NT

six proofs from the Infinities of prime

Euclid proof

finite set $P = \{p_1, p_2 \cdots, p_r\}$

$$n = p_1 p_2 \cdots p_r + 1$$

$Q = \{2, 3, 5, 7, 11, 13\}$
$p_1, p_2, p_3, p_4, p_5, p_6$

$Q = 2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031$

30031 can be divided by 59×509.

Fermat Number proof

- mathematic Induction

Base case: $n > k$

$$F_1 - 2 = 2^2 - 1 = F_0$$

$$3 \times 5 = 17 - 2$$

3
5
17
21

Induction step:

$$\prod_{k=0}^{n} F_k = \left(\prod_{k=0}^{n-1} F_k \right) F_n = \left(F_n - 2 \right) F_n =$$

$$= \left(2^{2^n} - 1 \right)\left(2^{2^n} + 1 \right) = 2^{2^{n+1}} - 2 = F_{n+1} - 2$$