omega$$

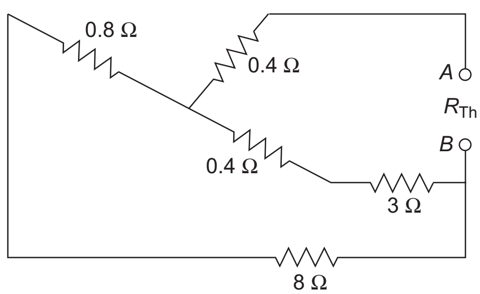


Fig. 2.598

Simplifying the network,

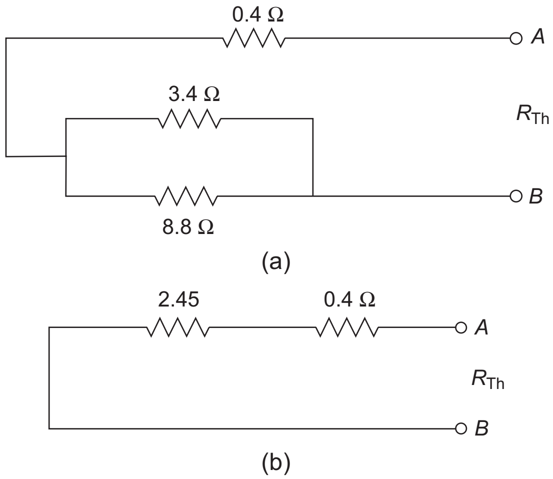


Fig. 2.599

$$R_{Th} = 2.85\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 2.85\ \Omega$$

Step IV: Calculation of P_{max}

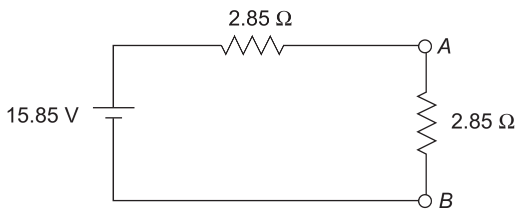


Fig. 2.600

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(15.85)^2}{4 \times 2.85} = 22.04\ \text{W}$$

Example 18

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

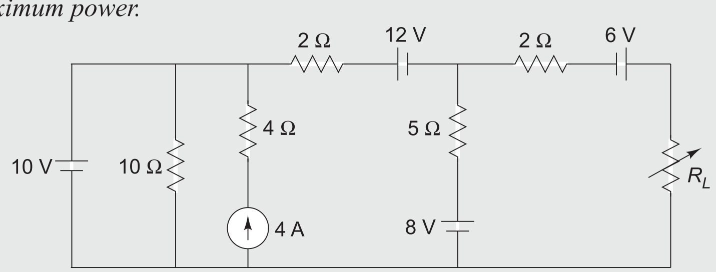


Fig. 2.601

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

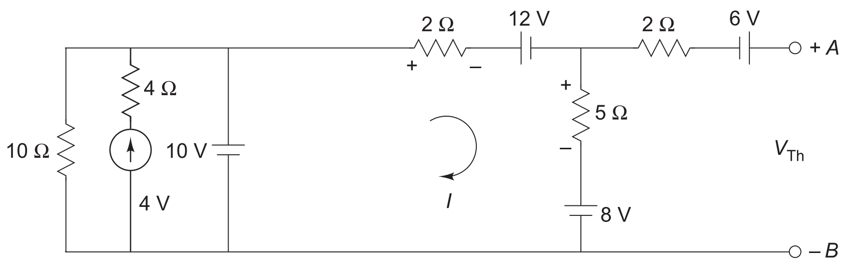


Fig. 2.602

Applying KVL to the outer path,

$$10 - 2I - 12 - 5I - 8 = 0$$

$$I = -\frac{10}{7} = -1.43 \text{ A}$$

Writing V_{Th} equation,

$$8 + 5I + 6 - V_{Th} = 0$$

$$V_{Th} = 8 + 6 + 5I$$

$$= 8 + 6 + 5(-1.43)$$

$$= 6.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits and current source by an open circuit,

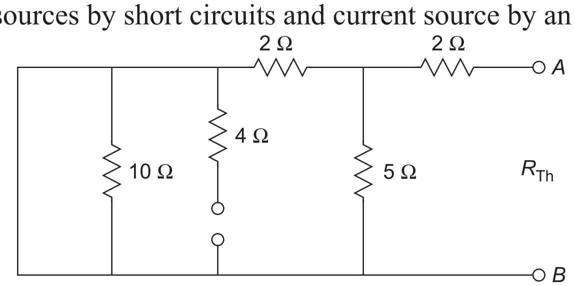


Fig. 2.603

$$R_{Th} = (2 \parallel 5) + 2$$
$$= 3.43\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.43\ \Omega$$

Step IV: Calculation of P_{max}

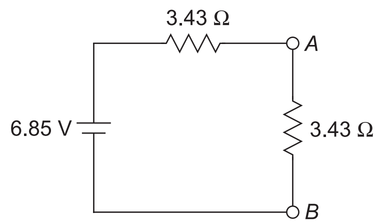


Fig. 2.604

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(6.85)^2}{4 \times 3.43} = 3.42\ \text{W}$$

Exercise 2.9

2.1 Find the value of the resistance R_L for maximum power transfer and calculate maximum power.

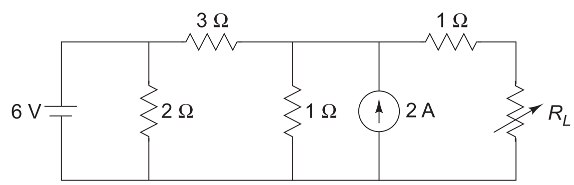


Fig. 2.605

[1.75 Ω, 1.29 W]

2.2 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

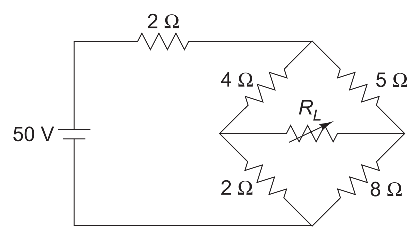


Fig. 2.606

[4.51 Ω, 4.95 W]

2.3 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

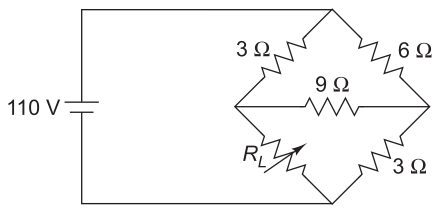


Fig. 2.607

[2.36 Ω, 940 W]

2.4 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

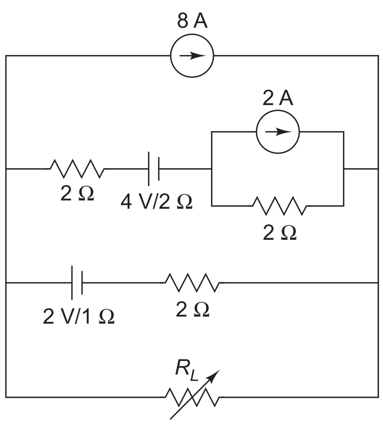


Fig. 2.608

[2.18 Ω, 29.35 W]

2.5 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

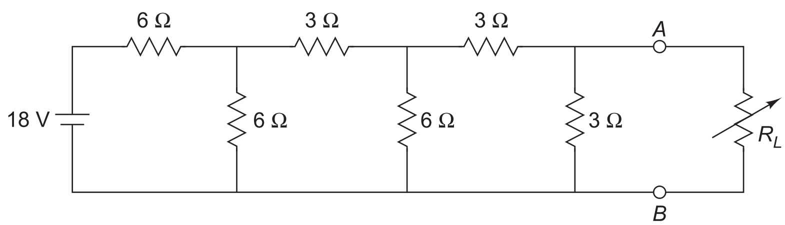


Fig. 2.609

[2 Ω, 0.281 W]

2.6 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

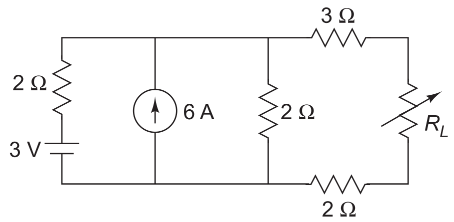


Fig. 2.610

[6 Ω, 2.52 W]

2.7 Find the value of the resistance R_L for maximum power transfer and calculate maximum power.

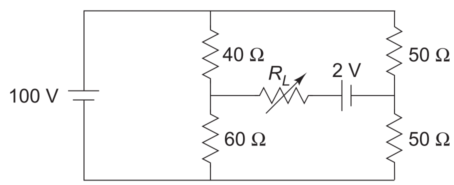


Fig. 2.611

[49 Ω, 0.32 W]

Review Questions

2.1 State and explain Kirchhoff's voltage and current law.
2.2 Define delta network and star network and derive the formulae to convert a delta network into its equivalent star network.
2.3 Derive the formulae to convert star connected network into its equivalent delta connected network.
2.4 State and explain superposition theorem.
2.5 State and explain Thevenin's theorem.
2.6 State and explain Norton's theorem.
2.7 State and prove maximum power transfer theorem.

Multiple Choice Questions

Choose the correct alternative in the following questions:

2.1 The nodal method of circuit analysis is based on
(a) KVL and Ohm's law (b) KCL and Ohm's law
(c) KCL and KVL (d) KCL, KVL and Ohm's law

2.2 A network contains only an independent current source and resistors. If the values of all resistors are doubled, the value of the node voltages will

(a) become half (b) remain unchanged
(c) become double (d) none of these

2.3 Superposition theorem is not applicable to networks containing

(a) nonlinear elements (b) dependent voltage source
(c) dependent current source (d) transformers

2.4 The value of R required for maximum power transfer in the network shown in Fig. 2.612 is

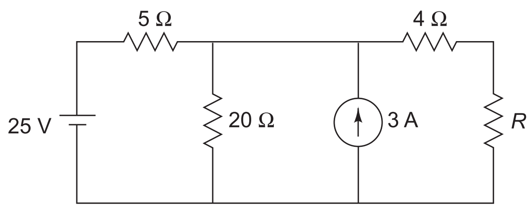


Fig. 2.612

(a) 2 Ω (b) 4 Ω (c) 8 Ω (d) 16 Ω

2.5 The maximum power that can be transferred to the load R_L from the voltage source in Fig. 2.613 is

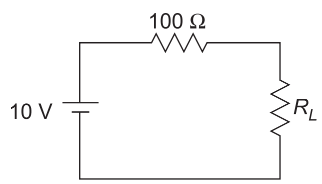


Fig. 2.613

(a) 1 W (b) 10 W (c) 0.25 W (d) 0.5 W

2.6 The value of R_L for maximum power transfer is

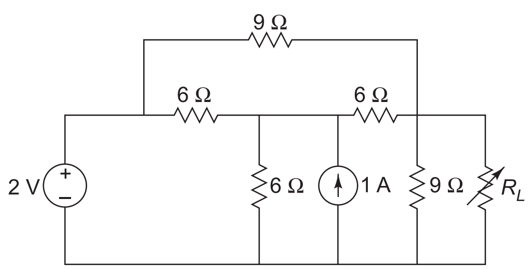


Fig. 2.614

(a) 3 Ω (b) 1.125 Ω (c) 4.1785 Ω (d) none of these

2.7 The Thevenin impedance across the terminal *AB* of Fig. 2.615 is

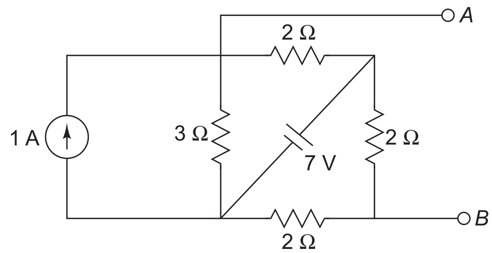


Fig. 2.615

(a) 2.2 Ω (b) $\frac{20}{9}$ Ω (c) 9 Ω (d) $\frac{11}{5}$ Ω

2.8 The Thevenin equivalent of the network shown in Fig. 2.616(a) is 10 V in series with a resistance of 2 Ω. If now, a resistance of 3 Ω is connected across *AB* as shown in Fig. 2.616(b), the Thevenin equivalent of the modified network across *AB* will be

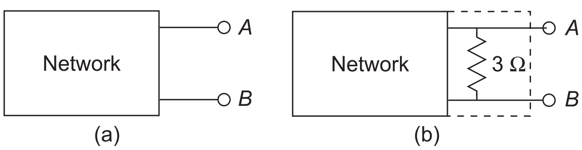


Fig. 2.616

(a) 10 V in series with 1.2 Ω resistance
(b) 6 V in series with 1.2 Ω resistance
(c) 10 V in series with 5 Ω resistance
(d) 6 V in series with 5 Ω resistance

2.9 The current I_4 in the circuit of Fig. 2.617 is equal to

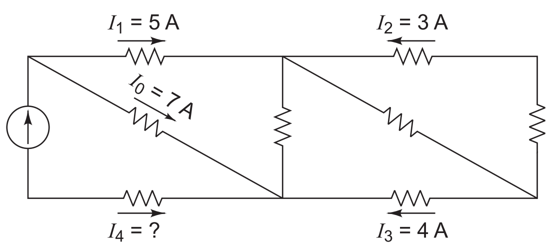


Fig. 2.617

(a) 12 A (b) –12 A (c) 4 A (d) none

2.10 The voltage V in Fig. 2.618 is equal to

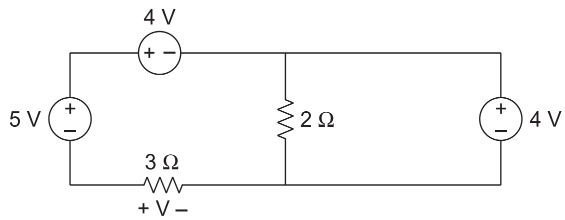


Fig. 2.618

(a) 3 V (b) –3 V (c) 5 V (d) none

2.11 The voltage V in Fig. 2.619 is equal to

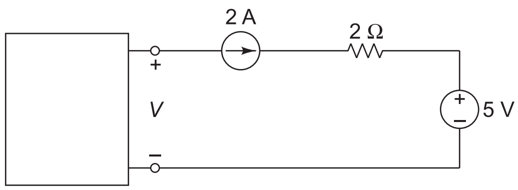


Fig. 2.619

(a) 9 V (b) 5 V (c) 1 V (d) none

2.12 The voltage V in Fig. 2.620 is

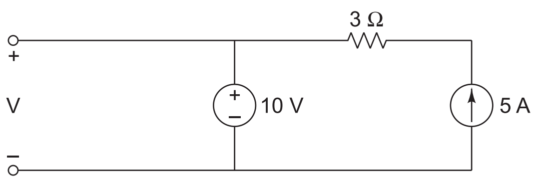


Fig. 2.620

(a) 10 V (b) 15 V (c) 5 V (d) none

2.13 In the circuit of Fig. 2.621, the value of the voltage source E is

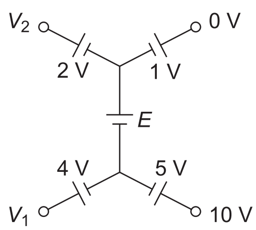


Fig. 2.621

(a) –16 V (b) 4 V (c) –6 V (d) 16 V

2.14 The voltage V_0 in Fig 2.622 is

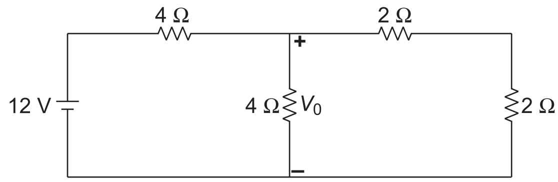


Fig. 2.622

(a) 2 V (b) $\frac{4}{3}$ V (c) 4 V (d) 8 V

2.15 If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1\ R$ in the bridge circuit shown in Fig. 2.623, then the reading in the ideal voltmeter connected between a and b is

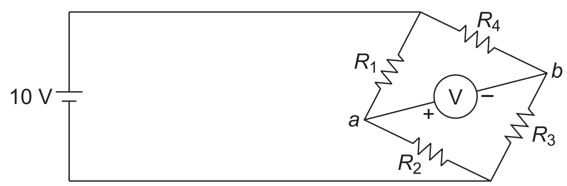


Fig. 2.623

(a) 0.238 V (b) 0.138 V (c) – 0.238 V (d) 1 V

2.16 The voltage across terminals a and b in Fig. 2.624 is

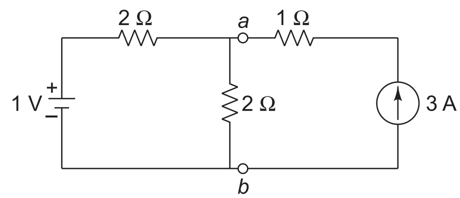


Fig. 2.624

(a) 0.5 V (b) 3 V (c) 3.5 V (d) 4 V

2.17 A delta-connected network with its wye-equivalent is shown in Fig. 2.625. The resistors R_1, R_2 and R_3 (in ohms) are respectively

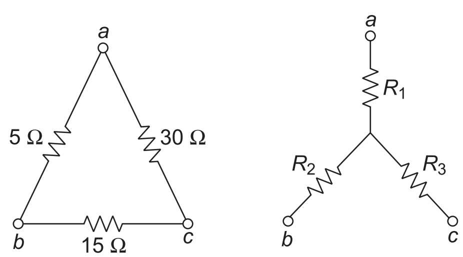


Fig. 2.625

(a) 1.5, 3 and 9 (b) 3, 9 and 1.5
(c) 9, 3 and 1.5 (d) 3, 1.5 and 9

2.18 If each branch of a delta circuit has resistance $\sqrt{3}R$, then each branch of the equivalent type circuit has resistance

(a) $\dfrac{R}{\sqrt{3}}$ (b) 3R (c) $3\sqrt{3}\,R$ (d) $\dfrac{R}{3}$

2.19 The voltage V_0 in the Fig. 2.626 is

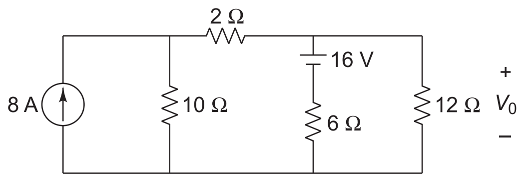


Fig. 2.626

(a) 48 V (b) 24 V (c) 36 V (d) 28 V

2.20 If $V = 4$ in Fig. 2.627 the value of I_S is given by

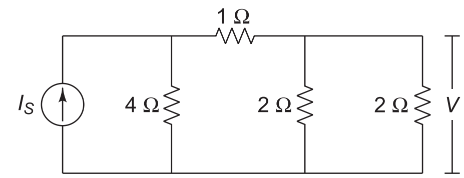


Fig. 2.627

(a) 6 A (b) 2.5 A (c) 12 A (d) none of these

2.21 The value of V_x, V_y and V_z in Fig. 2.628 shown are

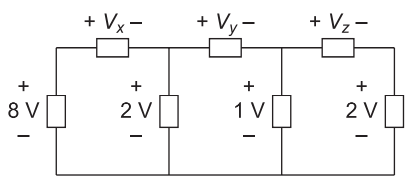


Fig. 2.628

(a) –6, 3, –3 (b) –6, –3, 1 (c) 6, 3, 3 (d) 6, 1, 3

2.22 Viewed from the terminal *AB*, the following circuit can be reduced to an equivalent circuit of a single voltage source in series with a single resistor with the following parameters:

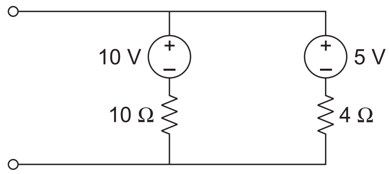


Fig. 2.629

(a) 5 volt source in series with a 10 Ω resistor
(b) 1 volt source in series with a 2.4 Ω resistor
(c) 15 volt source in series with a 2.4 Ω resistor
(d) 1 volt source in series with a 10 Ω resistor

2.23 Consider the star network shown in Fig. 2.630. The resistance between terminals *A* and *B* with *C* open is 6 Ω, between terminals *B* and *C* with *A* open is 11 Ω and between terminals *C* and *A* with *B* open is

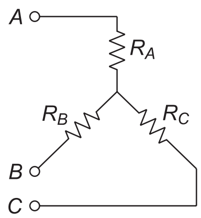


Fig. 2.630

(a) $R_A = 4\ \Omega,\quad R_B = 2\ \Omega,\quad R_C = 5\ \Omega$
(b) $R_A = 2\ \Omega,\quad R_B = 4\ \Omega,\quad R_C = 7\ \Omega$
(c) $R_A = 3\ \Omega,\quad R_B = 3\ \Omega,\quad R_C = 4\ \Omega$
(d) $R_A = 5\ \Omega,\quad R_B = 1\ \Omega,\quad R_C = 10\ \Omega$

2.24 In Fig. 2.631, the value of *R* is

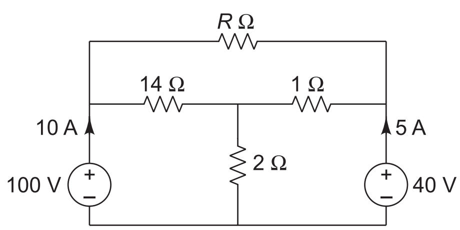
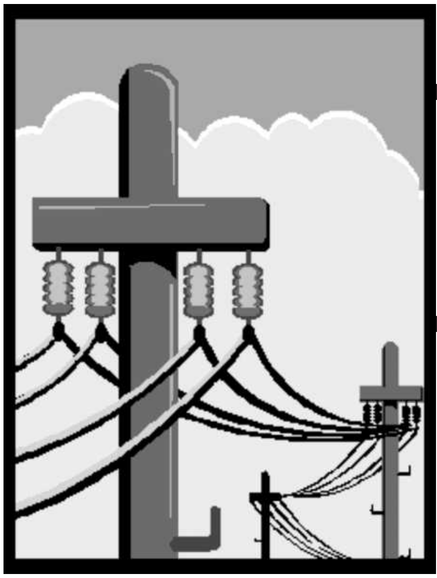


Fig. 2.631

(a) 10 Ω (b) 18 Ω (c) 24 Ω (d) 12 Ω

Answers to Multiple Choice Questions

2.1 (b)	**2.2** (c)	**2.3** (a)	**2.4** (c)	**2.5** (c)	**2.6** (a)
2.7 (a)	**2.8** (b)	**2.9** (b)	**2.10** (a)	**2.11** (d)	**2.12** (a)
2.13 (a)	**2.14** (c)	**2.15** (c)	**2.16** (c)	**2.17** (d)	**2.18** (a)
2.19 (d)	**2.20** (a)	**2.21** (d)	**2.22** (b)	**2.23** (b)	**2.24** (d)



Chapter 3

AC Fundamentals

Chapter Outline

3.1 GENERATION OF ALTERNATING VOLTAGES

An alternating voltage can be generated either by rotating a coil in a stationary magnetic field or by a rotating a magnetic field within a stationary coil. In both the cases, the magnetic field is cut by the conductors or coils and an emf is induced in the coil according to Faraday's laws of electromagnetic induction. The magnitude of the induced emf depends upon the number of turns of the coil, the strength of the magnetic field and the speed at which the coil or magnetic field rotates.

Consider a rectangular coil of N turns of area A m^2 and rotating in anti-clockwise direction with angular velocity of ω radians per second in a uniform magnetic field as shown in Fig. 3.1(a).

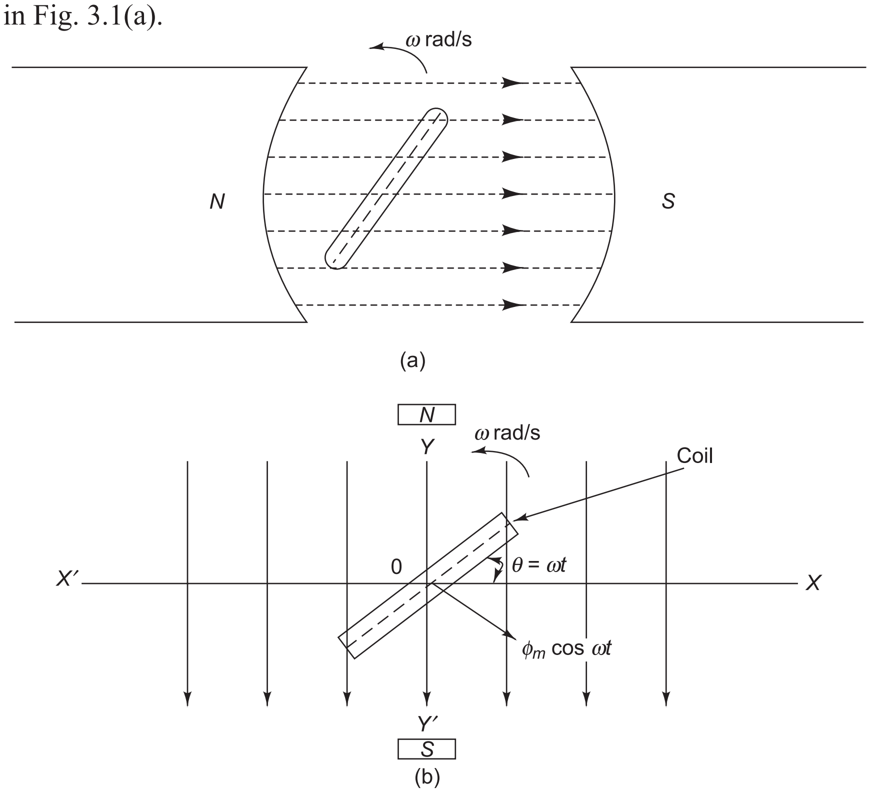


Fig. 3.1 *Generation of alternating voltage*

Let ϕ_m be the maximum flux cutting the coil when its axis coincides with the XX' axis (reference position of the coil). Thus when the coil is along XX', the flux linking with it is maximum, i.e., ϕ_m. When the coil is along YY', i.e., parallel to the lines of flux, the flux linking with it is zero.

The coil rotates through an angle $\theta = \omega t$ at any instant t.

At this instant, the flux linking with the coil is

$$\phi = \phi_m \cos \omega t$$

According to Faraday's laws of electromagnetic induction,

$$e = -N\frac{d\phi}{dt}$$
$$= -N\frac{d}{dt}(\phi_m \cos\omega t)$$
$$= N\,\phi_m\,\omega \sin\,\omega t$$
$$= E_m \sin\,\omega t$$

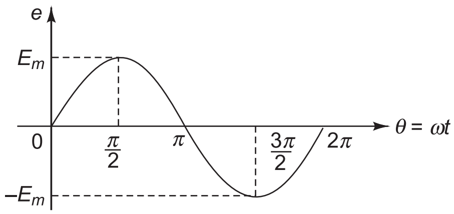


Fig. 3.2 *Sinusoidal waveform*

where $E_m = N\,\phi_m\omega$

$= $ maximum value of induced emf

When $\omega t = 0, \quad \sin\omega t = 0, \quad e = 0$

When $\omega t = \dfrac{\pi}{2}, \quad \sin\dfrac{\pi}{2} = 1 \quad e = E_m$

If the induced emf is plotted against time, a sinusoidal waveform is obtained.

3.2 TERMS RELATED TO ALTERNATING QUANTITIES

[May 2015]

Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Figure 3.3 shows a few waveforms.

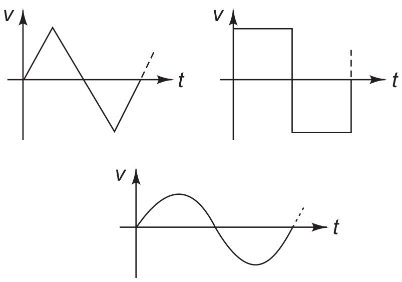


Fig. 3.3 *Alternating waveforms*

Cycle One complete set of positive and negative values of an alternating quantity is termed a cycle.

Frequency The number of cycles per second of an alternating quantity is known as its frequency. It is denoted by f and is measured in hertz (Hz) or cycles per second (c/s).

Time Period The time taken by an alternating quantity to complete one cycle is called its time period. It is denoted by T and is measured in seconds.

$$T = \frac{1}{f}$$

Amplitude The maximum positive or negative value of an alternating quantity is called the amplitude.

Phase The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.

Phase Difference This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier compared to the other quantity.

A lagging alternating quantity is one which attains its maximum or zero value later than the other quantity.

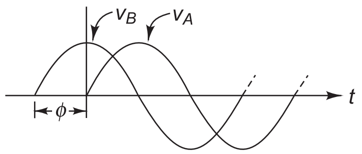


Fig. 3.4 *Phase difference*

A plus (+) sign, when used in connection with the phase difference, denotes 'lead' whereas a minus (–) sign denotes 'lag'.

$$v_A = V_m \sin \omega t$$

$$v_B = V_m \sin (\omega t + \phi)$$

Here, the quantity B leads A by a phase angle ϕ.

3.3 ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Normally, the current is measured by the amount of work it will do or the amount of heat it will produce. Hence, rms or effective value of alternating current is defined as that value of steady current (direct current) which will do the same amount of work in the same time or would produce the same heating effect as when the alternating current is applied for the same time.

Figure 3.5 shows the positive half cycle of a non-sinusoidal alternating current waveform. The waveform is divided in m equal intervals with the instantaneous currents, these intervals being $i_1, i_2, ..., i_m$. This waveform is applied to a circuit consisting of a resistance of R ohms.

Then work done in different intervals will be $\left(i_1^2 R \times \frac{t}{m}\right), \left(i_2^2 R \times \frac{t}{m}\right), ..., \left(i_m^2 R \times \frac{t}{m}\right)$ joules.

Thus, the total work done in t seconds on applying an alternating current waveform to a resistance $R = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$ joules

Let I be the value of the direct current that while flowing through the same resistance does the same amount of work in the same time t. Then

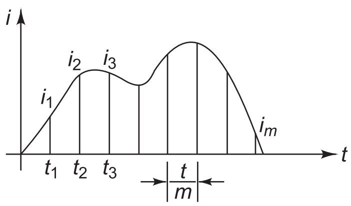


Fig. 3.5 *Mid-ordinate method*

$$I^2 Rt = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$$

$$I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}$$

Hence, rms value of the alternating current is given by

$$I_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}} = \sqrt{\text{Mean value of}\,(i)^2}$$

The rms value of any current $i(t)$ over the specified interval t_1 to t_2 is expressed mathematically as

$$I_{\text{rms}} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2(t)\,dt}$$

The rms value of an alternating current is of considerable importance in practice because the ammeters and voltmeters record the rms value of alternating current and voltage respectively.

3.3.1 RMS Value of Sinusoidal Waveform

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

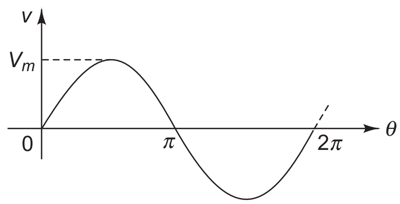


Fig. 3.6 *Sinusoidal waveform*

$$\begin{aligned}
V_{\text{rms}} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta} \\
&= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\,d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}} \\
&= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]} \\
&= \sqrt{\frac{V_m^2}{2}} \\
&= \frac{V_m}{\sqrt{2}} \\
&= 0.707\,V_m
\end{aligned}$$

Crest or Peak or Amplitude Factor It is defined as the ratio of maximum value to rms value of the given quantity.

$$\text{Peak factor } (k_p) = \frac{\text{Maximum value}}{\text{rms value}}$$

3.4 AVERAGE VALUE

The average value of an alternating quantity is defined as the arithmetic mean of all the values over one complete cycle.

In case of a symmetrical alternating waveform (whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in such a case, the average value is obtained over half the cycle only.

Referring to Fig. 3.5, the average value of the current is given by

$$I_{avg} = \frac{i_1 + i_2 + \ldots + i_m}{m}$$

The average value of any current $i(t)$ over the specified interval t_1 to t_2 is expressed mathematically as

$$I_{avg} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i(t)\,dt$$

3.4.1 Average Value of Sinusoidal Waveform [Dec 2013]

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

Since this is a symmetrical waveform, the average value is calculated over half the cycle.

$$\begin{aligned}
V_{avg} &= \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta \\
&= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\ d\theta \\
&= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\ d\theta \\
&= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi} \\
&= \frac{V_m}{\pi}[1+1] \\
&= \frac{2V_m}{\pi} \\
&= 0.637\ V_m
\end{aligned}$$

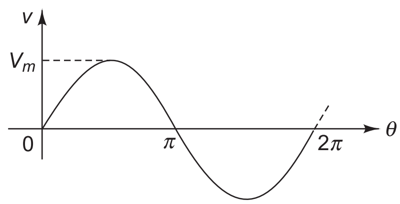


Fig. 3.7 *Sinusoidal waveform*

Form Factor It is defined as the ratio of rms value to the average value of the given quantity.

$$\text{Form factor } (k_f) = \frac{\text{rms value}}{\text{Average value}}$$

Example 1

An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of the current is 40 Hz. Find the maximum value of the alternating current. **[May 2014]**

Solution

$$i = 15 \text{ A}$$
$$t = 3.375 \text{ ms}$$
$$f = 40 \text{ Hz}$$
$$i = I_m \sin 2\pi ft$$
$$15 = I_m \sin (2 \times 180 \times 40 \times 3.375 \times 10^{-3}) \qquad \text{(angle in degrees)}$$
$$15 = I_m \times 0.75$$
$$I_m = 20 \text{ A}$$

Example 2

An alternating current of 50 c/s frequency has a maximum value of 100 A. (i) Calculate its value $\frac{1}{600}$ second and after the instant the current is zero. (ii) In how many seconds after the zero value will the current attain the value of 86.6 A?

Solution

$$f = 50 \text{ c/s}$$
$$I_m = 100 \text{ A}$$

(i) Value of current $\frac{1}{600}$ second after the instant the current is zero

$$i = I_m \sin 2\pi ft$$
$$= 100 \sin \left(2 \times 180 \times 50 \times \frac{1}{600}\right) \qquad \text{(angle in degrees)}$$
$$= 100 \sin (30^\circ)$$
$$= 50 \text{ A}$$

(ii) Time at which current will attain the value of 86.6 A after the zero value

$$i = I_m \sin 2\pi ft$$
$$86.6 = 100 \sin (2 \times 180 \times 50 \times t) \qquad \text{(angle in degrees)}$$
$$\sin (18000\, t) = 0.866$$
$$18000\, t = 60^\circ$$
$$t = \frac{1}{300} \text{ second}$$

Example 3

An alternating current varying sinusoidally with a frequency of 50 c/s has an rms value of 20 A. Write down the equation for the instantaneous value and find this value at (i) 0.0025 s, and (ii) 0.0125 s after passing through zero and increasing positively. (iii) At what time, measured from zero, will the value of the instantaneous current be 14.14 A?

Solution $f = 50$ c/s

$$I_{rms} = 20 \text{ A}$$

$$I_m = I_{rms} \times \sqrt{2} = 20\sqrt{2} = 28.28 \text{ A}$$

Equation of current, $i = I_m \sin 2\pi f t$

$$= 28.28 \sin (100\pi \times t)$$

$$= 28.28 \sin (100 \times 180 \times t) \qquad \text{(angle in degrees)}$$

(i) Value of current at $t = 0.0025$ second

$$i = 28.28 \sin (100 \times 180 \times 0.0025) \qquad \text{(angle in degrees)}$$

$$= 28.28 \sin (45^\circ)$$

$$= 20 \text{ A}$$

(ii) Value of current at $t = 0.0125$ second

$$i = 28.28 \sin (100 \times 180 \times 0.0125) \qquad \text{(angle in degrees)}$$

$$= 28.28 \sin (225^\circ)$$

$$= -20 \text{ A}$$

(iii) Time at which value of instantaneous current will be 14.14 A

$$i = 28.28 \sin 100\pi t$$

$$14.14 = 28.28 \sin 18000\, t \qquad \text{(angle in degrees)}$$

$$\sin 18000\, t = 0.5$$

$$18000\, t = 30^\circ$$

$$t = 1.66 \text{ ms}$$

Example 4

An alternating current of 60 Hz frequency has a maximum value of 110 A. Calculate (i) time required to reach 90 A after the instant current is zero and increasing positively, and (ii) its value $\frac{1}{600}$ second after the instant current is zero and its value decreasing thereafter.

Solution $f = 60$ Hz

$I_m = 110$ A

(i) Time required to reach 90 A after the instant current is zero and increasing positively.

$$i = I_m \sin 2\pi ft$$

$$90 = 110 \sin (2 \times 180 \times 60 \times t) \qquad \text{(angle in degrees)}$$

$$\sin 21600\, t = 0.818$$

$$21600\, t = 54.88°$$

$$t = 2.54 \text{ ms}$$

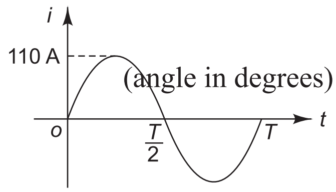


Fig. 3.8

(ii) Value of current $\dfrac{1}{600}$ second after the instant current is zero and decreasing thereafter.

From Fig. 3.8,

$$t = \frac{T}{2} + \frac{1}{600}$$

$$= \frac{1}{2f} + \frac{1}{600}$$

$$= \frac{1}{2 \times 60} + \frac{1}{600}$$

$$= 0.01 \text{ s}$$

$$i = I_m \sin 2\pi ft$$

$$= 110 \sin (2 \times 180 \times 60 \times 0.01) \qquad \text{(angle in degrees)}$$

$$= -64.66 \text{ A}$$

Example 5

A sinusoidal wave of 50 Hz frequency has its maximum value of 9.2 A. What will be its value at (i) 0.002 s after the wave passes through zero in the positive direction, and (ii) 0.0045 s after the wave passes through the positive maximum.

Solution $f = 50$ Hz

$I_m = 9.2$ A

(i) Value of current at 0.002 s after the wave passes through zero in the positive direction

$$i = I_m \sin 2\pi ft$$

$$= 9.2 \sin (2 \times 180 \times 50 \times 0.002) \qquad \text{(angle in degrees)}$$

$$= 5.41 \text{ A}$$

(ii) Value of current 0.0045 s after the wave passes through the positive maximum

From Fig. 3.9,

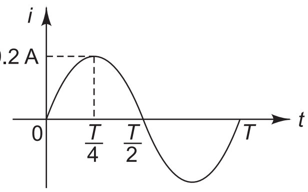


Fig. 3.9

$$t = \frac{T}{4} + 0.0045$$

$$= \frac{1}{4f} + 0.0045$$

$$= \frac{1}{4 \times 50} + 0.0045$$

$$= 9.5 \text{ ms}$$

$$i = I_m \sin 2\pi ft$$

$$= 9.2 \sin (2 \times 180 \times 50 \times 9.5 \times 10^{-3}) \quad \text{(angle in degrees)}$$

$$= 1.44 \text{ A}$$

Example 6

An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of current of 20 A. At what time measured from negative maximum value will the instantaneous current be $10\sqrt{2}$ A?

Solution

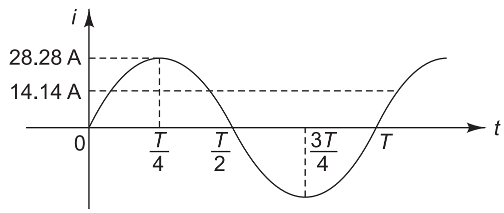


Fig. 3.10

$$f = 50 \text{ Hz}$$

$$I_{\text{rms}} = 20 \text{ A}$$

(i) Time at which instantaneous current will be $10\sqrt{2}$ A

$$i = 10\sqrt{2} \text{ A} = 14.14 \text{ A}$$

$$I_m = I_{\text{rms}} \times \sqrt{2} = 20\sqrt{2} = 28.28 \text{ A}$$

$$i = I_m \sin 2\pi ft$$

$$14.14 = 28.28 \sin (2 \times 180 \times 50 \times t) \quad \text{(angle in degrees)}$$

$$0.5 = \sin (18000\, t)$$

$$18000\, t = 30°$$

$$t = 1.67 \text{ ms}$$

(ii) Time, measured from negative maximum value, at which instantaneous current will be $10\sqrt{2}$ A

$$t = \frac{T}{4} + 1.67 \times 10^{-3}$$

$$= \frac{1}{4f} + 1.67 \times 10^{-3}$$

$$= \frac{1}{4 \times 50} + 1.67 \times 10^{-3}$$
$$= 6.67 \text{ ms}$$

Example 7

An alternating voltage is represented by v = 141.4 sin 377t. Find (i) max-value (ii) frequency (iii) time period. **[May 2016]**

Solution $v = 141.4 \sin 377\, t$

(i) Maximum value

Comparing with the equation $v = V_m \sin 2\pi ft$,

$$V_m = 141.4 \text{ V}$$

(ii) Frequency

$$2\pi f = 377$$
$$f = \frac{377}{2\pi} = 60 \text{ Hz}$$

(iii) Time period

$$T = \frac{1}{f} = \frac{1}{60} = 16.67 \text{ ms}$$

Example 8

An alternating voltage is given by v = 141.4 sin 314t. Find (i) frequency, (ii) rms value, (iii) average value, and (iv) instantaneous value of voltage, when t is 3 ms. **[Dec 2012]**

Solution $v = 141.4 \sin 314\, t$

(i) Frequency

Comparing with the equation $v = V_m \sin 2\pi ft$,

$$2\pi f = 314$$
$$f = \frac{314}{2\pi} = 49.97 \text{ Hz}$$

(ii) rms value

$$V_m = 141.4 \text{ V}$$
$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = 99.98 \text{ V}$$

(iii) Average value

$$V_{\text{avg}} = \frac{2V_m}{\pi} = \frac{2 \times 141.4}{\pi} = 90.02 \text{ V}$$

(iv) Instantaneous value of the voltage at $t = 3$ ms

$$v = 141.4 \sin 314 \times 3 \times 10^{-3} = 114.36 \text{ V}$$

Example 9

An alternating current is given by i = 14.14 sin 377 t. Find (i) rms value of the current, (ii) frequency, (iii) instantaneous value of the current when t = 3 ms, and (iv) time taken by the current to reach 10 A for first time after passing through zero.

Solution $i = 14.14 \sin 377\, t$

(i) The rms value of the current

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

(ii) Frequency

$$2\pi f = 377$$

$$f = \frac{377}{2\pi} = 60 \text{ Hz}$$

(iii) Instantaneous value of the current when $t = 3$ ms

$$i = 14.14 \sin (377 \times 3 \times 10^{-3}) \qquad \text{(angle in radians)}$$

$$= 12.79 \text{ A}$$

(iv) Time taken by the current to reach 10 A for the first time after passing through zero

$$i = 14.14 \sin 377\, t \qquad \text{(angle in radians)}$$

$$10 = 14.14 \sin 377\, t$$

$$\sin 377\, t = 0.707$$

$$377\, t = 0.79$$

$$t = 2.084 \text{ ms}$$

Example 10

An alternating current varying sinusoidally at 50 Hz has its rms value of 10 A. Write down an equation for the instantaneous value of the current. Find the value of the current at (i) 0.0025 second after passing through the positive maximum value, and (ii) 0.0075 second after passing through zero value and increasing negatively.

Solution $f = 50$ Hz

$$I_{\text{rms}} = 10 \text{ A}$$

(i) Equation for instantaneous value of the current

$$I_m = I_{\text{rms}} \times \sqrt{2} = 10\sqrt{2} = 14.14 \text{ A}$$

$$i = I_m \sin 2\pi f t$$

$$= 14.14 \sin (2 \times 180 \times 50 \times t) \qquad \text{(angle in degrees)}$$

$$= 14.14 \sin (18000\, t)$$

(ii) Value of the current at 0.0025 s after passing through the positive maximum value

From Fig. 3.11(a),

$$t = \frac{T}{4} + 0.0025$$

$$= \frac{1}{4f} + 0.0025$$

$$= \frac{1}{4 \times 50} + 0.0025$$

$$= 7.5 \text{ ms}$$

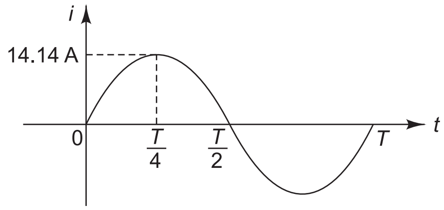


Fig. 3.11(a)

$$i = 14.14 \sin (18000 \times 7.5 \times 10^{-3}) \quad \text{(angle in degrees)}$$

$$= 10 \text{ A}$$

(ii) Value of the current 0.0075 s after passing through zero value and increasing negatively

From Fig. 3.11(b),

$$t = \frac{T}{2} + 0.0075$$

$$= \frac{1}{2f} + 0.0075$$

$$= \frac{1}{2 \times 50} + 0.0075$$

$$= 17.5 \text{ ms}$$

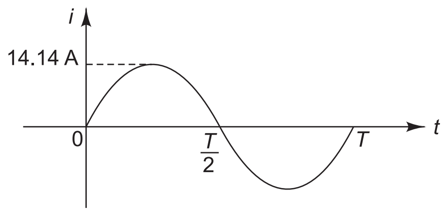


Fig. 3.11(b)

$$i = 14.14 \sin (18000 \times 17.5 \times 10^{-3}) \quad \text{(angle in degrees)}$$

$$= -10 \text{ A}$$

Example 11

Draw a neat sketch in each case of the waveform and write expressions of instantaneous value for the following:

(i) Sinusoidal current of amplitude 10 A, 50 Hz passing through its zero value at $\omega t = \frac{\pi}{3}$ *and increasing positively*

(ii) Sinusoidal current of amplitude 8 A, 50 Hz passing through its zero value at $\omega t = -\frac{\pi}{6}$ *and increasing positively.*

Solution (i) The current waveform is lagging in nature.

$$i = I_m \sin (2\pi ft - \phi)$$

$$= 10 \sin \left(2\pi \times 50 \times t - \frac{\pi}{3}\right)$$

$$= 10 \sin \left(100\pi t - \frac{\pi}{3}\right)$$

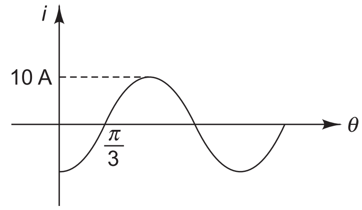


Fig. 3.12(a)

(ii) The current waveform is leading in nature.

$$i = I_m \sin(2\pi ft + \phi)$$
$$= 8 \sin\left(2\pi \times 50 \times t + \frac{\pi}{6}\right)$$
$$= 8 \sin\left(100\pi t + \frac{\pi}{6}\right)$$

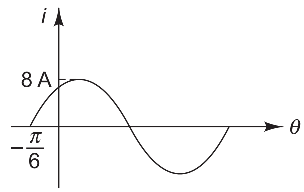


Fig. 3.12(b)

Example 12

The instantaneous current is given by $I = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the waveform from t = 0 over one complete cycle.

Solution $$i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$$

(i) Effective value

$$I_{eff} = I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{7.071}{\sqrt{2}} = 5 \text{ A}$$

(ii) Periodic time

$$2\pi f = 157.08$$
$$f = 25 \text{ Hz}$$
$$T = \frac{1}{f} = \frac{1}{25} = 0.04 \text{ s}$$

(iii) Instant at which the current reaches its positive maximum value, i.e., $i = 7.071$ A

$$7.071 = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right) \quad \text{(angle in radians)}$$
$$1 = \sin(157.08t - 0.785)$$
$$1.5708 = 157.08t - 0.785$$
$$t = 0.015 \text{ s}$$

(iv) The waveform is shown in Fig. 3.13.

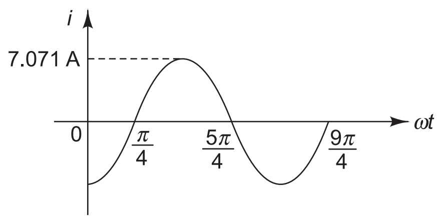


Fig. 3.13

Example 13

A 60 Hz sinusoidal current has an instantaneous value of 7.07 A at t = 0 and rms value of $10\sqrt{2}$ A. Assuming the current wave to enter positive half at t = 0, determine (i) expression for instantaneous current, (ii) magnitude of the current at t = 0.0125 second, and (iii) magnitude of the current at t = 0.025 second after t = 0.

Solution $$f = 60 \text{ Hz}$$

$$i(0) = 7.07 \text{ A}$$
$$I_{rms} = 10\sqrt{2} \text{ A}$$

(i) Expression for instantaneous current

$$I_m = I_{rms} \times \sqrt{2} = 10\sqrt{2} \times \sqrt{2} = 20 \text{ A}$$

Since $i = 7.07$ A at $t = 0$, the current is leading in nature.

At $t = 0$,

$$i = I_m \sin(2\pi ft + \phi)$$
$$7.07 = 20 \sin(2\pi \times 60 \times 0 + \phi)$$
$$\phi = 20.7°$$
$$i = 20 \sin(120\pi t + 20.7°)$$

(ii) Magnitude of current at $t = 0.0125$ second

$$i = 20 \sin(120 \times 180 \times 0.0125 + 20.7°) \qquad \text{(angle in degrees)}$$
$$= -18.7 \text{ A}$$

(iii) Magnitude of current at $t = 0.025$ second after $t = 0$

Time corresponding to a phase shift of 20.7°

$$= \frac{20.7°}{360°} \times T$$
$$= \frac{20.7°}{360°} \times \frac{1}{f}$$
$$= \frac{20.7°}{360°} \times \frac{1}{60}$$
$$= 0.958 \text{ ms}$$

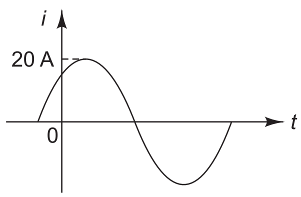


Fig. 3.14

Time 0.025 second after $t = 0$

$$t = 0.958 \times 10^{-3} + 0.025$$
$$= 25.958 \text{ ms}$$
$$i = 20 \sin(120 \times 180 \times 25.958 \times 10^{-3} + 20.7°) \qquad \text{(angle in degrees)}$$
$$= -13.22 \text{ A}$$

Example 14

A 50 Hz sinusoital voltage applied to a single-phase circuit has an rms value of 200 V. Its value at t = 0 is $(\sqrt{2} \times 200)$ V positive. The current drawn by the circuit is 5 A (rms) and lags behind the voltage by one-sixth of a cycle. Write the expressions for the instantaneous values of voltage and current. Sketch their waveforms and find their values at t = 0.0125 second.

Solution

$$f = 50 \text{ Hz}$$
$$V_{rms} = 200 \text{ V}$$
$$I_{rms} = 5 \text{ A}$$

$$v(0) = \sqrt{2} \times 200 = 282.84 \text{ V}$$

(i) Instantaneous value of voltage

$$V_m = V_{\text{rms}} \times \sqrt{2} = 200\sqrt{2} = 282.84 \text{ V}$$

$$v = V_m \sin(2\pi f t + \phi)$$

At $t = 0$,

$$282.84 = 282.84 \sin(0 + \phi)$$

$$\sin\phi = 1$$

$$\phi = 90°$$

$$v = 282.84 \sin(2\pi \times 50 \times t + 90°)$$

$$= 282.84 \sin(100\,\pi t + 90°)$$

(ii) Instantaneous value of current

The current lags behind the voltage by one-sixth of a cycle.

$$\phi = \frac{1}{6} \times 360 = 60°$$

$$I_m = I_{\text{rms}} \times \sqrt{2} = 5\sqrt{2} = 7.07 \text{ A}$$

$$i = I_m \sin(2\pi f t + 90° - 60°)$$

$$= 7.07 \sin(2\pi \times 50 \times t + 30°)$$

$$= 7.07 \sin(100\pi t + 30°)$$

(iii) Voltage and current waveforms are shown in Fig. 3.15.

(iv) Value of voltage at $t = 0.0125$ s

$$v = 282.84 \sin(100 \times 180 \times 0.0125 + 90°)$$

(angle in degrees)

$$= 200 \text{ V}$$

(v) Value of current at $t = 0.0125$ s

$$i = 7.07 \sin(100 \times 180 \times 0.0125 + 30°)$$

(angle in degrees)

$$= -6.83 \text{ A}$$

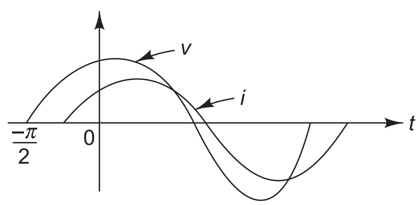


Fig. 3.15

Example 15

At the instant t = 0, the instantaneous value of a 50 Hz sinusoidal current is 5 A and increases in magnitude further. Its rms value is 10 A. (i) Write the expression for its instantaneous value. (ii) Find the current at t = 0.01 s and t = 0.015 s. (iii) Sketch the waveform indicating these values.

Solution

$$f = 50 \text{ Hz}$$

$$i(0) = 5 \text{ A}$$

$$I_{\text{rms}} = 10 \text{ A}$$

(i) Expression for instantaneous value of current

$$I_m = I_{\text{rms}} \times \sqrt{2} = 10\sqrt{2} = 14.14 \text{ A}$$

Since $i = 5$ A at $t = 0$, the current is leading in nature.

$$i = I_m \sin(2\pi f t + \phi)$$

At $t = 0$,

$$5 = 14.14 \sin(2 \times 180 \times 50 \times 0 + \phi) \qquad \text{(angle in degrees)}$$

$$5 = 14.14 \sin \phi$$

$$\phi = 20.7°$$

$$i = 14.14 \sin(100\pi t + 20.7°)$$

(ii) Current at $t = 0.01$ s

$$i = 14.14 \sin(100 \times 180 \times 0.01 + 20.7°) \qquad \text{(angle in degrees)}$$

$$= -5 \text{ A}$$

(iii) Current at $t = 0.015$ s

$$i = 14.14 \sin(100 \times 180 \times 0.015 + 20.7°) \qquad \text{(angle in degrees)}$$

$$= -13.23 \text{ A}$$

(iv) Waveform

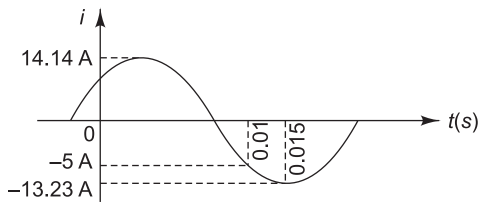


Fig. 3.16

Example 16

In a certain circuit supplied from 50 Hz mains, the potential difference has a maximum value of 500 V and the current has a maximum value of 10 A. At the instant t = 0, the instantaneous values of potential difference and current are 400 V and 4 A respectively, both increasing in the positive direction. State expressions for instantaneous values of potential difference and current at time t. Calculate the instantaneous values at time t = 0.015 second. Find the phase angle between potential difference and current.

Solution

$$f = 50 \text{ Hz}$$

$$V_m = 500 \text{ V}$$

$$I_m = 10 \text{ A}$$

$$v(0) = 400 \text{ V}$$

$$i(0) = 4 \text{ A}$$

(i) Expression for instantaneous values of potential difference and current.
Since $v = 400$ V and $i = 4$ A at $t = 0$, the voltage and current waveforms are leading in nature.

(a) $v = V_m \sin(2\pi ft + \phi_1)$

At $t = 0$,

$$400 = 500 \sin(2\pi \times 50 \times 0 + \phi_1)$$
$$\phi_1 = 53.13°$$
$$v = 500 \sin(100\pi t + 53.13°)$$

(b) $i = I_m \sin(2\pi ft + \phi_2)$

At $t = 0$,

$$4 = 10 \sin(2\pi \times 50 \times 0 + \phi_2)$$
$$\phi_2 = 23.58°$$
$$i = 10 \sin(100\pi t + 23.58°)$$

(ii) Instantaneous values of potential difference and current at $t = 0.015$ second

(a) $v = 500 \sin(100 \times 180 \times 0.015 + 53.13°)$ (angle in degrees)

$= -300$ V

(b) $i = 10 \sin(100 \times 180 \times 0.015 + 23.58°)$ (angle in degrees)

$= -9.17$ A

(iii) Phase angle between potential difference and current

$$\phi = \phi_1 - \phi_2 = 53.13° - 23.58° = 29.55°$$

Example 17

Find the following parameters of a voltage v = 200 sin 314 t:
(i) frequency, (ii) form factor, and (iii) crest factor.

Solution $v = 200 \sin 314\, t$

(i) Frequency

$$v = V_m \sin 2\pi ft$$
$$2\pi f = 314$$
$$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

For a sinusoidal waveform,

$$V_{\text{avg}} = \frac{2V_m}{\pi}$$
$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

(ii) Form factor

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = 1.11$$

(iii) Crest factor

$$k_p = \frac{V_m}{V_{\text{rms}}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$$

Example 18

A non-sinusoidal voltage has a form factor of 1.2 and peak factor of 1.5. If the average value of the voltage is 10 V, calculate (i) rms value, and (ii) maximum value.

Solution

$$k_f = 1.2$$
$$k_p = 1.5$$
$$V_{\text{avg}} = 10$$

(i) rms value

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}$$

$$1.2 = \frac{V_{\text{rms}}}{10}$$

$$V_{\text{rms}} = 12 \text{ V}$$

(ii) Maximum value

$$k_p = \frac{V_m}{V_{\text{rms}}}$$

$$1.5 = \frac{V_m}{12}$$

$$V_m = 18 \text{ V}$$

Example 19

The waveform of a voltage has a form factor of 1.15 and a peak factor of 1.5. If the maximum value of the voltage is 4500 V, calculate the average value and rms value of the voltage.

Solution

$$k_f = 1.15$$
$$k_p = 1.5$$
$$V_m = 4500 \text{ V}$$

(i) rms value of the voltage

$$k_p = \frac{V_m}{V_{\text{rms}}}$$

$$1.5 = \frac{4500}{V_{\text{rms}}}$$

$$V_{\text{rms}} = 3000 \text{ V}$$

(ii) Average value of the voltage

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}$$

$$1.15 = \frac{3000}{V_{\text{avg}}}$$

$$V_{\text{avg}} = 2608.7 \text{ V}$$

Example 20

A 50 Hz sinusoidal current has a peak factor of 1.4 and a form factor of 1.1. Its average value is 20 A. The instantaneous value of the current is 15 A at t = 0. Write the equation of the current and draw its waveform.

Solution

$$f = 50 \text{ Hz}$$

$$k_p = 1.4$$

$$k_f = 1.1$$

$$I_{\text{avg}} = 20 \text{ A}$$

$$i(0) = 15 \text{ A}$$

(i) Equation of current

$$k_f = \frac{I_{\text{rm}\tilde{\text{s}}}}{I_{\text{avg}}}$$

$$1.1 = \frac{I_{\text{rms}}}{20}$$

$$I_{\text{rms}} = 22 \text{ A}$$

$$k_p = \frac{I_m}{I_{\text{rms}}}$$

$$1.4 = \frac{I_m}{22}$$

$$I_m = 30.8 \text{ A}$$

Since $i = 15$ A at $t = 0$, the current is leading in nature.

$$i = I_m \sin(2\pi f t + \phi)$$

At $t = 0$,

$$15 = 30.8 \sin(2\pi \times 50 \times 0 + \phi)$$

$$\phi = 29.14°$$

$$i = 30.8 \sin(100\pi t + 29.14°)$$

(ii) The waveform is shown is Fig. 3.17.

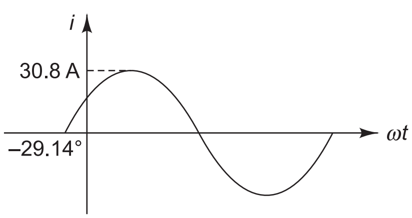


Fig. 3.17

Example 21

Find the average value and rms value of the waveform shown in Fig. 3.18.

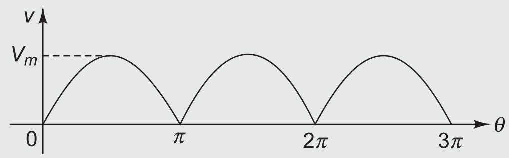


Fig. 3.18

Solution $v = V_m \sin\theta \qquad 0 < \theta < \pi$

(i) Average value of the waveform

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi}$$

$$= 0.637\, V_m$$

(ii) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^\pi}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$= 0.707\ V_m$$

Example 22

Find the average and rms values of the waveform shown in Fig. 3.19.

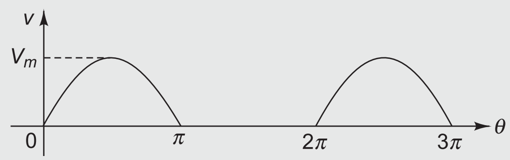


Fig. 3.19

Solution

$$v = V_m \sin \theta \qquad 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\ d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\ d\theta$$

$$= \frac{V_m}{2\pi}[-\cos\theta]_0^\pi$$

$$= \frac{V_m}{2\pi}[1+1]$$

$$= \frac{V_m}{\pi}$$

$$= 0.318\ V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\ d\theta + \int_{\pi}^{2\pi} 0\ d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\ d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\ d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{V_m^2}{4}}$$

$$= \frac{V_m}{2}$$

$$= 0.5\ V_m$$

Example 23

Find the average value and rms value of the waveform shown in Fig. 3.20.

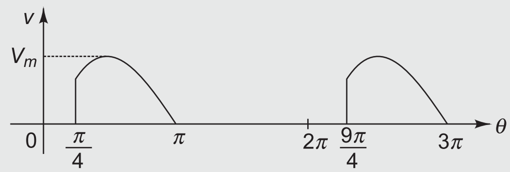


Fig. 3.20

Solution

$v = 0 \qquad 0 < \theta < \pi/4$

$= V_m \sin\theta \qquad \pi/4 < \theta < \pi$

$= 0 \qquad \pi < \theta < 2\pi$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{2\pi}[-\cos\theta]_{\pi/4}^{\pi}$$

$$= \frac{V_m}{2\pi}\,[1 + 0.707]$$

$$= 0.272\ V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \left(\frac{1-\cos 2\theta}{2}\right)} \, d\theta$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4}\right]}$$

$$= \sqrt{0.227 V_m^2}$$

$$= 0.476\ V_m$$

Example 24

A full-wave rectified wave is clipped at 70.7% of its maximum value as shown in Fig. 3.21. Find its average and rms values.

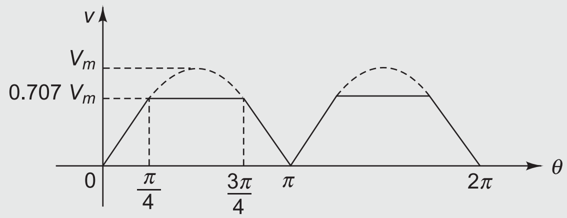


Fig. 3.21

Solution

$$\begin{aligned} v &= V_m \sin\theta && 0 < \theta < \pi/4 \\ &= 0.707\ V_m && \pi/4 < \theta < 3\pi/4 \\ &= V_m \sin\theta && 3\pi/4 < \theta < \pi \end{aligned}$$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\ d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m\, d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\ d\theta\right]$$

$$= \frac{V_m}{\pi}\left\{[-\cos\theta]_0^{\pi/4} + 0.707[\theta]_{\pi/4}^{3\pi/4} + [-\cos\theta]_{3\pi/4}^{\pi}\right\}$$

$$= \frac{V_m}{\pi}(0.293 + 1.11 + 0.293)$$

$$= 0.54\ V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2\theta\ d\theta + \int_{\pi/4}^{3\pi/4} (0.707 V_m)^2\, d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\ d\theta\right]}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/4} + 0.499[\theta]_{\pi/4}^{3\pi/4} + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{3\pi/4}^{\pi}\right\}}$$

$$= \sqrt{0.341\,V_m^2}$$
$$= 0.584\ V_m$$

Example 25

Find the rms value for the given waveform as shown in Fig. 3.22.

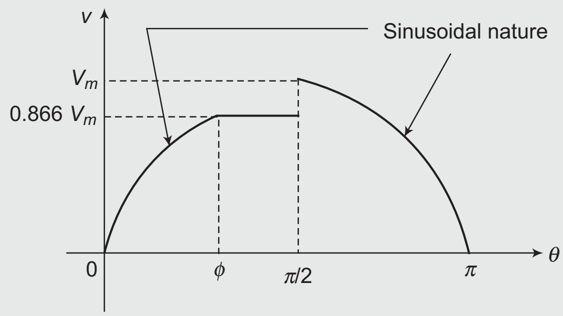


Fig. 3.22 [Dec 2014]

Solution

At $\theta = \phi$,
$$v = 0.866\ V_m$$
$$v = V_m \sin\theta$$
$$0.866\ V_m = V_m \sin\phi$$
$$\phi = \frac{\pi}{3}$$
$$v = V_m \sin\theta \qquad 0 < \theta < \frac{\pi}{3}$$
$$= 0.866\ V_m \qquad \frac{\pi}{3} < \theta < \frac{\pi}{2}$$
$$= V_m \sin\theta \qquad \frac{\pi}{2} < \theta < \pi$$
$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\,\mathrm{d}\theta}$$
$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/3} V_m^2 \sin^2\theta\,\mathrm{d}\theta + \int_{\pi/3}^{\pi/2} (0.866\,V_m)^2\,\mathrm{d}\theta + \int_{\pi/2}^{\pi} V_m^2 \sin^2\theta\,\mathrm{d}\theta\right]}$$
$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/3} + 0.75\left[\theta\right]_{\pi/3}^{\pi/2} + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi}\right\}}$$
$$= \sqrt{0.472\,V_m^2}$$
$$= 0.687\ V_m$$

Example 26

Find the rms value of the waveform shown in Fig. 3.23.

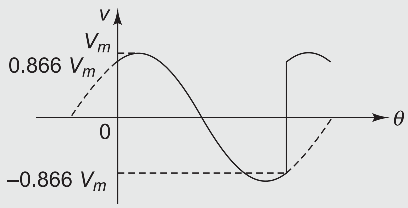


Fig. 3.23

Solution The equation of the waveform is given by $v = V_m \sin(\theta + \phi)$ where ϕ is the phase difference.

When $\theta = 0$, $v = 0.866\ V_m$.

$$0.866\ V_m = V_m \sin(0 + \phi)$$

$$\phi = \sin^{-1}(0.866) = \frac{\pi}{3}$$

$$v = V_m \sin\left(\theta + \frac{\pi}{3}\right)$$

The time period of a complete sine wave is always 2π. Since some part of the waveform is chopped from both the sides,

$$\text{Time period} = 2\pi - \frac{\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{4\pi/3}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta + \frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3}{4\pi}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta + \frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\int_0^{4\pi/3} \left[\frac{1 - \cos 2(\theta + \pi/3)}{2}\right] d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\left[\frac{\theta}{2} - \frac{\sin 2(\theta + \pi/3)}{4}\right]_0^{4\pi/3}}$$

$$= \sqrt{0.6031\, V_m^2}$$

$$= 0.776\ V_m$$

Example 27

Find the average and rms values of the waveform shown in Fig. 3.24.

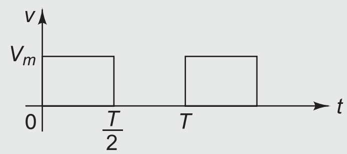


Fig. 3.24

Solution

$$v = V_m \qquad 0 < t < T/2$$
$$= 0 \qquad T/2 < t < T$$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,dt$$
$$= \frac{1}{T}\left[\int_0^{T/2} V_m\,dt + \int_{T/2}^{T} 0\,dt\right]$$
$$= \frac{1}{T}\int_0^{T/2} V_m\,dt$$
$$= \frac{V_m}{T}[t]_0^{T/2}$$
$$= \frac{V_m}{T}\cdot\frac{T}{2}$$
$$= 0.5\,V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$
$$= \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2\,dt}$$
$$= \sqrt{\frac{V_m^2}{T}[t]_0^{T/2}}$$
$$= \sqrt{\frac{V_m^2}{T}\cdot\frac{T}{2}}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= 0.707\ V_m$$

Example 28

Find the average value of the waveform shown in Fig. 3.25.

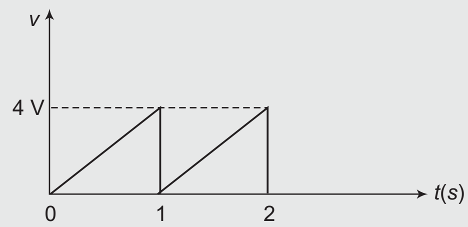


Fig. 3.25 **[Dec 2014]**

Solution

$$v = 4t$$

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,\mathrm{d}t$$

$$= \frac{1}{1}\int_0^1 4t\ \mathrm{d}t$$

$$= 4\left[\frac{t^2}{2}\right]_0^1$$

$$= 4\left(\frac{1}{2} - 0\right)$$

$$= 2\ \text{V}$$

Example 29

Find the rms value for the given waveform in Fig. 3.26.

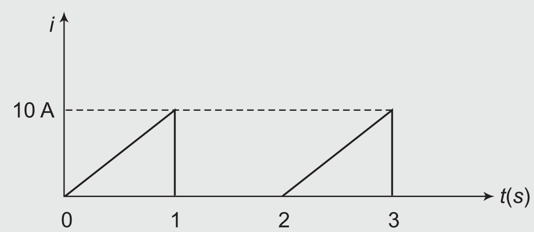


Fig. 3.26 **[May 2015]**

Solution

$$i = 10\,t \qquad 0 < t < 1$$
$$= 0 \qquad 1 < t < 2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,\mathrm{d}t}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 (10t)^2\,\mathrm{d}t + \int_1^2 0\,\mathrm{d}t\right]}$$

$$= \sqrt{\frac{1}{2}\times 100\left[\frac{t^3}{3}\right]_0^1}$$

$$= \sqrt{\frac{100}{2}\left[\frac{1}{3} - 0\right]}$$

$$= 4.084$$

Example 30

Determine the rms value of the voltage waveform shown in Fig. 3.27.

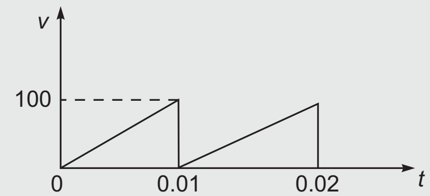


Fig. 3.27

[May 2013]

Solution

$$v(t) = \frac{100}{0.01}\,t = 10000\,t \qquad 0 < t < 0.01$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\frac{1}{0.01}\int_0^{0.01} (10000t)^2 dt}$$

$$= \sqrt{10^{10}\left[\frac{t^3}{3}\right]_0^{0.01}}$$

$$= \sqrt{10^{10}\left[\frac{(0.01)^3}{3} - 0\right]}$$

$$= 57.74 \text{ V}$$

Example 31

Find the average and rms values of the waveform shown in Fig. 3.28.

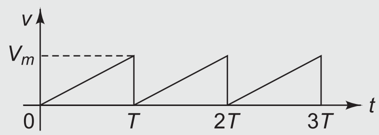


Fig. 3.28

Solution $\quad v = \dfrac{V_m}{T}t \qquad 0 < t < T$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m}{T}t\,dt$$

$$= \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{V_m}{T^2}\cdot\frac{T^2}{2}$$

$$= 0.5\,V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2}.t^2\,dt}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{T^3}{3}\right]}$$

$$= \sqrt{\frac{V_m^2}{3}}$$

$$= 0.577\ V_m$$

Example 32

Find the average and rms values of the waveform shown in Fig. 3.29.

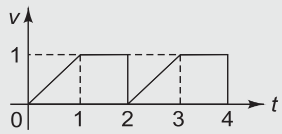


Fig. 3.29

[May 2016]

Solution $v = t \qquad 0 < t < 1$

$= 1 \qquad 1 < t < 2$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 1\,dt\right]$$

$$= \frac{1}{2}\left\{\left[\frac{t^2}{2}\right]_0^1 + [t]_1^2\right\}$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0 + 2 - 1\right]$$

$$= 0.75 \text{ V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (1)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left\{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2\right\}}$$

$$= \sqrt{\frac{1}{2}\left[\frac{1}{3} - 0 + 2 - 1\right]}$$

$$= 0.816 \text{ V}$$

Example 33

Find the average and rms values of the waveform shown in Fig. 3.30.

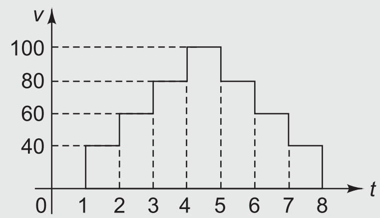


Fig. 3.30

Solution

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{0 + 40 + 60 + 80 + 100 + 80 + 60 + 40}{8} = 57.5 \text{ V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{0^2 + (40)^2 + (60)^2 + (80)^2 + (100)^2 + (80)^2 + (60)^2 + (40)^2}{8}} = 64.42 \text{ V}$$

Example 34

Find the average and rms values of the waveform shown in Fig. 3.31.

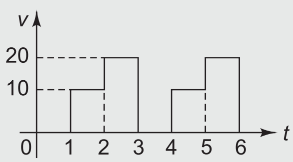


Fig. 3.31

Solution

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{0 + 10 + 20}{3} = 10 \text{ V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{0^2 + (10)^2 + (20)^2}{3}} = 12.9 \text{ V}$$

Example 35

Find the effective value of the resultant current which carries simultaneously a direct current of 10 A and a sinusoidally alternating current with a peak value of 10 A.

Solution

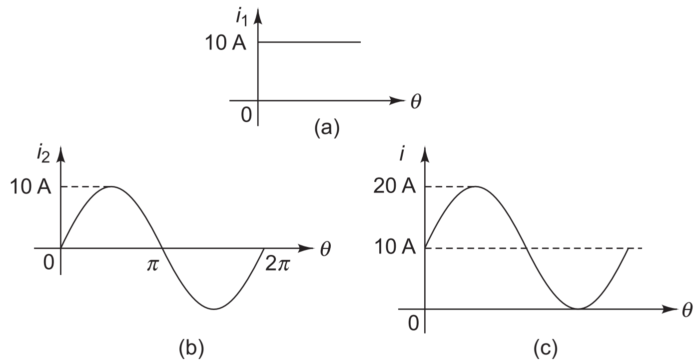


Fig. 3.32

$$i = i_1 + i_2 = 10 + 10 \sin \theta$$

$$I_{\text{eff}} = I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\theta)^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100 + 200\sin\theta + 100\sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1 + 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} \left[1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2\cos 2\pi + \frac{2\pi}{2} - \frac{\sin 4\pi}{4} - 0 + 2\cos 0 - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2 + \frac{2\pi}{2} + 2\right]}$$

$$= \sqrt{\frac{100}{2\pi} \times 3\pi}$$

$$= 12.25 \text{ A}$$

Example 36

Find the effective value of a resultant current in a wire which carries simultaneously a direct current of i_1 = 10 A and alternating current given by $i_2 = 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$.

Solution

$$i_1 = 10 \text{ A}$$

$$i_2 = 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$$

$$i = i_1 + i_2$$

$$= 10 + 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$$

$$= 10 + 12 \sin \theta + 6 \sin (3\theta - 30°) + 4 \sin (5\theta + 60°)$$

$$I_{\text{eff}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [10 + 12\sin\theta + 6\sin(3\theta - 30°) + 4\sin(5\theta + 60°)]^2\,d\theta}$$

$$= 14.07 \text{ A}$$

Example 37

Find the relative heating effects of two current waves of equal peak value, one sinusoidal and the other, rectangular in shape.

Solution

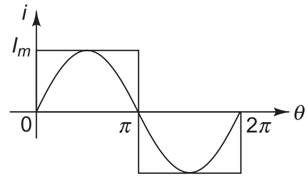


Fig. 3.33

$$\text{rms value of the rectangular wave} = I_m$$

$$\text{rms value of the sinusoidal current wave} = \frac{I_m}{\sqrt{2}}$$

$$\text{Heating effect due to the rectangular current wave} = (I_m)^2\, RT$$

$$\text{Heating effect due to the sinusoidal current wave} = \left(\frac{I_m}{\sqrt{2}}\right)^2 RT = \frac{(I_m)^2}{2}\; RT$$

$$\text{Relative heating effects} = \frac{(I_m)^2}{2} RT : (I_m)^2\, RT$$

$$= \frac{1}{2} : 1$$

$$= 1 : 2$$

Useful Formulae

Average Value and rms value

$$F_{\text{avg}} = \frac{1}{T}\int_0^T f(t)dt \qquad k_P = \frac{\text{max. value}}{\text{rms value}}$$

$$F_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt} \qquad k_f = \frac{\text{rms value}}{\text{avg. value}}$$

Exercise 3.1

3.1 An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 20 A. Write down the equation for the instantaneous value and find this value (i) 0.0025 second, and (ii) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A? *[i = 28.28 sin 100 π t, 20 A, –20 A, $\frac{1}{300}$ s]*

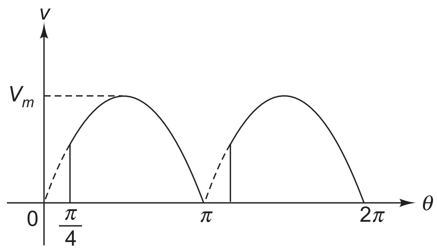
3.2 A certain waveform has a form factor of 1.2 and a peak factor of 1.5. If the maximum value is 100, find the rms value and average value. *[66.67 V, 55.67 V]*

3.3 Find the root mean square value, the average value and the form factor of the resultant current in a wire that carries simultaneously a direct current of 5 A and a sinusoidal alternating current with an amplitude of 5 A. *[6.12 A, 5 A, 1.224]*

3.4 Find the relative heating effects of two in-phase current waveforms of equal peak values and time periods, but one sinusoidal and the other triangular. *[3:2]*

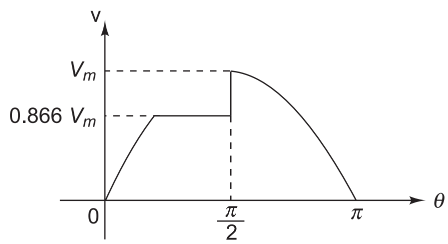
3.5 Prove that if a dc current of I amperes is superimposed in a conductor by an ac current of maximum value I amperes, the root mean square value of its resultant is $\sqrt{\frac{3}{2}}\, I$.

3.6 Find the average values and rms values of the waveforms shown in Fig. 3.34 to Fig. 3.37.



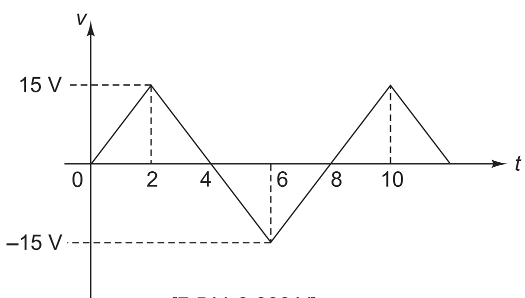
[0.543 V_m, 0.674 V_m]

Fig. 3.34



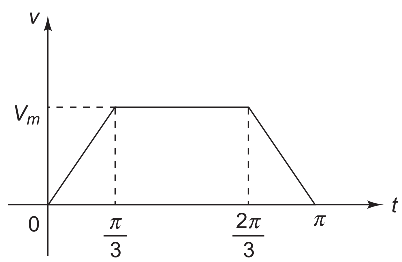
[0.622 V_m, 0.687 V_m]

Fig. 3.35



[7.5 V, 8.666 V]

Fig. 3.36



[0.67 V_m, 0.745 V_m]

Fig. 3.37

3.7 Find the rms value of the periodic waveform with time period T given in Fig. 3.38.

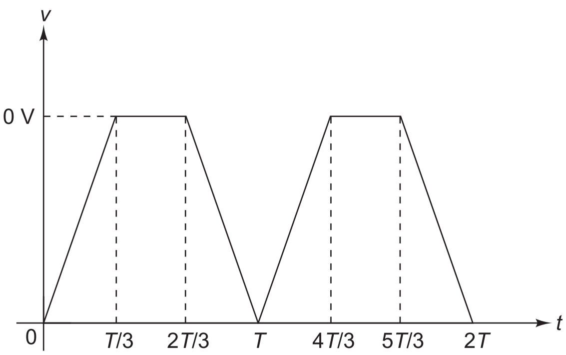


Fig. 3.38

[7.45 V]

3.8 A voltage wave has the variation shown in Fig. 3.39.

(i) Find the average and effective values of the voltage.

(ii) If this voltage is applied to a 10 Ω resistance, find the dissipated power.

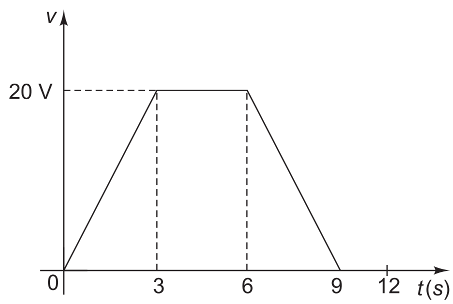


Fig. 3.39

[10 V, 12.91 V, 16.67 W]

3.5 PHASOR REPRESENTATIONS OF ALTERNATING QUANTITIES

The alternating quantities are represented by phasors. A phasor is a line of definite length rotating in an anticlockwise direction at a constant angular velocity ω. The length of a phasor is equal to the maximum value of the alternating quantity, and the angular velocity is equal to the angular velocity of alternating quantity.

As shown in Fig. 3.40(a), consider a phasor $OP = I_m$, where I_m is the maximum value of the alternating current. Let this phasor rotate in an anticlockwise direction at a uniform angular velocity of ω radians/second. The projection of the phasor OP on the Y-axis at any instant gives the instantaneous value of that alternating current.

$$\begin{aligned} OM &= OP \sin \omega t \\ &= I_m \sin \omega t = i \end{aligned}$$

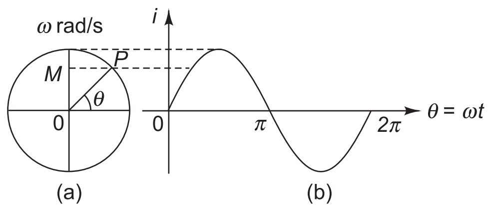


Fig. 3.40 *Representation of alternating quantities in terms of phasors*

Thus, if we plot the projections of the phasor on the *Y*-axis versus its angular position point by point, a sinusoidal alternating current waveform is obtained.

Phasor Diagram using rms Values Sinusoidal alternating currents and voltages can be represented by phasors. Electrical measuring instruments like ammeters and voltmeters are calibrated to read the rms values of ac quantities. Hence, instead of using maximum values, it is more convenient to draw phasor diagrams using rms values of alternating quantities. However, such a phasor diagram will not generate a sine wave of proper amplitude unless the length of the phasor is multiplied by $\sqrt{2}$.

Example 1

Two currents i_1 and i_2 are given by the expressions $i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$ and $i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$. Find (i) $i_1 + i_2$, and (ii) $i_1 - i_2$. Express the answers in the form $i = I_m \sin(\omega t \pm \phi)$.

Solution

$$i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$$

(i) Let phasors $\bar{I}_1$ and $\bar{I}_2$ represent the alternating currents i_1 and i_2 respectively in terms of their maximum values.

(a) Analytical method:

Resolving $\bar{I}_1$ and $\bar{I}_2$ into *x*- and *y*-components,

$$\Sigma x = 10 \cos(45°) + 8 \cos(-60°) = 11.07$$

$$\Sigma y = 10 \sin(45°) + 8 \sin(-60°) = 0.14$$

$$\text{Magnitude of } (\bar{I}_1 + \bar{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(11.07)^2 + (0.14)^2}$$

$$= 11.07 \text{ A}$$

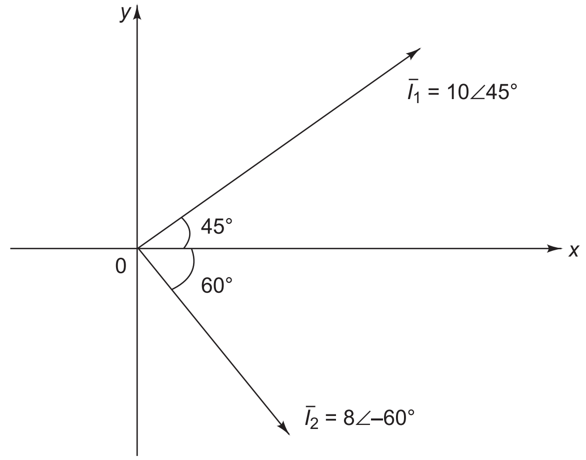


Fig. 3.41

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{0.14}{11.07}\right)$$

$$= 0.72°$$

$$i = i_1 + i_2 = 11.07 \sin(\omega t + 0.72°)$$

(b) Graphical method: The phasor sum $\bar{I}_1 + \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $\bar{I}_2$ by the parallelogram law.

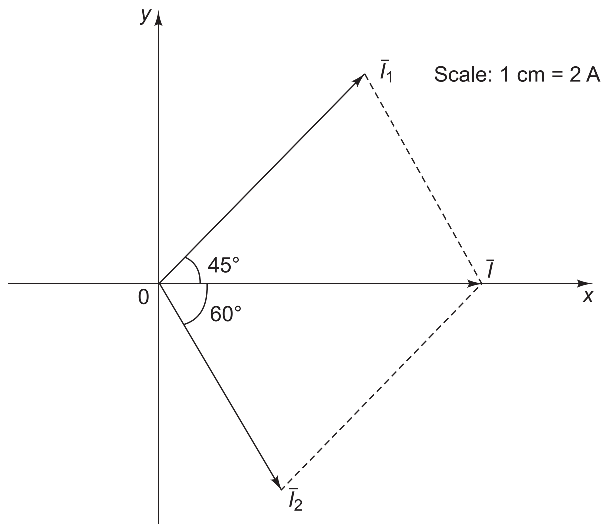


Fig. 3.42

(ii) (a) Analytical method:

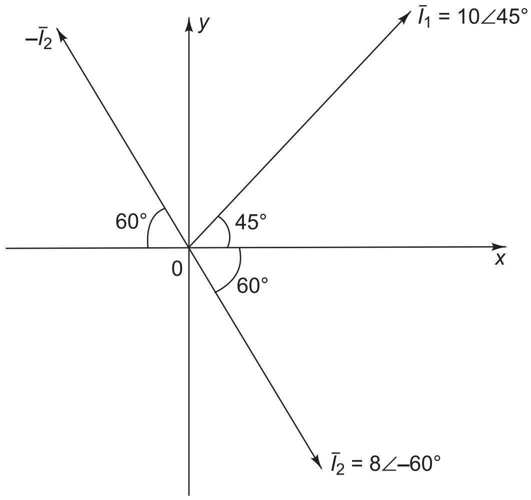


Fig. 3.43

Resolving $\bar{I}_1$ and $-\bar{I}_2$ into x- and y-components,

$$\Sigma x = 10 \cos (45°) - 8 \cos (-60°) = 3.07$$

$$\Sigma y = 10 \sin (45°) - 8 \sin (-60°) = 14$$

$$\text{Magnitude of } (\bar{I}_1 - \bar{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(3.07)^2 + (14)^2}$$

$$= 14.33 \text{ A}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{14}{3.07}\right)$$

$$= 77.63°$$

$$i = i_1 - i_2 = 14.33 \sin (\omega t + 77.63°)$$

(b) Graphical method: The phasor sum $\bar{I}_1 - \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $-\bar{I}_2$ by the parallelogram law.

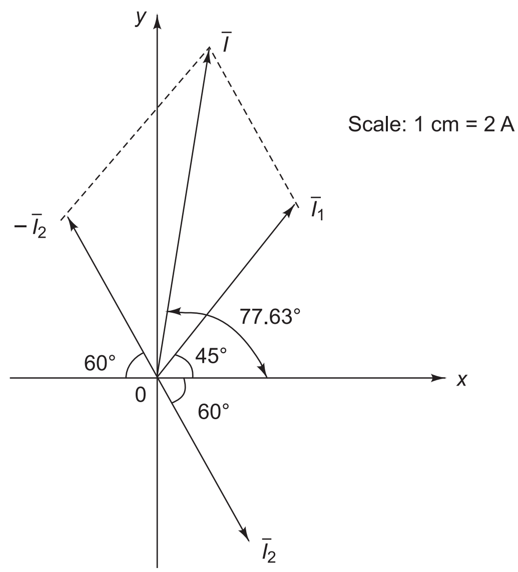


Fig. 3.44

Example 2

Three voltages are represented by $v_1 = 10 \sin \omega t$, $v_2 = 20 \sin \left(\omega t - \frac{\pi}{6}\right)$ and $v_3 = 30 \sin \left(\omega t + \frac{\pi}{4}\right)$. Find the magnitude and phase angle of the resultant voltage.

Solution

$$v_1 = 10 \sin \omega t$$

$$v_2 = 20 \sin \left(\omega t - \frac{\pi}{6}\right)$$

$$v_3 = 30 \sin \left(\omega t + \frac{\pi}{4}\right)$$

Let phasors $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ represent the alternating voltages v_1, v_2 and v_3 respectively in terms of their maximum values.

(a) Analytical method:

Resolving $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ into *x*- and *y*-components,

$$\Sigma x = 10 + 20 \cos (-30°) + 30 \cos (45°) = 48.53$$

$$\Sigma y = 20 \sin (-30°) + 30 \cos (45°) = 11.21$$

$$\text{Magnitude of } (\bar{V}_1 + \bar{V}_2 + \bar{V}_3) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(48.53)^2 + (11.21)^2}$$

$$= 49.81 \text{ V}$$

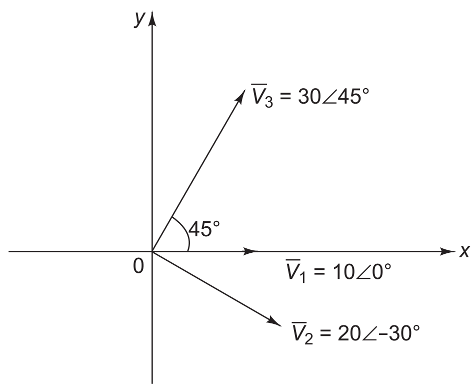


Fig. 3.45

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{11.21}{48.53}\right)$$

$$= 13°$$

$$v = 49.81 \sin(\omega t + 13°)$$

(b) Graphical method: The phasor sum $\bar{V}_1 + \bar{V}_2 + \bar{V}_3$ is obtained by first adding phasors $\bar{V}_1$ and $\bar{V}_2$ by the parallelogram law and then adding $\bar{V}_3$ to the resultant of $\bar{V}_1$ and $\bar{V}_2$ by the parallelogram law.

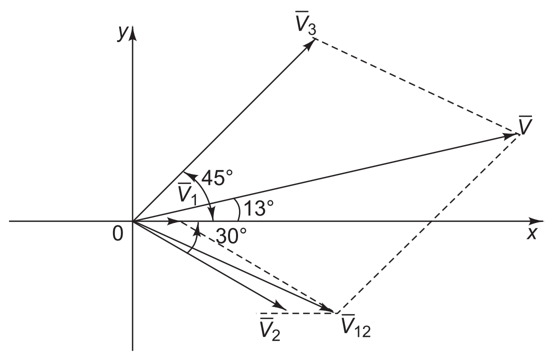


Fig. 3.46

Example 3

The instantaneous voltages across each of the four coils connected in series are given by

$$v_1 = 100 \sin \omega t, \quad v_2 = 250 \cos \omega t, \quad v_3 = 150 \sin\left(\omega t + \frac{\pi}{6}\right), \quad v_4 = 200 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Determine the resultant voltage by analytical method.

Solution
$$v_1 = 100 \sin \omega t$$
$$v_2 = 250 \cos \omega t = 250 \sin (\omega t + 90°)$$
$$v_3 = 150 \sin \left(\omega t + \frac{\pi}{6}\right)$$
$$v_4 = 200 \sin \left(\omega t - \frac{\pi}{4}\right)$$

Let phasors $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$ and $\overline{V}_4$ represent the instantaneous voltages v_1, v_2, v_3 and v_4 respectively in terms of their maximum values.

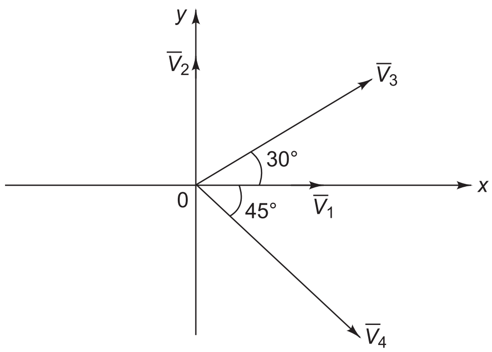


Fig. 3.47

Resolving $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$ and $\overline{V}_4$ into x- and y-components,
$$\Sigma x = 100 + 250 \cos (90°) + 150 \cos (30°) + 200 \cos (-45°) = 371.33$$
$$\Sigma y = 250 \sin (90°) + 150 \sin (30°) + 200 \sin (-45°) = 183.58$$

$$\text{Magnitude of } (\overline{V}_1 + \overline{V}_2 + \overline{V}_3 + \overline{V}_4) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$
$$= \sqrt{(371.33)^2 + (183.58)^2}$$
$$= 414.23 \text{ V}$$
$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$
$$= \tan^{-1}\left(\frac{183.58}{371.33}\right)$$
$$= 26.31°$$
$$v = v_1 + v_2 + v_3 + v_4 = 414.23 \sin (\omega t + 26.31°)$$

3.6 MATHEMATICAL REPRESENTATIONS OF PHASORS

A phasor can be represented in four forms.

(i) Rectangular form

$$\bar{V} = X \pm jY$$

Magnitude of phasor, $V = \sqrt{X^2 + Y^2}$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

(ii) Trigonometric form

$$\bar{V} = V(\cos\phi \pm j\sin\phi)$$

(iii) Exponential form

$$\bar{V} = V e^{\pm j\phi}$$

(iv) Polar form

$$\bar{V} = V \angle \pm \phi$$

Significance of Operator j The operator j is used in rectangular form. It is used to indicate anticlockwise rotation of a phasor through 90°. Mathematically,

$$j = \sqrt{-1}$$

Whenever a phasor is multiplied by j, the phasor is rotated once in the anticlockwise direction through 90°. The power of j represents the number of times the phasor should be rotated through 90° in the anticlockwise direction.

Example 1

Two sinusoidal currents are given as

$$i_1 = 10\sqrt{2}\ \sin\omega t,\ i_2 = 20\sqrt{2}\ \sin(\omega t + 60°).$$

Find the expression for the sum of these currents.

Solution

$$i_1 = 10\sqrt{2}\ \sin\omega t$$

$$i_2 = 20\sqrt{2}\ \sin(\omega t + 60°)$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}}\angle 0° = 10\angle 0°$$

$$\bar{I}_2 = \frac{20\sqrt{2}}{\sqrt{2}}\angle 60° = 20\angle 60°$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$
$$= 10\angle 0° + 20\angle 60°$$
$$= 26.46\ \angle 40.89°$$
$$i = 26.46\sqrt{2}\ \sin\ (\omega t + 40.89°)$$
$$= 37.42\ \sin\ (\omega t + 40.89°)$$

Example 2

The following three sinusoidal currents flow into the junction $i_1 = 3\sqrt{2}\ \sin\ \omega t$, $i_2 = 5\sqrt{2}\ \sin\ (\omega t + 30°)$ and $i_3 = 6\sqrt{2}\ \sin\ (\omega t - 120°)$. Find the expression for the resultant current which leaves the junction.

Solution

$$i_1 = 3\sqrt{2}\ \sin\ \omega t$$
$$i_2 = 5\sqrt{2}\ \sin\ (\omega t + 30°)$$
$$i_3 = 6\sqrt{2}\ \sin\ (\omega t - 120°)$$

Writing currents i_1, i_2 and i_3 in the phasor form,

$$\bar{I}_1 = \frac{3\sqrt{2}}{\sqrt{2}}\angle 0° = 3\angle 0°$$

$$\bar{I}_2 = \frac{5\sqrt{2}}{\sqrt{2}}\angle 30° = 5\angle 30°$$

$$\bar{I}_3 = \frac{6\sqrt{2}}{\sqrt{2}}\angle -120° = 6\angle -120°$$

The resultant current which leaves the junction is given by

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$
$$= 3\angle 0° + 5\angle 30° + 6\angle -120°$$
$$= 5.1\ \angle -31.9°$$
$$i = 5.1\sqrt{2}\ \sin\ (\omega t - 31.9°)$$
$$= 7.21\ \sin\ (\omega t - 31.9°)$$

Example 3

In a circuit, four currents as indicated below, are meeting at a point. Find the resultant current.

$i_1 = 5\ \sin\ \omega t$, $\quad i_2 = 10\ \sin\ (\omega t - 30°)$

$i_3 = 5\ \cos\ (\omega t - 30°)$ $\quad i_4 = -10\ \sin\ (\omega t + 45°)$

Solution

$$i_1 = 5\ \sin\ \omega t$$
$$i_2 = 10\ \sin\ (\omega t - 30°)$$

$$i_3 = 5\cos(\omega t - 30°) = 5\sin(\omega t + 60°)$$

$$i_4 = -10\sin(\omega t + 45°) = 10\sin(\omega t + 225°)$$

Writing currents i_1, i_2, i_3 and i_4 in the phasor form,

$$\bar{I}_1 = \frac{5}{\sqrt{2}}\angle 0° = 3.54\angle 0°$$

$$\bar{I}_2 = \frac{10}{\sqrt{2}}\angle -30° = 7.07\angle -30°$$

$$\bar{I}_3 = \frac{5}{\sqrt{2}}\angle 60° = 3.54\angle 60°$$

$$\bar{I}_4 = \frac{10}{\sqrt{2}}\angle 225° = 7.07\angle 225°$$

Resultant current $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4$

$$= 3.54\angle 0° + 7.07\angle -30° + 3.54\angle 60° + 7.07\angle 225°$$

$$= 8.44\angle -40.36°$$

$$i = 8.44\sqrt{2}\sin(\omega t - 40.36°)$$

$$= 11.94\sin(\omega t - 40.36°)$$

Example 4

Find the resultant voltage and its equation for the given voltages.

$$e_1 = 20\sin\omega t, \qquad e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right), \qquad e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right)$$

Solution $e_1 = 20\sin\omega t$

$$e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right) = 30\sin(\omega t - 45°)$$

$$e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right) = 40\sin(\omega t + 120°)$$

Writing voltages e_1, e_2 and e_3 in the phasor form,

$$\bar{E}_1 = \frac{20}{\sqrt{2}}\angle 0° = 14.14\angle 0°$$

$$\bar{E}_2 = \frac{30}{\sqrt{2}}\angle -45° = 21.21\angle -45°$$

$$\bar{E}_3 = \frac{40}{\sqrt{2}}\angle 120° = 28.28\angle 120$$

$$\text{Resultant voltage } \bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$
$$= 14.14\angle 0° + 21.21\angle -45° + 28.28\angle 120°$$
$$= 17.75\angle 32.33°$$
$$e = 17.75\sqrt{2}\sin(\omega t + 32.33°)$$
$$= 25.1\sin(\omega t + 32.33°)$$

Example 5

Obtain the sum of the three voltages.

$$v_1 = 147.3\cos(\omega t + 98.1°)$$
$$v_2 = 294.6\cos(\omega t - 45°)$$
$$v_3 = 88.4\sin(\omega t + 135°)$$

Solution

$$v_1 = 147.3\cos(\omega t + 98.1°) = 147.3\sin(\omega t + 188.1°)$$
$$v_2 = 294.6\cos(\omega t - 45°) = 294.6\sin(\omega t + 45°)$$
$$v_3 = 88.4\sin(\omega t + 135°)$$

Writing voltages v_1, v_2 and v_3 in the phasor form,

$$\bar{V}_1 = \frac{147.3}{\sqrt{2}}\angle 188.1° = 104.16\angle 188.1°$$
$$\bar{V}_2 = \frac{294.6}{\sqrt{2}}\angle 45° = 208.31\angle 45°$$
$$\bar{V}_3 = \frac{88.4}{\sqrt{2}}\angle 135° = 62.51\angle 135°$$

$$\text{Resultant voltage } \bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$$
$$= 104.16\angle 188.1° + 208.31\angle 45° + 62.51\angle 135°$$
$$= 176.82\angle 90°$$
$$v = 176.82\sqrt{2}\sin(\omega t + 90°)$$
$$= 250.06\sin(\omega t + 90°)$$

Example 6

Find vectorially the resultant of the following four voltages.

$$e_1 = 25\sin\omega t, \qquad e_2 = 30\sin\left(\omega t + \frac{\pi}{6}\right),$$
$$e_3 = 30\cos\omega t, \qquad e_4 = 20\sin\left(\omega t - \frac{\pi}{6}\right)$$

Obtain the answer in similar form.

Solution

$$e_1 = 25 \sin \omega t$$

$$e_2 = 30 \sin \left(\omega t + \frac{\pi}{6}\right) = 30 \sin (\omega t + 30°)$$

$$e_3 = 30 \cos \omega t = 30 \sin(\omega t + 90°)$$

$$e_4 = 20 \sin \left(\omega t - \frac{\pi}{6}\right) = 20 \sin (\omega t - 30°)$$

Writing voltages e_1, e_2, e_3 and e_4 in the phasor form,

$$\bar{E}_1 = \frac{25}{\sqrt{2}}\angle 0° = 17.68\angle 0°$$

$$\bar{E}_2 = \frac{30}{\sqrt{2}}\angle 30° = 21.21\angle 30°$$

$$\bar{E}_3 = \frac{30}{\sqrt{2}}\angle 90° = 21.21\angle 90°$$

$$\bar{E}_4 = \frac{20}{\sqrt{2}}\angle -30° = 14.14\angle -30°$$

Resultant voltage $\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4$

$$= 17.68\angle 0° + 21.21\angle 30° + 21.21\angle 90° + 14.14\angle -30°$$

$$= 54.26\angle 27.13°$$

$$e = 54.26\sqrt{2} \sin (\omega t + 27.13°)$$

$$= 76.74 \sin (\omega t + 27.13°)$$

Example 7

Two currents $i_1 = 100 \sin\left(\omega t + \frac{\pi}{4}\right)$ *and* $i_2 = 25 \sin\left(\omega t - \frac{\pi}{6}\right)$ *are fed to a common conductor. Find the total current. If the conductor resistance is 10 Ω, what is the power dissipated in the conductor.* **[Dec 2015]**

Solution

$$i_1 = 100 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = 25 \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$R = 10\ \Omega$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{100}{\sqrt{2}}\angle 45° = 70.71\angle 45°\text{A}$$

$$\bar{I}_2 = \frac{25}{\sqrt{2}} \angle -30° = 17.68 \angle -30° \text{A}$$

$$\text{Total current } \bar{I} = \bar{I}_1 + \bar{I}_2$$
$$= 70.71\angle 45° + 17.68 \angle -30°$$
$$= 77.2 \angle 32.22° \text{ A}$$
$$i = 77.2\sqrt{2} \sin(\omega t + 32.22°)$$
$$= 109.18 \sin(\omega t + 32.22°)$$

Power dissipated in conductor

$$P = I^2R = (77.2)^2 \times 10 = 59.59 \text{ kW}$$

Example 8

Two currents are represented by $i_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right)$ and $i_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$. These currents are fed into a common conductor. Find the total current in the form $i = I_m \sin(\omega t + \phi)$. If the conductor has a resistance of 10 Ω, what will be the energy loss in 24 hours? **[Dec 2013]**

Solution

$$i_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right)$$
$$i_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$$
$$R = 10\ \Omega$$
$$t = 24 \text{ hours} = 86400 \text{ seconds}$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{15}{\sqrt{2}} \angle 60° = 10.61\angle 60°$$
$$\bar{I}_2 = \frac{25}{\sqrt{2}} \angle 45° = 17.68\angle 45°$$

$$\text{Total current } \bar{I} = \bar{I}_1 + \bar{I}_2$$
$$= 10.61\angle 60° + 17.68\angle 45°$$
$$= 28.06 \angle 50.62°$$
$$i = 28.06\sqrt{2} \sin(\omega t + 50.62°)$$
$$= 39.68 \sin(\omega t + 50.62°)$$

Energy loss in 24 hours

$$E = I^2Rt \qquad \text{where } I \text{ is the rms value of the current}$$
$$= (28.06)^2 \times 10 \times 86400$$
$$= 6.8 \times 10^8 \text{ J}$$

Example 9

The voltage drops across four series-connected impedances are given:

$$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right) \qquad v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right),$$
$$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right), \qquad v_4 = V_{4m} \sin(\omega t + \phi_4)$$

Calculate the values of V_{4m} and ϕ_4 if the voltage applied across the series circuit is $140 \sin\left(\omega t + \frac{3\pi}{5}\right)$. **[May 2013]**

Solution

$$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right) = 60 \sin(\omega t + 30°)$$
$$v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right) = 75 \sin(\omega t - 150°)$$
$$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right) = 100 \sin(\omega t + 135°)$$
$$v = 140 \sin\left(\omega t + \frac{3\pi}{5}\right) = 140 \sin(\omega t + 108°)$$

Writing voltages v_1, v_2, v_3 and v in the phasor form,

$$\bar{V}_1 = \frac{60}{\sqrt{2}}\angle 30° = 42.43\angle 30°$$
$$\bar{V}_2 = \frac{75}{\sqrt{2}}\angle -150° = 53.03\angle -150°$$
$$\bar{V}_3 = \frac{100}{\sqrt{2}}\angle 135° = 70.71\angle 135°$$
$$\bar{V} = \frac{140}{\sqrt{2}}\angle 108° = 98.99\angle 108°$$

For series-connected impedances,

$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4$$
$$\bar{V}_4 = \bar{V} - \bar{V}_1 - \bar{V}_2 - \bar{V}_3$$

$$= 98.99\angle 108° - 42.43\angle 30° - 53.03\angle{-150°} - 70.71\angle 135°$$
$$= 57.13\ \angle 59.96°$$
$$v_4 = 57.13\sqrt{2}\ \sin(\omega t + 59.96°)$$
$$= 80.79\ \sin(\omega t + 59.96°)$$
$$V_{4m} = 80.79\ \text{V}$$
$$\phi_4 = 59.96°$$

Example 10

A circuit consists of three parallel branches. The branch currents are given as $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin(\omega t + 60°)$ and $i_3 = 7.5 \sin(\omega t - 30°)$. Find the resultant current and express it in the form $i = I_m \sin(\omega t + \phi)$. If the supply frequency is 50 Hz, calculate the resultant current when (i) $t = 0$ (ii) $t = 0.001$ s.

Solution

$$i_1 = 10 \sin \omega t$$
$$i_2 = 20 \sin(\omega t + 60°)$$
$$i_3 = 7.5 \sin(\omega t - 30°)$$

Writing currents i_1, i_2 and i_3 in the phasor form,

$$\bar{I}_1 = \frac{10}{\sqrt{2}}\angle 0° = 7.07\angle 0°$$
$$\bar{I}_2 = \frac{20}{\sqrt{2}}\angle 60° = 14.14\angle 60°$$
$$\bar{I}_3 = \frac{7.5}{\sqrt{2}}\angle -30° = 5.3\angle -30°$$

Resultant current $\bar{I}_1 = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

$$= 7.07\ \angle 0° + 14.14\ \angle 60° + 5.3\ \angle{-30°}$$
$$= 21.04\ \angle 27.13°$$
$$i = 21.04\sqrt{2}\ \sin(\omega t + 27.13°)$$
$$= 29.76\ \sin(\omega t + 27.13°)$$

(i) Resultant current at $t = 0$

$$i = 29.76\ \sin(0 + 27.13°)$$
$$= 13.57\ \text{A}$$

(ii) Resultant current at $t = 0.001$ s

$$i = 29.76\ \sin(2\pi f t + 27.13°)$$
$$= 29.76\ \sin(2 \times 180 \times 50 \times 0.001 + 27.13°) \qquad \text{(angle in degrees)}$$
$$= 21.09\ \text{A}$$

Example 11

Two currents, $\overline{I}_1 = 10\angle 50°$ A and $\overline{I}_2 = 5\angle -100°$ A, flow in a single-phase ac circuit. Estimate (i) $\overline{I}_1 + \overline{I}_2$ (ii) $\overline{I}_1 . \overline{I}_2$ (ii) $\dfrac{\overline{I}_1}{\overline{I}_2}$.

Solution $\overline{I}_1 = 10\angle 50°$ A

$\overline{I}_2 = 5\angle -100°$ A

(i) $\overline{I}_1 + \overline{I}_2 = 10\angle 50° + 5\angle -100° = 6.2\ \angle 26.21°$ A

(ii) $\overline{I}_1 . \overline{I}_2 = (10\angle 50°)\ (5\angle -100°) = 50\ \angle -50°$ A

(ii) $$\frac{\overline{I}_1}{\overline{I}_2} = \frac{10\angle 50°}{5\angle -100°} = 2\angle 150°\text{ A}$$

Example 12

Two voltages having rms values of 50 V and 75 V have a phase difference of 60°. Find the resultant sum of these two voltages.

Solution $V_1 = 50$ V

$V_2 = 75$ V

$\phi = 60°$

Let $\overline{V}_1 = 50\ \angle 0°$ V

$\overline{V}_2 = 75\ \angle -60°$ V

Resultant voltage $\overline{V} = \overline{V}_1 + \overline{V}_2$

$= 50\angle 0° + 75\angle -60°$

$= 108.97\ \angle -36.58°$ V

Example 13

Two single-phase alternators supply 300 A and 400 A respectively at a phase difference of 20° to a common load. Find the resultant current and its phase relation to its component.

Solution $I_1 = 300$ A

$I_2 = 400$ A

$\phi = 20°$

Let $\overline{I}_1 = 300\ \angle 0°$ A

$\overline{I}_2 = 400\ \angle -20°$ A

$$\text{Resultant current } \bar{I} = \bar{I}_1 + \bar{I}_2$$
$$= 300\angle 0° + 400\angle -20°$$
$$= 689.59 \angle -11.44° \text{ A}$$

Example 14

Two voltage sources have equal emfs and a phase difference α. When they are connected in series, the voltage is 200 V. When one source is reversed, the voltage is 15 V. Find their emfs and phase angle α.

Solution

$$\bar{E}_1 = E\angle 0°$$
$$\bar{E}_2 = E\angle \alpha°$$
$$E_1 = E_2 = E$$

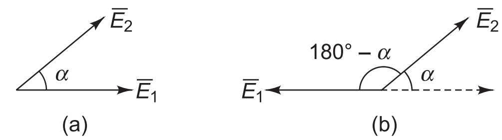


Fig. 3.48

When two sources are connected in series,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\alpha} = 200$$
$$\sqrt{E^2 + E^2 + 2E^2\cos\alpha} = 200$$
$$2E^2 + 2E^2 \cos\alpha = 40000 \tag{1}$$

When one source is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2\cos\alpha} = 15$$
$$\sqrt{E^2 + E^2 - 2E^2\cos\alpha} = 15$$
$$2E^2 - 2E^2 \cos\alpha = 225 \tag{2}$$

Adding Eqs (1) and (2),

$$4E^2 = 40225$$
$$E^2 = 10056.25$$
$$E = 100.28 \text{ V}$$
$$2E^2 + 2E^2\cos\alpha = 40000$$
$$20112.5 + 20112.5\cos\alpha = 40000$$
$$\cos\alpha = 0.988$$

$$\alpha = 8.58°$$

Example 15

Two sinusoidal sources of emf have rms values of E_1 and E_2 and a phase difference of α. When connected in series, the resultant voltage is 41.1 V. When one of the sources is reversed, the resultant emf is 17.52 V. When phase displacement is made zero, the resultant emf is 42.5 V. Calculate E_1, E_2 and α.

Solution

$$\bar{E}_1 = E_1 \angle 0°$$
$$\bar{E}_2 = E_2 \angle \alpha°$$

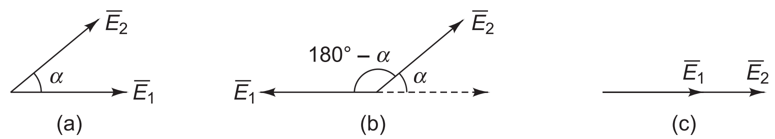


Fig. 3.49

When two sources are connected in series,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha} = 41.1$$
$$E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha = 1689.21 \tag{1}$$

When one of the sources is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha} = 17.52$$
$$E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha = 306.95 \tag{2}$$

When phase displacement is made zero,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos 0°} = 42.5$$
$$E_1 + E_2 = 42.5 \tag{3}$$

Adding Eqs (1) and (2),

$$2(E_1^2 + E_2^2) = 1996.16$$
$$E_1^2 + E_2^2 = 998.08$$
$$(42.5 - E_2)^2 + E_2^2 = 998.08$$
$$1806.25 - 85E_2 + E_2^2 + E_2^2 = 998.08$$
$$E_2^2 - 42.5E_2 + 404.09 = 0 \tag{4}$$

Solving Eq. (4),

$$E_2 = 28.14 \text{ V} \quad \text{or} \quad E_2 = 14.36 \text{ V}$$
$$E_1 = 14.36 \text{ V} \quad \text{or} \quad E_1 = 28.14 \text{ V}$$

Subtracting Eq. (2) from Eq. (1),

$$4E_1E_2 \cos \alpha = 1382.26$$
$$4 \times 14.37 \times 28.14 \cos \alpha = 1382.26$$
$$\cos \alpha = 0.855$$
$$\alpha = 31.24°$$

Exercise 3.2

3.1 Find the resultants of the following voltages:

(a) $v_1 = 60 \cos \theta$, $v_2 = 40 \sin\left(\theta - \frac{\pi}{3}\right)$, $v_3 = 15 \sin \theta$ *[43.22 sin (θ + 35.92°)]*

(b) $e_1 = 50 \sin \omega t$, $e_2 = 100 \sin (\omega t + 120°)$, $e_3 = 120 \sin (\omega t - 30°)$
[107.27 sin (ωt + 14.35°)]

(c) $v_1 = 4\sqrt{2} \sin (\omega t + 135°)$, $v_2 = -4\sqrt{3} \sin (\omega t + 60°)$, $v_3 = 4 \cos (\omega t - 150°)$
[7.73 sin (ωt – 135°)]

(d) $e_1 = 25 \sin \omega t$, $e_2 = 10 \sin\left(\omega t + \frac{\pi}{6}\right)$, $e_3 = 30 \cos \omega t$, $e_4 = 20 \sin\left(\omega t - \frac{\pi}{4}\right)$
[52.14 sin (ωt + 23.57°)]

(e) $v_1 = 10 \sin \omega t$, $v_2 = -10 \cos \omega t$ *[14.14 sin (ωt – 45°)]*

3.2 Three alternating currents $i_1 = 141 \sin\left(\omega t + \frac{\pi}{4}\right)$, $i_2 = 30 \sin\left(\omega t + \frac{\pi}{2}\right)$ and $i_3 = 20 \sin\left(\omega t + \frac{\pi}{6}\right)$ are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its rms value.
[167.5 sin (ωt + 45.6°), 118.54]

3.3 Four wires, p, q, r, s are connected to a common point. The currents in lines p, q and r are $6 \sin\left(\omega t + \frac{\pi}{3}\right)$, $5 \cos\left(\omega t + \frac{\pi}{3}\right)$ and $3 \cos\left(\omega t + \frac{\pi}{3}\right)$ respectively. Find the current in the wire s. *[9.99 sin (ωt – 66.84°)]*

3.4 A sinusoidal voltage $V_m \sin \omega t$ is applied to three parallel branches yielding branch currents as follows:

$i_1 = 14.14 \sin (\omega t - 45°)$ $\quad i_2 = 28.3 \cos (\omega t - 60°)$ $\quad i_3 = 7.07 \sin (\omega t + 60°)$

Find the complete expression for the source current. Draw the phasor diagram in terms of effective values. Use the voltage as reference. *[39.4 sin (ωt + 15.1°)]*

3.5 A sinusoidal voltage $V_m \sin \omega t$ is applied to three parallel branches. Two of the branch currents are given by

$$i_1 = 14.14 \sin(\omega t - 37°) \qquad i_2 = 28.28 \cos(\omega t - 143°)$$

The source current is found to be $i = 63.8 \sin(\omega t + 12.8°)$.

(i) Find the effective value of the current in the third branch.

(ii) Write the complete time expression for the instantaneous value of the current in part (i).

(iii) Draw the phasor diagram showing the source current and the three branch currents. Use voltage as the reference phasor.

[39.95A, 56.51 sin (ωt + 53.12°)]

3.6 A voltage wave $e(t) = 170 \sin 120\, t$ produces a net current of $14.14 \sin 120\, t + 7.07 \cos(120\, t + 30°)$.

(i) Express the effective value of the current as a single phasor quantity.

(ii) Draw the phasor diagram. Show the components of the current as well as the resultant.

(iii) Determine the power delivered by the source. *[8.66 ∠ 30°A, 901.7 W]*

3.7 The instantaneous values of two alternating voltages are represented by $v_1 = 60 \sin \theta$ and $v_2 = 40 \sin(\theta - \pi/3)$. Derive an expression for the instantaneous value of

(i) the sum, and

(ii) the difference of these voltages.

[87.17 sin (θ – 23.41°), 52.91 sin (θ + 40.89°)]

Review Questions

3.1 Define the following terms related to alternating quantities:

(i) Waveform	(ii) Cycle	(iii) Periodic time
(iv) rms value	(v) Average value	(vi) Form factor
(vii) Peak factor	(viii) Amplitude	(ix) Frequency

3.2 Explain the concepts of phase and phase difference in alternating quantities.

3.3 Derive an expression for the average value of a sinusoidally varying current in terms of peak value.

3.4 Find the rms value of a full-wave sinusoidal current whose maximum value is I_m.

3.5 Derive an expression for the rms value of a sinusoidally varying quantity in terms of its peak value.

3.6 Define phasors and explain how phasors can be used to represent a sinusoidal quantity.

3.7 What is the physical significance of the operator '*j*'.

3.8 What are the different mathematical representations of phasors?

Multiple Choice Questions

Choose the correct alternative in the following questions:

3.1 A current is said to be alternating when it changes in
(a) magnitude only (b) direction only
(c) both magnitude and direction (d) neither magnitude nor direction

3.2 An alternating current of 50 Hz frequency and 100 A maximum value is given as
(a) $i = 200 \sin 628\, t$ (b) $i = 100 \sin 314\, t$
(c) $i = 100\sqrt{2}\ \sin 314\, t$ (d) $i = 100\sqrt{2} \sin 157\, t$

3.3 An alternating current of 50 Hz frequency has a maximum value of 100 A. Its value $\frac{1}{600}$ second after the instant the current is zero will be
(a) 25 A (b) 12.5 A (c) 50 A (d) 75 A

3.4 A sinusoidal voltage varies from zero to a maximum of 250 V. The voltage at the instant of 60° of the cycle will be
(a) 150 V (b) 216.5 V (c) 125 V (d) 108.25 V

3.5 An alternating current is given by the expression $i = 200 \sin\left(314t + \frac{\pi}{3}\right)$ amperes. The maximum value and frequency of the current are
(a) 200 A, 50 Hz (b) $100\sqrt{2}$, 50 Hz
(c) 200 A, 100 Hz (d) 200 A, 25 Hz

3.6 The average value of the current $i = 200 \sin t$ from $t = 0$ to $t = \frac{\pi}{2}$ is
(a) $400\,\pi$ (b) $\frac{400}{\pi}$ (c) $\frac{1}{400}$ (d) $\frac{\pi}{400}$

3.7 The rms value of a half-wave rectified current is 50 A. Its rms value for full wave rectification would be
(a) 100 A (b) 70.7 A (c) $\frac{50}{\pi}$ A (d) $\frac{100}{\pi}$ A

3.8 When the two quantities are in quadrature, the phase angle between them will be
(a) 45° (b) 90° (c) 135° (d) 60°

3.9 The phase difference between the two waveforms can be compared when they
(a) have the same frequency (b) have the same peak value
(c) have the same effective value (d) are sinusoidal

3.10 If two sinusoids of the same frequency but of different amplitude and phase difference are added, the resultant is a
(a) sinusoid of the same frequency
(b) sinusoid of double the original frequency
(c) sinusoid of half the original frequency
(d) non-sinusoid

3.11 Two alternating currents represented as $i_1 = 4 \sin \omega t$ and $i_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ are fed into a common conductor. The rms value of the resultant current is
(a) 9.62 A (b) 8.83 A (c) 12.48 A (d) 13.6 A

3.12 A constant current of 2.8 A exists in a resistor. The rms value of the current is
(a) 2.8 A (b) 2 A (c) 1.4 A (d) undefined

3.13 The current through a resistor has a waveform as shown in Fig. 3.50. The reading shown by a moving coil ammeter will be

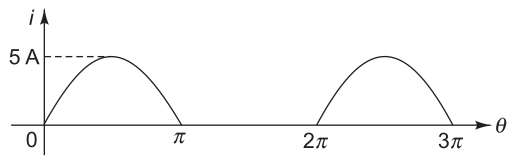


Fig. 3.50

(a) $\frac{5}{\sqrt{2}}$ A (b) $\frac{2.5}{\sqrt{2}}$ A (c) $\frac{5}{\pi}$ A (d) 0

3.14 Which of the following statements about the two alternating currents shown in Fig 3.51 is correct?

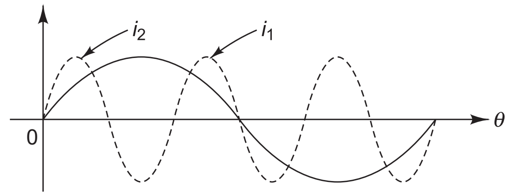


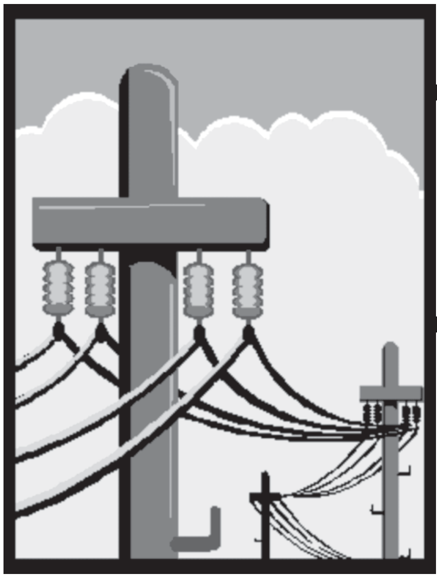
Fig. 3.51

(a) The peak values of i_1 and i_2 are different
(b) The rms values of i_1 and i_2 are different
(c) The time period of current i_1 is more than that of the current i_2
(d) The frequency of current i_1 is more than that of the current i_2.

3.15 The alternating voltage $e = 200 \sin 314\, t$ is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction while entirely preventing the flow in the opposite direction. The overage value of the current will be
(a) 5 A (b) 3.18 A (c) 1.57 A (d) 1.10 A

Answers to Multiple Choice Questions

3.1 (c)	**3.2** (b)	**3.3** (c)	**3.4** (b)	**3.5** (a)	**3.6** (b)
3.7 (b)	**3.8** (b)	**3.9** (a)	**3.10** (a)	**3.11** (c)	**3.12** (a)
3.13 (c)	**3.14** (c)	**3.15** (b)			



Chapter 4

Single-Phase AC Circuits

Chapter Outline

4.1 BEHAVIOUR OF A PURE RESISTOR IN AN AC CIRCUIT

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig. 4.1. Let the alternating voltage be $v = V_m \sin \omega t$.

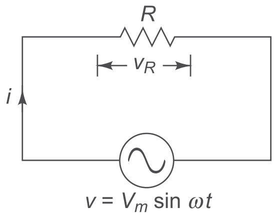


Fig. 4.1 *Purely resistive circuit*

Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Waveforms The voltage and current waveforms are shown in Fig. 4.2.

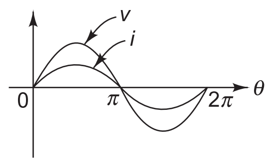


Fig. 4.2 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.3. The voltage and current phasors are drawn in phase and there is no phase difference.

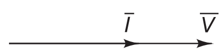


Fig. 4.3 *Phasor diagram*

Impedance It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

Power factor = $\cos\phi = \cos(0°) = 1$

Power Instantaneous power p is given by

$$\begin{aligned} p &= vi \\ &= V_m \sin \omega t\, I_m \sin \omega t \\ &= V_m I_m \sin^2 \omega t \\ &= \frac{V_m I_m}{2}(1-\cos 2\omega t) \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t \end{aligned}$$

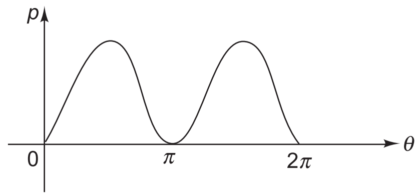


Fig. 4.4 *Power waveform*

The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$. The frequency of the fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

4.2 BEHAVIOUR OF A PURE INDUCTOR IN AN AC CIRCUIT

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. 4.5. Let the alternating voltage be $v = V_m \sin \omega t$.

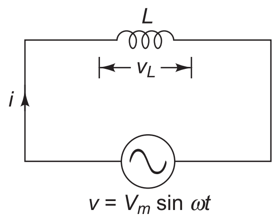


Fig. 4.5 *Purely inductive circuit*

Current The alternating current i is given by

$$\begin{aligned} i &= \frac{1}{L}\int v\,dt \\ &= \frac{1}{L}\int V_m \sin \omega t\, dt \\ &= \frac{V_m}{\omega L}(-\cos \omega t) \end{aligned}$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \cdots \left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms The voltage and current waveforms are shown in Fig. 4.6.

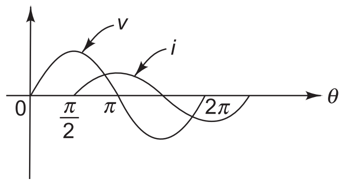


Fig. 4.6 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.7. Here, voltage $\bar{V}$ is chosen as reference phasor. Current $\bar{I}$ is drawn such that it lags behind $\bar{V}$ by 90°.

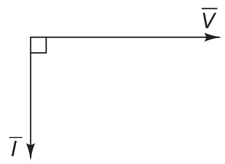


Fig. 4.7 *Phasor diagram*

Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$

Thus, an inductor acts as a short circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous powers p is given by

$$p = vi$$

$$= V_m \sin\omega t \, I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely inductive circuit is zero.

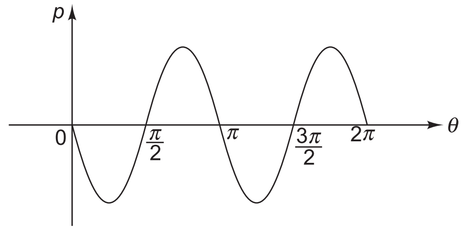


Fig. 4.8 *Power waveform*

During first $\frac{\pi}{2}$ duration of cycle, the power is negative and power flows from the inductor to source. During the $\frac{\pi}{2}$ to π duration of cycle, the power is positive and power flows from the source to the inductor. The same cycle repeats from π to 2π. Hence, the resultant power over one cycle (upto $\theta = 2\pi$) is zero, i.e., the pure inductor consumes no power.

When power is positive, the energy is supplied from the source to build up the magnetic field in the inductor. When power is negative, the energy is returned to the source and magnetic field collapses. Hence, power circulates in the purely inductive circuit. The circulating power is called as reactive power.

4.3 BEHAVIOUR OF A PURE CAPACITOR IN AN AC CIRCUIT

[Dec 2013]

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig. 4.9. Let the alternating voltage be $v = V_m \sin \omega t$.

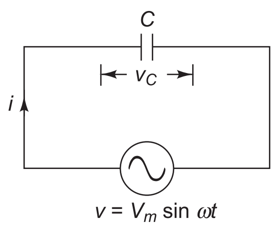


Fig. 4.9 *Purely capacitive circuit*

Current The alternating current i is given by

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}(V_m \sin \omega t)$$

$$= \omega C\, V_m \cos \omega t$$
$$= \omega C\, V_m \sin (\omega t + 90°)$$
$$= I_m \sin (\omega t + 90°) \quad \ldots (I_m = \omega C\, V_m)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

Waveforms The voltage and current waveforms are shown in Fig. 4.10.

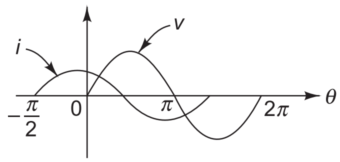


Fig. 4.10 *Waveforms*

Phasor Diagram The phasor diagram is shown in Fig. 4.11. Here, voltage $\overline{V}$ is chosen as reference phasor. Current $\overline{I}$ is drawn such that it leads $\overline{V}$ by 90°.

Fig. 4.11 *Phasor diagram*

Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_C = \infty$

Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

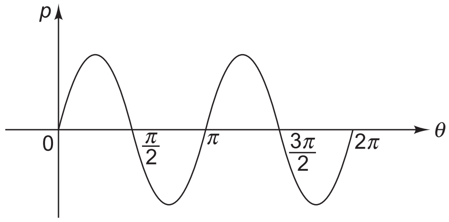


Fig. 4.12 *Power waveform*

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos \phi = \cos (90°) = 0$$

Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin \omega t\, I_m \sin (\omega t + 90)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely capacitive circuit is zero.

When power is positive, i.e., voltage increases across the plates of capacitor, energy is supplied form source to build up the electrostatic field between the plates of capacitor and the capacitor is energized. When power is negative, i.e., voltage decreases, the collapsing electrostatic field returns the stored energy to the source. This circulating power is called as reactive power.

Example 1

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution

$$R = 10\ \Omega$$
$$V = 230\text{ V}$$
$$f = 50\text{ Hz}$$

(i) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23\text{ A}$$

(ii) Power consumed

$$P = VI = 230 \times 23 = 5290\text{ W}$$

(iii) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$\text{pf} = \cos \phi = \cos (0°) = 1$$

(iv) Voltage and current equations

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 230 = 325.27\text{ V}$$
$$I_m = \sqrt{2}\, I = \sqrt{2} \times 23 = 32.53\text{ A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$
$$v = V_m \sin \omega t = 325.27 \sin 314.16\, t$$
$$i = I_m \sin \omega t = 32.53 \sin 314.16\, t$$

Example 2

A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current. **[Dec 2012]**

Solution

$$V_{\text{rms}} = 150\text{ V}$$
$$f = 50\text{ Hz}$$

$$L = 0.2 \text{ H}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83 \ \Omega$$
$$V_m = V_{\text{rms}} \sqrt{2} = 150 \sqrt{2} = 212.13 \text{ V}$$
$$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.38 \text{ A}$$
$$v = V_m \sin 2\pi f t = 212.13 \sin 2\pi \times 50 \times t = 212.13 \sin 100\pi t$$
$$i = I_m \sin(2\pi f t - 90°) = 3.38 \sin(100\pi t - 90°)$$

Example 3

An inductive coil having negligible resistance and 0.1 henry inductance is connected across a 200 V, 50 Hz supply. Find (i) inductive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$$L = 0.1 \text{ H}$$
$$V = 200 \text{ V}$$
$$f = 50 \text{ Hz}$$

(i) Inductive reactance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42 \ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_L} = \frac{200}{31.42} = 6.37 \text{ A}$$

(iii) Power

Since the current lags behind the voltage by 90° in purely inductive circuit, $\phi = 90°$

$$P = VI \cos \phi = 200 \times 6.37 \times \cos(90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos \phi = \cos(90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 200 = 282.84 \text{ V}$$
$$I_m = \sqrt{2}\, I = \sqrt{2} \times 6.37 = 9 \text{ A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$
$$v = V_m \sin \omega t = 282.84 \sin 314.16\, t$$
$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = 9 \sin\left(314.16\, t - \frac{\pi}{2}\right)$$

Example 4

The voltage and current through circuit elements are

$$v = 100 \sin(314\, t + 45°) \text{ volts}$$
$$i = 10 \sin(314\, t + 315°) \text{ amperes}$$

(i) Identify the circuit elements. (ii) Find the value of the elements. (iii) Obtain an expression for power.

Solution

$$v = 100 \sin (314\, t + 45°)$$
$$i = 10 \sin (314\, t + 315°)$$
$$= 10 \sin (314\, t + 315° - 360°)$$
$$= 10 \sin (314\, t - 45°)$$

(i) Identification of elements

From voltage and current equations, it is clear that the current i lags behind the voltage by 90°. Hence, the circuit element is an inductor.

(ii) Value of elements

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega$$
$$X_L = \omega L$$
$$10 = 314\, L$$
$$L = 31.8 \text{ mH}$$

(iii) Expression for power

$$p = -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{100 \times 10}{2} \sin(2 \times 314 t) = -500 \sin 628\, t$$

Example 5

A capacitor has a capacitance of 30 microfarads which is connected across a 230 V, 50 Hz supply. Find (i) capacitive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$$C = 30\ \mu\text{F}$$
$$V = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

(i) Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17 \text{ A}$$

(iii) Power

Since the current leads the voltage by 90° in purely capacitive circuit, $\phi = 90°$

$$P = VI \cos \phi = 230 \times 2.17 \times \cos (90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos \phi = \cos (90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 230 = 325.27 \text{ V}$$
$$I_m = \sqrt{2}\, I = \sqrt{2} \times 2.17 = 3.07 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$
$$v = V_m \sin \omega t = 325.27 \sin 314.16\, t$$
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

Example 6

Find the rms value of current flowing through a 314 μF capacitor when connected to a 230 V, 50 Hz, 1ϕ AC supply. **[Dec 2015]**

Solution

$$C = 314\ \mu\text{F}$$
$$V = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 314 \times 10^{-6}} = 10.14\ \Omega$$
$$I = \frac{V}{X_C} = \frac{230}{10.14} = 22.68 \text{ A}$$

4.4 SERIES *R-L* CIRCUIT

Figure 4.13 shows a pure resistor R connected in series with a pure inductor L across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90°.

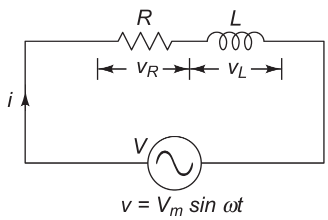


Fig. 4.13 *Series R-L circuit*

Phasor Diagram

Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\bar{I}$ is taken as reference phasor.
2. Draw $\bar{V}_R$ in phase with $\bar{I}$.
3. Draw $\bar{V}_L$ such that it leads $\bar{I}$ by 90°.
4. Add $\bar{V}_R$ and $\bar{V}_L$ by triangle law of vector addition such that
$$\bar{V} = \bar{V}_R + \bar{V}_L$$
5. Mark the angle between $\bar{I}$ and $\bar{V}$ as ϕ.

The phasor diagram is shown in Fig. 4.14.

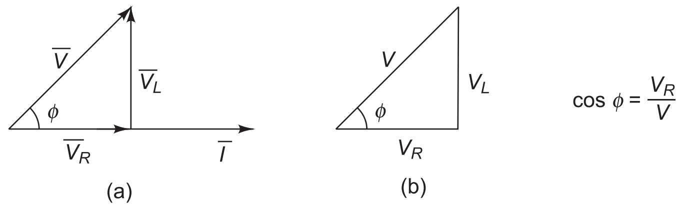


Fig. 4.14 *(a) Phasor diagram (b) Voltage triangle*

It is clear from phasor diagram that current $\overline{I}$ lags behind applied voltage $\overline{V}$ by an angle ϕ ($0° < \phi < 90°$).

Impedance

$$\overline{V} = \overline{V}_R + \overline{V}_L = R\overline{I} + jX_L\,\overline{I} = (R + jX_L)\,\overline{I}$$

$$\frac{\overline{V}}{\overline{I}} = R + jX_L = \overline{Z}$$

$$\overline{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the *R-L* circuit.

Impedance Triangle The impedance triangle is shown in Fig. 4.15.

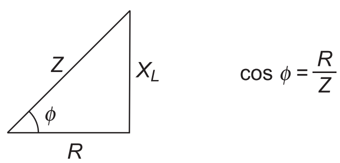


Fig. 4.15 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t - \phi)$$

where
$$I_m = \frac{V_m}{Z}$$

and
$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Waveforms The voltage and current waveforms are shown in Fig. 4.16.

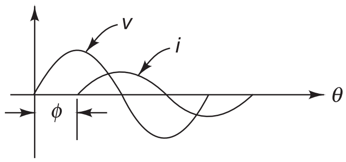


Fig. 4.16 *Waveforms*

Power Instantaneous power p is given by

$$p = v\, i$$
$$= V_m \sin \omega t\, I_m \sin (\omega t - \phi)$$
$$= V_m I_m \sin \omega t \sin (\omega t - \phi)$$
$$= V_m I_m \left[\frac{\cos\phi - \cos(2\omega t - \phi)}{2} \right]$$
$$= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t - \phi)$$

Thus, power consists of a constant part $\frac{V_m I_m}{2}$ cos ϕ and a fluctuating part $\frac{V_m I_m}{2}$ cos $(2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi = VI \cos \phi$$

Thus, power is dependent upon the in-phase component of the current. The average power is also called *active power* and is measured in watts.

We know that a pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called *reactive power*. It is a product of the voltage and reactive component of the current, i.e., $I \sin \phi$ and is measured in VAR (volt–ampere-reactive).

Reactive power $Q = VI \sin \phi$.

The product of voltage and current is known as *apparent power* (S) and is measured in volt–ampere (VA).

$$S = \sqrt{P^2 + Q^2}$$

Power Triangle In terms of circuit components,

$$\cos \phi = \frac{R}{Z}$$

and $\quad V = Z\, I$

$$P = VI\cos\phi = Z\,I\,I\,\frac{R}{Z} = I^2R$$

$$Q = VI\sin\phi = Z\,I\,I\,\frac{X_L}{Z} = I^2X_L$$

$$S = VI = Z\,I\,I = I^2Z$$

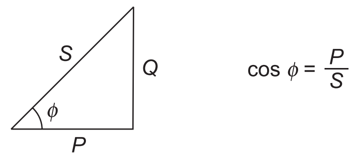


Fig. 4.17 *Power triangle*

The power triangle is shown in Fig. 4.17.

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle, $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an *R-L* series circuit, the power factor is lagging in nature since the current lags behind the voltage by an angle ϕ.

Example 1

An alternating voltage of 80 + j60 V is applied to a circuit and the current flowing is 4 – j2 A. Find the (i) impedance, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution $\bar{V} = 80 + j60$ V

$\bar{I} = 4 - j2$ A

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87°}{4.47\angle -26.56°} = 22.37\angle 63.43°\ \Omega$$

$$Z = 22.37\ \Omega$$

(ii) Phase angle

$$\phi = 63.43°$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(63.43°) = 0.447 \text{ (lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 100 \times 4.47 \times 0.447 = 199.81 \text{ W}$$

Example 2

The voltage and current in a circuit are given by $\bar{V} = 150\ \angle 30°$ V and $\bar{I} = 2\ \angle -15°$ A. If the circuit works on a 50 Hz supply, determines impedance, resistance, reactance, power factor and power loss considering the circuit as a simple series circuit.

Solution

$$\bar{V} = 150\ \angle 30°\ \text{V}$$

$$\bar{I} = 2\angle -15°\ \text{A}$$

$$f = 50\ \text{Hz}$$

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{150\angle 30°}{2\angle -15°} = 75\ \angle 45°\ \Omega = 53.03 + j53.03\ \Omega$$

$$Z = 75\ \Omega$$

(ii) Resistance

$$R = 53.03\ \Omega$$

(iii) Reactance

$$X = 53.03\ \Omega$$

(iv) Power factor

$$\phi = 45°$$

$$\text{pf} = \cos\phi = \cos(45°) = 0.707\ \text{(lagging)}$$

(v) Power loss

$$P = VI\cos\phi = 150 \times 2 \times 0.707 = 212.1\ \text{W}$$

Example 3

An rms voltage of 100 $\angle 0°$ V is applied to a series combination of Z_1 and Z_2 when $Z_1 = 20\ \angle 30°\,\Omega$. The effective voltage drop across Z_1 is known to be 40 $\angle -30°$ V. Find the reactive component of Z_2.

Solution

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{Z}_1 = 20\ \angle 30°\ \Omega$$

$$\bar{V}_1 = 40\ \angle -30°\ \text{V}$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{40\angle -30°}{20\angle 30°} = 2\angle -60°\ \text{A}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{100\angle 0°}{2\angle -60°} = 50\ \angle 60° = 25 + j43.3\ \Omega$$

$$\bar{Z}_1 = 20\ \angle 30° = 17.32 + j10\ \Omega$$

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$

$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 25 + j43.3 - 17.32 - j10 = 7.68 + j33.3\ \Omega$$

Reactive component of $\bar{Z}_2 = 33.3\ \Omega$

Example 4

A voltage v(t) = 177 sin (314t + 10°) is applied to a circuit. It causes a steady-state current to flow, which is described by i(t) = 14.14 sin (314t – 20°). Determine the power factor and average power delivered to the circuit.

Solution

$$v(t) = 177 \sin (314t + 10°)$$

$$i(t) = 14.14 \sin (314t - 20°)$$

(i) Power factor

Current $i(t)$ lags behind voltage $v(t)$ by 30°.

$$\phi = 30°$$

$$\text{pf} = \cos \phi = \cos (30°) = 0.866 \text{ (lagging)}$$

(ii) Average power

$$P = VI \cos \phi = \frac{177}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times 0.866 = 1083.7 \text{ W}$$

Example 5

When a sinusoidal voltage of 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by i(t) = 28.3 sin (314t – φ). Find the circuit resistance and inductance.

Solution

$$V = 120 \text{ V}$$

$$P = 1200 \text{ W}$$

$$i(t) = 28.3 \sin (314t - \phi)$$

(i) Resistance

$$I = \frac{28.3}{\sqrt{2}} = 20.01 \text{ A}$$

$$P = VI \cos \phi$$

$$1200 = 120 \times 20.01 \times \cos \phi$$

$$\cos \phi = 0.499$$

$$\phi = 60.02°$$

$$Z = \frac{V}{I} = \frac{120}{20.01} = 6 \ \Omega$$

$$\overline{Z} = Z \angle\phi = 6 \angle 60.02° = 3 + j5.2 \ \Omega$$

$$R = 3 \ \Omega$$

(ii) Inductance

$$X_L = 5.2 \ \Omega$$

$$X_L = \omega L$$

$$5.2 = 314 \times L$$

$$L = 0.0165 \text{ H}$$

Example 6

In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$ and $v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the average power drawn by the circuit?

Solution

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$

$$v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$$

(i) Impedance

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{20}{5} = 4\ \Omega$$

(ii) Power factor, resistance and inductance

Current $i(t)$ lags behind voltage $v(t)$ by an angle $\phi = 150° - 120° = 30°$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

$$\bar{Z} = 4\angle 30° = 3.464 + j2\ \Omega$$

$$R = 3.464\ \Omega$$

$$X_L = 2\ \Omega$$

$$X_L = \omega L$$

$$2 = 314 \times L$$

$$L = 6.37 \text{ mH}$$

(iii) Average power

$$P = VI\cos\phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3 \text{ W}$$

Example 7

A series circuit consists of a non-inductive resistance of 6 Ω and an inductive reactance of 10 Ω. When connected to a single-phase ac supply, it draws a current $i(t) = 27.89 \sin(628t - 45°)$. Calculate (i) the voltage applied to the series circuit in the form $V_m \sin(\omega t \pm \phi)$, (ii) inductance, and (iii) power drawn by the circuit.

Solution

$$R = 6\ \Omega$$

$$X_L = 10\ \Omega$$

$$i(t) = 27.89 \sin(628t - 45°)$$

(i) Voltage applied to the series circuit

$$\bar{Z} = R + jX_L = 6 + j10 = 11.66\angle 59.04°\ \Omega$$

$$\bar{I} = \frac{27.89}{\sqrt{2}} \angle{-45°} = 19.72 \angle{-45°} \text{ A}$$

$$\bar{V} = \bar{Z}\ \bar{I} = (11.66 \angle 59.04°)\ (19.72 \angle{-45°}) = 229.95 \angle 14.04° \text{ V}$$

$$v = 229.95\sqrt{2} \sin(\omega t + 14.04°) = 325.2 \sin(\omega t + 14.04°)$$

(ii) Inductance

$$X_L = \omega L$$

$$10 = 628 \times L$$

$$L = 15.9 \text{ mH}$$

(iii) Power drawn by the circuit

$$P = VI \cos\phi = 229.95 \times 19.72 \times \cos(59.04°) = 2332.78 \text{ W}$$

Example 8

A coil having a resistance of 10 Ω and an inductance of 40 mH is connected to a 200 V, 50 Hz supply. Calculate the impedance of the coil, current, power factor and power consumed. **[May 2015]**

Solution $r = 10\ \Omega$

$L = 40$ mH

$V = 200$ V

$f = 50$ Hz

$X_L = 2\pi f L = 2\pi \times 50 \times 40 \times 10^{-3} = 12.57\ \Omega$

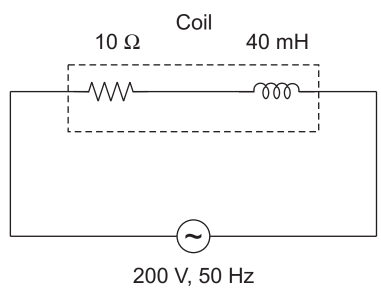


Fig. 4.18

(i) Impedance of coil

$$\bar{Z}_{\text{coil}} = r + jX_L = 10 + j\,12.57 = 16.06 \angle 51.5°\ \Omega$$

(ii) Current

$$I = \frac{V}{Z_{\text{coil}}} = \frac{200}{16.06} = 12.45 \text{ A}$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(51.5°) = 0.62 \text{ (lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 200 \times 16.06 \times 0.62 = 2 \text{ kW}$$

Example 9

When an inductive coil is connected to a dc supply at 240 V, the current in it is 16 A. When the same coil is connected to an ac supply at 240 V, 50 Hz, the current is 12.27 A. Calculate (i) resistance, (ii) impedance, (iii) reactance, and (iv) inductance of the coil.

Solution For dc: $V = 240$ V, $I = 16$ A

For ac: $V = 240$ V, $I = 12.27$ A

(i) Resistance

When an inductive coil is connected to a dc supply,

$$f = 0$$

$$X_L = 2\pi fL = 0$$

The coil behaves like a pure resistor.

$$R = \frac{V}{I} = \frac{240}{16} = 15\ \Omega$$

(ii) Impedance

When the coil is connected to an ac supply,

$$Z = \frac{V}{I} = \frac{240}{12.27} = 19.56\ \Omega$$

(iii) Reactance

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(19.56)^2 - (15)^2} = 12.55\ \Omega$$

(iv) Inductance

$$X_L = 2\pi fL$$

$$12.55 = 2\pi \times 50 \times L$$

$$L = 0.04\ \text{H}$$

Example 10

An inductive coil draws 10 A current and consumes 1 kW power from a 200 V, 50 Hz ac supply. Determine (i) impedance in Cartesian and polar forms, (ii) power factor, and (iii) reactive and apparent power.

Solution

$$I = 10\ \text{A}$$

$$P = 1\ \text{kW}$$

$$V = 200\ \text{V}$$

$$f = 50\ \text{Hz}$$

(i) Impedance in Cartesian and polar forms

$$Z = \frac{V}{I} = \frac{200}{10} = 20\ \Omega$$

$$P = VI\cos\phi$$
$$1000 = 200 \times 10 \times \cos\phi$$
$$\cos\phi = 0.5$$
$$\phi = 60°$$

Expressing $\bar{Z}$ in polar form,

$$\bar{Z} = Z\angle\phi° = 20\angle 60°\ \Omega$$

Expressing $\bar{Z}$ in Cartesian form,

$$\bar{Z} = 10 + j17.32\ \Omega$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(60°) = 0.5 \text{ (lagging)}$$

(iii) Reactive and apparent power

$$Q = VI\sin\phi = 200 \times 10 \times \sin(60°) = 1.73 \text{ kVAR}$$
$$S = VI = 200 \times 10 = 2 \text{ kVA}$$

Example 11

A coil connected across a 250 V, 50 Hz supply takes a current of 10 A at 0.8 lagging power factor. What will be the power taken by the choke coil when connected across a 200 V, 25 Hz supply? Also calculate resistance and inductance of the coil.

Solution

$$V_1 = 250 \text{ V}$$
$$f_1 = 50 \text{ Hz}$$
$$I_1 = 10 \text{ A}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$
$$V_2 = 200 \text{ V}$$
$$f_2 = 25 \text{ Hz}$$

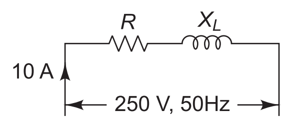


Fig. 4.19

(i) Resistance and inductance of the coil

$$Z_1 = \frac{V_1}{I_1} = \frac{250}{10} = 25\ \Omega$$
$$\phi_1 = \cos^{-1}(0.8) = 36.87°$$
$$\bar{Z}_1 = Z_1\angle\phi_1 = 25\angle 36.87° = 20 + j15\ \Omega$$
$$R = 20\ \Omega$$
$$X_{L_1} = 15\ \Omega$$
$$X_{L_1} = 2\pi f_1 L$$
$$15 = 2\pi \times 50 \times L$$
$$L = 0.0477 \text{ H}$$

(ii) Power taken by the choke coil when connected across a 200 V, 50 Hz supply.

$$X_{L_2} = 2\pi f_2 L = 2\pi \times 25 \times 0.0477 = 7.49\ \Omega$$

$$\bar{Z}_2 = R + jX_{L_2} = 20 + j\,7.49 = 21.36\,\angle 20.53°\ \Omega$$
$$\bar{Z}_2 = 21.36\ \Omega$$
$$\phi_2 = 20.53°$$
$$I_2 = \frac{V_2}{Z_2} = \frac{200}{21.36} = 9.36\ \text{A}$$
$$P = V_2 I_2 \cos\phi_2 = 200 \times 9.36 \times \cos(20.53°) = 1.753\ \text{kW}$$

Example 12

A load of 22 kW operates at 0.8 lagging power factor when connected to a 420 V, single-phase, 50 Hz source. Find (i) current in the load, (ii) power factor angle, (iii) impedance, (iv) resistance of load, (v) reactance of load, (vi) voltage and current equations.

Solution

$$P = 22\ \text{kW}$$
$$\text{pf} = 0.8\ \text{(lagging)}$$
$$V = 420\ \text{V}$$
$$f = 50\ \text{Hz}$$

(i) Current in the load

$$P = VI\cos\phi$$
$$22 \times 10^3 = 420 \times I \times 0.8$$
$$I = 65.48\ \text{A}$$

(ii) Power-factor angle

$$\phi = \cos^{-1}(0.8) = 36.87°$$

(iii) Impedance

$$Z = \frac{V}{I} = \frac{420}{65.48} = 6.41\ \Omega$$

(iv) Resistance of load

$$R = Z\cos\phi = 6.41 \times 0.8 = 5.13\ \Omega$$

(v) Reactance of load

$$X_L = Z\sin\phi = 6.41 \times \sin(36.87°) = 3.85\ \Omega$$

(vi) Voltage and current equations

$$v = V_m \sin 2\pi ft = 420\sqrt{2}\ \sin(2\pi \times 50)\,t = 593.97 \sin 100\pi t$$

The current lags behind the voltage by 36.87°

$$i = I_m \sin(2\pi ft - \phi) = 65.48\sqrt{2}\ \sin(2\pi \times 50\,t - 36.87°)$$
$$= 92.6 \sin(100\pi t - 36.87°)$$

Example 13

A choke coil is connected in series with a fixed resistor. A 240 V, 50 Hz supply is applied and a current of 2.5 A flows. If the voltage drops across the coil and fixed resistor are 140 V and 160 V respectively, calculate the value of the fixed resistance, the resistance and inductance of the coil, and power drawn by the coil.

Solution

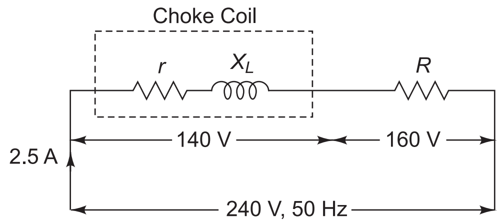


Fig. 4.20

$$V = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 2.5 \text{ A}$$
$$V_{\text{coil}} = 140 \text{ V}$$
$$V_R = 160 \text{ V}$$

(i) Resistance of fixed resistor

$$R = \frac{V_R}{I} = \frac{160}{2.5} = 64\ \Omega$$

(ii) Resistance and inductance of the coil

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{140}{2.5} = 56\ \Omega$$
$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = 56$$
$$r^2 + X_L^2 = 3136 \qquad (1)$$
$$Z = \frac{V}{I} = \frac{240}{2.5} = 96\ \Omega$$
$$\bar{Z} = (R + r) + jX_L$$
$$Z = \sqrt{(R + r)^2 + X_L^2}$$
$$96 = \sqrt{(64 + r)^2 + X_L^2}$$
$$(64 + r)^2 + X_L^2 = 9216 \qquad (2)$$

Subtracting Eq. (2) from Eq. (1),

$$(64 + r)^2 - r^2 = 6080$$
$$4096 + 128\, r + r^2 - r^2 = 6080$$
$$128\, r = 1984$$
$$r = 15.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(15.5)^2 + X_L^2 = 3136$$
$$X_L^2 = 2895.75$$

$$X_L = 53.81\ \Omega$$
$$X_L = 2\pi f L$$
$$53.81 = 2\pi \times 50 \times L$$
$$L = 0.17\ \text{H}$$

(iii) Power drawn by the coil

$$P_{\text{coil}} = I^2 r = (2.5)^2 \times 15.5 = 96.875\ \text{W}$$

Example 14

A 100 Ω resistor is connected in series with a choke coil. When a 400 V, 50 Hz supply is applied to this combination, the voltages across the resistance and the choke coil are 200 V and 300 V respectively. Find the power consumed by the choke coil. Also, calculate the power factor of the choke coil and the power factor of the circuit. **[Dec 2012]**

Solution

$$R = 100\ \Omega$$
$$V = 400\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$V_R = 200\ \text{V}$$
$$V_{\text{coil}} = 300\ \text{V}$$

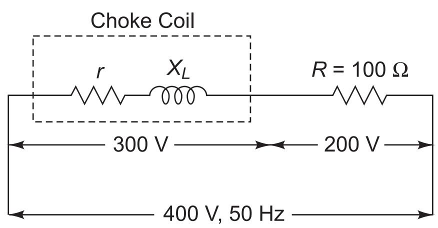


Fig. 4.21

(i) Power consumed by choke coil

$$I = \frac{V_R}{R} = \frac{200}{100} = 2\ \text{A}$$
$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{300}{2} = 150\ \Omega$$
$$\sqrt{r^2 + X_L^2} = 150$$
$$r^2 + X_L^2 = 22500 \quad (1)$$
$$Z = \frac{V}{I} = \frac{400}{2} = 200\ \Omega$$
$$\bar{Z} = (R + r) + jX_L$$
$$Z = \sqrt{(R + r)^2 + X_L^2} = 200$$
$$(100 + r)^2 + X_L^2 = 40000 \quad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$(100 + r)^2 - r^2 = 17500$$
$$10000 + 200r + r^2 - r^2 = 17500$$
$$200r = 7500$$

$$r = 37.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(37.5)^2 + X_L^2 = 22500$$
$$X_L^2 = 21093.75$$
$$X_L = 145.24\ \Omega$$
$$P_{\text{coil}} = I^2 r = (2)^2 \times 37.5 = 50\ \text{W}$$

(ii) Power factor of the choke coil

$$\text{pf}_{\text{coil}} = \frac{r}{Z_{\text{coil}}} = \frac{37.5}{150} = 0.25 \text{ (lagging)}$$

(iii) Power factor of the circuit

$$\text{pf}_{\text{circuit}} = \frac{R+r}{Z} = \frac{100+37.5}{200} = 0.6875 \text{ (lagging)}$$

Example 15

A resistor of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250 V, 50 Hz supply, draws a current of 4 A which lags behind the voltage by 65°. Calculate (i) resistance and inductance of the coil, (ii) total power, (iii) power consumed by resistance, and (iv) power consumed by choke coil. **[May 2014]**

Solution

$$R = 25\ \Omega$$
$$V = 250\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I = 4\ \text{A}$$
$$\phi = 65°$$

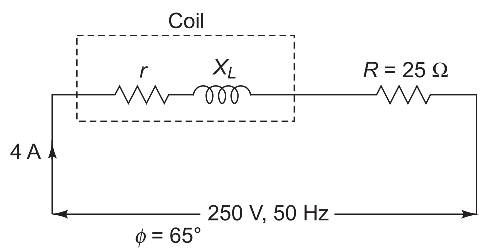


Fig. 4.22

(i) Resistance and inductance of the coil

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\ \Omega$$
$$\bar{Z} = Z\angle\phi = 62.5\angle 65° = 26.41 + j56.64\ \Omega$$

But
$$\bar{Z} = (R + r) + jX_L$$
$$X_L = 56.64\ \Omega$$
$$R + r = 26.41$$
$$r = 26.41 - 25 = 1.41\ \Omega$$
$$X_L = 2\pi f L$$
$$56.64 = 2\pi \times 50 \times L$$
$$L = 0.18\ \text{H}$$

(ii) Total power

$$P = I^2 (R + r) = (4)^2 \times 26.41 = 422.56 \text{ W}$$

(iii) Power consumed by resistance

$$P_R = I^2 R = (4)^2 \times 25 = 400 \text{ W}$$

(iv) Power consumed by choke coil

$$P_{\text{coil}} = I^2 r = (4)^2 \times 1.41 = 22.56 \text{ W}$$

Example 16

When a resistor and a coil in series are connected to a 240 V supply, a current of 3 A flows, lagging 37° behind the supply voltage. The voltage across the coil is 171 volts. Find the resistance and reactance of the coil, and the resistance of the resistor. **[May 2014, Dec 2015]**

Solution

$$V = 240 \text{ V}$$

$$I = 3 \text{ A}$$

$$\phi = 37°$$

$$V_{\text{coil}} = 171 \text{ V}$$

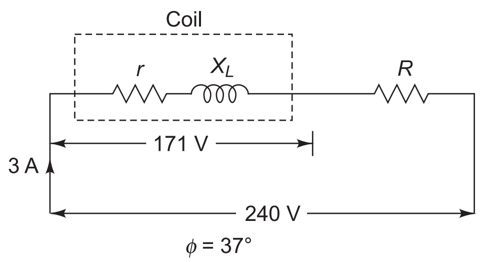


Fig. 4.23

(i) Resistance and reactance of the coil

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{171}{3} = 57\ \Omega$$

$$\sqrt{r^2 + X_L^2} = 57$$

$$r^2 + X_L^2 = 3249$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$\bar{Z} = Z\angle\phi = 80\ \angle 37° = 63.89 + j48.15\ \Omega$$

But

$$\bar{Z} = (R + r) + jX_L$$

$$X_L = 48.15\ \Omega$$

$$r^2 + X_L^2 = 3249$$

$$r^2 + (48.15)^2 = 3249$$

$$r^2 = 931.04$$

$$r = 30.51\ \Omega$$

(ii) Resistance of the resistor

$$R + r = 63.89$$

$$R + 30.51 = 63.89$$

$$R = 33.38\ \Omega$$

Example 17

A choke coil and a resistor are connected in series across a 230 V, 50 Hz ac supply. The circuit draws a current of 2 A at 0.866 lagging pf. The voltage drop across the resistor is 100 V. Calculate the power factor of the choke coil.

Solution

$$V = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 2 \text{ A}$$
$$\text{pf} = 0.866 \text{ (lagging)}$$
$$V_R = 100 \text{ V}$$

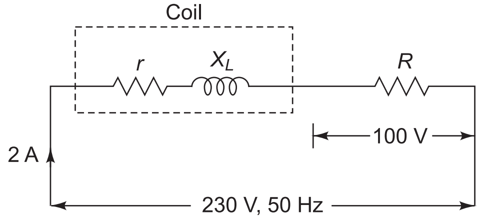


Fig. 4.24

$$R = \frac{V_R}{I} = \frac{100}{2} = 50\ \Omega$$
$$Z = \frac{V}{I} = \frac{230}{2} = 115\ \Omega$$
$$\text{pf} = 0.866 \text{ (lagging)}$$
$$\phi = \cos^{-1}(0.866) = 30°$$
$$\bar{Z} = Z\angle\phi = 115\angle 30° = 99.59 + j57.5\ \Omega$$
$$R + r = 99.59$$
$$50 + r = 99.59$$
$$r = 49.59\ \Omega$$
$$X_L = 57.5\ \Omega$$
$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = \sqrt{(49.59)^2 + (57.5)^2} = 75.93\ \Omega$$
$$\text{pf}_{\text{coil}} = \frac{r}{Z_{\text{coil}}} = \frac{49.59}{75.93} = 0.653 \text{ (lagging)}$$

Example 18

A circuit consists of a pure resistor and coil in series. Power dissipated in the resistor and in the coil are 1000 W and 250 W respectively. The voltage drops across the resistor and the coil are 200 V and 300 V respectively. Determine (i) value of pure resistance, (ii) resistance and reactance of the coil, (iii) combined resistance of the circuit, (iv) combined impedance, and (v) supply voltage.

Solution

$$P_R = 1000 \text{ W}$$
$$P_{\text{coil}} = 250 \text{ W}$$
$$V_R = 200 \text{ V}$$
$$V_{\text{coil}} = 300 \text{ V}$$

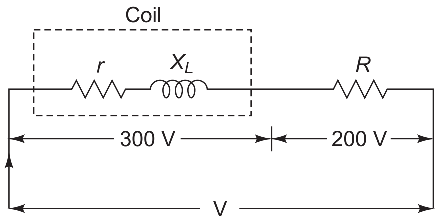


Fig. 4.25

(i) Value of pure resistance

$$P_R = \frac{V_R^2}{R}$$

$$1000 = \frac{(200)^2}{R}$$

$$R = 40\ \Omega$$

(ii) Resistance and reactance of the coil

$$V_R = RI$$

$$200 = 40I$$

$$I = 5\text{ A}$$

$$P_{\text{coil}} = I^2 r$$

$$250 = (5)^2 \times r$$

$$r = 10\ \Omega$$

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{300}{5} = 60\ \Omega$$

$$X_L = \sqrt{Z_{\text{coil}}^2 - r^2} = \sqrt{(60)^2 - (10)^2} = 59.2\ \Omega$$

(iii) Combined resistance of the circuit

$$R_T = R + r = 40 + 10 = 50\ \Omega$$

(iv) Combined impedance

$$Z = \sqrt{(R+r)^2 + X_L^2} = \sqrt{(50)^2 + (59.2)^2} = 77.5\ \Omega$$

(v) Supply voltage

$$V = Z\,I = 77.5 \times 5 = 387.5\text{ V}$$

Example 19

Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and inductance of B is 0.015 H. If the input from supply is 3 kW and 2 kVAR, find inductance of A and resistance of B. Calculate the voltage across each coil. **[May 2013]**

Solution

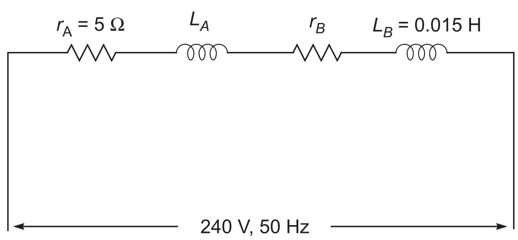


Fig. 4.26

$P = 3$ kW
$Q = 2$ kVAR

(i) Inductance of A and resistance of B

$$\tan\phi = \frac{Q}{P} = \frac{2}{3} = 0.67$$

$$\phi = 33.69°$$

$$P = VI\cos\phi$$

$$3000 = 240 \times I \times \cos(33.69°)$$

$$I = 15.02 \text{ A}$$

$$Z_T = \frac{V}{I} = \frac{240}{15.02} = 15.98\ \Omega$$

$$\bar{Z}_T = Z_T\angle\phi = 15.98\angle 33.69° = (13.3 + j8.86)\ \Omega$$

$$r_T = r_A + r_B = 13.3\ \Omega$$

$$r_B = 8.3\ \Omega$$

$$X_B = 2\pi f L_B = 2\pi \times 50 \times 0.015 = 4.71\ \Omega$$

$$X_T = X_A + X_B = 8.86$$

$$X_A = 4.15\ \Omega$$

$$X_A = 2\pi f L_A$$

$$4.15 = 2\pi \times 50 \times L_A$$

$$L_A = 0.013 \text{ H}$$

(ii) Voltage across each coil

$$Z_A = \sqrt{r_A^2 + X_A^2} = \sqrt{(5)^2 + (4.15)^2} = 6.5\ \Omega$$

$$Z_B = \sqrt{r_B^2 + X_B^2} = \sqrt{(8.3)^2 + (4.71)^2} = 9.54\ \Omega$$

$$V_A = Z_A I = 6.5 \times 15.02 = 97.63 \text{ V}$$

$$V_B = Z_B I = 9.54 \times 15.02 = 143.29 \text{ V}$$

Example 20

Two coils are connected in series across a 200 V, 50 Hz ac supply, The power input to the circuit is 2 kW and 1.15 kVAR. If the resistance and the reactance of the first coil are 5 Ω and 8 Ω respectively, calculate the resistance and reactance of the second coil. Calculate the active power and reactive power for both the coils individually. **[Dec 2014]**

Solution

$$P_T = 2 \text{ kW}$$

$$Q_T = 1.15 \text{ kVAR}$$

$$r_A = 5\ \Omega$$

$$X_A = 8\ \Omega$$

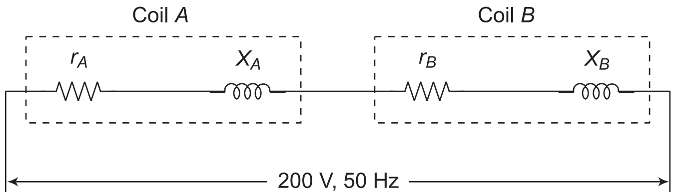


Fig. 4.27

(i) Resistance and reactance of the coil B

$$P_T = VI\cos\phi$$
$$Q_T = VI\sin\phi$$
$$\tan\phi = \frac{Q_T}{P_T} = \frac{1.15}{2} = 0.575$$
$$\phi = 29.9°$$
$$P_T = VI\cos\phi$$
$$2000 = 200 \times I \times \cos(29.9°)$$
$$I = 11.54 \text{ A}$$
$$P_T = I^2(r_A + r_B)$$
$$2000 = (11.54)^2(5 + r_B)$$
$$r_B = 10.02\ \Omega$$
$$Q_T = I^2(X_A + X_B)$$
$$1.15 \times 10^3 = (11.54)^2(8 + X_B)$$
$$X_B = 0.64\ \Omega$$

(ii) Active power and reactive power for both the coils individually

$$P_A = I^2 r_A = (11.54)^2 \times 5 = 665.86 \text{ W}$$
$$Q_A = I^2 X_A = (11.54)^2 \times 8 = 10.65.37 \text{ VAR}$$
$$P_B = I^2 r_B = (11.54)^2 \times 10.02 = 1334.38 \text{ W}$$
$$Q_B = I^2 X_B = (11.54)^2 \times 0.64 = 82.53 \text{ VAR}$$

Example 21

A coil A takes 2 A at a power factor of 0.8 lagging with an applied p.d. of 10 V. A second coil B takes 2 A with a power factor of 0.7 lagging with an applied voltage of 5 V. What voltage will be required to produce a total current of 2 A with coils A and B in series? Find the power factor in this case.

Solution

Coil A: $I_A = 2$ A, $\text{pf}_A = 0.8$ (lagging), $V_A = 10$ V

Coil B: $I_B = 2$ A, $\text{pf}_B = 0.7$ (lagging), $V_B = 5$ V

$I = 2$ A

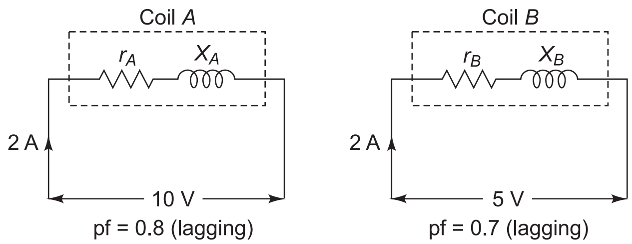


Fig. 4.28

For Coil *A*,

$$\phi_A = \cos^{-1}(0.8) = 36.87°$$

$$Z_A = \frac{V_A}{I_A} = \frac{10}{2} = 5\ \Omega$$

$$\bar{Z}_A = Z_A \angle \phi_A = 5\ \angle 36.87° = 4 + j3\ \Omega$$

$$r_A = 4\ \Omega$$

$$X_A = 3\ \Omega$$

For Coil *B*,

$$\phi_B = \cos^{-1}(0.7) = 45.57°$$

$$Z_B = \frac{V_B}{I_B} = \frac{5}{2} = 2.5\ \Omega$$

$$\bar{Z}_B = Z_B \angle \phi_B = 2.5\ \angle 45.57° = 1.75 + j1.78\ \Omega$$

$$r_B = 1.75\ \Omega$$

$$X_B = 1.78\ \Omega$$

When coils *A* and *B* are connected in series,

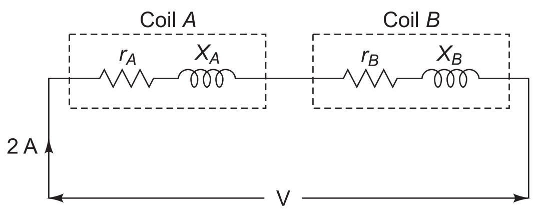


Fig. 4.29

$$\bar{Z} = r_A + jX_A + r_B + jX_B = 4 + j3 + 1.75 + j1.78$$

$$= 5.75 + j4.78 = 7.48\ \angle 39.74°\ \Omega$$

$$Z = 7.48\ \Omega$$

$$\phi = 39.74°$$

$$V = Z\,I = 7.48 \times 2 = 14.96\ \text{V}$$

$$\text{pf} = \cos\phi = \cos(39.74°) = 0.77 \text{ (lagging)}$$

Example 22

When a voltage of 100 V is applied to a coil A, the current taken is 8 A and the power is 120 W. When applied to a coil B, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series?

Solution

Coil A: $V_A = 100$ V, $I_A = 8$ A, $P_A = 120$ W

Coil B: $V_B = 100$ V, $I_B = 10$ A, $P_B = 500$ W

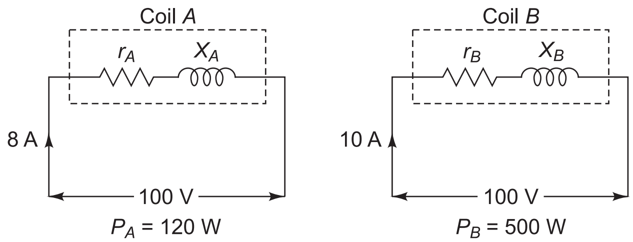


Fig. 4.30

For Coil A,

$$Z_A = \frac{V_A}{I_A} = \frac{100}{8} = 12.5\ \Omega$$

$$P_A = I_A^2\, r_A$$

$$120 = (8)^2 \times r_A$$

$$r_A = 1.875\ \Omega$$

$$X_A = \sqrt{Z_A^2 - r_A^2} = \sqrt{(12.5)^2 - (1.875)^2} = 12.36\ \Omega$$

For Coil B,

$$Z_B = \frac{V_B}{I_B} = \frac{100}{10} = 10\ \Omega$$

$$P_B = I_B^2\, r_B$$

$$500 = (10)^2 \times r_B$$

$$r_B = 5\ \Omega$$

$$X_B = \sqrt{Z_B^2 - r_B^2} = \sqrt{(10)^2 - (5)^2} = 8.66\ \Omega$$

When coils A and B are connected in series,

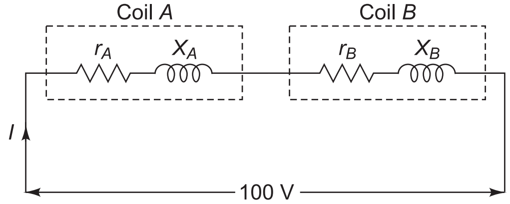


Fig. 4.31

$$\bar{Z} = r_A + jX_A + r_B + jX_B$$
$$= 1.875 + j12.36 + 5 + j8.66$$
$$= 6.875 + j21.02$$
$$= 22.11 \angle 71.89° \ \Omega$$
$$Z = 22.11 \ \Omega$$
$$\phi = 71.89°$$
$$I = \frac{V}{Z} = \frac{100}{22.11} = 4.52 \text{ A}$$
$$P = I^2 (r_A + r_B) = (4.52)^2 \times (6.875) = 140.64 \text{ W}$$

Example 23

In a particular circuit, a voltage of 10 V at 25 Hz produces 100 mA, while the same voltage at 75 Hz produces 60 mA. Find the values of components of the circuit.

Solution $V_1 = 10$ V, $f_1 = 25$ Hz, $I_1 = 100$ mA

$V_2 = 10$ V, $f_2 = 75$ Hz, $I_2 = 60$ mA

Case (i) $V_1 = 10$ V, $f_1 = 25$ Hz, $I_1 = 100$ mA

$$Z_1 = \frac{V_1}{I_1} = \frac{10}{100 \times 10^{-3}} = 100 \ \Omega$$

Case (ii) $V_2 = 10$ V, $f_1 = 75$ Hz, $I_2 = 60$ mA

$$Z_2 = \frac{V_2}{I_2} = \frac{10}{60 \times 10^{-3}} = 166.67 \ \Omega$$

As frequency increases, impedance of the circuit increases. In a series *R-L* circuit, inductive reactance X_L increases with frequency. Hence, impedance increases.

Hence, the circuit consists of a resistance R and an inductance L.

$$Z_1 = \sqrt{R^2 + X_{L_1}^2} = \sqrt{R^2 + (2\pi \times 25 \times L)^2} = 100 \ \Omega$$

$$R^2 + (50\pi L)^2 = 10000 \tag{1}$$

$$Z_2 = \sqrt{R^2 + X_{L_2}^2} = \sqrt{R^2 + (2\pi \times 75 \times L)^2} = 166.67 \ \Omega$$

$$R^2 + (150\pi L)^2 = 27778.89 \tag{2}$$

Solving Eqs (1) and (2),

$$R = 88.1 \ \Omega$$
$$L = 0.3 \text{ H}$$

Example 24

When 1 A is passed through three coils A, B and C in series, the voltages across them are 6 V, 3 V and 8 V respectively on a dc supply and 7 V, 5 V and 10 V respectively on an ac supply. Find the power factor and the power dissipated in each coil and the power factor of the whole circuit.

Solution $I = 1$ A

On dc supply, $V_A = 6$ V, $V_B = 3$ V, $V_C = 8$ V

On ac supply, $V_A = 7$ V, $V_B = 5$ V, $V_C = 10$ V

For dc supply, $f = 0$

$$X_L = 2\pi f L = 0$$

The coils behave like pure resistors.

$$R_A = \frac{V_A}{I} = \frac{6}{1} = 6\ \Omega$$

$$R_B = \frac{V_B}{I} = \frac{3}{1} = 3\ \Omega$$

$$R_C = \frac{V_C}{I} = \frac{8}{1} = 8\ \Omega$$

For ac supply,

$$Z_A = \frac{V_A}{I} = \frac{7}{1} = 7\ \Omega$$

$$Z_B = \frac{V_B}{I} = \frac{5}{1} = 5\ \Omega$$

$$Z_C = \frac{V_C}{I} = \frac{10}{1} = 10\ \Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(7)^2 - (6)^2} = 3.6\ \Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{(5)^2 - (3)^2} = 4\ \Omega$$

$$X_C = \sqrt{Z_C^2 - R_C^2} = \sqrt{(10)^2 - (8)^2} = 6\ \Omega$$

(i) Power factor of Coil A

$$\text{pf}_A = \frac{R_A}{Z_A} = \frac{6}{7} = 0.857 \text{ (lagging)}$$

(ii) Power factor of Coil B

$$\text{pf}_B = \frac{R_B}{Z_B} = \frac{3}{5} = 0.6 \text{ (lagging)}$$

(iii) Power factor of Coil C

$$\text{pf}_C = \frac{R_C}{Z_C} = \frac{8}{10} = 0.8 \text{ (lagging)}$$

(iv) Power dissipated in Coil *A*

$$P_A = I^2 R_A = (1)^2 \times 6 = 6 \text{ W}$$

(v) Power dissipated in Coil *B*

$$P_B = I^2 R_B = (1)^2 \times 3 = 3 \text{ W}$$

(vi) Power dissipated in Coil *C*

$$P_C = I^2 R_C = (1)^2 \times 8 = 8 \text{ W}$$

(vii) Power factor of the whole circuit

$$\begin{aligned} \bar{Z} &= R_A + jX_A + R_B + jX_B + R_C + jX_C \\ &= 6 + j3.6 + 3 + j4 + 8 + j6 \\ &= 17 + j13.6 = 21.77 \angle 38.68° \ \Omega \\ \text{pf}_T &= \cos(38.68°) = 0.78 \text{ (lagging)} \end{aligned}$$

Example 25

An air-cored coil takes 5 A of current and consumes 600 W of power when connected across a 200 V, 50 Hz ac supply. Calculate the value of the current drawn by the coil if the supply frequency increases to 60 Hz.

Solution

$$\begin{aligned} I &= 5 \text{ A} \\ P &= 600 \text{ W} \\ V &= 200 \text{ V} \end{aligned}$$

For

$$\begin{aligned} f &= 50 \text{ Hz} \\ Z &= \frac{V}{I} = \frac{200}{5} = 40 \ \Omega \\ P &= I^2 r \\ 600 &= (5)^2 \times r \\ r &= 24 \ \Omega \\ X_L &= \sqrt{Z^2 - r^2} = \sqrt{(40)^2 - (24)^2} = 32 \ \Omega \\ X_L &= 2\pi f L \\ 32 &= 2\pi \times 50 \times L \\ L &= 0.1019 \text{ H} \end{aligned}$$

For

$$\begin{aligned} f &= 60 \text{ Hz} \\ X_L &= 2\pi \times 60 \times 0.1019 = 38.4 \ \Omega \\ r &= 24 \ \Omega \\ Z &= \sqrt{r^2 + X_L^2} = \sqrt{(24)^2 + (38.4)^2} = 45.28 \ \Omega \end{aligned}$$

$$I = \frac{V}{Z} = \frac{200}{45.28} = 4.417 \text{ A}$$

Example 26

When an iron-cored choking coil is connected to a 12 V dc supply, it draws a current of 2.5 A and when it is connected to a 230 V, 50 Hz supply, it draws a 2 A current and consumes 50 W of power. Determine for this value of current (i) power loss in the iron core, (ii) inductance of the coil, (iii) power factor, and (iv) value of the series resistance which is equivalent to the effect of iron loss.

Solution For dc $V = 12$ V, $I = 2.5$ A

For ac $V = 230$ V, $I = 2$ A, $P = 50$ W

In an iron-cored coil, there are two types of losses.

(i) Losses in core known as core or iron loss

(ii) Losses in winding known as copper loss

$$P = I^2R + P_i$$

$$\frac{P}{I^2} = R + \frac{P_i}{I^2}$$

$$R_T = R + \frac{P_i}{I^2}$$

where R is the resistance of the coil and $\frac{P_i}{I^2}$ is the resistance which is equivalent to the effect of iron loss.

For dc supply, $f = 0$

$$X_L = 0$$

$$R = \frac{12}{2.5} = 4.8\ \Omega$$

For ac supply, $Z = \frac{230}{2} = 115\ \Omega$

(i) Iron loss

$$P_i = P - I^2R = 50 - (2)^2 \times 4.8 = 30.8 \text{ W}$$

$$R_T = \frac{P}{I^2} = \frac{50}{(2)^2} = 12.5\ \Omega$$

$$X_L = \sqrt{Z^2 - R_T^2} = \sqrt{(115)^2 - (12.5)^2} = 114.3\ \Omega$$

(ii) Inductance

$$X_L = 2\pi f L$$

$$114.3 = 2\pi \times 50 \times L$$

$$L = 0.363\ H$$

(iii) Power factor

$$\text{pf} = \frac{R_T}{Z} = \frac{12.5}{115} = 0.108 \text{ (lagging)}$$

(iv) The series resistance equivalent to the effect of iron loss

$$R_i = \frac{P_i}{I^2} = \frac{30.8}{(2)^2} = 7.7\ \Omega$$

Example 27

An iron-cored coil takes 4 A at a power factor of 0.5 when connected to a 200 V, 50 Hz supply. When the iron core is removed and the voltage is reduced to 40 V, the current rises to 5 A at a pf of 0.8. Find the iron loss in the core and inductance in each case.

Solution With iron core $I = 4$ A, pf = 0.5, $V = 200$ V
Without iron core $I = 5$ A, pf = 0.8, $V = 40$ V

(i) Inductance of the coil

(a) When the iron core is removed,

$$Z = \frac{V}{I} = \frac{40}{5} = 8\ \Omega$$

$$\text{pf} = \frac{R}{Z}$$

$$0.8 = \frac{R}{8}$$

$$R = 6.4\ \Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(8)^2 - (6.4)^2} = 4.8\ \Omega$$

$$X_L = 2\pi f L$$

$$4.8 = 2\pi \times 50 \times L$$

$$L = 0.0153 \text{ H}$$

(b) With iron core,

$$Z = \frac{V}{I} = \frac{200}{4} = 50\ \Omega$$

$$\text{pf} = \frac{R_T}{Z}$$

$$0.5 = \frac{R_T}{50}$$

$$R_T = 25\ \Omega$$

$$X_L = \sqrt{Z^2 - R_T^2} = \sqrt{(50)^2 - (25)^2} = 43.3\ \Omega$$

$$X_L = 2\pi f L$$
$$43.3 = 2\pi \times 50 \times L$$
$$L = 0.1378 \text{ H}$$

(ii) Iron loss

$$P_i = P - I^2R = VI\cos\phi - I^2R = 200 \times 4 \times 0.5 - (4)^2 \times 6.4 = 297.6 \text{ W}$$

4.5 SERIES R-C CIRCUIT

Figure 4.32 shows a pure resistor R connected in series with a pure capacitor C across an alternating voltage v.
Let V and I be the rms values of applied voltage and current.
Potential difference across the resistor = $V_R = R\,I$
Potential difference across the capacitor = $V_C = X_C\,I$

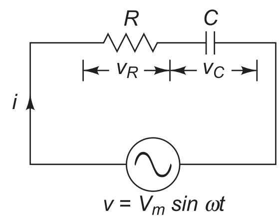


Fig. 4.32 *Series R-C circuit*

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

Phasor Diagram

Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\bar{I}$ is taken as reference phasor.
2. Draw $\bar{V}_R$ in phase with $\bar{I}$.
3. Draw $\bar{V}_C$ such that it lags behind $\bar{I}$ by 90°.
4. Add $\bar{V}_R$ and $\bar{V}_C$ by triangle law of addition such that
$$\bar{V} = \bar{V}_R + \bar{V}_C$$
5. Mark the angle $\bar{I}$ and $\bar{V}$ as ϕ.

The phasor diagram is shown in Fig. 4.33. It is clear from phasor diagram that current $\bar{I}$ leads applied voltage $\bar{V}$ by an angle ϕ (0° < ϕ < 90°).

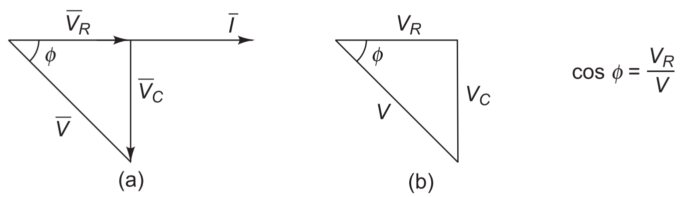


Fig. 4.33 *(a) Phasor diagram (b) Voltage triangle*

Impedance

$$\begin{aligned}\bar{V} &= \bar{V}_R + \bar{V}_C\\ &= R\bar{I} - jX_C\bar{I}\\ &= (R - jX_C)\bar{I}\\ \frac{\bar{V}}{\bar{I}} &= R - jX_C = \bar{Z}\\ \bar{Z} &= Z\angle -\phi\\ Z &= \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\\ \phi &= \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)\end{aligned}$$

The quantity $\bar{Z}$ is called the *complex impedance* of the *R*-*C* circuit.

Impedance Triangle The impedance triangle is shown in Fig. 4.34.

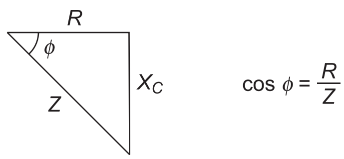


Fig. 4.34 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Waveforms The voltage and current waveforms are shown in Fig. 4.35.

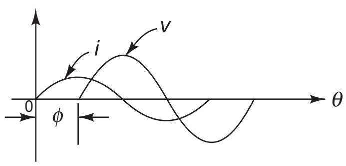


Fig. 4.35 *Waveforms*

Power

Active power $P = VI\cos\phi = I^2R$

Reactive power $Q = VI\sin\phi = I^2X_C$

Apparent power $S = VI = I^2Z$

Power Triangle The power triangle is shown in Fig. 4.36.

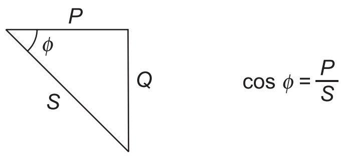


Fig. 4.36 *Power triangle*

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos \phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an R-C series circuit, the power factor is leading in nature since the current leads the voltage by an angle ϕ.

Example 1

The voltage applied to a circuit is e = 100 sin (ωt + 30°) and the current flowing in the circuit is i = 15 sin (ωt + 60°). Determine impedance, resistance, reactance, power factor and power.

Solution

$$e = 100 \sin(\omega t + 30°)$$

$$i = 15 \sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$

$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}} \angle 30°}{\frac{15}{\sqrt{2}} \angle 60°} = 6.67 \angle -30° = 5.77 - j3.33 = R - j\,X_C$$

$$Z = 6.67\ \Omega$$

(ii) Resistance

$$R = 5.77\ \Omega$$

(iii) Reactance

$$X_C = 3.33\ \Omega$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(30°) = 0.866\ \text{(leading)}$$

(v) Power

$$P = EI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5\ \text{W}$$

Example 2

The voltage and current in a circuit are given by e = 100 sin(ωt + 30°) and i = 50 sin (ωt + 60°). Determine the impedance of the circuit. Assuming the circuit to contain 2 elements in series find resistance, reactance and power factor of the circuit. **[Dec 2014]**

Solution $e = 100\sin(\omega t + 30°)$

$$i = 50\sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}}\angle 30°\ \text{V}$$

$$\bar{I} = \frac{50}{\sqrt{2}}\angle 60°\ \text{A}$$

$$\bar{Z} = \frac{\bar{E}}{I} = \frac{\frac{100}{\sqrt{2}}\angle 30°}{\frac{50}{\sqrt{2}}\angle 60°} = 2\angle -30° = 1.73 - j1 = R - jX_C$$

$$Z = 2\ \Omega$$

(ii) Resistance

$$R = 1.73\ \Omega$$

(iii) Reactance

$$X_C = 1\ \Omega$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(30°) = 0.866\ \text{(leading)}$$

Example 3

A series circuit consumes 2000 W at 0.5 leading power factor, when connected to 230 V, 50 Hz ac supply. Calculate (i) current, (ii) kVA, and (iii) kVAR.

Solution

$$P = 2000 \text{ W}$$
$$\text{pf} = 0.5 \text{ (leading)}$$
$$V = 230 \text{ V}$$

(i) Current

$$P = VI\cos\phi$$
$$2000 = 230 \times I \times 0.5$$
$$I = 17.39 \text{ A}$$

(ii) Apparent power

$$S = VI = \frac{P}{\cos\phi} = \frac{2000}{0.5} = 4 \text{ kVA}$$

(iii) Reactive power

$$\phi = \cos^{-1}(0.5) = 60°$$
$$Q = VI\sin\phi = 230 \times 17.39 \times \sin(60°) = 3.464 \text{ kVAR}$$

Example 4

A resistor R in series with a capacitor C is connected to a 240 V, 50 Hz ac supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum stored energy in C.

Solution

$$V = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$P = 300 \text{ W}$$
$$V_R = 100 \text{ V}$$

(i) Value of C

$$P = \frac{V_R^2}{R}$$
$$300 = \frac{(100)^2}{R}$$
$$R = 33.33\ \Omega$$
$$P = I^2R$$
$$300 = I^2 \times 33.33$$
$$I = 3 \text{ A}$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$72.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 43.77\ \mu\text{F}$$

(ii) Maximum charge

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17\ \text{V}$$

$$V_{C\max} = 218.17 \times \sqrt{2} = 308.54\ \text{V}$$

$$Q_{\max} = CV_{C\max} = 43.77 \times 10^{-6} \times 308.54 = 0.0135\ \text{C}$$

(iii) Maximum stored energy

$$E_{\max} = \frac{1}{2}\, C\,(V_{C\max})^2 = \frac{1}{2} \times 43.77 \times 10^{-6} \times (308.54)^2 = 2.08\ \text{J}$$

Example 5

A capacitor of 35 μF is connected in series with a variable resistor. The circuit is connected across 50 Hz mains. Find the value of the resistor for a condition when the voltage across the capacitor is half the supply voltage.

Solution

$$C = 35\ \mu\text{F}$$

$$f = 50\ \text{Hz}$$

$$V_C = \frac{1}{2}\text{V}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 35 \times 10^{-6}} = 90.946\ \Omega$$

$$V_C = \frac{1}{2}\text{V}$$

$$X_C I = \frac{1}{2}\, Z I$$

$$X_C = \frac{1}{2} Z$$

$$Z = 2X_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$(2X_C)^2 = R^2 + X_C^2$$

$$R^2 = 3X_C^2 = 3 \times (90.946)^2 = 24813.35$$

$$R = 157.5\ \Omega$$

Example 6

A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current is 2.2 A. The power loss in the resistor is 96.8 W. Calculate the resistance and capacitance.

Solution

$$V = 125\text{ V}$$

$$f = 50\text{ Hz}$$

$$I = 2.2\text{ A}$$

$$P = 96.8\text{ W}$$

(i) Resistance

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.82\text{ A}$$

$$P = I^2R$$

$$96.8 = (2.2)^2 \times R$$

$$R = 20\ \Omega$$

(ii) Capacitance

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 59.85\ \mu\text{F}$$

Example 7

A resistor and a capacitor are connected across a 250 V supply. When the supply frequency is 50 Hz, the current drawn is 5 A. When the frequency is increased to 60 Hz, it draws 5.8 A. Find the values of R and C and power drawn in the second case.

Solution

$$V = 250 \text{ V}$$
$$f_1 = 50 \text{ Hz}$$
$$I_1 = 5 \text{ A}$$
$$f_2 = 60 \text{ Hz}$$
$$I_2 = 5.8 \text{ A}$$

(i) Values of R and C

For $f_1 = 50$ Hz,

$$Z_1 = \frac{V}{I_1} = \frac{250}{5} = 50\ \Omega$$

$$Z_1 = \sqrt{R^2 + \left(\frac{1}{2\pi f_1 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{100\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 2500 \qquad (1)$$

For $f_2 = 60$ Hz,

$$Z_2 = \frac{V}{I_2} = \frac{250}{5.8} = 43.1\ \Omega$$

$$Z_2 = \sqrt{R^2 + \left(\frac{1}{2\pi f_2 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{120\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{120\pi C}\right)^2 = 1857.9\ \Omega \qquad (2)$$

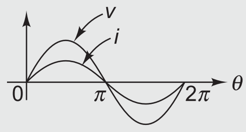
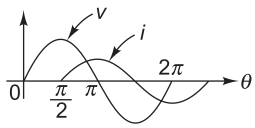
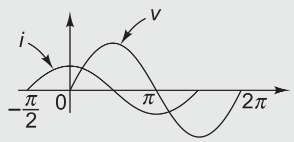
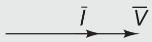
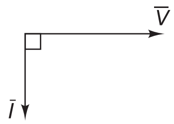
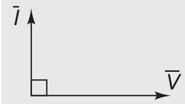
Solving Eqs (1) and (2),

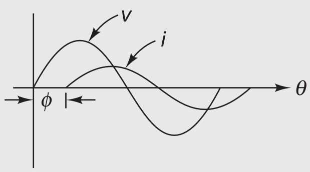
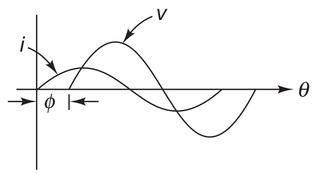
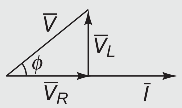
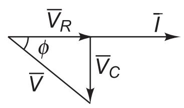
$$R = 19.96\ \Omega$$
$$C = 69.4\ \mu\text{F}$$

(ii) Power drawn in the second case

$$P_2 = I_2^2 R = (5.8)^2 \times 19.96 = 671.45 \text{ W}$$

Useful Formulae

	R	L	C
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin \omega t$	$I_m \sin (\omega t - 90°)$	$I_m \sin (\omega t + 90°)$
Waveform			
Phasor Diagram			
Impedance	R	$j\omega L$	$\dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C}$
Phase Difference	0°	90°	90°
Power Factor	1	0	0
Power	VI	0	0

	Series R-L Circuit	*Series R-C Circuit*
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin (\omega t - \phi)$	$I_m \sin (\omega t + \phi)$
Waveform		
Phasor Diagram		
Impedance	$R + jX_L,\ Z \angle \phi$	$R - jX_C,\ Z \angle -\phi$
Phase Difference	$0° < \phi < 90°$	$0° < \phi < 90°$
Power Factor	$\dfrac{V_R}{V} = \dfrac{R}{Z} = \dfrac{P}{S}$	$\dfrac{V_R}{V} = \dfrac{R}{Z} = \dfrac{P}{S}$
Power	$P = VI \cos \phi = I^2 R$ $Q = VI \sin \phi = I^2 X_L$ $S = VI = I^2 Z$	$P = VI \cos \phi = I^2 R$ $Q = VI \sin \phi = I^2 X_C$ $S = VI = I^2 Z$

Exercise 4.1

4.1 A 250 V, 50 Hz voltage is applied across a circuit consisting of a pure resistance of 20 Ω. Determine (i) the current flowing through the circuit, and (ii) power absorbed by the circuit. Give the expressions for the voltage and current.

[(i) 12.5 A (ii) 3.125 kW (iii) $v = 353.6 \sin 314\, t$, $i = 17.68 \sin 314\, t$]

4.2 A purely inductive circuit allows a current of 20 A to flow through a 230 V, 50 Hz supply. Find (i) inductive reactance, (ii) inductance of the coil, (iii) power absorbed, and (iv) equations for voltage and current.

$$\left[(i)\ 11.5\ \Omega\ (ii)\ 36.62\ mH\ (iii)\ 0\ (iv)\ v = 325.27 \sin 314\, t,\ i = 28.28 \sin\left(314\, t - \frac{\pi}{2}\right)\right]$$

4.3 A capacitor connected to a 230 V, 50 Hz supply draws a 15 A current. What current will it draw when the capacitance and frequency are both reduced to half?

[3.75 A]

4.4 Calculate the impedance of the circuit shown in Fig. 4.37.

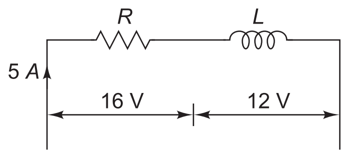


Fig. 4.37

[4 Ω]

4.5 An ac circuit has the following voltage and current: $v = 325 \sin 314\, t$, $i = 65 \sin (314\, t - 1.57)$. Find (i) frequency, (ii) rms value of voltage and current, (iii) impedance, and (iv) power factor.

[50 Hz, 229.81 V, 45.96 A, 5 Ω 0.99 (lagging)]

4.6 Two sources of an electromotive force represented respectively by $200 \sin \omega t$ and $200 \sin (\omega t + \pi/6)$ are in series. Express the resultant in vector notation with reference to $200 \sin \omega t$ and calculate the rms current and power supplied to a circuit of $8 + j6\ \Omega$ impedance. *[$273.2 \angle 15°$ V, $27.32 \angle -21.87°$ A, 5971 W]*

4.7 The voltage applied to a series circuit consisting of two pure elements is given by $v = 180 \sin \omega t$ and the resulting current is given by $i = 2.5 \sin (\omega t - 45°)$. Find the average power taken by the circuit and the values of the elements.

[159 W, 50.91 Ω, 50.91 Ω]

4.8 Find an expression for the current and calculate the power when a voltage represented by $v = 283 \sin 100\, \pi t$ is applied to a coil having $R = 50\ \Omega$ and $L = 0.159$ H. *[$4 \sin (100\, \pi t - \pi/4)$, 400 W]*

4.9 A voltage $V = (150 + j180)$ V is applied across an impedance and the current is found to be $I = (5 - j4)$ A. Determine (i) scalar impedance, (ii) reactance, and (iii) power consumed. *[36.6 Ω, 36.6 Ω, 30 W]*

4.10 The current in a series circuit of $R = 5\ \Omega$ and $L = 30$ mH lags the applied voltage by 72°. Determine the source frequency and the impedance Z. *[81.63 Hz, 16.18 Ω]*

4.11 Current flowing through an inductive circuit is 15 sin $(\omega t + \pi/4)$. When the voltage applied across it is 30 cos ωt, find the pf of the circuit. *[0.707 (lagging)]*

4.12 Voltage and current in an ac circuit are given by

$v = 200 \sin 377\, t \qquad i = 8 \sin (377\, t - \pi/6)$

Determine true power, reactive power and apparent power drawn by the circuit. *[692.82 W, 400 VAR, 800 VA]*

4.13 A current of 5 A flows through a non-inductive resistor in series with a choke coil when supplied at 250 V, 50 Hz. If the voltage drops across the coil and non-inductive resistor are 200 V and 125 V respectively, calculate the resistance and inductance of the impedance coil, value of non-inductive resistor and power drawn by the coil. Draw the vector diagram. *[5.5 Ω, 0.126 H, 25 Ω, 137.5 W]*

4.14 Two coils A and B are connected in series across a 200 V, 50 Hz ac supply. The power input to the circuit is 2 kW and 1.15 kVAR. If the resistance and reactance of the coil A are 5 Ω and 8 Ω respectively, calculate resistance and reactance of the coil B. Also, calculate the active power consumed by coils A and B. *[10.03 Ω, 0.642 Ω, 665. 3 W, 1334.7 W]*

4.15 A choking coil and a pure resistor are connected in series across a supply of 230 V, 50 Hz. The voltage drop across the resistor is 100 V and that across the chocking coil is 150 V. Find graphically the voltage drop across the inductance and resistance of the choking coil. Hence, find their values if the current is 1 A. *[109.98 V, 102 V, 0.366 H, 102 Ω]*

4.16 For Fig. 4.38, find R and L.

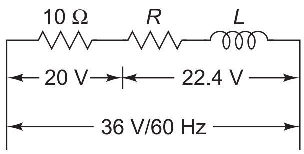


Fig. 4.38

[4.928 Ω, 0.0266 H]

4.17 Voltage and current for a circuit with two pure elements in series are expressed as follows:

$$v(t) = 170 \sin \left(6280\, t + \frac{\pi}{3}\right) \text{ volts}$$

$$i(t) = 8.5 \sin \left(6280\, t + \frac{\pi}{2}\right) \text{ amperes}$$

Sketch the two waveforms. Determine (i) frequency, (ii) power stating its nature, and (iii) values of the elements. *[(ii) 0.866 (leading) (iii) 17.32 Ω, 16 μF]*

4.18 A load consisting of a capacitor in series with a resistor has an impedance of 50 Ω and a pf of 0.707 leading. The load is connected in series with a 40 Ω resistor across an ac supply and the resulting current is of 3 A. Determine the supply voltage and overall phase angle. *[249.69 V, 25.135°]*

4.19 A capacitive load takes 10 kVA and 5 kVAR, when connected to a 200 V, 50 Hz ac supply. Calculate (i) resistance, (ii) capacitance, (iii) active power, and (iv) pf. *[3.464 Ω, 1.59×10^{-3} F, 8.66 kW, 0.866 (leading)]*

4.20 A resistor of 100 Ω is connected in series with a 50 μF capacitor to a 50 Hz, 200 V supply. Find:
(i) impedance (ii) current (iii) power factor (iv) phase angle
(v) voltage across the resistor and across the capacitor
[118.6 Ω, 1.69 A, 0.845 (leading), 32.48°, 168.712 V, 107.42 V]

4.6 SERIES *R-L-C* CIRCUIT

Figure 4.39 shows a pure resistor R, pure inductor L and pure capacitor C connected in series across an alternating voltage v.

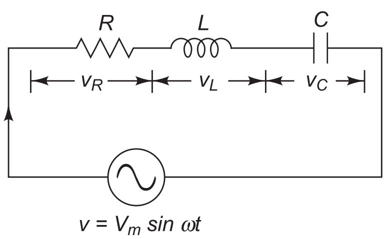


Fig. 4.39 *Series R-L-C circuit*

Let V and I be the rms values of the applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$, the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90° and the voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Phasor Diagram Since the same current flows through R, L and C, the current I is taken as a reference phasor.

Case (i) $X_L > X_C$

The reactance X will be inductive in nature and the circuit will behave like an R-L circuit.

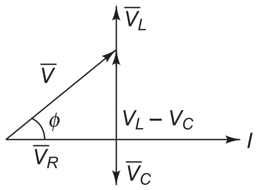


Fig. 4.40 *Phasor diagram*

Case (ii) $X_C > X_L$

The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit.

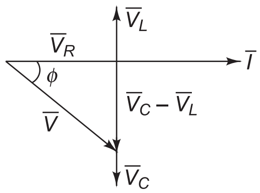


Fig. 4.41 *Phasor diagram*

Impedance

$$\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_C = R\overline{I} + jX_L I - jX_C\overline{I} = [R + j(X_L - X_C)]\,\overline{I}$$

$$\frac{\overline{V}}{\overline{I}} = R + j(X_L - X_C) = \overline{Z}$$

$$\overline{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance Triangles Impedance triangles are shown in Fig. 4.42.

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

Fig. 4.42 *Impedance triangles*

Current Equation If the applied voltage is given by $v = V_m \sin \omega t$ then current equation will be

$$i = I_m \sin(\omega t \pm \phi)$$

'–' sign is used when $X_L > X_C$.

'+' sign is used when $X_C > X_L$.

Waveforms The voltage and current waveforms are shown in Fig. 4.43.

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

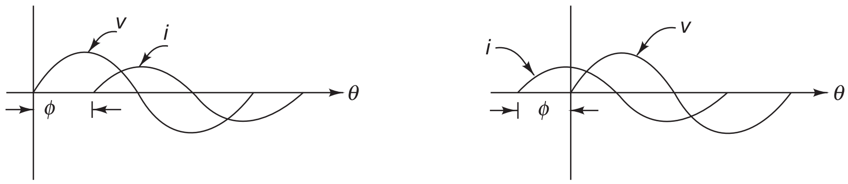


Fig. 4.43 *Waveforms*

Power

Average power $P = VI \cos \phi = I^2R$

Reactive power $Q = VI \sin \phi = I^2X$

Apparent power $S = VI = I^2Z$

Power Triangles Power triangles are shown in Fig. 4.44.

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

Fig. 4.44 *Power triangles*

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos \phi$$

$$\text{pf} = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

Example 1

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

Solution

$$R = 20\ \Omega$$
$$L = 0.05\ \text{H}$$
$$C = 50\ \mu\text{F}$$
$$V = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$

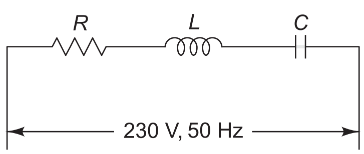


Fig. 4.45

(i) Impedance

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = R + jX_L - jX_C$$

$$= 20 + j15.71 - j63.66$$

$$= 51.95 \angle -67.36°\ \Omega$$

$$Z = 51.95\ \Omega$$

(ii) Phase difference

$$\phi = 67.36°$$

(iii) Current

$$I = \frac{V}{Z} = \frac{230}{51.95} = 4.43\ \text{A}$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(67.36°) = 0.385 \text{ (leading)}$$

(v) Active power

$$P = VI\cos\phi = 230 \times 4.43 \times 0.385 = 392.28\ \text{W}$$

(vi) Reactive power

$$Q = VI\sin\phi = 230 \times 4.43 \times \sin(67.36°) = 940.39\ \text{VAR}$$

Example 2

A circuit consists of a pure inductor, a pure resistor and a capacitor connected in series. When the circuit is supplied with 100 V, 50 Hz supply, the voltages across inductor and resistor are 240 V and 90 V respectively. If the circuit takes a 10 A leading current, calculate (i) value of inductance, resistance and capacitance, (ii) power factor of the circuit, and (iii) voltage across the capacitor.

Solution

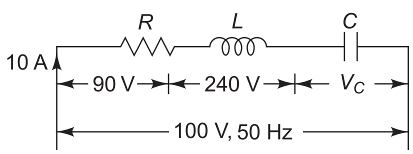


Fig. 4.46

$$V = 100\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$V_L = 240\ \text{V}$$
$$V_R = 90\ \text{V}$$
$$I = 10\ \text{A}$$

(i) Value of inductance, resistance and capacitance

$$R = \frac{V_R}{I} = \frac{90}{10} = 9\ \Omega$$

$$X_L = \frac{V_L}{I} = \frac{240}{10} = 24\ \Omega$$

$$Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

$$\bar{Z} = R + j\,X_L - j\,X_C = R - j(X_C - X_L)$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$10 = \sqrt{(9)^2 + (X_C - 24)^2}$$

$$X_C = 28.36\ \Omega$$

$$X_L = 2\pi f L$$

$$24 = 2\pi \times 50 \times L$$

$$L = 0.076\ \text{H}$$

$$X_C = \frac{1}{2\pi fC}$$

$$28.36 = \frac{1}{2\pi \times 50 \times C}$$

$$\text{C} = 112.24\ \mu\text{F}$$

(ii) Power factor of the circuit

$$\text{pf} = \frac{R}{Z} = \frac{9}{10} = 0.9 \text{ (leading)}$$

(iii) Voltage across the capacitor

$$V_C = X_C\,I = 28.36 \times 10 = 283.6\ \text{V}$$

Example 3

Two impedances $Z_1 = 40\angle 30°\ \Omega$ and $Z_2 = 30\ \angle 60°\ \Omega$ are connected in series across a single-phase 230 V, 50 Hz supply. Calculate the (i) current drawn, (ii) pf, and (iii) power consumed by the circuit.

Solution

$$\bar{Z}_1 = 40\ \angle 30°\ \Omega$$

$$\bar{Z}_2 = 30\ \angle 60°\ \Omega$$

$$V = 230\ \text{V}$$

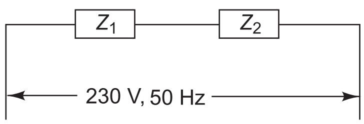


Fig. 4.47

(i) Current drawn

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 40\ \angle 30° + 30\ \angle 60° = 67.66\ \angle 42.81°\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{67.66} = 3.4\ \text{A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(42.81°) = 0.734 \text{ (lagging)}$$

(iii) Power consumed

$$P = VI\cos\phi = 230 \times 3.4 \times 0.734 = 573.99 \text{ W}$$

Example 4

A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to 115 V, 50 Hz supply. Another circuit takes a current of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz supply, calculate (i) current, (ii) power consumed, and (iii) power factor.

Solution Circuit 1: $I_1 = 3$ A, $\text{pf}_1 = 0.6$ (lagging), $V_1 = 115$ V

Circuit 2: $I_2 = 5$ A, $\text{pf}_2 = 0.707$ (leading), $V_2 = 115$ V

$V = 230$ V

For Circuit 1,

$$\phi_1 = \cos^{-1}(0.6) = 53.13°$$

$$Z_1 = \frac{V_1}{I_1} = \frac{115}{3} = 38.33\ \Omega$$

$$\bar{Z}_1 = Z_1\angle\phi_1 = 38.33\angle 53.13°\ \Omega$$

For Circuit 2,

$$\phi_2 = \cos^{-1}(0.707) = 45°$$

$$Z_2 = \frac{V_2}{I_2} = \frac{115}{5} = 23\ \Omega$$

$$\bar{Z}_2 = Z_2\angle\phi_2 = 23\angle{-45°}\ \Omega$$

When the two circuits are connected in series,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 38.33\angle 53.13° + 23\angle{-45°} = 41.82\angle 20.14°\ \Omega$$

(i) Current

$$I = \frac{V}{Z} = \frac{230}{41.82} = 5.5 \text{ A}$$

(ii) Power consumed

$$P = VI\cos\phi = 230 \times 5.5 \times \cos(20.14°) = 1.187 \text{ kW}$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(20.14°) = 0.939 \text{ (lagging)}$$

Example 5

Two impedances Z_1 and Z_2, having the same numerical value, are connected in series. If Z_1 has a pf of 0.866 lagging and Z_2 has a pf of 0.8 leading, calculate the pf of the series combination.

Solution

$$\text{pf}_1 = 0.866 \text{ (lagging)}$$

$$\text{pf}_2 = 0.8 \text{ (leading)}$$

$$Z_1 = Z_2 = Z$$
$$\phi_1 = \cos^{-1}(0.866) = 30°$$
$$\phi_2 = \cos^{-1}(0.8) = 36.87°$$
$$\bar{Z}_1 = Z\angle\phi_1 = Z\angle 30° = 0.866\, Z + j0.5\, Z\ \Omega$$
$$\bar{Z}_2 = Z\angle -\phi_2 = Z\angle -36.87° = 0.8\, Z - j0.6\, Z\ \Omega$$

For a series combination,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 0.866\, Z + j0.5\, Z + 0.8\, Z - j0.6\, Z$$
$$= 1.666\, Z - j0.1\, Z = Z(1.666 - j0.1) = 1.668\, Z\angle -3.43°\ \Omega$$
$$\text{pf} = \cos(3.43°) = 0.9982$$

Example 6

A coil of 3 Ω resistance and an inductance of 0.22 H is connected in series with an imperfect capacitor. When such a series circuit is connected across a 200 V, 50 Hz supply, it has been observed that their combined impedance is (3.8 + j6.4) Ω. Calculate the resistance and capacitance of the imperfect capacitor.

Solution

$$r = 3\ \Omega$$
$$L = 0.22\ \text{H}$$
$$V = 200\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$Z = 3.8 + j\,6.4\ \Omega$$

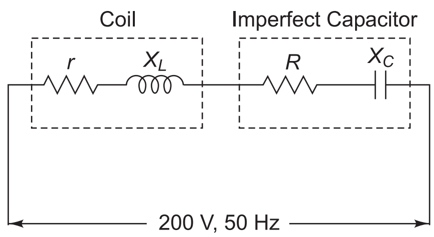


Fig. 4.48

(i) Resistance of the imperfect capacitor

$$\bar{Z} = 3.8 + j6.4\ \Omega$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.22 = 69.12\ \Omega$$
$$\bar{Z} = 3 + j69.12 + R - jX_C = (3 + R) + j(69.12 - X_C)$$
$$3 + R = 3.8$$
$$R = 0.8\ \Omega$$

(ii) Capacitance of the imperfect capacitor

$$69.12 - X_C = 6.4$$
$$X_C = 62.72\ \Omega$$
$$X_C = \frac{1}{2\pi fC}$$
$$62.72 = \frac{1}{2\pi \times 50 \times C}$$
$$C = 50.75\ \mu\text{F}$$

Example 7

An R-L-C series circuit has a current which lags the applied voltage by 45°. The voltage across the inductance has a maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin (1000t) and R = 20 Ω. Find the value of inductance and capacitance.

Solution

$$\phi = 45°$$

$$v_L = 300 \sin(1000t)$$

$$R = 20\ \Omega$$

(i) Value of inductance

$$V_{L(\text{max})} = 2V_{C(\text{max})}$$

$$V_L = 2V_C$$

$$IX_L = 2IX_C$$

$$X_L = 2X_C$$

$$\cos\phi = \frac{R}{Z}$$

$$\cos(45°) = \frac{20}{Z}$$

$$Z = 28.28\ \Omega$$

For a series R-L-C circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$28.28 = \sqrt{(20)^2 + (2X_C - X_C)^2}$$

$$= \sqrt{400 + X_C^2}$$

$$X_C = 20\ \Omega$$

$$X_L = 2X_C = 40\ \Omega$$

$$X_L = \omega L$$

$$40 = 1000 \times L$$

$$L = 0.04\ \text{H}$$

(ii) Value of capacitance

$$X_C = \frac{1}{\omega C}$$

$$20 = \frac{1}{1000 \times C}$$

$$C = 50\ \mu\text{F}$$

Example 8

A coil having a power factor of 0.5 is in series with a 79.57 μF capacitor and when connected across a 50 Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.

Solution

$$\text{pf}_{\text{coil}} = 0.5$$
$$C = 79.57\ \mu\text{F}$$
$$f = 50\ \text{Hz}$$
$$V_{\text{coil}} = V_C$$

(i) Resistance of the coil

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 79.57 \times 10^{-6}} = 40\ \Omega$$
$$V_{\text{coil}} = V_C$$
$$I\,Z_{\text{coil}} = I\,X_C$$
$$Z_{\text{coil}} = X_C = 40\ \Omega$$
$$\text{pf}_{\text{coil}} = \cos\phi = \frac{R}{Z_{\text{coil}}}$$
$$0.5 = \frac{R}{40}$$
$$R = 20\ \Omega$$

(ii) Inductance of the coil

$$X_L = \sqrt{Z_{\text{coil}}^2 - R^2} = \sqrt{(40)^2 - (20)^2} = 34.64\ \Omega$$
$$X_L = 2\pi f L$$
$$34.64 = 2\pi \times 50 \times L$$
$$L = 0.11\ \text{H}$$

Example 9

A 250 V, 50 Hz voltage is applied to a coil having a resistance of 5 Ω and an inductance of 9.55 H in series with a capacitor C. If the voltage across the coil is 300 V, find the value of C.

Solution

$$V = 250\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$R = 5\ \Omega$$
$$L = 9.55\ \text{H}$$
$$V_{\text{coil}} = 300\ \text{V}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 9.55 = 3000\ \Omega$$

$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (3000)^2} = 3000\ \Omega$$

$$I = \frac{V_{\text{coil}}}{Z_{\text{coil}}} = \frac{300}{3000} = 0.1\ \text{A}$$

$$Z = \frac{V}{I} = \frac{250}{0.1} = 2500\ \Omega$$

When $X_L > X_C$, $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$2500 = \sqrt{(5)^2 + (3000 - X_C)^2}$$

$$X_C = 500$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 500} = 6.37\ \mu\text{F}$$

When $X_C > X_L$, $\quad Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$2500 = \sqrt{(5)^2 + (X_C - 300)^2}$$

$$X_C = 5500$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 5500} = 0.58\ \mu\text{F}$$

Example 10

Draw the phasor diagram for the series circuit shown in Fig. 4.49 when the current in the circuit is 2 A. Find the values of V_1 and V_2 and show these voltages on the phasor diagram.

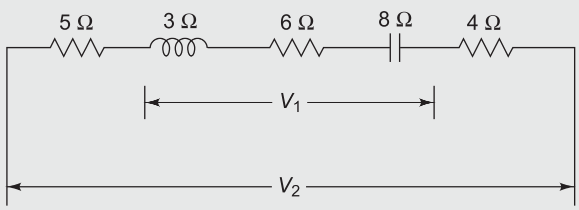


Fig. 4.49

Solution

$$\overline{Z}_1 = j3 + 6 - j8 = 6 - j5 = 7.81\ \angle{-39.8°}\ \Omega$$

$$\overline{Z}_2 = 5 + j3 + 6 - j8 + 4 = 15 - j5 = 15.81\ \angle{-18.43°}\ \Omega$$

$$I = 2\ \text{A}$$

(i) Values of V_1 and V_2

Let $\qquad \bar{I} = 2 \angle 0° \text{ A}$

$$\bar{V}_1 = \bar{Z}_1 \bar{I} = (7.81 \angle{-39.8°})\,(2 \angle 0°) = 15.62 \angle{-39.8°} \text{ V}$$

$$\bar{V}_2 = \bar{Z}_2 \bar{I} = (15.81 \angle{-18.43°})\,(2 \angle 0°) = 31.62 \angle{-18.43°} \text{ V}$$

(ii) Phasor diagram

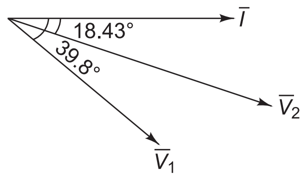


Fig. 4.50

Example 11

Draw a vector diagram for the circuit shown in Fig. 4.51 indicating terminal voltages V_1 and V_2 and the current. Find the value of (i) current, (ii) V_1 and V_2, and (iii) power factor.

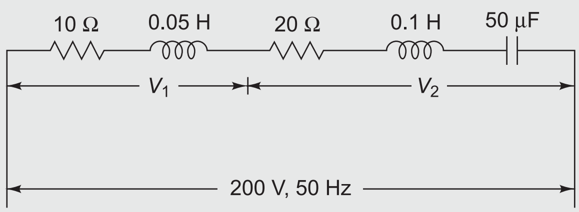


Fig. 4.51

Solution

(i) Current

$$X_{L_1} = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_{L_2} = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = 10 + j15.71 + 20 + j31.42 - j63.66$$

$$= 30 - j16.53 = 34.25 \angle{-28.85°}\ \Omega$$

$$I = \frac{V}{Z} = \frac{200}{34.25} = 5.84 \text{ A}$$

(ii) V_1 and V_2

Let $\qquad \bar{I} = 5.84 \angle 0° \text{A}$

$$\bar{Z}_1 = 10 + j15.71 = 18.62 \angle 57.52\ \Omega$$

$$\overline{V}_1 = \overline{Z}_1 \overline{I} = (18.62 \angle 57.52°)(5.84 \angle 0°) = 108.74 \angle 57.52° \text{ V}$$

$$\overline{Z}_2 = 20 + j31.42 - j63.66 = 20 - j32.24 = 37.94 \angle -58.19° \ \Omega$$

$$\overline{V}_2 = \overline{Z}_2 \overline{I} = (37.94 \angle -58.19°)(5.84 \angle 0°) = 221.57 \angle -58.19° \text{ V}$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(28.85°) = 0.875 \text{ (leading)}$$

(iv) Vector diagram

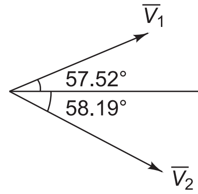


Fig. 4.52

Example 12

Find the values of R and C so that $V_x = 3V_y$, V_x and V_y are in quadrature.

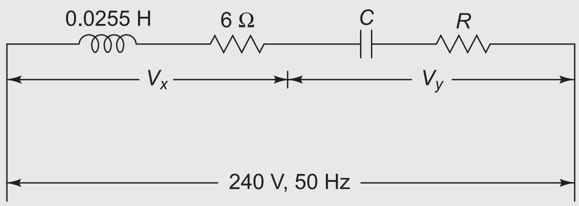


Fig. 4.53

Solution

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.0255 = 8 \ \Omega$$

$$\overline{Z}_x = 6 + j8 = 10 \angle 53.13° \ \Omega$$

$$V_x = 3V_y$$

$$I Z_x = 3I Z_y$$

$$Z_x = 3Z_y$$

V_x and V_y are in quadrature, i.e., phase angle between V_x and V_y is 90°. Hence, the angle between Z_x and Z_y will be 90°. The impedance Z_y is capacitive in nature.

$$\overline{Z}_y = Z_y \angle -\phi$$

$$\overline{Z}_y = \frac{10}{3} \angle (53.13 - 90)° = 3.33 \angle -36.87° = 2.66 - j2 \ \Omega$$

$$R = 2.66 \ \Omega$$

$$X_C = 2 \ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$2 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 1.59 \text{ mF}$$

Example 13

A pure resistor R, a choke coil and a pure capacitor of 15.91 μF are connected in series across a supply of V volts and carries a current of 0.25 A. The voltage across the choke coil is 40 V, the voltage across the capacitor is 50 V and the voltage across the resistor is 20 V. The voltage across the combination of R and the choke coil is 45 V. Calculate (i) supply voltage, (ii) frequency, and (iii) power loss in the choke coil.

Solution

$$C = 15.91\ \mu\text{F}$$
$$I = 0.25 \text{ A}$$
$$V_{\text{coil}} = 40 \text{ V}$$
$$V_C = 50 \text{ V}$$
$$V_R = 20 \text{ V}$$
$$V_{R\text{-coil}} = 45 \text{ V}$$

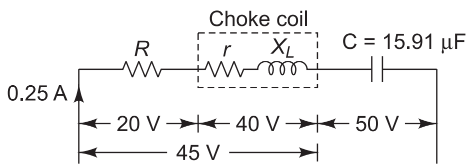


Fig. 4.54

(i) Supply voltage

$$R = \frac{V_R}{I} = \frac{20}{0.25} = 80\ \Omega$$

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{40}{0.25} = 160\ \Omega$$

$$X_c = \frac{V_C}{I} = \frac{50}{0.25} = 200\ \Omega$$

$$Z_{R\text{-coil}} = \frac{V_{R\text{-coil}}}{I} = \frac{45}{0.25} = 180\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = 160$$

$$r^2 + X_L{}^2 = 25600 \qquad (1)$$

$$\bar{Z}_{R\text{-coil}} = (R + r) + j\,X_L$$

$$Z_{R\text{-coil}} = \sqrt{(R+r)^2 + X_L^2}$$

$$180 = \sqrt{(80+r)^2 + X_L^2}$$

$$(80 + r)^2 + X_L{}^2 = 32400 \qquad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$(80 + r)^2 - r^2 = 6800$$

$$6400 + 160\,r + r^2 - r^2 = 6800$$

$$160\,r = 400$$
$$r = 2.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(2.5)^2 + X_L^2 = 25600$$
$$X_L^2 = 25593.75$$
$$X_L = 159.98\ \Omega$$
$$Z = R + r + j\,X_L - j\,X_C = 80 + 2.5 + j\,159.98 - j\,200$$
$$= 82.5 - j\,40.02 = 91.69\ \angle -25.88°\ \Omega$$
$$V = Z\,I = 91.69 \times 0.25 = 22.92\ \text{V}$$

(ii) Frequency

$$X_c = \frac{1}{2\pi fC}$$
$$200 = \frac{1}{2\pi f \times 15.91 \times 10^{-6}}$$
$$f = 50.02\ \text{Hz}$$

(iii) Power loss in the choke coil

$$P_{\text{coil}} = I^2 r = (0.25)^2 \times 2.5 = 0.156\ \text{W}$$

Example 14

Two impedances, one inductive and the other capacitive, are connected in series across the voltage of 120 ∠30° V and a frequency of 50 Hz. The current flowing in the circuit is 3∠–15°A. If one of the impedances is (10 + j 48.3) Ω, find the other. Also calculate the values of L and C in the impedances.

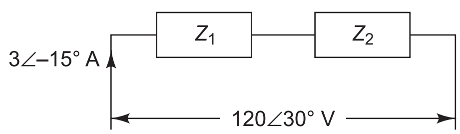


Fig. 4.55

Solution

$$V = 120\ \angle 30°\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I = 3\ \angle -15°\ \text{A}$$
$$\bar{Z}_1 = 10 + j\,48.3\ \Omega$$

(i) Impedance Z_2

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{120\angle 30°}{3\angle -15°} = 40\ \angle 45° = 28.28 + j\,28.28\ \Omega$$
$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$
$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 28.28 + j\,28.28 - 10 - j\,48.3 = 18.28 - j\,20.02\ \Omega$$

(ii) Value of L

$$\bar{Z}_1 = 10 + j\,48.3 = R_1 + j\,X_L$$
$$X_L = 48.3$$
$$X_L = 2\pi fL$$
$$48.3 = 2\pi \times 50 \times L$$
$$L = 0.1537 \text{ H}$$

(iii) Value of C

$$\bar{Z}_2 = 18.28 - j\,20.02 = R_2 - j\,X_C$$
$$X_C = 20.02$$
$$X_C = \frac{1}{2\pi fC}$$
$$20.02 = \frac{1}{2\pi \times 50 \times C}$$
$$C = 159\ \mu\text{F}$$

Exercise 4.2

4.1 In an *RLC* series circuit, the voltage across the resistor, inductor and capacitor are 10 V, 15 V and 10 V respectively. What is the supply voltage? *[11.18 V]*

4.2 A resistor R, inductor $L = 0.01$ H and capacitor C are connected in series. When a voltage of 400 sin (3000 t – 10°) is applied to a series combination, the current flowing is $10\sqrt{2}$ cos (3000 t – 55°) A. Find R and C. *[20 Ω, 6.66 μF]*

4.3 A resistor of 20 Ω, an inductor of 0.2 H and a capacitor of 100 μF are connected in series across a 220 V, 50 Hz supply. Determine

(i) impedance (ii) current flowing through the circuit
(iii) voltage drop across each element (iv) power consumed

[36.9 Ω, 5.96 A, 119.2 V, 374.52 V, 189.64 V, 710.43 W]

4.4 A series *R-L-C* circuit having a resistance of 8 Ω, an inductance of 80 mH and a capacitance of 100 μF is connected across a 150 V, 50 Hz supply. Calculate (i) current, (ii) power factor, and (iii) voltage across the inductance and capacitance. *[(i) 14.35 A (ii) 0.766 (leading) (iii) 360.47 V, 457.04 V]*

4.5 An emf of 100 V, 50 Hz is applied to an impedance $Z_1 = 8 + j6$ Ω. An impedance Z_2 is added in series with Z_1. The current becomes half of the original and leads it by 20°. Determine Z_2. *[(11.14 – j0.2) Ω]*

4.6 In a series *RLC* circuit, the inductor is imperfect and the capacitor has no leakage but has a lead resistance. When the circuit is connected to a 50 Hz supply, the voltage drop across the resistor and the inductor is 200 V. The voltage drop across the resistor and the capacitor is 250 V. The voltage drop across the resistor alone is 150 V. The voltage drop across the capacitor and inductor are 125 V and 100 V respectively. If the current flowing through the circuit is 1 A, find graphically

(i) capacitance (ii) lead resistance of the capacitor

(iii) inductance (iv) resistance of the inductor

(v) applied voltage

[33.5 μF, 81.25 Ω, 0.3 H, 25 Ω, 256.26 V]

4.7 Two impedances of $10 \angle 30° \ \Omega$ and $20 \angle -45° \ \Omega$ are connected in series. Calculate the power factor of the series combination. *[0.9281 (lagging)]*

4.8 Two impedances Z_1 and Z_2 are connected in series across a 230 V, 50 Hz ac supply. The total current drawn by the series combination is 2.3 A. The pf of Z_1 is 0.8 lagging. The voltage drop across Z_1 is twice the voltage drop across Z_2 and it is 90° out of phase with it. Determine the value of Z_2. *[44.719 Ω]*

4.9 In the arrangement shown in Fig. 4.56, $C = 20$ microfarads and the current flowing through the circuit is 0.345 A. If the voltages are as indicated, find the applied voltage, frequency and loss in the iron-cored inductor. Draw the phasor diagram.

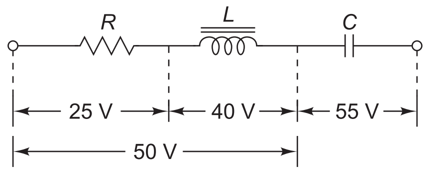


Fig. 4.56

[34.2 V, 50 Hz, 1.89 W]

4.7 PARALLEL AC CIRCUITS

In parallel circuits, resistor, inductor and capacitor or any combination of these elements are connected across same supply. Hence the voltage is same across each branch of the parallel ac circuit. The total current supplied to the circuit is equal to the phasor sum of the branch currents.

For the parallel ac circuit shown in Fig. 4.57,

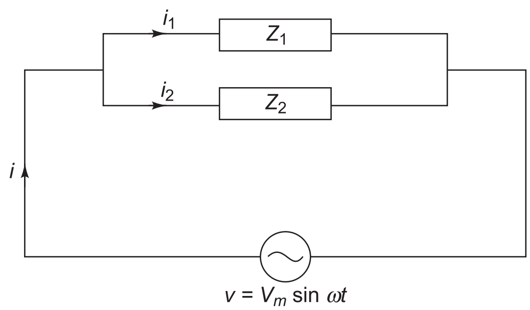


Fig. 4.57 *Parallel ac circuit*

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2$$

where Y represents the admittance of the circuit and is defined as the reciprocal of impedance. The real part of admittance is called conductance (G) and the imaginary part is called susceptance (B), and these are measured in mhos (℧) or siemens (S).

If $\quad \overline{Z}_1 = R + jX_L$, and $\overline{Z}_2 = -jX_C$

then,

$$\frac{1}{\overline{Z}} = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

$$= G + jB$$

where,

$$G = \frac{R}{R^2 + X_L^2}$$

$$B = \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}$$

The current in the parallel ac circuit can be found as the phasor sum of the branch currents,

i.e., $\quad \overline{I} = \overline{I}_1 + \overline{I}_2$

Note: 1. For a series *R-L* circuit,

$$\overline{Z} = R + jX_L$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$= \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2} = G - jB_L$$

where $\quad G = \dfrac{R}{R^2 + X_L^2} \quad$ and $\quad B_L = \dfrac{X_L}{R^2 + X_L^2}$

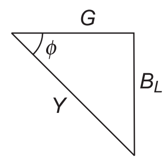


Fig. 4.58 *Admittance triangle*

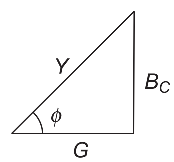


Fig. 4.59 *Admittance triangle*

2. For a series *R-C* circuit,

$$\overline{Z} = R - jX_C$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2} = \frac{R}{R^2 + X_C^2} + j\frac{X_C}{R^2 + X_C^2}$$

$$= G + jB_C$$

where $\quad G = \dfrac{R}{R^2 + X_C^2}$ and $B_C = \dfrac{X_C}{R^2 + X_C^2}$

Example 1

A resistance of 10 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 200 V, 50 Hz supply. Find the total current and power factor. **[May 2015]**

Solution

$$V = 200 \text{ V}$$
$$R = 10\ \Omega$$
$$L = 31.8 \text{ mH}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} \approx 10\ \Omega$$
$$I_R = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$
$$\bar{I}_L = \frac{V}{jX_L} = \frac{200}{j\,10} = \frac{200}{10\angle 90^\circ} = 20\angle -90^\circ \text{ A}$$

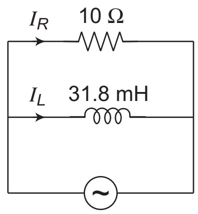


Fig. 4.60

(i) Total current

$$\bar{I} = \bar{I}_R + \bar{I}_L = 20\angle 0^\circ + 20\angle -90^\circ \text{ A} = 28.28\angle -45^\circ \text{A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(45^\circ) = 0.707 \text{ (lagging)}$$

Example 2

A coil having a resistance of 50 Ω and an inductance of 0.02 H is connected in parallel with a capacitor of 25 μF across a single-phase 200 V, 50 Hz supply. Calculate the current in coil and capacitance. Calculate also the total current drawn, total pf and total power consumed by the circuit.

Solution

$$R = 50\ \Omega$$
$$L = 0.02 \text{ H}$$
$$C = 25\ \mu\text{F}$$
$$V = 200 \text{ V}$$
$$f = 50 \text{ Hz}$$

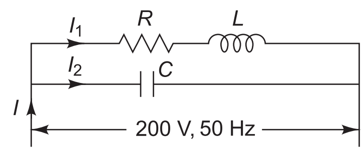


Fig. 4.61

(i) Current in coil and capacitance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\ \Omega$$
$$\bar{Z}_1 = R + jX_L = 50 + j\,6.28 = 50.39\angle 7.16^\circ\ \Omega$$
$$\bar{Z}_2 = -jX_C = -j127.32 = 127.32\angle -90^\circ\ \Omega$$
$$\bar{V} = 200\angle 0^\circ = 200 \text{ V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{50.39\angle 7.16^\circ} = 3.97\ \angle{-7.16^\circ}\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{127.32\angle -90^\circ} = 1.57\ \angle 90^\circ\ \text{A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 3.97\ \angle{-7.16^\circ} + 1.57\ \angle 90^\circ = 4.08\ \angle 15.27^\circ\ \text{A}$$

(iii) Total pf

$$\text{pf} = \cos\phi = \cos(15.27^\circ) = 0.965\ (\text{lagging})$$

(iv) Total power consumed

$$P = VI\cos\phi = 200 \times 4.08 \times 0.965 = 787.44\ \text{W}$$

Example 3

For the circuit shown in Fig.4.62, find supply current, current in each branch and total pf.

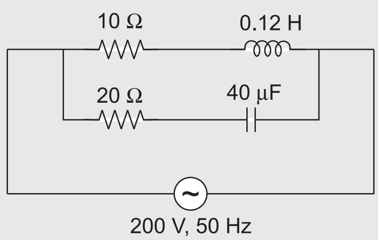


Fig. 4.62 **[May 2016]**

Solution (i) Supply current

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.12 = 37.7\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}} = 79.58\ \Omega$$

$$\bar{Z}_1 = 10 + j37.7 = 39\angle 75.14^\circ\ \Omega$$

$$\bar{Z}_2 = 20 - j79.58 = 82.05\ \angle -75.89^\circ\ \Omega$$

$$\bar{Z}_{eq} = \frac{\bar{Z}_1\,\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(39\angle 75.14^\circ)(82.05\angle -75.89^\circ)}{39\angle 75.14^\circ + 82.05\angle -75.89^\circ} = 62.11\ \angle 53.63^\circ\ \Omega$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \frac{200\angle 0^\circ}{62.11\angle 53.63^\circ} = 3.22\angle -53.63^\circ\,A$$

$$\bar{I}_1 = \bar{I}\cdot\frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = (3.22\angle -53.63^\circ)\frac{(82.05\angle -75.89^\circ)}{39\angle 75.14^\circ + 82.05\angle -75.89^\circ} = 5.13\ \angle{-75.14^\circ}\text{A}$$

$$\bar{I}_2 = \bar{I} - \bar{I}_1 = 3.22\angle-53.63° - 5.13\angle-75.14° = 2.43\angle 75.91° A$$

$$\text{pf} = \cos\phi = \cos(53.63°) = 0.593 \text{ (lagging)}$$

Example 4

Calculate the branch current I_1 and I_2 for the circuit shown in Fig. 4.61.

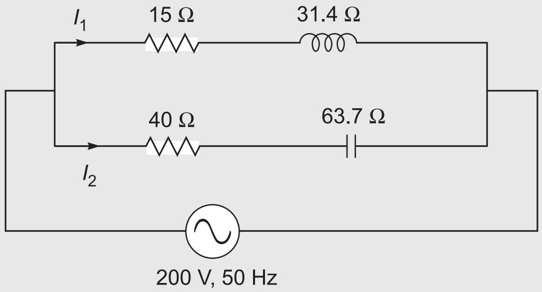


Fig. 4.63 **[Dec 2012]**

Solution

$$\bar{Z}_1 = 15 + j31.4 = 34.8\angle 64.47° \ \Omega$$

$$\bar{Z}_2 = 40 - j63.7 = 75.22\angle -57.87° \ \Omega$$

$$\bar{V} = 200 \text{ V}$$

(i) Branch current I_1

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{34.8\angle 64.47°} = 5.75\angle -64.47° \text{ A}$$

(i) Branch current I_2

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{75.22\angle -57.87°} = 2.66\angle 57.87° \text{ A}$$

Example 5

Two impedances $\bar{Z}_1 = 30\ \angle 45°\ \Omega$ and $\bar{Z}_2 = 45\angle 30°\ \Omega$ are connected in parallel across a single-phase 230 V, 50 Hz supply. Calculate (i) current drawn by each branch, (ii) total current, and (iii) overall power factor.

Also draw the phasor diagram indicating the current drawn by each branch and the total current, taking the supply voltage as reference.

Solution

$$\bar{Z}_1 = 30\ \angle 45°\ \Omega$$

$$\bar{Z}_2 = 45\ \angle 30°\ \Omega$$

$$V = 230 \text{ V}$$

(i) Current drawn by each branch

Let $\bar{V} = 230\ \angle 0° = 230$ V

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{230}{30\angle 45°} = 7.67\ \angle -45°\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{230}{45\angle 30°} = 5.11\ \angle -30°\ \text{A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 7.67\ \angle -45° + 5.11\ \angle -30° = 12.67\ \angle -39.01°\ \text{A}$$

(iii) Overall power factor

$$\text{pf} = \cos\phi = \cos(39.01°) = 0.777\ \text{(lagging)}$$

(iv) Phasor diagram

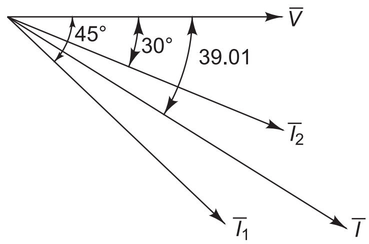


Fig. 4.64

Example 6

Two circuits, the impedances of which are given by $Z_1 = (10 + j15)$ ohms and $Z_2 = (6 - j8)$ ohms, are connected in parallel across an ac supply. If the total current supplied is 15 A, what is the power taken by each branch?

Solution $\bar{Z}_1 = (10 + j15)\ \Omega$

$\bar{Z}_2 = (6 - j8)\ \Omega$

$I = 15$ A

$$\bar{Z} = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(10 + j15)(6 - j8)}{10 + j15 + 6 - j8} = 10.32\ \angle -20.45°\ \Omega$$

$$V = ZI = 10.32 \times 15 = 154.8\ \text{V}$$

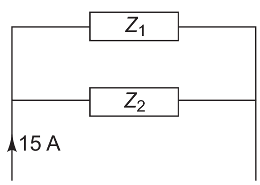


Fig. 4.65

Let $\bar{V} = 154.8\ \angle 0° = 154.8$ V

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{154.8}{10 + j15} = 8.59\ \angle -56.31°\text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{154.8}{6 - j8} = 15.48\ \angle 53.13°\text{A}$$

$$P_1 = I_1^2 R_1 = (8.59)^2 \times 10 = 737.88 \text{ W}$$
$$P_2 = I_2^2 R_2 = (15.48)^2 \times 6 = 1437.78 \text{ W}$$

Example 7

A circuit consists of a 25 Ω resistor, 64 mH inductor and 80 μF capacitor connected in parallel across a 110 V, 50 Hz single-phase supply. Calculate the individual currents drawn by each element, the total current drawn from the supply and the overall power factor of the circuit. Draw the phasor diagram.

Solution

$$R = 25\ \Omega$$
$$L = 64 \text{ mH}$$
$$C = 80\ \mu\text{F}$$
$$V = 110 \text{ V}$$
$$f = 50 \text{ Hz}$$

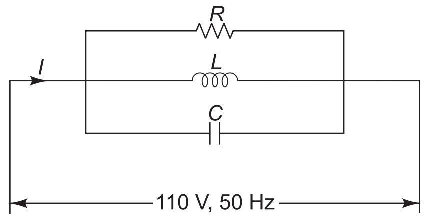


Fig. 4.66

(i) Individual currents

$$X_L = 2\pi fL = 2\pi \times 50 \times 64 \times 10^{-3} = 20.11\ \Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.79\ \Omega$$
$$\bar{Z}_1 = R = 25\ \Omega$$
$$\bar{Z}_2 = jX_L = j\,20.11\ \Omega$$
$$\bar{Z}_3 = -jX_C = -j\,39.79\ \Omega$$

Let $\bar{V} = 110\ \angle 0° = 110 \text{ V}$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{110}{25} = 4.4\ \angle 0° \text{ A}$$
$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{110}{j20.11} = 5.47\ \angle -90° \text{ A}$$
$$\bar{I}_3 = \frac{\bar{V}}{\bar{Z}_3} = \frac{110}{-j39.79} = 2.76\ \angle 90° \text{ A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 4.4\ \angle 0° + 5.47\ \angle -90° + 2.76\ \angle 90°$$
$$= 5.17\ \angle -31.63° \text{ A}$$

(iii) Overall power factor

$$\text{pf} = \cos\phi = \cos(31.63°) = 0.851 \text{ (lagging)}$$

(iv) Phasor diagram

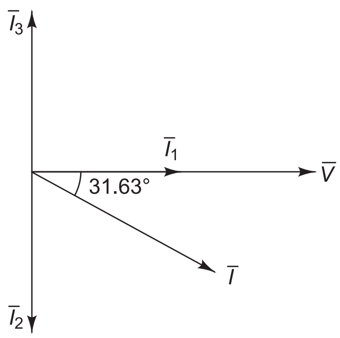


Fig. 4.67

Example 8

An ac circuit connected across a 200 V, 50 Hz, supply has two parallel branches A and B. Branch A draws a current of 4 A at 0.8 lagging power factor, while the total current drawn by the parallel combination is 5 A at unity power factor. Find (i) current and power factor of Branch B, and (ii) admittances of branches A and B, and their parallel combination both in polar and rectangular forms.

Solution

$$V = 200\text{ V},$$
$$f = 50\text{ Hz}$$
$$I_A = 4\text{ A}$$
$$\text{pf}_A = 0.8\text{ (lagging)}$$
$$I = 5\text{ A}$$
$$\text{pf} = 1$$

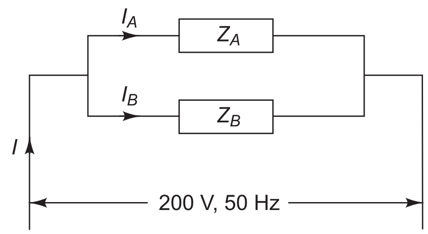


Fig. 4.68

(i) Current and power factor of Branch B

$$\phi_A = \cos^{-1}(0.8) = 36.87°$$
$$\phi = \cos^{-1}(1) = 0°$$
$$\bar{I}_A = I_A \angle -\phi_A = 4\angle -36.87°\text{ A}$$
$$\bar{I} = I\angle\phi = 5\angle 0°\text{ A}$$
$$\bar{I} = \bar{I}_A + \bar{I}_B$$
$$\bar{I}_B = \bar{I} - \bar{I}_A = 5\angle 0° - 4\angle -36.87° = 3\angle 53.13°\text{ A}$$
$$\text{pf}_B = \cos(53.13°) = 0.6\text{ (leading)}$$

(ii) Admittances of branches A and B and their parallel combination

Let $\bar{V} = 200\angle 0° = 200\text{ V}$

$$\bar{Y}_A = \frac{\bar{I}_A}{\bar{V}} = \frac{4\angle -36.87°}{200\angle 0°} = 0.02\angle -36.87°\ \mho$$

$$\overline{Y}_B = \frac{\overline{I}_B}{\overline{V}} = \frac{3\angle 53.13°}{200\angle 0°} = 0.015\ \angle 53.13°\ \mho$$

$$\overline{Y} = \overline{Y}_A + \overline{Y}_B = 0.02\ \angle -36.87° + 0.015\ \angle 53.13°\ \mho = 0.025\ \angle 0°\ \mho$$

Example 9

Two circuits A and B are connected in parallel to a 115 V, 50 Hz supply. The total current taken by the combination is 10 A at unity power factor. Circuit A consists of a 10 Ω resistor and 200 μF capacitor connected in series. Circuit B consists of a resistor and an inductor in series. Determine (i) current, (ii) power factor, (iii) impedance, (iv) resistance, and (v) reactance of the circuit B.

Solution

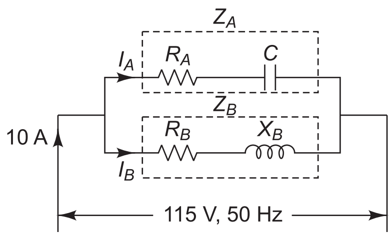


Fig. 4.69

$$V = 115 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 10 \text{ A}$$
$$\text{pf} = 1$$
$$R_A = 10\ \Omega$$
$$C = 200\ \mu\text{F}$$

(i) Current I_B

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$$

$$\overline{Z}_A = 10 - j\,15.92 = 18.8\ \angle -57.87°\ \Omega$$

$$\overline{I}_A = \frac{\overline{V}}{\overline{Z}_A} = \frac{115}{18.8\angle -57.87°} = 6.12\ \angle 57.87°\ \text{A}$$

$$\overline{I} = 10\ \angle 0°\ \text{A}$$

$$\overline{I}_B = \overline{I} - \overline{I}_A = 10\ \angle 0° - 6.12\ \angle 57.87° = 8.5\angle -37.54°\ \text{A}$$

(ii) Power factor of Circuit *B*

$$\text{pf}_B = \cos(37.54°) = 0.79 \text{ (lagging)}$$

(iii) Impedance of Circuit *B*

$$Z_B = \frac{\overline{V}}{\overline{I}_B} = \frac{115}{8.5\angle -37.54} = 13.53\ \angle 37.54°\ \Omega = 10.73 + j\,8.24\ \Omega$$

(iv) Resistance of Circuit *B*

$$R_B = 10.73\ \Omega$$

(v) Reactance of Circuit *B*

$$X_B = 8.24\ \Omega$$

Example 10

Two circuits, the impedances of which are given by $Z_1 = (6 + j8)\ \Omega$ and $Z_2 = (8 - j6)\ \Omega$, are connected in parallel. If the applied voltage to the combination is 100 V, find (i) current and pf of each branch, (ii) overall current and pf of the combination, and (iii) power consumed by each impedance.

Solution

$$\bar{Z}_1 = 6 + j8\ \Omega$$
$$\bar{Z}_2 = 8 - j6\ \Omega$$
$$V = 100\text{ V}$$

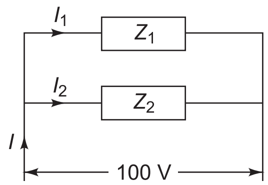


Fig. 4.70

(i) Current and pf of each branch

$$\bar{V} = 100\ \angle 0° = 100\text{ V}$$
$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100}{6 + j8} = 10\ \angle{-53.13°}\text{ A}$$
$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100}{8 - j6} = 10\ \angle 36.9°\text{ A}$$
$$\cos\phi_1 = \cos(53.13°) = 0.6\text{ (lagging)}$$
$$\cos\phi_2 = \cos(36.9°) = 0.8\text{ (leading)}$$

(ii) Overall current and pf of the combination

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\ \angle{-53.13°} + 10\ \angle 36.9° = 14.14\ \angle{-8.13°}\text{ A}$$
$$\text{pf} = \cos\phi = \cos(8.13°) = 0.989\text{ (lagging)}$$

(iii) Power consumed by each impedance

$$P_1 = I_1^2 R_1 = (10)^2 \times (6) = 600\text{ W}$$
$$P_2 = I^2 R_2 = (10)^2 \times (8) = 800\text{ W}$$

Example 11

Two impedances of $12 + j\,16\ \Omega$ and $10 - j\,20\ \Omega$ are connected in parallel across 230 V supply. Find the kW, kVA, kVAR and power factor of each branch. **[Dec 2015]**

Solution

$$\bar{Z}_1 = 12 + j16\ \Omega$$
$$\bar{Z}_2 = 10 - j20\ \Omega$$
$$V = 230\text{ V}$$

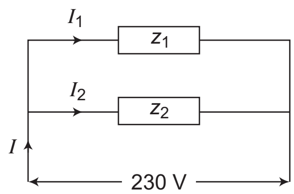


Fig. 4.71

Let $\bar{V} = 230\ \angle 0°$ V

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{230\angle 0°}{12 + j16} = 11.5\ \angle{-53.13°}\text{ A}$$
$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{230\angle 0°}{10 - j20} = 10.29\ \angle 63.43°\text{ A}$$

(i) kW, kVA, kVAR and power factor of branch 1

$$P_1 = V\,I_1 \cos\phi_1 = 230 \times 11.5 \times \cos(53.13°) = 1.59 \text{ kW}$$

$$S_1 = V\,I_1 = 230 \times 11.5 = 2.65 \text{ kVA}$$

$$Q_1 = V\,I_1 \sin\phi_1 = 230 \times 11.5 \times \sin(53.13°) = 2.12 \text{ kVAR}$$

$$\text{pf}_1 = \cos\phi_1 = \cos(53.13°) = 0.6 \text{ (lagging)}$$

(ii) kW, kVA, kVAR and power factor of branch 2

$$P_2 = V\,I_2 \cos\phi_2 = 230 \times 10.29 \times \cos(63.43°) = 1.06 \text{ kW}$$

$$S_2 = V\,I_2 = 230 \times 10.29 = 2.37 \text{ kVA}$$

$$Q_2 = V\,I_2 \sin\phi_2 = 230 \times 10.29 \times \sin(63.43°) = 2.12 \text{ kVAR}$$

$$\text{pf}_2 = \cos\phi_2 = \cos(63.43°) = 0.447 \text{ (leading)}$$

Example 12

Two impedances $R_1 - j\,X_{C_1}$ and $R_2 + jX_{L_2}$ are connected in parallel across a supply voltage $v = 100\sqrt{2}\,\sin 314\,t$. The current flowing through two impedances are $i_1 = 10\sqrt{2}\,\sin\left(314t + \frac{\pi}{4}\right)$ and $i_2 = 10\sqrt{2}\,\sin\left(314t - \frac{\pi}{4}\right)$ respectively. Find the equation for instantaneous value of total current drawn from the supply. Also find values of R_1, R_2, X_{C_1} and X_{L_2}.

Solution

$$\bar{Z}_1 = R_1 - j\,X_{C_1}$$

$$\bar{Z}_2 = R_2 + j\,X_{L_2}$$

$$v = 100\sqrt{2}\,\sin 314\,t$$

$$i_1 = 10\sqrt{2}\,\sin\left(314t + \frac{\pi}{4}\right)$$

$$i_2 = 10\sqrt{2}\,\sin\left(314t - \frac{\pi}{4}\right)$$

(i) Instantaneous value of total current

Writing v, i_1 and i_2 in polar form,

$$\bar{V} = 100\,\angle 0° \text{ V}$$

$$\bar{I}_1 = 10\,\angle 45° \text{ A}$$

$$\bar{I}_2 = 10\,\angle{-45°} \text{ A}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\angle 45° + 10\angle{-45°} = 14.14\angle 0° \text{ A}$$

$$i = I_m \sin 2\pi ft = 14.14\sqrt{2}\ \sin 314\, t = 20 \sin 314\, t$$

(ii) Values of R_1, R_2, X_{C_1} and X_{L_2}

$$\bar{Z}_1 = \frac{\bar{V}}{\bar{I}_1} = \frac{100\angle 0°}{10\angle 45°} = 10\ \angle{-45°} = 7.07 - j7.07\ \Omega$$

$$R_1 = 7.07\ \Omega$$

$$X_{C_1} = 7.07\ \Omega$$

$$\bar{Z}_2 = \frac{\bar{V}}{\bar{I}_2} = \frac{100\angle 0°}{10\angle -45°} = 10\ \angle 45° = 7.07 + j7.07\ \Omega$$

$$R_2 = 7.07\ \Omega$$

$$X_{L_2} = 7.07\ \Omega$$

Example 13

An impedance of (7 + j5) Ω is connected in parallel with another impedance of (10 – j8) Ω across a 230 V, 50 Hz supply. Calculate (i) admittance, conductance and susceptance of the combined circuit, and (ii) total current and power factor.

Solution

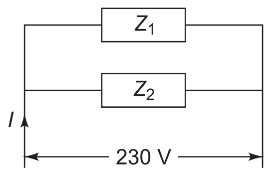


Fig. 4.72

$$\bar{Z}_1 = (7 + j5)\ \Omega$$

$$\bar{Z}_2 = (10 - j8)\ \Omega$$

$$V = 230\ \text{V}$$

(i) Admittance, conductance and susceptance of the combined circuit

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{7 + j5} = 0.12\angle{-35.54°}\ ℧$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10 - j8} = 0.08\angle 38.66°\ ℧$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.12\angle{-35.54°} + 0.08\angle 38.66°$$

$$= 0.16\ \angle{-7.04°}\ ℧$$

$$= 0.16 - j0.02\ ℧$$

$$Y = 0.16\ ℧$$

$$G = 0.16\ ℧$$

$$B = 0.02\ ℧$$

(ii) Total current and power factor

$$\bar{I} = \bar{V}\,\bar{Y} = (230\ \angle 0°)\ (0.16\ \angle -7.04°) = 36.8\ \angle -7.04°\ \text{A}$$

$$I = 36.8\ \text{A}$$

$$\text{pf} = \cos\phi = \cos(7.04°) = 0.99\ \text{(lagging)}$$

Example 14

Two impedances Z_1 and Z_2 are connected in parallel across a 200 V, 50 Hz ac supply. The current drawn by the impedance Z_1 is 4 A at 0.8 lagging pf. The total current drawn from the supply is 5 A at unity pf. Calculate the impedance Z_2.

Solution

$$V = 200\ \text{V}$$

$$I_1 = 4\ \text{A at 0.8 lagging pf}$$

$$I = 5\ \text{A at unity pf}$$

$$\bar{I}_1 = 4\ \angle -\cos^{-1}(0.8) = 4\ \angle -36.87°\ \text{A}$$

$$\bar{I} = 5\ \angle \cos^{-1}(1) = 5\ \angle 0°\ \text{A}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$\bar{I}_2 = \bar{I} - \bar{I}_1 = 5\angle 0° - 4\ \angle -36.87° = 3\ \angle 53.13°\ \text{A}$$

$$\bar{Z}_2 = \frac{\bar{V}}{\bar{I}_2} = \frac{200\angle 0°}{3\angle 53.13°} = 66.67\ \angle -53.13°\ \Omega$$

Example 15

Compute Z_{eq} and Y_{eq} for the circuit is shown in Fig. 4.66.

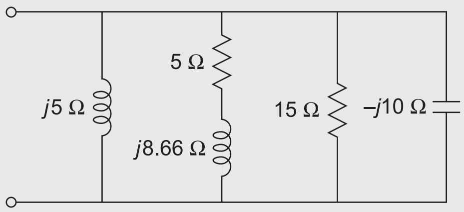


Fig. 4.73

Solution

$$\bar{Z}_1 = j5\ \Omega$$

$$\bar{Z}_2 = 5 + j8.66\ \Omega$$

$$\bar{Z}_3 = 15\ \Omega$$

$$\bar{Z}_4 = -j10\ \Omega$$

$$\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \bar{Y}_4$$

$$= \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} + \frac{1}{\bar{Z}_4}$$

$$= \frac{1}{j5} + \frac{1}{5 + j8.66} + \frac{1}{15} + \frac{1}{-j10}$$

$$= 0.22 \angle -57.99° \text{ ℧}$$

$$\overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \frac{1}{0.22\angle -57.99°} = 4.54 \angle 57.99° \ \Omega$$

Example 16

Find currents I_1 and I_2 in Fig. 4.74.

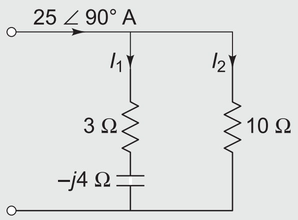


Fig. 4.74 **[May 2014]**

Solution

$$\overline{Z}_1 = 3 - j4 \ \Omega$$

$$\overline{Z}_2 = 10 \ \Omega$$

$$\overline{I} = 25 \angle 90° \text{ A}$$

By current-division rule,

$$\overline{I}_1 = \overline{I} \frac{\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} = (25 \angle 90°) \frac{10}{3 - j4 + 10} = 18.38 \angle 107.1° \text{ A}$$

$$\overline{I}_2 = \overline{I} - \overline{I}_1 = 25\angle 90° - 18.38 \angle 107.1° = 9.19 \angle 54° \text{ A}$$

Example 17

Three impedances of 25 ∠53.1° Ω, 5 ∠–53.1° Ω and 10 ∠36.9° Ω are connected in parallel. The combination is in series with another impedance of 14.14 ∠45° Ω. Calculate the equivalent impedance of the circuit.

Solution

$$\overline{Z}_1 = 25 \angle 53.1° \ \Omega$$

$$\overline{Z}_2 = 5 \angle -53.1 \ \Omega$$

$$\overline{Z}_3 = 10 \angle 36.9° \ \Omega$$

$$\overline{Z}_4 = 14.14 \angle 45° \ \Omega$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3} = \frac{1}{25\angle 53.1°} + \frac{1}{5\angle -53.1°} + \frac{1}{10\angle 36.9°}$$

$$= 0.23 \angle 16.86° \text{ ℧}$$

$$\overline{Z} = 4.27 \angle -16.86° \ \Omega$$

$$\overline{Z}_{eq} = \overline{Z} + \overline{Z}_4 = 4.27 \angle -16.86° + 14.14 \angle 45° = 16.58 \angle 31.87° \ \Omega$$

Example 18

A voltage of 200 ∠25° V is applied to a circuit composed of two parallel branches. If the branch currents are 10 ∠40° A and 20 ∠–30° A, determine the kVA, kVAR and kW in each branch. Also, calculate the pf of the combined load.

Solution

$$\overline{V} = 200 \angle 25° \text{ V}$$

$$\overline{I}_1 = 10 \angle 40° \text{ A}$$

$$\overline{I}_2 = 20 \angle -30° \text{ A}$$

Phase difference between V and I_1

$$\phi_1 = 40° - 25° = 15°$$

Phase difference between V and I_2

$$\phi_2 = 25° - (-30°) = 55°$$

$$\cos \phi_1 = \cos (15°) = 0.97 \text{ (leading)}$$

$$\cos \phi_2 = \cos (55°) = 0.57 \text{ (lagging)}$$

(i) kVA, kVAR and kW for the branch current of 10 ∠40° A

$$P_1 = VI_1 \cos \phi_1 = 200 \times 10 \times 0.97 = 1.94 \text{ kW}$$

$$Q_1 = VI_1 \sin \phi_1 = 200 \times 10 \times \sin (15°) = 0.52 \text{ kVAR}$$

$$S_1 = VI_1 = 200 \times 10 = 2 \text{ kVA}$$

(ii) kVA, kVAR and kW for the branch current of 20 ∠–30° A

$$P_2 = VI_2 \cos \phi_2 = 200 \times 20 \times 0.57 = 2.28 \text{ kW}$$

$$Q_2 = VI_2 \sin \phi_2 = 200 \times 20 \times \sin (55°) = 3.28 \text{ kVAR}$$

$$S_2 = VI_2 = 200 \times 20 = 4 \text{ kVA}$$

(iii) Power factor of the combined load

$$\overline{I}_1 = 10 \angle 40° \text{ A}$$

$$\overline{I}_2 = 20 \angle -30° \text{ A}$$

$$\overline{I} = \overline{I}_1 + \overline{I}_2 = 10 \angle 40° + 20 \angle -30° = 25.24 \angle -8.14° \text{ A}$$

$$\phi = 25° - (-8.14°) = 33.14°$$

$$\text{pf} = \cos (33.14°) = 0.84 \text{ (lagging)}$$

Example 19

The load taken from a supply consists of a (i) lamp load of 10 kW at unity power factor, (ii) motor load of 80 kVA at 0.8 power factor lagging, and (iii) motor load of 40 kVA at 0.7 power factor lagging. Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.

Solution Lamp load: $P_1 = 10$ kW, $\text{pf}_1 = 1$

Motor load: $S_2 = 80$ kVA, $\text{pf}_2 = 0.8$ (lagging)

Motor load: $S_3 = 40$ kVA, $\text{pf}_3 = 0.7$ (lagging)

(i) Total load in kW

For motor loads,

$$P_2 = S_2 \times \text{pf}_2 = 80 \times 0.8 = 64 \text{ kW}$$

$$P_3 = S_3 \times \text{pf}_3 = 40 \times 0.7 = 28 \text{ kW}$$

$$P = P_1 + P_2 + P_3 = 10 + 64 + 28 = 102 \text{ kW}$$

(ii) Total load in kVA

For lamp load,

$$S_1 = \frac{P_1}{\text{pf}_1} = \frac{10}{1} = 10 \text{ kVA}$$

$$S = S_1 + S_2 + S_3 = 10 + 80 + 40 = 130 \text{ kVA}$$

(iii) Power factor of the combined load

$$\text{pf} = \frac{P}{S} = \frac{102}{130} = 0.785 \text{ (lagging)}$$

Example 20

Two circuits have the same numerical value of impedance. The pf of one is 0.8 lagging and that of the other is 0.6 lagging. What is the pf of combination if they are connected in parallel?

Solution

$$\text{pf}_1 = 0.8 \text{ (lagging)}$$

$$\text{pf}_2 = 0.6 \text{ (lagging)}$$

$$\bar{Z}_1 = Z\angle\cos^{-1}(0.8) = Z\angle 36.87° \ \Omega$$

$$\bar{Z}_2 = Z\angle\cos^{-1}(0.6) = Z\angle 53.13° \ \Omega$$

For parallel combination,

$$\bar{Z} = \frac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(Z\angle 36.87°)(Z\angle 53.13°)}{Z\angle 36.87° + Z\angle 53.13°}$$

$$= \frac{Z^2\angle 90^\circ}{Z(1.4 + j1.4)}$$

$$= \frac{Z^2\angle 90^\circ}{1.98Z\angle 45^\circ}$$

$$= 0.505\ Z\ \angle 45^\circ\ \Omega$$

$$\text{pf} = \cos(45^\circ) = 0.707$$

Example 21

When a 240 V, 50 Hz supply is fed to a 15 Ω resistor in parallel with an inductor, the total current is 22.1 A. What value must the frequency have for the total current to be 34 A?

Solution

$$V = 240\text{ V}$$
$$R = 15\ \Omega$$
$$I = 22.1\text{ A}$$

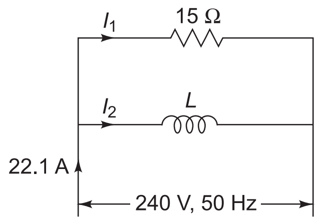


Fig. 4.75

Let

$$\bar{V} = 240\ \angle 0^\circ\text{ V}$$

$$\bar{I}_1 = \frac{\bar{V}}{R} = \frac{240\angle 0^\circ}{15\angle 0^\circ} = 16\ \angle 0^\circ = 16\text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{jX_L} = \frac{240\angle 0^\circ}{X_L\angle 90^\circ} = \frac{240}{X_L}\ \angle -90^\circ = -j\ \frac{240}{X_L}\ \text{A}$$

$$\bar{I} = 16 - j\ \frac{240}{X_L}$$

$$\sqrt{(16)^2 + \left(\frac{240}{X_L}\right)^2} = 22.1$$

$$256 + \frac{57600}{X_L^2} = 488.41$$

$$X_L = 15.74\ \Omega$$
$$X_L = 2\pi f L$$
$$15.74 = 2\pi \times 50 \times L$$
$$L = 0.05\text{ H}$$

Let the new frequency be f for the total current of 34 A.

$$\sqrt{(16)^2 + \left(\frac{240}{2\pi f \times 0.05}\right)^2} = 34$$

$$256 + \frac{57600}{0.0987 f^2} = 1156$$

$$f = 25.47\text{ Hz}$$

Example 22

Determine the current in the circuit of Fig. 4.76. Also, find the power consumed as well as pf.

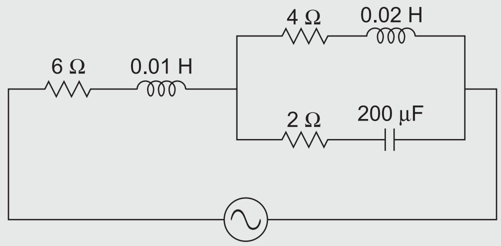


Fig. 4.76

Solution

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.01 = 3.14\ \Omega$$

$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$$

$$\overline{Z}_1 = 6 + j3.14\ \Omega$$

$$\overline{Z}_2 = 4 + j6.28\ \Omega$$

$$\overline{Z}_3 = 2 - j15.92\ \Omega$$

$$\overline{Z} = \overline{Z}_1 + \frac{\overline{Z}_2\,\overline{Z}_3}{\overline{Z}_2 + \overline{Z}_3}$$

$$= (6 + j3.14) + \frac{(4 + j6.28)(2 - j15.92)}{(4 + j6.28) + (2 - j15.92)} = 17.27\ \angle 30.75°\ \Omega$$

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{100\angle 0°}{17.27\angle 30.75°} = 5.79\ \angle{-30.75°}\ \text{A}$$

$$P = VI\cos\phi = 100 \times 5.79 \times \cos(30.75°) = 497.94\ \text{W}$$

$$\text{pf} = \cos\phi = \cos(30.75°) = 0.86\ \text{(lagging)}$$

Example 23

Two impedances $\overline{Z}_A = (4 + j3)\ \Omega$ and $\overline{Z}_B = (10 - j7)\ \Omega$ are connected in parallel and impedance $\overline{Z}_C = (6 + j5)\ \Omega$ is connected in series with parallel combination of $\overline{Z}_A$ and $\overline{Z}_B$. If the voltage applied across the circuit is 200 V at 59 Hz, calculate (i) currents flowing in Z_A, Z_B and Z_C, and (ii) total power factor of the circuit.

Solution

$$\overline{Z}_A = (4 + j3)\ \Omega$$

$$\overline{Z}_B = (10 - j7)\ \Omega$$

$$\overline{Z}_C = (6 + j5)\ \Omega$$

$$V = 200\ \text{V}$$

$$f = 59\ \text{Hz}$$

(i) Currents flowing in Z_A, Z_B and Z_C

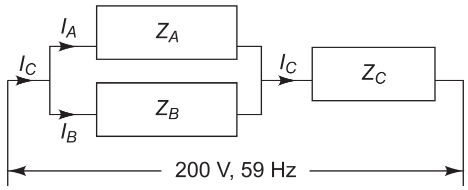


Fig. 4.77

$$\bar{Z}_A = 4 + j3 = 5\ \angle 36.87°\ \Omega$$
$$\bar{Z}_B = 10 - j7 = 12.21\ \angle{-35°}\ \Omega$$
$$\bar{Z}_C = 6 + j5 = 7.81\ \angle 39.81°\ \Omega$$
$$\bar{Z} = \frac{\bar{Z}_A \bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} + \bar{Z}_C$$
$$= \frac{(5\angle 36.87°)(12.21\angle -35°)}{5\angle 36.87° + 12.21\angle -35°} + 7.81\ \angle 39.81°$$
$$= 11.8\ \angle 32.17°\ \Omega$$
$$\bar{I}_C = \frac{\bar{V}}{\bar{Z}} = \frac{200\angle 0°}{11.8\angle 32.17°} = 16.95\ \angle{-32.17°}\ \text{A}$$
$$\bar{I}_A = \bar{I}_C\ \frac{\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = (16.95\angle -32.17°)\ \frac{(12.21\angle -35°)}{5\angle 36.87 + 12.21\angle -35°}$$
$$= 14.21\ \angle{-51.21°}\ \text{A}$$
$$\bar{I}_B = \bar{I}_C - \bar{I}_A = 16.95\ \angle{-32.17°} - 14.21\ \angle{-51.21°} = 5.82\ \angle 20.64\ \text{A}$$

(ii) Total power factor of the circuit

$$\text{pf} = \cos(32.17°) = 0.85\ \text{(lagging)}$$

Example 24

In a series–parallel circuit, the parallel branches A and B are in series with Branch C. The impedances are $Z_A = (4 + j3)\ \Omega$, $Z_B = \left(4 - j\frac{16}{3}\right)\ \Omega$ and $Z_C = (2 + j8)\ \Omega$. If the current $I_C = (25 + j0)$ A, determine the branch currents, voltages and the total voltage. Hence, calculate active and reactive powers for each branch and the whole circuit.

Solution

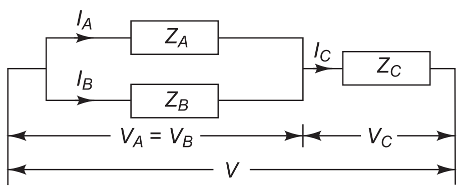


Fig. 4.78

$$\bar{Z}_A = (4 + j3)\ \Omega$$
$$\bar{Z}_B = \left(4 - j\frac{16}{3}\right)\Omega$$
$$\bar{Z}_C = (2 + j8)\ \Omega$$
$$\bar{I}_C = (25 + j0) = 25\angle 0°\ \text{A}$$

(i) Branch currents

By current-division rule,

$$\bar{I}_A = \bar{I}_C\ \frac{\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = (25 + j0)\frac{\left(4 - j\frac{16}{3}\right)}{4 + j3 + 4 - j\frac{16}{3}} = 20\ \angle{-36.87°}\ \text{A}$$
$$\bar{I}_B = \bar{I}_C - \bar{I}_A = 25\ \angle 0° - 20\ \angle{-36.87°} = 15\ \angle 53.13°\ \text{A}$$

(ii) Voltages and total voltage

$$\overline{V}_A = \overline{V}_B = \overline{Z}_A I_A = (4 + j3)\,(20\ \angle -36.87°) = 100\ \angle -1.02°\ \text{V}$$
$$\overline{V}_C = \overline{Z}_C \overline{I}_C = (2 + j8)\,(25\ \angle\ 0°) = 206.16\ \angle\ 75.96°\text{V}$$
$$\overline{V} = \overline{V}_A + \overline{V}_C = 100\ \angle -1.02° + 206.16\ \angle 75.16° = 249.7\ \angle 52.27°\ \text{V}$$

(iii) Active and reactive powers for each branch and the whole circuit

$$P_A = I_A^2\, R_A = (20)^2 \times 4 = 1600\ \text{W}$$
$$P_B = I_B^2\, R_B = (15)^2 \times 4 = 900\ \text{W}$$
$$P_C = I_C^2\, R_C = (25)^2 \times 2 = 1250\ \text{W}$$
$$P = P_A + P_B + P_C = 1600 + 900 + 1250 = 3750\ \text{W}$$
$$Q_A = I_A^2\, X_A = (20)^2 \times 3 = 1200\ \text{VAR}$$
$$Q_B = I_B^2\, X_B = (15)^2 \times \frac{16}{3} = 1200\ \text{VR}$$
$$Q_C = I_C^2\, X_C = (25)^2 \times 8 = 5000\ \text{VAR}$$
$$Q = Q_A + Q_B + Q_C = 1200 + 1200 + 5000 = 7400\ \text{VAR}$$

Example 25

Find the applied voltage V_{AB} so that a 10 A current may flow through the capacitor in Fig. 4.72.

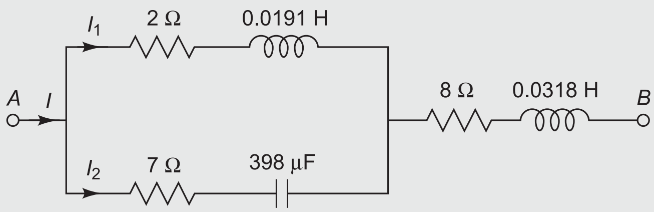


Fig. 4.79

Solution

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.0191 = 6\ \Omega$$
$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 0.0318 = 10\ \Omega$$
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 398 \times 10^{-6}} = 8\ \Omega$$
$$\overline{Z}_1 = 2 + j6 = 6.32\ \angle 71.56°\ \Omega$$
$$\overline{Z}_2 = 7 - j8 = 10.63\ \angle -48.8°$$
$$\overline{Z}_3 = 8 + j10 = 12.8\ \angle 51.34°\ \Omega$$
$$\overline{Z} = \frac{\overline{Z}_1\,\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} + \overline{Z}_3 = \frac{(2 + j6)(7 - j8)}{(2 + j6 + 7 - j8)} + (8 + j10) = 19.91\ \angle 45.53°\ \Omega$$

Let

$$\overline{I}_2 = 10\ \angle 0°\ \text{A}$$
$$\overline{V}_2 = \overline{Z}_2\,\overline{I}_2 = (10.63\ \angle -48.8°)\,(10\ \angle 0°) = 106.3\ \angle -48.8°\ \text{V}$$

$$\bar{V}_1 = \bar{V}_2 = 106.3\ \angle{-48.8°}\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{106.3\angle-48.8°}{6.32\angle 71.56°} = 16.82\ \angle{-120.36°}\ \text{A}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 16.82\ \angle{-120.36°} + 10\angle 0° = 14.58\ \angle{-84.09°}\ \text{A}$$

$$\bar{V}_{AB} = \bar{Z}\,\bar{I} = (19.91\ \angle 45.53°)(14.58\ \angle{-84.09°}) = 290.28\ \angle{-38.56°}\ \text{V}$$

Example 26

If a voltage of 150 V applied between terminals A and B produces a current of 32 A for the circuit shown in Fig. 4.80, calculate the value of the resistance R and pf of the circuit.

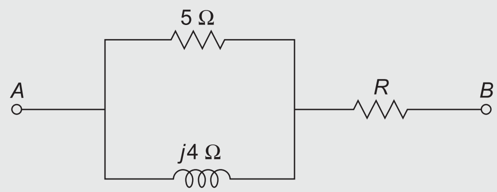


Fig. 4.80

Solution

$$V = 150\ \text{V}$$

$$\bar{I} = 32\ \text{A}$$

$$Z = \frac{150}{32} = 4.687\ \Omega$$

$$\bar{Z} = \frac{(5)(j4)}{5+j4} + R = \frac{(5)(4\angle 90°)}{64\angle 38.66°} + R$$

$$= 3.125\ \angle 51.34° + R = 1.95 + j2.44 + R$$

$$\sqrt{(1.95+R)^2 + (2.44)^2} = 4.687$$

$$(1.95 + R)^2 + (2.44)^2 = (4.687)^2$$

$$(1.95 + R)^2 = (4.687)^2 - (244)^2$$

$$1.95 + R = 4$$

$$R = 2.05\ \Omega$$

$$\text{pf} = \frac{\text{Total resistance}}{\text{Total impedance}} = \frac{1.95+2.05}{4.687} = 0.853\ \text{(lagging)}$$

Example 27

An impedance of R + jX ohms is connected in parallel with another impedance of –j5 ohms. The combination is then connected in series with a pure resistance of 2 Ω. When connected across a 100 V, 50 Hz ac supply, the total current drawn by the circuit is 20 A and the total power consumed by the circuit is 2 kW. Calculate (i) currents through parallel branches, and (ii) R and L.

Solution $P = 2\ \text{kW}$

$$V = 100 \text{ V}$$
$$I = 20 \text{ A}$$

(i) Currents through parallel branches

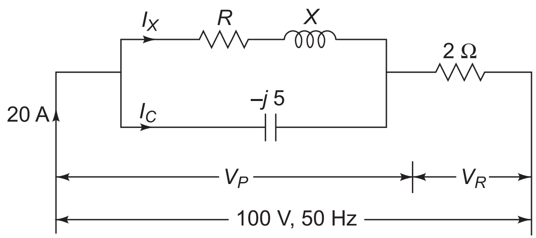


Fig. 4.81

$$P = VI \cos \phi$$
$$2000 = 100 \times 20 \times \cos \phi$$
$$\cos \phi = 1$$
$$\phi = 0°$$
$$\bar{V} = 100 \angle 0° \text{ V}$$
$$\bar{I} = 20\angle 0° \text{A}$$
$$\bar{V}_R = 2 \times 20° \angle 0° = 40 \angle 0° \text{ V}$$
$$\bar{V}_P = \bar{V} - \bar{V}_R = 100 \angle 0° - 40\angle 0° = 60 \angle 0° \text{ V}$$
$$\bar{I}_C = \frac{\bar{V}_P}{\bar{Z}_C} = \frac{60\angle 0°}{5\angle -90°} = 12 \angle 90° \text{ A}$$
$$\bar{I}_X = \bar{I} - \bar{I}_C = 20\angle 0° - 12 \angle 90° = 23.32 \angle -30.96° \text{ A}$$

(ii) Resistance and inductance

$$\bar{Z}_X = \frac{60\angle 0°}{23.32\angle -30.96°} = 2.57 \angle 30.96° \ \Omega = 2.2 + j1.32 \ \Omega$$
$$R = 2.2 \ \Omega$$
$$X_L = 1.32 \ \Omega$$
$$X_L = 2\pi f L$$
$$1.32 = 2\pi \times 50 \times L$$
$$L = 4.2 \text{ mH}$$

Example 28

The circuit of Fig. 4.82 takes 12 A at a lagging power factor and dissipates 1800 W. The reading of the voltmeter is 200 V. Find R_1, X_1 and X_2.

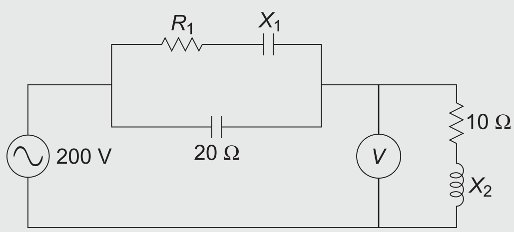


Fig. 4.82

Solution

$$I = 12 \text{ A}$$
$$P = 1800 \text{ W}$$

Let $\bar{I} = 12 \angle 0° \text{A}$

$$Z_2 = \frac{200}{12} = 16.67\ \Omega$$

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

$$16.67 = \sqrt{(10)^2 + X_2^2}$$

$$(16.67)^2 = 10^2 + X_2^2$$

$$277.88 = 100 + X_2^2$$

$$X_2 = 13.33\ \Omega$$

$$\bar{V}_2 = (12 \angle 0°)(10 + j13.33) = (12 \angle 0°)(16.67 \angle 53.13°) = 200 \angle 53.13° \text{ V}$$

$$P = VI \cos \phi$$

$$1800 = 200 \times 12 \times \cos \phi$$

$$\cos \phi = 0.75$$

$$\phi = 41.41°$$

Applied voltage $\bar{V}_{\text{req}} = 200 \angle 41.41° \text{ V}$

Voltage across parallel branches = $200 \angle 41.41° - 200 \angle 53.13° = 40.84 \angle -42.73° \text{ V}$

Current through capacitor = $\dfrac{40.84 \angle -42.73°}{20 \angle -90°} = 2.04 \angle 47.27° \text{ A}$

Current through R_1 and X_1 = $12 \angle 0° - 2.04 \angle 47.27° = 10.72 \angle -8.03° \text{ A}$

$$\bar{Z}_1 = \frac{40.84 \angle -42.73°}{10.72 \angle -8.03°} = 3.81 \angle -34.7°\ \Omega = 3.13 - j2.17\ \Omega$$

$$R_1 = 3.13\ \Omega$$

$$X_1 = 2.17\ \Omega$$

Example 29

For the circuit shown in Fig. 4.83, calculate (i) total admittance, total conductance and total susceptance, (ii) total current and total pf, and (iii) value of pure capacitance to be connected in parallel with the above combination to make the total pf unity.

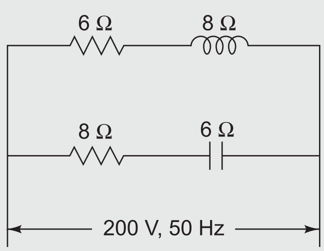


Fig. 4.83

Solution

(i) Total admittance, total conductance and total susceptance

$$\bar{Z}_1 = 6 + j8 = 10\ \angle 53.13°\ \Omega$$

$$\bar{Z}_2 = 8 - j6 = 10\ \angle -36.87°\ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{10\angle 53.13°} = 0.1\ \angle -53.13°\ \mho$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10\angle -36.87°} = 0.1\angle 36.87°\ \mho$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.1\ \angle -53.13° + 0.1\ \angle 36.87° = 0.14 - j0.02\ \mho = 0.14\ \angle -8.13°\ \mho$$

$$Y = 0.14\ \mho$$

$$G = 0.14\ \mho$$

$$B = 0.02\ \mho$$

(ii) Total current and total pf

Let $\bar{V} = 200\ \angle 0°\ \text{V}$

$$I = \bar{V}\,\bar{Y} = (200\ \angle 0°)\ (0.14\ \angle -8.13°) = 28\ \angle -8.13°\ \text{A}$$

$$\text{pf} = \cos\ (8.13°) = 0.989\ \text{(lagging)}$$

(iii) Value of pure capacitance to make total pf unity

Since the current lags behind voltage, the circuit is inductive in nature. In order to make the total pf unity, a pure capacitor is connected in parallel so that pf becomes unity and imaginary part of $\bar{Y}_{\text{req}}$ becomes zero.

$$\bar{Y}_{\text{req}} = Y_1 + Y_2 + Y_3$$

$$\bar{Y}_{\text{req}} = 0.14 - j0.02 + j0.02 = 0.14$$

$$Y_3 = Y_C = \frac{1}{X_C} = 0.02$$

$$X_C = 50\ \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 50} = 63.66\ \mu\text{F}$$

Exercise 4.3

4.1 Two impedances of $14 + j5\,\Omega$ and $18 + j10\,\Omega$ are connected in parallel across a 200 V, 50 Hz supply. Determine (i) admittance of each branch and the entire circuit, (ii) current in each branch and total current, (iii) power and power factor of each branch, (iv) total power factor, and (v) draw phasor diagram.

[0.067 $\angle -19.6°$ ℧, 0.048 $\angle -29.05°$ ℧, 0.115 $\angle -23.75°$ ℧, 13.45 $\angle -19.65°$ A, 9.6 $\angle -29.05°$ A, 22.91 $\angle -23.75°$ A, 2532.64 W, 1658.8 W, 0.94 (lagging), 0.874 (lagging), 0.916 (lagging)]

4.2 In a parallel *RC* circuit shown in Fig. 4.84, $i_R = 15 \cos (5000\,t - 30°)$ amperes. Obtain the current in capacitance.

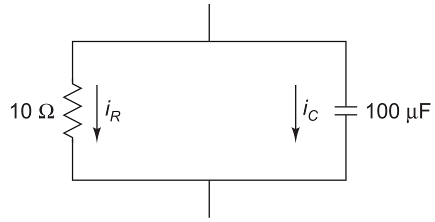


Fig. 4.84

[74.95 sin (5000 t + 150°)]

4.3 A voltage applied across a three-branch circuit is shown by $v = 100 \sin (5000\,t + \pi/4)$ in Fig. 4.85. Find the rms value of the current in the inductor.

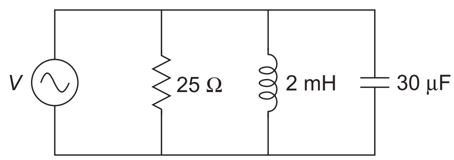


Fig. 4.85

[7.07 A]

4.4 A circuit comprises of a conductance *G* in parallel with a susceptance *B*. Calculate the admittance $G + jB$ if the impedance is $10 + j5\,\Omega$. *[0.08 S, 0.04 S]*

4.5 If an admittance of a circuit is $(8 + j6)$ ℧ and the circuit current is $2 \angle 30°$A, find the pf of the circuit. Also, calculate the apparent power in the circuit.

[0.8 (leading), 0.2 VA]

4.6 A resistor of 50 Ω, an inductor of 0.15 H and a capacitor of 100 μF are connected in parallel across a 100 V, 50 Hz ac supply. Calculate (i) current in each circuit, (ii) resultant current. Draw individual phasor diagrams and the overall phasor diagram. *[2 $\angle 0°$ A, 2.12 $\angle -90°$ A, 3.14 $\angle 90°$ A, 2.25 $\angle 27.02°$ A]*

4.7 The power dissipated in the coil A is 300 W and in the coil B is 400 W. Each coil takes a current of 5 A when connected to a 110 V, 50 Hz supply. Find the current drawn when the coils are connected in parallel. *[9.93 $\angle -50.11°$ A]*

4.8 Find the total impedance, supply current and pf of the entire circuit.

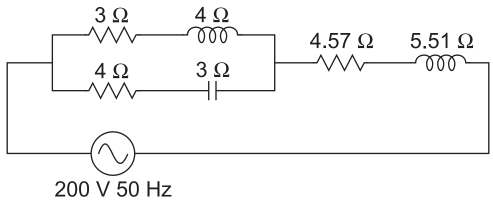


Fig. 4.86

[10.06 $\angle 36.68°$ Ω, 19.88 $\angle -36.68°$ A, 0.8 (lagging)]

4.9 Determine kVA, kVAR and kW consumed by the two impedances $\overline{Z}_1 = (20 + j37.7)$ Ω and $\overline{Z}_2 = (50 + j0)$ Ω, when connected in parallel across a 230 V, 50 Hz supply.

[1.971 kVA, 1.095 kVAR, 1.64 kW]

4.10 In the parallel circuit shown in Fig. 4.87, the power in the 3 Ω resistor is 666 W and the total volt-amperes taken by the circuit is 3370 VA. The power factor of the whole circuit is 0.937 leading. Find Z.

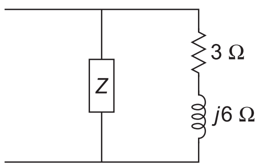


Fig. 4.87

[(2 – j2) Ω]

4.11 A coil is connected across a non-inductive resistor of 120 Ω. When a 240 V, 50 Hz supply is applied to this circuit, the coil draws a current of 5 A and the total current is 6 A. Determine the power and the power factor of (i) the coil, and (ii) the whole circuit. *[420 W, 0.35 (lagging), 900 W, 0.625 (lagging)]*

4.12 A coil takes a current of 10 A and dissipates 1410 W when connected to 220 V, 50 Hz supply. If another coil is connected in parallel with it, the total current taken from the supply is 20 A at a power factor of 0.868. Determine the current and the overall power factor when the coils are connected in series across the same supply. *[5.55 A, 0.8499 (lagging)]*

4.13 Draw an impedance triangle between terminals AB in Fig. 4.88, labelling its sides with appropriate values and units.

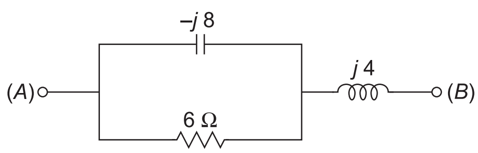


Fig. 4.88

[R = 3.84 Ω, X = 1.12 Ω (inductive), Z = 4 Ω]

4.14 Draw the impedance triangle between terminals *AB* in Fig. 4.89 for the following conditions:

(i) $X_C = 8\ \Omega, R = 6\ \Omega, X_L = 4\ \Omega$ (ii) $X_C = 4\ \Omega, R = 6\ \Omega, X_L = 6\ \Omega$

Label its side with appropriate values and units.

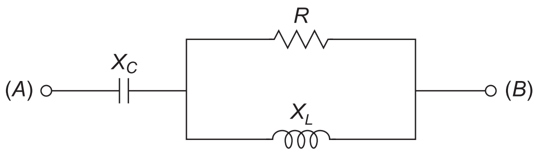


Fig. 4.89

[$R = 1.8461\ \Omega$, $X = 5.2308\ \Omega$ (capacitive), $Z = 5.547\ \Omega$, $R = 3\ \Omega$, $X = 1\ \Omega$ (capacitive), $Z = 3.1623\ \Omega$]

4.15 Draw an admittance triangle between terminals *AB* in Fig. 4.90, labelling its sides with appropriate values and units in case of

(i) $X_L = 4\ \Omega$ and $X_C = 8\ \Omega$ (ii) $X_L = 10\ \Omega$ and $X_C = 5\ \Omega$

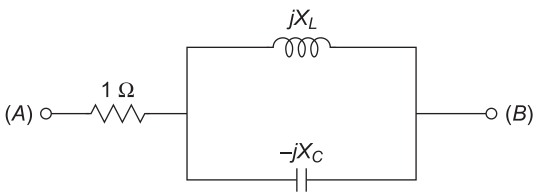


Fig. 4.90

[$G = 0.015\ \mho$, $B_L = 0.123\ \mho$, $Y = 0.124\ \mho$, $G = 0.0098\ \mho$, $B_C = 0.099\ \mho$, $Y = 0.1\ \mho$]

4.16 An inductive impedance *BC* is connected in series with a parallel combination *AB* consisting of a capacitor and a non-inductive resistor. The circuit constants are so adjusted that the current in the parallel branches *AB* are equal, and that the voltage across *AB* is equal to and in quadrature with the voltage across *BC*. When a voltage of 200 V is applied to *AC*, the total power absorbed is 1200 W. Calculate the circuit constants and draw a vector diagram.

[$AB = R = 33.32\ \Omega$ in parallel with $X_C = 33.32\ \Omega$, $BC = (16.66 + j16.66)\ \Omega$]

4.17 A 100 Ω resistor, shunted by a 0.4 H inductor is in series with a capacitor *C*. A voltage of 250 V at 50 Hz is applied to the circuit. Find

(i) the value of *C* to give unity power factor
(ii) the total current
(iii) current in the inductive branch *[65.4 μF, 4.08 A, 2.55 A]*

4.18 A non-inductive 10 Ω resistor is in series with a coil of 1.3 Ω resistance and 0.018 H inductance. If a voltage of maximum value of 100 V at a frequency of 100 Hz is applied to this circuit, what will be the voltage across the resistor? *[62.54 V]*

4.19 Two impedances $\overline{Z} = 10 - j15\ \Omega$ and $\overline{Z} = 4 + j8\ \Omega$ are connected in parallel. The supply voltage is 100 V, 25 Hz. Calculate (i) the admittance, conductance and susceptance of the combined circuit, and (ii) total current drawn and pf.

[$0.097\ \mho$, $0.081\ \mho$, $0.054\ \mho$, 9.7 A, 0.83 (lagging)]

4.20 A voltage of 200 $\angle$ 53.13° V is applied across two impedances in parallel. The values of the impedances are $(12 + j16)\ \Omega$ and $(10 - j20)\ \Omega$. Determine kVA, kVAR and kW in each branch and the pf of the whole circuit.

[2 kVA, 1.2 kW, 1.6 kVAR, 1.788 kVA, 0.8 kW, 1.6 kVAR, unity pf]

4.21 For the circuit shown in Fig. 4.91, evaluate the current through and voltage across each element.

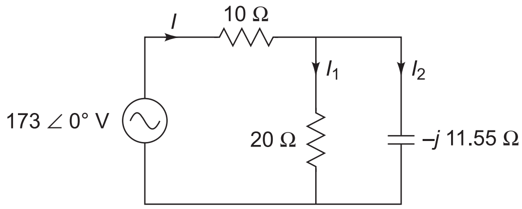


Fig. 4.91

$[\bar{I}_1 = 5 \angle -30°\ A,\ \bar{I}_2 = 8.66 \angle 60°\ A,\ \bar{V}_1 = 100 \angle -30°\ V,$
$\bar{V}_2 = 100 \angle -30°\ V,\ V_{10\,\Omega} = 100 \angle 30°\ V]$

4.22 Two impedances Z_1 and Z_2 are connected in parallel. The first branch takes a leading current of 16 A and has a resistance of 5 Ω, while the second branch takes a lagging current at a pf of 0.8. The total power supplied is 5 kW, the applied voltage being $(100 + j200)$ V. Determine branch currents and total current.

[16 ∠132.46° A, 20.8 ∠26.57° A, 22.49 A]

4.23 For the parallel branch shown in Fig. 4.92, find the value of R_2 when the overall power factor is 0.92 lag.

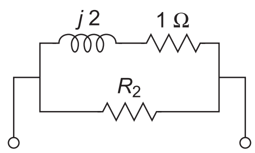


Fig. 4.92

[1.35 Ω]

4.24 In a series–parallel circuit, two parallel branches *A* and *B* are in series with *C*. The impedances are $Z_A = (10 + j8)\ \Omega$, $Z_B = (9 - j6)\ \Omega$ and $Z_C = (3 + j2)\ \Omega$. If the voltage across Z_C is 100 $\angle$ 0° V, determine the values I_A and I_B.

[15.7 ∠–73.39° A, 18.59 ∠– 1.04° A]

4.25 A capacitor is placed in parallel with two inductive loads. The current through the first inductor is 20 A at 30° lag and the current through the second is 40 A at 60° lag. What must be the current in the capacitor so that the current in the external circuit is of unity power factor? *[44.64 ∠ 90°A]*

4.26 A circuit of 15 Ω resistance and 12 Ω inductive reactance is connected in parallel with another circuit consisting of a resistor of 25 Ω in series with a capacitive reactance of 17 Ω. This combination is energized from a 200 V, 40 Hz mains. Find the branch currents, total current and power factor of the circuit. It is desired to raise the power factor of this circuit to unity by connecting a capacitor in parallel.

Determine the value of the capacitance of the capacitor.

[10.42 ∠– 38.56° A, 6.61 ∠34.21° A, 13.95 ∠– 11.56° A, 0.98 (lagging) 54.9 μF]

4.27 A resistor of 30 Ω and a capacitor of unknown value are connected in parallel across a 110 V, 50 Hz supply. The combination draws a current of 5 A form the supply. Find the value of the unknown capacitance of the capacitor. This combination is again connected across a 110 V supply of unknown frequency. It is observed that the total current drawn from the mains falls to 4 A. Determine the frequency of the supply. *[98.58 μF, 23.68 Hz]*

4.28 Two reactive circuits have an impedance of 20 Ω each. One of them has a lagging power factor of 0.8 and the other has a leading power factor of 0.6. Find (i) voltage necessary to send a current of 10 A through the two in series, and (ii) current drawn from 200 V supply if the two are connected in parallel. Draw a phasor diagram in each case. *[282.8 V, 14.14 A]*

4.29 Inductor loads of 0.8 kW and 1.2 kW at lagging power factors of 0.8 and 0.6 respectively are connected across a 200 V, 50 Hz supply. Find the total current, power factor and the value of the capacitor to be put in parallel to both to raise the overall power factor to 0.9 lagging. *[14.87 A, 0.673 (lagging), 98 μF]*

4.8 SERIES RESONANCE

[Dec 2013, May 2016]

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series *R-L-C* circuit as shown in Fig. 4.93. The impedance of the circuit is

$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

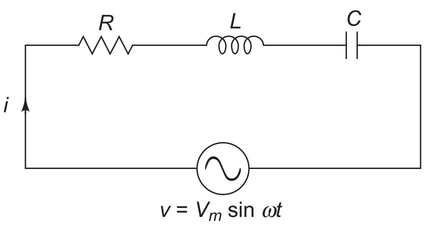


Fig. 4.93 *Series circuit*

At resonance, *Z* must be resistive. Therefore, the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Power Factor

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

At resonance $Z = R$

$$\text{Power factor} = \frac{R}{R} = 1$$

Current Since impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such, an *R-L-C* circuit under resonance is called an *acceptor circuit*.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

Voltage At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

Thus, potential difference across inductor equal to potential difference across capacitor being equal and opposite cancel each other. Also, since I_0 is maximum, V_{L_0} and V_{C_0} will also be maximum. Thus, voltage magnification takes place during resonance. Hence, it is also referred to as voltage magnification circuit.

Phasor Diagram The phasor diagram is shown in Fig. 4.94.

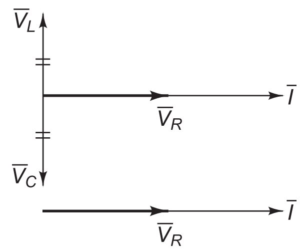


Fig. 4.94 *Phasor diagram*

Behaviour of R, L and C with Change in Frequency Resistance remains constant with the change in frequencies. Inductive reactance X_L is directly proportional to frequency f. It can be drawn as a straight line passing through the origin. Capacitive reactance X_C is inversely proportional to the frequency f. It can be drawn as a rectangular hyperbola in the fourth quadrant.

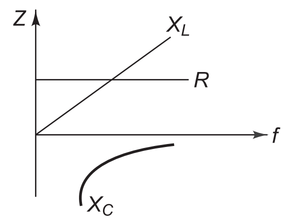


Fig. 4.95 *Behaviour of R, L and C with change in frequency*

Total impedance $\overline{Z} = R + j\,(X_L - X_C)$

(a) When $f < f_0$, impedance is capacitive and decreases up to f_0. The power factor is leading in nature.

(b) At $f = f_0$, impedance is resistive. The power factor is unity.

(c) When $f > f_0$, impedance is inductive and goes on increasing beyond f_0. The power factor is lagging in nature.

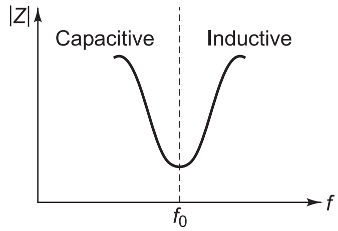


Fig. 4.96 *Impedance*

Bandwidth For the series *R-L-C* circuit, bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance. From the shape of the resonance curve, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance. For this reason, these frequencies are referred as those corresponding to the half-power points. The magnitude of the current at each half-power point is the same.

Hence, $$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point. It follows then that

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

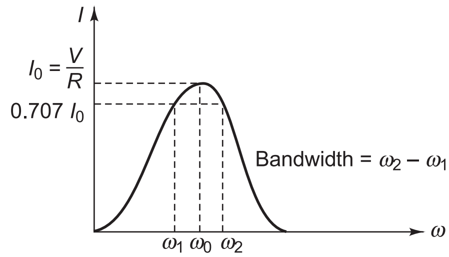


Fig. 4.97 *Resonance curve*

Accordingly, the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance. In Fig. 4.97, the bandwidth is $\omega_2 - \omega_1$.

Expression for Bandwidth Generally, at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{4.1}$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But $$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \tag{4.2}$$

From Eqs (4.1) and (4.2),

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both the sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of R, the term $\left(\dfrac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\dfrac{1}{LC}$.

Then ω is given by, $\omega = \pm\dfrac{R}{2L} \pm \sqrt{\dfrac{1}{LC}} = \pm\dfrac{R}{2L} \pm \dfrac{1}{\sqrt{LC}}$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm\frac{R}{2L} + \omega_0 \qquad \text{(considering only positive sign of } \omega_0\text{)}$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

and $$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

and $$f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

Quality Factor It is a measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R}$$

$$= \frac{X_{L_0}}{R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Example 1

A series R-L-C circuit has the following parameter values: R = 10 Ω, L = 0.01 H, C = 100 μF. Compute the resonant frequency, bandwidth, and lower and upper frequencies of the bandwidth.

Solution

$$R = 10\ \Omega$$
$$L = 0.01\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}} = 159.15\ \text{Hz}$$

(ii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.01} = 159.15\ \text{Hz}$$

(iii) Lower frequency of bandwidth

$$f_1 = f_0 - \frac{BW}{2} = 159.15 - \frac{159.15}{2} = 79.58\ \text{Hz}$$

(iv) Upper frequency of bandwidth

$$f_2 = f_0 + \frac{BW}{2} = 159.15 + \frac{159.15}{2} = 238.73\ \text{Hz}$$

Example 2

For a series RLC circuit having R = 10 Ω, L = 0.01 H and C = 100 μF, Find the resonant frequency, quality factor and bandwith. **[Dec 2014]**

Solution

$$R = 10\ \Omega$$
$$L = 0.01\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}} = 159.15\ \text{Hz}$$

(ii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.01}{100 \times 10^{-6}}} = 1$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.01} = 159.15\ \text{Hz}$$

Example 3

A series RLC circuit has the following parameter values: R = 10 Ω, L = 0.014 H, C = 100 μF. Compute the resonant frequency, quality factor, bandwidth, lower cut-off frequency and upper cut-off frequency. **[May 2015]**

Solution $R = 10\ \Omega$

$L = 0.014$ H

$C = 100\ \mu$F

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.014 \times 100 \times 10^{-6}}} = 134.51 \text{ kHz}$$

(ii) Qualify factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 1.18$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.014} = 113.68 \text{ Hz}$$

(iv) Lower cut-off frequency (f_1)

$$f_1 = f_0 - \frac{BW}{2} = 134.51 - \frac{113.68}{2} = 77.67 \text{ Hz}$$

(v) Upper cut-off frequency (f_2)

$$f_2 = f_0 + \frac{BW}{2} = 134.51 + \frac{113.68}{2} = 191.35 \text{ Hz}$$

Example 4

A series R-L-C circuit consists of R = 1000 Ω, L = 100 mH and C = 10 μF. The applied voltage across the circuit is 100 V.

(i) Find the resonance frequency of the circuit.

(ii) Find Q of the circuit at resonant frequency.

(iii) Calculate the bandwidth of the circuit. **[May 2016]**

Solution $R = 1000\ \Omega$

$L = 100$ mH

$C = 10\ \mu$F

$V = 100$ V

(i) Resonance frequency of the circuit

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}} = 159.15 \text{ kHz}$$

(ii) Q of the circuit at resonant frequency

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\sqrt{\frac{100 \times 10^{-3}}{10 \times 10^{-6}}} = 0.1$$

(iii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{1000}{2\pi \times 100 \times 10^{-3}} = 1591.55 \text{ Hz}$$

Example 5

An R-L-C series circuit with a resistance of 10 Ω, inductance of 0.2 H and a capacitance of 40 μF is supplied with a 100 V supply at variable frequency. Find the following w.r.t. the series resonant circuit:

(i) frequency at which resonance takes place
(ii) current
(iii) power
(iv) power factor
(v) voltage across R-L-C at that time
(vi) quality factor
(vii) half-power points
(viii) resonance and phasor diagrams

Solution

$$R = 10\ \Omega$$
$$L = 0.2 \text{ H}$$
$$C = 40\ \mu\text{F}$$
$$V = 100 \text{ V}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = 56.3 \text{ Hz}$$

(ii) Current

$$I_0 = \frac{V}{R} = \frac{100}{10} = 10 \text{ A}$$

(iii) Power

$$P_0 = I_0^2 R = (10)^2 \times 10 = 1000 \text{ W}$$

(iv) Power factor

$$\text{pf} = 1$$

(v) Voltage across R, L, C

$$V_{R_0} = R\, I_0 = 10 \times 10 = 100 \text{ V}$$
$$V_{L_0} = X_{L_0} I_0 = 2\pi f_0 L I_0 = 2\pi \times 56.3 \times 0.2 \times 10 = 707.5 \text{ V}$$
$$V_{C_0} = X_{C_0} I_0 = \frac{1}{2\pi f_0 C} I_0 = \frac{1}{2\pi \times 56.3 \times 40 \times 10^{-6}} \times 10 = 707.5 \text{ V}$$

(vi) Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.2}{40 \times 10^{-6}}} = 7.07$$

(vii) Half-power points

$$f_1 = f_0 - \frac{R}{4\pi L} = 56.3 - \frac{10}{4\pi \times 0.2} = 52.32\,\text{Hz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 56.3 + \frac{10}{4\pi \times 0.2} = 60.3\,\text{Hz}$$

(viii) Resonance and phasor diagram

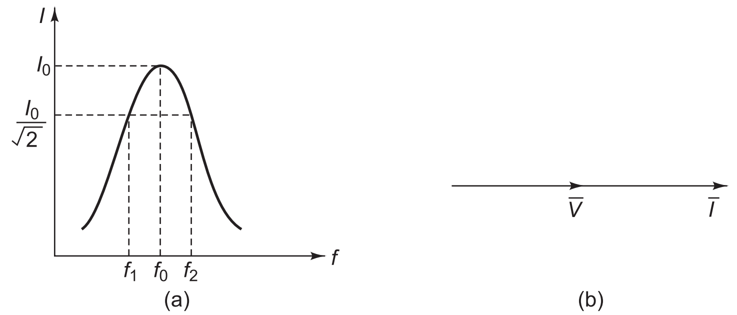


Fig. 4.98

Example 6

A series R-L-C circuit is supplied with v(t) = 10 sin 1000 t volts. If the maximum peak voltage across capacitor is 400 volts, find the quality factor of the circuit. **[Dec 2015]**

Solution

$$V_m = 10$$

$$V_{C_m} = 400\text{ V}$$

$$V = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}\text{ V}$$

$$V_{co} = \frac{V_{cm}}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 200\sqrt{2}\text{ V}$$

$$Q = \frac{V_{co}}{V} = \frac{200\sqrt{2}}{5\sqrt{2}} = 40$$

Example 7

A series R-L-C circuit is connected to a 200 V ac supply. The current drawn by the circuit at resonance is 20 A. The voltage drop across the capacitor is 5000 V at series resonance. Calculate resistance and inductance if capacitance is 4 μF. Also, calculate the resonant frequency.

Solution

$$V = 200\text{ V}$$

$$I_0 = 20\text{ A}$$

$$V_{C_0} = 5000\text{ V}$$

$$C = 4\ \mu\text{F}$$

(i) Resistance

$$R = \frac{V}{I_0} = \frac{200}{20} = 10\ \Omega$$

(ii) Resonant frequency

$$X_{C_0} = \frac{V_{C_0}}{I_0} = \frac{5000}{20} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times f_0 \times 4 \times 10^{-6}}$$

$$f_0 = 159.15\ \text{Hz}$$

(iii) Inductance

At resonance $X_{C_0} = X_{L_0} = 250\ \Omega$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 159.15 \times L$$

$$L = 0.25\ \text{H}$$

Example 8

A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 230 V, 50 Hz supply, the maximum current obtained by varying the inductance is 2 A. The voltage across the capacitor is 500 V. Calculate the resistance, inductance and capacitance of the circuit. **[Dec 2012]**

Solution

$$V = 230\ \text{V}$$

$$f_0 = 50\ \text{Hz}$$

$$I_0 = 2\ \text{A}$$

$$V_{C_0} = 500\ \text{V}$$

(i) Resistance

$$R = \frac{V}{I_0} = \frac{230}{2} = 115\ \Omega$$

(ii) Capacitance

$$X_{C_0} = \frac{V_{C0}}{I_0} = \frac{500}{2} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 12.73\ \mu\text{F}$$

(iii) Inductance

At resonance $X_{C_0} = X_{L_0} = 250\ \Omega$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 50 \times L$$

$$L = 0.795\ \text{H}$$

Example 9

A coil of 2 Ω resistance and 0.01 H inductance is connected in series with a capacitor across 200 V mains. What must be the capacitance in order that maximum current occurs at a frequency of 50 Hz? Find also the current and voltage across the capacitor.

Solution $R = 2\ \Omega$

$$L = 0.01\ \text{H}$$

$$V = 200\ \text{V}$$

$$f_0 = 50\ \text{Hz}$$

(i) Capacitance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$50 = \frac{1}{2\pi\sqrt{0.01 \times C}}$$

$$C = 1013.2\ \mu\text{F}$$

(ii) Current

$$I_0 = \frac{V}{R} = \frac{200}{2} = 100\ \text{A}$$

(iii) Voltage across capacitor

$$V_{C_0} = I_0\, X_{C_0} = I_0 \frac{1}{2\pi f_0 C} = 100 \times \frac{1}{2\pi \times 50 \times 1013.2 \times 10^{-6}} = 314.16\ \text{V}$$

Example 10

A voltage v(t) = 10 sin ωt is applied to a series R-L-C circuit. At the resonant frequency of the circuit, the voltage across the capacitor is found to be 500 V. The bandwidth of the circuit is known to be 400 rad/s and the impedance of the circuit at resonance is 100 Ω. Determine inductance and capacitance resonant frequency, upper and lower cut-off frequencies.

Solution $v(t) = 10 \sin \omega t$

$$V_{C_0} = 500 \text{ V}$$
$$BW = 400 \text{ rad/s}$$
$$R = 100\ \Omega$$

(i) Inductance and capacitance

$$V = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ V}$$

$$I_0 = \frac{V}{R} = \frac{7.07}{100} = 0.0707 \text{ A}$$

$$BW = \frac{R}{L}$$

$$400 = \frac{100}{L}$$

$$L = 0.25 \text{ H}$$

$$Q_0 = \frac{V_{C0}}{V} = \frac{500}{7.07} = 70.72$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$70.72 = \frac{1}{100}\sqrt{\frac{0.25}{C}}$$

$$C = 4.99 \text{ nF}$$

(ii) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.25 \times 4.99 \times 10^{-9}}} = 4506.09 \text{ Hz}$$

(iii) Lower cut-off frequency

$$f_1 = f_0 - \frac{R}{4\pi L} = 4506.09 - \frac{100}{4\pi \times 0.25} = 4474.26 \text{ Hz}$$

(iv) Upper cut-off frequency

$$f_2 = f_0 + \frac{R}{4\pi L} = 4506.09 + \frac{100}{4\pi \times 0.25} = 4537.92 \text{ Hz}$$

Example 11

An inductor having a resistance of 25 Ω and Q_0 of 10 at a resonant frequency of 10 kHz is fed from a 100 V supply. Calculate (i) value of series capacitance required to produce resonance with the coil, (ii) the inductance of the coil, (iii) Q_0 using L/C ratio, (iv) voltage across capacitor, and (v) voltage across the coil. **[May 2013]**

Solution

$$R = 25\ \Omega$$
$$Q_0 = 10$$
$$f_0 = 10\text{ kHz}$$
$$V = 100$$

(i) Value of series capacitance

$$Q_0 = \frac{V_{C0}}{V}$$
$$10 = \frac{V_{C0}}{100}$$
$$V_{C0} = 1000\text{ V}$$
$$I_0 = \frac{V}{R} = \frac{100}{25} = 4\text{ A}$$
$$V_{C0} = I_0 X_{C0}$$
$$1000 = 4X_{C0}$$
$$X_{C0} = 250\ \Omega$$
$$X_{C0} = \frac{1}{2\pi f_0 C}$$
$$250 = \frac{1}{2\pi \times 10 \times 10^3 \times C}$$
$$C = 6.37 \times 10^{-8}\text{ F}$$

(ii) Inductance of the coil

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$10 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 6.37 \times 10^{-8}}}$$
$$L = 3.98\text{ mH}$$

(iii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{3.98 \times 10^{-3}}{6.37 \times 10^{-8}}} = 10$$

(iv) Voltage across capacitor

$$V_{L0} = V_{C0} = 1000\text{ V}$$

(v) Voltage across coil

$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2} = \sqrt{(25)^2 + (250)^2} = 251.25\ \Omega$$

$$V_{\text{coil}} = IZ_{\text{coil}} = 4 \times 251.25 = 1005 \text{ V}$$

Example 12

A series resonant circuit has an impedance of 500 Ω at resonant frequency. Cut-off frequencies are 10 kHz and 100 Hz. Determine (i) resonant frequency, (ii) value of L, C, and (iii) quality factor at resonant frequency.

Solution

$$R = 500\ \Omega$$

$$f_1 = 100 \text{ Hz}$$

$$f_2 = 10 \text{ kHz}$$

(i) Resonant frequency

$$BW = f_2 - f_1 = 10000 - 100 = 9900 \text{ Hz}$$

$$f_1 = f_0 - \frac{R}{4\pi L} \tag{1}$$

$$f_2 = f_0 + \frac{R}{4\pi L} \tag{2}$$

Adding Eqs (1) and (2),

$$f_1 + f_2 = 2f_0$$

$$f_0 = \frac{f_1 + f_2}{2} = \frac{100 + 10000}{2} = 5050 \text{ Hz}$$

(ii) Values of L and C

$$BW = \frac{R}{2\pi L}$$

$$9900 = \frac{500}{2\pi L}$$

$$L = 8.038 \text{ mH}$$

$$X_{L_0} = 2\pi f_0 L = 2\pi \times 5050 \times 8.038 \times 10^{-3} = 255.05\ \Omega$$

At resonance

$$X_{L_0} = X_{C_0} = 255.05\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$255.05 = \frac{1}{2\pi \times 5050 \times C}$$

$$C = 0.12\ \mu\text{F}$$

(iii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{500}\sqrt{\frac{8.038 \times 10^{-3}}{0.12 \times 10^{-6}}} = 0.5176$$

Example 13

Impedance of a circuit is observed to be capacitive and decreasing from 1 Hz to 100 Hz. Beyond 100 Hz, the impedance starts increasing. Find the values of circuit elements if the power drawn by this circuit is 100 W at 100 Hz, when the current is 1 A. The power factor of the circuit at 70 Hz is 0.707.

Solution

$$f_0 = 100 \text{ Hz}$$
$$P_0 = 100 \text{ W}$$
$$I_0 = 1 \text{ A}$$
$$(\text{pf})_{70 \text{ Hz}} = 0.707$$

The impedance of the circuit is capacitive and decreasing from 1 Hz to 100 Hz. Beyond 100 Hz, the impedance starts increasing.

$$f_0 = 100 \text{ Hz}$$
$$P_0 = I_0^2 R$$
$$100 = (1)^2 \times R$$
$$R = 100\ \Omega$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$100 = \frac{1}{2\pi\sqrt{LC}}$$
$$LC = 2.53 \times 10^{-6} \tag{1}$$

Power factor at 70 Hz is 0.707.

$$\frac{R}{Z} = 0.707$$
$$\frac{100}{Z} = 0.707$$
$$Z = 141.44\ \Omega$$

$$\text{Impedance at 70 Hz} = Z_{70} = \sqrt{R^2 + (X_C - X_L)^2}$$
$$141.44 = \sqrt{(100)^2 + \left(\frac{1}{2\pi \times 70 \times C} - 2\pi \times 70 \times L\right)^2}$$
$$\frac{2.27 \times 10^{-3}}{C} - 439.82\, L = 100.02 \tag{2}$$

Solving Eqs (1) and (2),

$$L = 0.2187 \text{ H}$$
$$C = 11.58\ \mu\text{F}$$

Example 14

A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value while it is reduced to one-half when the capacitance is 600 pF. Find resistance, inductance and Q-factor of inductor.

Solution

$$f_0 = 1 \text{ MHz}$$
$$C_1 = 500 \text{ pF}$$
$$C_2 = 600 \text{ pF}$$

(i) Resistance and inductance of inductor

At resonance $C = 500$ pF

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$10^6 = \frac{1}{2\pi\sqrt{L \times 500 \times 10^{-12}}}$$

$$L = 0.05 \text{ mH}$$

$$X_L = 2\pi f_0 L = 2\pi \times 10^6 \times 0.05 \times 10^{-3} = 314.16\ \Omega$$

When capacitance is 600 pF, the current reduces to one-half of the current at resonance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}} = 265.26\ \Omega$$

$$I = \frac{1}{2} I_0$$

$$\frac{V}{Z} = \frac{1}{2}\frac{V}{R}$$

$$Z = 2R$$

$$\sqrt{R^2 + (X_L - X_C)^2} = 2R$$

$$R^2 + (314.16 - 265.26)^2 = 4R^2$$

$$3R^2 = 2391.21$$

$$R = 28.23\ \Omega$$

(ii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{28.23}\sqrt{\frac{0.05 \times 10^{-3}}{500 \times 10^{-12}}} = 11.2$$

4.9 PARALLEL RESONANCE

[Dec 2012, May 2014]

Consider a parallel circuit consisting of a coil and a capacitor as shown in Fig. 4.99. The impedances of two branches are

$$\bar{Z}_1 = R + jX_L$$

$$\bar{Z}_2 = -jX_C$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

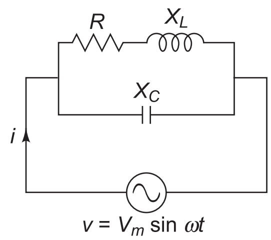


Fig. 4.99 *Parallel circuit*

Admittance of the circuit $\bar{Y} = \bar{Y}_1 + \bar{Y}_2$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore, the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L then

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Dynamic Impedance of a Parallel Circuit At resonance, the circuit is purely resistive. The real part of admittance is $\dfrac{R}{R^2 + X_L^2}$. Hence, the dynamic impedance at resonance is given by

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L X_C = \frac{L}{C}$$

$$Z_D = \frac{L}{CR}$$

Current Since impedance is maximum at resonance, the current is minimum at resonance.

$$I_0 = \frac{V}{Z_D} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

Phasor Diagram At resonance, power factor of the circuit is unity and the total current drawn by the circuit is in phase with the voltage. This will happen only when the current I_C is equal to the reactive component of the current in the inductive branch, i.e., $I_C = I_L \sin\phi$

Hence, at resonance

$$I_C = I_L \sin\phi$$

and $$I = I_L \cos\phi$$

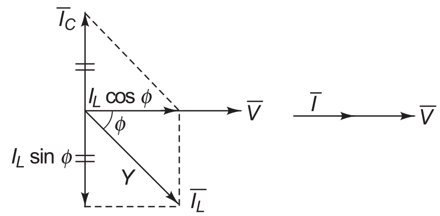


Fig. 4.100 *Phasor diagram*

Behaviour of Conductance G, Inductive Susceptance B_L and Capacitive Susceptance with Change in Frequency Conductance remains constant with the change in frequencies.

Inductive susceptance B_L is

$$B_L = \frac{1}{jX_L} = -j\frac{1}{X_L} = -j\frac{1}{2\pi f L}$$

It is inversely proportional to the frequency. Thus, it decreases with the increase in the frequency. Hence, it can be drawn as a rectangular hyperbola in the fourth quadrant.

Capacitive susceptance B_C is

$$B_C = \frac{1}{-jX_C} = j\frac{1}{X_C} = j2\pi fC$$

It is directly proportional to the frequency. It can be drawn as a straight line passing through the origin.

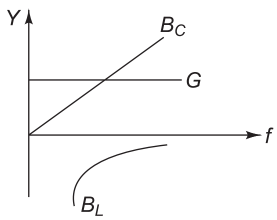


Fig. 4.101 *Behaviour of G, B_L and B_C with change in frequency*

(a) When $f < f_0$, inductive susceptance predominates. Hence, the current lags behind the voltage and the power factor is lagging in nature.

(b) When $f = f_0$, net susceptance is zero. Hence, the admittance is minimum and impedance is maximum. At f_0, the current is in phase with the voltage and the power factor is unity.

(c) When $f > f_0$, capacitive susceptance predominates. Hence, the current leads the voltage and power factor is leading in nature.

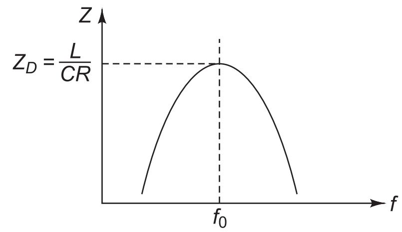


Fig. 4.102 *Impedance*

Bandwidth The bandwidth of a parallel resonant circuit is defined in the same way as that for a series resonant circuit.

Quality Factor It is a measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{Current through inductor or capacitor}}{\text{Current at resonance}} = \frac{I_{C_0}}{I_0}$$

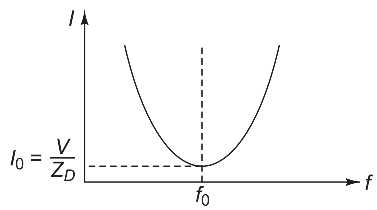


Fig. 4.103 *Resonance curve*

Substituting values of I_{C_0} and I_0,

$$Q_0 = \frac{\frac{V}{X_{C_0}}}{\frac{VCR}{L}} = \frac{\frac{1}{X_{C_0}}}{\frac{CR}{L}} = \frac{\omega_0 C}{\frac{CR}{L}} = \frac{\omega_0 L}{R}$$

Neglecting the resistance R, the resonant frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

4.10 COMPARISON OF SERIES AND PARALLEL RESONANT CIRCUITS

[May 2013, 2015]

Parameter	*Series Circuit*	*Parallel Circuit*
Current at resonance	$I = \frac{V}{R}$ and is maximum	$I = \frac{VCR}{L}$ and is minimum
Impedance at resonance	$Z = R$ and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q-factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$
It magnifies	Voltage across L and C	Current through L and C

Example 1

Derive the expression for resonant frequency for the parallel circuit shown in Fig. 4.97. Also calculate the impedance and current at resonance.

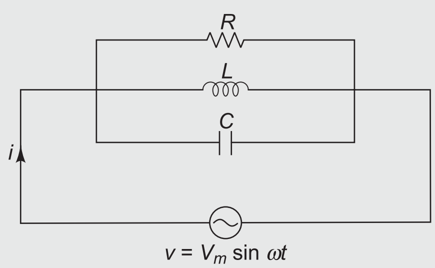


Fig. 4.104

Solution

$$\bar{Z}_1 = R$$

$$\bar{Z}_2 = jX_L$$

$$\bar{Z}_3 = -jX_C$$

(i) Resonant frequency of the circuit

For parallel circuit

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3}$$

$$\overline{Y} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

(ii) Impedance at resonance

At resonance, the circuit is purely resistive. Hence, the imaginary part of $\overline{Y}$ is zero.

$$Y_D = \frac{1}{R}$$

$$Z_D = R$$

(iii) Current at resonance

$$I_0 = \frac{V}{Z_D} = \frac{V}{R}$$

Example 2

Derive the expression for the resonant frequency of the parallel circuit shown in Fig. 4.105.

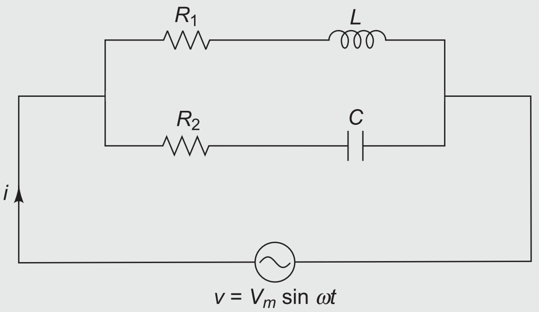


Fig. 4.105

Solution

$$\bar{Z}_1 = R_1 + jX_L$$

$$\bar{Z}_2 = R_2 - jX_C$$

For parallel circuit,

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}$$

$$\bar{Y} = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2} + \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} - j\left[\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right]$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{\dfrac{1}{\omega_0 C}}{R_2^2 + \dfrac{1}{\omega_0^2 C^2}}$$

$$\frac{\omega_0^2 LC}{R_1^2 + \omega_0^2 L^2} = \frac{\omega_0^2 C^2}{R_2^2 \omega_0^2 C^2 + 1}$$

$$LC\,(R_2^2\,\omega_0^2\,C^2 + 1) = C^2\,(R_1^2 + \omega_0^2 L^2)$$

$$\omega_0^2 R_2^2 LC^3 + LC = C^2 R_1^2 + \omega_0^2 L^2 C^2$$

$$\omega_0^2 R_2^2 LC^3 - \omega_0^2 L^2 C^2 = C^2 R_1^2 - LC$$

$$\omega_0^2 LC^2 (CR_2^2 - L) = C(CR_1^2 - L)$$

$$\omega_0^2 LC\,(CR_2^2 - L) = CR_1^2 - L$$

$$\omega_0^2 = \frac{CR_1^2 - L}{LC(CR_2^2 - L)}$$

$$\omega_0 = \sqrt{\frac{CR_1^2 - L}{LC(CR_2^2 - L)}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

where f_0 is called the resonant frequency of the circuit.

Example 3

A coil having an inductance of L henries and a resistance of 12 Ω is connected in parallel with a variable capacitor. At $\omega = 2.3 \times 10^6$ rad/s, resonance is achieved and at this instant, capacitance C = 0.021 μF. Find the inductance of the coil.

Solution

$$R = 12\ \Omega$$

$$\omega_0 = 2.3 \times 10^6 \text{ rad/s}$$

$$C = 0.021\ \mu\text{F}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2.3 \times 10^6 = \sqrt{\frac{1}{L \times 0.021 \times 10^{-6}} - \frac{(12)^2}{L^2}}$$

$$L = 89.7\ \mu\text{H}$$

Example 4

A coil of 31.8 mH inductances with a resistance of 12 Ω is connected in parallel with a capacitor across a 250-volt, 50 Hz supply. Determine the value of capacitance, if no reactive current is taken from the supply. **[May 2014]**

Solution

$$L = 31.8 \text{ mH}$$

$$R = 12\ \Omega$$

$$V = 250 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{R}{L}\right)^2}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{31.8\times10^{-3}\times C}-\left(\frac{12}{31.8\times10^{-3}}\right)^2}$$

$$C = 130.43\ \mu\text{F}$$

Example 5

A coil of 20 Ω resistance has an inductance of 0.2 H and is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which this circuit will behave as a non-inductive resistance. Find also the value of dynamic resistance.

Solution

$$R = 20\ \Omega$$
$$L = 0.2 \text{ H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.2\times100\times10^{-6}}-\left(\frac{20}{0.2}\right)^2} = 31.83 \text{ Hz}$$

(ii) Dynamic resistance

$$Z_D = \frac{L}{CR} = \frac{0.2}{100\times10^{-6}\times20} = 100\ \Omega$$

Example 6

An inductive coil having a resistance of 20 Ω and inductance of 0.2 H is connected in parallel with a 20 μF capacitor with variable frequency and 230 V supply. Find the frequency at which the total current drawn from supply is in phase with the supply voltage. Find the value of the current and the impedance of the circuit at this frequency. **[Dec 2014, 2015]**

Solution

$$R = 20\ \Omega$$
$$L = 0.2 \text{ H}$$
$$C = 20\ \mu\text{F}$$
$$V = 230 \text{ V}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 20 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2} = 77.97 \text{ Hz}$$

(ii) Dynamic Impedance

$$Z_D = \frac{L}{CR} = \frac{0.2}{20 \times 10^{-6} \times 20} = 500\ \Omega$$

(iii) Current

$$I = \frac{V}{Z_D} = \frac{230}{500} = 0.46 \text{ A}$$

Example 7

An inductive coil of 10 Ω resistance and 0.1 H inductance is connected in parallel with a 150 μF capacitor to a variable frequency, and 200 V supply. Find the resonance frequency at which the total current taken from the supply is in phase with the supply voltage. Also find value of this current. Draw the phasor diagram. **[Dec 2013]**

Solution

$$R = 10\ \Omega$$
$$L = 0.1 \text{ H}$$
$$C = 150\ \mu\text{F}$$
$$V = 200 \text{ V}$$

(i) Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 150 \times 10^{-6}} - \left(\frac{10}{0.1}\right)^2}$$

$$= 37.89 \text{ Hz}$$

(ii) Value of current

$$Z_D = \frac{L}{CR} = \frac{0.1}{150 \times 10^{-6} \times 10} = 66.67 \text{ A}$$

$$I = \frac{V}{Z_D} = \frac{200}{66.67} = 3\text{A}$$

(iii) Phasor diagram

$$\overline{Z}_{\text{coil}} = 10 + j2\pi \times 37.89 \times 0.1 = 25.82\ \angle 67.22°\ \Omega$$

$$\overline{Z}_C = -j\frac{1}{2\pi \times 37.89 \times 150 \times 10^{-6}} = -j28 = 28\angle -90°\ \Omega$$

$$\bar{I}_{coil} = \frac{200}{25.82\angle 67.22°} = 7.75\angle -67.22°\,\Omega$$

$$\bar{I}_C = \frac{200}{28\angle -90°} = 7.14\angle 90°\,\Omega$$

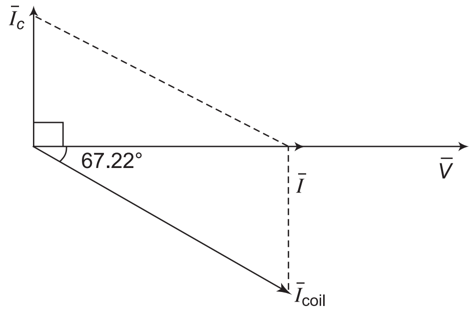


Fig. 4.106

Example 8

A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit. Calculate (i) the value of capacitance at resonance, (ii) Q factor of the circuit, (iii) dynamic impedance of the circuit, and (iv) total circuit current.

Solution

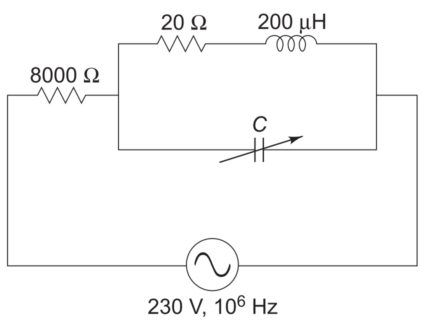


Fig. 4.107

$$R = 20\ \Omega$$
$$L = 200\ \mu\text{H}$$
$$f_0 = 10^6\ \text{Hz}$$
$$V = 230\ \text{V}$$
$$R_S = 8000\ \Omega$$

(i) Value of capacitance

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200\times 10^{-6}\times C} - \frac{(20)^2}{(200\times 10^{-6})^2}}$$

$$C = 126.65\ \text{pF}$$

(ii) Quality factor

$$Q_0 = \frac{2\pi f_0 L}{R} = \frac{2\pi\times 10^6\times 200\times 10^{-6}}{20} = 62.83$$

(iii) Dynamic impedance

$$Z_D = \frac{L}{CR} = \frac{200\times10^{-6}}{126.65\times10^{-12}\times20} = 78958\ \Omega$$

(iv) Current

Total impedance of the circuit at resonance = $Z_D + R_S$ = 78958 + 8000 = 86958 Ω

$$I = \frac{230}{86958} = 2.65\text{ mA}$$

Example 9

A coil of 400 Ω resistance and 318 μH inductance is connected in parallel with a capacitor and the circuit resonates at 1 MHz. If a second capacitor of 23.5 pF capacitance is connected in parallel with the first capacitor, find the frequency at which the circuit resonates.

Solution

$$R = 400\ \Omega$$
$$L = 318\ \mu\text{H}$$
$$f_0 = 1\text{ MHz}$$
$$C_2 = 23.5\text{ pF}$$
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1}-\frac{R^2}{L^2}}$$
$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{318\times10^{-6}\times C_1}-\frac{(400)^2}{(318\times10^{-6})^2}}$$
$$C_1 = 76.59\text{ pF}$$

When capacitor of 23.5 pF is connected in parallel with C_1 = 76.59 pF

$$C_T = C_1 + C_2 = 76.59 + 23.5 = 100.09\text{ pF}$$

The new resonant frequency $f_0' = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_T}-\dfrac{R^2}{L^2}}$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{318\times10^{-6}\times100.09\times10^{-12}}-\frac{(400)^2}{(318\times10^{-6})^2}}$$
$$= 869.34\text{ kHz}$$

Example 10

A 46 mH inductive coil has a resistance of 10 Ω. (i) How much current will it draw if connected across a 100 V, 60 Hz supply? (ii) What is the power factor of the coil? (iii) Determine the value of capacitance that must be connected across the coil to make the power factor of overall circuit unity. **[May 2013]**

Solution

$$L = 46 \text{ mH}$$
$$r = 10\ \Omega$$
$$V = 100 \text{ V}$$
$$f = 60 \text{ Hz}$$

(i) Current

$$X_L = 2\pi f L = 2\pi \times 60 \times 46 \times 10^{-3} = 17.34\ \Omega$$
$$Z = \sqrt{r^2 + X_L^2} = \sqrt{(10)^2 + (17.34)^2} = 20.02\ \Omega$$
$$I = \frac{V}{Z} = \frac{100}{20.02} = 5 \text{ A} = 60 \text{ Hz}$$

(ii) Power factor of the coil

$$\text{pf} = \cos\phi = \frac{r}{Z} = \frac{10}{20.02} = 0.5 \text{ (lagging)}$$

(iii) Value of capacitance

When a capacitor is connected across a coil to make power factor of overall circuit unity, the circuit is in resonance.

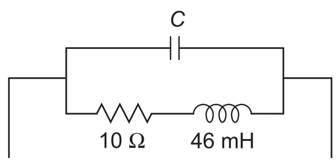


Fig. 4.108

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$
$$60 = \frac{1}{2\pi}\sqrt{\frac{1}{46 \times 10^{-3} \times C} - \frac{(10)^2}{(46 \times 10^{-3})^2}}$$
$$C = 114.79\ \mu\text{F}$$

Example 11

A coil having a resistance and inductance of 15 Ω and 8 mH respectively is connected in parallel with another coil having a resistance and inductance of 4 Ω and 18 mH. If this parallel combination is to be replaced by a single coil, calculate the value of resistance and inductance of that coil. What value of capacitance should be connected in parallel with this coil in order to get unity power factor? Assume operating frequency to be 50 Hz.

Solution

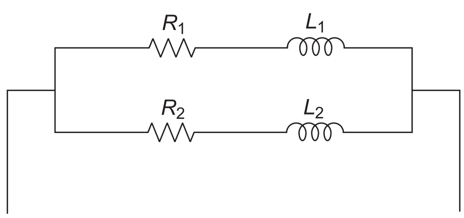


Fig. 4.109

$$R_1 = 15\ \Omega$$
$$L_1 = 8 \text{ mH}$$
$$R_2 = 4\ \Omega$$
$$L_2 = 18 \text{ mH}$$
$$f = 50 \text{ Hz}$$

(i) Value of resistance and inductance of the coil

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 8 \times 10^{-3} = 2.51\ \Omega$$

$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 18 \times 10^{-8} = 5.65\ \Omega$$

$$\bar{Z}_1 = R_1 + jX_{L_1} = 15 + j2.51 = 15.21\ \angle 9.5°\ \Omega$$

$$\bar{Z}_2 = R_2 + jX_{L_2} = 4 + j5.65 = 6.92\ \angle 54.7°\ \Omega$$

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(15.21\angle 9.5°)(6.92\angle 54.7°)}{15.21\angle 9.5° + 6.92\angle 54.7°} = 5.09\ \angle 40.96°\ \Omega$$

$$= 3.84 + j3.34\ \Omega$$

$$R = 3.84\ \Omega$$

$$X_L = 3.34\ \Omega$$

$$X_L = 2\pi f L$$

$$3.34 = 2\pi \times 50 \times L$$

$$L = 10.63\ \text{mH}$$

(ii) Value of capacitance

When a capacitance C is connected in parallel with this coil, power factor becomes unity. Hence, the circuit behaves like a parallel resonant circuit at 50 Hz.

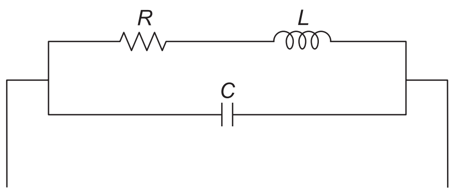


Fig. 4.110

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{10.63 \times 10^{-3} \times C} - \frac{(3.84)^2}{(10.63 \times 10^{-3})^2}}$$

$$C = 410.46\ \mu\text{F}$$

Example 12

A series circuit consisting of a 12 Ω resistor, 0.3 H inductor and a variable capacitor is connected across a 100 V, 50 Hz ac supply. The capacitance value is adjusted to obtain maximum current. Find, the capacitance value and the power drawn by the circuit under the condition. Now, the supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of the capacitance C_1 to be connected across the above series circuit so that current drawn from the supply is minimum.

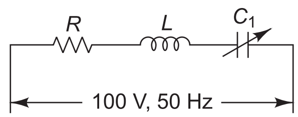


Fig. 4.111

Solution $R = 12\ \Omega$

$L = 0.3$ H

$V = 100$ V

$f = 50$ Hz

(i) Value of capacitance C_1

The resonance occurs at $f = 50$ Hz

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

$$50 = \frac{1}{2\pi\sqrt{0.3\times C_1}}$$

$$C_1 = 33.77\ \mu\text{F}$$

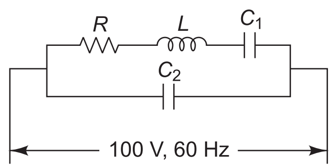


Fig. 4.112

(ii) Value of capacitance C_2

The resonance occurs at 60 Hz.

$$X_L = 2\pi f_0 L = 2\pi \times 60 \times 0.3 = 113.1\ \Omega$$

$$X_{C_1} = \frac{1}{2\pi f_0 C_1} = \frac{1}{2\pi \times 60 \times 33.77 \times 10^{-6}} = 78.55\ \Omega$$

$$\bar{Z}_1 = 12 + j\,113.1 - j\,78.55 = 36.57\ \angle 70.85\ \Omega$$

$$\bar{Y}_1 = \frac{1}{Z_1} = \frac{1}{36.57\angle 70.85^\circ} = 0.027\ \angle 70.85^\circ\ \mho = 8.86\times 10^{-3} - j\,0.0255\ \mho$$

$$\bar{Y}_{\text{req}} = \bar{Y}_1 + \bar{Y}_2 = 8.86\times 10^{-3} - j\,0.0255 + \bar{Y}_2$$

At resonance, the imaginary part of $\bar{Y}_{\text{req}}$ becomes zero.

$\therefore$ $$\bar{Y}_2 = j\,0.0255\ \mho$$

$$\bar{Y}_2 = \frac{1}{X_{C_2}} = 0.0255\ \mho$$

$$X_{C_2} = 39.22\ \Omega$$

$$X_{C_2} = \frac{1}{2\pi f_0 C_2}$$

$$39.22 = \frac{1}{2\pi \times 60 \times C_2}$$

$$C_2 = 67.63\ \mu\text{F}$$

Useful Formulae

Series Resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$I_0 = \frac{V}{R}$$

$$Z_0 = R$$

$$\text{pf} = 1$$

$$V_{L_0} = V_{C_0}$$

$$V_{L_0} = I_0 X_{L_0}$$

$$V_{C_0} = I_0 X_{C_0}$$

$$X_L = X_C$$

$$BW = f_2 - f_1 = \frac{R}{2\pi L} \text{ (Hz)}$$

$$= \omega_2 - \omega_1 = \frac{R}{L} \text{ (rad/s)}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonance (Coil in parallel with capacitor)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$Z_D = \frac{L}{CR}$$

$$I_0 = \frac{VCR}{L}$$

Exercise 4.4

4.1 A series *RLC* circuit has the following parameter values:
$R = 10\ \Omega$, $L = 0.014$ H, $C = 100\ \mu$F. Compute the following:
(i) resonant frequency in rad/s
(ii) quality factor of the circuit
(iii) bandwidth

(iv) lower and upper frequency points of the bandwidth

[845.2 rad/s, 1.185, 714.3 rad/s 487.9 rad/s, 1202.1 rad/s]

4.2 A series *RLC* circuit as $R = 500\ \Omega$, $L = 60$ mH and $C = 0.053\ \mu$F. Estimate

(i) circuit impedance at 50 Hz (ii) resonance frequency in rad/s

(iii) quality factor of the circuit (iv) bandwidth

(v) current at resonance frequency with an applied voltage of 30 V

[0.0041 Ω, 17733 rad/s, 2.128, 8333.33 rad/s 60 mA]

4.3 A choking coil of 10 Ω resistance and 0.1 H inductance is connected in series with a capacitor of 200 μF capacitance across a supply voltage of 230 V, 50 Hz ac. Find the following w.r.t. the resonant circuit.

(i) Neat circuit diagram indicating all the relevant parameters

(ii) At what frequency will the circuit resonate

(iii) Value of reactances at resonance condition

(iv) Impedance of the coil at resonance condition

(v) Voltage across the coil and voltage across the capacitance

(vi) Half-power frequencies f_1 and f_2

(vii) Q factor of the circuit

(viii) Power and power factor

[35.59 Hz, 22.36 Ω, 10 Ω, 744.28 V, 514.28 V, 27.63 Hz, 43.55 Hz, 2.24, 5.29 kW, 1]

4.4 A coil of 40 Ω resistance and 0.75 H inductance forms part of a series circuit for which the resonant frequency is 55 Hz. If the supply is 250 V, 50 Hz, find

(i) the line current (ii) the power factor

(iii) the voltage across the coil

[3.85 A, 0.616 (leading), 920.07 V]

4.5 An *RLC* series circuit of 10 Ω resistance should be designed to have a bandwidth of 100 Hz. Determine the values of *L* and *C* so that the circuit resonates at 250 Hz. *[0.0159 H, 25 μF]*

4.6 A resistor and capacitor are connected in series with a variable inductor. When the circuit is connected to a 220 V, 50 Hz supply, the maximum current obtained by varying the inductance is 0.314 A. The voltage across the capacitor is 800 V. Calculate the resistance, inductance and capacitance of the circuit.

[700.63 Ω, 1.25 μF, 8.10 H]

4.7 A coil of 2 Ω resistance and 0.01 H inductance is connected in series with a capacitor across 230 V mains. What must be the capacitance, in order that maximum current occurs at a frequency of 50 Hz? Find also the current and the voltage across the capacitor. *[1 mF, 115 A, 361.28 V]*

4.8 A choking coil of 10-ohm resistance and 0.1 H inductance is connected in series with a capacitor of 200 μF capacitance. Calculate the current, the coil voltage and the capacitor voltage. The supply voltage is 230 V at 50 Hz. At what frequency will the circuit resonate? Calculate the voltages at resonant frequency across the coil and capacitor. For this, assume that supply voltage is 230 V of variable frequency.

[12.47 A, 411.17, 198.47 V, 35.6 Hz, 563.3 V, 514.2 V]

4.9 An inductive coil having an inductance of 0.04 H and a resistance of 25 Ω has been connected in series with another inductive coil of 0.2 H inductance and 15 Ω resistance. The whole circuit has been energized from 230 V, 50 Hz mains. Calculate power dissipation in each coil and power factor of the whole circuit. Draw the phasor diagram. Suggest a suitable capacitor for the above circuit to resonate at 50 Hz. *[181.57 W, 108.95 W, 0.468 (lagging), 42.22 μF]*

4.10 A voltage $v(t) = 10\sqrt{2}\ \sin\ \omega t$ is applied to a series *RLC* circuit. At resonant frequency, voltage across the capacitor is found to be 500 V. The bandwidth of the circuit is known to be 400 rad/s and impedance at resonance is 10 Ω. Determine resonant frequency, upper and lower cut-off frequencies, *L* and *C*.
[3.183 kHz, 3.151 kHz, 3.214 kHz, 0.025 H, 0.1 μF]

4.11 An inductive coil having a resistance of 20 Ω and an inductance of 0.2 H is connected in parallel with a 20 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with the supply voltage. Also, find the value of this current. Draw the phasor diagram. *[19.49 Hz, 4.6 A]*

Review Questions

4.1 Show that current through pure inductor lags behind the applied sinusoidal voltage by 90°. Also show that pure inductance does not consume any power. Draw voltage, current and power waveforms.

4.2 Explain behaviour of a pure capacitor when connected across a single-phase ac supply.

4.3 Show that the average power absorbed by a capacitor is zero.

4.4 Draw a power triangle and name the sides with units and give their formulas.

4.5 If the impedance of a circuit is $z \angle -\phi$, what type of circuit is this? Which element should be added in series to bring the circuit under resonance condition? Explain with a phasor diagram.

4.6 Show the *Z*-triangle for an *R-L-C* series circuit, when inductive reactance is dominant over capacitive reactance.

4.7 Derive the condition for resonance in a series circuit.

4.8 Derive the relation for bandwidth of a series *R-L-C* circuit.

4.9 Define and find the expression for quality factor of a series *R-L-C* circuit.

4.10 Derive the condition for resonance in parallel circuit.

4.11 Mention the conditions under which a parallel *R-L-C* circuit is in electrical resonance.

4.12 Discuss graphical representation of series resonance and parallel resonance.

4.13 Compare series and parallel resonance.

Multiple Choice Questions

Choose the correct alternative in the following questions:

4.1 A 10 mH inductor carries a sinusoidal current of 1 A rms at a frequency of 50 Hz. The average power dissipated by the inductor is

(a) 0 (b) 0.25 W (c) 0.5 W (d) 1 W

4.2 The power factor of an ordinary bulb is

(a) zero (b) unity
(c) slightly more than unity (d) slightly less than unity

4.3 When a sinusoidal voltage is applied across *R-L* series circuit having $R = X_L$, the phase angle will be

(a) 90° (b) 45° lag (c) 45° lead (d) 90° lead

4.4 An ac source having voltage $\overline{e} = 110 \sin\left(\omega t + \frac{\pi}{3}\right)$ is connected in an ac circuit. If the current drawn from the circuit varies as $i = 5 \sin\left(\omega t - \frac{\pi}{3}\right)$, the impedance of the circuit will be

(a) 22 Ω (b) 16 Ω (c) 30.8 Ω (d) none of these

4.5 An ac source of 200 V rms supplies active power of 600 W and reactive power of 800 VAR. The rms current drawn from the source is

(a) 10 A (b) 5 A (c) 3.75 A (d) 2.5 A

4.6 The voltage phasor of a circuit is $10 \angle 15°$ V and the current phasor is $2 \angle -45°$ A. The active and reactive powers in the circuit are

(a) 10 W and 17.32 VAR (b) 5 W and 8.66 VAR
(c) 20 W and 60 VAR (d) $20\sqrt{2}$ and $10\sqrt{2}$ VAR

4.7 In a two element series circuit, the applied voltage and resultant current are respectively $v(t) = 50 + 50 \sin (5 \times 10^3\, t)$ and $i(t) = 11.2 \sin (5 \times 10^3\, t + 63.4°)$. The nature of the elements would be

(a) *R-L* (b) *R-C*
(c) *L-C* (d) neither *R*, nor *L*, nor *C*

4.8 A boiler at home is switched on to the ac mains supplying power at 230 V, 50 Hz. The frequency of instantaneous power consumed is

(a) 0 (b) 50 Hz (c) 100 Hz (d) 150 Hz

4.9 In a series *R-L-C* circuit, $V_R = 3$ V, $V_L = 14$ V, $V_C = 10$ V. The input voltage to the circuit is

(a) 10 V (b) 5 V (c) 27 V (d) 24 V

4.10 A_1, A_2 and A_3 are ideal ammeters, shown in Fig. 4.113. If A_1 reads 5 A, A_2 reads 12 A, then A_3 should read

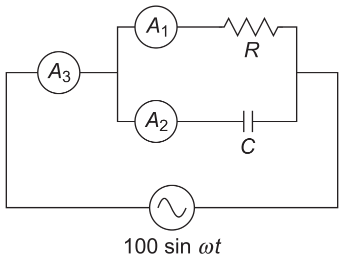


Fig. 4.113

(a) 7 A (b) 12 A (c) 13 A (d) 17 A

4.11 A series *R-L-C* circuit consisting of $R = 10\ \Omega$, $X_L = 20\ \Omega$ and $X_C = 20\ \Omega$ is connected across an ac supply of 200 V rms. The rms voltage across the capacitor is

(a) $200\ \angle{-90°}$ V (b) $200\ \angle 90°$ V

(c) $400\ \angle{-90°}$ V (d) $400\ \angle 90°$ V

4.12 In Fig. 4.114, A_1, A_2 and A_3 are ideal ammeters. If A_1 and A_3 read 5 A and 13 A respectively, the reading of A_2 will be

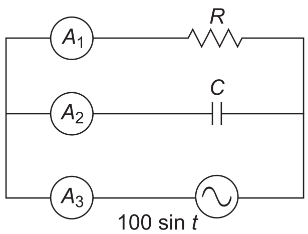


Fig. 4.114

(a) 8 A (b) 12 A (c) 18 A (d) 10 A

4.13 In Fig. 4.115, $i(t)$ under steady state is

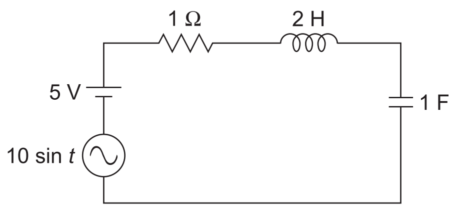


Fig. 4.115

(a) zero (b) 5

(c) 7.07 sin *t* (d) 7.07 sin (*t* – 45°)

4.14 The circuit shown with $R = \frac{1}{3}\ \Omega$, $L = \frac{1}{4}$ H, $C = 3$ F in Fig. 4.106 has input voltage $v(t)$ = sin 2*t*. The resulting current $i(t)$ is

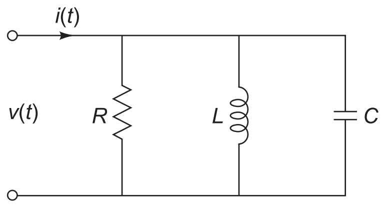


Fig. 4.116

(a) $5 \sin (2t + 53.1°)$ (b) $5 \sin (2t - 53.1°)$
(c) $25 \sin (2t + 53.1°)$ (d) $25 \sin (2t - 53.1°)$

4.15 A series *R-L-C* circuit will have unity power factor if operated at a frequency of

(a) $\dfrac{1}{LC}$ (b) $\dfrac{1}{\omega\sqrt{LC}}$ (c) $\dfrac{1}{\omega^2\sqrt{LC}}$ (d) $\dfrac{1}{2\pi\sqrt{LC}}$

4.16 For a series *R-L-C* resonant circuit, the total reactance at the lower half power frequency is

(a) $\sqrt{2}R\angle 45°$ (b) $\sqrt{2}R\angle -45°$ (c) R (d) $-R$

4.17 In a series *R-L-C* high Q circuit, the current peaks at a frequency

(a) equal to the resonant frequency (b) greater than the resonant frequency
(c) less than the resonant frequency (d) none of the above

4.18 In a series *R-L-C* circuit, $R = 2$ kΩ, $L = 1$ H, $C = \dfrac{1}{400}\,\mu$F . The resonant frequency is

(a) 2×10^4 Hz (b) $\dfrac{1}{\pi} \times 10^4$ Hz (c) 10^4 Hz (d) $2\pi \times 10^4$ Hz

4.19 For a series resonant circuit at low frequency, circuit impedance is __________ and at high frequency circuit impedance is __________ .

(a) capacitive, inductive (b) inductive, capacitive
(c) resistive, inductive (d) capacitive, resistive

4.20 A circuit with a resistor, inductor and capacitor in series is resonant of f_0 Hz. If all the component values are now doubled, the new resonant frequency is

(a) $2f_0$ (b) f_0 (c) $f_0/4$ (d) $f_0/2$

4.21 In a series *R-L-C* circuit at resonance, the magnitude of the voltage developed across the capacitor

(a) is always zero
(b) can never be greater than the input voltage
(c) can be greater than the input voltage, however it is 90° out of phase with the input voltage
(d) can be greater than the input voltage and is in phase with the input voltage

4.22 The power in a series *R-L-C* circuit will be half of that at resonance when the magnitude of the current is equal to

(a) $\dfrac{V}{2R}$ (b) $\dfrac{V}{\sqrt{3}R}$ (c) $\dfrac{V}{\sqrt{2}R}$ (d) $\dfrac{\sqrt{2}V}{R}$

4.23 A series *R*-*L*-*C* circuit has a resonance frequency of 1 kHz and a qualify factor *Q* of 100. If each of *R*, *L* and *C* is doubled from its original value, the new *Q* of the circuit is

(a) 25 (b) 50 (c) 100 (d) 200

4.24 A series *R*-*L*-*C* circuit has a *Q* of 100 and an impedance of $(100 + j0)\ \Omega$ at its resonant angular frequency of 10^7 rad/s. The values of *R* and *L* are

(a) $100\ \Omega$, 10^3 H (b) $100\ \Omega$, 10^{-3} H
(c) $10\ \Omega$, 10 H (d) $10\ \Omega$, 0.1 H

4.25 The following circuit in Fig. 4.117 resonates at

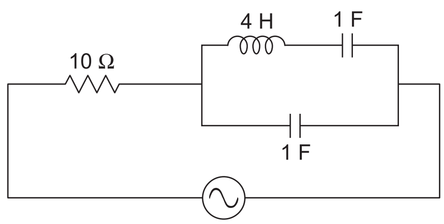


Fig. 4.117

(a) all frequencies (b) 0.5 rad/s
(c) 0.5 rad/s (d) 1 rad/s

4.26 At resonance, the parallel circuit shown in Fig. 4.118 behaves like

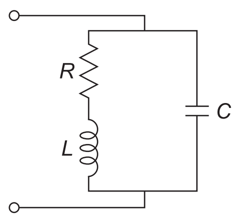
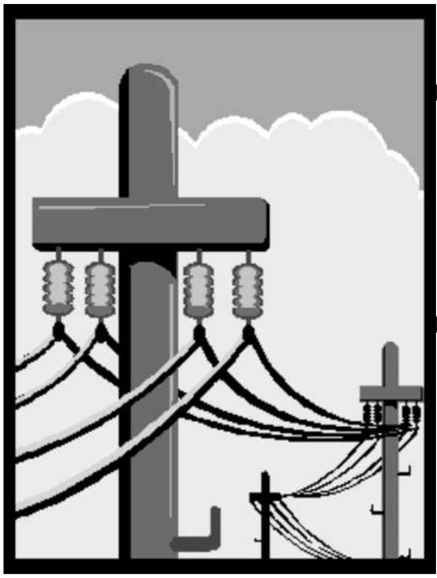


Fig. 4.118

(a) an open circuit
(b) a short circuit
(c) a pure resistor of value *R*
(d) a pure resistor of value much higher than *R*

Answers to Multiple Choice Questions

4.1 (a)	**4.2** (d)	**4.3** (b)	**4.4** (a)
4.5 (b)	**4.6** (a)	**4.7** (b)	**4.8** (c)
4.9 (b)	**4.10** (c)	**4.11** (c)	**4.12** (b)
4.13 (d)	**4.14** (a)	**4.15** (d)	**4.16** (d)
4.17 (a)	**4.18** (b)	**4.19** (a)	**4.20** (d)
4.21 (c)	**4.22** (c)	**4.23** (b)	**4.24** (b)
4.25 (b)	**4.26** (d)		



Chapter 5

Three-Phase Circuits

Chapter Outline

5.1 POLYPHASE SYSTEM

The type of alternating currents and voltages discussed in chapters 3 and 4 are termed as single-phase currents and voltages as they consist of single alternating current and voltage waves. Single-phase systems involving single-phase currents and voltages are quite satisfactory for domestic applications. The motors employed in domestic applications such as mixers, coolers, fans, air-conditioners, refrigerators, etc. are mostly single-phase. However, the single-phase system has its own limitations and hence, has been replaced by polyphase system. For supplying power to electric furnaces, two-phase system is generally used. Six-phase system is used in connection with converting machinery and apparatus. For generation, transmission and distribution of electric power, three-phase system is universally accepted as standard system. Systems with more than three phases, increase complexity and cost of transmission and utilisation hardware.

Polyphase system means the system which consists of many windings or circuits (*poly* means many and *phase* means windings or circuits). A polyphase system is essentially a combination of many single-phase voltages having same magnitude and frequency but are displaced by equal angle (electrical) from one another, which depends upon the number of phases.

$$\text{Electrical displacement} = \frac{360 \text{ electrical degrees}}{\text{Number of phases}}$$

The above relation does not hold good for two phase system in which windings are displaced by 90 electrical degrees apart.

5.2 GENERATION OF POLYPHASE VOLTAGES

(i) Generation of single-phase voltage A single-phase system utilizes single winding. When the winding is rotated in an anticlockwise direction with constant angular velocity ω rad/s in a uniform magnetic field, a voltage is induced in the winding. The equation of the induced voltage in the winding is given by

$$v_R = V_m \sin \theta$$

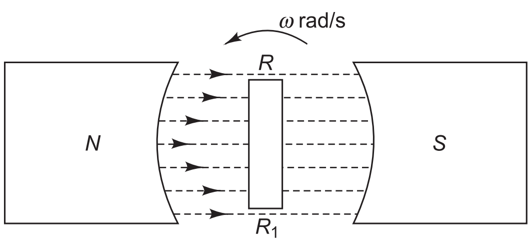


Fig 5.1 *Single-phase system*

The voltage waveform is shown in Fig. 5.2.

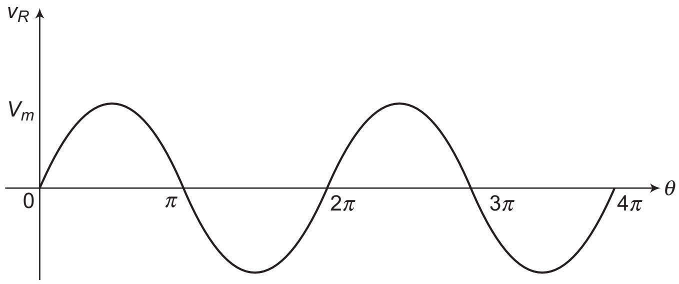


Fig. 5.2 *Voltage waveform*

Figure 5.3 shows the phasor diagram of the induced voltage.

Fig. 5.3 *Phasor diagram*

(ii) Generation of two-phase voltages A two-phase system utilizes two identical windings that are displaced by 90 electrical degrees apart from each other. When these two windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the voltages are induced in each winding which have the same magnitude and frequency but are displaced 90 electrical degrees from one another.

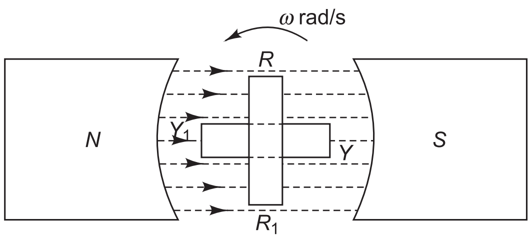


Fig. 5.4 *Two-phase system*

The instantaneous values of induced voltages in two windings RR_1 and YY_1 are given by

$$v_R = V_m \sin \theta$$

$$v_y = V_m (\theta - 90°)$$

The voltage waveform is shown in Fig. 5.5.

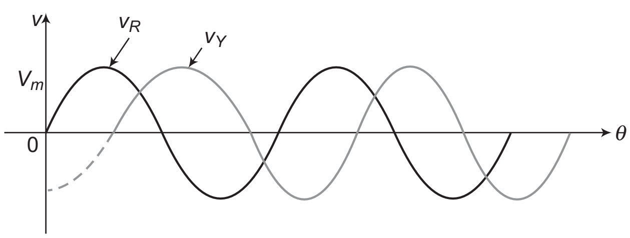


Fig. 5.5 *Voltage waveforms*

Figure 5.6 shows the phasor diagram of these two voltages

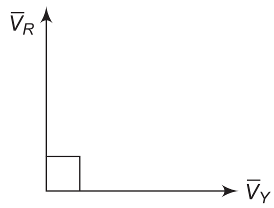


Fig. 5.6 *Phasor diagram*

(iii) Generation of three-phase voltages A three-phase system utilizes three separate but identical windings that are displaced by 120 electrical degrees from each other. When these three windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the voltages are induced in each winding which have the same magnitude and frequency but are displaced 120 electrical degrees from one another.

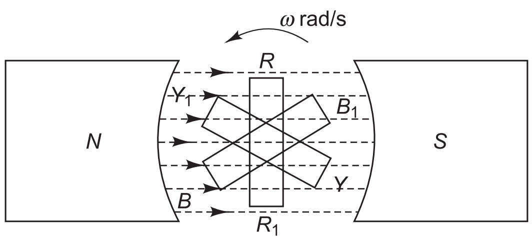


Fig. 5.7 *Three-phase system*

The instantaneous values of induced voltages in three windings RR_1, YY_1 and BB_1 are given by

$$v_R = V_m \sin \theta$$
$$v_Y = V_m \sin (\theta - 120°)$$
$$v_B = V_m \sin (\theta - 240°)$$

The induced voltage in winding YY_1 lags behind that in winding RR_1 by 120° and the induced voltage in winding BB_1 lags behind that in winding RR_1 by 240°. The waveforms of these three voltages are shown in Fig. 5.8.

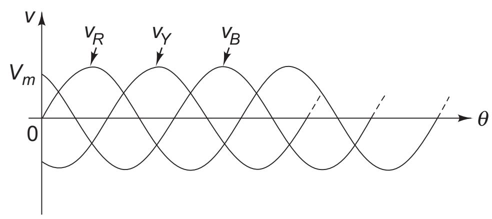


Fig. 5.8 *Voltage waveforms*

Figure 5.9 shows the phasor diagram of these induced voltages.

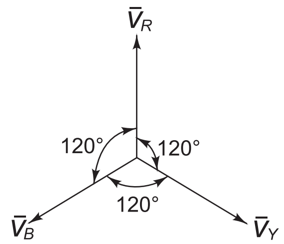


Fig. 5.9 *Phasor diagram*

5.3 ADVANTAGES OF A THREE-PHASE SYSTEM

1. In a single-phase system, the instantaneous power is fluctuating in nature. However, in a three-phase system, it is constant at all times.
2. The output of a three-phase system is greater than that of a single-phase system.
3. Transmission and distribution of a three-phase system is cheaper than that of a single-phase system.
4. Three-phase motors are more efficient and have higher power factors than single-phase motors of the same frequency.
5. Three-phase motors are self-starting whereas single-phase motors are not self-starting.

5.4 SOME DEFINITIONS

Phase Sequence The sequence in which the voltages in the three phases reach the maximum positive value is called the *phase sequence* or *phase order*. From the phasor diagram of a three-phase system, it is clear that the voltage in the coil R attains maximum positive value first, next in the coil Y and then in the coil B. Hence, the phase sequence is R-Y-B.

Phase Voltage The voltage induced in each winding is called the *phase voltage*.

Phase Current The current flowing through each winding is called the *phase current*.

Line Voltage The voltage available between any pair of terminals or lines is called the *line voltage*.

Line Current The current flowing through each line is called the *line current*.

Symmetrical or Balanced System A three-phase system is said to be balanced if the

(a) voltages in the three phases are equal in magnitude and differ in phase from one another by 120°, and
(b) currents in the three phases are equal in magnitude and differ in phase from one another by 120°.

Balanced Load The load is said to be balanced if loads connected across the three phases are identical, i.e., all the loads have the same magnitude and power factor.

5.5 INTERCONNECTION OF THREE PHASES

In a three-phase system, there are three windings. Each winding has two terminals, viz., 'start' and 'finish'. If a separate load is connected across each winding as shown in Fig. 5.10, six conductors are required to transmit and distribute power. This will make the system complicated and expensive.

In order to reduce the number of conductors, the three windings are connected in the following two ways:

1. Star, or Wye, connection
2. Delta, or Mesh, connection

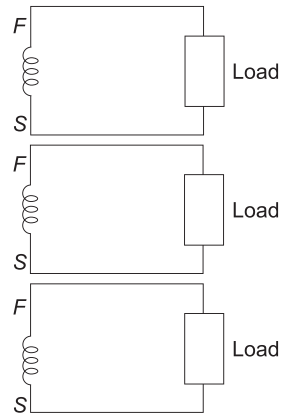


Fig. 5.10 *Non-interlinked three-phase system*

5.6 STAR OR WYE CONNECTION

In this method, similar terminals (start or finish) of the three windings are joined together as shown in Fig. 5.11. The common point is called *star* or *neutral point.*

Figure 5.12 shows a three-phase system in star connection.

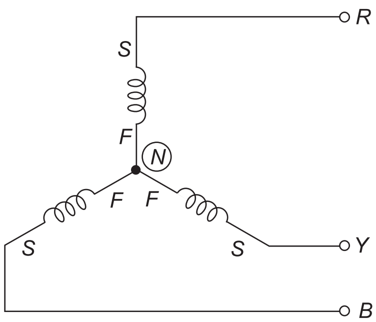


Fig. 5.11 *Three-phase star connection*

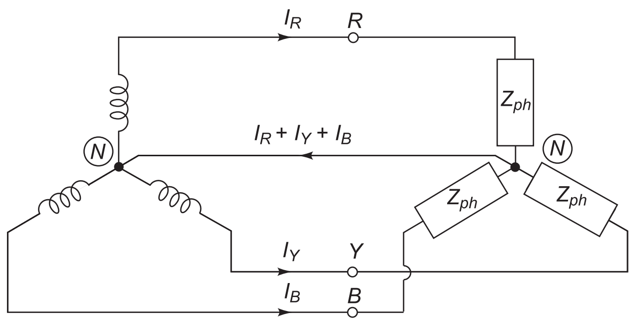


Fig. 5.12 *Three-phase, four-wire system*

This system is called a three-phase, four-wire system. If three identical loads are connected to each phase, the current flowing through the neutral wire is the sum of the three currents I_R, I_Y and I_B. Since the impedances are identical, the three currents are equal in magnitude but differ in phase from one another by 120°.

$$i_R = I_m \sin \theta$$

$$i_Y = I_m \sin (\theta - 120°)$$

$$i_B = I_m \sin (\theta - 240°)$$

$$i_R + i_Y + i_B = I_m \sin \theta + I_m \sin (\theta - 120°) + I_m \sin (\theta - 240°) = 0$$

Therefore, the neutral wire can be removed without any way affecting the voltages or currents in the circuit as shown in Fig. 5.13. This constitutes a three-phase, three-wire system. If the load is not balanced, the neutral wire carries some current.

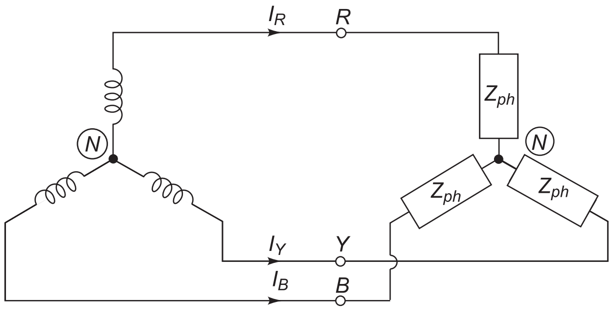


Fig. 5.13 *Three-phase, three-wire system*

5.7 DELTA OR MESH CONNECTION

In this method, dissimilar terminals of the three windings are joined together, i.e., the 'finish' terminal of one winding is connected to the 'start' terminal of the other winding, and so on, as shown in Fig. 5.14. This system is also called a three-phase, three-wire system.

For a balanced system, the sum of the three phase voltages round the closed mesh is zero. The three emfs are equal in magnitude but differ in phase from one another by 120°.

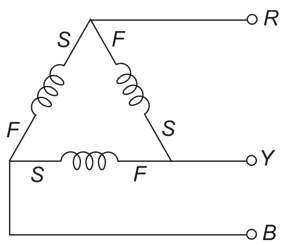


Fig. 5.14 *Three-phase delta connection*

$$v_R = V_m \sin \theta$$

$$v_Y = V_m \sin (\theta - 120°)$$

$$v_B = V_m \sin (\theta - 240°)$$

$$v_R + v_Y + v_B = V_m \sin \theta + V_m \sin (\theta - 120°) + V_m \sin (\theta - 240°) = 0$$

5.8 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED STAR-CONNECTED LOAD

[May 2013, Dec 2013]

5.8.1 Relation between Line Voltage and Phase Voltage

Since the system is balanced, the three-phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and differ in phase from one another by 120°.

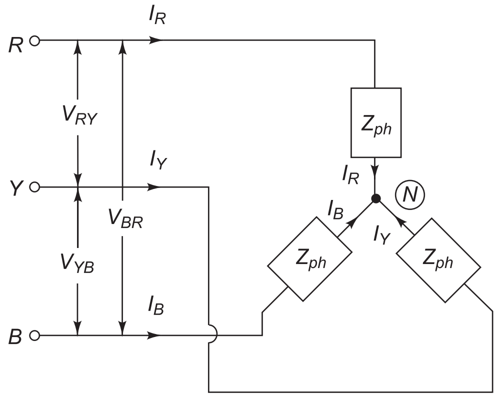


Fig. 5.15 *Star connection*

Let $V_{RN} = V_{YN} = V_{BN} = V_{ph}$

where V_{ph} indicates the rms value of phase voltage.

$$\overline{V}_{RN} = V_{ph} \angle 0°$$
$$\overline{V}_{YN} = V_{ph} \angle{-120°}$$
$$\overline{V}_{BN} = V_{ph} \angle{-240°}$$

Let $V_{RY} = V_{YB} = V_{BR} = V_L$

where V_L indicates the rms value of line voltage.

Applying Kirchhoff's voltage law,

$$\begin{aligned}\overline{V}_{RY} &= \overline{V}_{RN} + \overline{V}_{NY}\\ &= \overline{V}_{RN} - \overline{V}_{YN}\\ &= V_{ph} \angle 0° - V_{ph} \angle{-120°}\\ &= (V_{ph} + j0) - (-0.5\ V_{ph} - j0.866\ V_{ph})\\ &= 1.5\ V_{ph} + j0.866\ V_{ph}\\ &= \sqrt{3}\ V_{ph} \angle 30°\end{aligned}$$

Similarly,

$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB} = \sqrt{3}\,V_{ph} \angle 30°$$
$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR} = \sqrt{3}\,V_{ph} \angle 30°$$

Thus, in a star-connected, three-phase system, $V_L = \sqrt{3}\,V_{ph}$ and line voltages lead respective phase voltages by 30°.

5.8.2 Relation between Line Current and Phase Current

From Fig. 5.15, it is clear that line current is equal to the phase current.

$$I_L = I_{ph}$$

5.8.3 Phasor Diagram (Lagging Power Factor)

Steps for Drawing Phasor Diagram

1. First draw $\overline{V}_{RN}$ as reference voltage.
2. Since three phase voltages are equal in magnitude and differ in phase from one another by 120°, draw $\overline{V}_{YN}$ and $\overline{V}_{BN}$ lagging 120° behind w.r.t. each other.
3. Draw $\overline{V}_{NY}$ equal and opposite to $\overline{V}_{YN}$.
4. Add $\overline{V}_{RN}$ and $\overline{V}_{NY}$ using the parallelogram law of vector addition such that
$$\overline{V}_{RY} = \overline{V}_{RN} + \overline{V}_{NY}$$
5. Draw $\overline{V}_{NB}$ equal and opposite to $\overline{V}_{BN}$.

6. Add $\overline{V}_{YN}$ and $\overline{V}_{NB}$ using the parallelogram law of vector addition such that
$$\overline{V}_{YB} = \overline{V}_{YN} + \overline{V}_{NB}$$
7. Draw $\overline{V}_{NR}$ equal and opposite to $\overline{V}_{RN}$.
8. Add $\overline{V}_{BN}$ and $\overline{V}_{NR}$ using the parallelogram law of vector addition such that
$$\overline{V}_{BR} = \overline{V}_{BN} + \overline{V}_{NR}$$
9. Assuming inductive load, draw three phase currents $\overline{I}_R, \overline{I}_Y$ and $\overline{I}_B$ lagging behind its respective phase voltages by an angle ϕ. The phase currents are equal in magnitude and differ in phase from one another by 120°.
10. Line currents are same as the phase currents in star connected load. Hence, separate line currents are not drawn.
11. Since $\overline{V}_{NY}$ is antiphase with $\overline{V}_{YN}$, angle between $\overline{V}_{RN}$ and $\overline{V}_{NY}$ is 60°. The line voltage $\overline{V}_{RY}$ leads phase voltage $\overline{V}_{RN}$ by 30°. Similarly, line voltage $\overline{V}_{YB}$ leads phase voltage $\overline{V}_{YN}$ by 30° and line voltage $\overline{V}_{BR}$ leads phase voltage $\overline{V}_{BN}$ by 30°.

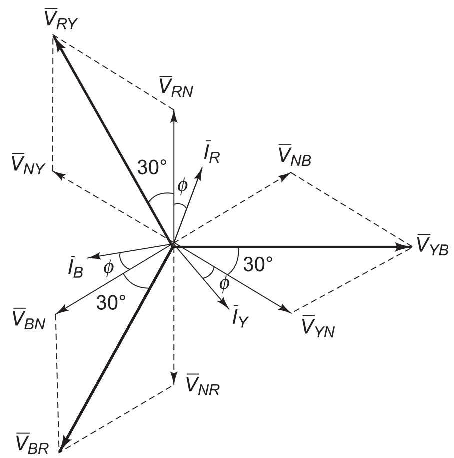


Fig. 5.16 *Phasor diagram*

5.8.4 Power

The total power in a three-phase system is the sum of powers in the three phases. For a balanced load, the power consumed in each load phase is the same.

Total active power $P = 3 \times$ power in each phase $= 3\ V_{ph}\ I_{ph} \cos \phi$

In a star-connected, three-phase system,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi = \sqrt{3}\ V_L\ I_L \cos \phi$$

where ϕ is the phase difference between phase voltage and corresponding phase current.

Similarly, total reactive power $Q = 3\ V_{ph}\ I_{ph}\ \sin\phi$

$$= \sqrt{3}\ V_L\ I_L\ \sin\phi$$

Total apparent power $S = 3V_{ph}\ I_{ph} = \sqrt{3}\ V_L\ I_L$

The power triangle for a three-phase system is shown in Fig. 5.17.

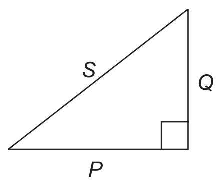


Fig. 5.17

5.9 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED DELTA-CONNECTED LOAD

[Dec 2012, 2013]

5.9.1 Relation between Line Voltage and Phase Voltage

From Fig. 5.18, it is clear that line voltage is equal to phase voltage.

$$V_L = V_{ph}$$

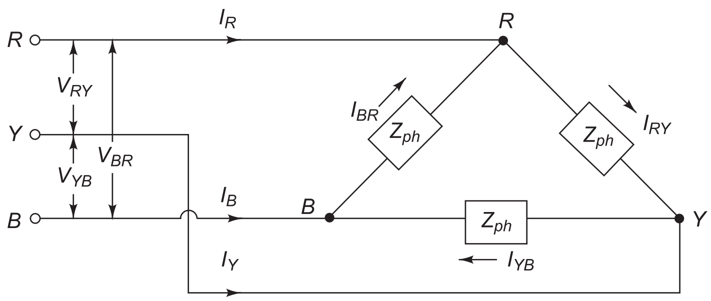


Fig. 5.18 *Delta connection*

5.9.2 Relation between Line Current and Phase Current

Since the system is balanced, the three-phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude but differ in phase from one another by 120°.

Let $\quad I_{RY} = I_{YB} = I_{BR} = I_{ph}$

where I_{ph} indicates rms value of the phase current.

$$\bar{I}_{RY} = I_{ph} \angle 0°$$

$$\bar{I}_{YB} = I_{ph} \angle -120°$$

$$\bar{I}_{BR} = I_{ph} \angle -240°$$

Let $\quad I_R = I_Y = I_B = I_L$

where I_L indicates rms value of the line current.

Applying Kirchhoff's current law,

$$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\begin{aligned}\bar{I}_R &= \bar{I}_{RY} - \bar{I}_{BR} = I_{ph}\angle 0° - I_{ph}\angle -240° \\ &= (I_{ph} + j0) - (-0.5\, I_{ph} + j0.866\, I_{ph}) \\ &= 1.5\, I_{ph} - j0.866\, I_{ph} \\ &= \sqrt{3}\ I_{ph}\angle{-30°}\end{aligned}$$

Similarly,

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = \sqrt{3}\ I_{ph}\angle -30°$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = \sqrt{3}\ I_{ph}\angle -30°$$

Thus, in a delta-connected, three-phase system, $I_L = \sqrt{3}\ I_{ph}$ and line currents are 30° behind the respective phase currents.

5.9.3 Phasor Diagram (Lagging Power Factor)

Steps for Drawing Phasor Diagram

1. First draw $\bar{V}_{RY}$ as reference voltage.
2. Since three phase voltages are equal in magnitude and differ in phase from one another by 120°, draw $\bar{V}_{YB}$ and $\bar{V}_{BR}$ lagging 120° behind w.r.t. each other.
3. Line voltages are same as the phase voltages for balanced delta connected load. Hence, separate line voltages are not drawn.
4. Assuming inductive load, draw three phase currents $\bar{I}_{RY}$, $\bar{I}_{YB}$ and $\bar{I}_{BR}$ lagging behind respective phase voltages by an angle ϕ. The phase currents are equal in magnitude and differ in phase from one another by 120°.
5. Draw $-\bar{I}_{BR}$ equal and opposite to $\bar{I}_{BR}$.
6. Add $\bar{I}_{RY}$ and $-\bar{I}_{BR}$ using the parallelogram law of vector addition such that
$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$
7. Draw $-\bar{I}_{RY}$ equal and opposite to $\bar{I}_{RY}$.
8. Add $\bar{I}_{YB}$ and $-\bar{I}_{RY}$ using the parallelogram law of vector addition such that
$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$$
9. Draw $-\bar{I}_{YB}$ equal and opposite to $\bar{I}_{YB}$.
10. Add $\bar{I}_{BR}$ and $-\bar{I}_{YB}$ using the parallelogram laws of vector addition such that
$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB}$$
11. Since $-\bar{I}_{BR}$ is antiphase with $\bar{I}_{BR}$, angle between $\bar{I}_{RY}$ and $-\bar{I}_{BR}$ is 60°. The line current $\bar{I}_R$ lags behind phase current $\bar{I}_{RY}$ by 30°. Similarly, the line current $\bar{I}_Y$ lags behind phase current $\bar{I}_{YB}$ by 30° and the line current $\bar{I}_B$ lags behind phase current $\bar{I}_{BR}$ by 30°.

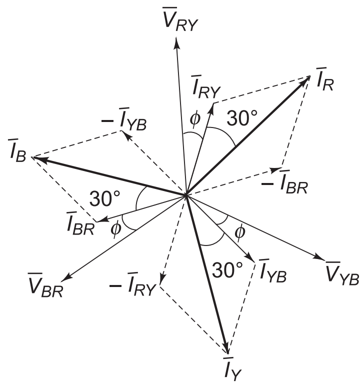


Fig. 5.19 *Phasor diagram*

5.9.4 Power

$$P = 3\ V_{ph}\ I_{ph} \cos \phi$$

In a delta-connected, three-phase system,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos \phi = \sqrt{3}\ V_L\ I_L \cos \phi$$

Total reactive power $Q = 3\ V_{ph}\ I_{ph} \sin \phi = \sqrt{3}\ V_L\ I_L \sin \phi$

Total apparent power $S = 3\ V_{ph}\ I_{ph} = \sqrt{3}\ V_L\ I_L$

The power triangle for a three-phase system is shown in Fig. 5.20.

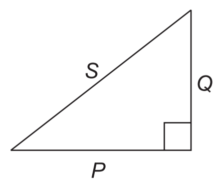


Fig. 5.20

5.10 BALANCED Y/Δ AND Δ/Y CONVERSIONS

Any balanced star-connected system can be converted into the equivalent delta-connected system and vice versa.

For a balanced star-connected load,

Line voltage $= V_L$

Line current $= I_L$

Impedance/phase $= Z_Y$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3}I_L}$$

For an equivalent delta-connected system, the line voltages and currents must have the same values as in the star-connected system, i.e.,

$$\text{Line voltage} = V_L$$

$$\text{Line current} = I_L$$

$$\text{Impedance/phase} = Z_\Delta$$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\frac{I_L}{\sqrt{3}}} = \sqrt{3}\frac{V_L}{I_L} = 3Z_Y$$

$$Z_Y = \frac{1}{3}Z_\Delta$$

Thus, when three equal phase impedances are connected in delta, the equivalent star impedance is one third of the delta impedance.

5.11 RELATION BETWEEN POWER IN DELTA AND STAR SYSTEMS

Let a balanced load be connected in star having impedance per phase as Z_{ph}.

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

$$I_L = I_{ph} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

$$P_Y = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times V_L \times \frac{V_L}{\sqrt{3}Z_{ph}} \times \cos\phi$$

$$= \frac{V_L^2}{Z_{ph}} \cos\phi$$

For a delta-connected load,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3}\frac{V_L}{Z_{ph}}$$

$$P_\Delta = \sqrt{3}\ V_L I_L \cos\phi$$

$$= \sqrt{3}\times V_L \times \sqrt{3}\ \frac{V_L}{Z_{ph}} \times \cos\phi$$

$$= 3\ \frac{V_L^2}{Z_{ph}}\ \cos\phi$$

$$= 3P_Y$$

$$P_Y = \frac{1}{3}P_\Delta$$

Thus, power consumed by a balanced star-connected load is one-third of that in the case of a delta-connected load.

5.12 COMPARISON BETWEEN STAR AND DELTA CONNECTIONS

Star Connection	*Delta Connection*
1. $V_L = \sqrt{3}\ V_{ph}$	1. $V_L = V_{ph}$
2. $I_L = I_{ph}$	2. $I_L = \sqrt{3}\ I_{ph}$
3. Line voltage leads the respective phase voltage by 30°.	3. Line current lags behind the respective phase current by 30°.
4. Power in star connection is one-third of power in delta connection.	4. Power in delta connection is 3 times of the power in star connection.
5. Three-phase, three-wire and three-phase, four-wire systems are possible.	5. Only three-phase, three-wire system is possible.
6. The phasor sum of all the phase currents is zero.	6. The phasor sum of all the phase voltages is zero.

Example 1

Three identical coils each of [4.2 + j5.6] ohms are connected in star across a 415 V, 3-phase, 50 Hz supply. Determine (i) V_{ph}, (ii) I_{ph}, and (iii) power factor. **[May 2014]**

Solution

$$\bar{Z}_{ph} = 4.2 + j5.6 = 7 \angle 53.13° \ \Omega$$

$$V_L = 415 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) $$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

(ii) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{7} = 34.23 \text{ A}$$

(iii) $$\text{pf} = \cos\phi = \cos(53.13°) = 0.6 \text{ (lagging)}$$

Example 2

Three equal impedances, each of 8 + j10 ohms, are connected in star. This is further connected to a 440 V, 50 Hz, three-phase supply. Calculate (i) phase voltage, (ii) phase angle, (iii) phase current, (iv) line current, (v) active power, and (vi) reactive power.

Solution

$$\bar{Z}_{ph} = 8 + j10 \ \Omega$$

$$V_L = 440 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

(ii) Phase angle

$$\bar{Z}_{ph} = 8 + j10 = 12.81 \angle 51.34° \ \Omega$$

$$Z_{ph} = 12.81 \ \Omega$$

$$\phi = 51.34°$$

(iii) Phase current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{12.81} = 19.83 \text{ A}$$

(iv) Line current

$$I_L = I_{ph} = 19.83 \text{ A}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 19.83 \times \cos(51.34°) = 9.44 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 19.83 \times \sin(51.34°) = 11.81 \text{ kVAR}$$

Example 3

A balanced delta-connected load of impedance (8 – j6) ohms per phase is connected to a three-phase, 230 V, 50 Hz supply. Calculate (i) power factor, (ii) line current, and (iii) reactive power.

Solution

$$\bar{Z}_{ph} = 8 - j6\ \Omega$$
$$V_L = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a delta-connected load,

(i) Power factor

$$\bar{Z}_{ph} = 8 - j6 = 10 \angle -36.87°\ \Omega$$
$$Z_{ph} = 10\ \Omega$$
$$\phi = 36.87°$$
$$\text{pf} = \cos\phi = \cos(36.87°) = 0.8 \text{ (leading)}$$

(ii) Line current

$$V_{ph} = V_L = 230 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10} = 23 \text{ A}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 23 = 39.84 \text{ A}$$

(iii) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 39.84 \times \sin(36.87°) = 9.52 \text{ kVAR}$$

Example 4

Three coils, each having a resistance and an inductance of 8 Ω and 0.02 H respectively, are connected in star across a three-phase, 230 V, 50 Hz supply. Find the (i) power factor, (ii) line current, (iii) power, (iv) reactive volt-amperes, and (v) total volt-amperes.

Solution

$$R = 8\ \Omega$$
$$L = 0.02 \text{ H}$$
$$V_L = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Power factor

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$\bar{Z}_{ph} = R + jX_L = 8 + j6.28 = 10.17\ \angle 38.13°\ \Omega$$
$$Z_{ph} = 10.17\ \Omega$$
$$\phi = 38.13°$$
$$\text{pf} = \cos\phi = \cos(38.13°) = 0.786 \text{ (lagging)}$$

(ii) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{10.17} = 13.05 \text{ A}$$
$$I_L = I_{ph} = 13.05 \text{ A}$$

(iii) Power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 230 \times 13.05 \times 0.786 = 4.088 \text{ kW}$$

(iv) Reactive volt-amperes

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 13.05 \times \sin(38.13°) = 3.21 \text{ kVAR}$$

(v) Total volt-ampere

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 230 \times 13.05 = 5.198 \text{ kVA}$$

Example 5

Three similar coils each having a resistance of 10 Ω and inductance of 0.04 H are connected in star across a 3 phase, 50 Hz, 200 V supply. Calculate the line current, total power absorbed, reactive volt amperes and total volt amperes. **[May 2015]**

Solution

$$R = 10\ \Omega$$
$$L = 0.04 \text{ H}$$
$$V_L = 200 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$X_L = 2\pi fL = 2\pi \times 50 \times 0.04 = 12.57\ \Omega$$
$$Z_{ph} = R + j\,X_L = 10 + j\,12.57 = 16.06\ \angle\ 51.5°\ \Omega$$

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115 \cdot 47 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{115 \cdot 47}{16.06} = 7.19 \text{ A}$$

$$I_L = I_{ph} = 7.19 \text{ A}$$

(ii) Total power absorbed

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 200 \times 7.19 \times \cos(51.5°) = 1550.5 \text{ W}$$

(iii) Reactive volt-ampere

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 200 \times 7.19 \times \sin(51.5°) = 1949.23 \text{ VAR}$$

(iv) Total volt ampere

$$S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 200 \times 7.19 = 2490.68 \text{ VA}$$

Example 6

Three coils, each having a resistance of 8 Ω and an inductance of 0.02 H, are connected in delta to a three-phase, 400 V, 50 Hz supply. Calculate the (i) line current, and (ii) power absorbed.

Solution

$$R = 8\ \Omega$$
$$L = 0.02 \text{ H}$$
$$V_L = 400 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a delta-connected load,

(i) Line current

$$V_L = V_{ph} = 400 \text{ V}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$\bar{Z}_{ph} = R + jX_L = 8 + j6.28 = 10.17 \angle 38.13°\ \Omega$$
$$Z_{ph} = 10.17\ \Omega$$
$$\phi = 38.13°$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10.17} = 651639.33 \text{ A}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 39.33 = 68.12 \text{ A}$$

(ii) Power absorbed

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 68.12 \times \cos(38.13°) = 37.12 \text{ kW}$$

Example 7

The three equal impedances of each of 10 ∠60° Ω, are connected in star across a three-phase, 400 V, 50 Hz supply. Calculate the (i) line voltage and phase voltage, (ii) power factor and active power consumed, (iii) If the same three impedances are connected in delta to the same source of supply, what is the active power consumed?

Solution
$$\bar{Z}_{ph} = 10 \angle 60° \ \Omega$$
$$V_L = 400 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Line voltage and phase voltage

$$V_L = 400 \text{ V}$$
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

(ii) Power factor and active power consumed

$$\phi = 60°$$
$$\text{pf} = \cos\phi = \cos(60°) = 0.5 \text{ (lagging)}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{10} = 23.094 \text{ A}$$
$$I_L = I_{ph} = 23.094 \text{ A}$$
$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.094 \times 0.5 = 8 \text{ kW}$$

(iii) Active power consumed for delta-connected load

$$V_L = 400 \text{ V}$$
$$Z_{ph} = 10 \ \Omega$$
$$V_{ph} = V_L = 400 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40 \text{ A}$$
$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 40 = 69.28 \text{ A}$$
$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 69.28 \times \cos(60°) = 24 \text{ kW}$$

Example 8

Three similar coils A, B, and C are available. Each coil has a 9 Ω resistance and a 12 Ω reactance. They are connected in delta to a three-phase, 440 V, 50 Hz supply. Calculate for this load, the (i) phase current, (ii) line current, (iii) power factor, (iv) total kVA, (v) active power, and (vi) reactive power. If these coils are connected in star across the same supply, calculate all the above quantities.

Solution
$$R = 9 \ \Omega$$
$$X_L = 12 \ \Omega$$
$$V_L = 440 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a delta-connected load,

(i) Phase current

$$V_L = V_{ph} = 440 \text{ V}$$

$$\bar{Z}_{ph} = 9 + j12 = 15 \angle 53.13° \ \Omega$$

$$Z_{ph} = 15 \ \Omega$$

$$\phi = 53.13°$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{15} = 29.33 \text{ A}$$

(ii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 29.33 = 50.8 \text{ A}$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(53.13°) = 0.6 \text{ (lagging)}$$

(iv) Total kVA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 50.8 = 38.71 \text{ kVA}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 50.8 \times 0.6 = 23.23 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 50.8 \times \sin(53.13°) = 30.97 \text{ kVAR}$$

If these coils are connected in star across the same supply,

(i) Phase current

$$V_L = 440 \text{ V}$$

$$Z_{ph} = 15 \ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{15} = 16.94 \text{ A}$$

(ii) Line current

$$I_L = I_{ph} = 16.94 \text{ A}$$

(iii) Power factor

$$\text{pf} = 0.6 \text{ (lagging)}$$

(iv) Total kVA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 16.94 = 12.91 \text{ kVA}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 16.94 \times 0.6 = 7.74 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 16.94 \times \sin(53.13°) = 12.33 \text{ kVAR}$$

Example 9

A balanced 3-phase load consists of 3 coils, each of resistance 4 Ω and inductance 0.02 H. It is connected to a 440 V, 50 Hz, 3 ϕ supply. Find the total power consumed when the load is connected in star and the total reactive power when the load is connected in delta. **[Dec 2014]**

Solution $R = 4\ \Omega$

$L = 0.02$ H

$V_L = 440$ V

$f = 50$ Hz

For a star-connected load,

(i) Total power consumed

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$\bar{Z}_{ph} = R + jX_L = 4 + j\,6.28 = 7.45\ \angle 57.51°\ \Omega$$

$$Z_{ph} = 7.45\ \Omega$$

$$\phi = 57.51°$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{7.45} = 34.1 \text{ A}$$

$$I_L = I_{ph} = 34.1 \text{ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 34.1 \times \cos(57.51°) = 13.96 \text{ kW}$$

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 34.1 \times \sin(57.51°) = 21.92 \text{ kVAR}$$

(ii) When the load is connected in delta across same supply

$$Q_\Delta = 3Q_Y = 3 \times 21.92 \times 10^3 = 65.76 \text{ kVAR}$$

Example 10

A 415 V, 50 Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15 Ω, a capacitance of 177 μF and an inductance of 0.1 henry in series. Find the (i) power factor, (ii) phase current, (iii) line current, (iv) active power, (v) reactive power, and (vi) total VA. Draw a neat phasor diagram. If the same impedances are connected in delta, find the (i) line current, and (ii) power consumed. **[Dec 2015]**

Solution
$$V_L = 415 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$R = 15\ \Omega$$
$$C = 177\ \mu\text{F}$$
$$L = 0.1 \text{ H}$$

For a star-connected load,

(i) Power factor

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 177 \times 10^{-6}} = 17.98\,\Omega$$
$$\bar{Z}_{ph} = R + jX_L - jX_C$$
$$= 15 + j31.42 - j17.98$$
$$= 15 + j13.44$$
$$= 20.14\ \angle 41.86°\ \Omega$$
$$Z_{ph} = 20.14\ \Omega$$
$$\phi = 41.86°$$
$$\text{pf} = \cos\phi = \cos(41.86°) = 0.744 \text{ (lagging)}$$

(ii) Phase current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\,\text{V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{20.14} = 11.9\,\text{A}$$

(iii) Line current

$$I_L = I_{ph} = 11.9 \text{ A}$$

(iv) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3}\ \times 415 \times 11.9 \times 0.744 = 6.36 \text{ kW}$$

(v) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3}\ \times 415 \times 11.9 \times \sin(41.86°) = 5.71 \text{ kVAR}$$

(vi) Total VA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3}\ \times 415 \times 11.9 = 8.55 \text{ kVA}$$

Phasor Diagram

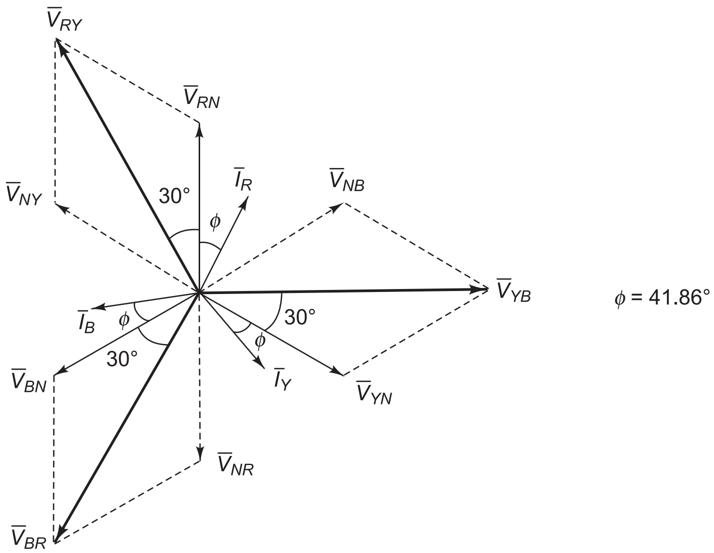


Fig. 5.21

If the same impedances are connected in delta,

(i) Line current

$$V_L = V_{ph} = 415 \text{ V}$$

$$Z_{ph} = 20.14 \ \Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{20.14} = 20.61 \text{ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 20.61 = 35.69 \text{ A}$$

(ii) Power consumed

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 415 \times 35.69 \times 0.744 = 19.09 \text{ kW}$$

Example 11

Each phase of a delta-connected load consists of a 50 mH inductor in series with a parallel combination of a 50 Ω resistor and a 50 µF capacitor. The load is connected to a three-phase, 550 V, 800 rad/s ac supply. Find the (i) power factor, (ii) phase current, (iii) line current, (iv) power drawn, (v) reactive power, and (vi) kVA rating of the load.

Solution

$$L = 50 \text{ mH}$$

$$R = 50 \ \Omega$$

$$C = 50 \ \mu\text{F}$$

$$V_L = 550 \text{ V}$$
$$\omega = 800 \text{ rad/s}$$

For a delta-connected load,

(i) Power factor

$$X_L = \omega L = 800 \times 50 \times 10^{-3} = 40\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{800 \times 50 \times 10^{-6}} = 25\,\Omega$$

$$\bar{Z}_{ph} = jX_L + \frac{R(-jX_C)}{R - jX_C}$$

$$= j40 + \frac{50(-j25)}{50 - j25}$$

$$= 10 + j20 = 22.36\ \angle 63.43°\ \Omega$$

$$Z_{ph} = 22.36\ \Omega$$

$$\phi = 63.43°$$

$$\text{pf} = \cos\phi = \cos(63.43°) = 0.447 \text{ (lagging)}$$

(ii) Phase current

$$V_L = V_{ph} = 550 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{550}{22.36} = 24.6 \text{ A}$$

(iii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 24.6 = 42.61 \text{ A}$$

(iv) Power drawn

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 550 \times 42.61 \times 0.447 = 18.14 \text{ kW}$$

(v) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 550 \times 42.61 \times \sin(63.43°) = 36.3 \text{ kVAR}$$

(vi) kVA rating of the load

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 550 \times 42.61 = 40.59 \text{ kVA}$$

Example 12

A balanced star-connected load is supplied from a symmetrical three-phase 400 volts, 50 Hz system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find the (i) phase voltage, (ii) resistance and reactance per phase, (iii) load inductance per phase, and (iv) total power consumed.

Solution $V_L = 400$ V

$$f = 50 \text{ Hz}$$
$$I_{ph} = 30 \text{ A}$$
$$\phi = 30°$$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

(ii) Resistance and reactance per phase

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{30} = 7.7\,\Omega$$
$$Z_{ph} = Z_{ph}\angle\phi = 7.7\angle 30° = (6.67 + j\,3.85)\,\Omega$$
$$R_{ph} = 6.67\,\Omega$$
$$X_{ph} = 3.85\,\Omega$$

(iii) Load inductance per phase

$$X_{ph} = 2\pi f L_{ph}$$
$$3.85 = 2\pi \times 50 \times L_{ph}$$
$$L_{ph} = 0.01225 \text{ H}$$

(iv) Total power consumed

$$P = 3V_{ph}I_{ph}\cos\phi = 3 \times 230.94 \times 30 \times \cos(30°) = 18 \text{ kW}$$

Example 13

A symmetrical three-phase 400 V system supplies a basic load of 0.8 lagging power factor and is connected in star. If the line current is 34.64 A, find the (i) impedance, (ii) resistance and reactance per phase, (iii) total power, and (iv) total reactive voltamperes.

Solution

$$V_L = 400 \text{ V}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$
$$I_L = 34.64 \text{ A}$$

For a star-connected load,

(i) Impedance

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$
$$I_{ph} = I_L = 34.64 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{34.64} = 6.67\,\Omega$$

(ii) Resistance and reactance per phase

$$\text{pf} = \cos\phi = 0.8$$

$$\phi = \cos^{-1}(0.8) = 36.87°$$

$$Z_{ph} = Z_{ph}\angle\phi = 6.67\angle 36.87° = (5.33 + j\,4)\ \Omega$$

$$R_{ph} = 5.33\ \Omega$$

$$X_{ph} = 4\ \Omega$$

(iii) Total power

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 400 \times 34.64 \times 0.8 = 19.19\text{ kW}$$

(iv) Total reactive volt-amperes

$$Q = \sqrt{3}\,V_L I_L \sin\phi = \sqrt{3} \times 400 \times 34.64 \times \sin(36.87°) = 14.4\text{ kVAR}$$

Example 14

A balanced star-connected load is supplied by a 415 V, 50 Hz three-phase system. Current in each phase is 20 A and lags 30° behind its phase voltage. Find the (i) phase voltage, (ii) power, and (iii) circuit parameters. Also, find power consumed when the same load is connected in delta across the same supply.

Solution

$$V_L = 415\text{ V}$$

$$f = 50\text{ Hz}$$

$$I_{ph} = 20\text{ A}$$

$$\phi = 30°$$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\text{ V}$$

(ii) Power

$$I_L = I_{ph} = 20\text{ A}$$

$$P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 415 \times 20 \times \cos(30°) = 12.45\text{ kW}$$

(iii) Circuit parameters

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{239.6}{20} = 11.98\ \Omega$$

$$\bar{Z}_{ph} = Z_{ph}\angle\phi = 11.98\angle 30° = (10.37 + j6)\ \Omega$$

$$R_{ph} = 10.37\ \Omega$$

$$X_{ph} = 6\ \Omega$$

$$X_{ph} = 2\pi f L_{ph}$$

$$6 = 2\pi \times 50 \times L_{ph}$$

$$L_{ph} = 19.1 \text{ mH}$$

(iv) Power consumed by same delta load across the same supply

$$P_{\Delta} = 3P_Y = 3 \times 12.45 \times 10^3 = 37.35 \text{ kW}$$

Example 15

Three identical coils connected in delta to a 440 V, three-phase supply take a total power of 50 kW and a line current of 90 A. Find the (i) phase current, (ii) power factor, and (iii) apparent power taken by the coils.

Solution

$$V_L = 440 \text{ V}$$

$$P = 50 \text{ kW}$$

$$I_L = 90 \text{ A}$$

For a delta-connected load,

(i) Phase current

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 51.96 \text{ A}$$

(ii) Power factor

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$50 \times 10^3 = \sqrt{3} \times 440 \times 90 \times \cos\phi$$

$$\text{pf} = \cos\phi = 0.73 \text{ (lagging)}$$

(iii) Apparent power

$$S = \sqrt{3}\, V_L\, I_L = \sqrt{3} \times 440 \times 90 = 68.59 \text{ kVA}$$

Example 16

Three similar choke coils are connected in star to a three-phase supply. If the line current is 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil. If these coils are now connected in delta to the same supply, calculate phase and line currents, active and reactive power.

Solution

$$I_L = 15 \text{ A}$$

$$P = 11 \text{ kW}$$

$$S = 15 \text{ kVA}$$

For a star-connected load,

(i) Line voltage

$$S = \sqrt{3}\, V_L\, I_L$$

$$15 \times 10^3 = \sqrt{3} \times V_L \times 15$$
$$V_L = 577.35 \text{ V}$$

(ii) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{577.35}{\sqrt{3}} = 333.33 \text{ V}$$

(iii) VAR input

$$\cos\phi = \frac{P}{S} = \frac{11 \times 10^3}{15 \times 10^3} = 0.733$$
$$\phi = 42.86°$$
$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 577.35 \times 15 \times \sin(42.86°) = 10.2 \text{ kVAR}$$

(iv) Reactance and resistance of coil

$$I_{ph} = I_L = 15 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{333.33}{15} = 22.22\ \Omega$$
$$R = Z_{ph}\cos\phi = 22.22 \times 0.733 = 16.29\ \Omega$$
$$X_L = Z_{ph}\sin\phi = 22.22 \times \sin(42.86°) = 15.11\ \Omega$$

If these coils are now connected in delta,

(i) Phase current

$$V_{ph} = V_L = 577.35 \text{ V}$$
$$Z_{ph} = 22.22\ \Omega$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{577.35}{22.22} = 25.98 \text{ A}$$

(ii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 25.98 = 45 \text{ A}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 577.35 \times 45 \times 0.733 = 32.98 \text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 577.35 \times 45 \times \sin(42.86°) = 30.61 \text{ kVAR}$$

Example 17

Three similar coils, connected in star, take a total power of 1.5 kW at p.f. of 0.2 lagging from a three-phase, 440 V, 50 Hz supply. Calculate the resistance and inductance of each coil. **[Dec 2012]**

Solution

$$P = 1.5 \text{ kW}$$
$$\text{pf} = 0.2 \text{ (lagging)}$$
$$V_L = 440 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$1.5 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.2$$

$$I_L = 9.84 \text{ A}$$
$$I_{ph} = I_L = 9.84 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254.03}{9.84} = 25.82\ \Omega$$

$$\phi = \cos^{-1}(0.2) = 78.46°$$
$$\bar{Z}_{ph} = Z_{ph}\angle\phi = 25.82\angle 78.46° = (5.17 + j25.3)\ \Omega$$
$$R_{ph} = 5.17\ \Omega$$
$$X_{L_{ph}} = 25.3\ \Omega$$
$$X_{L_{ph}} = 2\pi f L_{ph}$$
$$25.3 = 2\pi \times 50 \times L_{ph}$$
$$L_{ph} = 0.08 \text{ H}$$

Example 18

A three-phase, star-connected source feeds 1500 kW at 0.85 power factor lag to a balanced mesh-connected load. Calculate the current, its active and reactive components in each phase of the source and the load. The line voltage is 2.2 kV.

Solution

$$P = 1500 \text{ kW}$$
$$\text{pf} = 0.85 \text{ (lagging)}$$
$$V_L = 2.2 \text{ kV}$$

For a mesh or delta-connected load,

(i) Line current

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$1500 \times 10^3 = \sqrt{3} \times 2.2 \times 10^3 \times I_L \times 0.85$$

$$I_L = 463.12 \text{ A}$$

(ii) Active component of current in each phase of the load

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{463.12}{\sqrt{3}} = 267.38 \text{ A}$$

$$I_{ph} \cos \phi = 267.38 \times 0.85 = 227.27 \text{ A}$$

(iii) Reactive component of current in each phase of the load

$$\begin{aligned} I_{ph} \sin \phi &= 267.38 \times \sin (\cos^{-1} 0.85) \\ &= 267.38 \times 0.526 = 140.85 \text{ A} \end{aligned}$$

For a star-connected source, the phase current in the source will be the same as the line current drawn by the load.

(iv) Active component of this current in each phase of the source

$$I_L \cos \phi = 463.12 \times 0.85 = 393.65 \text{ A}$$

(v) Reactive component of this current in each phase of the source

$$I_L \sin \phi = 463.12 \times 0.526 = 243.6 \text{ A}$$

Example 19

A three-phase, 208-volt generator supplies a total of 1800 W at a line current of 10 A when three identical impedances are arranged in a Wye connection across the line terminals of the generator. Compute the resistive and reactive components of each phase impedance.

Solution

$$\begin{aligned} V_L &= 208 \text{ V} \\ P &= 1800 \text{ W} \\ I_L &= 10 \text{ A} \end{aligned}$$

For a Wye-connected load,

$$\begin{aligned} V_{ph} &= \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120.09 \text{ V} \\ I_{ph} &= I_L = 10 \text{ A} \\ Z_{ph} &= \frac{V_{ph}}{I_{ph}} = \frac{120.09}{10} = 12\ \Omega \\ P &= \sqrt{3}\ V_L I_L \cos \phi \\ 1800 &= \sqrt{3} \times 208 \times 10 \times \cos \phi \\ \cos \phi &= 0.5 \\ \phi &= 60° \\ R_{ph} &= Z_{ph} \cos \phi = 12 \times 0.5 = 6\ \Omega \\ X_{ph} &= Z_{ph} \sin \phi = 12 \times \sin (60°) = 10.39\ \Omega \end{aligned}$$

Example 20

A balanced, three-phase, star-connected load of 100 kW takes a leading current of 80 A, when connected across a three-phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.

Solution

$$P = 100 \text{ kW}$$
$$I_L = 80 \text{ A}$$
$$V_L = 1100 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 635.08 \text{ V}$$

$$I_{ph} = I_L = 80 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{635.08}{80} = 7.94\ \Omega$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$100 \times 10^3 = \sqrt{3} \times 1100 \times 80 \times \cos\phi$$

$$\cos\phi = 0.656 \text{ (leading)}$$

$$\phi = 49°$$

$$R_{ph} = Z_{ph}\cos\phi = 7.94 \times 0.656 = 5.21\ \Omega$$

$$X_{ph} = Z_{ph}\sin\phi = 7.94 \times \sin(49°) = 6\ \Omega$$

This reactance will be capacitive in nature as the current is leading.

$$X_C = \frac{1}{2\pi f C}$$

$$6 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 530.52\ \mu\text{F}$$

Example 21

Three identical impedances are connected in delta to a three-phase supply of 400 V. The line current is 34.65 A, and the total power taken from the supply is 14.4 kW. Calculate the resistance and reactance values of each impedance.

Solution

$$V_L = 400 \text{ V}$$
$$I_L = 34.65 \text{ A}$$

$$P = 14.4 \text{ kW}$$

For a delta-connected load,

$$V_L = V_{ph} = 400 \text{ V}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{34.65}{\sqrt{3}} = 20 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{20} = 20 \ \Omega$$

$$P = \sqrt{3} \ V_L I_L \cos \phi$$

$$14.4 \times 10^3 = \sqrt{3} \times 400 \times 34.65 \times \cos \phi$$

$$\cos \phi = 0.6$$

$$\phi = 53.13°$$

$$R_{ph} = Z_{ph} \cos \phi = 20 \times 0.6 = 12 \ \Omega$$

$$X_{ph} = Z_{ph} \sin \phi = 20 \times \sin (53.13°) = 16 \ \Omega$$

Example 22

Three similar coils, connected in star, take a total power of 18 kW at a power factor of 0.866 lagging from a three-phase, 400-volt, 50 Hz system. Calculate the resistance and inductance of each coil. **[May 2014]**

Solution

$$P = 18 \text{ kW}$$

$$\text{pf} = 0.866 \text{ (lagging)}$$

$$V_L = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$P = \sqrt{3} \, V_L \ I_L \cos \phi$$

$$18 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.866$$

$$I_L = 30 \text{ A}$$

$$I_{ph} = I_L = 30 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{30} = 7.7 \ \Omega$$

$$\phi = \cos^{-1}(0.866) = 30°$$
$$\bar{Z}_{ph} = Z_{ph} \angle \phi = 7.7 \angle 30° = 6.67 + j3.85\ \Omega$$
$$R_{ph} = 6.67\ \Omega$$
$$X_{ph} = 3.85\ \Omega$$
$$X_{ph} = 2\pi f L$$
$$3.85 = 2\pi \times 50 \times L$$
$$L = 12.25\ \text{mH}$$

Example 23

A balanced three-phase load connected in delta, draws a power of 10 kW at 440 V at a pf of 0.6 lead, find the values of circuit elements and reactive volt-amperes drawn. **[May 2016]**

Solution

$$P = 10\ \text{kW}$$
$$V_L = 440\ \text{V}$$
$$\text{pf} = 0.6\ \text{(lead)}$$

For a delta-connected load,

(i) Values of circuit elements

$$V_L = V_{ph} = 440\ \text{V}$$
$$P = \sqrt{3}\ V_L I_L \cos\phi$$
$$10 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.6$$
$$I_L = 21.87\ \text{A}$$
$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{21.87}{\sqrt{3}} = 12.63\ \text{A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{440}{12.63} = 34.84\ \Omega$$
$$\phi = \cos^{-1}(0.6) = 53.13°$$
$$R_{ph} = Z_{ph}\cos\phi = 34.84 \times 0.6 = 20.90\ \Omega$$
$$X_{ph} = Z_{ph}\sin\phi = 34.84 \times 0.8 = 27.87\ \Omega$$

(ii) Reactive volt-amperes drawn

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 21.87 \times 0.8 = 13.33\ \text{kVAR}$$

Example 24

Find the values of circuit elements and reactive volt-ampere drawn for a balanced 3-phase load connected in delta and drawing a power of 12 kW at 440 V. The power factor is 0.7 leading. **[Dec 2013]**

Solution

$$P = 12 \text{ kW}$$
$$V_L = 440 \text{ V}$$
$$\text{pf} = 0.7 \text{ (leading)}$$

For a delta-connected load,

(i) Values of circuit elements

$$V_L = V_{ph} = 440 \text{ V}$$
$$P = \sqrt{3} V_L I_L \cos\phi$$
$$12 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.7$$
$$I_L = 22.49 \text{ A}$$
$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{22.49}{\sqrt{3}} = 12.98 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{440}{12.98} = 33.9\ \Omega$$
$$R_{ph} = Z_{ph} \cos\phi = 33.9 \times 0.7 = 23.73\ \Omega$$
$$X_{ph} = Z_{ph} \sin\phi = 33.9 \times \sin(\cos^{-1}0.7) = 33.9 \times 0.71 = 24.07\ \Omega$$

(ii) Reactive volt-amperes drawn

$$Q = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 440 \times 22.49 \times 0.71 = 12.17 \text{ kVAR}$$

Example 25

Each leg of a balanced, delta-connected load consists of a 7 Ω resistance in series with a 4 Ω inductive reactance. The line-to-line voltages are

$$E_{ab} = 2360 \angle 0° \text{ V}$$
$$E_{bc} = 2360 \angle -120° \text{ V}$$
$$E_{ca} = 2360 \angle 120° \text{ V}$$

Determine (i) phase current I_{ab}, I_{bc} and I_{ca} (both magnitude and phase)

(ii) each line current and its associated phase angle

(iii) the load power factor

Solution

$$R = 7\ \Omega$$
$$X_L = 4\ \Omega$$
$$V_L = 2360 \text{ V}$$

For a delta-connected load,

(i) Phase current

$$V_{ph} = V_L = 2360 \text{ V}$$

$$\bar{Z}_{ph} = 7 + j4 = 8.06 \angle 29.74° \ \Omega$$

$$\bar{I}_{ab} = \frac{\bar{E}_{ab}}{\bar{Z}_{ph}} = \frac{2360\angle 0°}{8.06\angle 29.74°} = 292.8 \angle -29.74° \text{ A}$$

$$\bar{I}_{bc} = \frac{\bar{E}_{bc}}{\bar{Z}_{ph}} = \frac{2360\angle -120°}{8.06\angle 29.74°} = 292.8 \angle -149.71° \text{ A}$$

$$\bar{I}_{ca} = \frac{\bar{E}_{ca}}{\bar{Z}_{ph}} = \frac{2360\angle 120°}{8.06\angle 29.74°} = 292.8 \angle 90.26° \text{ A}$$

(ii) Line current

In a delta-connected, three-phase system, line currents lag behind respective phase currents by 30°.

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 292.8 = 507.14 \text{ A}$$

$$\bar{I}_{La} = 507.14 \angle -59.71° \text{ A}$$

$$\bar{I}_{Lb} = 507.14 \angle -179.71° \text{ A}$$

$$\bar{I}_{Lc} = 507.14 \angle 60.26° \text{ A}$$

(iii) Load power factor

$$\text{pf} = \cos(29.74°) = 0.868 \text{ (lagging)}$$

Example 26

A three-phase, 200 kW, 50 Hz, delta-connected induction motor is supplied from a three-phase, 440 V, 50 Hz supply system. The efficiency and power factor of the three-phase induction motor are 91% and 0.86 respectively. Calculate (i) line currents, (ii) currents in each phase of the motor, (iii) active, and (iv) reactive components of phase current.

Solution

$$P_o = 200 \text{ kW}$$

$$V_L = 440 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\eta = 91\%$$

$$\text{pf} = 0.86$$

For a delta-connected load (induction motor),

(i) Line current

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_o}{P_i}$$

$$0.91 = \frac{200 \times 10^3}{P_i}$$

$$P_i = 219.78 \text{ kW}$$

$$P_i = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$219.78 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.86$$

$$I_L = 335.3 \text{ A}$$

(ii) Currents in each phase of motor

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{335.3}{\sqrt{3}} = 193.6 \text{ A}$$

(iii) Active component of phase current

$$I_{ph} \cos\phi = 193.6 \times 0.86 = 166.5 \text{ A}$$

(iv) Reactive component of phase current

$$I_{ph} \sin\phi = 193.6 \times \sin(\cos^{-1} 0.86) = 193.6 \times 0.51 = 98.7 \text{ A}$$

Example 27

A three-phase, 400 V, star-connected alternator supplies a three-phase, 112 kW, mesh-connected induction motor of efficiency and power factor 0.88 and 0.86 respectively. Find the (i) current in each motor phase, (ii) current in each alternator phase, and (iii) active and reactive components of current in each case.

Solution

$$V_L = 400 \text{ V}$$

$$P_o = 112 \text{ kW}$$

$$\eta = 0.88$$

$$\text{pf} = 0.86$$

For a mesh-connected load (induction motor),

(i) Current in each motor phase

$$V_{ph} = V_L = 400 \text{ V}$$

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_o}{P_i}$$

$$0.88 = \frac{112 \times 10^3}{P_i}$$

$$P_i = 127.27 \text{ kW}$$

$$P_i = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$127.27 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.86$$

$$I_L = 213.6 \text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{213.6}{\sqrt{3}} = 123.32 \text{ A}$$

Current in a star-connected alternator phase will be same as the line current drawn by the motor.

(ii) Current in each alternator phase

$$I_L = 213.6 \text{ A}$$

(iii) Active component of current in each phase of motor

$$I_{ph} \cos\phi = 123.32 \times 0.86 = 105.06 \text{ A}$$

Reactive component of current in each phase of the motor

$$I_{ph} \sin\phi = 123.32 \times \sin(\cos^{-1} 0.86) = 123.32 \times 0.51 = 62.89 \text{ A}$$

(iv) Active component of current in each alternator phase

$$I_L \cos\phi = 213.6 \times 0.86 = 183.7 \text{ A}$$

Reactive component of current in each alternator phase

$$I_L \sin\phi = 213.6 \times \sin(\cos^{-1} 0.86) = 213.6 \times 0.51 = 108.94 \text{ A}$$

Example 28

Three similar resistors are connected in star across 400 V, three-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors connected in delta?

Solution $V_L = 400$ V

$$I_L = 5 \text{ A}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = I_L = 5 \text{ A}$$

$$Z_{ph} = R_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{5} = 46.19\ \Omega$$

For a delta-connected load,

$$I_L = 5 \text{ A}$$

$$R_{ph} = 46.19\ \Omega$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{5}{\sqrt{3}} \text{ A}$$

$$V_{ph} = I_{ph} R_{ph} = \frac{5}{\sqrt{3}} \times 46.19 = 133.33 \text{ V}$$

$$V_L = 133.33 \text{ V}$$

Voltage needed is one-third of the star value.

Example 29

Three 100 Ω, non-inductive resistors are connected in (a) star, and (b) delta across a 400 V, 50 Hz, three-phase supply. Calculate the power taken from the supply in each case. If one of the resistors is open circuited, what would be the value of total power taken from the mains in each of the two cases?

Solution

$$V_L = 400 \text{ V}$$
$$Z_{ph} = 100\ \Omega$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{100} = 2.31 \text{ A}$$
$$I_L = I_{ph} = 2.31 \text{ A}$$
$$\cos\phi = 1 \qquad \text{(For pure resistor, pf = 1)}$$
$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 2.31 \times 1 = 1600.41 \text{ W}$$

For a delta-connected load,

$$V_{ph} = V_L = 400 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{100} = 4 \text{ A}$$
$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 4 = 6.93 \text{ A}$$
$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 6.93 \times 1 = 4801.24 \text{ W}$$

When one of the resistors is open circuited

(i) Star connection The circuit consists of two 100 Ω resistors in series across a 400 V supply.

$$\text{Currents in lines } A \text{ and } C = \frac{400}{200} = 2 \text{ A}$$

$$\text{Power taken from the mains} = 400 \times 2 = 800 \text{ W}$$

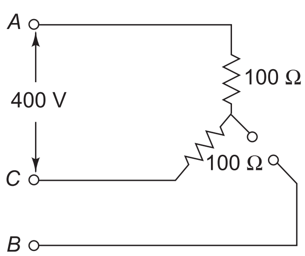


Fig. 5.22(a) *Star connection*

Hence, when one of the resistors is open circuited, the power consumption is reduced by half.

(ii) Delta connection In this case, currents in A and C remain as usual 120° out of phase with each other.

$$\text{Current in each phase} = \frac{400}{100} = 4 \text{ A}$$

Power taken from the mains $= 2 \times 4 \times 400 = 3200$ W

Hence, when one of the resistors is open circuited, the power consumption is reduced by one-third.

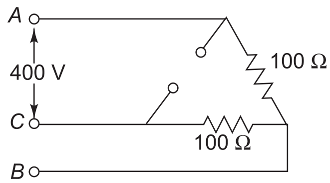


Fig. 5.22(b) *Delta connection*

Example 30

Three identical impedances of 10 ∠30° Ω each are connected in star and another set of three identical impedances of 18 ∠60° Ω are connected in delta. If both the sets of impedances are connected across a balanced, three-phase 400 V supply, find the line current, total volt-amperes, active power and reactive power.

Solution

$$\bar{Z}_Y = 10\ \angle 30°\ \Omega$$
$$\bar{Z}_\Delta = 18\ \angle 60°\ \Omega$$
$$V_L = 400\text{ V}$$

Three identical delta impedances can be converted into equivalent star impedances.

$$\bar{Z}_Y' = \frac{\bar{Z}_\Delta}{3} = \frac{18\angle 60°}{3} = 6\angle 60°\ \Omega$$

Now two star-connected impedances of 10 ∠30° Ω and 6 ∠60° Ω are connected in parallel across a three-phase supply.

$$\bar{Z}_{eq} = \frac{(10\angle 30°)(6\angle 60°)}{10\angle 30° + 6\angle 60°} = 3.87\ \angle 48.83°\ \Omega$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{ph}}{Z_{eq}} = \frac{230.94}{3.87} = 59.67\text{ A}$$

$$I_L = I_{ph} = 59.67\text{ A}$$

(ii) Total volt-amperes

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 400 \times 59.67 = 41.34\text{ kVA}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 59.67 \times \cos(48.83°) = 27.21\text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 400 \times 59.67 \times \sin(48.83°) = 31.12\text{ kVAR}$$

Example 31

Three star-connected impedances $Z_Y = (20 + j37.7)\ \Omega$ per phase are connected in parallel with three delta-connected impedances $Z_\Delta = (30 - j159.3)\ \Omega$ per phase. The line voltage is 398 V. Find the line current, pf, active and reactive power taken by the combination.

Solution

$$\bar{Z}_Y = 20 + j37.7 = 42.68\ \angle 62.05°\ \Omega$$

$$\bar{Z}_\Delta = 30 - j159.3 = 162.1\ \angle -79.3°\ \Omega$$

$$V_L = 398 \text{ V}$$

Three identical delta-connected impedances can be converted into equivalent star impedances.

$$\bar{Z}'_Y = \frac{162.1\angle -79.3°}{3} = 54.03\ \angle -79.3°\ \Omega$$

Now two star-connected impedances of 42.68 $\angle$62.05° Ω and 54.03 $\angle$–79.3° Ω are connected in parallel across the three-phase supply.

$$\bar{Z}_{\text{eq}} = \frac{(42.68\angle 62.05°)(54.03\angle -79.3°)}{42.68\angle 62.05° + 54.03\angle -79.3°)} = 68.33\angle 9.88°\ \Omega$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{398}{\sqrt{3}} = 229.79 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{ph}}{Z_{eq}} = \frac{229.79}{68.33} = 3.36 \text{ A}$$

$$I_L = I_{ph} = 3.36 \text{ A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(9.88°) = 0.99 \text{ (lagging)}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 398 \times 3.36 \times 0.99 = 2.29 \text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 398 \times 3.36 \times \sin(9.88°) = 397.43 \text{ VAR}$$

Example 32

Three coils, each having a resistance of 20 Ω and a reactance of 15 Ω, are connected in star to a 400 V, three-phase, 50 Hz supply. Calculate (i) line current, (ii) power supplied, and (iii) power factor. If three capacitors, each of same capacitance, are connected in delta to the same supply so as to form a parallel circuit with the above coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

Solution

$$R_{ph} = 20\ \Omega$$

$$X_{ph} = 15\ \Omega$$

$$V_L = 400 \text{ V}$$

For a star-connected load,

(i) Line current

$$\bar{Z}_{ph} = R_{ph} + jX_{ph} = 20 + j15 = 25 \angle 36.87° \ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{25} = 9.24 \text{ A}$$

$$I_L = I_{ph} = 9.24 \text{ A}$$

(ii) Power supplied

$$P_1 = \sqrt{3}\, V_L\, I_L \cos\phi_1 = \sqrt{3} \times 400 \times 9.24 \times \cos(36.87°) = 5.12 \text{ kW}$$

(iii) Power factor

$$\text{pf} = \cos\phi_1 = \cos(36.87°) = 0.8 \text{ (lagging)}$$

(iv) Value of capacitance of each capacitor

$$Q_1 = \sqrt{3}\, V_L\, I_L \sin\phi_1 = \sqrt{3} \times 400 \times 9.24 \times \sin(36.87°) = 3.84 \text{ kVAR}$$

When capacitors are connected in delta to the same supply

$$\text{pf} = 0.95$$

$$\phi_2 = \cos^{-1}(0.95) = 18.19°$$

$$\tan\phi_2 = \tan(18.19°) = 0.33$$

Since capacitors do not absorb any power, power remains the same even when capacitors are connected. But reactive power changes.

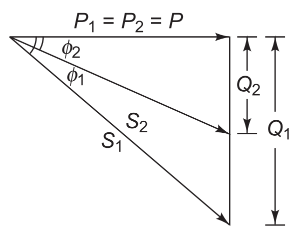


Fig. 5.23

$$P_2 = 5.12 \text{ kW}$$

$$Q_2 = P_2 \tan\phi_2 = 5.12 \times 0.33 = 1.69 \text{ kVAR}$$

Difference in reactive power is supplied by three capacitors.

$$Q = Q_1 - Q_2 = 3.84 - 1.69 = 2.15 \text{ kVAR}$$

$$Q = \sqrt{3}\, V_L\, I_L \sin\phi$$

$$2.15 \times 10^3 = \sqrt{3} \times 400 \times I_L \times \sin(90°)$$

$$I_L = 3.1 \text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 1.79 \text{ A}$$

$$I_{ph} = \frac{V_{ph}}{X_C} = V_{ph} \times 2\pi f C$$

$$C = \frac{I_{ph}}{V_{ph} \times 2\pi f} = \frac{1.79}{400 \times 2\pi \times 50} = 14.24 \ \mu\text{F}$$

Useful Formulae

Star Connection

$$V_L = \sqrt{3}\, V_{ph}$$
$$I_L = I_{ph}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}}$$
$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$
$$Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$$
$$S = 3V_{ph} I_{ph} = \sqrt{3}\, V_L I_L$$

Delta Connection

$$I_L = \sqrt{3}\, I_{ph}$$
$$V_L = V_{ph}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}}$$
$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$
$$Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$$
$$S = 3V_{ph} I_{ph} = \sqrt{3}\, V_L I_L$$

Exercise 5.1

5.1 Three coils, each of 5 Ω resistance, and 6 Ω inductive reactance are connected in closed delta and supplied from a 440 V, three-phase system. Calculate the line and phase currents, the power factor of the system and the intake in watts.
[97.58 A, 56.33 A, 0.64 (lagging), 47.61 kW]

5.2 Three coils each having a resistance of 10 Ω and inductance of 0.02 H are connected (i) in star, (ii) in delta to a three-phase, 50 Hz supply, the line voltage being 500 volts. Calculate for each case the line current and the total power taken from the supply. *[(i) star : 24.46 A, 17.94 kW, (ii) delta : 73.39 A, 53.83 kW]*

5.3 A balanced delta-connected load of (8 + *j*6) ohms per phase is connected to a three-phase, 230 V supply with phase sequence *R-Y-B*. Find the line current, power factor, power, reactive volt-amperes and the total volt-amperes. Draw the phasor diagram. *[39.85 A, 0.8 (lagging), 12.74 kW, 9.52 kVAR, 15.86 kVA]*

5.4 A balanced star-connected load with (6 + *j*8)Ω per phase is connected to a three-phase, 440 V supply. Find the line current, power factor, power, reactive volt-amperes and volt-amperes total.
[25.404 A, 0.6(lagging), 11.616 kW, 15.488 kVAR, 19.360 kVA]

5.5 Calculate the active and reactive components of the current in each phase of a star-connected 5000 V, 3-phase, alternator supplying 3000 kW at a power factor of 0.8.
[250 A, 150A]

5.6 Three similar coils, connected in star, take a total power of 1.5 kW at a power factor of 0.2 lagging from a 3 phase, 440 V, 50 Hz supply. Calculate the resistance and inductance of each coil. *[5.16 Ω, 0.08 H]*

5.7 A balanced three-phase load connected in delta draws a power of 10 kW at 440 V at a power factor of 0.6 lead. Find the values of the circuit elements and the reactive volt-amperes drawn. *[20.91 Ω, 27.88 Ω, 13.33 kVAR]*

5.8 A balanced star-connected load, connected to a 400 V, 50 Hz, three-phase ac supply draws a phase current of 50 A at 0.6 power factor lagging. Calculate (i) phase voltage, (ii) total power, and (iii) parameters in the star-connected load.
[230.94 V, 20.84 kW, (2.77 + j3.7) Ω]

5.9 Three equal star-connected inductors consume 8 kW power at 0.8 power factor when connected to 415 V, 3-phase, 3-wire, 50 Hz supply. Estimate the load parameters per phase and determine the line currents. *[6.2 Ω, 0.0386 H, 13.83A]*

5.10 A balanced three-phase, star-connected load of 150 kW takes a leading current of 100 A with a line voltage of 1100 V, 50 Hz. Find the circuit constant of the load per phase. *[5 Ω, 813 μF]*

5.11 Three pure elements connected in star draw *x* kVAR. What will be the value of elements that will draw the same kVAR, when connected in delta across the same supply? $[Z_\Delta = 3Z_Y]$

5.12 A balanced Wye-connected load with (10 + *j*20) ohms per phase is connected to a three-phase, 400 V supply. Determine the voltage across, current through and power dissipated in each resistor. Also, determine the total power.
[103.2 V, 10.32 V, 1067 W, 3201 W]

5.13 A delta-connected three-phase load is supplied from a 3-phase, 400 volts balanced supply system. The line current is 20 A and power taken by the load is 10 kW. Find (i) impedance in each branch, (ii) line current, power factor and power consumed if the same load is connected in star.
[(24.95 + j24.05) Ω, 6.66 A, 0.72(lagging), 3323.21 W]

5.14 A balanced star-connected load is supplied from a symmetrical 3-phase, 400 V system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find (i) phase voltage, (ii) the circuit elements, and (iii) draw the vector diagram showing the currents and the voltages. *[230.94 V, 6.67 Ω, 3.849Ω]*

5.15 A 3-phase, delta-connected load having a (3 + *j*4) ohms per phase is connected across a 230 V, 3-phase source. Calculate the magnitude of the line current.
[76.21 A]

5.16 A 220 V, 3-phase voltage is applied to a balanced delta-connected load. The rms value of the phase current is 20 ∠–30°A. Determine

(i) magnitude and phase of the line current
(ii) total power received by the three-phase load
(iii) value of the resistive portion of the phase impedance

Also, draw the phasor diagram showing clearly the line voltages, phase current and line currents. *[34.65 ∠–60°A, 11.43 kW, 9.53 Ω]*

5.17 A 3-phase, 37.3 kW, 440 V, 50 Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate total kVA rating of capacitance required to raise power factor at 0.95 lagging. What will be the value of capacitance/phase if capacitors are (i) delta connected? (ii) star connected?

[12.19 kVA, 66.8 μF, 200.4 μF]

5.18 Three coils each having impedance $(4 + j3)$ ohms are connected in star to a 440 V, three-phase, 50 Hz balanced supply. Calculate the line current and active power. Now if three pure capacitors, each of C farads, connected in delta, are connected across the same supply, it is found that the total power factor of the circuit becomes 0.96 lag. Find the value of C. Also, find the total line current.

[50.8 A, 30.976 kW, 77.75 μF, 42.34 A]

5.13 MEASUREMENT OF THREE-PHASE POWER

[May 2013]

In a three-phase system, total power is the sum of powers in three phases. The power is measured by wattmeter. It consists of two coils: (i) Current coil, and (ii) Voltage coil. Current coil is connected in series with the load and it senses current. Voltage coil is connected across supply terminals and it senses voltages.

There are three methods to measure three-phase power:

1. Three-wattmeter method
2. Two-wattmeter method
3. One-wattmeter method

5.13.1 Three-Wattmeter Method

This method is used for balanced as well as unbalanced loads. Three wattmeters are inserted in each of the three phases of the load whether star connected or delta connected as shown in Fig. 5.24. Each wattmeter will measure the power consumed in each phase.

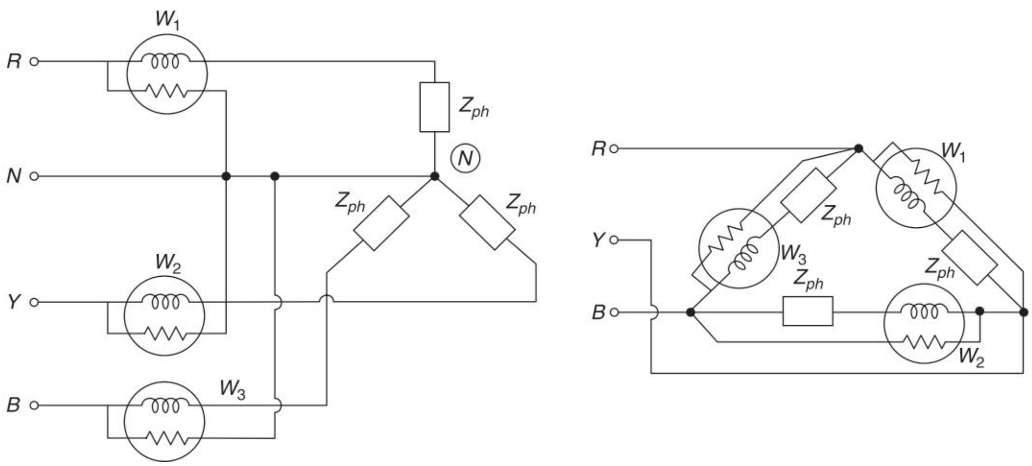


Fig. 5.24 *Three-wattmeter method*

For balanced load, $W_1 = W_2 = W_3$

For unbalanced load, $W_1 \neq W_2 \neq W_3$

Total power $P = W_1 + W_2 + W_3$

5.13.2 Two-Wattmeter Method

This method is used for balanced as well as unbalanced loads. The current coils of the two wattmeters are inserted in any two lines and the voltage coil of each wattmeter is joined to a third line. The load may be star or delta connected as shown in Fig. 5.25. The sum of the two wattmeter readings gives three-phase power.

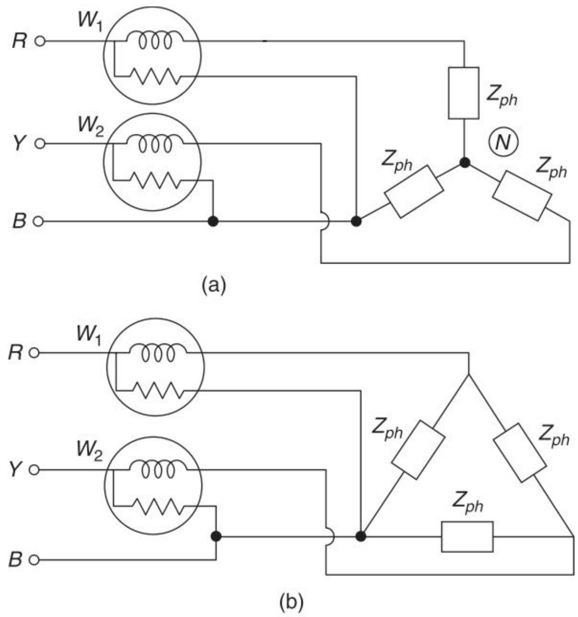


Fig. 5.25 *Two-wattmeter method*

` Total power $P = W_1 + W_2$

5.13.3 One-Wattmeter Method

This method is used for balanced loads only. When the load is balanced, total power is given by

$$P = 3\,V_{ph} I_{ph} \cos\phi$$

Hence, one wattmeter is used to measure power in one phase. The wattmeter reading is then multiplied by three to obtain three-phase power. The load may be star or delta connected as shown in Fig. 5.26.

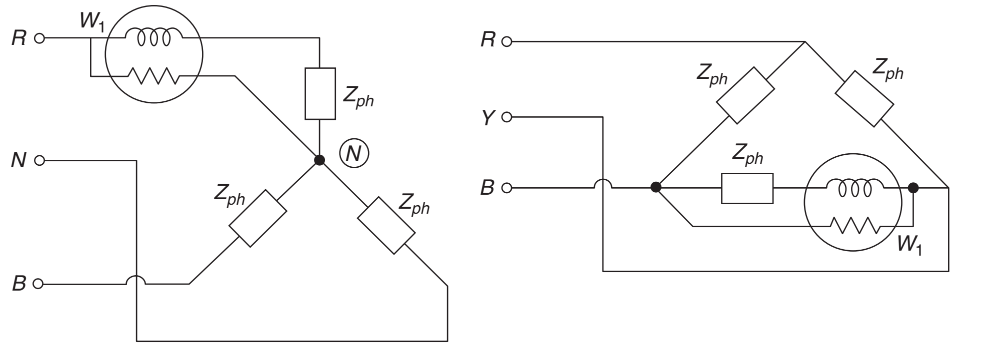


Fig. 5.26 *One-wattmeter method*

5.14 MEASUREMENT OF REACTIVE POWER BY ONE-WATTMETER METHOD

Figure 5.27 shows a balanced star-connected load and this load may be assumed to be inductive. Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages and I_R, I_Y and I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle ϕ.

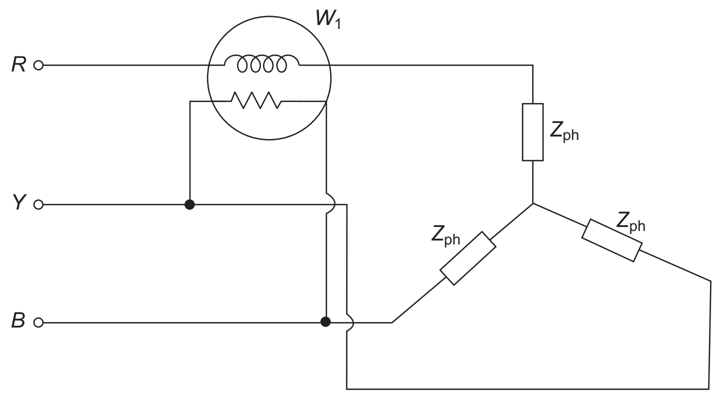


Fig. 5.27 *One-Wattmeter Method*

Current through current coil of $W_1 = I_R$

Voltage across voltage coil of $W_1 = V_{YB} = V_{YN} + V_{NB}$

Figure 5.28 shows the phasor diagram of a balanced star-connected inductive load.

From the phasor diagram, it is clear that the phase angle between V_{YB} and I_R is $(90° - \phi)$.

$$W_1 = V_{YB} I_R \cos (90° - \phi)$$

But, $I_R = I_L$

$$V_{\text{YB}} = V_L$$

$$W_1 = V_L I_L \cos (90° - \phi) = V_L I_L \sin \phi$$

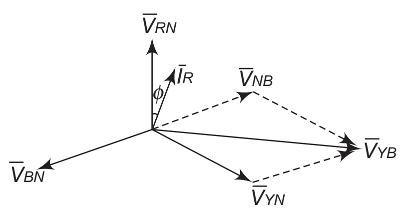


Fig. 5.28 *Phasor Diagram*

Thus, total reactive power of a three-phase system is obtained by multiplying the wattmeter reading by $\sqrt{3}$.

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\, W_1$$

5.15 MEASUREMENT OF ACTIVE POWER, REACTIVE POWER AND POWER FACTOR BY TWO-WATTMETER METHOD

5.15.1 Measurement of Active Power for Star Connected Load

[Dec 2012, 2015, May 2014, 2015]

Figure 5.24 shows a balanced star-connected load and this load may be assumed to be inductive. Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages and I_R, I_Y and I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle ϕ.

Current through current coil of $W_1 = I_R$

Voltage across voltage coil of $W_1 = V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

Figure 5.29 shows the phasor diagram of a balanced star-connected inductive load.

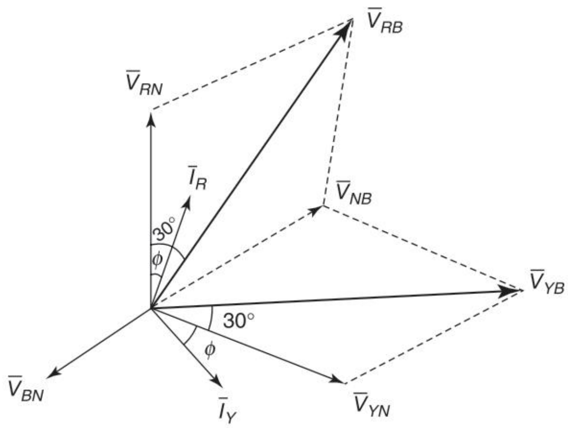


Fig. 5.29 *Phasor diagram*

From the phasor diagram, it is clear that the phase angle between V_{RB} and I_R is (30° – ϕ).

$$W_1 = V_{RB} I_R \cos(30° - \phi)$$

Current through current coil of $W_2 = I_Y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

From the phasor diagram, it is clear that phase angle between V_{YB} and I_Y is (30° + ϕ).

$$W_2 = V_{YB}\, I_Y \cos(30° + \phi)$$

But, $\quad I_R = I_Y = I_L$

$$V_{RB} = V_{YB} = V_L$$
$$W_1 = V_L I_L \cos(30° - \phi)$$
$$W_2 = V_L I_L \cos(30° + \phi)$$
$$W_1 + W_2 = V_L I_L [\cos(30° + \phi) + \cos(30° - \phi)]$$
$$= V_L I_L (2\cos 30° \cos\phi) = \sqrt{3}\, V_L I_L \cos\phi$$

Thus, the sum of two wattmeter readings gives three-phase power.

5.15.2 Measurement of Active Power for Delta Connected Load *[Dec 2014]*

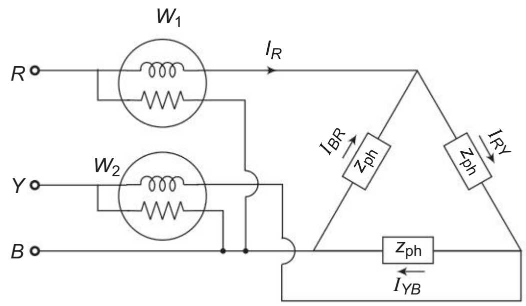


Fig. 5.30 *Two wattmeter method*

Current through current coil of $W_1 = \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$

Voltage across voltage coil of $W_1 = V_{RB}$

Current through current coil of $W_2 = I_Y = \bar{I}_{YB} - \bar{I}_{RY}$

Voltage across voltage coil of $W_2 = V_{YB}$

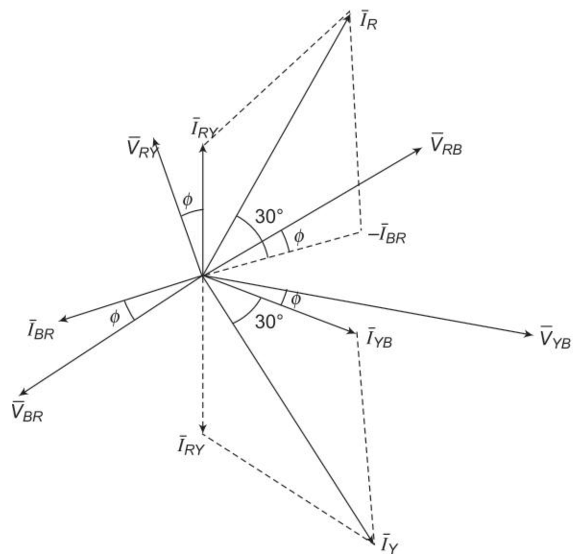


Fig. 5.31 *Phasor Diagram*

From phasor diagram, it is clear that the phase angle between I_R and V_{RB} is $(30° - \phi)$.

$$W_1 = I_R\, V_{RY} \cos(30° - \phi) = I_L\, V_L \cos(30° - \phi)$$

From phasor diagram, it is clear that the phase angle between I_Y and V_{YB} is $(30° + \phi)$

$$W_2 = I_Y\, V_{YB} \cos(30° + \phi) = I_L\, V_L \cos(30° + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30° - \phi) + \cos(30° + \phi)]$$

$$= V_L I_L (2 \cos 30° \cos \phi)$$

$$= \sqrt{3}\, V_L I_L \cos \phi$$

Thus, sum of two wattmeter readings gives three-phase power.

5.15.3 Measurement of Reactive Power

We know that,

$$W_1 - W_2 = V_L I_L \sin \phi$$

The total reactive power in a three-phase system is given by,

$$Q = \sqrt{3}\, V_L I_L \sin \phi = \sqrt{3}\,(W_1 - W_2)$$

Thus, total reactive power of a three-phase system is obtained by multiplying the difference of two-wattmeter readings by $\sqrt{3}$.

5.15.4 Measurement of Power Factor

1. Lagging Power Factor

$$W_1 = V_L I_L \cos(30° - \phi)$$

$$W_2 = V_L I_L \cos(30° + \phi)$$

$$\therefore \quad W_1 > W_2$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos \phi$$

$$W_1 - W_2 = V_L I_L [\cos(30° - \phi) - \cos(30° + \phi) = V_L I_L \sin \phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\tan \phi = \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left(\sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\text{pf} = \cos \phi = \cos\left\{\tan^{-1}\left(\sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

2. *Leading Power Factor*

Figure 3.32 shows the phasor diagram of a balanced star-connected capacitive load.

$$W_1 = V_L I_L \cos(30° + \phi)$$
$$W_2 = V_L I_L \cos(30° - \phi)$$
$$\therefore W_1 < W_2$$
$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$$
$$W_1 - W_2 = -V_L I_L \sin\phi$$
$$\tan\phi = -\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$
$$\phi = \tan^{-1}\left(-\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}\right)$$
$$\text{pf} = \cos\phi = \cos\left\{\tan^{-1}\left(-\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

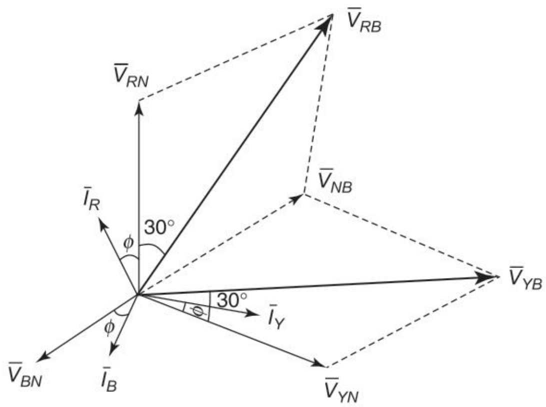


Fig. 5.32 *Phasor diagram*

5.16 EFFECT OF POWER FACTOR ON WATTMETER READINGS IN TWO WATTMETER METHOD

For a lagging power factor load,

$$W_1 = V_L I_L \cos(30° - \phi)$$
$$W_2 = V_L I_L \cos(30° + \phi)$$

Case I: $\phi = 0°$, $\quad$ $\text{pf} = \cos\phi = 1$

$$W_1 = V_L I_L \cos 30°$$
$$W_2 = V_L I_L \cos 30°$$

Hence, both wattmeter readings are equal and positive. For all power factors between 0.5 to 1, both wattmeter readings are positive.

Case II: $\phi = 60°$, $\quad$ $\text{pf} = \cos\phi = 0.5$ (lagging)

$$W_1 = V_L I_L \cos(30° - 60°) = V_L I_L \cos 30° \quad [\because \cos(-\theta) = \cos\theta]$$
$$W_2 = V_L I_L \cos(30° + 60°) = 0$$

Hence, wattmeter W_1 reading is positive and wattmeter W_2 reading is zero.

For all power factors between 0 to 0.5 (lagging), wattmeter W_1 reading is positive and wattmeter W_2 reading is negative.

Case III: $\phi = 90°$, $\quad$ pf = cos $\phi = 0$

$$W_1 = V_L I_L \cos(30° - 90°) = 0.5\, V_L I_L$$

$$W_2 = V_L I_L \cos(30° + 90°) = -0.5\, V_L I_L$$

Hence, $\quad W_1 = -W_2$

Negative reading indicates that the pointer deflects in negative direction i.e., to the left of zero. The readings can be converted to positive by interchanging either current coil or voltage coil terminals.

pf	ϕ	W_1 Reading	W_2 Reading	Remark
0	90°	Positive	Negative	$W_1 = -W_2$
0 < pf < 0.5	$90° < \phi < 60°$	Positive	Negative	
0.5	60°	Positive	0	
0.5 < pf < 1	$60° < \phi < 0°$	Positive	Positive	
1	0°	Positive	Positive	$W_1 = W_2$

Note: For leading power factor, readings of W_1 and W_2 are interchanged.

Example 1

Two wattmeters are used to measure power in a three-phase balanced load. Find the power factor if (i) two readings are equal and positive, (ii) two readings are equal and opposite, and (iii) one wattmeter reads zero. **[Dec 2013]**

Solution (i) $W_1 = W_2$

(ii) $W_2 = 0$ $W_1 = -W_2$

(i) Power factor if two readings are equal and positive

$$W_1 = W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,(0) = 0$$

$$\phi = 0°$$

$$\text{Power factor} = \cos\phi = \cos(0°) = 1$$

(ii) Power factor if two readings are equal and opposite

$$W_1 = -W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \infty$$

$$\phi = 90°$$

$$\text{Power factor} = \cos\phi = \cos(90°) = 0$$

(iii) Power factor if one wattmeter reads zero

$$W_2 = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{W_1}{W_1}\right) = \sqrt{3}$$

$$\phi = 60°$$

$$\text{Power factor} = \cos\phi = \cos(60°) = 0.5$$

Example 2

What will be the relation between readings on the wattmeter connected to measure power in a three-phase balanced circuit with (i) unity power factor, (ii) zero power factor, and (iii) power factor = 0.5.

Solution (i) pf = 1

(ii) pf = 0

(iii) pf = 0.5

(i) Relation between wattmeter readings with power factor = 1

$$\cos\phi = 1$$

$$\phi = 0°$$

$$\tan\phi = \tan(0°) = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$0 = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 = W_2$$

(ii) Relation between wattmeter readings with power factor = 0

$$\cos\phi = 0$$

$$\phi = 90°$$

$$\tan\phi = \tan(90°) = \infty$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 + W_2 = 0$$

$$W_1 = -W_2$$

(iii) Relation between wattmeter readings with power factor = 0.5

$$\cos\phi = 60°$$

$$\tan\phi = \tan(60°) = 1.732$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$1.732 = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 - W_2 = W_1 + W_2$$

$$W_2 = 0$$

Example 3

In a balanced three-phase circuit, power is measured by two wattmeters, the ratio of two wattmeter readings is 2 : 1. Determine the power factor of the system. **[Dec 2012]**

Solution

$$\frac{W_1}{W_2} = \frac{2}{1}$$

$$W_1 = 2W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{W_2}{3W_2} = \sqrt{3}\left(\frac{1}{3}\right) = 0.577$$

$$\phi = 30°$$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

Example 4

In a balanced three-phase system, the power is measured by two-wattmeter method and the ratio of two-wattmeter readings is 4:1. The load is inductive. Determine the load power factor.

Solution

$$\frac{W_1}{W_2} = \frac{4}{1}$$

$$W_1 = 4W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(3W_2)}{(5W_2)} = \sqrt{3}\left(\frac{3}{5}\right) = 1.039$$

$$\phi = 46.1°$$

$$\text{pf} = \cos\phi = \cos(46.1°) = 0.0693 \text{ (lagging)}$$

Example 5

Two wattmeters are used to measure power in a 3ϕ balanced delta connected load using two wattmeter method. The readings of the 2 wattmeters are 500 W and 2500 W respectively. Calculate the total power consumed by the 3ϕ load and the power factor. **[May 2015]**

Solution $W_1 = 500$ W

$W_2 = 2500$ W

(i) Total power

$$P = W_1 + W_2 = 500 + 2500 = 3 \text{ kW}$$

(ii) Power factor

The power factor is leading in nature since $W_1 < W_2$.

$$\tan\phi = -\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left[-\sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}\right] = \tan^{-1}\left[-\sqrt{3}\left(\frac{-2000}{3000}\right)\right] = 49.12$$

$$\text{pf} = \cos\phi = 0.65 \text{ (leading)}$$

Example 6

Find the power and power factor of the balanced circuit in which the wattmeter readings are 5 kW and 0.5 kW, the latter being obtained after the reversal of the current coil terminals of the wattmeter.

Solution $W_1 = 5$ kW

$W_2 = 0.5$ kW

(i) Power

When the latter reading is obtained after the reversal of the current coil terminals of the wattmeter,

$$W_1 = 5 \text{ kW}$$

$$W_2 = -0.5 \text{ kW}$$

$$\text{Power} = W_1 + W_2 = 5 + (-0.5) = 4.5 \text{ kW}$$

(ii) Power factor

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(5 + 0.5)}{(5 - 0.5)} = 2.12$$

$$\phi = 64.72°$$

$$\text{Power factor} = \cos\phi = \cos(64.72°) = 0.43$$

Example 7

Two wattmeters are used to measure power in a 3ϕ balanced star connected load using the two wattmeter method. The readings of the 2 wattmeters are 8 kW and 4 kW respectively. Calculate the total power consumed by the 3ϕ load and the power factor. **[Dec 2014]**

Solution $W_1 = 8$ kW

$W_2 = 4$ kW

(i) Total power

$$P = W_1 + W_2 = 8 + 4 = 12 \text{ kW}$$

(ii) Power factor

$$\tan\phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \frac{\sqrt{3}(8-4)}{(8+4)} = 0.577$$

$$\phi = 30°$$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (lagging)}$$

Example 8

The input power of a three-phase motor was measured by the two-wattmeter method. The readings of two wattmeters are 5.2 kW and –1.7 kW and the line voltage is 415 V. Calculate the total active power, power factor and line current. **[May 2013]**

Solution

$$W_1 = 5.2 \text{ kW}$$

$$W_2 = -1.7 \text{ kW}$$

$$V_L = 415 \text{ V}$$

(i) Total active power

$$P = W_1 + W_2 = 5.2 - 1.7 = 3.5 \text{ kW}$$

(ii) Power factor

$$\tan\phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{5.2 + 1.7}{5.2 - 1.7}\right) = 3.41$$

$$\phi = 73.68°$$

$$\text{pf} = \cos\phi = \cos(73.68°) = 0.28 \text{ (lagging)}$$

(ii) Line current

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$3.5 \times 10^3 = \sqrt{3} \times 415 \times I_L \times 0.28$$

$$I_L = 17.39 \text{ A}$$

Example 9

Two wattmeters connected to measure the input to a balanced, three-phase circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit (i) when both readings are positive and (ii) when the latter is obtained after reversing the connection to the current coil of one instrument.

Solution

$$W_1 = 2000 \text{ W}$$

$$W_2 = 500 \text{ W}$$

(i) Power factor of the circuit when both readings are positive

$$W_1 = 2000 \text{ W}$$

$$W_2 = 500 \text{ W}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(2000-500)}{(2000+500)} = 1.039$$

$$\phi = 46.102°$$

$$\text{Power factor} = \cos\phi = \cos(46.102°) = 0.693$$

(ii) Power factor of the circuit when the latter reading is obtained after reversing the connection to the current coil of one instrument

$$W_1 = 2000 \text{ W}$$

$$W_2 = -500 \text{ W}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(2000+500)}{(2000-500)} = 2.887$$

$$\phi = 70.89°$$

$$\text{Power factor} = \cos\phi = \cos(70.89°) = 0.33$$

Example 10

A three-phase, 10 kVA load has a power factor of 0.342. The power is measured by the two-wattmeter method. Find the reading of each wattmeter when the (i) power factor is leading, and the (ii) power factor is lagging. **[May 2014]**

Solution

$$S = 10 \text{ kVA}$$

$$\text{pf} = 0.342$$

$$S = \sqrt{3}\,V_L I_L$$

$$10 \times 10^3 = \sqrt{3}\,V_L I_L$$

$$V_L I_L = 5.77 \text{ kVA}$$

$$\cos\phi = 0.342$$

$$\phi = 72°$$

(i) Reading of each wattmeter when the power factor is leading

$$W_1 = V_L I_L \cos(30° + \phi) = 5.77 \cos(30° + 70°) = -1 \text{ kW}$$

$$W_2 = V_L I_L \cos(30° - \phi) = 5.77 \cos(30° - 70°) = 4.42 \text{ kW}$$

(ii) Reading of each wattmeter when the power factor is lagging

$$W_1 = V_L I_L \cos(30° - \phi) = 4.42 \text{ kW}$$

$$W_2 = V_L I_L \cos(30° + \phi) = -1 \text{ kW}$$

Example 11

A three-phase, star-connected load draws a line curent of 20 A. The load kVA and kW are 20 and 11 respectively. Find the readings on each of the two wattmeters used to measure the three-phase power.

Solution

$$I_L = 20 \text{ A}$$
$$S = 20 \text{ kVA}$$
$$P = 11 \text{ kW}$$
$$S = \sqrt{3}\ V_L I_L$$
$$20 \times 10^3 = \sqrt{3}\ V_L I_L$$
$$V_L I_L = 11.55 \text{ kVA}$$
$$P = \sqrt{3}\ V_L I_L \cos \phi$$
$$11 \times 10^3 = 20 \times 10^3 \times \cos \phi$$
$$\cos \phi = 0.55$$
$$\phi = 56.63°$$
$$W_1 = V_L I_L \cos (30° - \phi) = 11.55 \times \cos (30° - 56.63°) = 10.32 \text{ kW}$$
$$W_2 = V_L I_L \cos (30° + \phi) = 11.55 \times \cos (30° + 56.63) = 0.68 \text{ kW}$$

Example 12

Calculate the total power and readings of the two wattmeters connected to measure power in three-phase balanced load, if the reactive power is 15 kVAR and load pf is 0.8 lagging.

Solution

$$Q = 15 \text{ kVAR}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$

(i) Readings of the two wattmeters

$$\cos \phi = 0.8$$
$$\phi = 36.87°$$
$$Q = \sqrt{3}\ V_L I_L \sin \phi$$
$$15 \times 10^3 = \sqrt{3}\ V_L I_L \sin (36.87°)$$
$$V_L I_L = 14.43 \text{ kVA}$$
$$W_1 = V_L I_L \cos (30° - \phi) = 14.43 \times 10^3 \times \cos (30° - 36.87°) = 14.33 \text{ kW}$$
$$W_2 = V_L I_L \cos (30° + \phi) = 14.43 \times 10^3 \times \cos (30° + 36.87°) = 5.67 \text{ kW}$$

(ii) Total power

$$P = W_1 + W_2 = 14.03 + 5.67 = 19.7 \text{ kW}$$

Example 13

The power in a 3-ϕ circuit is measured by two wattmeters. If the total power is 50 kW and pf is 0.6 lagging, find the reading of each wattmeter. **[May 2016]**

Solution

$$W_1 + W_2 = 50 \text{ kW} \tag{1}$$
$$\text{pf} = 0.6 \text{ (lagging)}$$

$$\phi = \cos^{-1}(0.6) = 53.13°$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan(53.13°) = \sqrt{3}\,\frac{W_1 - W_2}{50}$$

$$W_1 - W_2 = 38.49 \text{ kW} \qquad (2)$$

Solving Eqs (1) and (2),

$$W_1 = 44.25 \text{ kW}$$

$$W_2 = 5.75 \text{ kW}$$

Example 14

Two wattmeters are connected to measure power in a three-phase circuit. The reading of one of the wattmeters is 5 kW when the load power factor is unity. If the power factor of the load is changed to 0.707 lagging without changing the total input power, calculate the readings of the two wattmeters.

Solution (i) pf = 1, $W_1 = 5$ kW

(ii) pf = 0.707 (lagging)

(i) When the power factor is unity,

$$W_1 = W_2 = 5 \text{ kW}$$

$$\text{Power} = W_1 + W_2 = 5 + 5 = 10 \text{ kW} \qquad (1)$$

(ii) When the power factor is changed to 0.707 lagging,

$$\text{pf} = \cos\phi = 0.707 \text{ (lagging)}$$

$$\phi = 45°$$

$$\tan\phi = \tan(45°) = 1$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$1 = \sqrt{3}\,\frac{W_1 - W_2}{10}$$

$$W_1 - W_2 = \frac{10}{\sqrt{3}} = 5.77 \text{ kW} \qquad (2)$$

Solving Eq. (1) and (2),

$$W_1 = 7.89 \text{ kW}$$

$$W_2 = 2.11 \text{ kW}$$

Example 15

Two wattmeters are connected to measure power in a three-phase circuit. The reading of one of the wattmeters is 7 kW when load power factor is unity. If the power factor of the load is changed to 0.707 lagging without changing the total input power, calculate the readings of the two wattmeters. **[Dec 2013]**

Solution $W_1 = 7$ kW

When power factor is unity,

$$W_1 = W_2 = 7 \text{ kW}$$

$$P = W_1 + W_2 = 7 + 7 = 14 \text{ kW}$$

When pf = 0.707 (lagging),

$$\phi = \cos^{-1}(0.707) = 45°$$

$$\tan\phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan 45° = \sqrt{3}\,\frac{W_1 - W_2}{14}$$

$$W_1 - W_2 = 8.08 \text{ kW} \tag{1}$$

$$W_1 + W_2 = 14 \text{ kW} \tag{2}$$

Solving Eqs (1) and (2),

$$W_1 = 11.04 \text{ kW}$$

$$W_2 = 2.96 \text{ kW}$$

Example 16

A three-phase RYB system has effective line voltage of 173.2 V. Wattmeters in line R and Y read 301 W and 1327 W respectively. Find the impedance of the balanced star-connected load.

Solution $V_L = 173.2$ V

$$W_1 = 301 \text{ W}$$

$$W_2 = 1327 \text{ W}$$

If the load is capacitive and pf is leading, then $W_1 < W_2$.

$$\tan\phi = -\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = -\sqrt{3}\frac{(301-1327)}{(301+1327)} = -1.09$$

$$\phi = 47.47°$$

$$P = W_1 + W_2 = 301 + 1327 = 1628 \text{ W}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

$$1628 = \sqrt{3} \times 173.2 \times I_L \times \cos(47.47°)$$

$$I_L = 8.03 \text{ A}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{173.2}{\sqrt{3}} = 100 \text{ V}$$

$$I_{ph} = I_L = 8.03 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{100}{8.03} = 12.45\ \Omega$$

Example 17

Each phase of a 3ϕ delta connected load has an impedance of $\bar{Z}_{ph} = 50\angle 60°\Omega$. The line voltage is 400 V. Calculate the total power. What will be the reading of two wattmeters connected to measure the power. **[Dec 2015]**

Solution

$$\bar{Z}_{ph} = 50\angle 60°\Omega$$

$$V_L = 400 \text{ V}$$

For a delta-connected load,

$$V_{ph} = V_L = 400 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{50} = 8 \text{ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 8 = 13.86 \text{ A}$$

(i) Total power

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 13.86 \times \cos(60°) = 4.8 \text{ kW}$$

(ii) Reading of two wattmeters

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 13.86 \times \cos(30° - 60°) = 4.8 \text{ kW}$$

$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 13.86 \times \cos(30° + 60°) = 0$$

Example 18

Three coils each with a resistance of 10 Ω and reactance of 10 Ω are connected in star across a three-phase, 50 Hz, 400 V supply. Calculate (i) line current, and (ii) readings on the two wattmeters connected to measure the power.

Solution

$$R = 10\ \Omega$$

$$X_L = 10\ \Omega$$

$$V_L = 400 \text{ V}$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V}$$

$$\bar{Z}_{ph} = R + jX_L = 10 + j10 = 14.14\angle 45^\circ\ \Omega$$

$$Z_{ph} = 14.14\ \Omega$$

$$\phi = 45^\circ$$

$$\text{Power factor} = \cos\phi = \cos(45^\circ) = 0.707\ \text{(lagging)}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{14.14} = 16.33\text{ A}$$

$$I_L = I_{ph} = 16.33\text{ A}$$

(ii) Readings on the two wattmeters

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 16.33 \times 0.707 = 7998.83\text{ W}$$

$$W_1 + W_2 = 7998.83\text{ W} \tag{1}$$

Also,

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan 45^\circ = \sqrt{3}\,\frac{W_1 - W_2}{7998.83}$$

$$W_1 - W_2 = 4618.13\text{ W} \tag{2}$$

Solving Eqs (1) and (2),

$$W_1 = 6308.48\text{ W}$$

$$W_2 = 1690.35\text{ W}$$

Example 19

Three coils each having a resistance of 20 Ω and reactance of 15 Ω are connected in (i) star, and (ii) delta, across a three-phase, 400 V, 50 Hz supply. Calculate in each case, the readings on two wattmeters connected to measure the power input.

Solution

$$R = 20\ \Omega$$

$$X_L = 15\ \Omega$$

$$V_L = 400\text{ V}$$

(i) Readings on two wattmeters for a star-connected load

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V}$$

$$\bar{Z}_{ph} = 20 + j15 = 25 \angle 36.87° \ \Omega$$
$$Z_{ph} = 25 \ \Omega$$
$$\phi = 36.87°$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{25} = 9.24 \text{ A}$$
$$I_L = I_{ph} = 9.24 \text{ A}$$
$$W_1 = V_L I_L \cos (30° - \phi) = 400 \times 9.24 \times \cos (30° - 36.87°) = 3669.46 \text{ W}$$
$$W_2 = V_L I_L \cos (30° + \phi) = 400 \times 9.24 \times \cos (30° + 36.87°) = 1451.86 \text{ W}$$

(ii) Readings on two wattmeters for a delta-connected load

$$V_{ph} = V_L = 400 \text{ V}$$
$$Z_{ph} = 25 \ \Omega$$
$$\phi = 36.87°$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16 \text{ A}$$
$$I_L = \sqrt{3} \ I_{ph} = \sqrt{3} \times 16 = 27.72 \text{ A}$$
$$W_1 = V_L I_L \cos (30° - \phi) = 400 \times 27.72 \times \cos (30° - 36.87°) = 11008.39 \text{ W}$$
$$W_2 = V_L I_L \cos (30° + \phi) = 400 \times 27.72 \times \cos (30° + 36.87°) = 4355.57 \text{ W}$$

Example 20

Two wattmeters connected to measure three-phase power for star-connected load reads 3 kW and 1 kW. The line current is 10 A. Calculate (i) line and phase voltage (ii) resistance and reactance per phase.

Solution

$$W_1 = 3 \text{ kW}$$
$$W_2 = 1 \text{ kW}$$
$$I_L = 10 \text{ A}$$

(i) Line and phase voltage

$$\tan \phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(3-1)}{(3+1)} = 0.866$$
$$\phi = 40.89°$$

Power factor = $\cos \phi = \cos (40.89°) = 0.756$

$$P = W_1 + W_2 = 3 + 1 = 4 \text{ kW}$$
$$P = \sqrt{3} \ V_L I_L \cos \phi$$
$$4 \times 10^3 = \sqrt{3} \times V_L \times 10 \times 0.756$$
$$V_L = 305.48 \text{ V}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{305.48}{\sqrt{3}} = 176.37 \text{ V}$$

(ii) Resistance and reactance per phase

$$I_{ph} = I_L = 10 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{176.37}{10} = 17.637 \ \Omega$$

$$R = Z_{ph} \cos \phi = 17.637 \times 0.756 = 13.33 \ \Omega$$

$$X_L = Z_{ph} \sin \phi = 17.637 \times \sin (40.89°) = 11.55 \ \Omega$$

Example 21

The power input to a 2000 V, 50 Hz, three-phase motor running on full load at an efficiency of 90% is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate the (i) input power, (ii) power factor, and (iii) line current.

Solution

$$V_L = 2000 \text{ V}$$

$$\eta = 0.9$$

$$W_1 = 300 \text{ kW}$$

$$W_2 = 100 \text{ kW}$$

(i) Input power

$$P = W_1 + W_2 = 300 + 100 = 400 \text{ kW}$$

(ii) Power factor

$$\tan \phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\frac{(300-100)}{(300+100)} = 0.866$$

$$\phi = 40.89°$$

$$\text{pf} = \cos \phi = \cos (40.89°) = 0.76 \text{ (lagging)}$$

(iii) Line current

$$P = \sqrt{3} \ V_L I_L \cos \phi$$

$$400 \times 10^3 = \sqrt{3} \times 2000 \times I_L \times 0.76$$

$$I_L = 151.93 \text{ A}$$

Example 22

A three-phase, 400 V, 50 Hz induction motor has a full load output of 14.9 kW at which the efficiency and power factor are 0.88 and 0.8 respectively. What is the line current? Find the readings on the two wattmeters connected to measure the power input to the motor.

Solution

$$V_L = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$
$$P_o = 14.9 \text{ kW}$$
$$\text{pf} = 0.8$$
$$\eta = 0.88$$

(i) Line current

$$\eta = \frac{P_o}{P_i}$$
$$0.88 = \frac{14.9 \times 10^3}{P_i}$$
$$P_i = 16.93 \text{ kW}$$
$$\text{pf} = \cos\phi = 0.8$$
$$\phi = 36.87°$$
$$P_i = \sqrt{3}\; V_L I_L \cos\phi$$
$$16.93 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.8$$
$$I_L = 30.55 \text{ A}$$

(ii) Readings on the two wattmeters

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 30.55 \times \cos(30° - 36.87°) = 12.13 \text{ kW}$$
$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 30.55 \times \cos(30° + 36.87°) = 4.8 \text{ kW}$$

Example 23

A three-phase, 220 V, 50 Hz, 11.2 kW induction motor has a full load efficiency of 88 per cent and draws a line current of 38 A under full load, when connected to a three-phase, 220 V supply. Determine power factor at which the motor is operating. Find the reading on two wattmeters connected in the circuit to measure the input to the motor.

Solution

$$V_L = 220 \text{ V}$$
$$P_o = 11.2 \text{ kW}$$
$$\eta = 88\%$$
$$I_L = 38 \text{ A}$$

(i) Power factor at which the motor is operating

$$\eta = \frac{P_o}{P_i}$$
$$0.88 = \frac{11.2 \times 10^3}{P_i}$$
$$P_i = 12.73 \text{ kW}$$

But
$$P_i = \sqrt{3}\ V_L I_L \cos\phi$$
$$12.73 \times 10^3 = \sqrt{3} \times 220 \times 38 \times \cos\phi$$
$$\text{pf} = \cos\phi = 0.88 \text{ (lagging)}$$

(ii) Reading on two wattmeters

$$\phi = 28.36°$$
$$W_1 = V_L I_L \cos(30° - \phi) = 220 \times 38 \times \cos(30° - 28.36°) = 8356.58 \text{ W}$$
$$W_2 = V_L I_L \cos(30° + \phi) = 220 \times 38 \times \cos(30° + 28.36°) = 4385.49 \text{ W}$$

Useful Formulae

Two-wattmeter method

For lagging power factor

$$W_1 = V_L I_L \cos(30° - \phi)$$
$$W_2 = V_L I_L \cos(30° + \phi)$$
$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

For leading power factor

$$W_1 = V_L I_L \cos(30° + \phi)$$
$$W_2 = V_L I_L \cos(30° - \phi)$$
$$\phi = \tan^{-1}\left[-\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

Exercise 5.2

5.1 Three identical coils each having a resistance of 8 Ω and inductance of 0.02 H are connected in (i) star, and (ii) delta across a 3ϕ, 400 V, 50 Hz supply. Draw a neat phasor diagram and calculate the reading of two wattmeters connected to measure power. Also, calculate pf of the circuit.
[8.99 kW, 3.3 kW, 0.7866 (lagging), 26.98 kW, 10.14 kW, 0.786 (lagging)]

5.2 Three identical coils each having a reactance of 20 Ω and resistance of 10 Ω are connected in (i) star, (ii) delta across a 440 V, 3-phase line. Calculate for each method of connection the line current and readings of each of the two wattmeters connected. *[11.36 A, 4.17 kW, –299.07 W, 34.08 A, 12.52 kW, –897.14 W]*

5.3 A 3-phase motor load has a pf of 0.397 lagging. Two wattmeters connected to measure power show the input as 30 kW. Find the reading on each wattmeter.
[35 kW, –5 kW]

5.4 Each of the wattmeters connected to measure the input to a 3-phase induction motor reads 10 kW. If the power factor of the motor be changed to 0.866 lagging, determine the readings of the two wattmeters, the total input power remaining unchanged. *[6.67 kW, 13.33 kW]*

5.5 A 3-ϕ, star-connected load draws a line current of 25 A. The load kVA and kW are 20 and 16 respectively. Find the readings on each of the two wattmeters used to measure the 3ϕ power. *[11.46 kW, 4.54 kW]*

5.6 Three similar coils are star-connected to a 3ϕ 50 Hz supply. The line current taken is 25 A and the two wattmeters connected to measure the input indicate 5.185 kW and 10.37 kW respectively. Calculate (i) the line and phase voltages, and (ii) the resistance and reactance of each coil. *[415 V, 240 V, 5.31 Ω, 4.8 Ω]*

5.7 A three-phase, 500 V motor load has a power factor of 0.4. Two wattmeters connected to measure power show the input to be 30 kW. Find the reading on each instrument. *[35 kW, – 5kW]*

5.8 The power in a three-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter? *[17.26 kW, 82.74 kW]*

5.9 Two wattmeters are connected to measure the input to a 400 V, 3-phase connected motor outputting 24.4 kW at a power factor of 0.4 lag and 80% efficiency. Calculate (i) resistance and reactance of motor per phase, (ii) reading of each wattmeter. *[2.55 Ω, 5.58 Ω, 34915W, – 4850 W]*

5.10 In a balanced 3-phase, 400 V circuit, the line current is 115.5 A. When power is measured by the two wattmeter method, one meter reads 40 kW and the other, zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter? *[0.5, 40 kW, 40 kW]*

5.11 A 440 V, 3-phase, delta-connected induction motor has an output of 14.92 kW at pf of 0.82 and efficiency of 85%. Calculate the readings on each of the two wattmeters connected to measure the input. If another star-connected load of 10 kW at 0.85 pf lagging is added in parallel to the motor, what will be the current drawn from the line and the power taken from the line? *[12.35 kW, 5.26 kW, 43.56 A, 27.6 kW]*

5.12 Balanced delta-connected impedances, each of $10 \angle 30°\ \Omega$ are connected across three-phase 400 V mains. Determine the two-wattmeter readings if the current coils of the two wattmeters are connected in lines *R* and *Y* and the pressure coils are connected between *R* and *B* and *Y* and *B* lines respectively. *[27.7 kW, 13.86 kW]*

5.13 A balanced star-connected load, each phase having a resistance of 10 Ω and the inductive reactance of 30 Ω is connected to 400 V, 50 Hz supply. The phase rotation is *R*, *Y*, and *B*. Wattmeters connected to read total power have their current coils in the red and blue lines respectively. Calculate the reading on each wattmeter. *[2190 W, – 583W]*

5.14 A balanced star-connected load is supplied from a symmetrical three-phase 400 V, 50 Hz supply system. The current in each phase is 20 A and lags behind its phase voltage by an angle of $2\pi/9$ radians. Calculate line voltage, phase voltage, current in each phase, load parameter, power in each phase, total power, readings of the wattmeters connected in the load circuit to measure the total power. Draw a neat circuit diagram and vector/phasor diagram.

[440 V, 254.034 V, 20 A, 9.73 Ω, 0.026 H, 3.892 kW, 11.676 kW, 8.666 kW, 3.009 kW]

Review Questions

5.1 Explain the following terms with reference to a polyphase system: (i) Balanced load (ii) Phase sequence (iii) Symmetrical system.

5.2 What are the advantages of a three-phase system over a single-phase system?

5.3 Prove that current in a neutral wire in a three-phase, four-wire balanced load system is zero.

5.4 Derive the relationship between phase and line quantities (voltage, current, power) for a balanced three-phase, delta-connected system. Also draw neat diagrams.

5.5 Deduce the relationship between phase and line quantities (voltage, current, power) for a balanced three-phase, star-connected system. Also draw neat diagrams.

5.6 Derive the relation between power in star and delta systems.

5.7 Explain merits of two-wattmeter method for power measurement, giving circuit diagram and phasor diagram.

5.8 Explain with phasor diagram of how two wattmeters can be used to measure power in a 3-phase system. Also explain the variations in the wattmeter readings with load power factors.

5.9 Derive the relation for total power and power factor in a 3-phase system with balanced load using two wattmeter method.

5.10 How do you measure power of a 3-phase balanced network by using a wattmeter with least number of wattmeters.

5.11 Explain the effect of power factor on wattmeter readings in three-phase power measurement by two-wattmeter method.

Multiple Choice Questions

Choose the correct alternative in the following questions:

5.1 In a three-phase system, voltages differ in phase by

(a) 30° (b) 60° (c) 90° (d) 120°

5.2 The rated voltage of a 3-phase power system is given as
(a) rms phase voltage (b) peak phase voltage
(c) rms line to line voltage (d) peak line to line voltage

5.3 Total instantaneous power supplied by a 3-phase ac supply to a balanced *R–C* load is
(a) zero (b) constant
(c) pulsating with zero average (d) pulsating with non-zero average

5.4 Which of the following equations is valid for a 3-phase, balanced star-connected system?
(a) $I_R + I_Y + I_B = 0$ (b) $I_R + I_B = I_Y$
(c) $I_R + I_Y - I_B = 0$ (d) $\dfrac{V_R + V_B + V_Y}{Z} = I_N$

5.5 A three-phase load is balanced if all the three phases have the same
(a) impedance (b) power factor
(c) impedance and power factor (d) none of the above

5.6 The phase sequence of a three-phase system system shown in Fig. 5.33 is

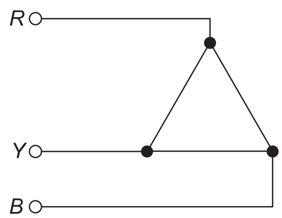


Fig. 5.33

(a) *RYB* (b) *RBY* (c) *BRY* (d) *YBR*

5.7 In a 3-phase system, $\overline{V}_{YN} = 100 \angle{-120°}$ V and $\overline{V}_{BN} = 100 \angle 120°$ V. Then $\overline{V}_{YB}$ will be
(a) $170 \angle 90°$ V (b) $173 \angle{-90°}$ V
(c) $200 \angle 60°$ V (d) none of the above

5.8 If a balanced delta load has an impedance of $(6 + j9)$ ohms per phase then the impedance of each phase in the equivalent star load is
(a) $(6 + j9)$ ohms (b) $(2 + j3)$ ohms
(c) $(2 + j8)$ ohms (d) $(3 + j4.5)$ ohms

5.9 Three equal impedances are first connected in delta across a 3-phase balanced supply. If the same impedances are connected in star across the same supply
(a) phase current will be one-third
(b) line current will be one-third
(c) power consumed will be one-third
(d) none of the above

5.10 Three identical resistors connected in star carry a line current of 12 A. If the same resistors are connected in delta across the same supply, the line current will be
(a) 12 A (b) 4 A (c) 8 A (d) 36 A

5.11 Three delta-connected resistors absorb 60 kW when connected to a 3-phase line. If the resistors are connected in star, the power absorbed is

(a) 60 kW (b) 20 kW (c) 40 kW (d) 180 kW

5.12 The power consumed in the star connected load shown in Fig. 5.34 is 690 W. The line current is

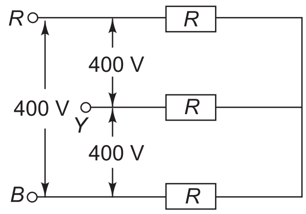


Fig. 5.34

(a) 2.5 A (b) 1 A (c) 1.725 A (d) none of the above

5.13 If the 3-phase balanced source in Fig. 5.35 delivers 1500 W at a leading power factor of 0.844 then the value of Z_L (in ohm) is approximately

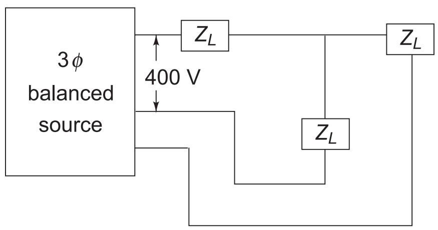


Fig. 5.35

(a) 90 ∠32.44° (b) 80 ∠32.44° (c) 80 ∠–32.44° (d) 90 ∠–32.44°

5.14 If one of the resistors in Fig. 5.36 is open circuited, power consumed in the circuit is

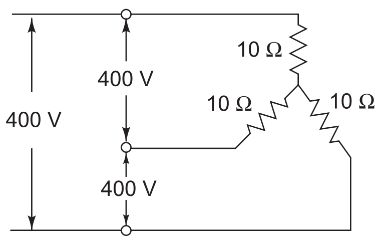


Fig. 5.36

(a) 8000 W (b) 4000 W (c) 16000 W (d) none of the above

5.15 For the three-phase circuit shown in Fig. 5.37, the ratio of the currents $I_R : I_Y : I_B$ is given by

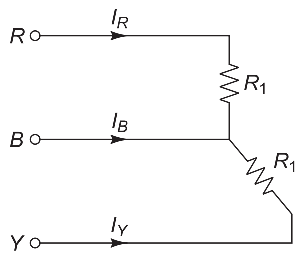


Fig. 5.37

(a) $1 : 1 : \sqrt{3}$ (b) 1 : 1 : 2 (c) 1 : 1 : 0 (d) $1 : 1 : \sqrt{\frac{3}{2}}$

5.16 The phase sequence *RYB* denotes that
(a) emf of phase *Y* lags behind that of phase *R* by 120°
(b) emf of phase *Y* leads that of phase *R* by 120°
(c) emf of phase *Y* and phase *R* are in phase
(d) none of the above

5.17 In the two wattmeter method of measurement, if one of the wattmeters reads zero, then power factor will be
(a) zero (b) unity (c) 0.5 (d) 0.866

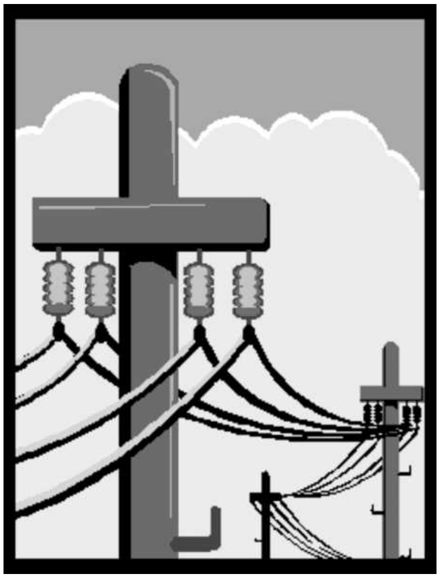
5.18 Two wattmeters, which are connected to measure the total power on a three-phase system, supplying a balanced load, read 10.5 kW and –2.5 kW, respectively. The total power and the power factor, respectively are
(a) 13 kW, 0.334 (b) 13 kW, 0.684
(c) 8 kW, 0.52 (d) 8 kW, 0.334

5.19 The minimum number of wattmeter(s) required to measure 3-phase, 3-wire balanced or unbalanced power is
(a) 1 (b) 2 (c) 3 (c) 4

5.20 One of the two wattmeters has read zero in the two-wattmeter method of power measurement. This indicated that the load phase angle is
(a) 0° (b) 30° (c) 60° (d) 90°

Answers to Multiple Choice Questions

5.1 (d)	**5.2** (c)	**5.3** (b)	**5.4** (a)	**5.5** (c)	**5.6** (b)
5.7 (b)	**5.8** (b)	**5.9** (c)	**5.10** (d)	**5.11** (b)	**5.12** (b)
5.13 (d)	**5.14** (a)	**5.15** (a)	**5.16** (a)	**5.17** (c)	**5.18** (d)
5.19 (b)	**5.20** (c)				



Chapter 6

Single-Phase Transformers

Chapter Outline

6.1 SINGLE-PHASE TRANSFORMERS

A transformer is a static device which can transfer electrical energy from one circuit to another circuit without change of frequency. It can increase or decrease the voltage but with a corresponding decrease or increase in current. It works on the principle of mutual induction. It must be used with an input voltage that varies in amplitude, i.e., an ac voltage. A major application of transformers is to increase voltage before transmitting electrical energy over long distances through wires and to reduce voltage at places where it is to be used. Transformers are also used in electronic circuits to step down the supply voltage to a level suitable for the low-voltage circuits they contain. Signal and audio transformers are used to couple stages of amplifiers and to match devices such as microphones to the input of amplifiers.

6.2 CONSTRUCTION

A transformer mainly consists of two coils or windings placed on a common core. With the increase in size (capacity) and operating voltage, it also needs other parts such as a suitable tank, bushing, conservator, breather, etc. We will discuss two basic parts—core and windings.

6.2.1 Core

The composition of a transformer core depends on voltage, current and frequency. Commonly used core materials are soft iron and steel. Generally, air-core transformers are used when the voltage source has a high frequency (above 20 kHz). Iron-core transformers are usually used when the source frequency is low (below 20 kHz). In most transformers, the core is constructed of laminated steel to provide a continuous magnetic path. The steel used for constructing the core is high-grade silicon steel where hysteresis loss is very low. Such steel is called soft steel. Due to alternating flux, certain currents are induced in the core, called as eddy currents. These currents cause considerable loss in the core, called eddy current loss. Silicon content in the steel increases its resistivity to eddy-current loss, thereby reducing eddy-current losses. To reduce eddy-current losses further, the core is laminated by a light coat of varnish or by an oxide layer on the surface. There are two main shapes of cores used in laminated steel-core transformers as shown in Fig 6.1 and Fig 6.2.

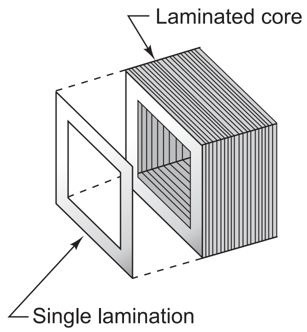


Fig. 6.1 *Hollow-core construction*

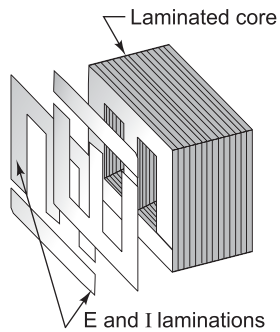


Fig. 6.2 *Shell-type core construction*

6.2.2 Transformer Windings

A transformer consists of two coils, called windings, which are wrapped around a core. The winding in which electrical energy is fed is called the *primary winding*. The winding which is connected to the load is called the *secondary winding*.

The primary and secondary windings are made up of an insulated copper conductor in the form of a round wire or strip. These windings are then placed around the limbs of the core. The windings are insulated from each other and the core, using cylinders of insulating material such as a press board or Bakelite.

For simplicity, the primary and secondary windings are shown on separate limbs of the core. If such an arrangement is used in actual practice, all the flux produced in the primary winding will not link with the secondary winding. Some of the flux will leak out through the air. Such flux is known as *leakage flux*. The more the value of leakage flux, poorer is the performance of the transformer. Hence, to reduce leakage flux, the windings are placed together on the same limb in actual transformers.

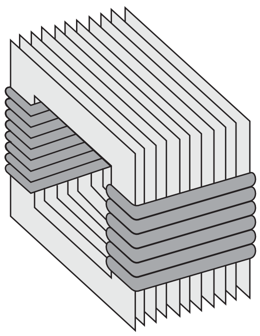


Fig. 6.3 *Core-type transformer*

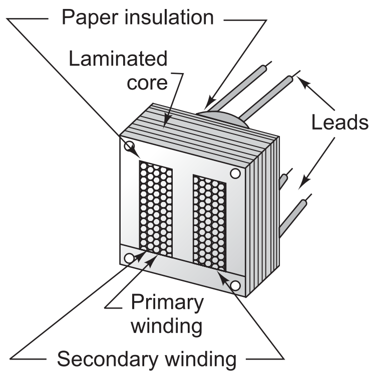


Fig. 6.4 *Shell-type transformer*

6.2.3 Comparison of Core-type and Shell-type Transformers

Core-type Transformer	*Shell-type Transformer*
1. It consists of a magnetic frame with two limbs.	It consists of a magnetic frame with three limbs.
2. It has a single magnetic circuit.	It has two magnetic circuits.
3. The winding encircles the core.	The core encircles most part of the winding.
4. It consists of cylindrical windings.	It consists of sandwich-type windings.
5. It is easy to repair.	It is not easy to repair.
6. It provides better cooling since windings are uniformly distributed on two limbs.	It does not provide effective cooling as the windings are surrounded by the core.
7. It is preferred for low-voltage transformers.	It is preferred for high-voltage transformers.

6.3 WORKING PRINCIPLE

[May 2013, Dec 2014]

When an alternating voltage V_1 is applied to a primary winding, an alternating current I_1 flows in it producing an alternating flux in the core. As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

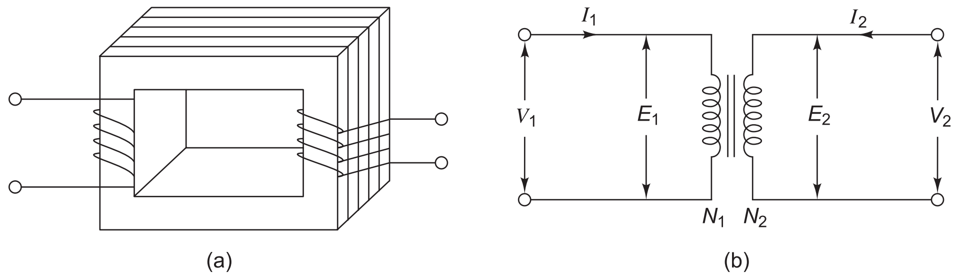


Fig. 6.5 *Working principle of a transformer*

$$e_1 = -N_1 \frac{d\phi}{dt}$$

where N_1 is the number of turns in the primary winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage V_1.

Assuming leakage flux to be negligible, almost the whole flux produced in primary winding links with the secondary winding. Hence, an emf e_2 is induced in the secondary winding.

$$e_2 = -N_2 \frac{d\phi}{dt}$$

where N_2 is the number of turns in the secondary winding. If the secondary circuit is closed through the load, a current I_2 flows in the secondary winding. Thus, energy is transferred from the primary winding to the secondary winding. The symbol of transformer is shown in Fig. 6.5(b). The lines between two windings represent iron core. If there is no line between two windings, then it represents air core transformer. If number of turns in the secondary winding N_2 is greater than the number of turns in the primary winding N_1, the transformer is called a step-up transformer. If N_2 is less than N_1, the transformer is called a step-down transformer. Thus, step-up transformer is used to increase the voltage at the output whereas step-down transformer is used to decrease the voltage at the output.

6.4 EMF EQUATION

[Dec 2012, 2014, May 2013, 2015]

As the primary winding is excited by a sinusoidal alternating voltage, an alternating current flows in the winding producing a sinusoidally varying flux ϕ in the core.

$$\phi = \phi_m \sin \omega t$$

As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

$$
\begin{aligned}
e_1 &= -N_1 \frac{d\phi}{dt} \\
&= -N_1 \frac{d}{dt}(\phi_m \sin \omega t) \\
&= -N_1\, \phi_m\, \omega \cos \omega t \\
&= N_1\, \phi_m\, \omega \sin(\omega t - 90°) \\
&= 2\pi f\, \phi_m\, N_1 \sin(\omega t - 90°)
\end{aligned}
$$

Maximum value of induced emf = $2\pi f\, \phi_m\, N_1$

Hence, rms value of induced emf in primary winding is given by

$$E_1 = \frac{E_{\max}}{\sqrt{2}} = \frac{2\pi f\, \phi_m\, N_1}{\sqrt{2}} = 4.44 f\, \phi_m\, N_1$$

Similarly, rms value of induced emf in the secondary winding is given by

$$E_2 = 4.44\, f\, \phi_m\, N_2$$

Also,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f\, \phi_m$$

Thus, emf per turn is same in primary and secondary windings and an equal emf is induced in each turn of the primary and secondary windings.

6.5 TRANSFORMATION RATIO (*K*)

We know that

$$
\begin{aligned}
E_1 &= 4.44\, f\, \phi_m\, N_1 \\
E_2 &= 4.44\, f\, \phi_m\, N_2 \\
\frac{E_2}{E_1} &= \frac{N_2}{N_1} = K
\end{aligned}
$$

where K is called the *transformation ratio*.

Neglecting small primary and secondary voltage drops,

$$
\begin{aligned}
V_1 &\approx E_1 \\
V_2 &\approx E_2 \\
\frac{E_2}{E_1} &= \frac{V_2}{V_1} = \frac{N_2}{N_1} = K
\end{aligned}
$$

In a transformer, losses are negligible. Hence, input and output can be approximately equated.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K$$

For step-up transformers,

$$N_2 > N_1 \qquad\qquad K > 1$$

For step-down transformers,

$$N_2 < N_1 \qquad\qquad K < 1$$

6.6 RATING OF A TRANSFORMER

[May 2014]

Rating of a transformer indicates the output power from it. But for a transformer, load is not fixed and its power factor goes on changing. Hence, rating is not expressed in terms of power but in terms of product of voltage and current, called VA rating. This rating is generally expressed in kVA.

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

We can calculate full-load currents of primary and secondary windings from kVA rating of a transformer. *Full-load current* is the maximum current which can flow through the winding without damaging it.

$$\text{Full-load primary current} \qquad I_1 = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$\text{Full-load secondary current} \qquad I_2 = \frac{\text{kVA rating} \times 1000}{V_2}$$

Example 1

What will be the secondary voltage at no load, if the primary of a 5 kVA, 220/110 V, 50 Hz transformer is fed at (i) 110 V, 50 Hz, and (ii) 220 V dc?

Solution kVA rating = 5 kVA

$$E_1 = 220 \text{ V}$$

$$E_2 = 110 \text{ V}$$

(i) Secondary voltage when $V_1 = 110$ V

For a transformer,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1}$$

$$\frac{V_2}{110} = \frac{110}{220}$$

$$V_2 = 55 \text{ V}$$

(ii) Secondary voltage when $V_1 = 220$ V dc

When the transformer is fed 220 V dc, no emf is induced in the primary winding.

$$V_2 = 0$$

Example 2

It is desired to have 4.13 mWb maximum flux in the core of a transformer operating at 110 V and 50 Hz. Determine the required number of turns in the primary.

Solution

$$\phi_m = 4.13 \text{ mWb}$$
$$V_1 = 110 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a transformer,

$$V_1 \approx E_1 = 110 \text{ V}$$
$$E_1 = 4.44\, f \phi_m N_1$$
$$110 = 4.44 \times 50 \times 4.13 \times 10^{-3} \times N_1$$
$$N_1 = 120$$

Example 3

A 3000/200 V, 50 Hz, single-phase transformer has a cross-sectional area of 150 cm² for the core. If the number of turns on the low-voltage winding is 80, determine the number of turns on the high-voltage winding and maximum value of flux density in the core. **[May 2013]**

Solution

$$E_1 = 3000 \text{ V}$$
$$E_2 = 200 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$A = 150 \text{ cm}^2 = 150 \times 10^{-4} \text{ m}^2$$
$$N_2 = 80$$

(i) Number of turns on the high-voltage winding

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{200}{3000} = \frac{80}{N_1}$$
$$N_1 = 1200$$

(ii) Maximum value of flux density

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$

$$3000 = 4.44 \times 50 \times B_m \times 150 \times 10^{-4} \times 1200$$
$$B_m = 0.75 \text{ Wb/m}^2$$

Example 4

A single-phase 50 Hz transformer has 80 turns on the primary winding and 280 turns in the secondary winding. The voltage applied across the primary winding is 240 V at 50 Hz. Calculate (i) maximum flux density in the core, and (ii) induced emf in the secondary. The net cross-sectional area of the core is 200 cm^2.

Solution

$$f = 50 \text{ Hz}$$
$$N_1 = 80$$
$$N_2 = 280$$
$$V_1 = 240 \text{ V}$$
$$A = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$$

(i) Maximum flux density in the core

For a transformer,

$$V_1 \approx E_1 = 240 \text{ V}$$
$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$
$$240 = 4.44 \times 50 \times B_m \times 200 \times 10^{-4} \times 80$$
$$B_m = 0.68 \text{ Wb/m}^2$$

(ii) Induced emf in the secondary

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{E_2}{240} = \frac{280}{80}$$
$$E_2 = 840 \text{ V}$$

Example 5

An 80 kVA, 3200/400 V, 50 Hz single-phase transformer has 111 turns on the secondary winding. Calculate (i) number of turns on primary winding, (ii) secondary current, and (iii) cross-sectional area of the core, if the maximum flux density is 1.2 teslas. **[Dec 2015]**

Solution

$$\text{kVA rating} = 80 \text{ kVA}$$
$$E_1 = 3200 \text{ V}$$
$$E_2 = 400 \text{ V}$$
$$f = 50 \text{ Hz}$$

$$N_2 = 111$$
$$B_m = 1.2 \text{ T}$$

(i) Number of turns of primary winding

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{400}{3200} = \frac{111}{N_1}$$

$$N_1 = 888$$

(ii) Secondary current

For a transformer,

$$V_2 \approx E_2 = 400 \text{ V}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{80 \times 1000}{400} = 200 \text{ A}$$

(iii) Cross-sectional area of the core

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$
$$400 = 4.44 \times 50 \times 1.2 \times A \times 111$$
$$A = 0.0135 \text{ m}^2 = 135 \text{ cm}^2$$

Example 6

A 5 kVA, 240/2400 V, 50 Hz single-phase transformer has the maximum value of flux density as 1.2 Teslas. If the emf per turn is 8, calculate (i) number of primary turns and secondary turns, (ii) cross-sectional area of the core, and (iii) primary and secondary current at full load.

[Dec 2014]

Solution kVA rating = 5 kVA

$$E_1 = 240 \text{ V}$$
$$E_2 = 2400 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$B_m = 1.2 \text{ T}$$
$$\frac{E_1}{N_1} = 8$$

(i) Number of primary and secondary turns

$$\frac{E_1}{N_1} = 8 = \frac{240}{N_1}$$

$$N_1 = 30$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{2400}{240} = \frac{N_2}{30}$$

$$N_2 = 300$$

(ii) Cross-sectional area of the core

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$

$$2400 = 4.44 \times 50 \times 1.2 \times A \times 300$$

$$A = 0.03 \text{ m}^2$$

(iii) Primary and secondary currents at full load

For a transformer,

$$V_1 \approx E_1 = 240 \text{ V}$$

$$V_2 \approx E_2 = 2400 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{5 \times 1000}{240} = 20.83 \text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{5 \times 1000}{2400} = 2.08 \text{ A}$$

Example 7

A 250 kVA, 50 Hz single-phase transformer has ratio of secondary to primary turns as 0.1. The secondary voltage at no-load condition is 240 V. Calculate (i) primary voltage, and (ii) full-load primary and secondary currents.

Solution kVA rating = 250 kVA

$$\frac{N_2}{N_1} = 0.1$$

$$E_2 = 240 \text{ V}$$

(i) Primary voltage

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = 0.1$$

$$E_1 = 2400 \text{ V}$$

(ii) Full-load primary and secondary current

For a transformer,

$$V_1 \approx E_1 = 2400 \text{ V}$$

$$V_2 \approx E_2 = 240 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{250 \times 1000}{2400} = 104.17 \text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{250 \times 1000}{240} = 1041.67 \text{ A}$$

Example 8

A 10 kVA, 3300/240 V, single-phase, 50 Hz transformer has a core area of 300 cm². The flux density is 1.3 T. Calculate (i) number of primary turns, (ii) number of secondary turns, and (iii) primary full-load current.

Solution

$$\text{kVA rating} = 10 \text{ kVA}$$
$$E_1 = 3300 \text{ V}$$
$$E_2 = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$A = 300 \text{ cm}^2 = 300 \times 10^{-4} \text{ m}^2$$
$$B_m = 1.3 \text{ T}$$

(i) Number of primary turns

$$E_1 = 4.44\, f\, \phi_m\, N_1 = 4.44\, f\, B_m\, A\, N_1$$
$$3300 = 4.44 \times 50 \times 1.3 \times 300 \times 10^{-4} \times N_1$$
$$N_1 = 381.15 \simeq 382$$

(ii) Number of secondary turns

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{240}{3300} = \frac{N_2}{382}$$

$$N_2 = 27.78 \simeq 28$$

(iii) Primary full-load current

For a transformer,

$$V_1 \approx E_1 = 3300 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{10 \times 1000}{3300} = 3.03 \text{ A}$$

Example 9

A 3300/250 V, 50 Hz, single-phase transformer has 125 cm² cross-sectional area of core and 70 turns on low-voltage side. Calculate (i) the value of maximum flux density, and (ii) number of turns on the high-voltage side.

Solution

$$E_1 = 3300 \text{ V}$$
$$E_2 = 250 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$A = 125 \text{ cm}^2 = 125 \times 10^{-4} \text{ m}^2$$
$$N_2 = 70$$

(i) Value of maximum flux density

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$
$$250 = 4.44 \times 50 \times B_m \times 125 \times 10^{-4} \times 70$$
$$B_m = 1.29 \text{ Wb/m}^2$$

(ii) Number of turns on high-voltage side

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{250}{3300} = \frac{70}{N_1}$$
$$N_1 = 924$$

Useful Formulae

$$E_1 = 4.44\, f\, \phi_m\, N_1$$
$$E_2 = 4.44\, f\, \phi_m\, N_2$$
$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$
$$\text{kVA rating} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$
$$I_1 \text{ (full load)} = \frac{\text{kVA rating} \times 1000}{V_1}$$
$$I_2 \text{ (full load)} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Exercise 6.1

6.1 The required no-load ratio in a single-phase, 50 Hz, core-type transformer is 6000/250 V. Find the number of turns in each winding if the flux is to be about 0.06 Wb. *[450, 20]*

6.2 A 6600/600 V, 50 Hz, 1 ϕ transformer has a maximum flux density of 1.35 Wb/m^2 in its core. If the net cross-sectional area of iron in the core is 200 cm^2, calculate the number of turns in the primary and secondary windings of the transformer.

[1101, 100]

6.3 A 1 ϕ, 50 Hz transformer has 500 turns on the primary and 1000 turns on the secondary. The voltage per turn in the primary winding is 0.2 volt. Calculate (i) voltage induced in primary and secondary windings, (ii) maximum value of flux density if the cross-sectional area of the core is 200 cm^2, (iii) kVA rating of the transformer if the current in primary at full load is 10 A.

[100 V, 200 V, 9.09 × 10^{-4} Wb, 0.045 Wb/m^2, 1 kVA]

6.4 A 40 kVA, 3300/240 V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine (i) the number of turns on the secondary, (ii) the maximum value of flux in the core, and (iii) the approximate value of primary and secondary full-load current. Internal drops in the windings are to be ignored.

[48, 0.02 Wb, 12.12 A, 166.67 A]

6.5 A single-phase transformer has 350 primary and 1050 secondary turns. The net cross-sectional area of the core is 55 cm^2. If the primary winding be connected to a 400 V, 50 Hz single-phase supply, calculate (i) maximum value of flux density in the core, and (ii) the voltage induced in the secondary winding.

[0.93 Wb/m^2, 1200 V]

6.6 A 25 kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to a 3000 V, 50 Hz supply. Find the full-load primary and secondary currents, the secondary emf and the maximum flux in the core. *[8.33 A, 83.3 A, 300 V, 27 mWb]*

6.7 LOSSES IN A TRANSFORMER

[May 2014]

There are two types of losses in a transformer:

(i) Iron or core loss
(ii) Copper loss

Iron Loss This loss is due to the reversal of flux in the core. The flux set-up in the core is nearly constant. Hence, iron loss is practically constant at all the loads, from no load to full load. The losses occuring under no-load condition are the iron losses because the copper losses in the primary winding due to no-load current are negligible. Iron losses can be subdivided into two losses:

(i) Hysteresis loss
(ii) Eddy-current loss

(1) *Hysteresis Loss* This loss occurs due to setting of an alternating flux in the core. It depends on the following factors:

(i) Area of the hysteresis loop of magnetic material which again depends upon the flux density
(ii) Volume of the core
(iii) Frequency of the magnetic flux reversal

(2) *Eddy–Current Loss* This loss occurs due to the flow of eddy currents in the core caused by induced emf in the core. It depends on the following factors:
(i) Thickness of lamination of core
(ii) Frequency of the magnetic flux reversal
(iii) Maximum value of flux density in the core
(iv) Volume of the core
(v) Quality of magnetic material used

Eddy-current losses are reduced by decreasing the thickness of lamination and by adding silicon to steel.

Copper Loss This loss is due to the resistances of primary and secondary windings.

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

where R_1 = primary winding resistance

R_2 = secondary winding resistance

Copper loss depends upon the load on the transformer and is proportional to square of load current of kVA rating of the transformer.

6.8 IDEAL AND PRACTICAL TRANSFORMERS

[Dec 2013]

For an ideal transformer, (i) there will be no core loss and copper loss, and (ii) winding resistance and leakage flux are zero. But in a practical transformer, the windings have some resistance and there is always some leakage flux.

In an ideal transformer, it is assumed that all the flux produced by the primary winding links both the primary and secondary windings. In practice, it is impossible to realize this condition. However, all the flux produced by the primary winding does not link with the secondary winding. Some part of the primary flux ϕ_{L_1} links with primary winding only. The flux ϕ_{L_1} is called primary leakage flux which links to primary winding and does not link to secondary winding. Similarly, some of the flux produced by the secondary winding links to secondary winding and does not link to primary winding. This flux is called secondary leakage flux and is represented by ϕ_{L_2}. The flux which does not pass completely through the core and links both the windings is known as the mutual flux and is represented by ϕ.

The primary leakage flux ϕ_{L_1} is in phase with I_1 and produces self-induced emf E_{L_1} in primary winding. Similarly, the secondary leakage flux ϕ_{L_2} is in phase with I_2 and produces self-induced emf E_{L_2} in secondary winding. The induced voltage E_{L_1} and E_{L_2} due to leakage fluxes ϕ_{L_1} and ϕ_{L_2} are different from induced voltages E_1 and E_2 caused by the main or mutual flux ϕ. Leakage fluxes produce self-induced emfs in their respective windings. It is, therefore, equivalent to an inductive coil in series with the respective winding such that voltage drop in each series coil is equal to that produced by leakage flux (Fig. 6.6).

$$E_{L_1} = I_1 X_1 \text{ and } E_{L_2} = I_2 X_2$$

The terms X_1 and X_2 are called primary and secondary leakage reactances respectively.

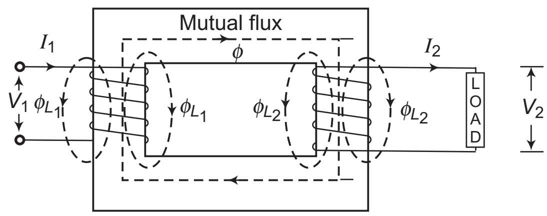


Fig. 6.6 *Magnetic fluxes in a transformer*

A transformer with winding resistance and magnetic leakage is equivalent to an ideal transformer (having no resistance and leakage reactance) having winding resistances and leakage reactances connected in series with each winding as shown in Fig. 6.7.

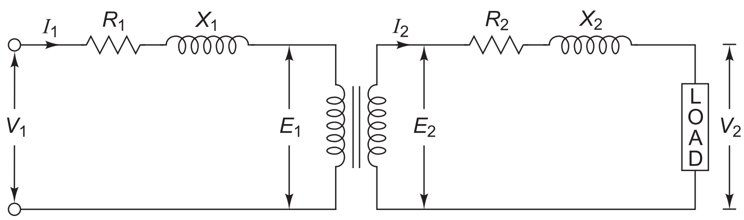


Fig. 6.7 *Winding resistances and leakage reactances of a practical transformer*

The following points should be kept in mind:

1. The leakage flux links one or the other winding but not both, hence, it in no way contributes to the transfer of energy from the primary to the secondary winding.
2. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly, the induced emf in the secondary winding E_2 will have to supply I_2R_2 and I_2X_2.

6.9 PHASOR DIAGRAM OF A TRANSFORMER ON NO LOAD

[Dec 2013, 2015, May 2015, 2016]

When the transformer is operating at no load, there is iron loss in the core and copper loss in the primary winding. Thus, primary input current I_0 has to supply iron loss in the core and a very small amount of copper loss in primary. Hence, the current I_0 has two components:

(i) a magnetising or reactive component I_μ, and
(ii) power or active component I_w.

The magnetising component I_μ is responsible for setting up flux in the core. It is in phase with the flux ϕ.

$$I_\mu = I_0 \sin \phi_0$$

The active component I_w is responsible for power loss in the transformer. It is in phase with V_1.

$$I_w = I_0 \cos \phi_0$$

Hence, no-load current I_0 is the phasor sum of I_μ and I_w.

$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

The no-load current I_0 is very small as compared to full-load current I_1. Hence, copper loss is negligible and no-load input power is practically equal to iron loss or core loss in the transformer.

Iron loss $W_i = V_1 I_0 \cos \phi_0$ where $\cos \phi_0$ is power factor at no load.

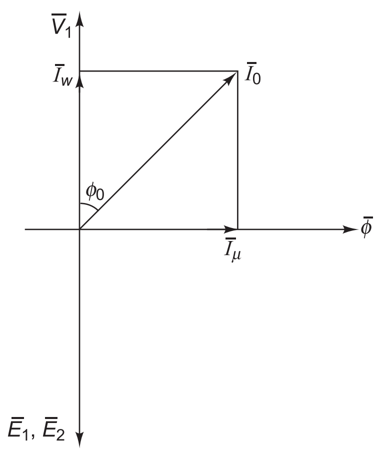


Fig. 6.8 *Phasor diagram*

Phasor Diagram Since the flux ϕ is common to both the windings, ϕ is chosen as a reference phasor. From emf equation of the transformer, it is clear that E_1 and E_2 lag the flux by 90°. Hence, emfs E_1 and E_2 are drawn such that these lag behind the flux ϕ by 90°. The magnetising component I_μ is drawn in phase with the flux ϕ. The applied voltage V_1 is drawn equal and opposite to E_1 as $V_1 \approx E_1$. The active component I_w is drawn in phase with voltage V_1. The phasor sum of I_μ and I_w gives the no-load current I_0.

Example 1

A 50 kVA, 2300/230 V, 50 Hz transformer takes 200 watts and 0.3 A at no load, when 2300 V are applied to the high-voltage side. The primary resistance is 3.5 Ω. Determine (i) core loss, and (ii) no-load pf.

Solution

$$W_i = 200 \text{ W}$$
$$I_0 = 0.3 \text{ A}$$
$$V_1 = 2300 \text{ V}$$
$$R_1 = 3.5\ \Omega$$

(i) Core loss

Copper loss in primary $= I_0^2 R_1 = (0.3)^2 \times 3.5 = 0.315$ W

Core loss $=$ Input power – Copper loss $= 200 - 0.315 = 199.685$ W

(ii) No load pf

$$W_i = V_1 I_0 \cos \phi_0$$
$$200 = 2300 \times 0.3 \times \cos \phi_0$$
$$\cos \phi_0 = 0.29 \text{ (lagging)}$$

Example 2

A single-phase transformer has a primary voltage of 230 V. No-load primary current is 5 A. No-load pf is 0.25. Number of primary turns are 200 and frequency is 50 Hz. Calculate (i) maximum value of flux in the core, (ii) core loss, and (iii) magnetising current.

Solution

$$V_1 = 230 \text{ V}$$
$$I_0 = 5 \text{ A}$$
$$\cos \phi_0 = 0.25$$
$$N_1 = 200$$
$$f = 50 \text{ Hz}$$

(i) Maximum value of flux in the core

For a transformer, $V_1 \approx E_1 = 230$ V

$$E_1 = 4.44\, f\, \phi_m\, N_1$$
$$230 = 4.44 \times 50 \times \phi_m \times 200$$
$$\phi_m = 5.18 \text{ mWb}$$

(ii) Core loss

Neglecting primary copper loss,

$$W_i = V_1 I_0 \cos \phi_0 = 230 \times 5 \times 0.25 = 287.5 \text{ W}$$

(iii) Magnetising current

$$\cos \phi_0 = 0.25$$

$$\sin\phi_0 = 0.97$$
$$I_\mu = I_0 \sin\phi_0 = 5 \times 0.97 = 4.85 \text{ A}$$

Example 3

A 230/110 V, single-phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find (i) no-load power factor, (ii) the iron loss component of no-load current, and (iii) magnetizing component of no-load current. **[Dec 2012]**

Solution

$$S = 350 \text{ VA}$$
$$W_i = 110 \text{ W}$$
$$E_1 = 230 \text{ V}$$

(i) No-load power factor

$$S = V_1 I_0 = E_1 I_0$$
$$350 = 230 \times I_0$$
$$I_0 = 1.52 \text{ A}$$
$$W_i = V_1 I_0 \cos\phi_0$$
$$110 = 350 \cos\phi_0$$
$$\cos\phi_0 = 0.314$$
$$\text{No-load pf} = 0.314$$

(ii) Iron loss component of no-load current

$$I_\omega = I_0 \cos\phi_0 = 1.52 \times 0.314 = 0.48 \text{ A}$$

(iii) Magnetizing component of no-load current

$$I_\mu = I_0 \sin\phi_0 = 1.52 \times \sin(\cos^{-1} 0.314) = 1.44 \text{ A}$$

6.10 PHASOR DIAGRAM OF A TRANSFORMER ON LOAD

[Dec 2012, 2013, May 2014]

When the transformer is loaded, a current I_2 will flow in the secondary winding. The secondary current I_2 sets up a secondary flux ϕ_2 that tends to reduce the flux ϕ produced by the primary current. Hence, induced emf E_1 in primary reduces. This causes more current to flow in the primary. Let the additional current in the primary be I_2'. This current I_2' is anti-phase with I_2 and sets up its own flux ϕ_2' which cancels the flux ϕ_2 produced by I_2.

Hence, the primary current I_1 is the phasor sum of the no-load current I_0 and the current I_2'.

$$\frac{N_2}{N_1} = \frac{I_1}{I_2} = \frac{I_0 + I_2'}{I_2} = \frac{I_2'}{I_2} = K$$
$$I_2' = K I_2$$

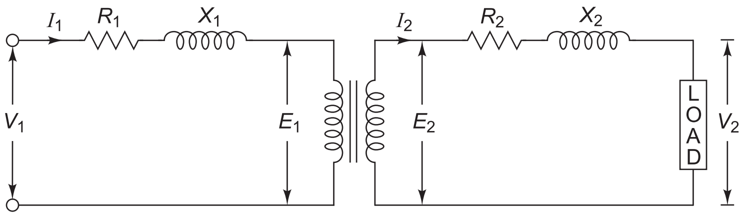


Fig. 6.9 *Practical transformer on load condition*

Figure 6.9 shows a practical transformer on load condition. When a transformer is loaded, the current I_2 flows in the secondary winding and the voltage V_2 appears across the load. Current I_2 is in phase with voltage V_2, if the load is resistive; it lags behind it, if load is inductive, and it leads, if load is capacitive.

Writing vector equations for primary and secondary sides,

$$\overline{V_1} = \overline{I_1R_1} + \overline{I_1X_1} + (-\overline{E_1})$$

$$\overline{E_2} = \overline{I_2R_2} + \overline{I_2X_2} + \overline{V_2}$$

where $$\overline{I_1} = \overline{I_0} + \overline{I_2'}$$

The phasor diagram of a transformer on load condition is drawn with the help of the above expressions.

Steps for Drawing Phasor Diagrams

1. First draw $\overline{V_2}$ and then $\overline{I_2}$. The phase angle between $\overline{I_2}$ and $\overline{V_2}$ will depend on the type of load.
2. To $\overline{V_2}$, add the resistive drop $\overline{I_2R_2}$, parallel to $\overline{I_2}$ and the inductive drop $\overline{I_2X_2}$, leading $\overline{I_2}$ by 90° such that
$$\overline{E_2} = \overline{V_2} + \overline{I_2R_2} + \overline{I_2X_2}$$
3. Draw $\overline{E_1}$ on the same side such that $E_1 = \dfrac{E_2}{K}$
4. Draw $-\overline{E_1}$ equal and opposite to $\overline{E_1}$.
5. For drawing $\overline{I_1}$, first draw $\overline{I_0}$ and $\overline{I_2'}$ such that
$$I_2' = K\,I_2$$
6. Add $\overline{I_0}$ and $\overline{I_2'}$ using the parallelogram law of vector addition.
$$\overline{I_1} = \overline{I_0} + \overline{I_2'}$$
7. To $-\overline{E_1}$, add the resistive drop $\overline{I_1R_1}$, parallel to $\overline{I_1}$ and the inductive drop $\overline{I_1X_1}$, leading $\overline{I_1}$ by 90° such that
$$\overline{V_1} = -\overline{E_1} + \overline{I_1R_1} + \overline{I_1X_1}$$

8. Draw flux ϕ such that ϕ leads $\overline{E_1}$ and $\overline{E_2}$ by 90°.

Case (i) Resistive load (unity power factor) *Case (ii)* Inductive load (lagging power factor)

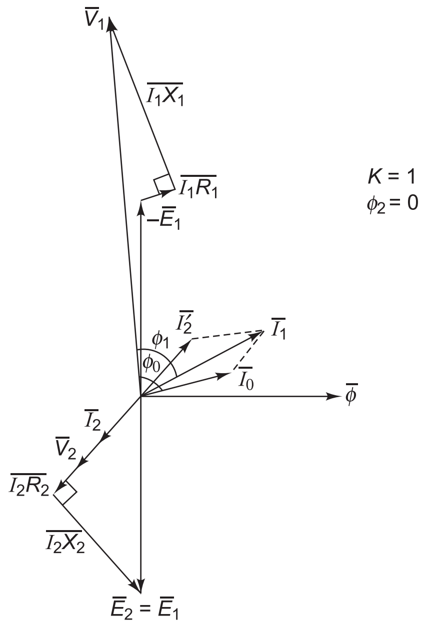


Fig. 6.10 *Phasor diagram for resistive load*

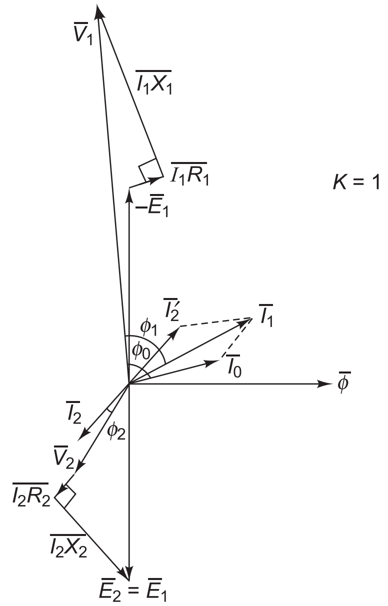


Fig. 6.11 *Phasor diagram for inductive load*

Case (iii) Capacitive load (leading power factor)

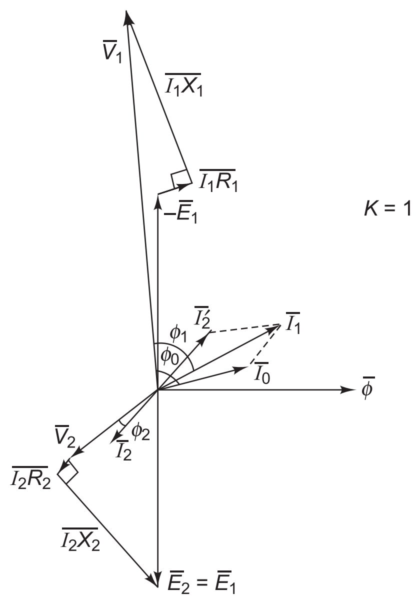


Fig. 6.12 *Phasor diagram for capacitive load*

6.11 EQUIVALENT CIRCUIT

Figure 6.13 shows a practical transformer. R_1 and R_2 represent the resistances of primary and secondary windings respectively. Similarly, X_1 and X_2 represent the leakage reactances of primary and secondary windings respectively.

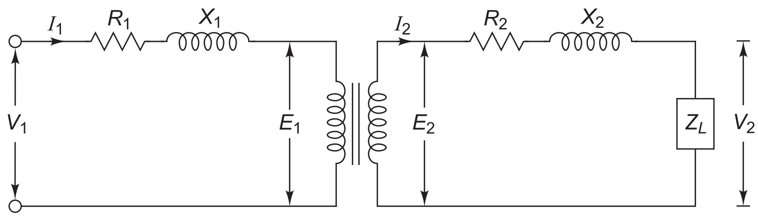


Fig. 6.13 *Practical transformer*

Figure 6.13 can be further modified to represent the no-load current I_0 and its component. The current I_0 is the phasor sum of currents I_w and I_μ. Hence, the current I_0 is simulated by the resistance R_0 taking working component I_w and inductance X_0, taking magnetising component I_μ connected in parallel across the primary circuit.

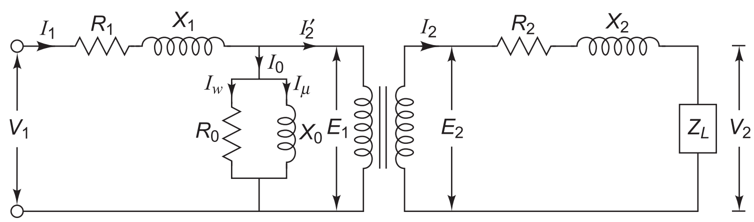


Fig. 6.14 *Practical transformer showing no-load current I_o and its component*

For convenience, all the quantities can be shown on only one side by transferring the quantities from one side to other without any power loss. The power loss in the secondary is $I_2^2\, R_2$. If R_2' is the resistance referred to primary which would have caused the same power loss as R_2 is secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \frac{R_2}{K^2}$$

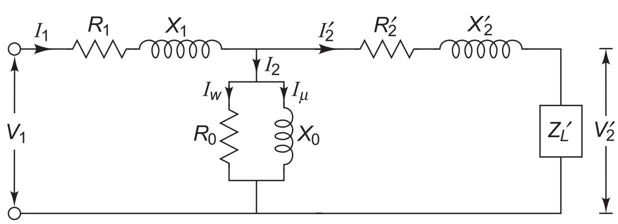


Fig. 6.15 *Modified circuit for primary winding*

Similarly, $$X_2' = \frac{X_2}{K^2}$$

Since all quantities are transferred to primary, the transformer need not be shown. The no-load current I_0 is very small compared to the full-load current I_1. Hence, drop across R_1 and X_1 due to I_0 can be neglected. Therefore, transferring R_0 and X_0 to the extreme left,

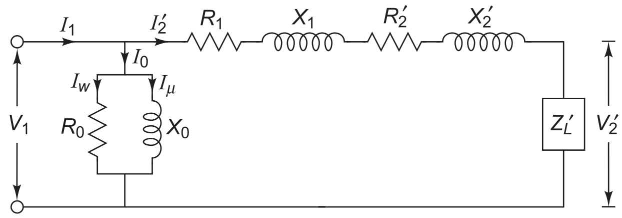


Fig. 6.16 *Modified circuit for primary winding*

The equivalent resistance referred to primary $R_{01} = R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$

The equivalent leakage reactance referred to primary $X_{01} = X_1 + X_2' = X_1 + \dfrac{X_2}{K^2}$

The equivalent impedance referred to primary $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

The equivalent circuit referred to primary is as shown in Fig. 6.18.

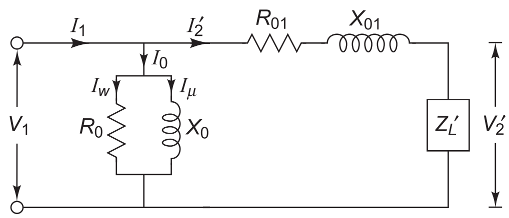


Fig. 6.17 *Equivalent circuit referred to primary winding*

Similarly, the equivalent circuit referred to secondary is as shown in Fig. 6.19.

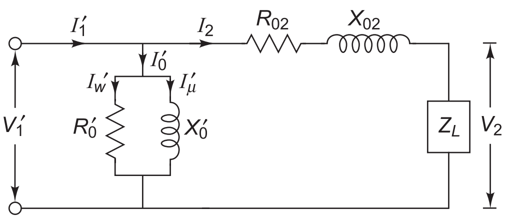


Fig. 6.18 *Equivalent circuit referred to secondary winding*

The equivalent resistance referred to secondary $R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = K^2 R_{01}$

The equivalent leakage reactance referred to secondary $X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = K^2 X_{01}$

The equivalent impedance referred to secondary $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$

Note:

(i) While shifting any primary resistance or reactance to the secondary, multiply it by K^2.

(ii) While shifting any secondary resistance or reactance to the primary, divide it by K^2.

Example 1

A transformer has a turn ratio N_1:N_2 of 4. If a 50 Ω resistance is connected across the secondary, what is the resistance referred to primary?

Solution

$$\frac{N_1}{N_2} = 4$$

$$R = 50\ \Omega$$

$$K = \frac{N_2}{N_1} = \frac{1}{4}$$

Let the resistance R be connected across the secondary.

Equivalent resistance referred to primary = $R' = \dfrac{R}{K^2} = \dfrac{50}{(1/4)^2} = 800\ \Omega$

Example 2

A resistance connected across the secondary of an ideal transformer has a value of 800 Ω as referred to the primary. The same resistance when connected across the primary has a value of 3.125 Ω as referred to the secondary. Find the ratio of the transformer.

Solution Let R be the resistance connected to the secondary.

Then equivalent resistance referred to the primary = $\dfrac{R}{K^2}$

$$\frac{R}{K^2} = 800\ \Omega$$

If the resistance R is connected across the primary, the equivalent resistance referred to the secondary = K^2R

$$K^2R = 3.125$$

$$\frac{K^2R}{R/K^2} = \frac{3.125}{800}$$

$$K^4 = 3.90624 \times 10^{-3}$$

$$K = 0.25$$

Example 3

A 6600/400 V transformer has a primary resistance of 2.5 Ω and a reactance of 3.9 Ω. The secondary resistance is 0.01 Ω and the reactance is 0.025 Ω. Determine the equivalent circuit parameters referred to primary and secondary.

Solution

$$E_1 = 6600 \text{ V}$$
$$E_2 = 400 \text{ V}$$
$$R_1 = 2.5\ \Omega$$
$$X_1 = 3.9\ \Omega$$
$$R_2 = 0.01\ \Omega$$
$$X_2 = 0.025\ \Omega$$
$$K = \frac{E_2}{E_1} = \frac{400}{6600} = 0.06$$

(i) Equivalent resistance referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 2.5 + \frac{0.01}{(0.06)^2} = 5.28\ \Omega$$

(ii) Equivalent reactance referred to primary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 3.9 + \frac{0.025}{(0.06)^2} = 10.84\ \Omega$$

(iii) Equivalent resistance referred to secondary

$$R_{02} = K^2 R_{01} = (0.06)^2 \times 5.28 = 0.02\ \Omega$$

(iv) Equivalent reactance referred to secondary

$$X_{02} = K^2 X_{01} = (0.06)^2 \times 10.84 = 0.04\ \Omega$$

Example 4

A 30 kVA, 2400/120 V, 50 Hz transformer has high-voltage winding resistance of 0.1 Ω and leakage reactance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and leakage reactance is 0.012 Ω. Calculate equivalent resistance as referred to primary and secondary, equivalent reactance as referred to primary and secondary, equivalent impedance as referred to primary and secondary, copper loss at full load and at 75% of full load. **[Dec 2014]**

Solution

kVA rating = 30 kVA

$$E_1 = 2400 \text{ V}$$
$$E_2 = 120 \text{ V}$$
$$R_1 = 0.1\ \Omega$$
$$X_1 = 0.22\ \Omega$$
$$R_2 = 0.035\ \Omega$$

$$X_2 = 0.012\ \Omega$$

$$K = \frac{E_2}{E_1} = \frac{120}{2400} = 0.05$$

(i) Equivalent resistance as referred to primary and secondary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.035}{(0.05)^2} = 14.1\ \Omega$$

$$R_{02} = K^2 R_{01} = (0.05)^2 \times 14.1 = 0.035\ \Omega$$

(ii) Equivalent reactance as referred to primary and secondary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.22 + \frac{0.012}{(0.05)^2} = 5.02\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.05)^2 \times 5.02 = 0.013\ \Omega$$

(iii) Equivalent impedance as referred to primary and secondary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(14.1)^2 + (5.02)^2} = 14.97\ \Omega$$

$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 14.97 = 0.037\ \Omega$$

(iv) Copper loss at full load and at 75% of full load

$$E_2 \simeq V_2 = 120\ \text{V}$$

Full load secondary current $I_2 = \dfrac{\text{kVA rating} \times 1000}{V_2} = \dfrac{30 \times 1000}{120} = 250\ \text{A}$

$$W_{Cu} = I_2^2\, R_{02} = (250)^2 \times 0.035 = 2.18\ \text{kW}$$

At 75% of full load

$$\text{Copper loss} = x^2\, W_{Cu} = (0.75)^2 \times 2.18 = 1.23\ \text{kW}$$

Example 5

A 30 kVA, 2400/120 V, 50 Hz, transformer has a high-voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low-voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Calculate for the transformer:

(i) Equivalent resistance as referred to both primary and secondary

(ii) Equivalent reactance as referred to both primary and secondary

(iii) Equivalent impedance as referred to both primary and secondary

(iv) Copper loss at full load **[Dec 2012, May 2013]**

Solution

$$\text{kVA rating} = 30\ \text{kVA}$$

$$E_1 = 2400\ \text{V}$$

$$E_2 = 120\ \text{V}$$

$$R_1 = 0.1\ \Omega$$

$$X_1 = 0.22\ \Omega$$

$$R_2 = 0.035\ \Omega$$
$$X_2 = 0.012\ \Omega$$
$$K = \frac{E_2}{E_1} = \frac{120}{2400} = 0.05$$

(i) Equivalent resistance as referred to both primary and secondary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.035}{(0.05)^2} = 14.1\ \Omega$$
$$R_{02} = K^2 R_{01} = (0.05)^2 \times 14.1 = 0.035\ \Omega$$

(ii) Equivalent reactance as referred to both primary and secondary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.22 + \frac{0.012}{(0.05)^2} = 5.02\ \Omega$$
$$X_{02} = K^2 X_{01} = (0.05)^2 \times 5.02 = 0.013\ \Omega$$

(iii) Equivalent impedance as referred to both primary and secondary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(14.1)^2 + (5.02)^2} = 14.97\ \Omega$$
$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 14.97 = 0.037\ \Omega$$

(iv) Copper loss at full load

$$V_1 \simeq E_1 = 2400\ \text{V}$$

Full-load current
$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{30 \times 1000}{2400} = 12.5\ \text{A}$$
$$W_{Cu} = I_1^2 R_{01} = (12.5)^2 \times 14.1 = 2.2\ \text{kW}$$

Example 6

A 50 kVA, 4400/220 V transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer, (i) full-load currents on primary and secondary side, (ii) equivalent resistances, reactances, impedances referred to primary side and secondary side, and (iii) total copper loss using individual resistances and equivalent resistances. **[May 2014]**

Solution kVA rating = 50 kVA

$$E_1 = 4400\ \text{V}$$
$$E_2 = 220\ \text{V}$$
$$R_1 = 3.45\ \Omega$$
$$R_2 = 0.009\ \Omega$$
$$X_1 = 5.2\ \Omega$$
$$X_2 = 0.015\ \Omega$$

$$K = \frac{E_2}{E_1} = \frac{220}{4400} = 0.05$$

(i) Full-load currents and primary and secondary side

For a transformer,

$$V_1 \simeq E_1 = 4400 \text{ V}$$

$$E_2 \simeq V_2 = 220 \text{ V}$$

$$\text{Full-load primary current } I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{50 \times 1000}{4400} = 11.36 \text{ A}$$

$$\text{Full-load secondary current } I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{50 \times 1000}{220} = 227.27 \text{ A}$$

(ii) Equivalent resistance, reactances, impedances referred to primary side and secondary side

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(0.05)^2} = 7.05 \ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{(0.05)^2} = 11.2 \ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(7.05)^2 + (11.2)^2} = 13.23 \ \Omega$$

$$R_{02} = K^2 R_{01} = (0.05)^2 \times 7.05 = 0.02 \ \Omega$$

$$X_{02} = K^2 X_{01} = (0.05)^2 \times 11.2 = 0.028 \ \Omega$$

$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 13.23 = 0.03 \ \Omega$$

(iii) Copper loss with individual resistances

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227.27)^2 \times 0.009$$

$$= 445.22 + 464.86 = 910.08 \text{ W}$$

(iv) Copper loss with equivalent resistances

$$W_{Cu} = I_1^2 R_{01} = I_2^2 R_{02} = (11.36)^2 \times 7.05 = 909.8 \text{ W}$$

6.12 VOLTAGE REGULATION

When a transformer is loaded, the secondary terminal voltage decreases due to a drop across secondary winding resistance and leakage reactance. This change in secondary terminal voltage from no load to full load conditions, expressed as a fraction of the no-load secondary voltage is called *regulation of the transformer*.

$$\text{Regulation} = \frac{\begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on no load}\end{pmatrix} - \begin{pmatrix}\text{Secondary terminal voltage}\\ \text{on full-load condition}\end{pmatrix}}{\text{Secondary terminal voltage on no load}}$$

$$= \frac{E_2 - V_2}{E_2}$$

$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

6.12.1 Expression for Voltage Regulation

Consider a phasor diagram of transformer referred to secondary side on load condition (load is assumed to be inductive). With O as centre and radius OC, draw an arc cutting OA produced at M. From the point B, draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

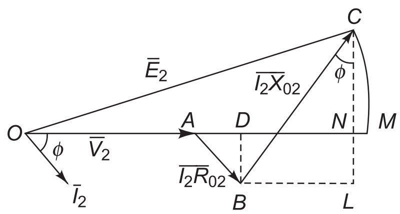


Fig. 6.19

$$\text{Total voltage drop} = E_2 - V_2 = OC - OA = OM - OA = AM = AN + NM$$

$$\text{Approximate voltage drop} \approx AN \qquad (\because NM \text{ is very small})$$

$$= AD + DN$$

$$= AD + BL$$

$$= I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$$

$$\%\ \text{regulation} = \frac{I_2\ R_{02}\ \cos\phi + I_2\ X_{02}\ \sin\phi}{E_2} \times 100$$

For leading pf,

$$\text{Approximate voltage drop} = I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$

$$\%\ \text{regulation} = \frac{I_2\ R_{02}\ \cos\phi - I_2\ X_{02}\ \sin\phi}{E_2} \times 100$$

Hence, in general,

$$\%\ \text{regulation} = \frac{I_2\ R_{02}\ \cos\phi \pm I_2\ X_{02}\ \sin\phi}{E_2} \times 100$$

'+' sign is used for lagging pf and '–' sign is used for leading pf.

On primary side, we can express regulation as,

$$\%\ \text{regulation} = \frac{I_1\ R_{01}\ \cos\phi \pm I_1\ X_{01}\ \sin\phi}{V_1} \times 100$$

We can also express percentage regulation as

$$\% \text{ regulation} = \frac{100\, I_2\, R_{02}}{E_2} \cos\phi \pm \frac{100\, I_2\, X_{02}}{E_2} \sin\phi$$
$$= v_r \cos\phi \pm v_x \sin\phi$$

where

$$v_r = \frac{100\, I_2\, R_{02}}{E_2} = \text{percentage resistive drop}$$

$$v_x = \frac{100\, I_2\, X_{02}}{E_2} = \text{percentage reactive drop}$$

Example 1

A 200 kVA, 2200/440 V, 50 Hz, single-phase transformer is operating at full load, 0.8 lagging pf. The voltage on secondary of the transformer at full load, 0.8 lagging pf is 400 V. Calculate voltage regulation of the transformer.

Solution $E_2 = 440$ V

$V_2 = 400$ V

$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100 = \frac{440 - 400}{440} \times 100 = 9.09\%$$

Example 2

A single-phase, 440/220 V, 10 kVA, 50 Hz transformer has a resistance of 0.2 Ω and reactance of 0.6 Ω on h.v. side. The corresponding values of l.v. side are 0.04 Ω and 0.14 Ω. Calculate the percentage regulation on full load for (i) 0.8 lagging pf (ii) 0.8 leading pf, (iii) unity pf.

Solution kVA rating = 10 kVA

$E_2 = 220$ V

$E_1 = 440$ V

$R_2 = 0.04\ \Omega$

$R_1 = 0.2\ \Omega$

$X_2 = 0.14\ \Omega$

$X_1 = 0.6\ \Omega$

For a transformer $V_2 \approx E_2 = 220$ V

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{10 \times 1000}{220} = 45.45 \text{ A}$$

$$K = \frac{E_2}{E_1} = \frac{220}{440} = 0.5$$

$$R_{02} = R_2 + K^2 R_1 = 0.04 + (0.05)^2 \times 0.2 = 0.09\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 0.14 + (0.5)^2 \times 0.6 = 0.29\ \Omega$$

(i) Percentage regulation on full load for 0.8 lagging pf

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

$$\%\text{ regulation} = \frac{I_2 (R_{02}\cos\phi + X_{02}\sin\phi)}{E_2} \times 100$$

$$= \frac{45.45(0.09 \times 0.8 + 0.29 \times 0.6)}{220} \times 100$$

$$= 5.08\%$$

(ii) Percentage regulation on full load for 0.8 leading pf

$$\%\text{ regulation} = \frac{I_2 (R_{02}\cos\phi - X_{02}\sin\phi)}{E_2} \times 100$$

$$= \frac{45.45(0.09 \times 0.8 - 0.29 \times 0.6)}{220} \times 100$$

$$= -2.11\%$$

(iii) Percentage regulation on full load for unity pf

$$\cos\phi = 1$$

$$\sin\phi = 0$$

$$\%\text{ regulation} = \frac{I_2 (R_{02}\cos\phi \pm X_{02}\sin\phi)}{E_2} \times 100$$

$$= \frac{45.45(0.09 \times 1 - 0.29 \times 0)}{220} \times 100$$

$$= 1.86\%$$

Example 3

Calculate the regulation of a transformer in which resistive drop is 1% of the output and reactive drop is 5 % of the output, when the pf is (a) 0.8 lagging, (b) unity, and (c) 0.8 leading.

Solution

$$v_r = 1$$

$$v_x = 5$$

$$\%\text{ regulation} = v_r \cos\phi \pm v_x \sin\phi$$

(a) Percentage regulation for 0.8 lagging pf

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

$$\% \text{ regulation} = 1 \times 0.8 + 5 \times 0.6 = 3.8\%$$

(b) Percentage regulation for unity pf

$$\cos \phi = 1$$
$$\sin \phi = 0$$
$$\% \text{ regulation} = 1 \times 1 + 5 \times 0 = 1\%$$

(c) Percentage regulation for 0.8 leading pf

$$\% \text{ regulation} = 1 \times 0.8 - 5 \times 0.6 = -2.2\%$$

Example 4

A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.

Solution

$$v_r = 5$$
$$v_x = 2.5$$
$$\% R = v_r \cos \phi + v_x \sin \phi \tag{1}$$

Differentiating Eq. (1),

$$\frac{dR}{d\phi} = -v_r \sin \phi + v_r \cos \phi$$

For regulation to be maximum,

$$\frac{dR}{d\phi} = 0$$

$$-V_r \sin \phi + V_x \cos \phi = 0$$

$$\tan \phi = \frac{v_x}{v_r} = \frac{5}{2.5} = 2$$

$$\phi = 63.43°$$

$$\text{pf} = \cos \phi = \cos (63.43°) = 0.45$$

$$\sin \phi = 0.89$$

Maximum percentage regulation = $v_r \cos \phi + v_x \sin \phi = 2.5 \times 0.45 + 5 \times 0.89 = 5.58\%$

Example 5

A 230/460 V transformer has a primary resistance of 0.2 Ω and a reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when 10 A is supplied at 0.8 pf lagging.

Solution

$$E_1 = 230 \text{ V}$$
$$E_2 = 460 \text{ V}$$
$$R_1 = 0.2\ \Omega$$

$$R_2 = 0.75\ \Omega$$
$$X_1 = 0.5\ \Omega$$
$$X_2 = 1.8\ \Omega$$
$$I_2 = 10\ \text{A}$$
$$\cos\phi = 0.8$$
$$K = \frac{E_2}{E_1} = \frac{460}{230} = 2$$
$$R_{02} = R_2 + K^2 R_1 = 0.75 + (2)^2 \times 0.2 = 1.55\ \Omega$$
$$X_{02} = X_2 + K^2 X_1 = 1.8 + (2)^2 \times 0.5 = 3.8\ \Omega$$
$$\cos\phi = 0.8$$
$$\sin\phi = 0.6$$

For lagging pf,

$$E_2 - V_2 = I_2\,(R_{02}\cos\phi + X_{02}\sin\phi)$$

Secondary terminal voltage

$$\begin{aligned} V_2 &= E_2 - I_2\,(R_{02}\cos\phi + X_{02}\sin\phi) \\ &= 460 - 10\,(1.55 \times 0.8 + 3.8 \times 0.6) \\ &= 424.8\ \text{V} \end{aligned}$$

Useful Formulae

$$W_i = V_1 I_0 \cos\phi_0$$
$$I_\mu = I_0 \sin\phi_0$$
$$I_w = I_0 \cos\phi_0$$
$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2$$
$$W_{Cu} = I_1^2 R_{01} = I_2^2 R_{02}$$
$$R_{01} = R_1 + \frac{R_2}{K^2}$$
$$R_{02} = R_2 + K^2 R_1 = K^2 R_{01}$$
$$X_{01} = X_1 + \frac{X_2}{K^2}$$
$$X_{02} = X_2 + K^2 X_1 = K^2 X_{01}$$
$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$
$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$$
$$\begin{aligned} \%\ \text{Regulation} &= \frac{E_2 - V_2}{E_2} \times 100 \\ &= \frac{I_2 R_{02}\cos\phi \pm I_2 X_{02}\sin\phi}{E_2} \times 100 \\ &= \frac{I_1 R_{01}\cos\phi \pm I_1\ R_{01}\sin\phi}{V_1} \times 100 \end{aligned}$$

Exercise 6.2

6.1 The no-load current of a transformer is 10 A at a pf of 0.25 lagging, when connected to a 400 V, 50 Hz supply. Calculate (a) magnetising component of no-load current, (b) iron loss, and (c) maximum value of flux in the core. Assume primary winding turns as 500. *[9.68 A, 1000 W, 3.6036 mWb]*

6.2 A 2200/250 V transformer takes 0.5 A at a pf of 0.3 on no load. Find magnetising and working components of no-load primary current. *[0.476 A, 0.15 A]*

6.3 The no-load current of a transformer is 4 A at 0.25 pf when supplied at 250 V, 50 Hz. The number of turns on the primary winding is 200. Calculate (i) rms value of flux in the core, (ii) core loss, and (iii) magnetising current. *[5.63 mWb, 250 W, 3.87 A]*

6.4 The values of the resistances of the primary and secondary windings of a 2200/ 200 V, 50 Hz single-phase transformer are 2.4 Ω and 0.02 Ω respectively. Find (i) equivalent resistance of primary referred to secondary, (ii) equivalent resistance of secondary referred to primary, (iii) total resistance referred to secondary, and (iv) total resistance referred to primary. *[0.0198 Ω, 2.42 Ω, 0.0398 Ω, 4.82 Ω]*

6.5 A 40 kVA transformer with a ratio of 2000/250 V has a primary resistance of 1.15 Ω and a secondary resistance of 0.0155 Ω. Calculate (i) the total resistance in terms of the secondary winding, (ii) total resistance drop on full load, and (iii) total copper loss on full load. *[0.0334 Ω, 5.35 V, 855.04 Ω]*

6.13 EFFICIENCY

[Dec 2013]

Efficiency is defined as the ratio of output power to input power.

Efficiency $$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}} = \frac{\text{Output}}{\text{Output + Copper loss + Iron loss}}$$

Also, $$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{\text{Input} - \text{Copper loss} - \text{Iron loss}}{\text{Input}}$$

Condition for Maximum Efficiency We know that,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

Considering secondary side of the transformer,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}}$$

Differentiating both the sides w.r.t. I_2,

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2I_2 R_{02})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})^2}$$

For maximum efficiency, $\frac{d\eta}{dI_2} = 0$

$$(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})V_2 \cos\phi_2 = V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2I_2 R_{02})$$

$$V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02} = V_2 I_2 \cos\phi_2 + 2I_2^2 R_{02}$$

$$W_i = I_2^2 R_{02}$$

Similarly on primary side,

$$W_i = I_1^2 R_{01}$$

Thus when copper loss = iron loss, the efficiency of the transformer is maximum.

Load Corresponding to Maximum Efficiency For maximum efficiency,

$$W_i = I_2^2 R_{02}$$

$$I_{2\,(\text{max. efficiency})} = \sqrt{\frac{W_i}{R_{02}}}$$

Multiplying both the sides by V_2,

$$V_2 I_{2\,(\text{max. efficiency})} = V_2 \sqrt{\frac{W_i}{R_{02}}}$$

$$\text{Load } VA_{(\text{max. efficiency})} = V_2 I_2 \sqrt{\frac{W_i}{I_2^2 R_{02}}} = V_2 I_2 \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\text{Load kVA}_{(\text{max. efficiency})} = \text{Full-load kVA} \sqrt{\frac{W_i}{W_{Cu}}}$$

where W_i = iron loss

W_{Cu} = full-load copper loss

Note: The efficiency at any load is given by

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

where x = ratio of actual to full load kVA

$$W_i = \text{iron loss in kW}$$
$$W_{Cu} = \text{full-load copper loss in kW}$$

Example 1

Iron loss of 80 kVA, 1000/250 V, single-phase, 50 Hz transformer is 500 W. The copper loss when the primary carries a current of 50 A is 400 W. Find (i) area of cross section of limb if working flux density is 1 T and there are 1000 turns on the primary, (ii) efficiency at full load and pf 0.8 lagging, and (iii) efficiency at 75% of full load and unity pf.

Solution Full load kVA = 80 kVA

$$E_1 = 1000 \text{ V}$$
$$E_2 = 250 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$W_i = 500 \text{ W} = 0.5 \text{ kW}$$
$$W_{Cu} = 400 \text{ W} = 0.4 \text{ kW}$$
$$I_1 = 50 \text{ A}$$
$$B_m = 1 \text{ T}$$
$$N_1 = 1000$$

(i) Area of cross section of limb

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$
$$1000 = 4.44 \times 50 \times 1 \times \text{A} \times 1000$$
$$A = 4.5 \times 10^{-3} \text{ m}^2$$

(ii) Efficiency at full load and 0.8 lagging pf

$$x = 1$$
$$\text{pf} = 0.8$$
$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$
$$= \frac{1 \times 80 \times 0.8}{1 \times 80 \times 0.8 + 0.5 + (1)^2 \times 0.4} \times 100$$
$$= 98.61\%$$

(iii) Efficiency at 75% of full load and unity pf

$$x = 0.75$$
$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.75 \times 80 \times 1}{0.75 \times 80 \times 1 + 0.5 + (0.75)^2 \times 0.4} \times 100$$

$$= 98.81\ \%$$

Example 2

A 100 kVA, single-phase transformer has iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (i) full load and 0.8 lagging pf, and (ii) half load and unity pf.

Solution Full load kVA = 100 kVA

$$W_i = 600\text{ W} = 0.6\text{ kW}$$

$$W_{Cu} = 1.5\text{ kW}$$

(i) Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.6 + (1)^2 \times 1.5} \times 100$$

$$= 97.44\ \%$$

(ii) Efficiency at half load and unity pf

$$x = 0.5$$

$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 100 \times 1}{0.5 \times 100 \times 1 + 0.6 + (0.5)^2 \times 1.5} \times 100$$

$$= 98.09\ \%.$$

Example 3

A 25 kVA, 2200/220 V, 50 Hz, single-phase transformer has a primary resistance of 1.8 Ω and a secondary resistance of 0.02 Ω. Calculate the efficiency of the transformer at (i) full load and unity pf, and (ii) half load and 0.8 lagging pf. Iron loss is 1000 watts.

Solution Full load kVA = 25 kVA

$$E_1 = 2200 \text{ V}$$
$$E_2 = 220 \text{ V}$$
$$R_1 = 1.8\ \Omega$$
$$R_2 = 0.02\ \Omega$$
$$W_i = 1000 \text{ W} = 1 \text{ kW}$$

For a transformer,

$$E_2 \simeq V_2 = 220 \text{ V}$$

$$I_2 = \frac{\text{Full load kVA} \times 1000}{V_2} = \frac{25 \times 1000}{220} = 113.64 \text{ A}$$

$$K = \frac{E_2}{E_1} = \frac{220}{2200} = 0.1$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (0.1)^2 \times 1.8 = 0.038\ \Omega$$

$$W_{Cu} = I_2^2 R_{02} = (113.64)^2 \times 0.038 = 0.49 \text{ kW}$$

(i) Efficiency at full load and unity pf

$$x = 1$$
$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 25 \times 1}{1 \times 25 \times 1 + 1 + (1)^2 \times 0.49} \times 100$$

$$= 94.38\%$$

(ii) Efficiency at half load and 0.8 lagging pf

$$x = 0.5$$
$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 25 \times 0.8}{0.5 \times 25 \times 0.8 + 1 + (0.5)^2 \times 0.49} \times 100$$

$$= 89.91\%.$$

Example 4

A 250 kVA, single-phase transformer has 98.135% efficiency at full load and 0.8 lagging pf. The efficiency at half load and 0.8 lagging pf is 97.751%. Calculate the iron loss and full load copper loss.

Solution Full load kVA = 250 kVA

$$\eta_1 = 98.135\%$$

$$\eta_2 = 97.751\%$$

(i) Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta_1 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$98.135 = \frac{1 \times 250 \times 0.8}{1 \times 250 \times 0.8 + W_i + (1)^2 W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 3.8 \text{ kW} \tag{1}$$

(ii) Efficiency at half load and 0.8 lagging pf

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\eta_2 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$97.751 = \frac{0.5 \times 250 \times 0.8}{0.5 \times 250 \times 0.8 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$$W_i + 0.25\, W_{Cu} = 2.3 \tag{2}$$

Solving Eqs (1) and (2),

$$W_{Cu} = 2 \text{ kW}$$

$$W_i = 1.8 \text{ kW}$$

Example 5

A 600 kVA, single-phase transformer has an efficiency of 92% at full load and also at half load, working at unity pf. Calculate the efficiency of the transformer at 60% full load and unity pf.

Solution Full load kVA = 600 kVA

$$\eta_1 = \eta_2 = 92\%$$

Efficiency at full load and unity pf

$$x = 1$$

$$\text{pf} = 1$$

$$\%\eta_1 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + (1)^2 W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 52.2 \qquad (1)$$

Efficiency at half load and unity pf

$$x = 0.5$$

$$\text{pf} = 1$$

$$\%\eta_2 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$92 = \frac{0.5 \times 600 \times 1}{0.5 \times 600 \times 1 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$$W_i + 0.25\, W_{Cu} = 26.1 \qquad (2)$$

Solving Eqs (1) and (2),

$$W_{Cu} = 34.8 \text{ kW}$$

$$W_i = 17.4 \text{ kW}$$

Efficiency at 60% full load and unity pf

$$x = 0.6$$

$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.6 \times 600 \times 1}{0.6 \times 600 \times 1 + 17.4 + (0.6)^2 \times 34.8} \times 100$$

$$= 92.32\ \%$$

Example 6

A 150 kVA, single-phase transformer has iron loss of 1.4 kW and full-load copper loss of 1.6 kW. Determine (i) the kVA load for maximum efficiency and the maximum efficiency at 0.8 lagging pf, and (ii) the efficiency at half full load and 0.8 lagging pf. **[May 2016]**

Solution Full load kVA = 150 kVA

$$W_i = 1.4 \text{ kW}$$

$$W_{Cu} = 1.6 \text{ kW}$$

(i) Load kVA for maximum efficiency and the maximum efficiency

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 150 \times \sqrt{\frac{1.4}{1.6}} = 140.31 \text{ kVA}$$

For maximum efficiency,

$$W_i = W_{Cu} = 1.4 \text{ kW}$$

$$\text{pf} = 0.8$$

$$\%\eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{140.31 \times 0.8}{140.31 \times 0.8 + 1.4 + 1.4} \times 100$$

$$= 97.57\%$$

(ii) Efficiency at half full load and 0.8 pf

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 150 \times 0.8}{0.5 \times 150 \times 0.8 + 1.4 + (0.5)^2 \times 1.6} \times 100$$

$$= 97.08\%.$$

Example 7

A 100 kVA, single-phase transformer has an efficiency of 97% at full load and 0.8 lagging pf. If the maximum efficiency occurs at 80% of full load at 0.8 lagging pf, calculate (i) iron loss and full load copper loss, (ii) maximum efficiency.

Solution Full load kVA = 100 kVA

$$\eta = 97\%$$

(i) Iron loss and full load copper loss

Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$97 = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + W_i + (1)^2 \times W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 2.474 \qquad (1)$$

(ii) Maximum efficiency occurs at 80 % of full load

$$W_i = (0.8)^2\, W_{Cu} = 0.64\, W_{Cu} \qquad (2)$$

Solving Eqs (1) and (2),

$$W_{Cu} = 1.508 \text{ kW}$$

$$W_i = 0.965 \text{ kW}$$

Maximum efficiency at 80 % full load and 0.8 lagging pf

$$\%\eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{0.8 \times 100 \times 0.8}{0.8 \times 100 \times 0.8 + 0.965 + 0.965} \times 100$$

$$= 97.07\%.$$

Example 8

The maximum efficiency of a 500 kVA, 3000/500 V, 50 Hz single-phase transformer is 98% and occurs at 3/4 full load, unity pf. If the impedance is 10%, calculate the regulation at full load, 0.8 lagging pf.

Solution $\%\,\eta_{\max} = 98\%$

Full load kVA = 500 kVA

$$E_1 = 3000 \text{ V}$$

$$E_2 = 500 \text{ V}$$

Since, maximum efficiency occurs at 3/4 of full load and unity pf, iron loss is equal to copper loss at this load.

$$\% \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$98 = \frac{3/4 \times 500 \times 1}{3/4 \times 500 \times 1 + 2W_i} \times 100$$

$$W_i = 3.826 \text{ kW}$$

Copper loss at 3/4 of full load = 3.826 kW

$$\text{Full-load copper loss} = \left(\frac{4}{3}\right)^2 \times 3.826 = 6.803 \text{ kW}$$

Percentage regulation at full load and 0.8 lagging pf

$$\% \text{ resistance} = v_r = \frac{I_2 R_{02}}{E_2} \times 100 = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \% \text{ Cu loss at full load}$$

$$= \frac{6.803}{500} \times 100$$

$$= 1.36\%$$

$$\% \text{ reactance} = v_x = \sqrt{\% Z^2 - \% R^2}$$

$$= \sqrt{(10)^2 - (1.36)^2}$$

$$= 9.91\%$$

$$\cos \phi = 0.8 \qquad \sin \phi = 0.6$$

$$\% \text{ regulation} = v_r \cos \phi + v_x \sin \phi$$

$$= 1.36 \times 0.8 + 9.91 \times 0.6$$

$$= 7.034\%$$

Example 9

The maximum efficiency of a 100 kVA, 6600/250 V single-phase transformer occurs at half load and is 98 % at unity power factor. If the percentage impedance is 8 %, calculate the percentage regulation and efficiency on full load at 0.8 lagging pf.

Solution

$$\eta_{\max} = 98\%$$

$$\text{Full load kVA} = 100 \text{ kVA}$$

$$E_1 = 6600 \text{ V}$$

$$E_2 = 250 \text{ V}$$

Since, maximum efficiency occurs at half load and unity pf, iron loss is equal to copper loss at this load.

$$\% \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$98 = \frac{1/2 \times 100 \times 1}{1/2 \times 100 \times 1 + 2W_i} \times 100$$

$$W_i = 0.51 \text{ kW}$$

Copper loss at half load = 0.51 kW

Full-load copper loss $= (2)^2 \times 0.51 = 2.04$ kW

Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.51 + (1)^2 \times 2.04} \times 100$$

$$= 96.91\%$$

Percentage regulation at full load and 0.8 lagging pf

$$\% R = v_r = \% \text{ Cu loss} = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100 = \frac{2.04}{100} \times 100 = 2.04\%$$

$$\% X = v_x = \sqrt{\% Z^2 - \% R^2} = \sqrt{(8)^2 - (2.04)^2} = 7.74 \%$$

$$\cos \phi = 0.8 \qquad \sin \phi = 0.6$$

$$\begin{aligned} \% \text{ regulation} &= v_r \cos \phi + v_x \sin \phi \\ &= 2.04 \times 0.8 + 7.74 \times 0.6 \\ &= 6.28\% \end{aligned}$$

Example 10

A 300 kVA, single-phase transformer has a percentage resistance of 1.5% and maximum efficiency occurs at a load of 173.2 kVA. Find the efficiency at full load and 0.8 lagging pf.

Solution Full load kVA = 300 kVA

Load kVA = 173.2 kVA

$$\% R = 1.5$$

$$\% \text{ resistance} = \% \text{ Cu loss}$$

$$= \frac{\text{Full-load copper loss}}{\text{Full-load kVA}} \times 100$$

$$1.5 = \frac{\text{Full-load copper loss}}{300} \times 100$$

Full-load copper loss $\quad W_{Cu} = 4.5 \text{ kW}$

Also, for maximum efficiency,

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$173.2 = 300 \times \sqrt{\frac{W_i}{4.5}}$$

$$W_i = 1.5 \text{ kW}$$

Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 300 \times 0.8}{1 \times 300 \times 0.8 + 1.5 + (1)^2 \times 4.5} \times 100$$

$$= 97.6\%$$

Example 11

The parameters of the equivalent circuit of 150 kVA, 2400/240 V transformer are as shown in the Fig. 6.20.

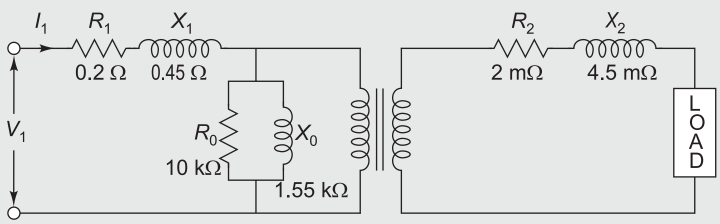


Fig. 6.20

Using the circuit referred to primary, determine voltage regulation and efficiency of the transformer operating at rated load with 0.8 lagging pf.

Solution

(i) Percentage regulation at rated load and 0.8 lagging pf

$$K = \frac{240}{2400} = 0.1$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.2 + \frac{2 \times 10^{-3}}{(0.1)^2} = 0.4\ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.45 + \frac{4.5 \times 10^{-3}}{(0.1)^2} = 0.9\ \Omega$$

$$I_1 = \frac{150 \times 1000}{2400} = 62.5\ \text{A}$$

$$\cos\phi = 0.8 \qquad \sin\phi = 0.6$$

$$\%\ \text{regulation} = \frac{I_1 (R_{01}\cos\phi + X_{01}\sin\phi)}{V_1} \times 100$$

$$= \frac{62.5(0.4 \times 0.8 + 0.9 \times 0.6)}{2400} \times 100$$

$$= 2.24\%$$

(ii) Efficiency at rated load and 0.8 lagging pf

$$I_w = \frac{2400}{10 \times 10^3} = 0.24\ \text{A}$$

$$I_w = I_0 \cos\phi_0 = 0.24\ \text{A}$$

$$W_i = V_1 I_0 \cos\phi_0 = 2400 \times 0.24 = 576\ \text{W} = 0.576\ \text{kW}$$

$$W_{Cu} = I_1^2 R_{01} = (62.5)^2 \times 0.4 = 1562.5\ \text{W} = 1.5625\ \text{kW}$$

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 150 \times 0.8}{1 \times 150 \times 0.8 + 0.576 + (1)^2 \times 1.5625} \times 100$$

$$= 98.25\ \%$$

Useful Formulae

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_i + W_{Cu}} \times 100$$

$$= \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

where W_i and W_{Cu} are in kW.

and $$x = \frac{\text{Actual kVA}}{\text{Full-load kVA}}$$

Condition for maximum efficiency,

$$W_i = W_{Cu}$$

Load kVA (maximum efficiency)

$$= \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\eta_{\max} = \frac{\text{Load kVA (max.efficiency)} \times \text{pf}}{\text{Load kVA(max.efficiency)} \times \text{pf} + 2W_i} \times 100$$

Exercise 6.3

6.1 A 100 kVA transformer has iron loss of 2 kW and full-load copper loss of 1 kW. Calculate the efficiency of the transformer at (i) full-load unity pf, and (ii) half-load unity pf. *[97.087%, 95.69%]*

6.2 Calculate the efficiency of a transformer at half load and quarter load for 0.71 lagging pf for a 800 kVA, 1100/250 V, 50 Hz single-phase transformer, whose losses are as follows: Iron loss = 800 W and copper loss at full load = 800 W. *[99.64%, 99.40%]*

6.3 A 50 kVA, 2300/230 V, 50 Hz, single-phase transformer has a primary resistance of 2 Ω and a secondary resistance of 0.02 Ω. Calculate the efficiency of the transformer at (i) full load, and (ii) half load when the pf of the load is 0.8. Given that the iron loss is 412 watts. *[94.56%, 95.76%]*

6.4 A 40 kVA transformer has iron loss of 450 W and full-load copper loss of 850 W. If power factor of the load is 0.8 lagging, calculate (i) full-load efficiency, (ii) the load at which maximum efficiency occurs, and (iii) the maximum efficiency. *[96.09%, 29.104 kVA, 96.278%]*

6.5 A 50 kVA transformer has an efficiency of 98% at full load 0.8 pf. and 97% at half load 0.8 pf. Determine full-load copper loss and iron loss. Find the load at which maximum efficiency occurs and also find the maximum efficiency.

[0.264 kW, 0.552 kW, 72.29 kVA, 98.12%]

6.6 The efficiency of a 220 kVA, 1100/220 V transformer is maximum of 98% at 50% of rated load. Calculate (i) core loss, and (ii) efficiency at rated load.

[1.12 kW, 97.51%]

6.7 Calculate the efficiencies at half, full and $1\frac{1}{4}$ load of a 100 kVA transformer for power factors of unity. The copper loss is 1000 W at full load and the iron loss is 1000 W. *[97.56%, 98.04%, 97.98%]*

6.8 In a 25 kVA, 2000/200 V transformer, the iron and copper losses are 350 W and 400 W respectively. Calculate the efficiency on unity power factor at (i) full load, (ii) half load, and (iii) determine the load for maximum efficiency and the iron and copper loss in this case. *[97.1%, 96.5%, 23.4 kVA, 350 W, 350 W]*

6.9 The efficiency of 400 kVA, 50 Hz, 1 phase transformer is 98.77% delivering FL at 0.8 pf and 99.13% at half load at UPF. Determine maximum efficiency at 0.8 pf.

[1.012 kW, 2.973 kW, 98.93%]

6.10 Calculate the efficiency at full load and one-fourth load at (i) unity pf, and (ii) 0.71 lagging pf, for a 80 kVA, 1100/250 V, 50 Hz single-phase transformer, whose losses are as follows:

Iron losses = 800 W

Total copper losses with 160 A in the low-voltage winding = 200 W

[(i) 98.04%, 97.57%, 95.92%, (ii) 97.25%, 96.61%, 94.36%]

6.11 A 200 kVA transformer has an efficiency of 98% at full load. If the maximum efficiency occurs at three quarters of full load, calculate the efficiency at half load. Assume negligible magnetising current and pf at all loads. *[97.9%]*

6.14 OPEN CIRCUIT (OC) TEST

[Dec 2014, 2015]

The purpose of this test is to determine (i) iron loss or core loss (W_i) (ii) magnetising resistance R_0, and (iii) magnetising reactance X_0.

Figure 6.21 shows the circuit diagram for conducting OC test on the transformer. In this test, one winding (usually high-voltage winding) is left open and the other winding is connected to a supply of normal voltage and frequency. An ammeter, voltmeter and wattmeter are connected on this side. The ammeter indicates no-load current drawn by the transformer. As the no-load current is usually 3 to 5% of the full-load current, copper losses are negligible and the wattmeter indicates iron loss.

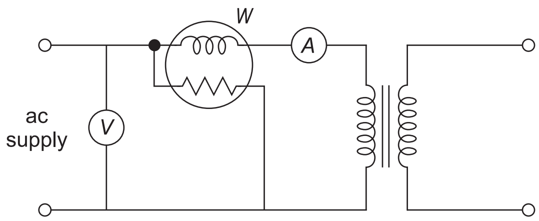


Fig. 6.21 *O.C. test*

Calculations

(i) When meters are connected on the primary side

Wattmeter reading = W_i

Voltmeter reading = V_1

Ammeter reading = I_0

$$W_i = V_1 I_0 \cos \phi_0$$

$$\cos \phi_0 = \frac{W_i}{V_1 I_0}$$

$$I_w = I_0 \cos \phi_0 \qquad I_\mu = I_0 \sin \phi_0$$

$$R_0 = \frac{V_1}{I_w}$$

$$X_0 = \frac{V_1}{I_\mu}$$

(ii) When meters are connected on the secondary side

Wattmeter reading = W_i

Voltmeter reading = V_2

Ammeter reading = I'_0

$$W_i = V_2 I'_0 \cos \phi'_0$$

$$\cos \phi'_0 = \frac{W_i}{V_2 I'_0}$$

$$I'_w = I'_0 \cos \phi'_0 \qquad I'_\mu = I'_0 \sin \phi'_0$$

$$R'_0 = \frac{V_2}{I'_w} \qquad X'_0 = \frac{V_2}{I'_\mu}$$

$$R_0 = \frac{R'_0}{K^2} \qquad X_0 = \frac{X'_0}{K^2}$$

6.15 SHORT-CIRCUIT (SC) TEST

[May 2015, Dec 2015]

The purpose of this test is to determine (i) full-load copper loss, (ii) equivalent resistance R_{01} or R_{02}, and (iii) equivalent reactance X_{01} or X_{02}.

Figure. 6.22 shows the circuit diagram for conducting an SC test on the transformer. In this test, one winding (usually low-voltage winding) is short circuited, while a low voltage is applied to the other winding. The applied voltage is slowly increased until full-load current flows in this winding and hence, through the other winding. Normally, the applied voltage is 5 to 10% of the rated voltage of this winding. Hence, fluxes produced in the core are small and the iron losses are very small. Thus, wattmeter indicates full-load copper loss.

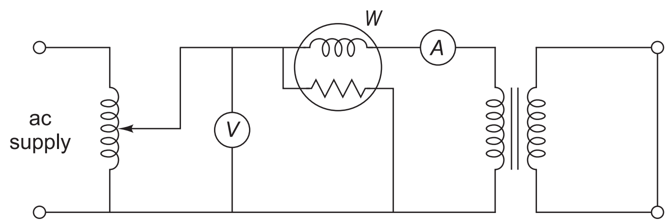


Fig. 6.22 *SC test*

Calculations

Wattmeter reading = W_{sc}
Voltmeter reading = V_{sc}
Ammeter reading = I_{sc}

(i) When meters are connected on the primary side

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2}$$

(ii) When meters are connected on the secondary side

$$Z_{02} = \frac{V_{sc}}{I_{sc}}$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2}$$

Example 1

A 5 kVA, 1000/200 V, 50 Hz, single-phase transformer gives the following test results:

OC test (LV side)	*200 V,*	*1.2 A,*	*90 W*
SC test (HV side)	*50 V,*	*5 A,*	*110 W*

Determine efficiency at half load at 0.8 pf lagging. **[May 2013]**

Solution From OC test (meters are connected on LV side, i.e., secondary),

$$W_i = 90\text{ W} = 0.09\text{ kW}$$

From SC test (meters are connected on HV side, i.e., primary),

$$W_{sc} = 110\text{ W}$$

$$\text{Full-load current } I_1 = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$I_1 = \frac{5 \times 1000}{1000} = 5\text{ A}$$

$$W_{Cu} = W_{sc} = 110\text{ W} = 0.11\text{ kW}$$

Efficiency at half load and 0.8 pf lagging

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\,\eta = \frac{x \times \text{Full-load kVA} \times \text{pf}}{x \times \text{Full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 5 \times 0.8}{0.5 \times 5 \times 0.8 \times 0.09 + (0.5)^2 + 0.11} \times 100$$

$$= 94.45\ \%$$

Example 2

A 5 kVA, 1000 V/200 V, 50 Hz, 1-ϕ transformer gave the following test results:

OC test (HV side)	*1000 V,*	*0.24 A,*	*90 W*
SC test (HV side)	*50 V,*	*5 A,*	*110 W*

Calculate

(i) Equivalent circuit parameters referred to LV side

(ii) Regulation at half load at 0.8 lagging power factor. **[May 2015, Dec 2015, May 2016]**

Solution

(i) Equivalent circuit parameters referred to LV side

From OC test (meters are connected on HV side, i.e., primary),

$$W_i = 90\text{ W} \qquad V_1 = 1000\text{ V} \qquad I_0 = 0.24\text{ A}$$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{90}{1000 \times 0.24} = 0.375$$

$$\sin\phi_0 = 0.927$$

$$I_w = I_0 \cos\phi_0 = 0.24 \times 0.375 = 0.09 \text{ A}$$
$$I_\mu = I_0 \sin\phi_0 = 0.24 \times 0.927 = 0.22 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{1000}{0.09} = 11111.11\ \Omega$$
$$X_0 = \frac{V_1}{I_\mu} = \frac{1000}{0.22} = 4545.45\ \Omega$$

From SC test (meters are connected on HV side, i.e., primary),

$W_{sc} = 110$ W $\quad V_{sc} = 50$ V $\quad I_{sc} = 5$ A

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{50}{5} = 10\ \Omega$$

$$R_{01} = \frac{V_{sc}}{I_{sc}^2} = \frac{110}{(5)^2} = 4.4\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2}$$
$$= \sqrt{(10)^2 - (4.4)^2} = 8.98\ \Omega$$

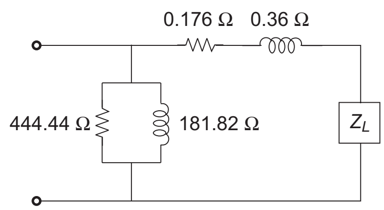


Fig. 6.23

$$K = \frac{E_2}{E_1} = \frac{200}{1000} = 0.2$$

$$R_0' = K^2 R_0 = (0.2)^2 \times 11111.11 = 444.44\ \Omega$$

$$X_0' = K^2 X_0 = (0.2)^2 \times 4545.45 = 181.82\ \Omega$$

$$R_{02} = K^2 R_{01} = (0.2)^2 \times 4.4 = 0.176\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.2)^2 \times 8.98 = 0.36\ \Omega$$

(ii) Regulation at half load at 0.8 lagging power factor

$$I_2 = \frac{5 \times 1000}{200} = 25 \text{ A}$$

At half load, $I_{2(HL)} = 12.5$ A

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$$\% \text{ Regulation} = \frac{I_{2(\text{HL})}\,(R_{02}\cos\phi + X_{02}\sin\phi)}{E_2} \times 100$$

$$= \frac{12.5(0.176 \times 0.8 + 0.36 \times 0.6)}{200} \times 100$$

$$= 2.23\%$$

Example 3

A 5 kVA, 200/400 V, 50 Hz, single-phase transformer gives the following test results:

OC test (L V side)	*200 V,*	*0.7 A,*	*60 W*
SC test (H V side)	*22 V,*	*16 A,*	*120 W*

(i) Draw the equivalent circuit of the transformer and insert all parameter values.
(ii) Find efficiency and regulation at 0.9 pf (lead) if operating at rated load.
(iii) Find current at which efficiency is maximum. **[May 2014]**

Solution

(i) Equivalent circuit of the transformer

From OC test (meters are connected on LV side, i.e., primary),

$$W_i = 60 \text{ W} \qquad V_1 = 200 \text{ V} \qquad I_0 = 0.7 \text{ A}$$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{200 \times 0.7} = 0.43$$

$$\sin\phi_0 = 0.9$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.43 = 0.3 \text{ A}$$

$$R_0 = \frac{V_1}{I_W} = \frac{200}{0.3} = 666.67\ \Omega$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.9 = 0.63 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.63} = 317.46\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$$W_{sc} = 120 \text{ W} \qquad V_{sc} = 22 \text{ V} \qquad I_{sc} = 16 \text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{22}{16} = 1.375\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(16)^2} = 0.47\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.375)^2 - (0.47)^2} = 1.29\ \Omega$$

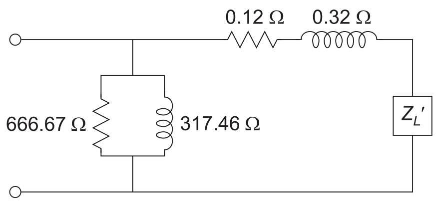


Fig. 6.24

$$K = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.47}{(2)^2} = 0.12\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.29}{(2)^2} = 0.32\ \Omega$$

(ii) Efficiency at rated load and 0.9 pf leading

$$W_i = 60 \text{ W} = 0.06 \text{ kW}$$

Since meters are connected on secondary in SC test,

$$I_2 = \frac{5 \times 1000}{400} = 12.5 \text{ A}$$

$$W_{Cu} = I_2^2 R_{02} = (12.5)^2 \times 0.47 = 73.43 \text{ W} = 0.073 \text{ kW}$$

$$x = 1$$

$$\text{pf} = 0.9$$

$$\%\ \eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 5 \times 0.9}{1 \times 5 \times 0.9 + 0.06 + (1)^2 \times 0.073} \times 100$$

$$= 97.13\%$$

Regulation at rated load and 0.9 pf lead

$$\cos\phi = 0.9$$

$$\sin\phi = 0.44$$

$$\%\ \text{regulation} = \frac{I_2 (R_{02} \cos\phi - X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{12.5(0.47 \times 0.9 - 1.29 \times 0.44)}{400} \times 100$$

$$= -0.45\ \%$$

(iii) Current at maximum efficiency

$$W_i = I_2^2 R_{02}$$

$$I_2 = \sqrt{\frac{W_i}{R_{02}}} = \sqrt{\frac{60}{0.47}} = 11.3 \text{ A}$$

Example 4

A 5 kVA, 250/500 V, 50 Hz, single-phase transformer gives the following test results:

No-load test (LV side)	*250 V,*	*0.75 A,*	*60 W*
Short Circuit test (HV side)	*9 V,*	*6 A,*	*21.6 W*

Calculate (i) The equivalent circuit constants and insert these on the equivalent circuit diagram

(ii) Efficiency at 60% of full-load unity pf

(iii) Maximum efficiency and the load at which it occurs

(iv) The secondary terminal voltage on full load at pf of 0.8 lagging, unity and 0.8 leading

Solution

(i) Equivalent circuit constants

From no-load test (meters are connected on LV side, i.e., primary),

$$W_i = 60\text{ W} \qquad V_1 = 250\text{ V} \qquad I_0 = 0.75\text{ A}$$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{250\times 0.75} = 0.32$$

$$\sin\phi_0 = 0.95$$

$$I_w = I_0\cos\phi_0 = 0.75\times 0.32 = 0.24\text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.24} = 1041.66\ \Omega$$

$$I_\mu = I_0\sin\phi_0 = 0.75\times 0.95 = 0.71\text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{250}{0.71} = 351.84\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$$W_{sc} = 21.6\text{ W} \qquad V_{sc} = 9\text{ V} \qquad I_{sc} = 6\text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{9}{6} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{21.6}{(6)^2} = 0.6\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.6)^2} = 1.37\ \Omega$$

$$K = \frac{500}{250} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.6}{(2)^2} = 0.15\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.37}{(2)^2} = 0.34\ \Omega$$

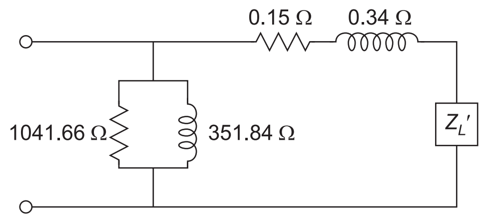


Fig. 6.25

(ii) Efficiency at 60 % of full load and unity factor

$$x = 0.6$$

$$W_i = 60 \text{ W} = 0.06 \text{ kW}$$

Since meters are connected on HV side in SC test,

$$I_2 = \frac{5 \times 1000}{500} = 10 \text{ A}$$

$$W_{Cu} = I_2^2 R_{02} = (10)^2 \times 0.6 = 60 \text{ W} = 0.06 \text{ kW}$$

$$\text{pf} = 1$$

$$\% \ \eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.6 \times 5 \times 1}{0.6 \times 5 \times 1 + 0.06 + (0.6)^2 \times 0.06} \times 100$$

$$= 97.35 \ \%$$

(iii) The load corresponding to maximum efficiency

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 5 \times \sqrt{\frac{60}{60}} = 5 \text{ kVA}$$

For maximum efficiency,

$$W_i = W_{Cu} = 60 \text{ W} = 0.06 \text{ kW}$$

$$\text{pf} = 1$$

$$\% \ \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{5 \times 1}{5 \times 1 + 0.06 + 0.06} \times 100$$

$$= 97.65 \ \%$$

(iv) Secondary terminal voltage

$$E_2 - V_2 = I_2 (R_{02} \cos \phi \pm X_{02} \sin \phi)$$

$$V_2 = E_2 - I_2 (R_{02} \cos \phi \pm X_{02} \sin \phi)$$

For $\quad \text{pf} = 0.8$ lagging

$$\cos \phi = 0.8$$

$$\sin\phi = 0.6$$

$$E_2 = 500 \text{ V}$$

$$V_2 = 500 - 10(0.6 \times 0.8 + 1.37 \times 0.6) = 486.98 \text{ V}$$

For pf = 0.8 leading,

$$V_2 = 500 - 10(0.6 \times 0.8 - 1.37 \times 0.6) = 503.42 \text{ V}$$

For unity pf,

$$V_2 = 500 - 10(0.6 \times 0.8 + 0) = 494 \text{ V}$$

Example 5

A single-phase, 50 kVA, 2400/120 V, 50 Hz transformer gives the following results:

OC test with instruments on LV side	*120 V,*	*9.65 A,*	*396 W*
SC test with instruments on HV side	*92 V,*	*20.8 A,*	*810 W*

Calculate (i) Equivalent circuit constants
(ii) Draw equivalent circuit
(iii) The efficiency when rated kVA is delivered to a load having a pf of 0.8 lagging
(iv) The voltage regulation
(v) kVA at maximum efficiency

Solution (i) Equivalent circuit constants

From OC test (meters are connected on LV side, i.e., secondary),

$$W_i = 396 \text{ W} \qquad V_2 = 120 \text{ V} \qquad I'_0 = 9.65 \text{ A}$$

$$\cos\phi'_0 = \frac{396}{120 \times 9.65} = 0.34$$

$$\sin\phi'_0 = 0.94$$

$$I'_w = I'_0 \cos\phi'_0 = 9.65 \times 0.34 = 3.28 \text{ A}$$

$$R'_0 = \frac{V_2}{I'_w} = \frac{120}{3.3} = 36.36\ \Omega$$

$$I'_\mu = I'_0 \sin\phi'_0 = 9.65 \times 0.94 = 9.07 \text{ A}$$

$$X'_0 = \frac{V_2}{I'_\mu} = \frac{120}{9.07} = 13.23\ \Omega$$

$$K = \frac{120}{2400} = 0.05$$

$$R_0 = \frac{R'_0}{K^2} = \frac{36.36}{(0.05)^2} = 14.54 \text{ k}\Omega$$

$$X_0 = \frac{X'_0}{K^2} = \frac{13.23}{(0.05)^2} = 5.29 \text{ k}\Omega$$

From SC test (meters are connected on primary),

$$W_{sc} = 810 \text{ W} \qquad V_{sc} = 92 \text{ V} \qquad I_{sc} = 20.8 \text{ A}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{92}{20.8} = 4.42\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{810}{(20.8)^2} = 1.87\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(4.42)^2 - (1.87)^2} = 4\ \Omega$$

(ii) Equivalent circuit

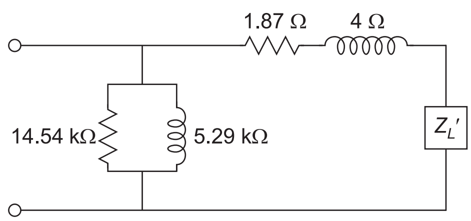


Fig. 6.26

(iii) Efficiency at full load and 0.8 pf lagging

$$x = 1$$

$$W_i = 396 \text{ W} = 0.396 \text{ kW}$$

Since meters are connected on the primary side in SC test,

$$I_1 = \frac{50 \times 1000}{2400} = 20.8 \text{ A}$$

$$W_{Cu} = I_1^2 R_{01} = (20.8)^2 \times 1.87 = 809 \text{ W} = 0.81 \text{ kW}$$

$$\text{pf} = 0.8$$

$$\%\ \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 50 \times 0.8}{1 \times 50 \times 0.8 + 0.396 + (1)^2 \times 0.81} \times 100$$

$$= 97.07\ \%$$

(iv) Voltage regulation

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

$$\% \text{ regulation} = \frac{I_1(R_{01}\cos\phi + X_{01}\sin\phi)}{V_1} \times 100$$

$$= \frac{20.8(1.87 \times 0.8 + 4 \times 0.6)}{2400} \times 100$$

$$= 3.38 \ \%$$

(v) Load kVA at maximum efficiency

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 50 \times \sqrt{\frac{0.396}{0.81}} = 34.96 \text{ kVA}$$

Example 6

The instrument readings obtained from open-circuit test and short-circuit test on a 10 kVA, 450/120 V, 50 Hz single-phase ac transformer are as follows:

OC test (LV side)	$V_0 = 120$ V	$I'_0 = 4.2$ A	$W_0 = 80$ W
SC test (HV side)	$V_{sc} = 9.65$ V	$I_{sc} = 22.2$ A	$W_{sc} = 120$ W

Compute the following: (i) Equivalent circuit constants

(ii) Draw the equivalent circuit

(iii) Efficiency and voltage regulation at 80 % lagging pf load

(iv) Efficiency at half full load at 80 % lagging pf load

(v) The maximum efficiency at 0.8 pf lagging

Solution (i) Equivalent circuit constants

From OC test (meters are connected on LV side, i.e., secondary),

$W_0 = 80$ W $\qquad V_0 = 120$ V $\qquad I'_0 = 4.2$ A

$$\cos\phi'_0 = \frac{W_0}{V_0 I'_0} = \frac{80}{120 \times 4.2} = 0.16$$

$$\sin\phi'_0 = 0.99$$

$$I'_w = I'_0 \cos\phi'_0 = 4.2 \times 0.16 = 0.67 \text{ A}$$

$$R'_0 = \frac{V_0}{I'_w} = \frac{120}{0.67} = 180.03 \ \Omega$$

$$I'_\mu = I'_0 \sin\phi'_0 = 4.2 \times 0.99 = 4.16 \text{ A}$$

$$X'_0 = \frac{V_0}{I'_\mu} = \frac{120}{4.16} = 28.85 \ \Omega$$

$$K = \frac{120}{450} = 0.27$$

$$R_0 = \frac{R'_0}{K^2} = \frac{180.03}{(0.27)^2} = 2469.55 \ \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{28.85}{(0.27)^2} = 395.75\ \Omega$$

From SC test (meters are connected on HV side, i.e., primary),

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{9.65}{22.2} = 0.43\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(22.2)^2} = 0.24\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(0.43)^2 - (0.24)^2} = 0.36\ \Omega$$

(ii) Equivalent circuit

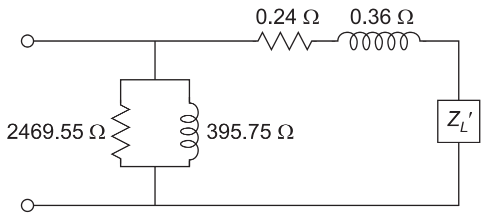


Fig. 6.29

(iii) Efficiency and voltage regulation at 80% lagging pf load

$$W_0 = 80\ \text{W} = 0.08\ \text{kW}$$

Since meters are connected on the primary side in SC test,

$$I_1 = \frac{10 \times 1000}{450} = 22.22\ \text{A}$$

$$W_{Cu} = I_1^2 R_{01} = (22.22)^2 \times 0.24 = 118.49 = 0.12\ \text{kW}$$

Efficiency at 80% lagging pf load

$$x = 1$$

$$\text{pf} = 0.8$$

$$\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 10 \times 0.8}{1 \times 10 \times 0.8 + 0.08 + (1)^2 \times 0.12} \times 100$$

$$= 97.56\ \%$$

Voltage regulation

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

$$\% \text{ regulation} = \frac{I_1 (R_{01} \cos\phi + X_{01} \sin\phi)}{V_1} \times 100$$

$$= \frac{22.22(0.24 \times 0.8 + 0.36 \times 0.6)}{450} \times 100$$

$$= 2.01\ \%$$

(iv) Efficiency at half full load for 80 % lagging pf load

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\% \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 10 \times 0.8}{0.5 \times 10 \times 0.8 + 0.08 + (0.5)^2 \times 0.12} \times 100$$

$$= 97.34\ \%$$

(v) Maximum efficiency at 0.8 pf lagging

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 10 \times \sqrt{\frac{80}{120}} = 8.16 \text{ kVA}$$

$$\% \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{8.16 \times 0.8}{8.16 \times 0.8 + 0.08 + 0.08} \times 100$$

$$= 97.61\ \%$$

Example 7

Obtain the equivalent circuit of a 200/400 V, 50 Hz, single-phase transformer from the following test:

OC test	*200 V,*	*0.7 A,*	*70 W*	*on LV side*
SC test	*15 V,*	*10 A,*	*85 W*	*on HV side*

Calculate the secondary voltage when delivering 5 kW, 0.8 pf lagging, the primary voltage being 200 V.

Solution From OC test (meters are connected on LV side, i.e., primary),

$$W_i = 70 \text{ W}, \quad V_1 = 200 \text{ V}, \quad I_0 = 0.7 \text{ A}$$

$$\cos\phi = \frac{W_i}{V_i I_0} = \frac{70}{200 \times 0.7} = 0.5$$

$$\sin\phi = 0.87$$

$$I_w = I_0 \cos \phi_0 = 0.7 \times 0.5 = 0.35 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.43\ \Omega$$

$$I_\mu = I_0 \sin \phi_0 = 0.7 \times 0.87 = 0.61 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.61} = 327.87\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$$W_{sc} = 85 \text{ W} \quad V_{sc} = 15 \text{ V} \quad I_{sc} = 10 \text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{85}{(10)^2} = 0.85\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.85)^2} = 1.24\ \Omega$$

$$K = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.85}{(2)^2} = 0.21\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.24}{(2)^2} = 0.31\ \Omega$$

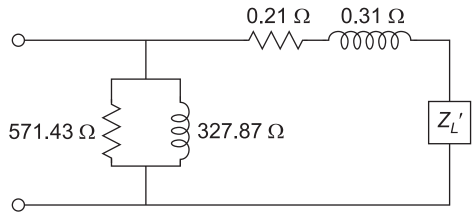


Fig. 6.28

$$I_2 = \frac{5000}{400 \times 0.8} = 15.63 \text{ A}$$

$$V_2 = E_2 - I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$$
$$= 400 - 15.63\ (0.85 \times 0.8 + 1.24 \times 0.6)$$
$$= 377.74 \text{ V}$$

Example 8

A 40 kVA single-phase transformer with voltages of 11 kV/440 V has the following test results:

OC test (Instruments on LV side)	*440 V,*	*1.1 A,*	*145 W*
SC test (Instruments on HV side)	*100 V,*	*3 A,*	*90 W*

Deduce an approximate circuit referred to the LV side. For this transformer, calculate maximum efficiency.

Solution (i) Approximate equivalent circuit

From OC test (meters are connected on LV side, i.e., secondary).

$$W_0 = 145 \text{ W} \qquad V_0 = 440 \text{ V} \qquad I_0' = 1.1 \text{ A}$$

$$\cos \phi'_0 = \frac{W_0}{V_0 I_0'} = \frac{145}{440 \times 1.1} = 0.3$$

$$\sin \phi'_0 = 0.95$$

$$I'_w = I'_0 \cos \phi'_0 = 1.1 \times 0.3 = 0.33 \text{ A}$$

$$R'_0 = \frac{V_0}{I'_w} = \frac{440}{0.33} = 1333.33\ \Omega$$

$$I'_\mu = I'_0 \sin \phi'_0 = 1.1 \times 0.95 = 1.05 \text{ A}$$

$$X'_0 = \frac{V_0}{I'_\mu} = \frac{440}{1.05} = 419.05\ \Omega$$

From SC test (meters are connected on the HV side, i.e., primary)

$$W_{sc} = 90 \text{ W} \qquad V_{sc} = 100 \text{ V} \qquad I_{sc} = 3 \text{ A}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{100}{3} = 33.33\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{90}{(3)^2} = 10\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(33.33)^2 - (10)^2} = 31.79\ \Omega$$

$$K = \frac{440}{11 \times 10^3} = 0.04$$

$$R_{02} = K^2 R_{01} = (0.04)^2 \times 10 = 0.016\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.04)^2 \times 31.79 = 0.05\ \Omega$$

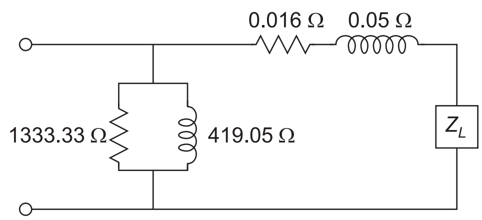


Fig. 6.29

(ii) Maximum efficiency

$$W_i = 145 \text{ W}$$

Since meters are connected on the primary in SC test,

$$I_1 = \frac{40 \times 1000}{11 \times 1000} = 3.64 \text{ A}$$

$$W_{Cu} = I_1^2 R_{01} = (3.64)^2 \times 10 = 132.5 \text{ W}$$

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 40 \times \sqrt{\frac{145}{132.5}} = 41.84 \text{ kVA}$$

$$\% \, \eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{41.84 \times 1}{41.84 \times 1 + 0.145 + 0.145} \times 100$$

$$= 99.31 \, \%$$

Example 9

The results of OC and SC test on a 25 kVA, 440/220V, 50 Hz transformer are on follows:

OC test	*220 V,*	*9.6 A,*	*710 W,*	*LV side*
SC test	*42 V,*	*57 A,*	*1030 W,*	*HV side*

Obtain the parameters of the exact equivalent circuit referred to the high-voltage side.

Solution From OC test (meters are connected on the LV side, i.e., secondary),

$$W_i = 710 \text{ W} \qquad V_2 = 220 \text{ V} \qquad I_0' = 9.6 \text{ A}$$

$$\cos \phi_0' = \frac{W_i}{V_2 I_0'} = \frac{710}{220 \times 9.6} = 0.34$$

$$\sin \phi_0' = 0.94$$

$$I_w' = I_0' \cos \phi_0' = 9.6 \times 0.34 = 3.26 \text{ A}$$

$$R_0' = \frac{V_2}{I_w'} = \frac{220}{3.26} = 67.48 \, \Omega$$

$$I_\mu' = I_0' \sin \phi_0' = 9.6 \times 0.94 = 9.02 \text{ A}$$

$$X_0' = \frac{V_2}{I_\mu'} = \frac{220}{9.02} = 24.39 \, \Omega$$

$$K = \frac{220}{440} = 0.5$$

$$R_0 = \frac{R_0'}{K^2} = \frac{67.48}{(0.5)^2} = 269.92 \, \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{24.39}{(0.5)^2} = 97.56 \, \Omega$$

From SC test (meters are connected on HV side, i.e., primary).

$$W_{sc} = 1030 \text{ W} \qquad V_{sc} = 42 \text{ V} \qquad I_{sc} = 57 \text{ A}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{42}{57} = 0.74 \, \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{1030}{(57)^2} = 0.32\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.74)^2 - (0.32)^2} = 0.67\ \Omega$$

Example 10

The two windings of a 2400/240 V, 48 kVA, 50 Hz transformer have resistances of 0.6 and 0.025 Ω for the high and low-voltage winding respectively. The transformer requires that 238 V be impressed on the high-voltage coil in order that the rated current be circulated in the short-circuit low-voltage winding.

(i) Calculate the equivalent leakage reactance referred to high-voltage side.

(ii) How much power is needed to circulate valid current on short circuit?

(iii) Compute the efficiency at full load when the pf is 0.8 lagging. Assume that core loss equals the copper loss.

Solution (i) The equivalent leakage reactance referred to high-voltage side.

$$R_1 = 0.6\ \Omega$$

$$R_2 = 0.025\ \Omega$$

$$K = \frac{240}{2400} = 0.1$$

Since rated current flows in short-circuited low voltage winding, i.e., primary winding,

$$I_1 = \frac{48000}{2400} = 20\text{ A}$$

$$I_{sc} = I_1 = 20\text{ A}$$

$$V_{sc} = 238\text{ V}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{238}{20} = 11.9\ \Omega$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.6 + \frac{0.025}{(0.1)^2} = 3.1\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(11.9)^2 - (3.1)^2} = 11.49\ \Omega$$

(ii) Power needed to circulate valid current on short circuit

$$W_{sc} = I^2{}_{sc}\, R_{01} = (20)^2 \times 3.1 = 1240\text{ W}$$

(iii) Efficiency at full load

$$x = 1$$

$$\text{pf} = 0.8$$

$$\% \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 48 \times 0.8}{1 \times 48 \times 0.8 + 1.24 + (1)^2 \times 1.24} \times 100$$

$$= 93.93\%$$

Exercise 6.4

6.1 A 4 kVA, 200/400 V, 50 Hz, single-phase transformer gave the following test results:

OC test (LV side)	200 V,	0.7 A,	60 W
SC test (HV side)	9 V,	6 A,	21.6 W

Calculate

(i) magnetising current and the component corresponding to iron loss at normal voltage and frequency

(ii) efficiency on full load at unity pf

(iii) secondary terminal voltage on full load at power factors of unity, 0.8 lagging and 0.8 leading

[(i) 0.63 A, 0.3 A, (ii) 97.08%, (iii) 394 V, 386.95 V, 403.44 V]

6.2 A 10 kVA, 500/2000 V, 50 Hz single-phase transformer gave the following results:

OC test : 500 V, 120 W on primary side

SC test : 15 V, 20 A, 100 W on primary side

Determine

(i) Efficiency on full-load unity power factor

(ii) Secondary terminal voltage on full load at unity pf, 0.8 lagging and 0.8 leading pf *[97.9%, 1980 V, 1950 V, 2018 V]*

6.3 A 100 kVA, 6600/330 V, 50 Hz, single-phase transformer took 10 A and 436 W at 100 V in a short-circuit test, the figures referring to the high-voltage side. Calculate the voltage to be applied to the high-voltage side on full load at power factor 0.8 lagging when the secondary terminal voltage is 330 V. *[6734 V]*

6.4 A 50 Hz, single-phase transformer has a turn ratio of 6. The resistances are 0.9 Ω and 0.03 Ω and the reactances are 5 Ω and 0.13 Ω for high-voltage and low-voltage winding respectively. Find (i) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short circuit, (ii) power factor in the short circuit. *[330 V, 0.2]*

6.5 A 5 kVA, 1000/200 V, 50 Hz, single-phase transformer gave the following test results:

OC test (LV side)	200 V,	1.2 A,	90 W
SC test (HV side)	50 V,	5 A,	110 W

Determine efficiency at half-load at 0.8 pf lagging. *[94.45%]*

6.6 Draw the equivalent circuit of transformer referred to primary side of 4 kVA, 200/400 V, 50 Hz transformer which gave the following test results:

OC test (on LV side)	200 V,	0.7 A,	70 W
SC test (on HV side)	15 V,	10 A,	80 W

[571 Ω, 330 Ω, 0.2 Ω, 0.317 Ω]

6.7 The following figures were obtained from tests on a 30 kVA, 3000/110 V transformer:

OC test	3000 V,	0.5 A,	350 W
SC test	150 V,	10 A,	500 W

Calculate the efficiency of the transformer at (a) full load 0.8 pf, (b) half-load unity pf. Also, calculate the kVA output at which the efficiency is maximum.

[96.56%, 97%, 25.1 kVA]

Review Questions

6.1 Explain the working principle of a transformer.

6.2 Explain what happens if a dc voltage is applied to a transformer.

6.3 Differentiate between shell-type and core-type transformers.

6.4 Derive an emf equation for a single-phase transformer and explain voltage and current ratio of an ideal transformer.

6.5 Show that the emf per turn in a transformer is 4.44 $f\,\phi_m$, where f is the frequency of supply and ϕ_m is maximum flux associated with transformer winding.

6.6 What do you understand by an ideal transformer?

6.7 Draw and explain phasor diagram of a transformer for

(a) Unity power factor or resistive load

(b) Lagging power factor or inductive load

(c) Leading power factor or capacitive load

6.8 Develop the approximate equivalent circuit of a transformer. How does it help in deciding the regulation of a transformer?

6.9 Define voltage regulation and derive its expression.

6.10 Explain various losses in a transformer.

6.11 What do you understand by efficiency of a transformer? Derive the condition for maximum efficiency.

Multiple Choice Questions

Choose the correct alternative in the following questions:

6.1 When the primary of a transformer is connected to a dc supply, the
- (a) primary draws small current
- (b) primary leakage reactance is increased
- (c) core losses are increased
- (d) primary may burn out

6.2 The function of oil in a transformer is
- (a) to provide insulation and cooling
- (b) to provide protection against lightning
- (c) to provide protection against short circuit
- (d) to provide lubrication

6.3 In a transformer, the primary and the secondary voltages are
- (a) 60° out of phase
- (b) 90° out of phase
- (c) 180° out of phase
- (d) always in phase

6.4 The core flux of a practical transformer with a resistive load
- (a) is strictly constant with load changes
- (b) increases linearly with load
- (c) increases the square root of the load
- (d) decreases with increase of load

6.5 The inductive reactance of a transformer depends on
- (a) electromotive force
- (b) magnetomotive force
- (c) magnetic flux
- (d) leakage flux

6.6 For an ideal transformer the windings should have
- (a) maximum resistance on primary side and least resistance on secondary side
- (b) least resistance on primary side and maximum resistance on secondary side
- (c) equal resistance on primary and secondary side
- (d) no ohmic resistance on either side

6.7 If the applied voltage to a primary transformer is increased by keeping the V/f ratio fixed, then the magnetising current and the core loss will respectively:
- (a) decrease and remain the same
- (b) increase and decrease
- (c) both remain the same
- (d) remain the same and increase

6.8 If the applied voltage to a certain transformer is increased by 50% and the frequency is reduced to 50% (assuming that the magnetic circuit remains unsaturated), the maximum core flux density will

(a) change to three times the original value
(b) change to 1.5 times the original value
(c) change to 0.5 times the original value
(d) remain the same as the original value

6.9 The low-voltage winding of a 400 V/230 V, 1 phase, 50 Hz transformer is to be connected to a 25 Hz supply. In order to keep the magnetisation current at the same level in both the cases, the voltage at 25 Hz should be

(a) 230 V (b) 460 V (c) 115 V (d) 65 V

6.10 A voltage $v = 400 \sin 314.16t$ is applied to a 1-phase transformer on no-load. If the no-load current of the transformer is $2 \sin (314.16\, t - 85°)$, the magnetisation branch impedance will be approximately equal to

(a) $141 \angle 90° \ \Omega$ (b) $200 \angle -85° \ \Omega$ (c) $200 \angle 85° \ \Omega$ (d) $282 \angle -80° \ \Omega$

6.11 The eddy current losses in a transformer is reduced

(a) if laminations are thick
(b) if the number of turns in primary winding is reduced
(c) if the number of turns in secondary winding is reduced
(d) if laminations are thin

6.12 The efficiency of a transformer will be maximum when

(a) leakage reactances of the two windings are equal
(b) resistances of the two windings are equal
(c) copper loss is equal to constant loss
(d) none of the above

6.13 The efficiency of a transformer is usually in the range of

(a) 50 to 60% (b) 60 to 80% (c) 80 to 90% (d) 90 to 98%

6.14 The full-load copper loss of a transformer is twice its core loss. The efficiency will be maximum at

(a) 25% of full load (b) 50% of full load
(c) 70.7% of full load (d) 141% of full load

6.15 The full-load copper loss and iron loss of a transformer are 6400 W and 5000 W respectively. The copper loss and iron loss at half load will be respectively

(a) 3200 W and 2500 W (b) 3200 W and 5200 W
(c) 1600 W and 1250 W (d) 1600 W and 5000 W

6.16 The load at which maximum efficiency occurs in case of a 100 kVA transformer with iron loss of 1 kW and full load copper loss of 2 kW is

(a) 100 kVA (b) 70.7 kVA (c) 50.5 kVA (d) 25.2 kVA

6.17 A 300 kVA transformer has 95% efficiency at full load, 0.8 pf lagging and 96% efficiency at half load, unity pf. The iron loss W_i and copper loss W_{cu} in kW, under full-load operation are

(a) 4.12, 8.15 (b) 6.59, 9.21 (c) 8.51, 4.12 (d) 12.72, 3.07

6.18 A single-phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load at the same power factor is

(a) 86.7 % (b) 88.26 % (c) 88.9 % (d) 87.9 %

6.19 The efficiency of a 100 kVA transformer is 0.98 at full as well as at half load. For this transformer at full load, the copper loss

(a) is less than core loss (b) is equal to core loss

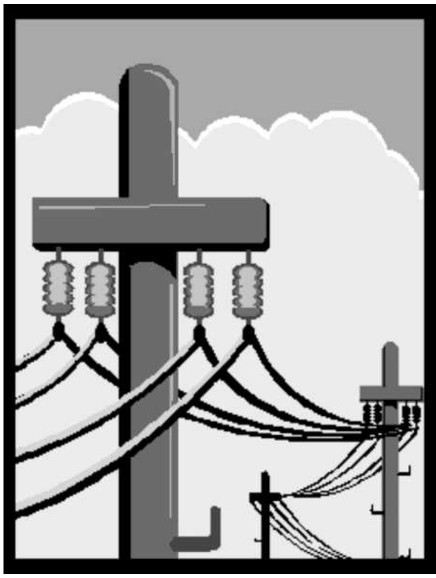
(c) is more than core loss (d) none of the above

6.20 If P_1 and P_2 be the iron and copper losses of a transformer at full-load and the maximum efficiency is at 75% of the full load, ratio of P_1 and P_2 will be

(a) $\frac{9}{16}$ (b) $\frac{10}{16}$ (c) $\frac{3}{4}$ (d) $\frac{3}{16}$

Answers to Multiple Choice Questions

6.1 (d)	**6.2** (a)	**6.3** (c)	**6.4** (a)	**6.5** (d)	**6.6** (d)
6.7 (d)	**6.8** (a)	**6.9** (c)	**6.10** (c)	**6.11** (d)	**6.12** (c)
6.13 (d)	**6.14** (c)	**6.15** (d)	**6.16** (b)	**6.17** (c)	**6.18** (d)
6.19 (c)	**6.20** (a)				



Chapter 7

DC Machines

Chapter Outline

7.1 DC MACHINES

A dc machine is a device that converts mechanical energy to electrical energy or vice versa. DC machines are divided into two categories:

(i) Generator
(ii) Motor

A generator is a machine that converts mechanical energy at its prime mover to produce electrical energy at its output. A motor is a machine that converts electrical energy at its input to produce mechanical energy.

7.2 PRINCIPLE OF OPERATIONS

A dc generator is a machine which converts mechanical energy into electrical energy, whereas a dc motor is a machine which converts electrical energy into mechanical energy. Construction wise there is no basic difference between a dc generator and a dc motor. Any dc machine can be used as a dc generator or as a dc motor.

Working Principle of a Generator When armature conductors are rotated externally in the magnetic field produced by field windings, an emf is induced in it according to Faraday's laws of electromagnetic induction. This emf causes a current to flow which is alternating in nature. It is converted in unidirectional current by the commutator.

Working Principal of a Motor When field winding is excited and armature conductors are connected across the supply, it experiences a mechanical force whose direction is given by Fleming's left-hand rule. Because of this force, the armature starts rotating. It cuts the magnetic field and an emf is induced in the armature winding. As per Lenz's law, this induced emf acts in the opposite direction to the armature supply voltage. This emf is known as back emf (E_b).

7.3 CONSTRUCTION

A dc machine, either a generator or a motor, essentially consists of the following main parts:

(i) Field windings
(ii) Armature core and armature windings
(iii) Commutator
(iv) Brushes

Field Windings The field windings are mounted on the pole core and they produce alternate north and south poles as seen in Fig. 7.1. The field windings form an electromagnet which provides the main magnetic field in the machine. When a current flows through the field winding, it establishes a magnetic flux. The windings are generally made of copper wire.

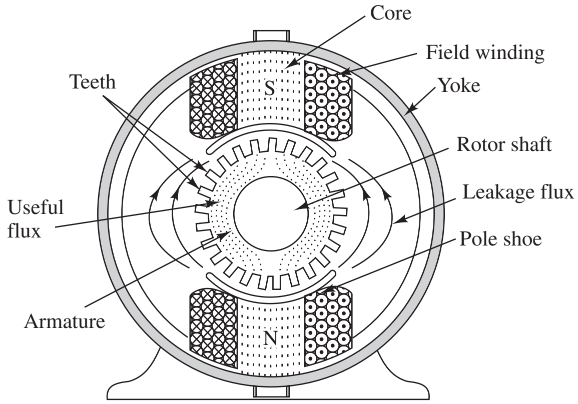


Fig. 7.1

Armature Core and Armature Windings The armature core shown in Fig. 7.2 is mounted on a shaft and rotates in the magnetic field. The armature core is made of laminations of sheet steel. The outer surface of the core is cylindrical in shape. It is provided with a large number of slots into which the armature conductors are placed. The ends of the armature windings are brought to the commutator segments as shown in Fig. 7.2.

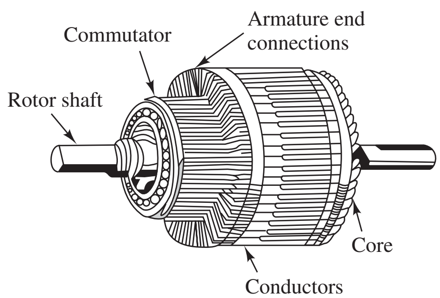


Fig. 7.2 Armature

Commutator A commutator is used for collecting current from the armature conductors. It is made of a number of wedge-shaped segments of copper. These segments are insulated from each other by thin layers of mica. Each commutator segment is connected to the armature conductor. The commutator converts the ac current induced in the armature conductors into unidirectional current across the brushes.

Brushes The brushes are used to collect current from the commutator and supply it to the external circuit. The brushes are usually made of carbon.

Types of Armature Winding

Depending upon the manner in which the armature conductors are connected to the commutator segments, there are the two types of armature winding:

Lap Winding In this arrangement, the armature conductors are connected in series through commutator segments in such a way that the armature winding is divided into as many parallel paths as the number of poles. If there are Z conductors and P poles, there will be P parallel paths, each containing Z/P conductors in series. The total emf is equal to the emf generated in any one of the parallel paths. The total armature current divides equally among the different parallel paths. It is used in low-voltage high-current machines.

Wave Winding In this arrangement, the armature conductors are connected in series through commutator segments in such a way that the armature winding is divided into two parallel paths irrespective of the number of poles. If there are Z conductors, $\frac{Z}{2}$ conductors will be in series in each parallel path. The total emf is equal to the emf generated in any one of the parallel paths. The total armature current divides equally between two parallel paths. It is used in high-voltage low-current machines.

7.4 CLASSIFICATION

Depending upon the method of excitation of field winding, dc machines are classified into two classes:

(i) Separately excited machines
(ii) Self-excited machines

Separately Excited Machines In a separately excited machine, the field winding is provided with a separate dc source to supply the field current as shown in Fig. 7.3.

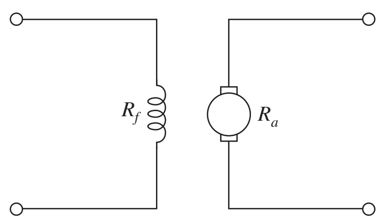


Fig. 7.3 Separately excited machine

Self-excited Machines In case of self-excited dc machines, no separate source is provided to drive the field current, but the field current is driven by its own emf generated across the armature terminals when the machine works as a generator. Self-excited machines are further classified into three types, depending upon the method in which the field winding is connected to the armature.

(a) Shunt-wound machines
(b) Series-wound machines
(c) Compound-wound machines

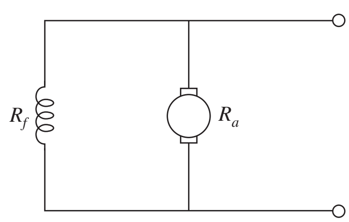


Fig. 7.4 Shunt wound machine

Shunt-wound Machines In this type of dc machines, field winding is connected in parallel with the armature as shown in Fig. 7.4. The number of turns of the field winding may range from 300 to 1000.

Series-wound Machines In this type of machines, the field winding is connected in series with the armature as shown in Fig. 7.5. This number of turns of the field winding is small (2 to 10 turns) and the field winding will have a heavy area of cross-section to carry the large armature current.

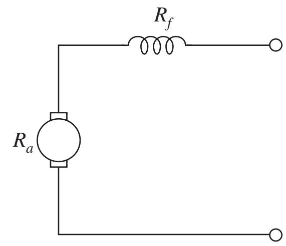


Fig. 7.5 Series-wound machine

Compound-wound Machines Compound machines carry both the shunt and series field windings. The compound machines are further classified into long shunt and short shunt, depending upon the direction of current flow in the two types of field windings as shown in Figs 7.6 and 7.7. Both the shunt field winding and the series field winding are generally wound on the same pole.

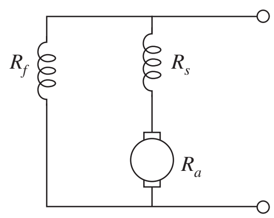


Fig. 7.6 Long-shunt compound wound machine

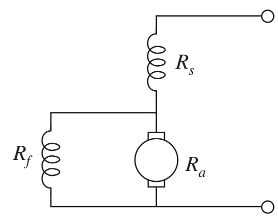


Fig. 7.7 Short-shunt compound machine

In case of a generator, load is connected across the armature whereas in case of a motor, dc supply is connected to the armature.

7.5 EMF EQUATION

Let ϕ = Flux per pole in webers

Z = Total number of armature conductors

N = Speed of the armature in revolutions per minute (rpm)

P = Number of poles

A = Number of parallel paths

When the armature completes one revolution, each conductor cuts the magnetic flux. Therefore, flux cut by one conductor in one revolution of the armature

$$= \text{Flux per pole} \times \text{Number of poles}$$

$$= \phi P \text{ webers}$$

Time taken to complete one revolution $= \dfrac{60}{N}$ seconds

Hence, average emf induced in one conductor

$$= \frac{\text{Flux cut}}{\text{Time taken}} = \frac{\phi P}{60/N}$$

$$= \frac{\phi PN}{60} \text{ volts}$$

Induced emf (E) = Resultant emf per parallel path

= Average emf per conductor × Number of conductors in series per parallel path

$$= \frac{\phi PN}{60} \times \frac{Z}{A}$$

$$= \frac{\phi ZN}{60} \frac{P}{A} \text{ volts}$$

In case of a dc generator, this emf is called generated emf (E_g). In the case of a dc motor, the induced emf opposes the applied emf and hence, it is called back emf (E_b).

7.6 VOLTAGE–CURRENT RELATIONSHIPS AND APPLICATIONS

(a) Generator When the machine runs as a generator, the generated emf (E_g) must be sufficient to supply both the terminal voltage (V) and the internal voltage drop. Voltage and current relationships for different types of generators are as follows:

Shunt generator

$$I_f = \frac{V}{R_f}$$

$$I_a = I_L + I_f$$

$$E_g = V + I_a R_a$$

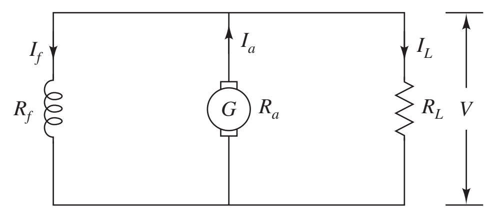


Fig. 7.8

Applications

(i) Ordinary lighting and power supply

(ii) Charging batteries

Series generator

$$I_a = I_s = I_L$$

$$E_g = V + I_a (R_a + R_s)$$

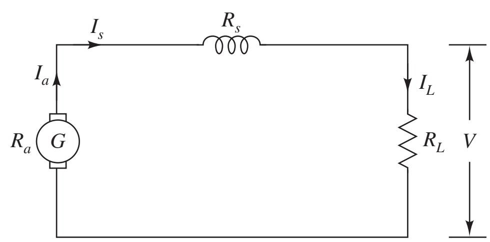


Fig. 7.9

Application As boosters in distribution systems

Compound generator

Short shunt

$$I_s = I_L$$

$$I_a = I_L + I_f$$

$$I_f = \frac{V + I_L R_s}{R_f}$$

$$E_g = V + I_a R_a + I_L R_s$$

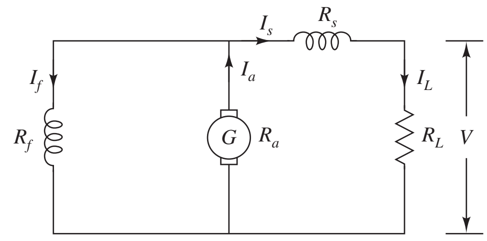


Fig. 7.10

Long shunt

$$I_a = I_s = I_L + I_f$$

$$I_f = \frac{V}{R_f}$$

$$E_g = V + I_a (R_a + R_s)$$

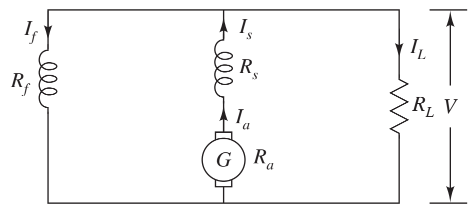


Fig. 7.11

Applications

(i) Lamp loads
(ii) Heavy power service such as electric railways
(iii) Arc welding

(b) Motor When the dc machine runs as a motor, the applied voltage (V) across its terminals must be sufficient to overcome the back emf (E_b) and supply the internal voltage drop. Voltage and current relationships for different types of motors are as follows:

Shunt motor

$$I_f = \frac{V}{R_f}$$

$$I_a = I_L - I_f$$

$$E_b = V - I_a R_a$$

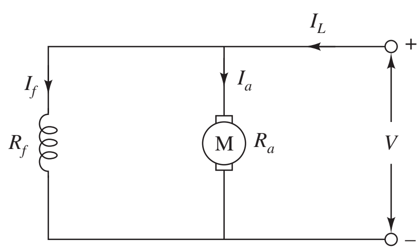


Fig. 7.12

Applications

(i) For driving lathes
(ii) Centrifugal pumps
(iii) Blowers and fans

Series motor

$$I_a = I_s = I_L$$

$$E_b = V - I_a (R_a + R_s)$$

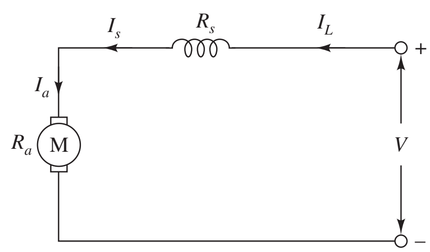


Fig. 7.13

Applications

(i) Electric locomotives

(ii) Cranes and hoists
(iii) Conveyors

Compound motor

Short shunt

$$I_s = I_L$$
$$I_a = I_L - I_f$$
$$I_f = \frac{V - I_L R_s}{R_f}$$
$$E_b = V - I_a R_a - I_L R_s$$

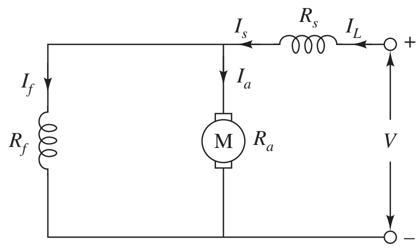


Fig. 7.14

Long shunt

$$I_a = I_s = I_L - I_f$$
$$I_f = \frac{V}{R_f}$$
$$E_b = V - I_a (R_a + R_s)$$

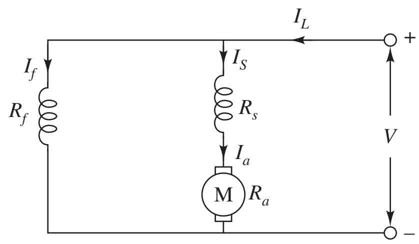


Fig. 7.15

Application
(i) Elevators
(ii) Conveyors
(iii) Rolling mills
(iv) Air compressors

Example 1

A six-pole lap-wound armature has 840 conductors and a flux per pole of 0.018 Wb. Calculate the emf generated, when the machine is running at 600 rpm.

Solution

$$P = 6$$
$$\phi = 0.018 \text{ Wb}$$
$$N = 600 \text{ rpm}$$
$$Z = 840$$

For lap-wound armature,

$$A = P = 6$$

Generated emf

$$E_g = \frac{\phi ZNP}{60A} = \frac{0.018 \times 840 \times 600 \times 6}{60 \times 6} = 151.2 \text{ V}$$

Example 2

A six-pole, 2-circuit wave connected armature has 300 conductors and runs at 1000 rpm. The emf generated on the open circuit is 400 V. Find the useful flux per pole.

Solution

$$P = 6$$
$$A = 2$$
$$Z = 300$$
$$N = 1000 \text{ rpm}$$
$$E_g = 400 \text{ V}$$
$$E_g = \frac{\phi ZNP}{60A}$$

Useful flux per pole

$$\phi = \frac{60 E_g A}{ZNP} = \frac{60 \times 400 \times 2}{300 \times 1000 \times 6} = 0.0267 \text{ Wb}$$

Example 3

A lap-wound dc shunt generator having 80 slots with 10 conductors per slot generates at no load an emf of 400 V, when running at 1000 rpm. At what speed should it be rotated to generate a voltage of 220 V on open circuit?

Solution Number of slots on armature = 80

Conductors per slot = 10

$$E_g = 400 \text{ V}$$
$$N = 1000 \text{ rpm}$$

(i) Total number of conductors on armature (Z) = 80 × 10 = 800

$$E_g = \frac{\phi ZNP}{60A}$$
$$A = P$$
$$400 = \frac{\phi \times 1000 \times 800}{60}$$
$$\phi = 0.03 \text{ Wb}$$

(ii)

$$E_g = 220 \text{ V}$$
$$\phi = 0.03 \text{ Wb}$$
$$220 = \frac{0.03 \times N \times 800}{60}$$
$$N = 550 \text{ rpm}$$

Example 4

A 4-pole dc shunt generator with lap-connected armature supplies a load of 100 A at 200 V. The armature resistance is 0.1 Ω and the shunt-field resistance is 80 Ω. Find (i) total armature current, (ii) current per armature path, and (iii) emf generated. Assume a brush contact drop of 2 V.

Solution

$$P = 4$$
$$V = 200 \text{ V}$$
$$R_f = 80\ \Omega$$
$$R_a = 0.1\ \Omega$$
$$I_L = 100 \text{ A}$$

(i) Total armature current

$$I_f = \frac{V}{R_f} = \frac{200}{80} = 2.5 \text{ A}$$

$$I_a = I_L + I_f = 100 + 2.5 = 102.5 \text{ A}$$

(ii) Current per armature path

For lap-wound shunt generator, $A = P = 4$

$$I_a = 102.5 \text{ A}$$

$$\text{Current per armature path} = \frac{102.5}{4} = 25.625 \text{ A}$$

(iii) EMF generated

$$E_g = V + I_a R_a + \text{Brush Drop} = V + I_a R_a + 0.1 \times 2 = 212.25 \text{ V}$$

Example 5

The armature of a four-pole, lap-wound shunt generator has 120 slots with 4 conductors per slot. The flux per pole is 0.05 Wb. The armature resistance is 0.05 Ω and the shunt-field resistance is 50 Ω. Find the speed of the machine when supplying 450 A at a terminal voltage of 250 V.

Solution

$$V = 250 \text{ V}$$
$$I_L = 450 \text{ A}$$
$$R_f = 50\ \Omega$$
$$R_a = 0.05\ \Omega$$
$$P = 4$$
$$\phi = 0.05 \text{ Wb}$$

$$I_f = \frac{V}{R_f} = \frac{250}{50} = 5 \text{ A}$$

$$I_a = I_L + I_f = 450 + 5 = 455 \text{ A}$$

$$E_g = V + I_a R_a = 250 + 455 \times 0.05 = 272.75 \text{ V}$$

Number of slots on armature = 120

Conductors per slot = 4

Total number of conductors on armature, $Z = 120 \times 4 = 480$

For lap wound generator, $A = P = 4$

$$E_g = \frac{\phi \text{ZNP}}{60\text{A}}$$

$$272.75 = \frac{0.05 \times 480 \times N \times 4}{60 \times 4}$$

$$N = 682 \text{ rpm}$$

Example 6

A 230 V dc shunt machine has an armature resistance of 0.5 Ω and a field resistance of 115 Ω. If this machine is connected to 230 V supply mains, find the ratio of speed as generator to the speed as a motor. The line current in each is 40 A.

Solution

$$V = 230 \text{ V}$$

$$I_L = 40 \text{ A}$$

$$R_a = 0.5\ \Omega$$

$$R_f = 115\ \Omega$$

Generator operation

$$I_L = 40 \text{ A}$$

$$I_f = \frac{V}{R_f} = \frac{230}{115} = 2\text{A}$$

$$I_a = I_L + I_f = 40 + 2 = 42 \text{ A}$$

$$E_g = V + I_a R_a = 230 + 42 \times 0.5 = 251 \text{ V}$$

Motor operation

$$I_L = 40 \text{ A}$$

$$I_f = 2 \text{ A}$$

$$I_a = I_L - I_f = 40 - 2 = 38 \text{ A}$$

$$E_b = V - I_a R_a = 230 - 38 \times 0.5 = 211 \text{ V}$$

$$E = \frac{\phi ZNP}{60}$$

$$E \propto N$$

$$\frac{E_b}{E_g} = \frac{N_2}{N_1}$$

$$\frac{211}{251} = \frac{N_2}{N_1}$$

$$\frac{N_2}{N_1} = 1.1896$$

Example 7

A short-shunt compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field is respectively, 0.04, 0.03 and 60 Ω. Find the emf generated.

Solution $V = 100$ V

$R_f = 60\ \Omega$

$I_L = 200$ A

$R_a = 0.04\ \Omega$

$R_s = 0.03\ \Omega$

Voltage drop in series field winding = $I_L R_s = 200 \times 0.03 = 6$ V

$$I_f = \frac{V + I_L R_s}{R_f} = \frac{100 + 6}{60}$$

$$= \frac{106}{60} = 1.77 \text{ A}$$

$$I_a = I_L + I_f$$

$$= 200 + 1.77 = 201.77 \text{ A}$$

Generated emf $E_g = V + I_L R_s + I_a R_a$

$$= 100 + 6 + 201.77 \times 0.04 = 114.07 \text{ V}$$

Example 8

A 120 V dc shunt motor draws a current of 200 A. The armature resistance is 0.02 Ω and shunt field resistance 30 Ω. Find the back emf. If the lap-wound armature has 90 slots with 4 conductors per slot at what speed will the motor run when the flux per pole is 0.04 Wb?

Solution

$$V = 120 \text{ V}$$
$$R_a = 0.02\ \Omega$$
$$I_L = 200 \text{ A}$$
$$\phi = 0.04 \text{ Wb}$$
$$R_f = 30\ \Omega$$

(i)
$$I_f = \frac{V}{R_f} = \frac{120}{30} = 4 \text{ A}$$
$$I_a = I_L - I_f = 200 - 4 = 196 \text{ A}$$
$$E_b = V - I_a R_a = 120 - 196 \times 0.02 = 116.08 \text{ V}$$

(ii) For-lap wound armature, $A = P$
Number of slots on the armature = 90
Total number of conductors on the armature, $Z = 90 \times 4 = 360$

$$E_b = \frac{\phi ZNP}{60\,A}$$
$$116.08 = \frac{0.04 \times 360 \times N}{60}$$
$$N = \frac{116.08 \times 60}{0.04 \times 360} = 484 \text{ rpm}$$

Example 9

A shunt generator delivers 40 kW at 240 V when running at 450 rpm. The armature and field resistances are 0.03 and 60 Ω respectively. Calculate the speed of the machine running as a shunt motor and taking 40 kW input at 420 V. Allow 1 V per brush for contact drop.

Solution

$$V = 240 \text{ V}$$
$$R_a = 0.03\ \Omega$$
$$R_f = 60\ \Omega$$

Generator Operation

$$I_L = \frac{40 \times 10^3}{240} = 166.7 \text{ A}$$
$$I_f = \frac{V}{R_f} = \frac{240}{60} = 4 \text{ A}$$
$$I_a = I_L + I_f = 166.7 + 4 = 170.7 \text{ A}$$
$$E_g = V + I_a R_a + \text{Brush drop}$$
$$= 240 + 170.7 \times 0.03 + 2 \times 1 = 247.1 \text{ V}$$

Speed as a generator at this load = 450 rpm

$$E_g = \frac{\phi ZNP}{60\,A}$$

$$E_g \propto \text{N}$$

$$E_g \propto 450$$

Motor Operation

Power drawn by the motor = 40 kW

$$I_L = \frac{40 \times 10^3}{240} = 166.7 \text{ A}$$

$$I_f = 4 \text{ A}$$

$$I_a = I_L - I_f = 166.7 - 4 = 162.7 \text{ A}$$

$$E_b = 240 - 162.7 \times 0.03 - 2 \times 1 = 233.1 \text{ V}$$

$$E_b = \frac{\phi ZNP}{60\,A}$$

$$E_b \propto N$$

$$233.1 \propto N$$

$$\frac{N}{450} = \frac{233.1}{247.1}$$

$$N = 450 \times \frac{233.1}{247.1} = 425 \text{ rpm}$$

Useful Formulae

$$E = \frac{\phi ZNP}{60\,A}$$

where $A = P$ for lap winding $A = 2$ for wave winding

Shunt Generator	**Series Generator**	**Compound Generator**	
		Short shunt	**Long shunt**
		$I_s = I_L$	
$I_a = I_L + I_f$	$I_a = I_s = I_L$	$I_a = I_L + I_f$	$I_a = I_s = I_L + I_f$
$I_f = \dfrac{V}{R_f}$		$I_f = \dfrac{V + I_L R_s}{R_f}$	$I_f = \dfrac{V}{R_f}$
$E_g = V + I_a R_a$	$E_g = V + I_a (R_a + R_s)$	$E_g = V + I_a R_a + I_L R_s$	$E_g = V + I_a (R_a + R_s)$

Shunt motor	Series motor	Compound motor	
		Short shunt	Long shunt
$I_a = I_L - I_f$	$I_a = I_s = I_L$	$I_s = I_L$	$I_a = I_s$
		$I_a = I_L - I_f$	$I_a = I_L - I_f$
$E_b = V - I_a R_a$	$E_b = V - I_a (R_a + R_s)$	$E_b = V - I_a R_a - I_L R_s$	$E_b = V - I_a (R_a + R_s)$
$I_f = \dfrac{V}{R_f}$		$I_f = \dfrac{V - I_L R_s}{R_f}$	$I_f = \dfrac{V}{R_f}$

Exercise 7.1

7.1 A 4-pole, dc generator has a wave-wound armature with 792 conductors. The flux per pole is 0.0121 Wb. Determine the speed at which it should be run to generate 240 V on no-load. *[751.3 rpm]*

7.2 A dc generator generates an emf of 520 V. It has 2,000 armature conductors, flux per pole of 0.013 Wb, speed of 1200 rpm and the armature winding has four parallel paths. Find the number of poles. *[4]*

7.3 When driven at 1000 rpm with a flux per pole of 0.02 Wb, a dc generator has an emf of 200 V. If the speed is increased to 1100 rpm and at the same time, the flux per pole is reduced to 0.019 Wb per pole, what is the induced emf? *[209 V]*

7.4 A 4-pole machine running at 1500 rpm has an armature with 90 slots and 6 conductors per slot. The flux per pole is 10 mWb. Determine the terminal emf as dc generator if the coils are lap connected. If the current per conductor is 100 A, determine the electrical power. *[810 V, 324 kW]*

7.5 An 8-pole lap wound dc generator has 120 slots having 4 conductors per slot. If each conductor can carry 250 A and if flux/pole is 0.05 Wb, calculate the speed of the generator for giving 240 V on open circuit. If the voltage drops to 220 V on full load, find the rated output of the machine. *[600 V, 440 kW]*

7.6 A 4-pole, dc shunt motor has a flux per pole of 0.04 Wb and the armature is lap wound with 720 conductors. The shunt field resistance is 240 Ω and the armature resistance is 0.2 Ω. Brush contact drop is 1 V per brush. Determine the speed of the machine when running (i) as a motor taking 60 A, and (ii) as a generator supplying 120 A. The terminal voltage in each case is 480 V. *[972 rpm; 1055 rpm]*

7.7 A 4-pole, dc motor has a wave-wound armature with 65 slots each containing 6 conductors. The flux per pole is 20 mWb and the armature has a resistance of 0.15 Ω. Calculate the motor speed when the machine is operating from a 250 V supply and taking a current of 60 A. *[927 rpm]*

7.8 A 500 V, dc shunt motor has armature and field resistances of 0.5 Ω and 200 Ω respectively. When loaded and taking a total input of 25 kW, it runs at 400 rpm. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. *[442 rpm]*

7.9 A shunt machine connected to 250 V mains has an armature resistance of 0.12 Ω and field resistance of 100 Ω. Find the ratio of the speed of the machine as a generator to the speed as motor, if the line current is 80 A in both cases. *[1.08]*

Review Questions

7.1 Explain the working principle of dc generator and dc motor.

7.2 What is the basic nature of the induced emf in a dc generator? What is the function of a commutator?

7.3 What is the difference between lap-type and wave-type of armature winding?

7.4 From first principle derive expression for the emf of a dc generator.

7.5 What is back emf? Explain the significance of back emf.

Multiple Choice Questions

Choose the correct alternative in the following questions:

7.1 Wave winding is employed in a dc machine of
(a) high current and low voltage rating (b) low current and high voltage rating
(c) high current and high voltage rating (d) low current and low voltage rating

7.2 If a 230 V dc series motor is connected to a 230 V ac supply
(a) the motor will vibrate violently
(b) the motor will run with less efficiency and more sparking
(c) the motor will not run
(d) the fuse will be blown

7.3 The value of back emf (E_b) in a dc motor in maximum at
(a) no load (b) full load
(c) half full load (d) none of the above

7.4 The function of a commutator in a dc machine is
(a) to improve commutation (b) to change ac current to dc current
(c) to change dc voltage to ac voltage (d) to provide easy speed control

7.5 If the number of poles in a lap-wound dc generator is increased by a factor of 2, the generated emf will
(a) increase by a factor of 2 (b) decrease by a factor of 2
(c) increase by factor of 4 (d) remain the same

7.6 As the speed of dc generator is increased, the generated emf

(a) increases (b) decreases

(c) remains constant (d) first increases and then becomes constant

7.7 Match list I (machine) with list II (characteristic/application) and select the correct answer:

List I	*List II*
A. dc shunt generator	1. electric traction
B. dc series motor	2. good voltage regulation
C. compound dc generator	3. must have residual flux
D. dc series generator	4. used as boosters

	A	B	C	D
(a)	4	1	2	3
(b)	3	2	1	4
(c)	4	2	1	3
(d)	3	1	2	4

7.8 Match list I which list II and select the correct answer:

List I (motor)	*List II (applications)*
A. dc series motor	1. shearing and pressing
B. squirrel cage induction motor	2. haulage and hoisting
C. dc shunt motor	3. rolling mill

	A	B	C
(a)	1	2	3
(b)	2	3	1
(c)	3	1	2
(d)	3	2	1

Answers to Multiple Choice Questions

7.1 (b) **7.2** (b) **7.3** (a) **7.4** (b) **7.5** (d) **7.6** (a)
7.7 (d) **7.8** (b)

APPENDIX

Additional Solved Mumbai University Examination Questions

May 2018

1. (a) What is the difference between an ideal source and an actual source? Illustrate the concept using the *V-I* characteristics of voltage and current source.

Ans. A voltage source is a two-terminal device whose voltage at any instant of time is constant and is independent of the current drawn from it. Such a voltage source is called an ideal voltage source and has zero internal resistance. The voltage source having some amount of internal resistance is called a practical or actual voltage source. Due to this internal resistance, voltage drop takes place and it causes terminal voltage to reduce.

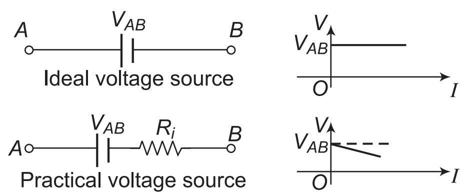


Fig. 1

An ideal current source is a two-terminal device which supplies constant current to any load resistance across its terminals and is independent of the voltage of source terminals. It has infinite resistance. A practical or actual current source is represented by an ideal current source in parallel with a resistance

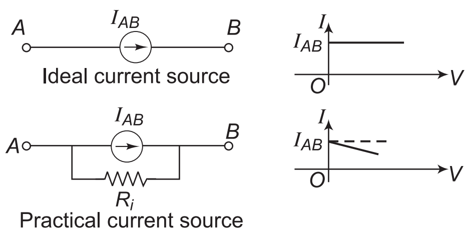


Fig. 2

1. (b) In a balanced three phase circuit the power factor is 0.866. What will be the ratio of two wattmeter readings if the power is measured using two wattmeters?

Ans.

$$\text{pf} = 0.866$$
$$\cos\phi = 0.866$$

$$\phi = 30^\circ$$

$$\tan \phi = \tan (30^\circ) = \frac{1}{\sqrt{3}}$$

$$\tan \phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\frac{1}{\sqrt{3}} = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 + W_2 = 3W_1 - 3W_2$$

$$4W_2 = 2W_1$$

$$\frac{W_1}{W_2} = \frac{2}{1}$$

1. (c) Calculate R_{AB}.

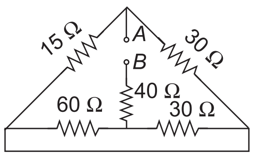


Fig. 3

Ans. Marking all the junctions and redrawing the network,

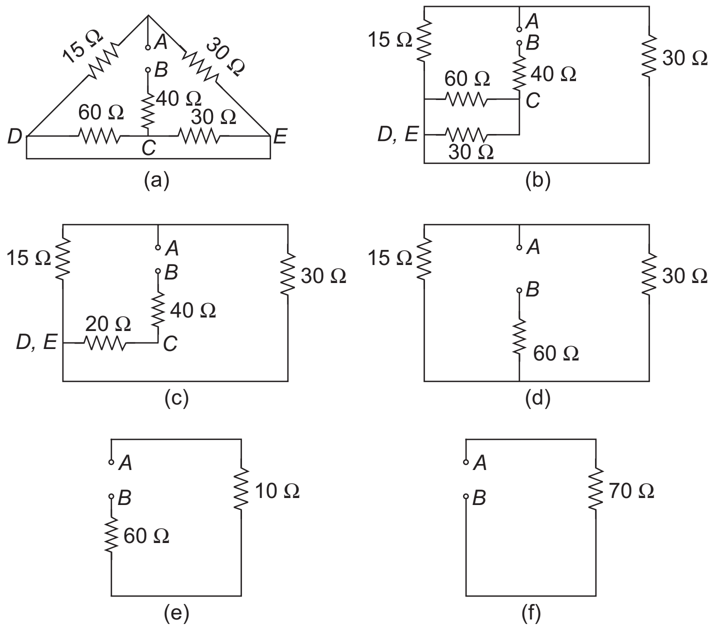


Fig. 4

$$R_{AB} = 70\ \Omega$$

1. (d) Derive the equation for resonance frequency of a parallel circuit in which a capacitor is connected in parallel with a coil having resistance R and inductive reactance X_L. What is the resonance frequency if inductor is ideal?

Ans. Refer Section 4.9 on page 4.106.

1. (e) What are the classifications of DC motors? Specify one application for each one.

Ans. Refer Section 7.4 on page 7.4 and Section 7.6 on page 7.6.

1. (f) Derive emf equation of a single phase transformer.

Ans. Refer Section 6.4 on page 6.4.

2. (a) Using mesh analysis find the current through 5 Ω resistor.

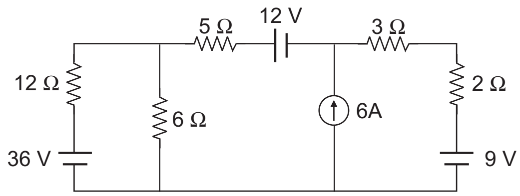


Fig. 5

Ans. Assigning clockwise currents in three meshes,

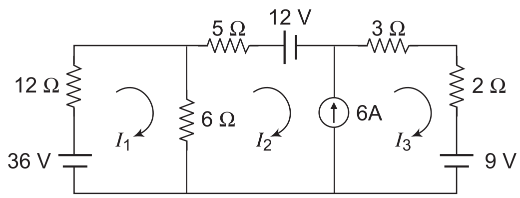


Fig. 6

Applying KVL to Mesh 1,

$$36 - 12I_1 - 6(I_1 - I_2) = 0$$

$$18I_1 - 6I_2 = 36 \qquad (1)$$

Meshes 2 and 3 will form a supermesh as these two meshes share a common current source of 6 A.

Writing current equation for the supermesh,

$$I_3 - I_2 = 6 \qquad (2)$$

Applying KVL to the outer path of the supermesh,

$$-6(I_2 - I_1) - 5I_2 - 12 - 3I_3 - 2I_3 - 9 = 0$$

$$6I_1 - 11I_2 - 5I_3 = 21 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.07 \text{ A}$$

$$I_2 = -2.79 \text{ A}$$

$$I_3 = 3.21 \text{ A}$$

$$I_{5\Omega} = I_2 = -2.79 \text{ A}$$

2. (b) An emf of 250 V is applied to an impedance $Z_1 = (12.5 + j20)\ \Omega$. When an impedance Z_2 is added in series with Z_1, the current becomes half of the original and leads the supply voltage by 20°. Determine Z_2.

Ans.

$$V = 250 \text{ V}$$

$$\overline{Z_1} = (12.5 + j20)\ \Omega$$

Case I:

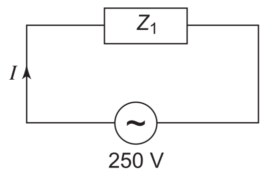


Fig. 7

$$\bar{I} = \frac{\overline{V}}{\overline{Z_1}} = \frac{250\angle 0°}{12.5 + j20} = 10.6\angle{-58°} \text{ A}$$

Case II:

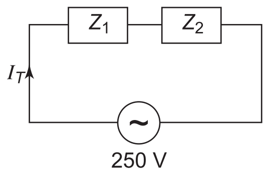


Fig. 8

$$\bar{I}_T = \frac{10.6\angle -58° + 20°}{2} = 5.3\angle{-38°} \text{ A}$$

$$\overline{Z}_T = \frac{\overline{V}}{\overline{I}_T} = \frac{250\angle 0°}{5.3\angle -38°} = 47.17\angle 38°\ \Omega$$

$$\overline{Z}_2 = \overline{Z}_T - \overline{Z}_1 = (47.17\angle 38°) - (12.5 + j20) = 26.27\angle 20.13°\ \Omega$$

2. (c) Determine the potential difference V_{AB} for the given network.

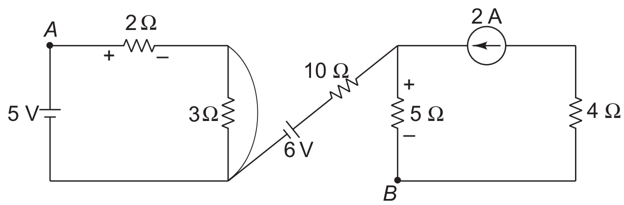


Fig. 9

Ans. The resistor of 3 Ω is connected across a short circuit. Hence, it gets shorted.

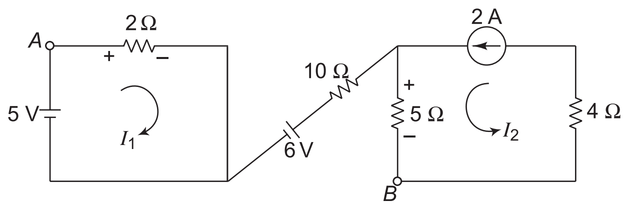


Fig. 10

$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Potential difference, $V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 6 - 5I_2 - V_B = 0$$

$$V_A - 2(2.5) + 6 - 5(2) - V_B = 0$$

$$V_A - V_B = 9$$

$$V_{AB} = 9 \text{ V}$$

3. (a) When a voltage of 100 V, 50 Hz is applied to an impedance A, current taken is 8 A lagging and power is 120 W. When it is connected to an impedance B, the current is 10 A leading and power is 500 W. What current and power will be taken if it is applied to the two impedances connected in series?

Ans. Impedance A: $V_A = 100$ V, $I_A = 8$ A (lagging), $P_A = 120$ W

Impedance B: $V_B = 100$ V, $I_B = 10$ A (leading), $P_B = 500$ W

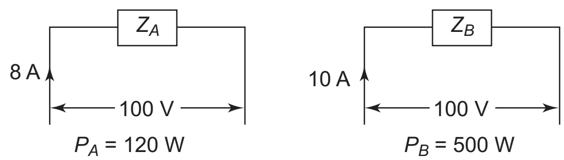


Fig. 11

For Impedance A,

$$Z_A = \frac{V_A}{I_A} = \frac{100}{8} = 12.5\ \Omega$$

$$P_A = I_A^2 R_A$$

$$120 = (8)^2 \times R_A$$

$$R_A = 1.875\ \Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(12.5)^2 - (1.875)^2} = 12.36\ \Omega$$

For Impedance B,

$$Z_B = \frac{V_B}{I_B} = \frac{100}{10} = 10\ \Omega$$

$$P_B = I_B^2 R_B$$

$$500 = (10)^2 \times R_B$$

$$R_B = 5\ \Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{(10)^2 - (5)^2} = 8.66\ \Omega$$

When two impedances are connected in series,

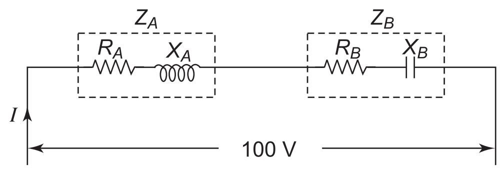


Fig. 12

$$\bar{Z} = R_A + jX_A + R_B - jX_B = 1.875 + j12.36 + 5 - j8.66 = 6.875 + j3.7 = 7.81\angle 28.29°\ \Omega$$

$$I = \frac{V}{Z} = \frac{100}{7.81} = 12.8\text{ A}$$

$$P = I^2(R_A + R_B) = (12.8)^2 \times 6.875 = 1.126\text{ kW}$$

3. (b) Find the current through 10 Ω using Thevenin's theorem.

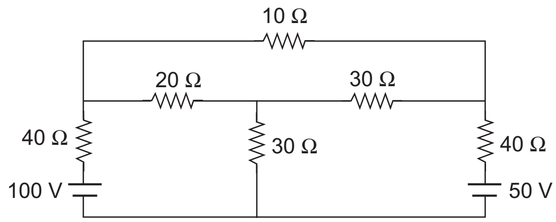


Fig. 13

Ans. ***Step I:*** Calculation of V_{Th}

Removing 10 Ω resistor from the network,

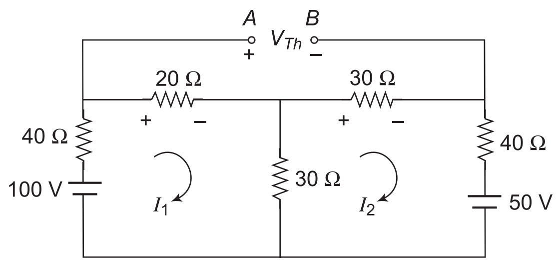


Fig. 14

Applying KVL to Mesh 1,

$$100 - 40I_1 - 20I_1 - 30(I_1 - I_2) = 0$$

$$90I_1 - 30I_2 = 100 \qquad (1)$$

Applying KVL to Mesh 2,

$$-30(I_2 - I_1) - 30I_2 - 40I_2 - 50 = 0$$

$$-30I_1 + 100I_2 = -50 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 1.05 \text{ A}$$
$$I_2 = -0.185 \text{ A}$$

Writing V_{Th} equation,

$$-V_{Th} + 30I_2 + 20I_1 = 0$$
$$V_{Th} = 30I_2 + 20I_1 = 30(-0.185) + 20(1.05) = 15.45 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

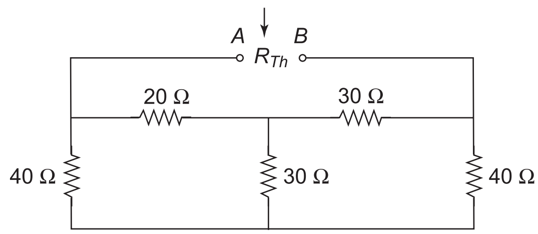


Fig. 14

Converting the star network formed by resistors of 20 Ω, 30 Ω and 30 Ω into an equivalent delta network,

$$R_1 = 20 + 30 + \frac{20 \times 30}{30} = 70 \ \Omega$$

$$R_2 = 20 + 30 + \frac{20 \times 30}{30} = 70 \ \Omega$$

$$R_3 = 30 + 30 + \frac{30 \times 30}{20} = 105 \ \Omega$$

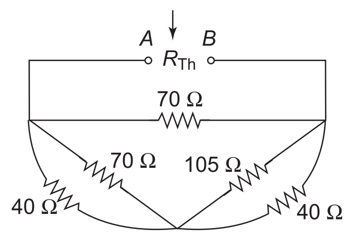


Fig. 15

Simplifying the network,

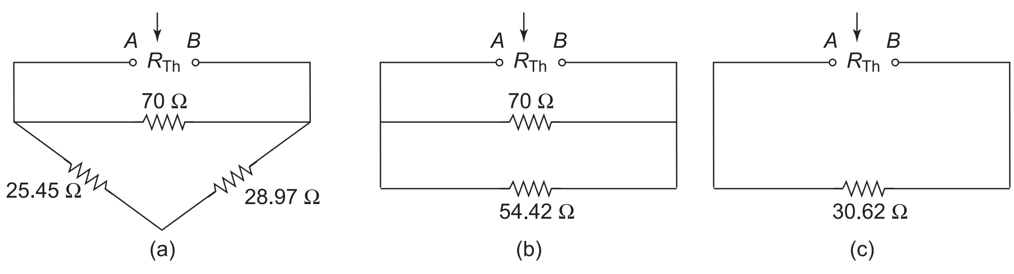


Fig. 16

$$R_{Th} = 30.62 \ \Omega$$

Step III: Calculation of I_L

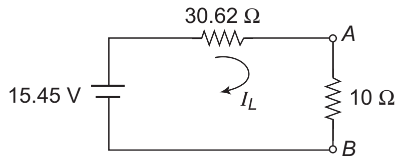


Fig. 17

$$I_L = \frac{15.45}{30.62 + 10} = 0.38 \text{ A}$$

3. **(c)** With the help of equivalent circuit of a single phase transformer show how total copper loss can be represented in primary of a transformer.

Ans. Refer Section 6.11 on page 6.21.

4. **(a)** Find V_1 using superposition theorem.

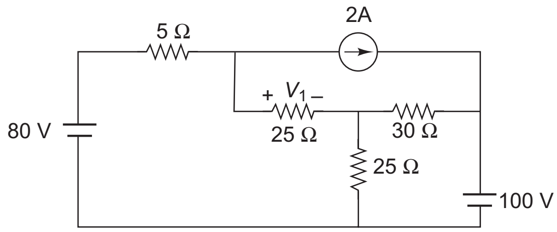


Fig. 18

Ans. ***Step I:*** When the 80 V source is acting alone

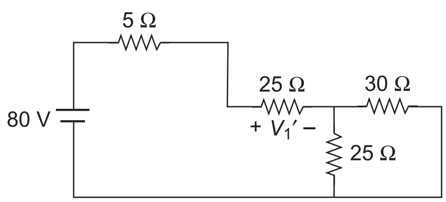


Fig. 19

Simplifying the network,

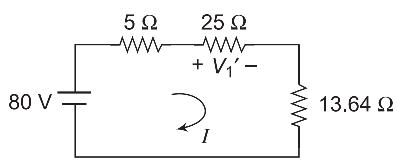


Fig. 20

$$I = \frac{80}{5 + 25 + 13.64} = 1.833 \text{ A}$$

$$V_1' = 25\, I = 25(1.833) = 45.83 \text{ V}$$

Step II: When the 2 A source is acting alone

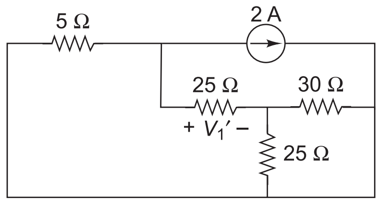


Fig. 21

Simplifying the network,

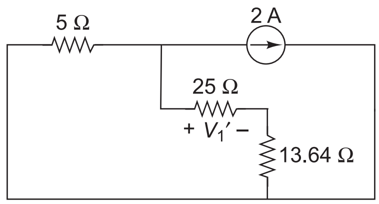


Fig. 22

By current-division rule,

$$I_{25\,\Omega} = 2 \times \frac{5}{5+25+13.64} = 0.23 \text{ A } (\uparrow) = -\,0.23 \text{ A } (\downarrow)$$

$$V_1'' = 25\, I_{25\,\Omega} = 25(-0.23) = -\,5.75 \text{ V}$$

Step III: When the 100 V source is acting alone

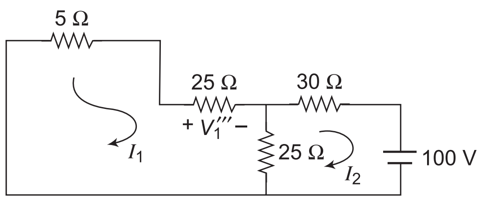


Fig. 23

Applying KVL to Mesh 1,

$$-5I_1 - 25I_1 - 25(I_1 - I_2) = 0$$

$$55I_1 - 25I_2 = 0 \tag{1}$$

Applying KVL to Mesh 2,

$$-25(I_2 - I_1) - 30I_2 - 100 = 0$$

$$-25I_1 + 55I_2 = -100 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = -1.042 \text{ A}$$

$$I_2 = -2.292 \text{ A}$$

$$V_1''' = 25I_1 = 25(-1.042) = -26.05 \text{ V}$$

Step IV: By superposition theorem,

$$V_1 = V_1' + V_1'' + V_1'''$$
$$= 45.83 - 5.75 - 26.05$$
$$= 14.03 \text{ V}$$

4. (b) In an *R-L-C* parallel circuit, the currents through the resistor, inductor (pure) and capacitor are 20 A, 15 A and 40 A, respectively. What is the current taken from the supply? Draw phasor diagram.

Ans.
$$\bar{I}_R = 20\angle 0° \text{ A}$$
$$\bar{I}_L = 15\angle -90° \text{ A}$$
$$\bar{I}_C = 40\angle 90° \text{ A}$$
$$\bar{I}_T = \bar{I}_R + \bar{I}_L + \bar{I}_C$$
$$= 20\angle 0° + 15\angle -90° + 40\angle 90°$$
$$= 32.02\angle 51.34° \text{ A}$$

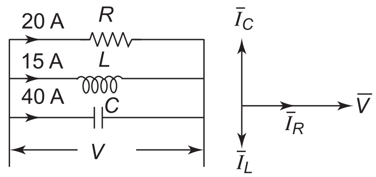


Fig. 24

4. (c) Two sinusoidal sources of emf have rms value E_1 and E_2. When connected in series, with a phase displacement α, the resultant voltage read on an electrodynamometer voltmeter is 41.1 V and with one source reversed is 17.52 V. When the phase displacement is made zero, a reading of 42.5 V is observed. Calculate E_1, E_2 and α.

Ans. Refer Ex. 15 on page 3.55.

5. (a) Prove that the power in a balanced three phase delta-connected circuit can be deduced from the readings of two wattmeters. Draw relevant connections and vector diagrams. Draw a table to show the effect of power factor on wattmeter

Ans. Refer Section 5.15.2 on page 5.48 and Section 5.16 on page 5.50

5. (b) A 5 kVA200/400,50 Hz single phase transformer gave the following test results.

OC test on LV side	200 V	0.7 A	60 W
SC test on HV side	22 V	0.16 A	120 W

(i) Draw the equivalent circuit of the transformer and insert all parameter values.
(ii) Efficiency at 0.9 pf lead and rated load.
(iii) Current at which efficiency is maximum.

Ans. Refer Ex. 3 on page 6.52.

5. (c) Prove that if the phase impedances are same, power drawn by a balanced delta connected load is three times the power drawn by the balanced star connected load.

Ans. Refer Section 5.11 on page 5.13.

6. (a) Three identical coils each having a reactance of 20 Ω and resistance of 10 Ω are connected in star across a 440 V three phase line. Calculate:
(i) Line current and phase current.
(ii) Active, reactive and apparent power.
(iii) Reading of each wattmeter connected to measure the power.

Ans.

$$R = 10\ \Omega$$
$$X_L = 20\ \Omega$$
$$V_L = 440\ \text{V}$$

For a star connected load,

(i) Line current and phase current

$$V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\ \text{V}$$

$$\overline{Z}_{\text{ph}} = 10 + j20 = 22.36\ \angle 63.43°\ \Omega$$

$$I_{\text{ph}} = \frac{V_{\text{ph}}}{Z_{\text{ph}}} = \frac{254.03}{22.36} = 11.36\ \text{A}$$

$$I_L = I_{\text{ph}} = 11.36\ \text{A}$$

(ii) Active, reactive and apparent power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 11.36 \times \cos(63.43°) = 3.87\ \text{kW}$$

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 11.36 \times \sin(63.43°) = 7.74\ \text{kVAR}$$

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 11.36 = 8.66\ \text{kVA}$$

(iii) Reading of each wattmeter

$$W_1 = V_L I_L \cos(30° + \phi) = 440 \times 11.36 \times \cos(30° + 63.43°) = -299.05\ \text{W}$$
$$W_2 = V_L I_L \cos(30° - \phi) = 440 \times 11.36 \times \cos(30° - 63.43°) = 4.17\ \text{kW}$$

6. (b) A series resonant circuit has an impedance of 500 Q at resonant frequency. The cut off frequency is observed are 10kHz and 100 Hz. Determine

(i) Resonant frequency.

(ii) Values of R, L and C.

(iii) Q factor at resonance.

Ans. Refer Ex. 12 on page 4.103.

6. (c) Draw and illustrate transformer phasor diagram for lagging power factor.

Ans. Refer Section 6.10 case (ii) on page 6.20.

December 2017

1. (a) A voltage $v(t) = 282.85 \sin 100\,\pi t$ is applied to a coil, having resistance of 20 Ω in series with inductance of 31.83 mH. Find:

(i) RMS value of voltage.

(ii) RMS value of current.

(iii) Power dissipated in the coil.

(iv) Power factor of the coil.

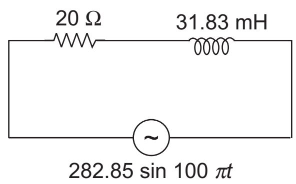


Fig. 1

Ans.

$$v(t) = 282.85 \sin 100\pi t$$
$$r = 20\ \Omega$$
$$L = 31.83\text{ mH}$$

(i) RMS value of voltage

$$V = \frac{V_m}{\sqrt{2}} = \frac{282.85}{\sqrt{2}} = 200.01\text{ V}$$

(ii) RMS value of current

$$X_L = \omega L = 100\pi \times 31.83 \times 10^{-3} = 10\ \Omega$$

$$\overline{Z} = r + jX_L = 20 + j10 = 22.36\angle 26.57^\circ\ \Omega$$

$$I = \frac{V}{Z} = \frac{200.01}{22.36} = 8.945\text{ A}$$

(iii) Power dissipated in the coil

$$P = VI\cos\phi = 200.01 \times 8.945 \times \cos(26.57^\circ) = 1.6\text{ kW}$$

(iv) Power factor of the coil

$$pf = \cos\phi = \cos(26.57^\circ) = 0.894\ \text{(lagging)}$$

1. (b) Derive the relation between line voltage and phase voltage in star connected three phase system.

Ans. Refer Section 5.8.1 on page 5.7.

1. (c) Find the node voltage V_2 by nodal analysis.

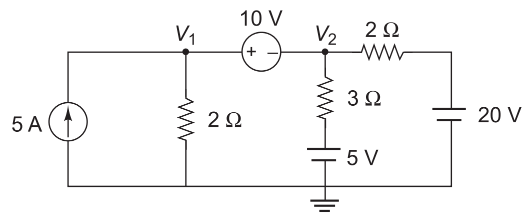


Fig. 2

Ans. Assume that the currents are moving away from the nodes. Nodes 1 and 2 will form a supernode.

Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \quad (1)$$

Applying KCL at the supernode,

$$5 = \frac{V_1}{2} + \frac{V_2 - 5}{3} + \frac{V_2 - 20}{2}$$

$$\frac{1}{2}V_1 + \left(\frac{1}{3} + \frac{1}{2}\right)V_2 = 5 + \frac{5}{3} + 10$$

$$0.5V_1 + 0.833V_2 = 16.67 \quad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 18.75 \text{ V}$$

$$V_2 = 8.75 \text{ V}$$

1. (d) A single phase transformer has a turn ratio (N_1/N_2) of 2:1 and is connected to a resistive load. Find the value of primary current (both magnitude and angle with reference to flux), if the magnetizing current is 1 A and the secondary current is 4 A. Neglect core losses and leakage reactance. Draw the corresponding phasor diagram.

Ans.

$$\frac{N_1}{N_2} = \frac{2}{1}$$

$$I_1 = 1 \text{ A}$$

$$I_2 = 4 \text{ A}$$

For a transformer

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{I_1}{4} = \frac{1}{2}$$

$$I_1 = 2 \text{ A}$$

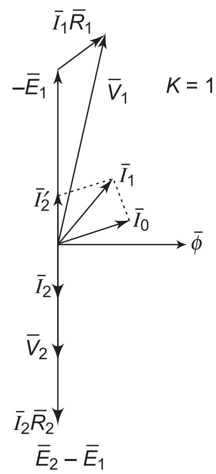


Fig. 3 *Phasor diagram*

1. (e) Find the Norton's equivalent of the given circuit across R_x.

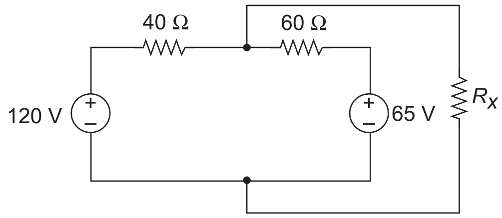


Fig. 4

Ans. ***Step I:*** Calculation of I_N

Replacing the resistor R_X by a short circuit,

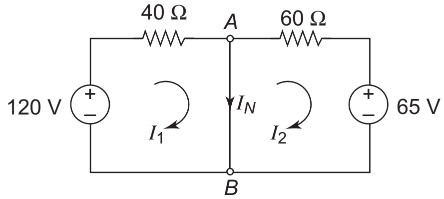


Fig. 5

Applying KVL to Mesh 1,

$$120 - 40I_1 = 0$$

$$I_1 = 3 \text{ A}$$

Applying KVL to Mesh 2,

$$-60I_2 - 65 = 0$$

$$I_2 = -1.08 \text{ A}$$

$$I_N = I_1 - I_2 = 3 - (-1.08) = 4.08 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage sources by short circuits,

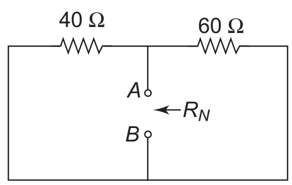


Fig. 6

$$R_N = 40 \,||\, 60 = 24 \ \Omega$$

Step III: Norton's equivalent network

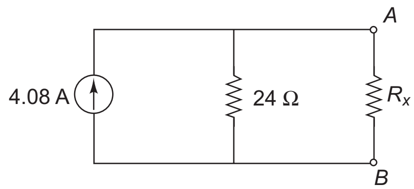


Fig. 7

1. (f) A coil having a resistance of 20 Ω and an inductance of 0.1 H is connected in series with a 50 μF capacitor. An alternating voltage of 250 V is applied to the circuit. At what value of frequency will the current in the circuit be maximum? What is the value of this current? Also find the voltage across the inductor and quality factor.

Ans.

$$R = 20\ \Omega$$
$$L = 0.1\ \text{H}$$
$$C = 50\ \mu\text{F}$$
$$V = 250\ \text{V}$$

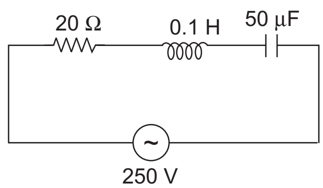


Fig. 8

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}} = 71.18\ \text{Hz}$$

(ii) Value of maximum current

$$I_0 = \frac{V}{R} = \frac{250}{20} = 12.5\ \text{A}$$

(iii) Voltage across inductor

$$X_L = 2\pi f_0 L = 2\pi \times 71.18 \times 0.1 = 44.72\ \Omega$$
$$V_L = I_0 X_L = 12.5 \times 44.72 = 559\ \text{V}$$

(iv) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{0.1}{50 \times 10^{-6}}} = 2.24$$

2. (a) With necessary diagrams, prove that three phase power can be measured by only two wattmeters. Also prove that reactive power can be measured from the wattmeter readings.

Ans. Refer Section 5.15 on page 5.47.

2. (b) A circuit having $L = 0.2$ H and inductive resistance = 20 Ω is connected in parallel with 200 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency and impedance at which the total current taken from the supply is in phase with supply voltage. Draw the phasor diagram and derive the formula used (both impedance and frequency). Also find the value of the supply current and the capacitor current.

Ans. For derivation, refer Section 4.9 on page 4.106.

$$L = 0.2 \text{ H}$$
$$R = 20\ \Omega$$
$$C = 200\ \mu\text{F}$$
$$V = 230 \text{ V}$$

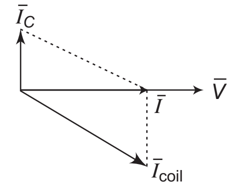


Fig. 9 *Phasor diagram*

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 200 \times 10^{-6}} - \frac{(20)^2}{(0.2)^2}} = 19.49 \text{ Hz}$$

$$Z_D = \frac{L}{CR} = \frac{0.2}{200 \times 10^{-6} \times 20} = 50\ \Omega$$

$$I = \frac{V}{Z_D} = \frac{230}{50} = 4.6 \text{ A}$$

$$X_L = 2\pi f L = 2\pi \times 19.49 \times 0.2$$

$$\bar{Z}_{coil} = R + jX_L = 20 + j24.49 = 31.62\angle 50.76°\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 19.49 \times 200 \times 10^{-6}} = 40.83\ \Omega$$

$$\bar{Z}_C = -jX_C = -j40.89 = 40.89\angle -90°\ \Omega$$

$$\bar{I}_C = \frac{\bar{V}}{\bar{Z}_C} = \frac{230\angle 0°}{40.89\angle -90°} = 5.63\angle 90° \text{ A}$$

$$\bar{I}_{coil} = \frac{\bar{V}}{\bar{Z}_{coil}} = \frac{230\angle 0°}{31.62\angle 50.76°} = 7.27\angle -50.76° \text{ A}$$

3. (a) Two impedances, $14 + j5\ \Omega$ and $18 + j10\ \Omega$, are connected in parallel across 200 V, 50 Hz, single phase supply. Determine:

(i) Admittance of each branch in polar form.
(ii) Current in each branch in polar form.
(iii) Power factor of each branch.
(iv) Active power in each branch.
(v) Reactive power in each branch.

Ans.

$$\bar{Z}_1 = (14 + j5)\ \Omega$$
$$\bar{Z}_2 = (18 + j10)\ \Omega$$
$$V = 200 \text{ V}$$

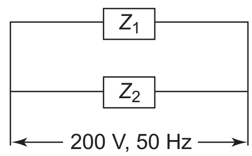


Fig. 10

(i) Admittance of each branch in polar form

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{14 + j5} = 0.067\angle -19.65°\ \mho$$

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{18 + j10} = 0.049\ \angle{-29.05°}\ ℧$$

(ii) Current in each branch in polar form

$$\overline{I}_1 = \overline{V}\,\overline{Y}_1 = 200\angle 0° \times 0.067\angle{-19.65°} = 13.4\angle{-19.65°}\text{ A}$$

$$\overline{I}_2 = \overline{V}\,\overline{Y}_2 = 200\angle 0° \times 0.049\angle{-29.05°} = 9.8\angle{-29.05°}\text{ A}$$

(iii) Power factor of each branch

$$\text{pf}_1 = \cos\phi_1 = \cos(19.65°) = 0.942 \text{ (lagging)}$$

$$\text{pf}_2 = \cos\phi_2 = \cos(29.05°) = 0.874 \text{ (lagging)}$$

(iv) Active power in each branch

$$P_1 = VI_1\cos\phi_1 = 200 \times 13.4 \times 0.942 = 2.52\text{ kW}$$

$$P_2 = VI_2\cos\phi_2 = 200 \times 9.8 \times 0.874 = 1.7\text{ kW}$$

(v) Reactive power in each branch

$$Q_1 = VI_1\sin\phi_1 = 200 \times 13.4 \times \sin(19.65°) = 0.901\text{ kVAR}$$

$$Q_2 = VI_2\sin\phi_2 = 200 \times 9.8 \times \sin(29.05°) = 0.952\text{ kVAR}$$

3. (b) Derive the emf equation of a single phase transformer. Find the value of the maximum flux in a 25 kVA, 3000/240 V, single phase transformer with 500 turns on the primary. The primary winding is connected to 3000 V, 50 Hz supply. Find primary and secondary currents. Neglect all voltage drops.

Ans. EMF equation: Refer Section 6.4 on page 6.3.

kVA rating = 25 kVA

$$E_1 = 3000\text{ V}$$

$$E_2 = 240\text{ V}$$

$$N_1 = 500$$

$$f = 50\text{ Hz}$$

(i) Value of maximum flux

$$E_1 = 4.44\, f\, \phi_m N_1$$

$$3000 = 4.44 \times 50 \times \phi_m \times 500$$

$$\phi_m = 0.027\text{ Wb}$$

(ii) Primary and secondary currents

For a transformer

$$V_1 \simeq E_1 = 3000\text{ V}$$

$$V_2 \simeq E_2 = 240\text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{25 \times 1000}{3000} = 8.33\text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{25 \times 1000}{240} = 104.17\text{ A}$$

3. (c) Compare core type and shell type transformer (any four points).

Ans. Refer Section 6.2.3 on page 6.3.

4. (a) An alternating voltage is represented by $v(t) = 141.4 \sin (377t)$V. Derive the RMS value of this voltage. Also find:

(i) Instantaneous value at $t = 3$ ms

(ii) Time taken for the voltage to reach 70.7 V for the first time.

Ans. For RMS value derivation, refer Section 3.3.1 on page 3.5.

$$v(t) = 141.4 \sin 377\, t$$

(i) Instantaneous value of voltage at $t = 3$ ms

$$v = 141.4 \sin 377 \times 3 \times 10^{-3} = 127.67 \text{ V}$$

(ii) Time taken for the voltage to reach 70.7 V for the first time

$$v = 141.4 \sin 377t$$
$$70.7 = 141.4 \sin 377t$$
$$\sin 377t = 0.5$$
$$377t = 0.524$$
$$t = 1.39 \text{ ms}$$

4. (b) State Superposition Theorem. Find I_x using Superposition Theorem without using source transformation technique.

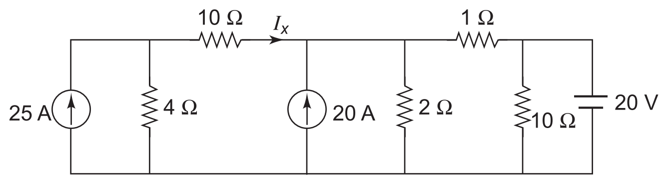


Fig. 11

Ans. Superposition Theorem: Refer Section 2.8 on page 2.116.

Step I: When the 25 A source is acting alone

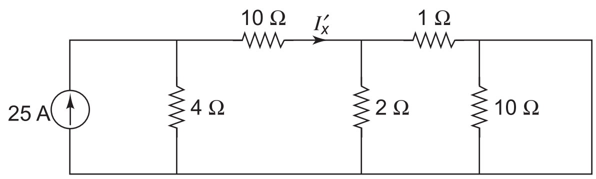


Fig. 12

Simplifying the network,

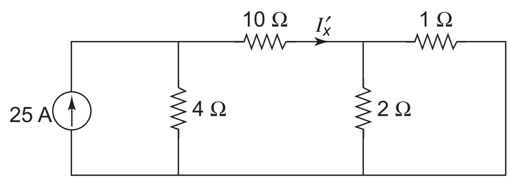


Fig. 13

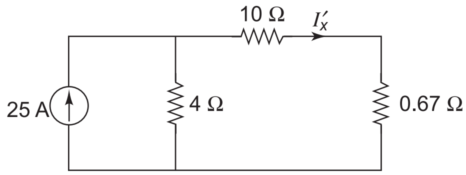


Fig. 14

By current-division rule,

$$I_x' = 25 \times \frac{4}{4+10+0.67} = 6.82 \text{ A } (\rightarrow)$$

Step II: When the 20 A source is acting alone

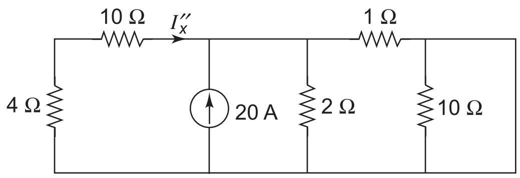


Fig. 15

Simplifying the network,

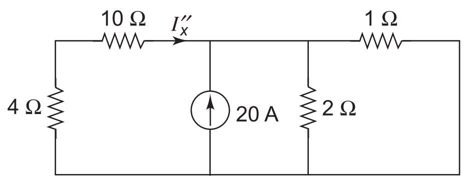


Fig. 16

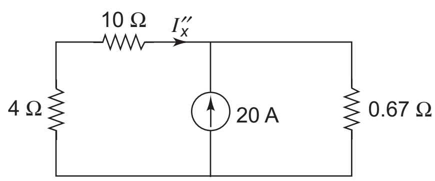


Fig. 17

By current-division rule,

$$I_x'' = 20 \times \frac{0.67}{0.64+10+4} = 0.913 \text{ A } (\leftarrow) = -0.913 \text{ A } (\rightarrow)$$

Step III: When the 20 V source is acting alone

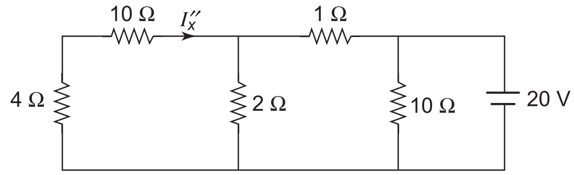


Fig. 18

Since the 10 Ω resistor is connected across the 20 V source, the resistor becomes redundant.

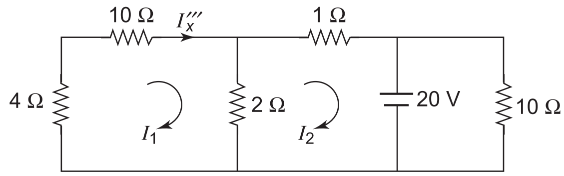


Fig. 19

Writing KVL equation in matrix form,

$$\begin{bmatrix} 16 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \end{bmatrix}$$

$$I_1 = -0.909 \text{ A}$$

$$I_2 = -7.27 \text{ A}$$

$$I_x'' = I_1 = -0.909 \text{ A } (\rightarrow)$$

Step IV: By superposition theorem,

$$I_x = I_x' + I_x'' + I_x'''$$

$$= 6.82 - 0.913 - 0.909$$

$$= 4.998 \text{ A } (\rightarrow)$$

5. (a) State and prove maximum power transfer theorem. Find the value of the resistance R_L using maximum power transfer theorem and find the value of maximum power transferred.

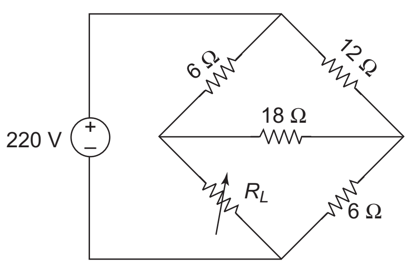


Fig. 20

Ans. Refer Section 2.11 on page 2.205.

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

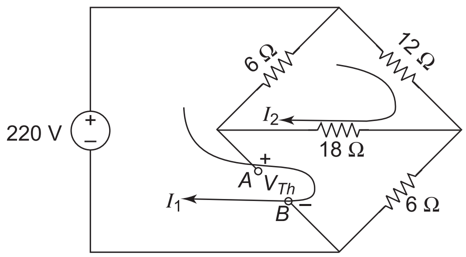


Fig. 21

Applying KVL to Mesh 1,

$$220 - 6(I_1 - I_2) - 18(I_1 - I_2) - 6I_1 = 0$$

$$30I_1 - 24\,I_2 = 220 \tag{1}$$

Applying KVL to Mesh 2,

$$-6(I_2 - I_1) - 12I_2 - 18(I_2 - I_1) = 0$$

$$-24I_1 + 36I_2 = 0 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 15.71\text{ A}$$

$$I_2 = 10.48\text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 18(I_1 - I_2) - 6I_1 = 0$$

$$V_{\text{Th}} = 18(I_1 - I_2) + 6I_1 = 18(15.71 - 10.48) + 6(15.71) = 188.4\text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit,

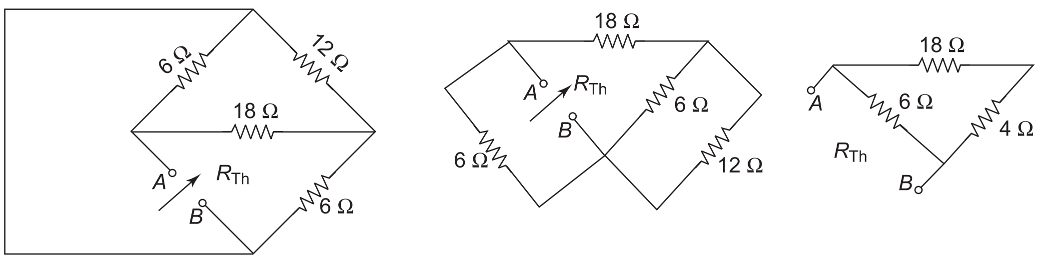


Fig. 22

$$R_{\text{Th}} = 22 \parallel 6 = 4.71\ \Omega$$

Step III: Value of R_L

For maximum power transfer,

$$R_L = R_{\text{Th}} = 4.71\ \Omega$$

Step IV: Calculation of $P_{\max}$

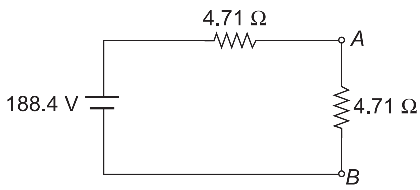


Fig. 23

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(188.4)^2}{4 \times 4.71} = 1884\ \text{W}$$

5. (b) A balanced load of phase impedance 100 Ω and power factor 0.8 (lag) is connected in delta to a 400 V, 3-phase supply. Calculate:

(i) Phase current and line current.

(ii) Active power and reactive power.

If the load is reconnected in star across the same supply, find

(iii) Phase voltage and line voltage.

(iv) Phase current and line current.

What will be the wattmeter readings if the power is measured by two wattmeter method (either star or delta).

Ans.

$$Z_{\text{ph}} = 100\ \Omega$$

$$\text{pf} = 0.8\ (\text{lag})$$

$$V_L = 400\ \text{V}$$

For a delta connected load,

$$\phi = \cos^{-1}(0.8) = 36.87°$$

(i) Phase current and line current

$$V_L = V_{\text{ph}} = 400\ \text{V}$$

$$I_{\text{ph}} = \frac{V_{\text{ph}}}{Z_{\text{ph}}} = \frac{400}{100} = 4\ \text{A}$$

$$I_L = \sqrt{3}\, I_{\text{ph}} = \sqrt{3} \times 4 = 6.93\ \text{A}$$

(ii) Active power and reactive power

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 6.93 \times 0.8 = 3.84\ \text{kW}$$

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 400 \times 6.93 \times \sin(36.87°) = 2.88\ \text{kVAR}$$

(iii) Phase voltage and line voltage for a star connected load

$$V_L = 400 \text{ V}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

(iv) Phase current and line current for a star connected load

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{100} = 2.31 \text{ A}$$

$$I_L = I_{ph} = 2.31 \text{ A}$$

(iv) Wattmeter readings for a star connected load

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 2.31 \times \cos(30° - 36.87°) = 917.37 \text{ W}$$

$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 2.31 \times \cos(30° + 36.87°) = 362.96 \text{ W}$$

6. (a) The readings when open circuit and short circuit tests are conducted on a 4kVA, 200/400 V, 50 Hz, single phase transformer are given below. Find the equivalent circuit parameters and draw the equivalent circuit referred to primary. Also find the transformer efficiency and regulation at full load and half load for 0.8 pf lagging.

OC test on LV side	200 V	0.7 A	70 W
SC test on H V side	15 V	10 A	85 W

Ans. (i) Equivalent circuit parameters

From OC test (meters are connected on LV side, i.e., primary)

$$W_i = 70 \text{ W}, \quad V_1 = 200 \text{ V}, \quad I_0 = 0.7 \text{ A}$$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{70}{200 \times 0.7} = 0.5$$

$$\sin\phi_0 = 0.867$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.43\ \Omega$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.867 = 0.61 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.61} = 327.87\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary)

$$W_{sc} = 85 \text{ W}, \quad V_{sc} = 15 \text{ V}, \quad I_{sc} = 10 \text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{85}{(10)^2} = 0.85\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.85)^2} = 1.24\ \Omega$$

$$K = \frac{E_2}{E_1} = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.85}{(2)^2} = 0.213\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.24}{(2)^2} = 0.31\ \Omega$$

(ii) Equivalent circuit referred to primary

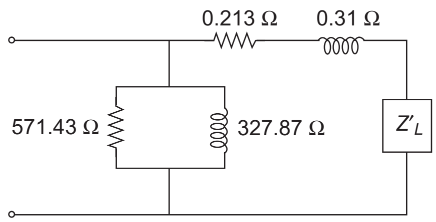


Fig. 24

(iii) Transformer efficiency at full load for 0.8 pf lagging

$$W_i = 70\ \text{W} = 0.07\ \text{kW}$$

Since meters are connected on secondary in SC test,

$$I_2 = \frac{\text{kVA} \times 1000}{V_2} = \frac{4 \times 1000}{400} = 10\ \text{A}$$

$$W_{cu} = I_2^2 R_{02} = (10)^2 \times 0.85 = 0.085\ \text{kW}$$

$$x = 1, \qquad \text{pf} = 0.8$$

$$\%\ \eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full load kVA} \times \text{pf} + W_i + x^2 W_{cu}} \times 100$$

$$= \frac{1 \times 4 \times 0.8}{1 \times 4 \times 0.8 + 0.07 + (1)^2 \times 0.085} \times 100$$

$$= 95.38\%$$

Transformer efficiency at half load for 0.8 pf lagging

$$x = 0.5, \qquad \text{pf} = 0.8$$

$$\%\ \eta = \frac{0.5 \times 4 \times 0.8}{0.5 \times 4 \times 0.8 + 0.07 + (0.5)^2 \times 0.085} \times 100$$

$$= 94.61\%$$

(iv) Regulation at full load for 0.8 pf lagging

$$I_2 = 10\ \text{A}$$

$$\cos \phi = 0.8$$

$$\sin \phi = 0.6$$

$$\% \text{ regulation} = \frac{I_2 (R_{02} \cos\phi + X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{10(0.85 \times 0.8 + 1.24 \times 0.6)}{400} \times 100$$

$$= 3.56\%$$

Regulation at half load for 0.8 pf lagging

$$I_2 = 5 \text{ A}$$

$$\cos \phi = 0.8$$

$$\sin \phi = 0.6$$

$$\% \text{ regulation} = \frac{I_2 (R_{02} \cos\phi + X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{5(0.85 \times 0.8 + 1.24 \times 0.6)}{400} \times 100$$

$$= 1.78\%$$

6. (b) With neat diagram explain the main parts of a dc machine? Mention the functions of each part.

Ans. Refer Section 7.3 on page 7.2.

May 2017

1. (a) Find the ratio V_L / V_S in the circuit shown below using Kirchhoff's laws.

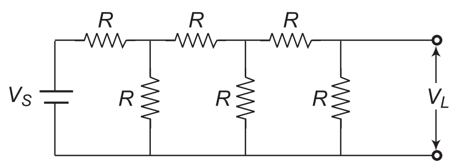


Fig. 1

Ans.

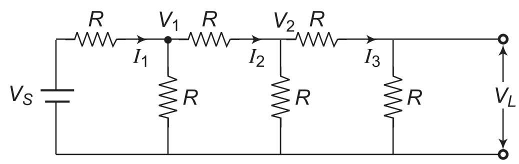


Fig. 2

$$I_3 = \frac{V_L}{R}$$

$$V_2 = RI_3 + V_L = V_L + V_L = 2V_L$$

$$I_2 = \frac{V_2}{R} + I_3 = \frac{2V_L}{R} + \frac{V_L}{R} = \frac{3V_L}{R}$$

$$V_1 = RI_2 + V_2 = R\left(\frac{3V_L}{R}\right) + 2V_L = 5V_L$$

$$I_1 = \frac{V_1}{R} + I_2 = \frac{5V_L}{R} + \frac{3V_L}{R} = \frac{8V_L}{R}$$

$$V_S = RI_1 + V_1 = R\left(\frac{8V_L}{R}\right) + 5V_L = 13V_L$$

$$\therefore \qquad \frac{V_L}{V_S} = \frac{1}{13}$$

(b) Find the rms value for the following waveform:

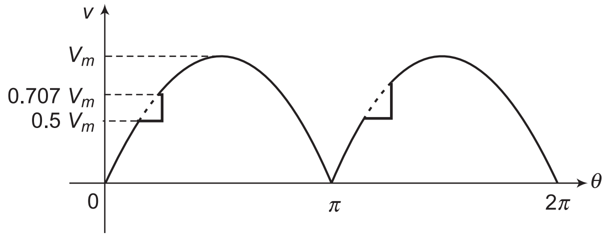


Fig. 3

Ans.

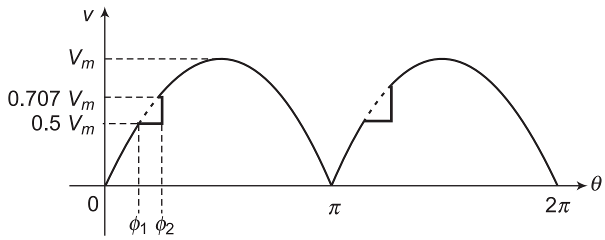


Fig. 4

$$v = V_m \sin \theta$$

At $\theta = \phi_1$, $\quad v = 0.5V_m$

$$0.5V_m = V_m \sin \phi_1$$

$$\phi_1 = \frac{\pi}{6}$$

At $\theta = \phi_2$, $\quad v = 0.707V_m$

$$0.707\ V_m = V_m \sin \phi_2$$

$$\phi_2 = \frac{\pi}{4}$$

$$\begin{aligned} v &= V_m \sin\theta && 0 < \theta < \frac{\pi}{6} \\ &= 0.5\, V_m && \frac{\pi}{6} < \theta < \frac{\pi}{4} \\ &= V_m \sin\theta && \frac{\pi}{4} < \theta < \pi \end{aligned}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\frac{\pi}{6}} V_m^2 \sin^2\theta \, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (0.5V_m)^2 \, d\theta + \int_{\frac{\pi}{4}}^{\pi} V_m^2 \sin^2\theta \, d\theta\right]}$$

$$= 0.699\, V_m$$

(c) Draw the phasor diagram for a three phase star connected load with leading power factor. Indicate all the line and phase voltages and currents.

Ans.

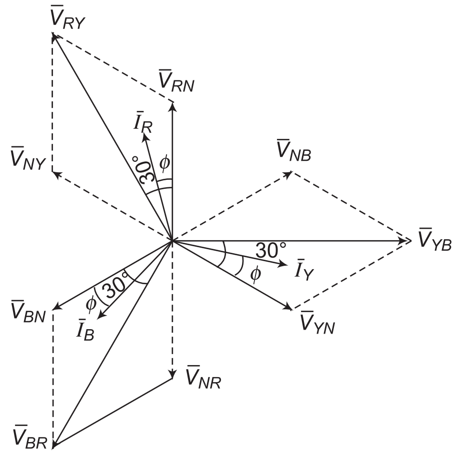


Fig. 5

(d) A 5 kVA, 240/2400 V, 50 Hz single phase transformer has the maximum value of flux density as 1 Tesla. If the emf per turn is 10, calculate the number of primary and secondary turns and the full load primary and secondary currents.

Ans.

kVA rating = 5 kVA

$E_1 = 240$ V

$E_2 = 2400$ V

$f = 50$ Hz

$B_m = 1$ T

$$\frac{E_1}{N_1} = 10$$

(i) Number of primary and secondary turns

$$\frac{E_1}{N_1} = 10 = \frac{240}{N_1}$$

$$N_1 = 24$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{2400}{240} = \frac{N_2}{24}$$

$$N_2 = 240$$

(ii) Full load primary and secondary currents

For a transformer,

$$V_1 \simeq E_1 = 240 \text{ V}$$

$$V_2 \simeq E_2 = 2400 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{5 \times 1000}{240} = 20.83 \text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{5 \times 1000}{2400} = 2.08 \text{ A}$$

(e) Explain the principle of operation of DC generator.

Ans. Refer Section 7.2 on page 7.2.

2. (a) Find the current through 3 Ω resistor by mesh analysis.

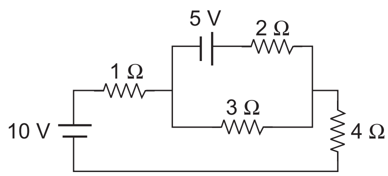


Fig. 6

Ans. Assigning clockwise currents in two meshes,

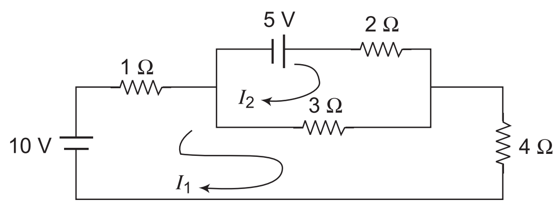


Fig. 7

Applying KVL to Mesh 1,

$$10 - 1I_1 - 3(I_1 - I_2) - 4I_1 = 0$$

$$8I_1 - 3I_2 = 10 \quad \text{(i)}$$

Applying KVL to Mesh 2,

$$5 - 2I_2 - 3(I_2 - I_1) = 0$$

$$-3I_1 + 5I_2 = 5 \quad \text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 2.097 \text{ A}$$

$$I_2 = 2.26 \text{ A}$$

$$I_{3\Omega} = I_2 - I_1 = 0.163 \text{ A}$$

(b) Find the current delivered by the source. **(8)**

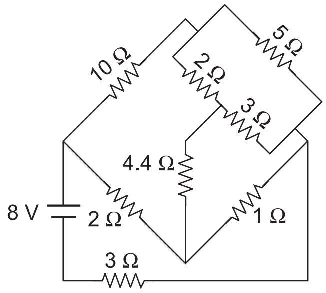


Fig. 8

Ans.

Converting the star network formed by resistors of 2 Ω, 3 Ω and 4.4 Ω into an equivalent delta network,

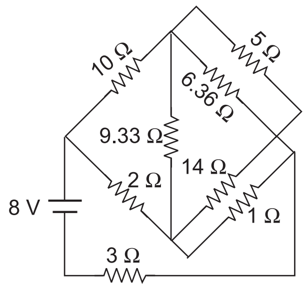


Fig. 9

Simplifying the network,

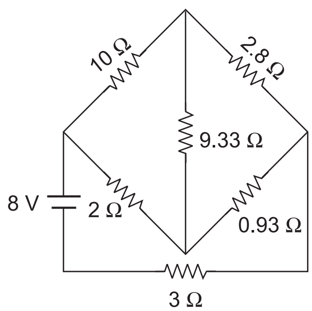


Fig. 10

Converting the delta network formed by resistors of 2.8 Ω, 9.33 Ω and 0.93 Ω into an equivalent star network,

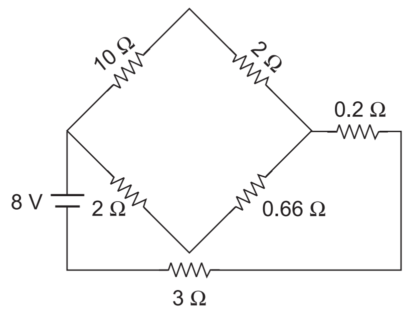


Fig. 11

Simplifying the network,

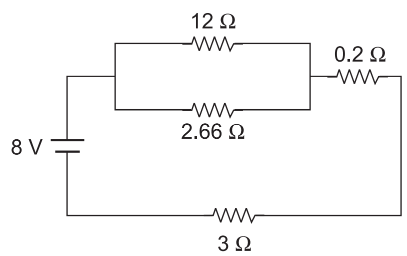


Fig. 12

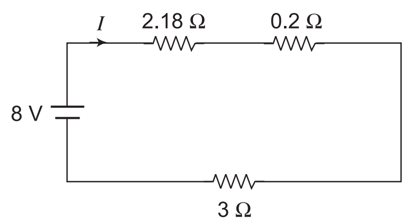


Fig. 13

$$I = \frac{8}{2.18 + 0.2 + 3} = 1.49 \text{ A}$$

2. (c) The voltage and current in a circuit are given by $\bar{V} = 12 \angle 30° \text{ V}$ and $\bar{I} = 3 \angle 60° \text{ A}$. The frequency of the supply is 50 Hz. Find

(i) Equation for voltage and current in both the rectangular and standard form.

(ii) Impedance, reactance and resistance.

(iii) Phase difference, power factor and power loss.

Draw the circuit diagram considering a simple series circuit of two elements indicating their values.

Ans.

$$\bar{V} = 12 \angle 30° \text{ V}$$

$$\bar{I} = 3 \angle 60° \text{ A}$$

$$f = 50 \text{ Hz}$$

(i) Equation for voltage and current in both the rectangular and standard form

$$\bar{V} = 12 \angle 30° = 10.39 + j\,6 \text{ V}$$

$$v = 12\sqrt{2}\sin(\omega t + 30°)$$

$$\bar{I} = 3 \angle 60° = 1.5 + j\,2.6 \text{ A}$$

$$i = 3\sqrt{2}\sin(\omega t + 60°)$$

(ii) Impedance, reactance and resistance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{12 \angle 30°}{3 \angle 60°} = 4 \angle -30° \,\Omega = 3.46 - j2\,\Omega$$

$$Z = 4\,\Omega$$

$$X_C = 2\,\Omega$$

$$R = 3.46\,\Omega$$

(iii) Phase difference, power factor and power loss

$$\phi = 30\,\Omega$$

$$\text{pf} = \cos\phi = 0.866 \text{ (leading)}$$

$$P = VI\cos\phi = 12 \times 3 \times 0.866 = 31.18 \text{ W}$$

(iv) Circuit diagram

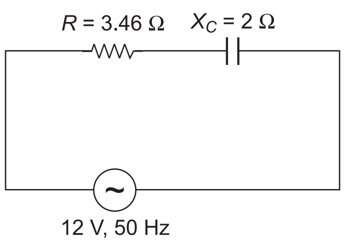


Fig. 14

$$X_C = 2\,\Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$2 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 1.59 \text{ mF}$$

3. (a) Find the resultant voltage and its equation for the given voltages which are connected in series.

$$e_1 = 2\sin\omega t,\ e_2 = -\cos\left(\omega t - \frac{\pi}{6}\right),\ e_3 = 2\cos\left(\omega t - \frac{\pi}{4}\right),\ e_4 = -2\sin\left(\omega t + \frac{\pi}{3}\right).$$

Ans.

$$e_1 = 2\sin\omega t$$

$$e_2 = -\cos\left(\omega t - \frac{\pi}{6}\right) = \sin\left(\omega t - \frac{\pi}{6} + \frac{5\pi}{4}\right) = \sin(\omega t + 240°)$$

$$e_3 = 2\cos\left(\omega t - \frac{\pi}{4}\right) = 2\sin\left(\omega t - \frac{\pi}{4} + \frac{\pi}{2}\right) = 2\sin(\omega t + 45°)$$

$$e_4 = -2\sin\left(\omega t + \frac{\pi}{3}\right) = 2\sin\left(\omega t + \frac{\pi}{3} + \pi\right) = 2\sin(\omega t + 240°)$$

Writing voltage e_1, e_2, e_3 and e_4 in the phasor form,

$$\bar{E}_1 = \frac{2}{\sqrt{2}}\angle 0° = 1.41\angle 0°$$

$$\bar{E}_2 = \frac{1}{\sqrt{2}}\angle 240° = 0.71\angle 240°$$

$$\bar{E}_3 = \frac{2}{\sqrt{2}}\angle 45° = 1.41\angle 45°$$

$$\bar{E}_4 = \frac{2}{\sqrt{2}}\angle 240° = 1.41\angle 240°$$

Resultant voltage $\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4$

$$= 1.4\angle 0° + 0.71\angle 240° + 1.41\angle 45° + 1.41\angle 240°$$

$$= 1.59\angle -31.92°$$

$$e = 1.59\sqrt{2}\sin(\omega t - 31.92°)$$

$$= 2.25\sin(\omega t - 31.92°)$$

(b) Find the current through 20 Ω resistor by using superposition theorem.

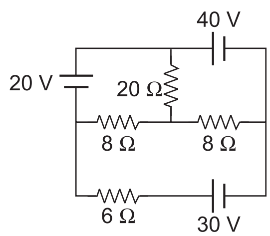


Fig. 15

Ans.

Step I: When the 20 V source is acting alone

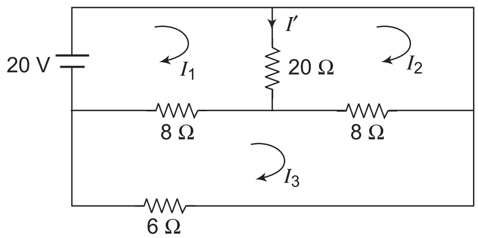


Fig. 16

Writing KVL equations in matrix form,

$$\begin{bmatrix} 28 & -20 & -8 \\ -20 & 28 & -8 \\ -8 & -8 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = 6.46 \text{ A}$$
$$I_2 = 6.04 \text{ A}$$
$$I' = I_1 - I_2 = 6.46 - 6.04 = 0.42 \text{ A} (\downarrow)$$

Step II: When the 40 V source is acting alone

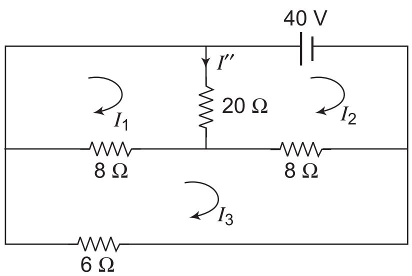


Fig. 17

Writing KVL equations in matrix form,

$$\begin{bmatrix} 28 & -20 & -8 \\ -20 & 28 & -8 \\ -8 & -8 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -40 \\ 0 \end{bmatrix}$$

$$I_1 = -12.08 \text{ A}$$
$$I_2 = -12.91 \text{ A}$$
$$I'' = I_1 - I_2 = -12.08 - (-12.91) = 0.83 \text{ A} (\downarrow)$$

Step III: When 30 V source is acting alone

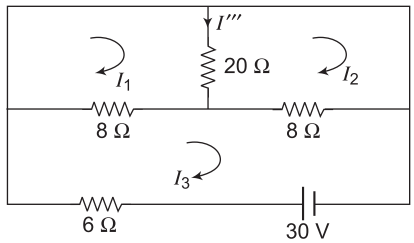


Fig. 18

Writing KVL equations in matrix form,

$$\begin{bmatrix} 28 & -20 & -8 \\ -20 & 28 & -8 \\ -8 & -8 & 20 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 30 \end{bmatrix}$$

$$I_1 = 7.5\text{A}$$
$$I_2 = 7.5\text{ A}$$
$$I''' = I_1 - I_2 = 7.5 - 7.5 = 0$$

Step IV: By superposition theorem,

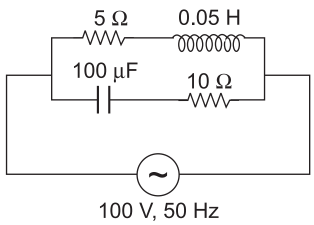
$$I = I' + I'' + I''' = 0.42 + 0.83 + 0 = 1.25\text{ A } (\downarrow)$$

(c) Two parallel branches of a circuit comprise respectively of (i) a coil having 5 Ω resistance and inductance of 0.05 H, (ii) a capacitor of capacitance 100 μF in series with a resistance of 10 Ω. The circuit is connected to a 100 V, 50 Hz supply. Find

(i) impedance and admittance and each branch,
(ii) equivalent admittance and impedance of the circuit,
(iii) the supply current and power factor of the circuit.

Draw its equivalent series circuit using two elements indicating their values.

Ans.



$R_1 = 5\ \Omega$
$L = 0.05\text{ H}$
$C = 100\ \mu\text{F}$
$R_2 = 10\ \Omega$
$\bar{V} = 100\ \angle 0°\text{ V}$

Fig. 19

(i) Impedance and admittance of each branch

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

$$\bar{Z}_1 = R_1 + jX_L = 5 + j15.71\ \Omega = 16.49\ \angle 72.35°\ \Omega$$

$$\bar{Z}_2 = -jX_C + R_2 = -j31.83 + 10 = 33.36 \angle -72.56°\ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{16.49 \angle 72.35°} = 0.06 \angle -72.35°$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{33.36 \angle -72.56°} = 0.03 \angle 72.56°$$

(ii) Equivalent admittance and impedance of the circuit

$$\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 = 0.06 \angle -72.35° + 0.03 \angle 72.56° = 0.04 \angle 46.41°$$

$$\bar{Z}_{eq} = \frac{1}{\bar{Y}_{eq}} = \frac{1}{0.04 \angle -46.41°} = 25 \angle 46.41°\ \Omega$$

$$= 17.24 + j18.11\ \Omega$$

(iii) Supply current and power factor of the circuit

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \frac{100 \angle 0°}{25 \angle 46.41°} = 4 \angle -46.41°\ \text{A}$$

pf = cos (46.41°) = 0.689 (lagging)

(iv) Circuit diagram

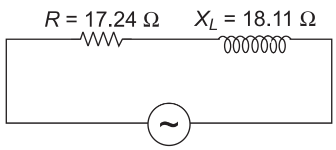


Fig. 20

4. (a) How are DC machines classified?

Ans. Refer Section 7.4 on page 7.4.

(b) Find the current through 10 Ω resistor by using Norton's theorem.

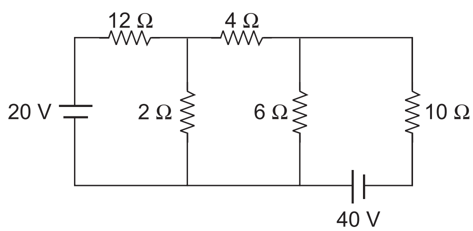


Fig. 21

Ans.

Step I: Calculation of I_N

Replacing 10 Ω resistor by short circuit,

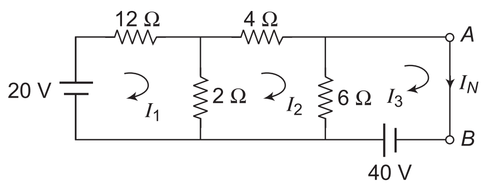


Fig. 22

Writing KVL equations in matrix form,

$$\begin{bmatrix} 14 & -2 & 0 \\ -2 & 12 & -6 \\ 0 & -6 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 40 \end{bmatrix}$$

$I_1 = 2.5$ A

$I_2 = 7.5$ A

$I_3 = 14.17$ A

$I_N = I_3 = 14.17$ A

Step II: Calculation of R_N

Replacing the voltage sources by short circuits,

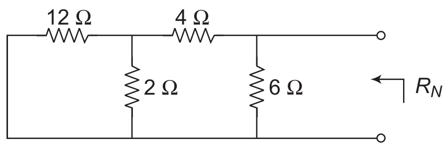


Fig. 23

$$R_N = [(12 \quad 2) + 4] \quad 6 = 2.93\ \Omega$$

Step III: Calculation of I_L

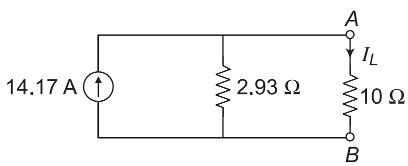


Fig. 24

$$I_L = 14.17 \times \frac{2.93}{2.93 + 10} = 3.2 \text{ A}$$

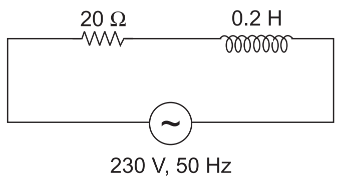
(c) An inductive coil has a resistance of 20 Ω and inductance of 0.2 H. It is connected in parallel with a capacitor of 20 μF. This combination is connected across a 230 V supply having variable frequency. Find the frequency at which the total current drawn from the supply is in phase with the supply voltage. What is this condition called? Find the values of total current drawn and the impedance of the circuit at this frequency. Draw the phasor diagram and indicate the various current and voltages in the circuit.

Ans. Refer Example 6 on page 4.113.

5. (a) A coil having a resistance of 20 Ω and inductance of 0.2 H is connected across a 230 V 50 Hz supply. Calculate

(i) circuit current
(ii) phase angle
(iii) power factor
(iv) power consumed

Ans.



$R = 20\ \Omega$
$L = 0.2$ H

Fig. 25

(i) Circuit current

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$

$$\bar{Z} = R + jX_L = 20 + j62.83 = 65.94 \angle 72.34°\ \text{A}$$

$$I = \frac{V}{Z} = \frac{230}{65.94} = 3.49\ \text{A}$$

(ii) Phase angle

$$\phi = 72.34°$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(72.34°) = 0.303\ \text{(lagging)}$$

(iv) Power consumed

$$P = VI\cos\phi = 230 \times 3.49 \times 0.303 = 243.22\ \text{W}$$

(b) A balanced three phase delta connected load draws a power of 10 kW, with a power factor of 0.6 leading when supplied with an ac supply of 440 V, 50 Hz. Find the circuit elements of the load per phase assuming a simple series circuit of two elements.

Ans. Refer Example 23 on page 5.33.

(c) Draw and explain the phasor diagram of a single phase transformer on no-load.

Ans. Refer Section 6.9 on page 6.16.

6. (a) Explain the various losses of a single phase transformer.

Ans. Refer Section 6.7 on page 6.13.

(b) Two wattmeters connected to measure power in three phase circuit using the two wattmeter method indicate 1250 W and 250 W respectively. Find the total power supplied and the power factor of the circuit when:

(i) both the reading are positive,
(ii) when the latter reading is obtained by reversing the connections of the pressure coil.

Ans.

$$W_1 = 1250 \text{ W}$$

$$W_2 = 250 \text{ W}$$

(i) Both the reading are positive

$$P = W_1 + W_2 = 1250 + 250 = 1500 \text{ W}$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \sqrt{3}\left(\frac{1250 - 250}{1250 + 250}\right) = 1.41$$

$$\phi = 54.74°$$

(ii) When the latter reading is obtained by reversing the connections of the pressure coil

$$W_2 = -250 \text{ W}$$

$$P = W_1 + W_2 = 1250 - 250 = 1000 \text{ W}$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \sqrt{3}\left\{\frac{1250 - (-250)}{1250 - 250}\right\} = 2.598$$

$$\phi = 68.95°$$

(c) A 200/400 V, 50 Hz single phase transformer gave the following test results:

OC test: 200 V 0.7 A 70 W (on lv side)

SC test: 15 V 10 A 85 W (on hv side)

Obtain the parameters and draw the equivalent circuit of the transformer as referred to the primary.

Ans. Refer Example 7 on page 6.60.

December 2016

1. (a) State maximum power transfer theorem.

Ans. Refer Section 2.11 on page 2.205.

(b) Derive the formula to convert a delta circuit into an equivalent star.

Ans. Refer Section 2.71 on page 2.86.

(c) Define average value and RMS value of an alternating quantity.

Ans. Refer Section 3.3 on page 3.4 and Section 3.4 on page 3.6.

(d) Prove that power in a 3-phase delta connected system is 3 times that of a star connected system.

Ans. Refer Section 5.13 on page 5.13.

(e) Explain the working principle of a single phase transformer.

Ans. Refer Section 6.3 on page 6.4.

(f) What is the use of commutator in a DC machine.

Ans. Refer Section 7.3 on page 7.3.

2. (a) Obtain current through 1 Ω resistor using Superposition Theorem in Fig. 1.

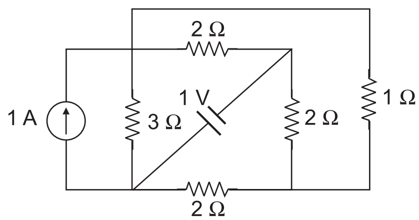


Fig. 1

Ans.

Step I: When the 1 A source is acting alone

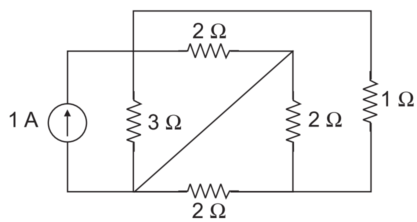


Fig. 2

Simplifying the network,

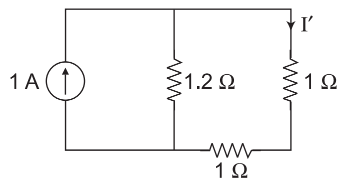


Fig. 3

By current division rule,

$$I' = 1 \times \frac{1.2}{1.2 + 1 + 1} = 0.375 \text{ A } (\downarrow)$$

Step II: When the 1 V source is acting alone

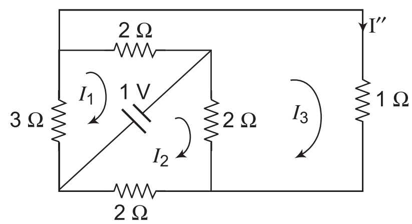


Fig. 4

Writing KVL equations in matrix form,

$$\begin{bmatrix} 5 & 0 & -2 \\ 0 & 4 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$I'' = I_3 = -0.03125 \text{ A } (\downarrow)$$

Step III: By superposition theorem,

$$I = I' + I'' = 0.375 - 0.3125 = 0.34375 \text{ A } (\downarrow)$$

(b) A coil is connected across a non-inductive resistance of 120 Ω. When a 240 V, 50 Hz supply is applied to this circuit the coil draws a current 5 A and total current is 6 A. Determine the power and power factor of

(i) the coil

(ii) the whole circuit

Ans.

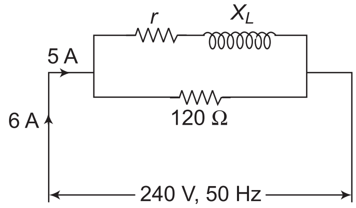


Fig. 5

$$Z_{\text{coil}} = \frac{240}{5} = 48\ \Omega$$

$$Z_T = \frac{240}{6} = 40\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = 48$$

$$r^2 + X_L^2 = 2304 \qquad \text{(i)}$$

$$\bar{Z}_T = \frac{(r + j\,X_L)120}{r + j\,X_L + 120} = \frac{120r + j120\,X_L}{(r+120) + j\,X_L}$$

$$Z_T = \frac{\sqrt{(120r)^2 + (120X_L)^2}}{(r+120)^2 + X_L^2} = 40$$

$$\frac{14400\,(r^2 + X_L^2)}{r^2 + 240r + 14400 + X_L^2} = 1600$$

$$\frac{14400(2304)}{2304 + 240r + 14400} = 1600$$

$$\therefore \quad r = 16.8\ \Omega$$

Substituting value of r in Eq. (i),

$$X_L = 44.96\ \Omega$$

$$\bar{Z}_T = \frac{(16.8 + j44.96)120}{16.8 + j44.96 + 120} = 25 + j31.22\ \Omega$$

$$P_{\text{coil}} = I_{\text{coil}}^2 \times r = (5)^2 \times 16.8 = 420\ \text{W}$$

$$\text{pf}_{\text{coil}} = \frac{r}{Z_{\text{coil}}} = \frac{16.8}{48} = 0.35 \ \ \text{(lagging)}$$

$$P_T = I_T^2 (R_T) = (6)^2 \times (25) = 900\ \text{W}$$

$$\text{pf}_T = \frac{R_T}{Z_T} = \frac{25}{40} = 0.625 \ \ \text{(lagging)}$$

3. (a) Obtain Norton's equivalent circuit of the network shown in Fig. 6, across the terminals A and B.

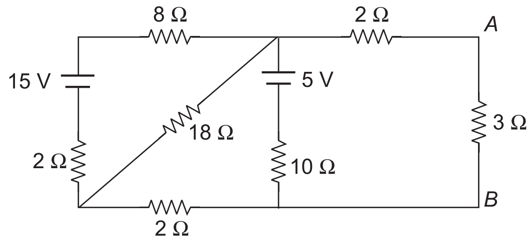


Fig. 6

Ans.

Step I: Calculation of I_N

Replacing the 3 Ω resistor by short circuit,

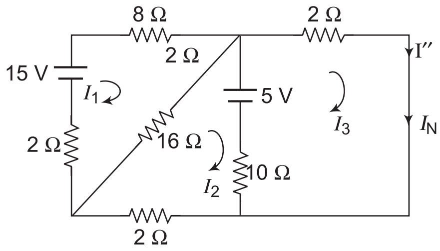


Fig. 7

Applying KVL to Mesh 1,

$$-2I_1 + 15 - 8I_1 - 16(I_1 - I_2) = 0$$

$$26I_1 - 16I_2 = 15 \qquad \text{(i)}$$

Applying KVL to Mesh 2,

$$-16(I_2 - I_1) - 5 - 10(I_2 - I_3) - 2I_2 = 0$$

$$-16I_1 + 28I_2 - 10I_3 = -5 \qquad \text{(ii)}$$

Applying KVL to Mesh 3,

$$-10(I_3 - I_2) + 5 - 2I_3 = 0$$

$$-10I_2 + 12I_3 = 5 \qquad \text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_N = I_3 = 1.13 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

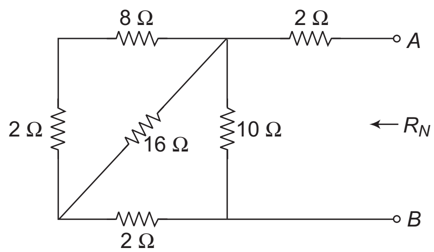


Fig. 8

By Series-Parallel reduction technique,

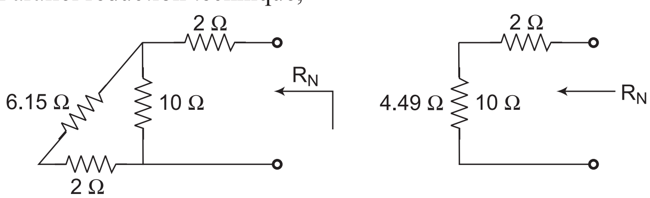


Fig. 9

$$R_N = 6.49 \ \Omega$$

Step III: Calculation of I_L

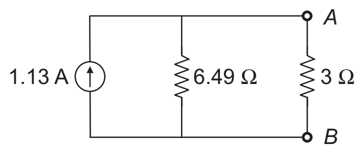


Fig. 10

$$I_L = 1.13 \times \frac{6.49}{6.49 + 3} = 0.77 \text{ A}$$

(b) A series RLC circuit, if ω_0 is the resonant frequency, ω_1 and ω_2 are the half power frequencies, prove that $\omega_0 = \sqrt{(\omega_1 \omega_2)}$.

Ans.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

$$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$\omega_1\omega_2 = \left(\omega_0 - \frac{R}{2L}\right)\left(\omega_0 + \frac{R}{2L}\right)$$

$$= \omega_0^2 - \frac{R^2}{4L^2}$$

For low values of R, the term $\frac{R^2}{4L^2}$ can be neglected in comparison with term ω_0^2 i.e., $\frac{1}{LC}$.

$$\omega_1\omega_2 = \omega_0^2$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

(c) Derive the equivalent circuit of a 1-phase transformer.

Ans. Refer Section 6.11 on page 6.21.

4. (a) Obtain current through 15 Ω resistance by nodal analysis in Fig. 11. Take reference node as marked.

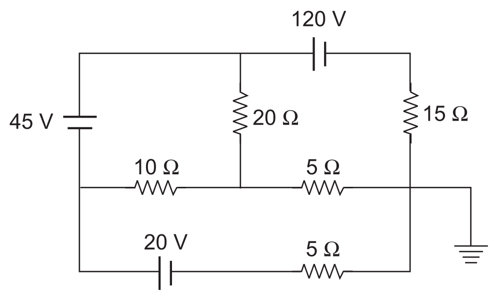


Fig. 11

Ans.

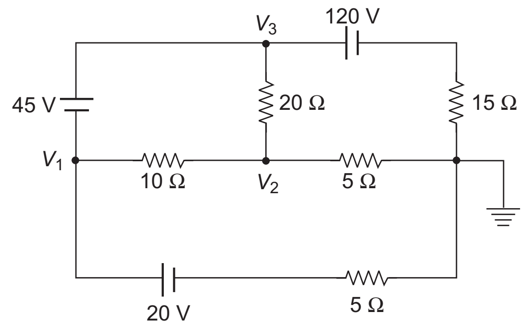


Fig. 12

Assume that the currents are moving away from the nodes. Nodes 1 and 3 will form a supernode.

Writing voltage equation for the supernode,

$$V_3 - V_1 = 45 \qquad \text{(i)}$$

Applying KCL at the supernode,

$$\frac{V_1 - 20}{5} + \frac{V_1 - V_2}{10} + \frac{V_3 - V_2}{20} + \frac{V_3 - 120}{15} = 0$$

$$\left(\frac{1}{5} + \frac{1}{10}\right)V_1 - \left(\frac{1}{10} + \frac{1}{20}\right)V_2 + \left(\frac{1}{20} + \frac{1}{15}\right)V_3 = \frac{120}{15} + \frac{20}{5}$$

$$0.3V_1 - 0.15V_2 + 0.12V_3 = 12 \qquad \text{(ii)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + \frac{V_2 - V_3}{20} = 0$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{20}\right)V_2 - \frac{1}{20}V_3 = 0$$

$$-0.1V_1 + 0.35V_2 - 0.05V_3 = 0 \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii),

$$V_1 = 21.27 \text{ V}$$

$$V_2 = 15.54 \text{ V}$$

$$V_3 = 66.27 \text{ V}$$

$$I_{15\Omega} = \frac{V_3 - 120}{15} = \frac{66.27 - 120}{15} = -3.58 \text{ A}$$

(b) In a balanced 3 phase, star connected system, a wattmeter is connected with is current coil in series with *Y* line and pressure coil between *Y* and *R* lines. Draw a neat diagram showing the above wattmeter connection. Assume a lagging power factor, draw the corresponding phasor diagram and derive the wattmeter reading in terms of line voltage, line current and phase angle.

Ans.

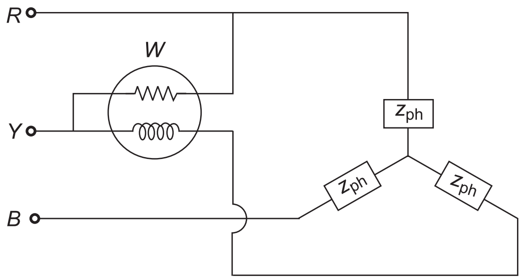


Fig. 13

Phasor diagram

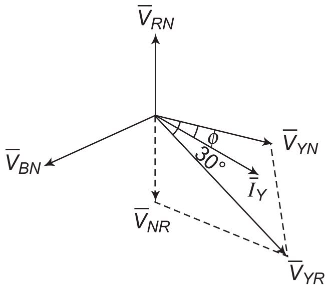


Fig. 14

From phasor diagram it is clear that the phase angle between V_{YR} and I_Y is $30° - \phi$.

$$W = V_{YR}\, I_Y \cos(30° - \phi)$$

5. (a) Obtain current through 60 Ω resistance by Mesh analysis in Fig. 15.

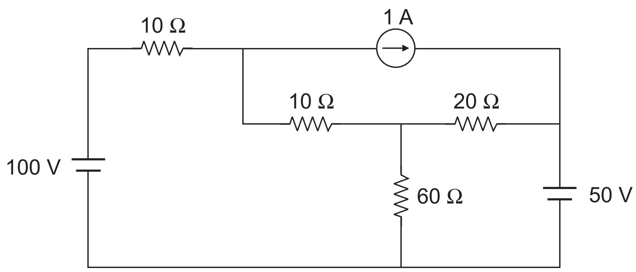


Fig. 15

Ans. Assigning clockwise current in three meshes,

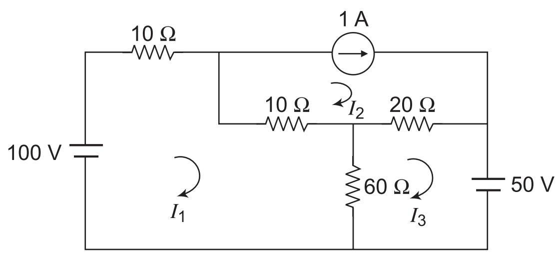


Fig. 16

Applying KVL to Mesh 1,

$$100 - 10I_1 - 10(I_1 - I_2) - 60(I_1 - I_3) = 0$$

$$80I_1 - 10I_2 - 60I_3 = 100 \qquad \text{(i)}$$

For Mesh 2,

$$I_2 = 1 \qquad \text{(ii)}$$

Applying KVL to Mesh 3,

$$-60(I_2 - I_3) - 20(I_3 - I_2) - 50 = 0$$

$$-60I_1 - 20I_2 + 80I_3 = -50 \qquad \text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_1 = 2.5 \text{ A}$$

$$I_2 = 1 \text{ A}$$

$$I_3 = 1.5 \text{ A}$$

$$I_{60\,\Omega} = I_1 - I_3 = 2.5 - 1.5 = 1 \text{ A}$$

(b) Develop the phasor diagram of a single phase transformer supplying to a resistive load.

Ans. Refer Section 6.10 on page 6.18.

(c) Derive the emf equation of a dc generator.

Ans. Refer Section 7.5 on page 7.5.

6. (a) A resistor and a pure reactance are connected in series across a 150 V ac supply. When the frequency is 40 Hz, the circuit draws 5 A. When the frequency is increased to 50 Hz, the circuit draws 6 A. Find the value of resistance and the element value of the reactance. Also find the power drawn in the second case.

Ans.

$$V_1 = 150 \text{ V}, \quad f_1 = 40 \text{ Hz}, \quad I_1 = 5 \text{ A}$$

$$V_2 = 150 \text{ V}, \quad f_2 = 50 \text{ Hz}, \quad I_2 = 6 \text{ A}$$

Case (i) $\quad V_1 = 150 \text{ V}, \quad f_1 = 40 \text{ Hz}, \quad I_1 = 5 \text{ A}$

$$Z_1 = \frac{V_1}{I_1} = \frac{150}{5} = 30\ \Omega$$

Case (ii) $\quad V_2 = 150 \text{ V}, \quad f_2 = 50 \text{ Hz}, \quad I_2 = 6 \text{ A}$

$$Z_2 = \frac{150}{6} = 25\ \Omega$$

As frequency increases, impedance of the circuit decreases. In a series R-C circuit, capacitive reactance decreases with increase in frequency. Hence, impedance decreases. Hence, the circuit consists of a resistor R and a capacitor C.

$$Z_1 = \sqrt{R^2 + X_{C1}^2} = \sqrt{R^2 \times \left(\frac{1}{2\pi \times 40 \times C}\right)^2} = 30$$

$$R^2 + \left(\frac{1}{80\pi C}\right)^2 = 900 \qquad \text{(i)}$$

$$Z_2 = \sqrt{R^2 + X_{C2}^2} = \sqrt{R^2 + \left(\frac{1}{2\pi \times 50 \times C}\right)^2} = 25$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 625 \qquad \text{(ii)}$$

Solving Eqs (i) and (ii),

$$R = 11.67\ \Omega$$

$$C = 143.96\ \mu\text{F}$$

$$P_2 = I_2^2\ R = (6)^2(11.67) = 420\ \text{W}$$

6. (b) A single phase 10 kVA, 500 V/250 V, 50 Hz transformer has the following constants:

Resistance	:	primary = 0.2 ohms,	secondary = 0.5 ohms
Reactance	:	primary = 0.4 ohms,	secondary = 0.1 ohms

Resistance of equivalent exciting circuit w.r.t. primary = 1500 ohms

Reactance of equivalent exciting circuit w.r.t. primary = 750 ohms

What will be the reading of the instruments placed on primary side when the transformer is connected for OC and SC test?

Ans. Full load kVA = 10 kVA

$$E_1 = 500\ \text{V}$$

$$E_2 = 250\ \text{V}$$

$$R_1 = 0.2\ \Omega$$

$$R_2 = 0.5\ \Omega$$

$$X_1 = 0.4\ \Omega$$

$$X_2 = 0.1\ \Omega$$

$$R_0 = 1500\ \Omega$$

$$X_0 = 750\ \Omega$$

For OC test,

$$V_1 \quad E_1 = 500\ \text{V}$$

$$I_\omega = \frac{V_1}{R_0} = \frac{500}{1500} = 0.33\ \text{A}$$

$$I_\mu = \frac{V_1}{X_0} = \frac{500}{750} = 0.67\ \text{A}$$

$$I_0 = \sqrt{I_\mu^2 + I_\omega^2} = \sqrt{(0.33)^2 + (0.67)^2} = 0.75\ \text{A}$$

$$\cos\phi_0 = \frac{I_\omega}{I_0} = \frac{0.33}{0.75} = 0.44$$

$$\phi_0 = \cos^{-1}(0.44) = 63.9°$$

$$W_i = V_1 I_0 \cos\phi_0 = 500 \times 0.75 \times 0.44 = 165\ \text{W}$$

For SC test,

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{10 \times 1000}{500} = 20 \text{ A}$$

Assuming full load current through primary winding,

$$I_{SC} = I_1 = 20 \text{ A}$$

$$K = \frac{E_2}{E_1} = \frac{250}{500} = 0.5$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.2 + \frac{0.5}{(0.5)^2} = 2.2\ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.4 + \frac{0.1}{(0.5)^2} = 0.8\ \Omega$$

$$W_{SC} = W_{Cu} = I_1^{\,2} R_{01} = (20)^2 (2.2) = 880 \text{ W}$$

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2} = \sqrt{(2.2)^2 + (0.8)^2} = 2.34\ \Omega$$

$$V_{SC} = I_{SC}\, Z_{01} = 20 \times 2.34 = 46.8 \text{ V}$$

Index

E

F

G

H

I

K

L

M

N

O

P